Innovation, Decentralization, and Planning in a Multi-Region Model of Schumpeterian Economic Growth[1]

by

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Abstract

We study innovation and the resulting Schumpeterian economic growth that this innovation gives rise to in a model with N heterogeneous regions. For each region i where $i = 1,\ldots,N$, our analysis leads to five findings. First, we define the balanced growth path (BGP) allocations and the equilibrium of interest. Second, we stipulate the form of the innovation possibilities frontier that is consistent with balanced economic growth. Third, we derive the growth rate of the ***ith*** region in the decentralized equilibrium and show that there are no transitional dynamics. Fourth, we solve the social planner's problem and derive the Pareto optimal growth rate for the ***ith*** region. Fifth, we compare the two preceding growth rates and then discuss the circumstances in which there is either too much or too little innovation in (i) the ***ith*** region, (ii) the aggregate economy of $N > 2$ regions and (iii) the specific case of an aggregate economy of $N = 2$ regions. Finally, we conclude and then offer suggestions for extending the research described here.

Keywords: Human Capital, Innovation, Multi-Region Economy, Schumpeterian Economic Growth

JEL Codes: R11, J24, O31

1. Introduction

1.1. Objective and rationale

The general objective of our paper is to study innovation and the resulting Schumpeterian economic growth that this innovation gives rise to in a dynamic model of an aggregate economy consisting of N heterogeneous regions. For each region i, $i=1,...,N$, our analysis focuses on five issues. First, we define the balanced growth path (BGP) allocations and the ensuing equilibrium that are of interest. Second, we stipulate the form of the so called innovation possibilities frontier that is consistent with balanced economic growth. Third, we derive the economic growth rate of the *ith* region in the decentralized equilibrium with *no* governmental or social planning and show that there are no transitional dynamics. Fourth, we solve the social planner's optimization problem and derive the Pareto optimal economic growth rate in the *ith* region. Finally, we compare the two preceding growth rates and discuss the circumstances in which there is either too much or too little innovation in (i) the *ith* region, (ii) the aggregate economy of $N>2$ regions and (iii) the specific case of an aggregate economy of $N=2$ regions.

The general objective stated in the preceding paragraph and the specific issues that our analysis concentrates on are interesting and relevant because of four reasons. First, our analysis formalizes and therefore helps us better understand the observation of researchers such as Fischer and Nijkamp (2009), Baumol (2010), and Batabyal and Nijkamp (2013a) who have emphasized that *innovation* is a significant driver of regional economic growth and development. In fact, this view is now widely accepted in regional science and hence policymakers understand that "the presence of successful entrepreneurship and of a favourable business and innovation climate will bring high benefits to the host region" (Fischer and Nijkamp, 2009, p. 186). Second, our analysis explicitly recognizes that innovative activities and processes are fundamentally *competitive* in nature and hence this analysis "operationalizes" for regions, a central insight of Joseph Schumpeter who argued that growth processes are characterized by *creative destruction* in which "economic growth is driven, at least in part, by new firms replacing incumbents and new machines and products replacing old ones" (Acemoglu, 2009, p. 458). Third, our analysis helps shed light on several aspects of the growth process including the impacts of the trinity of monopoly distortions, the profit stealing effect, and the replacement effect[4] on the extent of innovative activity in regions. Finally, the analysis we undertake helps shed light on firm dynamics and the reallocation of resources among incumbents and new R&D conducting entrants in regions.

1.2. Review of the literature

In an early paper, Leahy and McKee (1972) stated but did not explicitly model the idea that change in regional economies can be well understood by adopting a "Schumpeterian view" of the regional economy. Despite the appearance of this statement more than four decades ago, regional scientists have begun to use the ideas of Schumpeter to systematically investigate the nexus between innovation and economic growth in regions only since the early 1980s. Even so, there is now a fairly substantial empirical and case study based literature that has analyzed alternate aspects of Schumpeterian economic growth in regional economies.

Lodde (2008) uses sectoral data to compare the utility of the Schumpeterian approach in understanding the relationship between human capital and productivity growth in regions in Italy. He shows that there is qualified support for the Schumpeterian hypothesis. Crespi and Pianta (2008) focus on six nations in Europe and point out that the ideas of Schumpeter are useful in

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4 These terms are explained in detail in section 2.

comprehending the variety of innovation and what they call “innovation-performance relationships” in the six countries under study.

Qian (2007) uses the passage of national pharmaceutical patent law as a natural experiment to show that the implementation of patents stimulates innovation mostly in countries with higher market freedom. Similarly, Quatraro (2009) uses Italian patent data and shows that Schumpeter’s views about innovation and business cycles can be used to shed light on the diffusion of innovation capabilities in various Italian regions. These and other such studies provide empirical support for the idea that a complementarity exists between patent protection and product market competition in fostering innovation.

Aghion *et al.* (2009) use UK firm level panel data and show that there is empirical support for the idea that more intense competition enhances innovation among what they call “frontier” firms but that this kind of intense competition may actually discourage innovation in “non-frontier” firms. Focusing on innovative firms, Akcigit and Kerr (2010) and Haliwanger *et al.* (2010) show that firm size and firm age are positively correlated and that in innovative industries, small firms exit the industry more frequently and hence the surviving firms tend to grow relatively rapidly.

Focusing on 2,645 counties in the United States, Hodges and Ostbye (2010) find support for a Schumpeterian growth model because, in their empirical model, bigger firms are needed to carry out effective R&D which then leads to higher economic growth in the localities being studied. Finally, Saunoris and Payne (2011) use United States data from 1960 to 2007 and show that long run increases in R&D expenditures are necessary to offset lower R&D productivity due to the presence of product proliferation.

There are very few *theoretical* studies of the connections between innovation and

Schumpeterian economic growth in the context of regions. Recently, Batabyal and Nijkamp (2012) have theoretically analyzed a one-sector, discrete-time, Schumpeterian model of growth in a regional economy. These researchers show that the regional economy they study experiences bursts of unemployment followed by periods of full employment. There are two key differences between the Batabyal and Nijkamp (2012) paper and our paper. First, the Batabyal and Nijkamp (2012) paper studies a single region in discrete time, whereas we focus on a multi-region economy in continuous time. Second, the specific questions we study in our paper are different from the basic question studied by Batabyal and Nijkamp (2012). In particular, Batabyal and Nijkamp (2012) analyze the nature of labor dynamics in the presence of Schumpeterian economic growth. In contrast, the questions we address—see section 1.1—are very different and labor is not even a factor of production in the present paper.[5]

Our formal analysis departs from and "supersedes" the existing theoretical literature in four ways. First, the basic unit of analysis in our paper is a region and *not* a country. In this regard, the word "region" refers to a geographic entity that is smaller than a nation. Second, instead of working with labor, we work with *human capital* as a key factor of production. Third, the model we analyze

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In the last two years, in Batabyal and Nijkamp (2012, 2013a, 2013b, 2013c), we have analyzed various aspects of economic growth and development in stylized regions. What is common to these papers is that they all employ growth models to analyze the pertinent questions of interest. In addition, because of the nature of the theoretical modeling that is undertaken, all four papers share some similarities. For instance, human capital is a factor of production in more than one of these four papers. This commonality notwithstanding, it is important to understand that the growth models employed in these four papers are fundamentally *dissimilar*. We have already explained the differences between Batabyal and Nijkamp (2012) and the present paper in this paragraph. In Batabyal and Nijkamp (2013a), the basic focus is on optimal patent policy in a region and on the effects that alternate patent policies have on economic growth in this same region. In contrast, the present paper practically dispenses with patents and optimal patent policy by simply assuming that innovation generating firms receive perpetual patents on newly invented machines. The Batabyal and Nijkamp (2013b) paper is about the effect that the preferences of the creative class have on unbalanced growth in an urban economy. This paper analyzes a constant growth path (CGP) equilibrium. Once again in contrast, the present paper has nothing to do with the creative class and nor does it analyze unbalanced growth; instead, our paper analyzes balanced growth path (BGP) equilibria. Finally, in Batabyal and Nijkamp (2013c), we study the effect that negative externalities in innovation have on economic growth in multiple regions when the underlying growth is not based on the idea of creative destruction. We stress that the present paper has nothing to do with negative externalities in innovation. In addition, our paper is fundamentally about economic growth that arises from the process of creative destruction. Put differently, unlike the Batabyal and Nijkamp (2013c) paper, the present paper is about Schumpeterian economic growth.

is a model of multiple regions and *not* a model of a single country. Finally, unlike the existing literature, we show the impact that the trinity of monopoly distortions, the profit stealing effect, and the replacement effect have on the magnitude of innovations in multiple *regions*.

The remainder of this paper is organized as follows. Section 2 describes our theoretical model of an aggregate economy consisting of N heterogeneous regions that is adapted from Aghion and Howitt (1992) and Acemoglu (2009, pp. 459-472). Section 3 defines the balanced growth path (BGP) allocations and the resulting equilibrium that are of interest. Section 4 specifies the form of the innovation possibilities frontier that is consistent with balanced economic growth. Section 5 derives the economic growth rate of the ith region in the decentralized equilibrium without any governmental or social planning and shows that there are no transitional dynamics. Section 6 first solves the social planner's maximization problem and then derives the Pareto optimal economic growth rate in the ith region. Section 7 compares the decentralized equilibrium and the Pareto optimal growth rates and discusses the conditions in which there is either too much or too little innovation in (i) the ith region, (ii) the aggregate economy of $N>2$ regions and (iii) the specific case of an aggregate economy of $N=2$ regions. Finally, section 8 concludes and then discusses potential extensions of the research delineated in this paper.

2. The Theoretical Framework

2.1. Preliminaries

Consider an *aggregate economy* of N heterogeneous and non-overlapping regions. We index these regions with the subscript i where $i=1,2,...,N$. Each of these N regions is itself composed of J

distinct spatial units which we index with the superscript j where $j=1,2,...,J$.[6] The individual regions in the aggregate economy of N regions do not interact with each other and hence they are closed regions.[7] Therefore, except in section 7.2 (on which more below), we shall focus our analysis on the ith region without loss of generality. As we shall see, this focus on the ith region will not preclude us from discussing the spatial dimensions of many of our subsequent results.

The ith region has an infinite horizon economy. The representative household in region i displays constant relative risk aversion (CRRA) and its CRRA utility function is denoted by $\int_0^\infty \exp(-\rho_i t)[\{C_i(t)^{1-\theta_i}-1\}/(1-\theta_i)]dt,\ \theta_i \neq 1,$ where $C_i(t)$ is consumption in time t, ρ_i is the time discount rate, and $\theta_i \geq 0$ is the constant coefficient of relative risk aversion.[8] The ith region possesses human capital in each of its J distinct spatial units. The human capital in the jth spatial unit in region i at time t is denoted by $H_i^j(t)$. Clearly, the total stock of human capital in region i at time t is given by $H_i(t) = \Sigma_{j=1}^J H_i^j(t)$. There is no growth in the stock of human capital in the ith region or $H_i(t)$ and this $H_i(t)$ is supplied inelastically. The aggregate resource constraint in region i at time t is given by

$$C_i(t) + X_i(t) + Z_i(t) \leq Q_i(t), \tag{1}$$

where $C_i(t)$ is consumption, $X_i(t)$ is total spending on machines, $Z_i(t)$ is total spending on R&D,

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For concreteness, the reader may want to think of the aggregate economy as the European Union (EU), the regions as the various nations in the EU, and the spatial units as the provinces within these individual EU member nations. In an alternate interpretation, the aggregate economy would be the United States, the regions would correspond to the various US states, and the spatial units would denote the counties in the individual US states.

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The dynamic model of N regions we analyze is already quite complicated. If we allowed these N regions to, for instance, trade with each other then it would *not* be possible to obtain analytic solutions to this more complicated multi-region trade model. That is why we focus on the case of N closed regions. Having said this, the reader should note that for the case in which a region is a nation and for the case in which it is not, there is a tradition of studying closed economy models. See Obstfeld and Rogoff (1996, pp. 440-448) and Batabyal and Beladi (2013) for a more detailed corroboration of this claim.

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See Blanchard and Fisher (1989, pp. 43-45) for more on the properties of CRRA utility functions.

and $Q_i(t)$ is the output of the single final good for consumption that we shall think of as a knowledge good such as a smartphone or a laptop computer. In addition, note that the machines we have just referred to can also be thought of as inputs or as intermediate goods. The total expenditure on R&D in region i or $Z_i(t)$ is the sum of the R&D expenditures incurred by each one of the J distinct spatial units in region i. In symbols, this means that $Z_i(t)=\Sigma_{j=1}^{J} Z_i^j(t)$.

There is a continuum of machines that are used to produce the single final good $Q_i(t)$. We normalize the price of this final good to equal unity at all points in time. Each machine line or variety[9] is described by ν_i where $\nu_i \in [0,1]$. The source of economic growth in region i is *process innovations* that reduce the marginal cost of producing machines. To this end, let $M_i(\nu_i,t)$ be the marginal cost of producing the machine of variety ν_i at time t. We suppose that every process innovation reduces this marginal cost by a multiplicative factor $1/\alpha_i$ where $\alpha_i > 1$.

The single final good for consumption (the knowledge good) in region i or $Q_i(t)$ is produced competitively in a single location with the production function

$$Q_i(t)=\frac{1}{1-\beta_i}\left[\int_0^1 x_i(\nu_i,t)^{1-\beta_i} d\nu_i\right] H_i^{\beta_i}, \tag{2}$$

where H_i is the human capital input in region i, $x_i(\nu_i,t)$ is the total amount of the machine of variety ν_i that is used at time t, and $\beta_i \in (0,1)$ is a parameter of the production function. Let w_i denote the wage paid to the human capital input H_i and let r_i denote the interest rate. We know that $H_i(t)=\Sigma_{j=1}^{J} H_i^j(t)$ which means that each spatial unit in region i supplies its own human capital inelastically and thereby contributes additively and positively to the production of the final

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We shall use the words “line” and “variety” interchangeably in the remainder of this paper.

consumption good $Q_i(t)$ (see equation (2)). Because the final consumption good is produced in a single location, the human capital in each of the J spatial units migrates *over space* to that location. The sum of these migrating workers in this single location is denoted by $H_i(t)$, which produces the final consumption good. This is the first way in which the individual spatial units contribute to our analysis and this point also shows that there exists a spatial dimension in our analysis.

Note that for a given machine line ν_i, only the machine with the *lowest* marginal cost is used to produce the single final good in equilibrium. This feature of the model is the source of *creative destruction* in the sense of Joseph Schumpeter. In other words, when a machine with a lower marginal cost is invented, it replaces or destroys the previous higher marginal cost machine of the same line. Our next task is to discuss how new machines in the ith region are first invented and then produced and the two related notions of Arrow's (1962) *replacement effect* and the *profit stealing effect*.

2.2. Machine invention and production, replacement, and profit stealing effects

New machine varieties in our model are invented by R&D and this R&D builds on the knowledge about existing machines. More specifically, the R&D in region i gives rise to innovation and this innovation advances the existing knowledge about the various machine lines. There is free entry into R&D in region i. Hence, any firm can conduct research on the existing machine lines. The firm that makes an innovation receives a perpetual patent on the new machine it has invented. As such, this successful innovator has monopoly power in the market for machines. The cost of undertaking R&D is assumed to be the same for the incumbent monopolist and for new firms (potential entrants). The existing patent system in region i does not prevent other firms from undertaking research based on the newly invented machine just mentioned. Note that in contrast to the centralized production of the final consumption good in the *ith* region, the invention and production of machines in this region is *decentralized* and therefore can occur *in any* of the J spatial units. This is the second way in which these J spatial units contribute to our analysis. This point is the second demonstration of our claim that there is a spatial dimension in our analysis.

In the model of this paper, the *identity* of the firm that conducts R&D is salient. This is because as noted in section 2.1, existing machines can be improved upon and it is this process of creative destruction that is the source of economic growth in region i. This brings us to Arrow's (1962) well known *replacement effect*. Because the cost of undertaking R&D is identical for the incumbent monopolist and for potential entrants, following the seminal work of Arrow, Acemoglu (2009, p. 460, emphasis added) has rightly pointed out that the "incumbent has weaker incentives to innovate, since it would *replace* its own machine (thus destroying the profit that it is already making)." In contrast, a potential entrant has no similar replacement computation to make and hence, given that the cost of undertaking R&D is identical for the incumbent monopolist and a potential entrant, it is always the entrants who conduct R&D.

Note that by replacing the incumbent monopolist, an entrant is also stealing this monopolist's profit. This is the *profit stealing effect*. A key objective of ours in this paper—which we undertake in detail in section 7—is to study the nexuses between innovation, the replacement and the profit stealing effects, first, in the decentralized equilibrium and then in the socially planned equilibrium in single and in multiple regions. With this theoretical framework in place, our next task is to define the balanced growth path (BGP) allocations and the resulting equilibrium for our innovative *ith* region. While undertaking this exercise, we shall adapt some results in Peters and Simsek (2009, pp. 167-171) to our aggregate economy of N heterogeneous regions.

3. The BGP allocations and the equilibrium

As a prelude to defining the equilibrium, we will need to introduce some new notation. To this end, let $E_i(t)$ denote the average or mean productivity of a machine in the region i economy. Second, let $Z_i(\nu_i,t;M_i)$ denote the R&D expenditure on machine variety ν_i given that the marginal cost of producing this machine variety is M_i. Third, given that the marginal cost of producing the machine of variety ν_i is M_i, let $p_i(\nu_i,t;M_i)$ be the price of this machine and let its quantity be denoted by $x_i(\nu_i,t;M_i)$. Finally, let $V_i(\nu_i,t;M_i)$ denote the value function for the monopolist producing the machine of variety ν_i at marginal cost M_i.

The BGP equilibrium in the ith region is a *collection* of *time paths* of aggregate allocations $[Q_i(t),\ C_i(t),\ X_i(t),\ \{Z_i(\nu_i,t;M_i)\}_{\nu_i=0}^{\nu_i=1}]$, aggregate prices $[r_i(t),\ w_i(t)]$, innovation levels $[E_i(t)]$, machine prices and quantities $[\{p_i(\nu_i,t;M_i),\ x_i(\nu_i,t;M_i)\}_{\nu_i=0}^{\nu_i=1}]$, and the value function $V_i(\nu_i,t;M_i)_{\nu_i=0}^{\nu_i=1}$ such that the following seven conditions hold. First, the representative household in region i maximizes utility. Second, competitive producers of the sole final good maximize their profits taking the price of the final good as given. Third, the monopolistic machine producers select prices to maximize their profits. Fourth, there is free entry into the R&D sector in region i. Fifth, the distribution of the marginal costs (the technology) evolves over time in accordance with a R&D process described by equation (9) in section 4 below. Sixth, aggregate output of the sole final good $\{Q_i(t)\}$ and consumption $\{C_i(t)\}$ grow at the same rate as does the interest rate $r_i(t)$ and these three growth rates equal the constant r_i^S. Finally, innovations on all the machine varieties occur at a constant flow rate $z_i(\nu_i,t;M_i)=z_i^S$. The superscript S here denotes the steady or stationary state.

Since we are working with a model of endogenous technology, firms in region i must ultimately have a choice between different kinds of technologies and, in this regard, greater effort, investment, or R&D spending ought to lead to the invention of machines with lower marginal costs. This tells us that there must exist a *meta production function* or a "production function over production functions" which indicates how new technologies are generated in region i as a function of the various *inputs*. Following Acemoglu (2009, p. 413), we refer to this meta production function as the "innovation possibilities frontier." Our next task is to specify the form of the innovation possibilities frontier that is consistent with balanced economic growth in region i.

4. The innovation possibilities frontier

In order to specify the innovation possibilities frontier, let us begin by computing the value function for the monopolistic machine producers in region i. To do this, we must first specify the profit function for a machine producing monopolist with marginal cost M_i. Modifying equation (6) in Batabyal and Nijkamp (2012) to our case, we see that the demand for machines from the competitive producers of the final good in region i is isoelastic and given by

$$x_i(v_i,t;M_i) = p_i(\cdot,\cdot)^{-1/\beta_i} H_i. \tag{3}$$

Because $H_i(t) = \Sigma_{j=1}^{J} H_i^j(t)$, inspecting (3), we immediately see the *positive* impact of the *spatial distribution* of the human capital input on the demand for machines in region i. Specifically, the greater the human capital in any one of the J spatial units into which the ith region is divided, the higher is the demand for machines in this region. This point is the third demonstration of our claim that there is a spatial dimension in the analysis we undertake.

The price set by the machine producing monopolists will depend on the *magnitude* of the marginal cost reducing innovations for the various machine varieties. In this regard, there are two cases to consider.[10] If the innovations are "drastic" then, from equation (7) in Batabyal and Nijkamp

10 See Acemoglu (2009, pp. 418-419) for a more detailed textbook discussion of these two cases.

(2012), we infer that the monopoly price is $p_i^d(\cdot,\cdot)=1/(1-\beta_i)M_i$ where $1/(1-\beta_i)$ is the markup over the marginal cost M_i in this "drastic innovations" case. If the innovations are *not* drastic then we use the fact that the incumbent monopolist faces competition from a firm with marginal cost M_i/α_i to conclude that this monopolist will set a limit price whenever $p_i^d > M_i/\alpha_i$. Putting the information for the above two cases together, we conclude that the actual price set by our monopolistic machine producer will equal a markup times the marginal cost of production. In symbols, we have

$$p_i^{x_i}(\nu_i,t;M_i)=\min\left(\frac{1}{1-\beta_i},\frac{1}{\alpha_i}\right)M_i=\gamma_i M_i, \tag{4}$$

where $\gamma_i=\min\{1/(1-\beta_i),1/\alpha_i\}$ is the actual markup. Combining equations (3) and (4), we see that the current monopolist produces machines so that

$$x_i(\nu_i,t;M_i)=p_i^{x_i}(\nu_i,t;M_i)^{-1/\beta_i}H_i=(\gamma_i M_i)^{-1/\beta_i}H_i. \tag{5}$$

In addition, using the definition of profit, this monopolist's profit function is

$$\pi_i(\nu_i,t;M_i)=(\gamma_i-1)\gamma_i^{-1/\beta_i}M_i^{-(1-\beta_i)/\beta_i}H_i. \tag{6}$$

With this specification of the machine producing monopolist's profit function, we can now compute this monopolist's value function $V_i(\nu_i,t;M_i)$. Note that on a BGP, the interest rate and the flow rate of innovation[11] are both constant and equal to r_i^S and z_i^S respectively. Using this information and equation (6), the value function of interest is

$$V_i(\nu_i,t;M_i)=\frac{\pi_i(\nu_i,t;M_i)}{r_i^S+z_i^S}=\frac{(\gamma_i-1)\gamma_i^{-1/\beta_i}H_iM_i^{-(1-\beta_i)/\beta_i}}{r_i^S+z_i^S}. \tag{7}$$

[11] Note that the flow rate of innovation is also equal to the replacement rate of machines.

The innovations we are studying in this paper result in the *reduction* of the marginal cost of producing machines over time. As such, the value function in equation (7) is greater for machines with a lower marginal cost of production.

Having computed the monopolist's value function, let us now focus on an innovation possibilities frontier that permits innovation for each machine variety in region i. We know that the above value function is greater for machines with a lower marginal cost of production. This suggests that there will be more innovation for machine lines with lower marginal production costs. In turn, this means that for there to be a balanced rate of innovation across all machine lines—and hence a balanced rate of economic growth in region i —the cost of innovation must be higher for those machine lines that have a lower marginal production cost because these machine lines are the most sophisticated.

Let the function $h_i(M_i)$ denote the flow rate of innovation in region i that arises from a unit investment of R&D on a machine line with marginal production cost M_i. Some thought tells us that the function $h_i(\cdot)$ must be an increasing function. We now need to specify the precise functional form for the flow function $h_i(M_i)$. Because there is free entry into R&D in region i, we can adapt equation (14.14) in Acemoglu (2009, p. 463) and write the free entry condition in R&D as $h_i(M_i)V_i\{(1/\alpha_i)M_i\}=1$. Substituting from equation (7) into this last condition, the free entry condition becomes

$$h_i(M_i)\frac{(\gamma_i-1)\gamma_i^{-1/\beta_i}H_i(\alpha_i^{-1}M_i)^{-(1-\beta_i)/\beta_i}}{r_i^S+z_i^S}=1. \qquad (8)$$

Inspecting equation (8) it is clear that for innovation—and economic growth—in region i to be balanced, we must have

$$h_i(M_i) = \Phi_i M_i^{(1-\beta_i)/\beta_i}, \tag{9}$$

where the constant $\Phi_i = [(r_i^S + z_i^S)(\gamma_i - i)^{-1} \gamma_i^{1/\beta_i} H_i^{-1} \alpha_i^{-(1-\beta_i)/\beta_i}]$. We now derive the economic growth rate of the ith region in the decentralized equilibrium without any governmental or social planning and show that there are no transitional dynamics.

5. The decentralized equilibrium

Substituting the right-hand-side (RHS) of equation (5) in (2), the output of the final consumption good in region i can be expressed as

$$Q_i(t) = \frac{1}{1-\beta_i} \int_0^1 [(\gamma_i M_i)^{-1/\beta_i} H_i]^{1-\beta_i} d\nu_i H_i^{\beta_i} = \frac{\gamma_i^{-(1-\beta_i)/\beta_i} H_i}{1-\beta_i} \int_0^1 M_i^{-(1-\beta_i)/\beta_i} d\nu_i. \tag{10}$$

Adapting the logic behind equation (14.10) in Acemoglu (2009, p. 462) to our problem, we see that the integral on the RHS of equation (10) is the average or mean machine productivity in region i or the $E_i(t)$ we referred to in section 3 above. In symbols, we have

$$E_i(t) = \int_0^1 M_i^{-(1-\beta_i)/\beta_i} d\nu_i. \tag{11}$$

Substituting equation (11) in (10), we can express the output of the final consumption good in terms of the average machine productivity as

$$Q_i(t) = \frac{\gamma_i^{-(1-\beta_i)/\beta_i} H_i E_i(t)}{1-\beta_i} \tag{12}$$

where the return to human capital in region i or the wage $w_i(t) = \beta_i Q_i(t)/H_i(t)$.

We now want to compute the growth rate of the average machine productivity $E_i(t)$ or $\dot{E}_i(t)/E_i(t)$. To do this, let us first consider the change in $E_i(t)$ in a small time interval denoted by Δt. This change is

$$E(t+\Delta t) - E(t) = \int_0^1 z_i^S \Delta t[\{\frac{M_i}{\alpha_i}\}^{-(1-\beta_i)/\beta_i} - M_i^{-(1-\beta_i)/\beta_i}] d\nu_i = z_i^S \Delta t[\alpha_i^{(1-\beta_i)/\beta_i} - 1]E_i(t). \tag{13}$$

Now, dividing both sides of equation (13) by Δt and then taking the limit as $\Delta t \to 0$ we get

$$\frac{\dot{E}_i(t)}{E_i(t)} = z_i^S[\alpha_i^{(1-\beta_i)/\beta_i} - 1] = g_i, \tag{14}$$

where the growth rate g_i on the RHS of equation (14) is also the growth rate of the output $\{Q_i(t)\}$ of the final consumption good in region i. This last claim follows because equation (12) tells us that both $\{E_i(t)\}$ and $\{Q_i(t)\}$ grow at the same rate.

Maximizing the representative household's CRRA utility function (see section 2.1) gives us the standard consumption Euler equation. That equation is

$$\frac{\dot{C}_i(t)}{C_i(t)} = \frac{1}{\theta_i}(r_i^S - \rho_i) = g_i. \tag{15}$$

Given our depiction of the flow rate of innovation in region i with the $h_i(M_i)$ function in equation (9), the free entry condition in equation (8) tells us that

$$\Phi_i(\gamma_i - 1)\gamma_i^{-1/\beta_i}\alpha_i^{(1-\beta_i)/\beta_i} H_i = r_i^S + z_i^S. \tag{16}$$

Now, equations (14)-(16) are three equations in the three unknowns g_i, r_i^S, and z_i^S. Solving these three equations simultaneously, the growth rate of region i in the decentralized equilibrium or g_i^{DE} is

$$g_i^{DE} = \frac{\alpha_i^{(1-\beta_i)/\beta_i}(\gamma_i - 1)\gamma_i^{-1/\beta_i}\Phi_i H_i - \rho_i}{[1/\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}] + \theta_i}. \tag{17}$$

Inspecting equation (17), we see the *positive* impact that the *spatial distribution* of the human capital input in region i has on this region's growth rate. Specifically, since $H_i(t) = \Sigma_{j=1}^{J} H_i^j(t)$, there is a specific growth related outcome that is worth highlighting. In particular, the greater the human capital in any one of the J spatial units that together make up region i, the higher is the equilibrium economic growth rate in this region. This point is the fourth demonstration of our contention that there is a spatial dimension in our analysis in this paper.

In order to ensure that the growth rate in equation (17) is positive and that the transversality condition is satisfied, we adapt proposition 14.1 in Acemoglu (2009, p. 465) and suppose that the inequality

$$\frac{\{\alpha_i^{(1-\beta_i)/\beta_i}(\gamma_i - 1)\gamma_i^{-1/\beta_i}\Phi_i H_i - \rho_i\}(1-\theta_i)}{[1/\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}] + \theta_i} < \rho_i < \alpha_i^{(1-\beta_i)/\beta_i}(\gamma_i - 1)\gamma_i^{-1/\beta_i}\Phi_i H_i \tag{18}$$

holds. In (18), the first inequality ensures the satisfaction of the transversality condition and the second inequality guarantees that the economic growth rate in region i is positive.

Our next task is to derive an analytic expression for consumption in region i or $C_i(t)$. To do this, we use the fact that the aggregate resource constraint given in equation (1) binds at the optimum. Using equation (11), observe that the total expenditure on machines in region i is

$$X_i(t) = \int_0^1 x_i(\nu_i, t; M_i) M_i d\nu_i = \int_0^1 (\gamma_i M_i)^{-1/\beta_i} M_i H_i d\nu_i = \gamma_i^{-1/\beta_i} H_i E_i(t). \tag{19}$$

Similarly, using equation (14), the total expenditure on R&D in region i is

$$Z_i(t) = \int_0^1 Z_i(\nu_i, t) d\nu_i = \int_0^1 \frac{z_i^S}{h_i(M_i)} d\nu_i = \int_0^1 \frac{g_i M_i^{-(1-\beta_i)/\beta_i}}{\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}\Phi_i} d\nu_i = \frac{g_i E_i(t)}{\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}\Phi_i}. \tag{20}$$

Now, substituting the values of $Q_i(t)$, $X_i(t)$, and $Z_i(t)$ from equations (12), (19), and (20), in equation (1), we get an explicit expression for consumption and that expression is

$$C_i(t) = [\frac{\gamma_i^{-(1-\beta_i)/\beta_i} H_i}{1-\beta_i} - \gamma_i^{-1/\beta_i} H_i - \frac{g_i}{\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}\Phi_i}] E_i(t). \tag{21}$$

Equation (21) and some thought together tell us that like average machine productivity $E_i(t)$ and output $Q_i(t)$, consumption in region i also grows at the constant rate g_i^{DE}. Inspecting equations (19) and (21) we see the *positive* role played by the human capital input in determining the equilibrium levels of the spending on machines $\{X_i(t)\}$ and consumption $\{C_i(t)\}$. *Ceteris paribus*, the larger is H_i, the higher are the equilibrium values of both $X_i(t)$ and $C_i(t)$.

We have now described the decentralized economic equilibrium in region i completely. In this regard, two points are worth emphasizing. First, given the description of the R&D technology in equation (9), the satisfaction of the transversality condition, and the positivity of the growth rate in equation (17), there exists a BGP equilibrium in region i in which average machine productivity, consumption, and output all grow at the same constant rate. Second, adapting proposition 14.2 in Acemoglu (2009, p. 465) to our problem, the growth trajectory of the key model variables we have been discussing thus far constitutes an equilibrium starting with any initial distribution of marginal costs $\{M_i(\nu_i,0)\}_{\{\nu_i\in[0,1]\}}$. Put differently, there are no transitional dynamics in the model. We now discuss the region i economy when resource allocation in this economy is determined by a benevolent social planner. Specifically, we solve the social planner's maximization problem and then derive the Pareto optimal growth rate in region i.

6. The socially planned equilibrium

We first focus on the social planner's static resource allocation problem. We begin by supposing that the distribution of marginal costs $\{M_i(\nu_i,t)\}_{\{\nu_i\in[0,1]\}}$ at any time t is given. Unlike the decentralized equilibrium studied in section 5, there is now *no* monopoly markup, i.e., $\gamma_i=1$, in the price of the machines. This means that our social planner in region i allocates resources so that the price of machines equals their marginal cost or $p_i(\nu_i,t;M_i)=M_i$. Substituting $\gamma_i=1$ in equation (5), the output of each machine line is

$$x_i(\nu_i,t;M_i)=M_i^{-1/\beta_i}H_i. \tag{22}$$

Using equation (11) for the average machine productivity in region i and equation (22), the aggregate output of the final consumption good in this region is

$$Q_i(t)=\frac{1}{1-\beta_i}\int_0^1 (M_i^{-1/\beta_i}H_i)^{1-\beta_i}d\nu_i H_i^{\beta_i}=\frac{E_i(t)H_i}{1-\beta_i}. \tag{23}$$

Comparing equation (23) with (12), we see that for a given level of average machine productivity $E_i(t)$, the social planner produces *more* of the final consumption good because he accounts for the monopoly distortion in his decision making.

Moving on to the total expenditure on machines in region i or $X_i(t)$, we get

$$X_i(t)\int_0^1 x_i(\nu_i,t;M_i)M_i d\nu_i = E_i(t)H_i(t). \tag{24}$$

From the fact that (1) binds at the optimum, we know that $C_i(t)=Q_i(t)-X_i(t)-Z_i(t)$. Therefore, substituting the values of $Q_i(t)$ and $X_i(t)$ from equations (23) and (24) into the preceding equation, we get

$$C_i(t)=\frac{\beta_i E_i(t)H_i(t)}{1-\beta_i}-Z_i(t), \tag{25}$$

where $Z_i(t)$ is the total expenditure on R&D in region i. From section 2.1 we know that the total expenditure on R&D in region i or $Z_i(t)$ is the sum of the R&D expenditures incurred by each one of the J distinct spatial units in region i. In symbols, we have $Z_i(t)=\Sigma_{j=1}^{J}Z_i^j(t)$. Using this last relationship in equation (25) we see that there is a *negative* relationship between the *spatial distribution* of R&D expenditures and total consumption in region i. In particular, the higher the R&D expenditure in any one of the J spatial units comprising the ith region, the lower is the total consumption in this region. This point is the fifth demonstration of our claim that there is a spatial dimension in the analysis we undertake in this paper.

We now need to find an expression linking $Z_i(t)$ to the change over time in the average machine productivity or $\dot{E}_i(t)$. To do so, let us concentrate on the social planner's dynamic resource allocation problem. Note that when this social planner invests one unit on a machine line with marginal cost of production M_i, he engenders a flow rate of innovation (new machines) given by the function $h_i(M_i)$. This flow rate of innovation increases the contribution of this machine line to average machine productivity in region i by a certain amount. Using the logic behind equation (13), this increased contribution is given implicitly by

$$h_i(M_i)[\{\frac{M_i}{\alpha_i}\}^{-(1-\beta_i)/\beta_i} - M_i^{-(1-\beta_i)/\beta_i}] = \Phi_i[\alpha_i^{(1-\beta_i)/\beta_i} - 1]. \tag{26}$$

In equilibrium, our social planner must be indifferent between investing in the various machine lines. In addition, when this planner invests $Z_i(\nu_i,t)$ on R&D in region i, he increases average machine productivity in this region by

$$\dot{E}_i(t) = Z_i(t)\Phi_i[\alpha_i^{(1-\beta_i)/\beta_i} - 1]. \tag{27}$$

With equation (27) in place, we can now state our social planner's dynamic optimization problem. This planner solves

$$\max_{\{C_i(t),\ E_i(t),\ Z_i(t)\}} \int_0^\infty e^{-\rho_i t}\{\frac{C_i(t)^{1-\theta_i} - 1}{1-\theta_i}\}dt, \tag{28}$$

subject to equations (25) and (27). Substituting for $Z_i(t)$ from equation (25) into (27), the above problem becomes a maximization problem subject to a single constraint given by the re-written equation (27). The current value Hamiltonian function for this amended problem is

$$\mathrm{H}_i\{C_i(t),E_i(t),\zeta_i(t),t\} = \frac{C_i(t)^{1-\theta_i} - 1}{1-\theta_i} + \zeta_i[\Phi_i\{\frac{\beta_i E_i(t) H_i}{1-\beta_i} - C_i(t)\}\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}], \tag{29}$$

where ζ_i is the costate variable. Manipulating the first order necessary conditions for an optimum and then simplifying the resulting expressions gives us—like equation (15) in section 5—the consumption Euler or the growth rate of consumption equation. Let us denote this growth rate in the socially planned equilibrium in region i by g_i^{SP}. Then, we get

$$g_i^{SP} = \frac{\dot{C}_i}{C_i} = [(1)\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}\{\frac{\beta_i}{1-\beta_i}\}\Phi_i H_i - \rho_i]\{\frac{1}{\theta_i}\}. \qquad (30)$$

Inspecting equation (30), we see that as in the section 5 case, once again, the *spatial distribution* of the human capital input in region i or H_i has a *positive* effect on this region's growth rate. In particular, because $H_i(t) = \Sigma_{j=1}^{J} H_i^j(t)$, the following growth related implication is worth emphasizing: The greater the human capital in any one of the J spatial units into which region i is divided, the higher is the equilibrium growth rate in this region. We now compare the region i growth rate in the decentralized equilibrium with the corresponding growth rate in the socially planned equilibrium.

7. The decentralized versus the socially planned equilibrium

7.1. The single region case

The two equations of interest are (17) and (30). However, to ensure that this comparison is meaningful, recall the discussion preceding equation (4) in section 4 and note that the appropriate monopoly markup in equation (17) is the markup associated with the "drastic" innovations case. This means that $\gamma_i = 1/(1-\beta_i)$. Substituting this last expression in equation (17), we get an expression for the region i growth rate in the decentralized equilibrium with drastic innovations. Let us denote this growth rate by g_i^{DEd} where the last "d" in the superscript denotes drastic. In symbols, we get

$$g_i^{DEd} = [(1-\beta_i)^{1/\beta_i}\{\alpha_i^{(1-\beta_i)/\beta_i}\}\{\frac{\beta_i}{1-\beta_i}\}\Phi_i H_i - \rho_i]\{\frac{1}{[1/\{\alpha_i^{(1-\beta_i)/\beta_i}-1\}]+\theta_i}\}. \qquad (31)$$

Comparing equations (30) and (31), there are three key effects to comprehend. These three effects are described compactly in Table 1. Looking horizontally across the second row in Table 1,

Table 1 about here

the first effect concerns the static *monopoly distortions* denoted by MD. These distortions are accounted for in the socially planned equilibrium described by equation (30) but they are not in the decentralized equilibrium described by equation (31). This means that for a given number of machines in region i, there is *more* production of the final consumption good in the socially planned equilibrium than in the decentralized equilibrium. This effect is captured by the presence of the 1 in the RHS of equation (30) as opposed to the $(1-\beta_i)^{1/\beta_i}$ term in the RHS of equation (31). The impact of this additional production of the final good is to create an impetus that tends to *increase* the region i growth rate in the socially planned equilibrium relative to the growth rate in the decentralized equilibrium. This is shown by the $g_i^{SP} > g_i^{DEd}$ entry in the last cell of the second row in Table 1.

Looking horizontally across the third row in Table 1, the second effect is the *profit stealing effect* denoted by PS. In the decentralized equilibrium, potential entrants do not account for the fact that they are stealing the profit of the monopolist they are replacing but this accounting does occur in the socially planned equilibrium. This impact is captured by the $\{\alpha_i^{(1-\beta_i)/\beta_i}-1\}$ term in the RHS of equation (30) as opposed to the $\{\alpha_i^{(1-\beta_i)/\beta_i}\}$ term in equation (31). This creates a force that tends to *decrease* the growth rate in region i in the socially planned equilibrium relative to the growth rate in the decentralized equilibrium. This is demonstrated by the $g_i^{SP} < g_i^{DEd}$ entry in the last cell of the third row in Table 1.

Viewing the fourth row in Table 1 horizontally, the third and final effect is the *replacement effect* denoted by RE. In the decentralized equilibrium, monopolistic firms are concerned about the fact that they will be replaced by another monopolist at some future date but this fact is of no concern in the socially planned equilibrium. This effect is captured by the $[1/\{\alpha_i^{(1-\beta_i)/\beta_i}-1\}]+\theta_i$ term in the denominator on the RHS of equation (31) as compared to the θ_i term in the denominator on the RHS of equation (30). This gives rise to an impetus that tends to *increase* the growth rate in region i in the socially planned equilibrium relative to the growth rate in the decentralized equilibrium. This is shown by the $g_i^{SP}>g_i^{DEd}$ entry in the last cell of the fourth row in Table 1.

The net outcome of the above three effects in region i depends on their relative magnitudes and hence is, in general, indeterminate. However, our discussion thus far points to two interesting and opposite outcomes and a third knife-edge case. Specifically, suppose that the profit stealing effect dominates the sum of the monopoly distortions and the replacement effect. In symbols, we have $PS>MD+RE.$ Then Table 1 tells us that the growth rate in the socially planned equilibrium will be *lower* than the growth rate in the decentralized equilibrium. When this happens, there will be *excessive* innovation in the *ith* region. In contrast, suppose that the sum of the monopoly distortions and the replacement effect dominates the profit stealing effect. In symbols, we have $PS<MD+RE.$ In this case, Table 1 tells us that the growth rate in the socially planned equilibrium will be *greater* than the corresponding growth rate in the decentralized equilibrium. When this happens, there will be *too little* innovation in region $i.$ Finally, in the knife-edge case where the magnitude of the profit stealing effect equals the magnitude of the sum of the monopoly distortions and the replacement effect, i.e., when $PS=MD+RE,$ the decentralized equilibrium and the socially planned equilibrium

give rise to identical growth rates and hence both regimes give rise to the same (optimal) amount of innovation.

Generalizing from the discussion in the preceding paragraph to the aggregate economy of N regions is difficult. This is because the combined effect of innovative R&D in all the regions may result in either too many or in too few innovations. Having said this, the aggregate effect will depend, in part, on the heterogeneity of the individual regions. The greater the homogeneity of the individual regions, the more likely it is that the aggregate effect of R&D on the economic growth rate can be described by one of the two cases mentioned in the preceding paragraph. In contrast, the more heterogeneous the individual regions, the more likely it is that the aggregate impact of R&D on the economic growth rate will be completely unpredictable. When we analyze multiple regions explicitly, from the standpoint of both regional innovation and growth, *many outcomes* are logically possible. To give the reader a flavor for the *more interesting* outcomes,[12] we now concentrate on the case where the aggregate economy under study consists of two distinct regions.

7.2. The two regions case

The two regions are denoted by $i = 1,2.$ Table 2 describes the various forces that are now at

Table 2 about here

work in the aggregate economy. Let us first look horizontally across the second row in Table 2. The first column tells us that in region 1, the profit stealing effect dominates the sum of the monopoly distortions and the replacement effect. The three possible cases that might occur in region 2 are listed in the second column. The third column lists the growth rates that would prevail in the two

12 We reiterate that even when there are only two regions in the aggregate economy, the number of logically possible cases are many. Hence, our subsequent analysis in section 7.2 does *not* focus on every conceivable case. Instead, this section concentrates on what we believe are the more interesting cases that may occur in the aggregate economy of two regions.

regions in the socially planned and in the decentralized equilibria. Finally, the fourth column tells us whether innovation in the aggregate economy is excessive, insufficient, optimal, or if the comparison across the two regions results in an ambiguous outcome. So, for instance, when $PS > MD + RE$ in both regions 1 and 2, the economic growth rates of the two regions in the decentralized equilibrium exceed the growth rates in the socially planned equilibrium. As a result, there is *excessive* innovation in both the regions and hence also in the aggregate economy.

The results stated in the second and in the third rows of Table 2 can be interpreted in a similar manner. So, focusing on the second row, when the condition $PS = MD + RE$ holds in both the regions being studied, the economic growth rates in these two regions in the decentralized equilibrium are identical to the growth rates in the socially planned equilibrium. Put differently, these two growth rates are optimal and hence, looked at from this standpoint, the amount of innovation in the aggregate economy is also *optimal* and this is what the fourth column tells us. Finally, looking at the third row in Table 2, we see that when the condition $PS < MD + RE$ holds in both regions, the economic growth rates in the two regions under study are sub-optimal and hence there is *insufficient* innovation in both the regions and in the aggregate economy.[13] This concludes

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Our principal result in this paper is that in a model of Schumpeterian economic growth in multiple regions, the amount of innovation that occurs in a region or in the aggregate economy can be higher or lower than the innovation in the socially planned equilibrium. There is a superficial similarity between this result and results pertaining to excessive entry in the literature on monopolistic competition. This is because entry in this literature can be either excessive or sub-optimal depending on the details of the problem being studied. In an early paper, Chamberlin (1950) showed that firms set production to the left of the point of their minimum average cost and hence *too many* firms entered the industry. Spence (1976) showed that there was likely to be *excessive entry* in a monopolistically competitive industry. Relative to Spence (1976), in their influential paper employing constant elasticity of substitution (CES) preferences, Dixit and Stiglitz (1977) reached the opposite result—entry by firms was *below* the social optimum. The Chamberlin (1950) result arises largely from his focus on firms selling perfect substitutes. The Spence (1976) result follows from his focus on goods that are imperfect substitutes with high own-price and low cross-price elasticities. Finally, the Dixit and Stiglitz (1977) result is significantly dependent on the manner in which preferences are modeled. A good summary statement of this literature is provided by Picard and Toulemonde (2009, p. 1348) who note that "entry can be too large or too small according to the balance between consumers' preferences for variety and for individual consumption of each single variety." The reader should note that our paper has very little to do with these variety related factors. The factors that generate our basic "innovation can be excessive or sub-optimal" result are completely different from the factors that drive the excessive/sub-optimal entry result in the monopolistic competition literature. More specifically, what drives our central result is the way in which we model the invention and production of machines (see section 2.2), the subsequent monopoly distortions, and the profit stealing and the replacement effects.

our study of innovation, decentralization, and planning in a multi-region model of Schumpeterian economic growth.

8. Conclusions

In this paper, we studied innovation and the resulting Schumpeterian economic growth that this innovation gave rise to in a model of an aggregate economy made up of N heterogeneous regions. For each region i where $i=1,...,N$, our analysis led to five findings. First, we defined the BGP allocations and the equilibrium of interest. Second, we specified the form of the innovation possibilities frontier that was consistent with balanced economic growth. Third, we derived the growth rate in the *ith* region in the decentralized equilibrium and showed that there were no transitional dynamics. Fourth, we solved the social planner's optimization problem and derived the Pareto optimal growth rate in the *ith* region. Finally, we compared the two preceding growth rates and discussed the circumstances in which there was either too much or too little innovation in (i) the *ith* region, (ii) the aggregate economy of $N>2$ regions and (iii) the specific case of an aggregate economy of $N=2$ regions.

We reiterate that as noted in section 1.2, our formal analysis diverges from and extends the existing theoretical literature in four ways. First, the basic unit of analysis in our paper was a region and *not* a country. Here, the word "region" referred to a geographic entity that was smaller than a nation. Second, instead of working with labor, we worked with *human capital* as a major factor of production. Third, the model we analyzed was a model of multiple regions and *not* a model of a single country. Finally, unlike the existing literature, we demonstrated the effect that the trinity of monopoly distortions, the profit stealing effect, and the replacement effect had on the magnitude of innovations in multiple regions.

Here are three suggestions for extending the research delineated in this paper. First, an interesting extension that would also yield insights into phenomena occurring over space involves the numerical analysis of a multi-region model of the sort studied in this paper to determine what kinds of spatial interactions between regions can be studied meaningfully. Second, it would also be useful to study the impact that the taxation of R&D by new entrants has on incumbent firms specifically and on any given region in general. Finally, some innovations might prompt gradual changes in a regional economy whereas others might give rise to abrupt changes. This could be studied using the perspectives of complexity science. Studies that incorporate these aspects of the problem into the analysis will increase our understanding of the nexuses between innovation, activist policy, and Schumpeterian economic growth in aggregate economies made up of multiple regions.

Table 1: The decentralized versus the socially planned equilibrium in the *ith* region

Three key effects in the region i economy	Relevant terms to compare in (30) and (31)	Impact on growth in region i economy
Static monopoly distortions (MD): Accounted for in SP but not in DEd	1 in (30) versus $(1-\beta_i)^{1/\beta_i}$ in (31)	$g_i^{SP} > g_i^{DEd}$
Profit stealing effect (PS): Accounted for in SP but not in DEd	$\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}$ in (30) versus $\{\alpha_i^{(1-\beta_i)/\beta_i}\}$ in (31)	$g_i^{SP} < g_i^{DEd}$
Replacement effect (RE): Accounted for in DEd but not in SP	θ_i in (30) versus $[1/\{\alpha_i^{(1-\beta_i)/\beta_i} - 1\}]$ + θ_i in (31)	$g_i^{SP} > g_i^{DEd}$

Table 2: Growth and innovation in an aggregate economy consisting of two regions

Condition in region 1	Condition in region 2	Outcome in terms of growth rates	Outcome in terms of innovation
$PS > MD + RE$	$PS > MD + RE$	$g_1^{SP} < g_1^{DEd},\ g_2^{SP} < g_2^{DEd}$	Excessive innovation
	$PS = MD + RE$	$g_1^{SP} < g_1^{DEd},\ g_2^{SP} = g_2^{DEd}$	Excessive innovation
	$PS < MD + RE$	$g_1^{SP} < g_1^{DEd},\ g_2^{SP} > g_2^{DEd}$	Ambiguous outcome
$PS = MD + RE$	$PS > MD + RE$	$g_1^{SP} = g_1^{DEd},\ g_2^{SP} < g_2^{DEd}$	Excessive innovation
	$PS = MD + RE$	$g_1^{SP} = g_1^{DEd},\ g_2^{SP} = g_2^{DEd}$	Optimal innovation
	$PS < MD + RE$	$g_1^{SP} = g_1^{DEd},\ g_2^{SP} > g_2^{DEd}$	Insufficient innovation
$PS < MD + RE$	$PS > MD + RE$	$g_1^{SP} > g_1^{DEd},\ g_2^{SP} < g_2^{DEd}$	Ambiguous outcome
	$PS = MD + RE$	$g_1^{SP} > g_1^{DEd},\ g_2^{SP} = g_2^{DEd}$	Insufficient innovation
	$PS < MD + RE$	$g_1^{SP} > g_1^{DEd},\ g_2^{SP} > g_2^{DEd}$	Insufficient innovation

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