Unraveling versus Unraveling: A Memo on Competitive Equilibriums and Trade in Insurance Markets

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Abstract

Both Akerlof (1970) and Rothschild and Stiglitz (1976) show that insurance markets may "unravel". This memo clarifies the distinction between these two notions of unraveling in the context of a binary loss model of insurance. I show that the two concepts are mutually exclusive occurrences. Moreover, I provide a regularity condition under which the two concepts are exhaustive of the set of possible occurrences in the model. Akerlof unraveling characterizes when there are no gains to trade; Rothschild and Stiglitz unraveling shows that the standard notion of competition (pure strategy Nash equilibrium) is inadequate to describe the workings of insurance markets when there are gains to trade.

1 Introduction

Akerlof (1970) and Rothschild and Stiglitz (1976) have contributed greatly to the understanding of the potential problems posed by private information on the workings of insurance markets. Akerlof (1970) shows how private information can lead to an *equilibrium of market unraveling*, so that the only unique equilibrium is one in which only the worst quality good (i.e. the "lemons") are traded. Rothschild and Stiglitz (1976) show that private information can lead to an *unraveling of market equilibrium*, in which no (pure strategy) competitive equilibrium exists because insurance companies have the incentive to modify their contracts to cream skim the lower-risk agents from other firms.

While the term *unraveling* has been used to describe both of these phenomena, the distinction between these two concepts is often unclear, arguably a result of each paper's different approach to modeling the environment. Akerlof (1970) works in the context of a "supply and demand" environment with a fixed contract or asset (e.g. a used car), whereas Rothschild and Stiglitz (1976) work in the context of endogenous contracts in a stylized environment with only two types (e.g. high and low types).

This memo develops a generalized binary loss insurance model that incorporates the forces highlighted in both Akerlof (1970) and Rothschild and Stiglitz (1976). Using this unified model, I show that the *equilibrium of market unraveling* (in Akerlof) is a mutually exclusive occurrence from the *unraveling of market equilibrium* (in Rothschild and Stiglitz). Moreover, under the regularity condition that the type distribution has full support, one of these two events must occur: either there is a

Competitive (Nash) Equilibrium of no trade (Akerlof unraveling) or a Competitive (Nash) Equilibrium does not exist (Rothschild and Stiglitz unraveling). Thus, not only are these two concepts of unraveling different, but they are mutually exclusive and generically exhaustive of the potential occurrences in an insurance market with private information.

The mutual exclusivity result is more or less obvious in the canonical two-type binary loss model. The market unravels a la Rothschild and Stiglitz when the low type has an incentive to cross-subsidize the high type in order to obtain a more preferred allocation. This willingness of the good risk to subsidize the bad risk is precisely what ensures the market will not unravel a la Akerlof. Conversely, if the market unravels a la Akerlof, then the good risk is not willing to subsidize the bad risk, which implies an absence of the forces that drive non-existence in Rothschild and Stiglitz.

The intuition for the exhaustive result is also straightforward, but perhaps more difficult to see in the context of the stylized two-type model. When the type distribution has full support, trade *requires* cross-subsidization of types. If some low-risk agent is willing to cross-subsidize higher-risk agents, then the equilibrium will unravel a la Rothschild and Stiglitz (1976). Competitive (Nash) equilibriums cannot sustain cross-subsidization and break down when agents want to provide it. In contrast, if no agents are willing to cross-subsidize the worse risks in the population, then there exists a unique Nash equilibrium at the endowment: no one on the margin is willing to pay the average cost of worse risks, and any potential contract (or menu of contracts) unravels a la Akerlof (1970).

The logic can be seen in the canonical two-types case. Here, the analogue of the full support assumption is to assume the bad risk will experience the loss with certainty. The only way for the low type (good risk) to obtain an allocation other than her endowment is to subsidize the high type (bad risk) away from her endowment. If the low type is willing to do so, the equilibrium unravels a la Rothschild and Stiglitz. If the low type is unwilling to do so, the equilibrium unravels a la Akerlof.

In the two type model, the assumption that the bad risk experiences the loss with certainty is clearly restrictive. However, more generally the full support assumption is quite weak. It imposes no restriction on the mass of types anywhere in the distribution. For example, it is satisfied if insurers believe there is a one-in-a-million chance that an applicant might know s/he will experience the loss with arbitrarily high probability. Indeed, any finite type distribution (e.g. the two-type model of Rothschild and Stiglitz) can be approximated arbitrarily closely with type distributions that have full support. In this sense, the existence of pure strategy competitive equilibria of the type found by Rothschild and Stiglitz (1976) that yield outcomes other than the endowment is a knife-edge result. This highlights the importance of recent and future work to aid in our understanding of how best to model competition in insurance markets.

2 Model

Agents have wealth w and face a potential loss of size l which occurs with probability p, which is distributed in the population according to the c.d.f. $F(p)$ with support Ψ.[1] In contrast to Rothschild

[1]The model is adapted from Hendren (ming), which derives the no-trade condition analogue of Akerlof in the binary loss environment but does not provide any discussion of competitive equilibriums.

and Stiglitz (1976), I do not impose any restrictions on $F(p)$.[2] It may be continuous, discrete, or mixed. I let P denote the random variable with c.d.f. $F(p)$, so that realizations of P are denoted with lower-case p. Agents of type p have vNM preferences given by

$$pu(c_L) + (1-p)\,u(c_{NL})$$

where u is increasing and strictly concave, c_L (c_{NL}) is consumption in the event of (no) loss. I define an allocation to be a set of consumption bundles, c_L and c_{NL}, for each type $p \in \Psi$, $A = \{c_L(p), c_{NL}(p)\}_{p\in\Psi}$.

I assume there exists a large set of risk-neutral insurance companies, J, which each can offer menus of contracts $A_j = \left\{c_L^j(p), c_{NL}^j(p)\right\}_{p\in\Psi}$ to maximize expected profits[3]. Following Rothschild and Stiglitz (1976), I define a Competitive Nash Equilibrium as an equilibrium of a two stage game. In the first stage, insurance companies offer contract menus, A_j. In the second stage, agents observe the total set of consumption bundles offered in the market, $A^U = \cup_{j\in J} A_j$, and choose the bundle which maximizes their utility. The outcome of this game can be described as an allocation which satisfies the following constraints.

Definition 1. An allocation $A = \{c_L(p), c_{NL}(p)\}_{p\in\Psi}$ is a **Competitive Nash Equilibrium** if

1. A makes non-negative profits

$$\int_{p\in\Psi} \left[p\,(w - l - c_L(p)) + (1-p)\,(w - c_{NL}(p))\right] dF(p) \geq 0$$

2. A is incentive compatible

$$pu(c_L(p)) + (1-p)\,u(c_{NL}(p)) \geq pu(c_L(\tilde{p})) + (1-p)\,u(c_{NL}(\tilde{p})) \quad \forall p, \tilde{p} \in \Psi$$

3. A is individually rational

$$pu(c_L(p)) + (1-p)\,u(c_{NL}(p)) \geq pu(w-l) + (1-p)\,u(w) \quad \forall p \in \Psi$$

4. A has no profitable deviations: For any $\hat{A} = \{\hat{c}_L(p), \hat{c}_{NL}(p)\}_{p\in\Psi}$, it must be that

$$\int_{p\in D(\hat{A})} \left[p\,(w - l - c_L(p)) + (1-p)\,(w - c_{NL}(p))\right] dF(p) \leq 0$$

where

$$D\left(\hat{A}\right) = \left\{p \in \Psi \middle| \max_{\hat{p}} \left\{pu(\hat{c}_L(\hat{p})) + (1-p)\,u(\hat{c}_{NL}(\hat{p}))\right\} > pu(c_L(p)) + (1-p)\,u(c_{NL}(p))\right\}$$

[2] To my knowledge, Riley (1979) was the first paper to discuss this environment with a continuum of types.

[3] In contrast to Rothschild and Stiglitz (1976), I allow the insurance companies to offer menus of consumption bundles, consistent with the real-world observation that insurance companies offer applicants menus of premiums and deductibles.

The first three constraints require that a Competitive Nash Equilibrium must yield non-negative profits, must be incentive compatible, and must be individually rational. The last constraint rules out the existence of profitable deviations by insurance companies. For A to be a competitive equilibrium, there cannot exist another allocation that an insurance company could offer and make positive profits on the (sub)set of people who would select the new allocation (given by $D\left(\hat{A}\right)$).

2.1 Mutually Exclusive Occurrences

I first show that, in this model, the insurance market has the potential to unravel in the sense of Akerlof (1970).

Theorem 1. *The endowment, $\{(w, w-l)\}_{p\in\Psi}$, is the unique Competitive Nash Equilibrium if and only if*

$$\frac{p}{1-p}\frac{u'(w-l)}{u'(w)} \leq \frac{E[P|P\geq p]}{1-E[P|P\geq p]} \quad \forall p \in \Psi\backslash\{1\} \tag{1}$$

Proof. The no-trade theorem of Hendren (ming) shows that Condition (1) characterizes when the endowment is the only allocation satisfying incentive compatibility, individual rationality, and non-negative profits. Now, suppose $A = \{(w-l, l)\}$ and consider any allocation, $\hat{A} \neq \{(w-l, l)\}_{p\in\Psi}$. Suppose $\hat{A}$ delivers positive profits. Because A is the endowment, I can WLOG assume all agents choose $\hat{A}$ (since $\hat{A}$ can provide the endowment to types p at no cost). But then $\hat{A}$ would be an allocation other than A satisfying incentive compatibility, individual rationality, and non-negative profits, contradicting the no-trade theorem of Hendren (ming). $\square$

The market unravels a la Akerlof (1970) if and only if no one is willing to pay the pooled cost of worse risks in order to obtain some insurance. This is precisely the logic of Akerlof (1970) but provided in an environment with an endogenous contract space. When Condition (1) holds, no contract or menu of contracts can be traded because they would not deliver positive profits given the set of risks that would be attracted to the contract. This is precisely the unraveling intuition provided in Akerlof (1970) in which the demand curve lies everywhere below the average cost curve. Notice that when this no-trade condition holds, the endowment is indeed a Nash equilibrium. Since no one is willing to pay the pooled cost of worse risks to obtain insurance, there exist no profitable deviations for insurance companies to break the endowment as an equilibrium.

Theorem 1 also shows that whenever the no-trade condition holds, there must exist a Competitive Nash Equilibrium. Thus, whenever the market unravels a la Akerlof (1970), the competitive equilibrium cannot unravel a la Rothschild and Stiglitz (1976). Unraveling in the sense of Akerlof (1970) is a mutually exclusive occurrence from unraveling in the sense of Rothschild and Stiglitz (1976).

Two-type case To relate to previous literature, it is helpful to illustrate how Theorem 1 works in the canonical two-type model of Rothschild and Stiglitz (1976). So, let $\Psi = \left\{p^L, p^H\right\}$ with $p^H > p^L$ denote the type space and let λ denote the fraction of types p^H. When $p^H < 1$, Corollary 1 of Hendren (ming) shows that the market cannot unravel a la Akerlof.[4] Hence, the mutual exclusivity of

[4] If $p^H < 1$, then equation (1) would be violated at $p^H = 1$ by the assumption of strict concavity of u.

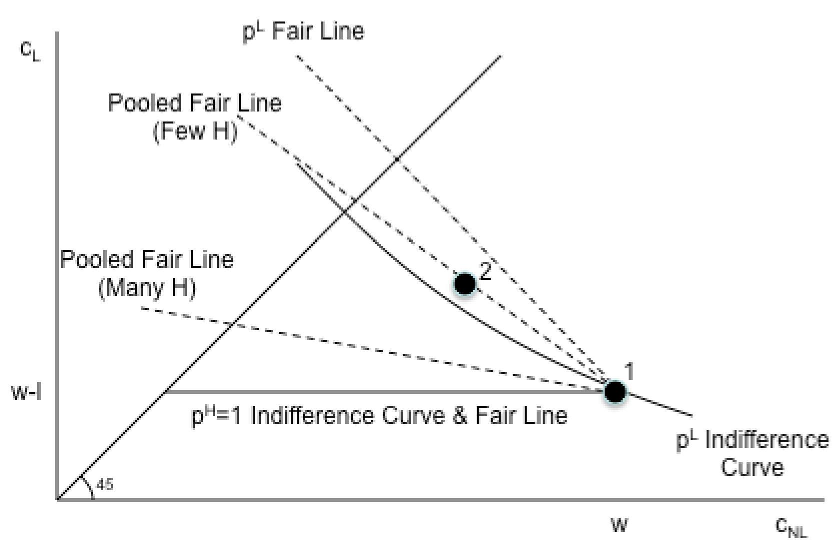
Akerlof and Rothschild and Stiglitz holds trivially. But, when $p^H = 1$, the situation is perhaps more interesting. To see this, Figure 1 replicates the canonical Rothschild and Stiglitz (1976) graphs in the case when $p^H = 1$.

The vertical axis is consumption in the event of a loss, c_L; the horizontal axis is consumption in the event of no loss, c_{NL}. Point 1 is the endowment $\{w - l, w\}$. Because $p^H = 1$, the horizontal line running through the endowment represents both the indifference curve of type p^H and the actuarially fair line for type p^H. Notice that type p^H prefers any allocation bundle that lies above this line (intuitively, she cares only about consumption in the event of a loss).

The low type indifference curve runs through the endowment (point 1) and intersects the 45-degree line parallel to her actuarially fair line. As noted by Rothschild and Stiglitz (1976), the outcomes in this environment depend crucially on the fraction of low versus high types. Figure 1 illustrates the two cases. If there are few p^H types (λ is large), then point 2 is a feasible pooling deviation from the endowment. When such a deviation is feasible, unraveling a la Akerlof does not occur: the low type is willing to pay the pooled cost of the worse risks. But, the existence of such a deviation is precisely what breaks the existence of a competitive equilibrium in Rothschild and Stiglitz (1976). Point 2 involves pooling across types and cannot be a competitive equilibrium. Hence, if the market unravels a la Akerlof, the endowment is the unique competitive equilibrium. If the market unravels a la Rothschild and Stiglitz, there exists implementable allocations other than the endowment and Akerlof's notion of unraveling does not occur.

As one might gather from Figure 1, when types are arbitrarily close to 1, the only feasible competitive equilibrium is the endowment. I now make this point in the more general setting that does not require any mass of types at $p^H = 1$.

Figure 1



2.2 Exhaustive Occurrences

I now show that no only are these two notions of unraveling mutually exclusive, but they are also, exhaustive of the possibilities that can occur in model environments when the type distribution satisfies the following regularity condition.

Assumption 1. *(Full support near $p = 1$) $F(p) < 1$ for all $p < 1$*

Assumption 1 assumes that one cannot rule out the chance of risks arbitrarily close to $p = 1$. In other words, I assume there does not exist a highest risk type, $\bar{p}$, such that $\bar{p} < 1$.[5] With this assumption, an insurance company cannot offer any insurance contract other than the endowment without being worried it will be selected by more than one type. Thus, in order to provide insurance, types must be cross-subsidized.

I now show that competitive equilibriums cannot sustain cross-subsidization, an insight initially provided in Rothschild and Stiglitz (1976).

Lemma 1. *(Rothschild and Stiglitz (1976)) Suppose A is a Competitive Nash Equilibrium. Then*

$$pc_L(p) + (1-p)\,c_{NL}(p) = w - pl \quad \forall p \in \Psi$$

Proof. Suppose there exists p such that $pc_L(p) + (1-p)\,c_{NL}(p) > w - pl$. Assume without loss of generality that at least two firms are offering point A. Then an insurance company could offer a new allocation, $\hat{A}$, which provides the endowment to type p, so that type p now chooses the allocation offered by remaining firms (essentially dumping type p onto other insurance companies). Therefore, $pc_L(p)+(1-p)\,c_{NL}(p) \leq w-pl$ for all $p \in \Psi$. So, condition (1) implies $pc_L(p)+(1-p)\,c_{NL}(p) = w-pl$ for all p. □

Given Lemma 1, it is straightforward to see that there cannot exist any Competitive Nash Equilibrium other than the endowment since trade requires cross-subsidization toward types near $p = 1$.

Theorem 2. *Suppose Assumption 1 holds. Then, there exists a Competitive Nash Equilibrium if and only if Condition* (1) *holds.*

Proof. Suppose Condition (1) does not hold. Then, clearly there exists a profitable deviation from the endowment. But, Lemma 1 and Assumption 1 ensure that no allocation other than the endowment can be a Competitive Nash Equilibrium. □

When Assumption1 holds, trade requires risk types to be willing to enter risk pools which pool ex-ante heterogeneous types. Such ex-ante pooling is not possible in a Competitive Nash Equilibrium. So, when the no-trade condition (1) does not hold, there does not exist any Competitive Nash Equilibrium: the equilibrium unravels a la Rothschild and Stiglitz (1976).

[5]Note that this assumption can be satisfied with any small amount of mass of types; it is only a condition on the support of the distribution.

3 Conclusion

This memo uses a generalized binary model of insurance to highlight the distinction between Akerlof's notion of unraveling, in which an equilibrium exists in which no trade can occur, and Rothschild and Stiglitz' notion of unraveling, in which a standard notion of competitive equilibrium (pure strategy Nash) cannot exist. In the latter case, there are (Pareto) gains to trade; but when the type distribution has full support near $p = 1$, the realization of these gains to trade require cross-subsidization of types. Such cross-subsidization cannot be sustained under the canonical notion of competition.[6] Hence, Akerlof unraveling shows when private information can lead to the absence of trade in insurance markets. Rothschild and Stiglitz unraveling shows that the canonical model of competition (Nash equilibrium) is inadequate to describe the behavior of insurance companies in settings where there are potential gains to trade.

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[6] In the modified models of competition, proposed by Miyazaki (1977), Wilson (1977), or Spence (1978), such gains to trade will be realized in equilibrium.