Partial enclosure of the commons☆

Christopher Costello[a], Nicolas Quérou[b], Agnes Tomini[c]

[a] *Corresponding Author: Bren School, 4410 Bren Hall, Santa Barbara, CA 93117, NBER, and Visiting Professor LAMETA, Montpellier, France. costello@bren.ucsb.edu. 805-893-5802.*
[b] *CNRS, LAMETA, Montpellier, France*
[c] *LAMETA, Montpellier, France*

Abstract

We examine the efficiency, distributional, and environmental consequences of assigning spatial property rights to *part* of a spatially-connected natural resource, a situation which we refer to as *partial enclosure of the commons.* Our theoretical approach addresses an empirically extensive class of institutions and natural resources for which complete enclosure by a sole owner may be desirable, but is institutionally impractical. When a sole owner is granted ownership of only a fraction of the spatial domain of the resource, so the remainder is competed for by an open access fringe, interesting spatial externalities arise. We obtain sharp analytical results regarding partial enclosure of the commons including: (1) While second best, it always improves welfare relative to no property rights, (2) all resource users are made better off, (3) positive rents arise in the open access area, and (4) the resource maintains higher stocks. Under spatial heterogeneity, we also characterize spatial regions that are ideal candidates for partial enclosure - typically, society should seek to enclose those patches with high environmental productivity and high self-retention, but whether high economic productivity promotes or relegates a patch may depend on one's objective. These results help inform a burgeoning trend around the world to partially enclose the commons.

Keywords: Incomplete property rights, natural resources, common property, spatial externalities, dynamic games

JEL classification: H2, H4, H7, Q2

☆We thank Gary Libecap, Robert Deacon, Martin Quaas, Vincent Martinet and seminar participants at Free University (Amsterdam), Lameta (Montpellier, France) and Institut Henri Poincaré (Paris) for helpful suggestions.

1. Introduction & Background

The economics profession has established incontrovertibly that open access or common pool management of natural resources leads to economic inefficiencies, and possibly environmental disaster (Hardin, 1968; Ciriacy-Wantrup and Bishop, 1975; Ostrom, 1990; Maler, 1990). A large body of evidence highlights mismanagement of fisheries, pastures, forests, groundwater, pollution sinks (Stavins, 2011), antibiotic resistance (Laxminarayan and Brown, 2001), among other tragedies of the commons. A massive literature has emerged that proposes economic instruments as solutions, including taxes (Rubio and Escriche, 2001; Rosenman, 1984; Weitzman, 2002), effort restrictions (Benchekroun, 2003; Quérou and Tomini, 2013), fully-delineated property rights (Costello and Deacon, 2007), tradeable permits (Montgomery, 1972; Stranlund and Dhanda, 1999; Raffensperger, 2011), and spatial zoning with taxation (Sanchirico and Wilen, 2005) or with unitization (Kim and Mahoney, 2002; Kaffine and Costello, 2011). Under certain conditions, each of these instruments has benefits and may even 'solve' the tragedy of the commons, and provide first-best outcomes.[1] Yet issues of wealth redistribution (Brito et al., 1997), heterogeneity (Karpoff, 1987), high political and economic costs (Johnson and Libecap, 1982), and other practical issues (Besley, 1995) may impede the performance of such instruments and may explain why we rarely observe these instruments being implemented in their pure form as economic models would suggest. Instead, we tend to observe hybrids where only part of the resource is subsumed within a market structure. Indeed the failure of many natural resource management institutions has been explained by the potential mismatch between the spatial scale of management and that at which environmental processes operate. For instance, Scott (1955) states that "the property must be allocated on a scale sufficient to insure that one management has complete control of the asset." Yet this is rarely the case for resources such as water, hunting game, fisheries, oil, and forests. A much more common institutional regime in both developing and developed countries is to assign property rights to a fraction of the natural resource, often leaving the remainder of the resource to be competed for by an open access fringe. In practice, decision makers are often able to partially enclose the commons, even when political, legal, or other constraints render it impossible to assign complete ownership of the entire resource to a sole owner.

We refer to this situation as 'partial enclosure of the commons,' and note as a starting point that this institution will not internalize all externalities, and will thus be a second best alternative to sole ownership of the entire resource domain.[2] While

[1]The OECD report (Le Gallic, 2006) provides a survey of many market-like instruments used to solve these problems.

[2]Taxes are a possible alternative, but a tax in only one region (analogous to partial enclosure)

the owner of the enclosed area may behave somewhat like a sole owner, resource mobility induces a spatial externality, so the open access fringe influences the enclosed area and may affect the enclosed owner's behavior and vice versa. Despite notable advances on the use of economic policies to internalize various externalities, the literature on *partial* property rights is sparse, and the use of partial enclosure remains an unresolved issue. We aim at filling this gap by analyzing the efficiency, distributional and environmental consequences of its application. Under what conditions will assigning rights in this way achieve economic, distributional, or environmental improvements over the pure open access case? And if we are to proceed with partial enclosure of the commons, what guiding principles can be generated to design these institutions? The remainder of this paper is devoted to addressing these, and related questions.

We illustrate the ubiquity of partial enclosure by providing some examples. The world's oceans are perhaps the most compelling illustration: 58% of the ocean constitutes the high seas, which are effectively open access. The remainder is delineated to individual states in exclusive economic zones; this is a classic partial enclosure of the commons. Even though rights are fully delineated within the exclusive economic zones, migrating fish such as tuna and billfish traverse these jurisdictions and are subject to harvest on the high seas by the open access fringe. But even species that are reasonably immobile (such as reef fish) are often exploited by different fleets (e.g. commercial vs. recreational), typically where one fleet has well-defined property rights and the other acts in an open access fashion. Partial enclosure of the commons is even more commonplace for groundwater and oil reserves facing uncoordinated exploitation levels in each area. Like fish, those resources are mobile (extraction in one location induces a flux). Rights to groundwater and oil stocks are often related to spatial property rights at the surface,[3] the spatial delineation of which almost never accords with the spatial domain of the underlying resource. Game, such as deer and waterfowl, have characteristics similar to fish - they migrate and are only partially enclosed on private lands. In Africa, great migrations of wildebeest, zebra, and other hunted species traverse private lands, public lands, and even natural reserves. Even some forest resources share these characteristics. Communities are often granted exclusive ownership

would be second best. Indeed, it is proven in Sanchirico and Wilen (2005) that a first best outcome would require the use of spatially differentiated taxes (one for each region).

[3]The "rule of capture" of groundwater and mineral resources in the United States is historically based on the concept that each landowner has complete ownership of resources under his land, and has an unlimited right to use them. This *Absolute Ownership Doctrine* has led to over-exploitation issues in areas where the number of users has grown so that the use of the resource, even if it is limited by land ownership, gets close to that of an open access outcome. It is now commonly rejected because of the existing diffusion/dispersal process of the resource.

over a tract of forest land, where the remainder of the forest is open to others. To the extent that actions outside the tract influence it (e.g. excessive harvest outside may reduce seed dispersal or increase erosion inside), or actions inside affect the open access area, the 'partial enclosure' prerequisites hold. This institutional arrangement is thus characterized by spatial externalities (external effects on adjacent areas) diffusing the institutional effect of partial enclosure by a single owner due to the mobility of the resource. The literature has not assessed the effect of such asymmetric property rights regimes on a spatially connected resource. Yet it seems that this arrangement addresses an empirically extensive array of cases. We focus on the effects of spatial externalities and heterogeneity on welfare, and in the characterization of the optimal siting of partial enclosure for renewable resources.

To address these questions we develop and analyze a model of partial enclosure of the commons. The model is simple enough to maintain analytical tractability, but contains all of the components essential to describe this ubiquitous institutional arrangement. It is meant to be generically applicable to a wide range of natural resources with certain characteristics. The dynamics of a natural resource are both temporal and spatial. Across time, the natural resource can grow (or shrink) depending on the level of extraction and the degree of regeneration which may, itself, depend on the level of resource stock. The resource also moves across space. We model space as a set of mutually-exclusive and exhaustive patches and keep track of natural resource stock in each patch.[4] Any given patch may be unregulated (i.e. open access - a situation in which current economic returns govern entry, exit, and extraction) or may be managed by an owner who maximizes her private benefits. Owing to spatial movement, behavior in the open access region has important consequences for the sole owner, and vice versa. The ensuing spatial and temporal externalities represent a potentially damaging market failure that induce a dynamic spatial game across patches with different characteristics. We solve this problem and explore its consequences.

The existing literature tends to consider natural resources as perfectly enclosed by one or more owners; thus property right definition is not an issue. Indeed, an extensive differential game literature analyzes non-cooperation between owners of common pool natural resources (a seminal paper is Levhari and Mirman (1980); a recent advance is Pintassilgo et al. (2010)). In that literature, a small number of fishermen compete against one another in a closed game (there is no open access fringe) with only implicit spatial dynamics, and fishermen behave non-cooperatively. In our model, we explicitly treat spatial externalities and resource dynamics, we implicitly assume perfect cooperation within the partial enclosure, but we allow for non-cooperation between the partial enclosure and a (spatially

[4] In the biological sciences, this is referred to as a "metapopulation."

connected) open access fringe of arbitrarily large size.

To the best of our knowledge, only a single existing paper addresses the issue of partial enclosure of natural resources (Fisher and Laxminarayan, 2010).[5] It focuses on uncertainty, instrument choice, and the congestion problem resulting from the enclosure of some resource pools on other open-access resource pools. By contrast, we investigate whether partial enclosure may increase (aggregate or patch-specific) resource stock levels and/or aggregate economic value (or individual profits). We highlight the influence of spatial externalities and environmental heterogeneity on the optimal assignment of partial property rights. The importance of spatial effects has been documented in the literature on learning externalities and agglomeration economies: it is emphasized how investment decisions made by one agent may influence others who learn from his experience (Lucas, 1988; Porter, 1998). In our setting, it is the physical diffusion or dispersal of the resource across space that gives rise to interesting spatial externalities. Given environmental heterogeneity, this diffusion effect may have different impacts from one region to another, emphasizing the importance of careful selection of the region in which property rights will be assigned; this is a central focus of our analysis.

The remainder of the paper is organized as follows. The model and results are provided in Section 2. Results are illustrated in an example in Section 3, and Section 4 concludes.

2. Model & Results

A natural resource stock (denoted by x) is distributed heterogeneously across a discrete spatial domain consisting of N patches. Patches may be heterogeneous in size, shape, economic, and environmental characteristics, and resource extraction can potentially occur in each patch. The only requirement for spatial delineation is that patches must be homogeneous intra-patch; all environmental and economic variables are constant within each patch. The resource is mobile and can migrate from patch to patch. In particular, denote by $D_{ij} \geq 0$ the (constant) fraction of the resource stock in patch i that migrates to patch j in a single time period. Time is treated in discrete steps. The resource may grow, and the growth conditions may be patch-specific denoted by the parameter α_j. This patch-specific parameter

[5] Colombo and Labrecciosa (2013) analyze the oligopolistic exploitation of a productive asset under private and common property. They assume that, under private exploitation, the resource is parceled out. Each firm owns and manages the assigned parcel over the entire planning horizon. Thus, fully delineated property rights exist over the entire domain of the resource. As such, they abstract from situations where the resource is fully mobile, and do not analyze (as we do) the impact of spatial externalities and environmental heterogeneity on the assignment of partial property rights.

reflects resource growth and has many possible interpretations including intrinsic rate of growth, carrying capacity, and the sheer size of the patch. Assimilating all of this information, the equation of motion, in the absence of extraction, is given as follows:

$$x_{it+1} = \sum_{j=1}^{N} D_{ji} g(x_{jt}, \alpha_j). \tag{1}$$

Here resource production in patch i is given by the general production function $g(x_i, \alpha_i)$; we follow the literature and require that $\frac{\partial g(x,\alpha)}{\partial x} > 0$, $\frac{\partial g(x,\alpha)}{\partial \alpha} > 0$, $\frac{\partial^2 g(x,\alpha)}{\partial x^2} < 0$, and $\frac{\partial^2 g(x,\alpha)}{\partial x \partial \alpha} > 0$. We also assume that extinction is absorbing ($g(0;\alpha) = 0$) and that the growth rate is finite ($\frac{\partial g(x,\alpha)}{\partial x}|_{x=0} < \infty$). All standard biological production functions are special cases of $g(x, \alpha)$.[6] The resource stock that is produced in patch i then disperses across the spatial domain: some fraction stays within patch i (D_{ii}) and some flows to other patches (D_{ij}). Indeed, some may flow out of the system entirely, so the dispersal fractions need not sum to one: $\sum_j D_{ij} \leq 1$.

Because this is a discrete-time model, we must specify the timing of events. For patch-i harvest h_{it}, the *residual stock*[7] left for reproduction is given by $e_{it} \equiv x_{it} - h_{it}$. Including harvest, the patch-i equation of motion becomes:

$$x_{it+1} = \sum_{j=1}^{N} D_{ji} g(e_{jt}, \alpha_j). \tag{2}$$

The timing is thus: the present period stock (x_{it}) is observed and then harvested (h_{it}) giving residual stock (e_{it}), which then grows ($g(e_{it}, \alpha_i)$), and disperses across the system (D_{ij}). This timing is consistent with the literature on discrete-time renewable resources (e.g. see Reed (1979) and Weitzman (2002)).[8] By the identity $e_{it} \equiv x_{it} - h_{it}$, there is a duality between choosing *harvest* and choosing *residual stock* as the decision variable. We use residual stock because it turns out to overcome many technical issues that arise when one uses harvest as the decision variable.[9] By adopting residual stock as the control, we are able to fully characterize the optimal policy; one can then back out the optimal harvest. This in turn will enable us to provide a precise assessment of partial enclosure as an institutional arrangement.

[6]The example in Section 3 provides a concrete illustration.

[7]In the fisheries literature, *residual stock* has been coined *escapement*.

[8]Specifying different timing may affect analytical tractability. For example, if dispersal occurred before growth, the problem would be much more difficult to solve analytically, but it would still be the case (as it is in our analysis) that optimal escapement in the enclosed patch will depend on escapement in other patches.

[9]This mathematical convenience was pointed out in Reed (1979) and has been adopted by several subsequent contributions (Costello and Polasky (2008), Kapaun and Quaas (2013) among others).

Patch-i harvesters earn constant price p_i per unit harvest and marginal harvest cost is a non-increasing function of resource stock in patch i. Harvesting patch i to residual stock level e_{it} results in profit given by:

$$\Pi_{it} = p_i(x_{it} - e_{it}) - \int_{e_{it}}^{x_{it}} c_i(s)ds \tag{3}$$

where $c_i'(s) \leq 0$. Total cost is given by the integral on the right hand side of Equation 3. It implies that the first units harvested in a period are the cheapest to access because the resource is at its densest prior to harvesting. It also implies (if $c'(s) < 0$) that marginal harvest costs increase in harvest, though this occurs not because units of harvest are increasingly costly per se, but because the resource becomes less dense and takes more effort to locate and extract (see Weitzman (2002) and Reed (1979)). We examine both the linear cost ($c'(s) = 0$) and the non-linear cost ($c'(s) < 0$) cases.

2.1. Open Access Benchmark

In the absence of property rights, resource users are able to costlessly access all patches to seek short-run profit, Π_{it}, given by Equation 3. As such, extraction effort will gravitate in any period to the patches with the highest marginal profit. If cost is linear, so $c'(s) = 0$, we will assume that $c_i(s) < p_i$ (otherwise, the problem is uninteresting because no harvest would ever occur, even under open access). In that case, it is profitable to extract the entire resource, so effort will enter until the stock is extinguished in a patch, so $\hat{e}_{it} = 0$. If this is the case for all patches, the entire resource will be mined to extinction. Instead, if marginal extraction cost depends on resource density, so $c'(s) < 0$, then we assume interior solutions, so effort will enter patch i until marginal profit is zero, i.e. until $p_i = c_i(\hat{e}_{it})$ were $\hat{e}_{it}$ denotes the residual stock in patch i when all patches are under open access. In this case the stock will never be completely exhausted, even under open access. Rather, in each patch the stock will be extracted down to a level where it becomes unprofitable to extract further. In patches for which this level is positive, stock will grow and redistribute spatially according to Equation 2. Thus, under pure open access of this spatial resource, we have the following benchmark results:

Proposition 1. *Open access residual stock level in patch i satisfies $p_i = c_i(\hat{e}_{it})$ when $c'(s) < 0$ and $\hat{e}_{it} = 0$ otherwise.*

Proof. All proofs are provided in the appendix □

In the next section we analyze the case in which exclusive property rights are assigned over part of the spatial domain of the resource.

2.2. Partial Assignment of Spatial Property Rights

We implement partial enclosure by assigning exclusive property rights over a single patch to a single owner while the other $N-1$ patches remain open access. Because the carrying capacity, or size, of the patch is arbitrary, this procedure allows us to examine enclosing an arbitrarily large fraction of the resource domain, leaving the remainder to the open access fringe. Partial enclosure induces a potentially complicated dynamic spatial game between the owner of the enclosed patch and the adjacent open access fringe areas, which are connected to the enclosed patch through the system dynamics. More specifically, assuming that patch i is the *enclosed* patch, the optimal policy function $e_{it} = \phi_i(t, x_{1t}, x_{2t}, ...)$ is potentially time and state dependent. The enclosed patch owner's economic objective is to maximize the expected net present value of harvest, expressed in terms of residual stock level, from patch i over an infinite horizon:

$$\max_{\{e_{it}\}} \sum_{t=1}^{\infty} \delta^t \left[p_i(x_{it} - e_{it}) - \int_{e_{it}}^{x_{it}} c_i(s) ds \right]. \tag{4}$$

Henceforth, all of the analysis will rely on solving the discrete-time difference game that is induced by the patchiness of spatial ownership. Because we focus on the residual stock as the control variable in each spatial patch, and owing to some useful characteristics of the economic environment, this problem can be solved analytically. We can immediately write down an implicit expression defining the residual stock level in every patch. These levels are summarized as follows:

Lemma 1. *When patch k is enclosed, and the other $N-1$ patches remain open access, the equilibrium residual stock levels are:*

$$\begin{aligned} &e_{it} \text{ given by Proposition 1 } \textit{for}\, i \neq k \\ &p_k - c_k(\bar{e}_{kt}) = \delta D_{kk} \left[p_k - c_k(\bar{x}_{kt+1}) \right] g_e(\bar{e}_{kt}, \alpha_k). \end{aligned}$$

We will adopt a notation convention that a bar over a variable (e.g. $\bar{e}_{kt}$) represents an enclosed patch while a variable without a bar represents an open access patch. Lemma 1 shows that the residual stock level takes just two possible forms. In the open access patches, *harvest* will respond to behavior in the other patches, but *residual stock* will not. Each open access patch is harvested, in each period, down to the open access residual stock level - i.e. where harvesting the next unit of resource stock would entail an economic loss. In the *enclosed* patch, the owner takes as given the behavior of the open access fringe and realizes that she will not be the residual claimant of any conservative harvesting behavior. Thus, she behaves as if any additional resource that disperses out of her patch will be lost. This is why the only dispersal term to enter the optimal residual stock term is D_{kk}: the fraction of the resource that remains in the enclosed patch.

As an extreme point of comparison, we can also consider the problem in which sole ownership is assigned over the entire spatial domain so there is no open access fringe. While this problem has been studied elsewhere, we provide the sole owner's first order conditions as a point of comparison for Lemma 1. Following Costello and Polasky (2008) the complete sole owner chooses residual stock in patch j as follows:

$$p_j - c_j\left(e^*_{jt}\right) = \delta \sum_{l=1}^{N} D_{jl}\left[p_l - c_l\left(x_{lt+1}\right)\right] g_e\left(e^*_{jt}, \alpha_j\right). \tag{5}$$

Equation 5 leads to a very different residual stock than is implied by Lemma 1. Indeed, the right hand side of the Equation 5 highlights that the sole owner would account for the spatial connection to all patches (D_{jl} $\forall l$) not just the fraction of the resource that stays in patch j (D_{jj}).

Now, relying on Lemma 1 and following Costello and Polasky (2008), it turns out that the residual stock level in every patch is constant; this is summarized by the following:

Lemma 2. *The equilibrium harvest strategy for all patches under* partial enclosure *is for all patches to harvest down to a pre-determined residual stock level that is time and state independent.*

Lemma 2 is extraordinarily useful. Normally, we would expect the optimal residual stock level to be a (possibly time varying) feedback control rule that mapped the state (all possible combinations of resource stock levels in all N patches) into the residual stock. Then finding the equilibrium across all patches would require solving a complicated N-dimensional system. Indeed, if we had specified *harvest* as the control variable, this would be the case. But since the marginal profit and marginal growth conditions depend only on the residual stock level, and not on resource stock, then Lemma 2 obtains, which dramatically simplifies the dynamic game. However, since growth, dispersal, and economic returns can vary across patches, the optimal choice will, in general, vary across space too.

2.3. Welfare & Distribution

Accounting for the behavior characterized in Section 2.2, we are initially interested in the consequences of partial enclosure on resource stocks. Because partial enclosure is second best, it is possible that perverse outcomes arise. Is it possible, for example, for partial enclosure, combined with spatial connectivity, to lead to lower natural resource stocks than under pure open access? We can unambiguously answer this question: it turns out that enclosing any patch will always increase resource stock. This is formalized below:

Proposition 2. *Partial enclosure of* any *patch (weakly) increases resource stock in all patches and strictly increases stock in at least one patch.*

Proposition 2 accords with economic intuition. When moving from a system of pure open access to a system in which some fraction of the resource is enclosed by a single owner, that owner will find it optimal to maintain a larger resource stock in her own patch. The adjacent open access patches clearly benefit from this behavior - to the extent that some of this increased resource spills over into adjacent patches, they are residual claimants of this behavior. Thus, the stock rises (weakly) everywhere.

Partial enclosure also has important consequences for profit as is formalized below:

Proposition 3. *Partial enclosure of* any *patch provides a strict Pareto-Improvement (i.e. increases profit in at least one patch without decreasing profit in any patch).*

Proposition 3 is a political economy result. It suggests that all resource users are made (weakly) better off from partial enclosure, even when their patch is not the one that is enclosed. It is instructive to consider three kinds of resource users. First, those users in the open access fringe whose stock is not spatially connected to the enclosed patch stock will be unaffected by the partial enclosure. Second, those users in the open access fringe whose patches are connected to the enclosed patch will see their stocks rise as a consequence of partial enclosure (Proposition 2). Finally, profit in the enclosed patch itself will increase. While we do not model the intra patch process of distributing this wealth among the (previously open access) users in the (now enclosed) patch, we note that each user can, in principle, be made strictly better off.[10]

An important corollary to Proposition 3 immediately emerges. Because the enclosed patch owner has the incentive to raise resource stock in every period, and because some of that resource spills over to the open access sector every period, the presence of *partial enclosure* guarantees positive profits in equilibrium for the open access sector, as is formalized below:

Corollary 1. *Profit to the open access fringe increases as a consequence of partial enclosure, even when extinction is optimal by the open access fringe.*

While Corollary 1 holds for any cost function, it is most striking when $c'(\cdot) = 0$ in which case all open access patches drive the stock extinct (see Proposition 1).

[10]On the other hand, it is also possible that the previous open access users will be left out without compensation, in which case they would suffer a loss from the partial enclosure.

Enclosure of any single patch k induces that patch owner to increase residual stock which bestows a positive externality on all patches j for which $D_{kj} \neq 0$. The result focuses on the positive rents accruing to open access patches when moving from a fully open access to a partial sole ownership management structure. This result is not surprising given our setup: Here, positive inframarginal rents can persist in open access. These accrue to resource extractors who intercept the stock while it is most dense.[11]

We have shown that even if exclusive ownership can be implemented only over a small part of the spatial domain, some positive effect is expected on resource stocks and all agents' profits. These results arise from behavioral adaptations: when part of the spatial domain is enclosed, behavioral changes (by the enclosed patch owner and the open access sector) ensue.

Before concluding this section, we examine the role of key spatial, environmental, and economic parameters on these behavioral changes. When patch i is enclosed, increasing self-retention (D_{ii}) or growth (α_i) causes the enclosed patch owner to increase her optimal residual stock. In contrast, increasing price (p_i) causes her to decrease residual stock. Increasing D_{ii} or α_i increases the rate of return to owner i from a larger residual stock, thus she favors a larger stock. The effect of price is more subtle: as it increases, the benefit of increased harvest (and thus lower residual stock) turns out to outweigh the future benefit of higher residual stock. These results are summarized below and we will make extensive use of them in the remainder of the analysis[12]:

Lemma 3. *When patch i is enclosed:*

$$\frac{d\bar{e}_i}{dD_{ii}} \geq 0, \quad \frac{d\bar{e}_i}{d\alpha_i} \geq 0, \quad \frac{d\bar{e}_i}{dp_i} \leq 0.$$

These results will prove useful in the optimal siting of partial enclosure of the commons.

2.4. Siting Partial Enclosure

Thus far we have focused on the welfare and environmental effects of partial enclosure of the commons. In many real-world contexts a government, NGO, or private agent has the opportunity to enclose part of the commons but for legal, institutional, or other practical reasons, cannot enclose the entire resource and grant it to a sole owner. For example, current initiatives by development banks

[11]We abstract away from the additional possibility of a *race* to intercept the stock early in the season; that possibility is addressed by Deacon et al. (2013).

[12]Due to Lemma 2, we have omitted the subscript t in the expressions of residual stocks and stock levels.

(such as the World Bank), NGOs (such as The Nature Conservancy or Rare), and countless local communities, often in developing countries with little or no existing formal governance, involve siting decisions where decision-makers must determine which areas to enclose. We have shown that even haphazard siting decisions will improve welfare and conservation. But there may be considerable differences across sites: enclosing the "right" patch may lead to substantially larger welfare gains, or may give rise to important distributional or conservation effects, than enclosing the "wrong" patch. This section is devoted to analyzing the characteristics of patches that are good candidates for enclosure.

Which patch to enclose may depend on one's objective. For example, many resource conservation groups may advocate partial enclosure, say of a fishery or a forest area, to protect the natural resource itself, often arguing that protecting the resource stock is a first step toward enhancing local, or aggregate, returns. If the objective is to site the enclosure to maximize equilibrium resource stock, we can use the model to derive conditions that define the optimal candidate for enclosure. Indeed, inspecting the expression characterizing the evolution of stock levels, it turns out that the difference $g(\bar{e_i}, \alpha_i) - g(e_i, \alpha_i)$ determines which patch will maximize aggregate resource stock. The decision-maker should enclose the patch with highest such difference (see Appendix for proof). Of course, calculating this difference requires calculating $\bar{e_i}$ and e_i which, in turn, are implicitly defined by economic returns and environmental parameters. Thus, even though this condition has a straightforward interpretation, it is difficult to use it in order to assess the effect of key model parameters on optimal enclosure siting. Furthermore, there is no guarantee that maximizing resource stock will coincide with maximizing profits.

Thus, we will seek to determine the role of key model parameters on the optimal siting of partial enclosure by adopting the following strategy: We define a benchmark situation in which all parameters are held equal across patches, thus enclosure does not favor any particular patch. Moving from this situation, we then increase the value of a single parameter in a particular patch, and assess the impact of the change on economic and environmental objectives across the entire spatial domain. Enclosing any given patch will give rise to dynamics throughout the system, which, in principle, affect all patches (including the enclosed patch), so the entire system's response to enclosure must be accounted for. This approach allows us to derive concrete conclusions about optimal enclosure siting while isolating the effects of any given parameter. We derive guiding principles for optimal enclosure under each possible objective.

In the benchmark case, $D_{ii} = D$, $D_{jk} = Q$, $\alpha_i = \alpha$, $p_i = p$, and $c_i(\cdot) = c(\cdot)\ \forall\ i$ and for $j \neq k$. Starting from this situation, patches are indistinguishable so there is no preference for enclosing any particular patch. Without loss of generality, we assume that a single parameter (either D_{11}, α_1, or p_1) increases in patch 1 and we calculate the comparative static effects given all possible enclosures. Because parameters are identical across all other patches j ($j \neq 1$), we need only calculate the effects of changing parameters in patch 1 when patch 1 is enclosed and when some other patch (we choose patch 2) is enclosed. Thus, under an increase in a single parameter in patch 1, we examine the following cases: (1) Patch 1 is enclosed and all $N-1$ other patches remain open access, and (2) Patch 2 is enclosed and all $N-1$ other patches remain open access.

This procedure allows us to determine, ceteris paribus, whether an increase in self retention (D), growth (α), or price (p) will favor, or relegate, a given patch for enclosure. We analyze optimal enclosure under four possible objectives: (1) maximize resource stock in the enclosed patch, (2) maximize aggregate stock, (3) maximize profit in the enclosed patch, and (4) maximize aggregate profit. For each objective, we compute the difference between the payoff when enclosing patch 1 and the payoff when enclosing patch 2. If that difference is positive, then a higher value of the parameter in patch 1 promotes the patch for enclosure (because enclosing patch 1 is preferred to enclosing patch 2). If the difference is negative, then a higher value of the parameter relegates the patch for enclosure (because enclosing patch 2 is preferred to enclosing patch 1).

Aside from the determination of optimal siting of an enclosure, we are interested in the conditions under which these four objectives are consistent or contradictory: If enclosure is sited to maximize local benefits, will this also maximize benefits system-wide? And if enclosure is sited to maximize resource stock, will this also maximize profit?

While our model permits any non-increasing $c(\cdot)$, we begin with the case of linear harvest cost, so $c'(\cdot) = 0$.[13] In that case, the open access patches always harvest the entire local resource stock in each period. Though by Proposition 2 the resource stock is positive in equilibrium - this is because the enclosed patch acts as a donor for all connected patches; provided that $Q > 0$, it supplies all patches with a surplus stock from which to harvest each period. While this case is not trivial to examine, the dynamics are somewhat muted because changes in system parameters do not change the optimal residual stock in the open access patches. Furthermore, the first order condition for the enclosed patch no longer relies on price or cost, which simplifies the comparative statics of behavioral changes.

2.4.1. Self-retention, D_{11}

We begin by assessing the effects of increasing the self-retention parameter D_{11} in patch 1 on the optimal siting of partial enclosure. Higher self-retention allows

[13] The linear cost case is extensively used in resource economics literature. While we find that most of the economic insight can be derived in a more straightforward manner by adopting this assumption, in Section 2.5 we re-derive all of our results when $c'(\cdot) < 0$.

patch 1 to retain a greater fraction of its growth. If patch 1 is open access (so patch 2 is enclosed), then there is no behavioral shift in patch 1 from the increase in D_{11} - all residual stocks are unchanged. But if patch 1 is enclosed, owner 1 will increase her residual stock to take advantage of higher retention (see Lemma 3). Proving that profits are larger when patch 1 is enclosed (compared to enclosing some other patch) relies on an envelope theorem: to analyze the total derivative of profit in patch 1 we can ignore the behavioral shift, and can instead focus only on the direct influence of D_{11} on profit. This effect is clearly positive since all non-enclosed patches have zero residual stock. Taking all of the dynamics into account, we find that if costs are linear, then across all four objectives, a higher value of D promotes a patch for enclosure. This result is formalized below:

Proposition 4. *Provided $c'(\cdot) = 0$, a higher value of self-retention (D_{11}) in patch* 1 *has the following effects:*

1. *(Enclosed Patch Stock): Patch 1 is the best candidate for enclosure.*
2. *(Aggregate Stock): Patch 1 is the best candidate for enclosure.*
3. *(Enclosed Patch Profit): Patch 1 is the best candidate for enclosure.*
4. *(Aggregate Profit): Patch 1 is the best candidate for enclosure.*

Proposition 4 shows that ceteris paribus, across all four objectives, the optimal patch to enclose is the patch with higher self-retention. Thus there is strong consistency between optimal enclosure siting for individual benefit and optimal enclosure siting for aggregate benefit. There is also strong consistency between conservation objectives (i.e. maximizing resource stock) and economic objectives (maximizing profit).

2.4.2. Growth, α_1

Our resource growth model from Section 2 is quite general and permits a wide range of interpretations for the parameter α_i. Regardless of the interpretation, we can think of higher α_1 as representing *improved* growth conditions in patch 1. Here we examine the role of α_1 on optimal enclosure siting: will improved growth conditions in a patch promote it as a candidate for enclosure? If patch 1 is open access (so patch 2 is enclosed), there will be no adjustment in residual stock in patch 1 because $p_1 = c_1(e_1)$. On the other hand, if patch 1 is enclosed, then the increase in α_1 leads to an unambiguous increase in residual stock in 1 (see Lemma 3). This also has positive externalities on adjacent patches. Again, in the case of linear cost we will prove that across all four objectives, a higher value of α_1 promotes patch 1 for enclosure. The result is formalized as follows:

Proposition 5. *Provided $c'(\cdot) = 0$, a higher value of growth (α_1) in patch* 1 *has the following effects:*

1. *(Enclosed Patch Stock): Patch 1 is the best candidate for enclosure.*
2. *(Aggregate Stock): Patch 1 is the best candidate for enclosure.*
3. *(Enclosed Patch Profit): Patch 1 is the best candidate for enclosure.*
4. *(Aggregate Profit): Patch 1 is the best candidate for enclosure.*

Proposition 5 reveals that α has the same effects as D on optimal siting of the enclosure: Higher growth always promotes a patch as an ideal candidate for partial enclosure regardless of one's objective.

We noted above that one convenient interpretatioin of α_i is *patch size*. Under that interpretation, the requirements stated below Equation 1 still hold and we can think of α as the carrying capacity of the underlying resource. When combined with Proposition 5, this observation yields a very useful insight: Regardless of one's objective, larger patches are always better candidates for partial enclosure (provided costs are linear). This makes intuitive sense because in the extreme, one would like to enclose the entire resource domain.

2.4.3. Market price, p_1

Finally we consider the effect of an increase in the price in patch 1, p_1, on optimal enclosure siting. A price increase in a single patch can have complex and far-reaching effects on stock and profit because both the enclosed patch owner and the open access patches may adjust residual stock. However, when $c'(\cdot) = 0$ (linear cost), neither the open access patch nor the enclosed patch will change residual stock following a price rise. Thus, residual stock is unaffected by the decision about which patch to enclose. However, profit is affected by the price rise, and thus the effects of higher price will depend on which patch is enclosed. Because resource stocks are unaffected, it is clear that to maximize enclosed patch profit, one should enclose the patch with the elevated price. But to maximize system-wide profit, it will depend on the spatial externality. When the patches are weakly connected (so Q is small), then the patch with high price should be enclosed. But when the patches are tightly connected, the patch with the elevated price should be left open access. These results are summarized as below.

Proposition 6. *Provided $c'(\cdot) = 0$, a higher price (p_1) in patch 1 has the following effects:*

1. *(Enclosed Patch Stock): All patches are equally desirable for enclosure.*
2. *(Aggregate Stock): All patches are equally desirable for enclosure.*
3. *(Enclosed Patch Profit): Patch 1 is the best candidate for enclosure.*
4. *(Aggregate Profit): Patch 1 is the worst candidate for enclosure if $D \leq Q$ or δ is small. Patch 1 is the best candidate for enclosure if Q is small and δ is large.*

Proposition 6 reveals an interesting tension between the environmental parameters and the economic parameters. While we found that higher growth parameters always promoted a patch for enclosure when the objective was to maximize aggregate profit, we find nearly the opposite for the key economic parameter.

Taken together, these results suggest that if the objective is to maximize local (i.e. enclosed patch) benefits from enclosure, it is typically optimal to enclose the patch with a high level of self-retention, a high growth parameter, or a high price. But if the goal is to site the enclosure to improve the system overall (whether system-wide profit or system-wide stock), the best candidate for enclosure may be a patch with high self-retention, high growth, or *low* price. This reveals an interesting tension between local and system-wide benefits. If the enclosure is to be sited by an agent who derives only local benefits, the enclosure may in fact *minimize* system-wide benefit, though this result can only occur if there is heterogeneity in economic returns across space. Conversely, if the enclosure is sited by a social planner who seeks to maximize aggregate benefits, payoffs in the local enclosure may suffer. Table 1 summarizes the results of Propositions 4-6, where a "+" indicates that the patch with elevated parameter is the best candidate for enclosure and a "-" indicates that the patch with the elevated parameter is the worst candidate for enclosure.

Table 1: Summary of results for linear cost ($c'(\cdot) = 0$)

Objective	D	α	p
Enclosed Stock	$+$	$+$	0
Aggregate Stock	$+$	$+$	0
Enclosed Profit	$+$	$+$	$+$
Aggregate Profit	$+$	$+$	$+/-$

2.5. Extension to non-linear cost

The siting results above assume harvest cost is linear ($c'(\cdot) = 0$). While this is a common assumption in natural resource models, it fails to capture the possibility of a stock effect, where harvest costs rise as resource scarcity sets in. In that case $c'(\cdot) < 0$ which has important economic and behavioral implications. First, this assumption implies that even in the open access patches, stocks will not be fully exhausted. As the resource becomes scarce, the costs rise to such a degree that, even under open access, the marginal profit eventually hits zero and harvesting ceases. Second, in the enclosed patch the optimal residual stock level will depend on both price and residual stock from the open access fringe. These facts link the system together in a more nuanced way than when costs are linear, rendering the spatial externalities more complex. Thus, one might expect there to be an enhanced role for connectivity (both self-retention, D and migration Q) driving results. Indeed, when $c'(\cdot) < 0$ we find that the result often hinges on connectivity. Table 2 summarizes our results for the case of $c'(\cdot) < 0$.

Table 2: Summary of results for nonlinear cost $c'(\cdot) < 0$

Objective	D	α	p
Enclosed Stock	$+$	$+$ if $D \geq Q$ $-$ if D small	$+$ if D small $-$ if Q small
Aggregate Stock	$+$	$+$ if $D \geq Q$ or D small	$+$ if D small
Enclosed Profit	$+$	$+$ if $D \geq Q$ $-$ if D small	$+$
Aggregate Profit	$+$ if $D \geq Q$	$+$ if $D \geq Q$ or D small	?

While these results are more nuanced than those derived when $c'(\cdot) = 0$, they are largely consistent. Three exceptions are worth pointing out. First, if the objective is to maximize enclosed patch stock or profit, then the effect of higher growth on the optimal patch to enclose can flip depending on the value of self-retention. When self retention is large (in particular, when $D \geq Q$) it is always optimal to enclose the patch with high growth (intuitively, because the high value of D allows the enclosed patch to capture most of the benefits of its larger α). But if self retention is sufficiently small, it is optimal to enclose a patch with lower self-retention: When self retention is small, the stock in a patch derives primarily from large values of residual stock in *other* patches. Thus, the enclosure owner would like his benefactor to have a high value of α (because he claims the spill-over). The other exceptions involve the parameter p. When $c'(\cdot) = 0$, resource stock is unaffected by price, so no preference is given for enclosure. But when $c'(\cdot) < 0$ the optimal enclosure again depends on the extent of the spatial externality. To maximize enclosed patch stock, when self retention is small, one would like to enclose the patch with high price. This is because a higher price causes the enclosed patch owner to decrease his residual stock (see Lemma 3), but since D is small, this has little effect on his own stock. Instead, if the high price patch is open access, it will cause the open access patch to reduce its residual stock and when D is small, this has a greater (negative) impact on the enclosed patch stock. Similar reasoning explains why enclosing a low price patch is optimal when out-of-patch migration (Q) is small. The third exception concerns the effects of price on aggregate resource stock which are explained by a similar argument. Finally, we note that the general case of price and its effect on aggregate profit cannot be signed, except in special cases.

3. Illustrative Example

To illustrate our analytical findings, we now present two versions of a simple example. We loosely base these examples on spatial analysis of fisheries near the Channel Islands, California (e.g. see White and Casselle (2008)); we focus on the 13 patches of roughly 210 km^2 each surrounding the Northern Channel Islands (see Figure 1). The equation of motion in patch i is given by a spatial version of the discrete-time logistic model:

$$x_{it+1} = \sum_{j=1}^{13} D_{ji} \underbrace{\left[e_{jt} + r_j e_{jt} \left(1 - \frac{e_{jt}}{K_j} \right) \right]}_{g(e_j,\alpha_j)}. \tag{6}$$

Under this functional specification, both the intrinsic growth rate r_j and the carrying capacity K_j conform to the requirements of the general parameter α_j from Section 2.[14]

3.1. Heterogeneous patches

In the first example, we allow the patches to be heterogeneous and calculate the benefits and optimal siting of enclosure. Dispersal (D_{ij}), growth (r_j), and carrying capacity (K_j) parameters are drawn loosely from real data in the region, though the example is meant to be illustrative only. Dispersal from patch j to i is given by the *dispersal kernel* (provided in the Appendix). We assume that both growth and carrying capacity are positively related to the fraction of the patch that is covered by kelp ($\mathcal{K}_j \leq 1$), according to these relationships:

$$r_j = 0.4 + \mathcal{K}_j^{1/2} \tag{7}$$
$$K_j = 100 + 1000\mathcal{K}_j \tag{8}$$

This produces intrinsic growth rates $r_j \in [0.40, 0.68]$ and carrying capacities $K_j \in [100, 177]$.[15] We assume marginal harvest cost is given by the function $c(s) = \theta/s$, and we use $\theta = 15$ (see White and Costello (2011)). Price is set to unity, and we use $\delta = .9$.

Under this model, the open access equilibrium residual stock is given by: $\hat{e} = 15$ in all patches (see Proposition 1), which is confirmed by this numerical application. Thus, under this parameterization, the open access residual stock is 8%-15% of

[14] Under this logistic growth model, $g(e) = e + re(1 - e/K)$. Both r and K adhere to requirements for α (within the relevant range of the variable, e): $\frac{\partial g}{\partial r} = e(1 - e/K) > 0$, $\frac{\partial g}{\partial K} = re^2/K^2 > 0$, $\frac{\partial^2 g}{\partial r \partial e} = 1 - \frac{2e}{K} > 0$ and $\frac{\partial^2 g}{\partial K \partial e} = \frac{2re}{K^2} > 0$, for $e < K/2$.

[15] We provide in the Appendix all input parameters necessary to replicate all of these results.

carrying capacity (depending on the patch). Enclosing patches one-by-one, while the other 12 patches remain open access, generates optimal residual stock level in the enclosed patch of between 18.7 (patch 1) and 47.1 (patch 7) with an average of 26.3 (see Lemma 1). It is not immediately obvious which patch to enclose to achieve different objectives. Using the guidance from Propositions 4-6, patch 12 has the highest self-retention and patch 6 has the most habitat ($\mathcal{K}_j$) and thus the largest values of r and K. However, while patch 7 does not have the largest value of any single parameter, it does have relatively large values of all parameters. Stock in all non-enclosed patches increases as a consequence of enclosure (see Proposition 2). In our numerical example, the increase in system-wide stock arising from partial enclosure depends on which patch is enclosed. It ranges from 4.1 (when enclosing patch 1, representing just a 2% increase in stock) to 38 (when enclosing patch 7, representing a 16% increase). Consistent with Proposition 3, equilibrium profit in all non-enclosed patches increases as a consequence of enclosure. As was the case with stock, the increase in system-wide profit arising from partial enclosure ranges from .2% (when enclosing patch 1) to 5% (when enclosing patch 6).

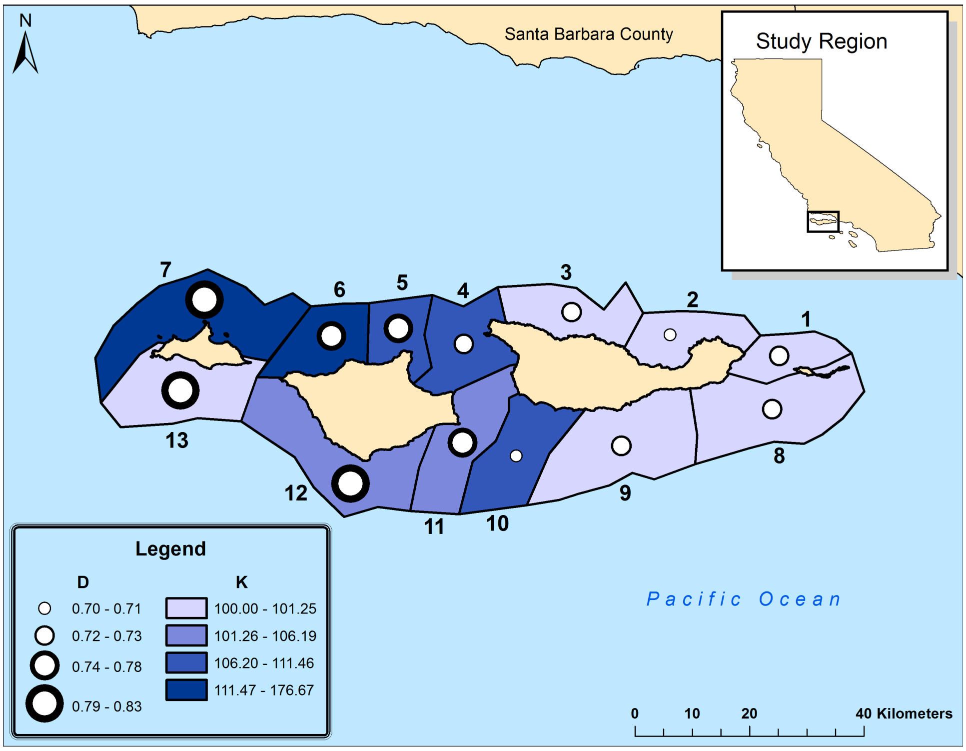
In this example, it turns out that for three of the four objectives (enclosed patch stock, aggregate stock, and enclosed patch profit), patch 7 is the optimal enclosure. The optimal enclosure to maximize aggregate profit is patch 6. Figure 1 shows the study area and displays the values of D in circles and K (which is correlated with r in this example) with shading. From this figure, it is clear why patches 6 and 7 are good candidates for enclosure.

3.2. Homogeneous patches

Our second example illustrates the comparative statics of optimal enclosure derived in Section 2.5. It builds on the first example, but adopts the starting point from Section 2.4 that all patches are symmetric. For r_i and K_i we assume that each patch has the average value of those parameters from the previous example (so $r = .49$, $K = 114.2$). For D_{ij} we assume that all off-diagonal terms are $Q = .06$ and that the diagonal terms are $D = .20$ (though we also explore values of $D \in [0, .30]$). We continue to assume $\delta = 0.9$, $p = 1$, and $\theta = 15$. Together, these assumptions yield a completely symmetric set of patches. From this starting point, it is equally desirable to enclose any one of the 13 patches; doing so increases both stock and profit.

Following the theoretical treatment, we numerically calculate the comparative statics associated with a 10% increase in each parameter in a single patch. We do so by incrementing a single parameter in patch 1 only, holding all other parameters at their initial values, and calculating the subsequent effects on the entire system. Our main focus is on how this change in a parameter will affect the optimal patch to enclose under the various objectives spelled out above.

Figure 1: Study region, patch delineation, and patch characteristics.



3.2.1. Comparative statics: D_{11}

When the self-retention parameter in patch 1 is increased, it implies that patch 1 is a stronger residual claimant of conservative harvesting behavior than are the other patches. Thus, consistent with Proposition 4 and with the extended results in Table 2, we find that patch 1 is the optimal enclosure. This result holds across all four objectives.

3.2.2. Comparative statics: r_1 *and* K_1

The parameters r_i and K_i are both special cases of the more general growth parameter α_i considered in the analytical model. Thus, the comparative statics in Section 2.5 apply to both r and K. To maximize enclosed patch stock, we find that a larger value of r or a larger value of K promotes a patch for enclosure, provided $D > .06$, and it relegates a patch for enclosure if $D < .06$. We obtain a very similar result for the objective of maximizing enclosed patch profit, though the cutoff values of D are slightly larger. To maximize aggregate stock we found a similar result for r (though the cutoff is $D = .01$), and we found that higher value of K always promotes a patch for enclosure. Finally, when the goal is to maximize aggregate profit, we found that higher values of r or K always promote a patch for enclosure. This result holds for all values of D.

3.2.3. Comparative statics: p_1

Finally we consider numerically the effects of an increase in price. When p_1 is increased, the optimal patch to enclose depends on the objective being pursued and on the extent of the spatial externality, via D and Q (see Table 2). If the objective is to maximize profit in the enclosed patch or aggregate stock, we find an unambiguous result that higher p promotes a patch for enclosure. If the goal is to maximize enclosed patch stock, we find a mixed result: when $D < .06$, we find that higher p promotes a patch for enclosure. But when $D > .06$, higher price relegates a patch. This is consistent with the theoretical finding reported in Table 2 that small D promotes a patch for enclosure and small Q relegates a patch for enclosure. Finally, if the objective is to maximize aggregate profit (a case we were unable to sign in the general model), we find the unambiguous result that higher p always relegates a patch for enclosure, regardless of the value of D.

These results are consistent with the theoretical results reported in Table 2. While the example is meant to be illustrative only, and not to provide specific policy guidance for the Channel Islands, it does demonstrate the ease and utility with which the model developed here can be applied in a real world context.

4. Conclusion

Partial enclosure of the commons is perhaps the most common institutional arrangement for governing renewable natural resources, yet it has received almost no attention in the literature. We define it as a circumstance in which *part* of a spatially-connected resource is assigned to a sole owner, and the remainder is competed for by an open access fringe. Because the resource is mobile, each group imposes an externality on others. We develop a spatial bioeconomic model to address questions such as: Under what conditions will *partial enclosure* lead to aggregate (or individual) welfare gains? What will be the consequences of *partial enclosure* on the open access fringe? What are the environmental effects of *partial enclosure*? And, for different objectives, in which patches should *partial enclosure* be undertaken? The framework allows us to make sharp analytical predictions, which are then illustrated with a numerical example of a spatially-connected fishery surrounding the Northern Channel Islands.

Perhaps the most salient welfare implication of partial enclosure is that it always leads to a strict Pareto Improvement over open access. This conclusion holds whether the agents' objectives are based solely on profit or are motivated by conservation, and it holds even when one assigns partial enclosure haphazardly. We also explored the spatial (via D_{ij}), environmental (via α_i), and economic (via p_i)

characteristics that make a patch a particularly good candidate for enclosure. We found that, ceteris paribus, if a patch has higher self-retention, it is a good candidate for enclosure regardless of one's objective. Here, there is strong consistency between individual and aggregate welfare and between stock and profit as objectives. But we found that patches with higher growth parameters or larger patch size may promote or relegate a patch for enclosure, depending on one's objective. In that case, the optimal enclosure location can be reversed depending on whether one is interested in enclosed patch outcomes or aggregate outcomes (though this result arises only under a stock effect). Finally, economic returns and resource growth have potentially opposite comparative effects.

Overall, these findings suggest that partial enclosure of the commons is a potentially valuable (though second best) institutional arrangement with positive economic and environmental consequences. Our comparative results emphasize that optimal siting of enclosures are often consistent between individual and societal objectives and between conservation and profit motives. But the analysis also illuminates interesting tensions where the optimal siting of partial enclosure can impact negatively on some agents. This latter class of problems may suggest interesting political economy extensions of our analysis.

Benchekroun, H., 2003. Unilateral production restrictions in a dynamic duopoly. Journal of Economic Theory 111, 214–239.

Besley, T., 1995. Property rights and investment incentives: Theory and evidence from Ghana. Journal of Political Economy 103, 903–937.

Brito, D., Intriligator, M., Sheshinski, E., 1997. Privatization and the distribution of income in the commons. Journal of Public Economics 64, 181–205.

Ciriacy-Wantrup, S. V., Bishop, R., 1975. Common property as a concept in natural resources policy. Natural Resources Journal 15, 713–727.

Colombo, L., Labrecciosa, P., 2013. Oligopoly exploitation of a private property productive asset. Journal of Economic Dynamics and Control (Forthcoming).

Costello, C., Deacon, R., 2007. The Efficiency Gains from Fully Delineating Rights in an ITQ Fishery. Marine Resource Economics 22, 347–361.

Costello, C., Polasky, S., 2008. Optimal harvesting of stochastic spatial resources. Journal of Environmental Economics and Management 56, 1–18.

Deacon, R., Parker, D., Costello, C., 2013. Reforming fisheries: lessons from a self-selected cooperative. Journal of Law and Economics (In Press).

Fisher, C., Laxminarayan, R., 2010. Managing partially protected resources under uncertainty. Journal of Environmental Economics and Management 59, 129–141.

Hardin, G. J., 1968. The tragedy of the commons. Science 162, 1243–8.

Johnson, R. N., Libecap, G. D., 1982. Contracting problems and regulation: The case of the fishery. American Economic Review 72, 1005–1022.

Kaffine, D. T., Costello, C., 2011. Unitization of spatially connected renewable resources. BE Journal of Economic Analysis and Policy 11 (1), 15.

Kapaun, U., Quaas, M., 2013. Does the optimal size of a fish stock increase with environmental uncertainties? Environmental and Resource Economics 54 (2), 293–310.

Karpoff, J. M., 1987. Suboptimal controls in common resource management: The case of the fishery. Journal of Political Economy 95 (1), 179–194.

Kim, J., Mahoney, J. T., 2002. Resource-based and property rights perspectives on value creation: the case of oil field unitization. Managerial and Decision Economics 23 (4-5), 225–245.

Laxminarayan, R., Brown, G. M., 2001. Economics of antibiotic resistance: a theory of optimal use. Journal of Environmental Economics and Management 42 (2), 183–206.

Le Gallic, B., 2006. Using Market Mechanisms to Manage Fisheries: Smoothing the Path. OECD.

Levhari, D., Mirman, L., 1980. The great fish war: an example using a dynamic cournot-nash solution. The Bell Journal of Economics 11 (1), 322–334.

Lucas, R. E., 1988. On the mechanics of economic development. Journal of Monetary Economics 22 (1), 3–42.

Maler, K. G., 1990. International environmental problems. Oxford Review of Economic Policy 6 (1), 80–108.

Montgomery, W. D., 1972. Markets in licences and efficient pollution control programs. Journal of Economic Theory 5, 395–418.

Ostrom, E., 1990. Governing the commons: The evolution of institutions for collective action. Cambridge; New York and Melbourne: Cambridge University Press.

Pintassilgo, P., Finus, M., Lindroos, M., Munro, G., 2010. Stability and success of regional fisheries management organizations. Environmental and Resource Economics 46 (3), 377–402.

Porter, M. E., 1998. Clusters and the new economics of competition. Harvard Business Review 76 (6), 77–90.

Quérou, N., Tomini, A., 2013. Managing interacting species in unassessed fisheries. Ecological Economics (Forthcoming).

Raffensperger, J. F., 2011. Matching user's rights to available groundwater. Ecological Economics 70, 1041–1050.

Reed, W., 1979. Optimal escapement levels in stochastic and deterministic harvesting models. Journal of Environmental Economics and Management 6, 350–363.

Rosenman, R. E., 1984. The optimal tax for maximum economic yield: Fishery regulation under rational expectations. Journal of Environmental Economics and Management 13 (4), 348–362.

Rubio, S., Escriche, L., 2001. Strategic pigouvian taxation, stock externalities and polluting non-renewable resources. Journal of Public Economics 79, 297–313.

Sanchirico, J. N., Wilen, J. E., 2005. Optimal spatial management of renewable resources: matching policy scope to ecosystem scale. Journal of Environmental Economics and Managemement 50 (1), 23–46.

Scott, A., 1955. The fishery: the objectives of sole ownership. Journal of Political Economy 63, 116–124.

Stavins, R., 2011. The problem of the commons: Still unsettled after 100 years. American Economic Review 101, 81–108.

Stranlund, J., Dhanda, K. K., 1999. Endogenous monitoring and enforcement of a transferable emissions permit system. Journal of Environmental Economics and Management 38 (3), 267–282.

Weitzman, M. L., 2002. Landing fees vs harvest quotas with uncertain fish stocks. Journal of Environmental Economics and Management 43, 325–338.

White, C., Costello, C., 2011. Matching spatial property rights fisheries with scales of fish dispersal. Ecological Applications 21 (2), 350–362.

White, W., Casselle, J., 2008. Scale dependent changes in the importance of larval supply and habitat to abundance of reef fish. Ecology 89 (5), 1323–1333.

5. Appendix

Proof of Proposition 1

Under open access, the patch i, period t objective is to maximize Π_{it} given in Equation 3. Taking the derivative and setting equal to zero provides first order condition: $p_i = c_i(\hat{e}_{it})$, which applies for $c_i'(s) < 0$. If $c_i'(s) = 0$, and adopting $c_i(s) < p_i$, the objective is downward-sloping in residual stock, so it is maximized by fulling extinguishing the patch i stock, in which case $\hat{e}_{it} = 0$.

Proof of Lemma 1

The first line (for $i \neq k$) follows directly from Proposition 1. The second line (for patch k) is the first order condition for the enclosed patch, found by taking the derivative of Equation 4 and setting equal to zero.

Proof of Lemma 2

By the first order condition in any period t, open access optimal residual stock levels are independent of all stock levels (the state vector) in that same period. The same result holds for the optimal residual stock level of the enclosed patch. Let patch j be enclosed. When the optimal residual stock level is positive, the optimality condition is necessary and sufficient. The term on the left hand side of this condition reflects the marginal contribution of residual stock to current period payoff, and is independent of x_t by inspection. The derivative of the payoff function in period $t+1$ depends on the period $t+1$ state, but is independent of the period t state. Since we know that an interior solution satisfies $\bar{e}_{jt} < \bar{x}_{jt}$ and using Expression 2, $\bar{x}_{jt+1}$ is a function of $\bar{e}_{jt}$ but not of $\bar{x}_{jt}$. Therefore, the term on the right hand side in the first order condition is independent of $\bar{x}_{jt}$, and the period t problem of the sole owner has state independent control.

Second, we must prove that optimal residual stock levels are time independent. Again, since economic returns are independent of time, open access optimal residual stock levels are time independent. Now, regarding patch j, we just proved that $\bar{e}_{jt}$ is independent of $\bar{x}_{jt}$. This implies that a change in stock in the next period affects the payoff function in $t+1$ only through the term relating to $\bar{x}_{jt+1}$. Since growth, dispersal, and economic returns are time-independent, the optimal choice, $\bar{e}_{jt}$, is also time-independent.

Proof of Proposition 2

Let $\hat{e}_i$ denote residual stock in patch i when all patches are open access and let e_i denote residual stock in patch i when patch j is enclosed, but all other patches i are open access. First, note that $\hat{e}_i = e_i$ for $i \neq j$ (because $p_i = c_i(\hat{e}_i) = c_i(e_i)$). For patch j, the residual stock under open access is $p_j - c_j(\hat{e}_j) = 0$. But under enclosure, residual stock in j is given by the FOC: $p_j - c_j(\bar{e}_j) = \delta D_{jj}\left[p_j - c_j(\bar{x}_j)\right] g_{\bar{e}}(\bar{e}_j, \alpha_j)$. The right hand side of this expression is > 0, thus $p_j - c_j(\bar{e}_j) > 0$, so $\bar{e}_j > \hat{e}_j$. The difference in patch i stock in the enclosed case minus the open access case is simply $D_{ji}g_j(\bar{e}_j) - D_{ji}g_j(\hat{e}_j)$. Because $g_e > 0$, stock is higher in all patches i such that $D_{ji} > 0$ and is unchanged in all patches i such that $D_{ji} = 0$.

Proof of Proposition 3

Without loss of generality, let patch j be enclosed. If patch j chooses the open access residual stock level, then all patches are indifferent to the enclosure. But if patch j chooses a different residual stock level, then patch j must do so to increase her profit. The proof to Proposition 2 shows that patch j chooses a residual stock larger than the open access level, so patch j must be better off under the enclosure. Again the proof to Proposition 2 shows that if $D_{ji} > 0$, then patch i receives a higher stock, and is thus better off under the enclosure. Instead, if $D_{ji} = 0$, then patch i stock is unchanged under the enclosure, and thus patch i is indifferent to the enclosure.

Proof of Corollary 1

Follows immediately from the evolution rule of the stock levels.

Proof of Lemma 3

Applying the implicit function theorem to the first order condition gives the relevant total derivatives:

$$\begin{aligned}
\frac{d\bar{e}_j}{dD_j} &= -\frac{\delta g_{\bar{e}}(e_j,\alpha_j)\left[p_j - c_j(\bar{x}_j) - D_{jj}c'_j(\bar{x}_j)\cdot g(\bar{e}_j,\alpha_j)\right]}{SOC} \\
\frac{d\bar{e}_j}{dp_j} &= \frac{1-\delta D_{jj} g_{\bar{e}}(\bar{e}_j,\alpha_j)}{SOC} \\
\frac{d\bar{e}_j}{d\alpha_j} &= -\frac{\delta D_{jj}\left[-g_{\bar{e}}(\bar{e}_j,\alpha_j)\cdot g_{\alpha}(\bar{e}_j,\alpha_j)c'_j(\bar{x}_j)D_{jj} + \left(p_j - c_j(\bar{x}_j)\right)g_{\bar{e}\alpha}(\bar{e}_j,\alpha_j)\right]}{SOC}
\end{aligned}$$

with $SOC = c'_j(\bar{e}_j) + \delta D_{jj}\left[-D_{jj}c'_j(\bar{x}_j)\left(g_{\bar{e}_j}(\bar{e}_j,\alpha_j)\right)^2 + \left(p_j - c_j(\bar{x}_j)\right)g_{\bar{e}_j\bar{e}_j}(\bar{e}_j,\alpha_j)\right] < 0$, which is the second order condition. The numerators of the fraction in the expressions of $\frac{d\bar{e}_j}{dD_{jj}}$ and $\frac{d\bar{e}_j}{d\alpha_j}$ are unambiguously non-negative. Therefore, $\frac{d\bar{e}_j}{dD_{jj}} \geq 0$ and $\frac{d\bar{e}_j}{d\alpha_j} \geq 0$. Finally, by the first order condition for $\bar{e}_j$, the numerator of $\frac{d\bar{e}_j}{dp_j}$ is non-negative, thus $\frac{d\bar{e}_j}{dp_j} \leq 0$.

Proof of the claim in Section 2.4

First, let us assume that patch j is enclosed while all other patches are under open access. Now, for $i \neq j$, we have :

$$\bar{x}_j \;=\; D_{jj}g\left(\bar{e}_j,\alpha_j\right) + \sum_{k\neq j}^{N} D_{kj}g\left(e_k,\alpha_k\right) \;\;;\;\; x_i = D_{ji}g\left(\bar{e}_j,\alpha_j\right) + \sum_{k\neq j}^{N} D_{ki}g\left(e_k,\alpha_k\right)$$

If enclosing patch j yields the highest value of the aggregate stock level, the following inequality is satisfied :

$$\bar{x}_j + \sum_{l\neq j}^{N} x_l \geq \max\left\{\bar{x}_1 + \sum_{k=2}^{N} x_k; \ldots ; \bar{x}_N + \sum_{l\neq N}^{N} x_l\right\}$$

Let us assume that this inequality holds. We are going to compare the expression of aggregate stock levels when patch j is enclosed and when another patch (say i) is enclosed (assuming in both cases that all other patches remain under open access). We obtain the equivalent inequalities :

$$\begin{aligned} & D_{jj} g\left(\bar{e}_j, \alpha_j\right) + \sum_{k \neq j}^{N} D_{kj} g\left(e_k, \alpha_k\right) + \sum_{l \neq j}^{N} \left(D_{jl} g\left(\bar{e}_j, \alpha_j\right) + \sum_{k \neq j}^{N} D_{kl} g\left(e_k, \alpha_k\right) \right) \\ > \quad & D_{ii} g\left(\bar{e}_i, \alpha_i\right) + \sum_{k \neq i}^{N} D_{ki} g\left(e_k, \alpha_k\right) + \sum_{l \neq i}^{N} \left(D_{il} g\left(\bar{e}_i, \alpha_i\right) + \sum_{k \neq i}^{N} D_{kl} g\left(e_k, \alpha_k\right) \right) \\ \Leftrightarrow \quad & g\left(\bar{e}_j, \alpha_j\right) + \sum_{l \neq j}^{N} g\left(e_l, \alpha_l\right) > g\left(\bar{e}_i, \alpha_i\right) + \sum_{l \neq i}^{N} g\left(e_l, \alpha_l\right) \\ \Leftrightarrow \quad & g\left(\bar{e}_j, \alpha_j\right) + g\left(e_i, \alpha_i\right) > g\left(\bar{e}_i, \alpha_i\right) + g\left(e_j, \alpha_j\right) \\ \Leftrightarrow \quad & g\left(\bar{e}_j, \alpha_j\right) - g\left(e_j, \alpha_j\right) > g\left(\bar{e}_i, \alpha_i\right) - g\left(e_i, \alpha_i\right) \end{aligned}$$

Setup of proofs to Propositions 4-6

Without loss of generality, we will assume that a single parameter is elevated in patch 1 and we explore the consequences of enclosing patch 1 or patch 2. We indicate the enclosed patch by placing a bar over its relevant variables ($\bar{x}$, $\hat{e}$, $\bar{\alpha}$, $\hat{D}$ and $\hat{p}$). We indicate an open access patch (which may have an elevated parameter) without a bar, e.g. x. Finally, we must also account for the other $N-2$ patches $3, 4, ..., N$ which neither have elevated parameters nor are enclosed. We denote the representative patch with a tilde, e.g. $\tilde{x}$. Prior to any change in parameters, the three equations of motion are given by:

$$\bar{x} = Dg(\bar{e}, \bar{\alpha}) + (N-1)Qg(e, \alpha) \tag{9}$$

$$x = \left[D + (N-2)Q\right] g(e, \alpha) + Qg(\bar{e}, \bar{\alpha}) \tag{10}$$

$$\tilde{x} = \left[D + (N-2)Q\right] g(e, \alpha) + Qg(\bar{e}, \bar{\alpha}) \tag{11}$$

By Lemma 3, the optimal residual stock is time-independent, which implies that the profit expressions are given by :

$$\bar{\Pi} = \bar{p}(x_0 - \bar{e}) - \int_{\bar{e}}^{x_0} c(s) ds + \frac{\delta}{1-\delta} \left[\bar{p}(\bar{x} - \bar{e}) - \int_{\bar{e}}^{\bar{x}} c(s) ds \right] \tag{12}$$

$$\Pi = p(x_0 - e) - \int_{e}^{x_0} c(s) ds + \frac{\delta}{1-\delta} \left[p(x - e) - \int_{e}^{x} c(s) ds \right] \tag{13}$$

$$\tilde{\Pi} = \tilde{p}(x_0 - \tilde{e}) - \int_{\tilde{e}}^{x_0} c(s) ds + \frac{\delta}{1-\delta} \left[\tilde{p}(\tilde{x} - \tilde{e}) - \int_{\tilde{e}}^{\tilde{x}} c(s) ds \right] \tag{14}$$

where x_0 is the initial stock level. We examine comparative statics for three parameters (D, α, and p). For any parameter $\theta \in \{D, \alpha, p\}$, total differentiation gives the expressions in Table 3.

To determine whether it is advantageous to enclose patch 1 (the patch with the elevated

Table 3: Comparative statics

Enclosed	Profit	Stock	# patches
1	1: $\frac{d\bar{\Pi}}{d\bar{\theta}} = \frac{\partial\bar{\Pi}}{\partial\bar{\theta}}$	1: $\frac{d\bar{x}}{d\bar{\theta}} = \frac{\partial\bar{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{\theta}} + \frac{\partial\bar{x}}{\partial\bar{\theta}}$	1
	2: $\frac{d\Pi}{d\bar{\theta}} = \frac{\partial\Pi}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{\theta}} + \frac{\partial\Pi}{\partial\bar{\theta}}$	2: $\frac{dx}{d\bar{\theta}} = \frac{\partial x}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{\theta}} + \frac{\partial x}{\partial\bar{\theta}}$	$N-1$
2	1: $\frac{d\Pi}{d\theta} = \frac{\partial\Pi}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\Pi}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial\Pi}{\partial\theta}$	1: $\frac{dx}{d\theta} = \frac{\partial x}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial x}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial x}{\partial\theta}$	1
	2: $\frac{d\bar{\Pi}}{d\theta} = \frac{\partial\bar{\Pi}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\bar{\Pi}}{\partial\theta}$	2: $\frac{d\bar{x}}{d\theta} = \frac{\partial\bar{x}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\bar{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial\bar{x}}{\partial\theta}$	1
	k: $\frac{d\tilde{\Pi}}{d\theta} = \frac{\partial\tilde{\Pi}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\tilde{\Pi}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial\tilde{\Pi}}{\partial\theta}$	k: $\frac{d\tilde{x}}{d\theta} = \frac{\partial\tilde{x}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\tilde{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial\tilde{x}}{\partial\theta}$	$N-2$

parameter) or patch 2 (a patch without the elevated parameter), we compute the differences:

$$\text{Enclosed Stock: } \Delta_1(\theta) \equiv \frac{d\bar{x}}{d\bar{\theta}} - \frac{d\bar{x}}{d\theta} \tag{15}$$

$$\text{Aggregate Stock: } \Delta_2(\theta) \equiv \frac{d\bar{x}}{d\bar{\theta}} + (N-1)\frac{dx}{d\bar{\theta}} - \left(\frac{d\bar{x}}{d\theta} + \frac{dx}{d\theta} + (N-2)\frac{d\tilde{x}}{d\theta}\right) \tag{16}$$

$$\text{Enclosed Profit: } \Delta_3(\theta) \equiv \frac{d\bar{\Pi}}{d\bar{\theta}} - \frac{d\bar{\Pi}}{d\theta} \tag{17}$$

$$\text{Aggregate Profit: } \Delta_4(\theta) \equiv \frac{d\bar{\Pi}}{d\bar{\theta}} + (N-1)\frac{d\Pi}{d\bar{\theta}} - \left(\frac{d\bar{\Pi}}{d\theta} + \frac{d\Pi}{d\theta} + (N-2)\frac{d\tilde{\Pi}}{d\theta}\right) \tag{18}$$

And we can use the total derivative calculations in the table 3 to re-write Equations 15-18 as partial derivatives. For example, $\Delta_3(\theta) = \frac{\partial\bar{\Pi}}{\partial\bar{\theta}} - \frac{\partial\bar{\Pi}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\bar{\Pi}}{\partial\theta}$.

Depending on the parameter being examined, many of these partial derivative terms are zero. For example, price in an open access patch has no direct influence on an enclosed patch stock, so $\frac{\partial\bar{x}}{\partial p} = 0$. All of the terms in the following table equal zero:

Table 4: Conditions

	D	α	p
Enclosed Stock	$\frac{\partial e}{\partial D}, \frac{\partial\bar{e}}{\partial D}, \frac{\partial\bar{x}}{\partial D}$	$\frac{\partial e}{\partial\alpha}$	$\frac{\partial\bar{x}}{\partial\bar{p}}, \frac{\partial\bar{x}}{\partial p}, \frac{\partial\bar{e}}{\partial p}, \left(\frac{\partial e}{\partial p}\right)$
Aggregate Stock	$\frac{\partial e}{\partial D}, \frac{\partial\bar{e}}{\partial D}, \frac{\partial\bar{x}}{\partial D}, \frac{\partial x}{\partial\bar{D}}, \frac{\partial\tilde{x}}{\partial D}$	$\frac{\partial e}{\partial\alpha}, \left(\frac{\partial\bar{e}}{\partial\alpha}\right), \left(\frac{\partial\bar{x}}{\partial\alpha}\right)$	$\frac{\partial\bar{x}}{\partial\bar{p}}, \frac{\partial\bar{x}}{\partial p}, \frac{\partial\bar{e}}{\partial p}, \left(\frac{\partial e}{\partial p}\right), \frac{\partial x}{\partial\bar{p}}, \frac{\partial x}{\partial p}, \frac{\partial\tilde{x}}{\partial p}$
Enclosed Profit	$\frac{\partial e}{\partial D}, \frac{\partial\bar{\Pi}}{\partial D}$	$\frac{\partial e}{\partial\alpha}$	$\frac{\partial\bar{\Pi}}{\partial p}$
Aggregate Profit	$\frac{\partial e}{\partial D}, \frac{\partial\bar{\Pi}}{\partial D}, \frac{\partial\bar{e}}{\partial D}, \frac{\partial\Pi}{\partial\bar{D}}$	$\frac{\partial e}{\partial\alpha}, \left(\frac{\partial\bar{e}}{\partial\alpha}\right), \left(\frac{\partial\bar{x}}{\partial\alpha}\right)$	$\frac{\partial\bar{\Pi}}{\partial p}, \frac{\partial\Pi}{\partial\bar{p}}, \frac{\partial\tilde{\Pi}}{\partial p}, \frac{\partial\bar{e}}{\partial p}$

The parenthetical terms (e.g. $\frac{\partial e}{\partial p}$) equal zero only if = 0. We will make extensive use of Table 4 in the proofs that follow. For each of the four objectives (enclosed stock, aggregate stock, enclosed profit, aggregate profit) and for each of the three parameters (D, α, and p) we analyze the effects of enclosing patch 1 minus patch 2. This difference is given in Equations 15-18.

Proof of Proposition 4 (self-retention, D)

1. **Enclosed patch stock**
   Adopting the conditions in Table 4, the difference 15 is $\Delta_1(D) = \frac{d\bar{x}}{d\bar{D}} > 0$.
2. **Aggregate stock**
   Adopting the conditions in Table 4, the difference 16 is:

   $$\Delta_2(D) \quad = \quad \frac{\partial \bar{e}}{\partial \bar{D}}\left(\frac{\partial \bar{x}}{\partial \bar{e}} + (N-1)\frac{\partial x}{\partial \bar{e}}\right) + \frac{\partial \bar{x}}{\partial \bar{D}} - \frac{\partial x}{\partial D} = \frac{\partial \bar{e}}{\partial \bar{D}}\left(\frac{\partial \bar{x}}{\partial \bar{e}} + (N-1)\frac{\partial x}{\partial \bar{e}}\right) + g(\bar{e},\bar{\alpha}) - g(e,\alpha)$$

   Each of these terms is positive since the growth rate function is increasing and $\bar{e} > e$ and $\bar{\alpha} = \alpha$ (because in this case the variable of interest is D), so $\Delta_2(D) > 0$.
3. **Enclosed patch profit**
   Adopting the conditions in Table 4, the difference 17 is $\Delta_3(D) = \frac{d\bar{\Pi}}{d\bar{D}} > 0$.
4. **Aggregate profit**
   Adopting the conditions in Table 4, the difference 18 is:

   $$\Delta_4(D) = \frac{\partial \bar{\Pi}}{\partial \bar{D}} - \frac{\partial \Pi}{\partial D} + (N-1)\frac{\partial \Pi}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \bar{D}}.$$

   First, note that the last term is always positive. Now, regarding the first term, we have :

   $$\frac{\partial \bar{\Pi}}{\partial \bar{D}} - \frac{\partial \Pi}{\partial D} = [p - c(\bar{x})]g(\bar{e},\bar{\alpha}) - [p - c(x)]g(e,\alpha)$$

   Note that if $c' = 0$, this term is $(p-c)[g(\bar{e},\bar{\alpha}) - g(e,\alpha)] > 0$ since g is increasing and $\bar{e} > e$. This implies that $\Delta_4(D) > 0$. If $c' < 0$, because $g_e > 0$, a sufficient condition for this term being positive is $\bar{x} > x$. The difference between Equations 9 and 10 is:

   $$\bar{x} - x = (D - Q)\,[g(\bar{e},\bar{\alpha}) - g(e,\alpha)]$$

   If $D \geq Q$ then the term is positive and $\Delta_4(D) > 0$.

Proof of Proposition 5 (growth, α)

We will use the following notations for the derivatives in the remainder of this appendix: $g_e = g_e(e,\alpha)$, $g_{\bar{e}} = g_e(\bar{e},\bar{\alpha})$, $g_\alpha = g_\alpha(e,\alpha)$, $g_{\bar{\alpha}} = g_\alpha(\bar{e},\bar{\alpha})$, and $g_{\bar{e},\bar{\alpha}} = g_{e\alpha}(\bar{e},\bar{\alpha})$.

1. **Enclosed patch stock**
   Adopting the conditions in Table 4 the difference 15 is:

   $$\Delta_1(\alpha) = \frac{\partial \bar{x}}{\partial \bar{e}}\underbrace{\left(\frac{\partial \bar{e}}{\partial \bar{\alpha}} - \frac{\partial \bar{e}}{\partial \alpha}\right)}_{\equiv A} + \left(\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha}\right). \qquad (19)$$

First note that if $c' = 0$, $\frac{\partial \bar{x}}{\partial \alpha} = \frac{\partial \bar{e}}{\partial \alpha} = 0$ and $\Delta_1(\alpha) > 0$. If $c' < 0$, then using Equation 9, we have $\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha} = Dg_{\bar{\alpha}} - Qg_{\alpha}$. This term is positive provided $D \geq Q$, and is negative for sufficiently small D. The other parenthetical term involves analyzing

$$\frac{\partial \bar{e}}{\partial \bar{\alpha}} = \frac{\delta D \left(g_{\hat{e}} c'(\bar{x}) \frac{\partial \bar{x}}{\partial \bar{\alpha}} - [p - c(\bar{x})] g_{\bar{e}\bar{\alpha}} \right)}{SOC} > 0 \tag{20}$$

$$\frac{\partial \bar{e}}{\partial \alpha} = \frac{\delta D \left(g_{\hat{e}} c'(\bar{x}) \frac{\partial \bar{x}}{\partial \alpha} \right)}{SOC} > 0 \tag{21}$$

The denominator of both terms is the second order condition, which is negative. Subtracting the expressions gives:

$$\frac{\partial \bar{e}}{\partial \bar{\alpha}} - \frac{\partial \bar{e}}{\partial \alpha} = \frac{\delta D \left(g_{\bar{e}} c'(\bar{x}) (\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha}) - [p - c(\bar{x})] g_{\bar{e}\bar{\alpha}} \right)}{SOC} \tag{22}$$

Because $g_{\bar{e}\bar{\alpha}} > 0$, a sufficient condition for this term being positive is $D \geq Q$. Instead, suppose $D = 0$. In that case, term $A = 0$ and $\frac{\partial \bar{x}}{\partial \bar{\alpha}} = 0$. The entire expression is negative. By a continuity argument, this implies that $\Delta_1(\alpha) < 0$ for sufficiently small values of D. To summarize, if $= 0$, we have $\Delta_1(\alpha) > 0$; if < 0 then if $D \geq Q$, we have $\Delta_1(\alpha) > 0$, and if D sufficiently small, we have $\Delta_1(\alpha) < 0$.

2. **Aggregate stock**
Adopting the conditions in Table 4, the difference 16 is:

$$\begin{aligned} \Delta_2(\alpha) &= \frac{\partial \bar{x}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{\alpha}} + \frac{\partial \bar{x}}{\partial \bar{\alpha}} + (N-1) \left(\frac{\partial x}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{\alpha}} + \frac{\partial x}{\partial \bar{\alpha}} \right) \\ &- \left[\frac{\partial x}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{\alpha}} + \frac{\partial x}{\partial \alpha} + \frac{\partial \bar{x}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \alpha} + \frac{\partial \bar{x}}{\partial \alpha} + (N-2) \left(\frac{\partial \tilde{x}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \alpha} + \frac{\partial \tilde{x}}{\partial \alpha} \right) \right] \end{aligned}$$

We use the fact that $\frac{\partial x}{\partial \bar{e}} = \frac{\partial \tilde{x}}{\partial \bar{e}}$ to rewrite $\Delta_2(\alpha)$:

$$\Delta_2(\alpha) = \underbrace{\left(\frac{\partial \bar{x}}{\partial \bar{e}} + (N-1) \frac{\partial x}{\partial \bar{e}} \right)}_{>0} \underbrace{\left(\frac{\partial \bar{e}}{\partial \bar{\alpha}} - \frac{\partial \bar{e}}{\partial \alpha} \right)}_{A} + (N-2) \left(\frac{\partial x}{\partial \bar{\alpha}} - \frac{\partial \tilde{x}}{\partial \alpha} \right) + \frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha} + \frac{\partial x}{\partial \bar{\alpha}} - \frac{\partial x}{\partial \alpha}$$

First note that the last term composed by the derivatives of stocks (w.r.t α) are unambiguously positive since it involves analyzing Equations 9 and 10 as follows:

$$\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha} + \frac{\partial x}{\partial \bar{\alpha}} - \frac{\partial x}{\partial \alpha} = (D + Q) \left(g_{\bar{\alpha}} - g_{\alpha} \right) > 0.$$

Then the other parenthetical term involves analyzing Equations 9 and 11 such that: $\frac{\partial x}{\partial \bar{\alpha}} - \frac{\partial \tilde{x}}{\partial \alpha} = Q \left(g_{\bar{\alpha}} - g_{\alpha} \right)$. We now provide sufficient conditions that ensure that term A is positive, which would enable us to conclude that $\Delta_2(\alpha)$ is positive. First, note that if $c' = 0$, $\frac{\partial \bar{x}}{\partial \alpha} = \frac{\partial \bar{e}}{\partial \alpha} = 0$, then the difference $\Delta_2(\alpha)$ is unambiguously positive. If $c' < 0$, then the difference $\Delta_2(\alpha)$ is positive provided $D \geq Q$ or $D = 0$ which, by a continuity argument, enables to conclude that $\Delta_2(\alpha) > 0$ for sufficiently small value of D.

3. **Enclosed patch profit**
Adopting the conditions in Table 4, the difference 17 is:

$$\Delta_3(\alpha) = \frac{d\bar{\Pi}}{d\bar{\alpha}} - \frac{\partial \bar{\Pi}}{\partial \alpha} = \frac{\delta}{1-\delta} \left(p - c(\bar{x}) \right) \left(\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha} \right) = \frac{\delta}{1-\delta} \left(p - c(\bar{x}) \right) \left(Dg_{\bar{\alpha}} - Qg_{\alpha} \right)$$

First note that if $c' = 0$ then $\frac{\partial \bar{x}}{\partial \alpha} = 0$ and $\Delta_3(\alpha)$ is unambiguously positive. If $c' < 0$, then this term is positive provided $D \geq Q$, and is negative for sufficiently small D.

4. **Aggregate profit**
   Adopting the conditions in Table 4, the difference 18 is:

$$\begin{aligned}\Delta_4(\alpha) &= \frac{\partial\bar{\Pi}}{\partial\bar{\alpha}} + (N-1)\left(\frac{\partial\Pi}{\partial\bar{e}} + \frac{\partial\bar{e}}{\partial\alpha} + \frac{\partial\Pi}{\partial\bar{\alpha}}\right) \\ &- \left[\frac{\partial\Pi}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\alpha} + \frac{\partial\Pi}{\partial\alpha} + \frac{\partial\bar{\Pi}}{\partial\alpha} + (N-2)\left(\frac{\partial\tilde{\Pi}}{\partial e}\frac{\partial e}{\partial\alpha} + \frac{\partial\tilde{\Pi}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\alpha} + \frac{\partial\tilde{\Pi}}{\partial\alpha}\right)\right]\end{aligned}$$

   We use the fact that $\frac{\partial\Pi}{\partial\bar{e}} = \frac{\partial\tilde{\Pi}}{\partial\bar{e}}$ to rewrite $\Delta_4(\alpha)$:

$$\Delta_4(\alpha) = (N-1)\frac{\partial\Pi}{\partial\bar{e}}\underbrace{\left(\frac{\partial\bar{e}}{\partial\bar{\alpha}} - \frac{\partial\bar{e}}{\partial\alpha}\right)}_{A} + (N-2)\underbrace{\left(\frac{\partial\Pi}{\partial\bar{\alpha}} - \frac{\partial\tilde{\Pi}}{\partial\alpha}\right)}_{=(p-c(x))Q(g_{\bar{\alpha}}-g_{\alpha})>0} + \frac{\partial\bar{\Pi}}{\partial\bar{\alpha}} - \frac{\partial\bar{\Pi}}{\partial\alpha} + \frac{\partial\Pi}{\partial\bar{\alpha}} - \frac{\partial\Pi}{\partial\alpha}$$

   The last term composed by the derivatives of profits (with respect to α) is unambiguously positive since it involves analyzing Equations 12 and 13 as follows:

$$\frac{\partial\bar{\Pi}}{\partial\bar{\alpha}} - \frac{\partial\bar{\Pi}}{\partial\alpha} + \frac{\partial\Pi}{\partial\bar{\alpha}} - \frac{\partial\Pi}{\partial\alpha} = (g_{\bar{\alpha}} - g_{\alpha})\left[D\left(p - c(\bar{x})\right) + Q\left(p - c(x)\right)\right] > 0.$$

   As previously, a sufficient condition to sign this difference depends on the term A. First, note that if $c'(\cdot) = 0$, the difference $\Delta_4(\alpha)$ is unambiguously positive. If $c'(\cdot) < 0$, if $D \geq Q$ then $\Delta_4(\alpha) > 0$. Instead, suppose $D = 0$. In that case, the term A is equal to zero and $\Delta_4(\alpha) > 0$ which, by a continuity argument, enables to conclude that $\Delta_4(\alpha) > 0$ for sufficiently small value of D.

Proof of Proposition 6 (price, p)

1. **Enclosed patch stock**
   Adopting the conditions in Table 4 the difference 15 is:

$$\Delta_1(p) = \frac{\partial\bar{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{p}} - \frac{\partial\bar{x}}{\partial e}\frac{\partial e}{\partial p} \tag{23}$$

   First note that if $c' = 0$, $\frac{\partial e}{\partial p} = \frac{\partial\bar{e}}{\partial\bar{p}} = 0$ so $\Delta_1(p) = 0$. If $c'(\cdot) < 0$, then the difference can be written:

$$\Delta_1(p) = Dg_{\bar{e}}\frac{\partial\bar{e}}{\partial\bar{p}} - Qg_e\frac{\partial e}{\partial p} \tag{24}$$

   This term is positive for sufficiently small D and negative for sufficiently small Q.
2. **Aggregate stock**
   Adopting the conditions in Table 4, the difference 16 is:

$$\begin{aligned}\Delta_2(p) &= \frac{\partial\bar{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{p}} + \frac{\partial\bar{x}}{\partial\bar{p}} + (N-1)\left(\frac{\partial x}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{p}} + \frac{\partial x}{\partial\bar{p}}\right) \\ &- \left[\frac{\partial x}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial x}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial p} + \frac{\partial x}{\partial p} + \frac{\partial\bar{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial p} + \frac{\partial\bar{x}}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial\bar{x}}{\partial p} + (N-2)\left(\frac{\partial\tilde{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial p} + \frac{\partial\tilde{x}}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial\tilde{x}}{\partial p}\right)\right]\end{aligned}$$

   We use the facts that $\frac{\partial\bar{x}}{\partial e} = \frac{\partial\tilde{x}}{\partial e}$ and $\frac{\partial\tilde{x}}{\partial\bar{e}} = \frac{\partial x}{\partial\bar{e}}$ to rewrite $\Delta_2(p)$:

$$\Delta_2(p) = [D + (N-1)Q]\left[g_{\bar{e}}\left(\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial \bar{e}}{\partial p}\right) - g_e\frac{\partial e}{\partial p}\right].$$

If $c'(\cdot) = 0$ then the first order condition for the enclosed patch is $1 = \delta D g_{\bar{e}}$, which is independent of the market price in patch 1 in any case. This implies that $\frac{\partial \bar{e}}{\partial \bar{p}} = \frac{\partial \bar{e}}{\partial p} = 0$. Thus, if $c'(\cdot) = 0$, $\Delta_2(p) = 0$.
Instead if $c'(\cdot) < 0$ we have

$$\frac{\partial \bar{e}}{\partial \bar{p}} = \frac{1 - \delta D g_{\bar{e}}}{SOC} < 0 \ ; \ \frac{\partial \bar{e}}{\partial p} = \frac{\delta D Q g_{\bar{e}} g_e c'(\bar{x})\frac{\partial e}{\partial p}}{SOC} < 0$$

Because $g_{\bar{e}} < g_e$ (as $\bar{\alpha} = \alpha$ and $g_{ee} < 0$), we have: $g_{\bar{e}}\left(\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial \bar{e}}{\partial p}\right) - g_e\frac{\partial e}{\partial p} > g_{\bar{e}}\left(\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial e}{\partial p}\right)$. When D gets close to zero, $\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial e}{\partial p}$ gets close to zero, which by a continuity argument, implies that $g_{\bar{e}}\left(\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial \bar{e}}{\partial p}\right) - g_e\frac{\partial e}{\partial p} \geq 0$ (and $\Delta_2(p) > 0$) for sufficiently small values of D.

3. **Enclosed patch profit**
   Adopting the conditions in Table 4 the difference 17 is:

$$\Delta_3(p) = \frac{\partial \bar{\Pi}}{\partial \bar{p}} - \frac{\partial \bar{\Pi}}{\partial e}\frac{\partial e}{\partial p} \tag{25}$$

   The first term is positive. If $c'(\cdot) = 0$, $\frac{\partial e}{\partial p} = 0$, so $\Delta_3(p) > 0$. If $c'(\cdot) < 0$, then the second term, equal to $\frac{(p - c(\bar{x}))Q g_e}{c'(e)}$, is negative. Thus the difference $\Delta_3(p) > 0$.

4. **Aggregate profit**
   Adopting the conditions in Table 4, the difference 18 is:

$$\begin{aligned}
\Delta_4(p) &= \frac{\partial \bar{\Pi}}{\partial \bar{p}} + (N-1)\frac{\partial \Pi}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \bar{p}} \\
&- \left[\frac{\partial \Pi}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial \Pi}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial p} + \frac{\partial \Pi}{\partial p} + \frac{\partial \bar{\Pi}}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial \bar{\Pi}}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial p} + (N-2)\left(\frac{\partial \tilde{\Pi}}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial \tilde{\Pi}}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial p}\right)\right]
\end{aligned}$$

   Which can be rewritten:

$$\begin{aligned}
\Delta_4(p) &= x_0 - \bar{e} + \frac{\delta}{1-\delta}(\bar{x} - \bar{e}) + (N-1)\frac{\delta}{1-\delta}(p - c(x))\,Q g_{\bar{e}}\frac{\partial \bar{e}}{\partial \bar{p}} \\
&- \left[x_0 - e + \frac{\delta}{1-\delta}(x - e) + \frac{\delta}{1-\delta}(p - c(x))\,D g_e\frac{\partial e}{\partial p}\right] \\
&- \left[\frac{\delta}{1-\delta}(p - c(x))\,Q g_{\bar{e}}\frac{\partial \bar{e}}{\partial p} + \frac{\delta}{1-\delta}(p - c(\bar{x}))\,Q g_e\frac{\partial e}{\partial p}\right] \\
&- (N-2)\left(\frac{\delta}{1-\delta}(p - c(\tilde{x}))\,Q g_e\frac{\partial e}{\partial p} + \frac{\delta}{1-\delta}(p - c(\tilde{x}))\,Q g_{\bar{e}}\frac{\partial \bar{e}}{\partial p}\right)
\end{aligned}$$

   If $c'(\cdot) = 0$, then we use the facts that $\frac{\partial \bar{e}}{\partial \bar{p}} = \frac{\partial \bar{e}}{\partial p} = \frac{\partial e}{\partial p} = 0$, which imply that

$$\begin{aligned}
\Delta_4(p) &= e - \bar{e} + \frac{\delta\,(\bar{x} - \bar{e} - x + e)}{1-\delta} = \frac{(e - \bar{e})}{1-\delta} + \frac{\delta\,(\bar{x} - x)}{1-\delta} \\
&= \frac{(e - \bar{e})}{1-\delta} + \frac{\delta\,(D - Q)\,[g(\bar{e}, \bar{\alpha}) - g(e, \alpha)]}{1-\delta}.
\end{aligned}$$

The first conclusion is that $\Delta_4(p) < 0$ as long as $Q \geq D$ or δ is small. Indeed, when $Q \geq D$, the second term in the expression of $\Delta_4(p)$ is negative, and the first term is obviously negative as $\bar{e} > e$. Now, when δ is small, the sign of $\Delta_4(p)$ is given by that of the following expression:
$e - \bar{e} + \delta(D-Q)\left[g(\bar{e},\bar{\alpha}) - g(e,\alpha)\right] = (\bar{e}-e)\left(-1 + \frac{\delta(D-Q)[g(\bar{e},\bar{\alpha})-g(e,\alpha)]}{\bar{e}-e}\right)$. When $\delta \to 0$, we have $\bar{e} \to e$: in this case, this implies that $\frac{g(\bar{e},\bar{\alpha})-g(e,\alpha)}{\bar{e}-e} \to g_e > 0$ (and finite). All together, when δ converges to zero, we have: $-1 + \frac{\delta(D-Q)[g(\bar{e},\bar{\alpha})-g(e,\alpha)]}{\bar{e}-e} \to -1 < 0$. Thus, a continuity argument enables us to conclude that $\Delta_4(p) \leq 0$ for sufficiently small values of δ.
Moreover, we know that $e = 0$, which implies that $g(e,\alpha) = g(0,\alpha) = 0$ and $\bar{x} = Dg(\bar{e},\bar{\alpha})$. Plugging these expressions into the above equality and simplifying, we obtain:

$$\Delta_4(p) = -\frac{\bar{e}}{1-\delta} + \frac{\delta\,(D-Q)\,g(\bar{e},\bar{\alpha})}{1-\delta} = \frac{1}{1-\delta}\left[-\bar{e} + \delta\left(1 - \frac{Q}{D}\right)\bar{x}\right].$$

If Q gets close to zero and δ gets close to one, then the sign of $\Delta_4(p)$ is that of $-\bar{e} + \bar{x}$, which is positive. By a continuity argument, we conclude that $\Delta_4(p) \geq 0$ for sufficiently small values of Q and large values of δ.

Input parameters for numerical example from Section 3.1

$D =$

$$\begin{pmatrix}
.728 & .013 & .039 & .008 & .002 & .005 & .016 & .018 & .011 & .005 & .002 & .003 & .002 \\
.008 & .697 & 0.056 & .007 & .002 & .005 & .025 & .026 & .013 & .004 & .002 & .004 & .002 \\
.010 & .025 & .722 & .007 & .002 & .005 & .028 & .026 & .013 & .005 & .002 & .004 & .002 \\
.004 & .015 & .039 & .717 & .002 & .006 & .020 & .018 & .014 & .007 & .001 & .004 & .002 \\
.002 & .010 & .022 & .003 & .765 & .004 & .013 & .015 & .006 & .005 & .001 & .003 & .001 \\
.002 & .009 & .013 & .002 & .001 & .777 & .015 & .015 & .006 & .003 & .002 & .004 & .001 \\
.003 & .004 & .008 & .003 & .001 & .003 & .803 & .010 & .005 & .004 & .002 & .004 & .001 \\
.009 & .016 & .046 & .008 & .002 & .005 & .015 & .717 & .015 & .007 & .003 & .003 & .003 \\
.005 & .015 & .044 & .005 & .002 & .004 & .013 & .019 & .731 & .006 & .003 & .002 & .002 \\
.008 & .014 & .032 & .011 & .002 & .005 & .014 & .020 & .021 & .711 & .007 & .002 & .002 \\
.005 & .007 & .013 & .004 & .001 & .003 & .010 & .012 & .013 & .009 & .772 & .002 & .001 \\
.003 & .002 & .001 & .001 & .000 & .000 & .003 & .004 & .007 & .002 & .001 & .825 & .001 \\
.004 & .003 & .003 & .001 & .000 & .001 & .003 & .005 & .004 & .001 & .001 & .001 & .824
\end{pmatrix}$$

$r = [.4297; .4254; .4329; .4980; .5034; .6769; .6460; .4000; .4353; .5070; .4758; .4787; .4000]$

$K = [100.88; 100.65; 101.08; 109.60; 110.68; 176.67; 160.53; 100.00; 101.25; 111.46; 105.75; 106.19; 100.00]$

$\delta = 0.90$, $p = 1.0$, $\theta = 15$.

Input parameters for numerical example from Section 3.2

$Q = .06$, $D = .20$ though we also examine values of $D \in [0, .3]$, $r = .485$, $K = 114.21$, $\delta = 0.90$, $p = 1.0$, $\theta = 15$.