The Impact of Government Debt and Taxation on Endogenous Growth in the Presence of a Debt Trigger

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Abstract

Relatively little is known about how government debt influences future growth or interest rates. This paper analyzes an infinite-horizon economy within which these issues can be studied. A novel characterization of the continuous-time model is developed, which is a solution to a Bernoulli differential equation. A "Debt Trigger" is introduced which prohibits the government from expanding its debt beyond some specific share of GDP. It is shown that the effect of the size of government debt on growth depends critically on the intertemporal elasticity of substitution of consumption. Higher initial levels of government debt may increase the resulting growth rate temporarily, but reduce it in the long term. It is shown that whether a reduction in the capital tax rate is growth-enhancing depends on certain parameter values, and particularly on the intertemporal elasticity of substitution of consumption. In contrast with existing models, it is shown that a temporary tax cut can have a *larger* impact on the growth rate than would a permanent tax cut of the same magnitude. For some parameter values the model exhibits some *Unpleasant Fiscal Arithmetic*: reducing the capital tax rate can cause a *drastic growth implosion*. Labor-leisure choice is then incorporated into to the model, and it is then shown that the *growth rate may be increasing in the rate at which the future labor tax is increasing*. The results presented here show that there may be a complex relationship between the size of the government debt, and the resulting growth and interest rates.

JEL codes: E2, E6, H6, O3

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1 Introduction

Little research seems to have been devoted to the study of how the magnitude of government debt influences the long-term balanced growth path. Relatedly, little is known about how the size of government debt will influence how the growth rate will react to a change in the tax rates. The government budget constraint imposes some discipline on how future tax rates need to be set, but obviously it does not pin down a unique path for these tax rates, and so the impact of the debt on current growth is unclear. This paper seeks to answer these, and related questions within a model in which both the long-term growth rate and the interest rate are determined endogenously.

One important question that this paper seeks to answer is: What is the effect that higher levels of government debt, held by domestic consumers, have on economic growth? If Ricardian Equivalence holds, the answer to this question is "nothing". This is true whether or not the growth rate is determined endogenously. Therefore, these circumstances are not particularly interesting. So what is the answer to this question in an environment where taxes are distortional, and growth is endogenous? The model explored below shows that the answer this question depends on the nature of the government budget constraint, and also on the consumer's intertemporal elasticity of substitution of consumption. It is shown that, ceteris paribus, higher levels of government debt do not necessarily reduce growth rates contemporaneously. In fact, and higher level of debt may even increase the short term growth rate. However, the effect of this debt is that *eventually* it will reduce the growth rate.

One of the earliest and foremost analyses of how the size of the government debt would influence aggregate income is that of Diamond [1]. In Diamond's paper higher government debt is shown to reduce saving as well as the capital stock, and therefore output is also reduced.[1] Despite the title of Diamond's paper, one drawback to his analysis is that there is no long-term endogenous growth in his model, and so this model cannot be used to determine how the government debt would influence the growth or interest rate. By contrast the model studied in this paper will do just that.

It is natural to think, as Diamond's work might suggest, that a higher level of government debt might reduce the growth rate. This might happen for different reasons than suggested by Diamond, such as the fact that the debt must eventually be paid off through higher distortional taxes, which would then reduce the growth rate. However, the model studied here shows that this relationship can be much more subtle than it first appears. This is because the higher future tax rates (or, the expectations thereof) can have a multitude of different effects on current consumption and investment decisions, which will then influence the current growth rate.

Other questions that this paper seeks to answer would be: What is the effect on the growth and interest rates of an economy, of a particular change in the tax rate? And how does the size of the government debt affect the answer to this last question? To ensure that these questions are coherent, or well-defined, it is necessary to make further assumptions about future government policies. This is because changes in current taxes can have many different implications for the future budget constraints of individuals and the government. That is, one cannot change just the tax revenue for a few periods in isolation from other

[1] Diamond uses an overlapping generations model.

government policy variables, because the government budget constraint will then imply that other things must also change: some combination of future taxes and or spending. Therefore, to investigate how a change in the tax rate will influence the growth rate, one must also make some assumptions about how other *future* government policies will be influenced as well.[2] In this paper it will be shown that the effect that a reduction in the capital tax rate can have on the growth may rate will depend critically on various policies and parameters of the economy. Of particular importance is the parameter determining the intertemporal elasticity of substitution of consumption.[3] Furthermore, this paper is a contribution to the literature explaining why capital tax rates seem to have such a negligible impact on the growth rate (see, for example, Stokey and Rebelo [12])

It is frequently assumed (or shown) that a permanent change in capital tax rates will have a larger impact than a temporary change because the former will have larger incentive effects. However, it is shown below that one implication of the model is that this result may not be the case: it depends on the parameters of the economy, and also on the implications that the tax change will have on *future* policies. But it is shown that it is certainly possible for a temporary change in capital taxes to have a larger impact on the growth rate than a permanent change.

An important economic feature that is introduced here is that of a "Debt Trigger." This idea is borrowed from Sargent and Wallace [11]. In the model it is assumed that there is some legal or constitutional impediment that prohibits the ratio of government debt to GDP from rising above some pre-specified upper bound. Once it reaches this upper bound, it is assumed that tax rates must immediately and permanently adjust so that the debt to GDP ratio does not exceed this upper bound.[4] With this mechanism in place, a cut in current taxes may then imply that in the future the tax rate must be increased to prevent the relative size of the government debt from escalating. This mechanism will then imply that for some paths for government taxes, this debt trigger will eventually (and predictably) kick-in, and then government taxes must be immediately imposed at some higher level to keep the debt to GDP ratio constant.

Further support of the existence of such a mechanism is obtained from the work of Kumar and Woo [6], Panizza and Presbitero [7] and also by Reinhart and Rogoff [9], [10]. These papers suggest that the economic growth rate tends to be fall as a government debt to GDP ratio reaches a relatively high level. This would provide support for why one might see a form of a debt trigger in an economy. This model will then provide an economic foundation to show *why* GDP growth may fall once a specific high value of the debt to GDP ratio is reached.

When studying how economies react to a change in taxes, it is usually assumed that any change in tax revenue will be directed into a corresponding change in future government spending or transfers. For example, Trabandt and Uhlig [13] consider both such possibilities

[2] Actually, to be more precise, what is important is what agents *expect* to be the effect that a change in the current tax rate will have on future government policies.

[3] Obviously this parameter has been known to play an important role in many dynamic models, particularly when studying issues related to business cycles. But until now this feature does not seem to have been identified as playing an important role in understanding how debt or taxes can influence growth.

[4] This is not entirely an ad-hoc assumption because if there were no such upper bound, then eventually the government budget constraint would be violated. Therefore, the government budget constraint implies that there must be some upper bound for the debt to GDP ratio.

in their study of the Laffer Curve. There may be valid economic and analytical reasons for this assumption. However, the assumption frequently seems to be one of convenience, since there does not seem be any essential reason why such an offsetting change in future government spending necessarily must be assumed. It is certainly not clear that this is a necessary characterization of the intent of actual policymakers. Therefore, in this paper an alternative, but no less plausible policy, will be assumed. Here a reduction in the tax rate in one period, which reduces government revenue, will necessarily imply an *increase* in future tax rates to make up for the lost revenue. In other words, rather than letting government spending or transfers fall because of the lost revenue, it will be assumed that government spending is unchanged, and so total tax revenue must be held fixed. Therefore future taxes (rather than spending) must adjust in this case. This analysis is made slightly more complicated because the initial change in taxes may also affect the interest rate, and this feature must also be taken into consideration when calculating how much the discounted government revenue must adjust.

It must be stated that the goal of this paper is not normative in nature, in that the objective is not to seek to characterize an optimal set of taxes. Instead, the analysis is more positive: the goal is to characterize the implications that alternative fiscal policies may have on an economy. Additionally, another objective is to understand the importance of identifying how current policy changes can influence the expectations of future policy changes.

The remainder of the paper is organized as follows. In the next section some data is presented on the magnitude of US tax rates and debt levels. In section 3 the key features are identified of a model that will be used below. Section 4 will present a simple version of a model without government. Next, section 5 will describe the behavior of the model when government is introduced. Section 6 will then derive the solution to the consumer's intertemporal optimization problem. Section 7 will describe the Debt Trigger mechanism, while section 8 will show how the parameters of the economy are selected. Section 9 illustrates the important role the intertemporal elasticity of substitution plays in this analysis. Section 10 shows how the size of the initial government debt affects the subsequent growth rate, while section 11 shows the complicated effects that a tax cut may have. Section 12 shows "Some Unpleasant Fiscal Arithmetic": a reduction in the capital tax rate may result in an immediate and continued fall in the growth rate. Section 13 shows how a labor-choice could be incorporated into the model. It is shown that the labor tax can influence the growth rate only if it is changing over time. Final remarks are listed in the final section.

2 Some Data on Debt and Taxes

For illustrative purposes, it may be illuminating to consider some data. In Figure 1, the lower line is the path of the growth rate of per-capita income since 1940. Also in this figure, there are several measures of tax rates that are illustrated. The line labeled "IT" is the ratio to federal income tax revenue to GDP. The line labeled "IT+SS" is the ratio to federal income tax plus social security tax revenue, to GDP. These are very rough measures of the tax rate over many decades. Obviously there has been some growth in this ratio over the past 80 years, but no trend over the past 40 years. The lower panel of this figure is the growth rate of per-capita GDP. As has been pointed out by Stokey and Rebelo [12], there is very little relationship between the tax rate, measured in this manner, and the growth rate. The correlation between the growth rate and the income tax revenue to GDP ratio is 0.11, while the correlation between the growth rate and the total tax revenue to GDP ratio is 0.03.

Now of course the top panel in the figure does not really represent a measure of statutory or effective marginal tax rates, since it would be difficult to get an effective measure of this. It is certainly possible to obtain the highest marginal income tax rates over this period. This is represented by the line labeled "Highest Tax Rate". Once more, there is very little relationship between this tax rate and the growth rate. The correlation between these two variables is .21, which obviously does not support the notion that higher taxes have a harnessing effect on growth. There are many reasons why there might not be a strong negative relationship between these two variables, and this is explored by Stokey and Rebelo.

One reason why there is not a tight relationship between the growth rate to the tax rate may be that other circumstances (or important variables) have been changing over this period. In particular, Figure 2 shows that the size of the federal government debt to GDP ratio, has certainly changed over this time frame.[5] This brings up an important issue or question: Might it be the case that the response of the growth rate, to a change in the tax rate would depend on the size of government debt? In other words, would a reduction in the tax rate affect the growth rate to a degree that depends on the size of government debt? The model studied below will be capable of suggesting some answers to these questions.

A related question also arises: What is the relationship between size of government debt, and the level of interest rates? The model studied below will be capable of answering this question. It will be shown that the size of government debt, combined with the level of tax rates, will have a complicated non-linear relationship. Furthermore, the effects may be very different at short, intermediate, and long-term horizons, depending on the parameter values of the economy.

3 Appropriate Features of a Model to be Studied

The model studied in this paper has to be sufficiently rich to be interesting, but also not too complex that it is difficult to characterize the behavior of the economy as it transitions from one growth path to another. That is, the model must be simple enough that it is possible to calculate exactly the response of variables, such as consumption, output, investment, government tax revenue, and the interest rate to a change in the tax rate. The desire here is to accomplish this without resorting to any approximation techniques which may not give precise answers. Precision is important here because the policy changes will impact the interest rate and future government revenue, and it is vital that this be calculated accurately, and not be loosely determined by some approximation methods. In a model with endogenous growth, any errors in the calculation of decision rules can be compounded over time and lead to misleading characterizations of present values.[6] Therefore it is important

[5] Of course this is only one very rudimentary measure of the indebtedness of the federal government. This does not capture the important obligations (or implicit promises) to individuals, such as through Social Security or Medicare, which are projected to balloon in the coming decades.

[6] This issue is complicated even further in this paper because agents will be making decisions that produce a growth path based partly on the *expectation policy changes that will be forthcoming in the future.*

here that in the model it must be possible to characterize the reaction of agent's decisions to a tax change, and in particular it must be possible to characterize the transition from one growth path to another. Some models are not well-suited for this task. For example, the two-sector model of Lucas [4] is very useful for some purposes, but not for the issues studied here. The reason is that in such a model a permanent change in the tax rate would have a non-trivial transition from one growth path to another, and it would be difficult to accurately characterize the behavior of the decision rules as the interest rate along this transition path.[7] Therefore the model employed here will only be sufficiently complex to exhibit all the necessary features - but no more complex.

The first model studied will be relatively primitive, and really just a version of the typical AK model (as described in Rebelo [8]). The reason for relying on such a model is that this is the simplest model imaginable that illustrates the relevant mechanisms at play. The model does not rely on potentially confusing complications such as externalities, imperfect competition, or multiple sectors. Additionally, the general equilibrium effects that (perceived or actual) government policies can have on features such as investment, or the interest rate, tend to be very transparent. Other complications, such as endogenous labor, for example, will be introduced and studied later. It is also important that the model deliver a balanced growth path, and therefore also an endogenous after-tax interest rate. With these characteristics, a change in government taxes will have non-trivial effects on the government's budget constraint.

4 The Primitive Model

The initial model presented here will be "stripped-down" version in which there is no government, so the basic features of the economy can be studied. The economy will be a one in which there are identical infinitely-lived representative agents. Time will be assumed to be continuous, and indexed by t. Each individual will have preferences of the following CRRA type:

$$\int_0^\infty e^{-\rho t}\left[\frac{c_t^{1-\sigma}}{1-\sigma}\right] dt,$$

where c_t is the flow of consumption. Here ρ is the rate of time preference, $(\frac{1}{\sigma})$ is the intertemporal elasticity of substitution of consumption, and it is assumed that $\sigma > 0$, and $\sigma \neq 1$. There is no uncertainty in the model, and so there is no reason that the parameter σ should be related to a measure of risk-aversion.

The technology is very simple, and one in which there is a technology for producing the consumption good, and another for producing the investment good. At any moment there is a stock of capital (k_t), and some fraction (ϕ_t) of this can be used to produce the consumption good, while the remainder is used to produce the investment good. Capital depreciates at the geometric rate of δ. Therefore, the constraints of the economy are then written as follows:

$$c_t = A\left(\phi_t k_t\right) \tag{1}$$

[7] This is the reason that in the models of Lucas [5] the transitions from one steady-state to another are not calculated, even though Lucas acknowledges that these transitions may be of vital importance in calculating things like welfare costs.

$$I_t = \lambda (1 - \phi_t) k_t \tag{2}$$

and

$$\dot{k}_t = I_t - \delta k_t$$

Here I_t is new investment The parameters A and λ are the fixed productivity parameters for the consumption and investment sector, respectively, while δ is the depreciation rate of capital.

4.1 Solution of the Model

It is easily seen, and will be checked below, that the solution for this model is that the key variable for determining the growth rate is ϕ, and this is given as

$$\phi = \frac{[\rho - (\lambda - \delta)(1 - \sigma)]}{\lambda\sigma}. \tag{3}$$

The growth rate of consumption is then given as

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\lambda - \delta - \rho}{\sigma}.$$

Of course, this solution is only sensible if the transversality condition is satisfied, which is true iff

$$(1 - \sigma)(\lambda - \delta) < \rho.$$

5 The Model With Government Taxation and Debt

Next, the model from the last section will be developed further. It will now be assumed that initially there is a fixed stock of real government debt that is outstanding, and this will be denoted as B_0. For convenience, it will be assumed that there is no future government spending to add to this existing stock of debt.[8] This debt must be financed through factor taxation, and since capital is the only factor of production, the government debt must be financed through capital taxation. There can be no default by the government, and this puts considerable discipline on what tax policies are feasible.

As is typical in such environments, there are multiple ways to describe the competitive equilibrium. The approach here, which may be the simplest, is to have a competitive equilibrium in which households own all the capital, and to undertake the investment.

In this competitive equilibrium there will be firms that use the capital to produce the consumption good and investment good. The instantaneous before-tax cost of capital to the firm is denoted by r_t. The fraction (ϕ_t) of capital (k_t) is used to produce the consumption good. Let P_t denote price of the consumption good, in units of the investment good. The profits received from the firm for this activity, measured in units of the investment good, are denoted by

$$\pi_{1t} = P_t [A(\phi_t k_t)] - (r_t + \delta)(\phi_t k_t).$$

[8] This makes the analysis of the model more manageable, and also makes it very apparent that changes in the tax rate cannot be absorbed by changes in government spending. This approach also removes the possibility that a modification of the tax rate alone may change the present value of government spending through a change in the interest rate.

The remaining capital $((1-\phi_t)\,k_t)$, is used to produce the investment good. The profits received from the firm for this activity, measured in units of the investment good, are denoted by

$$\pi_{2t} = \lambda\,(1-\phi_t)\,k_t - (r_t+\delta)\,(1-\phi_t)\,k_t.$$

It is easily seen that since the technologies have constant returns to scale, then $P_t = \left(\frac{\lambda}{A}\right)$, and in equilibrium profits of both firms will be zero. Also, in an equilibrium it must then be the case that

$$r_t = P_t A - \delta = \lambda - \delta. \tag{4}$$

The firm then pays the tax on the return to capital $(\tau_t r_t k_t)$, before paying the remainder after-tax income to the consumer $((1-\tau)_t\, r_t k_t)$. It is assumed that interest on government debt is taxable. Since there is no uncertainty, both capital and government bonds pay the same return. The consumer's instantaneous budget constraint can then be written as follows:

$$Pc_t + \left(\dot{k}_t + \dot{B}_t\right) = (k_t + B_t)\,r_t\,(1-\tau_t)\,.$$

5.1 Government

The government budget constraint specifies that the excess of interest paid on the debt $(B_t r_t)$ over the amount of tax revenue $((k_t+B_t)\,r_t\tau_t)$, must be financed by increased government borrowing. This is written as follows

$$\dot{B}_t = B_t r_t - (k_t + B_t)\,r_t\tau_t. \tag{5}$$

Alternatively, this states that the value of government debt at date t must equal the value of subsequent government revenue:

$$B_t = \int_t^\infty \exp\left(-\int_t^s r_z dz\right)(k_s + B_s)\,[r_s\tau_s]\,ds \tag{6}$$

$$= \int_t^\infty \exp\left(-\int_t^s r_z\,(1-\tau_z)\,dz\right)(k_s)\,[r_s\tau_s]\,ds. \tag{7}$$

Given an initial value for government debt (B_0), a feasible path for taxes is then a function $\{\tau_s\}_{s=0}^{s=\infty}$ that satisfies equation (6). The important feature here is that it is possible to evaluate the impact of alternative paths for taxes beginning at some initial date.

Note that this government budget constraint shows that a change in the tax rate does not just change the amount of tax revenue, but it also changes the after-tax interest rate. Furthermore, the relationship between the tax rate and discounted tax revenue is complicated by the fact that the future path of the tax rate obviously influences the path for the capital stock, and so this feature must be taken into consideration when considering the feasible path for the tax rate.

The analysis below will assume an extreme degree of government commitment, in the sense that the government budget constraint must be obeyed. However, this will not be a normative exercise in that these tax rates will not be chosen to solve any particular optimization problem. Instead, the impact of various alternative paths for the tax rate, which satisfy equation (6) will be considered. Obviously not all paths for the tax rate are feasible, in the sense that equation (6) is satisfied.

6 The Consumer's Problem in a Competitive Equilibrium

Since the government only has to finance its existing debt obligation, and government consumption is zero, tax revenue is actually rebated to individual's through the interest payments to holders of government debt. It is then of central importance to characterize the consumer's optimization problem. This problem is then one of maximizing the utility function

$$\int_0^\infty e^{-\rho t}\left[\frac{c_t^{1-\sigma}}{1-\sigma}\right]dt, \tag{8}$$

subject to the constraint that

$$Pc+\left(\dot{b}+\dot{k}\right)=(b+k)\,r\,(1-\tau_t) \tag{9}$$

where P is the price of the consumption good in units of the investment good.

6.1 Characterization of the Consumer's Problem

It is straightforward to see that the present value Hamiltonian for this problem is then written as

$$H=e^{-\rho t}\left[\frac{c_t^{1-\sigma}}{1-\sigma}\right]+e^{-\rho t}\mu\left[(b+k)\,r\,(1-\tau_t)-Pc\right],$$

where μ is the multiplier on the constraint, and has the interpretation of being the utility cost of investment. The optimization conditions for consumption for this problem is then

$$c^{-\sigma}=\mu P. \tag{10}$$

Next, there is the following optimization condition for capital or bond holdings, which must hold at each date

$$\frac{\dot{\mu}}{\mu}=\rho-r\,(1-\tau_t)\,. \tag{11}$$

Since the any government revenue is used to pay off the interest and perhaps principle of the government debt, it follows that all of the consumption good that is produced is consumed by consumers in equilibrium (i.e. from equation (1)). Next, equation (10) can be used to show that

$$\begin{aligned}\frac{\dot{\mu}}{\mu} &= -\sigma\left(\frac{\dot{c}}{c}\right) \\ &= -\sigma\left(\frac{\dot{\phi}}{\phi}+\frac{\dot{k}}{k}\right)\end{aligned} \tag{12}$$

since $\dot{A}=\dot{\lambda}=\dot{P}=0$. But equation (11) then implies that

$$-\sigma\left(\frac{\dot{\phi}}{\phi}+\frac{\dot{k}}{k}\right)=\rho-r\,(1-\tau_t)\,. \tag{13}$$

Now the before-tax net return to capital is $r = \lambda - \delta$, and the growth rate of capital is as follows:

$$\frac{\dot{k}}{k} = \lambda \left(1 - \phi_t\right) - \delta. \tag{14}$$

Then, using equations (13) and (14) implies that

$$-\sigma \left(\frac{\dot{\phi}}{\phi} + \lambda \left(1 - \phi_t\right) - \delta \right) = \rho - \left(\lambda - \delta\right)\left(1 - \tau_t\right). \tag{15}$$

This can be re-written as

$$\dot{\phi} = \frac{\left[\rho - \left(\lambda - \delta\right)\left(1 - \tau_t\right) + \sigma\left(\lambda - \delta\right)\right]}{-\sigma}\phi + \lambda\phi^2. \tag{16}$$

This is a non-linear differential equation. Fortunately, it is a Bernoulli differential equation, that has as solution shown in the following proposition. [9]

Proposition 1 *Equation (16) is characterized by the following solution*

$$\phi_t = \frac{\exp\left(\int_t^\infty \left[\frac{\left[\rho - (\lambda-\delta)(1-\tau_s-\sigma)\right]}{\sigma}\right] ds\right)}{\int_t^\infty \lambda \exp\left(\int_z^\infty \left[\frac{\left[\rho - (\lambda-\delta)(1-\tau_s-\sigma)\right]}{\sigma}\right] ds\right) dz}. \tag{17}$$

Proof. See Appendix B ■

It is useful to check that this is a sensible solution to the problem. First note that if the tax rate is constant ($\tau_t = \tau$), then it is possible to show that equation (17) converges to

$$\phi = \frac{\left[\rho - \left(\lambda - \delta\right)\left(1 - \tau - \sigma\right)\right]}{\sigma\lambda}, \tag{18}$$

and furthermore if $\tau = 0$, then this equation amounts to equation (3), as one would have hoped.[10]

Proposition 2 *i) The solution given by equation (17) has the property that ϕ_t is a function of all future taxes, and that $\frac{\partial\phi_t}{\partial\tau_s} > 0$, for $s > t$. ii) Furthermore, $\frac{\partial^2\phi_t}{\partial s\partial\tau_s} < 0$, for $s > t$, and $\frac{\partial\phi_t}{\partial\tau_s} \to 0$ as $s \to \infty$. iii) Along a path where the tax rate is constant ($\tau_s = \tau$), $\frac{\partial\phi_t}{\partial\tau_s}$ is decreasing in σ.*

[9] The discrete-time counterpart to this expression is a bit more cumbersonme to characterize. Nevertheless, this is derived in Appendix A. An alternative way to verify that equation (17) is the optimal decision rule is to substitute this equation into both sides of the intertemporal euler equation, and verify that this solution works.

[10] One way to check that equation (18) is the solution to equation (17) is to hold the tax rate τ_t constant, and then set the upper limit of equation (17) to be T, rather than ∞. Then the equation is easily solved, whereupon one can then let $T \to \infty$. It should also be noted that equation (16) has more than one solution. Equation (17) is the "forward solution", which depends on future tax rates. One could also obtain a "backward solution" that depends on past tax rates, but this solution is "unstable" and would not make much economic sense.

It should also be noted that equation (16) has other solutions, in addition to that given by equation (17). However, these are either partially or fully "backward-solutions" (or backwardy-looking), and additionally two of the solutions deliver negative values for ϕ, which is impermissible. Since it does not make a lot of sense to have investment decisions depend on past taxes, these other solutions are ignored.

Proof. See Appendix B ■

Corollary 3 *Since the growth rate of capital is related to* $(1-\phi_t)$*, this means that the current growth rate of capital is influenced negatively by the entire path of future tax rates* (τ_s).

The second proposition says that the effect of taxes on growth becomes reduced, the further out into the future the taxes are imposed.

A novelty of equation (17) is that this expression shows explicitly how future government policies affect the current savings rate, and therefore the growth rate. The reader will note that the size of the government debt does not appear explicitly in equation (17). However, it does appear implicitly. The size of the government debt determines the feasible paths for future taxes, through equation (6). And these tax rates influence the growth rate through equation (17). Needless to say, there does not appear to be a one-to-one mapping of the size of government debt (B_t) to the contemporaneous growth rate $(1-\phi_t)$ because there would not seem to be a unique set of taxes that finance any amount of initial government debt (B_0).[11]

Of course, the sensitivity of the growth rate $(1-\phi_t)$ to changes in taxes depends on the the value of σ. For higher values of σ, changes in the tax rate have a smaller impact on the growth rate. This will be explored below.

It is important to note the relationship between the growth rate and the interest rate in the analysis below. The growth rate of capital, is given by equation (14), and depends on ϕ_t. However, the growth rate of consumption is $\frac{\dot{c}}{c}=\frac{\dot{\phi}}{\phi}+\frac{\dot{k}}{k}$, and which depends on $\dot{\phi}$ and ϕ. Although both consumption and capital will grow at the same rate in the steady state (i.e. when $\dot{\phi}=0$), this will not be the case in some of the experiments below. Equations (17) and the right side of equation (16) then demonstrate *explicitly* how the growth rate would be a function of all future capital tax rates. Furthermore, in this model the before tax interest rate is constant (given by equation (4)). In contrast, the after-tax interest rate is given as $r(1-\tau_t)=(\lambda-\delta)(1-\tau_t)$, and so the path for the capital tax rate will influence the after tax interest rate. As is typical of a model in which preferences are given by a utility function like equation (8), the growth rate of consumption is determined by the after-tax interest rate, which here is influenced by the capital tax rate. This does not make the behavior of the consumption growth rate particularly interesting, and so in the analysis below the focus will be on the growth rate of capital.

As will be seen below, given a predetermined amount of government debt, any temporary change in the capital tax rate will influence the long-run after-tax interest rate.

6.2 Equilibrium

To this end, the following definition of a competitive equilibrium will be employed below.

Definition 4 *Given the initial stocks* k_0, B_0*, for this economy, a* Perfect Foresight Competitive Equilibrium *will be a collection of functions* $\{k_t, B_t, c_t, \phi_t, \tau_t,\}_{t=0}^{t=\infty}$*, such that for all* $t \geq 0$,

[11] Of course, one could consider just the set of policies in which the tax rate is held constant to finance a given feasible level of debt. In this case, a higher level of debt, that is feasibly financed, would entail a higher level of taxes, which in turn imply a lower growth rate.

1. *Consumers maximize utility (8) subject to their budget constraint (9). This implies that their consumption/saving decisions are then governed equation (17). Additionally, the transversality condition must hold for government debit and capital.*

2. *Factor prices are given by equation (4).*

3. *The government's budget constraint (6) is satisfied.*

As has been mentioned, not all paths government taxes and debt are feasible. In practice, when computing an equilibrium a path for taxes $\{\tau_t\}_{t=0}^{t=T}$ is determined up till some date T. Then some solution (τ^*) is sought for the tax rate thereafter ($t \geq T$) that will satisfy the government's budget constraint (6).

7 The Debt Trigger

The model will studied here will exhibit behavior pertinent to many current economies in that the taxes are set in such a way that the government is growing on a path that may be perceived to be unsustainable. There is some existing research which suggests that elevated levels of government debt-to-GDP may coincide with reductions in the growth rate.[12] Therefore, because of this empirical observation, and in the spirit of Sargent and Wallace [11], for some experiments it will be assumed that there is some upper bound beyond which government debt-to-GDP ratio is not permitted to exceed. Once this level of government debt is reached, it is assumed that the tax rate is then permanently increased to a constant level that will be sufficient to generate enough revenue to finance the current debt.[13] This mechanism will be referred to as a *"debt trigger"*. As a benchmark, it will be assumed this constraint becomes binding when the debt to GDP ratio hits 100%. The exact nature of why this mechanism exists, or why it kicks-in at a specific level, is not specified.[14] That is, it will be convenient to assume that this debt trigger is embedded in a fixed statute, or in a constitution, and is therefore unchanging, as is the fact that the government cannot default on the debt. The path of the government debt, after the debt trigger is imposed, is characterized in the following proposition.

Proposition 5 *If at some date T, a constant tax imposed to pay off the current debt, then the debt-to-output ratio will be constant thereafter.*

Proof. Equation (7) can be used to show that the government budget constraint at

[12] See Kumar and Woo [6], Panizza and Presbitero [7], and Reinhart and Rogoff [9], [10].

[13] Of course, this does not totally capture the budget difficulties faced by many countries in recent years. In many economies the debt has grown so rapidly not because the interest rate has exceeded the growth rate, but instead because government budget deficits have been so large that the government debt has grown much faster than income. Nevertheless, the key feature here is that individuals have to make consumption decisions while knowing that the government budget constraint dictates that at some future *known date*, the tax rate must be raised.

[14] Presumably a better economic foundation for this idea would be to utilize some politico-economic considerations to justify the equilibrium resulting debt to GDP ratio. That is, it might be interesting to have individual agents to vote on *feasible* (or consistent) policies that give rise to a debt to GDP ratio. While interesting, such an ambitious approach is well beyond the scope of the analysis of this paper.

date T can be written as

$$B_T = \frac{(r\tau)\, k_T}{r\,(1-\tau) - g}, \tag{19}$$

where g is the growth rate of capital $(\left(\dot{k}/k\right) = g)$. Then for $t \geq T$, equation (5) then can be written as

$$\frac{\dot{B}_t}{B_t} = r\,(1-\tau) - \left(\frac{k_t}{B_t}\right) r\tau. \tag{20}$$

Substituting equation (19) into (20) reveals that the growth rate of debt is g, which is the same rate as capital or output. ■

Since this is a perfect foresight economy, individuals will know, with certainty, the future path for government debt and tax rates, given the initial level of debt. Again, there is an extreme degree of commitment that is embedded here, in the sense that government is not permitted to renege (or default) on its stated policies. The point here is to study how the level of government debt and taxes in an economy will affect the decisions of agents, and thereby influence the present and future growth rates.

The imposition of a debt trigger in this environment is not completely without justification. After all, it is assumed that the government budget constraint must be satisfied. For the parameter values considered here, if government debt is permitted to grow without bound, then eventually the government budget constraint cannot hold because there is no way that the present value of future taxes will be sufficient to finance the government debt.

The use of such a debt trigger stands in contrast to much of the existing literature, which usually assumes that any changes in tax rates will be absorbed by a corresponding adjustment in future government spending or transfers. This is often a very convenient assumption, as the changes in government transfers ultimately are reflected in the consumer's budget constraint. For example, Trabandt and Uhlig [13] do just this in their study of the Laffer Curve. These authors are hardly alone, as this assumption seems to be a rather standard operating procedure. Nevertheless, it seems just as plausible to believe that when government taxes are reduced, that consumers will believe that this will necessitate a future increase in tax revenue, rather than a fall in government spending or transfers. Of course, modeling when the future increase in taxes will be implemented is problematic, since it is not evident when taxes *must* rise, or by what magnitude, because there will be a multitude of different paths for the tax rate that will satisfy the government's budget constraint. Nevertheless, the approach adopted here will be to assume that there will be a known future date at which the tax rate must adjust to a constant level, which will then satisfy the government's budget constraint.

8 Parameter Values

The model will be studied will not be calibrated, or designed to mimic, any particular economy, and instead the goal will be to illustrate the behavior of the model. In the model a period will correspond to a year. The other parameters value will then be set with this in mind. The discount factor ρ is chosen to be .05. Although this parameter does not have a significant effect on results of experiments conducted below, it affects the level of the growth rate (as shown by equations (3) and (17)). The value of the parameter A is not particularly relevant, as this parameter does not influence the growth rate, although it does determine the level of consumption and output. The annual depreciation rate is chosen to be $\delta = 10\%$, which is a typical number. Given this, the parameter λ is chosen to deliver the desired steady-state growth rate or a particular interest rate. In the benchmark model, a growth rate of 3% will be assumed. Also in the benchmark model, a tax rate of $\tau = 20\%$ will be employed.[15]

Various values will be assumed for the preference parameter σ. As will be shown below this parameter will be important because it is instrumental in determining the response of consumption and saving (or investment) to the change in current and future tax rates. As is well established, for higher value of σ, agents are less willing to substitute consumption intertemporally, and so in this case consumption and saving decisions are likely to be relatively unresponsive to changes in future tax rates. The opposite is true for lower values of σ. Following the work of Hall [3], it is generally assumed that values of σ well above 1.0 are appropriate.[16] Therefore, in the subsequent analysis the benchmark under consideration will be values of σ in excess of unity, although it will also be shown that there are some interesting things that will happen if σ is less than unity.

The numerical characterization of the model presented below, will be carried out in discrete time, as this is computationally simpler.

9 The Role of Intertemporal Substitution

At the outset it is useful to investigate the impact that intertemporal substitution parameter $\left(\frac{1}{\sigma}\right)$ has on the growth path, for a fixed level of initial debt, with the debt trigger. Therefore, let us consider fixing the initial debt to GDP ratio at 37%. As a benchmark, the initial capital tax rate will be zero, so initially the government debt is being entirely rolled over each period, until the debt trigger kicks in. The debt trigger is then implemented when the debt to GDP ratio hits 100%. For each separate value of σ, the parameter λ will be set to generate a growth rate of 3% in the initial benchmark economy. Of course, this means that for alternative values of σ will have different values for the equilibrium interest rate. In particular, economies with higher values of σ will have higher interest rates, for a given growth rate of consumption.

Figure 3 shows the growth rate of capital for three different values of this parameter (σ), beginning with a fixed amount of government debt. As is shown, the paths are non-monotonic. Initially (i.e. for the first few periods), the economy with higher values of σ has a *higher* growth path, and so it might be said that this parameter has a positive "short run" impact on the growth rate. But the economies with higher values of σ hit the debt trigger sooner, and so their growth rate then falls quicker as a result, than those economies

[15] This would seem to be somewhat lower than the typical US tax rate on capital. One reason for using this value is that with higher values, and for some versions of the model studied below, it is not possible to analyze the impact of a substantial tax cut (e.g. a cut of 10% for 10 years) because the resulting loss in government tax revenue is too large to be recovered by any feasible future increase in taxes. Alternatively, consideration must be given smaller tax changes, or changes over shorter periods of time. Therefore, for expository reasons a value of 20% is employed.

[16] There are many other studies that come to a similar conclusion. Also, Guvenen [2] uses elasticities as low as 0.1, which correspond to $\sigma = 10$.

with lower values of σ. Therefore the intermediate term impact is much harder to describe, and is certainly not *increasing* in σ. However, the long run impact (after the debt trigger is hit in all economies), is that the growth rate is increasing in the level of σ. This result is also suggested by noting that the growth rate is related to the size of the term $1-\phi$, and equation (18) (or 17) implies that

$$\frac{\partial^2 (1-\phi)}{\partial\sigma\partial\tau} = \frac{(\lambda-\delta)}{\lambda\sigma^2} > 0.$$

That is, for a given growth rate, a change in the capital tax rate will produce a smaller reduction in the growth rate if the value of σ is larger.

So what is going on here? As is well known, with these preferences, for higher values σ, of the income effect dominates any intertemporal substitution effect, as a reaction to the change in the tax rate. Then, for higher values of σ, in reaction to the anticipated future *increase* in taxes, the agent attempts to shift consumption into the future, when taxes are to rise. This raises the current savings rate, which increases the growth rate. Conversely, for very low values of σ, in reaction to the anticipated increase in taxes, the agent attempts to pull consumption into the present from the future, when taxes are to rise. This *lowers* the current savings or investment rate, which then reduces the growth rate. This latter effect is important, because the lower savings rate then reduces the discounted value of future tax revenue, necessitating a sharply higher future tax rate to balance the government's budget constraint.

One notable feature in this figure is that the economy with a higher value of σ hits the debt trigger earlier than those with a lower value of σ. The reason for this is that, for a given growth rate, a higher value of σ means a higher interest rate, which in turn implies that the government debt will grow faster, prompting the economy to hit the debt trigger earlier.[17] This also means that for economies with higher interest rates (and therefore higher values of σ), tax revenue in the immediate future is more important than that in the distant future. Therefore, a tax cut of a given size will then have a more dramatic impact on the government budget when the value of σ is higher.

A casual view of Figure 3 might lead one to conclude that the different reactions to the reduction in the capital tax rate are due to the fact that the savings or investment rate is more sensitive to the change in the tax when σ is higher. But this is not true, as is shown in Proposition 2, part iii. The curve labeled '$\sigma = 5$' displays a larger increase in growth not because this value generates a larger contemporaneous response to a change in current taxes. In fact the '$\sigma = .70$' economy is generally more responsive to a contemporaneous change in the tax rate. Instead, in the case of lower values of σ, although the current growth rate is more sensitive to a change in current taxes, *it is also sensitive to changes in future taxes*. In the figure, for low values of σ, the effect of the low current taxes is simply overwhelmed by the future increase in taxes, and this causes a very muted response of the current growth rate. In contrast, in the case in which $\sigma = 5$, the large increase in future taxes causes the current investment rate to rise, so as to partially offset the future increase in the tax rate.

[17] This feature is only partially offset by the fact that a higher interest rate will mean a higher amount of capital income tax revenue.

Note also that although the grow rate is at first increasing for $\sigma = 5$ and $\sigma = 2$, it is non-increasing for $\sigma = .70$. This would imply that the savings or investment rate is falling as the debt trigger is approached.

An alternative way to view the effect of alternative values of σ would be to fix the interest rate across alternative values of σ, rather than fixing the growth rate. Therefore, consider a similar environment in which the parameter λ is set to generate a before-tax interest rate of 6%, for each value of σ, and the initial debt to GDP ratio is initially 63%. Also, assume that the initial capital tax rate is zero. Figure 4 then shows how the growth rate behaves, for various values of σ. For each of these economies, the debt trigger is hit after 9 periods. This is because the time at which this target is hit is largely determined by the initial debt to GDP ratio, and the interest rate, and these are the same for all values of σ. As the figure shows, lower values of σ imply lower initial values for the growth rate, which seems rather obvious. But it is notable that the higher the value of σ, the greater will be the positive reaction of the growth rate to the anticipation of higher future taxes. Of course, for sufficiently low values of σ, the growth rate falls in reaction to higher future taxes.

10 The Impact of Debt on the Growth Rate.

It is now possible to address the issue of how the size of government debt can influence the equilibrium balanced growth path. To do so, again it will be assumed that there is a "Debt Trigger" in place, so that the debt to GDP ratio cannot exceed some level. The parameter values will be chosen so that the economy will be growing, but that the government debt will be growing faster, which will mean that the debt to GDP ratio will also be growing. Such an economy, if left unchanged, will not satisfy the transversality condition, and so if the Debt Trigger were not introduced, then some other policy would need to be instituted to make the resulting long-run allocations feasible.

In this experiment the parameters are chosen as described above, and as a benchmark, the preference parameter will be set at $\sigma = 5.0$. In the benchmark economy, with zero debt, the parameter λ will be set so that the growth rate will be 3%, when there is no government debt. Then, for this economy, three different scenarios will be considered. The initial government debt will then set at 67%, 37%, or 20%.[18]

The resulting growth rate for this experiment is illustrated in Figure 5. These three economies are identical in every respect with the exception of their initial debt to GDP ratios. As the figure shows, the higher is the initial debt level, the higher is the initial growth rate initially. Here, the higher value of the debt to GDP ratio, the closer is the economy to hitting the debt trigger, and so the nearer it is to having to raise taxes. This causes the agent to shift more consumption into the future, which means that the investment rate must rise (and ϕ must fall). Hence, the higher is the debt to GDP ratio, the higher is the growth rate. The punch line would seem to be that, holding other things constant, a higher level of government debt leads to higher initial (or short term) growth, but lower growth in the intermediate term because taxes will have to be raised to keep the government debt

[18] These levels are chosen because it is easy to calculate that they will then hit the debt trigger in periods 3, 6, and 9, respectively. In each of these instances the capital tax rate rises to 55% once the debt trigger is hit.

from escalating. The long term growth rate is the same for all scenarios because eventually the growth and tax rates assume their long-run levels, with the debt to GDP at 100%, and tax rates that will be sufficient to pay the interest on the debt.

This example illustrates why in the data it may be difficult to discover a tight relationship between the level of government debt and the corresponding growth rate. Suppose one were to observe a number of (otherwise identical) economies, of the sort studied here, that differed *only* in their level of government indebtedness. If one were to look at the corresponding relationship between the level of this debt and the resulting growth rate, this example shows why there might not be any close association, because the relationship can be quite nonlinear.

In some other contexts the fact that the growth rate is higher prior to the tax increase may be taken to mean that individuals are deliberately realizing capital gains income prior to the tax increase. However, this is clearly not happening here, since there are no capital gains in this model. Instead, in this model the individual can only shift consumption and investment intertemporally in response to the tax changes. So in this instance the expectation of higher future taxes is causing agents to shift their consumption/saving, and therefore income, across periods. For higher values of σ, the individual will raise saving prior to the rise in taxes, which raises growth. For very low values of σ, the individual will lower saving prior to the rise in taxes, which reduces growth.

Next, one might inquire about the role that the *level* of the debt trigger might play. That is, what would happen if the debt trigger were imposed at a higher level of debt? The answer to this is straightforward. If the debt trigger level were raised, then the paths in Figures 3-5 would all be identical prior to hitting the debt trigger. It is just that the economy would take longer to reach the debt trigger, and when it did so a higher level of the tax rate would be imposed (to finance this higher level of interest on the debt). This in turn would reduce the growth rate after the debt trigger was imposed. This suggests that what is important is not the level of the debt trigger, but instead whether or not there is such a mechanism.

Lastly, this experiment also has implications for the behavior of after-tax interest rates as well. Since the higher the level of debt implies that the capital tax will rise sooner, this implies that the higher level of debt will imply that after-tax interest rate will fall sooner. Hence there is an inverse relationship between the level of government debt, and future after-tax interest rates.

10.1 Implications for Government Spending Multipliers

This last set of results also has implications for the evaluation of the size of government spending multipliers, even though there is no explicit government spending in the model. This is a topic of some importance since there has been renewed interest in recent years for evaluating the size of these multipliers.

To proceed with this analysis, consider the following hypothetical experiment. Consider two economies that are identical in every respect, and have the technology and preference parameters specified for Figure 5. Now, suppose these economies are growing along identical paths until some specific date. Suppose that at that date, one government engages in some wasteful government spending (or just a pure transfer to consumers) that is financed entirely by debt issue. Because the spending itself is specious, it has no impact on the future growth path. Now the two economies are no longer identical, but instead one has a higher level of debt. The results from Figure 5 show that immediately after this event, the economy with the higher debt will grow faster than the one with lower debt. An observer might naturally conclude that the effect of the government spending was to generate higher economic growth, and may conclude that the spending had a relatively high multiplier. However, as the model shows, this is highly misleading. It really is not the government spending that causes growth to be higher. Instead, it is the higher expected taxes entailed by the higher debt which causes individual to invest in capital, that causes growth to be higher. It would be wrong to conclude that the government spending was "productive", and just as wrong to conclude that it was good use of resources. Such a policy must reduce welfare.

This experiment further illustrates that it is not possible to conclusively analyze the impact of a change in government spending without specifying how this will affect future government policies through the government's budget constraint.

11 The Impact of a Temporary Tax Cut

This model can also be used to evaluate the impact of a temporary tax rate reduction.[19] To do this, consider a model in which the initial capital tax rate is 20%, and in which the growth rate is then 3%. Suppose that the 20% tax rate is just sufficient to retire the existing stock of government debt. Beginning from this equilibrium, the tax rate will then be cut to 10%, for exactly 10 periods (years). At this time, the capital tax rate will then be raised to an amount that will raise the revenue necessary make up for the lost revenue.

Figure 6 shows the response of the growth rate of capital to this experiment when $\sigma = 2.0$. Here the dashed line indicates the initial growth rate, while the solid line indicates the actual equilibrium growth path. The growth rate immediately increases after the tax cut, and continues to rise until the higher tax rate is imposed, whereupon the growth rate plummets. Incidentally, the tax rate after period 10 is 27.6%. It is interesting to note that this temporary tax cut not only raises the growth rate, but makes the growth rate continue to rise. The reason for this is that the agent recognizes that the future tax rate is going to rise eventually, and so he must increase investment in order to finance these future tax payments, and also to pay for future consumption as well. This means that ϕ_t continues to fall (so that $(1 - \phi_t)$ can rise) between periods 1 and 10. After period 10, ϕ_t jumps back up because of the increase in the tax rate.

The path labeled '$\diamondsuit$' shows how the growth rate would react if the tax rate were reduced to 10% forever.[20] This experiment indicates that the short run impact of this tax cut is *much larger* than would be impact of a permanent tax reduction. This stands in contrast with much of the existing literature where it is generally shown that a permanent change in taxes will have a much larger impact than a temporary impact.

[19] Actually, strictly speaking, it is not absolutely necessary for there to be a debt trigger here, as long as there is *some* mechanism for generating a tax increase at some future date that is sufficiently large to capture all the revenue lost due to the tax cut.

[20] That is, this would be the path if there were no debt trigger, and it would be feasible to cut the tax rate to 10% forever.

Figure 7 shows the response of the growth rate of capital to this same experiment when $\sigma = 5.0$. Note that this figure has the same general configuration as Figure 6, but the magnitude of the impact is quite different. For a higher value of σ, the short run (between periods 1 and 10) effects are much larger. This means that the growth impact of a change in the tax rate is much larger for a higher value of σ. For comparison, in Figure 8, the tax rate falls from 20% to 10% in period 1, but then rises all the way back up to 37.8% in period 10. The tax rate must really rise a substantial degree to make up for the loss in revenue. Once again, the path labeled '$\diamondsuit$' shows how the growth rate would react if the tax rate were reduced to 10% forever. In contrast with Figure 6, this figure shows that there is an even greater disparity between the effect of a permanent change in the tax rate, and a temporary change.

The fact that temporary tax cuts may have larger growth effects than permanent tax cuts, may be a rather surprising result. This shows that in conducting such an experiment it is important to know exactly what is being held fixed, or allowed to change, in the government's budget constraint. Traditionally in the literature, a tax cut translates into a fall in government spending or transfers, and typically this would mean that a temporary tax cut would have a smaller impact than a permanent one. But here, with government spending and debt held fixed, and therefore total discounted government revenue held fixed, a temporary tax cut may have a larger short term impact on growth than would a permanent tax cut of the same magnitude.

It must be noted here though, that this result is *not* a necessary result of the presence of the debt trigger alone. It also requires a low intertemporal elasticity of substitution of consumption. In the next section it is shown that even with the debt trigger in place, the opposite result might hold.

Figure 8 shows these same experiments for different values of σ simultaneously. This illustrates the importance of the parameter σ, or alternatively the intertemporal elasticity of substitution $\left(\frac{1}{\sigma}\right)$ in studying the impact of tax reform.

Additionally, this experiment has implications for the behavior of interest rates, and the term structure of interest rates. Consider beginning from a steady-state in which there is a constant tax rate financing a stream of discounted revenue to equal the size of government debt. A temporary tax cut, will then imply a higher future tax rate. This implies that such a tax cut will then raise the current after-tax interest rate, but also imply that the after-tax interest rate will fall in the future. This implies that the current tax cut will cause the term structure of interest rate to become more downward sloping.

12 Some Unpleasant Fiscal Arithmetic?

In an influential paper, Sargent and Wallace [11] used a technique that is similar to the debt trigger to derive a striking result.[21] Their result was that a government that *fails* to sufficiently increase the money supply to finance some government spending may actually cause higher inflation today, because the failure to increase the money supply today signals an even higher increase in the money supply in the future. This latter effect would cause inflation today, provided agent's decisions are influenced by future expectations of prices.

[21] Actually, their idea is the progenitor to the debt trigger of this paper.

A very similar result can be shown to hold in the present model. Here, it is shown that a reduction in the capital tax rate, rather than increasing the growth rate, will immediately *lower* future growth path. The reason for this result is analogous to that of Sargent and Wallace. A cut in the capital tax rate, through the government budget constraint, must imply a large increase in future tax rates. Under certain circumstances, the impact on the growth rate of future tax rates dominates the effect of current taxes. It turns out that the circumstances necessary for this to take place are that the parameter (σ) cannot be too big, or the intertemporal elasticity of substitution $\left(\frac{1}{\sigma}\right)$ cannot be too small.

To illustrate this, consider an experiment identical to those in the previous section, wherein the tax rate is cut from 20% to 10% in the first period, but raised again after 10 periods to a level necessary to make up for all of the previous reduced revenue.[22] Figure 9 illustrates what happens if for a value of $\sigma = .515$. Immediately after the tax cut, the growth rate falls and continues to fall, until period 10, at which time the growth rate plummets further. In other words, rather than a tax cut raising the growth rate, it *reduces* growth. For low values of σ, in reaction to the anticipated increase in taxes, the agent really attempts to shift consumption into the present from the future, when taxes are to rise. This sharply lowers the savings or investment rate, which reduces the growth rate. Because of the lower level of investment reduces the growth rate of capital, this reduces the discounted value of future tax revenue, resulting in a much higher future tax rate to balance the government's budget constraint. In this experiment the tax rate rises from 10% to 44% after period 10.

The limits of this experiment needs to be explained. This last experiment cannot be studied for values of σ below .515. The reason is that for lower values of σ, a tax cut from 20% to 10% for 10 years reduces government revenue so much that there is no feasible way to make up for the lost revenue. Similarly, even for higher values of σ, it is possible to cut the tax rate for 10 years and then make up the loss in revenue, but it might not be possible to conduct this experiment for longer periods of time.

This experiment illustrates that for low values of σ, a reduction in the capital tax rate can have a dramatic negative impact on the growth rate. This carries over to interest rates as well. In this experiment the current tax cut will cause the future tax rate to rise dramatically. This implies that future after-tax interest rates will fall precipitously, after the higher tax rate kicks in.

In this experiment, with a low value of σ, there is a stark contrast illustrating how the effect of the tax depends on what this implies for future government policies. If the reduction in the tax rate could be made permanent, perhaps because lump-sum transfers to individuals were reduced, then the path of the tax rate would be given by the '$\diamondsuit$' line, which would imply an immediate and permanent rise in the growth rate. However, the solid line shows that when the tax cut necessarily implies a future rise in taxes, because of debt trigger, the future pattern for the growth rate is very different. This illustrates the importance of conveying to consumers how changes in current policies will influence future policies.

[22] Once again, suppose that the existing stock of government debt outstanding is equal to the present value of government revenue that would be yielded by a constant 20% tax rate.

13 The Model with Labor

An obvious question at this point would be to ask how the results would change if labor were added to the model. One needs to be careful in how this is done, because such a complication can easily make the model unmanageable, or at least, difficult to characterize. Therefore, one way to proceed is shown here. Suppose that n_t represents labor employed in the consumption sector, and $(1 - n_t)$ is leisure. Labor is not used in producing the investment good. Furthermore, suppose that preferences are now written as follows:

$$\int_0^\infty e^{-\rho t} \frac{\left[c_t^\theta (1 - n_t)^{1-\theta}\right]^{1-\sigma}}{1 - \sigma} dt. \tag{21}$$

Of course, if $\theta = 1$, then the preferences are identical to those in equation (8). It should also be noted that with these preferences, the intertemporal elasticity of substitution of consumption is now equal to $\left(\frac{1}{1-\theta(1-\sigma)}\right)$.

The technology is very similar to that in the previous section. The technology for producing the investment good is still given by equation (2). Again capital $(\phi_t k_t)$ is used to produce the consumption good, but labor is now also employed. The production function for this sector is now written as follows:

$$c_t = A (\phi_t k_t)^\alpha (n_t)^{1-\alpha}. \tag{22}$$

That is, labor will be used to produce the consumption good, but not the investment good. In the analysis below the focus will be on the growth rate of capital.

In Appendix C it is shown that the employment level will be strictly a decreasing function of the labor tax rate.

13.1 The Government Budget Constraint

The government budget constraint is now changed because both labor and capital are taxed. Let τ_{kt} and τ_{nt} denote the date-t capital and labor taxes at date t. Also, let w_t denote the date t wage measured in units of capital. Then the government budget constraint, which is measured in units of the capital good, can be written as

$$B_t = \int_t^\infty \exp\left(-\int_t^s r_z (1 - \tau_{kz}) dz\right) [k_s (r_s \tau_{ks}) + n_s (w_s \tau_{ns})] ds.$$

Here the equilibrium wage for the individual, measured in units of the consumption good, will be

$$\hat{w}_t = A (1 - \alpha) \left(\frac{\phi_t k_t}{n_t}\right)^\alpha,$$

while the price of the consumption good, in units of the capital good can be shown to be

$$P_t = \left(\frac{\lambda}{\alpha}\right) \left(\frac{\phi_t k_t}{n_t}\right)^{1-\alpha}.$$

Then the wage, measured in units of the capital good is then

$$w_t = P_t \hat{w}_t = \left(\frac{\lambda(1-\alpha)}{\alpha}\right)\left(\frac{\phi_t k_t}{n_t}\right)$$

In equilibrium the labor tax revenue at date t will be

$$n_t\left(w_t \tau_{nt}\right) = \left(\frac{\lambda(1-\alpha)}{\alpha}\right)\left(\phi_t k_t\right)\tau_{nt}.$$

13.2 Analysis of the Model

There is no need to go through the decentralization of this economy, since it is a straightforward extension of the framework of Section 6. The model can be re-cast in the form of a planning problem. For this problem the present value Hamiltonian is then written as

$$H = e^{-\rho t}\left[\frac{\left[c^{\theta}(1-n)^{1-\theta}\right]^{1-\sigma}}{1-\sigma}\right] + e^{-\rho t}\mu\left[(b+k)r\left(1-\tau_{kt}\right) + wn\left(1-\tau_{nt}\right) - Pc\right].$$

where the constraint is written in units of the capital good. Then the counterpart to equation (10) becomes

$$\theta\left[c^{\theta(1-\sigma)-1}\right]\left[(1-n)^{(1-\theta)(1-\sigma)}\right] = \mu P. \tag{23}$$

There is also the instantaneous condition for labor.

$$(1-\theta)\left[c^{\theta(1-\sigma)}\right]\left[(1-n)^{(1-\theta)(1-\sigma)-1}\right] = (\mu w)\left(1-\tau_{nt}\right)$$

Through the reasoning explored in Appendix C, equation (23) can also be written as

$$\eta\left(\tau_{nt}\right)[\phi k]^{\alpha\theta(1-\sigma)-1} = \mu.$$

Here the term $\eta(\tau_{nt})$ embodies how the labor tax rate influences *the marginal utility of consumption*, holding capital constant. In the Appendix C it is shown that $\eta'(\tau_{nt}) > 0$ if $\sigma > 1$, which is to say that an increase in the labor tax rate increases the marginal utility of consumption. The reason for this is that a rise in the tax rate reduces employment, and increases leisure. These effects are the crucial forces in raising the marginal utility of consumption. On the other hand, if $\sigma \in (0,1)$, then $\eta'(\tau_{nt}) < 0$, and an increase in the labor tax rate reduces the marginal utility of consumption.

Equation (12) then becomes

$$\frac{\dot{\mu}}{\mu} = (\alpha\theta(1-\sigma)-1)\left(\frac{\dot{\phi}}{\phi} + \frac{\dot{k}}{k}\right) + \frac{\eta'\left(\tau_{nt}\right)\dot{\tau}_{nt}}{\eta\left(\tau_{nt}\right)}$$

and equation (13) is then written as follows

$$(\alpha\theta(1-\sigma)-1)\left(\frac{\dot{\phi}}{\phi} + \frac{\dot{k}}{k}\right) + \frac{\eta'\left(\tau_{nt}\right)\dot{\tau}_{nt}}{\eta\left(\tau_{nt}\right)} = \rho - r\left(1-\tau_{kt}\right).$$

This implies that equation (15) is then written as

$$(\alpha\theta(1-\sigma)-1)\left(\frac{\dot{\phi}}{\phi}+\lambda(1-\phi)-\delta\right)+\frac{\eta'(\tau_{nt})\dot{\tau}_{nt}}{\eta(\tau_{nt})}=\rho-(\lambda-\delta)(1-\tau_{kt}).$$

Then finally, the non-linear differential equation (16) can be written as

$$\dot{\phi}=p_t\phi+\lambda\phi^2,$$

where

$$p_t=\frac{\left[\rho-(\lambda-\delta)(1-\tau_{kt})+(\alpha\theta(1-\sigma)-1)(\delta-\lambda)-\left(\frac{\eta'(\tau_{nt})\dot{\tau}_{nt}}{\eta(\tau_{nt})}\right)\right]}{(\alpha\theta(1-\sigma)-1)}. \tag{24}$$

The solution to this Bernoulli differential equation is then

$$\phi_t=\frac{\exp\left(-\int_t^\infty p_s ds\right)}{\int_t^\infty \lambda\exp\left(-\int_z^\infty p_s ds\right)dz}. \tag{25}$$

Now there is one obvious thing to note from this expression. If the labor tax rate is either constant, or changes only in "jumps" then $\dot{\tau}_{nt}=0$, and so this expression is identical to equation (17) when $\alpha=\theta=1$. It is only the continuous changes in the labor tax rate that will influence the growth rate through ϕ_t.

Proposition 6 *If $\sigma>1$, equations (25) and (24) imply that $\partial\phi_t/\partial\dot{\tau}_{ns}<0$, for $s>t$, so that ϕ_t is decreasing in the rate of change future labor tax rate. Furthermore, $\frac{\partial^2\phi_t}{\partial s\partial\dot{\tau}_s}<0$, for $s>t$, and $\frac{\partial\phi_t}{\partial\dot{\tau}_s}\to 0$ as $s\to\infty$. If $\sigma\in(0,1)$, then $\partial\phi_t/\partial\dot{\tau}_{ns}>0$, for $s>t$, then ϕ_t is increasing in the rate of change future labor tax rate. Also, $\frac{\partial^2\phi_t}{\partial s\partial\dot{\tau}_s}<0$, for $s>t$, and $\frac{\partial\phi_t}{\partial\dot{\tau}_s}\to 0$ as $s\to\infty$.*

Proof. Assume $\sigma>1$. Utilizing some calculus on equations (25) and (24), it is possible to see that ϕ_t is increasing in the size of the term in the exponent in this equation. It is shown in Appendix C that this exponent is positively related to $(\dot{\tau}_{ns})$ if $\sigma>1$. The opposite is the case if $\sigma\in(0,1)$. ■

Corollary 7 *Since the growth rate of capital is proportional to $(1-\phi_t)$, it follow then that if $\sigma>1$, the current growth rate of capital is positively related to the the rate of change future labor tax rate. If $\sigma\in(0,1)$, then the growth rate is negatively related to the rate of change in the labor tax rate.*

The reason for this result is as follows. Consider the case in which $\sigma>1$. If the labor tax is on a path where it is increasing, this causes the individual to gradually reduce employment and increase leisure. Both of these effects cause the marginal utility of consumption to increase over time. This *raises* the incentive to postpone some consumption, or raise the savings/investment rate. But this can only be done through raising the growth rate of capital. Once the labor tax ceases to rise (i.e. $\dot{\tau}_{nt} = 0$), this feature no longer influences the growth rate of capital.[23]

This last feature would be another possible explanation for why there might be surprisingly little correlation between the (labor) tax rate and the growth rate. People might expect, from other contexts or models, that this relationship would be negative, but this particular model predicts something quite different.

It should be noted that, in the case in which $\sigma > 1$, raising the labor tax certainly *does not* raise utility even if it does increase the growth rate, since this will distort the margin between the leisure-consumption choice. It only *raises the marginal utility* of consumption.

Another important issue that must be addressed is the fact that in this environment, consumption and investment are not naturally measured in the same units, and so some care must be taken to calculate GDP in the appropriate units. Because of this issue, capital and consumption do not grow at the same rate. A natural approach is to measure total income in units of the consumption good. It is possible to show that in this case GDP is the following:

$$A\left(\phi_t k_t\right)^{\alpha}\left(n_t\right)^{1-\alpha}\left[\frac{\alpha}{\phi_t}+1-\alpha\right].$$

The growth rate of income, measured in units of consumption, is then seen to be

$$-\alpha\left(\frac{\dot{\phi}_t}{\phi_t}\right)\left[\frac{(1-\alpha)\left(1-\phi_t\right)}{\alpha+\phi_t(1-\alpha)}\right]+\alpha\left[\left(1-\phi_t\right)\lambda-\delta\right]+(1-\alpha)\frac{\dot{n}_t}{n_t}.$$

The impact that a change in the capital tax rate has on this growth rate would appear to be ambiguous. The reason is that although a rise in the capital tax rate would raise ϕ, and lower the growth rate of capital $[(1-\phi)\lambda-\delta]$, this also lowers the relative price of investment goods, which works to lower the growth rate of income, measured in consumption units.

If GDP is measured in units of capital, then the growth rate of output will be proportional to the growth rate of the capital stock. This will be illustrated below.

13.3 Quantitative Effects of a Labor Tax

Because of the very stylized nature of the economic model, it is not straightforward to select the appropriate values for the parameters governing the preferences and technology. Therefore, a few different parameter values will be considered, to illustrate the behavior of the economy. As mentioned above, $\left(\frac{1}{\sigma}\right)$ is no longer the measure of the intertemporal elasticity of substitution for consumption, and so a higher value of σ, would now seem more appropriate.

Additionally, labor has a peculiar effect in particular case. Obviously, the parameter θ reflects the relative importance of consumption in utility. Additionally, if "labor's share" of output is chosen to be rather large (because α is small), then a change in the labor tax rate can have a significant impact on the government's budget, but virtually no impact on the long-run growth rate. Using values for α and θ that are substantially less than unity

[23] If $\sigma \in (0,1)$, then this still causes the individual to reduce employment and increase leisure. But this effect then lowers the marginal utility of consumption, and this must then result in a fall in the saving/investment rate.

puts more importance on the labor decision, and the labor income tax, relative to that of capital, since the technological channel for substituting consumption intertemporally is sharply muted. Therefore, to proceed with an illustration of the behavior of this economy, as a benchmark the parameters θ and α will both be chosen to be 0.40.[24]

In the following experiments, the initial tax rate for capital and labor will again be set at 20% initially, and the parameters $(\lambda - \delta)$ will be chosen to produce a benchmark growth rate of 3%. Again, the discount rate will be $\rho = .05$. In period $t = 1$, both tax rates will be reduced from 20% to 10% for precisely 10 periods, and then both tax rates will rise thereafter to such a level that will balance the government's budget constraint. That is, the amount of discounted revenue will be the same under this experiment as if the tax rate had been held at 20% forever.

Figures 10 and 11 show the behavior of this economy for $\sigma = 12$. In this experiment the tax rates are reduced from 20% to 10% initially in period 1, and then it rises to 59.5% in period 10. Figure 11 shows the behavior of the growth rate of output, denominated in units of consumption, in reaction to this rising labor tax rate. Figure 11 shows the how the growth rate of capital behaves in this same experiment.[25] The latter figure clearly illustrates the increased investment and saving, that was observed in the previous section, in reaction to the expected increase in the capital tax. The sharp fall in the growth rate seen in Figure 10 is due to the fall in output in that period that results from the sharp fall in employment, due to the large increase in the labor income tax rate in period 10.[26] Of course, in this stylized economy, labor has little impact on the growth rate unless the labor tax rate is changing. Again, in these figures the '$\diamondsuit$' symbol shows the behavior of the growth rates if the tax rate were kept at 10% forever. The higher growth rate of capital does not translate into a significantly higher growth rate of output, since also lowers the relative price of the investment or capital good.

Figures 12 and 13 illustrate the behavior of this economy for $\sigma = 3.0$. Again, in this instance the tax rates are reduced from 20% to 10% initially in period 1, and then it rises to 49% in period 10. Again, Figure 12 shows the growth rate of output, while Figure 13 shows the growth rate of capital. The growth rate of output exhibits behavior similar to that shown in the "Unpleasant Fiscal Arithmetic" of Section 12. This behavior is much more prevalent in this economy, in the sense that it can be seen for values of σ in excess of unity. Once again, the higher growth rate of the capital good lowers the relative price of this commodity, and so the growth rate of output, measured in units of the consumption good, does not display significant increased growth.

Additionally, it may be illuminating to further investigate the impact that a changing

[24] At this point it should be acknowledged that there is a disadvantage to conducting the numerical characterization of the model in continuous time. For reasons discussed above, if the labor tax rate were to instanteously change, then the model would imply that this would not affect the growth rate of capital (because ϕ would not change). But this *would* cause an instantaneous spike in output, since employment would suddenly. In a discrete time model this instantaneous change in employment would be spread out over a period. In light of this, it would seem be better to employ a discrete time model, in which a period corresponds to a year. This was the approach employed here.

[25] The growth rate of output, measured in units of capital, would behave identically to that of capital, as in Figure 12.

[26] Not only is the reduction in employment here reducing the quantity of the consumption good that is being produced, but it also lowers the price of capital, measured in units of the consumption. This further contributes to the fall in output, measured in units of consumption.

labor tax rate can have on the growth rate. That is, it would be interesting to quantify the impact that the term $\left(\frac{\eta'(\tau_{nt})\dot{\tau}_{nt}}{\eta(\tau_{nt})}\right)$ might have on the growth rate, through equations (24) and (25). Therefore, consider an economy in which the debt trigger is absent, since this will be a distraction from the experiment. Now consider a situation in which the capital and labor tax rates are fixed at 20%, and therefore the economy then grows at a 3% growth rate. Now, from this benchmark or steady-state, consider what would happen if, beginning at date $t = 1$, the labor tax rate were to rise continuously from 20% to 30%, from $t = 1$ until $t = 10$, and stays at 30% thereafter. The capital tax rate is held constant at 20% during the entire experiment. The parameter σ is held constant at 12.0, and the parameters α and θ will again be set equal to 0.40.

Figure 14 shows how the growth rate of capital behaves in reaction to this change in the labor tax. As can be seen, the expectation of higher labor taxes raises the growth rate of capital. This effect is dissipated as the economy approaches period 10, since at that time the labor tax will settle in at its new higher level.

14 Final Remarks

Several generations of research into dynamic models have taught us how vital it is to integrate expectations into the decision-making of agents. This is especially evident when studying issues related to tax reform. The model studied here makes it quite apparent that one cannot hope to capture the effects that government policies will have without also knowing what these also imply about future policies. This is not some fictional consideration of only theoretical consequence: because changes in policies at one date *must* have an influence on future policies.

In this model it has been shown that analyzing the effect of a change in the tax rate can be problematic because of the complicated dependence that future government policies will have on current policies. The government budget constraint presumably imposes discipline on what policies can be implemented. Agent's expectations must take into account how this constraint will affect future decisions. If agents believe that a reduction in current taxes necessarily implies a future reduction in spending, then the impact will be vastly different than if agents believe that spending is likely to be unchanged, and instead future taxes must necessarily be raised.

The model studied here assumed that a fixed amount of predetermined government debt had to be financed through distortional taxation. It was shown that the impact that the size of this debt had on the growth rate depended on some important parameters of the economy. In one extreme case, a *reduction* in the capital tax rate causes a growth *implosion* because of the expectation that the government budget constraint must imply that taxes will rise even higher in the future.

Although the analysis makes it very apparent that characterizing the impact of a policy change critically depends on understanding the expectations of agents, it is far from clear how to do this in practice. There are then other important related issues that also arise: It is also evident that the credibility of announced government policies is also of vital importance.

The model studied here is necessarily highly stylized, as there is only one technology in which individuals can invest. In practice, there are generally a multitude of vehicles that individuals can employ to shift consumption intertemporally. This may be another important feature that would affect the quantitative nature of the results.

References

[1] Diamond, Peter, "National Debt in a Neoclassical Growth Model", *American Economic Review*, 55, (December, 1965), pages 1126-1150.

[2] Guvenen, Fatih, "A Parsimonious Macroeconomic Model for Asset Pricing" *Econometrica*, 77, (November, 2009), pages 1711-1750.

[3] Hall, Robert, "Intertemporal Substitution in Consumption", *Journal of Political Economy*, 96(2), (1988), pages 339-357.

[4] Lucas, Robert E. Jr., "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, (July, 1988): pages 3-42.

[5] Lucas, Robert E. Jr. "Supply-Side Economics: An Analytical Review" *Oxford Economic Papers*, 42, (April, 1990), pages 293-316.

[6] Kumar, Manmohan S., and Jaejoon Woo, "Public Debt and Growth" IMF Working Paper, (WP/10/174), (July, 2010).

[7] Panizza, Ugo, and Andrea F. Presbitero, "Public Debt and Economic Growth: Is There a Causal Effect", Department of Public Policy and Public Choice – POLIS, Working Paper 198, (April 2012).

[8] Rebelo, Sergio, "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, 99, (June, 1991): 500-521.

[9] Reinhart, Carmen M., Kenneth S. Rogoff, "Growth in a Time of Debt", *American Economic Review* 100 (2), (May 2010), pages 573-578.

[10] Reinhart, Carmen M., Kenneth S. Rogoff, "Public Debt Overhangs: Advanced-Economy Episodes Since 1800" *Journal of Economic Perspectives*, 26(3), (Summer 2012), pages 69-86.

[11] Sargent, Thomas J., and Neil Wallace, "Some Unpleasant Monetarist Arithmetic", *Federal Reserve Bank of Minneapolis Quarterly Review*, 5, (Fall 1981), pages 1-17.

[12] Stokey, Nancy L., and Sergio Rebelo, "Growth Effects of Flat Taxes", *Journal of Political Economy*, 103(3), (June 1995): pages 519-550.

[13] Trabandt, Mathias, and Harald Uhlig,"The Laffer Curve Revisited", *Journal of Monetary Economics,* 58(4), (May 2011): pages 305-327.

15 Appendix A

In this Appendix the discrete-time counterpart to model section 5 is presented. The model is then re-written in the following manner. The preferences of the individuals are written as follows

$$\sum_{t=1}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} \right]$$

Let the technology be

$$c_t = A(\phi_t k_t)$$

along with the following

$$I_t = \lambda(1-\phi_t) k_t$$

and

$$k_{t+1} = (1-\delta) k_t + I_t$$

The budget constraint faced by the individual consumers is written as

$$c_t + (b_{t+1} + k_{t+1}) = (b_t + k_t)[1 + r_t(1-\tau_t)]$$

The euler equation for a consumer in this economy is of the usual form:

$$\left(c_t^{-\sigma}\right) = \beta \left(c_{t+1}^{-\sigma}\right)[1 + r_{t+1}(1-\tau_{t+1})]$$

Also, the interest rate can be seen to be $r = \lambda - \delta$. But since $c_t = A(\phi_t k_t)$, and $k_{t+1} = [(1-\delta) + \lambda(1-\phi_t)] k_t$, this equation can be written as

$$(A(\phi_t k_t))^{-\sigma} = \beta \left(A\left(\phi_{t+1} k_{t+1}\right)\right)^{-\sigma} [1 + r_{t+1}(1-\tau_{t+1})]$$

or

$$(\phi_t)^{-\sigma} = \beta \left(\phi_{t+1}\right)^{-\sigma} [(1-\delta) + \lambda(1-\phi_t)]^{-\sigma} [1 + (\lambda-\delta)(1-\tau_{t+1})]$$

which can be re-written as

$$\phi_t = \phi_{t+1} [(1-\delta) + \lambda(1-\phi_t)] \{\beta [1 + (\lambda-\delta)(1-\tau_{t+1})]\}^{-1/\sigma}$$

This equation is a non-linear difference equation, and is the discrete-time counterpart to the non-linear differential equation (16). In principle this difference equation could be "solved forward", but it would be a nasty mess. Nevertheless, this equation illustrates how the current value of ϕ_t is then going to be a function of all present and future values of the tax rate τ_{t+s}, for $s \geq 0$.

There is another approach to this problem that is illuminating. Imagine a fictitious planner who is maximizing the welfare of consumers in such an environment, while facing a series of intertemporal returns, denoted $1 + r_{t+1}(1-\tau_{t+1})$. Let the state variable for such a planner be k_t. It is straightforward to verify that the value function for the dynamic programming problem, that corresponds to the planning problem that corresponds to the competitive equilibrium, is of the form

$$v(k_t) = \Pi_t (k_t)^{1-\sigma}.$$

This value function is written as a function of the capital stock, and not the capital stock and the stock of bonds, because in this environment all output is consumed by consumers, and so the capital stock is a sufficient state variable to characterize the amount of consumption. It can be shown that the coefficient Π_t is a non-linear function of the parameters of the economy, as well as all the future capital tax rates $(\tau_s, s \geq t)$.

16 Appendix B

16.1 Proof of Proposition 1:

Equation (16) if of the following form:

$$\dot{\phi} = p_t \phi + \lambda \phi^2.$$

Writing the candidate solution as

$$\phi_t = \frac{\exp\left(-\int_t^\infty p_s ds\right)}{\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz}. \tag{26}$$

Taking the derivative of this expression with respect to t yields

$$\begin{aligned} \dot{\phi} &= \frac{p_t \exp\left(-\int_t^\infty p_s ds\right)}{\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz} - \frac{(-\lambda)\left[\exp\left(-\int_t^\infty p_s ds\right)\right]^2}{\left[\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz\right]^2} \\ &= p_t \phi + \lambda \phi^2 \end{aligned}$$

16.2 Proof of Proposition 2:

Using equation (26) it is possible to show that, for $j > t$,

$$\begin{aligned} \frac{\partial \phi_t}{\partial \tau_j} &= \frac{\partial \phi_t}{\partial p_j} \cdot \frac{\partial p_j}{\partial \tau_j} \\ &= \left[\frac{\exp\left(-\int_t^\infty p_s ds\right)}{\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz} - \frac{\exp\left(-\int_t^\infty p_s ds\right) \int_t^j \lambda \exp\left(-\int_z^\infty p_s ds\right) dz}{\left[\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz\right]^2}\right]\left[\frac{\lambda - \delta}{\sigma}\right] \\ &= \left[1 - \frac{\int_t^j \lambda \exp\left(-\int_z^\infty p_s ds\right) dz}{\left[\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz\right]}\right] \phi_t \left[\frac{\lambda - \delta}{\sigma}\right] > 0. \end{aligned} \tag{27}$$

Note that the first term in square brackets is unity when $j = t$, and goes to zero as $j \to \infty$, so that very distant taxes have a negligible effect. Then it is straightforward to show that

$$\frac{\partial^2 \phi_t}{\partial j \partial \tau_j} = \left[\frac{-\exp\left(-\int_j^\infty p_s ds\right)}{\left[\int_t^\infty \lambda \exp\left(-\int_z^\infty p_s ds\right) dz\right]}\right] \phi_t \left[\frac{\lambda - \delta}{\sigma}\right] < 0.$$

Equation (27) gives the change in ϕ_t, for a change in τ_j per unit of time. To obtain the effect of of changing the future tax rate over some interval if time (Ω) in the future $(> t)$, it is necessary to use the expression in equation (27) to calculate

$$\int_{j\in\Omega}\left(\frac{\partial\phi_t}{\partial\tau_j}\right)dj.$$

It is then possible to verify that a permanent change in the tax rate has the impact that would be calculated from equation (18).

Lastly, if the tax rate constant ($\tau_s=\tau$), then equation (27) shows that $\frac{\partial\phi_t}{\partial\tau_j}$ is decreasing in σ.

17 Appendix C (For Online Publication)

In this Appendix it is shown how the model with labor, and with labor taxes, can be re-written in a simpler manner. The utility function is given by equation (21). However, at any moment, the agent has an instantaneous utility function represented as follows:

$$U(c,l)=\frac{\left(c^{\theta}(1-n)^{1-\theta}\right)^{1-\sigma}}{1-\sigma} \tag{28}$$

The technology is given by equation (22). But in the competitive equilibrium the budget constraint is as follows:

$$c=\hat{w}n+\pi$$

where $\hat{w}$ is the after-tax wage ($w(1-\tau_n)$), and π is non-wage income. The optimization condition can then be re-written as follows:

$$\hat{w}U_1(c,1-n)=U_2(c,1-n)$$

or

$$\hat{w}=\left(\frac{1-\theta}{\theta}\right)\left(\frac{c}{1-n}\right). \tag{29}$$

Substituting this expression back into the utility function yields the following

$$\begin{aligned}
U(c,1-n) &= \frac{\left(c^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)\left(\frac{c}{\hat{w}}\right)\right]^{1-\theta}\right)^{1-\sigma}}{1-\sigma}\\
&= \frac{c^{1-\sigma}\left([\hat{w}]^{\theta-1}\left(\frac{1-\theta}{\theta}\right)^{1-\theta}\right)^{1-\sigma}}{1-\sigma}\\
&= \left[[\hat{w}]^{\theta-1}\left(\frac{1-\theta}{\theta}\right)^{1-\theta}\right]^{1-\sigma}\frac{c^{1-\sigma}}{1-\sigma}.
\end{aligned}$$

Noting that consumption is determined by the technology given by equation (22). This technology will then be used to determine the wage, or marginal product of labor, in a competitive equilibrium. This after-tax wage equals the after tax marginal product of labor as follows

$$\hat{w}=(1-\tau_n)\frac{dc}{dn}=(1-\tau_n)(1-\alpha)\frac{c}{n}=(1-\tau_n)(1-\alpha)A((\phi k)/n)^{\alpha}$$

and so equation (29) implies

$$(1-\tau_n)(1-\alpha)A((\phi k)/n)^\alpha = \left(\frac{1-\theta}{\theta}\right)\left(\frac{A(\phi k)^\alpha n^{1-\alpha}}{1-n}\right)$$

or

$$(1-\tau_n) = \left(\frac{1-\theta}{\theta(1-\alpha)}\right)\left(\frac{n}{1-n}\right)$$

which can be used to derive the following

$$n = \frac{(1-\tau_n)}{\left[\left(\frac{1-\theta}{\theta(1-\alpha)}\right)+(1-\tau_n)\right]} \tag{30}$$

and

$$1-n = \frac{\left(\frac{1-\theta}{\theta(1-\alpha)}\right)}{\left[\left(\frac{1-\theta}{\theta(1-\alpha)}\right)+(1-\tau_n)\right]}. \tag{31}$$

Now condition (23) can then be written as follows

$$\theta\left[c^{\theta(1-\sigma)-1}\right]\left[(1-n)^{(1-\theta)(1-\sigma)}\right] = \mu P.$$

But using the fact that $P = \left(\frac{\lambda}{\alpha}\right)\left(\frac{\phi k}{n}\right)^{1-\alpha}$, and the definition of consumption yields

$$\theta\left[A(\phi k)^\alpha (n)^{1-\alpha}\right]^{\theta(1-\sigma)-1}\left[(1-n)^{(1-\theta)(1-\sigma)}\right] = \mu\left(\frac{\lambda}{\alpha}\right)\left(\frac{\phi k}{n}\right)^{1-\alpha}$$

which can then be written as

$$\theta\left[A(\phi k)\right]^{\alpha\theta(1-\sigma)-1}\left[(n)^{1-\alpha}\right]^{\theta(1-\sigma)}\left[(1-n)^{(1-\theta)(1-\sigma)}\right] = \mu\left(\frac{\lambda}{\alpha}\right)$$

Therefore, the left side of this expression, which represents the marginal utility of consumption can now be written as follows

$$\theta\alpha\left[A(\phi k)\right]^{\alpha\theta(1-\sigma)-1}[n]^{\theta(1-\alpha)(1-\sigma)}[1-n]^{(1-\theta)(1-\sigma)} = \mu\lambda.$$

Using equations (30) and (31) this equation can be written as

$$\theta\alpha\left[A(\phi k)\right]^{\alpha\theta(1-\sigma)-1}\left[\frac{(1-\tau_n)}{\left[\left(\frac{1-\theta}{\theta(1-\alpha)}\right)+(1-\tau_n)\right]}\right]^{\theta(1-\alpha)(1-\sigma)}\left[\frac{\left(\frac{1-\theta}{\theta(1-\alpha)}\right)}{\left[\left(\frac{1-\theta}{\theta(1-\alpha)}\right)+(1-\tau_n)\right]}.\right]^{(1-\theta)(1-\sigma)} = \mu\lambda$$

or

$$\eta(\phi k)^{\alpha\theta(1-\sigma)-1} = \mu\lambda$$

where

$$\eta_t = \theta\alpha\left[A^{\alpha\theta(1-\sigma)-1}\right]\left[\frac{(1-\tau_{nt})}{\left[\left(\frac{1-\theta}{\theta(1-\alpha)}\right)+(1-\tau_{nt})\right]}\right]^{\theta(1-\alpha)(1-\sigma)}\left[\frac{\left(\frac{1-\theta}{\theta(1-\alpha)}\right)}{\left[\left(\frac{1-\theta}{\theta(1-\alpha)}\right)+(1-\tau_{nt})\right]}\right]^{(1-\theta)(1-\sigma)}$$

And so the preferences of equation (21) can be written in a manner identical with those of equation (8), as long as there is a preference shock (η_t), added, and the exponent $(1-\sigma)$ is replaced with $\alpha\theta(1-\sigma)$. It is then straightforward to verify that

$$\frac{\partial \eta_t}{\partial \tau_{nt}} = \frac{(\sigma - 1)}{(1-\tau_{nt})\left[\frac{1-\theta}{\theta(1-\alpha)} + (1-\tau_{nt})\right]}(1-\theta)\,\eta_t \tau_{nt}.$$

and it is easy to see that this expression is positive if $\sigma > 1$, but negative if $\sigma \in (0,1)$.

Figure 1

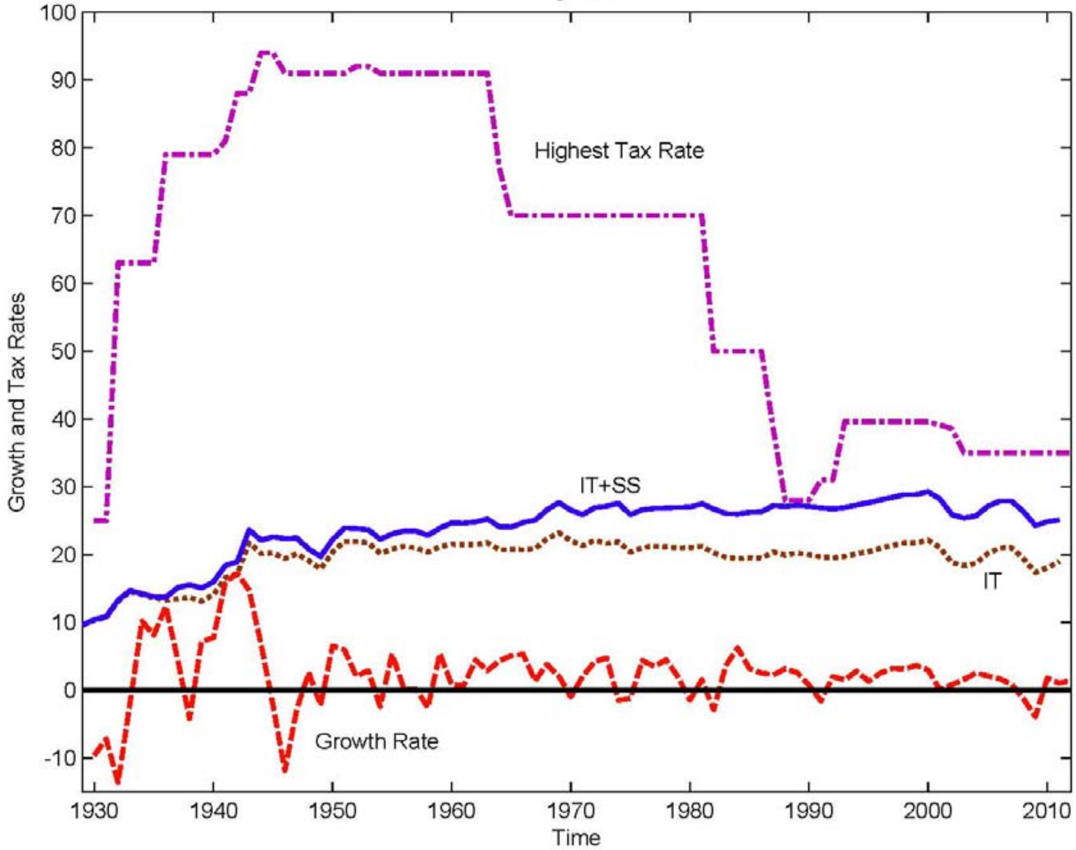


Figure 2

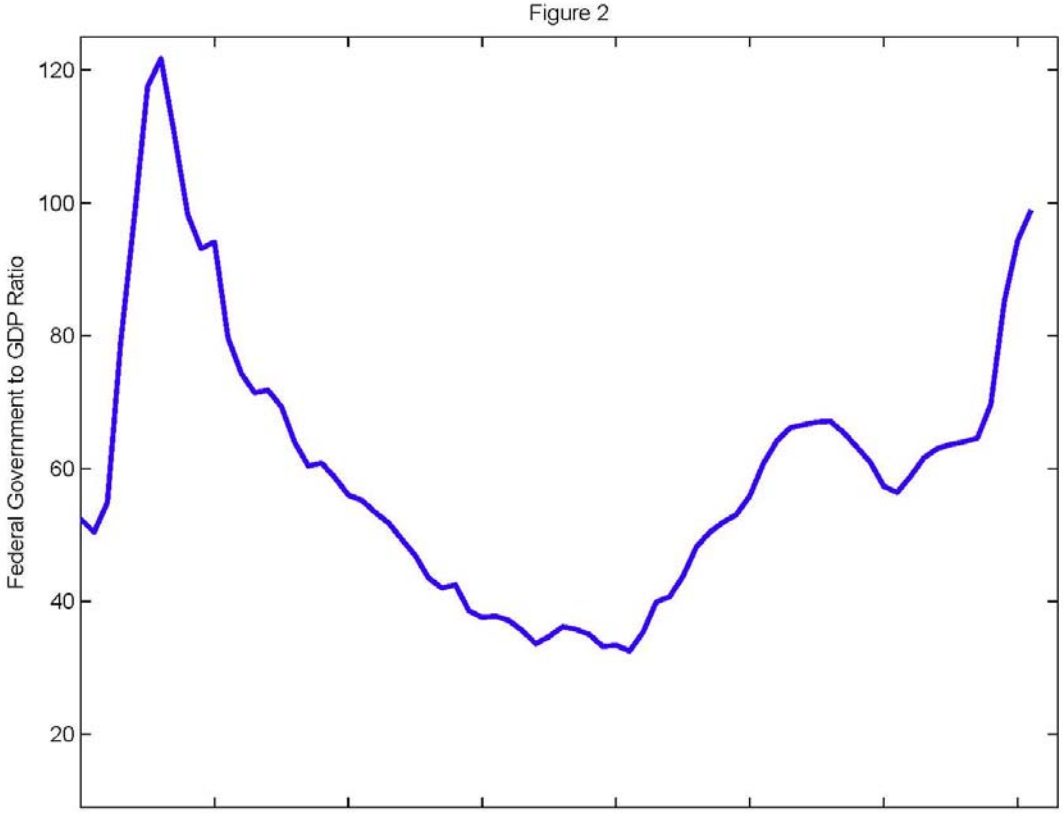
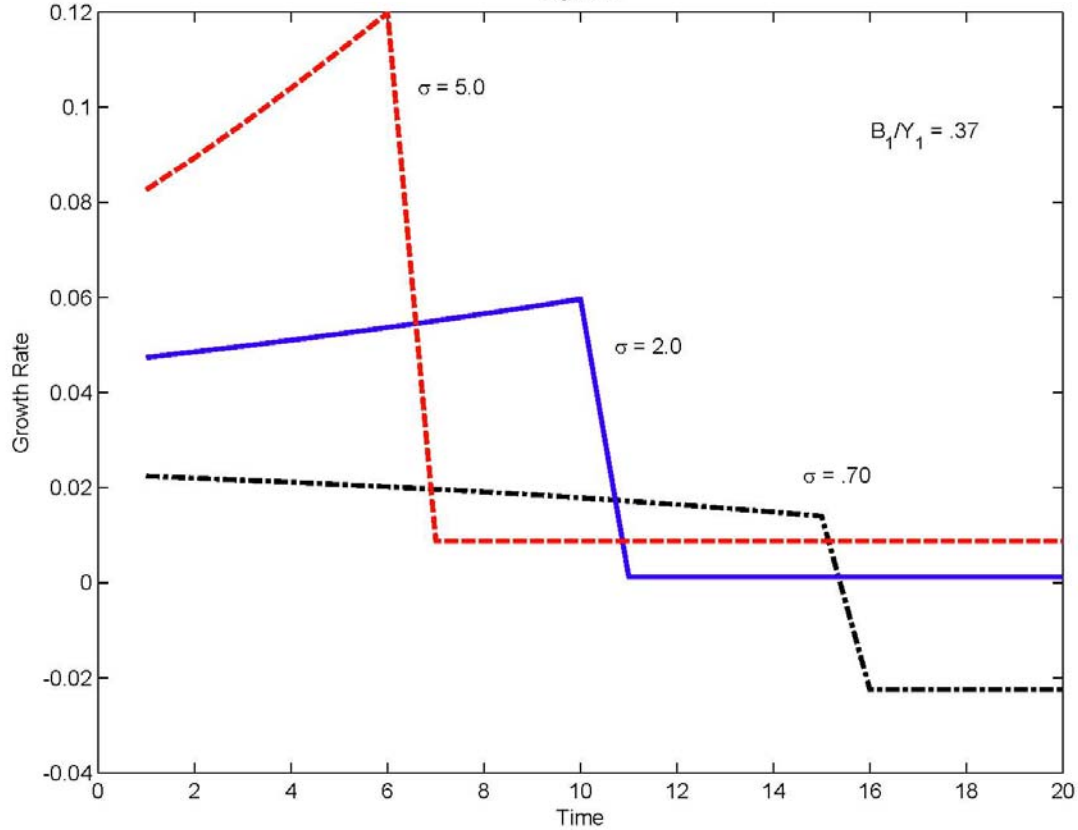


Figure 3



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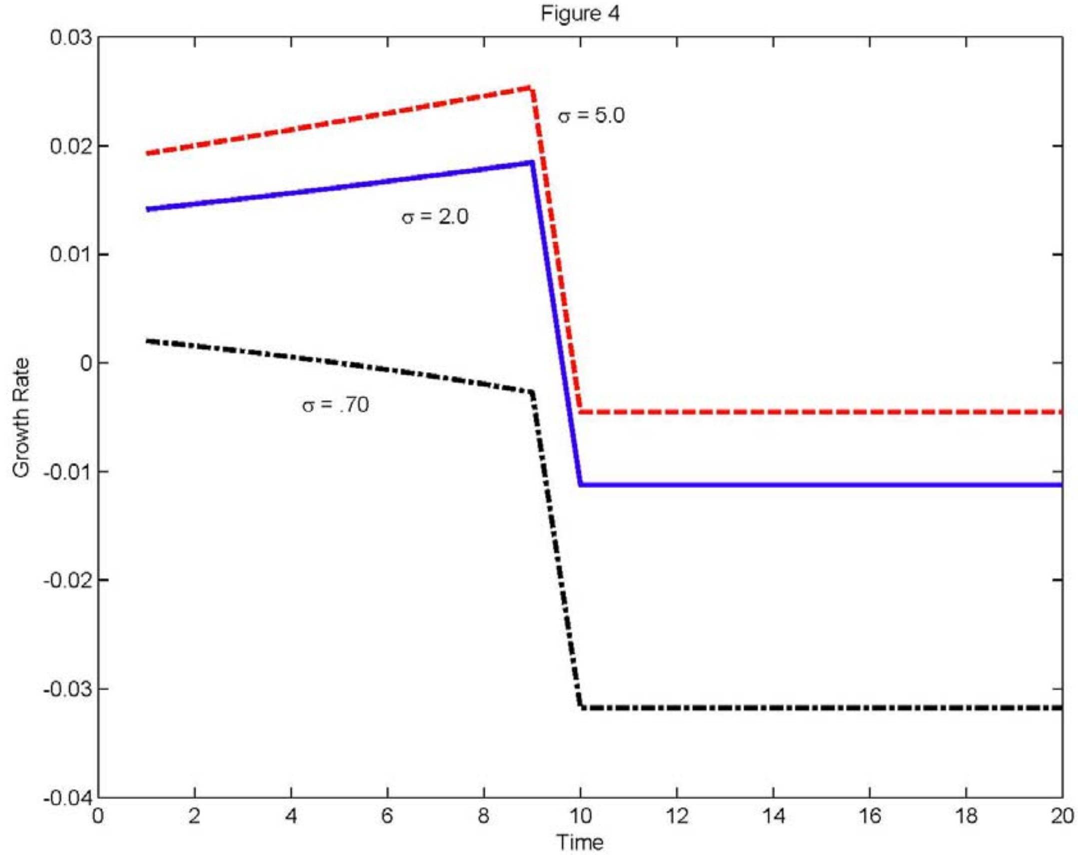

Figure 5

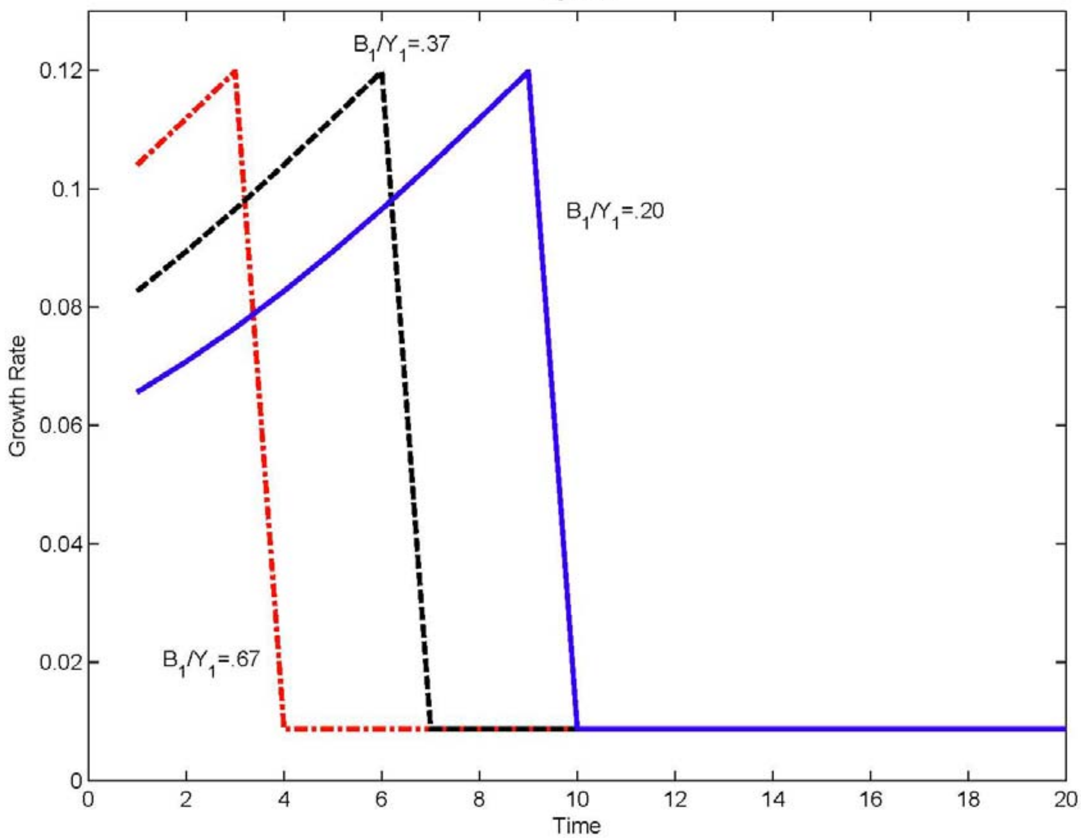

Figure 6

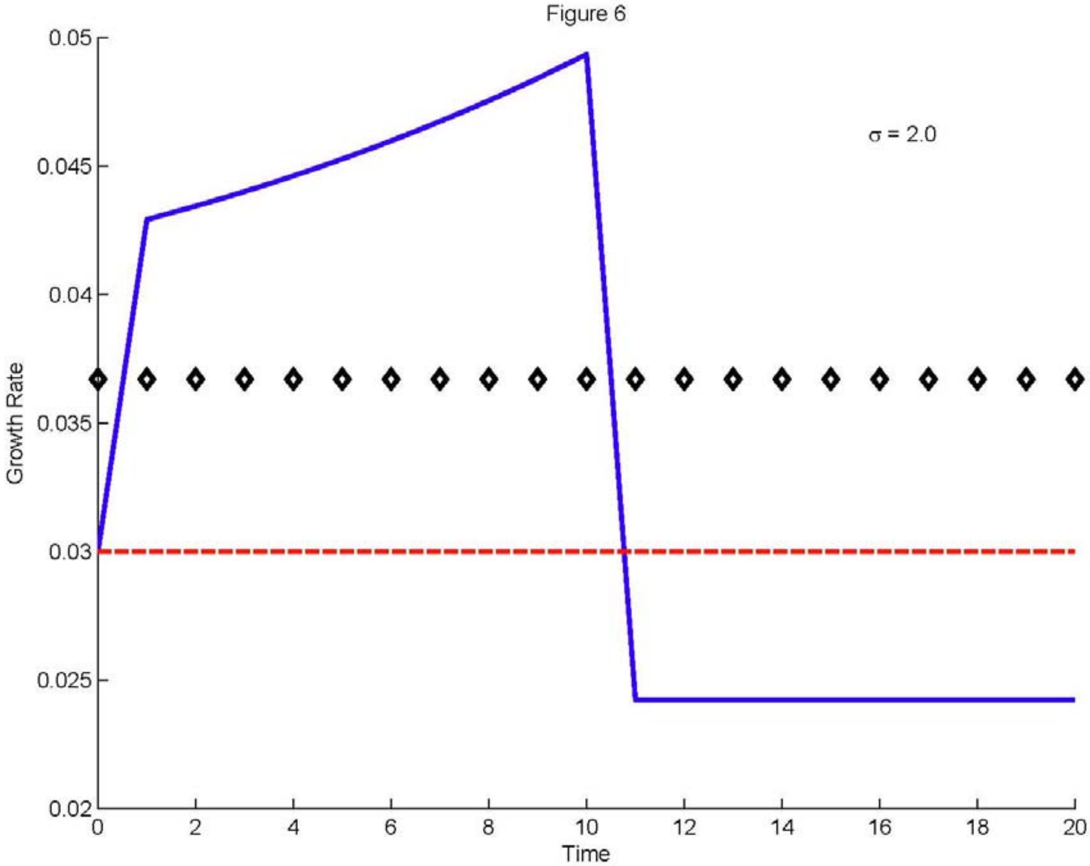

Figure 7

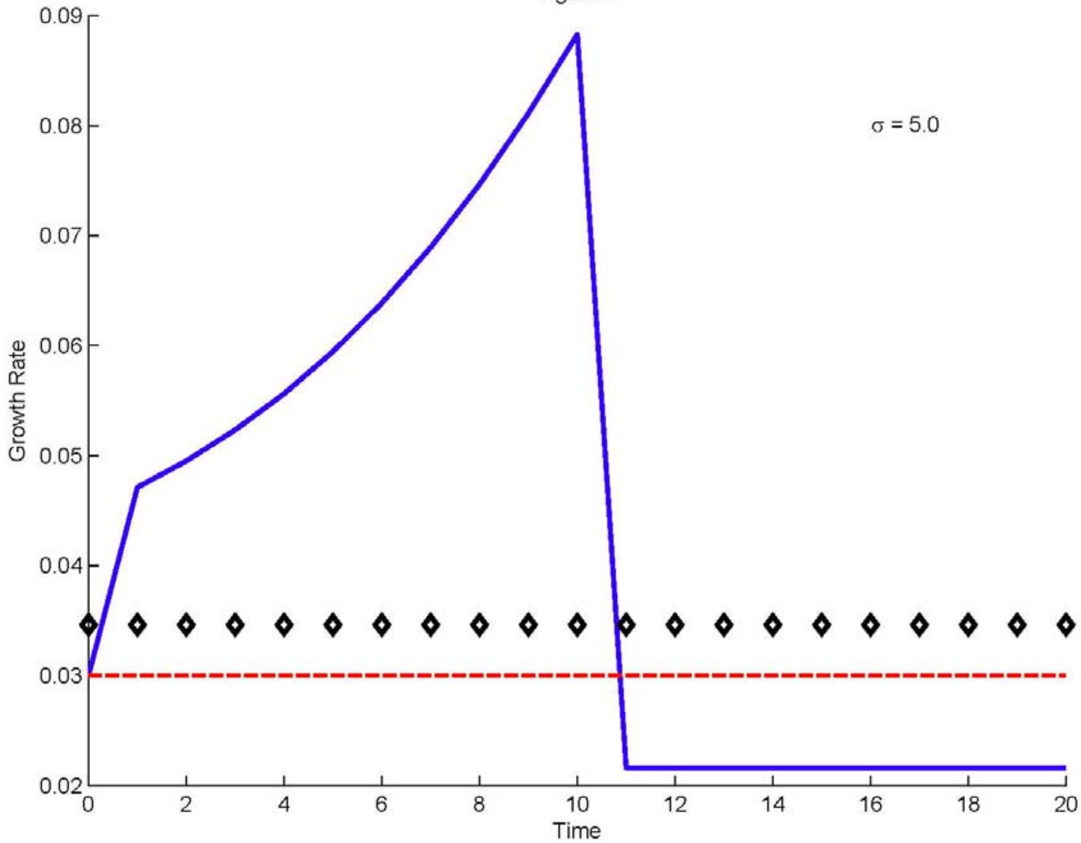

Figure 8

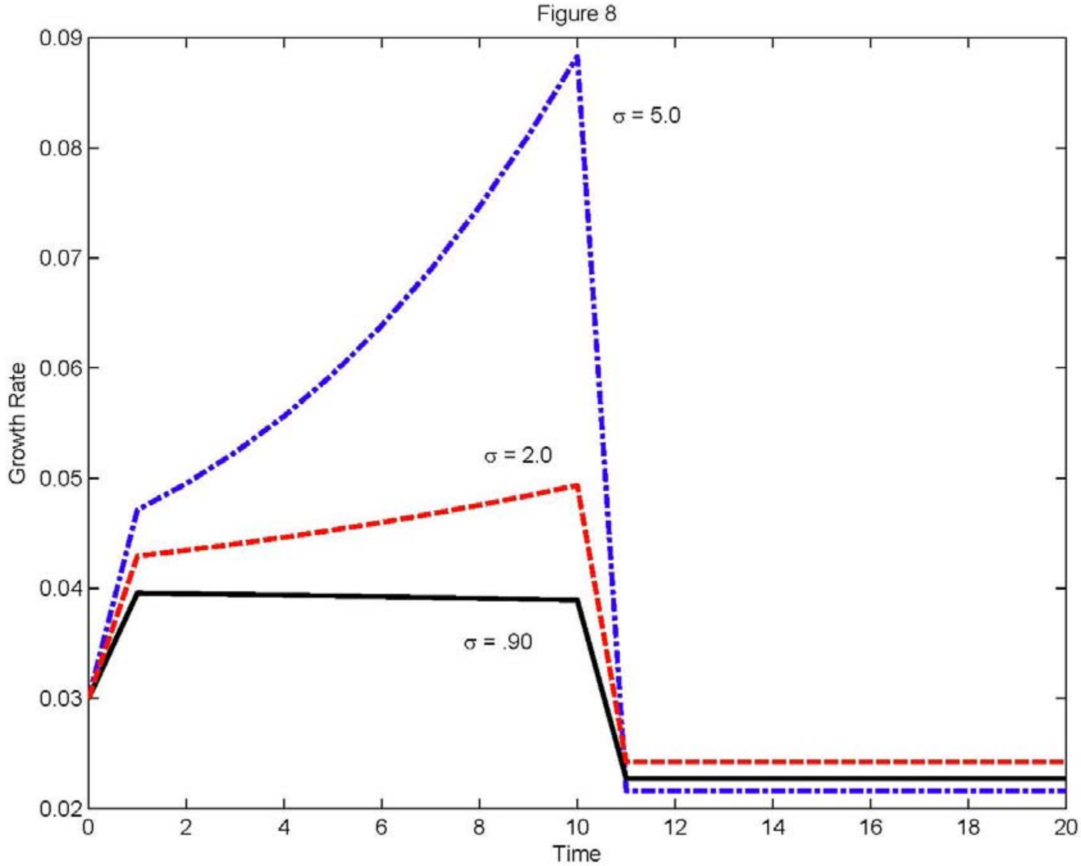

Figure 9

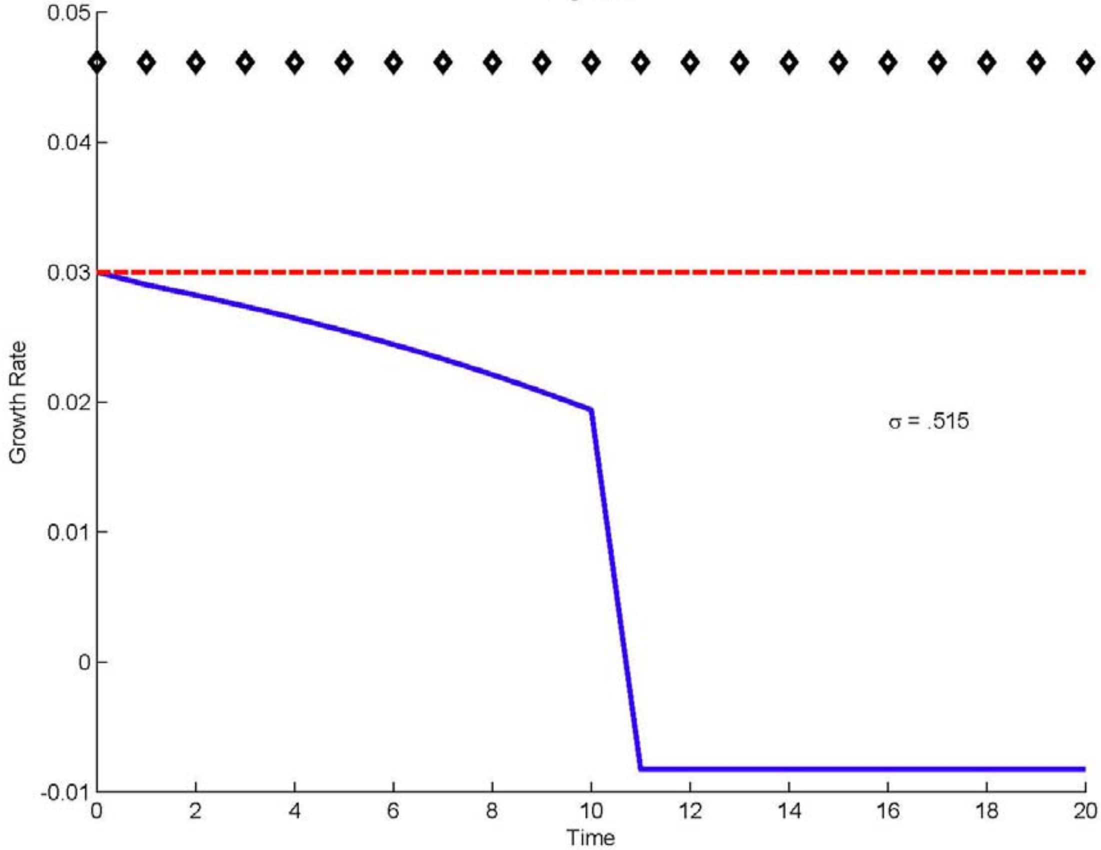

Figure 10

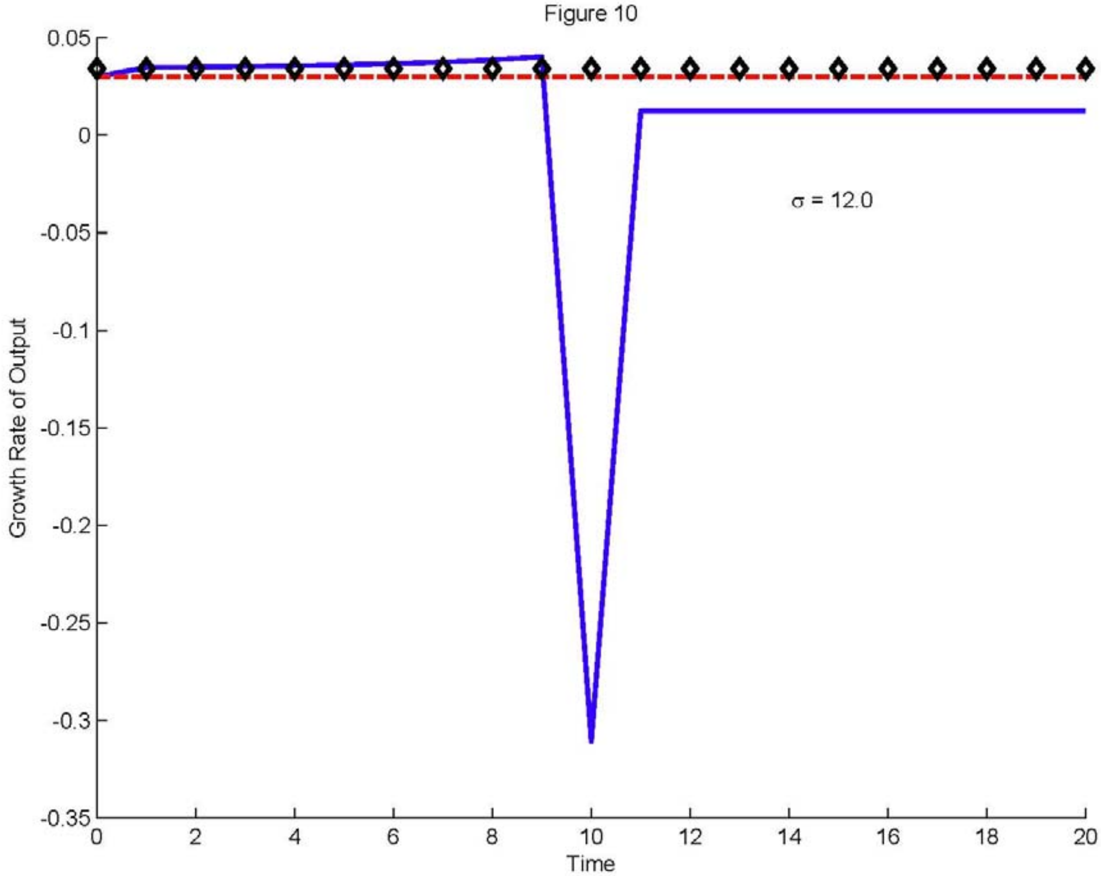

Figure 11

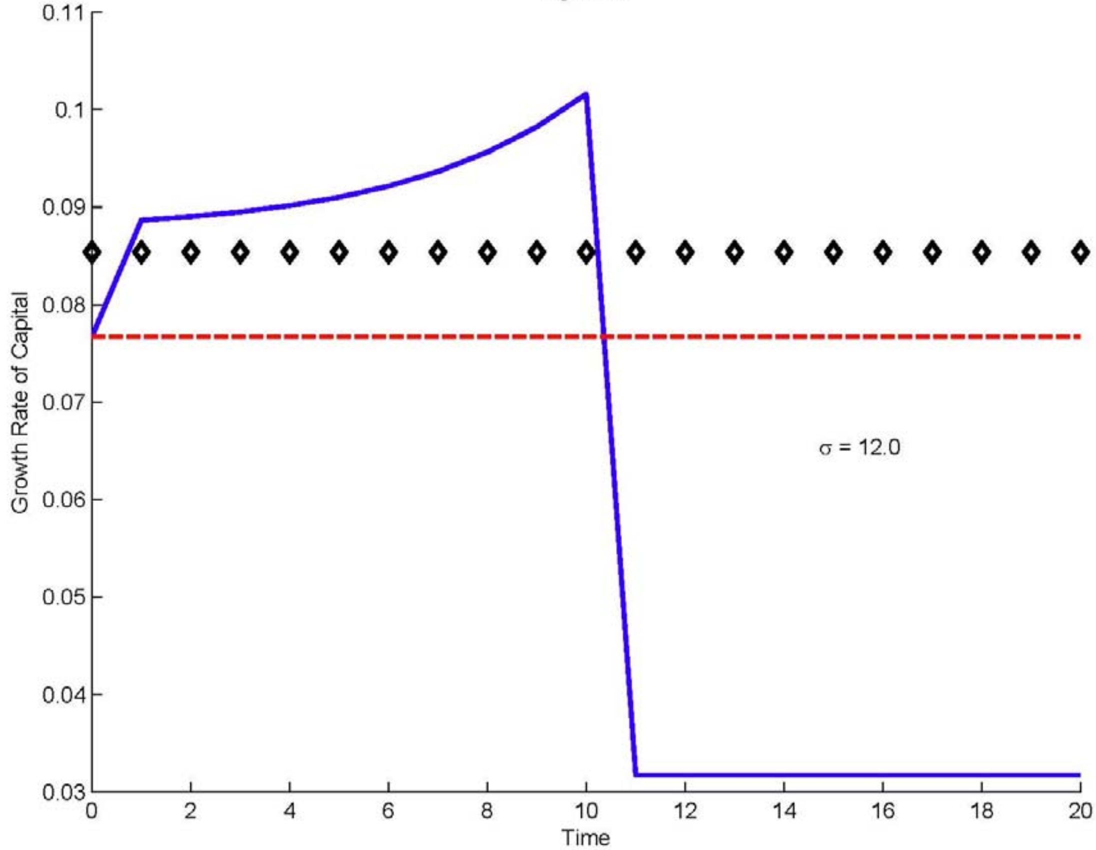

Figure 12

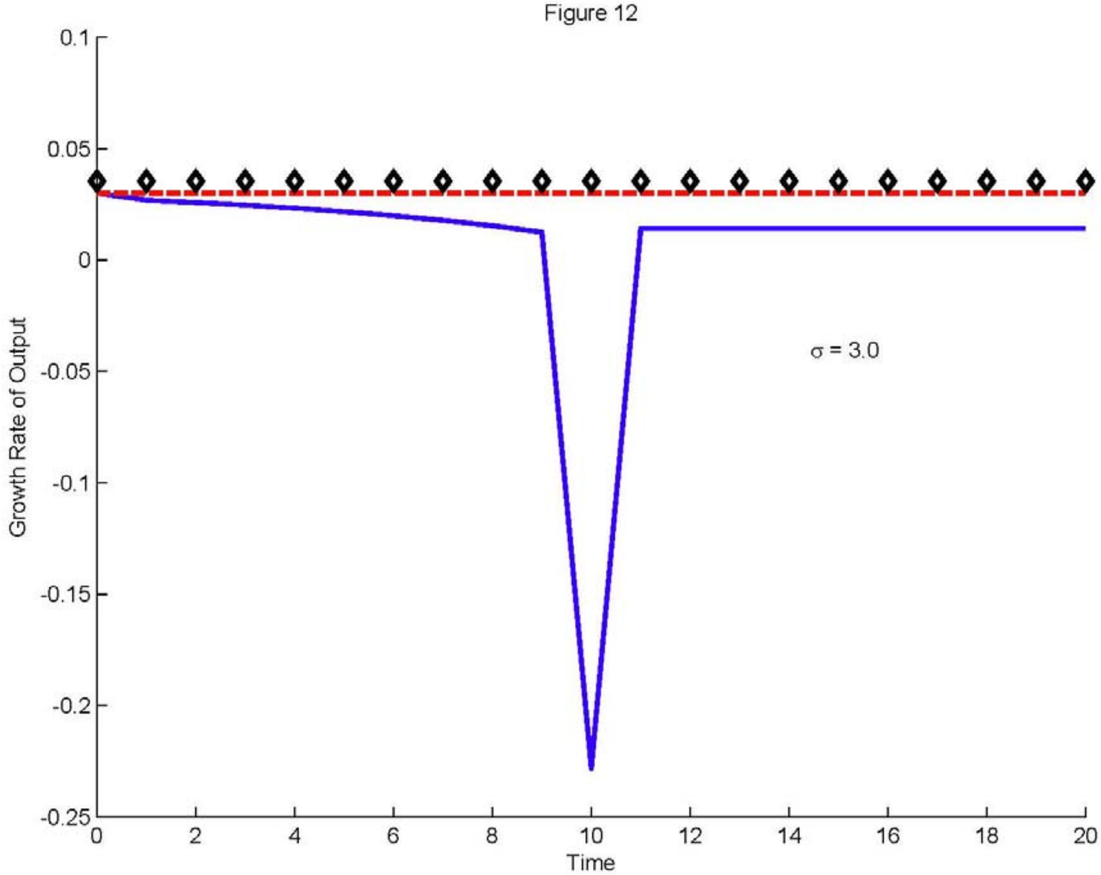

Figure 13

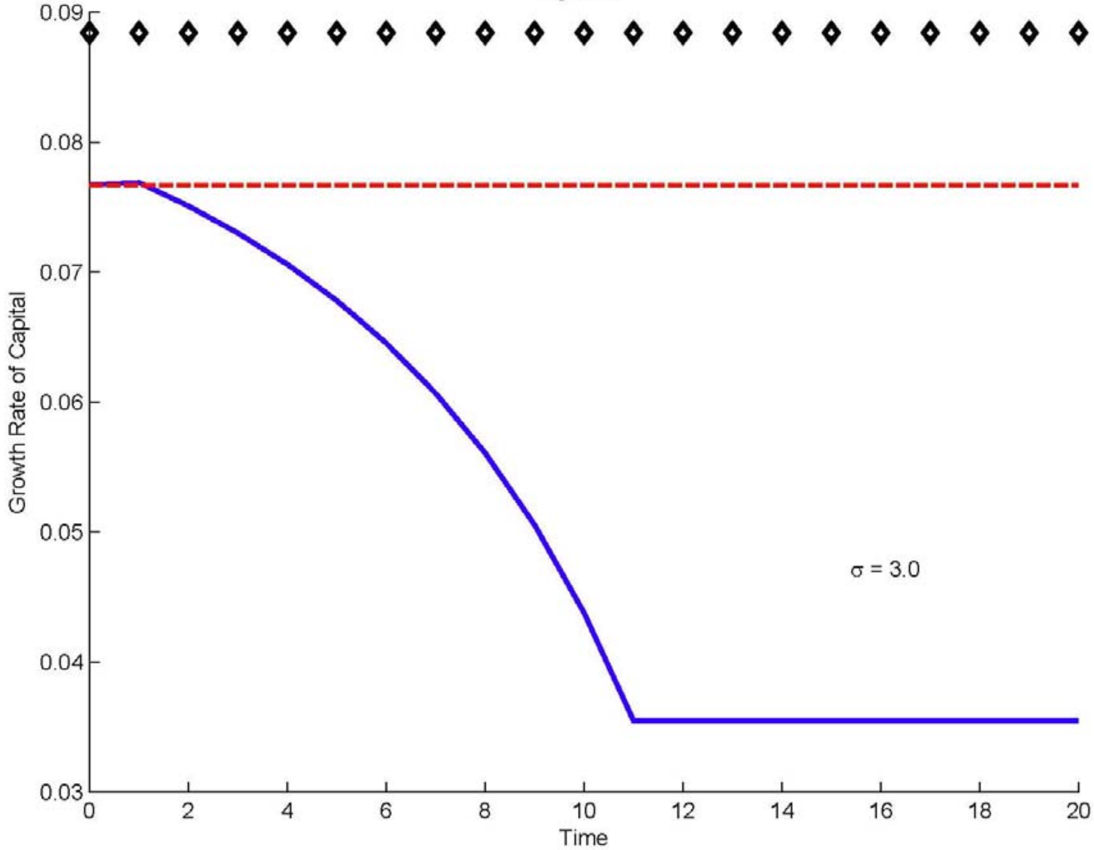

Figure 14