Rational Inattention, Multi-Product Firms and the Neutrality of Money*

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Abstract

Economies of scope in information processing naturally arise in a rational inattention model of multi-product firms: Processing information is difficult, but once information is internalized, it can be freely used for all decisions it concerns. Monetary information concerns all pricing decisions; good-specific information concerns only a few. Hence, in a quantitative model with good-specific shocks, attention to monetary shocks increases as firms produce more goods. Such good-specific shocks are necessary in our model to account for the dispersion of price changes within firms observed in both CPI and PPI data in the U.S.. Our model calibrated to CPI data predicts perfect neutrality of money while calibrated to PPI data it predicts little non-neutrality.

JEL codes: E3, E5, D8

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1 Introduction

The impact of monetary aggregates on the real economy, or whether money is "non-neutral," is one of the major question in macroeconomics. A general result in this respect is that monetary non-neutrality increases when prices become less responsive to monetary shocks. The friction that explains such unresponsiveness however varies among monetary theories. In Rational Inattention Theory (Sims (2003, 2010); Mackowiak and Wiederholt (2009)) – which is the focus of this paper – the friction is given by the limited capacity to process information ("attention") which firms must allocate to observe the realization of shocks with a certain precision. The argument for monetary non-neutrality then goes as follows: If firms are exposed to more volatile idiosyncratic shocks than aggregate shocks, they allocate most of their attention to idiosyncratic shocks. As a result, aggregate shocks – such as monetary shocks – are observed with large noise and hence prices react little to these shocks. Compared to alternative theories, attractive features of Rational Inattention are its very intuitive friction and its quantitative prediction of large and persistent monetary non-neutrality even if the friction is assumed to be "small."

This paper revisits this result to arrive at a substantially different quantitative conclusion once we augment a rational inattention model to capture two features in the data: First, firms sell or produce multiple goods, so firms take multiple pricing decisions simultaneously. Second, there is large dispersion of price changes even within firms, implying that idiosyncratic shocks have both a good- and a firm-specific component. We find, once these two features are added in, that money is fully neutral when we calibrate the model to stores, which price a large number of goods. Calibrating the model to goods producers, which price a much smaller number of goods, yields a trade-off between monetary non-neutrality and the assumed severity of the friction: To increase monetary non-neutrality by a factor of two (three), the friction must increase by a factor of two (three) as well. Overall, monetary non-neutrality is reduced by a factor of three relative to a benchmark of single-product firms when the friction is "small." When the friction is set in alternative ways, monetary non-neutrality is also much smaller than in the benchmark.

The main force behind our results is the existence of economies of scope in information processing: A firm must spend some of its attention to observe the realization of a shock with a certain precision, but then the firm can use such information at no additional cost in all its decisions. Hence, a firm that prices multiple goods has stronger incentives to pay attention to common shocks across its pricing decisions than a single-product firm. Aggregate and firm-specific shocks, but not good-specific shocks have this common shock property. In particular, these economies of scope are stronger for stores, which price a much larger number of goods than goods producers. Uncovering and studying these economies of scope are the main contribution of this paper.

To do so, we augment the model of Mackowiak and Wiederholt (2009) to allow for multi-product firms that are subject to good- and a firm-specific shocks, in addition to nominal aggregate demand shocks (in short, "monetary shocks"). We then present evidence regarding the two distinct features of our model by computing new empirical moments from the datasets used by the Bureau of Labor Statistics (BLS) to construct the Consumer Price Index (CPI) and the Producer Price Index (PPI). We calibrate our model separately to these two sets of moments, interpreting firms either as "stores" or "goods producers", and conduct a number of experiments. We separate our results into a theoretical, empirical and quantitative part.

Theoretical results. Assuming white noise shocks so that the model has a closed-form solution, we show that there are two other forces at work in addition to the economies of scope. First, firms must simply pay attention to a larger number of good-specific shocks as they price more goods. We call this the "income effect:" it resembles the situation of a consumer who faces a tighter budget constraint as her consumption basket expands to include more goods. This force makes the friction more severe as firms price more goods. A force that may go in opposite direction is the "aggregation effect:" firms' information-processing capacity may depend on the number of pricing decisions taken. Since there is no theory that models firms' decision of investing in information processing capacity, we discipline the aggregation effect through a number of alternative assumptions.

The interaction of these three forces determines the degree of monetary non-neutrality. If we assume away the aggregation effect – so that firms' information processing capacity is invariant to the number of goods – the income effect may dominate the economies of scope. This happens if firms price a small number of goods and good-specific shocks are very volatile relative to monetary and firm-specific shocks. In that case, monetary non-neutrality may be increasing in the number of goods priced. For a high enough number of goods, however, monetary non-neutrality is always decreasing in the number of goods. The reason is that the economies of scope become increasingly stronger relative to the income effect as the number of goods increases. At the same time, the severity of the friction unambiguously increases in the number of goods priced. This is true whether we measure the friction by the expected loss in per-good profits or the shadow price of information-processing capacity. This latter result also holds if we assume that the aggregation effect is such that firms' attention to monetary shocks is invariant to the number of goods. In contrast, if we choose to calibrate the aggregation effect such that the friction is equally binding regardless of the number of goods priced, using either of our two measures, monetary non-neutrality unambiguously decreases as firms price more goods.[1]

Empirical results. Our first fact – that most firms are multi-product producers – is well-established, for example by Bernard et al. (2010). The available evidence with respect to pricing, which we review in our empirical section, suggests an average number of goods of about 40,000 priced by stores and about 4 by goods producers. Even if firms price multiple goods, one may argue that prices decisions may be decentralized. However, this is not what empirical work suggests, for example Zbaracki et al. (2004). To establish our second key empirical fact and to provide further moments for calibrations, we compute the dispersion of price changes and other statistics from CPI and PPI micro data for the whole sample and after sorting firms into four bins according to the number of goods. This way, we can examine how key statistics change as firms price more goods.

We find that, in full CPI sample, 51.6% of the cross-sectional dispersion of log non-zero price changes is due to dispersion within firms. In the PPI data, we have sufficient variation in the number of goods to compute the ratio as a function of that number: it increases as firms price more goods, from 37% (for bin 1, where firms price between one and three goods) to 72.4% (for bin 4, where firms price more than seven goods). This finding is important from a modeling point of view: The model predicts that if there are no good-specific shocks, then the number of goods

[1] Among our three alternative specifications for the aggregation effect, keeping a constant severity of the friction is probably the most suitable one. The main reason is that it allows for a clean comparison among firms that price a different number of goods. It also makes our model internally consistent: In our model, the number of goods that firms price and firms' information processing capacity are exogenous. However, if the severity of the friction increases as firms price more goods, then firms have incentives to decentralize their pricing decisions in smaller decision units and/or firms that price more goods have stronger incentives to invest in information processing capacity.

that firms price has no effect on firms' attention allocation. But in this case, the model also predicts no within-firm dispersion of prices changes, which is not what we find in the data.

Regarding other statistics, we find that the average absolute size of price changes in the CPI data is 11.01%, which is in line with the findings of Klenow and Kryvtsov (2008), and in the PPI data this statistic decreases with the number of goods, from 8.5% for bin 1 to 6.5% for bin 4. We also find that good-level inflation has a negative serial correlation. Our estimate of an AR(1) coefficient is -0.29 in the CPI data and in the PPI data it ranges from -0.05 for bin 1 to -0.03 to bin 4.

Quantitative results. To set a benchmark, we replicate the findings of Mackowiak and Wiederholt (2009) who assume persistent shocks, single-product firms, one type of idiosyncratic shocks calibrated to match the average size of price changes in the CPI data, and a "small" friction. As in their work, we find large and long-lasting monetary non-neutrality.

When we depart from this benchmark, our main findings in this paper are as follows: (1) After calibrating the persistence of idiosyncratic shocks to that of good-level inflation in the CPI data, the cumulated effect of a monetary shock is cut by two relative to the benchmark for the same "small" friction. (2) When we additionally calibrate the volatility of firm-specific and good-specific shocks to match the ratio of within-dispersion of price changes, this cumulated effect is cut by three relative to the benchmark when firms produce two goods for the same "small" friction per good. Money is almost neutral when firms produce eight goods or more. (3) When we use a heterogeneous firm version of the model – different firms price a different number of goods in the same economy – we must separately calibrate the processes of idiosyncratic shocks to match moments in all four bins. Once we do so, monetary non-neutrality is cut by a factor of three relative to the benchmark. A monetary shock continues to have sizable effects on impact, but with much less persistence. (4) Using this calibration, we show that to increase monetary non-neutrality in the model, the loss of steady state revenues must increase almost linearly. Alternatively, if we calibrate our model to yield a loss of 0.34% of steady-state revenues assumed by Midrigan (2011), monetary non-neutrality is cut by five relative to the benchmark. [2] (5) Calibrating the aggregation effect from micro data is impracticable since the model's predicted micro moments show almost

[2] Our measure of monetary neutrality is difficult to compare with that of Midrigan (2011). In any case, contrasting the rational inattention model with the menu cost model is outside the scope of this paper.

no response to variations in information processing capacity while monetary non-neutrality is highly sensitive to such variations.

Literature review. The economies of scope in information processing studied in this paper have not been stressed before in the fast-growing literature of rational inattention, either applied to monetary economics as in Sims (2006), Woodford (2009, 2012), Mackowiak and Wiederholt (2009, 2011)), and Paciello and Wiederholt (2011) or in other applications such as asset pricing (Peng and Xiong (2006), portfolio choice (Mondria (2010)), rare disasters (Mackowiak and Wiederholt (2011)), consumption dynamics (Luo (2008)), home bias (Mondria and Wu (2010)), the current account (Luo et al. (2012)), discrete choice models (Matejka and McKay (2011)) and search (Cheremukhin et al. (2012)). Our paper is complementary to the study of multi-product firms and menu costs, as in Sheshinski and Weiss (1992), Midrigan (2011), Bhattarai and Schoenle (2011) and Alvarez and Lippi (2013). A key result in this literature is that introducing multi-product firms may increase monetary non-neutrality. We find the opposite result. The reason is that there is an extensive margin on price changes in menu cost models that does not exist in rational inattention models. Our empirical results are also novel since most empirical work views the data through the lens of menu cost models – for example, Bils and Klenow (2004), Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) – and does not provide the key statistics necessary to calibrate our rational inattention model. Finally, our paper shares its critical tone with Venkateswaran and Hellwig (2009), which, without using a rational inattention model, questions the assumption in Mackowiak and Wiederholt (2009) of independent sources of information for each type of shock.

Layout. Section 2 displays the model setup and solves it in closed form without frictions and with frictions after assuming white noise shocks. Section 3 uses this model to generate our theoretical results. Section 4 presents our empirical results from CPI and PPI data. Section 5 calibrates our model to CPI and PPI moments. Section 6 concludes and appendices collect tables, figures and material omitted in the main text.

2 A Model of Multi-Product, Rationally Inattentive Firms

Our model is an extension of Mackowiak and Wiederholt (2009) in which firms price an exogenous number of goods and where we allow for monetary, firm-specific and good-specific shocks. This section introduces this model and solves it analytically under the assumption of white noise shocks to obtain the main result of the Rational Inattention Theory regarding monetary non-neutrality.

2.1 Setup

Consider an economy with a continuum of goods of measure one indexed by $j \in [0,1]$, and a continuum of monopolist firms with measure $\frac{1}{N}$ indexed by $i \in \left[0, \frac{1}{N}\right]$ for $N \in \mathbb{N}$. Each firm i prices N goods which are randomly drawn without replacement from the set of goods. Denote $\aleph_i$ the set that collects the identity of the N goods produced by firm i.

Each good j contributes to the profits of its goods producer according to

$$\pi\left(P_{jt}, P_t, Y_t, F_{it}, Z_{jt}\right), \tag{1}$$

where P_{jt} is the fully flexible price of good j, P_t is the aggregate price, Y_t is real aggregate demand, and F_{it} and Z_{jt} are two idiosyncratic, exogenous random variables, the former specific to firm i and the latter specific to good j, all at time t. The function $\pi(\cdot)$ is assumed to be independent of which and how many goods the firm prices, twice continuously differentiable and homogenous of degree zero in the first two arguments. Idiosyncratic variables F_{it} and Z_{jt} satisfy

$$\int_0^{\frac{1}{N}} f_{it} di = 0, \tag{2}$$

$$\int_0^{1} z_{jt} dj = 0, \tag{3}$$

where small case generically denotes log-deviations from steady-state levels. Hence, f_{it} and z_{jt} have direct interpretation respectively as firm-specific and good-specific shocks.

Nominal aggregate demand Q_t is assumed to be exogenous and stochastic satisfying

$$Q_t = P_t Y_t, \tag{4}$$

where aggregate prices are obtained according to

$$p_t = \int_0^1 p_{jt} dj. \tag{5}$$

The total period profit function of firm i is

$$\sum_{n \in \aleph_i} \pi \left(P_{nt}, P_t, Y_t, F_{it}, Z_{nt} \right),$$

which sums up the contribution to profits of all goods produced by firm i.

The key assumption of rational inattention models is that firms are constrained in the "flow of information" that they can process at every period t:

$$I \left(\left\{ Q_t, F_{it}, \{Z_{nt}\}_{n \in \aleph_i} \right\}, \{s_{it}\} \right) \leq \kappa \left(N \right)$$

where $Q_t, F_{it}, \{Z_{nt}\}_{n \in \aleph_i}$ are variables of interest for firm i that are not directly observable, s_{it} is the vector of signals that firm i actually observes, the function $I(\cdot)$ measures the information flow between observed signals and variables of interest, and $\kappa(N)$ is an exogenous, limited capacity that without loss of generality is assumed to depend on the number N of goods the firm prices.

The information flow $I(\cdot)$ is a measure of how informative the observation of a signal is with respect to a given variable. This measure has been proposed by Shannon (1948) and has a complicated functional form that, as will become apparent below, does not need to be specified here except for computational purposes, so we relegate it to the appendix. However, to provide intuition, if one denotes as U_t an arbitrary unobservable variable of interest and as O_t an arbitrary observable signal, and assumes that U_t and O_t are Gaussian i.i.d. processes, then the information flow between U_t and O_t is given by

$$I\left(\{U_t\},\{O_t\}\right)=\frac{1}{2}\log_2\left(\frac{1}{1-\rho_{U,O}^2}\right), \tag{6}$$

which is increasing in $|\rho_{U,O}|$, the absolute correlation between U_t and O_t. Hence, a given information flow pins down the precision of signals with respect to the variables of interest.

We also assume that the vector of signals s_{it} may be partitioned into $N+1$ subvectors

$$\left\{s_{it}^a, s_{it}^f, \{s_{nt}^z\}_{n\in\aleph_i}\right\};$$

each subvector is correlated to one target variable such that $\{q_t, s_{it}^a\}$, $\left\{f_{it}, s_{it}^f\right\}$ and $\{z_{nt}, s_{nt}^z\}_{n\in\aleph_i}$ are independent of each other. Besides, we assume that all variables Gaussian, jointly stationary and there exists an initial infinite history of signals:

$$s_i^1=\{s_{i-\infty},\ldots,s_{i1}\}.$$

These assumptions imply that the information flow is additively separable according to

$$I\left(\left\{Q_t,F_{it},\{Z_{nt}\}_{n\in\aleph_i}\right\},\{s_{it}\}\right)=I\left(\{Q_t\},\{s_{it}^a\}\right)+I\left(\{F_{it}\},\left\{s_{it}^f\right\}\right)+\sum_{n\in\aleph_i}I\left(\{Z_{n\tau}\},s_{nt}^z\right).$$

Hence, the problem of the firm i may be represented as

$$\max_{\{s_{it}\}\in\Gamma}\mathbb{E}_{i0}\left[\sum_t^\infty\beta^t\left\{\sum_{n\in\aleph_i}\pi\left(P_{nt}^*,P_t,Y_t,F_{it},Z_{nt}\right)\right\}\right] \tag{7}$$

where

$$P_{nt}^*=\arg\max_{P_{nt}}\mathbb{E}\left[\pi\left(P_{nt},P_t,Y_t,F_{it},Z_{nt}\right)\mid s_{it}\right] \tag{8}$$

is subject to

$$I\left(\{P_t,Y_t\},\{s_{it}^a\}\right)+I\left(\{F_{it}\},\left\{s_{it}^f\right\}\right)+\sum_{n\in\aleph_i}I\left(\{Z_{nt}\},\{s_{nt}^z\}\right)\le\kappa\left(N\right)$$

$$\Leftrightarrow \kappa_a + \kappa_f + \sum_{n \in \aleph_i} \kappa_n \leq \kappa(N). \tag{9}$$

To abbreviate notation, we denote $I\left(\{P_t, Y_t\}, \{s_{it}^a\}\right)$, $I\left(\{F_t\}, \left\{s_{it}^f\right\}\right)$ and $I\left(\{Z_{nt}\}, \{s_{nt}^z\}\right)$ as κ_a, κ_f and κ_n, where $I\left(\{P_t, Y_t\}, \{s_{it}^a\}\right) = I\left(\{Q_t\}, \{s_{it}^a\}\right)$ since the only source of aggregate disturbances is Q_t. The absence of nominal rigidities implies that the pricing problem in (8) is static. The firm, however, must consider its whole discounted expected stream of profits to allocate its information flow capacity, its "attention", among a set Γ of signals. These signals are restricted to satisfy the above assumptions – being Gaussian, jointly stationary and independent – and must contain no information about future realizations of shocks. If a firm chooses more precise signals about, for instance, $\{P_t, Y_t\}$, then information flow $I\left(\{P_t, Y_t\}, \{s_{ait}\}\right)$ increases, reducing the information capacity to be allocated to other signals.

We define the equilibrium in this economy as follows:

Definition 1 *An equilibrium is a collection of signals $\{s_{it}\}$, prices $\{P_{jt}\}$, the aggregate price level $\{P_t\}$ and real aggregate demand $\{Y_t\}$ such that*

1. *Given $\{P_t\}, \{Y_t\}, \{F_{it}\}_{i \in \left[0, \frac{1}{N}\right]}$ and $\{Z_{jt}\}_{j \in [0,1]}$, all firms $i \in \left[0, \frac{1}{N}\right]$ choose the stochastic process of signals $\{s_{it}\}$ at $t = 0$ and the price of goods they produce, $\{P_{nt}\}_{n \in \aleph_i}$ for $t \geq 1$.*
2. *$\{P_t\}$ and $\{Y_t\}$ are consistent with equations (4) and (5) for $t \geq 1$.*

Discussion. A profit function $\pi(\cdot)$ independent across goods implies that the pricing problem in (8) is independent of N. However, N enters the attention allocation problem through three channels. First, the period objective in (7) sums up the contribution to profits of all goods produced by the firm. This is the source of economies of scope in information processing highlighted in this paper. Second, the firm has to pay attention to more signals regarding good-specific shocks as the firm produces more goods. We label this the "income effect" since it brings to mind a consumer whose basket of goods increases with N. This channel is captured by the left-hand side of (9). Finally, the capacity constraint $\kappa(N)$ in (9) may also depend on N; we call this channel the aggregation effect.

The aggregation effect simply acknowledges that firms may have different capacity to process information when they price a different number of goods. After all, this capacity should be endogenous to firms' internal organization or their investment in information technologies. However, there is no theory to guide us how to model this endogenous choice. As we show below, we can also not calibrate $\kappa(N)$ using micro moments in any straight-forward way. Hence, we take no stand on it. We simply make a variety of alternative assumptions to discipline the effect that we find illustrative or convenient for comparing economies, and study the implications of such assumptions.

We make some simplifying assumptions in our model economy: for example, we keep constant the number N of goods produced by all firms, that profits $\pi(\cdot)$ are independent across goods produced by the same firm, and that signals are informative only about one type of shocks. In section 3, we allow for heterogeneity in N in the same economy, and use this model for calibrations to PPI data in section 5. We relax the other assumptions in Appendix C to find either counterfactual predictions or no substantive effects.

We next solve this setup under the assumption that shocks are Gaussian i.i.d. This simplification allows for an analytical solution, but our setup also allows for more general specification of shocks. We solve this generalized problem in the Appendix B. We use such a solution in our quantitative analysis in section 5.

2.2 Solution for White Noise Shocks

Here, we present the key steps of the solution. As a main result, we derive the expression that relates monetary non-neutrality to information capacity.

First, when shocks are Gaussian and i.i.d., the firm's problem in (7) and (8) – up to a second-order approximation – is defined as the choice of attention to aggregate, firm- and good-specific shocks to minimize the discounted sum of firms' expected loss in profits due to the friction. After some algebra, this problem becomes

$$\min_{\kappa_a,\kappa_f,\{\kappa_n\}_{n\in\aleph_i}} \frac{\beta}{1-\beta}\frac{|\widehat{\pi}_{11}|}{2}\left[2^{-2\kappa_a}\sigma_\Delta^2 N+\left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}}\right)^2 2^{-2\kappa_f}\sigma_f^2 N+\left(\frac{\widehat{\pi}_{15}}{\widehat{\pi}_{11}}\right)^2\sum_{n\in\aleph_i}2^{-2\kappa_n}\sigma_z^2\right] \tag{10}$$

subject to the rational inattention constraint in (9).

In this expression, σ_{Δ}^{2} is the volatility of a compound aggregate variable

$$\Delta_t \equiv p_t + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} y_t \tag{11}$$

that linearly depends on monetary shocks q_t after we guess that the log-deviation of aggregate prices responds linearly to monetary shocks, $p_t = \alpha q_t$. We confirm this guess below. In addition, σ_f^2 and σ_z^2 are respectively the volatility of firm- and good-specific shocks. Parameters $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$, $\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}$ and $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}$ denote the sensitivity of frictionless prices to the log-deviation of real aggregate demand, firm- and good-specific shocks. Parameters $\widehat{\pi}_{11}$, $\widehat{\pi}_{13}$, $\widehat{\pi}_{14}$ and $\widehat{\pi}_{15}$ are the derivatives of the marginal effect of the good price on its own profits with respect to the good price, real aggregate demand, firm- and good-specific shocks, all evaluated at the non-stochastic steady state.

From the first order conditions of this problem, we obtain

$$\kappa_a^* = \kappa_f^* + \log_2(x_1), \tag{12}$$

$$\kappa_a^* = \kappa_n^* + \log_2\left(x_2\sqrt{N}\right), \quad \forall n \in \aleph_i \tag{13}$$

for $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{14}\sigma_f}$ and $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{15}\sigma_z}$. The assumption that all parameters are the same for all firms and goods along with the conditions in (12) and (13) has two implications: first, the attention paid to aggregate and firm-specific signals, κ_a^* and κ_f^*, is the same for all firms; second, the attention paid to good-specific signals is the same for all goods within all firms, $\kappa_n^* = \kappa_z^*$ for all $n \in \aleph_i$ and all i.

In addition, the conditions in (12) and (13) along with the constraint imply that

$$\kappa_a^* = \frac{1}{N+2}\left[\kappa(N) + \log_2(x_1) + N\log_2\left(x_2\sqrt{N}\right)\right] \tag{14}$$

if $x_1 x_2^N \in \left[\frac{2^{-\kappa(N)}}{\sqrt{N}}, \frac{2^{(N+1)\kappa(N)}}{\sqrt{N}}\right]$, which ensures that $\kappa_a^* \in [0, \kappa(N)]$.

In words, for a given N, the smaller is either capacity $\kappa(N)$ or parameters x_1 and x_2, the smaller is the attention to monetary shocks, or equivalently, the larger is the noise of firms' signals correlated to these shocks. A smaller x_1 comes out of this expression when the volatility σ_f of the firm-specific shocks is larger relative to the volatility σ_Δ of the aggregate compound variable in (23) and/or when frictionless prices are more responsive to firm-specific shocks, that is when $\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}$

is larger. Similarly, we obtain a smaller x_2 when $\frac{\sigma_z}{\sigma_\Delta}$ is larger and/or when $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}$ is larger.

Since all firms are identical, the price of any good $n \in \aleph_i$ for any firm i follows

$$p^*_{nt} = \left(1 - 2^{-2\kappa^*_a}\right)(\Delta_t + \varepsilon_{it}) + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\left(1 - 2^{-2\kappa^*_f}\right)(f_{it} + e_{it}) + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\left(1 - 2^{-2\kappa^*_z}\right)(z_{nt} + \psi_{nt}) \quad (15)$$

where ε_{it}, e_{it} are the realization of the noise of signals observed by firm i of a monetary shock and a shock specific to firm i, $\psi_{n,t}$. ψ_{nt} is the realization of the noise of signals of a shock specific to good n. Aggregating among all goods and firms by using (2), (3) and (5), the log-deviation of the aggregate prices with respect to the steady state is

$$p^*_t = \left(1 - 2^{-2\kappa^*_a}\right)\Delta_t = \left(1 - 2^{-2\kappa^*_a}\right)\left[\alpha + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}(1-\alpha)\right] q_t$$

which confirms the guess $p^*_t = \alpha q_t$ for

$$\alpha = \frac{\left(2^{2\kappa^*_a} - 1\right)\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}}{1 + \left(2^{2\kappa^*_a} - 1\right)\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}}. \quad (16)$$

This is the most important result of the Rational Inattention Theory. If firms have unlimited information-processing capacity, $\kappa(N) \to \infty$, then firms choose infinitely precise signals about monetary shocks (and all other shocks), so $\kappa^*_a \to \infty$ and $\alpha \to 1$. Money is fully neutral. In contrast, if firms have limited information-processing capacity, firms choose signals about shocks with finite precision, so κ^*_a is finite and thus $\alpha < 1$. Money becomes non-neutral. The more attention firms pays to idiosyncratic information – the higher are either κ^*_f or κ^*_z – the lower is κ^*_a, so monetary non-neutrality is stronger. Moreover, for a given κ^*_a, the stronger is complementarity in pricing decisions among firms – the smaller is $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} > 0$ – the stronger is monetary non-neutrality.

3 Theoretical Results

We next conduct a comparative statics analysis to illustrate in several propositions the implications of introducing multi-product firms and good-specific shocks. We have mentioned above three forces by which these ingredients affect firms' attention allocation. Before presenting our propositions, we build up intuition for the underlying mechanisms by going through these forces.

The first force is the economies of scope in information processing: The more goods a firm prices, the more pricing decisions can benefit from the information processed that is common to all goods. These economies of scope are captured in the first order conditions in (12) and (13), which we rewrite as

$$\begin{aligned} \kappa_a^* &= \kappa_z^* + \log_2\left(x_2\sqrt{N}\right), \\ \kappa_f^* &= \kappa_z^* + \log_2\left(\frac{x_2}{x_1}\sqrt{N}\right), \end{aligned}$$

after imposing that $\kappa_n^* = \kappa_z^*$ for all $n \in \aleph_i$ and all i. In words, the difference in attention paid by the firm to aggregate and good-specific shocks is increasing in N since $x_2 > 0$ while the difference in attention paid to firm-specific and good-specific shocks is increasing in N since $x_1, x_2 > 0$.

The second force is the income effect: Firms must pay attention to signals regarding more goods as firms price more goods. This force is captured by the N on the left-hand side of the constraint (9), which we rewrite as

$$\kappa_a + \kappa_f + N\kappa_z \leq \kappa(N)$$

after imposing that $\kappa_n^* = \kappa_z^*$ for all $n \in \aleph_i$ and all i. Just as a consumer whose consumption basket expands with N, if $\kappa(N)$ is kept constant, a firm pricing more goods has to distribute its attention amongst more shocks, so its information capacity becomes more binding. Thus, the income effect reduces firms' incentives to allocate attention to all shocks.

The third force is the aggregation effect, which is captured by the unspecified functional form of firms' information capacity $\kappa(N)$ in (9). In the following, we study the interaction of these forces after we make a number of alternative assumptions on the aggregation effect.

We start by assuming away any aggregation effect, $\kappa(N) = \kappa$ and dropping good-specific shocks:

Proposition 1 *If good-specific shocks do not exist, $\sigma_z = 0$, or are irrelevant for pricing decisions, $\pi_{15} = 0$, firms' allocation of attention is invariant to N if $\kappa(N) = \kappa$. Moreover, prices of goods produced by the same firm perfectly commove.*

Proof. When $\sigma_z = 0$ or $\pi_{15} = 0$, firms ignore signals s_t^z regarding firm-specific shocks, so $\kappa_z^* = 0$. Then κ_a^* is obtained from combining the condition in (12) and the constraint $\kappa_a + \kappa_f = \kappa$:

$$\kappa_a^* = \frac{1}{2}\left[\kappa + \log_2\left(x_1\right)\right].$$

which is constant in N. Moreover, the optimal pricing rule in (15) reduces to

$$p_{nt}^* = \left(1 - 2^{-2\kappa_a^*}\right)\left(\Delta_t + \varepsilon_{it}\right) + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\left(1 - 2^{-2\kappa_f^*}\right)\left(f_{it} + e_{it}\right)$$

which only varies with aggregate or firm-specific disturbances Δ_t, f_{it}, ε_{it} and e_{it}. ■

Intuitively, firms can equally exploit the economies of scope in information processing by paying attention to either aggregate or firm-specific shocks for any N. Further, there is no income effect since the number of shocks hitting firms is constant in N. Since $\kappa\left(N\right) = \kappa$, firms' constraint is invariant to N and thus N only affects the scale of the firms' objective.

Proposition 1 is useful for benchmarking. When all shocks hit the whole firm, a multi-product firm allocates its attention exactly as a single-product firm. However, the model in this case also predicts no dispersion of price changes within firms because prices do not respond to any good-specific disturbance. As anticipated in the introduction and documented in section 4, this prediction is strongly counterfactual according to both CPI data and PPI data. Hence, we focus on a setup with good-specific shocks.[3] None of our results below in this section relies on a specific process for these good-specific shocks but only on an interior allocation of firms' attention to all shocks.

Next, we keep our assumption of no aggregation effect, and further assume that the responsiveness α of aggregate prices to a monetary shock is exogenously constant.

Proposition 2 *If $\kappa\left(N\right) = \kappa$ and α is exogenously constant, firms' attention κ_a^* to monetary shocks is*

[3]The appendix solves for an extension of the model that specifies a common signal for all good-specific shocks that affect a given firm. We show that Proposition 1 holds. This result gives ground to our assumption that signals provide information about only one type of shocks.

increasing in N for $N > \hat{N}$ and $\kappa_a^* \in [0, \kappa]$, where $\hat{N}$ solves

$$\log \hat{N} + \frac{1}{2}\hat{N} = \kappa \log 2 - \log(x_2/x_1) - \log(x_2) - 1.$$

Proof. α constant implies that $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{14}\sigma_f}$ and $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{15}\sigma_z}$ are also constant. $\hat{N}$ solves $\frac{\partial \kappa_a^*}{\partial N} = 0$ for the interior solution of (14) after setting $\kappa(N) = \kappa$. ■

Proposition 2 states that the economies of scope in information processing dominates the income effect when $N > \hat{N}$. Note that a constant α allows to abstract from the feedback between firms' allocation of attention and the responsiveness of aggregate prices to shocks. We introduce such feedback in the next proposition. Here, since $N \in \mathbb{N}$, $\log 2 < 0$ and $x_1, x_2 > 0$, $\hat{N} \geq 1$ only holds if x_2/x_1 and/or x_2 are small enough. Since $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{14}\sigma_f}$ and $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{15}\sigma_z}$, x_2 is small either when $\frac{\sigma_\Delta}{\sigma_z}$ is small, that is, the volatility of good-specific shocks is high relative to the compound aggregate variable Δ_t, or when $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}$ is large, that is, frictionless prices are highly responsive to good-specific shocks. Similarly, x_2/x_1 is small either when $\frac{\sigma_\Delta}{\sigma_f}$ is small or when $\frac{\widehat{\pi}_{15}/|\widehat{\pi}_{11}|}{\widehat{\pi}_{14}/|\widehat{\pi}_{11}|}$ is high, that is, frictionless prices are highly responsive to good-specific shocks relative to firm-specific shocks.

To introduce endogeneity of α, we assume $\kappa(N)$ is unrestricted to state a general result.

Proposition 3 *The endogeneity of α amplifies the effect of N on κ_a^*. This amplification is stronger when the complementarity in pricing decisions is stronger, that is, when $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \leq 1$ is smaller.*

Proof. From equation (16), α is increasing in κ_a for $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \leq 1$, so Δ_t in (11) and thus σ_Δ are also increasing in κ_a^*; hence x_1 and x_2 are increasing in α. Besides, the interior solution of κ_a^* in (14) is increasing in x_1 and x_2; hence κ_a^* is increasing in α. As a result, the effect of N on κ_a^* in (14) gets amplified by the endogeneity of α captured in (16). According to (16), α is more increasing in κ_a as $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ is smaller, therefore this amplification effect is stronger. ■

Proposition 3 states that firms' optimal attention to monetary shocks (κ_a^*) and the degree of monetary non-neutrality (α) are jointly determined by a fixed point that solves equations (14) and (16). Visually, Figure 1 draws these two equations in the space (α, κ_a). The interior solution of (14) is drawn in red, while (16) is drawn in blue. In addition, the upper bounds of $\kappa_a \in [0, \kappa(N)]$ and

$\alpha \in [0,1]$ are represented by dashed lines. Equilibrium α is denoted as α_1^*.

Equation (16) is invariant to N but the intercept of (14) may decrease or increase while its slope is decreasing in N. The green line in Figure 1 depicts the case of a higher intercept of (14) as N increases, so equilibrium α is now α_2^*. As a result, the effect of N on κ_a^* in Proposition 2 is amplified by an indirect effect of κ_a^* on σ_Δ^2 in the same direction.

A key observation is that (16) is more flattened out for intermediate values of α than for high and low α. Hence, an increase in κ_a has a large effect on α when α is at an intermediate level. This result is stronger when $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ is smaller, that is, complementarity in pricing decisions is stronger. Visually, this is because (16) is flattened out for intermediate values of α as $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ is smaller. This result plays a crucial role in our quantitative analysis in section 5 when a small increase in firms' attention to monetary shocks yield a large reduction of monetary non-neutrality.

For our next result we assume that the aggregation effect is such that firms' attention to monetary shocks is invariant to N, that is $\kappa_a^*(N) = \bar{\kappa}_a$. This implies that $\kappa(N)$ is increasing for $N < \hat{N}$ and decreasing for $N > \hat{N}$, with $\hat{N}$ defined in Proposition 2. The next proposition shows that this assumption is equivalent to assuming that the friction is more binding as N increases as measured by two alternative indicators. One is the frictional cost, which is defined as the expected loss in profits due to the friction per-good and unit of time,

$$C(\kappa_a, \kappa_f, \kappa_n) = \frac{|\widehat{\pi}_{11}|}{2}\left[2^{-2\kappa_a}\sigma_\Delta^2 + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}}\right)^2 2^{-2\kappa_f}\sigma_f^2 + \left(\frac{\widehat{\pi}_{15}}{\widehat{\pi}_{11}}\right)^2 2^{-2\kappa_n}\sigma_z^2\right] \tag{17}$$

and the second is the shadow price of information-processing capacity, which is equal to the Lagrange multiplier of the constraint in (9).

Proposition 4 *When $\kappa(N)$ is set such that $\kappa_a^*(N) = \bar{\kappa}_a$, the friction is increasingly more binding, either measured by the frictional cost or the shadow price of information-processing capacity.*

Proof. Using the first order conditions in (12) and (13) and $\kappa_a^*(N) = \bar{\kappa}_a$, the frictional cost in (17) becomes

$$C_n(N) = \frac{|\widehat{\pi}_{11}|}{2} 2^{-2\bar{\kappa}_a}\sigma_\Delta^2 (N+2)$$

and the shadow price of information-processing capacity

$$\lambda\left(N\right)=-\frac{\beta}{1-\beta}\left|\widehat{\pi}_{11}\right|\log\left(2\right)2^{-2\bar{\kappa}_a}\sigma_\Delta^2 N$$

both of which increase linearly with N given σ_Δ^2 is also invariant to N because $\kappa_a(N)=\bar{\kappa}_a$. ■

The next proposition extends this result to any case in which firms' attention to aggregate shocks is decreasing in the number N of goods that firms price.

Proposition 5 *Any specification of the model such that $\kappa_a^*\left(N\right)$ is decreasing in N implies that the frictional cost and the shadow price of information-processing capacity are increasing in N.*

Proof. Proposition 4 states that the friction is increasingly binding with N for $N\leq\hat{N}$ even when $\kappa\left(N\right)$ is increased to preserve a constant κ_a^*. This is an lower bound for the case that this proposition refers to. ■

Proposition 4 and 5 predict that the severity of the friction must increase as firms price more goods to yield constant or increasing monetary non-neutrality. We confirm this prediction in our quantitative exercises in section 5. As a matter of fact, in section 5 we find that the severity of the friction must be much higher in our calibrated model with multi-product firms than in a comparable model of single-product firms such that both models yield the same monetary non-neutrality. This severity of the friction must be also higher than what is assumed in alternative models or what has been found in empirical studies.

A natural alternative assumption to discipline the aggregation effect is to assume that the severity of the friction is invariant to the number of goods that firms price. Since we use two measurements of friction, the exact condition we impose takes two alternative forms. In the first, we set the aggregation effect such that the frictional cost is invariant to N at the optimal allocation of attention, that is, the expected loss in profits due to the friction per good is the same in equilibrium for any firm regardless of the number of pricing decision it takes. In the second, the shadow price of information processing capacity is invariant to N at the optimal allocation of attention, that is, firms' incentives to increase their information processing capacity is the same in equilibrium for any firm regardless of the number of pricing decisions it takes.

Both of these alternative assumptions imply a concave relationship between information processing capacity and N. This is because firms can increasingly exploit the economies of scope in information processing as they take more pricing decisions. The next proposition states our main result for this case.

Proposition 6 *When the capacity function $\kappa(N)$ is restricted to preserve a severity of the friction invariant to N, firms' attention $\kappa_a^*(N)$ to monetary shocks is unambiguously increasing in N. When the frictional cost is invariant to N, $\kappa_a^*(N)$ becomes*

$$\kappa_a^*(N) = \kappa_a^*(1) + \frac{1}{2}\log_2\left(\frac{N+2}{3}\right) + \log_2\left[\frac{\sigma_\Delta\left(\kappa_a^*(N), \sigma_q\right)}{\sigma_\Delta\left(\kappa_a^*(1), \sigma_q\right)}\right].$$

When the shadow price of information processing is invariant to N, $\kappa_a^*(N)$ becomes

$$\kappa_a^*(N) = \kappa_a^*(1) + \frac{1}{2}\log_2(N) + \log_2\left[\frac{\sigma_\Delta\left(\kappa_a^*(N), \sigma_q\right)}{\sigma_\Delta\left(\kappa_a^*(1), \sigma_q\right)}\right].$$

Proof. These expressions follow from the definition of the frictional cost and the shadow price of information capacity along the optimal conditions for the allocation of attention. ■

This proposition provides the intuition for our main quantitative results. For a given extent of the friction, the model underestimates firms' attention to monetary shocks – and thus overstates monetary non-neutrality – under the assumption of single-product firms. This result holds for any specification of the model, even if the specification varies with N. It is only important to have a volatility of monetary shocks σ_q invariant to N and an interior solution for firms' allocation of attention.[4] In our quantitative exercises below, we pin down these parameters directly from the data.

The specification of the aggregation effect that leads to proposition 6 has two attractive features with respect to those previously imposed: The first is that it provides a natural discipline to compare among firms that price a different number of goods. The second is that it allows for internal consistency in our model. This requires further explanation:

[4]The exception is the degree of strategic complementarity, which is captured by $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ implicit in Δ_t. If one assumes that strategic complementarity of firms pricing decisions decreases strongly ($\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ increases) as N increases, then the monotonicity of $\kappa_a^*(N)$ may not hold for some values of N.

In our model, we assume that the number of pricing decisions that firms take is exogenous. If the aggregation effect is such that frictional cost were increasing in the number of goods priced by a firm, then firms would have incentives to delegate their pricing decisions to smaller decision units. Hence, if the number of pricing decisions were endogenous, our economy would collapse to one in which all goods are priced by single-product firms. Of course, there are more reasons for firms to produce a basket of goods than exploiting economies of scope in information processing. These other reasons are absent in our model due to the assumption that the contribution to profits of each good is independent of the number of goods and which goods a firm produces. This simplification implies that our firms are modeled as decision units, not as production units. We relax this assumption in Appendix C. We find that, given the productive structure of the firm, proposition 6 still applies: Decision units pay more attention to aggregate shocks as they prices more goods.

A similar argument applies to the shadow price of information-processing capacity. In our model, firms information processing capacity is exogenous. However, if the shadow price of such a capacity were increasing, firms pricing more goods would have larger incentives to invest in this capacity. By contrast, assuming that the shadow price of information processing is invariant to N is equivalent to assuming that the cost of building information capacity does not depend on the number of decisions that can benefit from the processed information. This result is not contaminated by the implicit assumption in our model that the scale of firms depends on N. We could instead assume the total measure of goods in the economy is N and the density of firms is always 1, so firms size is invariant to N. This modification has no effect on any of our results.

Heterogeneous firms. We now introduce a modification of our model that allows for firms that price a heterogeneous number of goods. This modified model produces the same qualitative results as the one studied above. However, it allows us to perform a more realistic calibration using several moments from PPI data which we compute according to the number of goods firms price.

Thus, consider G groups of firms such that firms in group $g = 1, ..., G$ produce N_g goods. Each group has measure ω_g satisfying $\sum_{g=1}^{G} \omega_g N_g = 1$. The processes for firm-specific and good-specific shocks are independent for each group, so these shocks still wash out when prices are aggregated. All parameters are the same for all groups.

For a given guess $p_t^* = \alpha q_t$, the solution of κ_a^* is still represented by (14) only replacing N by N_g. This guess is now confirmed for

$$\alpha = \frac{\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}\sum_{g=1}^{G}\omega_g N_g\left(1-2^{-2\kappa_a^*\left(N_g\right)}\right)}{1-\left(1-\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}\right)\sum_{g=1}^{G}\omega_g N_g\left(1-2^{-2\kappa_a^*\left(N_g\right)}\right)}$$

We find that Propositions 1 to 5 continue to hold in this setup. Proposition 6 gets modified to

$$\kappa_a^*\left(N\right) = \kappa_a^*\left(1\right) + \frac{1}{2}\log_2\left(\frac{N+2}{3}\right), \text{and}$$

$$\kappa_a^*\left(N\right) = \kappa_a^*\left(1\right) + \frac{1}{2}\log_2\left(N\right),$$

which is still increasing in N although there are two differences with respect to the above result. First, $\kappa_a^*\left(1\right)$ is now the attention paid to monetary shocks by single-product firms in the same economy – before it was firms' attention in an economy populated only by single-product firms. Second, the last term in the right-hand side of the equation for $\kappa_a^*\left(N\right)$ in Proposition 4 is zero since the volatility σ_Δ now is common to all firms. This modification implies that the effect of N on $\kappa_a^*\left(N\right)$ conditional on $\kappa_a^*\left(1\right)$ is now less steep. This does not mean that complementarity in pricing decisions plays no role: $\kappa_a^*\left(1\right)$ is higher than in an economy with only single-product firms. This is because $\kappa_a^*\left(1\right)$ is increasing in σ_Δ and σ_Δ is higher in an economy where there are multi-product firms.

4 Empirical Results

This section provides empirical support for two distinctive features of our rational inattention model: the multi-product nature of firms, and the existence of good-specific shocks. We also report several new facts about price setting in the CPI and PPI data. We generate these statistics explicitly with the calibration of our rational inattention model in mind.

4.1 Data description

We use confidential, monthly transaction-level micro price data collected by the U.S. Bureau Labor Statistics (BLS) to construct the Consumer Price Index (CPI) and the Producer Price Index (PPI).[5] We generate our results by computing statistics for the whole sample and for four 'bins.' We assign firms to these bins according to their number of goods in the data. Thus, we can track how key statistics change as the number of goods increases. All statistics, including standard deviations, are reported in Table 1.

Our CPI data span the time period from 1988 to 2009, containing approximately 125,782 outlets. An outlet usually corresponds to a non-producing retailer or, in colloquial terms, a store. Our main manipulation of the CPI data is to exclude sales and zero price changes. The exclusion of sales is common in studies of price setting: sales are usually considered practices of firms that are not necessarily related to the business cycles (for instance, see Guimaraes and Sheedy (2011)). We exclude zero price changes for two reasons: First, this is more consistent with our model, in which prices are fully flexible. Second, Mackowiak and Wiederholt (2009), which we use as benchmark in section 5, calibrate their model to statistics excluding sales and zero price changes.

Our PPI sample spans the time period from 1998 to 2005, containing approximately 28,575 firms. A "firm" is typically a goods producer which is defined as a decision unit that prices a given number of goods. Again, we exclude zero price changes, but we do not control for sales. This practice is sufficiently less common for firms in this dataset to leave results unchanged.[6]

4.2 Multi-Product Firms and Facts about Pricing

In our CPI sample, the median (mean) number of goods sampled from a single outlet is 1.39 (2.05) with a standard deviation of 2.03 goods.[7] In these data, 87% (75%) of outlets have less than 3 (2) goods in the sample. Given that outlets are usually stores, we conclude that the CPI data does not

[5]Nakamura and Steinsson (2008) or Bils and Klenow (2004) describe the CPI data in detail, while for example Bhattarai and Schoenle (2011) describe the PPI data.

[6]As work by Nakamura and Steinsson (2008) has shown, sales are not a factor in determining the behavior of PPI prices. We have computed our statistics excluding sales prices, and have found no significantly different results.

[7]The median is not integer because for the following reason: First, we compute for each outlet over time its mean number of goods. Due to exit and entry, this may not be an integer. Second, we take the median or mean across firms. The same reasoning applies to the PPI data.

provide reliable estimates of the number of goods that a single outlet prices. For this reason, we also do not use in our calibrations the moments computed by bin from this dataset; we only use moments computed for the whole sample.

To obtain an estimate of the number of goods in stores, we use the Food Marketing Institute (FMI) 2010 Report, which reports an average of 38,718 items per store.[8] The FMI is an industry-based institution that represents $1,500$ food retailers and wholesalers in the U.S.. Their members are large multi-store chains, regional firms and independent supermarkets, stores and drug stores of a large variety of classes with a combined annual sales volume of 680 billion.[9] Additional evidence is provided by Rebelo et al. (2010) which uses data from one particular store that prices about 60,000 items. In any case, as anticipated in the introduction and presented in detail in section 5, we do not need to pin down a precise estimate of the number of goods priced by stores. It suffices to establish that this number is large.

In contrast, we can use the PPI data to estimate variation in number of goods priced by productive firms. In turn, we can then relate that number to changes in key price statistics and calibrate our model accordingly. We note that the number of goods recorded in our data represents a lower bound on the actual number of goods per firm: the PPI sample is not the universe of all goods produced. This only makes our conclusions in section 5 stronger since we will thus work with a lower bound on the number of goods. However, what is important for representativeness is that there is a monotonic mapping from the actual number of goods to our sample, as Bhattarai and Schoenle (2011) discuss in detail. Moreover, our data contain substantial variation in the number of goods per firm such that we consider the statistics by bins as reliable. The median (mean) number of goods per firm is 4 (4.13) with a standard deviation of 2.55 goods. The median number of goods per firm are 2 (bin 1), 4 (bin 2), 6 (bin 3), and 8 (bin 4).

A rough alternative estimate for the number of goods priced by a firm is in Bernard et al. (2010). They define a product as a category of the five-digit Standard Industrial Classification in the US Manufacturing Census data, which is less thin than ours. Their research focus is very different from ours, but an informative result for our work is that average number of products produced

[8]http://www.fmi.org/research-resources/supermarket-facts

[9]http://www.fmi.org/about-us/who-we-are. For a detailed list of stores included, see http://www.fmi.org/about-us/our-members

by a multi-product firm is 3.5.

A final remark is that although the multi-product nature of firms is a well-established empirical fact, we cannot verify ourselves in our data whether prices are set by multi-product decision units or not. However, the data are carefully collected by the BLS such that a firm is a "price-forming unit." In addition, Zbaracki et al. (2004) conduct a case study that documents in great detail the decision process of setting prices of a productive firm. They report that the firm they study has multiple products, whose regular prices are decided at headquarters while sale prices are set by local managers in small geographical areas. However, at both levels there is a single decision unit setting prices for the whole portfolio of goods that this firm produces. We consider this as evidence that prices are indeed set by multi-product decision units.

Within-firm dispersion of price changes Separately using the CPI and PPI datasets, we construct a measure of the ratio of within-firm dispersion of log non-zero price changes relative to total cross-sectional dispersion of log non-zero price changes. We denote this statistics as r:

$$r = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{\sum_{i=1}^{I_t}\sum_{n\in\aleph_i}\left(\pi_{nt}-\overline{\pi}_{it}\right)^2}{\sum_{i=1}^{I_t}\sum_{n\in\aleph_i}\left(\pi_{nt}-\overline{\pi}_{t}\right)^2}\right]$$

where $\overline{\pi}_{it}$ is the mean absolute size of log price changes across all goods sampled for firm i at time t. It measures the ratio of within-firm cross-sectional variance relative to total cross-sectional variance.

Computation of this statistic leads to our most important empirical result. In the CPI data, 51.6% of the cross-sectional dispersion of log price changes is due to within-firm dispersion. In the PPI data, this ratio is increasing as firms produce more goods, from 36.5% (for bin 1, where firms produce between 1 and 3 goods) to 72.4% (for bin 4, where firms produce more than 7 goods). In the full PPI sample, 59.1% of the total variance is due to within-firm variance.

In the following discussions, we sometimes refer to this finding as imperfect co-movement of price changes. We interpret this result as evidence of the existence of good-specific shocks that firms must take into account in their pricing decisions.

4.3 Other relevant statistics

Absolute size of price changes. As a measure of the magnitude of price changes, we compute the average size of absolute price changes. We denote this statistic as $|\overline{\pi}|$. Labelling time as t, firms as i and goods produced by firm i at time t as $n \in \aleph_{it}$,

$$|\overline{\pi}| = \frac{1}{I}\sum_{i=1}^{I}\left[\frac{1}{N_i}\sum_{n\in\aleph_i}\left[\frac{1}{T_n}\sum_{t=1}^{T_n}|\pi_{nt}|\right]\right]$$

where $\pi_{nt} \equiv p_{nt} - p_{nt-1}$ is non-zero inflation for good n, T_n is the total number of periods for which inflation for good n can be computed, N_i is the number of goods produced by firm i in the sample, and I is the total number of firms in the sample. Thus, we first compute for each good the size of price changes, conditional on non-zero price changes. Second, we compute firm-level averages. Finally, we take the mean across all firms in the full sample, or within one of the bins.

In the CPI data, the mean (median) absolute size of regular price changes is 11.3% (9.6%), according to Klenow and Kryvtsov (2008). Our own computation gives us 11.01% (8.42%).[10] In the PPI data, the mean absolute size of price changes for the whole sample is 7.8%. For bins 1 to 4 the magnitudes are as follows: 8.5%, 7.9%, 6.8%, and 6.5%. This trend shows that the multi-product nature of price changes is strongly related to the size of price changes: as the number of goods increases, the magnitude of price changes becomes smaller. Various robustness checks in the PPI data, reported in Bhattarai and Schoenle (2011), leave this result unchanged.

Serial correlation of price changes. We denote this statistic by ρ for the whole sample and by ρ_k for bins $k \in (1,2,3,4)$. We obtain this statistic by computing median quantile estimates for the parameter of an $AR(1)$ coefficient for $\pi_{n,k,t}$, conditional on non-zero price changes. We compute the median quantile regression by estimating the following specification:

$$\hat{\rho}_k = argmin_{\rho_k} E[|\pi_{n,k,t} - \rho_k \pi_{n,k,t-1}|]$$

We find that the median estimate of the AR(1) coefficient is -0.29 in the CPI sample. Bils and

[10] The slight difference in results is due to our focus on outlets as the unit of analysis, which changes the aggregation approach.

Klenow (2004) estimate a comparable first-order serial correlation of -0.05.[11] In the PPI data, our estimate of the AR(1) coefficient is -0.04. It ranges from -0.05 in bin 1 to -0.03 in bin 4. All coefficients are statistically highly significant.

Cross-sectional dispersion of price changes. This statistic is denoted as $\widetilde{\sigma}$ and defined as

$$\widetilde{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{\sum_{i=1}^{I_t}\sum_{n\in\aleph_i}(\pi_{nt}-\overline{\pi}_t)^2}{\sum_{i=1}^{I_t}N_{it}-1}\right]$$

where $\overline{\pi}_t$ is the average of non-zero absolute log price changes π_{nt} of all goods sampled at time t, N_{it} is the total number of goods sampled for firm i at time t, I_t is the total number of firms at time t, and T is the total number of periods in our data. As Table 1 shows, the cross-sectional dispersion is 3.51% (2.65%) in the full PPI (CPI) sample. There is no clear trend in the PPI data.

5 Quantitative Results

In this section use a generalized version of our model that allows for persistent monetary, firm- and good-specific shocks[12] to tackle quantitatively two related questions: How sensitive is the prediction of the rational inattention model regarding monetary non-neutrality to the introduction of multiproduct firms? And, what is the degree of monetary non-neutrality predicted by a calibrated rational inattention model of multi-product firms?

5.1 Baseline Calibration

We start by replicating the results of Mackowiak and Wiederholt (2009) for an economy of single-product firms. We subsequently use this as a benchmark, to which we add calibration targets from the data and incorporate the multi-product nature of the firm. We nest the calibration of

[11] Bils and Klenow (2004) compute their estimate as the average of $AR(1)$ coefficients for inflation of 123 categories in the CPI data. They include sales and zero price changes, between 1995 and 1997. We differ in our methodology and by focusing on the period from 1989 to 2009. Qualitatively, both approaches give the same results.

[12] We discuss the problem of the firm and its numerical solution algorithm in the appendix.

Mackowiak and Wiederholt (2009) in our model by setting

$$N = 1; \kappa(1) = 3; \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} = 0.15; \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} = 0; \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} = 1.$$

First, setting capacity $\kappa(1) = 3$ implies a small frictional cost of 0.21% of firms' steady state revenues. Second, the complementarity in pricing decisions $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} = 0.15$ is in the lower bound of the range suggested by Woodford (2003). It is also exactly what Mackowiak and Wiederholt (2009) assume. Third, they refer to idiosyncratic shocks as firm-specific in their model. However, since their model has single-product firms, idiosyncratic shocks are indistinguishable between our firm-specific and good-specific shocks. To replicate their exercise, we must shut down one of them. We find it more appealing to start our analysis assuming that these shocks are good-specific, $\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} = 0$. Fourth, we set idiosyncratic and monetary shocks to be equally important in profits, so $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} = 1$. This parameter enters in the model through $x_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{15}\sigma_z}$, so setting $\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} = 1$ implies that $\frac{\sigma_\Delta}{\sigma_z}$ must be pinned down from the data.

To obtain σ_Δ, we also follow Mackowiak and Wiederholt (2009). We estimate an $AR(1)$ process for GNP quarterly data spanning 1959:1–2004:1 to obtain the volatility and persistence of q_t, $\sigma_q = 2.68\%$ and $\rho_q = .95$. Then, for computational simplicity, we approximate this process by a $MA(20)$:

$$q_t = \sum_{k=0}^{20} \left(1 - \frac{k}{20}\right) v_{t-k} \tag{18}$$

where $v_t \sim N(0,1)$ and coefficients decrease linearly with the order of lags up to 20 lags. Hence an innovation in nominal aggregate demand dies out after 21 periods. Given the process for q_t, the compound aggregate variable Δ_t also follows a $MA(20)$:

$$\Delta_t = \sum_{k=0}^{20} \left[\left(1 - \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}\right) \alpha_k + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \left(1 - \frac{k}{20}\right) \rho_q \right] v_{t-k}$$

where $\{\alpha_k\}$ are the parameters of the guessed process of aggregate prices, which is also $MA(20)$:

$$p_t = \sum_{k=0}^{20} \alpha_k v_{t-k} \tag{19}$$

such that $\{\alpha_k\}$ are found in equilibrium. We provide a detailed explanation in the appendix.

For idiosyncratic volatility σ_z, we assume that these shocks follow an $MA(20)$ similar to (18) with an adjusted scale of coefficients to match the 9.6% average absolute per-good inflation reported by Klenow and Kryvtsov (2008) for CPI data in the US. This implies $\sigma_z = 11.8\sigma_q$.

We then replicate the results in Mackowiak and Wiederholt (2009):[13] Firms' attention is $\kappa_a^*(1) = 0.09$ to monetary shocks and $\kappa_z^*(1) = 2.91$ to idiosyncratic shocks. This yields large and long-lasting monetary non-neutrality. Figure 2 depicts the response of aggregate prices after an innovation of 1% in q_t. The black line draws the response of frictionless prices; this response inherits the process assumed for q_t in (18). The blue line draws the response of aggregate prices under rational inattention. On impact, prices absorb only 2.8% of the innovation in q_t. Their response remains sluggish relative to the response of frictionless prices for 20 periods (the output deviation is less than 5% of the shock thereafter) and the cumulated response of prices is only 22% of the cumulated response of frictionless prices. As anticipated above, the frictional cost is 0.21% of the firm's steady state quarterly real revenue $\overline{Y}$. This cost is considered small. It gives little incentives for firms to increase their information capacity if such decision were endogenous. Importantly, these single-product results confirm Sims' statement about the ability of the Rational Inattention model to generate large macroeconomic effects even with a small friction.

5.2 Multi-Product Firms

We now extend the baseline calibration, allowing firms to produce $N > 1$ goods. As a result, we find that money becomes fully neutral. Again, we target an average absolute size of price changes of 9.6% and a frictional cost of $0.21\%\overline{Y}$. What is the right choice of N? We know from Section 4 that the CPI data does not provide sufficient variation to calibrate N. However, indirect estimates indicate $N \approx 40,000$ (see the Food Marketing Institute's 2010 report). Since the effect of multi-production is already very strong for $N = 2, 4$ and 8, we report results for these cases only.

We find that already for a two-good firm, monetary non-neutrality is cut by three in magnitude and duration. We illustrate in Figure 2 in red the resulting response of prices to a monetary shock,

[13] In our numerical algorithm, we use a tolerance of 2% for convergence, exactly as in Mackowiak and Wiederholt (2009). We keep this criterion for comparability with Mackowiak and Wiederholt (2009) in the following sections, but from section 5.5 on we replace it with a tighter tolerance of 0.01%. If we use the tighter convergence criterion in this and the next sections, we obtain even starker predictions from introducing multi-product firms.

when for $N = 2$ firms' attention is $\kappa_a^*(2) = 0.36$ and $\kappa_z^*(2) = 2.92$. Strikingly, the response of prices is almost identical to the frictionless price response after 7 periods (the output deviation is less than 5% of the shock thereafter), and their cumulated response is 74% that of frictionless prices. Prices absorb 29% of the innovation in q_t on impact. Note that this result holds when a firm's attention to monetary shocks is only a small fraction of the firm's total capacity. The strong effect is due to complementarity in pricing decisions, as stated by Proposition 3.

We also show the response of prices to an aggregate shock for a four-good firm in Figure 2 in green. We find that $\kappa_a^*(4) = 0.58$ and $\kappa_z^*(4) = 2.90$. Prices absorb 15% of the shock on impact. Overall, their response closely follows that of frictionless prices after 4 quarters with almost no real effects thereafter, and their cumulated response is 86% that of frictionless prices. For $N = 8$, in magenta in figure 2, results are even stronger: $\kappa_a^*(8) = 0.90$ and $\kappa_z^*(8) = 2.87$, prices absorb 49% of the shock on impact, the output deviation is less than 5% of the shock after 2 quarters and the cumulated response of prices is 93% that of frictionless prices. Given these results, we find it uninformative to report results for $N = 40,000$: Money is fully neutral.

5.3 Serial Correlation of Price Changes

We now calibrate the persistence of idiosyncratic shocks z_{jt} to match the persistence observed in the CPI data. Again, we find that this substantially reduces monetary non-neutrality. What do we calibrate good-level persistence to? So far we have followed Mackowiak and Wiederholt (2009) by assuming that z_{jt} is as persistent as q_t. Instead, we pick a much lower value for the persistence parameter. According to Bils and Klenow (2004), the first-order serial correlation of per-good inflation is -0.05. Our own computationis -0.29 (see Table 1). Both computations are methodologically different,[14] but both suggest that idiosyncratic shocks are substantially less persistent than monetary shocks.

We therefore set z_{jt} to follow an MA process for which the coefficients decrease linearly with the order of lags, as for q_t in equation (18). However, to match the -0.05 first-order serial correlation,

[14] Bils and Klenow (2004) compute this statistic by averaging the coefficient of $AR(1)$ regressions for inflation of 123 categories in the CPI data, including sales and zero price changes, between 1995 and 2007. We compute the coefficient from an $AR(1)$ quantile regressions for non-zero inflation of each item in the CPI data, excluding sales and zero price changes, between 1989 and 2009. Our computation is consistent with the other statistics we report.

z_{jt} must follow a $MA(5)$. We must also set $\sigma_z = 10.68\sigma_q$ to match the average absolute per good inflation. To generate -0.29 first-order serial correlation, z_{jt} must follow a $MA(1)$ with coefficient 0.33 and $\sigma_z = 9.74\sigma_q$. We also keep targeting $0.21\%\overline{Y}$ of per-good frictional cost.

We focus on results for the case $N = 1$. The reason is that results are qualitatively not different for $N > 1$ from those in the section above. However, attention allocated to the aggregate shock does change significantly when we calibrate the model to the serial correlation observed in the data for $N = 1$. Figure 3 summarizes the response of prices to a 1% innovation in q_t for $N = 1$. The black and blue lines show the response of frictionless prices, and prices under rational inattention for the benchmark calibration. The red line draws the response of prices calibrated to -0.05 serial correlation: We find that $\kappa_a^*(1) = 0.20$ and $\kappa_z^*(1) = 2.81$. This implies that the response of prices on impact is 7% of the shock and the deviation of output is less than 5% of the shock after 12 periods, and the cumulated response of prices is 52% of the frictionless price response. The green line shows the response of prices calibrated to -0.29 serial correlation of price changes. Now $\kappa_a^*(1) = 0.19$ and $\kappa_z^*(1) = 2.66$. This implies that the response of prices on impact is 7% of the shock, the output deviation is less than 5% of the shock after 12 periods, and the cumulated response of prices is 52% that of frictionless prices.[15]

We conclude that the monetary non-neutrality predicted by the model is substantially reduced for $N = 1$: When we calibrate the model to match the serial correlation of good-level price changes found in the data, the cumulated response of prices is now 52% of the frictionless price response, instead of 22% in the benchmark model. The intuition is straight-forward: When the process of a shock is less persistent, any mistakes of firms when tracking this shock have lower impact on future mistakes tracking this shock. Hence, firms pay less attention to this shock. We give a more detailed argument in the appendix.

5.4 Within-Firm Dispersion of Price Changes

We now introduce firm-specific shocks. This is necessary in order to match the imperfect comovement of price changes observed within firms in the data. Matching this additional target is only

[15] If we used OLS instead of quantile regressions to estimate an $AR(1)$ process for price changes, the estimate would be -0.22. The implied calibration results are very similar to those reported here.

possible in a model with all three kinds of shocks: firm-specific, good-specific and aggregate. Again, we find that monetary non-neutrality quickly vanishes as N increases.

To perform this exercise, we target the ratio of within-firm variance to total cross-sectional variance of non-zero absolute price changes in the CPI. In our exercises above with no firm-specific shocks, this statistic is 50% for $N = 2$, 86% for $N = 4$ and 93% for $N = 8$. Our target now is a ratio of 51.6%, as shown in Table 1.

To choose the relative volatility of firm-specific and good-specific shocks, we assume that $\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} = 1$; that is, firm-specific shocks f_{it} have the same weight in firms' profits as aggregate and good-specific shocks, q_t and z_{nt}.[16] For any number of goods N, we must set the process of firm-specific and good-specific shocks to follow an $MA(1)$ with parameter 0.33 to match -0.29 serial correlation of per-good inflation. The total volatility of these shocks to match 9.6% average absolute per-good inflation must be $9.75\sigma_q$ for $N = 2$, $11.7\sigma_q$ for $N = 4$, and $11.18\sigma_q$ for $N = 8$. To match the 51.6% ratio of within-firm dispersion, the calibration of σ_f/σ_z is also specific to the number of goods. For $N = 2$, we set $\sigma_f = 0$ since the highest within-firm dispersion ratio we can generate is 50%, so results for this case are the same as in section 5.3. We set $\sigma_f = 1.37\sigma_z$ for $N = 4$ and $\sigma_f = 1.90\sigma_z$ for $N = 8$. Finally, we calibrate $\kappa(N)$ to yield a $0.21\%\overline{Y}$ per-good frictional cost as in our previous exercises.

We find that for a four-good firm, the allocation of attention becomes $\kappa_a^*(4) = 0.61$, $\kappa_f^*(4) = 3.27$ and $\kappa_z^*(4) = 2.05$ respectively for monetary, firm-specific and good-specific shocks. This implies that aggregate prices absorb 30% of a monetary shock on impact. The deviation of output is less than 5% of the shock after 4 periods, and the cumulated response of prices is 87% of the frictionless price' response. For an eight-good firm, $\kappa_a^*(8) = 0.96$, $\kappa_f^*(8) = 3.85$ and $\kappa_z^*(4) = 1.90$, and prices absorb 52% on impact. The deviation of output is less than 5% of the shock after 2 periods, and the cumulated price response is 94% of the frictionless price response.

Discussion These results allow us to provide an answer to the two questions posed at the beginning of this section. Regarding the sensitivity of the monetary non-neutrality predicted by the model, we find that the model is very sensitive. The force behind this result is the economies of scope in information processing that multi-product firms can exploit by processing information

[16]Similarly than for x_2, $\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}$ enters in the model's predictions through $x_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta}{\widehat{\pi}_{14}\sigma_f}$.

about monetary and firm-specific shocks, but not good-specific shocks. This force is amplified by the complementarity in pricing decisions between firms, which is responsible for the large reduction in monetary non-neutrality in the model despite the fact that firms' attention to monetary shocks is only a small proportion of firms' capacity. We further study the role of complementarity in pricing in section 5.6.

Given this high sensitivity, our results suggest that there is no room for monetary non-neutrality in a rational inattention model calibrated to CPI data in which firms are interpreted as stores. This is because stores price a high number of goods priced. In addition, we find that even in a model of single-product firms monetary non-neutrality is smaller than in our benchmark model once the model matches the serial correlation of price changes at the good level.[17]

We conclude from sections 5.3 and 5.4 that our results do not depend on the relative volatility of firm-and good-specific shocks as long as firms pay some attention to good-specific shocks. This quantitative conclusion is the same as our theoretical one in section 3.

5.5 Calibration to PPI Data

Since this is a completely different dataset than the one used by Mackowiak and Wiederholt (2009), we abandon using their work as benchmark. Instead, we keep our target of 0.21% of steady state revenues for the frictional cost and we concentrate on the ability of the model to replicate moments from the data for our four bins and the monetary non-neutrality predicted once the model is calibrate to these moments. For this we use the version of our model introduced at the end of section 3, which allows for firms producing a heterogeneous number of goods, once it is generalized to include persistent monetary, firm- and good-specific shocks.

We now take the version of our model with heterogeneous firms and calibrate it to moments from the PPI sample. As discussed in section 4, our PPI sample has sufficient variation in N. This allows us to compute moments conditional on various N. Specifically, we assume that there are four groups of firms, producing $2, 4, 6$ and 8 goods – the median number of goods per firm in each

[17]These results are robust to varying the importance of good-specific shocks; it is only important that firms pay at least some attention to these types of shocks. These results also robust to using the tighter tolerance for convergence that we use in the following sections.

bin. The relative weights of these groups in the model economy are the shares of total employment in each bin. We keep our baseline calibration except for the processes of firm-specific and good-specific shocks. As above, we calibrate these processes to match three statistics: the mean absolute size of non-zero log price changes, the within-firm variation ratio of per-good inflation, and the first-order serial correlation of non-zero log price changes. We keep our target of $0.21\%\overline{Y}$ per-good frictional cost.

We start our analysis by asking if one set of parameters can explain trends in the data as the number of goods increases. To do so, we assume that the processes for firm-specific and good-specific shocks are the same in all bins. This means that we calibrate these two shock processes to match the three targets for bin 1 only. We then assume that the processes for the other bins follow the same calibrated processes. We report the model-predicted moments in italics in Table 3 and contrast them with the moments computed from the data. The model fails to account for the observed moments in other bins. While the average absolute size of log price changes and the serial correlation of non-zero log price changes are invariant to N in the model, they are decreasing in the data. Moreover, the ratio of within-firm variation is less increasing in N in the model than in the data.

Next, we calibrate firm-specific and good-specific shocks independently for each group to match the statistics for all bins. Figure 4 illustrates the response of prices to an aggregate shock. The response of prices is not very different across bins. This is due to strategic complementarity in pricing decisions within and across bins. Aggregate prices absorb on impact 16.7% of the monetary shock, output is less than 5% higher than its steady state after 7 periods, and the cumulated response of aggregate prices is 75% of the frictionless price response. We then conclude that monetary shocks have a strong effect on impact when firms in the model are interpreted as goods producers, but the persistence of these shocks is limited.

5.6 The Role of Complementarity in Pricing

To further highlight the importance of the complementarity in pricing decisions in our analysis, we reduce the complementarity in pricing decisions. We do so by increasing $\frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}$ from 0.15 to 0.85. This modification has two effects. On the one hand, an increase in firms' attention to monetary shocks has a milder effect on reducing monetary non-neutrality when $\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}$ is higher. This comes from Proposition 3. On the other hand, for a given level of firms' attention to monetary shocks, monetary non-neutrality is lower when the extent of complementarities is lower. This result comes from equation (16). Our model calibrated to all bins in the PPI data implies the following: Aggregate prices absorb 23% of the aggregate shock on impact, there are almost no real effects after only 6 periods, and the cumulated response of prices is 84% that of frictionless prices.

5.7 Calibrating Information Capacity

One key parameter in our model is the total information capacity $\kappa(N)$. So far, we have pinned it down by imposing a constant frictional cost of 0.21% per good in terms of expected profit losses. We now depart from this assumption: Since a direct attempt at calibrating $\kappa(N)$ reveals a high insensitivity to moments from the micro data, we instead use the shadow value of information-procession capacity to pin down $\kappa(N)$. As an alternative to calibrate the information friction, we also target the menu cost friction in Midrigan (2011).

First, we attempt to calibrate $\kappa(N)$ directly from the data by targeting an additional moment: We choose the total cross-sectional dispersion of absolute log price changes in the CPI sample. We then take our model from section 5.4 for $N = 2$ as well as $N = 4$ and solve it for a grid of $\kappa(N)$. We report results in Tables 4 and 5.[18] We find that we are unable to pin down $\kappa(N)$ through this approach. The reason is that the predictions of the model regarding moments that can be contrasted with the micro data are highly insensitive to changes in $\kappa(N)$. At the same time, the predictions regarding monetary non-neutrality are highly sensitive to changes in $\kappa(N)$. This invalidates this approach of calibrating $\kappa(N)$. While we could relax the assumption of Gaussian shocks to match our additional moment, this would would add new parameters to calibrate and would not solve the problem illustrated here that prevents us from calibrating $\kappa(N)$.

Indeed, as the above suggests, the trade-off between monetary non-neutrality and the severity of the friction is quite substantial: When we calibrate our model from section 5.4 to the PPI moments of the median four-good firm, we find that an increase of monetary non-neutrality by a

[18]Similar results hold when we try to pin down the lagrange multiplier $\lambda(N)$ this way, so we choose not to display these tables.

factor of 2 (3) is associated with an increase in the friction by a factor of approximately 2 (3) as well. We illustrate this trade-off for a wider range of the friction in Figure 5. Monetary non-neutrality is monotonically increasing in the friction.

For these reasons, we choose to pin down $\kappa(N)$ by keeping the shadow value of information capacity constant across goods. That is, we keep the Lagrange multiplier on the capacity constraint (9) constant across goods: $\lambda(N) = \bar{\lambda}$. We implement this in our model from section 5.4, again targeting an average of log absolute price changes of 9.6%, a serial correlation of -0.291 and a within-firm variance ratio of 51.6%. We find that the cumulated price response lies in between 52% and 83% of the frictionless price response while the friction ranges between 0.20% and 0.24%. Monetary neutrality is extremely high. It increases with the number of goods, exactly as predicted by our model.

As an alternative to pin down the information capacity, we force our model of section 5.5 to generate the same frictional cost as in the two-good menu cost model of Midrigan (2011) calibrated using the distribution of prices for a given store. There, the cost of the friction is 0.34% of steady state revenues. If we force our model, calibrated to PPI data, to generate this level of cost, aggregate prices absorb 8.44% of the shock on impact. The deviation of output then is less than 5% of the shock after 16 periods, and the cumulated response of aggregate prices is 48% that of frictionless prices.

6 Conclusion

In this paper, we have explored the impact of monetary policy on the real economy in a model of rational inattention. In our model, which accounts for the multi-product nature of firms, the real impact of monetary policy is much lower than in a less realistic model in which firms produce only one good. This result is due to economies of scope in information processing: As firms produce more goods, the return to gathering information on common monetary, rather than good-specific, shocks increases. When we calibrate our multi-product firm model to US CPI data, we find that monetary policy has minimal real effects. Calibrating our model to PPI data, in which firms price a much smaller number of goods, suggests only limited non-neutrality and aggregate inertia.

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Tables and Figures

Table 1: Multi-Product Firms and Moments from CPI and PPI data

CPI	1-3 Goods	3-5 Goods	5-7 Goods	>7 Goods	All
# goods, mean	1.47	3.89	6.02	10.82	2.05
# goods, median	1.00	3.85	6.00	9.00	1.39
Absolute size of price changes	10.87%	11.64%	11.69%	12.55%	11.01%
	(0.03%)	(0.09%)	(0.15%)	(0.11%)	(0.03%)
Within variance ratio of $\|\Delta p\|$	20.9%	55.8%	62.8%	79.0%	51.6%
	(0.3%)	(0.4%)	(0.4%)	(0.4%)	(0.6%)
Cross-sectional variance	1.93%	2.65%	3.60%	2.85%	2.65%
	(0.52%)	(0.70%)	(0.89%)	(0.50%)	(0.31%)
Serial correlation	-0.248	-0.307	-0.334	-0.355	-0.291
	(0.0008)	(0.0013)	(0.0022)	(0.0015)	(0.0006)
PPI					
# goods, mean	2.19	4.02	6.03	10.25	4.13
# goods, median	2	4	6	8	4
Absolute size of price changes	8.5%	7.9%	6.8%	6.5%	7.8%
	(0.13%)	(0.09%)	(0.14%)	(0.16%)	(0.10%)
Within variance ratio of $\|\Delta p\|$	36.5%	54.6%	67.2%	72.4%	59.1%
	(0.7%)	(0.6%)	(0.8%)	(1.0%)	(0.6%)
Cross-sectional variance	3.72%	3.60%	2.91%	3.64%	3.51%
	(0.20%)	(0.19%)	(0.15%)	(0.22%)	(0.10%)
Serial correlation	$-.050$	$-.057$	$-.033$	$-.032$	$-.043$
	(0.0024)	(0.0002)	(0.0001)	(0.0001)	(0.0001)
Share of total employment	25.0%	27.7%	16.0%	31.3%	100%

NOTE: We compute the above statistics using the monthly micro price data underlying the PPI and CPI. The time periods are from 1998 through 2005, and 1998 through 2009, respectively. We compute all statistics for firms with less than 3 goods (bin 1), with 3-5 goods (bin 2), with 5-7 goods (bin 3), >7 goods (bin 4), and the full sample. First, we compute the time-series mean of the number of goods per firm. We then report the mean (median) number of goods across all firms. Second, we start by computing the time-series mean of the absolute value of log price changes for each good in a firm. We take the median across goods within each firm, then report means across firms. Standard errors across firms are given in brackets. Third, we compute the monthly within variance ratio as the ratio of two statistics: first, the sum of squared deviations of the absolute value of individual log price changes from their average within each firm, summed across firms; second, the sum of squared deviations of the absolute value of individual log price changes from their cross-sectional average. We then report the time-series mean. Standard errors across monthly means are given in brackets. Fourth, we estimate the first-order auto-correlation coefficient of non-zero price changes using a median quantile regression. Fifth, we compute the monthly cross-sectional variance of absolute log price changes and then report standard errors of this monthly statistic. Finally, we compute the share of employment relative to total employment in each category at the time of re-sampling in 2005.

Table 2: Multi-Product Firms and Within-Firm Variance Ratio, Robustness

CPI	1-3 Goods	3-5 Goods	5-7 Goods	>7 Goods	All
Within variance ratio of Δp					
Mean	8.8%	32.8%	45.6%	64.7%	35.9%
Median	9.2%	32.7%	44.5%	62.0%	35.2%
Std. Error	(0.2%)	(0.3%)	(0.4%)	(0.5%)	(0.6%)
PPI					
Within variance ratio of Δp					
Mean	18.4%	31.2%	44.3%	54.0%	38.1%
Median	18.1%	30.1%	44.4%	53.3%	37.4%
Std. Error	(0.7%)	(0.9%)	(1.1%)	(1.0%)	(0.7%)

NOTE: We compute the above statistics using the monthly micro price data underlying the PPI and CPI. The time periods are from 1998 through 2005, and 1998 through 2009, respectively. We compute all statistics for firms with less than 3 goods (bin 1), with 3-5 goods (bin 2), with 5-7 goods (bin 3), >7 goods (bin 4), and the full sample. We compute the monthly within variance ratio as the ratio of two statistics: first, the sum of squared deviations of the individual log price changes, including zeros, from their average within each firm, summed across firms; second, the sum of squared deviations of individual log price changes, including zeros, from their cross-sectional average. We then report the time-series mean and medians. Standard errors across monthly means are given in brackets.

Table 3: Moments from the PPI and the Model

	1-3 Goods	3-5 Goods	5-7 Goods	>7 Goods	All
Absolute size of price changes, data	8.5%	7.9%	6.8%	6.5%	7.8%
Absolute size of price changes, model	*8.5%*	*8.5%*	*8.5%*	*8.5%*	*8.5%*
Serial correlation, data	$-.050$	$-.057$	$-.033$	$-.032$	$-.043$
Serial correlation, model	$-.050$	$-.050$	$-.050$	$-.050$	$-.050$
Within-firm variance ratio, data	36.5%	54.6%	67.2%	72.4%	59.1%
Within-firm variance ratio, model	*36.5%*	*54.5%*	*60.5%*	*63.5%*	*53.8%*

NOTE: We report moments predicted by the model in section 5.5 in italics. We contrast them with the moments from the data presented in Table 1.

Table 4: Moments from the CPI and the Model, N=2

	data	$\kappa = 5$	$\kappa = 6$	$\kappa = 7$	$\kappa = 8$	$\kappa = 9$	$\kappa = 10$	$\kappa = 30$
absolute size of price changes	9.6%	9.61%	9.65%	9.67%	9.70%	9.70%	9.73%	9.75%
serial correlation	-0.29	-0.291	-0.290	-0.290	-0.290	-0.289	-0.288	-0.289
within-firm var. ratio	51.6%	50.12%	50.04%	50.01%	50.01%	50.01%	50.04%	50.15%
cross-sectional variance	2.65%	7.22%	7.28%	7.31%	7.32%	7.33%	7.34%	7.36%
$\kappa_a^*(2)$		0.219	0.309	0.473	0.676	0.920	1.212	8.123
cumulated price response		51.67%	57.82%	71.97%	80.73%	86.14%	90.02%	97.98%
(relative to frictionless prices)								

NOTE: As discussed in section 5.7, the table shows moments computed from the data and their counterparts generated by the model for N=2 using different values for firms' capacity to process information.

Table 5: Moments from the CPI and the Model, N=4

	data	$\kappa = 10$	$\kappa = 11$	$\kappa = 12$	$\kappa = 13$	$\kappa = 14$	$\kappa = 15$	$\kappa = 30$
absolute size of price changes	9.60%	9.50%	9.54%	9.58%	9.60%	9.62%	9.66%	9.74%
serial correlation	-0.291	-0.292	-0.2908	-0.291	-0.2911	-0.2901	-0.2895	-0.2893
within-firm var. ratio	51.60%	50.99%	51.27%	51.53%	51.77%	51.85%	51.91%	52.11%
cross-sectional variance	2.65%	7.16%	7.21%	7.24%	7.25%	7.28%	7.29%	7.35%
$\kappa_a^*(4)$		0.31	0.37	0.44	0.52	0.62	0.72	3.31
cumulated price response		60.17%	64.74%	70.50%	75.32%	79.33%	82.60%	98.46%
(relative to frictionless prices)								

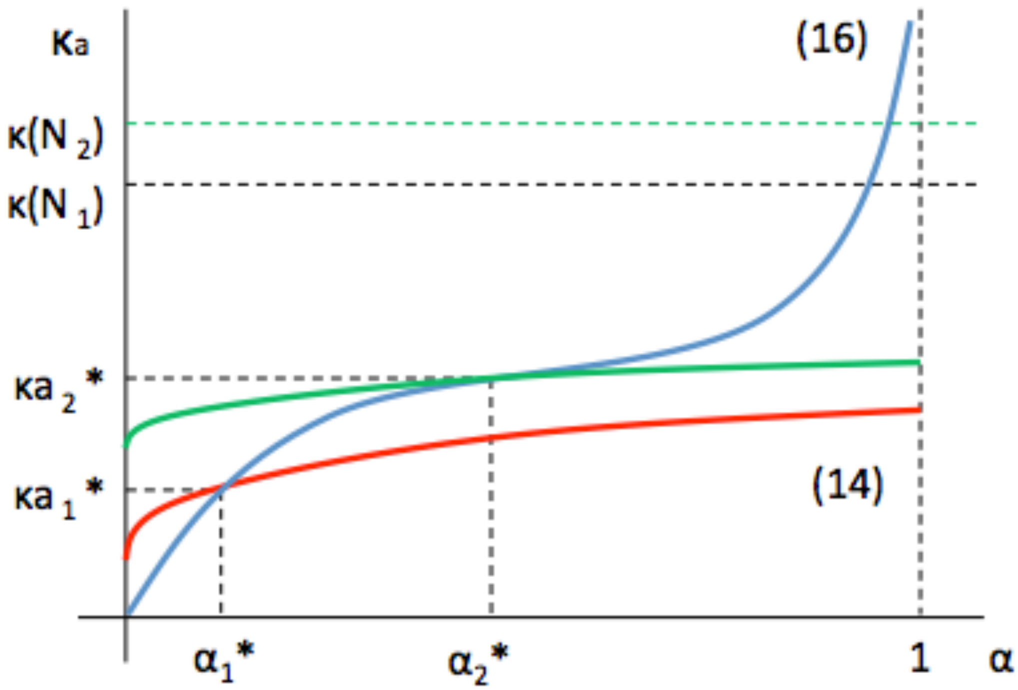
NOTE: As discussed in section 5.7, the table shows moments computed from the data and their counterparts generated by the model for N=2 using different values for firms' capacity to process information.

Table 6: Value of Information Capacity and the Number of Goods

	$N=1$	$N=2$	$N=4$	$N=8$
$\lambda(N)$	3.3348	3.3348	3.3348	3.3348
absolute size of price changes	9.62%	9.60%	9.60%	9.60%
serial correlation	-0.291	-0.291	-0.291	-0.291
within-firm variance ratio	0.00%	50.12%	51.59%	51.58%
cross-sectional variance	7.26%	7.25%	7.23%	7.25%
$\kappa_a(N)$	0.1935	0.2606	0.4429	0.6867
cumulated price response	51.81%	53.48%	72.05%	82.70%
(relative to frictionless prices)				
loss	0.21%	0.20%	0.24%	0.21%

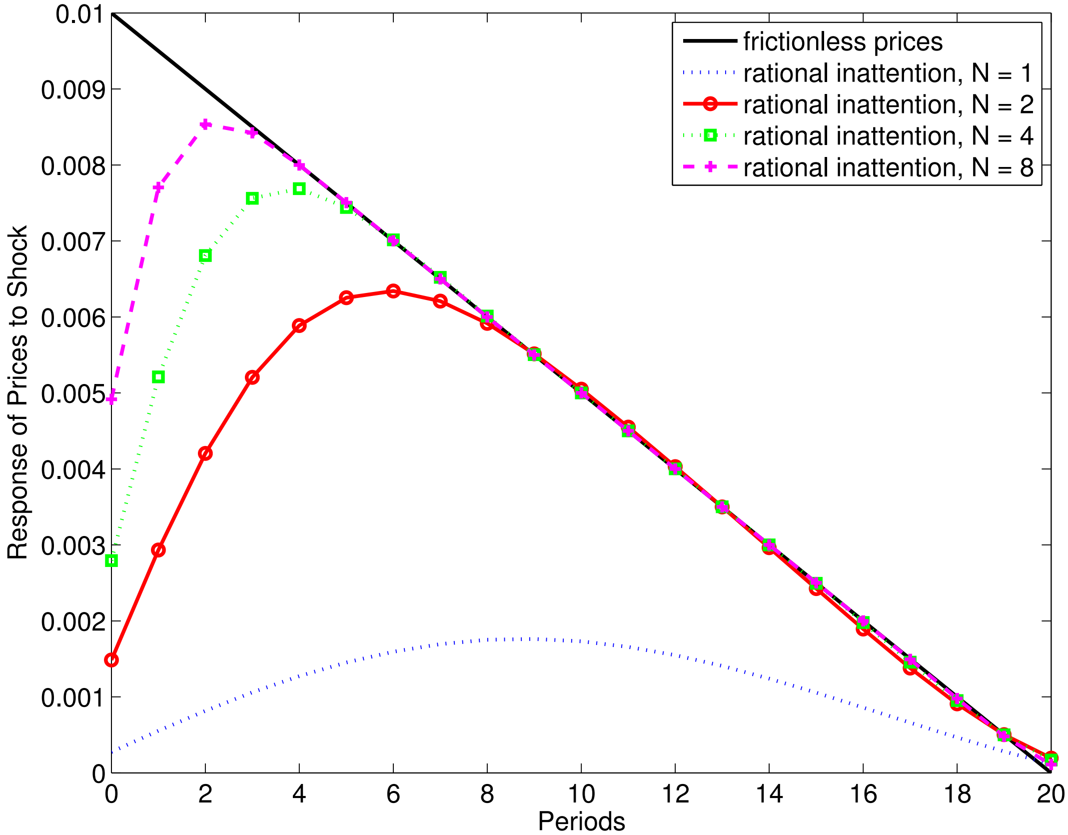
NOTE: We calibrate our model with homogeneous firms to moments for the whole sample of CPI data as we vary N. Firms' information processing capacity is calibrated such that its shadow price is invariant to N.

Figure 1: Equations (14) and (16) in the space (α, κ_a)



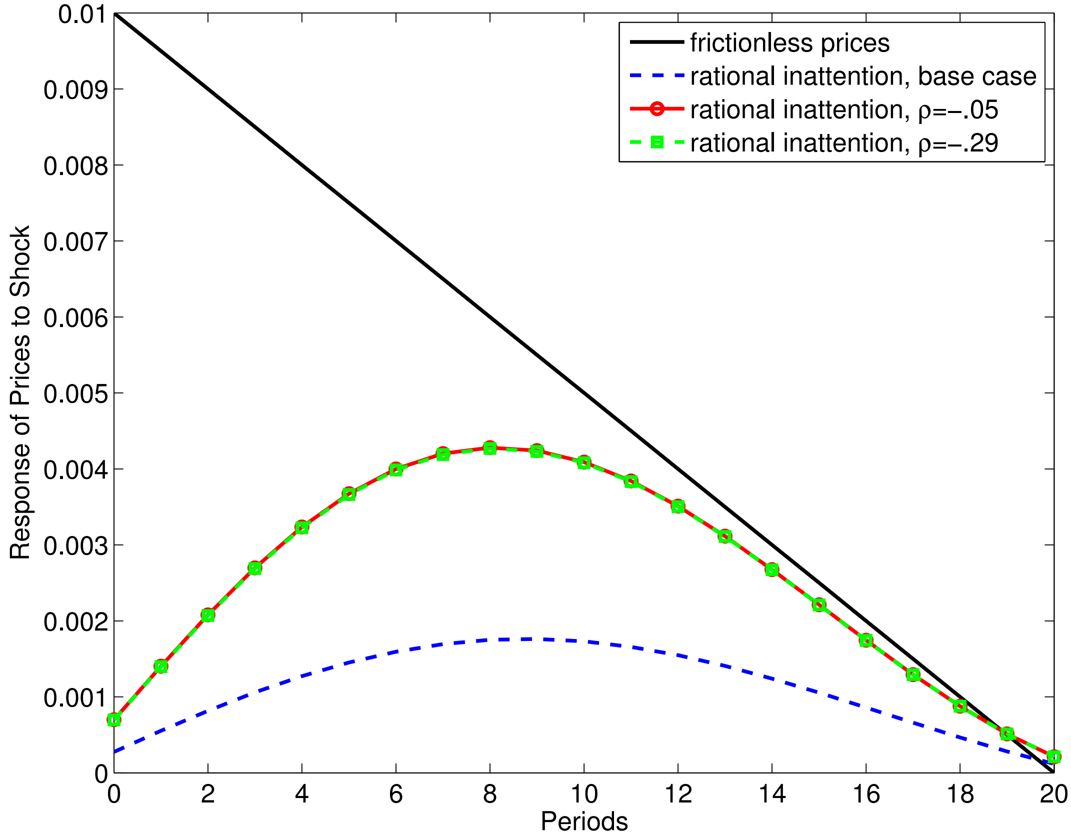
NOTE: The figure illustrates the fixed point problem of attention allocation given by equations (14) and (16). Equation (14) is drawn in red, while equation (16) is drawn in blue. Equation (16) is invariant to N, but N affects the drift and slope of equation (14). Under conditions described in Proposition 2 the drift of equation (14) is increasing in N. An upwards shift of this function is represented in green.

Figure 2: Response of prices to a 1% impulse in q_t for sections 5.1 and 5.2



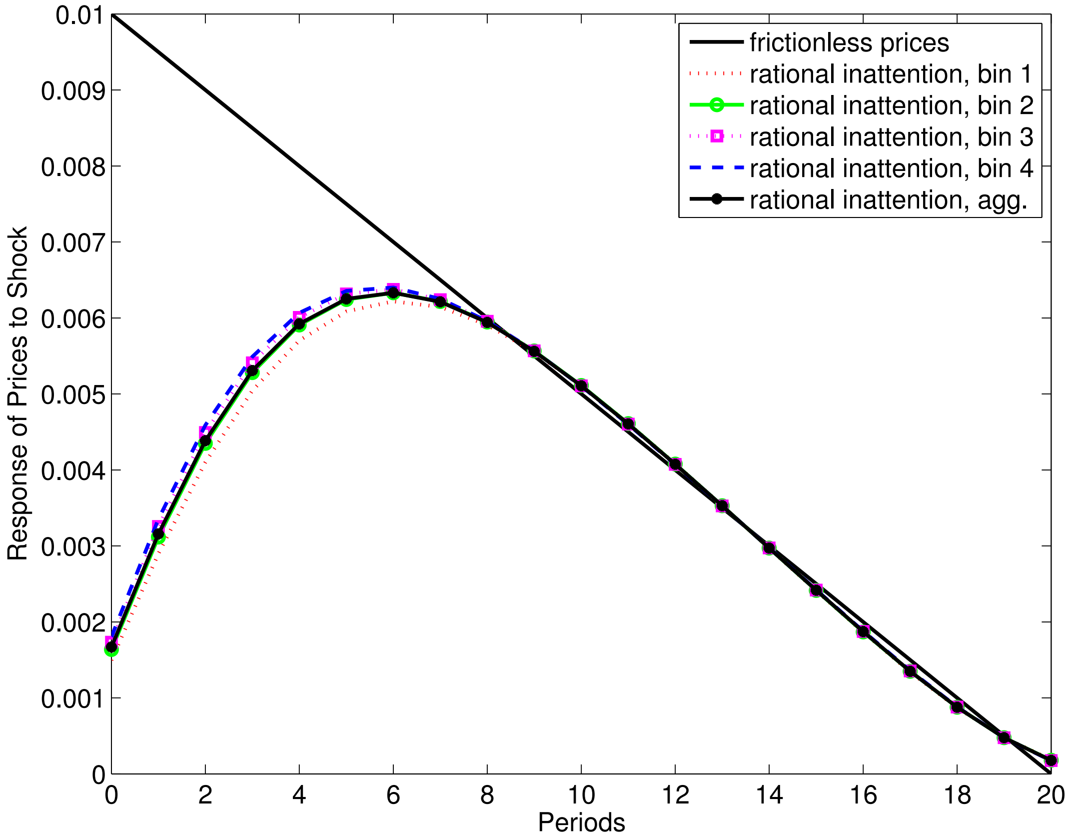
NOTE: We illustrate the response of prices to a 1% monetary shock as we vary N in our model calibrated to moments from the CPI data. The black line is for frictionless prices, the dashed blue line is for the benchmark of rationally inattentive prices with N=1, the red line with circles is for rationally inattentive prices with N=2, the dashed green line with squares is for rationally inattentive prices with N=4, and the dashed magenta line with dots is for is for rationally inattentive prices with N=8. The response of prices quickly becomes closer to that of frictionless prices as N increases. Details are given in sections 5.1 and 5.2.

Figure 3: Response of prices to a 1% impulse in q_t for section 5.3



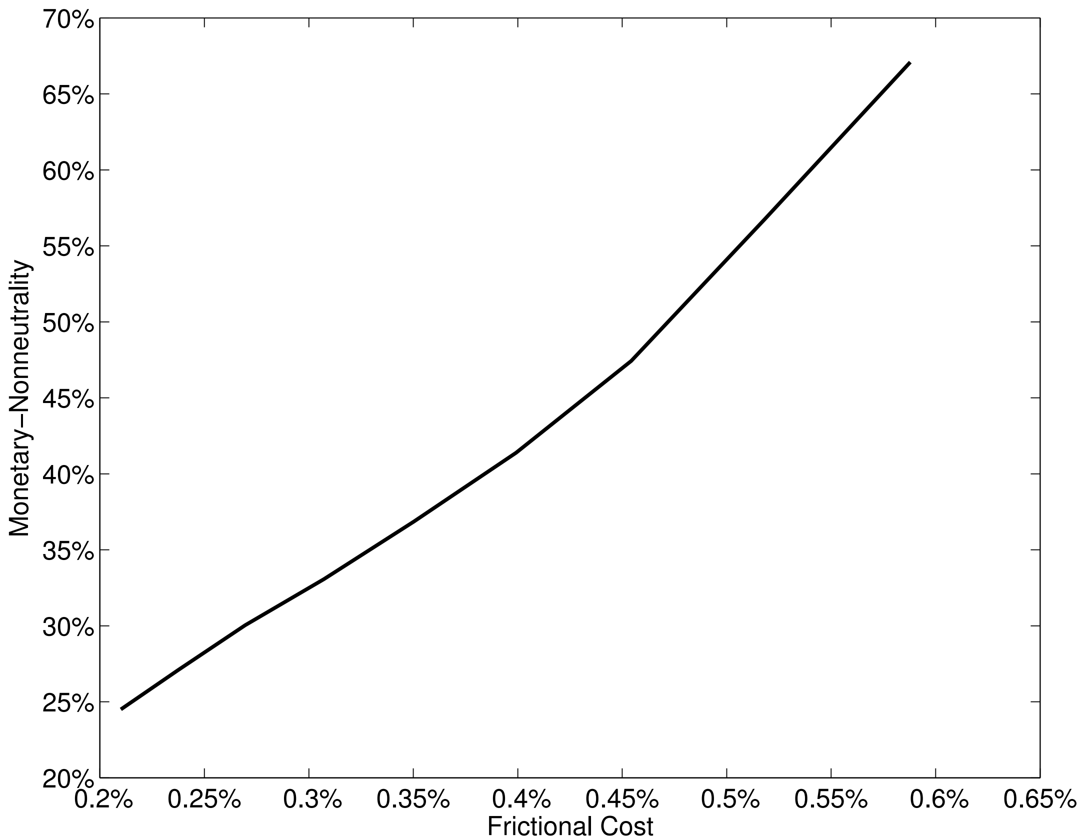
NOTE: We illustrate the response of prices to a 1% monetary shock as we vary the persistence of idiosyncratic shocks in our model calibrated to moments from the CPI data. The black line is for frictionless prices, the dashed blue line is for our benchmark with highly persistent idiosyncratic shocks, the red line with circles is for rationally inattentive prices that have serial correlation of -0.05, the dashed green line with squares is for rationally inattentive prices that have serial correlation of -0.29. Section 5.3 contains further details.

Figure 4: Response of prices to a 1% impulse in q_t for section 5.5



NOTE: We illustrate the response of prices to a 1% monetary shock as we vary N in our model calibrated to moments of the PPI data by bins. The black line is for frictionless prices, the dashed blue line is for rationally inattentive prices in bin 1, the red line with circles is for rationally inattentive prices in bin 2, the dashed green line with squares is for rationally inattentive prices in bin 3, and the dashed magenta line with dots is for is for rationally inattentive prices in bin 4, and the black solid line with dots is for aggregate rationally inattentive prices. Section 5.5 contains further details.

Figure 5: Trade-Off between Monetary Non-Neutrality and Frictional Cost



NOTE: We illustrate the relationship between monetary non-neutrality, measured as the cumulative response of rational inattentive prices relative to frictionless prices, and the frictional cost of as we vary firms' information processing capacity in our model calibrated to moments of the PPI data. Section 5.7 contains further details.

A APPENDIX: The Problem of the Firm with White Noise Shocks

We start by computing the frictionless non-stochastic steady state in this economy. Let $\bar{Q}$, $\bar{F}_i = \bar{F}$ $\forall i$ and $\bar{Z}_j = \bar{Z}$ $\forall j$ be the steady state level of these variables. Without frictions, it must hold that

$$\pi_1 \left(1, 1, Y_t, \bar{F}, \bar{Z}\right) = 0,$$

which follows from the optimality of prices. This equation solves for the steady-state level of real aggregate demand $\bar{Y}$, and equation (4) for the steady-state aggregate price level $\bar{P} = \bar{Q}/\bar{Y}$.

A second-order approximation of the problem of firm i around this steady-state is

$$\max_{\{p_{nt}\}_{n \in \aleph_i}} \sum_{n \in \aleph_i} \left\{ \begin{array}{c} \widehat{\pi}_1 p_{nt} + \frac{\widehat{\pi}_{11}}{2} p_{nt}^2 + \widehat{\pi}_{12} p_{nt} p_t + \widehat{\pi}_{13} p_{nt} y_t + \widehat{\pi}_{14} p_{nt} f_{it} + \widehat{\pi}_{15} p_{nt} z_{nt} \\ + \, terms \ independent \ of \ p_{nt}. \end{array} \right\}$$

with $\widehat{\pi}_1 = 0$, $\widehat{\pi}_{11} < 0$, $\widehat{\pi}_{12} = -\widehat{\pi}_{11}$ and all parameters identical for all goods and all firms.

The optimal frictionless pricing rule for each good $n \in \aleph_i$ for all i is

$$p_{nt}^{\diamondsuit} = p_t + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} y_t + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} f_{it} + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} z_{nt} \equiv \Delta_t + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} f_{it} + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} z_{nt} \tag{20}$$

where the compound variable Δ_t collects aggregate variables.

Since this is a linear pricing rule, the optimal price of good $n \in \aleph_i$ of an arbitrary firm i that solves (8) is

$$p_{nt}^{*} = \mathbb{E}\left[\Delta_t \mid s_{it}^{a}\right] + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} \mathbb{E}\left[f_{it} \mid s_{nt}^{f}\right] + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} \mathbb{E}\left[z_{jt} \mid s_{nt}^{z}\right]. \tag{21}$$

given the signal structure $\left\{s_{it}^{a}, s_{it}^{f}, s_{nt}^{z}\right\}$. We must solve now for firms' optimal choice of signals. To do so, we recast the firms' problem up to second order as the minimization the discounted sum of firms' expected loss in profits due to the friction (the "frictional costs" hereafter) for all goods produced by the firm:

$$\sum_{t=1}^{\infty} \beta^t \sum_{n \in \aleph_i} \left\{ \frac{|\widehat{\pi}_{11}|}{2} \mathbb{E}\left[\left(p_{nt}^{\diamondsuit} - p_{nt}^{*}\right)^2\right] \right\} \tag{22}$$

We assume now that shocks q_t, f_{it} and z_{jt} are white noise, with variances σ_q^2, σ_f^2 for any firm $i \in \left[0, \frac{1}{N}\right]$ and σ_z^2 for any good $j \in [0, 1]$. This assumption allows us to obtain analytical solution.[19]

Given this assumption, we guess that the log-deviation of aggregate prices respond linearly to a monetary shock, $p_t = \alpha q_t$, so the compound aggregate variable Δ_t is given by

$$\Delta_t = \left[\alpha + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} (1 - \alpha)\right] q_t. \tag{23}$$

[19] The appendix relaxes this assumption and presents the numerical algorithm used to solve for it. We use this general problem to obtain our quantitative results in Section 5.

In addition, signals chosen by firm $i \in \left[0, \frac{1}{N}\right]$ are restricted to have the structure

$$\begin{aligned} s_{it}^{a} &= \Delta_t + \varepsilon_{it}, \\ s_{it}^{f} &= f_{it} + e_{it}, \\ s_{nt}^{z} &= z_{nt} + \psi_{nt}, \end{aligned}$$

where $\sigma_{\varepsilon i}^2$, σ_{ei}^2 and $\sigma_{\psi n}^2$ are the variance of noise ε_{it}, e_{it} and ψ_{nt}.[20]

Therefore, given signals $\left\{s_{it}^{a}, s_{it}^{f}, s_{nt}^{z}\right\}$, the optimal pricing rule (21) solves as

$$p_{nt}^{*} = \frac{\sigma_{\Delta}^2}{\sigma_{\Delta}^2 + \sigma_{\varepsilon i}^2} s_{it}^{a} + \frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|} \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{ei}^2} s_{it}^{f} + \frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|} \frac{\sigma_z^2}{\sigma_z^2 + \sigma_{\psi n}^2} s_{nt}^{z}.$$

Replacing $p_{nt}^{\diamondsuit}$ and p_{nt}^{*} in (22) and using the functional form of information flow in (6) because shocks are Gaussian white noise, the problem of the firm becomes

$$\min_{\kappa_a, \kappa_f, \{\kappa_n\}_{n \in \aleph_i}} \frac{\beta}{1-\beta} \frac{|\widehat{\pi}_{11}|}{2} \left[2^{-2\kappa_a} \sigma_{\Delta}^2 N + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}}\right)^2 2^{-2\kappa_f} \sigma_f^2 N + \left(\frac{\widehat{\pi}_{15}}{\widehat{\pi}_{11}}\right)^2 \sum_{n \in \aleph_i} 2^{-2\kappa_n} \sigma_z^2 \right] \tag{24}$$

subject to

$$\kappa_a + \kappa_f + \sum_{n \in \aleph_i} \kappa_n \leq \kappa(N). \tag{25}$$

where

$$\begin{aligned} \kappa_a &\equiv \frac{1}{2} \log_2 \left(\frac{\sigma_{\Delta}^2}{\sigma_{\varepsilon i}^2} + 1 \right); \\ \kappa_f &\equiv \frac{1}{2} \log_2 \left(\frac{\sigma_f^2}{\sigma_{ei}^2} + 1 \right); \\ \kappa_n &= \log_2 \left(\frac{\sigma_z^2}{\sigma_{\psi n}^2} + 1 \right). \end{aligned}$$

B APPENDIX: The Problem of the Firm for a General Structure of Shocks

This appendix displays the analytical representation of firms' problem in the setup of section 5 and explains the numerical algorithm applied to solve it. This appendix adapts to our setup a similar presentation by Mackowiak and Wiederholt (2007). Assume that firms are exposed to three types of shocks:

$$q_t = \sum_{l=0}^{\infty} a_l v_{t-l},$$

[20] Mackowiak and Wiederholt (2009) show that this structure of signals is optimal. This result is not affected by the modifications to their model introduced here.

$$f_{it} = \sum_{l=0}^{\infty} b_l \xi_{t-l},$$

$$z_{jt} = \sum_{l=0}^{\infty} c_l \zeta_{t-l},$$

where q_t is a nominal aggregate demand shock (interpreted as a "monetary" shock), f_{it} is a shock idiosyncratic to each firm $i \in \left[0, \frac{1}{N}\right]$, z_{jt} is a shock idiosyncratic to each good $j \in [0,1]$, and $\{v_{t-l}, \xi_{t-l}, \zeta_{t-l}\}_{l=0}^{\infty}$ are innovations following Gaussian independent processes.

We guess that the log-deviation of aggregate prices follows

$$p_t = \sum_{l=0}^{\infty} \alpha_l v_{t-l}$$

which, given the definition of Δ_t in (20) and $y_t = q_t - p_t$, implies

$$\Delta_t = \left(1 - \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|}\right) \sum_{l=0}^{\infty} \alpha_l v_{t-l} + \frac{\widehat{\pi}_{13}}{|\widehat{\pi}_{11}|} \sum_{l=0}^{\infty} a_l v_{t-l} \equiv \sum_{l=0}^{\infty} d_l v_{t-l} \tag{26}$$

The problem of firm $i \in \left[0, \frac{1}{N}\right]$ has two stages. In the first stage, firms must choose conditional expectations for Δ_t , f_{it} and $\{z_{nt}\}_{n \in \aleph_i}$ to minimize the deviation of prices with respect to frictionless optimal prices subject to the information capacity constraint:

$$\min_{\hat{\Delta}_{it}, \{\hat{z}_{nt}\}_{n \in \aleph_i}} \sum_{n \in \aleph_i} \left\{ \sum_{t=1}^{\infty} \beta^t \frac{|\widehat{\pi}_{11}|}{2} \mathbb{E}\left[\left(p_{nt}^{\diamondsuit} - p_{nt}^{*}\right)^2\right] \right\}$$

which is equivalent to

$$\min_{\hat{\Delta}_{it}, \hat{f}_{it}, \{\hat{z}_{nt}\}_{n \in \aleph_i}} \left\{ \begin{array}{c} \mathbb{E}\left[\left(\Delta_t - \widehat{\Delta}_{it}\right)^2\right] N + \left(\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\right)^2 \mathbb{E}\left[\left(f_{it} - \widehat{f}_{it}\right)^2\right] N \\ + \left(\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\right)^2 \sum_{n \in \aleph_i} \mathbb{E}\left[\left(z_{nt} - \widehat{z}_{nt}\right)^2\right] \end{array} \right\}$$

subject to the process of Δ_t, f_{it} and $\{z_{nt}\}_{n \in \aleph_i}$ and the information capacity constraint

$$I\left(\Delta_t, \widehat{\Delta}_{it}\right) + I\left(f_{it}, \widehat{f}_{it}\right) + \sum_{n \in \aleph_{we}} I\left(z_{nt}, \widehat{z}_{nt}\right) \leq \kappa\left(N\right).$$

The function $I\left(\cdot\right)$ is the information flow. For instance, this function for Δ_t takes the form:

$$I\left(\Delta_t, \widehat{\Delta}_{it}\right) \equiv -\frac{1}{4\pi} \int_{-\pi}^{\pi} \log_2 \left[1 - C_{\Delta_t, \widehat{\Delta}_{it}}\left(\omega\right)\right] d\omega$$

where $C_{\Delta_t, \widehat{\Delta}_{it}}\left(\omega\right)$ is called coherence function, which is defined as follows. Let describe the conditional expectations $\widehat{\Delta}_{it}$ as

$$\widehat{\Delta}_{it} = \sum_{l=0}^{\infty} g_l v_{t-l} + \sum_{l=0}^{\infty} h_l \varepsilon_{t-l},$$

then

$$C_{\Delta,\widehat{\Delta}_{we}}(\omega) \equiv \frac{\frac{G(e^{-i\omega})G(e^{i\omega})}{H(e^{-i\omega})H(e^{i\omega})}}{\frac{G(e^{-i\omega})G(e^{i\omega})}{H(e^{-i\omega})H(e^{i\omega})}+1},$$

where $G\left(e^{i\omega}\right) = g_0 + g_1 e^{i\omega} + g_2 e^{i2\omega} + ...$ and $H\left(e^{i\omega}\right) = h_0 + h_1 e^{i\omega} + h_2 e^{i2\omega} + ...$

If the conditional expectations $\widehat{f}_{it}$ and $\{\widehat{z}_{nt}\}_{n\in\aleph_i}$ are described by

$$\begin{aligned}
\widehat{f}_{it}^{*} &= \sum_{l=0}^{\infty} r_l \xi_{t-l} + \sum_{l=0}^{\infty} s_l \varepsilon_{t-l}, \\
\widehat{z}_{nt}^{*} &= \sum_{l=0}^{\infty} w_{nl} \zeta_{t-l} + \sum_{l=0}^{\infty} x_{nl} e_{nt-l} \text{ for } n \in \aleph_i.
\end{aligned}$$

Then the problem may be represented as

$$\min_{g,h,r,s,\{w_n,x_n\}_{n\in\aleph_i}} \left\{ \begin{array}{c} \left[\sum_{l=0}^{\infty}(d_l-g_l)^2 + \sum_{l=0}^{\infty} h_l^2\right] N + \left(\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\right)^2 N \left[\sum_{l=0}^{\infty}(b_l-r_l)^2 + \sum_{l=0}^{\infty} s_l^2\right] \\ + \left(\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\right)^2 \sum_{n\in\aleph_{we}} \left[\sum_{l=0}^{\infty}(c_l-w_{nl})^2 + \sum_{l=0}^{\infty} x_{nl}^2\right] \end{array} \right\}$$

$$s.t. \;\; I\left(\Delta_t, \widehat{\Delta}_{it}\right) + I\left(f_{it}, \widehat{f}_{it}\right) + \sum_{n\in\aleph_{we}} I\left(z_{nt}, \widehat{z}_{nt}\right) \leq \kappa(N)$$

where $g, h, r, s, \{w_n, x_n\}_{n\in\aleph_i}$ represent vectors of coefficients. The first order conditions for g and h are

$$\begin{aligned}
g_l &: \; 2\left(d_l^* - g_l^*\right) N = -\frac{\mu_a}{4\pi\log(2)} \int_{-\pi}^{\pi} \frac{\partial \log\left[1 - C_{\Delta,\widehat{\Delta}_{we}^*}(\omega)\right]}{\partial g_l} d\omega, \\
h_l &: \; 2h_l^* N = \frac{\mu_a}{4\pi\log(2)} \int_{-\pi}^{\pi} \frac{\partial \log\left[1 - C_{\Delta,\widehat{\Delta}_{we}^*}(\omega)\right]}{\partial h_l} d\omega
\end{aligned}$$

where μ_a is the Lagrangian multiplier. Similar conditions must be satisfied by r^* and s^* and by $\{w_n^*, x_n^*\}_{n\in\aleph_i}$ but without N.

The second stage of the problem is to obtain optimal signals structures that deliver $\widehat{\Delta}_{it}^* = \widehat{\Delta}_{it}\left(\kappa_a^*(N), N\right)$ and $\widehat{z}_{nt}^* = \widehat{z}_{nt}\left(\kappa_n(N), N\right)$. Since we are interested in the aggregate implications of the model, we do not solve this part.

Numerically, we truncate the memory of all processes to 20 lags, which is the same order assumed for the MA process for q_t. Then we start from a guess for α to compute d, we find $g^*, h^*, r^*, s^*, \{w_n, x_n\}_{n\in\aleph_i}$ by using the Levenberg-Marquardt algorithm to solve the system of first-order conditions plus the information flow constraint after imposing symmetry in $\{w_n, x_n\}_{n\in\aleph_i}$. With these vectors, we compute $I\left(\Delta_t, \widehat{\Delta}_{it}\right) = \kappa_a^*(N)$, $I\left(f_{it}, \widehat{f}_{it}\right) = \kappa_f^*(N)$ and $I(z_{nt}, \widehat{z}_{nt}) = \kappa_z^*(N)$ and the vector α. We use this α as guess for a new iteration upon convergence on α.

C APPENDIX: Extensions

This appendix relaxes some expositional assumptions made in the set up studied in the main text. These extensions yield no substantive changes to our conclusions or counterfactual predictions.

Common Signals

In the main text we have assumed that there exists an independent signal for each good-specific idiosyncratic shock. We relax this assumption and instead we assume that there exists a signal

$$s_{it}^z = \sum_{n\in\aleph_i} z_{nt} + \psi_{it}.$$

In words, firms receive only one common signal regarding all its good-specific shocks. Under this assumption, we are in the same situation as in Proposition 1, where firms' attention to aggregate shocks is inviariant in the number of goods that this firm produces, but its prices perfectly comove. This latter result is clear from observing the form of optimal prices under rational inattention:

$$p_{nt}^* = \frac{\sigma_\Delta^2}{\sigma_\Delta^2 + \sigma_{\varepsilon i}^2} s_{it}^a + \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{ei}^2} s_{it}^f + \frac{\sigma_z^2}{N\sigma_z^2 + \sigma_{\psi i}^2} s_{it}^z$$

which only responds to aggregate and firm-specific components.

Persistent shocks

We now solve for a simplified version of our model that allows for persistent shocks and keeps at least partial closed solution. Assume that the process of q_t is such that Δ_t is $AR(1)$ with persistency ρ_Δ. Idiosyncratic shocks f_{it} and z_{jt} are also $AR(1)$ respectively with persistency ρ_f for all i and ρ_z for all j. The starting guess is now

$$p_t = \sum_{l=0}^{\infty} \alpha_l v_{t-l}, \tag{27}$$

where $\{v_{t-l}\}_{l=0}^{\infty}$ is the history of nominal aggregate demand innovations.

The firms' problem may be cast in two stages. In the first stage, firms choose

$$\min_{\hat{\Delta}_{it},\{\hat{z}_{nt}\}_{n\in\aleph_i}} \sum_{n\in\aleph_i} \left\{ \sum_{t=1}^{\infty} \beta^t \frac{|\widehat{\pi}_{11}|}{2} \mathbb{E}\left[\left(p_{nt}^{\diamondsuit} - p_{nt}^*\right)^2\right] \right\}$$

$$\to \min_{\hat{\Delta}_{it},\{\hat{z}_{nt}\}_{n\in\aleph_i}} \frac{\beta}{1-\beta} \frac{|\widehat{\pi}_{11}|}{2} \left\{ \begin{array}{c} \mathbb{E}\left(\Delta_t - \hat{\Delta}_{it}\right)^2 N + \left(\frac{\widehat{\pi}_{14}}{|\widehat{\pi}_{11}|}\right)^2 \mathbb{E}\left(f_{it} - \hat{f}_{it}\right)^2 N \\ + \left(\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}|}\right)^2 \sum_{n\in\aleph_i} \mathbb{E}\left(z_{nt} - \hat{z}_{nt}\right)^2 \end{array} \right\}$$

subject to

$$\begin{array}{rcl} I\left(\left\{\Delta_t, \hat{\Delta}_{it}\right\}\right) & \leq & \kappa_a, \\ I\left(\left\{f_{it}, \hat{f}_{it}\right\}\right) & \leq & \kappa_f, \\ I\left(\left\{z_{nt}, \hat{z}_{nt}\right\}\right) & \leq & \kappa_n, \ \ for\ n \in \aleph_i \\ \kappa_a + \sum_{n\in\aleph_{ie}} \kappa_n & \leq & \kappa(N) \end{array}$$

For the second stage, firms choose the signals that deliver $\hat{\Delta}_{it}^*, \{\hat{z}_{nt}^*\}_{n\in\aleph_{we}}$. As in Appendix B, we omit this stage. Our representation for the firm's problem follows from a result in Sims (2003): The solution of

$$\min_{b,c} \mathbb{E}\left(U_t - O_t\right)^2$$

where U_t is an unobservable and O_t is an observable variable, subject to

$$\begin{array}{rcl} U_t & = & \rho U_{t-1} + a u_t, \\ O_t & = & \sum_{l=0}^{\infty} b_l u_{t-l} + \sum_{l=0}^{\infty} c_l \varepsilon_{t-l}, \\ \kappa & \geq & I\left(\{U_t, O_t\}\right) \end{array}$$

yields

$$\mathbb{E}\left(U_t - O_t^*\right)^2 = \sigma_T^2 \frac{1-\rho^2}{2^{2\kappa} - \rho^2}.$$

Therefore, firms' problem may be represented as

$$\min_{\kappa_a,\kappa_f,\{\kappa_n\}_{n\in\aleph_i}} \frac{\beta}{1-\beta} \frac{|\widehat{\pi}_{11}|}{2} \left[\frac{1-\rho_\Delta^2}{2^{2\kappa_a} - \rho_\Delta^2} N\sigma_\Delta^2 + \left(\frac{\widehat{\pi}_{14}}{\widehat{\pi}_{11}}\right)^2 \frac{1-\rho_f^2}{2^{2\kappa_f} - \rho_f^2} N\sigma_f^2 + \left(\frac{\widehat{\pi}_{15}}{\widehat{\pi}_{11}}\right)^2 \sum_{n\in\aleph_i} \frac{1-\rho_z^2}{2^{2\kappa_z} - \rho_z^2} \sigma_z^2 \right]$$

subject to

$$\kappa_a + \kappa_f + \sum_{n\in\aleph_i} \kappa_n \leq \kappa(N).$$

This problem is identical to that solved in section 5 for $\rho_\Delta = \rho_z = 0$. Its first order conditions are

$$\begin{aligned}
\kappa_a^* + f(\rho_\Delta, \kappa_a^*) &= \kappa_f^* + f\left(\rho_f, \kappa_f^*\right) + \log_2 \tilde{x}_1 \\
\kappa_a^* + f(\rho_\Delta, \kappa_a^*) &= \kappa_z^* + f(\rho_z, \kappa_z^*) + \log_2 \tilde{x}_2 \sqrt{N}
\end{aligned}$$

where $\tilde{x}_1 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta(1-\rho_\Delta)}{\widehat{\pi}_{14}\sigma_f\left(1-\rho_f\right)}$, $\tilde{x}_2 \equiv \frac{|\widehat{\pi}_{11}|\sigma_\Delta(1-\rho_\Delta)}{\widehat{\pi}_{14}\sigma_z(1-\rho_z)}$ and $f(\rho, \kappa) = \log_2\left(1 - \rho^2 2^{-2\kappa}\right)$.

The function $f(\rho, \kappa)$ is weakly negative and increasing in κ, so the difference in attention to aggregate and good-specific shocks, $\kappa_a^* - \kappa_z^*$, is still increasing in N. As before, the difference $\kappa_a^* - \kappa_f^*$ remains invariant to N. The function $f(\rho, \kappa)$ is also decreasing in $|\rho|$. Hence, a decrease in persistency of idiosyncratic shocks ρ_f and ρ_z implies an increase of κ_a^* relative to κ_f^* and κ_z^*.

Interdependent Profits

We now depart from our assumption in the main text that firms are decision units but not production units. We now assume that firms' production or commercialization processes are integrated such that the pricing decision of goods produced by a single firm are interdependent. We capture this 'interdependence' by assuming that the contribution to profits of a given good $n \in \aleph_i$ to its producing firm i is now

$$\pi\left(P_{nt}, P_t, Y_t, F_{it}, Z_{nt}, \{P_{-nt}\}_{-n\in\aleph_i}\right).$$

Our notation remains identical to the main text for aggregate prices P_t, real aggregate demand Y_t, firm-specific shocks F_{it} and good-specific shocks Z_{nt}. The novelty comes in the last argument, $\{P_{-nt}\}_{-n\in\aleph_i}$, which represents the prices set by firm i for all its produced goods but good n.

Optimal frictionless prices now solve

$$P_{nt}^{\diamondsuit} = \arg\max_{P_{nt}} \mathbb{E}\left[\sum_n \pi\left(P_{nt}^*, P_t, Y_t, F_{it}, Z_{nt}, \{P_{-nt}^*\}_{-n\in\aleph_i}\right)\right]$$

This problem is identical to the one in the main text with the exception that optimal frictionless prices must take into account their effect on the contribution to profits of all goods produced by the same firm. The optimality of prices implies that in steady state prices must solve

$$\pi_1\left(1, 1, Y_t, \bar{F}, \bar{Z}, \{1\}_{-n\in\aleph_i}\right) + (N-1)\,\pi_6\left(1, 1, Y_t, \bar{F}, \bar{Z}, \{1\}_{-n\in\aleph_i}\right) = 0;$$

which implicitly assumes equal marginal effect of the price of any good on other good's profits.

A second order approximation of the total profits function is

$$\begin{aligned}&(\widehat{\pi}_1+\widehat{\pi}_6(N-1))\,p_{nt}+\tfrac{1}{2}(\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1))\,p_{nt}^2+(\widehat{\pi}_{12}+\widehat{\pi}_{62}(N-1))\,p_{nt}p_t\\&+(\widehat{\pi}_{13}+\widehat{\pi}_{63}(N-1))\,p_{nt}y_t+(\widehat{\pi}_{14}+\widehat{\pi}_{64}(N-1))\,p_{nt}f_{it}+\widehat{\pi}_{15}p_{nt}z_{nt}\\&+\sum_{-n\in\aleph_i}\widehat{\pi}_{65}p_{nt}z_{-nt}+2\sum_{-n\in\aleph_i}\widehat{\pi}_{16}p_{nt}p_{-nt}\\&+\textit{terms independent of } p_{nt}.\end{aligned}$$

Hence, the optimal frictionless price solves

$$\begin{aligned}p_{nt}^{\diamondsuit} &= \frac{\widehat{\pi}_{12}+\widehat{\pi}_{62}(N-1)}{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}p_t+\frac{\widehat{\pi}_{13}+\widehat{\pi}_{63}(N-1)}{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}y_t+\frac{\widehat{\pi}_{14}+\widehat{\pi}_{64}(N-1)}{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}f_{it}+\\&\quad\frac{\widehat{\pi}_{15}}{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}z_{nt}+\sum_{-n\in\aleph_i}\frac{\widehat{\pi}_{65}}{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}z_{-nt}+\sum_{-n\in\aleph_i}\frac{2\widehat{\pi}_{16}}{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}p_{-nt}^{\diamondsuit}.\end{aligned}$$

The interdependence between profit functions has two implications on optimal frictionless prices. First, frictionless prices respond to all good-specific shocks that hit a given firm. Second, frictionless prices respond to other prices set by the same firm. If we represent this linear pricing rule by

$$p_{nt}^{\diamondsuit}=b_0p_t+b_1y_t+b_2f_{it}+b_3z_{nt}+b_4\sum_{-n\in\aleph_i}z_{-nt}+b_5\sum_{-n\in\aleph_i}p_{-nt}^{\diamondsuit},$$

then a reduced form of this rule is

$$p_{nt}^{\diamondsuit}=\frac{1}{1-(N-1)\,b_5}\left[\begin{array}{c}b_0p_t+b_1y_t+b_2f_{it}+\left(b_3-\frac{(N-1)b_5(b_3-b_4)}{1+b_5}\right)z_{nt}\\+\left(b_4+\frac{b_5(b_3-b_4)}{1+b_5}\right)\sum_{-n\in\aleph_i}z_{-nt}\end{array}\right]$$

with a short-hand representation as

$$p_{nt}^{\diamondsuit}=a_0p_t+a_1y_t+a_2f_{it}+a_3z_{nt}+a_4\sum_{-n\in\aleph_i}z_{-nt}.$$

Note that a_0, a_1, a_2, a_3 and a_4 are functions of N. Further, to obtain neutrality of frictionless prices,

$$a_0=1$$

and to ensure that $a_1>0$, parameters must satisfy

$$1-(N-1)\,b_5\equiv|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|-2\,(N-1)\,\widehat{\pi}_{16}>0.$$

Turning to solve for optimal prices under rational inattention, we start by computing the second-order approximation for

$$\sum_{n,-n\in\aleph_i}\left\{\widetilde{\pi}\left(p_{nt}^{\diamondsuit},p_t,y_t,f_{it},z_{nt},\left\{p_{-nt}^{\diamondsuit}\right\}_{-n\in\aleph_i}\right)-\widetilde{\pi}\left(p_{nt}^{*},p_t,y_t,f_{it},z_{nt},\left\{p_{-nt}^{*}\right\}_{-n\in\aleph_i}\right)\right\}$$

which solves

$$\frac{\left|\widehat{\pi}_{11}+\widehat{\pi}_{66}\left(N-1\right)\right|}{2}\sum_{n\in\aleph_i}\left(p_{nt}^{\diamondsuit}-p_{nt}^{*}\right)^2-\widehat{\pi}_{16}\sum_{n\in\aleph_i}\sum_{-n\in\aleph_i}\left(p_{nt}^{\diamondsuit}-p_{nt}^{*}\right)\left(p_{-nt}^{\diamondsuit}-p_{-nt}^{*}\right).$$

Guessing $p_t=\alpha q_t$, defining $\Delta_t\equiv p_t+a_1y_t$, imposing

$$p_{nt}^{*}=\frac{\sigma_\Delta^2}{\sigma_\Delta^2+\sigma_{\varepsilon i}^2}s_{it}^a+a_2\frac{\sigma_f^2}{\sigma_f^2+\sigma_{ei}^2}s_{it}^f+a_3\frac{\sigma_z^2}{\sigma_z^2+\sigma_{\psi n}^2}s_{nt}^z+a_4\sum_{-n\in\aleph_i}\frac{\sigma_z^2}{\sigma_z^2+\sigma_{\psi n}^2}s_{nt}^z,$$

and using the definitions of κ_a,κ_f and $\{\kappa_n\}_{n\in\aleph_i}$, the problem of a decision unit taking $\widetilde{N}$ pricing decisions within a firm that produces N goods is

$$\min_{\kappa_a,\kappa_f,\{\kappa_n\}_{n\in\aleph_i}}\left\{\begin{array}{c}\frac{\left|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)\right|}{2}\left[\left(2^{-2\kappa_a}\sigma_\Delta^2+a_2^2 2^{-2\kappa_f}\sigma_f^2\right)\widetilde{N}+\left(a_3^2+\left(N-1\right)a_4^2\right)\sum_{n\in\aleph_i}2^{-2\kappa_n}\sigma_z^2\right]\\-\widehat{\pi}_{16}\left(N-1\right)\left[\left(2^{-2\kappa_a}\sigma_\Delta^2+a_2^2 2^{-2\kappa_f}\sigma_f^2\right)\widetilde{N}+\left(2a_3a_4+a_4^2\left(N-2\right)\right)\sum_{n\in\aleph_i}2^{-2\kappa_n}\sigma_z^2\right]\end{array}\right\}.$$

subject to

$$\kappa_a+\kappa_f+\sum_{n\in\aleph_i}\kappa_n\leq\kappa\left(\widetilde{N}\right)$$

We make the distinction between $\widetilde{N}$ and N because firms now are both production units and decision units. A firm that has an integrated productive process for its N goods may still decide to keep separated pricing processes such that a single decision unit decides $\widetilde{N}<N$ prices. A decision unit is endowed by information capacity $\kappa\left(\widetilde{N}\right)$ which, as in the main text, may depend on the number $\widetilde{N}$ of prices that this decision unit must set. To do so, a decision unit must take into account the cross effects of all prices set within the firm, which is captured by the optimal pricing rules for $p_{nt}^{\diamondsuit}$ and p_{nt}^{*} obtained above.

The first-order conditions for the allocation of attention are now

$$\begin{aligned}\kappa_a^* &= \kappa_f^*+\log_2\left(\widetilde{x}_1\left(N\right)\right),\\ \kappa_a^* &= \kappa_n^*+\log_2\left(\widetilde{x}_2\left(N\right)\sqrt{\widetilde{N}}\right),\ \ \forall n\in\aleph_i.\end{aligned}$$

As in the main text, the economies of scope in information processing are captured by $\sqrt{\widetilde{N}}$ in the second condition. The interdependence of profits introduced here are captured in $\widetilde{x}_1\left(N\right)$ and

$\widetilde{x}_2(N)$, which in the main text are parameters and here are functions of N:

$$
\begin{aligned}
\widetilde{x}_1 &\equiv \frac{\sigma_\Delta}{a_2\sigma_f}, \\
\widetilde{x}_2(N) &\equiv \left[\frac{\left(\frac{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}{2} - \widehat{\pi}_{16}(N-1)\right)\frac{\sigma_\Delta^2}{\sigma_z^2}}{\frac{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}{2}\left(a_3^2+(N-1)a_4^2\right) - \widehat{\pi}_{16}(N-1)\left(2a_3a_4+a_4^2(N-2)\right)}\right]^{\frac{1}{2}}
\end{aligned}
$$

We then follow a similar logic than in Proposition 6. We discipline $\kappa\left(\widetilde{N}\right)$ by assuming that the information capacity of decision units depends on the number $\widetilde{N}$ of decisions they take such that they have no incentives to merge or delegate their pricing decisions. This assumption is equivalent to assume that the frictional cost per good produced in a firm that produces N goods is independent of the number $\widetilde{N}$ of decisions taken by decision units within the firm. Under this assumption, we can establish that

$$
\kappa_a^*\left(\widetilde{N};N\right) = \kappa_a^*(1;N) + \frac{1}{2}\log_2\left(\frac{\widetilde{N}+2}{3}\right) + \frac{1}{2}\log_2\left(\frac{\sigma_\Delta^2\left(\widetilde{N};N\right)}{\sigma_\Delta^2(1;N)}\right).
$$

This expression is identical to Proposition 6, but its interpretation is more subtle. In an economy where firms produce N goods, the attention paid to aggregate shocks is increasing in the number $\widetilde{N}$ of pricing decisions that single decision units must take within firms. As in the main text, this result highlights the importance of economies of scope in information processing on the aggregate predictions of the rational inattention model. In the literature, these economies of scope are shut down by the assumption that firms produce only one good and decide only one price.

Finally, we drop the distinction between N and $\widetilde{N}$, that is, $N = \widetilde{N}$, to produce a version of proposition 4. This assumption is consistent with the evidence that a single decision unit prices all goods in the firm's portfolio of goods.

If we arrange parameters such that firms' attention to monetary shocks is invariant in N, $\kappa_a^*(N) = \overline{\kappa}_a$, then the frictional cost at the optimum is

$$
C_n(N) = \left(\frac{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}{2} - \widehat{\pi}_{16}(N-1)\right)(N+2)\,2^{-2\overline{\kappa}_a}\sigma_\Delta^2
$$

and the shadow price of information-processing capacity is

$$
\lambda(N) = -\frac{\beta}{1-\beta}\left(\frac{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}{2} - \widehat{\pi}_{16}(N-1)\right)N\log(2)\,2^{-2\overline{\kappa}_a}\sigma_\Delta^2
$$

which are both increasing in N unless $\widehat{\pi}_{16} > 0$ and high enough. If this is the case, then the term

$$
\frac{|\widehat{\pi}_{11}+\widehat{\pi}_{66}(N-1)|}{2} - \widehat{\pi}_{16}(N-1)
$$

is decreasing in N. However, this expression also governs the complementarity in pricing (a_1), so this complementarity would be increasing in N. As in the main text, the complementarity in pricing is deduced from aggregate data.

In addition, if this expression is decreasing in N, then the per-good expected profits of the firm falls as N increases. This contradicts our assumption that the number of produced goods by firms is exogenous and the observation that firms produce multiple goods.