Housing Markets with Construction, Screening, and Focused Search

Joseph Williams

Professors Capital
Williams@ProfessorsCapital.com

January 2014

Existing homes are sold at lower average prices than comparable new homes. Buyers search less intensively among existing homes than new housing projects, but buy more frequently after initial inspections. These and other properties of housing search are predicted in this paper by a model with the following properties. Buyers screen listings on electronic sites and search only among acceptable matches, controlling both the focus and intensity of their searches. Sellers of existing homes negotiate with buyers, whereas builders of new housing do not. Only new homes can be customized for buyers. Partial equilibria are calculated explicitly. Steady state is characterized for a housing market with entry by builders in some submarkets.

Key Words: Housing markets and construction, focused search

This paper was previously titled: Housing Prices, Sales, and Search with Construction. It has benefited from comments by participants in the real estate conference at U. Toronto Rotman in July 2013. I am extremely grateful to Peter Chinloy for many helpful conversations over many years. All errors are mine.

Housing Markets with Construction, Screening, and Focused Search

In large, modern markets, sellers advertise their homes for sale on public Web sites, but do not search for buyers. Buyers screen houses on those sites and then search only among acceptable homes. This truncation affects their distribution of acceptable matches. Also, buyers' search is not entirely random. Instead, buyers can focus their search on housing in distinguishable categories–for example, existing versus new homes in specific neighborhoods. Finally, buyers can control their intensities of search, spending more time and effort searching if, for example, they have more competition from other buyers.

Buyers focus their search, or not, on either existing or new housing depending on the differences between the two. Existing homes are sold largely as-is, subject only to repairs of minor deficiencies discovered during due-diligence. New homes currently under construction come with options. Buyers initially choose among lots and models in single-family subdivisions or floors and floor plans in multifamily condominiums. Later, they select fixtures and finishes. Thereby, buyers maximize the values of their matches, which, in turn, improves their efficiency of search. These options are costly. Builders bargain over prices far less frequently than sellers of existing homes. Also, builders commonly charge high prices with big margins on customized upgrades.

The above characteristics of housing markets are shown in this paper to affect housing prices, sales, and search. With initial screening by buyers of homes for sale, the truncated distribution of acceptable matches can be approximated asymptotically by a power-law distribution. This approximation, combined with an isoelastic opportunity cost of search, makes possible explicit, analytic solutions in partial equilibria, and thereby nearly explicit solutions in steady states, where values of all state variables are fixed or constant. In this paper the partial equilibria with explicit solutions have focused search with endogenous intensities, customization of new homes, Nash bargaining between buyers and sellers of existing homes, and no bargaining by builders of new homes. This contrasts with previous papers where analytic solutions for even the most basic search models without the above complications require strongly stylized assumptions: uniformly distributed match values or two-point distributions. In that literature solutions are frequently numerical with normally distributed match values. Normal distributions are poor approximations in models with screening and the resulting truncation of match values.

The principal properties of partial equilibria with construction follow from focused search combined with customization of new homes. Because buyers can distinguish from a distance existing from new homes, they focus their search on the type of housing with greater expected gains. In equilibrium they collectively search among both types, existing and new, if and only if they expect equal gains from trade with sellers of each. This condition constrains search, sales, and prices. For example, the expected gain from buying new housing is greater when buyers are offered more alternatives or options. Therefore, options to customize must be reflected in higher prices for new homes than otherwise comparable existing homes in any partial equilibrium with search among both types of housing.

Next, the properties of partial equilibria are used to identify properties of steady state for a set of submarkets. The best example is a metropolitan area with different neighborhoods, school districts, and suburbs, all with different commuting times to different clusters of employment. These submarkets are ordered on a continuum by the exogenous component of the average arrival rate of buyers relative to sellers of existing homes. Buyers enter with some inelasticity their most preferred markets. Sellers of existing homes enter their submarkets with the same constant elasticity. Builders, by contrast, develop with perfect elasticity in all submarkets where the value of new homes exceeds or equals the cost of construction. This generates a steady state with the following properties. Submarkets with more rapid relative entry by buyers have higher ratios of buyers to sellers of existing homes and, among markets with construction, lower ratios of buyers to new housing projects. Builders enter submarkets only with sufficiently rapid relative entry by buyers. Thereby, steady state has a range of ratios to which the previous properties of partial equilibrium can be applied. The properties of steady state are then used to calibrate the parameters of the power-law distribution for buyers' match values. That calibrated distribution has a much fatter tail than a normal distribution.

Partial equilibria and thereby steady state have additional properties. A new home sells at a higher price than the average price of otherwise comparable existing homes. Buyers search more intensively across developments of new homes than among listings of existing homes, but buy less frequently following their initial inspections. Both the premium paid and relative frequency of sales are greater with more options to customize. In this sense, the differences between new and existing homes are least for entry-level production homes and greatest for luxury custom homes.

The literature on housing search is lengthy. Examples include Wheaton (1990), Yavas (1992), Williams (1995), Arnold (1999), Krainer (2001), Novy-Marx (2009), Diaz and Jurez (2010), Carillo (2012), and Head, Lloyd-Ellis, and Sun (2012). Previous papers ignore either construction or its critical characteristics, customization and less bargaining, that distinguish it from existing housing. Screening that generates a power-law approximation of the distribution for acceptable matches appears only in Williams (2013). Only the latter paper has focused search driven by buyers' expected gains from trade, but with a different distinction: less and more motivated buyers can focus their searches on less or more motivated sellers. Finally, much of the recent literature on housing search relies on properties of two-sided search by both buyers and sellers. As argued in the next section, residential search is more accurately modeled as one-sided search by buyers only.

Housing search focused on submarkets also appears in the concurrent empirical paper: Piazzesi, Schneider, and Stroebel (2013). There, buyers have preferences over submarkets distinguished by the location, quality, and size of existing homes. Entry rates of buyers and sellers are shown to differ across submarkets with realistic consequences for turnover, inventory, time on the market, and discounts for illiquidity. Also related are recent theoretical papers on search for existing homes directed by sellers' listing prices: Arbrecht, Gautier, and Vroman (2010); and Han and Strange (2013). As shown there, listing prices can signal sellers' types even though sellers need not honor their listing prices when bidding wars between buyers generate higher offers.

The paper is organized as follows. The model is first motivated and then presented formally in the next two sections. In sections four and five, partial equilibrium is characterized and its main properties are identified. Steady state is characterized in the sixth section. Calibrated numerical results are presented in the seventh section. Empirical implications are described in the subsequent section and the main results are summarized in the final section. All derivations appear in the Appendix.

Motivation

The major assumptions of the model are motivated in this section. The assumptions include exogenous advertising by sellers of existing homes on electronic sites, screening by buyers of homes on those sites, and subsequent search by buyers only among acceptable houses. Buyers can control both the focus and intensity of their searches, subject to isoelastic opportunity costs. Existing homes are sold as-is with Nash bargaining between buyers and sellers. New homes are sold by identical builders, each with a fixed number of alternative, equally costly options at one fixed price for all options. That price maximizes each builder's expected gain from trade with a matched buyer. As argued below, these assumptions are analytically convenient approximations of common practice and institutions in large metropolitan markets for residential housing within the United States.

Search Only by Buyers: In large residential markets buyers search and sellers advertise. Buyers do not advertise and sellers do not search because their counter-parties can do so much more efficiently. Sellers of existing homes post on electronic sites both photos and written information available from public records.[1] Buyers can easily find that information, but cannot or do not disclose with similar precision on public sites their preferences or private information. Also, sellers cannot identify potential matches among a large population of almost entirely anonymous, infrequent buyers.[2] This anonymity of buyers distinguishes residential from commercial real estate. With the commercial real estate, potential buyers are easily identified among owners of comparable properties and frequently contacted by

[1] Listing agents often limit their potential liablility by posting only information that they acquire directly from public sources, such as tax assessors' Web sites.

[2] About 9% of all homes are now sold without the assistance of real estate agents. Among these sellers without agents, 40% know their buyers: NAR (2013).

listing agents. Finally, sellers of homes and their listing agents are less likely to contact buyers with common contractual constraints designed to minimize conflicts of interest.[3]

Advertising by sellers of existing homes is essentially exogenous. In practice, the vast majority of sellers of existing homes hire listing agents who within a few days must list their clients' properties on the local multiple listing service (MLS).[4] Buyers have access to listings through affiliated electronic sites or their real estate agents.[5] Because these public sites have wide reach, very low marginal costs, and limited formats, advertising to capacity constraints is almost nondiscretionary. Other, more costly marketing, like open houses, can be motivated in part by issues largely unrelated to advertising for sellers.[6] Under these conditions advertising is effectively exogenous and nearly identical for all sellers of existing homes. This advertising improves buyers' efficiency of search equally for all buyers and all sellers.

Power-Law Approximation: A large majority of buyers screen listings on electronic sites or receive similar services from their agents.[7] Thereby, buyers learn about the local housing market, which reduces their incentive to search with recall. Buyers also assess their matches with houses and eliminate unacceptable houses from their subsequent searches. In practice, this process of screening can be complicated. Noisy signals of match values are first observed on electronic sites with different precision by different buyers and then refined during initial, short, site visits. Here, this potentially complicated process is modeled simply as an exogenous truncation below of buyers' continuous distribution of match values with sellers' houses. This exogenous truncation can be interpreted as the solution to an unmodeled control problem in which tighter screens increase the expected value of search but impose costs on buyers and their agents. Buyers' distribution of match values is continuous or approximately so with many mass points because the summary statistic for buyers' match values is a composite of multiple attributes, each with multiple alternatives. In this context, buyers' distribution of match values, truncated by their initial screening, can be approximated by a power-law distribution.

[3] Listing agents have a fiduciary duty to their clients to present all offers. Also, they can act as dual agents only with their clients' written authorization. This restriction frequently extends to affiliated agents. Sellers also have limited incentive to find buyers because listing agreements commonly entitle listing agents to real estate commissions from buyers procured by sellers during listing periods.

[4] About 88% of all home sellers are now assisted by real estate agents: NAR (2013). Sellers can now hire at low cost listing agents with access to MLS. In return for lower commissions, discount brokerage firms offer limited services: access to MLS and assistance with contracts and closings. Sellers can avoid even the low listing fees of discount brokers by advertising directly on other electronic sites, like zillow.com, at very low cost.

[5] Realtors advertise their listings most frequently on realtor.com, their brokers' websites, and trulia.com: NAR (2013).

[6] Open houses are often run by junior members of listing agents' teams, who as buyers' agents, are motivated to find clients from among the visitors without agents. Open houses are also evidence to sellers that their listing agents are making their best efforts to sell their homes.

[7] The Internet and real estate agents are used respectively by 92% and 88% of buyers: NAR (2013).

Power-law approximations are motivated as follows. With economic and financial data, power-law approximations fit best in the upper tails of distributions, whereas other common approximations, including normals and lognormals, fit best in the bodies: Gabaix (2009). Also, all common continuous distributions used in economics and finance, again including normals and lognormals, can be approximated asymptotically in their upper tails by power-law distributions: Gabaix (2009). Finally, power-law approximations permit explicit analytical solutions for not only standard search equilibria, including those with endogenous search intensities, but also the extensions in this paper to focused search, customization only of new homes, and bargaining only by sellers of existing homes. These explicit solutions make it possible to decipher the complicated economic interactions between the many endogenous variables in search equilibria.

Power-law approximations should also be assessed in the context of the alternatives: numerical solutions with normal and lognormal distributions and analytic solutions with uniform distributions and discrete distributions with two possible match values, acceptable and unacceptable. Normals and lognormals are also approximations, best without truncations after initial screening on electronic sites. Uniform and two-point distributions have only one advantage: analytical convenience. Two-point distributions are not only stylized but also inconsistent with physical search after screening on electronic sites.

Controlled Search: In practice, buyers can focus their search on either existing homes or new homes or neither, all within specific neighborhoods. This focus is possible with truncated search because listings of existing homes and models within housing projects can be identified separately on public Web sites. Moreover, it can be done by jurisdiction or zip code. Also, buyers can control concurrently their intensity of search, subject to personal costs. In the model each buyer can control both the focus and intensity of her search subject to isoelastic opportunity costs. This power function is an analytical convenience. When combined with power-law matches, it makes possible an explicit solution even with controlled search. Focused search is a major contributor to the subsequent results.

Customization: Much like other multi-attributed products for heterogeneous consumers, new homes are commonly customized for buyers–more so for more expensive homes. Here, customized features include all alternatives offered by builders that allow buyers to maximize the value of their matches with their new homes: lots and models in single-family subdivisions, floors and floor plans in multifamily condominiums, and fixtures and finishes in both. For luxury homes in subdivisions or high-rises, builders can offer thousands of alternative upgrades, all supported by elaborate design studios staffed with professional designers. Those choices can add up to 30% to the base price of homes: WSJ (2013). Lots and models or floors and floor plans are selected when purchase and sale agreements are signed. Fixtures and finishes are selected, often months later, when buyers' homes are nearing completion.

In this paper the customization of new homes is modeled as simply as possible. The number of alternative customizations offered by each identical builder is exogenous. When a buyer arrives, she immediately observes all alteratives offered by the builder and focuses on her best match. The distribution of that best match value is also approximated by a power-law, but with a larger common or mean value that is increasing in the fixed number of customizations. The buyer either accepts the builder's price for that best match or resumes her search elsewhere.

Bargaining: In practice, buyers and sellers of existing homes bargain: Merlo and Ortalo-Magne (2004). In the model prices of existing homes are determined by Nash bargaining between matched buyers and sellers under complete information. This assumption is selected over its best alternative–a competitive search equilibrium–for the following reason. With competitive search the buyer's share of the surplus from trade equals the elasticity of the seller's matching function: Hoisos (1990) and Moen (1997). That elasticity equals one with one-sided search by buyers, so that buyers would take all the surplus. With nearly one-sided search, buyers would receive almost all of the surplus. In such situations, sellers would have little or no incentive to participate in the housing market. Extremely asymmetric allocations of surplus are not easily explained in steady state.

In practice, builders bargain with buyers far less frequently than sellers of existing homes. Unlike sellers of existing homes, builders have multiple housing units for sale in the same development. A discount negotiated with one buyer affects subsequent appraisals of comparable homes and thereby future buyers' ability and willingness to pay. Because those buyers may then demand similar discounts, builders disguise their discounts by offering, instead, free upgrades and no closing costs.[8] Builders also advertise aggressively competitive prices for basic models, but rarely report their relatively high prices for customized upgrades: WSJ (2013). Thereby, builders extract extra profits from matched buyers.[9] Excess profits for small, marginal builders are then eliminated by entry.[10] This combination of competitive, posted prices for basic products and obscured or shrouded prices for ancillary products or services is common in many industries. It can reflect an inability to commit through cost-effective advertising to a complicated mix of customized services in markets with non-repetitive, one-time buyers. Alternatively, it can reflect sellers' incentives to exploit myopic buyers who cannot correctly anticipate sellers' high prices for customized services. In markets with both rational and myopic buyers, it can be the only equilibrium: Gabaix and Laibson (2006). Shrouded prices are discussed in a subsequent section.

The above behavior by builders is approximated in the model as follows. Builders do not bargain. Instead, each builder quotes the same price for each customized home to each buyer who arrives at the builder's housing project. That price maximizes the builder's expected gain from trade in a symmetric Nash equilibrium before the builder observes the buyer's best

[8] This behavior by builders is reported by real estate agents in multiple Web sites identified under under the topic: negotiating with builders.

[9] Builders' margins on customized upgrades can be as high as 60%: WSJ (2013).

[10] This competition is consistent with the large fraction of small builders among all home builders and their rapid rates of entry and exit: NAHB (2010). Very large home builders have lower costs: Porter (2003).

match value. This simple pricing rule captures key elements of common practice. Each builder can compete by advertising the prices of its few basic models but not its complex mix of customized upgrades. Because the builder cannot commit to the latter prices, it acts opportunistically and exploits its local monopoly power when one-time buyers arrive at its housing project. One price is quoted by all builders for all customized alternatives only because, as an analytical convenience, all builders are identical and all customized homes are equally valuable ex-ante to all buyers. In the model all excess profits from new homes are then eliminated by builders' perfectly elastic entry in all submarkets where the value of entry is not less than the cost of entry.

Submarkets: A metropolitan housing market can have multiple submarkets distinguished by their local ratios of buyers to listings of existing homes, buyers to new housing projects, and thereby projects to listings. In a metropolitan area that grows from its core outwards, more central areas should have on average fewer new housing projects, measured relative to listings of existing houses, than more peripheral areas. An example–metropolitan Phoenix in mid 2013–appears in Table 1. As indicated, the ratio of projects to listings is lowest in the city, higher in the inner suburbs, highest in the outer suburbs, and high in the exurbs. This example motivates a model of steady state with a range of two ratios: buyers to listings and buyers to projects. Because all steady states are partial equilibria in which all state variables are constant, the explicit solution in this paper for partial equilibria can then be applied immediately to steady state in a market with a range of ratios across its submarkets.

Model

The housing market has buyers, sellers, and builders. Each buyer searches for one home to purchase for personal occupancy. Each seller offers one existing home for sale. Each builder of new homes has one housing development: a single-family subdivision or a multifamily condominium. With these expositional simplifications, sellers of existing and new homes are identified respectively by the index: $i = 0, 1$. The masses of buyers, $b > 0$, sellers, $s_0 > 0$, and builders, $s_1 > 0$, are measured relative to the stock of existing homes. Buyers and sellers of existing and new homes have the respective ratios: $r_i = b/s_i > 0$ for $i = 0, 1$. In the subsequent text these ratios are frequently identified as ratios of buyers to listings and buyers to builders. All notation is summarized in Table 2.

All search is one-sided. As explained in the previous section, buyers search while builders and other sellers advertise. Because that advertising is effectively exogenous, it is ignored in the subsequent analysis. Buyers search continuously in independent Poisson processes. For each buyer of housing i, this search has the intensity q_i. Given continuous search by buyers only, the aggregate matching rate between buyers and sellers of housing i must be proportional to the mass of buyers b and independent of the mass of sellers s_i. Thereby, each buyer of housing i meets sellers at the average rate q_i. In turn, each seller of housing

i is met by buyers at the average rate $q_i r_i$. These and all similarly identified relationships hold for both types of homes: $i = 0, 1$.

Each buyer controls both the focus and intensity of her search. She can split her search between existing and new homes: $q_i \geq 0$ for $i = 0, 1$. In so doing, she expends on search per unit of time the total effort: $q = q_0 + q_1$. Her personal effort has per unit of time the opportunity cost: γq^δ with the constants, $\gamma > 0$ and $\delta > 1$. This isoelastic cost, combined with the power-law matches identified below, makes possible the explicit, analytic solutions in this paper.

Before buyers visit houses for sale, they screen those houses as described in the previous section. This powerful preliminary screen eliminates all houses with values of matches x below a large lower bound: $\zeta_0 > 0$. Each buyer then visits houses for sale drawn independently from the truncated distribution F. This truncated distribution is calculated from the continuous distribution of match values without preliminary screening, which is assumed to have a tail index and thereby an asymptotic power-law approximation on its upper tail: $F(x) \approx 1 - (\zeta_0/x)^\eta$ for all $x > \zeta_0$ with $2 \leq \eta < \infty$. The common value of all screened houses is the mean of the truncated distribution: $\mu_0 = E(x) = \zeta_0 \eta/(\eta - 1) > 0$. Because buyers are self-selected, this truncated mean is greater than the corresponding common value to all buyers. The associated idiosyncratic value of the match x/μ_0 is an independent power-law variate with the mean: $E(x/\mu_0) = 1$. This power-law approximation $x^{-\eta}$ has fatter tails with smaller exponents η or, equivalently, larger tail indices $1/\eta$. The lower bound, $1/\eta > 0$, which precludes distributions with the thinnest tails, like normals and lognormals, simplifies the subsequent exposition.

New housing can also be customized for buyers as described in the previous section. Each identical builder offers to each buyer a fixed number, $\nu \geq 2$, of mutually exclusive, equally costly customizations. Each buyer then has a maximum match value: $x^* = \max(x_1, \ldots, x_\nu)$. The match values, $x_1, ..., x_\nu$, are drawn independently from the above truncated distribution F for the match values of existing houses. Each customized house is equally costly to build. The total cost per house κ is the sum of two constant unit costs: its starting cost κ_0 plus its finishing cost κ_1. For expositional simplicity the core of the house–foundation, framing, and such–must be constructed before the property is marketed for sale. The house is then finished when a contract is signed and customized upgrades are selected by the buyer.[11] To preclude peripheral complications, time to build is ignored.

Buyers search randomly among homes within their preferred categories that have survived their initial screens. Prices of existing homes are determined by Nash bargaining between matched buyers and sellers under complete information. One price for each, equally costly new home is quoted to all prospective buyers by each identical builder. That price maximizes

[11] In the United States during 2012, among new houses put into contract for sale with a deposit from the buyer, 30.4% had not been started, 34.0% were under construction, and 35.6% had been completed: census.gov.

each builder's expected gains from sales to matched buyers in a symmetric Nash equilibrium. This behavior by builders is motivated in the previous section. After transactions successful buyers and sellers of existing homes immediately leave the housing market. All new homes are delivered immediately to their buyers.

All buyers, sellers, and builders are risk-neutral over their possibly infinite horizons. Each has the same constant discount rate: $\iota > 0$. Each buyer always acts to maximize the expected value of her continued search v^B. Each seller of a home of type i waits for buyers and thereby realizes the expected present value v_i^S. These present values are identified in the next section as solutions to Bellman equations. As explained there, the present value for each builder v_1^S includes its deposit κ_1 for the subsequent completion of customized upgrades.

Partial Equilibrium

Existing Homes: Owners list their homes for sale after their matches with the houses break. Henceforth, sellers have no value in use for their homes. When a buyer arrives, both she and the seller observe her match with the house. The buyer and seller then negotiate a price under complete information that splits the total transactional surplus: $x - v^B - v_0^S$. With Nash bargaining under complete information, the buyer receives a constant share, $0 < \beta_0 < 1$, of that surplus and pays the price:

$$P_0(x) = (1-\beta_0)\big(x - v^B\big) + \beta_0 v_0^S. \tag{1}$$

The house is sold if and only if the surplus is positive, $x > x_0$, with the reservation value:

$$x_0 \equiv v^B + v_0^S. \tag{2}$$

The probability of a transaction conditional on a match is approximately power-law or Pareto:

$$y_0 \equiv \Pr\{x > x_0\} \approx \left(\frac{\zeta_0}{x_0}\right)^{\eta}, \tag{3}$$

with $\zeta_0 > 0$.

A matched buyer and seller expect the total gain from trade:

$$z_0 \equiv y_0 E(x - x_0 | x > x_0) \approx \frac{\zeta_0}{\eta - 1}\left(\frac{\zeta_0}{x_0}\right)^{\eta-1}. \tag{4}$$

This total gain is split between the buyer and seller by the Nash bargaining price (1). Thereby, the matched buyer expects the gain:

$$z_0^B \equiv y_0\big\{E[x - P_0(x)|x > x_0] - v^B\big\} = \beta_0 z_0. \tag{5}$$

Similarly, the seller expects the gain:

$$z_0^S \equiv y_0\{E[P_0(x)|x > x_0] - v_0^S\} = (1-\beta_0)z_0. \tag{6}$$

New Housing: Under the above assumptions the distribution of the maximum match, $x^* = \max(x_1, ..., x_\nu)$, is also asymptotically power-law with the same tail index $1/\eta$: Gabaix (2009), page 259. In this situation, a matched buyer and builder trade with the approximate probability:

$$y_1 \equiv \Pr\{x^* > x_1\} \approx \left(\frac{\zeta_1}{x_1}\right)^\eta. \tag{7}$$

for all $x_1 > \zeta_1 > 0$. As shown in the Appendix, the scaling parameter ζ_1 has the asymptotic approximation: $\zeta_1 \approx \nu^{1/\eta}\zeta_0$. Thereby, the common value of new housing, $\mu_1 = E(x^*) = \zeta_1\eta/(\eta-1)$, has the same property: $\mu_1 \approx \nu^{1/\eta}\mu_0$. This truncated mean μ_1 is increasing in the number of customizations ν at the rate $1/\eta\nu$.

With equally costly customizations and ex-ante identical buyers, each identical builder quotes the same price to all buyers. With one price for all new homes, all buyers of new homes search randomly among new homes that survive their initial screens on electronic sites. When a buyer arrives at a builder's home, she observes the values of her ν matches and thereby the maximum value x^*. She then buys from the builder her best match if its value exceeds her reservation value: $x^* > x_1 = p + v^B$. As indicated, the buyer's reservation value x_1 minus the builder's price of a new house p must equal the buyer's foregone expected present value of continued search v^B.

When a potential buyer arrives at a new development, she either buys or not at the builder's quoted price. As previously assumed, that asking price p_1 maximizes the builder's expected gain from trade with a matched buyer:

$$p_1 \approx \arg\max_{p\geq 0}\left\{\left(\frac{\zeta_1}{p+v^B}\right)^\eta (p - v_1^S)\right\} = \frac{v^B + \eta v_1^S}{\eta - 1}. \tag{8}$$

The maximand in (8) reflects the probability of trade in (7) with the buyer's reservation value: $p + v^B$. It also reflects the builder's net gain from a transaction: $p - v_1^S$. Here, the expected present value of future search v_1^S is defined to include the cost of completion κ_1. This definition is driven by the following assumption. When a builder starts a new house it must deposit sufficient funds to guarantee its completion. This deposit κ_1 can be interpreted as a requirement by the builder's unmodeled construction lender, which can be covered with a performance bond. When the house is customized and sold, the builder relinquishes both the deposit κ_1 and present value of future search $v_1^S - \kappa_1$. The assumption is required for an explicit solution in partial equilibrium. The somewhat awkward notation simplifies substantially the subsequent exposition by preserving the symmetry in notation between new houses with finishing costs and existing houses without those costs.

Given the price (8), each buyer of a new home has the reservation value:

$$x_1 = p_1 + v^B \approx \frac{\eta}{\eta-1}\left(v^B + v_1^S\right). \tag{9}$$

This reservation value is linear homogeneous in the expected present value of future search by the two counterparties, buyer and builder, as is corresponding reservation value (2) for existing homes. This linear homogeneity in both (2) and (9), combined with the above deposit by builders, makes possible the subsequent explicit solution of the partial equilibrium with both existing and new homes.

The expected gains from trade for both buyers and sellers are affected by both the power-law approximation (7) and the buyer's reservation value in (9). Thereby, each buyer matched to a builder expects the gain from trade:

$$z_1^B \equiv y_1 E(x^* - x_1 | x^* > x_1) \approx \frac{\zeta_1}{\eta-1}\left(\frac{\zeta_1}{x_1}\right)^{\eta-1}. \tag{10}$$

This is analogous to (4). Also, each matched builder expects in (8) with (9) the gain from trade:

$$z_1^S \approx \frac{\zeta_1}{\eta}\left(\frac{\zeta_1}{x_1}\right)^{\eta-1}. \tag{11}$$

The total expected gain from trade equals (10) plus (11). Of this total the buyer receives the fraction: $\beta_1 \equiv \eta/(2\eta-1)$. The buyer's share β_1 increases in the tail index $1/\eta$ and exceeds $1/2$ with the tail index, $1/\eta > 0$.

Search: Each buyer has the present value of continued search:

$$v^B = \max_{q_0, q_1 \geq 0} e^{-\iota \Delta t}\left\{v^B + \left[\sum_{i=0}^{1} q_i z_i^B - \gamma(q_0 + q_1)^\delta\right]\Delta t\right\} + o^2.$$

As indicated, the buyer can split her search between existing and new houses: $i = 0, 1$. If she searches among houses of type i with the intensity q_i, she meets one seller of type i during the next short interval of time Δt with the approximate probability $q_i \Delta t$ and two or more sellers of the same type with a much smaller probability o^2. Here, o^2 represents terms of order $(\Delta t)^2$. If the buyer meets some seller i, she expects the gain from trade: z_i^B. She then purchases one house i over the short interval of time Δt with the probability: $q_i y_i \Delta t + o^2$. As indicated, the buyer can allocate her effort among both types of sellers thereby incurring the total current cost from search of $\gamma(q_0+q_1)^\delta \Delta t$. The future value of this search at time $t + \Delta t$ is discounted to the present, time t, at the instantaneous discount rate ι.

The above recursion is simplified as follows. Construct the statistic: $z^B \equiv \max\left(z_0^B, z_1^B\right)$. Let $\Delta t \to 0$ and reorganize terms. This yields

$$\begin{aligned}\iota v^B &= \max_{q_0, q_1 \geq 0} \left\{ \sum_{i=0}^{1} q_i z_i^B - \gamma (q_0 + q_1)^\delta : q_0 + q_1 = q \right\} \\ &= \max_{q \geq 0} \left\{ q z^B - \gamma q^\delta \right\} \\ &= \gamma(\delta - 1) \left(\frac{z^B}{\gamma\delta} \right)^{\delta/(\delta-1)}. \end{aligned} \tag{12}$$

In (12) the buyer's problem is decomposed into two components. First, she splits her total search q between existing and new homes: $q_0 + q_1 = q$. That split maximizes the buyer's payoff, $z^B \equiv \max\left(z_0^B, z_1^B\right)$, subject to her total allocation q. Next, she optimizes her total search q by equalizing marginal benefits and costs.

This decomposition produces the following results. From the second equality in (12), the buyer searches only among houses i with the larger expected gain from trade:

$$\begin{array}{ll} q_0 = 0 & \text{if} \;\; z_0^B < z_1^B \\ q_0 \in [0, q] & \text{if} \;\; z_0^B = z_1^B \\ q_0 = q & \text{if} \;\; z_0^B > z_1^B, \end{array} \tag{13}$$

and $q_1 = q - q_0$. If the two alternatives have equal expected gains, then she allocates her efforts randomly among both. With the third equality in (12), the buyer maximizes her expected gain from trade minus her cost of effort. Thereby, she searches with the aggregate intensity:

$$q = \left(\frac{z^B}{\gamma\delta} \right)^{1/(\delta-1)}. \tag{14}$$

Again, each builder must make an initial deposit sufficient to cover the final costs of completing its house. By assumption, this deposit earns no interest. The lost interest can be interpreted as the cost of a commercial completion or performance bond. In this case, the seller of each type of home i has the expected present value of continued search:

$$v_i^S = e^{-\iota \Delta t}\left(v_i^S + q_i r_i z_i^S \Delta t\right) + o^2,$$

for $i = 1, 2$. During each small interval of time Δt, one buyer arrives with the probability $q_i r_i \Delta t + o^2$. This arrival rate depends on the ratio r_i of buyers to sellers of type i. Again, more than one buyer arrives concurrently with a much smaller probability o^2. If a buyer arrives, seller i expects the gain from trade z_i^S. The seller then sells his house i with the probability $q_i r_i \Delta t + o^2$. Again, this future payoff is discounted to the present at the instantaneous rate ι. Next, define the weights: $w_i \equiv q_i/q$ with $w_0 + w_1 = 1$. Again, let $\Delta t \to 0$ and reorganize terms. This yields

$$\iota v_i^S = q r_i w_i z_i^S. \tag{15}$$

These results hold for both types of sellers, $i = 0, 1$. The weights, w_0 and w_1, play a big part in the subsequent analysis.

Partial Equilibrium: A partial equilibrium in the housing market is a set of arrival rates q_i, pricing functions, P_0 and p_1, expected values of continued search, v^B and v_i^S, reservation values of matches x_i, probabilities of transactions conditional on matches y_i, and expected gains from trade, z_i^B and z_i^S, for both types of housing, $i = 0, 1$, that satisfy conditions (1) through (15) above. In this partial equilibrium the fractions of houses for sale s_i and ratios of buyers to sellers r_i are exogenous for both types of housing, $i = 0, 1$.

Properties of Partial Equilibrium

The previous problem, (1) through (15), is solved explicitly in the Appendix and its principal properties are presented in this section. For expositional ease construct the composite constant:

$$\omega \equiv \frac{\eta}{\eta - 1}\left(\frac{\beta_0}{\nu}\right)^{1/(\eta-1)} > 0. \tag{16}$$

with $0 < \beta_0 < 1$ and $2 \leq \eta, \nu < \infty$, as previously assumed. In this case, the parameter ω is everywhere decreasing in the number of customizations ν with the lower limit: $\omega \to 0$ as $\nu \to \infty$. It also satisfies the inequality: $\omega < 1$. Details appear in the Appendix.

The possible partial equilibria can now be identified. Define the weighting function:

$$W(r_0, r_1) \equiv \frac{\frac{1-\beta_0}{\beta_0} r_0 + \frac{\delta-1}{\delta}(1-\omega)}{\frac{1-\beta_0}{\beta_0} r_0 + \frac{1-\beta_1}{\beta_1}\omega r_1} > 0, \tag{17}$$

for all $r_0, r_1 > 0$. On this region the function W is decreasing everywhere in the ratio of buyers to builders r_1 with the property: $W(r_0, \rho_1) = 1$ at $\rho_1 \equiv \frac{\beta_1}{1-\beta_1}\frac{\delta-1}{\delta}\frac{1-\omega}{\omega} > 0$. As stated in the first proposition, the value of (17) determines the type of partial equilibrium.

Proposition 1: *With (1) through (15), buyers search either among both types of homes,* $0 < w_1 < 1$, *or only new homes,* $w_1 = 1$,

$$w_1 = \min\{1, W(r_0, r_1)\}, \tag{18}$$

with $w_0 = 1 - w_1$ *for all* $r_0, r_1 > 0$. *The type of equilibrium depends only on the ratio of buyers to builders:* $w_1 = 1$ *if and only if* $0 < r_1 \leq \rho_1$.

Proposition 1 is derived in the Appendix as follows. Buyers search in (14) among both types of homes, existing and new, $0 < w_1 < 1$, if and only if they expect equal gains from the two types: $z_0^B = z_1^B$. With (5) and (10) buyers expect equal gains from both types if and only if buyers' search for existing homes has in (18) the aggregate allocation or weight:

$w_1 = W(r_0, r_1)$. Similarly, buyers search only among new homes, $w_1 = 1$, if they expect the gains: $z_0^B < z_1^B$. With (5) and (10) these expectations require the aggregate allocation or weight: $w_1 < W(r_0, r_1)$ The latter inequality requires a ratio of buyers to builders: $0 \leq r_1 < \rho_1$. This ratio is feasible because $0 < \omega < 1$. Finally, buyers search only among existing homes if they expect the gains: $z_0^B > z_1^B$. With (5) and (10) the later expectations require the aggregate allocation or weight: $w_1 > W(r_0, r_1)$ In turn, this inequality requires the ratio of buyers to listings: $0 \leq r_0 < \frac{\beta_0}{1-\beta_0} \frac{\delta-1}{\delta} (\omega - 1)$. The latter constraint cannot be satisfied because $0 < \omega < 1$.

The above results have the following intuitive interpretation. At the reservation value x_0, buyers and sellers of existing homes are indifferent between transactions and continued search. At the reservation value x_1, buyers of new houses are also indifferent between transactions and continued search. Each reservation value x_i is higher with more rapid average arrival of buyers qw_i, in which case buyers' expected gain z_i^B is lower in (5) or (10). Because the two weights must sum to one, $w_0 + w_1 = 1$, the expected gains are then equalized, $z_0^B = z_1^B$, at an intermediate value w_1, which can only be (17). That intermediate value is feasible, $0 < w_1 < 1$, only with ratios of buyers to builders r_1 on the upper interval, $r_1 > \rho_1$.

In the analysis below buyers are assumed to search among both types of homes if both types are offered for sale. No search among new homes, $w_1 = 0$, is precluded in Proposition 1 by the sufficient but not necessary conditions: $\eta, \nu \geq 2$. No search among existing homes, $w_1 = 1$ in (18), is precluded in practice by large ratios r_1, measured as buyers to builders, buyers to new housing projects, or even buyers to distinct housing alternatives–mostly combinations of models and lots–offered by buyers across all projects. The latter, lowest ratio is used in the subsequent numerical solution. That calculation is calibrated with the ratio of listings to distinct housing alternatives: $s_0/s_1 = 8$. Standard models of housing markets with only existing homes have in steady state equal numbers of buyers and sellers: $r_0 = 1$. In this situation the ratio of buyers to distinct housing alternatives has the value: $r_1 = r_1/r_0 = s_0/s_1 = 8$. Even this low value greatly exceeds the calibrated lower bound, $\rho_1 = 0.812$, at which buyers begin to search only among new homes. For this reason the subsequent analysis is restricted to ratios, $r_1 > \rho_1$, at which buyers search among both existing and new houses: $0 < w_1 < 1$ in (18).

The arrival rate of buyers at existing homes relative to new housing projects follows immediately from Proposition 1. This relative arrival rate is the ratio:

$$\frac{q_0 r_0}{q_1 r_1} = \frac{1 - W(r_0, r_1)}{W(r_0, r_1)} \frac{r_0}{r_1}, \tag{19}$$

for $r_0 > 0$ and $r_1 \geq \rho_1 > 0$. The ratio (19) is decreasing in the number of customizations ν and increasing in the common value μ_0. It is less than one, $0 < q_0 r_0 < q_1 r_1$, under the following conditions. Builders offer a sufficient number of customizations ν such that $\omega < \frac{1-\beta_0}{\beta_0} \frac{\beta_1}{1-\beta_1}$. Since $\omega < 1$, this inequality is satisfied if buyers' expected share of the surplus from trade is no smaller with new homes: $0 < \beta_0 \leq \beta_1$. As indicated below (11), this inequality holds when, for example, buyers and sellers of existing homes have equal bargaining power: $\beta_0 = 1/2$. It holds in this situation because builders do not bargain with buyers, unlike sellers of existing homes. In this sense, more frequent arrival of buyers at new developments than existing homes follows from the two characteristics that distinguish existing from new homes: bargaining and customization.

When buyers and sellers meet, they either trade or not. Transactions occur with probability (3) for sellers of existing homes and probability (7) for builders of new homes. Given both types of transactions, all buyers must have in (13) the same expected gains: $z_0^B = z_1^B$. Divide (3) by (7); exploit the latter equality; and combine the result with (4), (5), and (10). This produces the ratio of probabilities:

$$\frac{y_0}{y_1} = \left(\frac{\nu}{\beta_0^{\eta}}\right)^{1/(\eta-1)} > 1, \tag{20}$$

for $r_0 > 0$ and $r_1 \geq \rho_1 > 0$. Both buyers and sellers of existing homes have at least some bargaining power: $0 < \beta_0 < 1$. With customization of new homes, $\nu \geq 2$, listers then sell to matched buyers more frequently than builders: $y_0 > y_1$. The ratio of probabilities (20) increases with the number of customizations ν.

The inequality in (20) reflects the two characteristics, bargaining and customization, that distinguish existing from new homes. Because builders do not bargain, they lose some transactions with positive total surplus that sellers of existing homes would make with their Nash bargains. This reduces buyers' efficiency of search for new houses. Customization also reduces the relative frequency of transactions between builders and matched buyers in (20), but for a different reason. It raises reservation values–more so for more customized homes. With more alternative customizations ν, the ratio of mean values μ_1/μ_0 is larger, but so is the ratio of reservation values x_1/x_0. The former effect reduces the ratio of probabilities (20), whereas the latter raises it. The latter effect must dominate the former to preserve in (5) and (10) buyers' equality of expected gains from a transaction, $z_0^B = z_1^B$, when buyers search in (13) among both existing and new houses.

The relative price of existing versus new homes is also determined in the partial equilibrium. In a very large market, the average price of existing homes equals the expected price: $\bar{p}_0 \equiv E\left[P_0(x)|x > x_0\right]$. As shown in the Appendix, the expected price of existing homes $\bar{p}_0$ is less than the price of new homes p_1.

Proposition 2: *With (1) through (15), the relative price $\bar{p}_0/p_1$ and relative value of continued search v_0^S/v_1^S satisfy the inequalities*:

$$\min\left\{(1-\beta_0)\omega, \frac{v_0^S}{v_1^S}\right\} \leq \frac{\bar{p}_0}{p_1} \leq \max\left\{(1-\beta_0)\omega, \frac{v_0^S}{v_1^S}\right\} < 1. \tag{21}$$

In (21) sellers of existing homes have relative to builders the value of continued search:

$$\frac{v_0^S}{v_1^S} = \frac{1-\beta_0}{\beta_0}\frac{\beta_1}{1-\beta_1}\frac{1-W(r_0,r_1)}{W(r_0,r_1)}\frac{r_0}{r_1} < 1. \tag{22}$$

The above holds for $r_0 > 0$ and $r_1 \geq \rho_1 > 0$.

With customization of new homes and no bargaining by builders, existing homes have lower average prices than otherwise comparable new homes. As shown in the Appendix, the relative price $\bar{p}_0/p_1$ is decreasing in the number of alternative customizations ν. Also, the expected value of continued search is less for sellers of existing homes than builders of new homes. Finally, the relative value of continued search, v_0^S/v_1^S in (22), is proportional to the relative arrival rate of buyers, $q_0 r_0/q_1 r_1$ in (19). The constant of proportionality is greater than one with the previously discussed buyers' shares of the surplus from trade: $0 < \beta_0 < \beta_1 < 0$.

Steady State

Henceforth, the housing market has a continuum of submarkets. This is an analytically convenient representation of a large metropolitan area with many local housing markets distinguished by jurisdictions, such as school districts, and commuting times to different clusters of employment. Each buyer enters her most preferred submarket. Each seller of an existing home enters the submarket surrounding his house. Only builders can enter multiple submarkets. The exogenous component of households' average entry rate is summarized by the variable: $a_0 \leq a \leq a_1$. As specified below, the parameter a affects the arrival rate of buyers relative to sellers of existing houses. Submarkets are distinguished only by the variable a and ordered by that variable. With this simple specification the exogenous average entry rate a is the metric for submarkets.

Different submarkets can have different ratios of buyers to listings and buyers to builders. As shown subsequently, each submarket a has in steady state a unique pair of ratios of buyers to both types of sellers: $R_0(a)$ and $R_1(a)$. With these ratios continued search in steady state has in submarket a for buyers and sellers of both types i the respective values, $V^B(a)$ and $V_i^S(a)$. These values are the previous solutions in partial equilibrium, v^B and v_i^S from (A.2) and (A.3), with the ratios, $r_i = R_i(a)$ for $i = 0, 1$.

Buyers and sellers of existing houses enter submarkets with some inelasticity. Measured per housing unit in the metropolitan area, the average rates of entry per unit of time by both buyers of all homes and sellers of existing homes are isoelastic functions of the values of entry: $aV^B(a)^\theta$ and $V_0^S(a)^\theta$ with the constant, $0 < \theta < \infty$. As in Novy-Marx (2009), this entry with equal elasticity θ by buyers and sellers of existing homes greatly simplifies the subsequent steady state. With $0 < \theta < \infty$, entry is neither perfectly inelastic nor perfectly elastic. Perfectly inelastic entry, $\theta = 0$, is the partial equilibrium of the previous section. In that situation the state variables, $B(a)$, $S_0(a)$, and $S_1(a)$, are exogenous constants. With perfectly elastic entry, $\theta = \infty$, the values of entry, $V^B(a)$ and $V_0^S(a)$, must equal in all submarkets the corresponding values of not entering. In the literature the latter values are specified as exogenous constants. Here, isoelastic entry is motivated by unmodeled costs and frictions that are heterogeneous across households. Alternatively, entry rates are isoelastic if entry rates are controlled subject to isoelastic costs.

Builders are different. Each identical builder can enter multiple submarkets. In each submarket it buys land and builds houses with constant returns to scale. In this case, both the cost and value of entry can be measured per new house. Each builder enters submarket a by starting a house at the constant unit cost κ_0 and depositing the unit completion cost κ_1. The value of that entry $V_1^S(a)$ is the builder's expected present value of waiting for a buyer plus the value of that deposit. The unit cost of entry, $\kappa = \kappa_0 + \kappa_1 > 0$, blocks building in all submarkets a with lesser values of entry: $0 < V_1^S(a) < \kappa$. With perfectly elastic entry by builders, their common unit value of entry must then equal their common unit cost, $V_1^S(a) = \kappa$, in any submarket a with construction. This relationship between construction and its cost holds for all submarkets: $a_0 \le a \le a_1$.

Buyers and sellers have the average rates of entry and exit indicated in Table 3. In the table these rates are measured relative to the stock of existing homes. With existing homes buyers and sellers exit in matched pairs immediately after transactions. With new homes sellers stay in the market and immediately start more houses. If $V_1^S(a) < \kappa$, then no new homes are built or sold in market a. Alternatively, if $V_1^S(a) = \kappa$, then any newly sold homes are immediately replaced with new homes for sale because no time is required to complete the core. Again, these properties hold for all submarkets: $a_0 \le a \le a_1$.

Steady state is a partial equilibrium, as defined below (15), combined with equal rates of entry and exit in Table 3. In steady state, the three state variables, $B(a)$, $S_0(a)$, and $S_1(a)$, and thereby the two ratios, $R_0(a)$ and $R_1(a)$, must be independent of calendar time for all submarkets, $a_0 \le a \le a_1$. From the previous paragraph steady state has two necessary conditions. Because buyers and sellers of existing homes exit each submarket at equal rates in matched pairs, they must enter each submarket at equal average rates. Also, with constant unit costs of entry, the value of entry must be strictly less than its cost, $V_1^S(a) < \kappa$, in almost all submarkets a without construction and equal to its cost in all other submarkets. These two necessary conditions are used repeatedly in the argument below.

Focus first on existing homes. Buyers enter submarket a at the exogenous relative rate a, and then allocate to existing houses the fraction of their search: $W_0(a) = 1 - W_1(a)$ with $W_1(a) = \min\{1, W[R_0(a), R_1(a)]\}$ from (18). Because buyers are atomistic and identical, this is equivalent to eventual buyers of existing homes entering at the relative rate $aW_0(a)$. In steady state these buyers must enter at the same average rates as sellers of existing homes:

$$aW_0(a)V^B(a)^\theta = V_0^S(a)^\theta. \tag{23}$$

in all submarkets, $a_0 \le a \le a_1$. With the expected present values of continued search, (A.2) and (A.3), the equality of average entry rates in (23) can be rewritten as

$$R_0(a)W_0(a) = \frac{\beta_0}{1-\beta_0}\frac{1-\delta}{\delta}\left[aW_0(a)\right]^{1/\theta}, \tag{24}$$

for all submarkets, $0 \leq a < \infty$. The variable $R_0(a)W_0(a)$ is the ratio of existing-home buyers to listings. It is driven in steady state by the relative entry rate of existing-home buyers $aW_0(a)$ with differences across submarkets that are decreasing in the elasticity θ.

Consider next new homes. Unlike existing homes, which are supplied with some inelasticity, new homes are either not supplied or supplied with perfect elasticity. In all submarkets a with construction, builders' common value of entry must then equal their common cost of entry:

$$\kappa = V_1^S(a) = \frac{\phi R_1(a)W_1(a)}{\left[\frac{1-\delta}{\delta} + \frac{1-\beta_1}{\beta_1}R_1(a)W_1(a)\right]^{\delta(\eta-1)/(\delta\eta-1)}} > 0, \tag{25}$$

with the constant of proportionality ϕ defined below (A.14) in the Appendix. Again, the common cost of entry κ includes the future finishing cost κ_1 because the common value of that entry $V_1^S(a)$ includes the initial deposit κ_1 for that future finishing cost. The function (25) follows from (A.3) and (A.6). The variable $R_1(a)W_1(a)$ is the ratio of new-home buyers to builders. It is the only variable in (25). Also, the right side of (25) is continuously increasing in that variable. Therefore, the ratio of new-home buyers to builders must be constant across all submarkets with construction: $R_1(a)W_1(a) = r_1^* > 0$. The value r_1^* is determined as part of the subsequent solution.

Steady state can now be characterized. With no new houses in the market, buyers allocate all their search to existing homes: $W_0(a) = 1$. In these markets each builder's value of waiting for buyers, $V_1^S(a)$, which depends only the the ratio of buyers to listings $R_0(a)$, increases through (24) with the entry rate of buyers a. Also, construction starts at the critical ratio of buyers to listings, $r_0^* = R_0(a^*)$, where builders' value of search for buyers first equals their unit cost of construction: $V_1^S(a_0^*) = \kappa$. Under these conditions construction is restricted to an upper interval of submarkets: $a^* < a \leq a_1$. Details appear in the Appendix. Building occurs only in some markets if $a_0 < a^* < a_1$. The subsequent analysis is largely limited to this case.

Collectively, the above results produce the third and final proposition. The proof is in the Appendix.

Proposition 3: *In steady state with (1) through (15), all submarkets without construction, $a_0 \leq a \leq a^*$, have the ratios of buyers to sellers of existing homes:*

$$R_0(a) = \frac{\beta_0}{1-\beta_0}\frac{1-\delta}{\delta}a^{1/\theta} > 0. \tag{26}$$

In all submarkets with construction, $a^* < a \leq a_1$, the ratios of buyers to listings and buyers to builders are

$$R_0(a) = r_0^*\frac{a}{a^*} \tag{27}$$

and

$$R_1(a) = \frac{r_1^* a}{a - a^*}. \tag{28}$$

The aggregate allocation of search on new housing is

$$W_1(a) = 1 - \frac{a^*}{a}. \tag{29}$$

The critical values, a^*, r_0^*, and r_1^*, are determined by (A.12) through (A.14).

In submarkets without construction, the ratio of buyers to sellers is (26). This is (24) with no search among new homes: $W_0(a) = 1$. It is also the main result of Novy-Marx (2009) extended to multiple submarkets. As indicated, the ratio of buyers to listings R_0 increases everywhere at a decreasing rate with more rapid relative entry of buyers a. With inelastic entry, $\theta < 1$, the rate of increase R_0'/R_0 can be very rapid. In turn, this affects the critical entry rate a^* at which builders begin construction. Here this result holds because builders begin construction at the critical ratio r_0^*. From (A.12) it follows that $\frac{da^*}{d\theta} \lesseqgtr 0$ as $r_0^* \lesseqgtr \frac{\beta_0}{1-\beta_0}\frac{1-\delta}{\delta}$.

In submarkets with construction, builders become the marginal suppliers of housing. As a result, the ratio of buyers to listings R_0 increases everywhere at a decreasing rate with more rapid relative entry a that does not depend on the elasticity θ. In the same submarkets the ratio of buyers to builders R_1 decreases at a decreasing rate with more rapid entry a that also does not depend on θ. Given the perfectly elastic entry buy builders in (25), the ratio of new-home buyers to builders must be constant across all submarkets with construction: $R_1(a)W_1(a) = r_1^*$ for $a^* < a \leq a_1$. Because buyers must expect in (13) equal gains from trading with either sellers of existing homes or builders of new homes, then, as shown in the Appendix, the ratio of existing-home buyers to listings must also remain constant across all submarkets with construction: $R_0(a)W_0(a) = r_0^*$ for $a^* < a \leq a_1$. As indicated in the Appendix, these constants must satisfy the inequality: $0 < r_0^* < r_1^*$ whenever $\beta_0 \leq \beta_1$. Finally, from (24) it follows that the relative entry rate of existing-home buyers $aW_0(a)$ must, in turn, remain constant across all submarkets with construction.

The above properties of steady state have strong implications for the previously identified properties of partial equilibria. In submarkets without construction, all variables, with one exception, increase with the relative entry rate of buyers a. The exception is the probability of a transaction between a matched buyer and seller, y_0 in (3), which is decreasing in the relative entry rate a. In submarkets with construction, all variables are constant, independent of the relative entry rate of buyers a. This strong result holds because all variables depend either directly or indirectly on the steady-state ratios of buyers to listings and buyers to builders, $R_0(a)$ and $R_1(a)$, only through the constant ratios of existing-home buyers to listings and new-home buyers to builders, r_0^* *and* r_1^*. Again, the latter result reflects two critical properties of the equilibrium: builders' perfectly elastic entry in all submarkets with construction and buyers' equal expected gains from trading with sellers of either type.

Conditions (24) and (25) are necessary but not sufficient for steady state. In submarkets without construction, the ratio of buyers to sellers of existing homes $R_0(a)$ is determined by (26). This ratio, combined with equal average rates of entry and exit for either buyers or sellers in Table 3, determines the two state variables: $B(a)$ and $S_0(a)$ for $a_0 \le a \le a^*$. In submarkets with construction, the ratios of buyers to sellers of existing and new homes, $R_0(a)$ and $R_1(a)$, are (27) and (28). These conditions, also combined with equal entry and exit rates for either buyers or sellers in Table 3, determine the three state variables: $B(a)$, $S_0(a)$, and $S_1(a)$ for $a^* \le a \le a_1$.

Numerical Results

Additional properties of partial equilibria and steady state are identified numerically in this section. Comparative statics are calculated for two cases: no bargaining by builders, as previously specified, and Nash bargaining by builders. The latter results are motivated by two considerations. Builders do, in fact, bargain with buyers, but less frequently than sellers of existing homes. Also, without bargaining by builders, the premium paid for new homes is excessive. As shown below, this excessive premium reverses with Nash bargaining by builders.

Calibration: Among the parameters of the model, two are difficult to calibrate: the tail index $1/\eta$ and the common value or mean value of matches μ_0 and. Both are dependent on buyers' unobserved screening or truncation of their match values. However, the two parameters can be calculated from the model if constraints are imposed on the parameters through plausible restrictions on steady state. To this end, the housing market must have in steady state among its many submarkets, $a_0 \le a \le a_1$, one with the properties identified below.

By the argument in the previous section, the ratios of new-home buyers to listings r_0^* and existing-home buyers to builders r_1^* must be constant in steady state. Thereby, the ratio r_1^*/r_0^* must be constant in steady state. In the calibration this constant ratio must have the value:

$$\frac{r_1^*}{r_0^*} = \frac{R_1 W_1}{R_0 W_0} = \frac{Q_1 R_1 S_1 Y_1}{Q_0 R_0 S_0 Y_0} \frac{S_0}{S_1} \frac{Y_0}{Y_1} = (.087)(8) \left(\frac{\nu}{\beta_0^\eta} \right)^{1/(\eta-1)}. \tag{30}$$

The capital letters identify functions in steady state of buyers' relative entry rate a. These functions replace the corresponding, lower-case constants in the previous partial equilibrium. In (30) the second equality follows from the definition of the aggregate weighting functions, W_0 and W_1. The final equality follows from the the ratio of transaction probabilities, Y_0/Y_1 in (20), and two empirical constraints. First, sales of new homes measured relative to existing homes have the value: $Q_1 R_1 S_1 Y_1 / Q_0 R_0 S_0 Y_0 = 8.7\%$. That ratio is reported for 2011 in the American Housing Survey: census.gov. The ratio, $S_0/S_1 = 8$, is motivated below. Again, the latter two ratios must hold in steady state for some submarket a.

In the model the ratio, $S_0/S_1 = R_1/R_0$, is identified as listings divided by builders of new homes. In fact, many builders have multiple housing projects. For each project builders must select their mix of model homes and housing lots. Each major alternative offered to buyers must satisfy a competitive entry condition like (25). Because (25) is critical in the subsequent calibration, the relevant ratio S_0/S_1 for this calibration is listings per major new-housing alternative offered to buyers. In Table 1 the average ratio of listings to new housing projects is approximately 100. That ratio divided by 12.5 major alternatives per project yields the ratio: $S_0/S_1 = 8$. The value 12.5 can be interpreted as 4-6 models on 2-3 types of lots with fractional contributions from fixtures, finishes, and other amenities.

Buyers screen new housing projects and then search only among projects that offer them one or more attractive alternatives. As a result, the average number of alternative customizations ν among which buyers choose need not equal the above 12.5 major alternatives per project. Instead, with efficient screening it is likely to be substantially less. In the subsequent numerical solutions, the parameter ν is set equal to 2. This value can be motivated as follows. In a survey of recent and prospective home buyers, 65% of respondents identified the most influential characteristics as "living space and number of rooms that met their needs:" NAHB (2013a). In the same survey 47% and 32% of respondents reported preferences for three and four bedrooms, respectively. Among new single-family homes completed during 2012, 46% and 43% had three and four or more bedrooms, respectively: U.S. Census Bureau. Among the active, single-family subdivisions in Table 1, the majority had 4-6 distinct models with a maximum of 3-5 bedrooms. Because extra bedrooms can have alternative configurations or uses, most home buyers then choose among 2-3 models per subdivision. Some of these alternatives are eliminated by initial screening on electronic sites, leaving 1-2 models per subdivision for careful inspection during site visits. Because buyers' alternatives also include other, less valuable amenities, like lots, fixtures, and finishes, the base case for the comparative statics has the alternative customizations: $\nu = 2$.

Builders' costs must be allocated between the core and customized upgrades. Construction costs are surveyed by the National Association of Homebuilders: NAHB (2011). Among construction costs an average of 30.2% is reported in categories commonly associated with fixtures, finishes, and other upgrades: appliances, cabinets and counter tops, carpet and tile, fixtures, interior doors and hardware, trim, and wood deck or patio. If other costs are allocated proportionally with construction costs, the cost of non-core, customizable components is about 30% of builders' total costs. Customized upgrades can cost substantially less because basic models include basic fixtures, finishes, and other amenities. The price of customized upgrades, which exceeds its cost to builders, reportedly ranges between about 5% and about 30% of the base price of a new house.[12] The lowest percentage applies to tract homes for first-time buyers with binding financial constraints, while the highest applies to semi-custom homes in luxury subdivisions. With higher margins on customized upgrades than core components, these are also upper bounds on the allocation of costs to

[12] For semi-custom homes in single-family subdivisions, the range is 10-30%: WSJ (2013). Starter production homes can have lower percentages: Chicago Tribune (2012). Upgrades are constrained by buyers' ability to pay. Price is a critical characteristic for 38% of first-time buyers and 26% of trade-up buyers: NAHB (2012b).

upgrades. In the numerical calculations the base case has the percentage costs: $\kappa_0 = .9$ and $\kappa_1 = .1$. These costs can be motivated further as follows. Builders' costs of customized upgrades equal to 10% of total construction costs are appropriate for trade-up, tract homes. Also, the distinction between finishing costs and costs of customized upgrades is moot in the model with its inital deposit for finishing costs. Finally, the normalization, $\kappa = \kappa_0 + \kappa_1 = 1$, is possible because the main results of the model are limited to ratios: arrival rates (19), transactions (20), and relative prices in (21).

Consider a category of trade-up, tract homes. That category could be determined by critical characteristics like total living space and numbers of bedrooms. Each model home for the category sits on a standard lot. It has a largely unobservable core–foundation, framing, and such–combined with a fixed floor plan. This common component can be upgraded with a premium lot and customized features. Suppose that each buyer's initial screening eliminates all houses not in her preferred category. Suppose also that her idiosyncratic preferences among new houses in her preferred category depend on their customizable components. In this case, the results from the model apply to the customizable components of a house. Also, the core has a common value equal to its price in a competitive market housing market. Again, metropolitan housing markets in the United States are extremely competitive for small builders: NAHB (2010).

For competitive American housing markets, prices are calibrated as follows. During 2011 the average profit of home builders in the United States was 7.5% of their total costs, excluding costs of financing: NAHB (2011). From the same source for the same period, average financing costs were about 3.3% of the same total costs. In the calibrated model builders' costs, other than financing costs, have the normalized sum: $\kappa_0 + \kappa_1 = \kappa = 1$. Also, builders of first-trade-up, tract houses in southwestern United States report privately margins on customized upgrades clustered around 35%. These statistics generate an average margin of 4.44% over the cost of the core κ_0, and thereby a mark-up on the core to cover financing costs of about 7.75%. In the calibrated model this produces for the core the common price: $1.0775\kappa_0 = .9698$ with $\kappa_0 = .9$.

With the above assumptions and this simple valuation of the core, the previous analysis applies only to the customized upgrades. In this situation the right side of the competitive entry condition (25) is replaced by the sum: $1.0775\kappa_0 + V_1^S(a)$. With the above normalization, $\kappa = 1$ and $\kappa_0 = .9$, this produces the calibrated competitive entry condition:

$$.0302 = V_1^S(a). \tag{31}$$

In (31) the right side is each builder's expected present value of waiting for a sale of the customized upgrades plus the cost, $\kappa_1 = .1$, of those upgrades.

With this calibration new homes have the common price: $1.0775\kappa_0 + p_1$ This includes each builder's price for customized upgrades p_1 from the model applied only to customized upgrades with the parameter value, $\kappa_0 = .9$, and the calibrated entry condition (31). An identical argument applies to existing houses after adjusting for economic depreciation. By that argument comparable existing houses have the average price: $1.0775\kappa_0 + \bar{p}_0$. This produces the relative price: $(1.0775\kappa_0 + \bar{p}_0)/(1.0775\kappa_0 + p_1)$. With $0 < \bar{p}_0 < p_1$ from (21), this relative price is positive and less than one. More importantly, it is increasing in the cost κ_0. Thereby, more costly common components reduce the premium paid for new homes.

The base case in the subsequent numerical solutions has the above parameter values and four others. With existing homes buyers and sellers have equal bargaining power: $\beta_0 = .5$. For buyers the cost of search is isoelastic with the unit cost and elasticity: $\gamma = .01$ and $\delta = 3$. Finally, both buyers and sellers have the discount rate: $\iota = .05$. Using these parameter values, the unspecified parameters, μ_0 and η, are calculated for submarket a with the steady-state ratios: $R_0(a) = 1$ and $R_1(a) = 8$. Again, the metropolitan housing market must have among all its submarkets, $a_0 \leq a \leq a_1$, one with this steady state. The first ratio, $R_0(a) = 1$, is standard in models with one steady state. The second ratio, $R_1(a) = 8$, follows with the ratio of listings to new housing alternatives: $R_1/R_0 = S_0/S_1 = 8$ in submarket a The calibration in the Appendix uses the aggregate weighting function (18) with both new and existing homes, $0 < w_1 < 1$, the steady-state condition (24), the ratios in steady state, (27) and (28), the empirical constraint (30), and the calibrated competitive entry condition (31).

With this steady state the truncated distribution of match values has the tail index: $1/\eta = .443$ Existing and new homes also have the respective common values: $\mu_0 = 0.0884$ and $\mu_1 = 0.120$. As illustrated in Table 4, both statistics are sensitive to the number of customizations ν. For the range of customizations ν reported in the table, all truncated distributions have fat tails: $1/\eta \geq 0.290$. Even this range is implausibly wide because the prices of existing homes relative to new homes in columns 5 and 6 are consistent with informal reports from real estate agents only in the middle of the range. By this measure normal distributions are poor approximations of buyers' match values when houses for sale are screened by buyers on electronic sites.

Focus on the relative prices in column 5 and 6. The values in column 5 come from the calibrated model with no bargaining by builders. The corresponding values in column 6 come from the calibrated model with Nash bargargaining between buyers and builders. The latter values are computed as indicated in the Appendix. As indicated, both ratios decrease monotonically with the number of customizations ν. With two customizations, $\nu = 2.0$, the ratios match most closely, as described below, informal reports by real estate agents. This match motivates the parameter values in the base case of the subsequent comparative statics.

Comparative Statics: Numerical results for partial equilibria surrounding the above steady state are reported in Table 5. In each pair of adjacent rows, the top row has the results with no bargaining by builders, while the bottom row has the corresponding results with Nash bargaining between buyers and builders. The base case has the parameter values for the above steady state with two customizations: $\nu = 2.0$.

In the base case existing homes are priced on average 17.75% less than comparable new houses with no bargaining by builders. This discount is larger than discounts reported informally by real estate agents for trade-up, tract homes. The difference can reflect bargaining by builders. It can also reflect unmodeled competition from contractors unrelated to the builder. In practice, buyers can close without fixtures, finishes, and ancillary features, like patios and pools, and then hire independent contractors to complete the work. Competition from those contractors is constrained by multiple issues, including difficulties of finding qualified contractors, financing improvements after closings, and living with work in process.

With Nash bargaining by builders, the discount for existing homes–9.3% in the base case–smaller than discounts reported informally by real estate agents for trade-up, tract homes. Combining this with the above discount without bargaining, suggests that builders do bargain with buyers, but less so than sellers of existing homes. Not surprisingly, matched buyers also purchase new homes with greater probability y_1 when builders bargain. Under the same circumstances they also also search with lower relative frequencies w_1 among new homes. The latter result suggests that less search is optimal when meetings are more likely to be successful.

The comparative statics in Table 5 are more compelling. Both the percentage discounts for existing homes for new houses and the aggregate fraction of search focused on new houses w_1 are increasing in the tail index $1/\eta$. For distributions of match values with fatter tails, customization is more valuable to buyers and thereby more frequently the focus of their searches. The same comparative statics also follow from increasing numbers of customizations ν. Also, additional customizations v decrease the probability y_1 that a matched buyer purchases a new home. Apparently, buyers demand better matches when builders offer more acceptable customizations. As the number v increases beyond the values reported in the table, the probability y_1 rapidly becomes implausibly small. In this sense, large numbers of acceptable customizations ν are inconsistent with initial screening on electronic sites by buyers with strong preferences for interior living space and numbers of bedrooms. The remaining results are straightforward. For example, the aggregate allocation of time or effort to search q is decreasing in the costs of search, as measured by its two components, γ and δ.

Discussion

In this section empirical implications are identified and related institutional issues are discussed. Topics in the first category are customization of new homes and submarkets within a metropolitan area. Topics in the second are contractual issues in construction and shrouded prices of upgrades.

Customization: Housing can be distinguished by the number of alternative customizations offered to buyers. Again, customizations are defined broadly to include models and lots in subdivisions and analogous alternatives in condominiums, as well as alternative fixtures and finishes. In practice, the number of alternatives is increasing in the size and quality of the housing unit, as measured by its common or hedonic value because more alternatives are both more costly for builders and more valuable to households with higher incomes or wealth. By this argument the number of customized alternatives should be the least for entry-level tract homes, larger for trade-up tract homes, even larger for semi-custom homes, all in single-family subdivisions, and largest for true custom homes that are built under contract for specific clients.

The comparative statics in the model for the number of customizations ν appear in Table 5. By the above argument, these comparative statics have cross-sectional implications for housing distinguished by its quality or value. Specifically, housing of higher quality has the following characteristics. Buyers focus more on new homes relative to existing homes. The aggregate allocation of time or effort spent searching for new homes increases in (17) and (18) with housing quality. This increased focus on new housing reduces in (19) the arrival rate of buyers at listings of existing homes relative to developments of new homes. This reduced arrival rate is partly offset by more frequent sales with matched buyers in (20). Also, sellers of existing homes are more motivated to trade in (22) relative to builders of new homes. Most importantly, the premium paid for new housing in (21) increases with the quality of that housing. The latter premium induces in Proposition 3 earlier and more rapid development of higher quality new housing.

Submarkets: In submarkets without construction the ratio of buyers to sellers $R_0(a)$ is driven in (26) by the entry rate of buyers relative to sellers of existing homes a. This ratio is lower in (26) with larger values of θ and thereby more elastic entry by both buyers and sellers. Also, the average price of existing homes $\bar{p}_0$ is increasing in the relative entry rate a from (26), (A.3) with $i = 0$, (A.6), and (A.7). As a result, the average price of existing homes in submarkets without builders can be below construction costs, while submarkets with builders can have new homes priced above construction costs. The slowest submarkets can have infrequent sales, while submarkets with construction have frequent sales of both existing and new homes at much higher prices.

Among submarkets with construction those with more rapid entry by buyers relative to sellers of existing homes have in steady state both more buyers relative to listings and fewer buyers relative to new housing projects. Thereby, these more preferred submarkets have fewer listings relative to new projects. With construction buyers' aggregate focus on new homes is measured by their aggregate allocation of time or effort in (17) and (18). This aggregate focus on new homes increases with the ratio of buyers to listings and decreases with the ratio of buyers to new projects. As a result, the aggregate focus on new housing (29) is higher in submarkets with more rapid relative entry by buyers.

The above monotonicity in both ratios of buyers to sellers follows from the assumption in the model that builders enter the housing market more elastically than sellers of existing homes. Additional results follow from the stronger assumptions of the model: perfectly elastic entry by builders and no relationship between the common value of housing and entry rates by buyers relative to sellers of existing homes. Those additional results are the constant ratios of existing-home buyers to listings and new-home buyers to new housing projects across all submarkets with construction. With these constant ratios all important variables–arrival rates, transaction frequencies, and average prices–are also constant across all submarkets with construction. The latter results survive with some realistic complications but disappear with others.

In practice, builders construct housing of multiple types ranging from entry-level tract homes to luxury customs. From the previous results the critical entry rate a^* of buyers relative to sellers of existing homes at which building begins decreases with the number of alternative customizations ν. Thereby, the mix of new construction becomes increasingly weighted toward homes with fewer customizations ν as the relative entry rate a increases. With increasing a and the constant ratios in the previous paragraph, the above ratios for existing relative to new homes of arrival, closings, and prices then shift, as shown in Table 6, in the opposite direction of the comparative statics for the number of customizations ν.

In the model construction costs are exogenous. This is not necessary. Endogeneity can be introduced through some inelasticity in the aggregate supply of inputs, like labor and lumber, that are not specific to submarkets. For example, builders could buy all inputs from subcontractors and other suppliers located across the metropolitan area. In this case, the common unit costs increase with the aggregate construction rate. Higher unit costs raise the ratio of buyers to existing sellers r_0^* at which building begins and thereby the critical entry rate a^*. Otherwise, the previous results are unchanged.

In the model the housing market is a continuum of submarkets distinguished only by the arrival rate of buyers relative to sellers. Alternatively, it could be a collection of neighborhoods, both discrete and finite. This minor change has no material impact on the previous results. However, these neighborhoods are also realistically distinguished by geography, broadly defined. In this case, neither the unit cost of production κ nor the common value of houses μ_0 need be constant across markets. Either can depend on the price of local inputs, like land, that capitalize commuting costs and the net value of local public goods. In this more realistic situation, the steady state no longer has the simple solution in Proposition 3. More importantly, the major variables of the model can vary across neighborhoods with construction.

Contracts: Once a new house is customized, the option to customize is lost. The resulting loss of market value affects the optimal ownership of new housing. For homes currently under construction, two alternatives are common practice. With true custom homes owner-occupiers buy lots and hire builders. Those builders are then paid in stages as construction is completed. In subdivisions of single-family homes, lots and homes under construction are owned by builders or developers, subject to residential purchase agreements with buyers. Nonrefundable deposits are often collected only when customized upgrades are ordered. In multifamily condominiums where initial construction costs are considerable and those costs are commonly covered by commercial construction loans, initial deposits are often nonrefundable, with exceptions for nonperformance by builders. Nonrefundable deposits are frequently due in stages as construction is completed and held in escrow until units can be occupied. Additional nonrefundable deposits are also demanded for upgraded fixtures and finishes.

Customized upgrades to new homes have idiosyncratic value only to owner-occupiers. Investors who sign purchase agreements for new housing currently under construction optimally do not take title. Instead, they assign or flip their contracts to owner-occupiers who then select their preferred upgrades. Investors in single-family, rental housing optimally avoid new construction and focus instead on existing homes. In such situations new single-family housing is sold to owner-occupiers.

Prices: Builders commonly advertise their competitive prices for basic models but not their high markups on customized upgrades: WSJ (2013). This behavior is consistent with builders' optimal obfuscation or shrouding of prices for upgrades in a competitive market with myopic buyers. Myopia is more plausible in markets with complicated products and nonrepetitive, one-time customers. Residential housing is a good example. It is also consistent with builders' local monopoly power over matched buyers in a market with one-time customers, combined with their inability to commit through advertisements to prices of complicated, customized upgrades. The latter is the approach in this paper.

With shrouded pricing builders advertise low prices for basic models and obscure high prices for upgrades because some buyers do not anticipate builders' high markups for upgrades. Builders have no incentive to educate these myopic buyers by advertising their competitors' high prices for upgrades because they attract no informed, rational buyers: Gabaix and Laibson (2006). Instead, rational buyers, including myopic buyers who become informed by the builder's advertising, prefer to buy from builders with shrouded prices and high markups for customizations because they can pay lower prices for basic models and reject the builders' customized upgrades. In practice, informed buyers can then buy better basic models and hire independent contractors to install other upgraded fixtures and finishes, as well as accessories like patios and pools.

Here, all buyers are equally informed and rational. In this context builders can commit through essentially free advertising on electronic sites to low prices for a few basic models. They cannot commit through advertising on the same sites to the many prices of customized upgrades, each with multiple options and complex contingencies. As predicted by the model, high prices for customized upgrades then follow from builders' optimal exploitation of their local monopoly power with matched, nonrepetitive buyers. Rational buyers anticipate this behavior and adjust accordingly their search across existing and new homes. Competitive entry by identical builders with constant returns to scale then eliminates all excess profits in submarkets with construction.

The results in this model can also be consistent with a combination of informed and uninformed buyers. Suppose that uninformed buyers can compute neither their reservation values for existing and new homes, (2) and (9), nor their expected gains from searching among either existing or new homes in (12) and (13). However, uninformed buyers can follow the advice of their real estate agents who commonly collect recent prices of comparable homes. Thereby, uninformed buyers can observe with noise the reservation prices of informed buyers. Also, some uninformed buyers can observe with noise the behavior of informed buyers and mimic that behavior. For example, uninformed buyers can follow flippers and look for long lines indicating hot housing projects. Ignore the above noise and suppose that all buyers have the same truncated distribution of match values. Assume also that some uninformed buyers allocate idiosyncratically their search across existing and new homes. This idiosyncratic search generates the aggregate fractional allocation w_1^U of their total time and effort to new houses. The idiosyncrate aggregate need not equal the optimal allocation, w_1 in (18) and (19). In response, informed buyers optimally focus on new houses if $w_1^U < w_1$ and existing houses if $w_1^U > w_1$ until the aggregate allocation equals w_1. Thereby, the aggregate allocation to new homes equals w_1 if the fraction of informed buyers and their mimics exceeds or equals the lower bound: $max\left(\frac{w_1-w_1^U}{1-w_1^U}, \frac{w_1^U-w_1}{w_1^U}\right)$. In this case, the previous partial equilibria persist with some uninformed buyers.

More generally, builders practice price discrimination. In competitive housing markets they advertise low prices of basic models, but not their high prices of customized upgrades, and thereby maximize their profits from myopic or uninformed buyers. To minimize their losses with rational or informed buyers, builders bargain privately with these buyers and offer concessions that subsequent buyers and their appraisers cannot observe. In this situation builders have no incentive either to attract informed buyers or to educate uninformed myopic buyers.

Conclusion

Partial equilibria and steady state are characterized for a housing market with the following properties. Sellers advertise their homes for sale on electronic sites, but do not search. Buyers screen houses on those sites and then search only among acceptable matches. Buyers can focus their searches on either existing or new houses or neither. Only new houses can be customized for buyers. Also, sellers of existing homes bargain with buyers, but builders do not.

Buyers' initial screening truncates their distribution of match values with prospective homes. That truncated distribution is approximately power law. With the power-law distribution and isoelastic search costs, partial equilibria can be calculated explicitly. Because search can be focused on either existing or new homes, buyers must be indifferent in equilibrium between searching among either type of home in all submarkets with construction. This indifference constrains the pricing and sales of new homes relative to existing homes.

It generates, for example, lower average prices for existing homes than otherwise comparable new homes because customization of new homes improves the quality of buyers' matches and thereby their efficiency of search. By the same argument, the premium paid for new homes must be greater with more options to customize.

These results in partial equilibrium are then extended to steady state for a housing market with a continuum of submarkets. Submarkets are ordered by the average entry rate of buyers relative to sellers of existing houses. Buyers enter their preferred submarket with imperfect price elasticity; sellers of existing houses enter with the same constant elasticity; whereas, builders can enter multiple submarkets with perfect elasticity. The unique steady state has a surprisingly simple, nearly explicit solution. Builders enter all submarkets with sufficiently rapid entry by buyers relative to sellers of existing homes. Across all submarkets the ratio of buyers to listings of existing homes increases with the relative entry rate of buyers. Across all submarkets with construction the ratio of buyers to new housing projects decreases with the same relative entry rate. These properties of steady state are used to calibrate the power-law distribution of buyers' match values. That calibrated distribution is far from normal.

Table 1: New housing projects relative to listings of existing homes

Metropolitan Phoenix, Arizona
May 14, 2013

Submarket	Population (000s)	Projects per listing	Cumulative average
Phoenix (city)	1602	0.49%	0.49%
Older suburbs	1586	0.63%	0.56%
Newer suburbs	551	3.08%	0.93%
Exurbs	113	1.55%	0.95%

Notes: Population measured in thousands, new housing projects measured relative to existing houses listed for sale, and cumulative averages are identified for 31 jurisdictions within metropolitan Phoenix grouped in four categories. The cumulative averages of housing projects per listing are weighted by relative population and measured from the center outwards. For example, the cell in the second row and third column is the weighted average of the city and older suburbs, whereas the adjacent cell below is the weighted average of the city, older suburbs, and newer suburbs. The older suburbs are Apache Junction, Avondale, Carefree, Cave Creek, El Mirage, Fountain Hills, Glendale, Guadalupe, Litchfield Park, Mesa, Paradise Valley, Peoria, Scottsdale, Surprise, Tempe, Tolleson, and Youngtown. The newer suburbs are Buckeye, Chandler, Gilbert, Goodyear, and Queen Creek. The exurbs are Casa Grande, Coolidge, Eloy, Florence, Gila Bend, Maricopa, New River, and Wikenburg.

Table 2: Notation

Variables:

a	Average exogenous entry rate of buyers relative to sellers.
i	Housing type: existing, $i = 0$, or new, $i = 1$.
$P(x)$	Price of existing home with match x.
$\bar{p}_0$	Average price of existing homes.
p_1	Price of each new house.
q	Total time or effort allocated to search: $q = q_0 + q_1$.
q_i	Average arrival rate of buyer at a house of some seller i.
r_i	Ratio of buyers to sellers of housing type i.
s_i	Ratio of sellers i to housing stock.
v^B	Expected present value of continued search by each buyer.
v_i^S	Expected present value of continued waiting by each seller i.
w_i	Fractional allocation of search to sellers i: $w_i = q_i/q$.
x	Value of match between buyer and house.
x^*	Maximum match among all alternatives offered by builder.
y_i	Probability of trade between a matched buyer and seller i.
z_i^B	Expected gain from trade with seller i.for buyer.
z_i^S	Expected gain from trade with buyer for seller i.

Constants:

β_0	Buyer's relative bargaining power with seller of existing home.
β_1	Buyer's share of expected surplus from purchasing new home.
γ	Unit opportunity cost of search.
δ	Elasticity of buyer's opportunity cost of search.
ζ_i	Scaling parameter of power-law distribution for match with homes i.
η	Exponent in power-law distribution for matches with tail index $1/\eta$.
θ	Elasticity of entry by buyers and sellers of existing homes.
ι	Discount rate of buyers and sellers.
κ	Unit cost of entry by builders.
μ_i	Expected value of match between buyer and homes i.
ν	Number of alternative customizations offered by builders.
ω	Composite parameter in (16).

Table 3: Average rates of entry and exit of buyers and homes for sale in each submarket

	Entry	**Exit**
No Construction		
Buyers	$aV^B(a)^\theta$	$Q(a)y_0B(a)$
Sellers:	$V_0^S(a)^\theta$	$Q(a)R_0(a)y_0S_0(a)$
Construction		
Existing Homes:		
Buyers	$aW_0(a)V^B(a)^\theta$	$Q(a)W_0(a)y_0B(a)$
Sellers:	$V_0^S(a)^\theta$	$Q(a)W_0(a)R_0(a)y_0S_0(a)$
New Homes:		
Buyers	$aW_1(a)V^B(a)^\theta$	$Q(a)W_1(a)y_1B(a)$
New homes:	perfectly elastic	$Q(a)W_1(a)R_1(a)y_1S_1(a)$

Notes: Each market a is identified by the exogenous average arrival rate a of buyers relative to sellers. Each buyer meets some seller i at the average rate: $Q_i(a) = Q(a)W_i(a)$ for $i = 0, 1$. With the ratio $R_i(a)$ of buyers to sellers i, each seller i is met at the average rate $Q(a)W_i(a)R_i(a)$. Between a matched buyer and seller i, a transaction occurs with probability y_i. For each buyer and seller of existing homes. the expected values of continued search are $V^B(a)$ and $V_1^S(a)$, respectively. Buyers and sellers i measured relative to the stock of existing homes are $B(a)$ and $S_i(a)$. In steady state the rate of entry in each row must equal the rate of exit in that row.

Table 4: Alternative calibrations

Custom-izations	Tail index	Mean value Old	New	Price ratio Bldrs bargain No	Yes
ν	$1/\eta$	μ_0	μ_1		
1.1	.671	.0158	.017	.961	.990
1.2	.631	.0220	.025	.945	.980
1.5	.539	.0453	.056	.893	.948
2.0	.443	.0884	.120	.823	.907
2.5	.384	.128	.182	.775	.882
3.0	.343	.162	.236	.742	.865
3.5	.313	.191	.283	.718	.854
4.0	.290	.216	.323	.699	.846

Notes: The truncated, asymptotic distribution of match values states is calculated for the calibrated model with different numbers of customizations ν. Its sufficient statistics are the tail index $1/\eta$ and the common or mean values, μ_0 and μ_1, for existing and new houses, respectively. The prices of existing homes relative to new homes are calculated for two cases: $\bar{p}_0/p_1$ with no bargaining by builders and $\bar{p}_0/\bar{p}_1$ with Nash bargaining by builders. These calculations use the exogenous parameter values: $\beta_0 = 0.5$, $\gamma = 0.01$, $\delta = 3.0$, and $\iota = 0.05$.

Table 5: Comparative statics

Para-meter value	Home price ratio	Search intensity Total q	Search intensity Weight w_1	Probability of sale Old y_0	Probability of sale New y_1
Base	.823	.507	.347	.188	.0641
	.907	.498	.229	.176	.101
β_0	Buyer's bargaining power				
0.4	.813	.480	.493	.229	.0417
	.906	.453	.212	.187	.108
0.6	.806	.537	.234	.166	.125
	.909	.538	.254	.167	.0964
γ	Unit opportunity cost of search				
.005	.806	.665	.347	.143	.0489
	.898	.653	.229	.134	.0771
.025	.832	.433	.347	.220	.0751
	.912	.425	.229	.206	.118
δ	Elasticity of opportunity cost				
2.0	.839	.444	.329	.242	.0633
	.916	.430	.216	.228	.131
4.0	.813	.561	.357	.161	.0633
	.901	.554	.235	.150	.0683
$1/\eta$	Tail index of power-law matches				
.343	.867	.448	.180	.467	.421
	.978	.446	.153	.461	.369
.543	.823	.507	.347	.188	.0642
	.907	.498	.229	.176	.101

Table 5: Comparative statics continued

Para- meter value	Home price ratio	Search intensity Total q	Search intensity Weight w_1	Probability of sale Old y_0	Probability of sale New y_1
Base	.823	.507	.347	.188	.0641
	.907	.498	.229	.176	.101
ι	Discount rate of buyers and sellers				
0.4	.777	.415	.347	.0917	.0313
	.880	.408	.229	.0858	.0494
0.6	.843	.562	.347	.271	.0926
	.919	.551	.229	.254	.146
ν	Number of customizations				
1.5	.855	.501	.269	.180	.0999
	.949	.494	.174	.171	.124
2.5	.790	.513	.414	.196	.0491
	.871	.501	.277	.180	.0870

Notes: The second column is the average price of existing homes divided by the price of new homes: $\bar{p}_0/p_1$without and $\bar{p}_0/\bar{p}_1$ with bargaining by builders. The third and fourth columns are the total intensity of search, $q = q_0 + q_1$, and fractional allocation of that search to new houses, $w_1 = q_1/q$, The fifth and sixth columns are the probabilities of transactions between matched buyers and sellers of existing and new homes, y_0 and y_1. The top two rows are the base case from the calibrated model calculated under two conditions. In the top row builders do not bargain with buyers; in the second row builders make Nash bargains. In the remaining rows the indicated parameter is changed from its value in the base case to the indicated value and no other parameters are changed. The two rows associated with each change are calculated with no bargaining by builders on the top and Nash bargaining on the bottom. The base case from the calibrated model has the parameter values: $\beta_0 = 0.5$, $\gamma = 0.01$, $\delta = 3.0$, $1/\eta = .443$, $\iota = 0.05$, $\kappa_0 = 0.9$, $\kappa_1 = 0.1$, $\mu_0 = 0.0884$, and $\nu = 2.0$. It also has the ratios: $r_0 = 1.0$ and $r_1 = 8.0$.

Table 6 : Empirical implications
for submarkets with builders

Impact on variable below from increase in parameter to right			**Relative entry rate** a	**Number custom-izations** ν
Aggregate focus	w_1	$0 < w < 1$	$+$	$+$
Listings/projects	s_0/s_1	> 1	$+$	
Arrival of buyers	q_0r_0/q_1r_1	< 1	$+$	$-$
Prob transactions	y_0/y_1	> 1	$-$	$+$
Prices	$\bar{p}_0/p_1$	< 1	$+$	$-$

Notes: Comparative statics with respect to the exogenous entry rate of buyers relative to sellers a and the acceptable number of customizations ν are calculated for five variables: the fractional allocation of aggregate search to new houses w_1, listings of existing homes relative to new housing projects s_0/s_1, the average arrival rate of buyers at listings relative to housing projects q_0r_0/q_1r_1, the probability of a transaction with an arriving buyer at a listing relative to a housing project y_0/y_1, and the average transaction price of an existing home relative to a comparable new house $\bar{p}_0/p_1$. These comparative statics without bargaining by builders hold in partial equilibria and thereby steady state.

References

Albrecht, J., P. Gautier, and S. Vroman. 2010. Directed Search in the Housing Market. Working Paper, Georgetown University.

Arnold, M. 1999. Search, Bargaining, and Optimal Asking Prices. *Real Estate Economics* 27: 453-481.

Carrillo, P. 2012. An Empirical Two-Sided Equilibrium Search Model of the Real Estate Market. *International Economic Review* 53: 203-231.

Chicago Tribune. Smart Upgrades for New Homes. Published December 20, 2012.

Diaz, A. and B. Jurez. 2010. House Prices, Sales, and Time on the Market: A Search-Theoretic Framework. Working Paper. Department of Economics. University Carlos III de Madrid.

Gabaix, X. 2009. Power Laws in Economics and Finance. *Annual Review of Economics* 1: 255-293.

Gabaix, X. and D. Laibson. 2006. Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets. *Quarterly Journal of Economics* 121: 505-540.

Han, L. and W. Strange. 2013. What is the Role of the Asking Price for a House? Working Paper, Rotman U. Toronto.

Head, A., H. Lloyd-Ellis, and H. Sun. 2012. Search, Liquidity, and the Dynamics of House Prices and Construction. Department of Economics. Queen's University.

Hosios, A. 1990. On the Efficiency of Matching and Related Models of Search and Unemployment. *Review of Economic Studies* 57: 279-298.

Krainer, J. 2001. A Theory of Liquidity in Residential Real Estate Markets. *Journal of Urban Economics* 49: 32-53.

Merlo, A. and F. Ortalo-Magne. 2004. Bargaining over Residential Real Estate: Evidence from England. *Journal of Urban Economics* 56: 192-216.

Moen, E. 1997. Competitive Search Equilibrium. *Journal of Political Economy* 105: 385-411.

National Association of Home Builders (NAHB). 2010. Structure of the Home Building Industry. HousingEconomics.com.

National Association of Home Builders (NAHB). 2011. New Construction Cost Breakdown. HousingEconomics.com.

National Association of Home Builders (NAHB). 2013a. What Home Buyers Really Want. HousingEconomics.com.

National Association of Home Builders (NAHB). 2013b. Characteristics of Home Buyers. HousingEconomics.com.

National Association of Realtors (NAR). 2013. Profile of Home Buyers and Sellers.

Novy-Marx, R. 2009. Hot and Cold Markets. *Real Estate Economics* 37:1-22.

Piazzesi, M., M. Schneider, and J. Stroebel. 2013. Segmented Housing Search. Working Paper, Stanford.

Porter, M. 2003. The US Home Building Industry and the Competitive Position of Large Builders. Harvard Business School.

Wall Street Journal (WSJ). That'll be Extra. Published February 22, 2012.

Wheaton, W. 1990. Vacancy, Search, and Prices in a Housing Market Matching Model. *Journal of Political Economy* 98: 1270-1292.

Williams, J. 1995. Pricing Real Assets with Costly Search. *Review of Financial Studies* 8: 55-90.

Williams, J. 2013. Housing Prices, Sales, and Search with Increasingly Motivated Buyers and Sellers. Working Paper. Professors Capital..

Yavas, A. 1992. A Simple Search Model of Real Estate Markets, *Real Estate Economics* 20: 533-548.

Appendix

Derivation of the Truncated Mean: The maximum value x^* has the distribution function:

$$\begin{aligned}
\Pr\{x^* \geq \xi\} &= 1 - \left[1 - \left(\frac{\zeta_0}{\xi}\right)^{\eta}\right]^{\nu} \\
&= 1 - \sum_{n=0}^{\nu} \binom{\nu}{n} (-1)^n \left(\frac{\zeta_0}{\xi}\right)^{\eta n} \\
&= \nu \left(\frac{\zeta_0}{\xi}\right)^{\eta} + O\left[\left(\frac{\zeta_0}{\xi}\right)^{2\eta}\right]
\end{aligned}$$

for $\xi \geq \zeta_0$. As $\xi \to \infty$ this has the asymptotic power-law approximation: $(\zeta_1/\xi)^{\eta}$ with $\zeta_1 = \zeta_0 \nu^{1/\eta} \zeta_0$ or, equivalently, $\mu_1 = \mu_0 \nu^{1/\eta}$.

Inequality below (16): Define

$$G(\eta) \equiv \frac{1}{\eta} + \left(\frac{\beta_0}{\nu}\right)^{1/(\eta-1)},$$

for $0 < \beta_0 < 1$ and ν, $\eta \geq 2$. Next, note that $\omega < 1$ if and only if $G < 1$ everywhere on its domain. The latter inequality holds because $G'' > 0$ everywhere with $G(2) < 1 = G(\infty)$.

Proof of Proposition 1: First, rewrite (15) as follows:

$$\iota v_i^S = q r_i w_i z_i^S = \frac{1-\beta_i}{\beta_i} \gamma \delta r_i w_i \left(\frac{z_i^B}{\gamma\delta}\right)^{\delta/(\delta-1)}, \tag{A.1}$$

for $i = 0, 1$. The second equality follows from (13) and (14), combined with (5) and (6) for existing homes, $i = 0$, and (10), (11), and the definition of β_1 for new homes, $i = 1$. From (13) there are two possibilities: either $z_i^B = z^B$ or, alternatively, $z_i^B < z^B$ with $w_i = 0$.

Construct the composite constants:

$$\lambda_i \equiv \left(\frac{\beta_0^{1-i}}{\gamma\delta} \frac{\zeta_i^{\eta}}{\eta-1}\right)^{\delta/(\delta-1)}, \qquad \chi \equiv \tfrac{\delta}{\delta-1}(\eta-1).$$

Next, note from (4), (5), and (10) that $\left(z_i^B/\gamma\delta\right)^{\delta/(\delta-1)} = \lambda_i x_i^{-\chi}$. With this new notation (12) and (A.1) can be rewritten as

$$v^B = \frac{\gamma}{\iota}(\delta-1)\max\{\lambda_0 x_0^{-\chi},\ \lambda_1 x_1^{-\chi}\} \tag{A.2}$$

and

$$v_i^S = \frac{\gamma\delta}{\iota}\frac{1-\beta_i}{\beta_i} r_i w_i \lambda_i x_i^{-\chi}. \tag{A.3}$$

All of the above hold for $i = 0, 1$.

Suppose that $z_0^B = z_1^B$, so that $\lambda_0 x_0^{-\chi} = \lambda_1 x_1^{-\chi}$ from the equality immediately above (A.2). In this case, it follows that

$$\frac{x_0}{x_1} = \left(\frac{\lambda_0}{\lambda_1}\right)^{1/\chi} = \left(\frac{\beta_0}{\nu}\right)^{1/(\eta-1)} = \frac{\eta-1}{\eta}\omega. \tag{A.4}$$

This calculation exploits (16) and the property: $\zeta_0/\zeta_1 = \mu_0/\mu_1$. Also, from (A.2) with $\lambda_0 x_0^{-\chi} = \lambda_1 x_1^{-\chi}$ and (A.3), the reservation values x_i in (2) and (9) satisfy

$$x_0 = \left[\lambda_0 \frac{\gamma}{\iota}\left(\delta - 1 + \frac{1-\beta_0}{\beta_0}\delta r_0 w_0\right)\right]^{1/(\chi+1)} \tag{A.5}$$

and

$$x_1 = \left\{\lambda_1 \frac{\gamma}{\iota}\left[\frac{\beta_1}{1-\beta_1}(\delta - 1) + \delta r_1 w_1\right]\right\}^{1/(\chi+1)} \tag{A.6}$$

The calculation of (A.6) uses the property: $\eta/(\eta-1) = \beta_1/(1-\beta_1)$.

Divide (A.5) by (A.6) and apply (A.4). This yields the ratio:

$$\omega = \frac{\eta}{\eta-1}\frac{x_0}{x_1} = \frac{\frac{\delta-1}{\delta} + \frac{1-\beta_0}{\beta_0} r_0 w_0}{\frac{\delta-1}{\delta} + \frac{1-\beta_1}{\beta_1} r_1 w_1}.$$

With $w_0 = 1 - w_1$, this linear equation has the solution (17).

The above result is feasible iff $0 \leq W \leq 1$. Alternatively, if $W > 1$, then $w_1 \leq 1 < W$, in which case $z_0^B < z_1^B$. The latter result holds because z_i^B decreases in w_i, which follows from the relationship immediately above (A.2) combined with (A.5) and (A.6). From (13) it then follows that $w_1 = 1$ if $W > 1$ or, equivalently, $0 \leq r_1 < \rho_1$ from (17). By the same argument, $w_1 = 0$ if $W < 0$ or, equivalently, $0 \leq r_0 < \rho_0 \equiv \frac{\beta_0}{1-\beta_0}\frac{1-\delta}{\delta}(\omega - 1)$. The latter inequality is impossible since $\omega < 1$. This yields (18).

Proof of Proposition 2: The expected price of an existing home is calculated from (3), (4), and (6):

$$\bar{p}_0 = E\left[P_0(x) | x > x_0\right] = \frac{1-\beta_0}{\eta-1} x_0 + v_0^S > 0. \tag{A.7}$$

The price of a new house is calculated from (8) and (9):

$$p_1 = \frac{1}{\eta} x_1 + v_1^S > 0. \tag{A.8}$$

This produces the relative price:

$$\frac{\bar{p}_0}{p_1} = \frac{\eta}{\eta-1}\frac{(1-\beta_0)x_0 + (\eta-1)v_0^S}{x_1 + \eta v_1^S}. \tag{A.9}$$

With $z_0^B = z_1^B$ then (A.4) holds with an equality. In this case, the ratio (22) follows from (17), (18), (A.3), and (A.4). That ratio can be rewritten as follows:

$$\frac{v_0^S}{v_1^S} = \frac{\omega b - \frac{\beta_1}{1-\beta_1}\frac{\delta-1}{\delta}(1-\omega)s_1}{b + \frac{\beta_0}{1-\beta_0}\frac{\delta-1}{\delta}(1-\omega)s_0} < 1 \tag{A.10}$$

with $0 < \omega < 1$. This is the inequality in (22). An equality in (A.4) is equivalent to $(1-\beta_0)\frac{\eta}{\eta-1}\frac{x_0}{x_1} = (1-\beta_0)\omega$. With this equality and (A.9), the inequalities (21) hold iff

$$\min\left\{(1-\beta_0)\omega, \frac{v_0^S}{v_1^S}\right\} \leq \max\left\{(1-\beta_0)\omega, \frac{v_0^S}{v_1^S}\right\} < 1.$$

The proof is complete if the last inequality above is satisfied. That inequality follows from the two inequalities: (A.10) and $(1-\beta_0)\omega < 1$.

From (A.4) the ratio x_0/x_1 decreases in ν and increases in μ_0. From (17) and (18), the weight, $w_0 = 1 - w_1$, has the same properties, while the weight w_1 has the reverse properties. In this case, the ratio v_0^S/v_1^S also has the same properties from (A.3). Thereby the ratio, $\bar{p}_0/p_1$ in (A.9), has the same properties: it decreases in in ν and increases in μ_0.

Proof of Proposition 3: The valuation function in (25) follows from (A.3) with $i = 1$ and (A.5). Next, note from (17) that

$$r_1 W(r_0, r_1)|_{s_1=0} = \frac{\frac{1-\beta_0}{\beta_0}r_0 + \frac{\delta-1}{\delta}(1-\omega)}{\frac{1-\beta_1}{\beta_1}\omega}. \tag{A.11}$$

Given the monotonicity of both (25) in its argument and (A.11) in its argument, the condition, $S_1(a) = 0$ with $\kappa > V_1^S(a)$, is satisfied only on some lower interval, $a_0 \leq a < a^*$. The alternative condition, $S_1(a) > 0$ with $\kappa = V_1^S(a)$, is satisfied on the upper interval, $a^* < a \leq a_1$. With the monotonicity in (24), the lower interval of entry rates is equivalent to the lower interval of ratios: $0 \leq R_0(a) < r_0^*$ with $r_0^* = R_0(a^*)$.

Focus on the upper interval with construction: $a^* < a \leq a_1$. With the competitive entry condition (25), the ratio R_1W_1 must be constant, independent of the relative entry rate a. In this situation, buyers' reservation value for new homes, x_1 in (A.6), which is increasing in its sole argument r_1w_1, must also be constant, independent of a. Because the ratio of reservation values x_0/x_1 is independent in (A.4) of the entry rate a, the reservation value for existing homes x_0 must also be independent of a. The latter reservation value, x_0 in (A.5), is also increasing in its only argument $r_0 w_0$. Therefore, the ratio $R_0 W_0$ must be independent of a. From (24) it then follows that aW_0 must be independent of a. Together, the last two results require that

$$\frac{R_0'}{R_0} = -\frac{W_0'}{W_0} = \frac{1}{a}$$

for $a^* \leq a \leq a_1$. This differential equation has the unique solution (27).

The remaining results, (28) and (29), follow almost immediately from the above properties. With $R_0 W_0$ constant on the upper interval, (27) and the boundary condition, $W_0(a^*) = 1$, require that

$$W_0(a) = \frac{a^*}{a}$$

and $R_0 W_0 = r_0^*$ for $a^* \leq a \leq a_1$. With the constraint, $W_1 = 1 - W_0$, this is (29). With $R_1 W_1$ constant on the upper interval, (29) requires (28).

The three values, a^*, r_0^*, and r_0^*, are jointly determined by three conditions. The first is the steady-state condition (24) evaluated at a^* with $W_0(a^*) = 1$:

$$a^* = \left(\frac{1-\beta_0}{\beta_0}\frac{\delta}{\delta-1}r_0^*\right)^{\theta}. \tag{A.12}$$

The second is partial equilibrium condition (18) with $W < 1$ or, equvalently, the equality between (A.4) and the ratio, (A.5) divided by (A.6). The latter equality is reorganized to yield

$$\frac{\delta-1}{\delta} + \frac{1-\beta_0}{\beta_0}r_0^* = \omega\left(\frac{\delta-1}{\delta} + \frac{\eta-1}{\eta}r_1^*\right), \tag{A.13}$$

since $R_i(a)W_i(a) = r_i^*$ for $i = 0, 1$ and all $a^* \leq a \leq a_1$. The values in (A.13) satisfy the inequality: $r_0^* < r_1^*$ if $\beta_0 \leq \beta_1$ or, equivalently, $(1-\beta_1)/\beta_1 \geq (\eta-1)/\eta$. The third is the steady-state condition (25) reorganized to yield

$$r_1^* = \frac{\kappa_0}{\phi}\left(\frac{\delta-1}{\delta} + \frac{\eta-1}{\eta}r_1^*\right)^{\delta(\eta-1)/(\delta\eta-1)} \tag{A.14}$$

with the constant value $r_1 w_1 = r_1^*$. In (A.14) the new constant is

$$\phi = \left(\frac{\eta-1}{\eta}\right)^2\left(\frac{\eta}{\eta-1}\frac{\gamma\delta}{\iota}\lambda_1\right)^{(\delta-1)/(\delta\eta-1)}$$

with

$$\lambda_1 = \left[\frac{\nu}{\gamma\delta\,(\eta-1)}\left(\frac{\eta-1}{\eta}\mu_0\right)^{\eta}\right]^{\delta/(\delta-1)}$$

from above (A.2) and $(1-\beta_1)/\beta_1 = (\eta-1)/\eta$.

Calibration: Set $\beta_0 = .5$, $\delta = 3$, and $s_0/s_1 = 8$. In this case, the empirical constraint (30) becomes

$$r_1^* = .695 r_0^* \left(\frac{\nu}{.5^\nu}\right)^{1/(\eta-1)}.$$

Insert this empirical constraint (30) into the partial equilibrium condition (A.13) and solve the resulting equation for r_0^*. This yields the value:

$$r_0^* = 1.7008\,(1-\omega)\,.$$

The selected steady state a has the ratios: $R_0(a) = 1$ and $R_1(a) = 8$. With these values the steady-state equations, (27) and (28), require that

$$r_0^* + \frac{r_1^*}{8} = 1.$$

Combining the above three equations produces the critical equation:

$$1 = 1.7008\,(1-\omega)\left[1 + .087\left(\frac{\nu}{.5^\nu}\right)^{1/(\eta-1)}\right].$$

It generates the values of the tail index $1/\eta$ reported in Table 4. The corresponding values for the truncated means, μ_0 and $\mu_1 = \mu_0 \nu^{1/\eta}$, follow from the calibrated entry condition (31).

Nash bargaining between buyers and builders: Add to the previous subscript, $i = 0, 1$, the new subscript, $j = 0, 1$, signifying, respectively, no bargaining by builders and Nash bargaining by builders. With this new notation the previous results are identified by the subscripts, 0 or $i0$. The analogous results with Nash bargaining by builders have the respective subscripts, 1 or $i1$. Set $\beta_1 = \beta_0$.

With Nash bargaining by builders, only the following results are altered. The reservation value in (9) is replaced by the analogue to (2):

$$x_{11} = v^B + v_{01}^S.$$

The expected value of search (A.3) with $i = 1$ is replaced by its analogue to (A.3) with $i = 0$:

$$v_{11}^S = \frac{\gamma\delta}{\iota}\frac{1-\beta_0}{\beta_0} r_1 w_{11} \lambda_1 x_{11}^{-\chi}.$$

The new constant in the above equation is

$$\lambda_{11} \equiv \left(\frac{\beta_0}{\gamma\delta}\frac{\zeta_1^\eta}{\eta-1}\right)^{\delta/(\delta-1)}.$$

The equalities, (A.4) and (A.6), are replaced by

$$\frac{x_{01}}{x_{11}} = \left(\frac{\lambda_0}{\lambda_1}\right)^{1/\chi} = \nu^{-1/(\eta-1)}.$$

and

$$x_{11} = \left[\lambda_1 \frac{\gamma}{\iota}\left(\delta - 1 + \frac{1-\beta_0}{\beta_0}\delta r_1 w_{11}\right)\right]^{1/(\chi+1)}.$$

With these substitutions the constant (16) and weighting function (17) have the replacements:

$$\omega_1 \equiv \nu^{-1/(\eta-1)}$$

satisfying $0 < \omega_1 < 1$ and

$$W_1(r_0, r_1) \equiv \frac{r_0 + \frac{\beta_0}{1-\beta_0}\frac{\delta-1}{\delta}(1-\omega_1)}{r_0 + \omega_1 r_1}.$$

The relative probability of trade (20) is replaced by the ratio:

$$\frac{y_{01}}{y_{11}} = \nu^{1/(\eta-1)} > 1.$$

Finally, the price (A.8) is replaced by the analogue to (A.7):

$$\bar{p}_{11} = \frac{1-\beta_0}{\eta-1} x_{11} + v_{11}^S > 0.$$

These values are used in the numerical calculations for the case with bargaining by builders.