

# Sector Biased Technical Change, Perpetual and Transient Structural Change

Pengfei Zhang\*

Peking University

**Abstract:** We contrast a two-sector growth model, which could not only generate sector unbiased technical change but also could generate sector biased technical change. Economic growth must be non-balanced at the sectoral level and perpetual structural change will take place when technical change is sector biased. However, when technical change is sector unbiased, economic growth must be balanced at the sectoral level and only transient structural change could occur between the two sectors. A model that features sector biased change and perpetual structural change could be more empirically based than that features sector unbiased technical change and transient structural change.

**Keywords:** aggregate balanced growth, non-balanced sectoral growth, perpetual structural change, sector biased technology, transient structural change.

**JEL classification:** O30, O33, O40, O41.

---

\* *Address for correspondence:* School of Economics, Peking University, Beijing, 100871, P. R. China. *Tel:* 86-10-6275-2016. *E-mail:* pku.pengfei@gmail.com. Precious helps and insight comments from Daron Acemoglu, Yi Chen, Takashi Kamihigashi, Justin Yifu Lin, Rachel L. Ngai, Danyang Xie, Xiaodong Zhu, seminar participants at 2012 Econometric Society Australasian Meeting, 2012 Tsinghua Workshop in Macroeconomics, 2014 North America Winter Meeting of the Econometric Society, Conference on New Structural Economics, Kobe University, and Peking University are much appreciated. Financial support from National Social Science Foundation of China (09AZD013), National Natural Science Foundation of China (71073033, 71273012, and 71350002), the Ministry of Education of the People's Republic of China (12JZD036), and Beijing Planning Office of Philosophy and Social Science (12JGB021) is gratefully acknowledged. Of course, the usual disclaimer applies.

### 1. Introduction

The well-known Kuznets facts (Kuznets, 1957 and 1973) concern the structural change in the process of economic growth. Structural change refers to the shifts in industrial employment shares taking place *over long periods of time* (Ngai and Pissarides, 2007, pp. 429, italic added). Recent theories stress two distinct economic mechanisms that can explain structural change: one emphasizes the demand-side reasons and uses non-homothetic preferences to introduce income-elasticity differentials across sectors as the main driving force behind structural change (e.g. Kongsamut, Rebelo, and Xie, 2001; Foellmi and Zweimüller, 2008; Buera and Kaboski, 2012b), whereas the other focuses on supply-side reasons and attribute structural change to productivity differentials across sectors (e.g. Baumol, 1967; Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008; Zhang and Liu, 2009; Buera and Kaboski, 2012a<sup>1</sup>).<sup>2</sup> However, there remains no consensus on the economic forces that drive the process of structural transformation (Herrendorf, Rogerson, and Valentinyi, forthcoming).

In order to understand the driving forces behind structural change, we will categorize it into transient structural change (e.g. structural change in Kongsamut, Rebelo, and Xie, 2001) and perpetual structural change (e.g. structural change in Ngai and Pissarides, 2007; and Acemoglu and Guerrieri, 2008). To distinguish two types of structural changes, we need to give a formal definition. Structural change is defined as the state in which the percent change rate of the

---

<sup>1</sup> Even though non-homothetic preferences of Matsuyama (2002) is adopted, Buera and Kaboski (2012a, pp. 685) still claims that the central contribution of their paper is to emphasize that scale technologies are the driving force of structural change.

<sup>2</sup> These two cases roughly correspond to what Herrendorf, Rogerson, and Valentinyi (forthcoming) calls “changes in aggregate income” and “changes in relative sectoral prices” explanations. For the references, see Matsuyama (2008) and Herrendorf, Rogerson, and Valentinyi (2013).

## Sector Biased Technical Change and Structural Change

fraction of labor used in at least one sector is not equal to zero.<sup>3</sup> In the case of perpetual structural change, the reallocation of labor between the two sectors will take place without end, and thus the percent change rate of the fraction of labor used in one of the two sectors will never converge to zero. Conversely, in the case of transient structural change, the reallocation of labor between the two sectors could only take place in the short-run, and the percent change rates of the fraction of labor used in both sectors will eventually converge to zero. In other words, perpetual structural change refers to structural change taking place in the long-run (in the steady state),<sup>4</sup> whereas transient structural change corresponds to structural change taking place in the short-run (during transitional dynamics).

By constructing a two-sector growth model with a constant elasticity of substitution (CES) aggregate production function, we find that structural change could take place if and only if economic growth takes place at an uneven rate between the two sectors. And thus, economic growth must be non-balanced at the sectoral level in the case of perpetual structural change. However, both factors, i.e., capital and labor, will eventually be allocated to the two sectors in constant proportions, when economic growth is balanced at the sectoral level. In other words, the structural change that occurs between the two sectors must be transient in the case of balanced sectoral growth. It is for this reason that we argue that the fundamental driving force of perpetual structural change is the same as that of non-balanced sectoral growth.

To identify the fundamental driving force of non-balanced sectoral growth, it is worthwhile to note that only purely labor augmenting (Harrod neutral) technical change is consistent with an

---

<sup>3</sup> The fraction of labor used in one sector in this paper refers to the labor share in the same sector in Ngai and Pissarides (2007, pp. 431). Unfortunately, the labor share is usually defined as the share of value added which is paid out to workers.

<sup>4</sup> The equilibrium steady state is synonymous with aggregate balanced growth path and is defined to be a situation that all quantities grow at constant (maybe zero) rates.

## Sector Biased Technical Change and Structural Change

equilibrium steady state (Uzawa, 1961; Jones and Scrimgeour, 2008; and others).<sup>5</sup> And a sector with a higher purely labor augmenting technical change rate will certainly experience a faster real output growth.<sup>6</sup> Therefore, when purely labor augmenting technical change takes place at an even rate between the two sectors, economic growth must be balanced at the sectoral level. In contrast, economic growth must be non-balanced at the sectoral level if and only if there is sectoral difference in purely labor augmenting technical change rates.

In the Cobb-Douglas production function, the purely labor augmenting technical change rate in one sector is equivalent to the Hicks-neutral technical change rate divided by the labor intensity of production in the same sector. Therefore, non-balanced sectoral growth is inevitable and perpetual structural change must take place, when production functions are identical in all sectors except for their total factor productivity (TFP) growth rates.<sup>7</sup> This conclusion is consistent with that in Ngai and Pissarides (2007).<sup>8</sup> However, the present paper also show that different sectoral TFP growth rates per se could not generate non-balanced sectoral growth and perpetual structural change, so long as the sectoral difference in labor intensities are large enough to guarantee that the purely labor augmenting technical change rates are equal among sectors. Our result is also in contrast with the conclusion in Acemoglu and Guerrieri (2008) in that differences in factor proportions (intensities) combined with capital deepening could not

---

<sup>5</sup> Another way to think of labor-augmenting technical change is as a rescaling of the measure of the labor input: each worker after the technical change functions as if her efforts were magnified by a factor representing the size of change (Foley and Michl, 1999, pp. 60). And thus human capital accumulation in Lucas (1988) could be thought of as labor augmenting technical change.

<sup>6</sup> The Rybczynski theorem runs into a serious problem in the two-sector growth model. To see this, assume that the labor intensive sector enjoys faster purely labor augmenting technical change. Then the growth rate of output will be higher in the labor intensive sector than in the capital intensive sector. In this case, capital deepening could not increase the relative output of the more capital-intensive sector.

<sup>7</sup> The TFP growth rate is referred to as Hicks-neutral technical change rate.

<sup>8</sup> Ngai and Pissarides (2007) derives the conditions for structural change and aggregate balanced growth by assuming Cobb-Douglas production functions are identical in all sectors except for their exogenous rates of TFP growth.

## Sector Biased Technical Change and Structural Change

lead to non-balanced sectoral growth and perpetual structural change. As a matter of fact, it is the sectoral difference in purely labor augmenting technical change rates, i.e.,  $m_1/\alpha_1 \neq m_2/\alpha_2$  in assumption 2 in Acemoglu and Guerrieri (2008, pp. 479),<sup>9</sup> that leads to non-balanced sectoral growth and perpetual structural change. However, if we assume  $m_1/\alpha_1 = m_2/\alpha_2$  instead of  $m_1/\alpha_1 \neq m_2/\alpha_2$ , economic growth must be balanced at the sectoral level in Acemoglu and Guerrieri (2008).

We define sector unbiased technical change as the state of purely labor augmenting technical change taking place at the same rate between the two sectors. And technical change is defined to be biased to a particular sector whose purely labor augmenting technical change rate is higher than that in the other sector. With the help of the above definitions, we argue that sector biased technical change is the only fundamental driving force of non-balanced sectoral growth and perpetual structural change. And what is more, when technical change is sector unbiased, economic growth must be balanced at the sectoral level and structural change that takes place between the two sectors must be transient.

It is worth pointing out that the model in the present paper does not really consider non-homothetic preferences. Nevertheless, we will show that the main conclusion of this paper is undoubtedly correct even when non-homothetic preferences are allowed in Kongsamut, Rebelo, and Xie (2001); Foellmi and Zweimüller (2008); and Buera and Kaboski (2012a and 2012b). Now let us present an intuitive explanation. The reallocation of labor between the two sectors could take place if and only if its marginal return is not equal between the two sectors. It is sector biased technical change, not anything else, that could *ceteris paribus* bring about permanent

---

<sup>9</sup> In Acemoglu and Guerrieri (2008),  $m_i$ ,  $\alpha_i$ , and  $m_i/\alpha_i$  are respectively the Hicks-neutral technical change rate, the labor intensity of production, and the purely labor augmenting technical change rate in sector  $i$ .

## Sector Biased Technical Change and Structural Change

sectoral difference in the marginal returns on labor. And thus, sector biased technical change requires a permanent reallocation of labor between the two sectors, so that the marginal returns on each factor could always be equalized in the two sectors. However, when technical change is sector unbiased, the marginal return on labor in one sector will eventually increase at a same rate as that in the other sector, and thus labor must be allocated to the two sectors in constant proportions in the long-run, regardless whether non-homothetic preferences are allowed.

The present model not only could generate endogenous sector biased technical change but also could generate endogenous sector unbiased technical change. The empirical finding in Duarte and Restuccia (2010) shows that most countries observe higher growth rates of labor productivity in agriculture and manufacturing than in services (Duarte and Restuccia, 2010). Even though we are not partial to the model that features sector biased technical change, we argue that a model which allows and considers sector biased change could be more empirically based than that could only generate sector unbiased technical change. When endogenous technical change is sector biased, our model will degenerate to that in Acemolgu and Guerrieri (2008). The numerical example in Acemolgu and Guerrieri (2008, pp. 489) implies that the economy takes a very long time, over 5,000 years, to reach the asymptotic equilibrium. By contrast, when technical change is sector unbiased, and thus transient structural change could only take place during transitional dynamics, the numerical example in Kongsamut, Rebelo, and Xie (2001, Figure 3) shows that almost all of transient structural changes take place in the first 100 years. Therefore, we think that a theory that features perpetual structural change could be more convincing than that features transient structural change, if structural change in the real world could date back at least to the industrial revolution and will not cease in the foreseeable future.

## Sector Biased Technical Change and Structural Change

Despite the generally agreed upon the importance of “productivity-based” theory (Matsuyama, 2009, pp. 482, footnote 6) for structural change, the existing literatures investigate biased technical change and non-balanced growth independently and mutually exclusively. On the one hand, the existing literatures on non-balanced sectoral growth and structural change rely heavily on the exogenous productivity growth differentials across sectors. On the other hand, almost all existing literatures on biased technical change focus on factor instead of sector biased technical change (e.g. Hicks, 1932; Kennedy, 1964; Acemoglu, 1998 and 2002; as well as Caselli and Coleman, 2006; et al). The two exceptions are Ngai and Samaniego (2011) and Cai and Li (2012). However, economic growth must be balanced at the sectoral level, and thus perpetual structural change is neglected in these two papers. The other contribution of this paper is to reconcile the literatures on structural change and the literatures on biased technical change. As far as we know, our paper is the first theoretical literature investigating the determinants of sector biased technical change based on a non-balanced sectoral growth model.

The rest of the paper is organized as follows. Section 2 describes the model setup and defines structural change. Section 3 investigates the driving force of perpetual and transient structural change. And in section 3, we also discuss the empirical relevance of sector biased technical change and perpetual structural change. Section 4 investigates the mechanism and determinants of endogenous sector biased and sector unbiased technical change. Finally, section 5 concludes the paper. To make more efficient use of pages, I place the dynamic equilibrium paths in the appendix. And in the appendix, I also show that the sustainable economic growth and the standard transversality condition (STVC, Kamihigashi, 2001) implies that the Jacobian has one positive real root and three negative real roots in the case of sector biased technical change. Therefore, the equilibrium dynamic system in the case of sector biased technical change

## Sector Biased Technical Change and Structural Change

will be locally indeterminate.

### 2. The model setup

We consider an economy with  $L(t)$  persons/workers at time  $t$ , supplying their labor without any disutility. The population size grows at a constant exponential growth rate  $\nu > 0$ , and the population size at time 0 is normalized to unity. The representative household's preferences over per-capita consumption streams at time  $t$ ,  $c(t)$ , is given by

$$V = \int_0^{\infty} e^{-(\rho-\nu)t} \frac{c(t)^{1-\sigma} - 1}{1-\sigma} dt \quad (1)$$

where  $c(t) = C(t)/L(t)$  and  $C(t)$  is total consumption at time  $t$ ,  $\rho > 0$  is a subjective discount rate, and  $\sigma \geq 0$  is the inverse of the intertemporal elasticity of substitution.<sup>10</sup>

There are two intermediate good sectors in the present economy. Both intermediate goods,  $Y_1$  and  $Y_2$ , are needed to produce final goods,  $Y$ , according to the following CES production function:

$$Y = \left[ \sum_{i=1}^2 \gamma_i (Y_i)^{(\varepsilon-1)/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)} \quad (2)$$

where  $\varepsilon \in (0, \infty)$ ,  $\varepsilon \neq 1$  is the elasticity of substitution between two intermediate goods,<sup>11</sup> and  $\gamma_i \in (0, 1)$  determines the relative importance of intermediate good  $i$  in the production of final goods. The final goods can be consumed, added to the existing capital stock, or devoted to the inventive activity (R&D) in the two intermediate good sectors, which implies that  $Y = C + \dot{K} + X_1 + X_2$ , where  $\dot{K} \geq 0$  is capital investment and  $X_i \geq 0$  is R&D expenditure in units of final goods in sector  $i$ .

<sup>10</sup> We omit time arguments to simplify the notations whenever this causes no confusion from now on.

<sup>11</sup> Standard Cobb-Douglas production function is adopted in the final good sector if  $\varepsilon = 1$ . And thus economic growth must be balanced at the sectoral level no matter whether technical change is sector biased or not.

## Sector Biased Technical Change and Structural Change

There are two primary factors of production, capital  $K$  and labor  $L$ , which are combined to produce intermediate goods according to Cobb-Douglas production function  $Y_i = A_i(K_i)^{\alpha_i}(L_i)^{1-\alpha_i}$ , where  $\alpha_i \in (0,1)$ ,  $A_i > 0$ ,  $K_i \geq 0$ , and  $L_i \geq 0$  are respectively the capital intensity, the state of art technology, capital and labor used in sector  $i$ . For an allocation of factors to be feasible at any time, it requires

$$K_i = u_i K, \quad L_i = n_i L, \quad u_1 + u_2 = 1, \quad \text{and} \quad n_1 + n_2 = 1 \quad (3)$$

where  $u_i \in [0,1]$  and  $n_i \in [0,1]$  are the fractions of capital and labor used in sector  $i$  respectively.

Following Shell (1966), we assume that the technological progress in sector  $i$  is fundamentally related to the amount of resources explicitly devoted to inventive activity in that sector. The deterministic relationship between the Hicks-neutral technical change rate in sector  $i$ ,  $\dot{A}_i/A_i$ , and R&D expenditures in that sector,  $X_i$ , is posited:

$$\dot{A}_i/A_i = A_i^{\varphi_i - 1} X_i \quad (4)$$

where  $\varphi_i \in (-\infty, 1)$  measures the degree of spillover effects from the current stock of technology in sector  $i$  to future technology invention within the same sector.<sup>12</sup>

We will solve the social planner's planning problem of maximizing the representative household's utility in equation (1) subject to the budget constraint, i.e.,  $Y = C + \dot{K} + X_1 + X_2$ , factor allocations in equation (3) and the evolution of technologies in equation (4).<sup>13</sup> After

---

<sup>12</sup> In the productivity literature,  $\varphi_i \in (-\infty, 1)$  corresponds to the case referred to as "fishing out," in which the rate of innovation decreases with the level of technology (Jones, 1995). Thus, the present model is a semi-endogenous growth model because a nonzero long-term growth rate of per capita consumption streams depends on a nonzero growth rate of population, i.e.,  $\nu > 0$ .

<sup>13</sup> The main conclusions of the present paper still apply in the competitive equilibrium setup in Zhang and Liu (2009). Unfortunately, the model in Zhang and Liu (2009) is much more complicated than that in the present paper, and as a result we could not successfully investigate the stability of the dynamic system in the competitive equilibrium setup.

## Sector Biased Technical Change and Structural Change

performing some simple arithmetic manipulations, the social planner's planning problem could be written as

$$\max_{C \geq 0, 1 \geq u_i \geq 0, 1 \geq n_i \geq 0, X_i \geq 0} \int_0^{\infty} \frac{C^{1-\sigma} e^{-(\rho-v\sigma)t}}{1-\sigma} dt$$

subject to

$$\dot{A}_1 = A_1^{\theta_1} X_1, \quad \dot{A}_2 = A_2^{\theta_2} X_2, \quad u_1 + u_2 = 1 \quad \text{and} \quad n_1 + n_2 = 1$$

$$C + \dot{K} + X_1 + X_2 = \left\{ \sum_{i=2}^2 \gamma_i \left[ A_i (u_i K)^{\alpha_i} (n_i L)^{1-\alpha_i} \right]^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

as well as the initial conditions  $K(0) > 0$ ,  $L(0) = 1$ .

The Hamiltonian is written as

$$H = \frac{C^{1-\sigma} e^{-(\rho-v\sigma)t}}{1-\sigma} + \lambda \left\{ \left[ \sum_{i=2}^2 \gamma_i \left( A_i (u_i K)^{\alpha_i} (n_i L)^{1-\alpha_i} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} - C - \dot{K} - X_1 - X_2 \right\} + \sum_{i=1}^2 \theta_i A_i^{\theta_i} X_i$$

and the STVC is given by  $\lim_{t \rightarrow \infty} (\lambda K + \theta_1 A_1 + \theta_2 A_2) = 0$ .

The derivatives of Hamiltonian with respect to the fractions of capital and labor used in the two sectors require

$$\frac{\alpha_2 u_1}{\alpha_1 u_2} = \frac{\gamma_1 (Y_1)^{(\varepsilon-1)/\varepsilon}}{\gamma_2 (Y_2)^{(\varepsilon-1)/\varepsilon}} = \frac{(1-\alpha_2) n_1}{(1-\alpha_1) n_2} \quad (5)$$

To obtain the intuition behind equation (5), note that the marginal returns on capital and labor used in sector  $i$  are respectively given by  $\partial Y / \partial K_i = Y^{1/\varepsilon} \gamma_i (Y_i)^{(\varepsilon-1)/\varepsilon} \alpha_i (K_i)^{-1}$  and  $\partial Y / \partial L_i = Y^{1/\varepsilon} \gamma_i (Y_i)^{(\varepsilon-1)/\varepsilon} (1-\alpha_i) (L_i)^{-1}$ . Thus, equation (5) just states that the marginal return on each factor used in sector 1 must be equal to that in sector 2 at any time.

Substituting  $u_1 + u_2 = 1$  and  $n_1 + n_2 = 1$  into equation (5) obtains

## Sector Biased Technical Change and Structural Change

$$\dot{n}_i/n_i = (1 - \alpha_{-i})\alpha_i[\alpha_i(1 - \alpha_{-i}) + (\alpha_{-i} - \alpha_i)u_i]^{-1} \dot{u}_i/u_i \quad (6)$$

Equation (6) implies that capital and labor would move in the same direction in the present model if they could. Thus, we could define structural change as follows.

**Definition 1.** Structural change is defined as the state in which the **percent change rate** of the fraction of labor used in at least one sector is not equal to zero, i.e.,  $\dot{n}_i/n_i \neq 0$  for at least some  $i$ .

It is obvious that the above definition of structural change is different from that in Ngai and Pissarides (2007, pp. 431), which defines structural change as the state in which at least some of the labor shares are changing over time, i.e.,  $\dot{n}_i \neq 0$  for at least some  $i$ . Unfortunately, when the limit cycles are ruled out in the multi-sector growth model, the fraction of labor used in either sector must be a constant in the long-run, which implies that we must have  $\lim_{t \rightarrow \infty} \dot{n}_i = 0$  for all  $i$  owing to  $\lim_{t \rightarrow \infty} n_i \in [0, 1]$ . Therefore, if we adopt the definition of structural change in Ngai and Pissarides (2007), any type of structural change could only take place in the short-run but not in the long-run. However, according to the definition in the present paper, structural change is still allowed to take place in the long-run, which implies that we must have  $\lim_{t \rightarrow \infty} \dot{n}_i/n_i < 0$  for at least some  $i$ . And  $\lim_{t \rightarrow \infty} \dot{n}_i/n_i < 0$  is consistent with  $\lim_{t \rightarrow \infty} \dot{n}_i = 0$  because  $\dot{n}_i$  could be an infinitesimal of the same order as  $n_i$  in the long-run. To distinguish structural change in the short-run from that in the long-run, we give the following definitions.

**Definition 2.** (a) Perpetual structural change is defined to be the state in which the percent change rate of the fraction of labor used in at least one sector is not equal to zero in the long-run, i.e.,  $\lim_{t \rightarrow \infty} \dot{n}_i/n_i \neq 0$  for some  $i$ . (b) Transient structural change is defined to be the state in which the percent change rate of the fraction of labor used in the two sectors are not equal to zero in the

## Sector Biased Technical Change and Structural Change

short-run, but both of them will eventually converge to zero in the long-run, i.e.,  $\lim_{t \rightarrow \infty} \dot{n}_1/n_1 = 0$  and  $\lim_{t \rightarrow \infty} \dot{n}_2/n_2 = 0$ .

### 3. The driving force of perpetual and transient structural change

#### 3.1 The supply-side reasons

Simple arithmetic manipulation in equation (5) obtains

$$\dot{u}_1/u_1 - \dot{u}_2/u_2 = (\varepsilon - 1)(\dot{Y}_1/Y_1 - \dot{Y}_2/Y_2)/\varepsilon = \dot{n}_1/n_1 - \dot{n}_2/n_2 \quad (7)$$

Equations (5) and (7) imply that both  $\dot{u}_i/u_i = 0$  and  $\dot{n}_i/n_i = 0$  for  $i = 1, 2$  if and only if  $\dot{Y}_1/Y_1 = \dot{Y}_2/Y_2$  owing to  $\dot{u}_1 + \dot{u}_2 = 0$  and  $\dot{n}_1 + \dot{n}_2 = 0$ . Therefore, we could establish the following lemma.

**Lemma 1.** (a) In the present model, structural change, i.e.,  $\dot{u}_i/u_i \neq 0$  for at least some  $i$ , will take place if and only if economic growth takes place at an uneven rate between the two sectors, i.e.,  $\dot{Y}_1/Y_1 \neq \dot{Y}_2/Y_2$ . More explicitly, when the elasticity of substitution is less (greater) than unity, both factors would move in the direction of the slower-growing (faster-growing) sector. (b) However, when economic growth takes place at an even rate between the two sectors, i.e.,  $\dot{Y}_1/Y_1 = \dot{Y}_2/Y_2$ , structural change could not take place anymore, i.e.,  $\dot{u}_i/u_i = 0$  and  $\dot{n}_i/n_i = 0$  for  $i = 1, 2$ .

Differentiating  $Y_i = A_i(u_i K)^{\alpha_i} (n_i L)^{1-\alpha_i}$  with respect to time  $t$  yields

$$\dot{Y}_i/Y_i = \dot{A}_i/A_i + \alpha_i \dot{K}/K + \alpha_i \dot{u}_i/u_i + (1-\alpha_i) \dot{n}_i/n_i + (1-\alpha_i) \nu \quad (8)$$

**Proposition 1.** Economic growth must be balanced at the sectoral level, i.e.,  $\lim_{t \rightarrow \infty} \dot{Y}_1/Y_1 = \lim_{t \rightarrow \infty} \dot{Y}_2/Y_2$ , and structural change could only take place during transitional dynamics,

## Sector Biased Technical Change and Structural Change

i.e.,  $\lim_{t \rightarrow \infty} \dot{u}_i/u_i = 0$  and  $\lim_{t \rightarrow \infty} \dot{n}_i/n_i = 0$  for  $i=1,2$ , if and only if the purely labor augmenting technical change rates are equal in the two sectors, i.e.,  $(1-\alpha_1)^{-1} \dot{A}_1/A_1 = (1-\alpha_2)^{-1} \dot{A}_2/A_2$ .

**Proof.** When economic growth is balanced at the sectoral level, i.e.,  $\lim_{t \rightarrow \infty} \dot{Y}_1/Y_1 = \lim_{t \rightarrow \infty} \dot{Y}_2/Y_2$ , equations (7) and (8) imply that  $(1-\alpha_1)^{-1} \dot{A}_1/A_1 = (1-\alpha_2)^{-1} \dot{A}_2/A_2$  owing to  $\lim_{t \rightarrow \infty} \dot{Y}/Y = \lim_{t \rightarrow \infty} \dot{K}/K$ .<sup>14</sup> And we will use proof by contradiction to show that not only economic growth must be balanced at the sectoral level but also there is no perpetual structural change, when the purely labor augmenting technical change rates are equal in the two sectors. To do this, we will assume  $\lim_{t \rightarrow \infty} \dot{Y}_i/Y_i > \lim_{t \rightarrow \infty} \dot{Y}_{-i}/Y_{-i}$  without loss of generality.

When the elasticity of substitution is less than unity, i.e.,  $\varepsilon \in (0,1)$ , faster growth in sector  $i$  implies that  $\lim_{t \rightarrow \infty} \dot{u}_i/u_i < 0$ ,  $\lim_{t \rightarrow \infty} \dot{u}_{-i}/u_{-i} = 0$ ,  $\lim_{t \rightarrow \infty} \dot{n}_i/n_i < 0$ , and  $\lim_{t \rightarrow \infty} \dot{n}_{-i}/n_{-i} = 0$  from lemma 1. Moreover,  $\varepsilon \in (0,1)$  implies that the slower growing sector  $-i$  will determine the growth rate of the aggregate output in the steady state, i.e.,  $\lim_{t \rightarrow \infty} \dot{Y}/Y = \lim_{t \rightarrow \infty} \dot{Y}_{-i}/Y_{-i}$ . Thus, equation (8) requires that  $\lim_{t \rightarrow \infty} \dot{Y}/Y - \nu < (1-\alpha_i)^{-1} \dot{A}_i/A_i$  and  $(1-\alpha_{-i})^{-1} \dot{A}_{-i}/A_{-i} < \lim_{t \rightarrow \infty} \dot{Y}/Y - \nu$ , which is inconsistent with the assumption of  $(1-\alpha_i)^{-1} \dot{A}_i/A_i = (1-\alpha_{-i})^{-1} \dot{A}_{-i}/A_{-i}$ . By the same logic, when the elasticity of substitution is greater than unity, i.e.,  $\varepsilon > 1$ , the necessity of the condition in proposition 1 is also obvious.  $\square$

It is worth noting that proposition 1 in our paper involves dynamic analysis, while the conclusions in proposition 1 of Acemoglu and Guerrieri (2008, pp. 475) are just comparative

---

<sup>14</sup> One of the stylized facts listed in Kaldor (1961) is that the ratio of physical capital to output is nearly constant in the process of economic growth.

## Sector Biased Technical Change and Structural Change

statics, the latter of which states that the fraction of capital allocated to the labor-intensive sector increases with the stock of capital if  $\varepsilon < 1$  and decreases if  $\varepsilon > 1$ . Proposition 1 in Acemoglu and Guerrieri (2008) is certainly correct. Nevertheless, to derive this comparative statics conclusion, Acemoglu and Guerrieri (2008) must assume that technological levels in both sectors are fixed. Unfortunately, capital deepening could not take place in the long-run without technical change, which implies that the combination of factor proportion (intensity) differences across sectors and capital deepening alone could not successfully lead to non-balanced sectoral growth and perpetual structural change in Acemoglu and Guerrieri (2008).

The intuition for proposition 1 in this paper is straightforward. In the long run, the growth rate of sector  $i$ 's real output per capita, i.e.,  $\lim_{t \rightarrow \infty} \dot{Y}_i / Y_i - \nu$ , will be equal to the purely labor augmenting technical change rate in this sector, if we leave out structural change temporarily. Thus, if both factors were allocated to the two sectors in constant proportions, the higher the purely labor augmenting technical change rate in one sector, the greater the real output growth rate in this sector. And the uneven growth between the two sectors will induce factors to flow away from one sector to the other sector so that the marginal returns on both factors could always be equalized in the two sectors. In other words, when there is sectoral difference in purely labor augmenting technical change rates, the reallocation of factors between the two sectors is necessary and inevitable.<sup>15</sup> In contrast, when the purely labor augmenting technical change rates are equal in the two sectors, economic growth must be balanced at the sectoral level, and structural change could not take place between the two sectors in the long-run.

---

<sup>15</sup> Even though capital and labor inflows will help to accelerate output growth in the recipient sector, which is a slower growing sector if the elasticity of substitution is less than unity, the effect of sectoral difference in the rates of purely labor augmenting technical change on the sectoral difference in the real output growth rates could not be completely offset by the reallocation of factors between the two sectors.

## Sector Biased Technical Change and Structural Change

**Definition 3.** Sector unbiased technical change between sector  $i$  and sector  $-i$  is defined as the state in which the purely labor augmenting technical change takes place at an equal rate between these two sectors, i.e.,  $(1-\alpha_i)^{-1} \dot{A}_i/A_i = (1-\alpha_{-i})^{-1} \dot{A}_{-i}/A_{-i}$ . On the contrary, when the purely labor augmenting technical change rate in sector  $i$  is greater than that in sector  $-i$ , i.e.,  $(1-\alpha_i)^{-1} \dot{A}_i/A_i > (1-\alpha_{-i})^{-1} \dot{A}_{-i}/A_{-i}$ , technical change is defined to be biased to sector  $i$ .

With the help of the above definitions, if we focus on supply-side reasons, we find that the fundamental driving force of perpetual structural change and non-balanced sectoral growth comes only from sector biased technical change. And in the following subsection, we will show that this conclusion is still correct when we focus on demand-side reasons.

### 3.2 *The demand-side reasons*

According to Foellmi and Zweimüller (2008), the first paper that has explicitly addressed the issue of how to reconcile the huge structural changes with the Kaldor facts of economic growth is Kongsamut, Rebelo, and Xie (2001).<sup>16</sup> They study a three-sector model where consumers have Stone-Geary specification of non-homothetic preferences over an agricultural good,  $A$ , a manufactured good,  $M$ , and services,  $S$ . To highlight the demand-channel and to keep things as simple as possible, Kongsamut, Rebelo, and Xie (2001) assumes that the production functions in the three sectors are identical and argues that sectoral movement must originate from differences in the income elasticity of demand for the different goods. The identical Cobb-Douglas production function in the three sectors in Kongsamut, Rebelo, and Xie (2001) implies that technical change must be sector unbiased, and thus economic growth must be balanced in the sectoral level and structural change could only take place during transitional

---

<sup>16</sup> The purely labor augmenting technical change rate in the agricultural sector must be assumed to be less than that in the manufacturing sector in Laitner (2000, pp. 549, equations (4)-(7)), otherwise the consumer whose preferences manifest Engel's law in Laitner (2000, pp. 548, equations (2)) must be irrational.

## Sector Biased Technical Change and Structural Change

dynamics.<sup>17</sup> As a matter of fact, the real output growth rate in the long-run in the agricultural sector, the manufacturing sector, and the services sector must be equal to the purely labor augmenting technical change rate in Kongsamut, Rebelo, and Xie (2001). Thus, it is obvious that sectoral differences in income elasticities per se could not bring about perpetual structural change and non-balanced sectoral growth in Kongsamut, Rebelo, and Xie (2001).

Foellmi and Zweimüller (2008) considers a representative agent economy with infinitely many potentially producible goods and services ranked by an index  $i$ . Figure 1 in Foellmi and Zweimüller (2008) implies that the path of the demand for some good  $i$  will approach the saturation level  $s$  in the long run. In other words, in Foellmi and Zweimüller (2008), the economic growth rates of all existing goods must be zero in the long run.<sup>18</sup> This is consistent with our main conclusion that sector biased technical change is the fundamental driving force of non-balanced sectoral growth because technical change is indeed sector unbiased in Foellmi and Zweimüller (2008).<sup>19</sup> Foellmi and Zweimüller (2008, Proposition 1) claims that structural change occurs because income elasticities of demand are different across sectors. Because our model just considers two sectors, we must be more cautious and precise in defining and explaining perpetual and transient structural change in Foellmi and Zweimüller (2008). In the long run, the labor employed to produce all existing goods must converge to zero, which implies that there is no reallocation of labor among all existing sectors in Foellmi and Zweimüller (2008). In other

---

<sup>17</sup> In Kongsamut, Rebelo, and Xie (2001, pp. 874)'s notation, we must have  $\lim_{t \rightarrow \infty} \dot{N}_t^A / N_t^A = 0$ ,  $\lim_{t \rightarrow \infty} \dot{N}_t^M / N_t^M = 0$ , and  $\lim_{t \rightarrow \infty} \dot{N}_t^S / N_t^S = 0$ , in which  $N^i$  denotes the fraction of labor devoted to sector  $i = A, M, S$ .

<sup>18</sup> This conclusion is consistent with "Consumption of good  $i$  increases over time but at a decreasing rate and approaches the saturation level as the relative position in the consumption hierarchy approaches 0." in Foellmi and Zweimüller (2008, pp. 1322).

<sup>19</sup> The implicit assumption that one unit of output good should be used to produce one unit of consumption good for  $i \in [0, \infty)$  is made in Foellmi and Zweimüller (2008).

## Sector Biased Technical Change and Structural Change

words, perpetual structural change cannot occur across all existing sectors in Foellmi and Zweimüller (2008), which is consistent with our conclusion.

Nevertheless, there are still reallocations of labor from the existing sectors to the latest introduced sector in the long-run in Foellmi and Zweimüller (2008). In order to identify the fundamental driving force of structural change between the existing sectors and the latest introduced sector in Foellmi and Zweimüller (2008), we need to give a reasonable explanation why new goods could be continuously introduced and consumed therein. It is obvious that new goods could be continuously introduced and consumed in Foellmi and Zweimüller (2008), i.e.,  $\lim_{t \rightarrow \infty} \dot{N}/N > 0$ , just because the positive purely labor augmenting technical change rate, i.e.,  $\dot{A}/A > 0$ . However, if there is no labor augmenting technical change, i.e.,  $\dot{A}/A = 0$ , new goods could not be continuously introduced and consumed in Foellmi and Zweimüller (2008), and thus there is no structural change of any type in the long-run. This conclusion must hold not only in the neoclassical growth mode but also in the R&D-based growth model in Foellmi and Zweimüller (2008).<sup>20</sup>

There are three sectors, i.e., traditional agricultural sector, modern service (modern market service and modern home service) sector and manufacturing sector in Buera and Kaboski (2012a). The traditional agricultural sector is stagnant in the sense that there is no labor augmenting technical change in this sector.<sup>21</sup> However, both the modern service sector and the manufacturing sector experience identical labor augmenting technical change at an exogenous rate  $\gamma$ . According to the conclusion of our paper, technical change is sector unbiased between

---

<sup>20</sup> In the hierarchical model instead there has to be technical progress otherwise innovations come to a halt because consumers are not willing to reduce consumption on high-priority goods if new goods come along (Foellmi and Zweimüller, 2008, pp. 1323, footnote 13).

<sup>21</sup> Unfortunately, in the United States, over the century farm productivity, on average, grew faster than nonfarm productivity (Caselli and Coleman, 2001, pp. 594).

## Sector Biased Technical Change and Structural Change

the modern service sector and the manufacturing sector, and thus only transient structural change could take place between these two sectors.<sup>22</sup> Nevertheless, technical change is sector biased between the agricultural sector and the non-agricultural sector, and as a result agriculture declines continually in terms of both value-added and consumption.

Based on the non-homothetic preferences of Matsuyama (2002), Buera and Kaboski (2012b) provides a theoretical framework for understanding the connection between skill accumulation and the growth of the service sector, and argues that the growth in services is driven by the movement of consumption into more skill-intensive output. Buera and Kaboski (2012b, pp. 2553, line 1-2 from the bottom) assumes that the neutral labor-augmenting productivity parameter  $A > 0$  is common across technologies in the goods sector and the service sector. According to the conclusion in our paper, technical change is sector unbiased between the goods sector and the service sector, and thus perpetual structural change could not take place between these two sectors in Buera and Kaboski (2012b).<sup>23</sup>

As a matter of fact, without income changes, non-homothetic preferences per se could not successfully lead to structural change. And it is purely labor augmenting technical change that bring about perpetual income changes, but not vice versa. Therefore, income changes combined with sectoral difference in income elasticities could not successfully generate perpetual structural change and non-balanced sectoral growth.

**Theorem 1.** The fundamental driving force of perpetual structural change and non-balanced sectoral growth comes only from the sector biased technical change.

It is worthy noting that theorem 1 still holds in the open economy in Matsuyama (2009) as

---

<sup>22</sup> This conclusion is consistent with “Asymptotically, the model converges to a constant share of manufacturing and services, in terms of both value-added and consumption.” (Buera and Kaboski, 2012a, pp. 700).

<sup>23</sup> This conclusion is consistent with “In the limit, as productivity increases, the share of services in value-added is bounded below one.” (Buera and Kaboski, 2012b, pp. 2542).

well as Uy, Yi and Zhang (2013).

### ***3.3 Discussion and Implication***

According to Duarte and Restuccia (2010, pp. 136), technical change in the United States is indeed sector biased and the annualized growth rate of labor productivity between 1956 and 2004 has been highest in agriculture (3.8%), second in industry (2.4%), and lowest in services (1.3%). When technical change is sector biased and the elasticity of substitution between the intermediate goods in the two sectors is less (respectively greater) than one,<sup>24</sup> we also show that the economy will **continually** reallocate factors towards (respectively away from) a sector with a lower purely labor augmenting technical change rate, regardless of whether there is sectoral difference in income elasticities or not. Therefore, if agricultural goods, manufacturing goods, and services are complements to each other, according the conclusion in the present model, we find that: (a) the economy will continually reallocate factors from the agricultural sector towards the manufacturing sector, provided that the purely labor augmenting technical change rate in the agricultural sector is greater than that in the manufacturing sector; (b) the economy will continually reallocate factors from the manufacturing sector towards the service sector, provided that the purely labor augmenting technical change rate in the manufacturing sector is greater than that in the service sector. These two conclusions are consistent with the basic fact that countries follow a common process of structural transformation characterized by a declining share of hours in agriculture over time, an increasing share of hours in services, and a hump-shaped share of hours in industry (Duarte and Restuccia, 2010, pp. 132).

It is worth pointing out that we by no means show too much favor to perpetual structural

---

<sup>24</sup> If the elasticity of substitution between the intermediate goods in the two sectors is less (respectively greater) than one, the outputs in the two sectors are complements (respectively substitutes). And Acemoglu and Guerrieri (2008) argues that the elasticity of substitution being less than one is empirically relevant.

## Sector Biased Technical Change and Structural Change

change. Nevertheless, the numerical example in Acemolgu and Guerrieri (2008, pp. 489) implies that perpetual structural change could take place over 5,000 years. However, the numerical example in Kongsamut, Rebelo, and Xie (2001, Figure 3) shows that almost all of transient structural changes take place in the first 100 years.<sup>25</sup> Generally, when one tries to explain sustained economic growth with transitional dynamics, there are extremely counterfactual implications (King and Rebelo, 1993). And thus, we think that a theory that features perpetual structural change could be more convincing than that features transient structural change, if structural change in the real world could date back at least to the industrial revolution and will not cease in the foreseeable future.

Finally, the present paper could also provide a reasonable answer to the question why do so many people in poor countries work in the extremely unproductive agricultural sector. When the agricultural goods are complements to the other consumption goods and services, the larger the gap between the purely labor augmenting technical change rate in the agricultural sector and that in the non-agricultural sector, the higher the speed of labor reallocation away from the agricultural sector to the non-agricultural sector. Unfortunately, the fact is that poor countries are much less productive in agriculture than in non-agriculture in comparison to rich countries (Restuccia, Yang, and Zhu, 2008). And our paper also implies that the poor countries will experience substantial amounts of labor reallocation from the agricultural sector to the non-agricultural sector when farmers adopt modern technology to accelerate labor augmenting technical change in the agricultural sector.<sup>26</sup>

---

<sup>25</sup> The numerical estimates of the dynamic paths in the neoclassical growth model in King and Rebelo (1993) as well as Barro and Sala-i-Martin (2003, pp. 117) implies that all of dynamics never take place more than 200 years.

<sup>26</sup> This conclusion is consistent with “The decline in the relative price of industrial output not only reflects technological progress in industry but also induce farmers to adopt modern technology that relies on industry-supplied inputs.” in Yang and Zhu (2013). As a matter of fact, over time, productivity gaps between rich and poor

#### 4. The mechanism and determinants of sector biased and sector unbiased technical change

The derivative of Hamiltonian with respect to R&D expenditures in sector  $i$  yields

$$\theta_i A_i^{\varphi_i} = \lambda \quad (9)$$

where  $\theta_i A_i^{\varphi_i}$  is the marginal return on R&D resources in sector  $i$ .

The derivative of Hamiltonian with respect to the stock of capital,  $K$ , obtains

$$-\dot{\lambda}/\lambda = \sum_{i=1}^2 \gamma_i (Y/Y_i)^{1/\varepsilon} \alpha_i (Y_i/K) \quad (10)$$

Finally, the derivative of Hamiltonian with respect to the state of art technology in sector  $i$  provides

$$\dot{\theta}_i = -\gamma_i \lambda (Y/Y_i)^{1/\varepsilon} (Y_i/A_i) - \varphi_i \theta_i A_i^{\varphi_i - 1} X_i \quad (11)$$

##### 4.1 Sector unbiased technical change and transient structural change

Combining the results in equations (4), (5), (9) and (10) with that in equation (11), the state of art technology in sector  $i$  can respectively be expressed as

$$(A_i)^{1-\varphi_i} = (\alpha_i)^{-1} u_i K \quad (12)$$

Simple arithmetic manipulation in equations (9) and (12) gives

$$(1-\varphi_1) \dot{A}_1/A_1 - (1-\varphi_2) \dot{A}_2/A_2 = (\dot{\theta}_1/\theta_1 + \dot{A}_1/A_1) - (\dot{\theta}_2/\theta_2 + \dot{A}_2/A_2) = \dot{u}_1/u_1 - \dot{u}_2/u_2 \quad (13)$$

The current value of the state of art technology in sector  $i$  at time  $t$  is  $\theta_i(t)A_i(t)$ , and thus  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i$  measures the percentage change rate of the state of art technology's current value in sector  $i$ . Proposition 1 and equation (13) imply that the percentage change rates of the state of art technology's current value in the two sectors are equal, i.e.,  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i = \dot{\theta}_{-i}/\theta_{-i} + \dot{A}_{-i}/A_{-i}$ ,

---

countries have been substantially reduced in agriculture and industry but not nearly as much in services (Duarte and Restuccia, 2010, pp. 131).

## Sector Biased Technical Change and Structural Change

if and only if technical change is sector unbiased, i.e.,  $(1-\alpha_1)^{-1} \dot{A}_1/A_1 = (1-\alpha_2)^{-1} \dot{A}_2/A_2$ . Thus, we could establish the following proposition.

**Proposition 2.** The necessary and sufficient condition for sector unbiased technical change is  $(1-\varphi_1)(1-\alpha_1) = (1-\varphi_2)(1-\alpha_2)$ .

The percentage change rate of the state of art technology's current value in sector  $i$ , i.e.,  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i$ , is endogenously related to R&D expenditures in that sector, and we need to investigate the explicit relationship between them. Substituting equation (12) into equation (4), the R&D expenditures in sector  $i$  is given by

$$X_i = [\alpha_i(1-\varphi_i)]^{-1} u_i K (\dot{K}/K + \dot{u}_i/u_i) \quad (14)$$

When the percentage change rates of the state of art technology's current value are equal in the two sectors, i.e.,  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i = \dot{\theta}_{-i}/\theta_{-i} + \dot{A}_{-i}/A_{-i}$ , substituting  $\dot{u}_i/u_i = 0$  and  $\dot{u}_{-i}/u_{-i} = 0$  into equation (14), we find that the ratio of the R&D resource used in sector  $i$  to that in sector  $-i$ , i.e.,  $X_i/X_{-i}$ , must be a constant. Summarizing the analysis above, we could establish the following lemma.

**Lemma 2.** The percentage change rates of the state of art technology's current value will be equalized in the two sectors if and only if both R&D resources and two factors are allocated to the two sectors equiproportionately, i.e.,  $\dot{X}_i/X_i = \dot{X}_{-i}/X_{-i}$ ,  $\dot{u}_i/u_i = 0$ , and  $\dot{n}_i/n_i = 0$  for  $i=1,2$ .

Finally, when  $(1-\varphi_1)(1-\alpha_1) = (1-\varphi_2)(1-\alpha_2)$ , the steady state of the present economy, in which technical change is sector unbiased, could be described by the following proposition.

**Proposition 3.** Suppose that  $\alpha_1 \neq \alpha_2$ ,<sup>27</sup>  $(1-\alpha_1)(1-\varphi_1) = (1-\alpha_2)(1-\varphi_2)$ ,  $(\alpha_i - \alpha_{-i})\xi_i^{-1} > 1$ , and  $(1-\sigma)(\alpha_i - \alpha_{-i})\xi_i^{-1} < (\rho - \nu\sigma)$ , where  $\xi_i \equiv (1-\varphi_i)^{-1} - (1-\varphi_{-i})^{-1} + \alpha_i - \alpha_{-i}$ , then the present model could generate endogenous sector unbiased technical change, i.e.,  $\lim_{t \rightarrow \infty} \dot{A}_i/A_i = (1-\varphi_i)^{-1}(\alpha_i - \alpha_{-i})\xi_i^{-1}\nu$ , in which economic growth must be balanced at the sectoral level, i.e.,  $\lim_{t \rightarrow \infty} \dot{Y}_i/Y_i = \lim_{t \rightarrow \infty} \dot{Y}_{-i}/Y_{-i} = (\alpha_i - \alpha_{-i})\xi_i^{-1}\nu$ , and only transient structural change could take place during the transition process, i.e.,  $\lim_{t \rightarrow \infty} \dot{u}_i/u_i = 0$  and  $\lim_{t \rightarrow \infty} \dot{n}_i/n_i = 0$ , for  $i = 1, 2$ .

**Proof.** See appendix A.1.

#### 4.2 Sector biased technical change and perpetual structural change

On the contrary, when the percentage change rates of the state of art technology's current value are not equal in the two sectors, i.e.,  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i \neq \dot{\theta}_{-i}/\theta_{-i} + \dot{A}_{-i}/A_{-i}$ , equation (13) implies that economic growth must be non-balanced at the sectoral level, which further implies that technical change must be sector biased according to proposition 1. More specifically, when the percentage change rate of the state of art technology's current value in sector  $i$  is greater than in sector  $-i$ , i.e.,  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i > \dot{\theta}_{-i}/\theta_{-i} + \dot{A}_{-i}/A_{-i}$ , equation (13) implies that both factors would move in the direction of sector  $i$ . And according to lemma 1, when the elasticity of substitution between the two sectors is less (respectively greater) than unity, both factors flowing out from sector  $-i$  to sector  $i$  implies that the real output growth rate in sector  $-i$  must be greater (respectively less) than that in sector  $i$ , and thus technical change will be biased to sector  $-i$  (respectively sector  $i$ ). Thus, we could establish the following proposition.

---

<sup>27</sup> It should be noted that  $\alpha_1 \neq \alpha_2$  is the implicit assumption in the case of sector unbiased technical change, whereas the production function of the final goods takes the standard Cobb-Douglas form if  $\alpha_1 = \alpha_2$  and  $\varphi_1 = \varphi_2$ .

**Proposition 4.** When the percentage change rate of the state of art technology's current value in sector  $i$  is greater than in sector  $-i$ , technical change must be biased to sector  $-i$  (respectively sector  $i$ ) if the elasticity of substitution between intermediate goods in the two sectors is less (respectively greater) than unity, so that both factors would move in the direction of sector  $i$ , i.e.,  $\dot{u}_{-i}/u_{-i} < 0$  and  $\dot{n}_{-i}/n_{-i} < 0$ .

When the percentage change rate of the state of art technology's current value in sector  $i$  is greater than in sector  $-i$ ,  $\dot{u}_{-i}/u_{-i} < 0$  in equation (14) implies that the ratio of the R&D resource used in sector  $i$  to that in sector  $-i$  will increase over time, i.e.,  $X_i/X_{-i}$  is an increasing function of time  $t$ .

**Lemma 3.** The fraction of R&D resources allocated to sector  $i$  will increase over time, i.e.,  $\dot{X}_i/X_i > \dot{X}_{-i}/X_{-i}$ , when the percentage change rate of the state of art technology's current value in sector  $i$  is greater than that in sector  $-i$ .

To obtain the intuition behind lemma 3, note that equations (9) and (13) imply that the marginal return on R&D investments in sector  $i$  is an increasing function of the percentage change rate of the state of art technology's current value in this sector, i.e.,  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i$ , but a decreasing function of the product of the Hicks-neutral technical change rate and one minus the degree of spillover effects in the same sector, i.e.,  $(1 - \varphi_i) \dot{A}_i/A_i$ . If the percentage change rate of the state of art technology's current value in sector  $i$  is greater than that in sector  $-i$ , the marginal return on R&D investments in sector  $i$  will ceteris paribus increase relative to that in sector  $-i$ , which leads to more R&D resources being allocated in sector  $i$  than those in sector  $-i$ , i.e.,  $\dot{X}_i/X_i > \dot{X}_{-i}/X_{-i}$ .

## Sector Biased Technical Change and Structural Change

The increase of relative R&D resources in sector  $i$  is *ceteris paribus* more conducive to the Hicks-neutral technical change in this sector than that in sector  $-i$ , and therefore the product of the Hicks-neutral technical change rate in sector  $i$  and one minus the degree of spillover effects in the same sector will also increase relative to that in sector  $-i$ , i.e.,  $(1-\varphi_i)\dot{A}_i/A_i > (1-\varphi_{-i})\dot{A}_{-i}/A_{-i}$ . Thus, according to equation (9), the impact of the sectoral difference in the percentage change rates of the state of art technology's current value on the sectoral difference in marginal returns on R&D investments will completely offset by the sectoral difference in the products of the Hicks-neutral technical change rate and one minus the degree of spillover effects in the same sector.

If the percentage change rate of the state of art technology's current value in sector  $i$  is greater than that in sector  $-i$ , according to equation (13), the fractions of both factors used in sector  $-i$  will converge to zero in the steady state, i.e.,  $\lim_{t \rightarrow \infty} u_{-i} = 0$  and  $\lim_{t \rightarrow \infty} n_{-i} = 0$ . Moreover, proposition 1 implies that, when the elasticity of substitution between intermediate goods in the two sectors is less (greater) than unity, the real output growth rate in sector  $-i$  will be greater (less) than that in sector  $i$ , which further implies that  $\lim_{t \rightarrow \infty} Y/Y_i = \gamma_i^{\frac{\varepsilon}{\varepsilon-1}}$  from the final good production function in equation (2). Summarizing the analysis above yields the following lemma.

**Lemma 4.** The long-run GDP growth rate, i.e., the real output growth rate in the final good sector, will be determined by the sector whose percentage change rate of the state of art technology's current value is greater than that in the other sector. That is, if  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i > \dot{\theta}_{-i}/\theta_{-i} + \dot{A}_{-i}/A_{-i}$ , then we must have  $\lim_{t \rightarrow \infty} \dot{Y}/Y = \lim_{t \rightarrow \infty} \dot{Y}_i/Y_i$ .

Substituting  $\lim_{t \rightarrow \infty} \dot{Y}/Y = \lim_{t \rightarrow \infty} \dot{K}/K = \lim_{t \rightarrow \infty} \dot{Y}_i/Y_i$ ,  $\lim_{t \rightarrow \infty} \dot{u}_i/u_i = 0$ ,  $\lim_{t \rightarrow \infty} \dot{n}_i/n_i = 0$ , and the state of art

## Sector Biased Technical Change and Structural Change

technology in sector  $i$  in equation (12) into equation (8), we know that the real output growth rate and the purely labor augmenting technical change rate in sector  $i$  in infinite horizon are respectively given by

$$\begin{aligned} \lim_{t \rightarrow \infty} \dot{Y}/Y = \lim_{t \rightarrow \infty} \dot{K}/K = \lim_{t \rightarrow \infty} \dot{Y}_i/Y_i &= [(1-\alpha_i)(1-\varphi_i)-1]^{-1}(1-\alpha_i)(1-\varphi_i)\nu \\ (1-\alpha_i)^{-1} \lim_{t \rightarrow \infty} \dot{A}_i/A_i &= [(1-\alpha_i)(1-\varphi_i)-1]^{-1}\nu \end{aligned} \quad (15)$$

Substituting equations (7), (12), and (15) into equation (8), the purely labor augmenting technical change rate and the percent change rate of the fraction of each factor used in sector  $-i$  in infinite horizon are respectively given by

$$\begin{aligned} (1-\alpha_{-i})^{-1} \lim_{t \rightarrow \infty} \frac{\dot{A}_{-i}}{A_{-i}} &= \left[ \frac{(\varepsilon-1)(1-\alpha_{-i})-1}{\varepsilon-2+\varphi_{-i}} \frac{(1-\alpha_i)(1-\varphi_i)(1-\alpha_{-i})^{-1}}{(1-\alpha_i)(1-\varphi_i)-1} - \frac{\varepsilon-1}{\varepsilon-2+\varphi_{-i}} \right] \nu \\ \lim_{t \rightarrow \infty} \frac{\dot{u}_{-i}}{u_{-i}} = \lim_{t \rightarrow \infty} \frac{\dot{n}_{-i}}{n_{-i}} &= (\varepsilon-1) \left\{ \frac{[(1-\alpha_{-i})(1-\varphi_{-i})-1](1-\alpha_i)(1-\varphi_i)}{[(1-\alpha_i)(1-\varphi_i)-1](\varepsilon-2+\varphi_{-i})} - \frac{(1-\alpha_{-i})(1-\varphi_{-i})}{\varepsilon-2+\varphi_{-i}} \right\} \nu \end{aligned} \quad (16)$$

In the steady state, both capital and labor reallocating away from sector  $-i$  to sector  $i$  without end requires  $\lim_{t \rightarrow \infty} \dot{n}_{-i}/n_{-i} = \lim_{t \rightarrow \infty} \dot{u}_{-i}/u_{-i} < 0$ . And thus, simple arithmetic manipulation in equation (16) implies that we have the following lemma.

**Lemma 5.** The necessary condition for sector biased technical change, in which the percentage change rate of the state of art technology's current value in sector  $i$  is greater than that in sector  $-i$  is: (a)  $(1-\alpha_i)(1-\varphi_i) > (1-\alpha_{-i})(1-\varphi_{-i})$  if  $0 < \varepsilon < 1$ ; (b)  $(1-\alpha_{-i})(1-\varphi_{-i}) > (1-\alpha_i)(1-\varphi_i)$  if  $1 < \varepsilon < 2 - \varphi_{-i}$ ; or (c)  $(1-\alpha_i)(1-\varphi_i) > (1-\alpha_{-i})(1-\varphi_{-i})$  if  $\varepsilon > 2 - \varphi_{-i}$ .

Moreover, the ratio of the purely labor augmenting technical change rate in infinite horizon in sector  $-i$  to that in sector  $i$  is given by

## Sector Biased Technical Change and Structural Change

$$\frac{(1-\alpha_{-i})^{-1} \lim_{t \rightarrow \infty} \dot{A}_{-i}/A_{-i}}{(1-\alpha_i)^{-1} \lim_{t \rightarrow \infty} \dot{A}_i/A_i} = \frac{\varepsilon - 1}{\varepsilon - 2 + \varphi_{-i}} + \frac{(1-\alpha_i)(1-\varphi_i)}{(1-\alpha_{-i})(1-\varphi_{-i})} \frac{\varphi_{-i} - 1}{\varepsilon - 2 + \varphi_{-i}} \quad (17)$$

Simple arithmetic manipulation in equation (17) yields the following lemma.

**Lemma 6.** The sufficient condition for sector biased technical change, in which the percentage change rate of the state of art technology's current value in sector  $i$  is greater than that in sector  $-i$  is: (a)  $(1-\alpha_i)(1-\varphi_i) > (1-\alpha_{-i})(1-\varphi_{-i})$  if  $0 < \varepsilon < 1$ ; (b)  $(1-\alpha_{-i})(1-\varphi_{-i}) > (1-\alpha_i)(1-\varphi_i)$  if  $1 < \varepsilon < 2 - \varphi_{-i}$ ; or (c)  $(1-\alpha_i)(1-\varphi_i) > (1-\alpha_{-i})(1-\varphi_{-i})$  if  $\varepsilon > 2 - \varphi_{-i}$ .

Finally, when  $(1-\varphi_1)(1-\alpha_1) \neq (1-\varphi_2)(1-\alpha_2)$ , the steady state of the present economy in the case of sector biased technical could be described by the following proposition.

**Proposition 5.** Suppose that  $(1-\alpha_i)(1-\varphi_i) > 1$ ,  $(1-\varphi_1)(1-\alpha_1) \neq (1-\varphi_2)(1-\alpha_2)$ , and  $(1-\sigma)(1-\alpha_i)(1-\varphi_i)[(1-\alpha_i)(1-\varphi_i)-1]^{-1} \nu < (\rho - \nu\sigma)$ , then the present model could generate endogenous sector biased technical change, in which (a)  $0 < \varepsilon < 1$  and  $(1-\alpha_i)(1-\varphi_i) > (1-\alpha_{-i})(1-\varphi_{-i})$ , technical change will be biased to sector  $-i$ ; (b)  $1 < \varepsilon < 2 - \varphi_{-i}$  and  $(1-\alpha_{-i})(1-\varphi_{-i}) > (1-\alpha_i)(1-\varphi_i)$ , technical change will be biased to sector  $i$ ; and (c)  $\varepsilon > 2 - \varphi_{-i}$  and  $(1-\alpha_i)(1-\varphi_i) > (1-\alpha_{-i})(1-\varphi_{-i})$ , technical change will be biased to sector  $i$ .

**Proof.** See appendix A.2.

### 4.3 Discussion and extension

Note that given the amount of resources allocated to inventive activity in each sector, the larger the degree of R&D spillover effects in one sector, the faster the Hicks-neutral technical change in this sector. Furthermore, given the Hicks-neutral technical change rate in one sector, the less the labor intensity, the greater the purely labor augmenting technical change rate in this

## Sector Biased Technical Change and Structural Change

sector. From what has been discussed above, given the R&D resource allocation between the two sectors, we may safely draw a conclusion that the less the product of one minus the degree of R&D spillover effects and the same sector's labor intensity in one sector, the faster the purely labor augmenting technical change in this sector. Nevertheless, the R&D resource allocation between the two sectors in the present model is endogenously determined in the present model so that the marginal returns on R&D resources in the two sectors could be equalized at any time.

The marginal return on R&D investments in sector  $i$  is an increasing function of the percentage change rate of the state of art technology's current value in this sector, i.e.,  $\dot{\theta}_i/\theta_i + \dot{A}_i/A_i$ , but a decreasing function of the product of the Hicks-neutral technical change rate and one minus the degree of spillover effects in the same sector, i.e.,  $(1-\varphi_i)\dot{A}_i/A_i$ . And the percentage change rates of the state of art technology's current value are equal in the two sectors if and only there exists no sectoral difference in the products of the Hicks-neutral technical change rate and one minus the degree of spillover effects in the same sector. Moreover, when the percentage change rates of the state of art technology's current value are equal in the two sectors, R&D resources will be allocated to the two sectors in constant proportion. Given the constant proportion of R&D resource allocation between the two sectors, technical change will definitely be sector unbiased. Thus, when the products of the labor intensity and one minus the degree of R&D spillover effects in the same sector are equal in the two sectors, i.e.,  $(1-\varphi_1)(1-\alpha_1) = (1-\varphi_2)(1-\alpha_2)$ , technical change must be sector unbiased, whatever the elasticity of substitution between intermediate good in the two sectors is.<sup>28</sup>

---

<sup>28</sup> It is consistent with "Notice that differences in demand parameters affect neither comparisons of productivity growth rates nor of R&D intensity when  $l_i$  are constants, where  $l_i$  is the fraction of labor allocated to research in sector  $i$ , (Ngai and Samaniego, 2011, pp. 482)".

## Sector Biased Technical Change and Structural Change

In contrast, when the products of the labor intensity and one minus the degree of R&D spillover effects in the same sector are not equal in the two sectors, say  $(1 - \varphi_i)(1 - \alpha_i) < (1 - \varphi_{-i})(1 - \alpha_{-i})$ , whether technical change will be biased to sector  $i$  or to sector  $-i$ , will depend on the elasticity of substitution between the two sectors. More specifically, when  $(1 - \alpha_i)(1 - \varphi_i) < (1 - \alpha_{-i})(1 - \varphi_{-i})$ , technical change will be biased to sector  $i$  only if the elasticity of substitution between the two sectors satisfies  $0 < \varepsilon < 2 - \varphi_{-i}$ ; or else technical change will be biased to sector  $-i$  when the elasticity of substitution between the two sectors is large than the threshold value, i.e.,  $\varepsilon > 2 - \varphi_{-i}$ .

Now we will discuss the determinants of sector biased technical change based on the more general innovation possibility curve (IPC) such as that in Ngai and Samaniego (2011) as well as Cai and Li (2012).<sup>29</sup> Careful contrast of the IPC in equation (4) in the present paper with that in equation (2) in Ngai and Samaniego (2011, pp. 478) shows up the key differences:  $Z_i$ ,  $\eta_i$ , and  $\psi_i$  in Ngai and Samaniego (2011) being all normalized to 1 in the present paper, and  $\rho_i = \kappa_i + \sigma_i$  in Ngai and Samaniego (2011) being replaced by  $\varphi_i$  in the present paper.<sup>30</sup> Proposition 2 in Ngai and Samaniego (2011) shows that along the balanced growth path, cross-industry comparisons of productivity growth depend only on the technological opportunity factors  $\rho_i$ ,  $\psi_i$  and  $\eta_i$ . Thus,

---

<sup>29</sup> IPC was first introduced by Kennedy (1964), which is called as the knowledge production function in Ngai and Samaniego (2011) and the innovation production function in Cai and Li (2012).

<sup>30</sup> The implicit assumption that one unit of final good could be used to produce one unit of capital implies that the IPC in our paper does allow for capital being used in the production of knowledge. In Ngai and Samaniego (2011), parameter  $Z_i$  is an efficiency parameter for carrying out research in industry  $i$ , the total knowledge spillover  $\rho_i$  is the extent to which the production of new knowledge in sector  $i$  benefits from prior knowledge, parameter  $\psi_i$  indicates decreasing returns to research inputs, and parameter  $\eta_i$  captures the share of capital in R&D spending.

## Sector Biased Technical Change and Structural Change

$Z_i$  could be normalized to one without loss of generality in our paper.<sup>31</sup>

Contrary to the conclusion in Ngai and Samaniego (2011), the **new finding in our paper** is that both the capital intensity in the production, i.e.,  $\alpha_i$  in our paper, and the elasticity of substitution between intermediate goods in the two sectors, i.e.,  $\varepsilon$  in our paper, will also determine whether technical change is biased to the particular sector instead of the other sector. In our paper, sector biased technical change implies that economic growth must be non-balanced at the sectoral level and the fraction of labor allocated to one sector will tend to zero in infinite horizon, i.e.,  $\lim_{t \rightarrow \infty} n_i(t) = 0$  for some  $i$ . In contrast, economic growth is balanced at the sectoral level in Ngai and Samaniego (2011, pp. 482, Proposition 1), in which a Cobb-Douglas utility function is used, and the fraction of labor allocated to production in all sectors in the long-run is greater than zero, i.e.,  $\lim_{t \rightarrow \infty} n_i(t) \in (0,1)$  for all  $i$ . And it is non-balanced sectoral growth and perpetual structural change that lead us to obtain the different conclusion in our paper from that in Ngai and Samaniego (2011).

Cai and Li (2012) studies the impact of inter-sectoral knowledge linkages on aggregate innovation and growth and find that barriers to diversity significantly reduce technological progress and prevent firms from fully internalizing spillovers from sectors with high knowledge applicability and investing in research in these sectors. Cobb-Douglas production function is assumed in the final good sector in equation (4) in Cai and Li (2012), and thus they abstract non-

---

<sup>31</sup> Zhang and Liu (2009, pp. 9) assumes that the cost to create a new type of intermediate good in sector  $j$ , which depends on the number of varieties previously invented in this sector,  $M_j$ , is  $M_j^{\phi_j} / b_j$  units of final goods  $Y$ , where  $b_j$  is a strictly positive constant measuring the technical difficulty of creating new blueprints in sector  $j$ . Thus,  $b_j$  in Zhang and Liu (2009) is the same as  $Z_j$  in Ngai and Samaniego (2011). And the sectoral difference in purely labor augmenting technical change rates in Zhang and Liu (2009) really does not depend on  $b_j$ .

balanced sectoral growth and structural change in our paper. Nevertheless, we think that the extension of the IPC in our paper to that in equation (12) in Cai and Li (2012) and allow inter-sectoral knowledge spillover as a potential determinant of sector biased technical change and non-balanced sectoral growth will be a fruitful area for future research.

### **5. Concluding remarks**

In this paper, we categorize structural change into perpetual structural change and transient structural change to investigate the driving force behind it. We find that economic growth must be non-balanced at the sectoral level and perpetual structural change will take place when technical change is sector biased. More specifically, if the elasticity of substitution between intermediate goods in the two sectors is less (respectively greater) than one, the economy will continually reallocate factors towards (respectively away from) a sector with a lower purely labor augmenting technical change rate. However, when technical change is sector unbiased, we show that economic growth must be balanced at the sectoral level and only transient structural change could occur between the two sectors. It is worth noting that these conclusions are certainly correct regardless of whether there is sectoral difference in income elasticities or not.

Perpetual structural change implies that the reallocation of labor among sectors will take place without end, and thus the fraction of labor used in at least one sector will converge to zero in the long-run. Nevertheless, sustainable economic growth in the present model implies that the real outputs in both sectors will eventually tend to infinity, and thus neither sector will completely disappear in the long-run. Conversely, transient structural change implies that the reallocation of labor between the two sectors could only take place during the transitional dynamics, and thus the fraction of labor allocated to both sectors should be a constant lying on

## Sector Biased Technical Change and Structural Change

the unity interval in the long-run.<sup>32</sup> Unfortunately, the numerical estimates of the dynamic paths in the neoclassical growth model in King and Rebelo (1993), Barro and Sala-i-Martin (2003, pp. 117) as well as Kongsamut, Rebelo, and Xie (2001) implies that the transitional dynamics take place not more than 200 years. And thus, transient structural change alone could not successfully explain the large scales reallocation of resources across agriculture, manufacturing, and services sectors since the industrial revolution.

The two-sector growth model in this paper not only could generate sector biased technical change but also could generate sector unbiased technical change.<sup>33</sup> Therefore, owing to endogenous sector unbiased technical change being allowed and considered in our paper, our model does not prevent transient structural change from occurring at all. Our paper is the very first one that addresses sector biased technical change and non-balanced sectoral growth, and in fact builds a previously-ignored causal relation between the two. It is non-balanced sectoral growth and perpetual structural change that lead us to find new determinants that underlies sector biased technical change, which are neglected by Ngai and Samaniego (2011) and Cai and Li (2012). More specifically, besides the sectoral difference in the degree of the spillover effects from the current stock of technology to future technology invention within the same sector, we find that both the elasticity of substitution between intermediate good in the two sectors and the sectoral difference in labor intensities will also determine the direction of sector biased technical change.

---

<sup>32</sup> According to the data from the World Bank (World Development Indicators), the employment in agriculture (% of total employment) in the United Kingdom showed a gradual decrease from 3 in 1980 to 1 in 2010, and a similar downtrend was observed in the United States, in which the employments in agriculture are 4 in 1980 and 2 in 2010 respectively. Therefore, it would be difficult to argue that the employment in agriculture in both United Kingdom and United States will be a positive constant in the long-run.

<sup>33</sup> Acemoglu and Guerrieri (2006) assumes that the degrees of spillover effects of current technology stock to the future technology creation within the same sector are equal in the two sectors, and thus technical change must be sector biased when labor intensities are not equal in the two sectors.

## Appendix

In the appendix, we will analyze the steady state and the dynamic equilibrium paths of the present model, in which the following lemma is useful.

**Lemma A.1.** When  $\lim_{t \rightarrow \infty} u_{-i} > 0$ , the optimal trajectory of the present model could be described by an autonomous nonlinear dynamical system consisting of the dynamic trajectory of  $C$ ,  $K$ ,  $u_i$ , and  $L$  in equations (A.1), (A.2), (A.3) as well as  $L(t) = \exp(\nu t)$ .

$$\frac{\dot{C}}{C} = -\frac{(\rho - \nu\sigma)}{\sigma} + \left( \sum_{i=1}^2 \alpha_i u_{-i} \right)^{\frac{1}{\varepsilon-1}} \sigma^{-1} \sum_{i=1}^2 \gamma_i^{\frac{\varepsilon}{\varepsilon-1}} \alpha_i \alpha_{-i}^{\frac{1}{1-\varepsilon}} [u_i^{1-\varepsilon} (Y_i/K)] \quad (\text{A.1})$$

$$\frac{\dot{K}}{K} = \xi_i^{-1} \left[ \frac{\varepsilon}{(\varepsilon-1)u_{-i}} - \mathbb{F}_i(u_i) \right] \frac{\dot{u}_i}{u_i} - (\alpha_{-i} - \alpha_i) \xi_i^{-1} \nu \quad (\text{A.2})$$

$$\begin{aligned} \frac{C}{K} + \left\{ \mathbb{S}(u_i) \xi_i^{-1} \left[ \frac{\varepsilon}{(\varepsilon-1)u_{-i}} - \mathbb{F}_i(u_i) \right] + [\alpha_i(1-\varphi_i)]^{-1} u_i - [\alpha_{-i}(1-\varphi_{-i})]^{-1} u_i \right\} \frac{\dot{u}_i}{u_i} \\ = \left[ \sum_{i=1}^2 \gamma_i (Y_i/K)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} + \mathbb{S}(u_i) (\alpha_{-i} - \alpha_i) \xi_i^{-1} \nu \end{aligned} \quad (\text{A.3})$$

where

$$Y_i/K = A_i u_i \left( \frac{(1-\alpha_i)\alpha_{-i}}{(1-\alpha_{-i})\alpha_i + (\alpha_{-i} - \alpha_i)u_i} \right)^{1-\alpha_i} (L/K)^{1-\alpha_i}$$

$$\begin{aligned} \mathbb{F}_i(u_i) &\equiv \frac{1}{1-\varphi_i} + \alpha_i + \frac{(1-\alpha_i)(1-\alpha_{-i})\alpha_i}{(1-\alpha_{-i})\alpha_i + (\alpha_{-i} - \alpha_i)u_i} \\ &+ \left[ \frac{1}{1-\varphi_{-i}} + \alpha_{-i} + \frac{(1-\alpha_{-i})(1-\alpha_i)\alpha_{-i}}{(1-\alpha_{-i})\alpha_i + (\alpha_{-i} - \alpha_i)u_i} \right] \frac{u_i}{u_{-i}} \end{aligned}$$

$$\mathbb{S}(u_i) \equiv 1 + \sum_{i=1}^2 \frac{u_i}{\alpha_i(1-\varphi_i)}$$

**Proof.** The derivative of Hamiltonian with respect to total consumption,  $C$ , implies that

## Sector Biased Technical Change and Structural Change

$$\dot{C}/C = -\sigma^{-1}[\dot{\lambda}/\lambda + (\rho - \nu\sigma)] \quad (\text{A.4})$$

Substituting  $\dot{\lambda}/\lambda$  in equation (10) into equation (A.4) obtains equation (A.1). Substituting  $\dot{n}_i/n_i$  in equation (6) and the state of art technology in sector  $i$  in equation (12) into equation (8), the growth rate of real output in sector  $i$  is given by

$$\frac{\dot{Y}_i}{Y_i} = \left[ \frac{1}{1-\varphi_i} + \alpha_i \right] \frac{\dot{K}}{K} + \left[ \frac{1}{1-\varphi_i} + \alpha_i + \frac{(1-\alpha_i)(1-\alpha_{-i})\alpha_i}{\alpha_i(1-\alpha_{-i}) + (\alpha_{-i} - \alpha_i)u_i} \right] \frac{\dot{u}_i}{u_i} + (1-\alpha_i)\nu \quad (\text{A.5})$$

Substituting  $\dot{Y}_i/Y_i$  in equation (A.5) into equation (7) obtain equation (A.2).

Finally, combining the R&D expenditures in sector  $i$  in equation (14) and capital accumulation rate in equation (A.2) with the budget constraint in the present economy, i.e.,  $Y = C + \dot{K} + X_1 + X_2$ , gives equation (A.3).  $\square$

It is worthy noting that if  $\lim_{t \rightarrow \infty} u_{-i} = 0$ , then  $u_{-i}$  could not alone appear in the denominator in  $\mathbb{F}_i(u_i)$  and equation (A.3) in lemma A.1 with the exception of  $\dot{u}_{-i}/u_{-i}$ . Thus, when  $\lim_{t \rightarrow \infty} u_{-i} = 0$ , the optimal trajectory of the present model could be described by an autonomous nonlinear dynamical system consisting of the dynamic trajectory of  $C$ ,  $K$ ,  $u_{-i}$ , and  $L$ , with  $i$  and  $-i$  being switched in lemma A.1.

### ***A.1 The dynamic equilibrium paths in the case of sector unbiased technical change***

When technical change is sector unbiased, proposition 1 implies that balanced sectoral growth is inevitable, i.e.,  $\lim_{t \rightarrow \infty} \dot{Y}_1/Y_1 = \lim_{t \rightarrow \infty} \dot{Y}_2/Y_2$ , in which only transient structural change could take place during transitional process, which further implies that the fractions of both factors allocated in sector  $i = 1, 2$  must lie on the unit interval, i.e.,  $\lim_{t \rightarrow \infty} u_i \in (0, 1)$  and  $\lim_{t \rightarrow \infty} n_i \in (0, 1)$  for

## Sector Biased Technical Change and Structural Change

$i = 1, 2$ . Substituting  $\lim_{t \rightarrow \infty} \dot{u}_i/u_i = 0$  and  $\lim_{t \rightarrow \infty} \dot{n}_i/n_i = 0$  into equation (A.2), the capital accumulation rate in the steady state in the case of sector unbiased technical change is given by  $\lim_{t \rightarrow \infty} \dot{K}/K = (\alpha_i - \alpha_{-i})\xi_i^{-1}\nu$ . Sustainable economic growth in the case of sector unbiased technical change requires that  $(\alpha_i - \alpha_{-i})\xi_i^{-1} > 1$ .

The aggregate balanced growth requires that  $\lim_{t \rightarrow \infty} \dot{K}/K = \lim_{t \rightarrow \infty} \dot{Y}/Y$ , which further implies that

$$\lim_{t \rightarrow \infty} \dot{Y}_1/Y_1 = \lim_{t \rightarrow \infty} \dot{Y}_2/Y_2 = \lim_{t \rightarrow \infty} \dot{C}/C = (\alpha_i - \alpha_{-i})\xi_i^{-1}\nu$$

Substituting  $\lim_{t \rightarrow \infty} \dot{K}/K = (\alpha_i - \alpha_{-i})\xi_i^{-1}\nu$  into equation (12), the purely labor augmenting technical change rate in sector  $i = 1, 2$  in the case of sector unbiased technical change is given by

$$(1 - \alpha_i)^{-1} \lim_{t \rightarrow \infty} \dot{A}_i/A_i = (1 - \alpha_{-i})^{-1} \lim_{t \rightarrow \infty} \dot{A}_{-i}/A_{-i} = (1 - \alpha_i)^{-1} (1 - \varphi_i)^{-1} (\alpha_i - \alpha_{-i})\xi_i^{-1}\nu$$

Substituting  $\lim_{t \rightarrow \infty} \dot{C}/C = (\alpha_i - \alpha_{-i})\xi_i^{-1}\nu$  into equation (A.4), the growth rate for the co-state variable on physical capital in infinite horizon in the case of sector unbiased technical change, i.e.,  $\lim_{t \rightarrow \infty} \dot{\lambda}/\lambda$ , can be expressed as

$$\lim_{t \rightarrow \infty} \dot{\lambda}/\lambda = -\sigma(\alpha_i - \alpha_{-i})\xi_i^{-1}\nu - (\rho - \nu\sigma)$$

And the growth rate for the co-state variable on the state of the art technology in sector  $i$ , i.e.,  $\lim_{t \rightarrow \infty} \dot{\theta}_i/\theta_i$ , could be calculated from equation (9). Finally, the STVC in the case of sector unbiased technical could be satisfied provided that  $(1 - \sigma)(\alpha_i - \alpha_{-i})\xi_i^{-1}\nu < (\rho - \nu\sigma)$ . Summarizing the analysis above provides proposition 3 in the main text.  $\square$

Now we will turn to the dynamic equilibrium paths of the present model in the case of sector unbiased technical change. Sustainable economic growth implies that both  $C$  and  $K$  will

## Sector Biased Technical Change and Structural Change

grow without bound in the present model, thus we need to transform these two variables into constants in the steady state, and then investigate the evolution of the transformed variables. The foregoing analysis implies  $C/K$ ,  $u_1^{\frac{1}{1-\varepsilon}} Y_1/K$ ,  $u_2^{\frac{1}{1-\varepsilon}} Y_2/K$ , and  $u_i$  are all positive constants in the steady state in the case of sector unbiased technical change.

Let us define  $\Theta \equiv C/K$ ,  $\Phi_1 \equiv u_1^{\frac{1}{1-\varepsilon}} Y_1/K$ , and  $\Phi_2 \equiv u_2^{\frac{1}{1-\varepsilon}} Y_2/K$ . Lemma A.1 implies that the dynamic equilibrium paths of the present model in the case of sector unbiased technical change could be described as the evolution of  $\Theta$ ,  $\Phi_1$ ,  $\Phi_2$ , and  $u_i$  (or  $u_2$ ), which are respectively given by:

$$\frac{\dot{\Theta}}{\Theta} = -\frac{(\rho - \nu\sigma)}{\sigma} + \left( \sum_{i=1}^2 \alpha_i u_{-i} \right)^{\frac{1}{\varepsilon-1}} \sigma^{-1} \sum_{i=1}^2 \gamma_i^{\frac{\varepsilon}{\varepsilon-1}} \alpha_i \alpha_{-i}^{\frac{1}{1-\varepsilon}} \Phi_i - \frac{\dot{K}}{K} \quad (\text{A.6})$$

$$\begin{aligned} \frac{\dot{\Phi}_1}{\Phi_1} &= \left[ \frac{1}{1-\varphi_1} + \alpha_1 - 1 \right] \frac{\dot{K}}{K} + (1-\alpha_1)\nu + \\ &\left[ \frac{1}{1-\varepsilon} + \frac{1}{1-\varphi_1} + \alpha_1 + \frac{(1-\alpha_1)(1-\alpha_2)\alpha_1}{\alpha_1(1-\alpha_2) + (\alpha_2 - \alpha_1)u_1} \right] \frac{\dot{u}_1}{u_1} \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \frac{\dot{\Phi}_2}{\Phi_2} &= \left[ \frac{1}{1-\varphi_2} + \alpha_2 - 1 \right] \frac{\dot{K}}{K} + (1-\alpha_2)\nu + \\ &\left[ \frac{1}{1-\varepsilon} + \frac{1}{1-\varphi_2} + \alpha_2 + \frac{(1-\alpha_2)(1-\alpha_1)\alpha_2}{\alpha_2(1-\alpha_1) + (\alpha_1 - \alpha_2)u_2} \right] \frac{\dot{u}_2}{u_2} \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} \Theta + \left\{ \mathbb{S}(u_i) \xi_i^{-1} \left[ \frac{\varepsilon}{(\varepsilon-1)u_{-i}} - \mathbb{F}_i(u_i) \right] + [\alpha_i(1-\varphi_i)]^{-1} u_i - [\alpha_{-i}(1-\varphi_{-i})]^{-1} u_i \right\} \frac{\dot{u}_i}{u_i} \\ = \left[ \sum_{i=1}^2 \gamma_i (Y_i/K)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} + \mathbb{S}(u_i) (\alpha_{-i} - \alpha_i) \xi_i^{-1} \nu \end{aligned} \quad (\text{A.9})$$

where  $\dot{K}/K$  is given by equation (A.2).

## Sector Biased Technical Change and Structural Change

Summarizing the analysis above, we could obtain the following proposition.

**Proposition A.1.** Suppose that the conditions in proposition 3 in the main text are satisfied, given any initial conditions  $K(0) > 0$  and  $L_0 = 1$ , then the present economy in the case of sector unbiased technical change could be described either by: (a) a four-dimensional autonomous dynamical system consisting of the evolution of  $\Theta$  in equation (A.6), the evolution of  $\Phi_1$  in equation (A.7), the evolution of  $\Phi_2$  in equation (A.8), and the evolution of  $u_1$  in equation (A.9) with  $u_2$  being replaced by  $u_2 = 1 - u_1$ ; or equivalently by: (b) a four-dimensional autonomous dynamical system consisting of the evolution of  $\Theta$  in equation (A.6), the evolution of  $\Phi_1$  in equation (A.7), the evolution of  $\Phi_2$  in equation (A.8), and the evolution of  $u_2$  in equation (A.9) with  $u_1$  being replaced by  $u_1 = 1 - u_2$ .

### *A.2 The dynamic equilibrium paths in the case of sector biased technical change*

Without loss of generality, we will assume that the conditions in proposition 5 in the main text are satisfied, so that both factors could flow away from sector  $-i$  to sector  $i$  without cession, i.e.,  $\lim_{t \rightarrow \infty} u_{-i} = 0$  and  $\lim_{t \rightarrow \infty} n_{-i} = 0$ .

When technical change is sector biased, in which both factors will flow away from sector  $-i$  to sector  $i$  without cession, equation (15) implies that the real output growth rate and the purely labor augmenting technical change rate in sector  $i$  in infinite horizon are respectively given by

$$\lim_{t \rightarrow \infty} \dot{Y}/Y = \lim_{t \rightarrow \infty} \dot{K}/K = \lim_{t \rightarrow \infty} \dot{Y}_i/Y_i = \lim_{t \rightarrow \infty} \dot{C}/C = (1 - \alpha_i)(1 - \varphi_i)[(1 - \alpha_i)(1 - \varphi_i) - 1]^{-1} \nu \quad (\text{A.10})$$

Sustainable economic growth in the case of sector biased technical change, in which both factors will flow away from sector  $-i$  to sector  $i$  without cession, requires that

## Sector Biased Technical Change and Structural Change

$$(1-\alpha_i)(1-\varphi_i)-1 > 0.$$

When technical change is sector biased, in which both factors will flow away from sector  $-i$  to sector  $i$  without cession, equation (16) implies that the purely labor augmenting technical change rate and the percent change rate of the fraction of each factor used in sector  $-i$  in infinite horizon are respectively given by

$$(1-\alpha_{-i})^{-1} \lim_{t \rightarrow \infty} \frac{\dot{A}_{-i}}{A_{-i}} = \left[ \frac{(\varepsilon-1)(1-\alpha_{-i})-1}{\varepsilon-2+\varphi_{-i}} \frac{(1-\alpha_i)(1-\varphi_i)(1-\alpha_{-i})^{-1}}{(1-\alpha_i)(1-\varphi_i)-1} - \frac{\varepsilon-1}{\varepsilon-2+\varphi_{-i}} \right] \nu$$

$$\lim_{t \rightarrow \infty} \frac{\dot{u}_{-i}}{u_{-i}} = (\varepsilon-1) \left\{ \frac{[(1-\alpha_{-i})(1-\varphi_{-i})-1](1-\alpha_i)(1-\varphi_i)}{[(1-\alpha_i)(1-\varphi_i)-1](\varepsilon-2+\varphi_{-i})} - \frac{(1-\alpha_{-i})(1-\varphi_{-i})}{\varepsilon-2+\varphi_{-i}} \right\} \nu \quad (\text{A.11})$$

When technical change is sector biased, in which both factors will flow away from sector  $-i$  to sector  $i$  without cession, equation (7) implies that the real output growth rate in sector  $-i$  in infinite horizon is given by

$$\lim_{t \rightarrow \infty} \dot{Y}_{-i}/Y_{-i} = \lim_{t \rightarrow \infty} \dot{K}/K + \varepsilon/(\varepsilon-1) \lim_{t \rightarrow \infty} \dot{u}_{-i}/u_{-i}$$

When technical change is sector biased, in which both factors will flow away from sector  $-i$  to sector  $i$  without cession, substituting  $\lim_{t \rightarrow \infty} \dot{C}/C = (1-\alpha_i)(1-\varphi_i)[(1-\alpha_i)(1-\varphi_i)-1]^{-1} \nu$  into equation (A.4), the growth rate for the co-state variable on physical capital in infinite horizon, i.e.,  $\lim_{t \rightarrow \infty} \dot{\lambda}/\lambda$ , can be expressed as

$$\lim_{t \rightarrow \infty} \dot{\lambda}/\lambda = -\sigma(1-\alpha_i)(1-\varphi_i)[(1-\alpha_i)(1-\varphi_i)-1]^{-1} \nu - (\rho - \nu\sigma)$$

Thus, the STVC in the case of sector biased technical change, in which both factors will flow away from sector  $-i$  to sector  $i$  without cession, could be satisfied provided that  $(1-\sigma)(1-\alpha_i)(1-\varphi_i)[(1-\alpha_i)(1-\varphi_i)-1]^{-1} \nu < (\rho - \nu\sigma)$ . Summarizing the analysis above provides

## Sector Biased Technical Change and Structural Change

proposition 5 in the main text.  $\square$

Now we will turn to dynamic equilibrium paths of the present model in the case of sector biased technical change, in which both factors will flow away from sector  $-i$  to sector  $i$  without cession. The foregoing analysis implies both  $\lim_{t \rightarrow \infty} C/K$  and  $\lim_{t \rightarrow \infty} u_i^{1-\varepsilon} Y_i/K$  are positive constants.

Moreover,  $\lim_{t \rightarrow \infty} \dot{Y}_{-i}/Y_{-i} = \lim_{t \rightarrow \infty} \dot{K}/K + \varepsilon/(\varepsilon-1) \lim_{t \rightarrow \infty} \dot{u}_{-i}/u_{-i}$  implies that  $\lim_{t \rightarrow \infty} u_{-i}^{1-\varepsilon} Y_{-i}/K = 0$ . Let us

define  $\Theta \equiv C/K$ ,  $\Phi_i \equiv u_i^{1-\varepsilon} Y_i/K$ , and  $\Phi_{-i} \equiv u_{-i}^{1-\varepsilon} Y_{-i}/K$ .<sup>A.1</sup> Lemma A.1 implies that the dynamic equilibrium paths of the present model in the case of sector biased technical change, in which both factors will flow away from sector  $-i$  to sector  $i$  without cession, could be described as the evolution of  $\Theta$ ,  $\Phi_i$ ,  $\Phi_{-i}$ , and  $u_{-i}$ , which are respectively given by:

$$\frac{\dot{\Theta}}{\Theta} = -\frac{(\rho - v\sigma)}{\sigma} + \left( \sum_{i=1}^2 \alpha_i u_{-i} \right)^{\frac{1}{\varepsilon-1}} \sigma^{-1} \sum_{i=1}^2 \gamma_i^{\frac{\varepsilon}{\varepsilon-1}} \alpha_i \alpha_{-i}^{1-\varepsilon} \Phi_i - \frac{\dot{K}}{K} \quad (\text{A.12})$$

$$\begin{aligned} \frac{\dot{\Phi}_i}{\Phi_i} &= \left[ \frac{1}{1-\varphi_i} + \alpha_i - 1 \right] \frac{\dot{K}}{K} + (1-\alpha_i)v - \\ \frac{u_{-i}}{u_i} \left[ \frac{1}{1-\varepsilon} + \frac{1}{1-\varphi_i} + \alpha_i + \frac{(1-\alpha_{-i})(1-\alpha_i)\alpha_i}{\alpha_i(1-\alpha_{-i}) + (\alpha_{-i}-\alpha_i)(1-u_{-i})} \right] \frac{\dot{u}_{-i}}{u_{-i}} \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} \frac{\dot{\Phi}_{-i}}{\Phi_{-i}} &= \left[ \frac{1}{1-\varphi_{-i}} + \alpha_{-i} - 1 \right] \frac{\dot{K}}{K} + (1-\alpha_{-i})v + \\ \left[ \frac{1}{1-\varepsilon} + \frac{1}{1-\varphi_{-i}} + \alpha_{-i} + \frac{(1-\alpha_{-i})(1-\alpha_i)\alpha_{-i}}{\alpha_{-i}(1-\alpha_i) + (\alpha_i-\alpha_{-i})u_{-i}} \right] \frac{\dot{u}_{-i}}{u_{-i}} \end{aligned} \quad (\text{A.14})$$

---

<sup>A.1</sup>  $\lim_{t \rightarrow \infty} \dot{Y}_{-i}/Y_{-i} = \lim_{t \rightarrow \infty} \dot{K}/K + \varepsilon/(\varepsilon-1) \lim_{t \rightarrow \infty} \dot{u}_{-i}/u_{-i}$  implies that  $\lim_{t \rightarrow \infty} \dot{\Phi}_{-i}/\Phi_{-i} = \lim_{t \rightarrow \infty} \dot{u}_{-i}/u_{-i}$ .

## Sector Biased Technical Change and Structural Change

$$\begin{aligned} \Theta + \left\{ \mathbb{S}(u_{-i}) \xi_{-i}^{-1} \left[ \frac{\varepsilon}{(\varepsilon-1)u_i} - \mathbb{F}_{-i}(u_{-i}) \right] + [\alpha_{-i}(1-\varphi_{-i})]^{-1} u_{-i} - [\alpha_i(1-\varphi_i)]^{-1} u_{-i} \right\} \frac{\dot{u}_{-i}}{u_{-i}} \\ = \left[ \sum_{j=1}^2 \gamma_j u_j^{\frac{1}{\varepsilon}} (\Phi_j)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} + \mathbb{S}(u_{-i}) (\alpha_i - \alpha_{-i}) \xi_{-i}^{-1} \nu \end{aligned} \quad (\text{A.15})$$

where  $\dot{K}/K$  is given by

$$\frac{\dot{K}}{K} = \xi_{-i}^{-1} \left[ \frac{\varepsilon}{(\varepsilon-1)u_i} - \mathbb{F}_{-i}(u_{-i}) \right] \frac{\dot{u}_{-i}}{u_{-i}} - (\alpha_i - \alpha_{-i}) \xi_{-i}^{-1} \nu \quad (\text{A.16})$$

Summarizing the analysis above, we could obtain the following proposition.

**Proposition A.2.** Suppose that the conditions in proposition 5 in the main text are satisfied, given any initial conditions  $K(0) > 0$  and  $L(0) = 1$ , then the present economy in the case of sector biased technical change, in which both factors will flow away from sector  $-i$  to sector  $i$  without secession, could be described by a four-dimensional autonomous dynamical system consisting of the evolution of  $\Theta$  in equation (A.12), the evolution of  $\Phi_i$  in equation (A.13), the evolution of  $\Phi_{-i}$  in equation (A.14), and the evolution of  $u_{-i}$  in equation (A.15).

### A.3 The Jacobian of the dynamic system in the case of sector biased technical change

When technical change is sector biased, in which both factors will flow away from sector  $-i$  to sector  $i$ , we have  $\Theta^* > 0$ ,  $\Phi_i^* > 0$ ,  $\Phi_{-i}^* = 0$ , and  $u_{-i}^* = 0$ .<sup>A.2</sup> Substituting  $\lim_{t \rightarrow \infty} \dot{\Theta}/\Theta = 0$ ,  $u_{-i}^* = 0$ , and  $\Phi_{-i}^* = 0$  into equations (A.12) and (A.15), the steady-state value of  $\Theta^*$  and  $\Phi_i^*$  are given by the following linear system of equations.

$$0 = -(\rho - \nu\sigma)\sigma^{-1} - \lim_{t \rightarrow \infty} \dot{K}/K + \sigma^{-1} (\gamma_i)^{\frac{\varepsilon}{\varepsilon-1}} \alpha_i \Phi_i^*$$

---

<sup>A.2</sup> We use \* as a superscript to indicate the constant value of a variable in infinite horizon in the appendix.

## Sector Biased Technical Change and Structural Change

$$\Theta^* + \lim_{u_{-i} \rightarrow 0} \mathbb{S}(u_{-i}) \xi_{-i}^{-1} \left[ \frac{\varepsilon}{(\varepsilon - 1)u_{-i}} - \mathbb{F}_{-i}(u_{-i}) \right] \frac{\dot{u}_{-i}}{u_{-i}} = (\gamma_i)^{\frac{\varepsilon}{\varepsilon - 1}} \Phi_i^* + \lim_{u_{-i} \rightarrow 0} \mathbb{S}(u_{-i}) (\alpha_i - \alpha_{-i}) \xi_{-i}^{-1} \nu$$

Substituting  $\lim_{u_{-i} \rightarrow 0} \mathbb{F}_{-i}(u_{-i}) = (1 - \varphi_{-i})^{-1} + 1$ ,  $\lim_{u_{-i} \rightarrow 0} \mathbb{S}(u_{-i}) \equiv 1 + [\alpha_i(1 - \varphi_i)]^{-1}$ ,  $\lim_{t \rightarrow \infty} \dot{K}/K$  in equation (A.10),  $\xi_{-i} = (1 - \varphi_{-i})^{-1} - (1 - \varphi_i)^{-1} + \alpha_i - \alpha_{-i}$ , and  $\lim_{t \rightarrow \infty} \dot{u}_{-i}/u_{-i}$  in equation (A.11) into the

above linear system of equations, the explicit solutions of  $\Phi_i^*$  and  $\Theta^*$  are respectively given by

$$(\rho - \nu\sigma) + (1 - \alpha_i)(1 - \varphi_i)\sigma[(1 - \alpha_i)(1 - \varphi_i) - 1]^{-1} \nu = (\gamma_i)^{\frac{\varepsilon}{\varepsilon - 1}} \alpha_i \Phi_i^*$$

$$\alpha_i \Theta^* = (1 - \alpha_i) \left\{ (1 - \varphi_i)\sigma - [\alpha_i(1 - \varphi_i) + 1] \right\} [(1 - \alpha_i)(1 - \varphi_i) - 1]^{-1} \nu + (\rho - \nu\sigma)$$

The Jacobian of the dynamical system in proposition A.2 is given by

$$\begin{bmatrix} J_{11}^0 & J_{12}^0 & J_{13}^0 & J_{14}^0 \\ J_{21}^0 & J_{22}^0 & J_{23}^0 & J_{24}^0 \\ J_{31}^0 & J_{32}^0 & J_{33}^0 & J_{34}^0 \\ J_{41}^0 & J_{42}^0 & J_{43}^0 & J_{44}^0 \end{bmatrix} = \lim_{t \rightarrow \infty} \begin{bmatrix} \partial \dot{\Theta} / \partial \Theta & \partial \dot{\Theta} / \partial u_{-i} & \partial \dot{\Theta} / \partial \Phi_{-i} & \partial \dot{\Theta} / \partial \Phi_i \\ \partial \dot{u}_{-i} / \partial \Theta & \partial \dot{u}_{-i} / \partial u_{-i} & \partial \dot{u}_{-i} / \partial \Phi_{-i} & \partial \dot{u}_{-i} / \partial \Phi_i \\ \partial \dot{\Phi}_{-i} / \partial \Theta & \partial \dot{\Phi}_{-i} / \partial u_{-i} & \partial \dot{\Phi}_{-i} / \partial \Phi_{-i} & \partial \dot{\Phi}_{-i} / \partial \Phi_i \\ \partial \dot{\Phi}_i / \partial \Theta & \partial \dot{\Phi}_i / \partial u_{-i} & \partial \dot{\Phi}_i / \partial \Phi_{-i} & \partial \dot{\Phi}_i / \partial \Phi_i \end{bmatrix}$$

It is obvious that  $J_{21}^0 = 0$ ,  $J_{23}^0 = 0$ ,  $J_{24}^0 = 0$ ,  $J_{31}^0 = 0$ ,  $J_{32}^0 = 0$ , and  $J_{34}^0 = 0$  owing to  $u_{-i}^* = 0$  and  $\Phi_{-i}^* = 0$ . Moreover,  $\lim_{t \rightarrow \infty} \dot{\Phi}_{-i} / \Phi_{-i} = \lim_{t \rightarrow \infty} \dot{u}_{-i} / u_{-i} < 0$  implies that  $J_{22}^0 = J_{33}^0 < 0$ .

Equation (A.12) implies that  $J_{11}^0$  and  $J_{14}^0$  are respectively given by

$$J_{11}^0 = -\Theta^* \lim_{t \rightarrow \infty} \frac{\partial(\dot{K}/K)}{\partial \Theta} \quad \text{and} \quad J_{14}^0 = \Theta^* \left[ -\lim_{t \rightarrow \infty} \frac{\partial(\dot{K}/K)}{\partial \Phi_i} + \sigma^{-1} (\gamma_i)^{\frac{\varepsilon}{\varepsilon - 1}} \alpha_i \right]$$

where  $\lim_{t \rightarrow \infty} \frac{\partial(\dot{K}/K)}{\partial \Theta} = -\frac{\alpha_i(1 - \varphi_i)}{1 + \alpha_i(1 - \varphi_i)}$  and  $\lim_{t \rightarrow \infty} \frac{\partial(\dot{K}/K)}{\partial \Phi_i} = \frac{\alpha_i(1 - \varphi_i)(\gamma_i)^{\frac{\varepsilon}{\varepsilon - 1}}}{\alpha_i(1 - \varphi_i) + 1}$  are the direct results of

equations (A.15) and (A.16).

Equation (A.13) implies that  $J_{41}^0$  and  $J_{44}^0$  are respectively given by

## Sector Biased Technical Change and Structural Change

$$J_{41}^0 = \Phi_i^* \left[ \frac{1}{1-\varphi_i} + \alpha_i - 1 \right] \lim_{t \rightarrow \infty} \frac{\partial(\dot{K}/K)}{\partial \Theta} \quad \text{and} \quad J_{44}^0 = \Psi_i^* \left[ \frac{1}{1-\varphi_i} + \alpha_i - 1 \right] \lim_{t \rightarrow \infty} \frac{\partial(\dot{K}/K)}{\partial \Phi_i}$$

It is obvious that  $J_{11}^0 + J_{44}^0 = (\sigma - 1)(1 - \alpha_i)(1 - \varphi_i)[(1 - \alpha_i)(1 - \varphi_i) - 1]^{-1} \nu + (\rho - \nu\sigma) > 0$ , which is the direct result of the STVC in the case of sector biased technical change, in which both factors will flow out from  $-i$  to sector  $i$  without cession. And  $J_{11}^0 J_{44}^0 - J_{41}^0 J_{14}^0$  is given by

$$J_{11}^0 J_{44}^0 - J_{41}^0 J_{14}^0 = -[(1 - \alpha_i)(1 - \varphi_i) - 1] \sigma^{-1} [1 + \alpha_i(1 - \varphi_i)]^{-1} (\alpha_i)^2 \Theta^* \Phi_i^* (\gamma_i)^{\frac{\varepsilon}{\varepsilon-1}}$$

The condition that guarantees sustainable economic growth in the present economy,  $(1 - \alpha_i)(1 - \varphi_i) > 1$ , implies that  $J_{11}^0 J_{44}^0 - J_{41}^0 J_{14}^0 < 0$ . The Jacobian of the dynamical system in proposition A.2 has three negative real roots and a positive real root. Therefore, the dynamical system in the case of sector biased technical change will be locally indeterminate.

### References

- Acemoglu, D., 1998. Why do new technologies complement skills? Directed technical change and wage inequality. *Quarterly Journal of Economics* 113, 1055-89.
- Acemoglu, D., 2002. Directed technical change. *Review of Economic Studies* 69, 781-809.
- Acemoglu, D., Guerrieri V., 2006. Capital deepening and non-balanced economic growth. NBER Working Paper, No. W12475, Available at SSRN: <http://ssrn.com/abstract=926054>.
- Acemoglu, D., Guerrieri V., 2008. Capital deepening and non-balanced economic growth. *Journal of Political Economy* 116, 467-98.
- Barro, R.J., Sala-i-Martin, X., 2003. *Economic Growth*, 2nd Edition, Cambridge, Mass: MIT Press.
- Baumol, W.J., 1967. Macroeconomics of unbalanced growth: the anatomy of urban crisis. *American Economic Review* 57, 415-26.

## Sector Biased Technical Change and Structural Change

Buera, F.J., Joseph, K.P., 2012a. Scale and the origins of structural change. *Journal of Economic Theory* 147, 684-712.

Buera, F.J., Joseph, K.P., 2012b. The rise of the service economy. *American Economic Review* 102, 2540-69.

Cai, J., Li, N., 2012. Growth through intersectoral knowledge linkages, mimeo.

Caselli, F., Coleman II, W.J., 2001. The U.S. structural transformation and regional convergence: a reinterpretation. *Journal of Political Economy* 109, 584-616.

Caselli, F., Coleman II, W.J., 2006. The world technology frontier. *American Economic Review* 96, 499-522.

Duarte, M., Restuccia, D., 2010. The role of the structural transformation in aggregate productivity. *Quarterly Journal of Economics*, 125, 129-73.

Foellmi, R., Zweimüller, J., 2008. Structural change, Engel's consumption cycles and Kaldor's facts of economic growth. *Journal of Monetary Economics* 55, 1317-28.

Foley, D.K., Michl, T.R., 1999. *Growth and Distribution*, Cambridge, Mass: Harvard University Press.

Herrendorf, B., Rogerson, R., Valentinyi, A., 2013. Growth and structural transformation. In: Aghion, P., Durlauf, S.N. (Eds.), *Handbook of Economic Growth*, Elsevier B.V.

Herrendorf, B., Rogerson, R., Valentinyi, A., forthcoming. Two perspectives on preferences and structural transformation. *American Economic Review*.

Hicks, J.R., 1932. *The Theory of Wages*, London: Macmillan.

Jones, C.I., 1995. R&D-based models of economic growth. *Journal of Political Economy* 103, 759-84.

Jones, C.I., Scrimgeour, D., 2008. A new proof of Uzawa's steady-state growth theorem.

## Sector Biased Technical Change and Structural Change

Review of Economics and Statistics 90, 180-182.

Kaldor, N., 1961. Capital accumulation and economic growth, .In: Lutz, F., Hague, D.C. (Eds.), *The Theory of Capital*. St. Martin's Press, New York, pp.177-222.

Kamihigashi, T., 2001. Necessity of transversality conditions for infinite horizon problems. *Econometrica* 69, 995-1012.

Kennedy, C., 1964. Induced bias in innovation and the theory of distribution. *Economic Journal* 74, 541-47.

King, R.G., Rebelo, S.T., 1993. Transitional dynamics and economic growth in the neoclassical model. *American Economic Review* 83, 908-31.

Kongsamut, P., Rebelo, S., Xie, D., 2001. Beyond balanced growth. *Review of Economic Studies* 68, 869-82.

Kuznets, S., 1957. Quantitative aspects of the economic growth of nations: II. industrial distribution of national product and labor force. *Economic Development and Cultural Change*, 5, 1-111.

Kuznets, S., 1973. Modern economic growth: findings and reflections. *American Economic Review* 63, 247-58.

Laitner, J., 2000. Structural change and economic growth. *Review of Economic Studies* 67, 545-61.

Lucas, R., 1988, On the mechanics of economic development. *Journal of Monetary Economics* 22, 3-42.

Matsuyama, K., 2002. The rise of mass consumption societies. *Journal of Political Economy* 110, 1035-70.

Matsuyama, K., 2008. Structural change, In: Durlauf, S.N., Blume, L.E. (Eds.), *The New*

## Sector Biased Technical Change and Structural Change

Palgrave Dictionary of Economics. 2nd ed. Macmillan, London.

Matsuyama, K., 2009. Structural change in an interdependent world: a global view of manufacturing decline. *Journal of the European Economic Association* 7, 478-86.

Ngai, R.L., Pissarides, C.A., 2007. Structural change in a multisector model of growth. *American Economic Review* 97, 429-43.

Ngai R.L., Samaniego, R.M., 2011. Accounting for research and productivity growth across industries. *Review of Economic Dynamics* 14, 475-95.

Restuccia, D., Yang, D.T., Zhu, X., 2008. Agriculture and aggregate productivity: a quantitative cross-country analysis. *Journal of Monetary Economics* 55, 234-50.

Shell, K., 1966. Toward a theory of inventive activity and capital accumulation. *American Economic Review* 56, 62-8.

Uzawa H., 1961. Neutral inventions and the stability of growth equilibrium. *Review of Economic Studies* 28, 117-24.

Xie, D., 1994. Divergence in economic performance: transitional dynamics with multiple equilibria. *Journal of Economic Theory* 63, 97-112.

Yang, D.T., Zhu, X., 2013. Modernization of agriculture and long-term growth. *Journal of Monetary Economics* 60, 367-382.

Uy, T., Yi, K., Zhang, J., 2013. Structural change in an open economy. *Journal of Monetary Economics* 60, 667-682.

Zhang, P., Liu, M., 2009. Structural change in the two-sector model with endogenous technological progress. Unpublished, Available at SSRN: <http://ssrn.com/abstract=1523602>.