# Aging and Monetary Policy*

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November 16, 2013

**Abstract**

This paper studies the impact of aging on effectiveness of monetary policy. To do so, it introduces an OLG-DNK framework where the demand side is represented by a two period overlapping generations setup and the supply side of the economy follows a New Keynesian framework. The model enables the study of the interaction of monetary policy with demographics in a coherent general equilibrium model. The main finding is that this merger of two basic strands of the macroeconomics literature implies monetary policy should be expected to be less effective as societies age since the interest rate sensitivity of real economic activity declines as the population ages.

**JEL codes:** E32, E52, J11
**Keywords:** aging, monetary policy, heterogenous agents, overlapping generations model, New Keynesian model

*I thank Refet Gürkaynak and Jaume Ventura for their guidance. I am indebted to Jordi Galí for his valuable help and comments. I also thank participants of Macroeconomics Breakfast and International Lunch Seminars in Universitat Pompeu Fabra, VI. Winter Workshop in Economics in Koç University, Macroeconomics Seminars in Bilkent University and EEA-ESEM Gothenburg 2013 for helpful comments and suggestions. All errors are mine.

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## 1 Motivation

Aging populations are important policy concerns and a correspondingly important academic research area. The economic and the policy impact of aging have been extensively studied in the fiscal policy context, especially with respect to its social security implications. The literature, however, surprisingly kept the studies of monetary policy and demographics separate, effectively treating these as independent issues. This paper argues that demographics should be expected to have important bearing on the effectiveness and hence conduct of monetary policy.

The contribution of this paper is not only to address the proposed policy concern, but also to provide a tractable model for analysis. This paper constructs a simple setup which merges two period overlapping generations (OLG) and dynamic new Keynesian (DNK) frameworks. The demand side of the model assumes an OLG structure which enables us to introduce aging into the model economy. Furthermore, differently from the existing OLG literature with aging, I introduce a DNK setup to analyze the effectiveness of monetary policy. Following the standard DNK setup of Galí (2008), the introduction of price rigidities to the model allows monetary policy to influence interest rate and the real economy. This highly stylized model enables us to study the impact and transmission of monetary policy in economies with different demographic profiles.

A number of papers have focused on the relation between societal aging and monetary policy.[1] The closest papers to this work are Fujiwara and Teranishi (2005) and Fujiwara and Teranishi (2008). The structure of these two papers are similar to this paper. Both[2] employ the same theoretical model by incorporating capital producers, financial intermediaries and

[1] Miles (2002) is the first study that discusses the aging and monetary policy with various specifications for pension systems. He utilizes three different OLG models by assuming consumption smoothing over time and forward looking behavior of agents. Findings of the paper are highly sensitive to the existence and generosity of the pension system.

[2] Fujiwara and Teranishi (2005) is the welfare implication of the Fujiwara and Teranishi (2008)

government. They utilize a dynamic stochastic general equilibrium model that integrates life-cycle behavior a la Gertler (1997), to study whether structural shocks to the economy have asymmetric effects on workers and retirees. Both papers suggest that in an older economy, tightening monetary policy shock has much larger negative effect on aggregate demand compared to younger and no cycle economies. However, this result depends on the assumption which allows retired agents to rejoin to the labor force. The findings of the paper would be consistent with the results of this paper if the old agents are unable to rejoin.

In addition to the different structure of the model, the comparative advantage of this paper is its simplicity. I believe that studying the proposed question in a basic and a tractable theoretical framework is an intermediate and also a required step for full understanding of the dynamics of such economy. This paper evaluates and analyzes the effects of integration of OLG setup and aging to key equations of the standard DNK framework: the forward looking IS equation, the dynamic Phillips equation and the Taylor rule. Differently from the Fujiwara and Teranishi (2008), this paper constructs the demand side of the model with two period OLG setup which is consistent with life cycle theory of Modigliani and Brumberg (1954). The life cycle theory suggests that individuals try to smooth their consumption over time and their savings follow a hump-shaped pattern, with higher savings during their working age. Moreover, in this simple model economy I assume retired agents do not rejoin to the labor force and do not participate actively in financial markets. Findings of the paper can be summarized as follows:

First, the natural rate of interest decreases monotonically as the ratio of population of old to young agents increases which is consistent with the earlier applied studies. Bean (2004) suggests that both along the transition path and the steady state, societal aging may lead to a sharp decline in saving behavior of agents, supply of labor and the natural rate of interest. There are other papers which have findings in line with this result. Auerbach et al. (1991) and Auerbach et al. (1989) quantitatively show the negative impact of aging on saving rate. Rios-Rull (2001) finds a similar result by using Spanish data. Miles (1999) points out that there will be radical changes in saving rates as a result of an increase in the old age dependency ratio which is the ratio of population of retired agents to workers. Simulations of the paper indicate that private saving rates are likely to fall in long term in line with aging populations.

Second, the effectiveness of monetary policy on output gap (output) decreases due to decreasing interest rate sensitivity of the society as the population ages. The model also suggests that it is even possible to see positive response of output to a tightening monetary policy shock if the economy becomes old enough. Third, the trade-off between inflation and output decreases as the economy ages. Hence, results of the paper suggests that the policymakers should account for the demographic profile of the country when conducting monetary policies.

This highly stylized model may not provide an accurate quantitative responses to an unexpected monetary policy shock, since it assumes an extreme case where retired agents are not active in the financial market and consume all their wealth. However, the model utilized in this paper is an intermediate and also a required step to analyze and understand the dynamics of the monetary policy in an aging economy.

The paper is organized as follows: Section 2 sets up the model which incorporates two period OLG model into basic New Keynesian framework of Clarida et al. (1999). In section 3, I conduct experiments with the proposed theoretical model by assuming economies with different demographic profiles. Then, I interpret and compare the responses of these economies to an unexpected monetary policy shock. Finally, section 4 concludes.

# 2 The Model

This section introduces the model which incorporates a two period overlapping generations (OLG) setup to standard New Keynesian framework (DNK). OLG setup enables us to introduce aging to the model and DNK framework has the convenient environment to study the effectiveness of monetary policy.

The demand side of the economy, the households' problem, is modeled as an OLG setup, introduced by Samuelson (1958) and Diamond (1965). It contains two generations, workers and retirees, who are born at different dates and have finite lifetimes, even though the economy lasts forever. All agents in the economy are born as workers. In the first period of their lifetime, agents earn wage income by supplying labor and decide how much to consume and save. Workers can save in two types of assets: one-period nominally riskless discount bonds yielding a nominal return and equity shares of the firms which are infinite-lived assets. It is crucial to have stock market in this setup because it links the short-lived agents to infinitely lived firms. The ownership of the firms is transferred through equity market. All agents retire in the second period of their lifetime. In the retirement period, households stop supplying labor and consume all their wealth and at the end of the period they die. Supply side of the economy has the basic New Keynesian framework a lá Clarida et al. (1999). Monetary policy follows a Taylor (1993) rule, where the central bank reacts to the output gap and inflation.

## 2.1 Demographics

This section describes how we introduce aging into the model. The number of workers and retired agents at time $t$ are denoted by $N_t^w$ and $N_t^r$, respectively. Only workers are fertile so that $N_t^w = (1+n)N_{t-1}^w$, where $n$ is the fertility rate of workers. The total population at time $t$ is

$$\underbrace{N_{t-1}^w}_{\text{Number of Retirees}} + \overbrace{(1+n)N_{t-1}^w}^{\text{Number of Workers}}$$

where $N_{t-1}^w = N_t^r$. Agents can work only in the first period of their lifetime. Hence, the labor supply at time $t$ corresponds to the the number of workers at time $t$.

Finally, it is useful, to define an indicator for aging, which is the old-age dependency ratio, denoted by $\varphi$. It is the ratio of retired to employed agents in period $t$.

$$\frac{N_t^r}{N_t^w} = \frac{N_{t-1}^w}{(1+n)N_{t-1}^w} = \frac{1}{1+n} = \varphi$$

Old age dependency ratio decreases as the population growth rate increases. This study compares old and relatively younger societies at steady state, where the population growth rates are constant over time, but different across the economies. In other words, the old age dependency ratio does not change across time in either economy.

## 2.2 Households

In the life cycle economy assumed in this model, there are two types of households; workers and retirees. Agents live for two periods.

### 2.2.1 Retirees

The representative retiree $j$ consumes all his wealth and then dies. The budget constraint of retired agent $j$ at time $t+1$ is given below. $D_{t+1}(j)$ denotes consumption of a retired agent at time $t+1$. $P_{t+1}$ refers to price of a consumption good at time $t+1$. $Div_{t+1}(i)$ and $Q_{t+1}(i)$ represent dividend paid by the monopolistically competitive firm $i$ and price of share of firm $i$ at time $t+1$, respectively. $S_t(i,j)$ shows the amount of shares of firm $i$

held by agent $j$. Formally,

$$P_{t+1}D_{t+1}(j) = B_t^n(j)(1+i_t) + \int_0^1 \left(Div_{t+1}(i) + Q_{t+1}(i)\right) S_t(i,j)di$$

where $B_t^n(j)$ represents nominal bond holdings of agent $j$ and $i_t$ refers nominal interest rate at time $t$. Differently from the standard DNK model, we have an equity market in this setup, which enables us to combine the short-lived agents to infinite living firms. The ownership of the firm is transferred through the equity market, that is to say when agents are young they buy stocks of the firms and in the next period they become the owner of it.

### 2.2.2 Workers

The representative worker $j$ maximizes in period $t$ the objective function,

$$V_t^j = \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \frac{L_t(j)^{1+\psi}}{1+\psi} + \beta E_t\left(\frac{D_{t+1}(j)^{1-\sigma}}{1-\sigma}\right)$$

subject to

$$P_tC_t(j) + B_t^n(j) + \int_0^1 Q_t(i)S_t(i,j)di = W_tL_t(j)$$

and

$$P_{t+1}D_{t+1}(j) = B_t^n(j)(1+i_t) + \int_0^1 \left(Div_{t+1}(i) + Q_{t+1}(i)\right) S_t(i,j)di$$

where $C_t(j)$ and $L_t(j)$, respectively, denote consumption and labor supply of agent $j$. The parameters $\psi$ and $\sigma$ represent the elasticity of labor supply and intertemporal elasticity of substitution, respectively. The representative worker faces the nominal wage rate $W_t$ at time $t$. The saving occurs in form of equity shares of firms or one-period nominally riskless discount bonds. The first order conditions of the household's problem are

$$C_t(j)^{-\sigma}\frac{W_t}{P_t} = L_t(j)^{\psi} \tag{1}$$

$$C_t(j)^{-\sigma} = \beta E_t \left\{(1+i_t)\frac{P_t}{P_{t+1}}D_{t+1}(j)^{-\sigma}\right\} \tag{2}$$

$$C_t(j)^{-\sigma} = \beta E_t \left\{\left[\frac{Q_{t+1}(i) + Div_{t+1}(i)}{Q_t(i)}\right]\frac{P_t}{P_{t+1}}D_{t+1}(j)^{-\sigma}\right\}. \tag{3}$$

That is,

$$\Lambda_{t,t+1} = \frac{1}{1+i_t} = \beta E_t \left\{\left(\frac{C_t(j)}{D_{t+1}(j)}\right)^{\sigma}\frac{P_t}{P_{t+1}}\right\} \tag{4}$$

where $\Lambda_{t,t+1}$ represents the stochastic discount factor.

Rearranging equations (2) and (3), we get the no arbitrage condition,

$$E_t\left\{\left(\frac{D_{t+1}(j)}{C_t(j)}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\left((1+i_t) - \left(\frac{Div_{t+1}(i) + Q_{t+1}(i)}{Q_t(i)}\right)\right)\right\} = 0 \tag{5}$$

which suggests that it does not matter whether workers save in riskless bonds or stocks in equity market. Both yield the same return in such a riskless economy.

## 2.3 Firm side

The supply side of the economy is modeled based on the basic New Keynesian framework following Clarida et al. (1999). There are two types of firms which are consumption and intermediate goods producers. There is imperfect competition in the intermediate goods market due to the assumption that each firm produces a differentiated good. We follow a staggered price setting a la Calvo (1983), in which every period a random fraction of firms is optimally setting prices.

### 2.3.1 Consumption Good Firm

There is a continuum of intermediate goods indexed by $i \in [0,1]$, which are transformed into a homogenous consumption good according to the constant returns to scale production function

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $Y_t(i)$ is the quantity of the intermediate good $i$ and $\epsilon > 1$ denotes the price elasticity of demand. The consumption good sector is subject to perfect competition which determines the demand function for the representative intermediate good $i$

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t$$

where $P_t(i)$ and $P_t$ denote the price of good $i$ and the average price level, respectively. Reflecting the CES-structure of the technology in the final goods sector, $P_t$, is given by

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} .$$

### 2.3.2 Intermediate Good Firms

In the intermediate good sector there is a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, $i$, with the production function

$$Y_t(i) = N_t(i) \tag{6}$$

where $Y_t(i)$ and $N_t(i)$, respectively, denote the output of firm $i$ and the labor input. Labor market is competitive, i.e. the nominal wage rate $W_t$ is taken as given in the production of good $i$. Intermediate firms are owned by the equity holders (retired agents) and are managed to maximize the profit to the current owners. Through the final goods producing sector, intermediate firm $i$ faces a downward sloping demand curve. At time $t$ nominal profits (dividends) are:

$$P_t Div_t(i) = P_t(i)Y_t(i) - W_t N_t(i)$$

Under flexible prices, after observing the shock, firms choose price $P_t^*$,

$$P_t^* = \frac{\epsilon}{\epsilon - 1}\frac{W_t}{P_t}P_t = \mathcal{M}MC_t$$

where $MC_t$ and $\frac{\epsilon}{\epsilon-1} = \mathcal{M}$, respectively, denote the marginal cost and the desired mark up value. In a symmetric equilibrium, $P_t^* = P_t$ and following that $\frac{W_t}{P_t} = \frac{1}{\mathcal{M}}$.

Following Calvo (1983), nominal price rigidity is modeled by allowing random intervals between price changes. At each period a firm adjusts its price with a constant probability $(1-\theta)$ and keeps its price fixed with probability $\theta$. The reoptimizing firm solves

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{\Lambda_{t,t+k}(P_t^* Y_{t+k}(i) - W_{t+k}Y_{t+k}(i))\right\}$$

subject to the demand function of the intermediate good. The first order condition is:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{\Lambda_{t,t+k} Y_{t+k}(i) \left(P_t^* - \frac{\epsilon}{\epsilon - 1}W_{t+k}\right)\right\} = 0$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor formalized in equation 4.

The reoptimizing firms owners set their price with the above equality considering the expected profit of the firm. The existence of equity market is crucial in this setup. If we do not have the equity market, then the owner of the firm will just maximize the profit of the current period because she dies at the end of the period. However, the equity market enables us to utilize the standard firm side problem as in the DNK model. Following the no arbitrage condition, to maximize the price of the stocks today, owners of the firms (retired agents) should maximize the expected profit of the firm.

### 2.4 The Central Bank

The Monetary policy authority follows a standard Taylor (1993) type feedback rule:

$$i_t = i^* + \phi_\pi(\hat{\pi}_t) + \phi_x(\hat{x}_t) + v_t$$

where $i^*$ denotes the steady state value of interest rate, $\phi_\pi$ and $\phi_x$ are positive feedback parameters, $\hat{\pi}_t$ is the deviation of rate of inflation from its steady state value, and $\hat{x}_t$ is the deviation of real GDP from potential output. The exogenous component of the monetary policy is denoted by $v_t$ and follows an AR(1) process

$$v_t = \rho_v v_{t-1} + \epsilon_t^v$$

where $\epsilon_t^v$ denotes the monetary policy shock and $\rho_v \in [0, 1)$ shows the persistence of the shock.

### 2.5 Equilibrium Conditions

This section presents the market clearing conditions for the model economy. It is worth emphasizing that, in the standard DNK setup aggregate, per worker and per capita variables are all same. However, in this analysis they are all different and have to be kept track of.

Goods market clearing condition requires that $Y_t = N_t^w C_t(j) + N_t^r D_t(j)$ for all $t$. I normalize the equations in terms of per worker, that is:

$$Y_t^p = C_t(j) + \varphi D_t(j)$$

where $Y_t^p$ refers to per worker output. Labor market clearing implies

$$L_t(j)N_t^w = \int_0^1 N_t(i)di = N_t.$$

In per worker terms, it is

$$L_t(j) = \frac{N_t}{N_t^w} = Y_t^p.$$

At equilibrium, agents do not trade any assets among themselves and there is no room for transactions between households from different cohorts, therefore $B_t = 0$. The aggregate stock outstanding equity, $S_t(i)$, for each intermediate good producing firm must equal to the corresponding total amount of issued shares normalized to 1 $\forall i \in [0, 1]$.

Combining the budget constraints of the two cohorts in period $t$ at equilibrium we get,

$$\begin{aligned} Y_t^p &= C_t(j) + \varphi D_t(j) \\ &= \frac{W_t}{P_t} L_t(j) + Div_t. \end{aligned}$$

Finally, the total real dividend payments by the intermediate firms and total stock price index are given:

$$Div_t = \int_0^1 Div_t(i)di \qquad\qquad Q_t = \int_0^1 Q_t(i)di. \tag{7}$$

In order to analyze the dynamics of the model, this paper focuses on the log-linearized system around steady state values.

#### 2.5.1 Steady State

The following equations describe the steady state conditions of the model. Starred variables denote steady state values.

$$y^* = c^* + \varphi d^* \tag{8}$$

$$w^* - p^* = \frac{1}{\mathcal{M}} \tag{9}$$

$$c^{*-\sigma} = \mathcal{M} y^{*\psi} \tag{10}$$

$$\left(\frac{d^*}{c^*}\right)^{\sigma} = \beta(1+i^*) \tag{11}$$

$$\varphi d^* = y^* \left(1 - \frac{1}{\mathcal{M}}\right)\left(\frac{1+i^*}{i^*}\right) \tag{12}$$

$$div^* = y^* \left(\frac{\mathcal{M}-1}{\mathcal{M}}\right) \tag{13}$$

Equation (8) demonstrates the steady state of goods market clearing condition in per worker terms. Equation (9) shows the steady state value of real marginal cost, which is real wage in this setup and it equals to the markup in a frictionless environment. Steady state of the labor supply condition is given in Equation (10).

In standard DNK setup steady state interest rate depends on the discount factor, $\beta$. However, in this setup it is different. Equation (11) suggests that the steady state interest rate also depends on the demographic structure of the economy[3]. It is worth emphasizing the relation between steady state interest rate and the old age dependency ratio.

***Remark:*** When $\sigma = 1$, the steady state interest rate equation is as follows:

$$1 + i^* = \frac{\mathcal{M} + \sqrt{\mathcal{M}^2 + 4\frac{\mathcal{M}-1}{\varphi\beta}}}{2}.$$

[3] $\beta(1+i^*) = \left(\frac{d^*}{c*}\right)^{\sigma} = \left(\frac{1}{\varphi}\frac{(\mathcal{M}-1)(1+i^*)}{1+i^*-\mathcal{M}}\right)^{\sigma}$

The above equation suggests that as the old age dependency increases, steady state interest rate decreases monotonically. For every $\sigma > 1$, this statement is still valid. Figure 1 shows the relation between old age dependency and the steady state interest rate with the assumed parameter values. The relation suggested by the model is consistent with the empirical literature and captures the fact that, a decrease in the number of workers implies scarcer labor compared to capital and this leads to a decrease in the real interest rate.

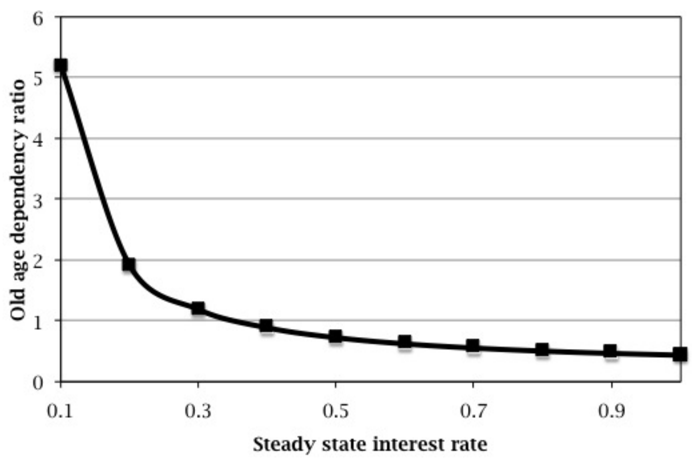


Figure 1: Steady state interest rate

Equations (12) and (13) show, respectively, the steady state values of per worker old age consumption and dividends.

### 2.6 Log-linearized Dynamics

This section provides the log-linearized equations around the zero inflation steady state. I use lower case letters to show the log of the variable and hat to indicate the deviation from its steady state. The demand side equations are as follows:

$$\hat{w}_t - \hat{p}_t = \psi \hat{l}_t + \sigma \hat{c}_t \tag{14}$$

$$\hat{c}_t = E_t\{\hat{d}_{t+1}\} - \frac{1}{\sigma}[\hat{\imath}_t - E_t\{\pi_{t+1}\}] \tag{15}$$

$$-\sigma\hat{c}_t(j) = -\sigma\hat{d}_{t+1} + \frac{i^*}{1+i^*}\hat{div}_{t+1} + \frac{1}{1+i^*}\hat{q}_{t+1} - \pi_{t+1} - \hat{q}_t \tag{16}$$

$$\hat{div}_t = \hat{y} + \frac{1}{\mathcal{M}-1}(\hat{w}_t - p_t) \tag{17}$$

Equation (14) denotes the labor supply decision of a young agent at time $t$. Equation (15) is the linear Euler equation for a young agent at time $t$. Differently from the standard DNK setup, Euler equation in this setup shows the relationship between consumption of a representative young agent at time $t$ and her old age consumption at time $t+1$. Therefore, in this model one can not say Euler equation denotes the relation between total production (consumption) at time $t$ and $t+1$, namely the *dynamic IS equation*. The total consumption (production) in this setup at time $t$ is (in linearized terms):

$$\hat{y}_t = \frac{1+i^*-\mathcal{M}}{\mathcal{M}i^*}\hat{c}_t(j) + \varphi\frac{(\mathcal{M}-1)(1+i^*)}{\mathcal{M}i^*}\hat{d}_t(j) \tag{18}$$

Equation (18) refers to *OLG–DNK IS equation*. To get the total consumption, additional calculations are made differently from the derivation of standard DNK IS equation.[4] The OLG–IS equation not only depends on the current period's interest rate and expected inflation but also the previous period's interest rate and expected inflation rate and realized inflation. This is due to the fact that, at time $t$ there are two types of agents, optimizing according to the available information to them. Young agents at time $t$, decide their consumption using the current period's information. However, retired agents at time $t$, have chosen their old age consumption in the previous period using the information that is available in the previous period. Therefore, compared to the standard DNK IS equation, a richer dynamic system is achieved. Finally, Equation (16) shows the stock price dynamics

[4] See appendix A for the detailed derivation.

implied by the model.

The supply side equations are:

$$\hat{y}_t = \hat{l}_t \tag{19}$$

The forward looking Phillips equation is as follows.

$$\begin{aligned} \pi_t &= \tilde{\beta} E_t\{\pi_{t+1}\} + \tilde{\kappa}\left[\frac{\varphi i^*(1+\psi)\sigma}{(\mathcal{M}-1)(1-\varphi)-\varphi i^*(\sigma-1)} + \psi\right]\hat{x}_t \\ &+ \tilde{\kappa}\left[\frac{(\mathcal{M}-1)\sigma}{(\mathcal{M}-1)(1-\varphi)-\varphi i^*(\sigma-1)}\left(\frac{\sigma-1}{\sigma}\hat{i}_t - \pi_{t+1} + \frac{1}{\sigma}E_t\{\pi_{t+1}\} - \rho\right)\right] \end{aligned} \tag{20}$$

where $\tilde{\beta} = \beta J,\ J = \left[\left(\varphi\frac{1+i^*-\mathcal{M}}{(\mathcal{M}-1)(1+i^*)}\right)^{\sigma}\right]$ and $\tilde{\kappa} = \frac{(1-\theta)(1-\theta\tilde{\beta})}{\theta}$.

Due to the OLG setup in the demand side, we have an unconventional Phillips equation[5]. We can express the inflation as the discounted sum of current and expected future deviations of marginal costs from steady state by solving the above equation forward. Differently from the standard Phillips equation, we weight the marginal cost with the modified discount factor, $\tilde{\beta}$. Notice that an older economy implies a higher weight on the marginal cost compared to younger economy. Hence, inflation becomes more sensitive to marginal cost changes as the society ages.

The equilibrium is characterized by equations (14)–(20), together with a description of monetary policy.

# 3 Experiments

This section compares the dynamics of younger and relatively older economies after an unexpected monetary policy shock. First, to provide a clear insight, I will be discussing the baseline scenario in which old age dependency is assumed to be zero. Then I will

[5]See appendix B for the detailed derivation

conduct the same analysis with a positive old age dependency ratio. Finally, I will compare and examine the responses of economies with different demographic profiles to the same monetary policy shock. Figures in this section illustrate the responses[6] of young and old age consumption per worker, output gap per worker, wage, labor, inflation, dividend, asset price and nominal interest rates to a tightening (positive) monetary policy shock.

### 3.1 *The Effects of a Monetary Policy Shock in a Baseline Economy*

Figure 2 shows the dynamic effects of a tightening monetary policy shock in the baseline model. The shock corresponds to an increase of 100 basis points in the exogenous component of the monetary policy, $v_t$. In the baseline model, old age dependency ratio is assumed to be zero implying that all agents in the economy are workers and active in the financial market.

After a positive monetary policy shock, to smooth their consumption, agents decrease their consumption levels followed by the output gap (output) falls. Thus, the labor market tightens and the real wage decreases. In DNK setup, marginal cost is the source of inflation. Therefore, a decline in real wage leads to a decrease in marginal cost. Hence, we observe deflation in the baseline economy.

[6] See appendix C for the values of the preference parameters.

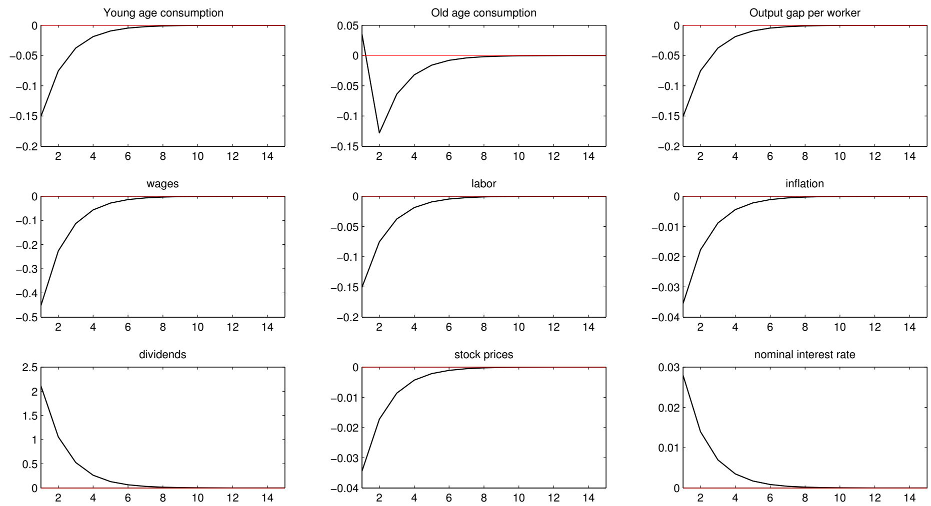


Figure 2: Impulse Responses to a Tightening Monetary Policy Shock in the Baseline Economy

## 3.2 *The Effects of a Monetary Policy Shock in a Model with a Positive Old Age Dependency Ratio*

Before comparing the responses of variables to a monetary policy shock in economies with different old age dependency ratios, in this section, I display and interpret the responses to a tightening monetary policy shock in the proposed model economy. For this analysis, I set the old age dependency ratio to be 20%.

It is worth emphasizing that the evaluation of the impulse responses in this study and in the standard models is different. In the standard setup, there is only one type of representative agent and the response of her consumption to a shock is reflected along one graph. However, in this paper, the consumption decision of an agent is demonstrated in two graphs. Suppose that a shock hits the economy at time $t$. First graph (young age consumption) demonstrates the response of an agent to the shock when she is young at time $t$, and the second graph (old age consumption) reflects the ongoing response to the shock when the agent becomes retired at time $t + 1$. Therefore, the response that we observe at $t + 1$ in young age consumption graph belongs to an agent who is born at time $t + 1$, and she perceives the decreasing effect of a monetary policy shock as a new shock.

In Figure 3 we see that a tightening monetary policy shock leads young agents at the time of the shock to decrease their consumption similar to the baseline scenario. The effect of the monetary policy shock on old agents (at the time of the shock) is indirect; their consumption is affected only from the deflation. This is because the old agents at the time of the shock decided their consumption in the previous period using the previous period's information. Hence, old agent's consumption level is not affected by the interest rate change at the time of the shock. However, we observe a peak in the old age consumption graph at time $t+1$. The reason is that old agents decided their consumption level at the time of the shock when they were young by decreasing their consumption level and increasing their savings. When they become retired, they consume all of their savings and their returns. In other words, the assets they bought when they were young gain value due to an increase in interest rate, and they sell them at a higher price. Hence, their wealth and consumption level increase. Since output is a combination of young and old age consumption, in such a young economy, we observe a decline in output similar to the baseline scenario.

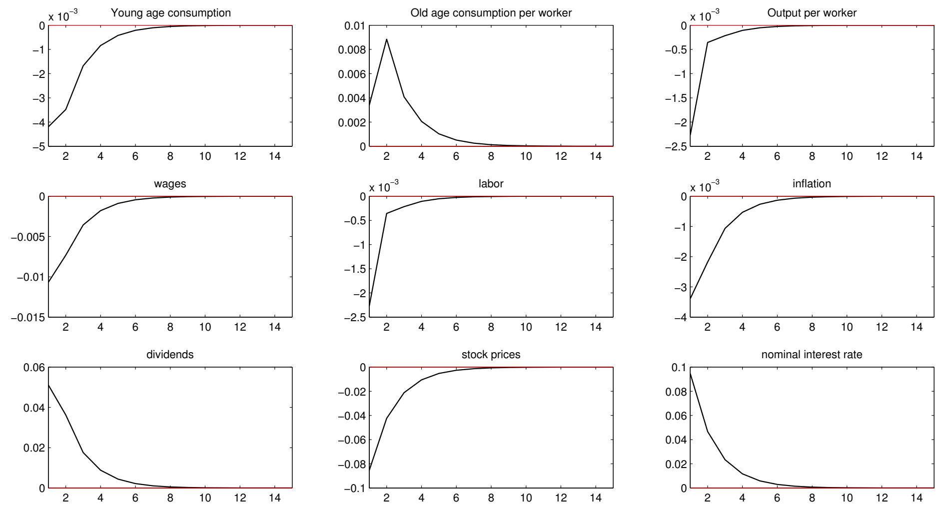


Figure 3: Impulse Responses to a Tightening Monetary Policy Shock in an Economy with 20% Old Age Dependency Ratio.

### 3.3 *Comparison of The Effects of a Monetary Policy Shock with Different Demographic Profiles*

To analyze the impact of aging on the effectiveness of monetary policy, I carry out the model with different old age dependency ratios. Dashed line with dot, solid line with star and solid line show the responses in a young ($\varphi = 20\%$), relatively older ($\varphi = 50\%$) and oldest economy ($\varphi = 80\%$), respectively. Percentages refer to old age dependency ratios[7].

Similar to the previous section's responses, Figure 4 shows that in all types of economies, a tightening monetary policy shock leads to a fall in young consumption and to an increase in old consumption. However, the magnitudes of the responses vary. Notice that young agents' interest rate sensitivity increases as the old age dependency ratio increases. The reason is that there is a lender–borrower relation between old and young agents. Old

[7]For example, 20% old age dependency ratio represents an economy where 100 workers take care of 20 retired agent.

agents' consumption at the time of the shock increases more due to stronger deflation. Hence, the value of assets that old agents lend to young agents in an old economy is higher compared to younger economies. Therefore, young agents' consumption level drops more. Since young agents in an old economy (at the time of the shock) save more, we observe a peak in the old age consumption one period ahead of the shock.

On the supply side, output decreases due to a fall in total demand at the time of the shock in all types of economies. However, one period after the shock, differently from the baseline case, the response of output (output gap) becomes weaker as the economy ages. Moreover, the response graph of output indicates that if the economy is old enough, we may observe an increase in the output after a contractionary monetary policy shock.

In the proposed model, labor is the sole input in production, hence response of labor follows the response of output. Differences in the labor supply decisions of young agents in different types of economies are stemmed from the variances in the responses of wages. Young agents in older economies want to save more for the retirement period, thus they supply more labor, and it leads to a more decline in wages compared to other types of economies. The response of inflation follows wages, hence we observe higher deflation in older economies. Therefore, we can conclude that the sacrifice ratio between inflation and output gap decreases as the economy ages.

To sum up, (1) the effectiveness of monetary policy on output gap (output) decreases as the population ages. In other words, monetary policy authority becomes less effective in controlling output as the population ages. This can be an explanation for the zero lower bound implication in Japan. (2) A tightening monetary policy shock may lead to an increase in output in an older economy due to the rise in old age consumption. (3) The sacrifice ratio between inflation and output decreases as the population ages.

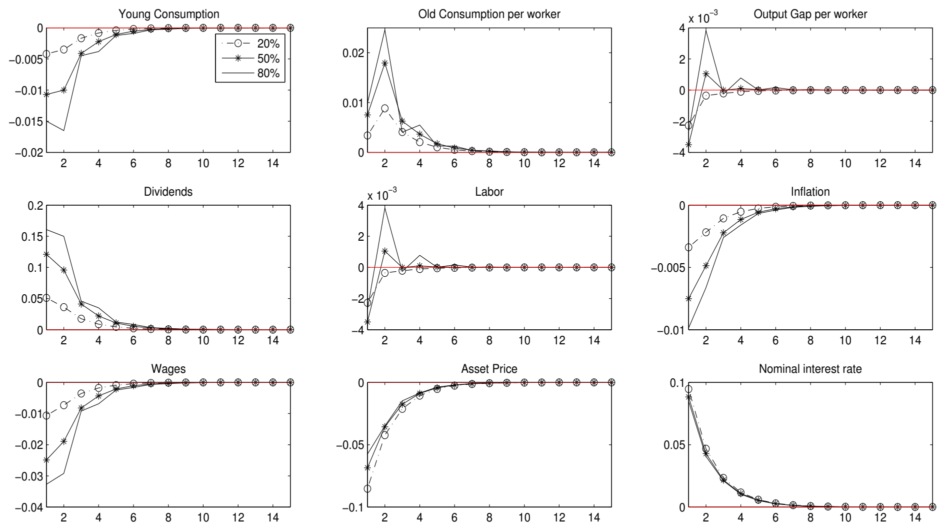


Figure 4: Impulse Responses to a Tightening Monetary Policy Shock in Economies with Different Old Age Dependency Ratios.

# 4 Conclusion

All developed countries are aging. Demographic structure of developed countries has been changing permanently due to decline in the fertility and increase in the longevity rates. In few years, baby boom generation will retire and reinforce the permanent effect on demographic structure by increasing the ratio of old to young people[8]. Aging can be related to economics through the saving behaviors of societies. Therefore, aging populations are important policy concerns and a correspondingly important academic research area. This paper argues that demographics should be expected to have an impact on the effectiveness

[8]According to demographic projections by the United Nations, http://data.un.org, the share of the old-aged population is anticipated to double on average in 40 years. For example, Japanese old-dependency ratio, defined as the ratio of adults aged 65 and above to the working-age population of adults aged 15 to 65, is expected to increase from three elderly persons to 10 working-age adults in 2010, to seven elderly persons to every 10 working-age adults by 2050. Similarly, German old-dependency ratio is expected to increase from three elderly persons to 10 working-age adults in 2010, to six elderly persons to every 10 working-age adults by 2050.

and hence conduct of the monetary policy. In other words, this paper answers how should the Central Bankers perceive a grayer society and how should they react to it?

This paper provides a model which combines the standard New Keynesian framework of Clarida et al. (1999) and OLG setup of Samuelson (1958) and Diamond (1965) with aging. The overlapping generations structure of the demand side of the economy allows one to introduce aging into the model. On the supply side, a New Keynesian setup with nominal rigidities is required to analyze the effect of monetary policy. By utilizing the proposed model, this paper studies the effectiveness of monetary policy in an aging economy.

Main results of the experiments can be described as follows: First, as the population ages, the natural rate of interest decreases monotonically. Secondly, the effectiveness of the monetary policy on output gap (output) decreases due to decreasing interest rate sensitivity of the society. The model also suggests that it would not even be a surprise if the economy becomes old enough we may see positive response of output to a tightening monetary policy shock. Finally, the sacrifice ratio between inflation and output decreases as the economy ages. Hence, results of the paper suggest that the policymakers should account for the demographic profile of the country when conducting monetary policies.

# Appendices

## A Derivation of the IS curve

This section of the paper shows the derivation of modified dynamic IS equation step by step in detail. Inter temporal budget constraint (IBC) is

$$D_{t+1} = \left[\frac{W_t}{P_t} L_t(j) - C_t(j)\right] \frac{(1 + i_t)}{1 + \pi_{t+1}}.$$

Log-linearize the above equation around steady state

$$\hat{d}_{t+1} = (1 + i^*)\frac{1}{\mu}\frac{y^*}{d^*}(\hat{w}_t - \hat{p}_t + \hat{l}_t(j)) - (1 + i^*)\frac{c^*}{d^*}\hat{c}_t(j) + (\hat{i}_t - \pi_{t+1}).$$

Plug Euler equation into IBC.

$$\hat{c}_t + \frac{1}{\sigma}(\hat{i}_t - E_t\{\pi_{t+1}\}) = (1 + i^*)\frac{1}{\mu}\frac{y^*}{d^*}(\hat{w}_t - \hat{p}_t + \hat{l}_t(j)) - (1 + i^*)\frac{c^*}{d^*}\hat{c}_t(j) + (\hat{i}_t - \pi_{t+1}) \quad (21)$$

At steady state,

$$y^* = c^*(j) + \varphi d^*(j)$$

$$\varphi d^* = y^* \left(1 - \frac{1}{\mu}\right)\left(\frac{1 + i^*}{i^*}\right)$$

$$\frac{y^*}{d^*} = \varphi \frac{\mu i^*}{(\mu - 1)(1 + i^*)}$$

and

$$\frac{c^*}{d^*} = \varphi \frac{1 + i^* - \mu}{(\mu - 1)(1 + i^*)}.$$

Plug the steady state values into (21). We have

$$\hat{c}_t\left[1-\frac{\sigma\varphi i^*}{(\mu-1)}+\frac{\varphi(1+i^*-\mu)}{(\mu-1)}\right]=\frac{\varphi i^*(1+\psi)}{(\mu-1)}\hat{y}_t+\left(1-\frac{1}{\sigma}\right)\left(\hat{i}_t-\pi_{t+1}+\frac{1}{\sigma}E_t\{\pi_{t+1}\}\right).$$

The equations for the young age and old age consumptions at time $t$ are

$$\hat{c}_t=\frac{\varphi i^*(1+\psi)}{(\mu-1)(1-\varphi)-\varphi i^*(\sigma-1)}\hat{y}_t+\frac{(\mu-1)}{(\mu-1)(1-\varphi)-\varphi i^*(\sigma-1)}\left(\frac{\sigma-1}{\sigma}\hat{i}_t-\pi_{t+1}+\frac{1}{\sigma}E_t\{\pi_{t+1}\}\right)$$

$$\hat{d}_t(j)=\hat{c}_{t-1}(j)+\frac{1}{\sigma}[\hat{i}_{t-1}-\pi_t]-\pi_t+E_{t-1}\{\pi_t\}$$

At time $t$ total consumption per worker equals to total production per worker.

$$\hat{y}_t=\frac{c^*}{y^*}\hat{c}_t(j)+\varphi\frac{d^*}{y^*}\hat{d}_t(j) \tag{22}$$

The above equation shows the OLG–Dynamic New Keynesian IS curve. Different from the standard DNK IS curve, now we have a richer dynamic system. OLG-DNK IS equation not only depends on current periods nominal interest rate and next period's inflation expectation and also previous period's nominal interest rate and inflation expectation and the realized inflation.

# B Derivation of the Phillips Curve

Evolution of the aggregate price index :

$$P_t=\left[\theta(P_{t-1})^{1-\epsilon}+(1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

Divide both sides by $P_{t-1}$.

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

Log-linearize around zero inflation steady state

$$\pi_t = (1-\theta)[p_t^* - p_{t-1}] \tag{23}$$

equivalently,

$$p_t = (1-\theta)p_t^* + \theta p_{t-1}.$$

Re-optimizers problem is

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{\Lambda_{t,t+k}(P_t^* Y_{t+k}(i) - W_{t+k}Y_{t+k}(i))\right\}$$

subject to

$$Y_{t+k}(i) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

FOC:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{\Lambda_{t,t+k}(1-\epsilon)(P_t^*)^{-\epsilon}(P_{t+k})^{\epsilon} Y_{t+k} + \epsilon W_{t+k}(P_t^*)^{-\epsilon-1}(P_{t+k})^{\epsilon} Y_{t+k}\right\} = 0$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{\Lambda_{t,t+k} Y_{t+k}(i)\left(P_t^* - \frac{\epsilon}{\epsilon-1} W_{t+k}\right)\right\} = 0$$

Divide both sides by $P_{t-1}$.

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{\Lambda_{t,t+k}Y_{t+k}(i)\left(\frac{P_t^*}{P_{t-1}}-\frac{\epsilon}{\epsilon-1}W_{t+k}^r\frac{P_{t+k}}{P_{t-1}}\right)\right\}=0$$

Perfect foresight, zero inflation steady state condition is:

$$\frac{P_t^*}{P_{t-1}}=1 \quad \pi_{t-1,t+k}=1 \qquad Y_{t+k}(i)=y^* \quad W^*=\frac{1}{\mathcal{M}} \qquad \Lambda_{t,t+k}=\beta^k J^k \quad J=\left[\left(\varphi\frac{1+i^*-\mathcal{M}}{(\mathcal{M}-1)(1+i^*)}\right)^{\sigma}\right].$$

Log-linearize the above expression around zero inflation steady state,

$$p_t^*-p_{t-1}=(1-\theta\beta J)\sum_{k=0}^{\infty}(\theta\beta J)^k E_t\{\hat{m}c_{t+k}+p_{t+k}-p_{t-1}\}.$$

Rewriting the optimal price setting rule in recursive form

$$p_t^*=(1-\theta\beta J)(\hat{w}_t+p_t)+(1-\theta\beta J)(\theta\beta J)E_t\{\hat{m}c_{t+1}+p_{t+1}\}+\dots$$

$$p_t^*=\theta\beta J E_t\{p_{t+1}^*\}+(1-\theta\beta J)\hat{m}c_t+(1-\theta\beta J)p_t. \tag{24}$$

Combine (23) and (24),

$$\pi_t=(1-\theta)(p_t^*-p_{t-1})$$

$$\pi_t=(1-\theta)[\theta\beta J E_t\{p_{t+1}^*\}+(1-\theta\beta J)\hat{m}c_t+(1-\theta\beta J)p_t-p_{t-1}].$$

Plug $p_{t+1}^*$ and rewrite the above expression.

$$\pi_t=(1-\theta)\left[\theta\beta J E_t\left[\frac{\pi_{t+1}}{(1-\theta)}+p_t\right]+(1-\theta\beta J)\hat{m}c_t+(1-\theta\beta J)p_t-p_{t-1}\right]$$

$$\pi_t=\tilde{\beta}E_t\{\pi_{t+1}\}+\tilde{\kappa}\hat{m}c_t$$

where $\tilde{\beta} = \beta J$ and $\tilde{\kappa} = \frac{(1-\theta)(1-\theta\tilde{\beta})}{\theta}$.

$$
\begin{aligned}
mc_t &= w_t - p_t \\
&= \sigma c_t + \psi l_t \\
&= \sigma c_t + \psi y_t \\
&= \left[\frac{\varphi i^*(1+\psi)\sigma}{(\mathcal{M}-1)(1-\varphi) - \varphi i^*(\sigma-1)} + \psi\right] y_t^p + \frac{(\mathcal{M}-1)\sigma}{(\mathcal{M}-1)(1-\varphi) - \varphi i^*(\sigma-1)} \left(\frac{\sigma-1}{\sigma}\hat{i}_t - \pi_{t+1} + \frac{1}{\sigma}E_t\{\pi_{t+1}\} - \rho\right)
\end{aligned}
$$

where $\rho = \log\tilde{\beta}$. Natural level of output is

$$
-log(\mathcal{M}) = \left[\frac{\varphi i^*(1+\psi)\sigma}{(\mathcal{M}-1)(1-\varphi) - \varphi i^*(\sigma-1)} + \psi\right] y_t^n.
$$

The deviation of real marginal cost from the flexible price case is

$$
\hat{mc}_t = \left[\frac{\varphi i^*(1+\psi)\sigma}{(\mathcal{M}-1)(1-\varphi) - \varphi i^*(\sigma-1)} + \psi\right] x_t + \frac{(\mathcal{M}-1)\sigma}{(\mathcal{M}-1)(1-\varphi) - \varphi i^*(\sigma-1)} \left(\frac{\sigma-1}{\sigma}\hat{i}_t - \pi_{t+1} + \frac{1}{\sigma}E_t\{\pi_{t+1}\} - \rho\right).
$$

Hence, the dynamic Phillips curve is

$$
\begin{aligned}
\pi_t &= \tilde{\beta}E_t\{\pi_{t+1}\} + \tilde{\kappa}\left[\frac{\varphi i^*(1+\psi)\sigma}{(\mathcal{M}-1)(1-\varphi) - \varphi i^*(\sigma-1)} + \psi\right]\hat{x}_t \\
&+ \tilde{\kappa}\left[\frac{(\mathcal{M}-1)\sigma}{(\mathcal{M}-1)(1-\varphi) - \varphi i^*(\sigma-1)} \left(\frac{\sigma-1}{\sigma}\hat{i}_t - \pi_{t+1} + \frac{1}{\sigma}E_t\{\pi_{t+1}\} - \rho\right)\right].
\end{aligned}
$$

## C Simulation Values

Table 1: Preference Parameters

| Parameter | Name | Value |
|---|---|---|
| $\varphi$ | old-age dependency ratio | 0.2 0.5 0.8 |
| $\sigma$ | intertemporal elasticity of substitution | 2 |
| $\psi$ | elasticity of labor supply | 1 |
| $\beta$ | discount factor | 0.98 |
| $\mathcal{M}$ | markup | 1.2 |
| $\theta$ | price stickiness | 0.75 |
| $\phi_x$ | feedback parameter of output gap | 0.125 |
| $\phi_\pi$ | feedback parameter of inflation | 1.5 |
| $\rho_v$ | persistence of monetary policy shock | 0.5 |

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