# Dynamic Corporate Liquidity[*]

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## Abstract

When external finance is costly, liquid funds provide corporations with instruments to absorb and react to shocks. Making optimal use of liquid funds means transferring them to times and states where they are most valuable. We examine the determinants of corporate liquidity management in a dynamic model where stochastic investment opportunities and cash shortfalls provide liquidity needs. Firms can transfer liquidity across time using cash and across states drawing on credit lines subject to debt capacity constraints. We generate empirical and quantitative predictions by means of calibration. Small and constrained firms use cash to provide liquidity to fund investment opportunities, while large and unconstrained firms manage their liquidity needs by means of credit lines. In the time series, equity issuances are used to replenish cash balances, a credit lines to fund unanticipated investment opportunities. We find strong support for our predictions in the data. Overall, the model thus provides a quantitatively and empirically successful framework explaining corporate investment, financing and liquidity policies and the joint occurrence of cash, debt and credit lines in the presence of capital market imperfections.

**Keywords:** Corporate liquidity, cash, credit lines, debt capacity, leverage, corporate investment, hedging

# 1 Introduction

When external finance is costly, liquid funds provide corporations with instruments to absorb and react to shocks. Making optimal use of liquid funds means transferring them to times and states where they are most valuable. Liquid funds may be valuable because they aid financing of a profitable investment opportunity, or because they help covering cash shortfalls. Anticipations of such future states thus provide a rationale for corporate liquidity management and renders it inherently dynamic. One way to implement liquidity management is using uncontingent instruments, such as holding cash, which transfers liquid funds across all states symmetrically. We will refer to such policies as unconditional liquidity management. Alternative instruments, such as credit lines or derivatives, have a more state-contingent flavor in that corporations may draw on them to transfer funds to specific states only. We will refer to such policies as conditional liquidity management.

In practice, we see firms engaging in many combinations of conditional and unconditional liquidity management policies, yet there is relatively little work attempting to understand the determinants of these choices. In this paper, our objective is to take a step towards filling this gap. We do so by proposing a dynamic model of corporate policies that explicitly allows corporations to transfer liquid funds unconditionally using cash and conditionally by drawing on credit lines. In the model, liquidity needs arise from stochastic investment opportunities and cash shortfalls in the context of high leverage. By solving the model numerically, we provide novel empirical predictions on the cross-sectional and time-series determinants of corporations' liquidity policies. We test these predictions empirically using data on credit lines from CapitalIQ and find strong support for them. The model thus provides a quantitatively and empirically successful framework explaining corporate investment, financing and liquidity policies and the joint occurrence of cash, debt and credit lines in the presence of capital market imperfections.

In the model, firms attempt to take advantage of profitable investment opportunities that arise stochastically. However, due to capital market imperfections, issuing equity entails costs such that firms will find it beneficial to exploit the tax benefits of leverage by issuing debt. However, we assume that debt needs to be collateralized by capital so that all debt is secured. This means that firms' debt capacity is endogenously bounded. In this context, a rationale for liquidity management arises. Firms can transfer liquidity unconditionally across all states by saving, that is, by holding cash. On the other hand, firms can preserve debt capacity in a state-contingent way by drawing on their credit lines as economic conditions dictate. This allows firms to transfer liquidity conditionally to specific states only. We show that the model predicts that firms will exploit conditional and unconditional liquidity management highly differentially both in the cross-section and in the time series. Calibrating the model, we find that such differential use of liquidity management provides a coherent explanation for many stylized facts about firms joint investment, financing and liquidity policies.

Our model rationalizes the empirical evidence that firms simultaneously hold cash and debt, hence corroborating the notion that cash is not negative debt. Within the context of our model, the intuition is simple. While debt and credit lines jointly allow for state-contingency within the limits of debt capacity, holding cash allows to transfer liquidity beyond collateral constraints in case of high financing needs. Such high financing needs most likely arise when firms have many profitable investment opportunities. In this context, the model predicts that small firms and constrained firms (as measured by net worth) hold more cash, all else equal. This is a pattern well documented in the data, indicating that such firms mostly manage liquidity by means of unconditional instruments. On the other hand, large firms and relatively unconstrained firms are predicted to hold less cash and have more undrawn credit, indicating that they rely more conditional policies for liquidity management. We confirm this prediction using data on credit lines from CapitalIQ. Our model also replicates the well documented positive relationship between leverage and size.

An important implication of the model is that empirically we carefully need to distinguish between small firms (as measured by the capital stock) and constrained firms (as measured by net worth). Indeed, these variables are the two relevant (endogenous) state variables in the model. While the two variables are indeed somewhat correlated, we document the need of distinguishing them by means of two way sorts on relevant variables on capital and undrawn credit. These sorts suggest that the main driver of cash holdings is capital, while financial constraints matter less. Since low capital implies valuable growth opportunities (in a model with decreasing returns to scale), this suggests that unconditional liquidity management mostly serves to transfer funds to states with high investment opportunities. On the other hand, the amount of undrawn credit mostly varies with net worth, controlling for capital. Indeed, unconstrained firms have more slack on their credit lines, so that the transfer more funds to valuable states conditionally. Symmetrically, constrained firms mostly exhaust their debt capacity. This is consistent with the notion, developed in Rampini and Viswanathan (2010), and Rampini and Viswanathan (2012a), that constrained firms hedge less, and that if they do, they do it unconditionally using cash. We find strong support for these predictions in the data, suggesting the need to distinguish between size and financial constraints, in contrast to most commonly used financial constraint indicators in empirical work. Moreover, these findings suggest that cross-sectionally we can distinguish firms whose liquidity management is mostly dictated by preserving liquidity for investment opportunities, which we label 'upstate hedging', as opposed to firms preserving liquidity in order to cover cash shortfalls, which we label 'downstate hedging'. In particular, our findings suggest that different instruments serve such liquidity needs better. Figure 1 illustrates our results.

Our analysis points to the importance of examining financing and liquidity policies in the context of investment opportunities, and in particular, investment frictions. While it is well known that financing policies in dynamic investment models exhibit considerable sensitivity to the specification of investment technologies, we reinforce such results in the context of measures of firms' liquidity management. Obstructions to frictionless adjustment of the capital stock in

dynamic corporate models are most commonly represented by means of a convex (quadratic) adjustment cost. Our results clearly indicate that fixed costs of adjustment are important to understand liquidity management at the firm level, and cash holdings in particular.

Our paper is at the intersection of several converging lines of literature. In particular we interpret the quantitative literature on dynamic investment and financing (as started by Gomes (2001), Hennessy and Whited (2005), and Hennessy and Whited (2007)) further in light of the recently emerged literature on dynamic risk management and hedging in the context of collateralized debt (Rampini and Viswanathan (2010), Rampini and Viswanathan (2012a)). We build on Rampini and Viswanathan by modeling state-contingent debt subject to collateral constraints. While Rampini and Viswanathan operate in a dynamic optimal contracting framework, we take the form of the contracts as exogenously given and interpret them in the wider context of commonly used frictions in the dynamic financing literature, such as equity issuance costs and investment frictions. Most importantly, we allow firms to use cash as a form of liquidity management. While these leads to a distinct set of empirical predictions, we moreover view our paper as contributing more to the quantitative and empirical literature rather than the one on optimal security design.

Our paper is closely related to the emerging literature on firm policies and cash holdings. A non-exhaustive list includes Nikolov and Whited (2009), Morellec and Nikolov (2009), Hugonnier, Malamud and Morellec (2011), Bolton, Chen, and Wang (2011), Falato, Kadyrzhanova, and Sim (2013), Bolton, Chen, and Wang (2012), and Eisfeldt and Muir (2013). Our main departure from this line of literature is that we allow for conditional liquidity management that we interpret in the context of credit lines. Our empirical results suggest that this is a relevant model feature. In this context, our paper is most closely related to Bolton, Chen and Wang (2011, 2012), who allow firms to access credit lines and hedge aggregate shocks using derivatives. On the other hand, for tractability, these authors operate within an AK-framework which allows to reduce the number of state variables and to obtain analytical solutions up to an ordinary differ-

ential equation. However, our empirical results suggest that distinguishing between the capital stock and net worth as state variables is empirically relevant.

From a computational viewpoint, we introduce linear programming methods into dynamic corporate finance. Accounting for conditional liquidity management by means of state-contingent policies introduces a large number of control variables into our setup which would render our model subject to the curse of dimensionality for standard computational methods. We exploit and extend linear programming methods to circumvent this problem and efficiently solve for the value and policy functions in this class of problems. Linear programming methods, while common in operations research, have been introduced into economics and finance by Trick and Zin (1993, 1997). We extend their methods to setups common in corporate finance. More specifically, we exploit a separation oracle, an auxiliary mixed integer programming problem, to deal with large state spaces and find efficient implementations of Trick and Zin's constraint generation algorithm.

This paper is structured as follows. After presenting some stylized empirical evidence on corporate liquidity management in section 2, we present our model in section 3. We qualitatively examine the determinants of corporations' joint investment, financing and liquidity policies in section 4. After detailing our approach to calibration and identification of our quantitative model in section 5, we present cross-sectional implications in section 6 and time-series implications by means of generalized nonlinear impulse response functions in section 7. Section 8 concludes.

[Insert Figure 1 Here]

# 2 Stylized Facts on Corporate Liquidity

In this section we revisit the key empirical facts about firms' joint liquidity management, cash, and capital structure decisions. This evidence can be rationalized and interpreted within our model. In Table 1, we present stylized evidence by sorting firms on the empirical counterpart of the two state variables of our model, namely net worth, and capital stock. The sorts in table 1 are based on a sample of manufacturing firms from the merged Compustat Annual and Capital IQ datasets, for the period 2001-2011. As in the models of Rampini and Viswanathan (2010), and Rampini and Viswanathan (2012a), net worth determines the amount of resources that are available to the firm in a certain state of the world. Net worth is the sum of realized cash flows from current investment, capital net of depreciation, and cash holdings, net of debt repayments. Intuitively, net worth is the firm's counterpart of household's wealth. Therefore, net worth captures how constrained a company is with respect to funds to allocate to investment, risk management, and distributions. Consistent with the definition in our model, we proxy net worth as the book value of shareholder equity as in Rampini, Sufi, and Viswanathan (2012). Capital stock is measured as the book value of property, plant, and equipment. For each year, firms that are above (below) the 67th (33th) percentile of net worth are classified as relatively unconstrained (constrained). Using the same procedure, firms are classified as large or small on the basis of their capital stock.

An important caveat that limits empirical evidence on corporate risk management is that firm's hedging is unobservable. Existing studies focus on specific industries and types of hedging to draw inference. For example, Tufano (1996) considers hedging of output price in the gold mining industry, while Rampini, Sufi, and Viswanathan (2012) investigate hedging of input (fuel) price for airlines. In our model, firms can transfer conditional liquidity by keeping slack on their collateral constraints, that is by saving debt capacity in a state-contingent way. As Rampini and Viswanathan (2012a) discuss, an important implementation of conditional liquid-

ity management relies on loan commitments. This implementation appears to be important in practice, because credit lines play a first-order role for firm's financing. As Sufi (2009) points out, over 80 percent of bank debt held by public firms is in the form of lines of credit. Moreover, Colla, Ippolito, and Li (2013) report that the drawn part alone of credit lines accounts for more than 20 percent of the debt structure of US listed firms. On the contrary, the overall quantitative importance of risk management based on derivatives is debatable. For instance, Guay and Kothari (2003) find that even large firms implement little hedging through financial derivatives. In table 1, we report the undrawn fraction of credits from lines of credit from the Capital IQ dataset. For the aforementioned reasons, and because of data limitations, we consider this indicator as a proxy of how much firms are slack on their collateral constraints for providing stylized evidence. This choice is consistent with the definition of conditional liquidity in our model. Despite there are reasons other than hedging for which firms do not fully draw from their credit lines, such as limited investment needs, we expect to observe cross-sectional differences in the fraction of undrawn debt capacity across net worth and capital clusters.

Panel A shows one-way sorts by net worth and capital. We report mean cash-to-asset and debt-to-asset ratios, and the average fraction of undrawn credit from credit lines. More constrained and smaller firms have larger cash holdings, consistent with existing empirical studies, such as Denis and Sibilkov (2009), and Almeida, Campello, and Weisbach (2004). Consistent with some constrained firms having low cash holdings, as in Denis and Sibilkov (2009), the pattern is more pronounced for the sort on capital. Small firms in our sample have an average cash-to-asset ratio of 23.6 percent, compared to 9.6 percent for large firms. Constrained firms instead hold 16.7% of their asset in cash and cash equivalents, while the ratio falls to 11.7% for unconstrained firms. Regarding leverage, the cross-sectional patterns for sorts on net worth and capital have opposite directions. The sort on capital highlights the well-known positive relationship between leverage and size, that several studies document. Firms with low net worth appear to have more debt than those with high net worth, namely 35.1% versus 23.1% of total

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assets. Finally, relatively unconstrained firms appear to have more undrawn credit, in line with the result in Rampini, Sufi, and Viswanathan (2012) that firms with high net worth hedge more (0.922 versus 0.716). Large firms also appear to keep more slack on their credit lines, despite the pattern is not as clear as for the sort on net worth.

As Rampini, Sufi, and Viswanathan (2012) discuss, patterns that relate the corporate policy to net worth are largely unexplored. In our framework, net worth measures how constrained is a firm with respect to the amount of available resources. Other proxies of financial constraints used in the empirical literature capture different dimensions. For example, bond ratings proxy for distance to default. Remarkably, size, typically measured as the book value of total assets, is one of those proxies. In our model, net worth and capital are two different state variables. Therefore, in panel B, we report two-way sorts to revisit and provide new insights about the key stylized facts on debt, cash, and risk management with respect to these two variables. Distinguishing between net worth and capital allows to uncover stylized evidence that can be useful to understand firms' conditional and unconditional liquidity and hedging policies.

Concerning cash holdings, our two-way sorts show that capital is the main variable that influences cash. Small firms hold more cash than large firms for each cluster of new worth. The "Cash holdings" panel shows that the cash-to-asset ratio of small firms is around three times higher than that of large firms. Remarkably, after controlling for capital, unconstrained firms appear to hold more cash than constrained firms. This evidence is consistent with the finding in Denis and Sibilkov (2009) that some constrained firms have low cash holdings, despite small firms hold more cash than large firms. The "Leverage" panel highlights that large constrained firms have very high debt ratios (69.6% of total assets), much higher than large firms with high net worth (25.1% of total assets). A similar pattern, but less strong, can be observed for less constrained firms, which are likely to have more internal resources (38.1% versus 8.9% for the second cluster, and 25.1% versus 10.5% for unconstrained firms). Finally, the joint effect of net worth and capital on undrawn credit suggest that unconstrained firms are more slack on

their credit lines, while capital does not appear to play a very important role. This pattern is consistent with the evidence in Rampini, Sufi, and Viswanathan (2012), and emphasizes the importance to distinguish between net worth and size per se.

Finally, as Strebulaev and Whited (2012) point out, an interesting piece of evidence, which existing dynamic models of investment and financing are generally unable to rationalize, is that firms simultaneously hold cash and debt.[1]

[Insert Table 1 Here]

In summary, the key stylized facts on corporate liquidity, financing, and hedging can be summarized as follows:

- Firms with low capital and high net worth have higher cash holdings;

- Firms with high capital and low net worth have higher leverage;

- Firms with high net worth are more slack on their lines of credit;

- Firms simultaneously hold cash and debt.

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[1]An exception is Gamba and Triantis (2008).

# 3 The Model

This section provides a dynamic neoclassical model of investment, financing, and corporate liquidity. Managers decide at each period in a infinite-horizon environment. This ensures that they take into account the expected consequences of today's decisions for the feasibility of future decisions. They jointly decide over (i) investment in real capital, (ii) debt and equity issues, (iii) cash holdings, and (iv) state-contingent hedging in order to maximize shareholders' wealth. The feasible set of managers' decisions is limited by the presence of real and financial frictions. As in Rampini and Viswanathan (2010), dynamic debt financing is subject to collateral constraints that limit firms' debt capacity. Collateral constraints reflect limited enforcement problems that prevent creditors from accurately assessing the firms ability to repay debt. State-contingent hedging can hence be interpreted as conserving debt capacity to finance future investments, in presence of uncertainty and limited debt capacity. State-contingent liquidity management can be implemented, for example, by loan commitments, or by purchasing traded securities to hedge shocks which can affect firms' cash flows and investment opportunities. On the real side, adjusting the real capital stock entails both fixed and smooth costs, as in Cooper and Haltiwanger (2006). In addition, following the existing literature, firms face costly equity issues, and costs of maintaining cash balances.

## 3.1 Technology and Investment

We consider the problem of a value-maximizing firm in a perfectly competitive environment. Time is discrete. The operating profit for firm $i$ in period $t$ depends upon the capital stock $k_{i,t}$ and a shock $z_{i,t}$, as described by the expression

$$\Pi(k_{i,t}, z_{i,t}) = (1 - \tau)z_{i,t}k_{i,t}^{\alpha} - f \tag{1}$$

The production function exhibits decreasing returns to scale with $0 < \alpha < 1$. As in Gomes (2001), we assume there is a per-period fixed production cost $f \geq 0$. $\tau \geq 0$ is the corporate tax rate. The variable $z_{i,t}$ reflects shocks to demand, input prices, or productivity. $z_{i,t}$ is assumed to be lognormal and to obey the Markovian law of motion

$$\log(z_{i,t+1}) = \mu_z(1 - \rho_z) + \rho_z \log(z_{i,t}) + \sigma_z \varepsilon_{i,t+1} \tag{2}$$

where $\varepsilon_{i,t+1}$ is a truncated standard normally distributed random variable. The parametrization in equation (2) ensures that the transition probability has the Feller property. In addition, we require that $z_{i,t}$ lies in a close and bounded (therefore compact) set by imposing large bounds on the values of $\varepsilon_{i,t+1}$. $k_{i,t}$ falls into the compact set $[0, \bar{K}]$ without loss of generality. Following Gomes (2001), $\bar{K}$ can be defined as

$$\Pi(\bar{K}, \bar{Z}) = \delta \bar{K} \tag{3}$$

where $\bar{Z}$ in the upper bound for $z_{i,t}$, and $\delta$ is the depreciation rate of capital. Hence, a capital stock larger than $\bar{K}$ cannot be observed, because not economically profitable. The compactness of the state space for $k_{i,t}$ and $z_{i,t}$, and the continuity of $\Pi(k_{i,t}, z_{i,t})$, ensure that $\Pi(k_{i,t}, z_{i,t})$ is bounded. This is a necessary condition for the existence of a solution for the firm's problem. At the beginning of each period the firm is allowed to scale its operations by choosing next period capital stock $k_{i,t+1}$. This is accomplished through investment $i_{i,t}$, which is defined by the standard capital accumulation rule

$$k_{i,t+1} = k_{i,t}(1 - \delta) + i_{i,t} \tag{4}$$

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Investment is subject to capital adjustment costs. Following Cooper and Haltiwanger (2006), we include both fixed and convex adjustment cost components. We parametrize capital adjustment costs with the following functional form:

$$\Psi(k_{i,t+1},k_{i,t}) \equiv \left( \psi_0^+ k_{i,t} + \frac{1}{2}\psi^+ \left( \frac{i_{i,t}}{k_{i,t}} \right)^2 k_{i,t} \right) \mathbf{1}_{\{k_{i,t+1}>(1-\delta)k_{i,t}\}} + \left( \psi_0^- k_{i,t} + \frac{1}{2}\psi^- \left( \frac{i_{i,t}}{k_{i,t}} \right)^2 k_{i,t} \right) \mathbf{1}_{\{k_{i,t+1}<(1-\delta)k_{i,t}\}} \tag{5}$$

where $\mathbf{1}_{\{.\}}$ is an indicator function, and the parameters $\psi_0^+$ and $\psi_0^-$ govern fixed-adjustment costs of investing and disinvesting respectively. Non-convex costs of adjustment are typically intended to capture indivisibilities in capital, increasing returns to the installation of new capital, and increasing returns to retraining and restructuring of production activity. $\psi^+$ and $\psi^-$ instead drive the convex component of adjustment costs. We consider asymmetric adjustment costs because, as in Zhang (2005), disinvesting is typically more costly than increasing capital. Both convex and non-convex costs are proportional to the initial capital stock $k_{i,t}$ to eliminate any size effect.

## 3.2 Financing and Liquidity Management

Investment and distributions to shareholders can be financed with three potential sources: internally generated cash flows, riskfree debt (net of repayments), and external equity. In addition, firms have the option to hoard cash for future investments. As in Rampini and Viswanathan (2010), we model one-period state-contingent debt. Formally, $(1+r)b_{i,t+1}(z(i,t+1))$ represents the face value to be repaid at time $t+1$ in the state of the world $s(t+1)$ corresponding to the realization of the shock $z(i,t+1)$, where $r$ is the one-period rate of return.[2] In other words, the firm is borrowing from deep-pocket lenders who are willing to lend in all states and

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[2]Because our focus in not on endogenous costs of distress, as in Hennessy and Whited (2005) we make the assumption of riskfree debt in the interest of tractability. Given the high number of decision variables and the presence of occasionally non-binding constraints and non-convex costs, solving the model is computationally intensive. The introduction of endogenous default costs would disproportionately increase the computational burden.

dates at the rate of return $r$. To simplify notation, we introduce the shorthand $b_i(s(t+1))$ for the decision variables $b_{i,t+1}(z(i,t+1))$. The value of new debt issues at time $t$ in state $s(t)$ is

$$E_t[b_i(s(t+1))] - (1 + r(1-\tau))b_i(s(t)) \tag{6}$$

where the operator $E_t[\cdot]$ denotes the expectation under the manager's probability measure conditional to her information set at time $t$. In equation (6), the term $E_t[b_i(s(t+1))]$ represents the observed debt stock on the firm balance sheet in period $t$, which is determined by risk-neutral security pricing in the capital market.[3] $1 + r(1-\tau)$ is the effective interest rate paid by the firm, after accounting for the tax shield of debt. Firms are subject to collateral constraints, that impose an upper bound on the amount of one-period state-contingent debt that a firm can issue. Assuming that future cash flows are not pledgeable, collateral constraints take the form:

$$(1 + r(1-\tau))b_i(s(t+1)) \le \theta(1-\delta)k_{i,t+1} \tag{7}$$

Up to a fraction $\theta$ of the resale value of the firm's tangible capital can be used as collateral for state-contingent debt at time $t+1$ in state $s(t+1)$. Rampini and Viswanathan (2012a) prove that collateral constraints of this form are equivalent to limited enforcement constraints. Intuitively, in a dynamic agency problem, the enterpreneur can abscond with a part of tangible capital. The lender is mindful of this possibility, and she cannot precisely gauge the firm's ability to support debt. Therefore, he imposes participation and enforcement constraints that limit the share of capital she is willing to finance. We characterize risk management and the conditional

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[3]To see this, one can apply the basic asset pricing formula to the state-contingent claim with payoffs $(1+r)b_i(s(t+1))$ at time $t+1$. Today's market valuation of the debt stock under the measure of deep-pocket investors is therefore

$$E_t[M(s(t+1))(1+r)b_i(s(t+1))]$$

where $M(s(t+1))$ is the stochastic discount factor. Under risk neutrality, $M(s(t+1)) = \frac{1}{1+r}$. As a consequence:

$$E_t\left[\frac{1}{1+r}(1+r)b_i(s(t+1))\right] = E_t[b_i(s(t+1))]$$

corporate liquidity policy by defining *conditional hedging* $h_i^C(s(t+1))$ as the slacks on the state-contingent collateral contraints:

$$h_i^C(s(t+1)) \equiv \theta(1-\delta)k_{i,t+1} - (1+r(1-\tau))b_i(s(t+1)) \tag{8}$$

The higher $h_i^C(s(t+1))$, the larger the amount of debt capacity the firm is preserving for possible investment opportunities that may arise conditionally on the realization of the state $s(t+1)$. This means that firms can *conditionally* manage its liquidity, that is they can preserve their ability to raise debt and support investment in states in which their cash flows are low, and they have less internally generated resources. There is a clear-cut tradeoff between *conditional hedging* against future income shortfalls, and available funds for current investment. The amount of raised debt $E_t[b_i(s(t+1))]$ in equation (8) is supported by the promised payments in future states. Therefore, the higher $h_i^C(s(t+1))$, the more firms are tranferring resources from today to future states, and the lower $E_t[b_i(s(t+1))]$. As Rampini and Viswanathan (2010) discuss, state-contingent debt contracts can be implemented in practice by arranging loan commitments or purchasing derivative securities. The model with state-contingent debt $b_i(s(t+1))$ is also equivalent to a model in which debt is not state-contingent, but the firm can conditionally transfer liquidity by purchasing Arrow-Debreu securities.[4]

*Conditional hedging* is not the only way firms can transfer liquid funds. Firms can hoard cash and implement *unconditional hedging*. Hoarding cash is equivalent to unconditionally transferring resources from today to *all* future states, including those in which investment can be financed by internally generated funds. As for *conditional hedging*, there is a tradeoff between current investment and saving resources for the future. However, as we are going to discuss in section 4, *conditional hedging* is preferable to *unconditional hedging* because it allows to tranfer resources to the future states where they are needed the most. Nevertheless, the presence

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[4]Technically, recalling that collateral constraints are equivalent to limited enforcement constraints, this interpretation is possible because the market is complete in the set of enforceable payoffs.

of capital adjustment costs as in equation (5) makes cash hoarding optimal for smaller firms that would not otherwise be able to invest to an economically profitable scale, even if they exhaust their debt capacity. For this reason, and consistent with empirical evidence, our model predicts that firms can simultaneously hold debt and cash instead of using cash for repaying debt. This mechanism corroborates the intuition in Acharya, Almeida, and Campello (2007) that cash is not negative debt. We denote cash holdings in period $t$ as $c_{i,t}$. Firms earn the after-tax riskfree interest rate $r(1-\tau)$ on their cash balances, but also bear costs for holding them. Previous studies motivate the costs of holding cash by agency costs, and different lending and borrowing rates. Following DeAngelo, DeAngelo, and Whited (2011), we model these costs through an "agency parameter" $0 \leq \gamma \leq 1$. We interpret $\gamma$ as the one-period rate to which cash holdings deteriorate in value. Accordingly, the total hedging for firm $i$ at time $t+1$ in state $s(t+1)$ is the amount of resources available from both *conditional hedging* and *unconditional hedging*, that is:

$$h_i^T(s(t+1)) \equiv h_i^C(s(t+1)) + (1 + r(1-\tau) - \gamma)c_{i,t+1} \tag{9}$$

Finally, the firm can raise external equity. We assume seasoned equity offers are costly, so that it is never optimal for the firm to simultaneously pay dividends and issue equity. Following Hennessy and Whited (2005), we model equity flotation costs with a fixed and a proportional component. We indicate *net equity payout* at time $t$ as $e_{i,t}$. When $e_{i,t} < 0$ the firm is raising equity, while $e_{i,t} \geq 0$ means that the firm is making distributions to shareholders. Equity issuance costs are given by:

$$(\lambda_0 + \lambda_1 |e_{i,t}|)\mathbf{1}_{\{e_{i,t}<0\}} \tag{10}$$

The parameters $\lambda_0 \geq 0$ and $\lambda_1 \geq 0$ drive the fixed and the proportional component, respectively. The indicator function denotes that the firm faces these costs only in the region where the the net payout is negative. Accordingly, distributions to shareholders $d_{i,t}$ are the equity payout net of issuance costs:

$$d_{i,t} = e_{i,t} - (\lambda_0 + \lambda_1 |e_{i,t}|)\mathbf{1}_{\{e_{i,t}<0\}} \tag{11}$$

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## 3.3 The Firm Problem

Managers determine investment, financing, and risk management to maximize the wealth of shareholders, which is the risk-neutral security price in the capital market. Hence, in period $t$, they decide over real capital $k_{i,t+1}$, cash $c_{i,t+1}$, and state-contingent debt $b_i(s(t+1))$, for each state $s(t+1)$. As we discuss in section 3.1, the choice set for capital is compact. Collateral constraints in equation (7) imply that state contingent debt variables are bounded between 0 and $\frac{\theta(1-\delta)k_{i,t+1}}{1+r(1-\tau)}$. To ensure compactness of the feasible set for $c_{i,t+1}$, we impose an arbitrarily high bound $\bar{C}$ on cash holdings. This bound is imposed without loss of generality because of the assumption of costly cash balances. Intuitively, even when the marginal productivity of real capital is low, it is never optimal for the firm to invest in liquid assets and have unbounded savings. Cash can be distributed as dividends right away, and shareholders discount future dividends at a rate $r$ per period, while the rate of return for each unit of cash is $r(1-\tau)-\gamma$. The overall choice set is therefore compact.

Despite the large number of choice variables in the firm problem, the current state can be more efficiently summarized by introducing *realized net worth* as a state variable. Realized net worth at time $t$ in the (realized) state $s(t)$ for firm $i$ is given by:

$$w_{i,t} \equiv \Pi(k_{i,t}, z_{i,t}) + k_{i,t}(1-\delta) - (1+r(1-\tau))b_i(s(t)) + (1+r(1-\tau)-\gamma)c_{i,t} + \tau\delta k_{i,t} \quad (12)$$

As in Rampini and Viswanathan (2012a), net worth measures the amount of resources that are available to the firm in a certain state. It includes cash flows from current investment, value of capital net of depreciation, and value of cash holdings, all net of due debt payments. Intuitively, net worth is the corporate counterpart of household's wealth (Rampini and Viswanathan (2012b)). Therefore, net worth is a measure of how constrained a firm is in terms of available funds to allocate to investment, risk management, and distributions to shareholders. In our model, the presence of capital adjustment costs implies that the current stock of capital $k_{i,t}$ is

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also a relevant state variable. In fact, the knowledge of net worth and of the choice variables does not suffice to determine distributions to shareholders $d_{i,t}$ that appear in the objective function, because the adjustment costs $\Psi(k_{i,t+1}, k_{i,t})$ also directly depend on the current stock of capital. The current state is therefore summarized by the vector $(w_{i,t}, k_{i,t}, z_{i,t})$. The set of state variables is compact because $k_{i,t}$ and $z_{i,t}$ are bounded, and from equation (12) it is straightforward that net worth lies in a closed and bounded interval $[\underline{W}, \bar{W}]$.

Investment, financing, and liquidity management decisions are intimately related. They should satisfy the following budget identities between sources and uses of funds both at time $t$, and for each state at time $t+1$:

$$w_{i,t} + E_t[b_i(s(t+1))] = e_{i,t} + k_{i,t+1} + \Psi(k_{i,t+1}, k_{i,t}) + c_{i,t+1} \tag{13a}$$

$$w_i(s(t+1)) = \Pi(k_{i,t+1}, z_{i,t+1}) + k_{i,t+1}(1-\delta) - (1 + r(1-\tau))b_i(s(t+1)) +$$
$$+ (1 + r(1-\tau) - \gamma)c_{i,t+1} + \tau\delta k_{i,t} \tag{13b}$$

where $w_i(s(t+1))$ denotes net worth at time $t+1$ is state $s(t+1)$.

The firm objective function is to maximize the equity value $V(k_{i,t}, w_{i,t}, z_{i,t})$, that is the discounted value of distributions to shareholders. By the Bellman's principle of optimality, the equity value be computed as the solution to the dynamic programming problem

$$V(k_{i,t}, w_{i,t}, z_{i,t}) = \max\left\{0, \max_{k_{i,t+1}, c_{i,t+1}, b_i(s(t+1))}\left\{d_{i,t} + \frac{1}{1+r}E_t[V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})]\right\}\right\} \tag{14}$$

subject to the constraints in (4), (5), (7), (11), and (13). In equation (14), $V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})$ denotes the continuation value for equity, which depends on the future state $(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})$ and on the values of the choice variables at time $t$. The first maximum captures instead the pos-

sibility of default in current period, in which case the shareholders get nothing. To sum up, the complete firm problem is the following:

$$V(k_{i,t}, w_{i,t}, z_{i,t}) = \max \left\{ 0, \max_{k_{i,t+1}, c_{i,t+1}, b_i(s(t+1))} \left\{ d_{i,t} + \frac{1}{1+r} E_t [V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})] \right\} \right\} \quad (15)$$

s.t.

$$w_{i,t} + E_t[b_i(s(t+1))] \geq e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1}) \tag{16a}$$

$$w_i(s(t+1)) \leq (1-\tau)\Pi(k_{i,t+1}, z_{i,t+1}) + k_{i,t+1}(1-\delta) - (1+r(1-\tau))b_i(s(t+1)) +$$
$$+ (1 + r(1-\tau) - \gamma)c_{i,t+1} + \tau\delta k_{i,t+1} \qquad \forall s(t+1) \tag{16b}$$

$$(1+r(1-\tau))b_i(s(t+1)) \leq \theta(1-\delta)k_{i,t+1} \qquad \forall s(t+1) \tag{16c}$$

$$b_i(s(t+1)) \geq 0 \qquad \forall s(t+1) \tag{16d}$$

$$c_{i,t+1} \geq 0 \tag{16e}$$

## 3.4  Model Solution

Because of the presence of occasionally non-binding collateral constraints, and because costs of equity issues and capital adjustment depend on indicator functions, the model cannot be solved numerically by interior points methods. In principle, the model could be solved on a discrete grid by value function iteration or policy function iteration. The Bellman operator in equation (14) is indeed a contraction mapping, in that Blackwell's sufficient conditions hold in this framework. Therefore, the fixed point of the functional equation (14) is well-defined.

For a standard formal proof in a similar framework, we refer to Hennessy and Whited (2005). Unfortunately, there is a computational hurdle that prevents the solution of the model with standard techniques. Due to the large number of control variables (capital, cash, and one debt variable for each future state), value function iteration and policy iteration cannot be practically implemented. In particular, the maximization step is critical. Determining for each state the combination of control variables that maximizes the sum of distributions and the continuation value implies to store and maximize over a vector of $nk \times nc \times nb^{nz}$ elements, where $nk$, $nc$, $nb$, and $nz$ are the number of grid points for capital, cash, debt, and the shock. As in Rust (1997), this problem is plagued by a curse of dimensionality, since the amount of computer memory and CPU time required increases exponentially with the number of control variables. As a consequence, even for modest values for $nz$, such a vector becomes too large even to be stored.

We overcome this difficulty by exploiting the linear programming representation of dynamic programming problems with infinite horizon (Ross (1983)), as in Trick and Zin (1993), and Trick and Zin (1997). This technique has not been historically widely used. Despite it often allows to achieve significant speed gains over iterative methods, it requires in turn to store huge matrices and arrays that make it impractical for complex enough models. Specifically, we extend the constraint generation algorithm developed by Trick and Zin (1993), and we rely on a *separation oracle*, an auxiliary mixed integer programming problem, to avoid dealing with large vectors at all. As in Trick and Zin (1993), the contrained generation algorithm converges to the fixed point faster than traditional iterative methods. Moreover, the separation oracle allows to efficiently implement the maximization step because of a remarkable feature of our model, namely the relatively small number of state variables in spite of the large number of control variables. With this method, we manage to solve the model in a reasonable time (around ten minutes on our workstation).

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# 4 Investment, Financing, and Liquidity Management

## 4.1 Hedging Formulation

**Lemma 4.1** (Hedging formulation)

*The constrained optimization problem (15) is equivalent to:*

$$V(k_{i,t}, w_{i,t}, z_{i,t}) = \max \left\{ 0, \max_{k_{i,t+1}, h_{i,t+1}^U, h_i^C(s(t+1))} \left\{ e_{i,t} - \Lambda(e_{i,t}) + \frac{1}{1+r} E_t [V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})] \right\} \right\} \tag{17}$$

*s.t.*

$$w_{i,t} \geq e_{i,t} + E_t \left[ \frac{h_i^C(s(t+1))}{1+r(1-\tau)} \right] + \frac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma} + P k_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1}) \tag{18a}$$

$$w_i(s(t+1)) \leq (1-\tau)\Pi(k_{i,t+1}, z_{i,t+1}) + (1-\theta)(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h_i^T(s(t+1)) \quad \forall s(t+1) \tag{18b}$$

$$h_i^C(s(t+1)) \geq 0 \qquad\qquad \forall s(t+1) \tag{18c}$$

$$h_i^C(s(t+1)) \leq \theta(1-\delta)k_{i,t+1} \qquad\qquad \forall s(t+1) \tag{18d}$$

$$h_{i,t+1}^U \geq 0 \tag{18e}$$

*where $P \equiv 1 - \frac{\theta(1-\delta)}{1+r(1-\tau)}$ is the fraction of each unit of capital paid down by the firm at time t, $h_i^C(s(t+1)) \equiv \theta(1-\delta)k_{i,t+1} - (1+r(1-\tau))b_i(s(t+1))$ is conditional hedging for state $s(t+1)$, $h_i^U(s(t+1)) \equiv h_{i,t+1}^U = (1+r(1-\tau)-\gamma)c_{i,t+1}$ is unconditional hedging for all states at time $t+1$, and $h_i^T(s(t+1)) \equiv h_i^C(s(t+1)) + h_{i,t+1}^U$ is total hedging.*

The hedging formulation is particularly instructive because it emphasizes the role of dynamic liquidity management. The problem (17) can be equivalently interpreted as a problem where

firms pledge all their collateral, and transfer resources (net worth) from $t$ to $t+1$ both condition-ally, to specific states, and unconditionally, to all future states. Regarding conditional liquidity, firms decide to purchase $\frac{h_i^C(s(t+1))}{1+r(1-\tau)}$ Arrow-Debreu securities at time $t$ in order to obtain a pay-off of $h_i^C(s(t+1))$ is state $s(t+1)$ next period. Constraints (18c) and (18d) impose bounds on the amount of conditional hedging the firm can implement. The collateral constraint imposes a lower bound, that corresponds to exhausting all debt capacity. Constraint (18d) states that the maximum amount of liquid funds that a firm can transfer to state $s(t+1)$ corresponds to its debt capacity, that is to the firm having zero debt due in state $s(t+1)$. Unconditional hedging instead consists of hoarding an amount of cash $\frac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma}$, in order to get to obtain a payoff $h_{i,t+1}^U$ in all future states at time $t+1$. The hedging formulation provides a preliminary intuition on the different nature of conditional and unconditional liquidity management. Equations (18a) and (18b) hint that transferring liquid funds conditionally is more efficient than doing so uncon-ditionally if the firm needs to transfer resources only to some states (for example to bad states). Transferring funds to future states involves subtracting resources available to be distributed to shareholders $e_{i,t}$ and to be paid down to make investment possible $Pk_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$. If, for example, a firms needs to transfer an amount $M$ only to the specific state $s(t+1)$ (for ex-ample the lowest state), the amount of resources it needs at time $t$ is $\pi(s(t), s(t+1))\frac{M}{1+r(1-\tau)}$, where $0 \leq \pi(s(t), s(t+1)) < 1$ is the transition probability from state $s(t)$ to state $s(t+1)$. On the contrary, implementing unconditional hedging for the same purpose would require to subtract $\frac{M}{1+r(1-\tau)-\gamma}$. So, why should firms engage in unconditional liquidity management at all? Constraint (18d) states that the maximum amount of liquid funds that a firm can trans-fer conditionally is bounded by its total debt capacity $\theta(1-\delta)k_{i,t+1}$. Therefore, whenever it is optimal for the firm to have total hedging greater than this amount, hoarding cash becomes necessary. As a result, endougenously, cash is not negative debt, and consistent with empiri-cal evidence we can observe firms simultaneously holding cash and debt.[5] As the quantitative

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[5]As DeAngelo, DeAngelo, and Whited (2011) discuss, in frameworks in which firms never optimally hold cash and debt together, it is not necessary to model them using two separate positive control variables. In our model,

analysis in section 5 emphasizes, capital adjustment costs $\Psi(k_{i,t}, k_{i,t+1})$ play an important role, both qualitatively and quantitatively. Specifically, they allow to differentiate between firms that are constrained in terms of net worth, and small firms, and rationalize patterns that are observed in the data. Equation (18a) points up that different current and future investment needs yield to different needs of tranferring net worth to future states. This creates sharp differences in corporate liquidity policy of large and small firms. Suppose, for example, that adjustment costs are quadratic in the investment-to-capital ratio. With decreasing returns to scale, small firms with high investment needs would be better off in spreading investment over multiple periods to avoid incurring disproportionately high adjustment costs. Therefore, they may find optimal to hedge more, by saving debt capacity in a state contingent and possibly by hoarding cash. This creates a dependence between investment and liquidity needs, and, as a consequence, between size and risk management.

## 4.2 Optimal Policy

**Proposition 4.2** (Optimality conditions)

*Denote by $\lambda^w$, $\frac{\pi(s(t),s(t+1))\lambda^w_{s(t+1)}}{1+r}$, $\frac{\pi(s(t),s(t+1))\underline{\lambda}^C_{s(t+1)}}{1+r}$, $\frac{\pi(s(t),s(t+1))\overline{\lambda}^C_{s(t+1)}}{1+r}$, and $\underline{\lambda}^U$ the multipliers on constraints (18a), (18b), (18c), (18d), and (18e) respectively, where $\pi(s(t),s(t+1))$ is the Markovian transition probability from state $s(t)$ to state $s(t+1)$. Assume that the equity*

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letting negative debt being cash by relaxing constraint (18d) would not only prevent firms from simultanously holding cash and debt, but also assume that state-contingent cash securities exist, which is unrealistic.

*cost function $\Lambda(e_{i,t})$ is differentiable in $e(i,t)$.*[6] *Then, the first order conditions for the hedging formulation (17) can be expressed as follows:*

$$\lambda^w = 1 - \frac{\partial \Lambda(e_{i,t})}{\partial e_{i,t}} \tag{19a}$$

$$\lambda^w \left( P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}} \right) = \frac{1}{1+r} E_t [\lambda^w_{s(t+1)} V^k(s(t+1)) + \overline{\lambda}^C_{s(t+1)} H^k] \tag{19b}$$

$$\lambda^w \frac{1}{1+r(1-\tau)-\gamma} = \frac{1}{1+r} E_t [\lambda^w_{s(t+1)}] + \underline{\lambda}^U \tag{19c}$$

$$\frac{1}{1+r(1-\tau)} \lambda^w = [(\underline{\lambda}^C_{s(t+1)} - \overline{\lambda}^C_{s(t+1)}) + \lambda^w_{s(t+1)}] \frac{1}{1+r} \qquad \forall s(t+1) \tag{19d}$$

*where*

$$V^k(s(t+1)) = (1-\tau) \frac{\partial \Pi(k_{i,t+1}, z_{i,t+1})}{\partial k_{i,t+1}} + \tau \delta + (1-\theta)(1-\delta) \qquad \forall s(t+1) \tag{20a}$$

$$H^k = \theta(1-\delta) \tag{20b}$$

*The envelope conditions imply:*

$$\frac{\partial V(w_{i,t}, z_{i,t})}{\partial w_{i,t}} = \lambda^w \tag{21a}$$

$$\frac{\partial V(w_{i,t+1}, z_{i,t+1})}{\partial w_{i,t+1}} = \lambda^w_{s(t+1)} \qquad \forall s(t+1) \tag{21b}$$

*Moreover, the investment Euler equation is:*

$$P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}} = E_t[M^w(s(t), s(t+1)) V^k(s(t+1))] + E_t[M^h(s(t), s(t+1)) H^k] \tag{22}$$

---

[6]In our model, we choose a functional form for equity flotation costs with a fixed and a proportional component, which is non-differentiable for $e(i,t) = 0$ (its derivative at zero exists only in a distributional sense). This assumption is not critical for our qualitative analysis. Alternatively, one can approximate $\Lambda(e_{i,t})$ with $0.5(1 + tanh(Ne(i,t)))$, with $N$ large enough, in the neighborhood of zero. A similar argument applies to the adjustment cost function $\Psi(k_{i,t}, k_{i,t+1})$ in case fixed costs are included.

*where $M^w(s(t),s(t+1)) \equiv \frac{1}{1+r}\frac{\lambda^w_{s(t+1)}}{\lambda^w}$ and $M^h(s(t),s(t+1)) \equiv \frac{1}{1+r}\frac{\overline{\lambda}^C_{s(t+1)}}{\lambda^w}$ are stochastic discount factors. In addition:*

$$M^w(s(t),s(t+1)) = \frac{1}{1+r(1-\tau)} - \frac{1}{1+r}\frac{\lambda^C_{s(t+1)} + \overline{\lambda}^C_{s(t+1)}}{\lambda^w} \tag{23}$$

The optimality conditions illustrate how investment, financing, liquidity and payout policies are intimately related, and shed light on the qualitative mechanism that drive firm's decisions. Moreover, they allow to understand the rationale for liquidity management, and which future states firms optimally hedge. Since the problem has no closed-form solution, the following analysis relies on the economic interpretation of the Lagrange multipliers as shadow values.

Equation (19b) relates the costs and benefits of investing an additional unit of real capital at time $t + 1$. The left hand side represent the marginal cost of investing. An additional unit of capital requires that the firm puts $P$ money down and pays capital adjustment costs. The cost of doing so is $(P + \frac{\partial \Psi(k_{i,t},k_{i,t+1})}{\partial k_{i,t+1}})\lambda^w$. The multiplier $\lambda^w$ accounts for the shadow loss in firm value of relaxing the resource constraint (18a) at time $t$ (resource constraints are always binding). The right hand side is the marginal benefit of an additional unit of investment, discounted back to time $t$ by the shareholders' discount factor $\frac{1}{1+r}$. The benefits correspond to the two terms on the right hand side. First, the expected value of the additional investment $V^k(s(t+1))$ across all future possible states, that consists of marginal changes in profits, of tax benefits, and of the liquidation value of the share of capital not pledged to lenders. Second, the expected increase in debt capacity available for conditional hedging $H^k$ in all states. The multipliers $\lambda^w_{s(t+1)}$ and $\overline{\lambda}^C_{s(t+1)}$ instead account respectively for the additional future net worth (constraint (18b)), and

for the additional debt capacity (constraint (18d)) available to transfer conditional liquidity to state $s(t+1)$ because of the additional unit capital installed (in case this constraint is binding).

$$\overbrace{\lambda^w(P+\frac{\partial\Psi(k_{i,t},k_{i,t+1})}{\partial k_{i,t+1}})}^{\text{Marginal cost of investment}} = \overbrace{\frac{1}{1+r}E_t[\underbrace{\lambda^w_{s(t+1)}V^k(s(t+1))}_{\text{Net worth}}+\underbrace{\overline{\lambda}^C_{s(t+1)}H^k}_{\text{Debt capacity}}]}^{\text{Marginal benefit of investment}} \tag{24}$$

Equation (19c) describes the unconditional liquidity policy of the firm. Similar to equation (19b), the left-hand side $\lambda^w\frac{1}{1+r(1-\tau)-\gamma}$ is the cost of allocating a unit of current net worth to cash hoarding, in order to tranfer one unit of cash to all future states at $t+1$. The right-hand side is the value of this additional unit of net worth available in all states $\frac{1}{1+r}E_t[\lambda^w_{s(t+1)}]$. In addition, the term $\underline{\lambda}^U$ accounts for the possibility that the constraint on positive cash is binding.[7]

$$\overbrace{\lambda^w\frac{1}{1+r(1-\tau)-\gamma}}^{\text{Marginal cost of unconditional liquidity}} = \overbrace{\underbrace{\frac{1}{1+r}E_t[\lambda^w_{s(t+1)}]}_{\text{Net worth}}+\underbrace{\underline{\lambda}^U}_{\text{Positive cash holdings}}}^{\text{Marginal benefit of unconditional liquidity}} \tag{25}$$

Equation (19d) describes the conditional liquidity policy of the firm. As for unconditional liquidity management, the marginal cost of allocating one unit of net worth to risk management is $\lambda^w\frac{1}{1+r(1-\tau)}$ (the agency parameter $\gamma > 0$ makes it more costly for unconditional hedging). It is however more interesting to examine the right-hand side, and to compare it to the optimality conditions for unconditional liquidity management in equation (19c). As for cash hoarding, the benefits are discounted to time $t$ through the manager's discount factor $\frac{1}{1+r}$. However, the value of additional net worth potentially available for the state $s(t+1)$ is $\lambda^w_{s(t+1)}$. In equation equa-

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[7]This term is more meaningful in case we interpret the first-order condition on unconditional hedging for a reduction of one unit. In this case, the marginal benefit is the additional amount $\lambda^w\frac{1}{1+r(1-\tau)-\gamma}$ available at time $t$ for investment, distributions, and conditional hedging, and the marginal cost is the sum of the value of one less unit of net worth available in all states, and of the shadow value of being able to reduce further cash if constraint (18e) binds.

tion (19c), the value of the net worth tranferred to state $s(t+1)$ is only $\pi(s(t),s(t+1))\lambda^w_{s(t+1)}$.[8] This supports the statement in section 4.1 that conditional liquidity management is preferrable to unconditional liquidity management because with the same amount of net worth at time $t$ it allows to transfer more resources to a *specific* state $s(t+1)$. The term $\underline{\lambda}^C_{s(t+1)} - \overline{\lambda}^C_{s(t+1)}$ instead illustrates why firms may be interested in managing its liquidity both conditionally and unconditionally at the same time. Specifically, since in our model conditional hedging can be implemented only saving debt capacity in a state contingent way, the amount of conditional liquidity is limited by the constraints (18c) and (18d). The term $\underline{\lambda}^C_{s(t+1)}$ accounts for the presence of occasionally binding state-contingent collateral constraints, that may become active and limit the amount of state-contingent debt that a firm can hold given the amount of pleadgeable capital $k_{i,t+1}$. Simmetrically, the multiplier $\overline{\lambda}^C_{s(t+1)}$ is different from zero in case the firm would like to transfer more resources conditionally, but its amount is limited because the firm has alredy zero debt due in state $s(t+1)$. The limited amount of implementable conditional hedging through liquidity management implies that firms can simultaneously hold cash and debt. To see this, suppose that the firm is interested in hedging a specific state, such as the lowest state $\underline{s}$, as much as possible. *Ceteris paribus*, the maximum amount of resources that the firm can tranfer to $\underline{s}$ corresponds to exhausting all debt capacity in all states except $\underline{s}$. This implies that no debt is due in state $\underline{s}$. Moreover, the firm can transfer the net worth raised by the state-contingent debt issues in all states excluding $\underline{s}$, to all future states, including $\underline{s}$, by hoarding cash. As a result, the firm would hold cash and debt together.

$$\overbrace{\frac{1}{1+r(1-\tau)}\lambda^w}^{\text{Marginal cost of conditional hedging}} = [\ \underbrace{(\underline{\lambda}^C_{s(t+1)} - \overline{\lambda}^C_{s(t+1)})}_{\text{Limited conditional liquidity}} \overbrace{+ \underbrace{\lambda^w_{s(t+1)}}_{\text{Net worth}}]\frac{1}{1+r}}^{\text{Marginal benefit of conditional liquidity}} \qquad \forall s(t+1)$$

$$(26)$$

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[8]To better see this, notice that the expectation in equation (19c) is $\sum_{s=1}^{S} \pi(s(t),s)\lambda^w_s$, where $S$ is the total number of states.

The payout policy instead balances the marginal cost of allocating a unit of net worth to dividend distributions or, viceversa, to issue equity to increment net worth by one unit. In case of equity issues, there is not a one-to-one correspondence between raised equity and increased net worth because of equity flotation costs.

$$
\underbrace{\overbrace{\lambda^w}^{\text{Marginal benefit of issuing equity}}}_{\text{Marginal cost of paying dividends}} = \underbrace{1}_{\text{Marginal benefit of paying dividends}} \overbrace{- \frac{\partial \Lambda(e_{i,t})}{\partial e}}^{\text{Marginal cost of issuing equity}} \tag{27}
$$

The Euler condition (22) clarifies the important matter of the firm's rationale for liquidity management, and of which states it is optimal to hedge. The Euler equation can be interpreted as a pricing relationship. The left-hand side can be seen as the valuation of the paid down share $P +$ $\frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}}$ per unit of capital. The right-hand hand side shows that this value is supported by two terms. The term $E_t[M^w(s(t), s(t+1))V^k(s(t+1))]$ is the stochastically discounted valuation of the benefits $V^k(s(t+1))$ of investing an additional unit. $M^w(s(t), s(t+1))$ is the firm's stochastic discount factor, and is equal to $\frac{1}{1+r} \frac{\lambda^w_{s(t+1)}}{\lambda^w}$. The concavity properties of the value function imply that the marginal value of a certain level of net worth is higher in bad times, so that the stochastic discount factor puts more weight on bad states through the Lagrange multipliers. Indeed, envelope conditions (21a) and (21b) show how Langrange multipliers are related to the shape of the value function, so that $\lambda^w_{s(t+1)}$ is decreasing in $w_i(s(t+1))$. In a valuation perspective, since a larger share of $P$ is supported by those states, the firm behaves as if it were risk-averse. This provides incentives to implement liquidity management by preserving net worth for investments and distributions for bad future states, where internally generated cash flows and *future realized* net worth are, other conditions equal, lower. Viceversa, the payoff from investments $V^k(s(t+1))$ suggests that the firm may want to hedge good states as well. If the law of motion of shocks to capital productivity $z_{i,t}$ is persistent enough, the payoff of investing in good (bad) times is higher (lower) because the firm expects a sequence of good (bad) shock realizations. The firm will therefore save resources for good states and boost investment

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in good times. If this is the case, the marginal value of net worth is not necessarily lower in bad states anymore. An instructive benchmark case is the case with independent productivity shocks. In such a scenario, the expected productivity of capital $\frac{\partial \Pi(k_{i,t+1},z_{i,t+1})}{\partial k_{i,t+1}}$ is independent of the current state. As a consequence, firms only hedge bad states because of the properties of the discount factor $M^w(s(t),s(t+1))$. In practice, however, the productivity process in quite persistent. Therefore, the matter of whether firms hedge good or bad states (or both), and how much, is a purely quantitative question. Also, it is a quantitative question whether firms hedge at all. As in Rampini and Viswanathan (2012a), firms that are particularly constrained may not hedge, and prefer to allocate their scarce resources to current investment and distributions. The second term on the right-hand side instead $H^k$ reflects that capital is valuable also because it serves as collateral, it increases debt capacity and, as a consequence, the amount of conditional liquidity management implementable in all states. The stochastic discount factor $M^h(s(t),s(t+1))$ depends on the multiplier $\overline{\lambda}^C_{s(t+1)}$. Therefore, the value of increased debt capacity is higher is states where firms hold no debt because conditional liquidity is more valuable.

$$
P + \overbrace{\frac{\partial \Psi(k_{i,t},k_{i,t+1})}{\partial k_{i,t+1}}}^{\text{Value of paid-down capital}} = \overbrace{E_t[M^w(s(t),s(t+1))V^k(s(t+1))]}^{\text{Discounted investment profits}} + \overbrace{E_t[M^h(s(t),s(t+1))H^k]}^{\text{Debt capacity}} \quad (28)
$$

Finally, equation (23) explicitly relates the stochastic discount factor $M^w(s(t),s(t+1))$, which appears in the investment Euler equation, to the hedging policy of the firm. The multipliers $\underline{\lambda}^C_{s(t+1)}$, and $\overline{\lambda}^C_{s(t+1)}$ differ from zero respectively when the firm exhausts all its debt capacity in state $s(t+1)$, and when the firm has zero debt in state $s(t+1)$. These multipliers enter the Euler equation because of market incompleteness. Given the stochastic nature of the model, firms anticipate that colleteral and debt positivity constraints may bind in the future, and this affect their investment and liquidity management policy. By transferring liquid funds conditionally, the firm can therefore influence the relative importance of different states for determining the value of paid-down capital. For example, if a company borrows constrained in the low state

$\underline{s}$ and saves all its debt capacity for future investment in the high state $\bar{s}$, the stochastic discount factor puts more weight on the high state, namely $\frac{1}{1+r(1-\tau)} + \frac{1}{1+r}\frac{\bar{\lambda}^C_{\bar{s}}}{\lambda^w}$ versus $\frac{1}{1+r(1-\tau)} - \frac{1}{1+r}\frac{\lambda^C_{\underline{s}}}{\lambda^w}$.

$$M^w(s(t),s(t+1)) = \underbrace{\frac{1}{1+r(1-\tau)}}_{\text{Unconditional component}} - \underbrace{\frac{1}{1+r}\frac{\overbrace{\underline{\lambda}^C_{s(t+1)}}^{\text{Debt capacity}} + \overbrace{\bar{\lambda}^C_{s(t+1)}}^{\text{Positive debt}}}{\lambda^w}}_{\text{State-contingent component}} \tag{29}$$

## 4.3  Numerical Illustration

We provide numerical examples to illustrate the analytical analysis in section 4.2, and to better understand the qualitative importance of different types of capital adjustment costs for corporate investment and liquidity policy. In the interest of clarity, in all the examples we solve the model with three possible states and in absence of equity issues, and report the policy for the middle state. The details of the parametrizations are reported in the captions of figures 2 to 7.

Figure 2 refers to the case with no adjustment costs and independent investment opportunities. Specifically, Markovian transition probabilities are uniform (equal to one third for each pair of states), so that the expected capital productivity is the same for every state at time $t$. Panels A and B depict investment and payout as a function of current net worth. Similar to Rampini and Viswanathan (2012a), there esist a threshold of net worth below which investment is increasing, and dividends are zero. Above the threshold investment is constant and dividends are linear. Panel C shows that the value function is weakly concave in net worth. This is an important property, because the firm's stochastic discount factor in equation (28) is equal to $\frac{1}{1+r}\frac{\lambda^w_{s(t+1)}}{\lambda^w}$. As a consequence, the firm behaves as if risk averse with respect to net worth. Such a behavior is clearly visible in panel F. As we pointed out in the previous section, with independent productivity, the firm implements downstate hedging. In this example, it saves all its debt capacity for the low state for almost all levels of net worth. The dashed line (conditional hedging for

the low state), and the thin line (available debt capacity) are indeed very close. The amount of hedging decreases for the middle states (solid line), and is equal to zero for the high state (dashed-dotted line). Panel E shows the cash policy of the firm. When hedging needs exceed the available debt capacity, that is the amount of implementable conditional hedging, and the firm is unconstrained enough in terms of net worth, it implements unconditional hedging too. This way, additional resources are transferred to the low state. As a consequence, as panel D depicts, cash is not negative debt, and it is optimal for the firm to simultaneously hold them.

[Insert Figure 2 Here]

Figure 3 removes the assumption of independent investment opportunities, and introduces some persistence. In particular, the firm has now a probability of one half to stay in the current state, and of one quarter to move to another state. The policy is generally similar to that in figure 2, except for conditional liquidity management. The dashed-dotted line in Panel F is no longer equal to zero, meaning that the firm hedges upstate as well. Intuitively, with independent investment opportunities, the firm has no incentive to hedge the state where the marginal value of future net worth is lower. However, as equation (28) states, if there is a high probability that periods of high profits are followed by periods of high profits, expected future productivity is higher in good states. Therefore, the firm may rationally save resources for future investments in states where investment opportunities are likely to remain good.

[Insert Figure 3 Here]

Figures 4 to 7 emphasize the importance of capital adjustment costs to disentangle net worth from capital, and rationalize the patterns in table 1. We consider, one at a time, the four types of adjustment costs in the general functional form (5), namely convex investment costs, fixed investment costs, convex disinvestment costs, and fixed disinvestment costs. This approach

allows to see how the firm implements conditional and unconditional liquidity management for investment and disinvestment motives. Moreover, we can assess how the investment, liquidity, and risk management policy differs if we consider either fixed or smooth costs.

Figure 4 illustrates investment and liquidity management in presence of smooth investment costs. Panels A to C show how, for some values of the current capital stock, the policy is similar to the case with no adjustment costs. *Conditional on capital*, unconstrained firms transfer more liquidity, both conditionally and unconditionally. However, Panels D to F depict how the level of current capital now influences investment and hedging decisions, *conditional on net worth*. Panel D reports the optimal investment-to-capital ratio as a function of firm's size. Because of decreasing returns to scale in the production function, capital installment is relatively more profitable for small firms, which have higher investment needs. Because adjustment costs are quandratically increasing in the investment-to-capital ratio, smaller firms cannot istantaneously adjust to the desired capital level. Partial adjustment is hence optimal, and small firms transfer net worth for (costly) investment to future states, both conditionally (panel F), and unconditionally (panel E). This behavior results in small firms having more cash. Remarkably, these patterns are qualitatively consistent with the stylized facts we revisit in the two-way sorts of table 1.

[Insert Figure 4 Here]

Figure 5 shows instead the case of liquidity management for investment in presence of fixed capital adjustment costs. As panel D clearly shows, the firm has a standard (S,s) policy as a function of current capital.[9] In the figure, $k^*$ denotes the "frictionless" level of capital in absence of investment adjustment costs, defined as in Caballero, Engel, and Haltiwanger (1995), and Caballero and Engel (1999). Intuitively, the more the firm deviates from the "target" level, the higher the cost it bears. As a consequence, when the disequilibrium $|k_{i,t} - k^*|$ is large, it is

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[9]For an exhaustive treatment of models with fixed costs we refer to Stokey (2008).

optimal to pay the fixed cost and to re-adjust the capital level to $k^*$. This policy determines an inaction region bounded by the low barrier $k^D$, and by the high barrier $k^U$. In this region, optimal investment is zero. Panels E and F emphasize how firms transfers conditional and unconditional liquid funds precisely in the inaction region. Intuitively, since they are not currently investing, they transfer some net worth to future states, instead of paying it off as dividends.

[Insert Figure 5 Here]

Finally, figures 6 and 7 analyze the case of costly disinvestment with convex and fixed costs respectively. In these cases, firms implement conditional and unconditional liquidity management to cover future costs of disinvestment. This mechanism is similar to the one in Gamba and Triantis (2008), where firms hold cash and debt together because of the presence of transaction costs of issuing debt. Panel F of figure 6 shows how the firm hedges the low state, where disinvestment needs, and costs, are higher. In addition, if the firm is small, current investment needs are high, as panel D depicts. As a consequence, the firm borrows constrained against the middle and the good state, and hoards cash (Panel E) to transfer additional net worth to the bad state as well.

[Insert Figure 6 Here]

In the case with fixed disinvestment costs, the firm still transfers resources to the low state (Panel F). In the inaction region, collateral constraints imply that the firm's debt capacity is higher because of the capital in excess to the "frictionless" target capital stock. Therefore, in this region, firms are able to save more conditional liquidity for the bad state, and they need to hoard less cash, as Panel E shows. In this example, also large firms hold cash. Different from the investment case, disinvestment generates internal resources from capital liquidation. Firms can keep part of these resources as cash reserves, and hedge future investment and disinvestment needs.

[Insert Figure 7 Here]

In the full model, all these types of adjustment costs are present. Therefore, it is a quantitative question how much each type of cost is important, and whether firms hedge mainly for either investment or disinvestment reasons. In sections 5 and 6 we analyze the quantitative implications of the model.

# 5  Calibration and Identification

In order to assess the quantitative implications of the model, and to perform counterfactual comparative statics, we calibrate the model to match a set of data moments. In this process, it is important to understand how the parameters of the model can be identified. Ideally, a one-to-one mapping between the structural parameters and a set of data moments provides a sufficient condition for identification. Such a close mapping in difficult to obtain in every economic model, and all the model parameters affect all the data moments to some extent. However, although firm's investment, financing, and liquidity management decisions are intertwined, we can still classify the moments roughly as representing the firm's investment, financing, and hedging decisions. We first discuss the implementation of state-contingent debt with credit lines. This provides a mapping between the concept of conditional liquidity in the model, and the data of leverage an lines of credits from Compustat and Capital IQ. Then we describe how parameter values are set, and discuss the quantitative performance of the model.

## 5.1  Implementation with Lines of Credit

As we discuss in Section 2, conditional liquidity management entails transfers of net worth to specific states, which are inherently unobservable. This feature renders our structural approach particularly suitable to investigate corporate liquidity management. To identify conditional liquidity management, we take advantage of data on credit lines from the Capital IQ dataset. Capital IQ reports the drawn fraction of funds from firms' credit lines. This metric is particularly useful because, consistent with the model, reflects differences in the fraction of debt capacity that firms preserve to conditionally transfer liquidity. The following proposition shows how state-contingent debt can be implemented in the model with a combination of traditional state-uncontingent debt intruments, such as bank loans or corporate bonds, and lines of credit.

**Proposition 5.1 (Implementation with Credit Lines)** *State-contingent debt $b_i(s(t+1))$ can be implemented by the following combination of securities: state-uncontingent debt $D_{i,t+1} \geq 0$, and a secured line of credit $C_i^L(s(t+1))$, with interest rate $r$, and limit $\overline{C_{i,t+1}}$. The firm arranges a loan $L_{i,t}$ at time $t$ of size*

$$L_{i,t} = E_t \left[ \frac{D_{i,t+1}}{1 + r(1-\tau)} \right] \tag{30}$$

*where the uncontingent debt claim is*

$$D_{i,t+1} = (1 + r(1-\tau))E_t[b_i(s(t+1))] \tag{31}$$

*and saves state-contingent debt capacity by drawing $(1 + r(1-\tau))(E_t[b_i(s(t+1))] - b_i(s(t+1)))$ from the credit line in each state $s(t+1) \in S$, that is:*

$$C_i^L(s(t+1)) = (1 + r(1-\tau))(E_t[b_i(s(t+1))] - b_i(s(t+1))) \tag{32}$$

*The limit of the credit line is defined as*

$$\overline{C_{i,t+1}} = (1 + r(1-\tau))E_t[b_i(s(t+1))]$$

The proposition illustrates how firms can implement conditional liquidity management combining available securities, namely standard debt and credit lines. This provides a mapping between the variables in the model and the corresponding data moments. We use this mapping to compare the mean, the variance, and the serial correlation of undrawn debt capacity in the model and in the data. More precisely, in this implementation firms borrow the expected amount of required debt financing $E_t[b_i(s(t+1))]$ using standard uncontingent debt. Liquidity is then drawn from credit lines to fulfill unanticipated funding needs in the amount $(1 + r(1-\tau))(E_t[b_i(s(t+1))] - b_i(s(t+1)))$ in each future state $s(t+1)$. The limit $\overline{C_{i,t+1}}$ on

the credit line is set such that the total amount borrowed never exceeds the firm's debt capacity $\theta(1-\delta)k_{i,t+1}$.

Of course, the implementation with credit lines is not the only possibility for firms to engage in conditional liquidity management. For example, as Rampini and Viswanathan (2010) discuss, other possibilities involve the use of forwards and futures. In general, the state-contingent debt variables $b_i(s(t+1))$ in the model encompass different possible implementations. However, in quantitative analyses, taking a stand on a specific implementation provides a closer mapping between the model and the data. In this respect, as we discuss in Section 2, credit lines appear to be very important in practice, while even larger firms appear to implement little hedging through financial derivatives. For these reasons, and because of data limitations, we rely on the implementation in Proposition 5.1 in the following quantitative analysis.

## 5.2 Parameter Values and Model Fit

The model parameters we set in order to obtain a close match between the simulated data moments from the model, and the real data moments, are the production function curvature $\alpha$, the operating leverage parameter $f$, the shock serial correlation $\rho_z$, the shock standard deviation $\sigma_z$, the fixed and convex physical adjustment cost parameters $\psi_0^+$, $\psi_0^-$, $\psi^+$, and $\psi^-$, the debt capacity parameter $\theta$, the agency parameter $\gamma$, and the equity issuance fixed and proportional unit costs $\lambda_0$, and $\lambda_1$.[10]

We pick 19 moments to match. On the investment side, we choose moments that relate to operating income, investment, and Tobin's Q. Average operating income is primarily affected

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[10]Following DeAngelo, DeAngelo, and Whited (2011), we instead fix the tax rate parameter $\tau$ to the the statutory tax rate in the United States (0.35), the interest rate $r$ to be approximately equal to the real interest rate in the 20th century (0.015), and $\delta$ to be approximately equal to the depreciation rate in our sample (0.15). Finally, we set the unconditional mean $\mu_z$ of the shock process such that the steady-state stock of capital is normalized to the value of two. This choice allows to obtain a sufficient precision on the grid for capital without significantly increasing the computational burden with a finer grid choice.

by the curvature of the production function $\alpha$, and by the operating leverage parameter $f$. The variance of operating income and its first-order autocorrelation instead capture the parameters $\sigma_z$ and $\rho_z$ that govern the dynamics of the shock process $z_{i,t}$. The investment moments we match are the mean, the variance, the serial correlation, and the skewness of investment. These moments are not only affected by the parameters $\alpha$, $\sigma_z$, and $\rho_z$, but also help pin down the capital adjustment cost parameters $\psi_0^+$, $\psi^+$, $\psi_0^-$, and $\psi^-$. Higher values of $\psi_0^-$, and $\psi_0^+$ lead to more volatile, less autocorrelated, and more skewed investment. Higher $\psi^+$ and $\psi^-$ result in less volatile, and more serially autocorrelated investment. Also, the debt capacity parameter $\theta$ has an impact on investment variance and skewness, because financing and investment are linked through state-contingent collateral constraints. Finally, average Tobin's Q is affected by all the parameters in the models, especially by $\sigma_z$ and $\rho_z$, by the adjustment cost parameters, and by the fixed operating costs $f$.

On the financing side, we consider mean, average, and serial correlation of leverage, average equity issues, and their variance. The leverage moments are affected by all parameters in the model, and especially by $\theta$. The mean and variance of equity issues help identify $\lambda_0$ and $\lambda_1$. The remaining moments pertain to the conditional, and unconditional hedging policy. We choose to match mean, variance and serial correlation for both cash holdings, and undrawn credit from firms' credit lines. As we illustrate in section 4, all these moments are affected by the dynamics of the shock process, and by the capital adjustment cost parameters. Moreover, the agency parameter $\gamma$ affects average cash holdings. Finally, $\theta$ plays a very important role for the tradeoff between conditional, and unconditional liquidity management. Higher values of $\theta$ imply that the amount of liquid funds which can be transferred conditionally is higher. As a consequence, the higher $\theta$, the higher the average undrawn debt capacity, and the lower the average cash holdings.

The calibrated parameters in table 2 are comparable to those of existing studies. The curvature of the profit function $\alpha$ is close to the estimated values in Hennessy and Whited (2005),

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and Hennessy and Whited (2007). The fixed cost parameter $f$, on an annual basis, is in line with the calibration of Gomes and Schmid (2010). The parameters $\sigma_z$, and $\rho_z$, that govern the shock dynamics, are less than one standard error from the estimates in Hennessy and Whited (2005). The external equity cost parameters $\lambda_0$ and $\lambda_1$ are also very close to the point estimates of Hennessy and Whited (2005), who use the same functional form. The value of the cash hoarding cost parameter $\gamma$ is similar to the one in DeAngelo, DeAngelo, and Whited (2011). Our values for the capital adjustment cost parameters exhibit a similar patterns to Cooper and Haltiwanger (2006), and DeAngelo, DeAngelo, and Whited (2011) as far as the relative magnitude of the fixed and convex component is concerned. Different from these studies, we also allow for asymmetries in capital adjustment costs for investment and disinvestment. Our parameters provide support to the calibration in Zhang (2005), who requires that disinvestment is by far more costly than investment to rationalize the value premium. Finally, to the best of our knowledge there is no direct quantitative term of comparison for the parameter $\theta$ in state-contingent collateral constraints. However, our calibrated value is extremely close to the share of pledgeable steady-state capital estimated by DeAngelo, DeAngelo, and Whited (2011).

Table 2 shows that, overall, the model provides a good fit to the data. Remarkably, with only one exogenous shock process, the model manages to endogenously generate very different variances for operating income on one hand, and investment, leverage, and undrawn debt capacity on the other hand. In contrast, in existing models (e.g. Hennessy and Whited (2007), DeAngelo, DeAngelo, and Whited (2011), Nikolov and Schmid (2012)) simulated variances are typically much lower than real data variances. This leads to the need to either remove firm and time fixed effects from the data, or to add noise to the simulation, in order to make volatilities of simulated and actual moments comparable. We attribute this result to the presence of additional frictions in comparison to these models, and in particular to state-contingent collateral constraints, and to our flexible adjustment cost function for physical capital. In addition, the model is able to replicate fairly well the relative differences in serial correlations for operating

income, investment, leverage, cash, and undrawn credit that are observed in the data. Specifically, data moments for these variables are approximately 0.79, 0.37, 0.91, 0.89, and 0.63, while their simulated counterparts are around 0.63, 0.22, 0.68, 0.72, and 0.68.

The model appears to be slightly on the variance of cash holdings, and on the mean of undrawn debt capacity. The former is too high because in our model the only motive for which firms hold cash is hedging. Therefore, firms with no hedging needs, or firms that can satisfy all their hedging needs with conditional liquidity only, hold exactly zero cash. In reality, firms also hold cash for other reasons, for example for operating purposes. The lower mean of undrawn debt capacity with respect to the data is the result of the assumption of relative impatience of managers because of tax benefits of debt. As in Rampini and Viswanathan (2012a), firms are never completely unconstrained, and even large unconstrained firms issue debt. The fit may be probably further improved by introducing additional frictions. However, we do not include them to make the tradeoff between unconditional and conditional hedging clearly driven by limited conditional hedging in presence of collateral constraints, and investment adjustment costs.

[Insert Table 2 Here]

# 6 Empirical Implications and Comparative Statics

## 6.1 Stylized evidence under the baseline calibration

In this section, we evaluate the model performance by reproducing the stylized empirical evidence on corporate liquidity we revisit in section 2. Table 3 is a replica of table 1 with a simulated panel of observations from our model. All parameters are set to the baseline values in table 2, and data are simulated using the same procedure.

A comparison of tables 1 and 3 shows that the model conforms with the key patterns that are observed in the data, and that we summarize in section 2. The patterns of simulated evidence are generally sharper that those in actual data. This is primarily because of the higher variance of cash, and the lower undrawn debt capacity, as we discuss in section 5.

Panel A of table 3 reports simulated evidence for one-way sorts on net worth and capital. The row labeled "Cash Holdings" shows that smaller and more constrained firms hoard more cash, as Almeida, Campello, and Weisbach (2004), and Denis and Sibilkov (2009) document. The "Leverage" row reproduces the well-known positive relation between size and leverage. In addition, firms with low net worth are more levered than firms with high net worth. Finally, the row labeled "Undrawn Credit" reproduces the finding that unconstrained firms are more slack on their credit lines. While the evidence in panel A provides a crude assessment of the model, the two-way sorts in panels B and C are definitely more informative. Indeed, they allow to effectively interpret empirical patterns within our framework of our model, and better understand why these patterns are observed in actual data.

The sub-panel labeled "Cash Holdings" emphasizes that the main variable that drives firms cash policy is capital, rather than net worth. This can be rationalized within our model, and is consistent with the graphical representation in figure 1. Conditional on some level of net worth,

hence on some *total* liquidity need, capital essentially determines the optimal mix between conditional and unconditional liquidity. Transferring resources in a state-contingent way is more efficient, but a firm's ability to implement conditional hedging is limited by collateral constraints. As a consequence, smaller firms also need to transfer resources unconditionally, to all future states, and hoarding more cash than large firms. In addition, consistent with net worth being the main determinant of total corporate liquidity (figure 1), less constrained firms appear to have more cash than more constrained firms after controlling for capital. As in the data, the pattern is less pronounced than on the capital dimension. Unconstrained firms implement more total hedging and, *ceteris paribus*, also hoard more cash. This piece of evidence relates to the result in Denis and Sibilkov (2009) that some constrained firms have surprisingly low cash holdings.

The "Undrawn Credit" sub-panel replicates the stylized fact that firms with high net worth implement more total and, consequently, more conditional hedging. At a first glance, this result may look at odds with our key message that capital is the main determinant of the composition of corporate liquidity as conditional versus unconditional, as figure 1 shows. However, an important caveat is needed in interpreting this reduced-form evidence. In table 3, we compute undrawn credit as a fraction of debt capacity, while the mix of conditional and unconditional liquidity must account for how much cash firms hoard. Large firms have also less cash than small firms, and the ratio of conditional-to-unconditional liquidity is higher for more capitalized, hence more collateralized, firms. Panel C addresses this point and provides additional evidence by computing the ratio of conditional-to-total liquidity for simulated data, and the ratio of undrawn credit to the sum of undrawn credit and cash for the sample of table 1. Clearly, panel C shows that capital determines the mix of conditional and unconditional liquidity as the model predicts, and empirical proxies support this prediction.

Finally, the "Leverage" sub-panel in panel B provides substantial support for the hedging view of capital structure in Rampini and Viswanathan (2012a). Similar to them, in our model

capital stucture and conditional hedging are intimately related. For the same level of capital, the more a firm raises debt, the less resources it allocates to risk management. For this reason, *within every capital group*, we observe an opposite pattern with respect to the "Undrawn Credit" panel. Conditional on capital, which is determined endougenously, the more a firm keeps slack on its collateral contraints, the higher observed leverage is. Because in practice one important way to tranfer conditional liquidity is based on loan commitments (Rampini and Viswanathan (2010)), this pattern is also reflected in data on credit lines. Undrawn credit therefore appears to be a good proxy for conditional hedging.

We believe these results are informative in three ways. First, our dynamic model of corporate liquidity provides a unified framework to rationalize and interpret existing empirical evidence on cash, risk management, leverage, and lines of credit.

Second, our simulated results have implications for empirical work, and specifically for how to proxy financial constraints. Our model shows that net worth, that we proxy as the book value of equity, and capital, capture different aspects of financial constraints for corporate liquidity. A common practice in empirical studies is to use both capital and book value of equity as proxies for how a firm is constrained. In constrast, recognizing that net worth is a theoretically grounded state variable in models of financial constraints, and that it plays a different role from capital for liquidity and risk management decisions, appears to be a necessary condition for most empirical studies to be informative.

Third, our findings suggest an empirical proxy for hedging, namely undrawn credit from credit lines. As we discuss in section 2, empirical studies on risk management are plagued because hedging is unobservable. Despite there is not a one-to-one mapping between undrawn credit and conditional hedging, our results suggest that the former is a reasonable proxy for the latter. This appears plausible if one considers the widespread use of lines of credits, as Sufi (2009) points out. Data on credit lines are nowadays available for large cross sections of firms

in commercial datasets. Therefore, they may help extend and complement existing studies that, while based on specific data that are more closely mapped into hedging, are limited in scope.

[Insert Table 3 Here]

## 6.2  Comparative Statics: Debt Capacity

Table 4 summarizes the predicted impact of variations in the fraction of collateralizable capital $\theta$ on firms' policy. The rows of the table refer to investment, leverage, equity issues, cash holdings, and hedging through conditional liquidity. The columns report average values for all firms, and for firms that differ in terms of the two state variables of our model, namely net worth and capital.

Panel A refers to low values of $\theta$, panel B to moderate values, and panel C to high values. Different levels of $\theta$ can be interpreted as cross-industry predictions. Intuitively, the information technology industry relies on more intangible assets, that cannot usually be pledged as collateral.[11] In contrast, steel manufacturing companies typically operate with collateralizable capital such as properties, plants, and equipments. In our framework, industries with less pledgeable assets can be associated to lower values of $\theta$.

Table 4 illustrates the hedging view of capital structure of Rampini and Viswanathan (2012a), and the tradeoff between conditional and unconditional liquidity management. Firms with a low fraction of collateralizable assets have both lower leverage, and residual debt capacity. Our model predicts that firms with $\theta$ equal to 10% have a debt-to-asset ratio of 4.7%, compared to 30.7% for firms with $\theta$ equal to 90%. Residual debt capacity is ranging from 43.8% for firms with a low fraction of pledgeable capital, to almost 60% for firms with a high fraction. The latter can implement more conditional liquidity management, and therefore face less needs to

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[11]However, Amable, Chatelain, and Ralf (2010) argue that a recent common practice is to pledge patents as collateral.

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resort to cash hoarding to hedge against income shortfalls. In addition, firms with lower debt capacity have less needs for costly external equity financing.

Consistent with the patterns we illustrate in section 6.1, firms that differ in terms of the endogenous state variables of our model have a different expected leverage and liquidity management policy across different levels of θ. In particular, since debt capacity is a fraction of capital, the latter is the variable that interacts more with θ to determine the firm's policy. Smaller firms hoard more cash and implement less conditional liquidity management in all panels A, B, and C, but their liquidity is disproportionally more state-contingent for high values of θ. For instance, small firms in panel A have a 91.7% cash-to-assets ratio, and 53.6% undrawn debt capacity. Panel C instead predicts a cash-to-asset ratio of 38.2%, and a fraction of undrawn credit equal above 75% when θ = 0.9. In addition, our model predicts that the positive relationship between leverage and capital is steeper in industries with more tangible assets, reflecting higher opportunities to secure debt financing with collateral.

[Insert Table 4 Here]

## 6.3 Comparative Statics: Capital Adjustment Costs

As we discuss in section 4 and illustrate in figures 4 through 7, the presence of investment and disinvestment adjustment costs has a qualitative and quantitative impact on the type of liquidity management firms implement. In this section, we examine how predicted liquidity management and financing policy vary across firms with different magnitudes for adjustment costs of physical capital.

Table 5 examines the case of convex disinvestment adjustment costs. As figure 6 illustrates, firms with higher smooth adjustment costs of disinvestment have more liquidity needs for bad states. These needs reflect the necessity to bear these expected costs in future periods, and to

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be able to gradually adjust their capital stock. Panels A to C show how firms with higher adjustment cost of disinvestment implement more liquidity management, both conditionally and unconditionally. Average cash holdings vary from about 10% to over 20% if disinvestment adjustment costs increase from low to high values. Analogously, undrawn debt capacity approximately ranges from 48% to 60%. As a consequence, firms with lower values for $\psi^-$ need to save less debt capacity, and are more levered.

[Insert Table 5 Here]

Table 6 performes counterfactual analysis for firms that are associate to different smooth investment adjustment costs $\psi^+$. Firms with higher values for $\psi^+$ are more levered, invest less, and implement less liquidity management. Intuitively, investment is less profitable if associated to higher costs, and companies that are more exposed to these costs raise more debt finance to pay out more dividends, and take advantage of the tax benefits of debt. As figure 3 illustrates, the persistence in investment opportunities creates a need to transfer liquidity to good states. However, when adjustment costs are too high, investment needs decrease in such states, and so do liquidity needs. As a consequence, firms with high $\psi^+$ hoard less cash than firms with low $\psi^+$ (2% versus 31%), and save less debt capacity (32% versus 60%).

[Insert Table 6 Here]

# 7 Impulse Response Functions

In this section, we investigate the dynamics of investment, leverage, conditional, and unconditional liquidity management for firms that differ in terms of net worth and capital. To this end, figures 8 to 11 depict impulse response functions for the model to a positive shock (dashed

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lines), and to a negative shock (dashed-dotted lines). In all panels, the solid lines represents the benchmark case, that is the case in which the representative firm is exposed to neutral shocks.

Firms are classified as relatively constrained/unconstrained, and relatively small/large on the basis on their initial values for the two endogenous state variables of the models. Accordingly, figure 8 plots impulse response functions for firms with initial low net worth and median capital stock (constrained), figure 9 refers to firms with with initial high net worth and median capital stock (unconstrained), figure 10 refers to firms with with initial low capital stock and median net worth (small), and figure 11 refers to firms with with initial high capital stock and median net worth (large).

Because the model is nonlinear, we construct generalized impulse response functions following Potter (2000), to which we refer for an exhaustive treatment. Effectively, impulse response functions are computed as the averages of 5000 draws of sequences of shocks from $z_{i,t}$ for 30 periods under the baseline parametrization of table 2. In the benchmark case, the shock process is initalized to the mean shock $\mu_z$ for all draws, while for the positive (negative) response cases the process is initialized to values above (below) $\mu_z$. The exact definitions of positive and negative shocks, small and high initial net worth and capital, and of the variables on the graphs are provided in the caption of the figures.

A comparison of figure 8 and figure 9 highlights how the dynamics of investment, leverage, and hedging differ between relatively constrained and unconstrained firms in response to positive and negative shocks to investment opportunities. Panel A shows that both types of firms increase investment when a positive shock occurs, and decrease investment when a negative shock occurs. This result is due to the high persistence of exogenous shock process. However, the dynamics of both unconditional and conditional hedging deeply differ, as panels D, E, and F depict. Specifically, constrained firms have less resources to allocate to risk management when the shock realizes. Therefore, their adjustment to cash holdings (panel D), hedging for good

states (panel E), and hedging for bad states (panel F) are low than for unconstrained firms. This implies that the dynamics of leverage, net worth, and capital differ in that the effects of the shocks are more persistent for more constrained firms. Remarkably, the response is asymmetric. After a negative shock constrained firms become even more constrained, as the dynamics of net worth in panel C show. They can allocate little resources to conditional downstate hedging and to cash hoarding for future bad states, that are more likely to occur due to the persistence of the shock process. After a positive shock, instead, constrained firms benefit from additional cash flow, and have more net worth to tranfer to future states in the form of both conditional and unconditional liquidity. To sum up, relatively constrained firms have a lower capacity to implement total hedging the relatively unconstrained firms, and they are sluggish in reacting to negative shocks.

Figures 10 and 11 illustrate how the dynamics of corporate policy differs between small and large firms. As figure 1 indicates, the capital stock primarily affects the composition of corporate liquidity (conditional versus unconditional), rather than the amount of total liquidity. The hump-shaped investment dynamics in panel A suggest that small firms are slower in adjust their capital stock after the shocks. Indeed, large firms can transfer larger amounts of state-contingent liquidity (panels E and F), and be more efficient in boosting their investment in good times, and reducing their capital stock in bad times. Small firms, because on collateral constraints, can pledge less capital and need to hedge by hoarding cash (panel D). As a consequence they are forced to tranfer net worth to all future states and, *ceteris paribus*, they can tranfer less resources to the states where they are needed the most. Overall, large firms can take advantage of more pleadgeable capital for conditional liquidity management, and be more efficient in adjusting their investment policy. As a consequence, they benefit more than small firms from improved investment opportunities, and they reduce the impact of bad shocks on their value.

[Insert Figure 8 Here]

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[Insert Figure 9 Here]

[Insert Figure 10 Here]

[Insert Figure 11 Here]

# 8   Conclusions

In the presence of capital market imperfections expectations of future investment opportuni-
ties or cash shortfalls provide a rationale for dynamic liquidity management. We develop a
quantitative model to examine the cross-sectional and time-series determinants of corporations'
liquidity management. Our model identifies unconditional liquidity management using cash and
conditional liquidity management by means of drawing on credit lines as important instruments
of corporate policy. In particular, our model predicts substantial cross-sectional variation in the
relative usage of these instruments for liquidity purposes across firms, for which we find strong
empirical support. Similarly, the model successfully rationalizes time-series patterns in corpora-
tions' liquidity management. Overall, the model thus provides a quantitatively and empirically
successful framework explaining corporate investment, financing and liquidity policies and the
joint occurrence of cash, debt and credit lines in the presence of capital market imperfections.

A large literature has recently attempted to rationalize the apparent secular trend in firms'
cash holdings. It has been widely documented that in the US, firms' cash-to-asset ratios have
increased dramatically since the 1970's. While in this paper we focus on stationary properties of
firms' liquidity policies, we think it would be interesting to examine the possible determinants
of this trend through the lens of our model. We leave this important question for future research.

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Figure 1: Dynamic Corporate Liquidity

The figure illustrates the relationship between the different types of corporate liquidity, and the state variables of the model. In every period, firms are sorted independently by net worth $w_{i,t}$, capital $k_{i,t}$, and productivity $z_{i,t}$. Firms whose net worth is above the median of the cross-sectional distribution are labeled as unconstrained ('Unc'), and firms whose net worth is below the median of the cross-sectional distribution are labeled as constrained ('Con'). Firms whose capital is above the median of the cross-sectional distribution are labeled as large ('Lar'), and firms whose capital is below the median of the cross-sectional distribution are labeled as small ('Sm'). Firms whose realized productivity is above the middle state are labeled as profitable ('Pr'), and firms whose productivity is below the middle state are labeled as unprofitable ('Unp'). For each bin, we compute total hedging, the fraction of conditional to total hedging (on the horizontal axis), and the fraction of upstate to total hedging (on the vertical vertical axis). In the figure, the radious of the circle is proportional to total hedging. Data are simulated from the model with the baseline parametrization in Table X, for a panel of 1000 firms and 100 time periods.
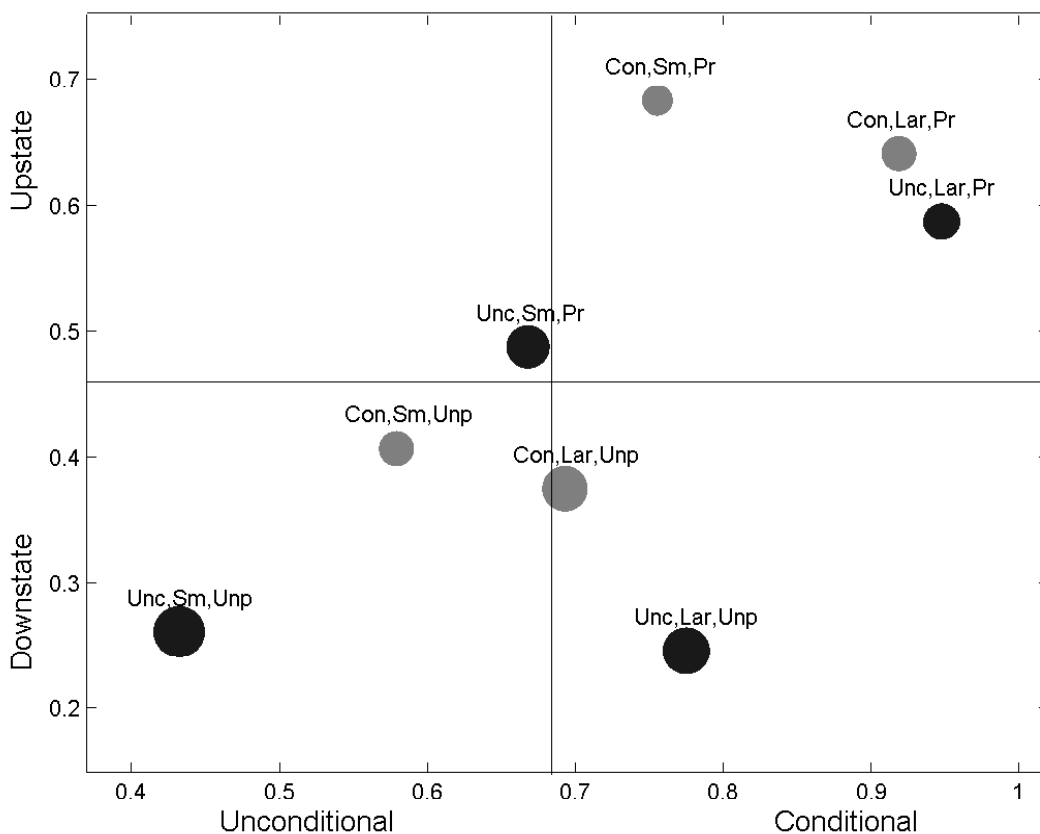
Figure 2: Firm's policy with no persistence and no adjustment costs

The figure illustrates the investment, financing, and risk management policy of the firm as a function of current net worth $w_{i,t}$. For illustrative purposes, the model is solved with a number of states equal to three, with uniform transition probabilities, and with all adjustment costs parameters set to zero. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to -0.3000 for the low state, to 0.5000 for the middle state, and to 1.7000 for the high state. Panels A through F show: the future capital stock $k_{i,t+1}$, the net equity payout $e_{i,t}$ the equity value $V_{i,t}$, the observed debt stock $E[b_i(s(t+1))]$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$. In panel F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.5000$, $\gamma = 0.0010$, $r = 0.0100$.
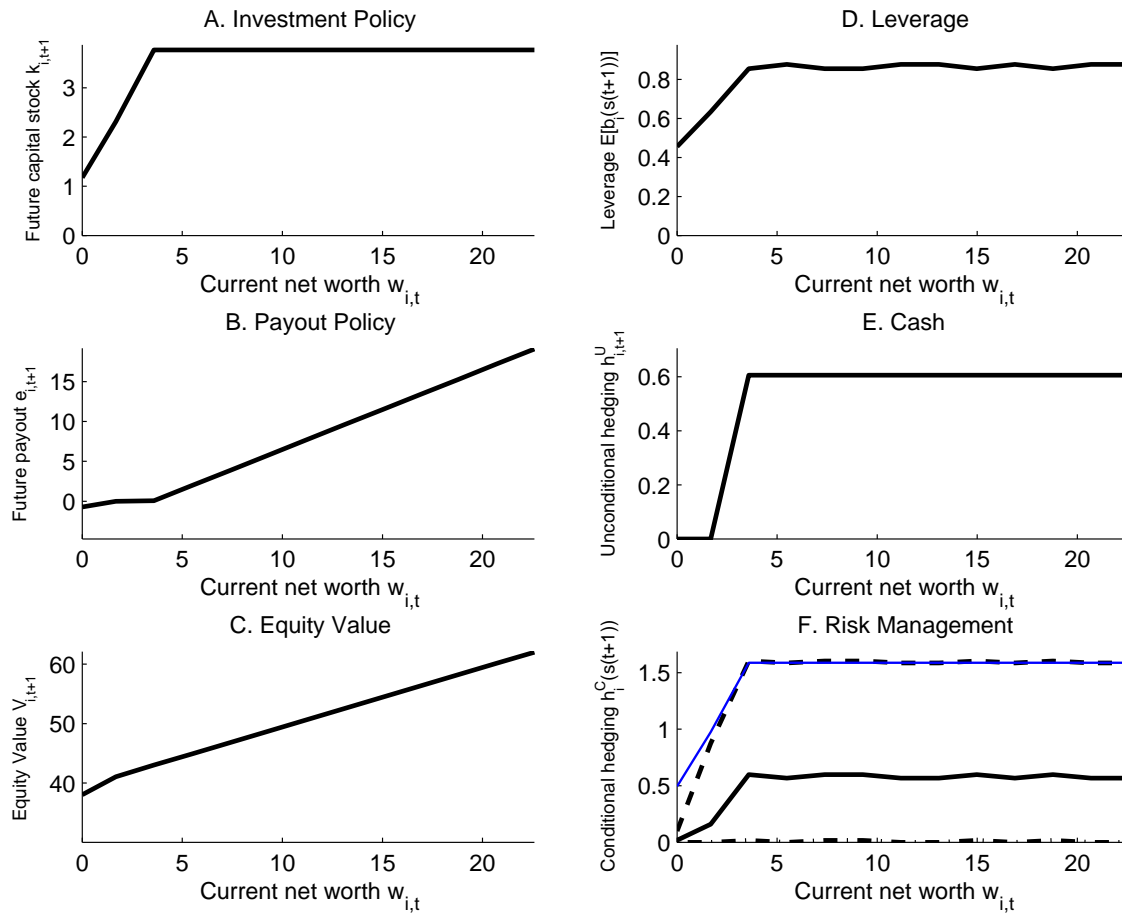


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## Figure 3: Firm's policy with persistence and no adjustment costs

The figure illustrates the investment, financing, and risk management policy of the firm as a function of current net worth $w_{i,t}$. For illustrative purposes, the model is solved with a number of states equal to three, and with all adjustment costs parameters set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.2000 for the low state, to 0.5000 for the middle state, and to 0.8000 for the high state. Panels A through F show: the future capital stock $k_{i,t+1}$, the net equity payout $e_{i,t}$ the equity value $V_{i,t}$, the observed debt stock $E[b_i(s(t+1))]$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$. In panel F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.6000$, $\gamma = 0.0010$, $r = 0.0100$.
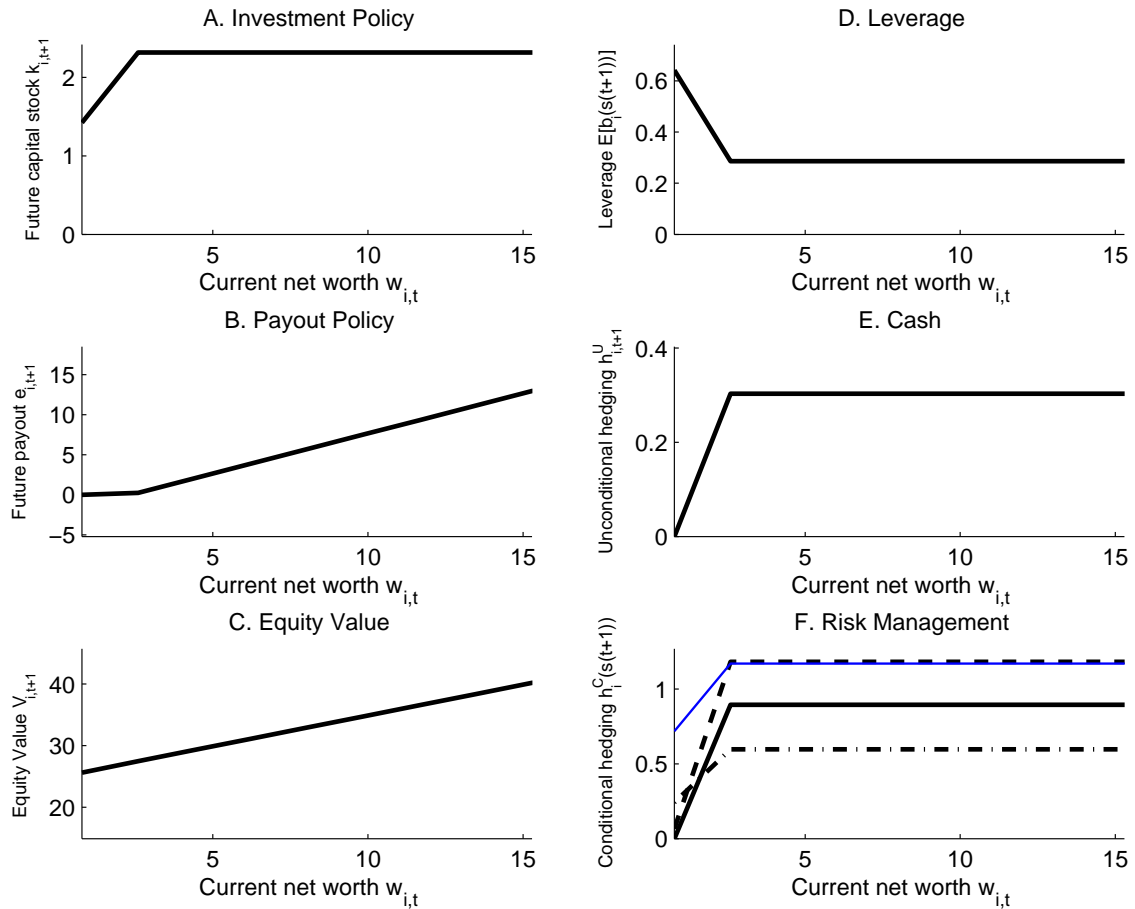


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## Figure 4: Firm's policy with convex investment adjustment costs

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex investment adjustment cost parameter $\psi^+$ is set to 1.0000. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.3000 for the low state, to 0.7000 for the middle state, and to 1.1000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$ as a function of current net worth. Panels D through F show: the investment-to-capital ratio $i_{i,t}/k_{i,t}$, unconditional hedging (cash) $h^U_{i,t+1}$, and conditional hedging $h^C_i(s(t+1))$ as a function of the current capital stock. In panels C and F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.6000$, $\gamma = 0.0010$, $r = 0.0100$.
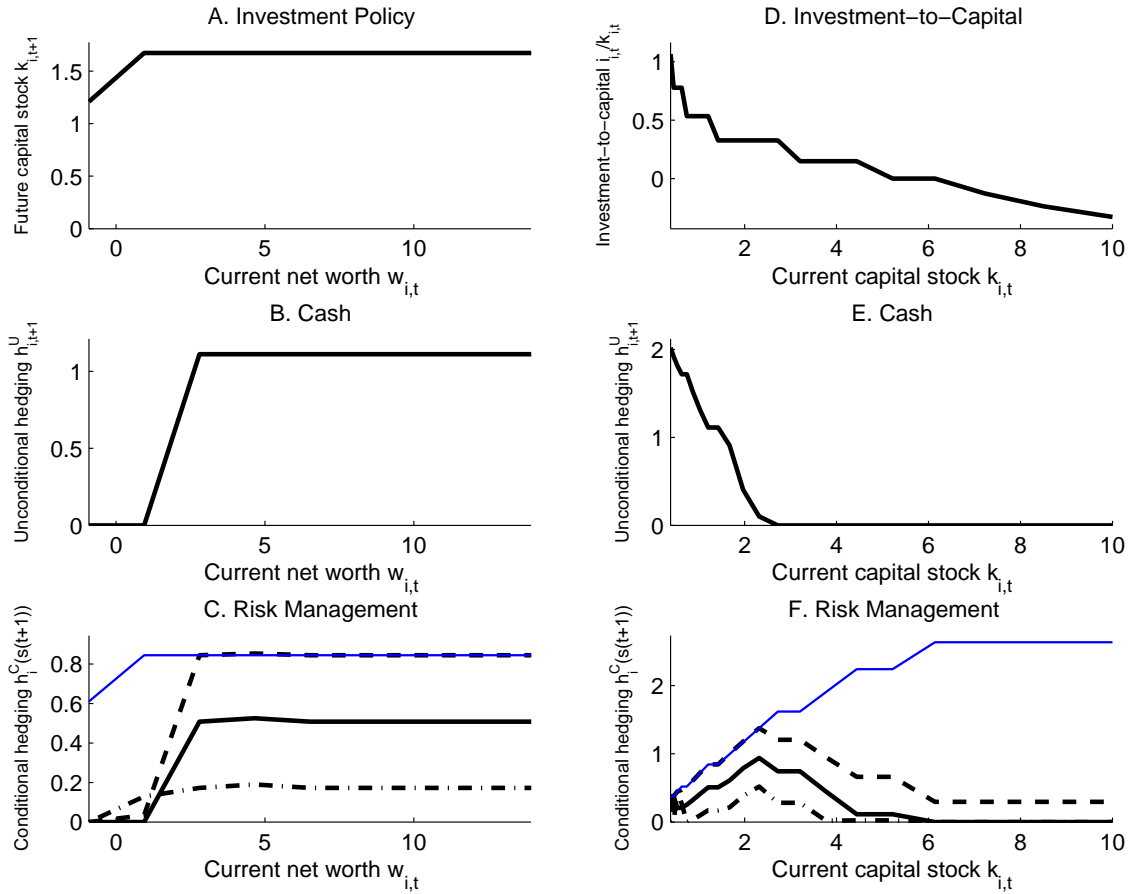
## Figure 5: Firm's policy with fixed investment adjustment costs

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex investment adjustment cost parameter $\psi_0^+$ is set to 0.0750. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.3000 for the low state, to 0.7000 for the middle state, and to 0.9000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of current net worth. Panels D through F show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of the current capital stock. In panel D, $k^*$ denotes the "frictionless" level of capital with $\psi_0^+ = 0$, while $k^D$ and $k^U$ are the bounds of the inaction region. In panels C and F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.3500$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.4000$, $\gamma = 0.0010$, $r = 0.0100$.
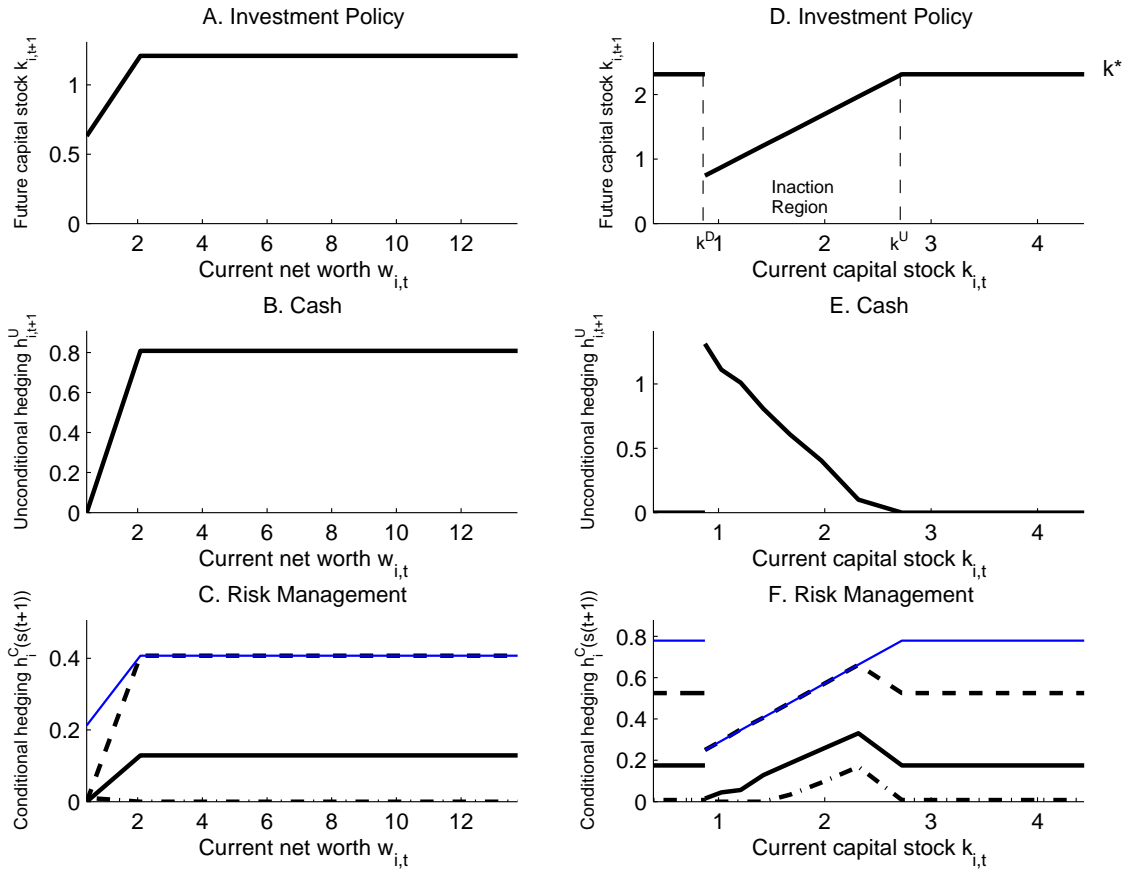
## Figure 6: Firm's policy with convex disinvestment adjustment costs

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex disinvestment adjustment cost parameter $\psi^-$ is set to 0.4000. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to -0.1000 for the low state, to 0.5000 for the middle state, and to 0.6000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of current net worth. Panels D through F show: the investment-to-capital ratio $i_{i,t}/k_{i,t}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of the current capital stock. In panels C and F, the solid blue line represents total debt capacity $\theta \delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.3750$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.4000$, $\gamma = 0.0010$, $r = 0.0100$.
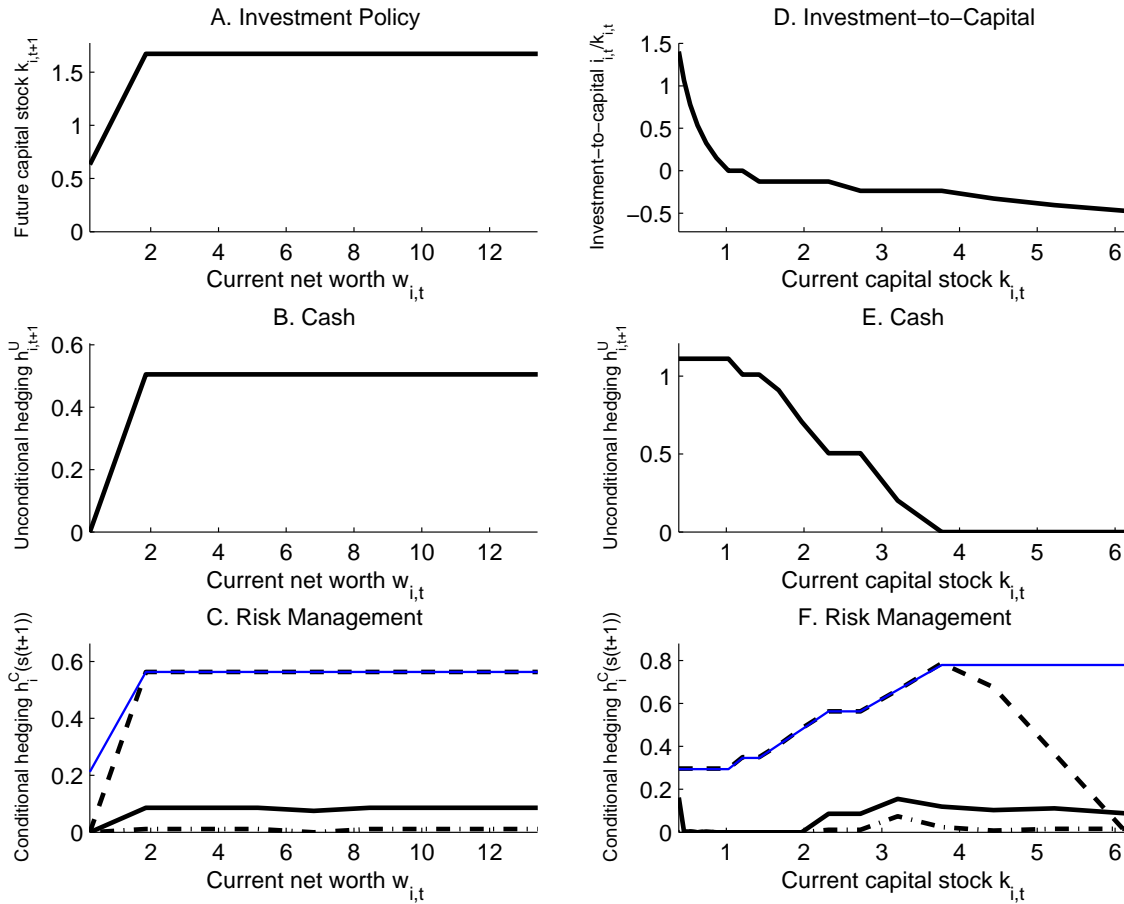
# Figure 7: Firm's policy with fixed disinvestment adjustment costs

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex investment adjustment cost parameter $\psi_0^-$ is set to 0.0250. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to -0.1000 for the low state, to 0.6000 for the middle state, and to 0.7000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of current net worth. Panels D through F show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of the current capital stock. In panel D, $k^*$ denotes the "frictionless" level of capital with $\psi_0^- = 0$, while $k^D$ and $k^U$ are the bounds of the inaction region. In panels C and F, the solid blue line represents total debt capacity $\theta\delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.4000$, $f = 0.0000$, $\tau = 0.3500$, $\theta = 0.4000$, $\gamma = 0.0010$, $r = 0.0100$.



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Figure 8: Impulse Response Functions for Relatively Constrained Firms

Panels A through F illustrate the generalized impulse response functions of capital, leverage, net worth, cash, conditional upstate hedging (one state up), and conditional downstate hedging (one state down). The impulse response functions are computed as averages 5000 draws of 30 periods for the shock $z_{i,t}$. For each draw, the process $z_{i,t}$ is initialized at the middle state for the benchmark case (solid line), to the second highest state for the response to a positive shock (dashed line), and to the second loweest state for the response to a negative shock (dashed-dotted line). The initial values for capital are set to the corrisponding median point on the grid for $k_{i,t}$, and the initial values for net worth are set to the corresponding value for the point on the grid for $w_{i,t}$ that leaves one fifth of grid points to its left. All parameters values are set to the baseline values reported in table 2.

Figure 9: Impulse Response Functions for Relatively Unconstrained Firms

Panels A through F illustrate the generalized impulse response functions of capital, leverage, net worth, cash, conditional upstate hedging (one state up), and conditional downstate hedging (one state down). The impulse response functions are computed as averages 5000 draws of 30 periods for the shock $z_{i,t}$. For each draw, the process $z_{i,t}$ is initialized at the middle state for the benchmark case (solid line), to the second highest state for the response to a positive shock (dashed line), and to the second loweest state for the response to a negative shock (dashed-dotted line). The initial values for capital are set to the corrisponding median point on the grid for $k_{i,t}$, and the initial values for net worth are set to the corresponding value for the point on the grid for $w_{i,t}$ that leaves one fifth of grid points to its right. All parameters values are set to the baseline values reported in table 2.
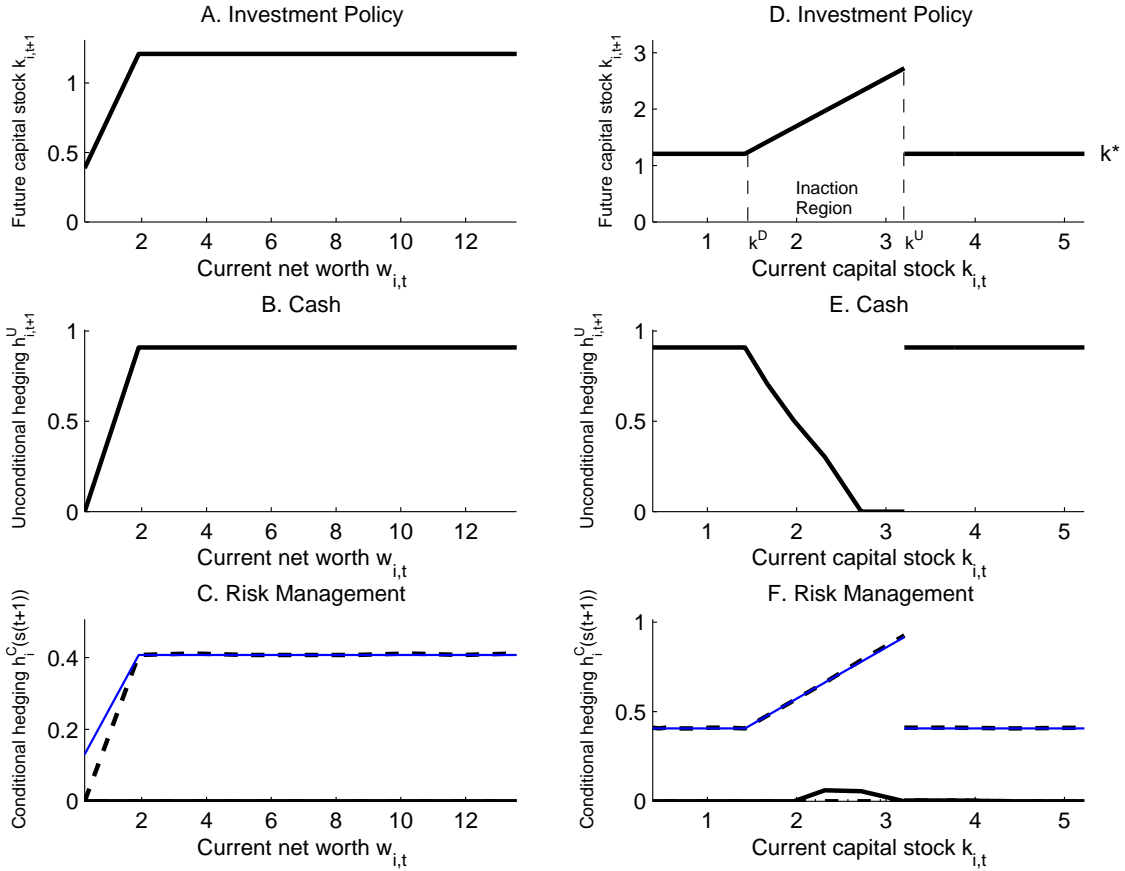


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# Figure 10: Impulse Response Functions for Relatively Small Firms

Panels A through F illustrate the generalized impulse response functions of capital, leverage, net worth, cash, conditional upstate hedging (one state up), and conditional downstate hedging (one state down). The impulse response functions are computed as averages 5000 draws of 30 periods for the shock $z_{i,t}$. For each draw, the process $z_{i,t}$ is initialized at the middle state for the benchmark case (solid line), to the second highest state for the response to a positive shock (dashed line), and to the second loweest state for the response to a negative shock (dashed-dotted line). The initial values for net worth are set to the corrisponding median point on the grid for $w_{i,t}$, and the initial values for capital are set to the corresponding value for the point on the grid for $k_{i,t}$ that leaves one fifth of grid points to its left. All parameters values are set to the baseline values reported in table 2.
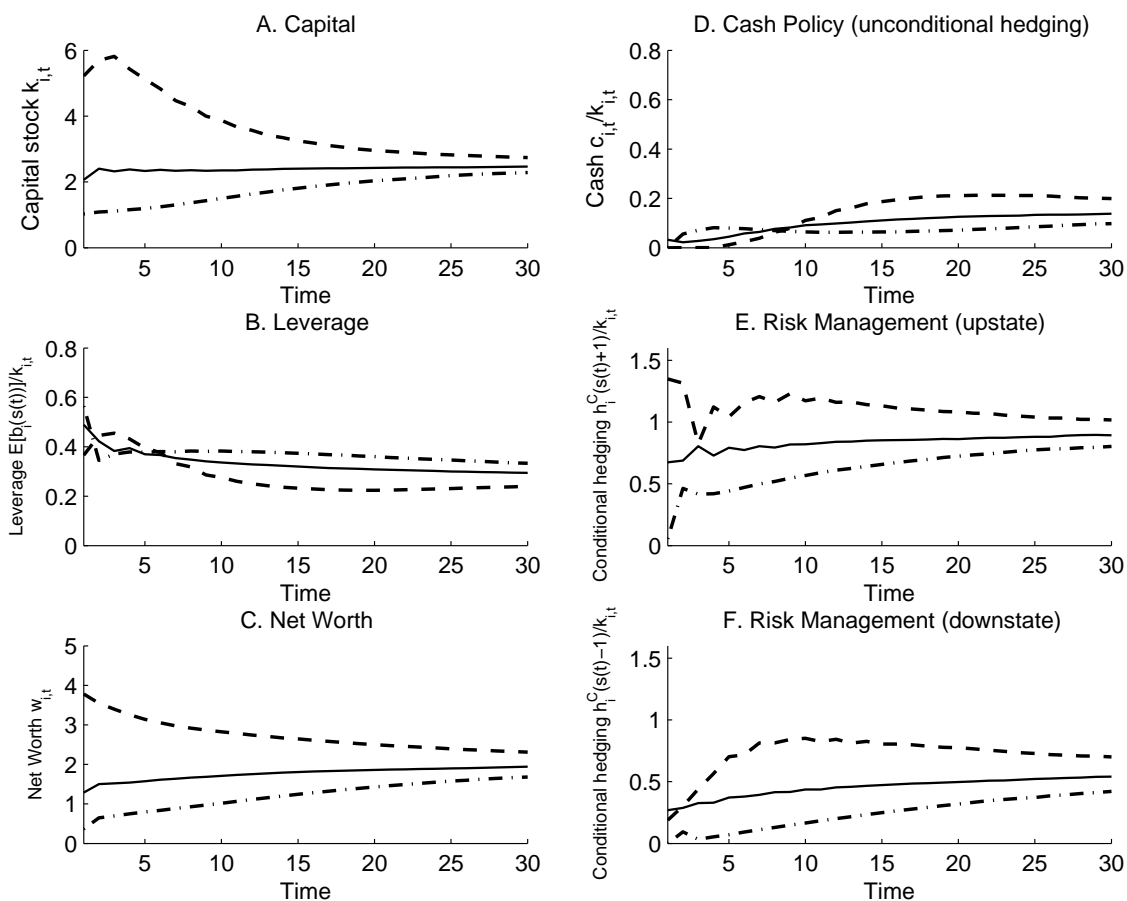
## Figure 11: Impulse Response Functions for Relatively Large Firms

Panels A through F illustrate the generalized impulse response functions of capital, leverage, net worth, cash, conditional upstate hedging (one state up), and conditional downstate hedging (one state down). The impulse response functions are computed as averages 5000 draws of 30 periods for the shock $z_{i,t}$. For each draw, the process $z_{i,t}$ is initialized at the middle state for the benchmark case (solid line), to the second highest state for the response to a positive shock (dashed line), and to the second loweest state for the response to a negative shock (dashed-dotted line). The initial values for net worth are set to the corrisponding median point on the grid for $w_{i,t}$, and the initial values for capital are set to the corresponding value for the point on the grid for $k_{i,t}$ that leaves one fifth of grid points to its right. All parameters values are set to the baseline values reported in table 2.
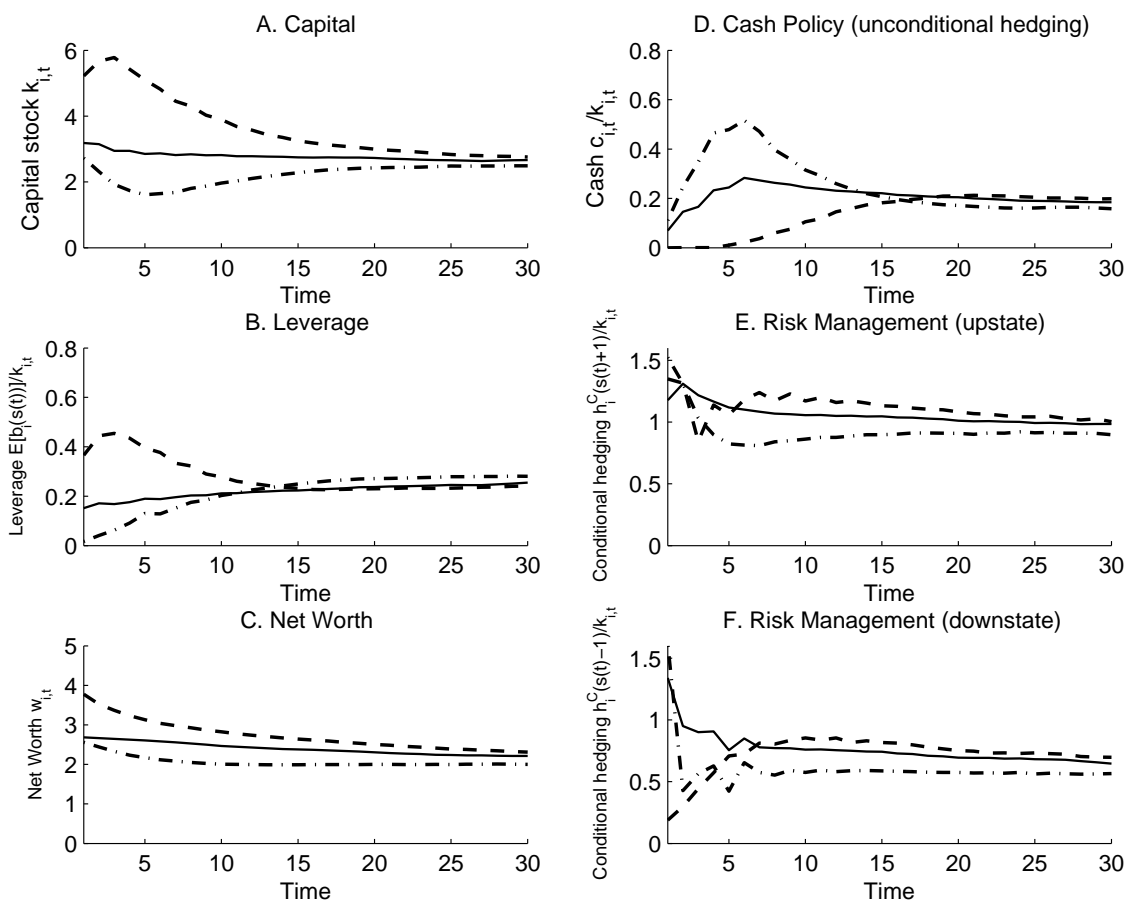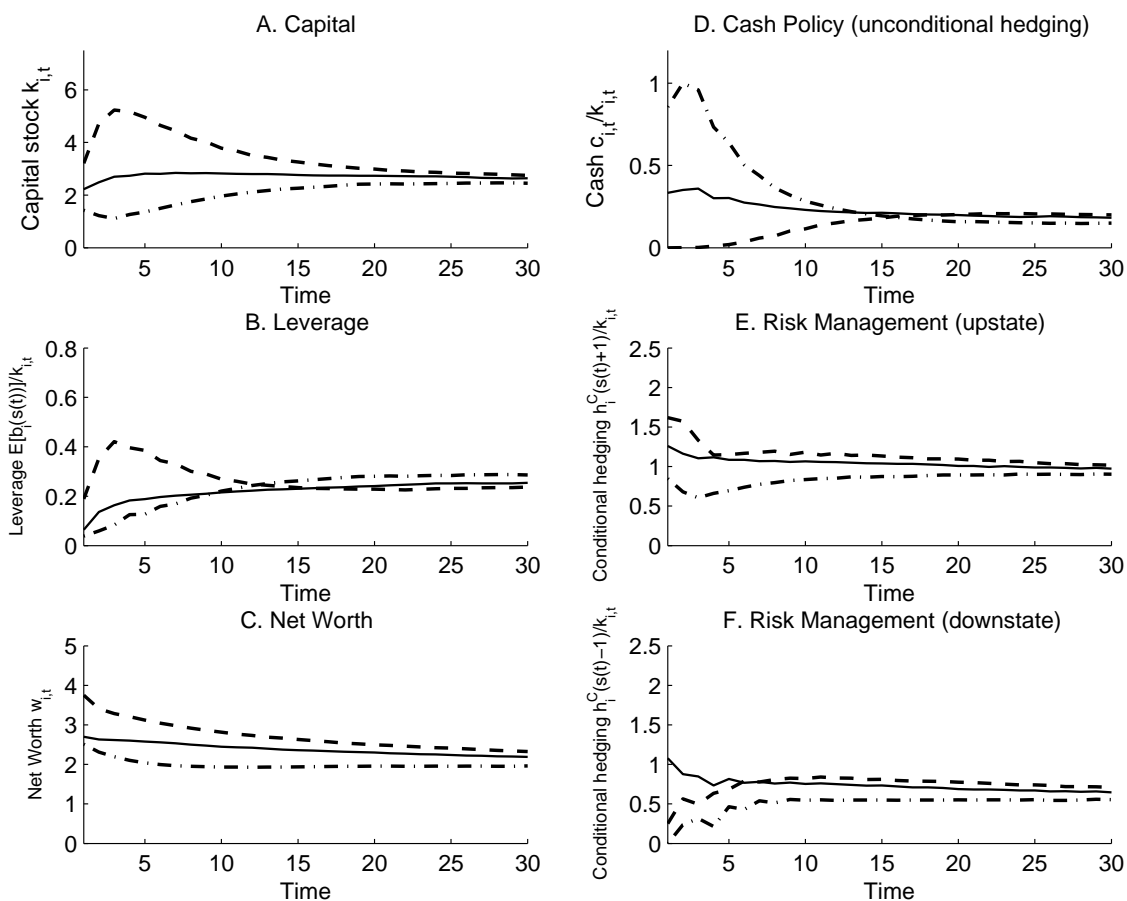
Table 1: Leverage, Cash, and Conditional Liquidity: Stylized Evidence.

The table reports stylized evidence from sorts of companies by net worth and capital (the state variables of our model). Data are from Compustat and Capital IQ for the period 2001-2011. Net worth is measured as the book value of equity, in line with Rampini, Sufi, and Viswanathan (2012), and capital is the book value of property, plant and equipment. The breakpoints for defining relative constrained and unconstrained firms for the sorts on net worth, and relatively small and large firms for the sorts on capital are the 33th and the 66th percentile of the cross-sectional distribution for every fiscal year. We exclude financials (SIC 4900-4099), utilities (SIC 6000-6999), and firms from other regulated industries (SIC greater than 9000). The final sample consists of 14220 firm-year observations. Panel A reports average cash holdings and debt as a fraction of total assets, and the fraction of undrawn credit from credit lines for one-way sorts, while panel B reports the same variables for two-way sorts.

| | Panel A: Univariate Sorts | | | | | |
|---|---|---|---|---|---|---|
| | Net Worth | | | Capital | | |
| | Constr. | 2 | Unconstr. | Small | 2 | Large |
| Cash Holdings | 0.187 | 0.178 | 0.126 | 0.240 | 0.153 | 0.098 |
| Leverage | 0.216 | 0.192 | 0.228 | 0.118 | 0.220 | 0.296 |
| Undrawn Credit | 0.754 | 0.811 | 0.861 | 0.793 | 0.793 | 0.842 |

| | | Panel B: Bivariate Sorts | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Cash Holdings | | | Leverage | | | Undrawn Credit | | |
| | Capital | Small | 2 | Large | Small | 2 | Large | Small | 2 | Large |
| | Constr. | 0.220 | 0.103 | 0.085 | 0.128 | 0.385 | 0.697 | 0.770 | 0.712 | 0.748 |
| Net Worth | 2 | 0.297 | 0.156 | 0.075 | 0.089 | 0.180 | 0.378 | 0.864 | 0.810 | 0.761 |
| | Unconstr. | 0.269 | 0.193 | 0.105 | 0.109 | 0.161 | 0.249 | 0.878 | 0.833 | 0.867 |

Table 2: Model Calibration.

The table reports actual and simulated moments, together with the corresponding choice of structural parameters. Calculations of data moments are based on a sample of nonfinancial, unregulated firms from the annual 2011 Compustat Industrial database merged to the Capital IQ dataset. The sample period is from 2001 to 2011. Panel A reports the moments from a simulated panel of firms, and the corresponding moments from the data. Operating income is defined as $(zk^\alpha - f)/k$, investment as $i = k_{t+1} - (1 - \delta)k_t$, leverage as $E[b(z_{t+1})]/k$, equity issues as $|min(d, 0)|/k$, cash holdings as $c/k$, undrawn debt capacity as $h^c(z_{t+1})/(\theta(1 - \delta)k)$, and Tobin's Q as $(V + E[b(z_{t+1})])/k$. Panel B reports the chosen values for structural parameters. $\alpha$ is the curvature of the production function, $f$ is the per-period fixed production cost, $\theta$ is the collateralizable fraction of assets, $\gamma$ is the agency cost cash parameter, $\psi_0^+$ and $\psi^+$ are the fixed and convex investment adjustment costs parameters, $\psi_0^-$ and $\psi^-$ are the fixed and convex disinvestment adjustment costs parameters, $\rho_z$ and $\sigma_z$ are the serial correlation and the standard deviation to innovations of $ln(z)$, where $z$ is the shock to the revenue function, $\lambda_0$ and $\lambda_1$ are the fixed and the proportional equity flotation costs. The remaining parameters are $r$, the interest rate, $\tau$, the tax rate, and $\delta$, the depreciation rate. They are set to 0.015, 0.35, and 0.15, to be approximately equal to the real interest rate in the 20th century, to the statutory tax rate in the United States, and to the average depreciation in our sample.

| Panel A: Moments | | |
|---|---|---|
| | Simulated Moments | Data Moments |
| Mean of operating income | 0.1201 | 0.1387 |
| Variance of operating income | 0.0056 | 0.0068 |
| Serial correlation of operating income | 0.6270 | 0.7920 |
| Mean of investment | 0.1723 | 0.2018 |
| Skewness of investment | 1.3465 | 1.9872 |
| Variance of investment | 0.0531 | 0.0516 |
| Serial correlation of investment | 0.2167 | 0.3655 |
| Mean of leverage | 0.2965 | 0.2121 |
| Variance of leverage | 0.0365 | 0.0461 |
| Serial correlation of leverage | 0.6813 | 0.9173 |
| Mean of equity issues | 0.0107 | 0.0205 |
| Variance of equity issues | 0.0042 | 0.0028 |
| Mean of cash holdings | 0.1277 | 0.1632 |
| Variance of cash holdings | 0.0650 | 0.0294 |
| Serial correlation of cash holdings | 0.7298 | 0.8859 |
| Mean of undrawn debt capacity | 0.5278 | 0.8114 |
| Variance of undrawn debt capacity | 0.0926 | 0.0744 |
| Serial correlation of undrawn debt capacity | 0.6813 | 0.6344 |
| Mean Tobin's Q | 2.0666 | 1.5594 |

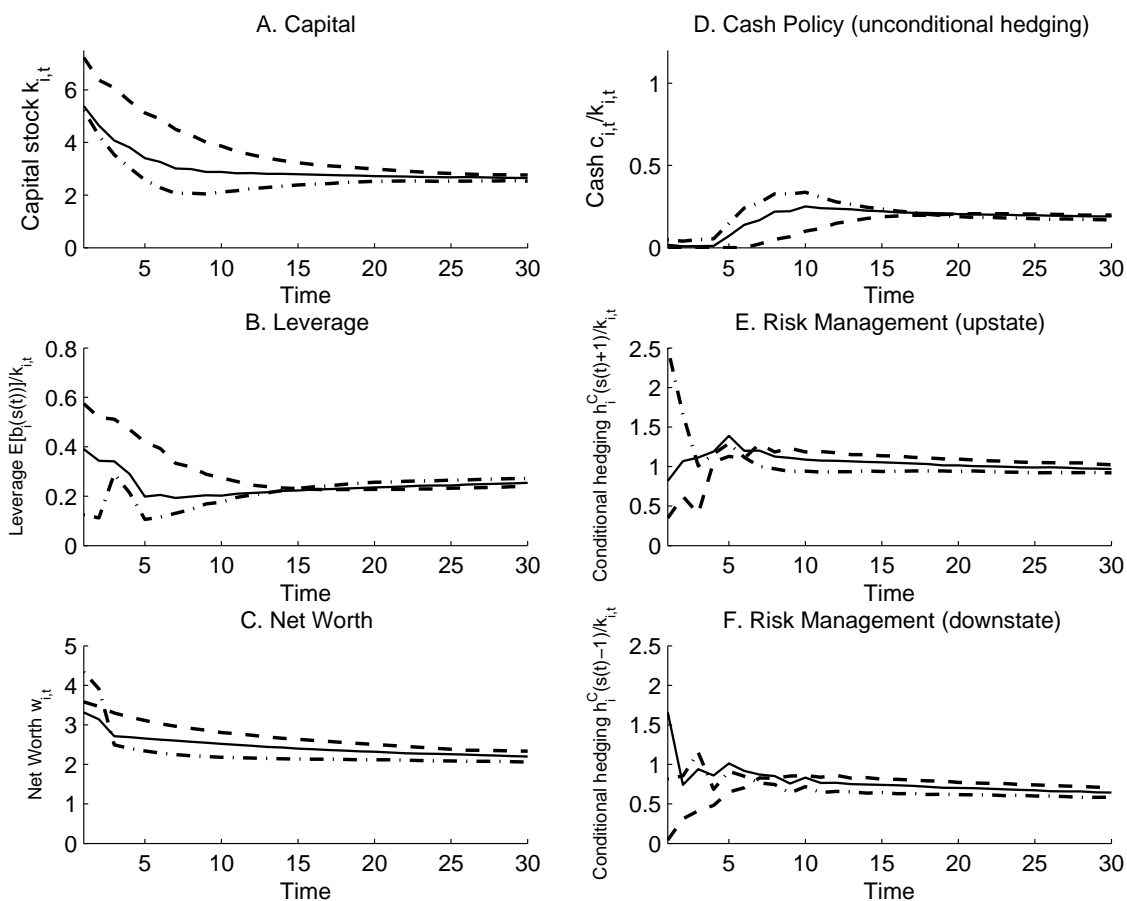| Panel B: Calibrated Parameters | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $f$ | $\theta$ | $\gamma$ | $\psi_0^+$ | $\psi^+$ | $\psi_0^-$ | $\psi^-$ | $\rho_z$ | $\sigma_z$ | $\lambda_0$ | $\lambda_1$ |
| 0.6800 | 0.1000 | 0.7500 | 0.0020 | 0.0075 | 0.1100 | 0.0400 | 0.2000 | 0.7100 | 0.2900 | 0.6000 | 0.0080 |

Table 3: Leverage, Cash, and Conditional Liquidity: Simulated Stylized Evidence.

The table reports stylized evidence from sorts of companies by net worth and capital (the state variables of our model). Data are simulated from the model under the baseline parametrization in table 2 for a panel of 1000 firms and 100 time periods. The breakpoints for defining relative constrained and unconstrained firms for the sorts on net worth, and relatively small and large firms for the sorts on capital are the 33th and the 66th percentile of the cross-sectional distribution. Panel A reports average cash holdings and debt as a fraction of capital, and the fraction of undrawn debt capacity for one-way sorts, panel B reports the same variables for two-way sorts, and panel C reports the fraction of conditional to total liquidity for both simulated, and actual data.

| | Panel A: Univariate Sorts | | | | | |
|---|---|---|---|---|---|---|
| | Net Worth | | | Capital | | |
| | Constr. | 2 | Unconstr. | Small | 2 | Large |
| Cash Holdings | 0.129 | 0.122 | 0.133 | 0.334 | 0.051 | 0.000 |
| Leverage | 0.303 | 0.282 | 0.305 | 0.162 | 0.281 | 0.450 |
| Undrawn Credit | 0.518 | 0.550 | 0.515 | 0.742 | 0.553 | 0.283 |

| | | Panel B: Bivariate Sorts | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Cash Holdings | | | Leverage | | | Undrawn Credit | | |
| | Capital | Small | 2 | Large | Small | 2 | Large | Small | 2 | Large |
| | Constr. | 0.228 | 0.025 | 0.000 | 0.211 | 0.357 | 0.560 | 0.664 | 0.432 | 0.108 |
| Net Worth | 2 | 0.310 | 0.035 | 0.000 | 0.119 | 0.291 | 0.531 | 0.811 | 0.537 | 0.155 |
| | Unconstr. | 0.872 | 0.132 | 0.000 | 0.072 | 0.120 | 0.404 | 0.885 | 0.808 | 0.357 |

| | | Panel C: Liquidity Composition | | | | | |
|---|---|---|---|---|---|---|---|
| | | Conditional Liquidity (model) | | | Conditional Liquidity (data) | | |
| | Capital | Small | 2 | Large | Small | 2 | Large |
| | Constr. | 0.681 | 0.922 | 1.000 | 0.377 | 0.551 | 0.497 |
| Net Worth | 2 | 0.679 | 0.916 | 1.000 | 0.258 | 0.479 | 0.621 |
| | Unconstr. | 0.430 | 0.824 | 1.000 | 0.250 | 0.390 | 0.534 |

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Table 4: Comparative Statics: Debt Capacity.

The table reports simulated evidence from the model for the same panel of table 2. All parameters except $\theta$ are set to the baseline values of table 2. In each panel, we report average investment, leverage, equity issses, cash, and residual debt capacity (conditional liquidity). Averages are reported both for all firms, and for firms with low, medium, and high capital and net worth. Breakpoints are set as in panel A of tables 1 and 3. All variables are measured as in table 2. Panel A reports simulated moments for $\theta = 0.1$, panel B for $\theta = 0.5$, and panel C for $\theta = 0.9$.

| A. $\theta = 0.1000$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
| Investment | 0.170 | 0.215 | 0.173 | 0.123 | 0.191 | 0.166 | 0.153 |
| Leverage | 0.047 | 0.039 | 0.046 | 0.056 | 0.044 | 0.048 | 0.049 |
| Equity Issues | 0.007 | 0.016 | 0.003 | 0.001 | 0.017 | 0.002 | 0.002 |
| Cash | 0.513 | 0.917 | 0.492 | 0.129 | 0.547 | 0.702 | 0.282 |
| Residual Debt Capacity | 0.438 | 0.536 | 0.452 | 0.327 | 0.472 | 0.425 | 0.419 |

| B. $\theta = 0.5000$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
| Investment | 0.169 | 0.210 | 0.179 | 0.119 | 0.195 | 0.166 | 0.148 |
| Leverage | 0.195 | 0.121 | 0.157 | 0.309 | 0.176 | 0.158 | 0.253 |
| Equity Issues | 0.007 | 0.012 | 0.004 | 0.006 | 0.013 | 0.005 | 0.005 |
| Cash | 0.159 | 0.409 | 0.069 | 0.003 | 0.239 | 0.215 | 0.022 |
| Residual Debt Capacity | 0.534 | 0.711 | 0.624 | 0.263 | 0.580 | 0.622 | 0.397 |

| C. $\theta = 0.9000$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
| Investment | 0.171 | 0.216 | 0.172 | 0.124 | 0.213 | 0.179 | 0.120 |
| Leverage | 0.307 | 0.174 | 0.234 | 0.514 | 0.350 | 0.202 | 0.372 |
| Equity Issues | 0.005 | 0.009 | 0.005 | 0.002 | 0.010 | 0.004 | 0.002 |
| Cash | 0.134 | 0.382 | 0.020 | 0.003 | 0.198 | 0.190 | 0.011 |
| Residual Debt Capacity | 0.593 | 0.769 | 0.690 | 0.318 | 0.535 | 0.733 | 0.507 |

Table 5: Comparative Statics: Smooth Disinvestment Adjustment Costs.

The table reports simulated evidence from the model for the same panel of table 2. All parameters except $\psi^-$ are set to the baseline values of table 2. In each panel, we report average investment, leverage, equity issses, cash, and residual debt capacity (conditional liquidity). Averages are reported both for all firms, and for firms with low, medium, and high capital and net worth. Breakpoints are set as in panel A of tables 1 and 3. All variables are measured as in table 2. Panel A reports simulated moments for $\psi^- = 0$, panel B for $\psi^- = 0.5$, and panel C for $\psi^- = 1$.

| | A. $\psi^- = 0.0000$ | | | | | | |
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
|---|---|---|---|---|---|---|---|
| Investment | 0.174 | 0.219 | 0.186 | 0.116 | 0.217 | 0.159 | 0.146 |
| Leverage | 0.310 | 0.231 | 0.250 | 0.450 | 0.269 | 0.328 | 0.333 |
| Equity Issues | 0.015 | 0.031 | 0.012 | 0.003 | 0.035 | 0.008 | 0.002 |
| | | | | | | | |
| Cash | 0.095 | 0.251 | 0.035 | 0.000 | 0.171 | 0.044 | 0.071 |
| Residual Debt Capacity | 0.479 | 0.611 | 0.579 | 0.243 | 0.547 | 0.449 | 0.441 |

| | B. $\psi^- = 0.5000$ | | | | | | |
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
|---|---|---|---|---|---|---|---|
| Investment | 0.172 | 0.228 | 0.174 | 0.115 | 0.233 | 0.148 | 0.137 |
| Leverage | 0.300 | 0.198 | 0.257 | 0.445 | 0.279 | 0.289 | 0.331 |
| Equity Issues | 0.011 | 0.022 | 0.008 | 0.002 | 0.027 | 0.003 | 0.003 |
| | | | | | | | |
| Cash | 0.108 | 0.276 | 0.050 | 0.001 | 0.147 | 0.093 | 0.086 |
| Residual Debt Capacity | 0.496 | 0.666 | 0.568 | 0.252 | 0.531 | 0.514 | 0.443 |

| | C. $\psi^- = 1.0000$ | | | | | | |
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
|---|---|---|---|---|---|---|---|
| Investment | 0.172 | 0.227 | 0.166 | 0.121 | 0.195 | 0.186 | 0.134 |
| Leverage | 0.241 | 0.147 | 0.197 | 0.391 | 0.250 | 0.202 | 0.274 |
| Equity Issues | 0.007 | 0.015 | 0.005 | 0.002 | 0.016 | 0.003 | 0.003 |
| | | | | | | | |
| Cash | 0.212 | 0.529 | 0.102 | 0.003 | 0.179 | 0.323 | 0.130 |
| Residual Debt Capacity | 0.594 | 0.753 | 0.668 | 0.342 | 0.580 | 0.660 | 0.540 |

Table 6: Comparative Statics: Smooth Investment Adjustment Costs.

The table reports simulated evidence from the model for the same panel of table 2. All parameters except $\psi^+$ are set to the baseline values of table 2. In each panel, we report average investment, leverage, equity issses, cash, and residual debt capacity (conditional liquidity). Averages are reported both for all firms, and for firms with low, medium, and high capital and net worth. Breakpoints are set as in panel A of tables 1 and 3. All variables are measured as in table 2. Panel A reports simulated moments for $\psi^+ = 0$, panel B for $\psi^+ = 0.5$, and panel C for $\psi^+ = 1$.

| A. $\psi^+ = 0.0000$ | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
| Investment | 0.224 | 0.422 | 0.197 | 0.050 | 0.275 | 0.226 | 0.172 |
| Leverage | 0.240 | 0.140 | 0.164 | 0.419 | 0.251 | 0.150 | 0.322 |
| Equity Issues | 0.007 | 0.010 | 0.007 | 0.005 | 0.014 | 0.006 | 0.003 |
| Cash | 0.314 | 0.814 | 0.123 | 0.001 | 0.412 | 0.457 | 0.070 |
| Residual Debt Capacity | 0.596 | 0.765 | 0.724 | 0.295 | 0.578 | 0.747 | 0.458 |

| B. $\psi^+ = 0.5000$ | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
| Investment | 0.158 | 0.177 | 0.158 | 0.134 | 0.157 | 0.169 | 0.147 |
| Leverage | 0.321 | 0.353 | 0.211 | 0.458 | 0.386 | 0.242 | 0.338 |
| Equity Issues | 0.016 | 0.040 | 0.004 | 0.005 | 0.040 | 0.006 | 0.002 |
| Cash | 0.069 | 0.148 | 0.047 | 0.003 | 0.090 | 0.095 | 0.021 |
| Residual Debt Capacity | 0.461 | 0.407 | 0.646 | 0.230 | 0.351 | 0.594 | 0.432 |

| C. $\psi^+ = 1.0000$ | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | Capital | | | Net Worth | | |
| | All | Low | 2 | High | Low | 2 | High |
| Investment | 0.155 | 0.183 | 0.140 | 0.142 | 0.094 | 0.195 | 0.175 |
| Leverage | 0.403 | 0.424 | 0.483 | 0.301 | 0.514 | 0.444 | 0.251 |
| Equity Issues | 0.054 | 0.120 | 0.036 | 0.006 | 0.161 | 0.001 | 0.001 |
| Cash | 0.022 | 0.005 | 0.024 | 0.036 | 0.004 | 0.017 | 0.045 |
| Residual Debt Capacity | 0.322 | 0.286 | 0.188 | 0.495 | 0.135 | 0.254 | 0.578 |

# Appendix A  Proofs of Propositions

**Proof of Lemma 4.1.** From the definition of $h_i^C(s(t+1))$ we obtain:

$$b_i(s(t+1)) = \frac{\theta(1-\delta)k_{i,t+1} - h_i^C(s(t+1))}{1+r(1-\tau)} \tag{A.1}$$

Substituting (A.1) and the definition of $h_{i,t+1}^U$ into the original problem yields the result. ∎

**Proof of Proposition 4.2.** Denote the total number of states by $S$. The Lagrangian function for the constrained optimization problem is:

$$\mathcal{L}(e_{i,t}, k_{i,t+1}, h_{i,t+1}^U, \{h_i^C(s(t+1))\}, \{w_i(s(t+1))\}, \lambda^w, \left\{\frac{\pi(s(t),s(t+1))\lambda_{s(t+1)}^w}{1+r}\right\}, \left\{\frac{\pi(s(t),s(t+1))\underline{\lambda}_{s(t+1)}^C}{1+r}\right\}, \left\{\frac{\pi(s(t),s(t+1))\overline{\lambda}_{s(t+1)}^C}{1+r}\right\}, \underline{\lambda}^U) \equiv$$

$$e_{i,t} - \Lambda(e_{i,t}) + \frac{1}{1+r}E_t[V(w_{i,t+1}, z_{i,t+1})] + \lambda^w\left(w_{i,t} - e_{i,t} - E_t\left[\frac{h_i^C(s(t+1))}{1+r(1-\tau)}\right] - \frac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma} - Pk_{i,t+1} - \Psi(k_{i,t}, k_{i,t+1})\right) +$$

$$+\sum_{s=1}^{S}\frac{\pi(s(t),s)\lambda_s^w}{1+r}\left((1-\tau)\Pi(k_{i,t+1}, z_{i,s}) + (1-\theta)(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h_i^T(s) - w_i(s)\right) +$$

$$+\sum_{s=1}^{S}\frac{\pi(s(t),s)\underline{\lambda}_s^C}{1+r}\left(h_i^C(s)\right) + \sum_{s=1}^{S}\frac{\pi(s(t),s)\overline{\lambda}_s^C}{1+r}\left(\theta(1-\delta)k_{i,t+1} - h_i^C(s)\right) + \underline{\lambda}^U(h_{i,t+1}^U)$$

Differentiating the Lagrangian with rispect to $e_{i,t}$, $k_{i,t+1}$, $h_{i,t+1}^U$, $\{h_i^C(s(t+1))\}$, and $\{w_i(s(t+1))\}$ yields equations (19a), (19b), (19c), (19d), (21b) after some algebraic manipulation. Because the Slater condition holds, the envelope theorem can expressed as:

$$\frac{\partial V(w_{i,t}, z_{i,t})}{\partial w_{i,t}} = \frac{\partial e_{i,t} - \Lambda(e_{i,t})}{\partial w_{i,t}} + \lambda^w\frac{\partial\left(w_{i,t} - e_{i,t} - E_t\left[\frac{h_i^C(s(t+1))}{1+r(1-\tau)}\right] - \frac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma} - Pk_{i,t+1} - \Psi(k_{i,t}, k_{i,t+1})\right)}{\partial w_{i,t}} +$$

$$+\sum_{s=1}^{S}\frac{\pi(s(t),s)\lambda_s^w}{1+r}\frac{\partial\left((1-\tau)\Pi(k_{i,t+1}, z_{i,s}) + (1-\theta)(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h_i^T(s) - w_i(s)\right)}{\partial w_{i,t}} + \sum_{s=1}^{S}\frac{\pi(s(t),s)\underline{\lambda}_s^C}{1+r}\frac{\partial(h_i^C(s))}{\partial w_{i,t}} +$$

$$+\sum_{s=1}^{S}\frac{\pi(s(t),s)\overline{\lambda}_s^C}{1+r}\frac{\partial(\theta(1-\delta)k_{i,t+1} - h_i^C(s))}{\partial w_{i,t}} + \underline{\lambda}^U\frac{\partial(h_{i,t+1}^U)}{\partial w_{i,t}}$$

which immediately yields (21a). The Euler equation (22) can be simply obtained, by dividing both sides of (19b) by $\lambda^w$. The division is well-defined because the resource constraint at time $t$ is always binding. Finally, equation (23) can be derived by substituting $\lambda_{s(t+1)}^w$ from (19d) into the definition of $M^w(s(t), s(t+1))$. ∎

**Proof of Proposition 5.1.** To prove the claim, we proceed in two steps. First, we show that the payoff $b_i(s(t+1))$ can be replicated with the combination of securities described above. Second, we verify that the recursive problem with the new securities is equivalent to the original one in terms of constraints. First, in the recursive problem, at time $t+1$ in each state $s(t+1)$ the firm pays back $D_{i,t+1} - C_i^L(s(t+1))$ Therefore, using (31) and (32) we obtain

$$D_{i,t+1} - C_i^L(s(t+1)) = (1+r(1-\tau))b_i(s(t+1)) \tag{A.2}$$

from which the replication result follows:

$$b_i(s(t+1)) = \frac{D_{i,t+1} - C_i^L(s(t+1))}{1 + r(1-\tau)} \tag{A.3}$$

Because state-contingent debt can be directly expressed as the combination in (A.3) of state-uncontingent debt and the credit line, the replicating strategy is trivially budget feasible at time $t+1$. The resource constraint at time $t$ is also unchanged, because

$$w_{i,t} + L_{i,t} \geq e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$

can be rewritten as

$$w_{i,t} + E_t[b_i(s(t+1))] \geq e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$

using (30) and (A.3). Finally, we shall show that the limits for the feasible set for $b_i(s(t+1))$ implied by collateral and debt positivity constraints are preserved by the replicating portfolio of debt and lines of credit, namely that:

$$0 \leq \frac{D_{i,t+1} - C_i^L(s(t))}{1 + r(1-\tau)} \leq \theta(1-\delta)k_{i,t+1}$$

The debt positivity constraints can be rewritten as:

$$C_i^L(s(t+1)) \leq (1 + r(1-\tau))E_t[b(s(t+1))]$$

which is consistent with the feasible set for $C_i^L(s(t+1))$ because:

$$\max C_i^L(s(t+1)) = \overline{C_{i,t+1}}$$

∎