A Panel Data Study of Student Knowledge Growth: Application of an Economic Empirical Growth Model

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ABSTRACT. This paper explores factors that contribute to growth in undergraduate knowledge of economics. An endogenous growth empirical model was applied to estimate students' knowledge growth. Sources of knowledge growth were extracted to identify each factor's contribution. Analysis indicated that in-classroom factors (instructional skill and attendance) accounted for over 50% of knowledge growth. A complementarity test showed that in- and out-of-classroom efforts were not crowding out each other; moreover, instructional skill/progress and in-classroom efforts were complementary—the more frequently the student attended class, the greater the effect of the instructor's instructional skills on student progress. Findings implied the importance of traditional classroom learning. In addition to offering online classes to raise enrollments and revenues, school authorities should be aware of the contribution and importance of traditional face-to-face classes and continue to improve the quality of traditional classroom learning. (A20; A22; I20; I21; C30)

I. Introduction

While the endogenous growth model was launched by Paul Romer (1986) and Robert Lucas Jr. (1988), a number of researchers have broadly applied their models to build up empirical models of endogenous growth[1] to estimate a country's (or cross-country) economic growth over the past two decades (e.g., Lau, Jamison, Liu, and Rivkin, 1993; Tallman and Wang, 1994; McMahon, 1998; Lloyd-Ellis and Roberts, 2002; Monteils, 2002; Lin, 2004, and Jones and Romer, 2010). However, these researchers may not be aware that the empirical model of endogenous growth may also be used to estimate a student's performance growth (i.e., knowledge growth). The process followed by a student in his/her pursuit of knowledge is quite similar to that followed by a nation in its economic production process.

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The main determinants or factors that affect a country's economic output are technological progress, physical capital, labor, and human capital. Similarly, the main factors that seem to cause variations in student's knowledge output are instructional skill/progress, in- and out-of-classroom efforts, and human capital. Examination and estimation of a student's knowledge growth are important because such growth reflects a student's potential during his/her learning process. Positive knowledge growth implies that the student is making progress; a high rate of growth means that the student has substantial potential. Some students may initially function at a low-performance level but end up at high performance, implying that these students have substantial learning potential. All in all, this issue needs to be raised for discussion and investigation because educators and school administrators, due to the nature of their employment, must assist in maximizing students' potential and enhancing their academic performance in order to achieve success.

For that reason, in this study an empirical model of endogenous growth was applied to an investigation and estimation of students' knowledge growth in a sample of 203 students enrolled in four introductory microeconomics classes during the spring semesters of 2007 and 2009. The endogenous growth model was selected because endogenous growth theory suggests that investment in human capital significantly contributes to growth, while the exogenous growth model explains long-run growth by looking at productivity, capital accumulation, population growth, and technological progress. Since one of this study's purposes was to establish the extent to which students' human capital contributes to their knowledge growth in a class, use of the endogenous growth model was appropriate.

In this research, students' knowledge growth was estimated and the sources of their knowledge growth were identified. The contribution of each source was numerically accounted for so that we might estimate each factor's contribution to students' knowledge growth. Moreover, a complementarity test was used to verify whether instructional skill/progress and in-classroom effort (i.e., attendance) are complements, and whether in- and out-of-classroom efforts substitute for one another. More importantly, this paper extends the literature on both endogenous growth empirical models used in economic growth theories and student performance used in higher education.

II. Model

Students pursuing knowledge can be treated as producing their knowledge products. Four main factors are involved in producing knowledge products: instructional skill/progress, in-classroom effort, out-of-classroom effort, and human capital (i.e., the student's quality). Therefore, knowledge output is modeled as a function of these four factors using the multiplicative Cobb-Douglas production. The knowledge production function is expressed as:

$$Y_t = T_t A_t^{\alpha} E_t^{\beta} H_t^{\gamma}, \tag{1}$$

where Y stands for knowledge output, A stands for in-classroom effort (i.e., attendance), E stands for out-of-classroom effort, H stands for human capital (i.e., the student's quality), T is an exogenous instructional skill and progress factor, and α, β, and γ are the in-classroom effort, out-of-classroom effort, and human capital shares, respectively, and t is time trend. The parameters that need to be estimated in Equation (1) are T, α, β, and γ. If $\alpha + \beta + \gamma = 1$, there are constant returns to scale in in-classroom effort, out-of-classroom effort, and human capital; and the model can generate perpetual growth. Taking natural logarithms of both sides of Equation (1), the knowledge production function becomes linear:

$$\ln Y_t = \ln T_t + \alpha \ln A_t + \beta \ln E_t + \gamma \ln H_t \ . \tag{2}$$

It should be noted that the knowledge level gained (or produced) by a student in a class represents the student's performance in the class. The higher the knowledge level achieved, the better the student performance. For that reason, Y now stands for a student's performance output (i.e., knowledge product). We take the first difference for both sides of Equation (2). Thus, the regression model can be created as follows:

$$\ln Y_{it} - \ln Y_{it-1} = C_0 + b_A(\ln A_{it} - \ln A_{it-1}) + b_E(\ln E_{it} - \ln E_{it-1}) + b_H(\ln H_{it} - \ln H_{it-1}) + \varepsilon_t, \quad (3)$$

where t = 2 and 3, and ε_t is stochastic in disturbance terms and assuming a mean 0 and a variance σ^2. It is assumed that the independent variables—the first difference in in-classroom effort, out-of-classroom effort, and human capital—are uncorrelated with the stochastic disturbance terms. Equation (3) is known as an empirical model of endogenous growth. $\ln Y_{it} - \ln Y_{it-1}$ stands for growth in performance (i.e., knowledge), $\ln A_{it} - \ln A_{it-1}$ stands for growth in in-classroom effort, $\ln E_{it} - \ln E_{it-1}$ stands for growth in out-of-classroom effort, $\ln H_{it} - \ln H_{it-1}$ stands for growth in human capital, and the constant term (C_0) identifies the teacher's contribution, which is the teacher's instructional progress/skill. In addition, the use of differencing data can eliminate the unobserved individual effect. Those individual unobserved heterogeneity variables are all time-invariant variables. Hence, the act of differencing the data removes the unobserved individual effect. Therefore, behavioral variables (e.g., motivation to complete one's degree, motivation to get a good grade, graduate school aspirations, importance of success in life, student perception of instructor, perception of grading difficulty, perception of teaching and learning style congruence, self-conception of one's ability, etc.) are not shown in the model.

III. Data

Four factors need to be held constant to conduct this experiment. These factors are:

(1) *Teacher's instructional style and teaching materials*. There were four classes (two sections in each semester), so a teacher's instructional style and teaching materials must be held constant. For that reason, only one teacher was chosen to ensure the same instructional style and teaching materials.

(2) *Incentive to attend class*. Students were given complete freedom to make their own choice to attend or not to attend class. Hence, there were no mandatory attendance policies, no attendance bonus, and no quizzes. Both mandatory attendance policies and quizzes enforced students' class attendance while an attendance bonus encouraged students to attend class. In addition, both punishment due to mandatory attendance policies and a bonus changed students' original grades, which led to bias.

(3) *Quality of classroom*. The same classroom was used for two different sections each semester so that the instructor could maintain the same instructional style. The classroom had a chalkboard, an over-head projector, and high-tech equipment, including a computer.

(4) *Same exams for all sections*. The same exams (including midterm exams and final exam) were used with all sections so that results were consistent. All exams were collected when students turned in their answers. Hence, it was impossible for students to get information from a previous year's exams.

In spring 2007 and spring 2009, 203 students in introductory microeconomics classes were the participants in this case study. There were two sections in each semester, and these two sections met twice a week. No additional weekly review/tutorial classes were provided by graduate students for this course. There were three exams in a semester (i.e., two midterm exams and one final exam). Each exam was one-third of the final grade.

The following variables were used in this study:

1. *Performance output (Knowledge output)*. Three exam scores were used to proxy this variable. Each student's three exam scores were recorded and the scores were on a 100-point scale. Due to each exam's differing difficulties, the score for each exam had to be adjusted. To adjust the score, the average for each exam was set at 75.[2] Therefore, each individual's adjusted exam score can be specified as follows: $Aexam_{it} = \frac{x_{it}}{\bar{x}_t} \times 75$, where $t = 1, 2, 3$; $i = 1, \cdots, 203$, $Aexam_{it}$ = student i's adjusted exam score at exam t; x_{it} = student i's initial exam score at exam t; $\bar{x}_t$ = initial average exam score at exam t.

2. *In-classroom effort*. Daily attendance was used to proxy this variable. Daily attendance for each exam period was taken by the instructor. There were three exam periods and ten classes (including the exam day) between exam periods.

3. *Out-of-classroom effort*. Frequency of studying for the class was used to proxy this variable. To collect the data, a questionnaire was developed at each exam. Five minutes before each exam began the questionnaire was handed out to each student. Since no question was confidential, all students were required to write down their names so that these self-reported data could be matched with non-self-reported data. Students were asked: Overall, approximately how long did you study for the class during this exam period?[3] There were five choices for these questions. *1* = I study 1–5 hours before the test; *2* = I study 6–10 hours before the test; *3* = I study 11–15 hours before the test; *4* = I study 16–20 hours before the test; *5* = I study more than 20 hours before the test..

4. *Human capital*. A student's human capital represents his/her quality. Both GPA (Grade Point Average) and SAT (Scholastic Aptitude Test) have been used by researchers to proxy student quality. As a matter of fact, both measures indicate two different dimensions of a student. The GPA, regardless of a student's major, is a measure of a student's motivation and scholarly ability. The SAT score, on the other hand, is a measure of a student's innate ability. Hence, the measurement of both innate ability and motivation and scholarly ability is necessary in addressing this issue. Therefore, we define a student's quality (*QUA*) as $SAT \times GPA$. Students' SAT scores were provided by the admissions office, while students' GPAs were offered by the registrar's office. In this study, the *QUA* scores were converted into an index that may be described as follows: $IQUA_i = \frac{QUA_i}{\max QUA} \times 100$, where $IQUA_i$ = student *i*'s *QUA* index, QUA_i = student *i*'s initial *QUA* scores, and $\max QUA =$ the maximum *QUA* scores among all students. In addition, since human capital is accumulated, each student's human capital can change over exam periods, depending on the student's performance in the previous exam period. A student's human capital is accumulated via efforts devoted to in-classroom/out-of-classroom, human capital in the last period, and exogenous instructional skill/progress. Thus, the student's human capital is given by $H_t = T_t A_{t-1}^{\alpha} E_{t-1}^{\beta} H_{t-1}^{\gamma}$. Based upon this idea, a student's human capital in the second and third periods can be estimated as follows.

<u>*In the second period*</u>:

$$\ln Aexam_1 = \kappa_0 + \kappa_1 \ln A_1 + \kappa_2 \ln E_1 + \kappa_3 \ln IQUA + u_1, \tag{4}$$

where $\ln IQUA = \ln H_1 =$ human capital in the first period; A_1 = in-classroom effort in the first period; E_1 = out-of-classroom effort in the first period; u_1 is stochastic disturbance terms and assuming with a mean 0 and a variance σ^2. We then save $\ln \hat{Aexam_1}$, the predicted value of $\ln Aexam_1$ as obtained from the reduced form estimates. $\ln \hat{Aexam_1}$ can be used to proxy students' human capital in the second period (i.e., $\ln H_2 = \ln \hat{Aexam_1}$).

<u>In the third period</u>:

$$\ln Aexam_2 = \omega_0 + \omega_1 \ln A_2 + \omega_2 \ln E_2 + \omega_3 \ln \hat{Aexam_1} + u_2, \tag{5}$$

where $\ln \ln \hat{Aexam_1} = \ln H_2$ = human capital in the second period; A_2 = in-classroom effort in the second period; E_2 = out-of-classroom effort in the second period; u_2 is stochastic disturbance terms and assuming with a mean 0 and a variance σ^2. We then save $\ln \hat{Aexam_2}$, the predicted value of $\ln Aexam_2$ as obtained from the reduced form estimates. $\ln \hat{Aexam_2}$ can be used to proxy students' human capital in the third period (i.e., $\ln H_3 = \ln \hat{Aexam_2}$). The empirical results of Equations (4) and (5) are presented in Table A.1 in Appendix 1. It should be noted that a student's human capital growth could be positive or negative. Positive growth implies that the student's quality is improving; while negative growth means that student quality is decreasing.

Table 1 reports means and standard deviations for the variables used in this study. Table 2 presents summary statistical elements of growth.

IV. Empirical Results and Sources of Knowledge Growth

4.1 Empirical Results

The results from estimating Equation (3) are reported in Table 3. As shown in that table, growth in in-classroom effort, out-of-classroom effort, and human capital all exerted positive and statistically significant effects on students' performance growth at the 5%, 1%, and 10% significance levels, respectively. The instructional skill/progress also exerted a positive and statistically significant effect on students' knowledge growth at the 10% significance level. Estimates of in-classroom effort share, out-of-classroom effort share, human capital share, and constant term (i.e., instructional skill/progress) were 7.6%, 43%, 35.78%, and 0.854%, respectively.

Moreover, the R-square had approximately 26.7% explanatory power for the independent variables. The equality of all means was tested. Based upon the *F*-statistic, the null hypothesis that all means are equal was rejected. In addition, whether or not autocorrelation exists was tested. According to the Durbin-Watson (DW) statistic, there was no evidence of autocorrelation. The hypothesis of constant returns to scale was also tested. Based upon the results in Appendix 2, the null hypothesis was rejected at the 1% significance level, implying that the knowledge production function does not display constant returns to scale. Furthermore, the hypothesis of heteroskedasticity was tested. Results indicated no evidence of heteroskedasticity in the model.

4.2 Sources of Knowledge Growth

The knowledge growth of a student can be generally attributed to four factors: growth in attendance (i.e., in-classroom effort), growth in out-of-classroom effort, growth in human capital, and instructional skill/progress. Hence, the knowledge output can be specified by an aggregate production function:

$$Y = F(A, E, H, t), \qquad (6)$$

where Y, A, E, and H are knowledge output, in-classroom effort, out-of-classroom effort, and human capital, respectively, and t is an index of chronological time. After differentiating

Equation (6), the rate of growth of knowledge can be expressed in the following equation of growth accounting:

$$\frac{d\ln Y}{dt} = \frac{\partial \ln F}{\partial t} + \frac{\partial \ln F}{\partial \ln A}\frac{d\ln A}{dt} + \frac{\partial \ln F}{\partial \ln E}\frac{d\ln E}{dt} + \frac{\partial \ln F}{\partial H}\frac{dH}{dt}. \tag{7}$$

The four terms on the right-hand-side of Equation (7) can be identified as the contributions of instructional skill/progress, in-classroom effort, out-of-classroom effort, and human capital, respectively, to knowledge growth.

As a matter of fact, Equation (7) is the same as the estimated regression of Equation (3). The estimated constant term for Equation (3) is instructional skill/progress. According to the coefficients for Equation (3), knowledge growth may be split into its proximate sources to obtain the average percentage of distribution for instructional skill/progress, in-classroom effort, out-of-classroom effort, and human capital.

The results of the decomposition are presented in Table 4 (and Figure 1). Findings show that instructional skill/progress accounts for 29.13% on average as a whole during a semester. In-classroom effort, out-of-classroom effort, and human capital account for 22.55%, 27.52%, and 20.80%, respectively. A 29.13% contribution in instructional skill/progress does not appear to be extraordinarily high although it is the largest percentage among these four factors. However, if we combine both instructional skill/progress and in-classroom effort, the total contribution from these two factors becomes 51.68% (= 29.13% + 22.55%), which is over 50%. That is, more than 50% of the sources of a student's knowledge growth are mainly attributed to in-classroom efforts. The reason is obvious. If the student does not attend class, instructional skill/progress cannot contribute to the student's knowledge growth. This result also explains why a student's exam performance suffers in relation to the number of times he/she skips class (Schmidt, 1983; Jones, 1984; Brocato, 1989; Park and Kerr, 1990; Van Blerkom, 1992; Gunn, 1993; Durden and

Ellis, 1995; Douglas and Sulock, 1995; Devadoss and Foltz, 1996; Marburger, 2001; Rodgers, 2001; Rocca, 2003; Krohn and O'Connor, 2005; Cohn and Johnson, 2006; and Stanca, 2006).

V. Further Exploration

Study results revealed that instructional skill/progress and in-classroom effort may be complementary. In addition, students who frequently skip class may still perform well in that class. For this reason, it is also possible that in-classroom effort and out-of-classroom effort may substitute for one another. Therefore, in this section a complementarity test is provided to investigate whether a relationship (i.e., complements or substitutes) exists between in-classroom effort and out-of-classroom effort as well as between instructional skill/progress and in-classroom effort.

The complementarity formation may be modeled using a generalization of the Cobb-Douglas model, the transcendental logarithmic production function, which is:

$$\ln Y_t - \ln Y_{t-1} = D_0 + a_A\left[\ln A_t - \ln A_{t-1}\right] + a_S\left[\ln S_t - \ln S_{t-1}\right] + a_H\left[\ln H_t - \ln H_{t-1}\right]$$

$$\begin{aligned} &+ a_{AA}\left[(\ln A_t)^2 - (\ln A_{t-1})^2\right]/2 + a_{SS}\left[(\ln S_t)^2 - (\ln S_{t-1})^2\right]/2 + a_{HH}\left[(\ln H_t)^2 - (\ln H_{t-1})^2\right]/2 \\ &+ a_{AS}\left[\ln A_t \ln S_t - \ln A_{t-1} \ln S_{t-1}\right] + a_{AH}\left[\ln A_t \cdot H_t - \ln A_{t-1} \cdot H_{t-1}\right] \\ &+ a_{SH}\left[\ln S_t \cdot \ln H_t - \ln S_{t-1} \cdot \ln H_{t-1}\right] + u_t \end{aligned}, \quad (8)$$

where u_t is stochastic disturbance terms assuming a mean 0 and a variance σ^2. If in-classroom effort and out-of-classroom effort are substitutes, $a_{AS} < 0$ and the effect should be significant. According to the results shown in Table 5, unfortunately, no remarkable relationship is found between in-classroom effort and out-of-classroom effort (neither substitutes nor complements), implying that they are not significantly related. That is, students' in-classroom efforts do not crowd out their out-of-classroom efforts. Students who frequently skipped class might not necessarily study harder out-of-classroom. On the other hand, students who always attended class might also study hard out-of-classroom.

In addition, are in-classroom effort and human capital substitutes; similarly, are out-of-classroom effort and human capital substitutes? If they are substitutes, it should be possible to estimate $a_{AH} < 0$, $a_{SH} < 0$ and the effects should be significant. Nevertheless, as shown in the results in Table 5, there are no remarkable relationships between in-classroom effort and human capital as well as out-of-classroom effort and human capital, implying that they are not significantly related and they will not crowd out each other. A student who is smart (has high human capital) might not necessarily skip class frequently or study hard out-of-classroom.

Furthermore, it may be possible that in-classroom effort and instructional skill/progress are complements. Thus, a simple model for in-classroom effort and instructional skill/progress complementarity can be constructed as follows:

$$\ln Y_t - \ln Y_{t-1} = G_0 + d_A\left[\ln A_t - \ln A_{t-1}\right] + d_S\left[\ln S_t - \ln S_{t-1}\right] + d_H\left[\ln H_t - \ln H_{t-1}\right] + d_{At}\left[A_t \cdot t - A_{t-1} \cdot (t-1)\right] + \nu_t, \qquad (9)$$

where t = 2 and 3, and ν_t is stochastic disturbance terms assuming a mean 0 and a variance σ^2. If instructional skill/progress and in-classroom effort are complements, an estimate of $d_{At} > 0$ should be feasible and the effect should be significant. Consequently, based upon results shown in Table 5, a positive and remarkable relationship was found between in-classroom effort and instructional skill/progress, implying that they are complements. That is, students who do not attend class do not benefit from the instructor's instructional skills/progress. The more often the student attends class, the greater will be the effect of the instructor's instructional skill/progress.

VI. Conclusion

The techniques involved in modeling assistance to a student as she/he produces knowledge are quite similar to those experienced by a country engaging in economic production modeling techniques. In this paper, an empirical model of endogenous growth was applied to estimate and investigate students' knowledge growth. Results revealed that growth in in-classroom effort, out-of-classroom effort, and human capital all exerted positive and statistically significant effects on students' knowledge growth at the 5%, 1%, and 10% levels, respectively. Instructional skill/progress also exerted a positive and statistically significant effect on students' knowledge growth.

In addition, sources of knowledge growth were extracted to identify each factor's contribution to students' knowledge growth. The evidence suggested that the total contribution from a combination of both instructional skill/progress and in-classroom effort was over 50%, implying that more than half of sources were primarily attributed to in-classroom efforts. This is because instructional skill/progress does not contribute to a student's knowledge growth if that student does not attend class. Therefore, this paper offers another alternative approach to investigating the issue of attendance and exam performance—a positive and significant correlation between the two.

Moreover, a complementarity test was conducted to verify whether in-classroom effort and out-of-classroom effort are substitutes, and whether instructional skill/progress and in-classroom efforts are complements. Findings revealed that in-classroom efforts and out-of-classroom efforts are not significantly related, which means that they will not crowd out each other. However, findings show that instructional skill/progress and in-classroom efforts are significantly related and are complements. Students attend class to hear the instructors' lectures.

Therefore, the more frequently the student attends class, the greater will be the effect on the instructor's instructional skill/progress.

Finally, this study has an important finding—over 50% of student knowledge growth occurred in the traditional face-to-face classroom. The information tells us that the importance of traditional classroom learning cannot be simply ignored. The process of this type learning is important to our personal as well as professional growth. Due to technology improvements, online classes have been widely offered by almost every university/college in the United States, and have been significantly enhancing student enrollments and thus increasing school revenues. While online classes have many advantages (e.g., convenience, flexibility, learning at home, etc.), traditional face-to-face classes also have a number of advantages that cannot be perfectly replaced by online classes. This includes teacher-student interaction and instant feedback—aspects that cannot be provided via online components. In addition, the social and communication aspects of traditional classroom learning are another advantage that cannot be ignored. Therefore, in addition to offering online classes to increase student enrollments and raise school revenues, school authorities should be aware of the contribution and importance of traditional face-to-face classes and continue to improve the quality of traditional classroom learning.

TABLE 1–Mean and Standard Deviation of Variables

Variables	Mean	Standard Deviation
Scores for exam I	67.121	16.553
Scores for exam II	77.515	15.175
Scores for exam III	50.877	14.431
SAT scores	1006.503	139.000
GPA	2.749	0.596
Frequency of studying for first exam	2.916	1.120
Frequency of studying for second exam	3.192	1.155
Frequency of studying for third exam	2.921	1.224
Number of attendance in the first exam period	9.118	1.249
Number of attendance in the second exam period	8.498	1.876
Number of attendance in the third exam period	8.621	1.975

TABLE 2–Summary Statistics Elements of Growth

	Growth Rate of Adjusted Exam Performance (%)	Growth Rate of In-classroom Effort (%)	Growth Rate of Out-of-classroom Effort (%)	Growth Rate of Human Capital Index (%)
Mean	2.16	-1.24	3.01	1.74
Standard Deviation	25.1	27.13	31.77	4.38

TABLE 3–Estimate of $\ln Y_t - \ln Y_{t-1}$

Explanatory Variables	**OLS** Explained Variable: $\ln Y_t - \ln Y_{t-1}$
Constant	0.00854* (1.80)
$\ln A_t - \ln A_{t-1}$	0.07607** (2.13)
$\ln S_t - \ln S_{t-1}$	0.43055*** (11.41)
$\ln H_t - \ln H_{t-1}$	0.3578* (1.65)
R^2	0.267
$\overline{R}^2$	0.262
F-Statistic	48.89
Durbin-Watson Statistic	1.92931
Observations	406

(*t*-value) *** denotes statistical significance of the *t*-statistic at the 0.01 level; ** denotes statistical significance of the *t*-statistic at the 0.05 level; * denotes statistical significance of the *t*-statistic at the 0.10 level.

TABLE 4–Average Percentage of Distribution of Performance Growth

Instructional Skill/Progress	In-classroom Effort	Out-of-classroom Effort	Human Capital
29.13%	22.55%	27.52%	20.80%

TABLE 5–Estimate of $\ln Y_t - \ln Y_{t-1}$ (Complementarity Test)

Explanatory Variables	**OLS** Explained Variable: $\ln Y_t - \ln Y_{t-1}$	**OLS** Explained Variable: $\ln Y_t - \ln Y_{t-1}$
Constant	0.01960*	-0.20086***
	(1.70)	(-2.63)
$\ln A_t - \ln A_{t-1}$	1.210	-0.20086*
	(0.85)	(-1.71)
$\ln S_t - \ln S_{t-1}$	0.648	0.41738***
	(0.50)	(11.06)
$\ln H_t - \ln H_{t-1}$	-18.398**	0.4561**
	(-2.48)	(1.98)
$\frac{(\ln A_t)^2 - (\ln A_{t-1})^2}{2}$	0.19757***	
	(2.93)	
$\frac{(\ln S_t)^2 - (\ln S_{t-1})^2}{2}$	0.4480***	
	(4.04)	
$\frac{(\ln H_t)^2 - (\ln H_{t-1})^2}{2}$	4.514***	
	(2.62)	
$\ln A_t \ln S_t - \ln A_{t-1} \ln S_{t-1}$	-0.05185	
	(-0.84)	
$\ln A_t \ln H_t - \ln A_{t-1} \ln H_{t-1}$	-0.3224	
	(-0.99)	
$\ln S_t \ln H_t - \ln S_{t-1} \ln H_{t-1}$	-0.1139	
	(-0.37)	
$\ln A_t \cdot t - \ln A_{t-1} \cdot (t-1)$		0.09731***
		(2.77)
R^2	0.325	0.281
$\overline{R}^2$	0.31	0.274
F-Statistics	21.19	38.19
Observations	406	406

(t-value) *** denotes statistical significance of the t-statistic at the 0.01 level; ** denotes statistical significance of the t-statistic at the 0.05 level; * denotes statistical significance of the t-statistic at the 0.10 level.

Appendix 1

TABLE A.1– Estimates of $\ln Aexam_1$ and $\ln Aexam_2$

Explanatory Variables	**OLS** Explained Variable: $\ln Aexam_1$	**2SLS** Explained Variable: $\ln Aexam_2$
Constant	0.5073	1.9904***
	(0.92)	(4.03)
$\ln A_1$	0.2662***	
	(2.60)	
$\ln A_2$		0.18187***
		(4.30)
$\ln E_1$	0.14847***	
	(3.86)	
$\ln E_2$		0.03255
		(0.89)
$\ln IQUA$ $(= \ln H_1)$	0.7028***	
	(5.86)	
$\ln \hat{Aexam}_1$ $(= \ln H_2)$		0.4408***
		(3.55)
R^2	0.243	0.239
$\bar{R}^2$	0.232	0.227
F-Statistics	21.35	20.78
Observations	203	203

(t-value) *** denotes statistical significance of the t-statistic at the 0.01 level.

Appendix 2: Testing for Constant Returns to Scale

In this appendix section, we test whether the performance function displays constant returns to scale. As shown in Equation (1) in section 2, the knowledge production function is the general form of the Cobb-Douglas production function. We take the natural log of both sides of Equation (1) and define the regression equation, as shown in the following:

$$f(A,E,H) = Y_t = TA_t^{\alpha} E_t^{\beta} H_t^{\gamma} e^{\varepsilon_t}$$

$$\Rightarrow \ln Y_t = \ln T + \alpha \ln A_t + \beta \ln E_t + \gamma \ln H_t + \varepsilon_t$$

$$\Rightarrow \ln Y_t = C_0 + \alpha \ln A_t + \beta \ln E_t + \gamma \ln H_t + \varepsilon_t, \qquad \text{(A.1)}$$

where $C_0 = \ln T$ and ε_t is stochastic disturbance terms and assuming with a mean 0 and a variance σ^2. We are interested in determining whether the performance function exhibits constant returns to scale and test against the alternative hypothesis that the returns are not constant. Hence, the null hypothesis is $H_0 : \alpha + \beta + \gamma = 1$ and the alternative hypothesis is $H_0 : \alpha + \beta + \gamma \neq 1$. We now define $\gamma_2 = \alpha + \beta + \gamma - 1$. Under the null hypothesis, $\gamma_2 = 0$. Solving γ, we get $\gamma = \gamma_2 - \alpha - \beta + 1$. Substituting this in Equation (A.1), we obtain

$$\ln Y_t = C_0 + \alpha \ln A_t + \beta \ln E_t + (\gamma_2 - \alpha - \beta + 1)\ln H_t + \varepsilon_t$$

$$\Rightarrow \ln Y_t = C_0 + \alpha(\ln A_t - \ln H_t) + \beta(\ln E_t - \ln H_t) + \ln H_t + \gamma_2 \ln H_t + \varepsilon_t$$

$$\Rightarrow \ln Y_t - \ln H_t = C_0 + \alpha(\ln A_t - \ln H_t) + \beta(\ln E_t - \ln H_t) + \gamma_2 \ln H_t + \varepsilon_t. \qquad \text{(A.2)}$$

The results of the estimation from Equation (A.2) are reported in Table A.2. As that table shows, the coefficient of $\ln H_t$ (i.e., γ_2) is statistically significantly different from zero at the 1% significance level, implying that $\gamma_2 \neq 0$. That is, the null hypothesis is rejected. Therefore, we conclude that the performance function does not display constant returns to scale.

TABLE A.2–Estimate of $\ln Y_t - \ln H_t$

Explanatory Variable	OLS
Constant	-0.2700
	(-0.71)
$\ln A_t - \ln H_t$	0.30019***
	(10.59)
$\ln E_t - \ln H_t$	0.10698***
	(4.74)
$\ln H_t$	0.29135***
	(3.18)
R^2	0.218
$\overline{R}^2$	0.214
F-Statistics	56.28
Observations	608

(t-value) *** denotes statistical significance of the t-statistic at the 0.01 level.

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Endnotes

1. In the empirical model of endogenous growth, human capital is the engine of growth.
2. There is no particular reason to set the average at 75 beyond the fact that 75 is between grade B and grade C.
3. Students were not asked to write down the number of hours devoted to studying for the class because they might not precisely remember how many hours they studied for the class in each exam period, but they may be able to recall an extent they studied for the class.