Quantifying Liquidity and Default Risks of Corporate Bonds over the Business Cycle*

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Abstract

We introduce procyclical liquidity frictions into a structural model of corporate bond pricing with countercyclical macroeconomic fundamentals. When calibrated to the historical moments of default probabilities and empirical measures of bond liquidity, our model matches the observed credit spreads of corporate bonds as well as measures of non-default components including Bond-CDS spreads and bid-ask spreads. Our calibration focuses on both cross-sectional matching across credit ratings, and time-series matching over business cycle. A novel structural decomposition scheme is proposed to capture the interaction between liquidity and default in corporate bond pricing. We use this framework to quantitatively evaluate the effects of liquidity-provision policies during recession, and identify important economic forces that the previous reduced-form approach overlooked before.

Keywords: Positive Liquidity-Default Feedback, Procyclical Liquidity, Rollover Risk, Search in Over-The-Counter Market, Endogenous Default, Structural Models

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1. Introduction

It is well known that default risk only accounts for part of the pricing of corporate bonds. For example, Longstaff, Mithal, and Neis (2005) estimate that the default component explains about 50% of the spread between the yields of Aaa/Aa-rated bonds and Treasury bonds. Furthermore, Longstaff, Mithal, and Neis (2005) find that the non-default component of credit spreads is weakly related to the differential state tax treatment on corporate bonds and Treasury bonds. Rather, consistent with the fact that the secondary corporate bond market being illiquid (e.g., Edwards, Harris, and Piwowar (2007), Bao, Pan, and Wang (2011)), the non-default component is strongly related to measures of bond liquidity.

The literature on structural credit risk modeling has mostly focused on understanding the "default component" of credit spreads only. The "credit spread puzzle," first discussed by Huang and Huang (2012), refers to the finding that, when calibrated to match the observed default rates and recovery rates, traditional structural models have difficulty explaining the credit spreads for bonds rated investment grade and above. By introducing time-varying macroeconomic risks into the structural models, Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) are able to explain the default components of the credit spreads for investment-grade corporate bonds.[1] However, the significant non-default components in credit spreads still remain to be explained.

This paper attempts to provide a full resolution of the credit spread puzzle by quantitatively explaining both the default and non-default components of the credit spreads. It is commonly accepted that the non-default component of credit spreads is a premium to compensate investors for the liquidity risk when holding corporate bonds. There are two general empirical patterns for liquidity of corporate bonds. First, cross-sectionally, corporate bonds tend to be more liquid for bonds with higher credit ratings (e.g., Edwards, Harris, and Piwowar (2007); Bao, Pan, and Wang (2011)). Second, over business cycle, corporate bonds are less liquid

[1] Chen (2010) relies on the estimates of Longstaff, Mithal, and Neis (2005) to obtain the default component of the credit spread for Baa rated bonds, while Bhamra, Kuehn, and Strebulaev (2010) focus on the difference between Baa and Aaa rated bonds. The difference of spreads between Baa and Aaa rated bonds presumably takes out the common liquidity component, which is a widely used practice in the literature. This treatment is accurate only if the liquidity components for both bonds are the same, which is at odds with existing literatures on liquidity of corporate bonds, e.g., Edwards, Harris, and Piwowar (2007), Bao, Pan, and Wang (2011).

during economic downturns, and more so for riskier bonds (e.g., Dick-Nielsen, Feldhütter, and Lando (2011); Friewald, Jankowitsch, and Subrahmanyam (2012)). The cross-sectional pattern implies the importance of *endogenous liquidity* in modeling the non-default component of corporate bonds.

We follow He and Milbradt (2012) by introducing a secondary over-the-counter market search friction (a la Duffie, Gârleanu, and Pedersen (2005)) into a structural credit models with aggregate macroeconomic fluctuations (e.g., Chen (2010)). In our model, bond investors face the risk of uninsurable idiosyncratic liquidity shocks that drive up their costs for holding the bonds. Market illiquidity arises endogenously because to sell their bonds, investors have to search for dealers to intermediate transactions with other investors not yet hit by liquidity shocks. The dealers set bid-ask spreads to capture part of trading surplus, and default risk affects the liquidity discount of corporate bonds by influencing the outside option of the illiquid bond investors in the ensuing bargaining.

The endogenous liquidity is further amplified by the *endogenous default* decision of the equity holders, as shown in Leland and Toft (1996) and emphasized by He and Xiong (2012). A default-liquidity spiral arises in He and Milbradt (2012): when secondary market liquidity deteriorates, equity holders suffer heavier rollover losses in refinancing their maturing bonds and will consequently default earlier. This earlier default in turn worsens secondary bond market liquidity even further, and so on so forth. In contrast to He and Milbradt (2012) with constant parameters for secondary market liquidity, in this paper we explicitly allow for procyclical liquidity, which interacts with the cyclical variation in the firm's cash flows growth and aggregate risk prices.

As the goal of our structural model is to deliver quantitative results, allowing for time-varying macroeconomic risk is important in explaining the credit risk puzzle, as shown by Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010). We introduce state-dependent liquidity risk, which interacts with countercyclical macroeconomic risk prices and goes a long way in explaining the credit spread of corporate bonds. The fact that the economy spends considerably longer time in the good state than in the bad state, and therefore most bond transactions driven by liquidity shocks occur in the good state with a fairly liquid secondary bond market, does not necessarily imply a low liquidity risk of holding such bonds. This is because investors are most likely to get stuck with the illiquid bond precisely in recessions during which prices of risk are high, low recovery values prevail, and it takes a long time to sell the bond.

We follow the literature in calibrating the pricing kernel parameters over binary macroeconomic states (normal and recession) to fit key moments of asset prices. The parameters governing secondary bond market liquidity over macroeconomic states are calibrated based on existing empirical studies and TRACE (e.g., bond turnovers and bid-ask spreads). In our model, liquidity of corporate bonds requires compensation, either because of the liquidity premium where investors face uninsurable idiosyncratic liquidity shocks on holding costs, or because of the liquidity risk premium so that the secondary market liquidity worsens (e.g., the meeting intensity with dealers goes down) in recession during which the marginal utility is high.

We apply our model to corporate bonds across four credit rating classes (Aaa/Aa, A, Baa, and Ba) and two different time-to-maturities (both 5-year and 10-year bonds). In addition to the two common measures — cumulative default probabilities and credit spreads — that the previous literature on corporate bonds calibration (e.g., Huang and Huang (2012)) has focused on, modeling bond market liquidity allows us to investigate the model's quantitative performance in matching two empirical measures of non-default risk for corporate bonds. The first measure is Bond-CDS spreads, defined as the bond's credit spread minus the Credit Derivative Swap (CDS) spread; this is motivated by Longstaff, Mithal, and Neis (2005) who argue that CDS contracts mostly price the default risk of bonds because of their more liquid secondary market relative to that of corporate bonds. The second measure is bid-ask spreads for bonds of different ratings, and we compare our model implied bid-ask spreads to those documented in Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2011). These two measures crucially rely on secondary market illiquidity: in a model with a perfectly liquid bond market, both the implied Bond-CDS spread and bid-ask spread will be zero.

By adopting the over-the-counter search modeling, our model focuses on trading liquidity of corporate bonds while missing the funding liquidity, i.e., the ability of using bonds as collateral in securing financing. Indeed, one leading concern for using Bond-CDS spreads, which adopts Treasuries as the default-free and illiquidity-free benchmark,[2] is that Treasuries enjoy certain liquidity premia that are not captured by our search-based model (e.g., Treasuries have the lowest hair cut in collateralized financing). To address this concern, we separate the risk-free rate and the Treasury yield by allowing for state-dependent liquidity premium for Treasuries. In calibration, we proxy this liquidity premium by repo-Treasury spreads observed in the data.

Since it is well-known that CDS market is most liquid for 5-year contracts, our calibration focuses on bonds with 5-year maturity. We are able to match the empirical pattern of credit spreads for 5-year bonds, both cross-sectionally across credit ratings and time-series matching over business cycle.[3] On the dimension of non-default risk, endogenously linking bond liquidity to a firm's distance-to-default allows us to generate the cross-sectional and business-cycle patterns in both Bond-CDS spreads and bid-ask spreads. Overall, relative to the data, our model produces less variation in Bond-CDS spreads across rating classes, and future research incorporating heterogenous funding liquidity across rating classes should help in this regard. Finally, the matching on 10-year bonds is less satisfactory, in that our model features a steeper term structure of credit spreads and Bond-CDS spreads than the data suggests.

Our model has important implications in understanding the role of default and liquidity in determining a firm's borrowing cost. A common practice in the empirical literature is to decompose credit spreads into a liquidity and a default component, which naturally leads to the interpretation that these components are independent of each other. Our model suggests that both liquidity and default are endogenously linked, and thus there can be economically significant interaction terms. These dynamic interactions are difficult to capture

[2]Our *Bond-CDS spread* is defined as the corporate bond yield minus the Treasury yield with the same maturity, and then minus its corresponding CDS spread. Another closely related measure widely used among practitioners and academic researchers is *Bond-CDS basis*. The only difference is on the choice of risk-free benchmark: our Bond-CDS spread takes the treasury rate as the benchmark, while Bond-CDS basis takes the interest rate swap rate. Interest rate swap gives a more accurate measure of an arbitrager's financing cost, and recent studies on Bond-CDS basis focus on limits-to-arbitrage (e.g., Gârleanu and Pedersen (2011), Bai and Collin-Dufresne (2012).) For our paper, treasury is a better default-free benchmark because the interest rate swap is contaminated by the default risk of LIBOR. Treasury also serves as the illiquidity-free benchmark, where "liquidity" are trading liquidity and market illiquidity captured by our model.

[3]Our calibration on aggregate macroeconomic states focuses on normal expansions and recessions, but not crises. As a result, in constructing empirical moments for recessions, we exclude the 2008 crises period from October 2008 to March 2009 throughout.

using reduced-form models with *exogenously* imposed liquidity premia.

We propose a structural decomposition that nests the common additive default-liquidity decomposition to quantify the interaction between default and liquidity for corporate bonds. Motivated by Longstaff, Mithal, and Neis (2005) who use CDS spread to proxy for default risk, we identify the "default" part by pricing a bond in a counterfactually perfectly liquid market but with the model implied default threshold. After subtracting this "default" part, we identify the remaining credit spread as the "liquidity" part. We further decompose the "default" ("liquidity") part into a "pure default" ("pure liquidity") component and a "liquidity-driven-default" ("default-driven liquidity") component, where the "pure default" or "pure liquidity" part is the spread implied by a counterfactual model where either the bond market is perfectly liquid as in Leland and Toft (1996) hence equity holders default less often, or only the over-the-counter search friction is at work for default-free bonds as in Duffie, Gârleanu, and Pedersen (2005), respectively. The two interaction terms that emerge, i.e., the "liquidity-driven default" and the "default-driven liquidity" components, capture the endogenous positive spiral between default and liquidity. For instance, "liquidity-driven-default" is driven by the rollover risk mechanism in that firms relying on finite-maturity debt financing will default earlier when facing worsening secondary market liquidity.

Besides giving a more complete picture of how the default and liquidity forces affect credit spreads, our structural decomposition also offers important insight on evaluating hypothetical government policies, as it is important to fully take into account of how an individual firm's default responds to liquidity conditions. Imagine a policy that makes the secondary market in recession as liquid as in normal times, which lowers the credit spread of Ba rated bonds in recession by about 137 bps (about 29% of the spread). The liquidity-driven default part, which captures lower default risk from firms with mitigated rollover losses, can explain 27% of this drop. The default-driven liquidity part, which captures the endogenous reduction of liquidity premium for safer bonds, can explain about 17%. The prevailing view in the literature masks this interdependence between default and liquidity and thus tends to miss these interaction terms.

The paper is structured as follows. Section 2 introduces the model, which is solved in Section 3. Section 4 presents the main calibration. Section 5 discusses the model-based default-liquidity decomposition, and analyzes the effectiveness of a policy geared towards liquidity provision from the perspective of our decomposition. Section 6 concludes. The appendix provides proofs and a more general formulation of the model.

2. The Model

2.1 Aggregate States and the Firm

The following model elements are similar to Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010), except that we study the case in which firms issue bonds with an average finite maturity a la Leland (1998) so that rollover risk in He and Xiong (2012) is present.

2.1.1 Aggregate states and stochastic discount factor

The aggregate state of the economy is described by a continuous time Markov chain, with the current Markov state denoted by s_t and the physical transition density between state i and state j denoted by $\zeta_{ij}^{\mathcal{P}}$. We assume an exogenous stochastic discount factor (SDF):

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s_t)dt - \eta\left(s_t\right)dZ_t^m + \sum_{s_t \neq s_{t-}} \left(e^{\kappa\left(s_{t-}, s_t\right)} - 1\right) dM_t^{\left(s_{t-}, s_t\right)}, \tag{1}$$

where $\eta\left(\cdot\right)$ is the state-dependent price of risk for Brownian shocks, $dM_t^{(j,k)}$ is a compensated Poison process capturing switches between states, and $\kappa\left(i,j\right)$ embeds the jump risk premia such that in the risk neutral measure, the distorted jump intensity between states is $\zeta_{ij}^{\mathcal{Q}} = e^{\kappa(i,j)}\zeta_{ij}^{\mathcal{P}}$.

In this paper we focus on the case with binary aggregate states to capture the notion of economic expansions and recessions, i.e., $s_t \in \{G, B\}$. In the Appendix we provide the general setup for the case with $n > 2$ aggregate states.

2.1.2 Firm cash flows and risk neutral measure

A firm has assets in place that generate cash flows at the rate of Y_t. Under the physical measure $\mathcal{P}$, the cash-flow rate Y_t follows, given the aggregate state s_t,

$$\frac{dY_t}{Y_t} = \mu_{\mathcal{P}}(s_t)\,dt + \sigma_m(s_t)\,dZ_t^m + \sigma_f dZ_t^f. \tag{2}$$

Here, dZ_t^m captures aggregate Brownian risk, while dZ_t^f captures idiosyncratic Brownian risk. Given the stochastic discount factor Λ_t, risk neutral cash-flow dynamics under the risk neutral measure $\mathcal{Q}$ follow

$$\frac{dY_t}{Y_t} \quad = \quad \mu_{\mathcal{Q}}(s)\,dt + \sigma(s)\,dZ_t^{\mathcal{Q}},$$

where $Z_t^{\mathcal{Q}}$ is a Brownian Motion under the risk-neutral measure $\mathcal{Q}$. The state-dependent risk-neutral cash-flow drift and volatility are given by

$$\mu_{\mathcal{Q}}^s \equiv \mu_{\mathcal{P}}(s) - \sigma_m(s)\,\eta(s), \text{ and } \sigma_s \equiv \sqrt{\sigma_m^2(s) + \sigma_f^2}.$$

For ease of notation, we work with log cash flows $y \equiv \log(Y)$ throughout. Define

$$\mu_s \equiv \mu_{\mathcal{Q}}^s - \frac{1}{2}\sigma_s^2 = \mu_{\mathcal{P}}(s) - \sigma_m(s)\,\eta(s) - \frac{1}{2}\left(\sigma_m^2(s) + \sigma_f^2\right)$$

so that we have

$$dy_t = \mu_s dt + \sigma_s dZ_t^{\mathcal{Q}}. \tag{3}$$

From now on we work under measure $\mathcal{Q}$ unless otherwise stated, so we drop the superscript $\mathcal{Q}$ in $dZ_t^{\mathcal{Q}}$ and $\zeta_{ij}^{\mathcal{Q}}$ to simply write dZ_t and ζ_{ij} where no confusion can arise.

As standard in the asset pricing literature, we can obtain valuations for any asset as the expected discounted cash flows under the risk neutral measure $\mathcal{Q}$. The unlevered firm value, given the aggregate state s and the cash-flow rate y, is

$$\mathbf{v}_U(y) \equiv \begin{bmatrix} r_G - \mu_G + \zeta_G & -\zeta_G \\ -\zeta_B & r_B - \mu_B + \zeta_B \end{bmatrix}^{-1} \mathbf{1} \exp(y). \tag{4}$$

We will use v_U^s to denote the element of $\mathbf{v}_U$ in state s.

There is one caveat in applying the risk neutral pricing to bond valuations, as later we will introduce undiversifiable idiosyncratic liquidity shocks to bond investors. Because we model liquidity shocks as holding costs which can be interpreted as negative dividends, the risk neutral pricing for bonds with holding-cost adjusted cash flows is still valid provided that the bond holding is infinitesimal in the representative investor's portfolio.[4]

2.1.3 Firm's debt maturity structure and rollover frequency

The firm has bonds in place of measure 1 which are identical except for their time to maturity, and thus the aggregate and individual bond coupon (face value) is c (p). As in Leland (1998), equity holders commit to keeping the aggregate coupon and outstanding face-value constant before default, and thus issue new bonds of the same average maturity as the bonds maturing.

Each bond matures with intensity m, and the maturity event is i.i.d. across individual bonds. Thus, by law of large numbers over $[t, t+dt)$ the firm retires a fraction $m \cdot dt$ of its bonds. This implies an expected average debt maturity of $\frac{1}{m}$. The deeper implication of this assumption is that the firm adopts a "smooth" debt maturity structure with an average refinancing/rollover frequency of m. As shown later, the rollover frequency (at the firm level) is important for secondary market liquidity to affect a firm's endogenous default decisions.

2.2 Secondary Over-the-Counter Corporate Bond Market

We follow He and Milbradt (2012) and Duffie, Gârleanu, and Pedersen (2005) in modeling the over-the-counter corporate bond market. Individual bond holders are subject to liquidity shocks that entail a positive holding cost. Bond holders hit by liquidity shocks will try to sell

[4] Intuitively, if the representative agent's consumption pattern is not affected by the idiosyncratic shock brought on by the bond holdings (which is true if the bond holding is infinitesimal relative to the rest of the portfolio), then the representative agent's pricing kernel is independent of idiosyncratic undiversified shocks.

by searching for dealers in the over-the-counter secondary market, and transaction prices are determined by bargaining with a dealer once a contact is established. Bond investors can hold either zero or one unit of the bond. They start in the H state without any holding cost when purchasing corporate bonds in the primary market. As time passes by, H-type bond holders are hit independently by idiosyncratic liquidity shocks with intensity ξ_s, which leads them to become L-types who bear a positive holding cost χ_s per unit of time.

There is a trading friction in moving the bonds from L-type sellers to H-type buyers without bond holdings, in that trades have to be intermediated by dealers in the over-the-counter market. Sellers meet dealers with intensity λ_s, which we interpret as the intermediation intensity of the financial sector. For simplicity, we assume that after L-type investors sell their holdings, they exit the market forever. The H-type buyers on the sideline currently not holding the bond also contact dealers with intensity λ_s. We follow Duffie, Gârleanu, and Pedersen (2007) to assume Nash-bargaining weights β for the investor and $1-\beta$ for the dealer across all dealer-investor pairs.

Dealers use the competitive (and instantaneous) interdealer market to sell or buy bonds. When a contact between a type L seller and a dealer occurs, the dealer can instantaneously sell a bond at a price M to another dealer who is in contact with an H investor via the interdealer market. If he does so, the bond travels from an L investor to an H investor via the help of the two dealers who are connected in the inter-dealer market.

Fixing any aggregate state s, denote by D_l^s the individual bond valuation for the investor with type $l \in \{H, L\}$. Denote by B^s the bid price at which the L type is selling his bond, by A^s the ask price at which the H type is purchasing this bond, and by M^s the inter-dealer market price.

Following He and Milbradt (2012) we assume that the flow of H-type buyers contacting dealers is greater than the flows of L-type sellers contacting dealers; in other words, the secondary market is a *seller's market*. Similar to Duffie, Gârleanu, and Pedersen (2005) and He and Milbradt (2012), we have the following proposition. Essentially, Bertrand competition, the holding restriction, and excess demand from buyer-dealer pairs in the interdealer market drive the surplus of buyer-dealer pairs to zero.[5]

[5]This further implies that the value function of buyers without bond holdings who are sitting on the

Proposition 1. *Fix valuations D_H^s and D_L^s, and denote the surplus from trade by $\Pi^s = D_H^s - D_L^s > 0$. In equilibrium, the ask price A^s and inter-dealer market price M^s are equal to D_H^s, and the bid price is given by $B^s = \beta D_H^s + (1-\beta) D_L^s$. The dollar bid ask spread is $A^s - B^s = (1-\beta)(D_H^s - D_L^s) = (1-\beta)\Pi^s$.*

Empirical studies focus on the proportional bid-ask spread which is defined as the dollar bid-ask spread divided by the mid price, i.e.,

$$\Delta^s(y,\tau) = \frac{2(1-\beta)(D_H^s - D_L^s)}{(1+\beta)D_H^s + (1-\beta)D_L^s} = \frac{(1-\beta)\Pi^s}{D_H^s - \frac{1-\beta}{2}\Pi^s}. \tag{5}$$

2.3 State Transition

As notational conventions, we use capitalized bold-faced letters (e.g., $\mathbf{X}$) to denote matrices, lower case bold face letters (e.g. $\mathbf{x}$) to denote vectors, and non-bold face letters denote scalars (e.g. x). The only exceptions are the value functions for debt and equity, $\mathbf{D}, \mathbf{E}$ respectively, which will be vectors, and the (diagonal) matrix of drifts, $\boldsymbol{\mu}$. Dimensions for most objects are given underneath the expression. While we focus on 2-aggregate-state case where $s \in \{G, B\}$, the Appendix presents general results for an arbitrary number of (Markov) aggregate states.

Denote by $\mathbf{Q}$ the Markov-transition matrix for both individual and aggregate states, where each entry $q_{ls \to l's'}$ is the intensity of transitioning from (individual) liquidity state l to l' where $l, l' \in \{H, L\}$ and from aggregate state s to s' where $s, s' \in \{G, B\}$. The transition matrix $\mathbf{Q}$ can be written as:[6]

$$\underbrace{\mathbf{Q}}_{4\times 4} \equiv \begin{bmatrix} -\xi_G - \zeta_G & \xi_G & \zeta_G & 0 \\ \beta\lambda_G & -\beta\lambda_G - \zeta_G & 0 & \zeta_G \\ \zeta_B & 0 & -\xi_B - \zeta_B & \xi_B \\ 0 & \zeta_B & \beta\lambda_B & -\beta\lambda_B - \zeta_B \end{bmatrix}. \tag{6}$$

sideline is identically zero, which makes the model tractable. Introducing for example direct bilateral trades or assuming a *buyer's market* would both entail tracking the value functions of investors on the sideline but would not add additional economic insights pertaining to credit risk in particular.

[6]Our intensity-based modeling rules out the possibility of coinciding jumps in the aggregate and individual states, so that $q_{ls \to l's'} = 0$ if $l \neq l'$ and $s \neq s'$). Economically, this implies that the adverse aggregate shock can bring about more liquidity shocks to individual bond holders given any time interval, although these shocks are still i.i.d across individuals.

The entry $q_{Ls\to Hs} = \lambda_s\beta$ in the above transition matrix requires further explanation. Given the aggregate state s, recall that we have assumed that the intensity of switching from state-H to state-L is ξ_s, and the L-state is absorbing, i.e., those L-type investors leave the market forever. However, an L-type bond holder meets a dealer with intensity λ_s and sells the bond for $B^s = \beta D_H^s + (1-\beta) D_L^s$ that he himself values at D_L^s (see Proposition 1). Then the L-type's intensity-modulated surplus when meeting the dealer can be rewritten as

$$\lambda_s \left(B^s - D_L^s\right) = \lambda_s\beta \left(D_H^s - D_L^s\right).$$

As a result, for the purpose of pricing, the "effective" transitioning intensity from L-type to H-type is $q_{Ls\to Hs} = \lambda_s\beta$ where λ_s is the state-dependent intermediation intensity and β is the investor's bargaining power.

2.4 Delayed Bankruptcy Payouts and Effective Recovery Rates

In Leland-type frameworks, when the firm's cash flow deteriorates, equity holders are willing to repay the maturing debt holders only when the equity value is still positive, i.e. the option value of keeping the firm alive justifies absorbing rollover losses and coupon payments. The firm defaults when its equity value drops to zero at some default threshold y_{def}, which is endogenously chosen by equity holders. As in Chen (2010), we will impose bankruptcy costs as a fraction $1-\hat{\alpha}_s$ of the value from unlevered assets $v_U^s\left(y_{def}\right)$ given in (4), where the debt holder's bankruptcy recovery $\hat{\alpha}_s$ may depend on the aggregate state s.

As emphasized in He and Milbradt (2012), because the driving force of liquidity in our model is that agents value receiving cash early, our bankruptcy treatment has to be careful in this regard (and different from typical Leland models). If bankruptcy leads investors to receive the bankruptcy proceeds immediately, then bankruptcy confers a "liquidity" benefit similar to a bond maturing. This "expedited payment" benefit runs counter to the fact that in practice bankruptcy leads to the freezing of assets within the company and a delay in the payout of any cash depending on court proceeding.[7] Moreover, bond investors with

[7] For evidence on inefficient delay of bankruptcy resolution, see Gilson, John, and Lang (1990) and Ivashina, Smith, and Iverson (2013). In addition, the Lehman Brothers bankruptcy in September 2008 is a good case in point. After much legal uncertainty, payouts to the debt holders only started trickling out after about

defaulted bonds may face a much more illiquid secondary market (e.g., Jankowitsch, Nagler, and Subrahmanyam (2013)), and potentially a much higher holding cost once liquidity shocks hit due to regulatory or charter restrictions which prohibit institutions to hold defaulted bonds.

To capture above features, we assume that a bankruptcy court delay leads the bankruptcy cash payout $\hat{\alpha}_s v_U^s < p$ to occur at a Poisson arrival time with intensity θ,[8] where we simply denote $v_U^s(y_{def})$ by v_U^s. The holding cost of defaulted bonds for L-type investors is $\chi_{def}^s v_U^s$ where $\chi_{def}^s > 0$, and the secondary over-the-counter market for defaulted bonds is illiquid with contact intensity λ_{def}^s. Denote the values of defaulted bonds by $D_H^{s,def}$ and $D_L^{s,def}$, which satisfy

$$\begin{aligned} r_s D_H^{s,def} &= \theta\left[\hat{\alpha}_s v_U^s - D_H^{s,def}\right] + \xi_s\left[D_L^{s,def} - D_H^{s,def}\right] + \zeta_s\left[D_H^{s',def} - D_H^{s,def}\right] \\ r_s D_L^{s,def} &= -\chi_{def}^s v_U^s + \theta\left[\hat{\alpha}_s v_U^s - D_L^{s,def}\right] + \lambda_{def}^s \beta\left[D_H^{s,def} - D_L^{s,def}\right] + \zeta_s\left[D_L^{s',def} - D_L^{s,def}\right] \end{aligned} \quad (7)$$

Take $D_L^{s,def}$ for example: the first term is the illiquidity holding cost, the second term captures the bankruptcy payout, the third term captures trading the defaulted bonds with dealers, and the last term captures the jump of the aggregate state.

In equation (7) we have assumed that the cash-flow rate y remains constant at y_{def} (through $v_U^s(y_{def})$) during bankruptcy procedures, a simplifying assumption that can be easily relaxed.[9] Defining $\mathbf{D}^{def} \equiv \left[D_H^{G,def}, D_L^{G,def}, D_H^{B,def}, D_L^{B,def}\right]^\top$, it is easy to show that[10]

$$\underbrace{\mathbf{D}^{def}(y)}_{4\times 1} = \mathrm{diag}\left(\left[\begin{array}{cccc} v_U^G(y) & v_U^G(y) & v_U^B(y) & v_U^B(y) \end{array}\right]\right) \underbrace{\left(\mathbf{R} - \mathbf{Q}_{def} + \theta\mathbf{I}\right)^{-1}\left(\theta\hat{\boldsymbol{\alpha}} - \boldsymbol{\chi}_{def}\right)}_{\equiv\boldsymbol{\alpha}}, \quad (8)$$

where $\mathbf{R} \equiv \mathrm{diag}([r_G\ r_B])$, $\boldsymbol{\chi}_{def} \equiv [0, \chi_{def}(G), 0, \chi_{def}(B)]^\top$, and where $\mathbf{Q}_{def}$ is the post-

three and a half years.

[8]We could allow for a state-dependent bankruptcy court delay, i.e., $\theta(s)$; but the Moody's Ultimate Recovery Dataset reveals that there is little difference between the recovery time in good time versus bad time.

[9]We have identical results if instead we assume that y evolves as in (3), and debt holders receive the entire payout (net bankruptcy cost) of αv_U^s eventually. The values of defaulted bonds will be slightly lower if we take into account that equity holders receive some payouts in the event of $\alpha v_U^s > p$, but one can derive the formula of $D_H^{s,def}$ and $D_L^{s,def}$ in closed form.

[10]Throughout, diag$(\cdot)$ is the diagonalization operator mapping any row or column vector into a diagonal matrix (in which all off-diagonal elements are identically zero).

default counterpart of $\mathbf{Q}$ in (6).

In equation (8), for easier comparison to existing Leland-type models where debt recovery at bankruptcy is simply $\hat{\alpha} v_U$, we denote the (bold face) vector $\boldsymbol{\alpha} \equiv \left[\alpha_H^G, \alpha_L^G, \alpha_H^B, \alpha_L^B\right]^\top$ as the *effective* bankruptcy recovery rates at the time of default. We will have $\alpha_H^s > \alpha_L^s$ to capture the fact that default is more costly to L-type investors.

These effective bankruptcy recovery factors $\boldsymbol{\alpha}$ are determined by the post-default corporate bond market structures;[11] and they are the only critical ingredients for us to solve for the pre-default bond valuations, as well as its secondary market liquidity. In calibration, we will not rely on deeper structural parameters (say, post-default holding cost χ_{def}). Instead, we choose these effective recovery rates $\boldsymbol{\alpha}$ to target both the market price of defaulted bonds observed immediately after default (which are close to L-type valuations) and the associated empirical bid-ask spreads.

3. Model Solutions and Bond-CDS Spread

Denote by $D_l^{(s)}$ the l-type bond value in aggregate state s, $E_l^{(s)}$ the equity value in aggregate state s, and $\boldsymbol{y}_{def} = \left[y_{def}^G, y_{def}^B\right]^\top$ the vector of endogenous default boundaries. We derive the closed-form solution for debt and equity valuations in this section as a function of a given $\boldsymbol{y}_{def}$, along with the characterization of endogenous default boundaries $\boldsymbol{y}_{def}$.

3.1 Debt Valuations

Because equity holders will default earlier in state B, i.e., $y_{def}^G < y_{def}^B$, the domains of debt valuations change when the aggregate state switches. We deal with this issue by the following treatment; see the Appendix for the generalization of this analysis.

Define two intervals $I_1 = \left[y_{def}^G, y_{def}^B\right]$ and $I_2 = \left[y_{def}^B, \infty\right)$, and denote by $D_l^{s,i}$ the restriction of D_l^s to the interval I_i, i.e., $D_l^{s,i}(y) = D_l^s(y)$ for $y \in I_i$. Clearly, $D_l^{B,1}(y) = \alpha_l^B v_U^B(y)$ is in the "dead" state, so that the firm immediately defaults in interval I_1 when switching into

[11] Interestingly, as emphasized in He and Milbradt (2012), because $\mathbf{v}_U(y_{def})$ depends on the endogenously determined bankruptcy boundary y_{def}, the dollar bid-ask spread of defaulted bonds is higher if the firm defaults earlier. Thus, the illiquidity of defaulted bonds relative to that of default-free bonds depends on the firm's pre-default parameters, exactly through the channel of endogenous default.

state B (from state G). In light of this observation, on interval $I_2 = \left[y_{def}^B, \infty\right)$ all bond valuations denoted by $\mathbf{D}^{(2)} = \left[D_H^{G,2}, D_L^{G,2}, D_H^{B,2}, D_L^{B,2}\right]^\top$ are "alive."

For bond valuation equations we simply treat holding costs given liquidity shocks as negative dividends, which effectively lower the coupon flows that investors are receiving. Moreover, we directly apply the pricing kernel (pricing under risk neutral measure $\mathcal{Q}$) given in (1) and make no risk adjustments on the liquidity shocks, which is justified by the assumption that the illiquid bond holding is infinitesimal in the representative investor's portfolio. For further discussions, see footnote 4 and the end of Section 2.1.2.

Proposition 2. *The bond values on interval i are given by*

$$\underbrace{\mathbf{D}^{(i)}}_{2i\times 1} = \underbrace{\mathbf{G}^{(i)}}_{2i\times 4i} \cdot \underbrace{\exp\left(\mathbf{\Gamma}^{(i)} y\right)}_{4i\times 4i} \cdot \underbrace{\mathbf{b}^{(i)}}_{4i\times 1} + \underbrace{\mathbf{k}_0^{(i)}}_{2i\times 1} + \underbrace{\mathbf{k}_1^{(i)}}_{2i\times 1} \exp(y), \tag{9}$$

where the matrices $\mathbf{G}^{(i)}$, $\mathbf{\Gamma}^{(i)}$ and the vectors $\mathbf{k}_0^{(i)}$, $\mathbf{k}_1^{(i)}$ and $\mathbf{b}^{(i)}$ are given in the Appendix 1.3.

3.2 Equity Valuations and Default Boundaries

When the firm refinances its maturing bonds, we assume that the firm can place newly issued bonds with H investors in a competitive primary market.[12] This implies that there are rollover gains/losses of $m\left[\mathbf{S}^{(i)} \cdot \mathbf{D}^{(i)}(y) - p\mathbf{1}_i\right]$ at each instant as a mass $m \cdot dt$ of debt holders matures on $[t, t+dt]$, where $\mathbf{S}^{(i)}$ is a $i\times 2i$ matrix that selects the appropriate D_H as we assumed the firm issues to H-type investors in the primary market. For instance, for $y \in I_2 = [y_{def}(B), \infty)$, we have $\mathbf{D}^{(2)} = \left[D_H^{G,2}, D_L^{G,2}, D_H^{B,2}, D_L^{B,2}\right]^\top$ and $\mathbf{S}^{(2)} = (1-\omega)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, where $\omega \in (0,1)$ is the proportional issuance costs in the primary corporate bond market.

The rollover term due to bond repricing enters the equity valuation. For ease of exposition, we denote by double letters (e.g. $\mathbf{xx}$) a constant for equity that takes a similar place as a single letter (i.e. $\mathbf{x}$) constant for debt. We can write down the valuation equation for equity

[12] This is consistent with our seller's market assumption in Section 2.2, i.e., there are sufficient H-type buyers waiting on the sidelines.

on interval I_i. For instance, on interval I_2 we have

$$\underbrace{\left(\mathbf{R}-\mathbf{QQ}^{(2)}\right)}_{2\times 2}\underbrace{\mathbf{E}^{(2)}(y)}_{2\times 1} = \underbrace{\boldsymbol{\mu\mu}^{(2)}}_{2\times 2}\underbrace{\left(\mathbf{E}^{(2)}\right)'(y)}_{2\times 1}+\frac{1}{2}\underbrace{\boldsymbol{\Sigma\Sigma}^{(2)}}_{2\times 2}\underbrace{\left(\mathbf{E}^{(2)}\right)''(y)}_{2\times 1}$$
$$+\underbrace{\mathbf{1}_2\exp(y)}_{Cashflow,2\times 1}-\underbrace{(1-\pi)c\mathbf{1}_2}_{Coupon,2\times 1}+\underbrace{m\left[\mathbf{S}^{(2)}\cdot\mathbf{D}^{(2)}(y)-p\mathbf{1}_2\right]}_{Rollover,2\times 1} \quad (10)$$

where the matrices $\boldsymbol{\mu\mu}$, $\boldsymbol{\Sigma\Sigma}$, $\mathbf{QQ}$ summarize the drifts, volatilities and transition probabilities of the system and are defined in the Appendix.

Proposition 3. *The equity value is given by*

$$\underbrace{\mathbf{E}^{(i)}(y)}_{i\times 1}=\underbrace{\mathbf{GG}^{(i)}}_{i\times 2i}\cdot\underbrace{\exp\left(\boldsymbol{\Gamma\Gamma}^{(i)}y\right)}_{2i\times 2i}\cdot\underbrace{\mathbf{bb}^{(i)}}_{2i\times 1}+\underbrace{\mathbf{KK}^{(i)}}_{i\times 4i}\underbrace{\exp\left(\boldsymbol{\Gamma}^{(i)}y\right)}_{4i\times 4i}\underbrace{\mathbf{b}^{(i)}}_{4i\times i}+\underbrace{\mathbf{kk}_0^{(i)}}_{i\times 1}+\underbrace{\mathbf{kk}_1^{(i)}}_{i\times 1}\exp(y) \text{ for } y\in I_i \quad (11)$$

where the matrices $\mathbf{GG}^{(i)}$, $\boldsymbol{\Gamma\Gamma}^{(i)}$, $\mathbf{KK}^{(i)}$, $\boldsymbol{\Gamma}^{(i)}$ *and the vectors* $\mathbf{kk}_0^{(i)}$, $\mathbf{kk}_1^{(i)}$ *and* $\mathbf{b}^{(i)}$ *are given in the Appendix 1.3.*

Finally, the endogenous bankruptcy boundaries $\boldsymbol{y}_{def}=\left[y_{def}^G,y_{def}^B\right]^\top$ are given by the standard smooth-pasting condition:

$$\left(\mathbf{E}^{(1)}\right)'\left(y_{def}^G\right)_{[1]} = 0, \text{ and } \left(\mathbf{E}^{(2)}\right)'\left(y_{def}^B\right)_{[2]}=0. \quad (12)$$

3.3 Model Implied Credit Default Swap

One of key empirical moments for bond liquidity used in this paper is the Bond-CDS spread, defined as Bond credit spread minus the spread of the corresponding Credit Default Swap (CDS). Since the CDS market is much more liquid than that of corporate bonds, following Longstaff, Mithal, and Neis (2005) we compute the model implied CDS spread under the assumption that the CDS market is perfectly liquid.[13]

Let τ (in years from today) be the time of default. Formally, $\tau\equiv\inf\{t: y_t\le y_{def}^{s_t}\}$ can be either the first time at which the cash-flow rate y_t reaches the default boundary y_{def}^s in state

[13] Arguably, the presence of CDS market will in general affect the liquidity of corporate bond market; but we do not consider this effect. A recent theoretical investigation by Oehmke and Zawadowski (2013) shows ambiguous results on this regard.

s, or when $y^G_{def} < y_t < y^B_{def}$ so that a change of state from G to B triggers default. Thus, for a T-year CDS contract, the required flow payment f is the solution to the following equation:

$$\mathbb{E}^{\mathcal{Q}}\left[\int_0^{\min[\tau,T]} \exp\left(-rt\right) f dt\right] = \mathbb{E}^{\mathcal{Q}}\left[\exp\left(-r\tau 1_{\{\tau\leq T\}}\right) LGD_{\tau}\right], \tag{13}$$

where LGD_{τ} is the loss-given-default when the default occurs at time τ. If there is no default, no loss-given-default is paid out by the CDS seller. The loss-given-default LGD is defined as the bond face value p minus its recovery value, and we follow the practice to define the recovery value as the transaction price right after default (with the mid price when the firm defaults at y^s_{def}). We calculate the required flow payment f that solves (13) using a simulation method. Finally, the CDS spread, f/p, is defined as the ratio between the flow payment f and the bond's face value p.

3.4 Liquidity Premium of Treasury

It has been widely recognized (e.g., Duffie (1996), Krishnamurthy (2002), Longstaff (2004)) that Treasuries, due to their special role in financial markets, are earning returns that are significantly lower than the risk-free rate, which in our model is represented by r_s in equation (1). The risk-free rate is the discount rate for future deterministic cash flows, whereas treasury yields also reflect the additional benefits of holding Treasuries relative to a generic default-free and easy-to-transact bond. The wedge between the two rates, which we term "liquidity premium of Treasuries", represents the convenience yield that is specific to Treasury bonds, e.g., the ability to post Treasuries as collateral with a significantly lower haircut than other financial securities. Although this broad collateral-related effect is empirically relevant, our model is not designed to capture this economic force.

Motivated by the above consideration, we assume that there are (exogenous) state-dependent liquidity premia Δ_s for Treasuries. Specifically, given the risk-free rate r_s in state s, the yield of Treasury bonds is simply $r_s - \Delta_s$. When calculating credit spreads of corporate bonds, we can use either the risk-free rate or the Treasury yield as the benchmark. In the latter case, there will be a baseline state-dependent Bond-CDS spread of Δ_s even for those illiquidity-free and default-free bonds.

As explained shortly, we calibrate Δ_s using the spread between 3-month general collateral repo rates and 3-month Treasury yields observed in the data, which is close to the values reported in Longstaff (2004).[14] Longstaff (2004) shows that the liquidity premium does not feature any significant term structure effect. Thus, we use this 3-month repo-Treasury spread to proxy for both 5-year and 10-year Treasury liquidity premium. Last but not least, from the perspective of our model, our ideal measure of liquidity premium should only capture the convenience yield of Treasuries for its ability of being posted as collateral. If we believe that repo contracts are not perfectly liquid in trading, the repo-Treasury spread might reflect the trading friction and thus probably gives an overestimate of the liquidity premium.

4. Calibration

4.1 Benchmark Parameters

We calibrate the parameters governing firm fundamentals and pricing kernels to the key moments of the aggregate economy and asset pricing. Parameters governing time-varying liquidity conditions are calibrated to their empirical counterparts on bond turnover, dealer's bargaining power, and observed bid-ask spreads.

[TABLE 1 ABOUT HERE]

4.1.1 SDF and cash flows liquidity parameters

We follow Chen, Xu, and Yang (2012) in calibrating firm fundamentals and investors' pricing kernel. Table 1 reports the benchmark parameters we use, which are standard in the literature. Start from investors' pricing kernel. The risk free rate is $r_G = r_B = 2\%$ in both aggregate states, so that we abstract from the standard term structure effect. Transition intensities give the duration of the business cycle (10 years for expansions and 2 years for recessions). Jump

[14] There are a few alternative ways to identify the Treasury liquidity premium. One could use Refcorp as a proxy for the risk-free rate as in Longstaff (2004), but that data is unavailable. By imposing a multi-factor affine model of Treasury bonds, corporate bonds, and swap rates, Feldhütter and Lando (2008) arrive at an estimate of the risk-free rate after taking out the default component in swap rates.

risk premium $\exp(\kappa) = 2$ in state G (and the state B jump risk premium is the reciprocal of that of state G) is consistent with a long-run risk model with Markov-switching conditional moments and calibrated to match the equity premium (Chen (2010)). The risk price η is the product of relative risk aversion γ and consumption volatility σ_c: $\eta = 0.165$ (0.255) in state G (state B) requires $\gamma = 10$ and $\sigma_c = 1.65\%$ ($\sigma_c = 2.55\%$).

On the firm side, the cash-flow growth is matched to the average growth rate of aggregate corporate profits. State-dependent systematic volatilities σ_m^s are chosen to match equity return volatilities. We set $m = 0.2$ so that the average debt maturity is about $1/m = 5$ years. This is close to the empirical median debt maturity (including bank loans and public bonds) reported in Chen, Xu, and Yang (2012). We set the debt issuance cost ω in the primary corporate bond market to be 1% as in Chen (2010). And, the idiosyncratic volatility σ_i is chosen to match the default probability of Baa firms. There is no state-dependence of σ_i as we do not have data counterparts for state-dependent Baa default probabilities. Finally, as explained later, the firm's cash-flow is determined from empirical leverage observed in the data.

Chen, Collin-Dufresne, and Goldstein (2009) argue that generating a reasonable equity Sharpe ratio is an important criterion for a model that tries to simultaneously match the default rates and credit spreads, for otherwise one can simply raise credit spreads by imposing unrealistically high systematic volatility and prices of risk. Based on our calibration (especially the choices of σ_m, σ_i, κ, and η), we obtain the equity Sharpe ratio of 0.11 in state G and 0.20 in state B, which is close to the mean firm-level Sharpe ratio for the whole universe of the CRSP firms (0.17) reported in Chen, Collin-Dufresne, and Goldstein (2009).

4.1.2 Bond market illiquidity

We set the state-dependent liquidity premium Δ_s for Treasuries based on observed repo-Treasuries spread. This spread is measured as the difference between the 3-month general collateral repo rate and the 3-month Treasury rate. This is because the repo rate can be interpreted as the true "risk-free" rate, i.e., the discount rate for future deterministic cash flows. The daily average of the repo-Treasury spread is 15 bps in the non-recession period from October 2005 to September 2013 and 40 bps in the recession period, which lead us to set

$\Delta_G = 15bps$ and $\Delta_B = 40bps$.[15] These estimates are consistent with to the average liquidity premium reported in Longstaff (2004) based on Refcorp curve.

The other liquidity parameters in secondary corporate bond market are less standard in the literature. We first fix the state-dependent intermediary meeting intensity based on anecdotal evidence, so that it takes a bond holder on average a week ($\lambda_G = 50$) in the good state and 2.6 weeks ($\lambda_B = 20$) in the bad state to find an intermediary to sell all bond holdings.[16] We interpret the lower λ in state B as a weakening of the financial system and its ability to intermediate trades. We then set bond holders bargaining power $\beta = 0.05$ independent of the aggregate state, based on the empirical work that estimates search frictions in secondary corporate bond markets (Feldhütter (2012)).

We choose intensity of liquidity shocks, ξ_s, based on observed bond turnovers in the secondary market. In our model, all turnovers in secondary corporate bond markets are driven by liquidity reasons. Apparently, in practice investors trade corporate bonds for reasons other than liquidity, and recent turmoil during 2007/08 financial crisis suggests that during recession institutional investors are more likely to be hit by liquidity shocks and hence trade their bond holdings. We thus rely on the empirical turnover frequency during recessions to set $\xi_B = 1$.[17] Given the state B meeting intensity of $\lambda_B = 20$ with dealers, the model implied turnover year in recession, which is $\frac{\xi_B \lambda_B}{\xi_B + \lambda_B}$, is about 1.05 years.

The state-G liquidity intensity $\xi_G = 0.5$ is then chosen to target a good overall fit of state-G Bond-CDS spread in the investment grade (A/Baa). With a meeting intensity of $\lambda_G = 50$, the model implied state-G turnover is about 2.02 years.[18] Procyclicality of ξ_s over the business cycle captures the important time-varying liquidity conditions in the secondary corporate bond markets. In our model, adverse macroeconomic conditions (prices of risk) coincide and interact with weaker firm fundamentals and worsened secondary market liquidity,

[15] We exclude crisis period of October 2008 to March 2009 throughout the paper. Also, over a given horizon, state-dependent instantaneous liquidity premium suggests that the average liquidity premium is horizon-dependent, but we ignore this effect for simplicity.

[16] Ideally one can infer λ using the total time the corporate bond funds take to complete a sale, which is a challenging task empirically.

[17] In TRACE, the value-weighted turnover of corporate bonds during NBER recessions is about 1.4 years.

[18] In TRACE, the value-weighted turnover of corporate bonds during non-recessions is about 1.4 years, similar to the turnover during recessions. As explained we decide not to set ξ_G based on non-recession bond turnover years. This is because, in normal times, bond trading is more likely to be driven by reasons (say, speculative) other than liquidity shocks.

which help generate quantitatively important implications for the pricing of defaultable bonds.

The holding costs χ_s are central parameters that determine the bid-ask spread in the secondary market of corporate bonds. Since there is no direct empirical counterpart for holding costs, we calibrate χ_s to target the bid-ask spread for bonds with investment grade in both aggregate states.

4.1.3 Effective recovery rates

As explained in Section 2.4, our model features type- and state-dependent recovery rates α_l^s for $l \in \{L, H\}$ and $s \in \{G, B\}$. We first borrow from the existing structural credit risk literature (say, Chen (2010)) who treats the traded prices right after default as recovery rates, and estimates recovery rates of $57.6\% \cdot v_U^G$ in normal times and $30.6\% \cdot v_u^B$ in recessions (recall v_U^s is the unlevered firm value at state s).

Assuming that post-default prices are bid prices at which investors are selling, then Proposition 1 implies:

$$0.5755 \quad = \quad \alpha_L^G + \beta(\alpha_H^G - \alpha_L^G), \text{ and } 0.3060 = \alpha_L^B + \beta(\alpha_H^B - \alpha_L^B). \tag{14}$$

We need two more pieces of bid-ask information for defaulted bonds to pin down the α_l^s's. Edwards, Harris, and Piwowar (2007) report that in normal times, the transaction cost for defaulted bonds for median-sized trades is about $200bps$. To gauge the bid-ask spread for defaulted bonds during recessions, we take the following approach. Using TRACE, we first follow Bao, Pan, and Wang (2011) to calculate the implied bid-ask spreads for low rated bonds (C and below) for both non-recession and recession periods. We find that relative to the non-recession period, during recessions the implied bid-ask spread is about 3.1 times higher. Given a bid-ask spread of $200bps$ for defaulted bonds, this multiplier implies that the bid-ask spread for defaulted bonds during recessions is about $620bps$. Hence we have

$$2\% \quad = \quad \frac{2\,(1-\beta)\left(\alpha_H^G - \alpha_L^G\right)}{\alpha_L^G + \beta(\alpha_H^G - \alpha_L^G) + \alpha_H^G}, \text{ and } 6.2\% = \frac{2\,(1-\beta)\left(\alpha_H^B - \alpha_L^B\right)}{\alpha_L^B + \beta(\alpha_H^B - \alpha_L^B) + \alpha_H^B}. \tag{15}$$

Solving (14) and (15) gives us the estimates of:[19]

$$\boldsymbol{\alpha} = \left[\alpha_H^G = 0.5871, \alpha_L^G = 0.5749, \alpha_H^B = 0.3256, \alpha_L^B = 0.3050\right]. \tag{16}$$

4.1.4 Degree of freedom in calibration

We summarize our calibration parameters in Table 1. Although there are a total of 31 parameters, most of them are in Panel A "pre-fixed parameters" of Table 1, which are set either using the existing literature or based on moments other than the corporate bond pricing moments. We only pick (calibrate) four parameters freely to target the empirical moments that our model aims to explain, which are highlighted in bold fonts in Panel B "calibrated parameters" in Table 1. In summary, we pick σ_i to target Baa firm default probabilities, ξ_G to target state-G Bond-CDS spreads for investment grade (A/Baa) firms, and χ_G and χ_B to target investment grade bid-ask spread in both states. As shown shortly, this degree of freedom (four) is far below the number of our empirical moments that we aim to match.[20]

We point out that in our model, the quantitative performance along the dimension of business cycles is less surprising, simply because our model takes (and sometimes, chooses) exogenous parameters in two aggregate macroeconomic states. Because our model links the secondary bond market liquidity to the firm's distance-to-default, our model's quantitative strength is more reflected on its cross-sectional performance (say, matching the total credit spreads over four ratings). And, recall that ξ_B is chosen based on empirical bond turnovers in state B; hence matching state-B Bond-CDS spreads can also be considered as a success of our model.

4.2 Empirical Moments

We consider four rating classes: Aaa/Aa , A, Baa, and Ba; the first three rating classes are investment grade, while Ba is speculative grade. We combine Aaa and Aa together because

[19]This calculation assumes that bond transactions at default occur at the bid price. If we assume that transactions occur at the mid price, these estimates are $\alpha_H^G = 0.5813, \alpha_L^G = 0.5691, \alpha_H^B = 0.3140, \alpha_L^B = 0.2972$.

[20]We have cumulative default probabilities over four credit ratings (4), total credit spreads and Bond-CDS spreads over four ratings and two aggregate states ($2 \times 4 \times 2 = 16$), and bid-ask spreads over three rating classes and two aggregate states ($3 \times 2 = 6$).

there are few observations for Aaa firms. We emphasize that previous calibration studies on corporate bonds focus on the *difference* between Baa and Aaa only, while we are aiming to explain the *level* of credit spreads across a wide range of rating classes. Furthermore, we report the model performance *conditional* on macroeconomic states, while typical existing literature only focus on *unconditional* model performance (Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010)). We classify each quarter as either in "state G" or "state B" based on NBER recession. As the "B" state in our model only aims to capture normal recessions in business cycles, we exclude two quarters during the 2008 financial crisis, which are 2008Q3 and 2009Q1, to mitigate the effect caused by the unprecedented disruption in financial markets during crisis.[21]

4.2.1 Default Probabilities

The default probabilities for 5-year and 10-year bonds in the data column of Panel A in Table 2 are taken from Exhibit 33 of Moody's annual report on corporate default and recovery rates (2012), which gives the cumulative default probabilities over the period of 1920-2011. Unfortunately, the state-dependent measurement on default probabilities over business cycles are unavailable.

[TABLE 2 ABOUT HERE]

4.2.2 Bond Spreads

Our data of bond spreads is obtained using Mergent Fixed Income Securities Database (FISD) trading prices from January 1994 to December 2004, and TRACE data from January 2005 to June 2012. We follow the standard data cleaning process, e.g. excluding utility and financial firms.[22] For each transaction, we calculate the bond credit spread by taking the difference between the bond yield and the treasury yield with corresponding maturity. Within each rating class, we average these observations in each month to form a monthly time series

[21]For recent empirical research that focuses on the behaviors of corporate bonds market during the 2007/08 crisis, see Dick-Nielsen, Feldutter, and Lando (2011), and Friewald, Jankowitsch, and Subrahmanyam (2011).

[22]For FISD data, we follow Collin-Dufresne, Goldstein,and Martin (2001). For TRACE data, we follow Dick-Nielsen (2009).

of credit spreads for that rating. We then calculate the time-series average for each rating conditional on the macroeconomic state (whether the month is classified as NBER recession), and provide the conditional standard deviation for the conditional mean. To account for the autocorrelation of these monthly series, we calculate the standard deviation using Newey-West procedure with 15 lags.

We report the conditional means for each rating and their corresponding conditional standard deviations for both 5-year and 10-year bonds in the data column in Panel B of Table 2. In the existing literature, Huang and Huang (2012) cover the period from the 1970's to the 1990's, and report an (unconditional) average credit spread of 55 bps for 4-year Aaa rated bonds, 65 bps for Aa, 96 for A, 158 for Baa, and 320 for Ba. Our unconditional 5-year average credit spreads are fairly close, which is the weighted average across conditional means reported in Panel B of Table 2: 65 bps for Aaa/Aa, 100 bps for A, 167 for Baa, and 349 for Ba.

4.2.3 Bond-CDS spreads

Longstaff, Mithal, and Neis (2005) argue that because the market for CDS contracts is much more liquid than the secondary market for corporate bonds, the CDS spread should mainly reflect the default risk of a bond, while the credit spread also includes liquidity premium to compensate for the illiquidity in the corporate bond market. Following Longstaff, Mithal, and Neis (2005), we take the difference between the bond credit spread and the corresponding CDS spread. This Bond-CDS spread is our first empirical measure for the non-default risk of corporate bonds.

We construct Bond-CDS spreads as follows. We first match FISD bond transaction data with CDS prices from Markit, and then follow the same procedure as above, with two caveats. First, the data sample period only starts from 2005 when CDS data become available. Second, to address the potential selection issue, we follow Chen, Xu, and Yang (2012) and focus on firms that have both 5-year and 10-year bonds outstanding. The results are reported in the data column in Panel A in Table 2.

One issue is worth further discussing. Our *Bond-CDS spread* is defined as the corporate bond yield minus the treasury yield with matching maturity, and then minus its corresponding

CDS spread. Another closely related measure, *Bond-CDS basis*, is of great interest to both practitioners and academic researchers. The only difference is on the risk-free benchmark: our Bond-CDS spread takes the Treasury yield as the benchmark, while Bond-CDS bases takes interest rate swap rate as the benchmark. For recent studies on Bond-CDS basis, see Gârleanu and Pedersen (2011) and Bai and Collin-Dufresne (2012).

The study of Bond-CDS basis mostly focuses on limits-to-arbitrage during the turmoil of financial market. Because interest rate swap gives a more accurate measure of an arbitrager's financing cost, the choice of interest rate swap is more appropriate when studying Bond-CDS basis.

In contrast, our paper aims to explain the credit spread, and we follow the corporate bond pricing literature in setting the Treasury yield as our benchmark. The credit spread includes both the default and liquidity components. Treasuries are a better default-free benchmark, because the interest rate swap rate is the fixed leg of LIBOR, which is contaminated by default risk. Treasuries also serve as the illiquidity-free benchmark, where "liquidity" can be interpreted broadly to include trading liquidity and market liquidity that are captured by our model. Finally, as explained in Section 3.4, our calibration allows for state-dependent liquidity premia Δ_s for Treasuries to capture other liquidity benefits (of holding Treasury bonds) that are missing from our model.

4.2.4 Bid Ask Spreads

The second non-default measure that we study is bid-ask spreads in the secondary market for corporate bonds, whose model counterpart is given in (5). Previous empirical studies have uncovered rich patterns of bid-ask spreads across aggregate states and rating classes. More specifically, we combine Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2011) to construct the data counterparts for the bid-ask spread, as Edwards, Harris, and Piwowar (2007) only report the average bid-ask spread across ratings in normal times (2003-2005). The ratings considered in Edwards, Harris, and Piwowar (2007) are superior grade (Aaa/Aa) with an bid-ask spread of 40 bps, investment grade (A/Baa) with an bid-ask spread of 50 bps, and junk grade (below Ba) with a bid-ask spread of 70 bps.[23] For each grade, we then compute the measure of liquidity in Roll (1985) as in Bao, Pan, and Wang (2011), which we use to back out the bid-ask spread ratio between B-state and G-state. We multiply this ratio by the bid-ask spread estimated by Edwards, Harris, and Piwowar (2007) in normal times (2003-2005) to arrive at bid-ask spread in B state. These empirical estimates are reported in Panel B in Table 3.

4.3 Model Performance on Default Risk and Credit Spreads

4.3.1 Calibration method

For any given cash-flow y, which links one-to-one to the firm's market leverage, we can compute the default probability and credit spread of bonds at 5 and 10 year maturity using Monte-Carlo methods. [24] As typical in structural corporate bond pricing models, we find that the model implied default probability and total credit spread are highly nonlinear in market leverage (see Figure 3). The non-linearity inherent in the model implies that the average credit spreads are higher than the spreads at average market leverage. We thus follow David (2008) in computing model implied aggregate moments. Specifically, we compute the market leverage (i.e., book debt over the sum of market equity and book debt) of all Compustat firms (excluding financial and utility firms and other standard filters) for which we have ratings data between 1994 and 2012.[25] We then match each firm-quarter observed in Compustat to its model counterpart based on the observed market leverage, compute the average across aggregate states, and repeat the procedure for each rating class and each maturity (5 or 10

[23]We take the median size trade around 240K. Edwards, Harris, and Piwowar (2007) show that trade size is an important determinants for transaction costs of corporate bonds. But, for tractability reasons, we have abstracted away from the trade size.

[24]Recall that for tractability we assume that bonds are with random maturity. In calibration, we study bonds with deterministic maturities, which can be viewed as some infinitesimal bonds in the firm's aggregate debt structure analyzed in Section 2.1.3. Since the debt valuation derived in Proposition 2 does not apply, we rely on Monte-Carlo methods. Specifically, we simulate the cash flow of the firm and aggregate state for 50,000 times for a fixed duration of 5 or 10 years and count the times where the cash flow cross the state dependent default boundary and also record the cash flow received by bond holders of either H or L type. Following the literature, we consider bonds that are issued at par.

[25]A similar point is made in Bhamra, Kuehn, and Strebulaev (2010). For empirical distribution of market leverage for each rating, see Figure 2. Market leverage is defined as the ratio of book debt over book debt plus market equity.

years). Hence our model always exactly matches the data counterpart on the dimension of leverage.

Relative to the existing literature, our calibration aims at explaining the level of credit spread across ratings, rather than differences between ratings. For instance, (Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010)) focus on explaining the difference between Baa and Aaa rated bonds, which is considered as the default component of Baa rated bonds under the assumption that the observed spreads for Aaa rated bonds are mostly driven by liquidity premium. Because our framework endogenously models bond liquidity, we are able to match the credit spreads that we observe in the data across the superior ratings (Aaa/Aa) and the high end of speculative rating bonds (say Ba).

Another important dimension that our paper improves over the existing literature is on the matching of conditional means of credit spreads. Because the success of Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010) hinges on the idea that the bond's payoff is lower in recessions with a higher marginal utility of consumption, checking whether the model implied bond spreads during recessions matches empirical counterpart can be viewed as a disciplinary test for the mechanism proposed by those papers.

4.3.2 Calibration results

Table 2 presents our calibration results on aggregate default probability (Panel A) and credit spread for bonds of four rating classes (Panel B), for both 5-year and 10-year bonds.

5-year default probabilities and credit spreads On the maturity end of 5-year, our quantitative model is able to deliver a decent matching for both cross-sectional and state-dependent patterns in default probabilities and credit spreads. For instance, the model implied default probability is 2.9%, matching quite well with 3.1% reported by Moody's. The model implied credit spread in state G (state B) is 148 (235) bps. They are close to 149 (275) bps in the data, taking into account of the standard deviation of the conditional sample means.

On the superior grade bonds with Aaa/Aa ratings, our model gives an almost perfect matching for 5-year credit spreads: in state G (state B), the model predicts 61.4 (117) bps while the data counterpart is 55.7 (107) bps. Thanks to introducing liquidity into the structural corporate bond pricing model, we are able to produce reasonable credit spreads for Aaa/Aa bonds conditional on empirically observed default probabilities. In fact, somewhat interestingly, our model implied 5-year default probability for Aaa/Aa firms (0.4%) *undershoots* the data counterpart (0.7%) a bit. This suggests that our model may *overshoot* the non-default component for superior grade bonds, which is indeed our finding in Section 4.4.1.

Our calibration exercise puts more emphasis on the 5-year horizon.[26] The reason that we focus more on 5-year, rather than 10-year, is that this paper aims to explain the non-default component of corporate bonds. The Bond-CDS spreads require the input of observed CDS spreads. Motivated by the fact that CDS market is more liquid than corporate bond market (e.g., Longstaff, Mithal, and Neis (2005)), our model assumes a perfectly liquid CDS market. In practice, it is well-known that the most liquid CDS contracts are those with a 5-year maturity. Hence, focusing on the 5-year end mitigates the potential liquidity effect of the CDS market in biasing our calibration results.

10-year default probabilities and credit spreads While our model is able to quantitatively match the cross-sectional and state-dependent pattern for the credit spreads of 5-year bonds, the matching for 10-year bonds is less satisfactory. The general pattern is that although the default probability matches the data counterpart reasonably well, the model implied conditional credit spread for 10-year bonds overshoots the empirical moments, suggesting that the model implied term structure of credit spreads is steeper than the data suggests. In unreported results, we find that the method of David (2008) which addresses the nonlinearity in the data (caused by the diverse distribution in leverage) has helped our model greatly to deliver a flatter term structure. This finding is consistent with Bhamra, Kuehn, and Strebulaev (2010). Nevertheless, this treatment is not strong enough to get the

[26] More specifically, within the reasonable range used in the literature, we have chosen the state-dependent risk price η and systematic volatility σ_m to deliver an overall good match for *5-year* Baa rated bonds.

term structure right. Certain interesting extensions of our model (e.g., introducing jumps in cash flows that are more likely to occur in state B) should help in this dimension, and we leave future research to address this issue.

Bond recovery rates As emphasized by Huang and Huang (2012), in order for a model to explain the corporate bond spreads, it should not only be able to match the observed spreads, but also generate default probabilities and bond recovery rates that are consistent with the data. In our model, the bond recovery rate is 49.7% in state G and 24.5% in state B. The unconditional average recovery rate is 44.6%. These values are consistent with the average issuer-weighted bond recovery rate of 42% in Moody's recovery data over 1982-2012, and they capture the cyclical variations in recovery rates as documented in Chen (2010).

4.4 Model Performance on Non-Default Risk

Our model features an illiquid secondary market for corporate bonds, which implies that the equilibrium credit spread must compensate the bond investors for bearing not only default risk but also liquidity risk. This new element allows us to investigate the model's quantitative performance on dimensions specific to bond market liquidity, i.e., Bond-CDS spreads and bid-ask spreads, in addition to cumulative default probabilities and credit spreads on which the previous literature has focused.

4.4.1 Bond-CDS Spread

Recall that we assume a perfectly liquid CDS market in Section 3.3. In practice, although much more liquid than the secondary corporate bond market, the CDS market is still not perfectly liquid. As explained above, to mitigate this effect we have focused on bonds with 5-year maturity, because 5-year CDS contracts are traded with the most liquidity.[27]

5-year Bond-CDS spread Similar to the above procedure in Section 4.3, following David (2008) we obtain our model-implied aggregated moments by first calculate the Bond-

[27] Because the CDS market is a zero-net-supply derivative market, how the secondary market liquidity of CDS market affects the pricing of CDS depends on market details. Bongaerts, De Jong, and Driessen (2011) show that indeed, the sellers of CDS contract earn a liquidity premium.

CDS spread for each firm-quarter observation in Compustat based on its market leverage, conditional on the macroeconomic state. To be consistent with data counterpart constructed in 4.2 which is only available after 2005, we report our model implied Bond-CDS spreads in Table 3 using empirical leverage distribution from 2005 to 2012.

[TABLE 3 ABOUT HERE]

The quantitative matching of Bond-CDS spreads, which is reported in Panel A in Table 3, is reasonably good for 5-year bonds, including both cross-sectional and state-dependent dimensions. For 5-year Baa bonds, our model implies a Bond-CDS spread of 72.1 bps in state G, while the data has a mean of 74.6 bps and a standard deviation of 8.7 bps. In recession, the model implied Bond-CDS spread for 5-year Baa bonds is 135 bps, which undershoots the data counterpart 182.3 bps (with a standard deviation of 18.0 bps). The matching of Ba bonds is similar.

On the end of superior grade bonds with Aaa/Aa ratings, the model implied Bond-CDS spread overshoots the data counterpart in both states: 50.4 bps (model) vs 27.7 bps (data) in state G, and 105 bps (model) vs 76 bps (data) in state B).[28] This overshooting in non-default component is consistent with Table 2 where the model undershoots in default probabilities for Aaa/Aa rated bonds but delivers an almost perfect match in credit spreads.

Overall, our model delivers the right magnitude for the empirically observed Bond-CDS spreads, especially those in state-B which we do not choose parameters to target on. The matching across macroeconomic states performs better than the matching across four rating categories, in that our model seems to produce too little variation ranging from superior grade bonds (Aaa/Aa) to speculative grade bonds (Ba). One possible explanation is that our model misses another important cross-sectional liquidity effect in that safer bonds receives better terms in funding liquidity (say, lower haircut), which helps in generating a steeper Bond-CDS spread across rating classes. We await future research on this interesting topic.

10-year Bond-CDS spreads and term structure Moving on to 10-year bonds, the model is doing a fair job in quantitatively matching the observed Bond-CDS spread in state

[28]Recall that even for illiquidity-free ($\chi = 0$) and default-free ($y = \infty$) bonds, the model implied Bond-CDS spreads will be $\Delta_G = 15bps$ and $\Delta_B = 40bps$.

B, while the performance in state G is poor. More importantly, similar to the discussion at the end of Section 4.3, one area our model clearly fails is to replicate a slightly downward sloping Bond-CDS term structure in the data. In our data from 2005 to 2012, the 5-year Bond-CDS spread is slightly higher than the 10-year counterpart across all rating categories, subject to the caveat that the difference may not be statistically significant taking standard deviations into account. It is worth noting that this downward sloping term structure in Bond-CDS spreads is *inconsistent* with the robust finding of longer-maturity bonds being less liquid documented in the empirical literature (e.g., Edwards, Harris, and Piwowar (2007); Bao, Pan, and Wang (2011)). In fact, Longstaff, Mithal, and Neis (2005) report a *positive* relation between Bond-CDS spread and maturity in their sample.

From the theoretical perspective, the model implied term structure for Bond-CDS spreads is upward sloping for investment grade bonds, but may turn downward sloping or flat for speculative grade bonds. In Table 3 state B model row, the Bond-CDS spread difference between 10-year and 5-year is about 3 bps for Aaa/Aa bonds, while the difference turns -10 bps for Ba bonds. In our model, this is because bonds with shorter maturity are more liquid due to a better outside option of bond sellers who can sit out waiting for the principal payment (He and Milbradt (2012)). For bonds that are close to default, the bond's stated maturity matters little, thus 5-year and 10-year bonds face similar illiquidity. Thus, the illiquidity discount per year (which is stated maturity) is higher for 5-year bonds, leading to downward sloping curve for Bond-CDS spreads for risky bonds.[29]

One possible explanation for the downward sloping Bond-CDS spreads, which is outside our model, is that the CDS spreads at different maturities are affected by liquidity differently. It is well recognized that CDS contracts are most liquid at the 5-year horizon when measured by the number of dealers offering quotes. If dealers are mainly selling CDS protections to regular investors and they possess market power (consistent with the empirical evidence in Bongaerts, De Jong, and Driessen (2011)), then the price of 10-year CDS contracts that are only offered by a small number of dealers tend to be higher than the price of 5-year CDS contracts with more competitive dealers. This may contribute to the relatively lower 10-year

[29] This is similar to the inverted term structure of credit spreads for bonds with lower distance-to-default in standard default-driven models.

Bond-CDS spreads.

4.4.2 Bid-Ask Spread

Now we move on to the bid-ask spread as the second measure of non-default component. Recall that on the data side we combine both Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2011) to obtain the estimates of bid-ask spreads for corporate bonds, both across credit ratings and over business cycle. On the model side, again we rely on the empirical leverage distribution in Compustat firms across ratings and aggregate states to calculate the average of model implied bid-ask spreads. Since the average maturity in TRACE data is around 8.3 years, the model implied bid-ask spread is calculated as the weighted average between the bid-ask spread of a 5-year bond and a 10-year bond.

The model implied bid-ask spreads are reported in Panel B of Table 3, together with their empirical counterparts. The model is able to generate both cross-sectional and state-dependent patterns that quantitatively match what we observe in the data, especially in normal time. As mentioned before, we calibrate two state-dependent holding cost parameters (χ_G and χ_B) to match the bid-ask spread of investment grade bonds over macroeconomic states. Thus, it is less surprising that we are able to match the state-dependent pattern that bid-ask spreads more than double when the economy switches from state G to state B. However, in our model the bond's secondary market liquidity is endogenously linked to the firm's distance-to-default, which allows us to deliver the cross-sectional matching across three ratings. In normal times, the average bid-ask spread is 43 bps for superior grade bonds, 50 bps for investment grade bonds, and 73 bps for junk grade bonds, which are close to the data row in Table 3 taken from Edwards, Harris, and Piwowar (2007). The quantitative matching during recession is also satisfactory. Although not reported here, the model-implied bid-ask spread of longer-maturity bonds are higher than that of shorter-maturity bonds, which is consistent with previous empirical studies (eg. Edwards, Harris, and Piwowar (2007); Bao, Pan, and Wang (2011)). Finally, the implied bid-ask spread for the case of $\chi = 0$ is zero by definition (unreported in Table 3).

4.5 What if Pre-default Secondary Market is Perfectly Liquid?

Compared to earlier credit risk models that also incorporate macroeconomic risks, such as Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010), our model adds an illiquid secondary market for corporate bonds. By setting either the holding cost for type L investors, or the liquidity shock intensity, to zero (i.e., either $\chi_s = 0$ or ξ_s=0), we see what our model calibration implies about default risk and credit spreads in the absence of liquidity frictions, which helps isolate the effects of pre-default secondary market illiquidity.

The results are reported in the rows "$\chi = 0$" in Table 2 and Table 3. With $\chi_s = 0$, the credit spreads become significantly lower. For Aaa/Aa rated firms, in state G the 5-year spread falls from 61.4 bps to 23.4 bps (compared to the average spread of 55.7 bps in the data), while in state B it falls from 117 bps to 49.1 bps (compared to 107 bps in the data).[30] For 10-year spread, the unconditional credit spread falls from 93.8 bps to 43.7 bps (compared to 61.2 bps in the data). Credit spreads for low-rated firms also fall, but by less in relative terms compared to highly-rated firms. Though not reported in Table 3, once we set $\chi_s = 0$ the model implied Bond-CDS spreads is close to $\Delta_G = 15bps$ in state G and $\Delta_B = 40bps$ in state B.

Besides the credit spreads, shutting off the pre-default secondary market illiquidity lowers default probabilities as well. Quantity wise, the reduction for bonds with high rating is about 25% lower, while it is about 15% for speculative grade bonds. It is because the secondary market illiquidity raises the rollover risk for firms, which in turn raises the probability of default. More importantly, this result illustrates that in order to obtain a precise decomposition of the default and liquidity components in credit spreads, we need to take into account the interactions between default risk and liquidity risk. We propose such a decomposition in the next section.

[30] Recall that we impose a liquidity premium for treasury, which is 15 bps in state G and 40 bps in state B, independent of default. In light of this, when setting $\chi = 0$, the potential default of Aaa/Aa firms contributes 23.4bps-15bps=8.4bps in state G while 49.1bps-40bps=9.1bps in state B.

5. Structural Default-Liquidity Decomposition

Our structural model of corporate bonds features a full interaction between default and liquidity in determining the credit spreads of corporate bonds. It has been a common practice in the empirical literature to decompose the credit spread into liquidity and default components in an additive way, such as in Longstaff, Mithal, and Neis (2005). From the perspective of our model, this "intuitively appealing" decomposition tends to over-simplify the role of liquidity in determining the credit spread. More importantly, the additive structure often leads to a somewhat misguided interpretation that liquidity or default is the cause of the corresponding component, and each component would be the resulting credit spread if we were to shut down the other channel.

This interpretation may give rise to misleading answers in certain policy related questions. For instance, as our decomposition indicates, part of the default risk comes from the illiquidity in the secondary market. Thus, when the government is considering providing liquidity to the market, besides the direct effect on the credit spread by improving liquidity, there is also an indirect effect in lowering the default risk via the rollover channel and the firm's endogenous default decision. The traditional perspective with an additive structure often overlooks this indirect effect, a quantitatively important effect according to our study.

5.1 Decomposition Scheme

We propose a more detailed structural decomposition, which nests the additive default-liquidity decomposition common in the literature. We first isolate liquidity premium for Treasury bonds, which comes from the specialness of Treasuries (in serving as collateral) but not so much about trading illiquidity of corporate bonds. The remaining spread, which can be considered as credit spread relative to the risk-free rate (instead of the Treasury rate), consists of the default and liquidity parts. We further decompose the default part into the pure-default and liquidity-driven-default parts, and similarly decompose the liquidity part into the pure-liquidity and default-driven-liquidity parts:

$$\hat{cs} \;=\Delta \;+\; \overbrace{\hat{cs}_{pureDEF}+\hat{cs}_{LIQ\to DEF}}^{\text{Default Component } \hat{cs}_{DEF}} \;+\; \overbrace{\hat{cs}_{pureLIQ}+\hat{cs}_{DEF\to LIQ}}^{\text{Liquidity Component } \hat{cs}_{LIQ}} \tag{17}$$

This way, we separate *causes* from *consequences*, and emphasize that lower liquidity (higher default) risk can lead to a rise in the credit spread via the default (liquidity) channel. Recognizing and further quantifying this endogenous interaction between liquidity and default is important in evaluating the economic consequence of policies that are either improving market liquidity (e.g., Term Auction Facilities or discount window loans) or alleviating default issues (e.g, direct bailouts).

Start with the default component. Imagine a hypothetical investor who is not subject to liquidity frictions, both pre- and post- default, and consider the spread that this investor demands over the risk-free rate. The resulting spread, denoted by $\hat{cs}_{DEF}$, only prices the default event given default threshold y_{def}, in line with Longstaff, Mithal, and Neis (2005) who use information from the relatively liquid CDS market to back out the default premium. Importantly, the default boundaries y^s_{def}'s in calculating $\hat{cs}_{DEF}$ remains the same given liquidity frictions as derived in equation (12).[31]

In contrast, we define the "Pure-Default" component $\hat{cs}_{pureDEF}$ as the spread implied by the benchmark Leland model without secondary market liquidity frictions at all (e.g., setting $\xi = 0$ or $\chi = 0$ for both pre- and post-default). Because the liquidity of the bond market leads to less rollover losses, equity holders default less often, i.e., $y^{Leland,s}_{def} < y^s_{def}$, where $y^{Leland,s}_{def}$ denotes the endogenous default boundary in Leland and Toft (1996) with time-varying aggregate states. The distinction between default boundaries implies a smaller pure-default component $\hat{cs}_{pureDEF}$ than the default component $\hat{cs}_{DEF}$. The difference $\hat{cs}_{DEF} - \hat{cs}_{pureDEF}$ gives the novel "Liquidity-Driven Default" component, which quantifies the effect that the illiquidity of secondary bond markets makes default more likely.

Now we move on the liquidity side. The liquidity component, in line with Longstaff, Mithal, and Neis (2005), is defined as $\hat{cs}_{LIQ} \equiv \hat{cs} - \hat{cs}_{DEF}$. That is to say, the liquidity component is

[31] Hypothetically, this is the situation where all other bond investors are still facing liquidity frictions as modeled. Hence, equity holders' default decision is not be affected.

the difference between the credit spread $\widehat{cs}$ implied by our model, and that required by a hypothetical investor without liquidity frictions, i.e., the spread $\hat{cs}_{DEF}$. Following a similar treatment to the default component, we further decompose $\hat{cs}_{LIQ}$ into a "Pure-Liquidity" component and a "Default-driven Liquidity" component. Let $\hat{cs}_{pureLIQ}$ be the spread (relative to the risk-free rate) of a bond that is subject to liquidity frictions as in Duffie, Gârleanu, and Pedersen (2005) but does not feature any default risk; this is the spread implied by our model as $y \to \infty$ so that the bond becomes default free. The residual $\hat{cs}_{LIQ} - \hat{cs}_{pureLIQ}$ is what we term the default-driven liquidity part of our credit spread. Economically, the default-driven liquidity part arises because default leads to a more illiquid post-default secondary market, which endogenously worsens the pre-default secondary market liquidity.

5.2 Ultimate Recovery Rates

The proposed default-liquidity decomposition requires us to estimate the benchmark "pure-default" Leland model by removing liquidity frictions in the secondary corporate bond market, not only for pre-default but also for post-default. This affects the hypothetical Leland default recovery rates, which we denote by α^G_{Leland} and α^B_{Leland}. It is worth noting that these recovery rates under Leland setting do not affect our previous calibration exercise.

In the context of our model, α^s_{Leland}'s are valuations of hypothetical bond investors at default who can costlessly wait for the ultimate recovery from bankruptcy payout. Following this idea, we first estimate the ultimate recovery rate $\hat{\alpha}_s$, which is the debt holders' final payout (as a fraction of the unlevered firm value) from bankruptcy settlement. Using *Moody's default and recovery database* that covers defaulted corporate bonds between 1987 and 2011, we track the price path for each defaulted bond from the default date to the settlement (or emergence) date.[32] The average time from credit event to ultimate resolution is 501 days, implying a bankruptcy payout intensity of $\theta = 0.73$. And, this duration varies little across recession and non-recession periods.

To adjust risk, we borrow from the empirical literature on venture capital / private equity

[32]We follow Moody's preferred method in choosing the emergence price. For each bond, Moody's calculates the emergence price using three methods: trading price, settlement price or liquidity price and indicates which one is preferred.

(e.g. Kaplan and Schoar (2005)) by discounting the return for each defaulted bond by a public market reference return over the same horizon (from default date to emergence date). We use the S&P500 total return (including dividends) as the relevant benchmark, and the resulting excess returns are called "Public Market Equivalent" (PME).[33] It is also convenient to account for state dependence in risk premium under this simple method. Sorting our sample into two groups based on whether the default month is classified as recession by NBER,[34] we find that the average risk-adjusted buy-and-hold return when default occurs in recession is about 212%, and when default occurs in non-recession is about 153%.

Suppose that right after default the trading prices is $p_s v_U^s$ in state s. Then, *Moody's default and recovery database* implies that the expected final payouts are $p_G v_u^G \cdot 153\%$ if default occurs at state G, and $p_B v_u^B \cdot 212\%$ if default occurs at state B. Because the aggregate states are switching before eventual bankruptcy payout, the ultimate recovery rates $\widehat{\alpha}_G$ and $\widehat{\alpha}_B$ satisfy

$$\begin{bmatrix} p_G \cdot 153\% \\ p_B \cdot 212\% \end{bmatrix} = \begin{bmatrix} 0.5755 \cdot 153\% \\ 0.3060 \cdot 212\% \end{bmatrix} = \begin{bmatrix} \pi_{GG} & \pi_{GB}\frac{x_B}{x_G} \\ \pi_{BG}\frac{x_G}{x_B} & \pi_{BB} \end{bmatrix} \begin{bmatrix} \widehat{\alpha}_G \\ \widehat{\alpha}_B \end{bmatrix}$$

where $\pi_{ij} = \Pr\left(s_{\hat{\tau}} = j \middle| s_0 = i\right)$, and $\hat{\tau}$ is the random payout time following an exponential distribution with intensity $\theta = 0.73$, and x_G and x_B are state-dependent price-dividend ratios.[35] Finally, we discount these ultimate recovery rates back to the date of default, taking switching aggregates states in to account:

$$\begin{bmatrix} \alpha_G^{Leland} \\ \alpha_B^{Leland} \end{bmatrix} = \begin{bmatrix} r+\theta+\zeta_G & -\zeta_G\frac{x_B}{x_G} \\ -\zeta_B\frac{x_G}{x_B} & r+\theta+\zeta_B \end{bmatrix}^{-1} \begin{bmatrix} \theta\widehat{\alpha}_G \\ \theta\widehat{\alpha}_B \end{bmatrix}.$$

Under our calibrations, we have $\alpha_G^{Leland} = 0.8583$ and $\alpha_B^{Leland} = 0.6297$.

[33] We use this approach because it is difficult to estimate *beta* for this investment strategy due to unbalanced panels and unknown interim returns before emergence date, a well-known problem in the VC/PE literature. Moreover, we need to calculate the state-dependent excess return, which makes the estimation of *beta* even harder.

[34] There are 130 defaults occurring in recession among a total of 642 defaults. Table 6 in the Appendix provides summary statistics on our excess return matric and Figure 1 plots its empirical distribution.

[35] We compute x_s as $\begin{bmatrix} x_G \\ x_B \end{bmatrix} = \begin{bmatrix} r-\mu_G+\zeta_G & -\zeta_G \\ -\zeta_B & r-\mu_B+\zeta_B \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

5.3 Default-Liquidity Decomposition

We perform the above default-liquidity decomposition for typical 5-year bonds. The firm's cash-flow rate y is set at state G to match the average credit spreads of different ratings observed in non-recession periods. We then imagine that the economy switches to state B, and investigate the change of decomposition due to the change of aggregate state. Importantly, because we fix the firm's cash-flow y across two states, the resulting credit spread in B typically is below the data counterpart in Table 2.[36]

The decomposition results for both aggregate states are presented in Panel I in Table 4. We give both the credit spread relative to the Treasury rate and the credit spread relative to the risk-free rate; their difference is just the Treasury liquidity premium Δ_s (15bps in state G and 40bps in state B) exogenously specified by our calibration. Because the liquidity-default decomposition applies to the credit spread relative to the risk-free rate, in this subsection we place more emphasis on this credit spread measure. For this reason, in this subsection the term "credit spread" refers to the credit spread relative to the risk-free rate.

5.3.1 Level of credit spreads

For each component, Table 4 reports its absolute level in bps, as well as the percentage contribution to the credit spread relative to the risk-free rate. As expected, the "pure liquidity" component accounts for a greater fraction of credit spread for higher rated bonds. For instance, for Aaa/Aa rated bonds about 75% of the credit spread comes from the "pure liquidity" component, and the aggregate state matters little. In contrast, the "pure liquidity" component only accounts for 21% (26%) of the credit spread of Ba rated bonds in state G (state B). A similar intuitive pattern holds for the "pure default" component across credit ratings. The fraction of credit spreads that can be explained by the "pure default" component starts from around 12% for Aaa/Aa rated bonds, and monotonically increases to about 45% for Ba rated bonds.

The remaining part of the observed credit spreads, which is around 12%~30% depending on the rating, can be attributed to the novel interaction terms, either "liquidity-driven default"

[36] In Table 2, we identify the firm's cash-flow in each state. based on the corresponding empirical leverage distribution in Compustat.

or "default-driven liquidity." The "liquidity-driven default" part captures how corporate endogenous default decisions are affected by secondary market liquidity frictions via the rollover channel, which is non-negligible even for the highest rating firms (about 8% for Aaa/Aa rated bonds). As expected, its quantitative importance rises for low rating bonds: for Ba rated bonds, the liquidity-driven default accounts for about 21% of observed credit spreads.

The second interaction term, i.e., the "default-driven liquidity" component, captures how secondary market liquidity endogenously worsens when a bond is closer to default. Given a more illiquid secondary market for defaulted bonds, a lower distance-to-default leads to a worse pre-default secondary market liquidity because of the reduced outside option of L investors when bargaining with dealers. Similar to "liquidity-driven default," the "default-driven liquidity" component becomes larger for bonds of lower rating classes. In our calibration, this component is a bit smaller than the "liquidity-driven default" part, but is significant for low rated bonds (about 10% of the credit spread for Ba rated bonds).

Comparison to Longstaff, Mithal, and Neis (2005) How do our decomposition results compare to those documented in Longstaff, Mithal, and Neis (2005)? Since we are decomposing the credit spread relative to the risk-free rate, the more appropriate benchmark in Longstaff, Mithal, and Neis (2005) is the results based on the Refcorp curve. Based on CDS spreads and their structural model, Longstaff, Mithal, and Neis (2005) estimate that for 5-year Aaa/Aa rated bonds, the default component is about 62% of their credit spreads. For lower ratings, they report 63% for A, 77% for Baa, and 86% for Ba.

Overall, our decomposition in Table 4 gives a much lower default component compared to Longstaff, Mithal, and Neis (2005). More specifically, by adding up the "pure-default" and "liquidity-driven default" components, we have a default component of 20% for Aaa/Aa rated bonds, 40% for A, 51% for Baa, and 65% for Ba. The main driving force behind this difference is that we calibrate our model to match a much lower empirical ratio between CDS spread and credit spreads, especially for investment grade bonds. More specifically, in our data, the CDS ratio is only about 51% for 5-year Baa rated bonds, compared to 74%

reported in Longstaff, Mithal, and Neis (2005).[37] A lower targeted CDS spread implies a smaller default component, which leads to a calibrated model with a relatively high liquidity component.

5.3.2 The change of credit spreads over aggregate states

This subsection focuses on a long-standing question that has interested empirical researchers, e.g., Dick-Nielsen, Feldhütter, and Lando (2011) and Friewald, Jankowitsch, and Subrahmanyam (2012): How much of the soaring credit spread when the economy switches from boom to recession is due to increased credit risk, and how much is due to worsened secondary market liquidity? Our novel default-liquidity decomposition in (17) acknowledges that both liquidity and default risks for corporate bonds are endogenous and may affect each other. Given this feature, structural answers that rely on well-accepted economic structures are more appropriate than reduced-form approaches.

We report results in Table 4. As suggested by Panel II, increased default risks constitute a large fraction of the jump in credit spreads. The pure liquidity component is also quantitatively significant in explaining the rise of credit spreads: even for Ba rated bonds, about 34% of the rise when entering recessions is due to the lower secondary market illiquidity in state B.

When the economy encounters a recession, the higher default risk lowers secondary market liquidity further, giving rise to a greater "default-driven liquidity" part. Since worse liquidity in state B also pushes firms to default earlier, bond spreads rise because of a larger "liquidity-driven default" part. For low rated (say Ba) bonds, the "default-driven liquidity" channel (13%) is slightly less important than that of "liquidity-driven default" (18%).

[TABLE 4 ABOUT HERE]

[37] In Longstaff, Mithal, and Neis (2005) whose sample period is from March 2001 to October 2002, the CDS spreads for Aaa/Aa rated bonds are about 59% of their corresponding credit spreads; 60% for A, 74% for Baa, and 87% for Ba. In our data with a much longer sample period, these moments are 63% for Aaa/Aa, 52% for A, 51% for Baa, and 68% for B.

5.4 Implications on Evaluating Liquidity Provision Policy

Our decomposition and its quantitative results are informative for evaluating the effect of policies that target lowering the borrowing cost of corporations in recession by injecting liquidity into the secondary market. As argued before, a full analysis of the effectiveness of such a policy must take account of how firms' default policies respond to liquidity conditions and how liquidity conditions respond to the default risks. These endogenous forces are what our model is aiming to capture.

Suppose that the government is committed to launching certain liquidity enhancing programs (e.g., Term Auction Facilities or discount window loans) whenever the economy falls into a recession, envisioning that the improved funding environment for financial intermediaries alleviates the worsening liquidity in the secondary bond market. Suppose that the policy is effective in making the secondary market in state B as liquid as that of state G. More precisely, the policy helps increase the meeting intensity between L investors and dealers in state B, so that noneλ_B rises from 20 to $\lambda_G = 50$; reduce the state B holding cost χ_B from 2.35 to $\chi_G = 1.25$; reduce the liquidity intensity of $\xi_G = 1$ in state B to $\xi_G = 0.5$ as in state G; and finally make the post-default secondary market in state B to be as liquid as state G.[38]

In Table 5 we take the same cash flow levels for each rating class as in Table 4, and calculate the credit spreads with and without the state-B liquidity provision policy. We find that a state-B liquidity provision policy lowers state-B credit spreads by about 21 (137) bps for Aaa/Aa (Ba) rated bonds, which is about 29% (29%) of the corresponding credit spreads. Moreover, given the dynamic nature of our model, the state-B-only liquidity provision also affects firms' borrowing costs in state G: the state-G credit spreads for Aaa/Aa (Ba) rated bonds go down by 13 (59) bps, or about 31% (20%) of the corresponding credit spreads.

Our structural decomposition further allows us to investigate the underlying driving force for the effectiveness of this liquidity provision policy. By definition, the "pure default" component remains unchanged given any policy that only affects the secondary market liquidity.[39] In Table 5, we observe that the pure-liquidity component accounts for above 83%

[38]This implies that hypothetically, the state B buy-and-hold return is the same as state G, which is 152%. Hence, we keep $p_G = 0.5755$ but set $p_B = 0.3060 \times 212\%/153\% = 0.4240$. We then obtain the hypothetical α_l^s's by imposing the state-G bid-ask spread at default (2%) for both states.

[39]The "pure default" component is defined by Leland and Toft (1996) which is independent of the secondary

(90%) of the drop in spread for Aaa/Aa rated bonds in state G (state B). However, the quantitative importance of the pure-liquidity component goes down significantly when we walk down the rating spectrum: for Ba rated bonds, it only accounts for about 44∼56% of the decrease in the credit spread.

The market-wide liquidity provision not only reduces investors' required compensation for bearing liquidity risk, but also alleviates some default risk faced by bond investors. A better functioning financial market helps mitigate a firm's rollover risk and thus its default risk, and this force is captured by the "liquidity-driven default" part. The importance of this mechanism goes up for lower rated bonds (around 27∼38%), but it remains quantitatively significant even for Aaa/Aa rated bonds (around 7∼12%).

Given that the hypothetical policy was limited to only improving secondary market liquidity, the channel of "default-driven liquidity" is more intriguing, which only exists in our model with endogenous liquidity featuring a positive feedback loop between corporate default and secondary market liquidity. Not surprisingly, the contribution through "default-driven liquidity" is smaller; however, this interaction component is quantitatively significant. It explains about 17% of policy effect for Ba rated bonds, although only about 5% for Aaa/Aa rated bonds.

[TABLE 5 ABOUT HERE]

6. Concluding Remarks

We build over-the-counter search frictions into a structural model of corporate bonds. In the model, firms default decisions interact with time varying macroeconomic and secondary market liquidity conditions. We calibrate the model to historical moments of default probability and empirical measures of liquidity. The model is able to match the observed credit spreads for corporate bonds with different rating classes, as well as various measures of non-default component studied in the previous literature. We propose a structural decomposition that captures the interaction of liquidity and default risks of corporate bonds over the business

market liquidity.

cycle and use this framework to evaluate the effects of liquidity provision policies during recessions. Our results identifies quantitatively important economic forces that were previously overlooked in empirical researches on corporate bonds.

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A Appendix

1.1 Appendix for Section 5.2 in Estimating Ultimate Recovery

[TABLE 6 ABOUT HERE]

[FIGURE 1 ABOUT HERE]

1.2 Appendix for Section 4.3: Empirical Leverage Distribution across Ratings

[FIGURE 2 ABOUT HERE]

[FIGURE 3 ABOUT HERE]

1.3 Omitted formulas from the main text

Define $\mathbf{Q}^{(1)} \equiv \begin{bmatrix} -\xi_G - \zeta_G & \xi_G \\ \beta\lambda_G & -\beta\lambda_G - \zeta_G \end{bmatrix}$ and $\tilde{\mathbf{Q}}^{(1)} \equiv \begin{bmatrix} \zeta_G & 0 \\ 0 & \zeta_G \end{bmatrix}$. The $\mathcal{Q}$-measure Brownian motion is given by

$$dZ_t^{\mathcal{Q}} = \frac{\sigma_m(s)}{\sqrt{\sigma_m^2(s_t) + \sigma_f^2}} dZ_t^m + \sqrt{1 - \frac{\sigma_m^2(s)}{\sigma_m^2(s) + \sigma_f^2}} dZ_t^f + \frac{\sigma_m(s)}{\sigma(s)} \eta(s)\, dt,$$

We have the following system of ODEs for $\mathbf{D}^{(2)}$ when $y \in I_2 = [y_{def}(B), \infty)$:

$$[(r+m)\mathbf{I}_4 - \mathbf{Q}]\mathbf{D}^{(2)} = \left(c\mathbf{1}_4 - \boldsymbol{\chi}^{(2)}\right) + \boldsymbol{\mu}^{(2)}\left(\mathbf{D}^{(2)}\right)' + \frac{1}{2}\boldsymbol{\Sigma}^{(2)}\left(\mathbf{D}^{(2)}\right)'' + m \cdot p\mathbf{1}_4, \tag{18}$$

where $\mathbf{Q}^{(2)} = \mathbf{Q}$,

$$\boldsymbol{\mu}^{(2)} = \mathrm{diag}\left([\mu_G, \mu_G, \mu_B, \mu_B]\right), \boldsymbol{\Sigma}^{(2)} = \mathrm{diag}\left(\left[\sigma_G^2, \sigma_G^2, \sigma_B^2, \sigma_B^2\right]\right).$$

In contrast, on interval $I_1 = [y_{def}(G), y_{def}(B)]$, the bond is "dead" in state B, and the alive bonds $\mathbf{D}^{(1)} = \left[D_H^{(G,1)}, D_L^{(G,1)}\right]^\top$ solve

$$\left[(r+m)\mathbf{I}_2 - \mathbf{Q}^{(1)}\right]\mathbf{D}^{(1)} = \left(c\mathbf{1}_2 - \boldsymbol{\chi}^{(1)}\right) + \boldsymbol{\mu}^{(1)}\left(\mathbf{D}^{(1)}\right)' + \frac{1}{2}\boldsymbol{\Sigma}^{(1)}\left(\mathbf{D}^{(1)}\right)'' + m \cdot p\mathbf{1}_2 + \zeta_G \begin{bmatrix} \alpha_H^B \\ \alpha_L^B \end{bmatrix} v_U^B(y) \tag{19}$$

for

$$y \in I_1 = [y_{def}(G), y_{def}(B)],$$

where the last term is the recovery value in case of a jump to default brought about by a state jump.

The boundary conditions at $y = \infty$ and $y = y_{def}(G)$ are standard:

$$\lim_{y\to\infty}\left|\mathbf{D}^{(2)}(y)\right| < \infty, \text{ and } \mathbf{D}^{(1)}\left(y_{def}^G\right) = \begin{bmatrix} \alpha_H^G \\ \alpha_L^G \end{bmatrix} v_U^G\left(y_{def}^G\right) \tag{20}$$

For the boundary y_{def}^B, we must have value matching conditions for all functions across $y_{def}(B)$:

$$\mathbf{D}^{(2)}\left(y_{def}^B\right) = \begin{bmatrix} \mathbf{D}^{(1)}\left(y_{def}^B\right) \\ \begin{bmatrix} \alpha_H^B \\ \alpha_L^B \end{bmatrix} v_U^B\left(y_{def}^B\right) \end{bmatrix} \tag{21}$$

and smooth pasting conditions for functions that are alive across y_{def}^{B} ($\mathbf{x}_{[1,2]}$ selects the first 2 rows of vector $\mathbf{x}$):

$$\left(\mathbf{D}^{(2)}\right)'\left(y_{def}^{B}\right)_{[1,2]} = \left(\mathbf{D}^{(1)}\right)'\left(y_{def}^{B}\right). \tag{22}$$

$$\boldsymbol{\mu\mu}^{(2)} \;=\; \mathrm{diag}\left(\left[\mu_G, \mu_B\right]\right), \boldsymbol{\Sigma\Sigma}^{(2)} = \mathrm{diag}\left(\left[\sigma_G^2, \sigma_B^2\right]\right), \mathbf{QQ}^{(2)} = \begin{bmatrix} -\zeta_G & \zeta_G \\ \zeta_B & -\zeta_B \end{bmatrix} \tag{23}$$

The particular solution is

$$\begin{aligned}
\underbrace{\mathbf{E}^{(2)}(y)}_{2\times1} &= \underbrace{\mathbf{GG}^{(2)}}_{2\times4} \cdot \underbrace{\exp\left(\boldsymbol{\Gamma\Gamma}^{(2)} y\right)}_{4\times4} \cdot \underbrace{\mathbf{bb}^{(2)}}_{4\times1} + \underbrace{\mathbf{KK}^{(2)}}_{2\times8} \underbrace{\exp\left(\boldsymbol{\Gamma}^{(2)} y\right)}_{8\times8} \underbrace{\mathbf{b}^{(2)}}_{4\times2} + \underbrace{\mathbf{kk}_0^{(2)}}_{2\times1} + \underbrace{\mathbf{kk}_1^{(2)}}_{2\times1} \exp(y) \text{ for} y \in I_2 \\
\underbrace{\mathbf{E}^{(1)}(y)}_{1\times1} &= \underbrace{\mathbf{GG}^{(1)}}_{1\times2} \cdot \underbrace{\exp\left(\boldsymbol{\Gamma\Gamma}^{(1)} y\right)}_{2\times2} \cdot \underbrace{\mathbf{bb}^{(1)}}_{2\times1} + \underbrace{\mathbf{KK}^{(1)}}_{1\times4} \underbrace{\exp\left(\boldsymbol{\Gamma}^{(1)} y\right)}_{4\times4} \underbrace{\mathbf{b}^{(1)}}_{4\times1} + \underbrace{\mathbf{kk}_0^{(1)}}_{1\times1} + \underbrace{\mathbf{kk}_1^{(1)}}_{1\times1} \exp(y) \text{ for } y \in I_1
\end{aligned}$$

where $\mathbf{GG}^{(i)}$,$\boldsymbol{\Gamma\Gamma}^{(i)}$, $\mathbf{bb}^{(i)}$,$\mathbf{KK}^{(i)}$, $\mathbf{kk}_0^{(i)}$ and $\mathbf{kk}_1^{(i)}$ for $i \in \{1,2\}$ are given below. In particular, the constant vector $\mathbf{bb}^{(i)}$ is determined by boundary conditions similar to those for debt.

1.3.1 Generalization to n aggregate states

We follow the Markov-modulated dynamics approach of Jobert and Rogers (2006).

We note that there are multiple possible bankruptcy boundaries, $y_b(s)$, for each aggregate state s one boundary. Order states s such that $s > s'$ implies that $y_b(s) > y_b(s')$ and denote the intervals $I_s = [y_b(s), y_b(s+1)]$ where $y_b(n+1) = \infty$, so that $I_s \cap I_{s+1} = y_b(s+1)$. Finally, let $\boldsymbol{y}_b = [y_b(1), ..., y_b(n)]^\top$ be the vector of bankruptcy boundaries.

It is important to have a clean notational arrangement to handle the proliferation of states. Let $D_l^{(s)}$ denote the value of debt for an creditor in individual liquidity state l and with aggregate state s. We will use the following notation: $D_l^{(s,i)} \equiv D_l^{(s)}, y \in I_i$, that is $D_l^{(s,i)}$ is the restriction of $D_l^{(s)}$

to the interval I_i. It is now clear that $D_l^{(s,i)} = 0$

for any $i < s$, as it would imply that the company immediately defaults in interval I_i for state s. Let us, for future reference, call debt in states $i < s$ dead and in states $i \geq s$ alive. Finally, let us stack the alive functions along states s but still restricted to interval i so that $\mathbf{D}^{(i)} = \left[D_H^{(1,i)}, D_L^{(1,i)}, ..., D_H^{(i,i)}, D_L^{(i,i)}\right]^\top$ where $D_l^{(s,i)}$ has s denoting the state, i denotes the interval and l denotes the individual liquidity state. The separation of s and i will clarify the pasting arguments that apply when y crosses from one interval to the next. Let

$$\underbrace{\mathbf{I}_i}_{i\times i} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \tag{24}$$

i.e. a 2x2 diagonal identity matrix, and let

$$\underbrace{\mathbf{1}_i}_{i\times1} = [1, ..., 1]^\top \tag{25}$$

be a column vector of just ones.

Fundamental parameters. For a 2x2 case, we have a transition matrix $\mathbf{Q}$ that looks like

$$\underbrace{\mathbf{Q}}_{2n\times 2n} = \begin{bmatrix} -\sum_{ls\neq H1}\xi_{H1\to ls} & \xi_{H1\to L1} & \xi_{H1\to H2} & \xi_{H1\to L2} \\ \xi_{L1\to H1} & -\sum_{ls\neq L1}\xi_{L1\to ls} & \xi_{L1\to H2} & \xi_{L1\to L2} \\ \xi_{H2\to H1} & \xi_{H2\to L1} & -\sum_{ls\neq H2}\xi_{H2\to ls} & \xi_{H2\to L2} \\ \xi_{L2\to H1} & \xi_{L2\to L1} & \xi_{L2\to H2} & -\sum_{ls\neq L2}\xi_{L2\to ls} \end{bmatrix} \tag{26}$$

Further, define the possibly state-dependent discount rates

$$\underbrace{\mathbf{R}}_{2n\times 2n} = \begin{bmatrix} \operatorname{diag}\left(\begin{bmatrix} r_H(1) \\ r_L(1) \end{bmatrix}\right) & \cdots & \mathbf{0}_2 \\ \vdots & \ddots & \vdots \\ \mathbf{0}_2 & \cdots & \operatorname{diag}\left(\begin{bmatrix} r_H(n) \\ r_L(n) \end{bmatrix}\right) \end{bmatrix} + m\mathbf{I}_{2n} \tag{27}$$

where we are including the intensity of the random maturity in the definition of $\mathbf{R}$ for notational convenience and brevity.

Building blocks for interval I_i. We now decompose the matrix $\mathbf{Q}$. Let $\mathbf{Q}^{(i)}$ be the transition matrix of jumping into an alive state $s' \leq i$ when currently in interval i and in an alive state $s \leq i$. Let $\tilde{\mathbf{Q}}^{(i)}$ be the transition matrix of jumping into a default state $s' > i$ when currently in interval i and in an alive state $s \leq i$.

Let $\mathbf{v}^{(i)}$ be the recovery or salvage value of the firm when default is declared in states $s > i$ when currently in interval i, where $v_l^{(s,i)} \exp(y) = \alpha_{(s,l)} \frac{\exp(y)}{r_H}$. Thus, $\mathbf{v}^{(i)}$ is a vector containing recovery values for states $(i+1, ..., n) \times (H, L)$ (i.e., it is of dimension $2(n-i) \times 1$).

Let $\boldsymbol{\chi}^{(i)}$ be a vector of holding costs in states $(1, ..., i) \times (H, L)$ (i.e, it is of dimension $2i \times 1$). The holding costs are all positive, and are deducted from the coupon payment. Higher holding costs indicate more severe liquidity states L for the agent.

First, let us start with the interval $i = n$. On this interval, all debt $D_l^{(s,n)}$ is alive. Let

$$\underbrace{\boldsymbol{\mu}^{(n)}}_{2n\times 2n} = \begin{bmatrix} \mu(1)\mathbf{I}_2 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mu(n)\mathbf{I}_2 \end{bmatrix} \tag{28}$$

and similarly let

$$\underbrace{\boldsymbol{\Sigma}^{(n)}}_{2n\times 2n} = \begin{bmatrix} \sigma^2(1)\mathbf{I}_2 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \sigma^2(n)\mathbf{I}_2 \end{bmatrix} \tag{29}$$

and let

$$\mathbf{Q}^{(n)} = \mathbf{Q} \tag{30}$$
$$\mathbf{R}^{(n)} = \mathbf{R} \tag{31}$$
$$\tilde{\mathbf{Q}}^{(n)} = 0 \tag{32}$$

Next, for the interval $i = n-1$ we drop the last two rows and columns (i.e. rows and columns $2n$ and $2n-1$) (because they account for different liquidity states) of $\boldsymbol{\mu}^{(n)}, \boldsymbol{\Sigma}^{(n)}, \mathbf{Q}^{(n)}, \mathbf{R}^{(n)}$ to form $\boldsymbol{\mu}^{(n-1)}, \boldsymbol{\Sigma}^{(n-1)}, \mathbf{Q}^{(n-1)}, \mathbf{R}^{(n-1)}$ which are all $2(n-1) \times 2(n-1)$ matrices. In contrast, we form $\tilde{\mathbf{Q}}^{(n-1)}$ by dropping the last two rows and the first $2(n-1)$ columns of $\mathbf{Q}^{(n)}$ to form a $2(n-1) \times 2$ matrix.

We repeat this procedure, dropping rows and columns and thus shrinking the matrices, step by step all the all the way down to $i = 1$.

Debt valuation within an interval I_i. Debt valuation follows the following differential equation on interval I_i:

$$\left(\mathbf{R}^{(i)}-\mathbf{Q}^{(i)}\right)\mathbf{D}^{(i)}=\left(c\mathbf{1}_{2i}-\boldsymbol{\chi}^{(i)}\right)+\boldsymbol{\mu}^{(i)}\left(\mathbf{D}^{(i)}\right)'+\frac{1}{2}\boldsymbol{\Sigma}^{(i)}\left(\mathbf{D}^{(i)}\right)''+\tilde{\mathbf{Q}}^{(i)}\mathbf{v}^{(i)}\exp(y)+m\cdot p\mathbf{1}_{2i} \tag{33}$$

where $\tilde{\mathbf{Q}}^{(i)}\mathbf{v}^{(i)}\exp(y)$ represents the intensity of jumping into default times the recovery in the default state and $m\cdot p\mathbf{1}_{2i}$ represents the intensity of randomly maturing times the payoff in the maturity state. Next, let us conjecture a solution of the kind $\mathbf{g}\exp(\gamma y)+\mathbf{k}_0^{(i)}+\mathbf{k}_1^{(i)}\exp(y)$ where $\mathbf{g}$ is a vector and γ is a scalar. The particular part stemming from $\mathbf{c}^{(i)}$ is solved by a term $\mathbf{k}_0^{(i)}$ with

$$\underbrace{\mathbf{k}_0^{(i)}}_{2i\times 1}=\underbrace{\left(\mathbf{R}^{(i)}-\mathbf{Q}^{(i)}\right)^{-1}}_{2i\times 2i}\underbrace{(c+m\cdot p)\,\mathbf{1}_{2i}-\boldsymbol{\chi}^{(i)}}_{2i\times 1} \tag{34}$$

and the particular part stemming from $\tilde{\mathbf{Q}}^{(i)}\mathbf{v}^{(i)}$ is solved by a term $\mathbf{k}_1^{(i)}\exp(y)$ with

$$\underbrace{\mathbf{k}_1^{(i)}}_{2i\times 1}=\underbrace{\left(\mathbf{R}^{(i)}-\mathbf{Q}^{(i)}-\boldsymbol{\mu}^{(i)}-\frac{1}{2}\boldsymbol{\Sigma}^{(i)}\right)^{-1}}_{2i\times 2i}\underbrace{\tilde{\mathbf{Q}}^{(i)}}_{2i\times 2(n-i)}\underbrace{\mathbf{v}^{(i)}}_{2(n-i)\times 1} \tag{35}$$

It should be clear that $\mathbf{k}_1^{(n)}=\mathbf{0}$ as on I_n there is no jump in the aggregate state that would result in immediate default. Plugging in, dropping the $\mathbf{c}^{(i)}$ and $\tilde{\mathbf{Q}}^{(i)}\mathbf{v}^{(i)}\exp(y)$ terms, canceling out $\exp(\gamma y)>0$, we have

$$\mathbf{0}_{2i}=\left(\mathbf{Q}^{(i)}-\mathbf{R}^{(i)}\right)\mathbf{g}+\boldsymbol{\mu}^{(i)}\gamma\mathbf{g}+\frac{1}{2}\boldsymbol{\Sigma}^{(i)}\gamma^2\mathbf{g} \tag{36}$$

Following JR06, we premultiply by $2\left(\boldsymbol{\Sigma}^{(i)}\right)^{-1}$ and define $\mathbf{h}=\gamma\mathbf{g}$ to get

$$\gamma\mathbf{g} = \mathbf{h} \tag{37}$$

$$\gamma\mathbf{h} = -2\left(\boldsymbol{\Sigma}^{(i)}\right)^{-1}\boldsymbol{\mu}^{(i)}\mathbf{h}+2\left(\boldsymbol{\Sigma}^{(i)}\right)^{-1}\left(\mathbf{R}^{(i)}-\mathbf{Q}^{(i)}\right)\mathbf{g} \tag{38}$$

Stacking the vectors $\mathbf{j}=\begin{bmatrix}\mathbf{g}\\ \mathbf{h}\end{bmatrix}$ we have

$$\gamma\mathbf{j}=\begin{bmatrix}\mathbf{0}_{2i} & \mathbf{I}_{2i}\\ 2\left(\boldsymbol{\Sigma}^{(i)}\right)^{-1}\left(\mathbf{R}^{(i)}-\mathbf{Q}^{(i)}\right) & -2\left(\boldsymbol{\Sigma}^{(i)}\right)^{-1}\boldsymbol{\mu}^{(i)}\end{bmatrix}\mathbf{j}=\underbrace{\mathbf{A}^{(i)}}_{4i\times 4i}\mathbf{j} \tag{39}$$

where $\mathbf{I}$ is of appropriate dimensions. The problem is now a simple eigenvalue-eigenvector problem and each solution j is a pair $\left(\underbrace{\gamma_j^{(i)}}_{1\times 1},\underbrace{\mathbf{j}_j^{(i)}}_{4i\times 1}\right)$ (or rather $\left(\underbrace{\gamma_j^{(i)}}_{1\times 1},\underbrace{\mathbf{g}_j^{(i)}}_{2i\times 1}\right)$, as the vector $\mathbf{j}_j^{(i)}$ contains the same information as $\mathbf{g}_j^{(i)}$ when we know $\gamma_j^{(i)}$, so we discard the lower half of $\mathbf{j}_j^{(i)}$). The number of solutions j to this eigenvector-eigenvalue problem is $4i$. Let

$$\mathbf{G}^{(i)}\equiv\left[\mathbf{g}_1^{(i)},...,\mathbf{g}_{2\times 2\times i}^{(i)}\right] \tag{40}$$

be the matrix of eigenvectors, and let

$$\boldsymbol{\gamma}^{(i)} \equiv \left[\gamma_1^{(i)},...,\gamma_{2\times 2\times i}^{(i)}\right]' \tag{41}$$

$$\boldsymbol{\Gamma}^{(i)} \equiv \mathrm{diag}\left[\boldsymbol{\gamma}^{(i)}\right] \tag{42}$$

be the corresponding vector and diagonal matrix, respectively, of eigenvalues.

The general solution on interval i is thus

$$\underbrace{\mathbf{D}^{(i)}}_{2i\times 1} = \underbrace{\mathbf{G}^{(i)}}_{2i\times 4i} \cdot \underbrace{\exp\left(\mathbf{\Gamma}^{(i)} y\right)}_{4i\times 4i} \cdot \underbrace{\mathbf{c}^{(i)}}_{4i\times 1} + \underbrace{\mathbf{k}_0^{(i)}}_{2i\times 1} + \underbrace{\mathbf{k}_1^{(i)}}_{2i\times 1} \exp(y) \tag{43}$$

where the constants $\mathbf{c}^{(i)} = \left[c_1^{(i)}, ..., c_{4i}^{(i)}\right]^\top$ will have to be determined via conditions at the boundaries of interval I_i (**NOTE:** $c_j^{(i)} \neq c$ where c is the coupon payment).

Boundary conditions. The different value functions $\mathbf{D}^{(i)}$ for $i \in \{1, ..., n\}$ are linked at the boundaries of their domains I_i. Note that $I_i \cap I_{i+1} = \{y_B(i+1)\}$ for $i < n$.

For $i = n$, we can immediately rule out all positive solutions to γ as debt has to be finite and bounded as $y \to \infty$, so that the entries of $\mathbf{C}^{(n)}$ corresponding to positive eigenvalues will be zero:[40]

$$\lim_{y\to\infty} \left|\mathbf{D}^{(n)}(y)\right| < \infty \tag{44}$$

For $i < n$, we must have value matching of the value functions that are alive across the boundary, and we must have value matching of the value functions that die across the boundary:

$$\mathbf{D}^{(i+1)}(y_B(i+1)) = \begin{bmatrix} \mathbf{D}^{(i)}(y_B(i+1)) \\ \begin{bmatrix} v_H^{i+1} \\ v_L^{i+1} \end{bmatrix} \exp(y_B(i+1)) \end{bmatrix} \tag{45}$$

For $i < n$, we must have mechanical (i.e. non-optimal) smooth pasting of the value functions that are alive across the boundary:

$$\left(\mathbf{D}^{(i+1)}\right)'(y_B(i+1))_{[1...2i]} = \left(\mathbf{D}^{(i)}\right)'(y_B(i+1)) \tag{46}$$

where $\mathbf{x}_{[1...2i]}$ selects the first $2i$ rows of vector $\mathbf{x}$.

Lastly, for $i = 1$, we must have

$$\mathbf{D}^{(1)}(y_B(1)) = \begin{bmatrix} v_H^1 \\ v_L^1 \end{bmatrix} \exp(y_B(1)) \tag{47}$$

Full solution. We can now state the full solution to the debt valuation given cut-off strategies:

[40] According to JR06, there are exactly $2 \times |S| = 2n$ eigenvalues of $\mathbf{A}$ in the left open half plane (i.e. negative) and $2n$ eigenvalues in the right open half plane (i.e. positive) (actually, they only argue that this holds if $\boldsymbol{\mu} = \mathbf{R} - \frac{1}{2}\mathbf{\Sigma}$, but maybe not for general $\boldsymbol{\mu}$).

Proposition 4. *The debt value functions* $\mathbf{D}$ *for a given default vector* $\boldsymbol{y}_B$ *are*

$$\mathbf{D}(y)=\begin{cases}\underbrace{\mathbf{D}^{(n)}(y)}_{2n\times 1}=\mathbf{G}^{(n)}\cdot\exp\left(\boldsymbol{\Gamma}^{(n)}y\right)\cdot\mathbf{c}^{(n)}+\mathbf{k}_0^{(n)} & y\in I_n\\ \vdots & \vdots\\ \underbrace{\mathbf{D}^{(i)}(y)}_{2i\times 1}=\mathbf{G}^{(i)}\cdot\exp\left(\boldsymbol{\Gamma}^{(i)}y\right)\cdot\mathbf{c}^{(i)}+\mathbf{k}_0^{(i)}+\mathbf{k}_1^{(i)}\exp(y) & y\in I_i\\ \vdots & \vdots\\ \underbrace{\mathbf{D}^{(1)}(y)}_{2\times 1}=\mathbf{G}^{(1)}\cdot\exp\left(\boldsymbol{\Gamma}^{(1)}y\right)\cdot\mathbf{c}^{(1)}+\mathbf{k}_0^{(1)}+\mathbf{k}_1^{(1)}\exp(y) & y\in I_1\end{cases}$$

with the following boundary conditions to pin down vectors $\mathbf{c}^{(i)}$*:*

$$\lim_{y\to\infty}\left|\underbrace{\mathbf{D}^{(n)}(y)}_{2n\times 1}\right| < \infty \tag{48}$$

$$\underbrace{\mathbf{D}^{(i+1)}(y_B(i+1))}_{2(i+1)\times 1} = \underbrace{\begin{bmatrix}\mathbf{D}^{(i)}(y_B(i+1))\\ \begin{bmatrix}v_H^{i+1}\\ v_L^{i+1}\end{bmatrix}\exp(y_B(i+1))\end{bmatrix}}_{2(i+1)\times 1} \tag{49}$$

$$\underbrace{\left(\mathbf{D}^{(i+1)}\right)'(y_B(i+1))_{[1\ldots 2i]}}_{2i\times 1} = \underbrace{\left(\mathbf{D}^{(i)}\right)'(y_B(i+1))}_{2i\times 1} \tag{50}$$

$$\underbrace{\mathbf{D}^{(1)}(y_B(1))}_{2\times 1} = \underbrace{\begin{bmatrix}v_H^{1}\\ v_L^{1}\end{bmatrix}\exp(y_B(1))}_{2\times 1} \tag{51}$$

where $\mathbf{x}_{[1..2i]}$ *selects the first* $2i$ *rows of vector* $\mathbf{x}$*.*

.

Note that the derivative of the debt value vector is

$$\underbrace{\left(\mathbf{D}^{(i)}\right)'(y)}_{2i\times 1}=\mathbf{G}^{(i)}\boldsymbol{\Gamma}^{(i)}\cdot\exp\left(\boldsymbol{\Gamma}^{(i)}y\right)\cdot\mathbf{c}^{(i)}+\mathbf{k}_1^{(i)}\exp(y) \tag{52}$$

where we note that $\boldsymbol{\Gamma}^{(i)}\cdot\exp\left(\boldsymbol{\Gamma}^{(i)}y\right)=\exp\left(\boldsymbol{\Gamma}^{(i)}y\right)\cdot\boldsymbol{\Gamma}^{(i)}$

as both are diagonal matrices (although this interchangeability only is important when $s=1$ as it then helps collapse some equations).

The first boundary condition (48) essentially implies that we can discard any positive entries of $\boldsymbol{\gamma}^{(n)}$ by setting the appropriate coefficients of $\mathbf{C}^{(n)}$ to 0. The second boundary condition (49) implies that we have value matching at any boundary $y_B(i+1)$ for $i<n$, be it to a continuation state or a bankruptcy state . The third boundary condition (50) implies that we also have smooth pasting at the boundary $y_B(i+1)$ for those states in which the firm stays alive on both sides of the boundary. Finally, the fourth boundary condition (51) implies value matching at the boundary $y_B(1)$, but of course only for those states in which the firm is still alive.

Thus, let us summarize the solution steps:

1. Order states so that the most restrictive/illiquid states are with the highest indices, such that $y_B(i)<y_B(j)$ implies $i<j$ (i.e. they appear in the lowest rows/columns in the following matrices).

2. Define the suitable matrices $\mathbf{R}, \mathbf{Q}$ for the transitions, and of course $\boldsymbol{\mu}, \boldsymbol{\Sigma}$ for drift and variance. These apply on the highest interval I_n.

3. Set up the eigenvalue-eigenvector problem and solve for (the matrix of) eigenvectors $\mathbf{G}^{(n)}$ and (the vector of) eigenvalues $\boldsymbol{\gamma}^{(n)}$. Solve for the constant $\mathbf{k}_0^{(n)}$ on this interval.

4. For intervals I_{n-i} we drop for each increment i the last pair of rows and columns of the appropriate matrices, with the following exception. We define $\mathbf{Q}^{(n-i)}$ as the matrix that arises out of $\mathbf{Q}$ when we drop the last i pair of rows and columns, i.e. rows 1-2 and columns 1-2 survive in the 4x4 case. We similarly define $\mathbf{R}^{(n-i)}, \boldsymbol{\mu}^{(n-i)}, \boldsymbol{\Sigma}^{(n-i)}$. We define $\tilde{\mathbf{Q}}^{(n-i)}$ as the matrix that arises out of $\mathbf{Q}$ when we drop the last i pair of rows and the first $n-i$ pairs of columns, i.e. rows 1-2 and columns 3-4 survive in the 4x4 case.

5. Set up the eigenvalue-eigenvector problem for interval I_{n-i} and solve for (the matrix of) eigenvectors $\mathbf{G}^{(n-1)}$ and (the vector of) eigenvalues $\boldsymbol{\gamma}^{(n-1)}$. Solve for the constant $\mathbf{k}_0^{(n-1)}$ on this interval and also for the particular part $\mathbf{k}_1^{(n-1)} \exp(y)$.

6. Build the system of boundary conditions via the matrix definitions of the debt to solve for the linear coefficients $\mathbf{c}^{(i)}$. To impose boundary condition (48), it is probably easiest to just use those entries of $\boldsymbol{\gamma}^{(n)}$ that are negative. Thus, the appropriate $\mathbf{C}^{(n)}$ for I_n is only a $2n \times 1$ vector, and not a $4n \times 1$ vector.

1.4 Equity

The equity holders are unaffected by the individual liquidity shocks the debt holders are exposed to. The only shocks the equity holders are directly exposed to are the shifts in $\mu(s)$ and $\sigma(s)$, i.e. shifts to the cash-flow process.

However, as debt has maturity and is rolled over, equity holders are indirectly affected by liquidity shocks in the market through the effect it has on debt prices. Thus, when debt matures, it is either rolled over if the debt holders are of type H, or it is reissued to different debt holders in the case that the former debt holder is of type L. Either way, there is a cash flow (inflow or outflow) of $m\left[\mathbf{S}^{(i)} \cdot \mathbf{D}^{(i)}(y) - p\mathbf{1}_i\right]$ at each instant as a mass $m \cdot dt$ of debt holders matures on $[t, t+dt]$.

For notational ease, we will denote by double letters (e.g. $\mathbf{xx}$) a constant for equity that takes a similar place as a single letter (i.e. $\mathbf{x}$) constant for debt. Then, the HJB for equity on interval I_i is given by

$$\begin{aligned}\left(\mathbf{RR}^{(i)} - \mathbf{QQ}^{(i)}\right)\mathbf{E}^{(i)}(y) &= \boldsymbol{\mu\mu}^{(i)}\left(\mathbf{E}^{(i)}\right)'(y) + \frac{1}{2}\boldsymbol{\Sigma\Sigma}^{(i)}\left(\mathbf{E}^{(i)}\right)''(y) \\ &\quad + \underbrace{\mathbf{1}_i \exp(y)}_{Cashflow} - \underbrace{(1-\pi)\, c\mathbf{1}_i}_{Coupon} + \underbrace{m\left[\mathbf{S}^{(i)} \cdot \mathbf{D}^{(i)}(y) - p\mathbf{1}_i\right]}_{Rollover} \qquad (53)\end{aligned}$$

where

$$\mathbf{RR}^{(i)} = \operatorname{diag}\left([r_H(1), ..., r_H(i)]\right) \qquad (54)$$
$$\boldsymbol{\mu\mu}^{(i)} = \operatorname{diag}\left([\mu(1), ..., \mu(i)]\right) \qquad (55)$$
$$\boldsymbol{\Sigma\Sigma}^{(i)} = \operatorname{diag}\left(\left[\sigma^2(1), ..., \sigma^2(i)\right]\right) \qquad (56)$$

are $i \times i$ square matrices, $\mathbf{QQ}^{(i)}$is the transition matrix only between aggregate states that is also an $i \times i$ square matrix, and $\mathbf{S}^{(i)}$ is a $i \times 2i$ matrix that selects which debt values the firm is able to issue (each row has to sum to 1), and m is a scalar (**NOTE**: In contrast to $\mathbf{R}$, the matrix $\mathbf{RR}$ does not contain the maturity intensity m). For example, for $i = 2$, if the company is able to place debt only to H types, then $\mathbf{S}^{(2)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. It is important that for reach row i only entries $2i-1$ and $2i$ are possibly nonzero, whereas all other entries are identically zero (otherwise, one would issue bonds belonging to a different state).

[TABLE 1.4 ABOUT HERE]

Writing out $\mathbf{D}^{(i)}(y) = \mathbf{G}^{(i)} \exp\left(\mathbf{\Gamma}^{(i)} y\right) \mathbf{c}^{(i)}$

and conjecturing a solution to the particular, non-constant part $\underbrace{\mathbf{KK}^{(i)}}_{i\times 4i}\underbrace{\exp\left(\mathbf{\Gamma}^{(i)} y\right)}_{4i\times 4i}\underbrace{\mathbf{c}^{(i)}}_{4i\times 1}$, we have

$$\begin{aligned}&\left(\mathbf{RR}^{(i)} - \mathbf{QQ}^{(i)}\right)\mathbf{KK}^{(i)} \exp\left(\mathbf{\Gamma}^{(i)} y\right)\mathbf{c}^{(i)} \\ = \ &\left[\boldsymbol{\mu\mu}^{(i)} \cdot \mathbf{KK}^{(i)} \cdot \mathbf{\Gamma}^{(i)} + \frac{1}{2}\mathbf{\Sigma\Sigma}^{(i)}\mathbf{KK}^{(i)} \cdot \left(\mathbf{\Gamma}^{(i)}\right)^2 + m \cdot \mathbf{S}^{(i)} \cdot \mathbf{G}^{(i)}\right]\exp\left(\mathbf{\Gamma}^{(i)} y\right)\mathbf{c}^{(i)}\end{aligned} \tag{57}$$

We can solve this by considering each $\gamma_j^{(i)}$ separately — recall that $\mathbf{c}^{(i)}$ is a vector and $\exp\left(\mathbf{\Gamma}^{(i)} y\right)$ is a *diagonal* matrix and in total there are $4i$ different roots. Consider the part of the particular part $\mathbf{S}^{(i)} \cdot \mathbf{g}_j^{(i)} \exp\left(\gamma_j^{(i)} y\right) \cdot c_j^{(i)}$ and our conjecture gives $\underbrace{\mathbf{KK}_j^{(i)}}_{i\times 1}\underbrace{\exp\left(\gamma_j^{(i)} y\right)}_{1\times 1} \cdot \underbrace{c_j^{(i)}}_{1\times 1}$ for each root $j \in [1, ..., 4i]$. Plugging in and multiplying out the scalar $\exp\left(\gamma_j^{(i)} y\right) c_j^{(i)}$, we find that

$$\left(\mathbf{RR}^{(i)} - \mathbf{QQ}^{(i)}\right)\mathbf{KK}_j^{(i)} = \boldsymbol{\mu\mu}^{(i)} \cdot \mathbf{KK}_j^{(i)} \cdot \gamma_j^{(i)} + \frac{1}{2}\mathbf{\Sigma\Sigma}^{(i)}\mathbf{KK}_j^{(i)} \cdot \left(\gamma_j^{(i)}\right)^2 + m \cdot \mathbf{S}^{(i)} \cdot \mathbf{g}_j^{(i)} \tag{58}$$

Solving for $\mathbf{KK}_j^{(i)}$, we have

$$\underbrace{\mathbf{KK}_j^{(i)}}_{i\times 1} = \underbrace{\left[\mathbf{RR}^{(i)} - \mathbf{QQ}^{(i)} - \boldsymbol{\mu\mu}^{(i)} \cdot \gamma_j^{(i)} - \frac{1}{2}\mathbf{\Sigma\Sigma}^{(i)} \cdot \left(\gamma_j^{(i)}\right)^2\right]^{-1}}_{i\times i} m \cdot \underbrace{\mathbf{S}^{(i)}}_{i\times 2i}\underbrace{\mathbf{g}_j^{(i)}}_{2i\times 1} \tag{59}$$

Finally, for the homogenous part we use the same approach as above, but now we have less states as the individual liquidity state drops out. Thus, we conjecture $\mathbf{gg}\exp(\gamma\gamma y)$ to get

$$\mathbf{0}_i = \left(\mathbf{QQ}^{(i)} - \mathbf{RR}^{(i)}\right)\mathbf{gg} + \boldsymbol{\mu\mu}^{(i)}\gamma\gamma\mathbf{gg} + \frac{1}{2}\mathbf{\Sigma\Sigma}^{(i)}\gamma\gamma\mathbf{gg} \tag{60}$$

so that, again, we have the following eigenvector eigenvalue problem

$$\gamma\gamma\mathbf{jj} = \begin{bmatrix} \mathbf{0}_i & \mathbf{I}_i \\ 2\left(\mathbf{\Sigma\Sigma}^{(i)}\right)^{-1}\left(\mathbf{RR}^{(i)} - \mathbf{QQ}^{(i)}\right) & -2\left(\mathbf{\Sigma\Sigma}^{(i)}\right)^{-1}\boldsymbol{\mu\mu}^{(i)}\end{bmatrix}\mathbf{jj} = \underbrace{\mathbf{AA}^{(i)}}_{2i\times 2i}\mathbf{jj} \tag{61}$$

which gives $\left(\gamma\gamma_j^{(i)}, \mathbf{gg}_j^{(i)}\right)$ for $j \in [1, ..., 2i]$ solutions. We stack these into a matrix of eigenvectors $\mathbf{GG}^{(i)}$ and a vector of eigenvalues $\boldsymbol{\gamma\gamma}^{(i)}$, from which we define the diagonal matrix of eigenvalues $\mathbf{\Gamma\Gamma}^{(i)} \equiv \operatorname{diag}\left(\boldsymbol{\gamma\gamma}^{(i)}\right)$. What remains is to solve for $\mathbf{kk}_0^{(i)}$ and $\mathbf{kk}_1^{(i)}$. We have

$$\mathbf{kk}_0^{(i)} = \left[\mathbf{RR}^{(i)} - \mathbf{QQ}^{(i)}\right]^{-1}\left[-(1-\pi)c\mathbf{1}_i + m\left(\mathbf{S}^{(i)}\mathbf{k}_0^{(i)} - p\mathbf{1}_i\right)\right] \tag{62}$$

and

$$\mathbf{kk}_1^{(i)} = \left[\mathbf{RR}^{(i)} - \mathbf{QQ}^{(i)} - \boldsymbol{\mu\mu}^{(i)} - \frac{1}{2}\mathbf{\Sigma\Sigma}^{(i)}\right]^{-1}\left(\mathbf{1}_i + m \cdot \mathbf{S}^{(i)}\mathbf{k}_1^{(i)}\right) \tag{63}$$

with $\mathbf{k}_1^{(n)} = \mathbf{0}$.

We are left with the following proposition.

Proposition 5. *The equity value functions* $\mathbf{E}$ *for a given default vector* $\boldsymbol{y}_B$ *are*

$$\mathbf{E}(y) = \begin{cases} \underbrace{\mathbf{E}^{(n)}(y)}_{n\times 1} = \mathbf{GG}^{(n)} \cdot \exp\left(\mathbf{\Gamma\Gamma}^{(n)} y\right) \cdot \mathbf{cc}^{(n)} + \mathbf{KK}^{(n)} \exp\left(\mathbf{\Gamma}^{(n)} y\right) \mathbf{c}^{(n)} + \mathbf{kk}_0^{(n)} + \mathbf{kk}_1^{(n)} \exp(y) & y \in I_n \\ \vdots & \vdots \\ \underbrace{\mathbf{E}^{(i)}(y)}_{i\times 1} = \mathbf{GG}^{(i)} \cdot \exp\left(\mathbf{\Gamma\Gamma}^{(i)} y\right) \cdot \mathbf{cc}^{(i)} + \mathbf{KK}^{(i)} \exp\left(\mathbf{\Gamma}^{(i)} y\right) \mathbf{c}^{(i)} + \mathbf{kk}_0^{(i)} + \mathbf{kk}_1^{(i)} \exp(y) & y \in I_i \\ \vdots & \vdots \\ \underbrace{\mathbf{E}^{(1)}(y)}_{1\times 1} = \mathbf{GG}^{(1)} \cdot \exp\left(\mathbf{\Gamma\Gamma}^{(1)} y\right) \cdot \mathbf{cc}^{(1)} + \mathbf{KK}^{(1)} \exp\left(\mathbf{\Gamma}^{(1)} y\right) \mathbf{c}^{(1)} + \mathbf{kk}_0^{(1)} + \mathbf{kk}_1^{(1)} \exp(y) & y \in I_1 \end{cases}$$

with the following boundary conditions to pin down the vector $\mathbf{cc}^{(i)}$*:*

$$\lim_{y\to\infty} \left| \underbrace{\mathbf{E}^{(n)}(y) \exp(-y)}_{n\times 1} \right| < \infty \tag{64}$$

$$\underbrace{\mathbf{E}^{(i+1)}(y_B(i+1))}_{(i+1)\times 1} = \underbrace{\begin{bmatrix} \mathbf{E}^{(i)}(y_B(i+1)) \\ 0 \end{bmatrix}}_{(i+1)\times 1} \tag{65}$$

$$\underbrace{\left(\mathbf{E}^{(i+1)}\right)'(y_B(i+1))_{[1\ldots i]}}_{i\times 1} = \underbrace{\left(\mathbf{E}^{(i)}\right)'(y_B(i+1))}_{i\times 1} \tag{66}$$

$$\underbrace{\mathbf{E}^{(i)}(y_B(1))}_{i\times 1} = 0 \tag{67}$$

where $\mathbf{x}_{[1\ldots i]}$ *selects the first* i *rows of vector* $\mathbf{x}$*.*

Note first the dimensionalities: $\underbrace{\mathbf{\Gamma\Gamma}^{(i)}}_{2i\times 2i}, \underbrace{\mathbf{GG}^{(i)}}_{i\times 2i}$ and $\underbrace{\mathbf{\Gamma}^{(i)}}_{4i\times 4i}, \underbrace{\boldsymbol{G}^{(i)}}_{2i\times 4i}$. Note second the derivative of the equity value vector is

$$\underbrace{\left(\mathbf{E}^{(i)}\right)'(y)}_{i\times 1} = \mathbf{GG}^{(i)}\mathbf{\Gamma\Gamma}^{(i)} \cdot \exp\left(\mathbf{\Gamma\Gamma}^{(i)} y\right) \cdot \mathbf{cc}^{(i)} + \mathbf{KK}^{(i)}\mathbf{\Gamma}^{(i)} \exp\left(\mathbf{\Gamma}^{(i)} y\right) \mathbf{c}^{(i)} + \mathbf{kk}_1^{(i)} \exp(y) \tag{68}$$

where we note that $\mathbf{\Gamma}^{(i)} \cdot \exp\left(\mathbf{\Gamma}^{(i)} y\right) = \exp\left(\mathbf{\Gamma}^{(i)} y\right) \cdot \mathbf{\Gamma}^{(i)}$ and $\mathbf{\Gamma\Gamma}^{(i)} \cdot \exp\left(\mathbf{\Gamma\Gamma}^{(i)} y\right) = \exp\left(\mathbf{\Gamma\Gamma}^{(i)} y\right) \cdot \mathbf{\Gamma\Gamma}^{(i)}$

as both are diagonal matrices (although this interchangeability only is important when $s = 1$ as it then helps collapse some equations).

The optimality conditions for bankruptcy boundaries $\{y_B(i)\}_i$ are given by

$$\left(\mathbf{E}^{(i)}\right)'(y_B(i))_{[i]} = 0 \tag{69}$$

i.e., a smooth pasting condition at the boundaries at which default is declared.

Table 1: **Baseline Parameters used in calibration.** Unreported parameters are tax rate $\pi = 0.35$. For pre-fixed parameters, transition density $\zeta^{\mathbb{P}}$, jump risk premium $\exp(\kappa)$, risk price η, risk-free rate r, cash flow growth $\mu_{\mathbb{P}}$, primary bond market issuance cost κ, and inverse of debt maturity m are taken from literature (e.g., Chen, Xu, and Yang (2012)). Systematic volatility σ_m is chosen to match equity volatility. Treasury liquidity premium Δ is from 3-month repo-treasury spread. Meeting intensity λ between low-type investors and dealers are set so that selling holdings takes one week (2.5 weeks) in normal (recession) period. Investors' bargaining power β is from Feldhütter (2012). State- and type- dependent recovery rates α_l^s's are calculated using existing literature on credit risk models and observed bid-ask spreads of defaulted bonds. In Section 5.2, the ultimate recovery rate $\hat{\alpha}$ is based on risk-adjusted holding period returns of post-default corporate bonds in Moody's Default and Recovery Databases from 1984-2010. For calibrated parameters (in bold face), the idiosyncratic volatility σ_i is chosen to target the default probability of Baa firms. The state-B liquidity shock intensity ξ_B is also pre-fixed to match corporate bond turnover in recession, while ξ_G is chosen to target the investment-grade Bond-CDS spread in normal time. Holding cost χ are chosen to target the investment grade bid-ask spread.

Symbol	Description	State G	State B	Justification / Target
		A. Pre-fixed parameters		
$\zeta^{\mathbb{P}}$	Transition density	0.1	0.5	literature
$\exp(\kappa)$	Jump risk premium	2.0	0.5	literature
η	Risk price	0.165	0.255	literature
r	Risk free rate	0.02	0.02	literature
$\mu_{\mathbb{P}}$	Cash flow growth	0.045	0.015	literature
σ_m	Systematic vol	0.1	0.11	equity vol
ω	Primary market cost	0.01		literature
m	Average maturity intensity	0.2		literature
Δ	Treasury liquidity premium	15 bps	40 bps	repo spread
λ	Meeting intensity	50	20	anecdotal evidence
β	Investor's bargaining power	0.05		literature
α_H	Recovery rate of H type	58.71%	32.56%	literature
α_L	Recovery rate of L type	57.49%	30.50%	literature
$\hat{\alpha}$	Ultimate recovery rate	87.96%	64.68%	literature
		B. Calibrated parameters		
σ_i	Idiosyncratic vol	**0.225**		Baa default prob
ξ	Liquidity shock intensity	**0.5**	1.0	Bond-CDS in G, turnover
χ	Holding cost	**1.25**	**2.35**	Investment bid-ask sprd

Table 2: **Default probabilities and credit spreads across credit ratings.** Default probabilities are cumulative default probabilities over 1920-2011 from Moody's investors service (2012), and credit spreads are from FISD transaction data over 1994-2010. We report the time series mean, with the standard deviation (reported underneath) being calculated using Newey-West procedure with 15 lags. The standard deviation of default probabilities are calculated based on the sample post 1970's due to data availability issue. On model part, we first calculate the quasi market leverage for Compustat firms (excluding financial and utility firms) for each rating over 1994-2010, then match observed quasi market leverage by locating the corresponding cash flow level y. We then calculate the time series average of model-implied credit spreads and Bond-CDS spreads across these firm-quarter observations. This procedure implies that our model-implied leverages exact match the empirical counterpart.

	Maturity = 5 years				Maturity = 10 years			
	Aaa/Aa	A	Baa	Ba	Aaa/Aa	A	Baa	Ba
	Panel A. Default probability (%)							
data	0.7	1.3	3.1	9.8	2.1	3.4	7.0	19.0
model	0.4	1.1	2.9	8.6	1.7	4.1	8.4	16.8
$\chi = 0$	0.3	0.9	2.5	7.4	1.5	3.7	7.6	15.5
	Panel B. Credit spreads (bps)							
	State G							
data	55.7	85.7	149	315	61.2	90.2	150	303
	(3.7)	(6.6)	(15.5)	(33.8)	(4.4)	(6.3)	(12.8)	(22.7)
model	61.4	87.2	148	317	93.8	139	216	372
$\chi = 0$	23.4	42.2	88.1	226	43.7	79.1	141	270
	State B							
data	107	171	275	542	106	159	262	454
	(5.8)	(10.5)	(23.9)	(29.8)	(6.7)	(13.8)	(29.3)	(44.4)
model	117	155	235	434	142	201	292	465
$\chi = 0$	49.1	74.8	132	283	71.6	116	186	320

Table 3: **Bond-CDS spreads and bid-ask spreads across credit ratings.** In Panel A, the sample to construct Bond-CDS spreads are firms with both 5-year and 10-year bonds, over the sample period from 2005 to 2012. We report the time series mean for both including and excluding crisis (from October 2008 to March 2009), with the standard deviation (reported underneath) being calculated using Newey-West procedure with 15 lags. On the model side, we calculate the quasi market leverage for Compustat firms (excluding financial and utility firms) for each rating classes. We match the observed quasi market leverage by locating the corresponding cash flow level y, and calculate the time series average of model-implied credit spreads and Bond-CDS spreads across these firm-quarter observations. This procedure implies that our model-implied leverages exactly match the empirical counterpart. The row of $\chi = 0$ gives the model implied moments when there is no liquidity frictions in pre-default market under our baseline parameters. In Panel B, the normal time bid-ask spread are taken from Edwards, Harris, and Piwowar (2007) for a median size trade. The recession time numbers are normal time numbers multiplied by the ratio of bid-ask spread implied by Roll's measure of illiquidity (following Bao, Pan, and Wang (2011)) in recession time to normal time. The model counterpart is computed for a bond with time to maturity of 8.3 years, which is the mean time-to-maturity of frequently traded bonds (where we can compute a Roll's measure) in the TRACE sample. The model implied bid-ask spread for $\chi = 0$ is zero by definition.

	Panel A. Bond-CDS spreads (bps)							
	Maturity = 5 years				Maturity = 10 years			
	Aaa/Aa	A	Baa	Ba	Aaa/Aa	A	Baa	Ba
	State G							
data	27.7	44.4	74.6	104	23.2	37.2	58.5	67.8
	(6.6)	(5.8)	(8.7)	(11.2)	(9.9)	(6.1)	(9.0)	(16.1)
model	50.4	56.3	72.1	113	63.5	71.8	89.0	119
	State B							
data	76.0	125	182	227	72.2	104	162	191
	(5.1)	(2.1)	(18.0)	(39.2)	(3.4)	(6.1)	(22.0)	(36.5)
model	105	114	135	182	108	119	140	172

	Panel B. Bid-Ask spreads (bps)					
	State G			State B		
	Superior	Investment	Junk	Superior	Investment	Junk
data	40	50	70	77	125	218
model	43	50	73	109	127	187

Table 4: **Structural Liquidity-Default Decomposition for 5-Year Bonds Across Ratings.** For each rating, we locate the cash flow y that corresponds to the historical total credit spread of a ten year bond at this rating. We perform the structural liquidity-default decomposition following the procedure discussed in the text across aggregate states. We quantitatively evaluate the channels that give rise to the observed level of credit spreads and their changes when the economy shifts from normal time to recession. As a comparison to previous literature (e.g. Longstaff, Mithal, and Neis (2005)) , we also report the CDS spread implied by the model across ratings and aggregate states.

Rating	State	Spread (treasury)	Spread (rf)	Default-Liquidity Decomposition *Pure Def*	*Pure Liq*	*Liq* → *Def*	*Def* → *Liq*
				Panel I: Explaining Credit Spread Levels			
Aaa/Aa	G (bps)	56.5	41.5	4.8	31.1	3.5	2.1
	(%)		100	12	75	9	5
	B (bps)	114	73.8	9.6	55.8	5.4	2.9
	(%)		100	13	76	7	4
A	G (bps)	85.3	70.3	16.2	40.2	9.2	4.7
	(%)		100	23	57	13	7
	B (bps)	165	125	26.9	75.7	15.3	6.6
	(%)		100	22	61	12	5
Baa	G (bps)	151	136	48.1	50.8	24.9	11.7
	(%)		100	35	37	18	9
	B (bps)	267	227	76.4	93.7	39.2	17.9
	(%)		100	34	41	17	8
Ba	G (bps)	318	303	145	64.0	66.2	27.4
	(%)		100	48	21	22	9
	B (bps)	522	482	207	126	98.3	50.6
	(%)		100	43	26	20	10
				Panel II: Explaining Credit Spread Changes			
Aaa/Aa	$G \to B$ (bps)	57.3	32.3	4.8	24.7	1.9	0.9
	(%)		100	15	76	6	3
A	$G \to B$ (bps)	79.3	54.3	10.7	35.5	6.1	2.0
	(%)		100	20	65	11	4
Baa	$G \to B$ (bps)	117	91.7	28.3	42.9	14.3	6.2
	(%)		100	31	47	16	7
Ba	$G \to B$ (bps)	204	179	62.1	61.8	32.1	23.2
	(%)		100	35	34	18	13

Table 5: **Effect of Liquidity Provision Policy on 5-Year Bonds Across Ratings.** We consider a policy experiment that improves the liquidity condition (χ, λ) in the B state to be as good as G state. We compute the credit spread under the policy for both G and B state, and perform the structural liquidity-default decomposition to examine the channels that are responsible for the reduced borrowing cost.

Rating	State	Credit Spread (rf)		Contribution of Each Component		
		w/o. policy	w. policy	*pure LIQ* (%)	*LIQ→DEF* (%)	*DEF→LIQ* (%)
Aaa/Aa	*G*	41.5	28.6	83	12	5
	B	73.8	52.7	89	7	4
A	*G*	70.3	47.0	71	12	17
	B	125	67.3	80	10	10
Baa	*G*	136	99.4	61	25	14
	B	227	146	71	16	13
Ba	*G*	303	244	44	38	18
	B	482	344	56	27	17

Table 6: **Summary Statistics for Annualized Net PME on Defaulted Bond by Default Time** Data on holding period return of post-default bonds are from Moody's Default and Recovery Database 1984-2010. We adjust for risk by discounting the return of holding defaulted bonds by a public market benchmark over the same investment horizon. The resulting measure is called "Public Market Equivalent" as reported below.

Default Time	# of Def. Bond	Mean Annual Net PME	Mean Annual Net Return
Non-Recession	512	0.3126	0.3922
Recession	130	0.5537	0.4672
Full Sample	642	0.3613	0.4074

Table 7: Matrix & Vector Dimensions.

Debt Parameters			Equity Parameters		
Symbol	Interpretation	Dimension	Symbol	Interpretation	Dimension
$\mathbf{D}^{(i)}(y)$	Debt Value Function	$2i \times 1$	$\mathbf{E}^{(i)}(y)$	Equity Value Function	$i \times 1$
$\boldsymbol{\mu}^{(i)}$	(Log)-Drifts	$2i \times 2i$	$\boldsymbol{\mu\mu}^{(i)}$	(Log-)Drifts	$i \times i$
$\boldsymbol{\Sigma}^{(i)}$	Volatilities	$2i \times 2i$	$\boldsymbol{\Sigma\Sigma}^{(i)}$	Volatilities	$i \times i$
$\mathbf{R}^{(i)}$	Discount rates and maturity	$2i \times 2i$	$\mathbf{RR}^{(i)}$	Discount rates	$i \times i$
$\boldsymbol{\chi}^{(i)}$	Holding costs	$2i \times 1$	c	Coupon	1×1
$\mathbf{Q}^{(i)}$	Transition to cont. states	$2i \times 2i$	$\mathbf{QQ}^{(i)}$	Transition to cont. states	$i \times i$
$\tilde{\mathbf{Q}}^{(i)}$	Transition to default states	$2i \times 2(n-i)$	$\mathbf{AA}^{(i)}$	Matrix to be decomposed	$2i \times 2i$
$\mathbf{v}^{(i)}$	Vector of recovery values	$2(n-i) \times 1$	$\boldsymbol{\Gamma\Gamma}^{(i)}$	Diag matrix of eigenvalues	$2i \times 2i$
$\mathbf{A}^{(i)}$	Matrix to be decomposed	$4i \times 4i$	$\mathbf{GG}^{(i)}$	Matrix of eigenvectors	$i \times 2i$
$\boldsymbol{\Gamma}^{(i)}$	Diag matrix of eigenvalues	$4i \times 4i$	$\mathbf{kk}_0^{(i)}, \mathbf{kk}_1^{(i)}$	Coeff. of particular sol.	$i \times 1$
$\mathbf{G}^{(i)}$	Matrix of eigenvectors	$2i \times 4i$	$\mathbf{S}^{(i)}$	Issuance matrix	$i \times 2i$
$\mathbf{k}_0^{(i)}, \mathbf{k}_1^{(i)}$	Coeff. of particular sol.	$2i \times 1$	$\mathbf{KK}^{(i)}$	Coeff. of particular sol.	$i \times 4i$
$\mathbf{c}^{(i)}$	Vector of constants	$4i \times 1$	$\mathbf{cc}^{(i)}$	Vector of constants	$2i \times 1$

Figure 1: Distribution of Annualized Net Return (left) and Public Market-Adjusted Return (right) of Defaulted Bonds

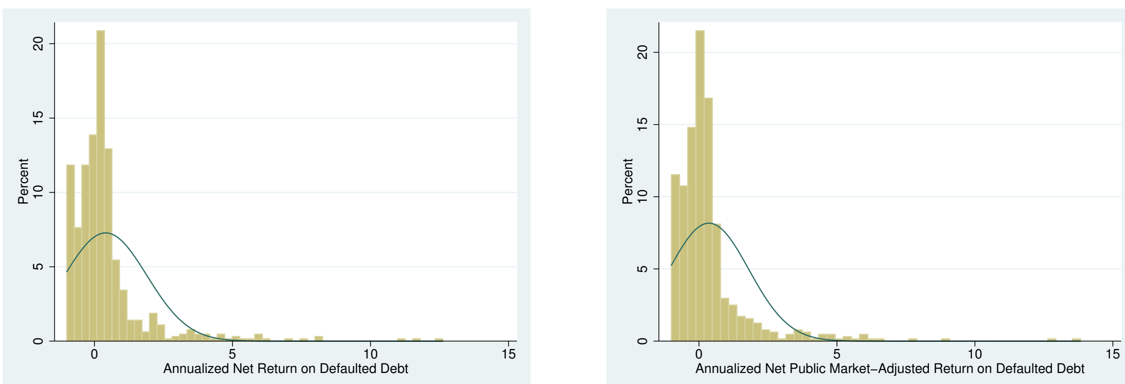

Figure 2: **Empirical Distribution of Market Leverage for Compustat Firms by Aggregate State and Rating classes.** We compute quasi-market leverage for each firm-quarter observation in the Compustat database from 1997-2012. The B state is defined as quarters for which at least two months are classified as NBER recession month. The remaining quarters are G state. We drop financial and utility firms in our sample. We also exclude firms with zero leverage.

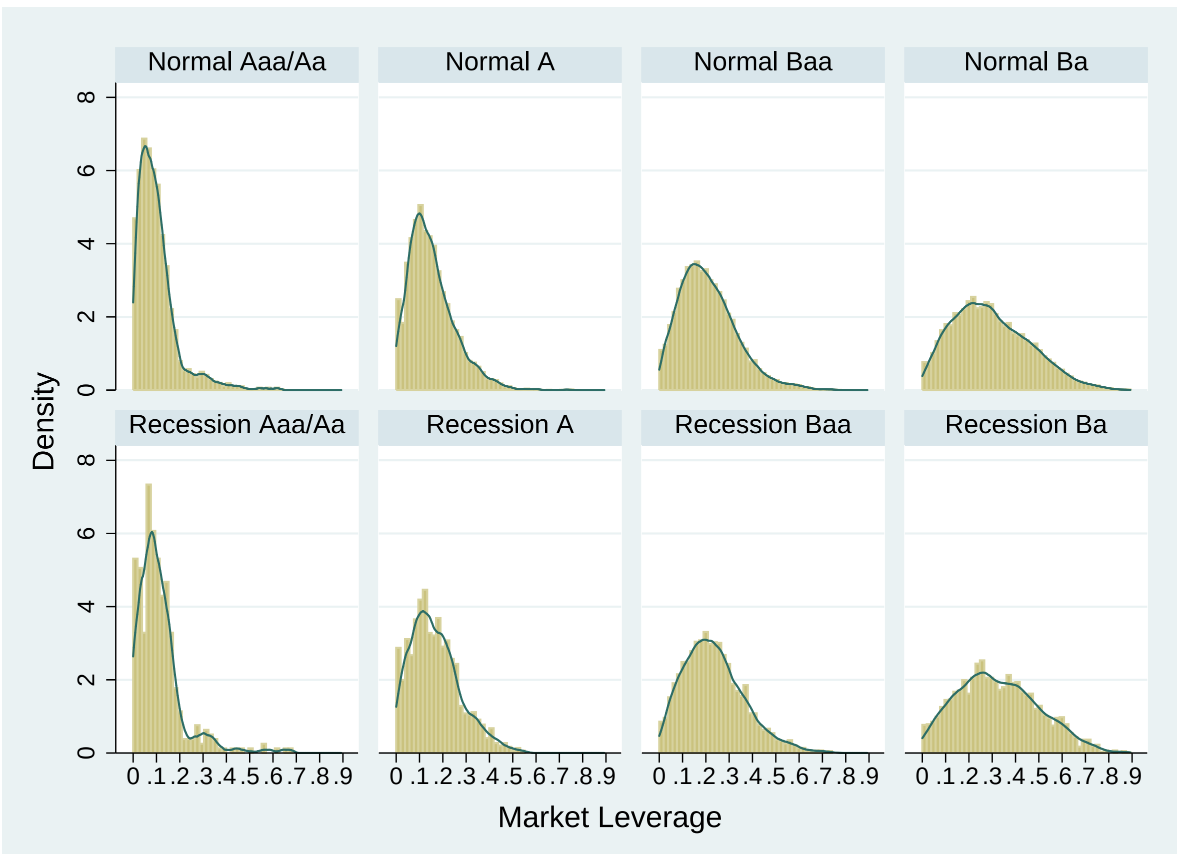

Figure 3: Model Implied Nonlinearity between Market Leverage, Default Rates and Total Credit Spread