Infectious Disease Detection with Private Information

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Abstract

This paper studies the allocation of the diagnostic capacity in an environment where disease endogenously stochastically spreads among a fixed number of non-cooperative agents who have private information about the probability of being infected. When the diagnostic capacity is allocated by the health authority, agents with high and low infection risks are assigned different testing priorities. Prioritized testing may or may not require a payment schedule to incentivize the reporting of clinically suspect situations. When the diagnostic capacity is allocated by a profit-maximizing firm, the efficient usage is achieved under a price policy contingent on testing priority and diagnosis if the disease is not too infectious or the cost of disease control is not too great. Otherwise, the firm can test too few or too many agents with suspicions about being infected.

Keywords: infectious disease, reporting, diagnostic testing, private information

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1. Introduction

Early detection of an infectious disease outbreak can reduce its social costs and spread but requires an effective disease surveillance program and diagnostic testing. Although individuals may know more about their probability of carrying disease than the health authorities, when deciding whether to seek diagnosis individuals presumably do not take into account both the full social value and the opportunity cost of utilization of the diagnostic capacity. This raises a number of questions for the economic analysis of disease surveillance and management programs. Does targeted diagnostic testing of high-risk individuals perform better than random testing? Do participation in the diagnostic testing program and reporting of clinically suspect situations require payments? Is it easier to sustain truthful reporting in a joint disease surveillance and management program? How does a profit-maximizing testing policy compare with the efficient one?

Here we consider the allocation of the diagnostic capacity in an environment where disease endogenously stochastically spreads among a fixed number of non-cooperative agents who have private information about their probability of carrying disease. In our model, prioritized testing based on prior infection risk outperforms random testing, which brings to the fore the credibility of communication of suspicions about carrying disease. When the diagnostic capacity is allocated by the health authority, credible reporting does not require monetary transfers if (i) the participation in the diagnostic testing program is mandatory, (ii) disease is sufficiently rare but infectious, (iii) private information about the probability of being infected is sufficiently precise, and (iv) there are not too many agents.¹ If these conditions are not satisfied or the cost of treatment is either sufficiently small or sufficiently large compared with the potential loss from untreated infection, an efficient test allocation requires a schedule of payments for each testing priority and test outcome.

In our setting, diagnostic testing has three effects on welfare. First, a test provides an informational benefit to the tested agent who learns her current disease status. Second, testing reduces the risk of disease transmission from the tested agent to the other agents since treating an ongoing infection is privately optimal. Third, testing increases the risk of disease transmission from the remaining untested agents. This happens because in equilibrium the untested agents exercise less care in controlling the disease as they are less likely to be infected in the future when there are fewer untested agents. Thus, whether agents with or without suspicions should receive the testing priority is determined by comparing the informational benefits from testing, likely exposures to disease transmission risk, and the levels of infectiousness if an agent remains untested across different types of agents.

When communicating their private information, non-cooperative agents are not concerned with their own level of infectiousness, but they take into account that a test provides them with an informational

¹Transfers may consist of subsidies or taxes for participation in the diagnostic testing program and reporting information about a possible outbreak or indemnity payments for losses from the disease outbreak.

benefit and that a tested agent "crowds out" the diagnostic capacity allocation to other potentially infectious agents. Whether this partial internalization of the social benefits and opportunity costs of using the scarce diagnostic capacity is sufficient to satisfy the condition for screening agents according to their prior infection risk without payments depends on the parameters of the model. When the number of agents is sufficiently great, the overall disease transmission risk is almost beyond the control of a single agent. Then agents only consider the informational benefits of testing and view the risk of disease transmission from other agents as fixed, and contingent transfers are always necessary to screen out agents with or without suspicions. All else equal, the truth-telling incentive compatibility constraints are easier to satisfy under a joint disease surveillance and management program. Indeed, the costs of compliance with the mandatory disease control policy may obviate the need for payments to achieve the efficient utilization of the diagnostic capacity even in large populations.

We also show that when the diagnostic capacity is allocated by a profit-maximizing firm, the efficient usage is achieved under a pricing policy contingent on testing priority and diagnosis if the disease is not too infectious or the cost of disease control is sufficiently small.² However, if the total number of agents is not too great and the cost of disease control is sufficiently high, the firm tests too few agents with suspicions compared with the efficient testing policy. Then each agent prefers not to crowd out the allocation of the diagnostic capacity to other agents, and in equilibrium some agents stay out of the market for diagnostic services. On the other hand, the firm tests too many agents with suspicions if the total number of agents is sufficiently large and the cost of disease control is not too small or too large. This happens because the firm cannot extract the surplus generated by diagnostic testing for the remaining untested agents. The firm assigns a higher-than-efficient testing priority to agents with suspicions since they derive a greater informational benefit from testing and are less concerned with the risk of disease transmission from the remaining untested agents without suspicions.³

Concerns with early detection of outbreaks of communicable diseases such as malaria or tuberculosis frequently arise in low-income country contexts where diagnostic testing is provided by public health authorities or for-profit firms with limited human resources, infrastructure, and laboratory facilities (Mikanatha et al. 2007, Das, Hammer, and Leonard 2008).⁴ Our formal representation of tensions underlying disease reporting, detection, and management, and characterization of optimal testing priority and payment

²A firm can offer packages of diagnostic test and diagnosis-contingent treatments at different prices (Cohen and Dickens 2012). ³In our setting, a for-profit firm may not be able to extract the total social surplus generated by the diagnostic capacity in equilibrium. This differs from the finding in Cohen and Dickens (2012) that profit-maximizing drug shop owners will provide diagnostic testing under the same circumstances that a social welfare maximizing planner would, because they assume that consumers do not have private information, the marginal cost of diagnostic testing is constant, and there are no negative externalities generated by the spread of disease.

⁴Cohen, Dupas, and Schaner (2012) find empirical evidence of over-treatment and under-treatment of malaria and conclude that subsidizing diagnostic tests could improve outcomes. This paper focuses on diseases which can be treated or contained without a professional diagnostic effort as in the case of certain over-the-counter treatments for malaria or preemptive depopulation of birds suspected of carrying avian influenza in a developing country (Adhvaryu 2012; Catley, Alders, and Wood 2011).

schedule may help evaluate the design of the disease surveillance programs and provision of diagnostic services in the real world (World Health Organization 2012).

Delayed reporting and detection of outbreaks also happens in the case of animal and plant infectious diseases such as highly pathogenic avian influenza (HPAI) and wheat rust for which agricultural producers are often the first to observe early warning signs such as increased animal mortality or deviations from normal plant growth (Palmer, Fozdar, and Sully 2009, Azhar et al. 2010, Elbers et al. 2010).⁵ The problem of incentivizing reporting of animal and plant diseases is recognized by policy makers (Institute of Medicine and National Research Council 2009, Chan et al. 2010).⁶ For example, World Bank guidelines regarding the control of HPAI state that enhancing "early reporting and complete culling of diseased or suspected birds is ... the first objective of compensation schemes" (World Bank 2006, ix). Our model identifies circumstances under which a health authority cannot rely on voluntary reporting and participation in a disease surveillance program and a compensation scheme is appropriate.

There are several other papers concerned with disease surveillance and control in the presence of private information.⁷ Gramig, Horan, and Wolf (2009) explore the design of indemnity payments that incentivize agents to invest in biosecurity measures and to report infection to the government in an environment with a single agent and perfect private information. Sheriff and Osgood (2010) assume that private information is imperfect and consider the effects of cash transfers, testing, and forecast on the seller's incentives to disclose exogenous food safety. Malani and Laxminarayan (2011) study incentives for a country to report imperfect information about a disease outbreak to its trading partner, resulting in a trade-off between medical assistance and trade sanctions. In contrast, we study incentives to report in an environment where the disease spreads stochastically in multiple directions and the allocation of diagnostic tests and disease control efforts are endogenous.⁸

More broadly, our model contributes to the literature on screening with externalities by endogenizing the benefits and costs of reporting private information in a novel environment (Weber 2012). Following the literature on incentivizing disclosure of private information with probabilistic auditing (Kaplow and

⁵Kuchler and Hamm (2000) find empirical evidence that farmers delayed reporting infected animals. Elbers et al. (1999, 2006) and Cuenot et al. (2003) identify instances of underreporting by comparing the quantities of officially notified and expected clinically suspect situations.

⁶Livestock owners receive indemnity payments for animals culled by the government in developed countries, and similar programs are at various stages of implementation in developing countries (Ott 2006, Wilkinson et al. 2011).

⁷Gramig and Horan (2011) explore the effects of government testing and disease surveillance on private biosecurity incentives in a dynamic model of infectious livestock disease with non-Bayesian updating of beliefs and public information about disease incidence. Cacho and Hester (2011) discuss the efficient allocation of surveillance resources in a spatio-temporal model of invasive species management but without strategic agents.

⁸ Mesnard and Seabright (2009) study the effects of restricting migration in a model of an infectious human disease with private information about health, but they do not consider the effects of reporting and diagnostic testing. Adhvaryu (2012) and Cohen, Dupas, and Schaner (2012) study the effects of diagnostic tests on the treatment of malaria in the presence of learning about therapeutic effectiveness and subsidies for treatment and diagnostics but do not explicitly take into account the effects of the spread of disease.

Shavell 1994, Pfaff and Sanchirico 2000), we assume that the diagnostic capacity is not sufficient to test each member of a susceptible population but we also allow for externalities among members. Our paper is also related to the model of cheap talk with multiple senders and partial, non-overlapping, and complementary private information in McGee and Yang (2009). Here we allow for two-way communication since in our model individuals report and receive information about disease incidences.

2. Model

There is a population of $n \ge 2$ identical risk-neutral potentially infected agents and a fixed diagnostic capacity m < n, where *m* is the maximum number of agents that can be tested for disease. In the case of a human disease, an agent can represent a single individual or a household. In the case of an animal or plant disease, an agent can represent a farm or a larger collective decision-making unit such as a village. Figure 1 depicts the order of play of the game.

Figure 1: Timing of events

| | | | Fachagar | at | Disassa spraads and asah |
|---------------|------------------------|-----------------------------|------------|---------------|----------------------------------|
| draws initial | Each agent ob- | t_s agents who reported s | chooses d | ll isease | agent draws final disease |
| disease state | serves signal y_i | and t_h agents who re- | control ef | fort <i>e</i> | state A and gets utility |
| A . | and reports s or h | ported h are tested, and | control ch | | state $O_{i,1}$ and gets utility |
| $v_{i,0}$ | or opts out of di- | payments τ_i are made | | | $v - d\theta_{i,1} - c(1 - e_i)$ |
| | agnostic testing | 1 | | | |

Let $\theta_{i,0}$ and $\theta_{i,1}$ denote the random initial and final disease states of agent *i*, where $\theta_{i,0}, \theta_{i,1} \in \{S, H\}$, , $S \equiv 1$ means that agent *i* is infected and $H \equiv 0$ means that the agent is not infected with the disease in question. Initially, each agent *i* is infected with probability

$$\Pr(\theta_{i,0}=S)=1-\alpha\in(0,1),$$

where $\theta_{1,0},...,\theta_{n,0}$ are drawn independently.

Agents do not know the true initial disease states. Instead, each agent *i* privately observes a random signal that is imperfectly correlated with her initial disease state, denoted by $Y_i \in \{s, h\}$ with typical realization y_i .⁹ The signals are independently and identically distributed, and are correct with probability $\beta \in (\frac{1}{2}, 1)$ and incorrect with probability $1 - \beta$, that is,

$$Pr(Y_{i} = h | \theta_{i,0} = H) = Pr(Y_{i} = s | \theta_{i,0} = S) = \beta,$$

$$Pr(Y_{i} = s | \theta_{i,0} = H) = Pr(Y_{i} = h | \theta_{i,0} = S) = 1 - \beta.$$

Signal s (respectively, signal h) indicates that the agent is more likely to be initially infected (respectively,

⁹ For some diseases symptoms or signs of a late-stage disease can be observed publicly.

less likely) than according to the prior. For example, an agent observes signal *s* ("suspicions") when she notices symptoms or knows of the prior exposure to the disease, otherwise signal *h* is observed. Let $\lambda_y \equiv \Pr(\theta_{i,0} = S \mid Y_i = y)$ denote the posterior belief, where $\lambda_s = (1-\alpha)\beta/g_s$, $\lambda_h = (1-\alpha)(1-\beta)/g_h$, $g_s \equiv \Pr(Y_i = s) = \alpha(1-\beta) + (1-\alpha)\beta$, and $g_h \equiv \Pr(Y_i = h) = \alpha\beta + (1-\alpha)(1-\beta)$.

In addition, each agent can be tested for disease and learn her actual initial disease state $\theta_{i,0} \in \{H, S\}$. It will be convenient to let $\lambda_{\theta} = \theta = \Pr(\theta_{i,0} = S | \theta_{i,0} = \theta)$ denote the updated belief for tested agents.¹⁰ We will assume for now that the diagnostic capacity is allocated by the health authority such as a government health care facility in the case of human disease or a veterinary office in the case of an animal disease. In Section 4.3, we will consider the allocation of the diagnostic capacity by a for-profit firm.

Upon observing y_i , each agent simultaneously decides whether to opt out of the diagnostic testing program $r_i = o$ or to participate and report $r_i = s$ or h to the health authority. We assume that only participating agents can be tested, and the allocation of the diagnostic capacity is not directly constrained by the participating agents' reports and disease states.¹¹ Let n_r denote the number of agents and t_r denote the number of tested agents who report r, and $p_r = t_r / n_r$ denote the testing priority, that is, the probability with which an agent who reported r is selected for testing, where $p_q = 0$, $t_r \le n_r$, $t_s + t_h \le m$, $r \in \{s, h, o\}$.

We assume for the moment that all information collected by the health authority is public: all the participation decisions, reports, and test results are observed by both the health authority and all agents. In Section 4.2, we consider a private information regime where the health authority observes all the participation decisions, reports, and test results, each agent i observes her private signal Y_i and the outcome of her test, but does not observe the participation decisions, reports, and test results.

After testing, each agent *i* receives contingent payment $\tau(r,\theta) \ge 0$, where $\theta \in \{S,H\}$ is a test outcome if the agent has been tested and $\theta = U$ if the agent has not been tested (*U* stands for "untested"). These subsidies do not directly affect welfare that is measured as the sum of agents' payoffs net of cash transfers, but they can influence actions. In the scenario where the diagnostic capacity is allocated by a profit-maximizing firm, agents pay for diagnostic testing and $\tau(r,\theta)$ represents the price schedule that assigns payments to testing priorities p_r and test outcomes θ .

Each agent *i* uses her private signal and the available public information to update her beliefs about the initial disease states of all agents before choosing the level of effort to control disease $1 - e_i \in [0,1]$ at a

¹⁰ The analysis does not change as long as diagnostic tests are sufficiently precise.

¹¹ For example, a health authority may be constrained to test agents who report suspicious signs.

constant marginal cost c, where e_i is the probability that an infected agent continues to carry disease.¹² We first consider equilibrium with voluntary disease control where agents are free to choose any effort. In Appendix B, we consider the design of a joint disease surveillance and management program, where the health authority both allocates tests and assigns disease control efforts to all agents.

The final probability that agent i is infected is given by

$$\Pr(\theta_{i,1} = S \mid \theta_{1,1}, ..., \theta_{n,1}, e_1, ..., e_n) = \begin{cases} e_i, \text{ if } \theta_{i,0} = S \\ e_i \frac{\delta}{n-1} \sum_{j=1, j \neq i}^n e_j \theta_{j,0} \text{ if } \theta_{i,0} = H \end{cases}$$

where $\delta \in [0,1]$ is the maximum infectiousness of the disease. In accordance with (1), an agent who is initially infected remains infected with probability e_i . In addition, the infection can spread from initially infected agents to initially uninfected agents. Consistent with the traditional models of infectious diseases, we assume that the probability that initially uninfected agents become infected is proportional to the share of infectious agents.¹³

At the end of the game, agent *i* privately learns her final disease state $\theta_{i,1}$, and earns $v - d\theta_{i,1} - c(1-e_i)$, where v > v - c > v - d.¹⁴ The parameter *v* is the utility of an agent who is not infected and *d* is the disutility from disease expressed in monetary terms. *d* can represent the loss of individual health capital and wage income due to illness in the case of a human disease or the agent-specific production loss in the case of an agricultural disease. Efforts $1 - e_i$ can be interpreted as preemptive culling of animals or unplanned crop rotation in the case of an animal or plant disease for v = c. We assume that an agent who is not infected and saves on the disease control costs is better off than an agent who incurs the full cost of disease control, and that an agent who incurs the full cost of disease control is better off than an agent who is infected in the end of the game. Let $\rho \equiv \frac{c}{d} \in (0,1)$ denote the ratio of the marginal cost of disease control and the loss from the disease.

2.1. Equilibrium Concept

Note that exercising full disease control effort is the dominant strategy for any agent who tested positive since the marginal benefits and costs of disease control are constant and the test determines the actual initial disease state. Consequently, in any equilibrium each tested agent is not infectious, and as we will verify

¹² In our model diagnostic testing does not *directly* determine the set of possible disease control choices. For example, this is not the case for treatment options that require a prescription written by a medical professional.

¹³ We could also allow agents to become infected at the end of the game if none are initially infected.

¹⁴ The setting in this paper does not accommodate distinctions between the nature of harm and epidemiology of human and animal or plant diseases. Note that we ignore negative externalities from the spread of disease outside of the population of agents.

later, there is no loss of generality if we assume that the disease control strategies are given by $e_{yr}(n_s, n_h, t_s, t_h)$, , where $y \in \{s, h, S, H\}$, $r \in \{s, h, o\}$. The test allocation policy is given by $(t_s(n_s, n_h), t_h(n_s, n_h))$. Let η_y denote the out-of-equilibrium beliefs about the private information of each non-participating agent. We focus on a symmetric perfect Bayesian equilibrium (PBE) of this game where (i) test allocation maximizes welfare and (ii) transfers are used as little as possible. A PBE consists of the disease control strategies, e_{yr}^* , the testing policy, (t_s^*, t_h^*) , the transfer policy τ^* , and beliefs η^* such that

- following testing agents choose the efforts simultaneously and non-cooperatively

$$e_{yr}^{*}(n_{s}, n_{h}, t_{s}, t_{h}) \in \arg\max_{e \in [0,1]} \begin{cases} v - (\lambda_{y} + (1 - \lambda_{y})R_{r}^{*}(n_{s}, n_{h}, t_{s}, t_{h}))ed - (1 - e)c, \text{ if } y \in \{s, h\} \\ v - (\lambda_{y} + (1 - \lambda_{y})R_{y}^{*}(n_{s}, n_{h}, t_{s}, t_{h}))ed - (1 - e)c, \text{ if } y \in \{S, H\} \end{cases}$$
(1)

for all y, r, n_s, n_h, t_s, t_h , where $R_b^*(n_s, n_h, t_s, t_h) = \frac{\delta}{n-1} \sum_{y \in \{s,h\}} (n_y - t_y - 1_{b=y}) \lambda_y e_{yy}^*(n_s, n_h, t_s, t_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_s, n_h) + (n - n_s - n_h) \lambda_y e_{yy}^*(n_h) + ($

 $(-1_{b=o})\lambda_y e_{yo}^* (n_s, n_h, t_s, t_h)\eta_y^*$ is the aggregate infectiousness of all agents except for a given agent; - the test allocation maximizes the expected welfare (that is, minimizes the aggregate expected loss from infection and disease control costs) conditional on the report profile (n_s, n_h)

$$(t_{s}^{*}(n_{s},n_{h}),t_{h}^{*}(n_{s},n_{h})) \in \arg\max_{t_{s} \leq n_{s},t_{h} \leq n-n_{s},t_{s}+t_{s} \leq m} W^{*}(n_{s},n_{h},t_{s},t_{h}),$$
(2)

where $W^*(n_s, n_h, t_s, t_h) = \sum_{y \in \{s,h\}} (n_y - t_y) \pi^*_{yy}(n_s, n_h, t_s, t_h) + t_y(\pi^*_{Sy}(n_s, n_h, t_s, t_h) \lambda_y + \pi^*_{Hy}(n_s, n_h, t_s, t_h)(1 - \lambda_y)) + (n - n_s - n_h) \pi^*_{yo}(n_s, n_h, t_s, t_h) \eta^*_y$, and $\pi^*_{yr}(n_s, n_h, t_s, t_h) = v - (\lambda_y + (1 - \lambda_y)R^*_r(n_s, n_h, t_s, t_h))e^*_{yr}(n_s, n_h, t_s, t_h)d - c(1 - e^*_{yr}(n_s, n_h, t_s, t_h))$ is the equilibrium post-testing expected payoff for agents with signal y and report r; - the transfer policy minimizes the expected amount paid out

$$\tau^* \in \underset{\tau \ge 0}{\operatorname{arg\,min}} n \sum_{y \in \{s,h\}} T^*_{yy} g_y \tag{3}$$

subject to the truth-telling incentive compatibility (IC) and participation constraints

$$\Pi_{yy}^{*} + T_{yy}^{*} \ge \Pi_{yr}^{*} + T_{yr}^{*} \text{ for all } y \in \{s, h\}, r \in \{s, h, o\},$$
(4)

where $\Pi_{yr}^* = \sum_{n_s=0}^{n-1} (p_r^*(n_s)(\lambda_y f_{Sr}(n_s) + (1-\lambda_y) f_{Hr}(n_s)) + (1-p_r^*(n_s)) f_{yr}(n_s)) B_{n_s,n-1}$ and $T_{yr}^* = \sum_{n_s=0}^{n-1} (p_r^*(n_s) + (\lambda_y \tau(r, S) + (1-\lambda_y) \tau(r, H)) + (1-p_r^*(n_s)) \tau(r, U)) B_{n_s,n-1}$ are agent's expected equilibrium utility and transfer, respectively, following a history of signal $y \in \{s,h\}$ and report $r \in \{s,h,o\}$, $f_{yr}(n_s) = \pi_{yr}^*(n_s + 1_{r=s}, n - n_s - 1_{r \in \{s,o\}}), t_s^*(n_s + 1_{r=s}, n - n_s - 1_{r \in \{s,o\}}))$ and $p_r^*(n_s) = t_r^*(n_s + 1_{r=s}, n - n_s - 1_{r=s}) / (1_{r=s}(n_s + 1) + 1_{r=h}(n - n_s))$ are the payoff and the probability of being selected for testing after observing

y and reporting r if $n_s \le n-1$ agents report s and $n-1-n_s$ agents report h, respectively, and $B_{n_s,n} = \frac{n!}{n_s!(n-n_s)!} g_s^{n_s} g_h^{n-n_s}$ is the probability of a signal profile $(n_s, n-n_s)$, where n_s agents observe signal s and $n-n_s$ agents observe h. Also, let $P_r^* = \sum_{n_s=0}^{n-1} p_r^*(n_s) B_{n_s,n-1}$ denote the expected equilibrium probability of being selected for testing after reporting r.

Since in equilibrium all agents reveal their private information, the agents' and health authority's beliefs regarding the private information of the other agents are degenerate in the public information regime. However, when an agent deviates from truthful reporting, her beliefs can differ from those of the other agents and the health authority, and consistently with a PBE concept, we allow the agents' disease control strategies to depend on their private information. Also, note that our focus on equilibrium where agents reveal their private information needs to be justified. In our setting, the revelation principle cannot be applied directly since the set of equilibrium payoffs when reports are not informative is, in general, not a subset of the set of equilibrium payoffs when all agents report truthfully. Nonetheless, we will show that our focus on equilibrium where screening conditions (4) are satisfied is without loss of generality since the overall welfare is greater in a PBE with informative reporting.

We will first consider equilibrium where participation in the disease surveillance program is mandatory, that is, agents cannot opt out of diagnostic testing if selected, $n_o = 0$ and $n_s + n_h = n$. In Section 4.1, we will consider the design of the disease surveillance program with voluntary participation.

3. Analysis

Here we suppose that the health authority allocates diagnostic tests, participation in the testing program is mandatory, disease control is voluntary, and all information collected by the authority is public.

3.1. Post-testing voluntary disease control

When all agents report their signals truthfully, by Lemma 1, the equilibrium disease control strategies are given by

$$e_{yr}^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = 1 - y, \text{ if } y \in \{S, H\}, r \in \{s, h\},$$
(5a)

$$e_{ss}^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = \begin{cases} 1, \text{if } \lambda_{s} + (1 - \lambda_{s})R_{s}(1, 1, n_{s}, n_{h}, t_{s}, t_{h}) < \rho \\ \widetilde{e}_{ss}^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = \begin{cases} 1, \text{if } \lambda_{s} + (1 - \lambda_{s})R_{s}(0, 1, n_{s}, n_{h}, t_{s}, t_{h}) < \rho \le \lambda_{s} + (1 - \lambda_{s})R_{s}(1, 1, n_{s}, n_{h}, t_{s}, t_{h}) \\ 0, \text{if } \lambda_{s} + (1 - \lambda_{s})R_{s}(0, 1, n_{s}, n_{h}, t_{s}, t_{h}) \ge \rho \end{cases}$$
(5b)

$$e_{hh}^{*}(n_{s},n_{h},t_{s},t_{h}) = \begin{cases} 1, 1, \lambda_{h} + (1-\lambda_{h})R_{h}(0,1,n_{s},n_{h},t_{s},t_{h}) < \rho \\ \tilde{e}_{h} \in (0,1), \text{ if } \lambda_{h} + (1-\lambda_{h})R_{h}(0,0,n_{s},n_{h},t_{s},t_{h}) < \rho \leq \lambda_{h} + (1-\lambda_{h})R_{h}(0,1,n_{s},n_{h},t_{s},t_{h}), \quad (5c) \\ 0, \text{ if } \lambda_{h} + (1-\lambda_{h})R_{h}(0,0,n_{s},n_{h},t_{s},t_{h}) \geq \rho \end{cases}$$

where \tilde{e}_s and \tilde{e}_h solve the indifference conditions $\lambda_s + (1 - \lambda_s)R_s(\tilde{e}_s, 1, n_s, n_h, t_s, t_h) = \rho$ and $\lambda_h + (1 - \lambda_h)$

 $R_{h}(0, \tilde{e}_{h}, n_{s}, n_{h}, t_{s}, t_{h}) = \rho, \text{ respectively, and } R_{r}(e_{s}, e_{h}, n_{s}, n_{h}, t_{s}, t_{h}) = \frac{\delta}{n-1} \sum_{y \in \{s,h\}} (n_{y} - t_{y} - 1_{r=y}) \lambda_{y} e_{y} \text{ is the probability of becoming infected by other agents for an agent with updated signal } y \in \{s, h, S, H\}$ when untested agents with and without suspicions choose susceptibilities to infection e_{s} and e_{h} , respectively. By (1), the out-of-equilibrium disease control efforts of untested agents who deviate from truthful reporting are given by $e_{yr}^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = 1_{\lambda_{y}+(1-\lambda_{y})R_{r}^{*}(n_{s}, n_{h}, t_{s}, t_{h}) < \rho$ for $y \in \{s, h\}, r \in \{s, h, o\}, y \neq r$.

Note that the probability of carrying disease, $q_y(e_s, e_h, n_s, n_h, t_s, t_h) = \lambda_y + (1 - \lambda_y) R_y(e_s, e_h, n_s, n_h, t_s, t_h)$, is increasing in the aggregate levels of susceptibility e_{ss} and e_{hh} for each $y \in \{s, h\}$. Because untested agents with suspicions are initially more likely to carry disease, $q_h(0, 1, n_s, n_h, t_s, t_h) < q_s(0, 1, n_s, n_h, t_s, t_h)$, in equilibrium untested agents with suspicions exert a (weakly) greater disease control effort than the untested agents without suspicions. The comparative statics for the equilibrium disease control efforts for untested agents, $e_{yy}^*(n_s, n_h, t_s, t_h)$, follow from the properties of the function $q_y(e_s, e_h, n_s, n_h, t_s, t_h)$. All else equal, agents with signal y exercise (weakly) greater care in controlling disease (smaller e_{yy}^*) when the relative cost of disease control is smaller (smaller ρ), the disease is more infectious (greater δ), and there are fewer tested agents (smaller t_s or t_h).

We are now in a position to characterize the optimal test allocation and transfer policy. We begin with the case where n = 2 and m = 1.

3.2. Disease surveillance with two agents

For n = 2 and m = 1, by Lemmas 1 and 2, the equilibrium efforts and test allocation policy are given by $e_{yr}^*(n_s, n_h, t_s, t_h) = 1_{\rho > \lambda_y}$ and $(t_s^*(n_s, n_h), t_h^*(n_s, n_h))$ in Table 1, where $t_s + t_h = m$, $k = \lambda_h (1 + (1 - \lambda_s)\delta)/(1 - \lambda_s + \lambda_h)$. Table 1: Efficient test allocation for n = 2, m = 1

| (n_s, n_h) | $t_s^*(n_s,n_h)$ | $t_h^*(n_s,n_h)$ |
|--------------|------------------------------------|---------------------------------|
| 2,0 | 1 | 0 |
| 1,1 | $1_{\rho \geq \min[k, \lambda_s]}$ | $1_{\rho < \min[k, \lambda_s]}$ |
| 0,2 | 0 | 1 |

If the relative cost of disease control ρ is sufficiently large, the health authority assigns the testing priority to an agent with suspected infection. Then an untested agent with suspected infection either poses a greater disease transmission risk or gains more from learning about her initial disease state or both. Otherwise, it is optimal to test an agent who did not observe suspicious signs. This happens because an untested agent with suspected infection controls the disease while an untested agent without suspicions either remains potentially infectious or is more likely to incur unnecessary treatment expenditures.¹⁵

By Lemma 3, the *incremental* expected (before reporting but after observation of private signals) payoff net of transfers when an agent with signal y reports signal s rather than signal h is given by

$$\Delta_{y}^{*} = \Pi_{ys}^{*} - \Pi_{yh}^{*} = (P_{s}^{*} - P_{h}^{*})(V_{y} - D_{y}), \qquad (6)$$

where $P_s^* - P_h^* = t_s^*(1,1) - \frac{1}{2}$ is the incremental probability of being selected for testing, $V_y = \min[\lambda_y(1-\rho), (1-\lambda_y)\rho]d$ is the option value generated by information about the agent's initial disease state keeping the disease transmission risk constant at the level that is faced by an untested agent, and $D_y = \delta(1-\lambda_y)$. $\sum_{y \in \{s,h\}} g_y \mathbf{1}_{\lambda_y < \rho} \lambda_y d$ is the incremental disease transmission risk from changing one's testing priority.

The informational benefit from testing, $V_y = (\min[\lambda_y, \rho] - \lambda_y \rho)d$, is greatest when the probability of being infected early, λ_y , and the relative cost of treatment, ρ , are not too high or too low. The option value generated by diagnosis arises because the agent avoids over-exertion or under-exertion of the disease control effort depending on whether $\lambda_y \ge (<)\rho$. As illustrated in Figure 2, the option value of knowing one's initial disease state is greater or smaller for agents with suspicions than for those without suspicions, $V_s \ge (<)V_h$, depending on whether ρ is high or low.

Figure 2: Option values of diagnosis and cost of disease control for $\alpha = 0.5$, $\beta = 0.85$, d = 1



¹⁵ The result that in equilibrium agents who do not have suspicions about being infected receive the testing priority when the marginal relative cost of disease control is sufficiently small is not general. Suppose that the cost of disease control effort is given by a sufficiently convex function, c(1-e), where c', c'' > 0. Then in equilibrium untested agents with suspicions about being infected will be more infectious than untested agents without suspicions, $\lambda_h e_{hr}^* < \lambda_s e_{sr}^*$, and the health authority will tend to assign a higher testing priority to agents with suspicions in order to reduce the risk of disease transmission.

The tested agent is also exposed to a risk of disease transmission from the untested agent when disease is not controlled. The expected incremental loss from disease transmission conditional on signal y, D_y , is the loss from disease, d, multiplied by the probability that the tested agent is initially uninfected, $1-\lambda_y$, times the increase in the probability of disease transmission, $\delta \sum_{y \in \{s,h\}} g_y \mathbf{1}_{\lambda_y < \rho} \lambda_y$, since in equilibrium only the untested agent can be infectious. Conversely, the untested agent foregoes the gain from becoming informed about one's own initial disease state but is also shielded from the incremental transmission risk since the tested agent is not infectious.

Thus, whether the expected payoff net of transfers after reporting *s* is smaller or greater than that after reporting *h* depends on the testing priority and the relative magnitudes of the expected gain from better information about one's own initial disease state and the expected loss from disease transmission. It will be convenient to let $\Omega_0^* \equiv [\max[1 - \delta \alpha \beta, k], \lambda_s] \cup [\max[1 - \delta \alpha \frac{\beta}{1-\beta}, \lambda_s], 1 - \delta \alpha \frac{1-\beta}{\beta}].$

| | $	au^*(s,S)$ | $	au^*(s,H)$ | $\tau^*(h,S)$ | $	au^*(h,H)$ | $	au^*(.,U)$ |
|---|---|--|--------------------------------------|---|--------------|
| $ \rho \in (1 - \frac{\delta \alpha (1 - \beta)}{\beta}, 1) $ | $\frac{-\Delta_s^*}{\frac{1}{2}g_s + g_h}$ | $\frac{-\Delta_s^*}{\frac{1}{2}g_s + g_h}$ | 0 | 0 | 0 |
| $\rho \in \Omega_0^*$ | 0 | 0 | 0 | 0 | 0 |
| $\rho \in [k, \max[\lambda_s, 1 - \delta \alpha \beta]]$ | 0 | 0 | $\frac{\Delta_h^*}{\frac{1}{2} g_h}$ | $\frac{\Delta_h^*}{\frac{1}{2} g_h}$ | 0 |
| $\rho \in (1-\alpha, \min[\lambda_s, k]]$ | $\frac{(1-\lambda_s)\Delta_h^*-(1-\lambda_h)\Delta_s^*}{\frac{1}{2}g_s(\lambda_s-\lambda_h)}$ | 0 | 0 | $\frac{\lambda_h \Delta_s^* - \lambda_s \Delta_h^*}{(g_s + \frac{1}{2}g_h)(\lambda_s - \lambda_h)}$ | 0 |
| $ \rho \in (0, \min[1-\alpha, k]] $ | $-\frac{\Delta_s^*}{\frac{1}{2}g_s\lambda_s}$ | 0 | 0 | 0 | 0 |

Table 2: Optimal transfers with two agents

Proposition 1 Suppose that n = 2, m = 1. An optimal transfer policy is given by function τ^* in Table 2. Proof: See the Appendix.

In accordance with Table 2, when ρ is sufficiently small or sufficiently large, agents who report suspected infection must be compensated to sustain credible reporting. When the relative cost of disease control is large, $\rho > 1 - \frac{\delta \alpha (1-\beta)}{\beta}$, both types of agents prefer to have the other agent tested for infection. This happens because the incremental disease transmission risk is greater than the option value that arises from the possibility of undergoing treatment for the discovered existing infection. Because an agent who observed signal s values this option more and is less likely to contract infection from a neighbor than an agent who observed signal h, a subsidy for reporting signal s provides appropriate incentives for reporting s even if doing so increases the likelihood of being selected for testing.

When treatment is cheap, $\rho \leq \min[1-\alpha, k]$, both types of agents prefer to be selected for testing because the incremental disease transmission risk is smaller than the option value that arises from the possibility of saving the cost of treatment if the agent has not been infected in the beginning of the game. An agent with signal *s* values the testing priority less or more than an agent with *h* depending on whether $\rho \leq (>)z$, where $z \equiv (\lambda_h - \delta(1-\alpha)(1-\beta)(\lambda_s - \lambda_h))/(1-\lambda_s + \lambda_h)$. The subsidy for agents who reported *s* and tested positive in Table 1 provides the appropriate incentives for truthful reporting even if doing so can decrease the probability of being selected for testing since the expected amount paid out is greater for agents who suspect infection. For $\rho \leq z$ transfers that are contingent only on the report and not the test outcome are sufficient to satisfy the screening condition: $\tau^*(h,\theta) = 0$, $\tau^*(s,\theta) = -\Delta_s^*/(\frac{1}{2}g_s)$ for $\theta \in \{S,H\}$.

The optimal compensation policy rewards agents who report a lack of suspicions for $\rho \in [k, \max[\lambda_s, 1 - \delta\alpha\beta]]$, or pays out only if the report correctly predicts the outcome of the diagnostic test for $\rho \in (1 - \alpha, \min[\lambda_s, k]]$. In the former case, only an untested agent who suspects infection controls the disease and agents who report suspected infection receive the testing priority. Then the agents who are not suspicious need to be given an additional incentive to reveal their type as they face a relatively small disease transmission risk and have a relatively large option value that arises from the possibility of undergoing treatment if the agent is currently infected. In the latter case, agents without suspicions receive the testing priority, and although both types of agents prefer to be selected for testing, the contingent subsidy assures that the screening condition is satisfied.

A noteworthy feature of the optimal transfer scheme is that credible reporting does not require transfers for $\rho \in \Omega_0^*$. The condition such that the individual and social preferences over the allocation of diagnostic tests are aligned is easier to satisfy when the private information is more precise (β is larger), the prior probability of infection is smaller (α is larger), or disease is more infectious (δ is larger). As β \rightarrow 1, an agent who suspects that he has already been infected is less concerned about the disease transmission risk than an agent who does not suspect infection. As a result, when the disease is highly infectious but rare, $\delta, \alpha \rightarrow 1$, the screening condition is easy to satisfy since the health authority also assigns the testing priority to agents who are more concerned about the existing infection than about being infected by other agents in the future. Since $\Omega_0^* \rightarrow (0,1)$ as $\delta, \alpha, \beta \rightarrow 1$, we obtain:

Corollary For sufficiently large α , β , and δ , the efficient test allocation does not require transfers.

This suggests that health authorities may not need to offer rewards to rely on unverifiable reports to implement a risk-based test allocation in an environment where participation in the disease surveillance program is mandatory, the number of agents is small, disease incursion happens infrequently, there is good knowledge of what the clinical signs of the disease look like, and the disease spreads rapidly.¹⁶

3.3. Disease surveillance with many agents

We now consider problems (2) and (3) when there are more than two agents. We start with two simple cases where the equilibrium disease control effort of each untested agent is independent of the signal profile (the private information of the other agents). Again, for concreteness, we assume that only tested agents receive transfers in equilibrium. By (5), for $\rho \le \lambda_h$ each untested agent controls the disease under any test allocation, and consequently, there is no disease transmission in equilibrium. Therefore, agents who report lack of suspicions receive the testing priority, $(t_s^*(n_s, n_h), t_h^*(n_s, n_h)) = (\min[0, m - n_h], \min[m, n_h])$ as they are more likely to incur unnecessary expenditures on disease control, $V_s = (1 - \lambda_s)c < (1 - \lambda_h)c = V_h$, $n_s + n_h = n$. In this case, the incremental payoff net of transfers from reporting signal *s* rather than *h* for an agent with signal *y* is given by (6), where the incremental probability of selection for testing and disease transmission risk are now given by $P_s^* - P_h^* = -(\sum_{n_s=0}^{n-m-1} B_{n_s,n-1} \frac{m}{n-n_s} + \sum_{n_s=n-m}^{n-1} B_{n_s,n-1}(1 - \frac{m-(n-1-n_s)}{n_s+1})) < 0$ and $D_y = 0$, respectively. With probability $B_{n_s,n-1}$, n_s other agents observe signal *s*, in which case the probability of selection for testing decreases from min $[1, \frac{m}{n-n_s}]$ to max $[\frac{m-(n-1-n_s)}{n_s+1}, 0]$. Since $\Delta_h^* < \Delta_s^* < 0$, by (4), an optimal transfer policy is given by $\tau^*(s, \theta) = -\Delta_s^* / P_s^* > 0 = \tau^*(h, \theta) = \tau^*(y, U)$, y = s, h, $\theta = S, H$, where $P_s^* = \sum_{n_s=n-m}^{n-1} \frac{m-(n-1-n_s)}{n_s+1} B_{n_s,n-1}$.

Now suppose that $\rho > \lambda_s (1 + \delta(1 - \lambda_s))$, so that, by (5), in equilibrium only the tested and initially infected agents control the disease since $\lambda_s (1 + \delta(1 - \lambda_s)) \ge q_y (e_s, e_h, n_s, n_h, t_s, t_h)$ for all $y, e_s, e_h, n_s, n_h, t_s, t_h$. In Lemma 4, it is shown that welfare is maximized when the agents who observe suspicious signs receive the testing priority $(t_s^*(n_s, n_h), t_h^*(n_s, n_h)) = (\min[m, n_s], \max[0, m - n_s])$ because (i) their option value that arises from treating infection when definitely necessary is greater than that for the agents with signal h,

¹⁶ Credible reporting is important because efficient targeted testing dominates *random* testing and *no* testing. In a working paper we show that when reports are *uninformative* testing *decreases* expected welfare relative to the level of expected welfare in equilibrium without testing for n = 2, m = 1, and $\rho \in (\lambda_s, 1 - \max[\alpha \frac{2^{(1-\beta)} - \delta(1-\alpha)(1-2\beta(1-\beta))}{3\alpha+2\beta-4\alpha\beta-1}], \alpha(1-\beta) \frac{2^{-\delta(1-\alpha)}}{3\alpha+3\beta-5\alpha\beta-1}]]$. For such parameter values agents with suspicions too often stop exerting disease control efforts when selection for testing is random.

 $V_{s} = \lambda_{s}(d-c) > \lambda_{h}(d-c) = V_{h}, \text{ and (ii) they are more likely to infect other agents. The incremental payoff net of transfers from reporting$ *s*rather than*h*for an agent with signal*y* $is given by (6), where <math>P_{s}^{*} - P_{h}^{*} = \sum_{m_{s}=0}^{m-1} B_{n_{s},n-1}(1 - \frac{m-n_{s}}{n-n_{s}}) + \sum_{n_{s}=m}^{n-1} B_{n_{s},n-1} \frac{m}{n_{s}+1} \text{ and } D_{y} = \delta \frac{1}{n-1} (\sum_{m_{s}=0}^{m-1} B_{n_{s},n-1}(1 - \frac{m-n_{s}}{n-n_{s}})\lambda_{h} + \sum_{n_{s}=m}^{n-1} B_{n_{s},n-1} \frac{m}{n_{s}+1}\lambda_{s})$ (1 $-\lambda_{y}$) $d/(P_{s}^{*} - P_{h}^{*})$. In this case, when n_{s} other agents observe signal *s* the probability of being selected for testing increases from max[$\frac{m-n_{s}}{n-n_{s}}$, 0] to min[1, $\frac{m}{n_{s}+1}$]. The incremental disease transmission risk, D_{y} , arises since the marginal untested agent that is crowded out when an agent reports suspicions, is infectious. All else equal, as *n* increases, the truth-telling constraint for the agent without suspected infection is more difficult to satisfy because the reporting decision of any single agent has a smaller influence on the incremental risk of disease transmission, D_{y} , while the option value of diagnosis, V_{y} , remains bounded away from zero. Therefore, all else equal, for sufficiently large *n* each agent gains from receiving the testing priority, $\Delta_{s}^{*} > \Delta_{h}^{*} > 0$, and an optimal transfer policy is given by $\tau^{*}(h,\theta) = \Delta_{h}^{*}/P_{h}^{*} > 0 = \tau^{*}(s,\theta) = \tau^{*}(y,U)$, $y = s, h, \theta = H, S$, where $P_{h}^{*} = \sum_{n_{s}=0}^{m-1} \frac{m-n_{s}}{n_{s},n-1}$.

Next we characterize the optimal test allocation and transfer policy for all levels of relative disease control cost in a model with a continuum of agents where there is no uncertainty about the aggregate distribution of signals and each agent takes the disease transmission risk as given and beyond his control. We now let $m \in (0,1)$ denote the maximum share of the population that can be tested, t_y denote the share of the tested agents with signal y, and

$$V_{y}(e_{s}, e_{h}, t_{s}, t_{h}) = \min[\lambda_{y}(1-\rho), (1-\lambda_{y})(\rho - R(e_{s}, e_{h}, t_{s}, t_{h}))]d, \ y \in \{s, h\},$$
(7)

denote the informational benefit of knowing one's current disease state, where $R(e_s, e_h, t_s, t_h) = \delta \sum_{y \in \{s,h\}} (g_y - t_y) \lambda_y e_y$ is the probability that infection spreads times the average disease incidence given that untested agents with and without suspicions exert efforts e_s and e_h , respectively. Note that, by Lemma 1, in equilibrium an agent controls the disease if he tested positive and an agent exerts no efforts if he tested negative. Also, let $q_y(e_s, e_h, t_s, t_h) = \lambda_y + (1 - \lambda_y) R(e_s, e_h, t_s, t_h)$ denote the total conditional probability of becoming infected, let $k(x) = \lambda_h (1 + \delta g_h (2 - \lambda_h - \lambda_s) + (1 - \lambda_s) \delta x) / (1 - \lambda_s + \lambda_h)$, $\hat{k}(x) = \min[q_s(0,1, \min[g_s, m], \max[m - g_s, 0]), k(x)]$, and $\tilde{k} = \lambda_h (1 + \delta \max[g_h - m, 0](1 - \lambda_s)) / (1 - \lambda_s + \lambda_h)$ denote the threshold levels of the relative cost of disease control that determine the switching points of the equilibrium testing and payment policies.

Proposition 2 In a model with a continuum of agents, the equilibrium disease control efforts of the untested agents are given by

$$(e_{s}^{*}, e_{h}^{*}) = \begin{cases} (1,1), \text{ if } \rho > q_{s}(1,1,\min[g_{s},m],\max[m-g_{s},0]) \\ (\frac{\rho-\lambda_{s}-(1-\lambda_{s})\delta(g_{h},\lambda_{h}}{(1-\lambda_{s})\delta(g_{s}-m)\lambda_{s}},1), \text{ if } q_{s}(0,1,\min[g_{s},m],\max[m-g_{s},0]) < \rho \le q_{s}(1,1,\min[g_{s},m],\max[m-g_{s},0]) \\ (0,1), \text{ if } q_{h}(0,1,\max[m-g_{h},0],\min[m,g_{h}]) < \rho \le q_{s}(0,1,\min[g_{s},m],\max[m-g_{s},0]) \\ (0,\frac{\rho-\lambda_{h}}{(1-\lambda_{h})\delta(g_{h}-m)\lambda_{h}}), \text{ if } q_{h}(0,0,\max[m-g_{h},0],\min[m,g_{h}]) < \rho \le q_{h}(0,1,\max[m-g_{h},0],\min[m,g_{h}]) \\ (0,0), \text{ if } \rho \le q_{h}(0,0,\max[m-g_{h},0],\min[m,g_{h}]) \end{cases}$$
(8)

the optimal test allocation policy is given by

$$(t_{s}^{*}, t_{h}^{*}) = \begin{cases} (\min[g_{s}, m], \max[m - g_{s}, 0]), \text{ if } \rho \ge \hat{k}(\min[m, 2g_{s} - m]) \\ (\frac{1}{2}(m + \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}}), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}}))), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{s})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{h})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h} - \lambda_{h})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h})}{(1 - \lambda_{s})\delta \lambda_{h}})), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda_{h})}{(1 - \lambda_{s})\delta \lambda_{h}}))), \frac{1}{2}(m - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho) - \delta g_{h}\lambda_{h}(2 - \lambda$$

and the optimal transfer policy is given by $\tau^*(s,H) = \tau^*(h,S) = 0$ and transfers in Table 3, where $V_y^* = V_y(e_s^*, e_h^*, t_s^*, t_h^*)$, and $P_y^* = t_y^* / g_y$, $y \in \{s, h\}$.

| | $	au^*(s,S)$ | $	au^*(h,H)$ | $	au^*(s,U)$ | $	au^*(h,U)$ |
|--|--|---|---|---|
| $\rho \in (\hat{k}(m(g_s - g_h), \mathbf{l}))$ | 0 | 0 | 0 | $\frac{P_s^* - P_h^*}{1 - P_h^*} V_h^*$ |
| $\rho \in (\tilde{k}, \hat{k}(m(g_s - g_h)))$ if $t_s^* > 0$ then | $(\frac{P_h^*}{P_s^*}-1)\frac{V_s^*(1-\lambda_h)-V_h^*(1-\lambda_s)}{\lambda_s-\lambda_h}$ | $(1-\frac{P_s^*}{P_h^*})rac{V_s^*\lambda_h-V_h^*\lambda_s}{\lambda_s-\lambda_h}$ | 0 | 0 |
| if $t_s^* = 0$ then | 0 | $\frac{\overline{V_s^*-V_h^*}}{\lambda_s-\lambda_h}$ | $P_h^* \frac{V_s^*(1-\lambda_h) - V_h^*(1-\lambda_s)}{\lambda_s - \lambda_h}$ | 0 |
| $\rho \in (0, \widetilde{k})$ | 0 | 0 | $\frac{\boldsymbol{P}_h^* - \boldsymbol{P}_s^*}{1 - \boldsymbol{P}_s^*} \boldsymbol{V}_s^*$ | 0 |

| Table 3: Optimal | transfers with | a continuum | of agents |
|------------------|----------------|-------------|-----------|
|------------------|----------------|-------------|-----------|

Agents with or without suspicions receive the testing priority, $P_s^* > (\leq) P_h^*$, depending on whether $\rho > (\leq) \hat{k}(m(g_s - g_h))$. Because function $\hat{k}(x)$ is non-decreasing it follows that agents with suspicions are more or less likely to receive the testing priority as the diagnostic capacity increases depending on whether $g_s < (>)g_h$. The incremental payoff net of transfers from reporting *s* rather than *h* for an agent with signal *y* is again given by (6), where $V_y = V_y(e_s^*, e_h^*, t_s^*, t_h^*)$ and $D_y = 0$. When agents with suspicions receive the

testing priority, agents without suspicions receive a subsidy for truthful reporting. Because agents with suspicions get a greater informational benefit from testing they are not tempted to misrepresent their signal in order to become eligible for this transfer. On the other hand, when agents without suspicions receive the testing priority and the relative cost of disease control is sufficiently small, agents with suspicions receive a subsidy for truthful reporting. In this case, agents without suspicions get a greater informational benefit from testing and are not tempted by the transfer.

However, if the informational benefit of diagnostic testing for the agents with suspicions is slightly greater than that for the agents without suspicions, screening condition requires transfers contingent on the outcome of the test. Then untested agents without suspicions do not control disease and impose a negative externality as they remain potentially infectious while untested agents with suspicions control the disease and are not infectious in equilibrium. The health authority takes this into account and assigns the testing priority to the agents without suspicions in order to reduce the risk of disease transmission. In this case, credible reporting requires a payment for reporting a lack of suspicions and testing negative that, on average, pays out more to the agents without suspicions as well as a payment for reporting suspicions that may be contingent on testing positive.

It is also of interest to explore how the characterization of the optimal test allocation and transfer policy depends on the disease management policy. Consider a joint disease surveillance and management program where the health authority collects reports, allocates tests, and regulates disease control efforts for all agents. In Appendix B, we show that the set of parameter values for which credible reporting can be sustained without transfers expands under the joint efficient testing and disease control policy. There are two reasons. First, under the efficient disease control policy the revelation of private information may not be necessary to achieve a socially efficient test allocation. Second, the agent's report now influences not only the test allocation but also the *regulatory constraints* on the disease control efforts if the agent remains untested. These constraints on the agent's disease control choices depend on the agent's report and tend to align the individual and social preferences over the test allocation. As a result, agents gain less from the misrepresentation of their private information and truthful reporting may not require transfers even when there are many agents.

4. Extensions

4.1. Voluntary Participation in Disease Surveillance Program

So far we assumed that the participation in the disease surveillance program is mandatory, that is, an agent reports either h or s and cannot opt out of diagnostic testing if he has been selected for testing by the health authority. In reality, the participation in public health monitoring programs may be voluntary, and the health authority may not be able to test an individual or one's property for carrying disease without consent for

diagnostic testing. So we now suppose that agents can opt out of the diagnostic testing program. The ex ante participation constraint is written as one of the incentive compatibility conditions in (4) for r = o: $\Pi_{yy}^* + T_{yy}^* \ge \Pi_{yo}^* + T_{yo}^*$ for all $y \in \{s, h\}$.

In the case with two agents (n = 2, m = 1), as shown in Lemmas 1 and 2, (a) the equilibrium outcome does not depend on the beliefs of the tested agent about the private information of the untested (or non-participating) agent, and (b) the health authority, that is constrained to test the single agent who chose to participate in the disease surveillance program, can do no better than test the participating agent with probability one. Therefore, the participation constraints become

$$\Pi_{yy}^* + T_{yy} \ge v - d \min[\rho, \lambda_y] \text{ for } y = s, h,$$
(10)

where, by Lemma 1 and 2, $\Pi_{yy}^* + T_{yy} = v - \sum_{y' \in \{s,h\}} (p_y^*(1_{y'=s})((c - \tau(y, S))\lambda_y + (\delta 1_{\lambda_{y'} < \rho} \lambda_{y'} d - \tau(y, H)) (1 - \lambda_y))$ + $(1 - p_y^*(1_{y'=s})) (\min[c, \lambda_y d] - \tau(y, U))g_{y'}$ is the equilibrium payoff when all agents participate in the disease surveillance program. The right-hand side is the payoff for an agent who unilaterally opts out of the diagnostic testing program and is ineligible for payments. A transfer policy that satisfies the incentive compatibility and participation constraints is given by

$$\tau(r,\theta) = \begin{cases} 0, \text{if } r = o \\ \tau^*(r,\theta), \text{if } r \in \{s,h\} \text{ and } \rho \le \max[\min[k,\lambda_s], 1 - \delta(1 - \lambda_h)] \\ \hat{\tau}(r,\theta) + \tau^*(r,\theta), \text{if } r \in \{s,h\} \text{ and } \rho > \max[\min[k,\lambda_s], 1 - \delta(1 - \lambda_h)] \end{cases}$$
(11a)

where $\tau^*(r,\theta)$ is given in Table 2, and

$$\hat{\tau}(r,\theta) = \begin{cases} 0, \text{if } (r,\theta) \in \{(s,S), (h,S), (s,U), (h,U)\} \\ \frac{\delta(1-\lambda_h)\lambda_h - \lambda_h(1-\rho)}{1-\lambda_h} d, \text{if } (r,\theta) = (h,H) \\ \frac{\frac{1}{2}g_h}{g_h + \frac{1}{2}g_s} \frac{\delta(1-\lambda_h)\lambda_h - \lambda_h(1-\rho)}{1-\lambda_h} d, \text{if } (r,\theta) = (s,H) \end{cases}$$
(11b)

The expected additional payment in (11), $E[\hat{\tau}(h,\theta_{i,0}) | Y_i = h] = \frac{1}{2}g_h(\delta(1-\lambda_h)\lambda_h - \lambda_h(1-\rho))d$, equals the expected incremental payoff from opting out of the diagnostic testing program for an agent who does not observe suspicious signs when agents with suspected infection receive the testing priority.

Note that the participation constraint in (10) binds at optimum for y = h when $\rho \in \Omega_0^*$, where the parameters are such that the truth-telling incentive compatibility conditions are slack under mandatory participation in diagnostic testing. This happens because, by opting out of the diagnostic testing program, an agent without suspected infection can avoid the disease transmission risk (and forego the informational benefit of testing) more frequently than he can by participating in the program. On the other hand, when

the health authority assigns the testing priority to agents without suspected infection and the relative cost of effort is sufficiently small, the participation constraints do not bind. Then the equilibrium probability that the untested agent is infectious is so small that both types of agents prefer to be selected for testing.

For sufficiently large n, the participation constraints also do not bind in equilibrium since agents have little concern about the effect of their participation in the disease surveillance program on the disease transmission risk in a large population. Then agents cannot increase their payoffs by being excluded from the testing program, since it entails completely foregoing the informational benefits of testing and possible eligibility for transfers. To recap, when the participation in the disease surveillance program is voluntary, efficient test allocation always requires some transfers since *all* of the participation and the incentive compatibility constraints cannot be simultaneously slack in equilibrium.

4.2. Privately Collected Information about Disease Incidences

We now consider equilibrium in the private information regime where the health authority observes all the reports and test results, each tested agent only observes the outcome of her own test, and none of the agents observe the reports and test outcomes of the other agents. It is easy to show that the equilibrium outcome is the same in the private and public information regimes as long as (i) the number of agents is sufficiently small or sufficiently large, or (ii) the relative cost of disease control is sufficiently small or sufficiently large.

For n = 2, m = 1, a lack of public disclosure of the test outcomes and reports has no effect on the equilibrium outcome, because, by (5), the equilibrium disease control strategies for the tested agent and untested agent are independent of one another's private information, $e_{yr}^*(n_s, n_h, t_s, t_h) = 1_{\lambda_y < \rho}$, where $n_s + n_h = 2$ and $t_s + t_h = 1$. On the other hand, when the number of agents is large, nondisclosure of signal profile and test outcomes is also inconsequential since the idiosyncrasies in the aggregate level of suspicions and test outcomes among the agents are washed away by the law of large numbers. In a model with a continuum of agents there is no uncertainty about the shares of tested and untested agents with and without suspicions and about the average disease incidence among untested agents. Consequently, the equilibrium outcome in the private information regime where agents do not observe each other's report and test results converges in probability to the equilibrium outcome in the public information regime as $n \rightarrow \infty$. Similarly, by Lemma 1, when ρ is sufficiently small or large, the disease control strategy for each agent is independent of the private information of the other agents, and the equilibrium outcomes in the regimes with public and private information about the disease incidences coincide as well. However, when $2 < n < \infty$ and ρ is in an intermediate range, in order to evaluate the effect of withholding the reports and test results, we need

to specify agents' equilibrium beliefs about one another's private information that take into account information about the aggregate report profile that is signaled to each agent by her own selection or non-selection for testing.

Another plausible scenario is that the health authority can choose what information about disease incidences that it has collected to disclose to each agent. In this case, the health authority can attempt to convince agents to exercise more care in controlling the disease. However, applying the unraveling argument (Milgrom 1981), it is easy to show that the health authority cannot raise social welfare by withholding the agents' reports and test outcomes from the agents. Suppose that the health authority sends agent-specific messages. Then in equilibrium the health authority will fully disclose all information about the initial disease states and reports based on the following considerations. First, withholding the test result and the reports in the messages sent to the tested agents is not sequentially optimal because each agent who learns his actual initial disease status chooses the socially optimal disease if each untested agent expects that the tests will be allocated in accordance with the efficient testing policy $(t_s^*(n_s, n_h), t_h^*(n_s, n_h))$ and believes that any agent with an undisclosed report or test result poses the least possible disease transmission risk.

We also did not consider the possibility that agents are allowed to communicate amongst themselves. If the relative cost of disease control is not too small, each agent benefits when the other agents exert greater efforts to control the disease. As a result, communication among agents will not be credible because each agent will try to convince the other agents that she is potentially infectious in order to incentivize them to exercise greater care in controlling the disease and reduce its own risk of future infection.

4.3. Allocation of Diagnostic Capacity by a For-Profit Firm

In the previous analysis we have assumed that the diagnostic capacity is allocated by the public health organization and that agents do not pay for diagnostic testing themselves. We now suppose that a for-profit firm allocates the diagnostic capacity. For concreteness, we also assume that reports and test results are not publicly disclosed and the cost of the diagnostic capacity is sunk.¹⁷ The model remains unchanged in other aspects. After agents observe signals, the firm offers each agent a diagnostic "priority service" contract with a two-dimensional price schedule $(\tau(r,\theta), P_r)_{r\in\{s,h\},\theta\in\{S,H,U\}}$, and each agent simultaneously decides whether to accept or reject it. The agent's report (that is, the contract choice) determines the expected probability of being selected for testing $P_r = \sum_{n_s=0}^{n-1} p_r(n_s)B_{n_s,n-1}$. The price schedule, $\tau(r,\theta)$, assigns a payment to each possible combination of the report (suspected infection or no suspected infection) and the outcome of the test (not tested, tested negative, or tested positive). Then the firm allocates diagnostic tests

¹⁷ All results in this section continue to hold if reports and test results are public information.

among the agents who accepted a diagnostic service contract, the firm and each tested agent observe the outcome of the test $\theta \in \{S, H, U\}$, and each participating agent pays $\tau(r, \theta)$ to the firm.¹⁸ If an agent rejects the contract, he does not pay anything for any test allocation and does not get tested. Finally, as in the model in Section 2, agents non-cooperatively choose disease control efforts and the final disease states are realized.

Note that the test outcome is *both* correlated with the customer's private information about valuation for the testing priority and is publicly observed by the firm and the customer (and is verifiable and contractible). Consequently, with an appropriate price schedule the firm can extract the full informational benefit of diagnostic testing since an agent who rejects a contract is not tested with probability one. The next proposition establishes that whenever the disease transmission risk is sufficiently small, the profitmaximizing and socially efficient test allocations coincide.

Proposition 3 (i) For each set of values α, β, n, ρ , there exists a threshold infectiousness $\hat{\delta} > 0$ such that for all $\delta \leq \hat{\delta}$ the equilibrium outcome is efficient. (ii) For each set of values α, β, n, δ , there exists a threshold relative disease control cost $\hat{\rho} > \lambda_h$ such that for all $\rho \leq \hat{\rho}$ the equilibrium outcome is efficient. Proof: See the Appendix.

Suppose that the disease is not infectious or the relative cost of disease control is smaller than the initial probability of carrying disease by an agent without suspicions. Then there is no disease transmission risk in equilibrium, and the social value of testing, which only consists of the informational benefit of diagnosis, $V_y = \min[\lambda_y(1-\rho), (1-\lambda_y)\rho]d$, is fully internalized by each tested agent. By offering the price schedule $\tau^f(o,U) = \tau^f(s,U) = \tau^f(h,U) = 0$,

$$\tau^{f}(y,S) = \frac{(1-\lambda_{h})V_{s}-(1-\lambda_{s})V_{h}}{\lambda_{s}-\lambda_{h}}, \text{ and } \tau^{f}(y,H) = \frac{\lambda_{s}V_{h}-\lambda_{h}V_{s}}{\lambda_{s}-\lambda_{h}} \text{ for } y = s,h,$$
(12)

the firm is able to extract the entire social surplus from testing, where superscript f denotes the equilibrium where the firm allocates the diagnostic capacity. Consequently, the firm chooses the test allocation that maximizes social welfare, and the equilibrium outcome is efficient.

However, the firm cannot extract the full social value of the reduction in the disease transmission risk unless the firm is able to commit not to offer diagnostic tests to the agents who accept the contract if some agents reject the contract. As a result, the profit-maximizing and efficient test allocation may diverge if the disease transmission risk is significant.

¹⁸ Our results will not change if the test outcomes are publicly observed because tested agents are not infectious in equilibrium.

Proposition 4 If $\rho \ge \max[1 - \frac{\delta}{n-1}(1 - \lambda_s), \lambda_s(1 + \delta(1 - \lambda_s))]$, the firm tests too few agents with suspicions. Proof: See the Appendix.

Suppose that the disease is sufficiently infectious, disease control is sufficiently costly, and the number of agents is not too great. Then the option value that arises from the possibility of applying treatment if the disease is discovered early is small, and each agent prefers not to crowd out the allocation of the diagnostic capacity to other agents in order to reduce the number of infectious agents. Therefore, the firm cannot attract all agents to participate in the market for diagnostic services without net subsidies for participation for some agents. However, the firm achieves greater profits in equilibrium where a subset of agents stays out of the market, so that too few agents *with* suspected infection are tested with positive probability since under the socially efficient solution the agents with suspected infection receive the testing priority.

On the other hand, comparing the profit-maximizing and the socially efficient test allocation as $n \rightarrow \infty$ (see Proposition 2), we establish the following.

Proposition 5 In a model with a continuum of agents, the firm tests too many agents with suspicions for $\hat{k}(-\min[m,2g_h-m]) - g_h(1-\lambda_h)\delta\lambda_h d/(1-\lambda_s+\lambda_h) < \rho < \hat{k}(\min[m,2g_s-m])$. Otherwise, the profitmaximizing and socially efficient test allocations coincide. Proof: See the Appendix.

The profit-maximizing test allocation maximizes the total informational benefit from testing that accrues to the tested agents without taking into account the welfare of the untested agents. Nonetheless, in a model with a continuum of agents where each agent prefers to be selected for testing, the profit-maximizing and efficient test allocations coincide whenever the relative disease control cost is either sufficiently small or sufficiently large. For the intermediate values of the disease control cost the firm tests too few agents *without* suspicions as the firm only takes into account the disease transmission risk faced by the initially uninfected tested agents. The firm assigns an excessively high testing priority to the agents with suspicions because they derive a greater informational benefit from testing even though in equilibrium the untested agents with suspicions are not infectious as they exert the full disease.

When the test outcome or testing priority are not contractible, the result that the firm allocates diagnostic tests inefficiently or undersupplies the market, becomes generic (Laffont and Martimort 2002). As in the standard model of monopolistic screening subject to capacity constraints, the allocation (testing priority) tends to be distorted in order to reduce agents' information rents (Wilson 1989, Spulber 1994). Endogenous type-dependent payoff externalities further compound the firm's rent extraction problem since strategic agents take into account that their use of the diagnostic capacity crowds out its utilization by other

agents, which increases the disease transmission risk when the untested agents do not exert full effort to control the disease (Weber 2012).

4.4. Asymmetric Agents

In the previous analysis of test allocation by the health authority, we assumed that agents are identical in all aspects except for heterogeneous private beliefs about the probability of being infected. In an agricultural disease or cooperative community context, it is of interest to understand how incentives to report suspected infection differ among small and large agents. So we now assume that agents can comprise one or more units of a susceptible population. Suppose that there are just two agents: S (small) and L (large). Agent S comprises 1 unit as in the basic model and agent L comprises n-1 units of the susceptible population, where $n \ge 3$. This has two effects on the overall efficiency of the disease control decisions. On the one hand, the large agent is more likely to control the disease when it is socially efficient to do so. On the other hand, the small agent faces a smaller transmission risk from the large agent than from a group of independent n-1 small agents and is, therefore, less likely to efficiently control the disease.

We now turn to the reporting incentives. Note that the small agent tends to take the overall transmission risk as given since the average disease incidence in the large agent's units is more stable than her own disease incidence and a single test is unlikely to significantly reduce the disease transmission risk from the large agent to the small agent. Therefore, the small agent prefers to be selected for testing. The large agent may also prefer to educate the small agent about her own initial state of health so as to decrease the probability of becoming infected by the small agent who would otherwise tend to exercise too little care in controlling the disease. On the other hand, the large agent may value information about the initial state of health of her units more than the small agent because the large agent can coordinate disease control across multiple units more efficiently than n-1 non-cooperative small agents.

A new feature of the model with asymmetric agents is the agent-specific report credibility whereas some, but not all, agents send informative reports in equilibrium. For sufficiently large n, in equilibrium, without transfers it is efficient to rely on the reports submitted by the large agent and ignore the report submitted by the small agent. This happens because the interests of the large agent are almost perfectly aligned with that of the social planner and the large agent is appropriately incentivized to reveal her observations of local health conditions, whereas the small agent may benefit from misrepresenting her probability of being infected in order to manipulate the assignment of testing priorities.

4.5. Costly Reporting Procedure

So far we also ignored the cost of the reporting procedure itself. For example, such costs may arise when an individual needs to visit the health facility to report suspicions of an illness or a farmer needs to suspend his production operations during a veterinary inspection. So now we suppose that each agent incurs cost A_y to report signal y, where $A_s > 0 = A_h$. The incremental payoff net of transfers for reporting signal s rather than signal h for an agent with signal y is now given by $\Delta_y^* - A_s$. If the cost of the reporting procedure is sufficiently small, a transfer policy $\tau(r,\theta) + A_r$ will make truthful reporting incentive compatible and satisfy the participation constraints. Also note that an increase in the direct exogenous costs of reporting suspected infection can make it less or more difficult to sustain truthful reporting without transfers. Although the reporting cost makes it easier to satisfy the truth-telling constraint for an agent who observed signal h, $\Delta_h^* - A_s \leq 0$, it makes more difficult to satisfy the truth-telling constraint for an agent who observed signal s, $\Delta_h^* - A_s \geq 0$. If the cost of the reporting procedure is too great, it may be socially optimal to implement a random or no testing policy in lieu of targeted testing to reduce the costs of collecting information about prior infection risks.

5. Conclusions

This paper has studied how certain environmental and institutional characteristics shape the design of a disease surveillance program. Credible reporting of clinically suspect cases does not require transfers from the health authority when testing is mandatory for selected agents and the agents who are concerned more about the risk of prior infection than becoming infected by their neighbors in the future are also the ones who generate the greatest social benefit from utilizing the diagnostic capacity. In general, the circumstances under which the agents' and health authority's preferences over the testing priorities coincide are rather restrictive. However, it is also shown that credible reporting of suspected infection is easier to sustain when the health authority administers a joint diagnostic testing and disease management policy. Furthermore, a for-profit firm will not necessarily allocate the diagnostic capacity inefficiently if it can offer a price schedule for different testing priorities and test outcomes.

Our model can be extended in several additional ways. In our setting, the negative externalities arise solely due to uncertainty about initial disease incidence among the untested agents. By allowing for a convex cost of disease control, we could study the design of a disease surveillance program when not only untested agents but also agents who test positive for infection impose negative externalities on the other agents (Olmstead and Rhode 2004). Also, by allowing agents to control their initial disease status, we could study the effects of early detection on the preventive actions. Another interesting extension would be to consider a more general private information structure and reports that are partially informative. Also left for future research is the design of a dynamic surveillance program that incentivizes reporting of clinically suspect cases and the allocation of diagnostic resources in a continuous time susceptible–infected–recovered model of an infectious disease (Anderson and May 1979).

Appendix A: Proofs

Lemma 1 Suppose that all agents participate in the diagnostic testing program and reporting is truthful. Then the equilibrium disease control strategies are given by (5).

Proof of Lemma 1 Note that fully protecting against the disease is the dominant strategy for an agent who tested positive, that is, $e_{yr}^*(n_s, n_h, t_s, t_h) = 0$ for y = S, and an agent who tested negative is not infectious. By (1), in a symmetric equilibrium where agents with the same information adopt the same strategy, the optimality condition for an untested agent with signal y=s,h is given by

 $\lambda_{y} + (1 - \lambda_{y})R_{y}(e_{ss}^{*}(n_{s}, n_{h}, t_{s}, t_{h}), e_{hh}^{*}(n_{s}, n_{h}, t_{s}, t_{h}), n_{s}, n_{h}, t_{s}, t_{h}) > (=, <)\rho \text{ as } e_{yr}^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = 0 (\in (0, 1), 1), \quad (A1)$ where $R_{y}(e_{s}, e_{h}, n_{s}, n_{h}, t_{s}, t_{h}) = \frac{\delta}{n-1} (\sum_{y' \in \{s,h\}} (n_{y'} - t_{y'} - 1_{y=y'}) \lambda_{y'} e_{y'}$. Therefore, the strategies in (5) simultaneously satisfy the optimality conditions for all untested agents.

An agent who tested negative chooses the disease control effort e_H to maximize $v - R_H(e_{ss}^*(n_s, n_h, t_s, t_h), e_{hh}^*(n_s, n_h, t_s, t_h), n_s, n_h, t_s, t_h)e_H d - c(1 - e_H)$, or $v - \frac{\delta}{n-1} \left(\sum_{y \in \{s,h\}} (n_y - t_y)\lambda_y e_{yy}^*(n_s, n_h, t_s, t_h)\right)e_H d - (1 - e_H)c$. Differentiating with respect to $1 - e_H$ yields

$$(\frac{\delta}{n-1}\sum_{y\in\{s,h\}}(n_{y}-t_{y})\lambda_{y}e_{yy}^{*}(n_{s},n_{h},t_{s},t_{h})-\rho)d<0.$$
(A2)

The last inequality can be shown as follows. Suppose that (A2) does not hold. Then it must that $(n_y - t_y)\lambda_y e_{yy}^*(n_s, n_h, t_s, t_h) > 0$ for some $y \in \{s, h\}$, so that, by (A1), we have $\lambda_y + \frac{\delta}{n-1}\sum_{y' \in \{s,h\}}(n_{y'} - t_{y'} - 1_{y'=y})$ $\lambda_{y'}e_{y'y'}^*(n_s, n_h, t_s, t_h) - \rho \leq 0$ for some $y \in \{s, h\}$. But this yields a contradiction since $\frac{\delta}{n-1}\sum_{y' \in \{s,h\}}(n_{y'} - t_{y'})$ $\lambda_{y'}e_{y'y'}^*(n_s, n_h, t_s, t_h) \leq \lambda_y + \frac{\delta}{n-1}\sum_{y' \in \{s,h\}}(n_{y'} - t_{y'} - 1_{y'=y})\lambda_{y'}e_{y'y'}^*(n_s, n_h, t_s, t_h)$. Therefore, $e_{Hr}^*(n_s, n_h, t_s, t_h) = 1$ is optimal for any r, n_s, n_h, t_s, t_h . Agents who tested negative exert no effort because they have the smallest expected return to disease control and the equilibrium risk of getting infected in the future is too small to warrant preemptive treatment.

Note that the test outcomes do not influence the levels of disease control efforts of the untested agents that satisfy (A1). Therefore, the assumption that the equilibrium disease control strategy is a function of the agent's signal, y, report, r, the number of agents with and without suspicions, n_s , n_h , and the number of untested and tested agents of each type, t_s , t_h , is without loss of generality.

Lemma 2 Suppose that n = 2, m = 1, and agents reveal their signals to the health authority. Then the test allocation policy in Table 1 maximizes welfare.

Proof of Lemma 2 First, we determine an optimal test allocation provided that one of the agents is tested, $\max[t_s, t_h] = 1$. Second, we establish an upper bound on the expected payoffs when none of the agents are tested. Third, we show that the expected welfare is greater under targeted testing than under no testing. Step 1. By Lemma 1, in equilibrium the sum of the expected profits net of transfers *before* testing but after the agents revealed their signals is

$$W^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = 2\nu - \begin{cases} (1 + \lambda_{y_{1}})c, \text{ if } \lambda_{y_{2}} > \rho \\ \lambda_{y_{1}}(c + \lambda_{y_{2}}d) + (1 - \lambda_{y_{1}})(\delta + 1)\lambda_{y_{2}}d, \text{ if } \lambda_{y_{2}} \le \rho \end{cases},$$
(A3)

where $(y_1, y_2) = (s, s)$ if $(n_s, n_h, t_s, t_h) = (2,0,1,0)$, $(y_1, y_2) = (s, h)$ if $(n_s, n_h, t_s, t_h) = (1,1,1,0)$, $(y_1, y_2) = (h, s)$ if $(n_s, n_h, t_s, t_h) = (1,1,0,1)$, and $(y_1, y_2) = (h, h)$ if $(n_s, n_h, t_s, t_h) = (0,2,0,1)$. Therefore, $(t_s^*(1,1), t_h^*(1,1)) = (1,0)$ or (0,1) depending on whether $W^*(1,1,1,0) \ge (\le)W^*(1,1,0,1)$, which is equivalent to $\min[k, \lambda_s] \le (>)\rho$. Step 2. If the social planner knows the realizations of the private signals, does not test, and assigns efficient disease control efforts to each agent, welfare is given by

$$W^{M}(n_{s}, n_{h}, 0, 0) = \max_{e_{1}, e_{2} \in [0, 1]} \sum_{i=1, 2} v - d(\lambda_{y_{i}} + (1 - \lambda_{y_{i}}) \delta e_{-i} \lambda_{y_{-i}}) e_{i} - c(1 - e_{i})$$

= 2v - d min[2\rho, \rho + min[\lambda_{y_{1}}, \lambda_{y_{2}}], (\lambda_{y_{1}} + \lambda_{y_{2}})(1 + \delta) - 2\lambda_{y_{1}} \lambda_{y_{2}} \delta], (A4)

where -i=1 if i=2 and -i=2 if i=1, $(y_1, y_2) = (s, s)$ if $(n_s, n_h, t_s, t_h) = (2,0,0,0)$, $(y_1, y_2) = (s, h)$ if $(n_s, n_h, t_s, t_h) = (1,1,0,0)$, and $(y_1, y_2) = (h, h)$ if $(n_s, n_h, t_s, t_h) = (0,2,0,0)$.

Step 3. Substituting from (A3) it follows that

$$W^{M}(n_{s}, n_{h}, 0, 0) \leq \max_{\substack{t_{s} \leq n_{s}, t_{h} \leq n_{h} \\ t_{s} + t_{s} = 1}} W^{*}(n_{s}, n_{h}, t_{s}, t_{h}).$$
(A5)

But if none of the agents are tested, welfare is bounded by $W^{M}(n_{s}, n_{h}, 0, 0)$, that is,

$$W^*(n_s, n_h, 0, 0) \le W^{\mathrm{M}}(n_s, n_h, 0, 0).$$
 (A6)

Combining (A5) and (A6) completes the proof.

Note that we showed a bit more since from (A4) and (A5) it follows that welfare in equilibrium with credible reporting and optimal testing is greater than welfare in any equilibrium where reports are not informative (agents do not reveal their private information) and none of the agents are tested. \blacksquare

Lemma 3 For n = 2 and m = 1, we have $\Delta_y^* = (P_s^* - P_h^*)(V_y - D_y)$, where $P_s^* - P_h^* = t_s^*(1,1) - \frac{1}{2}$, $V_y = \min[\lambda_y(1-\rho), (1-\lambda_y)\rho]d$, and $D_y = \delta(1-\lambda_y) \sum_{y \in \{s,h\}} g_y \mathbf{1}_{\lambda_y < \rho} \lambda_y d$.

Proof of Lemma 3 By Lemmas 1 and 2, the expected payoff net of transfers when an agent with signal y reports r can be written as

$$\Pi_{yr}^{*} = v - d \sum_{y' \in \{s,h\}} (p_{r}^{*}(1_{y'=s})(\rho\lambda_{y} + \delta 1_{\lambda_{y'} < \rho}\lambda_{y'}(1 - \lambda_{y})) + (1 - p_{r}^{*}(1_{y'=s}))(1_{\lambda_{y} \ge \rho}\rho + 1_{\lambda_{y} < \rho}\lambda_{y}))g_{y'}.$$
 (A7)

Substituting (A7) the difference between the expected payoffs from reporting signals s and h for an agent with signal y is given by

$$\Delta_{y}^{*} = \Pi_{ys}^{*} - \Pi_{yh}^{*} = d \sum_{y' \in \{s,h\}} (p_{h}^{*}(1_{y'=s}) - p_{s}^{*}(1_{y'=s}))(\rho\lambda_{y} + \delta 1_{\lambda_{y'} < \rho} \lambda_{y'}(1 - \lambda_{y}))$$

$$- (p_{h}^{*}(1_{y'=s}) - p_{s}^{*}(1_{y'=s}))(1_{\lambda_{y} \ge \rho} \rho + 1_{\lambda_{y} < \rho} \lambda_{y})$$

$$= d \sum_{y' \in \{s,h\}} (p_{h}^{*}(1_{y'=s}) - p_{s}^{*}(1_{y'=s}))(\rho(\lambda_{y} - 1_{\lambda_{y} \ge \rho}) - 1_{\lambda_{y} < \rho} \lambda_{y} + \delta 1_{\lambda_{y'} < \rho} \lambda_{y'}(1 - \lambda_{y}))g_{y'}$$

$$= d (\frac{1}{2} - p_{s}^{*}(0))(\rho(\lambda_{y} - 1_{\lambda_{y} \ge \rho}) - 1_{\lambda_{y} < \rho} \lambda_{y} + \delta (1 - \lambda_{y})(1_{\lambda_{h} < \rho} \lambda_{h} g_{h} + 1_{\lambda_{s} < \rho} \lambda_{s} g_{s})) = (P_{s}^{*} - P_{h}^{*})(V_{y} - D_{y}).$$

Proof of Proposition 1 Since agents are risk-neutral and by Lemma 2, $P_r^* = \sum_{y \in \{s,h\}} p_r^* (1_{y=s}) g_y > 0$ for

r = s, h, without loss, we assume that only tested agents receive transfers in equilibrium, that is, $\tau^*(r,U) = 0$ for r = s, h. By Lemmas 1, 2, and 3, the social planner's problem (3) now becomes

$$\min_{\tau(.,.)\geq 0} 2 \sum_{y \in \{s,h\}} T_{yy}^* g_y$$
(A9a)

subject to the IC constraints (screening condition)

$$\Delta_s^* + L_s^* \ge 0 \ge \Delta_h^* + L_h^*, \tag{A9b}$$

where $T_{yr}^* = (\lambda_y \tau(r, S) + (1 - \lambda_y) \tau(r, H)) P_r^*$ and $L_y^* = T_{ys}^* - T_{yh}^*$ is the *incremental* expected transfer from reporting signal *s* rather than *h* for an agent who observed signal *y*. We will consider the following three cases that are differentiated by the equilibrium level of the disease control effort exercised by an untested agent. Case 1. $\lambda_h \ge \rho$. In this case, the incremental payoffs (net of transfers) from reporting *s* for an agent who observed signal *y* is $\Delta_y^* = -\frac{1}{2}(1-\lambda_y)c < 0$, y = s, h, because agents who report signal *h* receive the testing priority and both types of untested agents are not infectious. Hence, only agents who observed signal *s* need to be rewarded for truth-telling. By (A9b), the smallest transfers that satisfy the IC constraints are

$$\tau^{*}(h,S) = \tau^{*}(h,H) = 0, \\ \tau^{*}(s,S) = \frac{-\Delta_{s}^{*}(1-\varepsilon)}{\frac{1}{2}g_{s}\lambda_{s}}, \\ \tau^{*}(s,H) = \frac{-\Delta_{s}^{*}\varepsilon}{\frac{1}{2}g_{s}(1-\lambda_{s})} \text{ for any } \varepsilon \in [0,1].$$
(A10)

Under the transfer scheme in (A10), the incentive compatibility constraint for an agent who observed signal *s* binds while an agent who observed signal *h* strictly prefers to report *h*: $\Delta_s^* + L_s^* = 0 > \Delta_h^* + L_h^*$, where $L_s^* = L_h^* = -\Delta_s^*$. Note that for $\varepsilon = 1 - \lambda_s$ the transfer policy in (A10) becomes

$$\tau^*(h,S) = \tau^*(h,H) = 0, \ \tau^*(s,S) = \tau^*(s,H) = -\frac{1}{\frac{1}{2}g_s}\Delta_s^*,$$
(A11)

so that optimal transfers do not need to be contingent on the test result. Case 2. $\lambda_h < \rho \leq \lambda_s$. First, we suppose that the agents who report *h* receive the testing priority, $\rho < k$. Then we have $\Delta_s^* = \frac{1}{2} (\delta(1-\alpha)(1-\beta)(1-\lambda_s)d - (1-\lambda_s)c) < 0$ and $\Delta_h^* = \frac{1}{2} (\delta(1-\alpha)(1-\beta)(1-\lambda_h)d - \lambda_h(d-c)) < 0$, where $\Delta_h^* \leq (>)\Delta_s^*$ depending on whether $z \geq (<)\rho$. For $\rho \leq z$, only agents with signal *s* need to be rewarded for truth-telling, and an optimal transfer policy is given by (A11). For $z < \rho < \min[1-\alpha, k]$, as in the previous case, only agents who observe signal *s* need to be rewarded for truth-telling. However, an optimal transfer scheme is now given by (A10) for sufficiently small ε to assure that $\Delta_s^* + L_s^* = 0 \ge \Delta_h^* + L_h^*$. The expected diagnosis-contingent transfer after reporting *s* is greater for an agent with signal *s* than for an agent with signal *h*, and at optimum the IC constraint binds only for an agent with signal *s*. For $1-\alpha < \rho \le \min[\lambda_s, k]$, the IC constraints for both types of agents bind, $\Delta_s^* + L_s^* = 0 = \Delta_h^* + L_h^*$, and the optimal solution is given by $\tau^*(s, H) = \tau^*(h, S) = 0$, $\tau^*(s, S) = ((1-\lambda_s)\Delta_h^* - (1-\lambda_h)\Delta_s^*)/(\frac{1}{2}g_s(\lambda_s - \lambda_h))$, $\tau^*(h, H) = (\lambda_h \Delta_s^* - \lambda_s \Delta_h^*)/((g_s + \frac{1}{2}g_h)(\lambda_s - \lambda_h))$. In this case, at optimum transfers are received by the tested agents only when their reports match the test results.

Now we suppose that $\rho \ge k$. Then agents who report signal *s* receive the testing priority, and $\Delta_s^* = \frac{1}{2}((1-\lambda_s)c - \delta(1-\alpha)(1-\beta)(1-\lambda_s)d)) > 0$ and $\Delta_h^* = \frac{1}{2}(\lambda_h(d-c) - \delta(1-\alpha)(1-\beta)(1-\lambda_h)d) \le (>)0$ as $1-\delta\alpha\beta \le (>)\rho$. For $\rho < \max[\lambda_s, 1-\delta\alpha\beta]$, at optimum only the IC constraint for agents with signal *h* binds, $\Delta_s^* + L_s^* > 0 = \Delta_h^* + L_h^*$, and the smallest transfers that do not depend on the test result and satisfy both IC constraints are given by

$$\tau^*(h,S) = \tau^*(h,H) = \frac{\Delta_h^*}{\frac{1}{2}g_h} > 0 = \tau^*(s,S) = \tau^*(s,H).$$
(A12)

For $\max[1 - \delta \alpha \beta, k] \le \rho < \lambda_s$, the IC constraints do not bind, $\Delta_s^* > 0 > \Delta_h^*$, and transfers are not needed to incentivize truthful reporting: $\tau^*(s, S) = \tau^*(s, H) = \tau^*(h, S) = \tau^*(h, H) = 0$.

Case 3. $\lambda_s < \rho$. In this case, agents who observed signal *s* receive the testing priority, and the incremental payoffs (net of transfers) from reporting *s* are given by $\Delta_h^* = \frac{1}{2}(\lambda_h(d-c) - \delta(1-\alpha)(1-\lambda_h)d) \le (>)0$ as $1 - \delta \alpha \frac{\beta}{1-\beta} \le (>)\rho$, and $\Delta_s^* = \frac{1}{2}(\lambda_s(d-c) - \delta(1-\alpha)(1-\lambda_s)d) \ge (<)0$ as $1 - \delta \alpha \frac{1-\beta}{\beta} \ge (<)\rho$. So for $\max[1 - \delta \alpha \frac{\beta}{1-\beta}, \lambda_s] < \rho \le 1 - \delta \alpha \frac{1-\beta}{\beta}$, agents can credibly communicate their signals without transfers since both IC constraints are slack, $\Delta_s^* > 0 > \Delta_h^*$. For $\lambda_s < \rho \le 1 - \delta \alpha \frac{\beta}{1-\beta}$, only the IC constraint for an agent

with signal *h* binds, and an optimal transfer policy is given by (A12). Finally, for $1 - \delta \alpha \frac{1-\beta}{\beta} < \rho$, the IC constraint for agent with signal *s* binds, $\Delta_s^* + L_s^* = 0 \ge \Delta_h^* + L_h^*$, and the optimal transfer policy that is not contingent on the test result is given by $\tau^*(s, S) = \tau^*(s, H) = -\Delta_s^* / (\frac{1}{2}g_s + g_h) > 0$ = $\tau^*(h, S) = \tau^*(h, H)$.

Lemma 4 Suppose that $\rho > \lambda_s (1 + \delta(1 - \lambda_s))$. Then the agents who report signal *s* receive the testing priority, $(t_s^*(n_s, n_h), t_h^*(n_s, n_h)) = (\min[m, n_s], \max[m - n_s, 0])$ for any *n* and m < n.

Proof of Lemma 4 Since, by (5), only agents who tested positive control the disease, the expected (post-reporting, pre-testing) welfare is given by

$$W^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = nv - \sum_{y \in \{s,h\}} t_{y} (\lambda_{y}c + (1 - \lambda_{y})\frac{\delta}{n-1} \sum_{y' \in \{s,h\}} \lambda_{y'}(n_{y'} - t_{y'})d) \quad (A13)$$
$$+ (n_{y} - t_{y})(\lambda_{y} + (1 - \lambda_{y})\frac{\delta}{n-1} \sum_{y' \in \{s,h\}} \lambda_{y'}(n_{y'} - t_{y'} - 1_{y=y'}))d.$$

Treating the numbers of tested agents of each type as continuous, differentiation yields

$$\frac{\partial W(n_s, n_h, t_s, t_h)}{\partial t_s} = (\lambda_s (1 - \rho) + (n_s (\lambda_h - \lambda_s) + n(1 - \lambda_h) - (1 - \lambda_s)) \frac{\delta}{n - 1} \lambda_s) d$$
(A14)

$$> (\lambda_h(1-\rho) + (n_s(\lambda_h - \lambda_s) + n(1-\lambda_h) - (1-\lambda_h))\frac{\delta}{n-1}\lambda_h)d = \frac{\partial W^*(n_s, n_h, t_s, t_h)}{\partial t_h}.$$

Therefore, agents with signal *s* receive the testing priority.

Proof of Proposition 2 It will be convenient to let

$$w(e_{s}, e_{h}, t_{s}, t_{h}) = v - d \sum_{y \in \{s,h\}} t_{y} (\lambda_{y} \rho + (1 - \lambda_{y}) R(e_{s}, e_{h}, t_{s}, t_{h})) + (g_{y} - t_{y}) ((\lambda_{y} + (1 - \lambda_{y}) R(e_{s}, e_{h}, t_{s}, t_{h})) e_{y} + \rho(1 - e_{y}))$$
(A15)

denote the average payoff $(w = \text{plim}_{n \to \infty} \frac{1}{n}W)$ for given disease control efforts of untested agents and the shares of tested agents with and without suspicions, e_s, e_h, t_s, t_h . By Lemma 1, the equilibrium can be recovered as the solution to this continuous maximization problem

$$\min_{\tau \ge 0} \sum_{y \in \{s,h\}} t_y(\lambda_y \tau(y,S) + (1 - \lambda_y)\tau(y,H)) + (g_y - t_y)\tau(y,U)$$
(A16)

subject to

$$(e_s, e_h, t_s, t_h) \in \arg\max w(e_s, e_h, t_s, t_h)$$
(A17)

(diagnostic capacity constraints)

$$0 \le t_y \le g_y, t_s + t_h \le m, \tag{A18}$$

(disease control effort optimality conditions)

$$e_{s} = \begin{cases} 1, \text{if } q_{s}(1, t_{s}, t_{h}) < \rho \\ \tilde{e}_{s}, \text{if } q_{s}(0, t_{s}, t_{h}) < \rho \le q_{s}(1, t_{s}, t_{h}), \\ 0, \text{if } q_{s}(0, t_{s}, t_{h}) \ge \rho \end{cases}$$
(A19a)

$$e_{h} = \begin{cases} 1, \text{ if } q_{h}(0, 1, t_{s}, t_{h}) < \rho \\ \widetilde{e}_{h}, \text{ if } q_{h}(0, 0, t_{s}, t_{h}) < \rho \le q_{h}(0, 1, t_{s}, t_{h}), \\ 0, \text{ if } q_{h}(0, 0, t_{s}, t_{h}) > \rho \end{cases}$$
(A19b)

where \tilde{e}_s and \tilde{e}_h satisfy the indifference conditions $q_s(\tilde{e}_s, \mathbf{l}, t_s, t_h) = \rho$ and $q_h(0, \tilde{e}_h, t_s, t_h) = \rho$, (truth-telling incentive compatibility constraints)

$$P_{y}(V_{y}(e_{s},e_{h},t_{s},t_{h})+\lambda_{y}\tau(y,S)+(1-\lambda_{y})\tau(y,H))+(1-P_{y})\tau(y,U)$$

 $\geq P_{y'}(V_y(e_s, e_h, t_s, t_h) + \lambda_y \tau(y', S) + (1 - \lambda_y)\tau(y', H)) + (1 - P_{y'})\tau(y', U) \text{ for all } y, y' \in \{s, h\}, \text{ (A20a)}$ (participation constraints)

$$P_{y}(V_{y}(e_{s},e_{h},t_{s},t_{h})+\lambda_{y}\tau(y,S)+(1-\lambda_{y})\tau(y,H))+(1-P_{y})\tau(y,U)\geq 0, \ y \in \{s,h\},$$
(A20b)

where $P_y = t_y / g_y$, $y \in \{s, h\}$. Note that the truth-telling incentive compatibility constraints in (A20a) take into account that the average disease incidence, $R(e_s, e_h, t_s, t_h)$, does not vary across agents and is beyond the control of any single agent. Also, note that the equilibrium outcome is the same under mandatory and voluntary participation in the diagnostic testing program since the participation constraints in (A20b), which are considered in Section 4.1, do not bind at optimum.

In accordance with the disease control effort optimality conditions in (A19), there are six cases to consider. Suppose that at optimum $(e_s^*, e_h^*) = (1,1)$. Then from (A19) it follows that $\lambda_s + (1-\lambda_s)R(1,1,t_s,t_h) < \rho$, and differentiation yields

$$\frac{\partial w(\mathbf{l},\mathbf{l},t_{s},t_{h})}{\partial t_{s}} = (\lambda_{s}(1-\rho) + \delta \lambda_{s} \sum_{y \in \{s,h\}} g_{y}(1-\lambda_{y}))d$$

$$> (\lambda_{h}(1-\rho) + \delta \lambda_{h} \sum_{y \in \{s,h\}} g_{y}(1-\lambda_{y}))d = \frac{\partial w(\mathbf{l},\mathbf{l},t_{s},t_{h})}{\partial t_{s}} > 0.$$
(A21)

So from the binding constraints in (A18) it follows that the optimal test allocation is given by

$$(t_s^*, t_h^*) = (\min[g_s, m], \max[m - g_s, 0])$$
(A22)

for $q_s(1,1,\min[g_s,m],\max[m-g_s,0]) < \rho$. Suppose that at optimum $e_s^* \in (0,1), e_h^* = 1$. Then from (A19) it follows that $\lambda_s + (1-\lambda_s)R(e_s^*,1,t_s,t_h) = \rho$. Substituting this indifference condition into (A15), and upon some manipulation, the average welfare can be written as

$$w(e_s^*, \mathbf{l}, t_s, t_h) = v - d(t_s \lambda_s(\rho - 1) + g_s \rho + t_h \lambda_h(\rho - 1) + g_h(\lambda_h + (1 - \lambda_h) \frac{\rho - \lambda_s}{1 - \lambda_s})).$$
(A23)

Differentiation yields

$$\frac{\partial w(e_s^*, \mathbf{l}, t_s, t_h)}{\partial t_s} = \lambda_s (1 - \rho) d > \lambda_h (1 - \rho) d = \frac{\partial w(e_s^*, \mathbf{l}, t_s, t_h)}{\partial t_s} > 0.$$
(A24)

So for $q_s(0,1,\min[g_s,m],\max[m-g_s,0]) < \rho \le q_s(1,1,\min[g_s,m],\max[m-g_s,0])$, the optimal test allocation is given by (A22).

Suppose that at optimum $(e_s^*, e_h^*) = (0,1)$. By (A19), we have $q_h(0,1,t_s,t_h) < \rho \le q_s(0,1,t_s,t_h)$. Substituting $R(0,1,t_s,t_h) = \delta(g_h - t_h)\lambda_h$, the average welfare becomes

 $w(t_s, t_h, 0, 1) = v - d(t_s((\lambda_s - 1)\rho + (1 - \lambda_s)\delta(g_h - t_h)\lambda_h) + g_s\rho + t_h\lambda_h(\rho - 1) + g_h(\lambda_h + (1 - \lambda_h)\delta(g_h - t_h)\lambda_h)).$ (A25) Differentiation yields

$$\frac{\partial w(0,1,t_s,t_h)}{\partial t_s} = ((1-\lambda_s)\rho - (1-\lambda_s)\delta(g_h - t_h)\lambda_h)d > 0 \quad \text{and}$$
(A26a)

$$\frac{\partial w(0,1,t_s,t_h)}{\partial t_h} = (t_s(1-\lambda_s)\delta\lambda_h + \lambda_h(1-\rho) + g_h(1-\lambda_h)\delta\lambda_h)d > 0, \quad (A26b)$$

where

$$\frac{\partial w(0,1,t_s,t_h)}{\partial t_s} > (\leq) \frac{\partial w(0,1,t_s,t_h)}{\partial t_h} \text{ as } \rho > (\leq) k(t_s - t_h).$$
(A27)

Since $k(t_s - t_h)$ is increasing in $t_s - t_h$, by (A27), for $k(\min[m, 2g_s - m]) \le \rho \le q_s(0, 1, \min[m, g_s], \max[m - g_s, 0])$ the optimal test allocation is given by (A22). For $k(-\min[m, 2g_h - m]) < \rho < \min[k(\min[m, 2g_s - m]), q_s(0, 1, \min[m, g_s], \max[m - g_s, 0])]$, the first-order conditions become $\partial w(0, 1, t_s^*, t_h^*) / \partial t_s = \partial w(0, 1, t_s^*, t_h^*) / \partial t_h$ and $t_s^* + t_h^* = m$, and the optimal solution is given by

$$t_{s}^{*} = \frac{1}{2} \left(m + \frac{(1-\lambda_{s})\rho - \lambda_{h}(1-\rho) - \delta g_{h}\lambda_{h}(2-\lambda_{h}-\lambda_{s})}{(1-\lambda_{s})\delta \lambda_{h}} \right) \text{ and } t_{h}^{*} = \frac{1}{2} \left(m - \frac{(1-\lambda_{s})\rho - \lambda_{h}(1-\rho) - \delta g_{h}\lambda_{h}(2-\lambda_{h}-\lambda_{s})}{(1-\lambda_{s})\delta \lambda_{h}} \right).$$
(A28)

For $q_h(0,1, \max[0, m-g_h], \min[m, g_h]) < \rho < \min[q_s(0,1, \min[m, g_s], \max[m-g_s, 0]), k(-\min[m, 2g_h - m])]$, we have $\partial w(0,1, t_s, t_h) / \partial t_s < \partial w(0,1, t_s, t_h) / \partial t_h$ for any t_s, t_h such that the capacity constraints in (A18) are satisfied, and the optimal test allocation is given by

$$(t_s^*, t_h^*) = (\max[m - g_h, 0], \min[m, g_h]).$$
(A29)

Suppose that at optimum $e_s^* = 0, e_h^* \in (0,1)$. Then from (A19) it follows that $\lambda_h + (1 - \lambda_h)R(0, e_h^*, t_s, t_h) = \rho$. Substituting this indifference condition into (A15), the average welfare becomes

$$w(0, e_h^*, t_s, t_h) = v + d(\frac{(1-\lambda_s)\lambda_h}{1-\lambda_h}(1-\rho)t_s + \lambda_h(1-\rho)t_h - \rho).$$
(A30)

Differentiation yields

$$0 < \frac{\partial w(0, e_h^*, t_s, t_h)}{\partial t_s} = d \frac{(1 - \lambda_s)\lambda_h}{1 - \lambda_h} (1 - \rho) < d\lambda_h (1 - \rho) = \frac{\partial w(0, e_h^*, t_s, t_h)}{\partial t_h}.$$
 (A31)

Therefore, for $q_h(0,0, \max[0, m-g_h], \min[m, g_h]) < \rho < q_h(0,1, \max[0, m-g_h], \min[m, g_h])$ the optimal test allocation is given by (A29). Finally, for $q_h(0,0, \max[0, m-g_h], \min[m, g_h]) \ge \rho$, we have $(e_s^*, e_h^*) = (0,0)$, and since $R(0,0, \max[0, m-g_h], \min[m, g_h]) = 0$, the average welfare is given by $w(0,0,t_s,t_h) = v - d(t_s\lambda_s\rho + (g_s - t_s)\rho + t_h\lambda_h\rho + (g_h - t_h)\rho)$. (A32)

Differentiation yields

$$0 < \frac{\partial w(0,0,t_s,t_h)}{\partial t_s} = (1 - \lambda_s)\rho d < (1 - \lambda_h)\rho d = \frac{\partial w(0,0,t_s,t_h)}{\partial t_h},$$
(A33)

and the optimal test allocation is given by (A29).

To determine the optimal transfer policy, note that in equilibrium agents with signal *s* or *h* receive the testing priority, $P_s^* = t_s^* / g_s > (\leq)t_h^* / g_h = P_h^*$, depending on whether $\rho > (\leq)k(m(g_s - g_h))$. Also, note that in equilibrium $V_s^* = V_s(e_s^*, e_h^*, t_s^*, t_h^*) > (\leq)V_h(e_s^*, e_h^*, t_s^*, t_h^*) = V_h^*$ depending on whether $\rho > (\leq)\lambda_h(1 + \delta(g_h - t_h^*)(1 - \lambda_s))/(1 - \lambda_s + \lambda_h)$. Therefore, from the IC constraints (A20a) it follows that only the truth-telling constraint for agents without suspicions binds in equilibrium for $\rho \in (k(m(g_s - g_h), 1)$ since in this case $V_s^* > V_h^*$ and $P_s^* > P_h^*$, and, by (A20a), the optimal non-zero transfer satisfies:

$$P_{s}^{*}V_{h}^{*} = P_{h}^{*}V_{h}^{*} + (1 - P_{h}^{*})\tau^{*}(h, U).$$
(A34)

Both IC constraints bind in equilibrium for $\rho \in (\tilde{k}, \hat{k}(m(g_s - g_h)))$ since in this case $V_s^* > V_h^*$ and $P_s^* < P_h^*$, and by (A20a) at optimum the optimal transfers satisfy the following equations:

$$P_{s}^{*}(V_{s}^{*} + \lambda_{s}\tau^{*}(s,S)) + (1 - P_{s}^{*})\tau^{*}(s,U) = P_{h}^{*}(V_{s}^{*} + (1 - \lambda_{s})\tau^{*}(h,H))$$
(A35a)
$$P_{s}^{*}(U_{s}^{*} + \lambda_{s}\tau^{*}(s,S)) + (1 - P_{s}^{*})\tau^{*}(s,U) = P_{h}^{*}(V_{s}^{*} + (1 - \lambda_{s})\tau^{*}(h,H))$$
(A35a)

$$P_{s}^{*}(V_{h}^{*}+\lambda_{h}\tau^{*}(s,S))+(1-P_{s}^{*})\tau^{*}(s,U) = P_{h}^{*}(V_{h}^{*}+(1-\lambda_{h})\tau^{*}(h,H)).$$
(A35b)
zero payments are given by

Hence, the optimal non-zero payments are given by

$$\tau^{*}(s,S) = \frac{P_{h}^{*} - P_{s}^{*}}{P_{s}^{*}} \frac{V_{s}^{*}(1 - \lambda_{h}) - V_{h}^{*}(1 - \lambda_{s})}{\lambda_{s} - \lambda_{h}} \text{ and } \tau^{*}(h,H) = \frac{P_{h}^{*} - P_{s}^{*}}{P_{h}^{*}} \frac{V_{s}^{*}\lambda_{h} - V_{h}^{*}\lambda_{s}}{\lambda_{s} - \lambda_{h}} \text{ for } t_{s}^{*} > 0, (A36)$$

and

$$\tau^{*}(s,U) = \frac{m}{g_{h}} \frac{V_{s}^{*}(1-\lambda_{h}) - V_{h}^{*}(1-\lambda_{s})}{\lambda_{s} - \lambda_{h}} \text{ and } \tau^{*}(h,H) = \frac{V_{s}^{*} - V_{h}^{*}}{\lambda_{s} - \lambda_{h}} \text{ for } t_{s}^{*} = 0.$$
 (A37)

Finally, for $\rho < \tilde{k}$ only the truth-telling constraint for agents with suspicions binds in equilibrium since in this case $V_s^* < V_h^*$ and $P_s^* = \max[m - g_h, 0]/g_s < \min[m, g_h]/g_h = P_h^*$, and the non-zero payment satisfies:

$$P_h^* V_s^* = P_s^* V_s^* + (1 - P_s^*) \tau^*(s, U).$$
(A38)

Proof of Proposition 3 Part (i). For $\delta = 0$, by (5), the post-testing disease control efforts are given by $e_{yr}^*(n_s, n_h, t_s, t_h) = 1_{\rho > \lambda_y}, y \in \{s, h, H, S\}, r \in \{s, h, o\},$ (A39)

and the socially efficient test allocation policy assigns the testing priority to agents with the greatest informational benefit from testing:

$$(t_{\bar{y}}^{*}(n_{s},n_{h}),t_{\underline{y}}^{*}(n_{s},n_{h})) = (\min[m,n_{\bar{y}}],\max[m-n_{\bar{y}},0]) \text{ for any } n_{s},n_{h},$$
(A40)

where $(\underline{y}, \overline{y}) = (s, h)$ if $V_s < V_h$ and $(\underline{y}, \overline{y}) = (h, s)$ if $V_s \ge V_h$, $V_y = \min[\lambda_y(1-\rho), (1-\lambda_y)\rho)]d$.

By (A39), the incentive compatibility and participation constraints are given by, respectively,

$$\sum_{n_s=0}^{n-1} p_y(n_s)(v - \lambda_y c - E[\tau(y, \theta_{i,0}) | Y_i = y]) + (1 - p_y(n_s))(v - \min[c, \lambda_y d]))B_{n_s, n-1}$$

$$\geq \sum_{n_s=0}^{n-1} p_r(n_s)(v - \lambda_y c - E[\tau(r, \theta_{i,0}) | Y_i = y]) + (1 - p_r(n_s))(v - \min[c, \lambda_y d]))B_{n_s, n-1}$$
(A41)

and

$$\sum_{n_s=0}^{n-1} p_y(n_s)(v - \lambda_y c - E[\tau(y, \theta_{i,0}) | Y_i = y]) + (1 - p_y(n_s))(v - \min[c, \lambda_y d]))B_{n_s, n-1} \ge v - \min[c, \lambda_y d] \quad (A42)$$

for all $y, r \in \{s, h\}$, where $p_y(n_s) = t_r(n_s + 1_{r=s}, n - n_s - 1_{r=s}) / (1_{r=s}(n_s + 1) + 1_{r=h}(n - n_s))$. Applying the revelation principle (recall that the equilibrium disease control strategy in (A39) is independent of the private information of the other agents), the firm's pricing and test allocation policies solve

$$\max_{\tau, t_s, t_h} \sum_{n_s=0}^{n} \sum_{y \in \{s,h\}} t_y(n_s, n-n_s) (\lambda_y \tau(y, S) + (1-\lambda_y) \tau(y, H)) B_{n_s, n} \text{ subject to (A41) and (A42).}$$
(A43)

Upon simplification, the participation and incentive compatibility constraints can be rewritten as

$$P_{y}(V_{y} - E[\tau(y, \theta_{i,0}) | Y_{i} = y]) \ge P_{y'}(V_{y} - E[\tau(y', \theta_{i,0}) | Y_{i} = y]),$$
(A41')

$$P_{y}(V_{y} - E[\tau(y, \theta_{i,0}) | Y_{i} = y]) \ge 0$$
 (A42')

for all $y, y' \in \{s, h\}$, where $P_y = \sum_{n_s=0}^{n-1} p_y(n_s) B_{n_s, n-1}$. Each agent is indifferent between reporting *h* or *s* or opting out of the diagnostic program if the expected price for any testing priority equals the expected informational benefit from being selected for testing

$$\lambda_{y}\tau(r,S) + (1 - \lambda_{y})\tau(r,H) = V_{y} \text{ for all } y, r \in \{s,h\}.$$
(A44)

Solving the system of equations in (A44) yields the price policy in (12). Substituting (12) into the firm's objective function yields

$$\sum_{e \in \{s,h\}} t_y(n_s, n_h) V_y = W^*(n_s, n_h, t_s(n_s, n_h), t_h(n_s, n_h)) - v + \sum_{y \in \{s,h\}} n_y \min[\rho, \lambda_y] d, \qquad (A45)$$

so that (43) can rewritten as

$$\max_{t_s, t_h} \sum_{n_s=0}^{n} (W^*(n_s, n-n_s, t_s, t_h) - v + \sum_{y \in \{s,h\}} n_y \min[\rho, \lambda_y] d) B_{n,n}.$$
(A46)

Therefore, the testing policy in (A40) and the price policy in (12) solve the program (A43). By continuity, the firm allocates tests efficiently for all δ sufficiently close to 0.

Part (ii). When the relative cost of disease control is sufficiently small, the proof is analogous since in the post-testing equilibrium there is small disease transmission risk under any test allocation.

Proof of Proposition 4 To obtain a contradiction, suppose that in equilibrium the firm implements an efficient testing policy, which, by Lemma 4, is given by $(t_s^*(n_s, n_h), t_h^*(n_s, n_h)) = (\min[m, n_s], \min[\max[m-n_s, 0], n_h])$ for any signal profile (n_s, n_h) . Since agents' initial disease states and signals are independently and identically distributed, and in accordance with the PBE concept, we do not allow for correlated choices of diagnostic testing contracts and disease control strategies, it must be that in equilibrium each agent accepts the offered contract with probability one. From condition $\rho > \lambda_s + (1 - \lambda_s) \delta \lambda_s$ and (5) it follows that

 $e_{yr}^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = 1 - 1_{y=S}$ for any $y \in \{s, h, S, H\}$, $r \in \{s, h, o\}$, signal profile (n_{s}, n_{h}) , and test allocation (t_{s}, t_{h}) . Thus, the expected payoff for agents with signal y gross of the contract price is given by

$$\Pi_{yy}^{*} = v - \sum_{n_{s}=0}^{n-1} (p_{y}^{*}(n_{s})(c\lambda_{y} + (1-\lambda_{y})\frac{\delta}{n-1}\sum_{y'\in\{s,h\}} (1_{y'=s}n_{s} + 1_{y'=h}(n-1-n_{s}) + 1_{y'=y} - t_{y'}^{*}(n_{s} + 1_{y=s}, n-n_{s} - 1_{y=s}))\lambda_{y'}d)$$

$$+ (1 - p_{y}^{*}(n_{s}))(\lambda_{y} + (1-\lambda_{y})\frac{\delta}{n-1}\sum_{y'\in\{s,h\}} (1_{y'=s}n_{s} + 1_{y'=h}(n-1-n_{s}) - t_{y'}^{*}(n_{s} + 1_{y=s}, n-n_{s} - 1_{y=s}))\lambda_{y'}d)B_{n_{s},n-1},$$
(A47)

where $p_y^*(n_s) = t_y^*(n_s + 1_{y=s}, n - n_s - 1_{y=s}) / (1_{y=s}(n_s + 1) + 1_{y=h}(n - n_s))$ for any $y \in \{s, h\}$. Given that all agents accept the contract in equilibrium, the participation constraint is given by

$$\Pi_{yy}^{*} - \sum_{n_{s}=0}^{n-1} (p_{y}^{*}(n_{s})(\lambda_{y}\tau(y,S) + (1-\lambda_{y})\tau(y,H)) + (1-p_{y}^{*}(n_{s}))\tau(y,U))B_{n_{s},n-1} \quad (A48)$$

$$\geq v - d\sum_{n_{s}=0}^{n-1} (\lambda_{y} + (1-\lambda_{y})\frac{\delta}{n-1}\sum_{y' \in \{s,h\}} (1_{y'=s}n_{s} + 1_{y'=h}(n-1-n_{s}) - t_{y'}^{*}(n_{s},n-n_{s}-1))\lambda_{y'})B_{n_{s},n-1}.$$

The left-hand side is the expected payoff minus the price paid under the terms of the contract. The righthand side is the expected payoff when the agent rejects the contract. In this case, the agent pays nothing to the firm, expects that he will remain untested with probability one, and as a result, faces a smaller disease transmission risk. Substituting (A47) into (48) and manipulating terms, condition (A48) becomes

$$\sum_{n_s=0}^{n-1} (p_y^*(n_s)(\lambda_y \tau(y, S) + (1 - \lambda_y)\tau(y, H)) + (1 - p_y^*(n_s))\tau(y, U))B_{n_s, n-1}$$
(A49)

$$\leq d \sum_{n_s=0}^{n-1} (p_y^*(n_s)(\lambda_y(1-\rho) - (1-\lambda_y)\frac{\delta}{n-1}\lambda_y) + (1-\lambda_y)\frac{\delta}{n-1}\sum_{y'\in\{s,h\}} (t_{y'}^*(n_s, n-n_s-1) - t_{y'}^*(n_s+1_{y=s}, n-n_s-1_{y=s}))\lambda_{y'})B_{n_s,n-1}$$

$$=d\sum_{n_{s}=0}^{n-1}(p_{y}^{*}(n_{s})(\lambda_{y}(1-\rho)-\frac{\delta}{n-1}\lambda_{y}(1-\lambda_{y}))+(1-\lambda_{y})\frac{\delta}{n-1}((\min[n_{s},m]-\min[n_{s}+1_{y=s},m])\lambda_{s})+(\max[m-n_{s},0]-\max[m-n_{s}-1_{y=s},0])\lambda_{h})B_{n_{s},n-1}\leq 0,$$

where the second inequality follows by assumption that $\rho \ge 1 - \frac{\delta}{n-1}(1-\lambda_s)$. Therefore, in equilibrium the firm earns a non-positive profit.

But this yields a contradiction, because the firm can earn a strictly positive profit if it offers contract $\{\tau^{f}(r,\theta), P_{r}\}_{r\in\{s,h\},\theta\in\{S,H\}}$ to a fixed set of agents T that contains m agents and makes no offers to the other agents, where $\tau^{f}(r,\theta)$ is given by the price policy in (12) for $V_{y} = \lambda_{y}(1-\rho)$ and $P_{r} = 1$. In equilibrium each agent $i \in T$ accepts the contract to be tested with probability one and pays $E[\tau^{f}(r,\theta) | Y_{i} = y] = V_{v}$, because the disease transmission risk, $\frac{\delta}{n-1}\sum_{n=0}^{n-m}(n_s\lambda_s+(n-m-n_s)\lambda_h)B_{n_s,n-m}d$, is the same whether or not she rejects the contract, which leaves her indifferent between accepting and rejecting the contract. Each agent $i \notin T$ rejects any contract that assigns him a positive probability of being selected for testing $P_r > 0$ at a non-negative expected price $P_r E[\tau(r, \theta_{i,0}) | Y_i = y] + (1 - P_r)\tau(r, U) \ge 0$, because accepting it implies an increase in the disease transmission risk that is greater than the informational benefit from testing:

$$\Pi_{yy} = v - P_r (c\lambda_y + (1 - \lambda_y) \frac{\delta}{n-1} \sum_{n_s=0}^{n-m} (n_s \lambda_s + (n - m - n_s) \lambda_h) B_{n_s, n-m} d)$$
(A50)
$$- (1 - P_r) (\lambda_y + (1 - \lambda_y) \frac{\delta}{n-1} \sum_{n_s=0}^{n-1-m} (n_s \lambda_s + (n - 1 - m - n_s) \lambda_h) B_{n_s, n-1-m}) d$$
$$< v - (\lambda_y + (1 - \lambda_y) \frac{\delta}{n-1} \sum_{n_s=0}^{n-1-m} (n_s \lambda_s + (n - 1 - m - n_s) \lambda_h) B_{n_s, n-1-m} d) = \Pi_{yo},$$

or

$$\lambda_{y}(1-\rho) < (1-\lambda_{y})\frac{\delta}{n-1} \left(\sum_{n_{s}=0}^{n-m} (n_{s}\lambda_{s} + (n-m-n_{s})\lambda_{h})B_{n_{s},n-m} - \sum_{n_{s}=0}^{n-1-m} (n_{s}\lambda_{s} + (n-1-m-n_{s})\lambda_{h})B_{n_{s},n-1-m}\right), \quad (A51)$$
where the last inequality follows from $\rho \ge 1 - \frac{\delta}{n-1}(1-\lambda_{s})$.

Proof of Proposition 5 Using the notation in the proof of Proposition 2, in a model with a continuum of agents, the firm's problem can be written as

$$\max_{\tau, t_s, t_h} \sum_{y \in \{s,h\}} t_y(\lambda_y \tau(y, S) + (1 - \lambda_y) \tau(y, H)) + (g_y - t_y) \tau(y, U)$$
(A52)

subject to (A18), (A19),

(truth-telling incentive compatibility constraints)

$$P_{v}(V_{v}(e_{s},e_{h},t_{s},t_{h})-\lambda_{v}\tau(y,S)-(1-\lambda_{v})\tau(y,H))-(1-P_{v})\tau(y,U)$$

 $\geq P_{y'}(V_{y}(e_{s},e_{h},t_{s},t_{h}) - \lambda_{y}\tau(y',S) - (1-\lambda_{y})\tau(y',H)) - (1-P_{y'})\tau(y',U) \text{ for all } y,y' \in \{s,h\}, \text{ and } (A53a)$ (participation constraints)

$$P_{y}(V_{y}(e_{s},e_{h},t_{s},t_{h}) - \lambda_{y}\tau(y,S) - (1 - \lambda_{y})\tau(y,H)) - (1 - P_{y})\tau(y,U) \ge 0, \ y \in \{s,h\}.$$
(A53b)

Because the disease transmission risk is beyond the control of a single agent, the profit-maximizing price policy extracts the full informational benefit of testing and is again given by (12):

$$\tau^{f}(y,S) = \frac{(1-\lambda_{h})V_{s}(e_{s},e_{h},t_{s},t_{h}) - (1-\lambda_{s})V_{h}(e_{s},e_{h},t_{s},t_{h})}{\lambda_{s} - \lambda_{h}}$$
 and (A54)

$$\tau^{f}(y,H) = \frac{\lambda_{s}V_{h}(e_{s},e_{h},t_{s},t_{h}) - \lambda_{h}V_{s}(e_{s},e_{h},t_{s},t_{h})}{\lambda_{s} - \lambda_{h}} \text{ for } y = s,h,$$

and $\tau^{f}(o,U) = \tau^{f}(s,U) = \tau^{f}(h,U) = 0$. Hence, the firm's problem becomes max $\pi^{f}(e_{s},e_{h},t_{s},t_{h})$ subject to (A18) and (A19),

where
$$\pi^{f}(e_{s},e_{h},t_{s},t_{h}) = \sum_{y \in \{s,h\}} t_{y} V_{y}(e_{s},e_{h},t_{s},t_{h}) = w(e_{s},e_{h},t_{s},t_{h}) - v + \sum_{y \in \{s,h\}} g_{y}((\lambda_{y}+(1-\lambda_{y})R(e_{s},e_{h},t_{s},t_{h}))e_{y})$$

 $+\rho(1-e_y)d$. We next show that the profit-maximizing and efficient solutions tend to diverge because the firm cannot extract the total value of the reduction in the disease transmission risk due to diagnostic testing.

Following the same steps as in the proof of Proposition 2, for $\rho \ge q_s(0,1,\min[g_s,m],\max[m-g_s,0])$ at optimum we have $e_s^* \in (0,1], e_h^* = 1$. Then the firm's profit become

$$\pi^{f}(e_{s}^{*},\mathbf{l},t_{s},t_{h}) = (t_{s}\lambda_{s}(1-\rho) + t_{h}\lambda_{h}(1-\rho))d, \qquad (A56)$$

(A55)

and differentiation yields

$$\frac{\partial \pi^{f}(e_{s}^{*},\mathbf{l},t_{s},t_{h})}{\partial t_{s}} = \lambda_{s}(1-\rho)d > \lambda_{h}(1-\rho)d = \frac{\partial \pi^{f}(e_{s}^{*},\mathbf{l},t_{s},t_{h})}{\partial t_{s}} > 0.$$
(A57)

Hence, the optimal test allocation is given by

 $(t_s^f, t_h^f) = (\min[g_s, m], \max[m - g_s, 0]).$ (A58)

For $q_h(0,1, \max[m-g_h,0], \min[m,g_h]) < \rho < q_s(0,1, \min[g_s,m], \max[m-g_s,0])$, at optimum we have $(e_s^*, e_h^*) = (0,1)$, and the firm's profit becomes

$$\pi^{f}(0,1,t_{s},t_{h}) = (t_{s}((1-\lambda_{s})\rho - (1-\lambda_{s})\delta(g_{h}-t_{h})\lambda_{h}) + t_{h}\lambda_{h}(1-\rho))d.$$
(A59)

Differentiation yields

$$\frac{\partial \pi^{f}(0,1,t_{s},t_{h})}{\partial t_{s}} = ((1-\lambda_{s})\rho - (1-\lambda_{s})\delta(g_{h}-t_{h})\lambda_{h})d = \frac{\partial w(0,1,t_{s},t_{h})}{\partial t_{s}} \quad \text{and}$$
(A60a)

$$\frac{\partial \pi^{f}(0,1,t_{s},t_{h})}{\partial t_{h}} = (t_{s}(1-\lambda_{s})\delta\lambda_{h} + \lambda_{h}(1-\rho))d = \frac{\partial w(0,1,t_{s},t_{h})}{\partial t_{h}} - g_{h}(1-\lambda_{h})\delta\lambda_{h}d.$$
(A60b)

Note that

$$\frac{\partial \pi^{f}(0,lt_{s},t_{h})}{\partial t_{s}} > (\leq) \frac{\partial \pi^{f}(0,l,t_{s},t_{h})}{\partial t_{h}} \text{ as } \rho > (\leq) \hat{k}^{f}(t_{s}-t_{h}), \quad (A61)$$

where $k^{f}(t_{s} - t_{h}) = k(t_{s} - t_{h}) - g_{h}(1 - \lambda_{h})\delta\lambda_{h}d/(1 - \lambda_{s} + \lambda_{h})$. Since $k^{f}(t_{s} - t_{h})$ is increasing in $t_{s} - t_{h}$, by (A61), for $k^{f}(\min[m, 2g_{s} - m]) \leq \rho \leq q_{s}(0, 1, \min[m, g_{s}], \max[m - g_{s}, 0])$ the optimal test allocation is given by (A58). For $k^{f}(-\min[m, 2g_{h} - m]) < \rho < \min[k^{f}(\min[m, 2g_{s} - m]), q_{s}(0, 1, \min[m, g_{s}], \max[m - g_{s}, 0])]$, the first-order conditions become $\partial \pi^{f}(0, 1, t_{s}^{f}, t_{h}^{f})/\partial t_{s} = \partial \pi^{f}(0, 1, t_{s}^{f}, t_{h}^{f})/\partial t_{h}$ and $t_{s}^{f} + t_{h}^{f} = m$, and the optimal shares of tested agents are given by

$$t_{s}^{f} = \frac{1}{2} \left(m - g_{h} + \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho)}{(1 - \lambda_{s})\delta\lambda_{h}} \right) \text{ and } t_{h}^{f} = \frac{1}{2} \left(m + g_{h} - \frac{(1 - \lambda_{s})\rho - \lambda_{h}(1 - \rho)}{(1 - \lambda_{s})\delta\lambda_{h}} \right).$$
(A62)

For $q_h(0,1, \max[m-g_h, 0], \min[m, g_h]) < \rho \le k^f (-\min[m, 2g_h - m])$, we have $\partial \pi^f(0,1, t_s, t_h) / \partial t_s < \partial \pi^f(0,1, t_s, t_h) / \partial t_h$ for any t_s, t_h such that the capacity constraints in (A18) are satisfied, and the optimal test allocation is given by

$$(t_s^f, t_h^f) = (\max[m - g_h, 0], \min[m, g_h]).$$
(A63)

For $q_h(0,0, \max[m-g_h,0], \min[m,g_h]) < \rho < q_h(0,1, \max[m-g_h,0], \min[m,g_h])$, we have $e_s^* = 0$, $e_h^* \in (0,1)$. Substituting the indifference condition $\lambda_h + (1-\lambda_h)R(0,e_h^*,t_s,t_h) = \rho$, the firm's profit becomes $\pi^f(0,e_h^*,t_s,t_s) = (t_s(1-\lambda_s)\lambda_s(1-\rho)/(1-\lambda_s) + t_s\lambda_s(1-\rho))d$. (A64)

$$\pi^{s}(0, e_{h}, t_{s}, t_{h}) = (t_{s}(1 - \lambda_{s})\lambda_{h}(1 - \rho)/(1 - \lambda_{h}) + t_{h}\lambda_{h}(1 - \rho))d, \qquad (A64)$$

yields

and differentiation yields

$$\frac{\partial \pi^{f}(0, e_{h}^{*}, t_{s}, t_{h})}{\partial t_{s}} = ((1 - \lambda_{s})\lambda_{h}(1 - \rho)/(1 - \lambda_{h}))d < \lambda_{h}(1 - \rho)d = \frac{\partial \pi^{f}(0, e_{h}^{*}, t_{s}, t_{h})}{\partial t_{h}}.$$
 (A65)

Hence, the optimal test allocation is given by (A63). Finally, for $\rho \le q_h(0,0, \max[m-g_h,0], \min[m,g_h])$, we have $(e_s^*, e_h^*) = (0,0)$. Substituting $R(0,0,t_s,t_h) = 0$, the firm's profit becomes

$$\pi^{f}(0,0,t_{s},t_{h}) = (t_{s}(1-\lambda_{s})\rho + t_{h}(1-\lambda_{h})\rho)d, \qquad (A66)$$

and differentiation yields

$$\frac{\partial \pi^{f}(0,0,t_{s},t_{h})}{\partial t_{s}} = (1-\lambda_{s})\rho d < (1-\lambda_{h})\rho d = \frac{\partial \pi^{f}(0,0,t_{s},t_{h})}{\partial t_{h}}.$$
(A67)

Hence, the optimal test allocation is also given by (A63).

The profit-maximizing firm tends to test too many agents with suspicions compared with the socially efficient test allocation since $t_s^f \ge t_s^*$ and $t_h^f \le t_h^*$, where t_s^* and t_h^* are the socially efficient shares of tested agents with and without suspicions in Proposition 2. Specifically, $t_s^f > t_s^*$ and $t_h^f < t_h^*$ if $k^f(-\min[m, 2g_h - m]) < \rho < \min[k(\min[m, 2g_s - m]), q_s(0, 1, \min[m, g_s], \max[m - g_s, 0])]$.

Appendix B: Joint Disease Surveillance and Management Program

In this Appendix we consider a scenario where the participation in the disease surveillance program is mandatory, $n_s + n_h = n$, and the health authority (constrained social planner) allocates both (i) the diagnostic capacity *and* (ii) disease control efforts. The disease control efforts $e_y^*(n_s, n_h, t_s, t_h, n_s)$ now maximize the total expected welfare conditional on the signal profile (n_s, n_h) , test allocation (t_s, t_h) , and the number of agents that tested positive for infection, n_s , $0 \le n_s \le t_s + t_h$:

$$\{e_{y}^{*}(n_{s},n_{h},t_{s},t_{h},n_{S})\}_{y=s,h,S,H} \in \underset{e_{s},e_{h},e_{S},e_{H}\in[0,1]}{\arg\max}W(n_{s},n_{h},t_{s},t_{h},n_{S},e_{s},e_{h},e_{S},e_{H}),$$
(B1)

where $W(n_s, n_h, t_s, t_h, n_S, e_s, e_h, e_S, e_H) = \sum_{y \in \{s,h\}} (n_y - t_y) \pi_{yy}(n_s, n_h, t_s, t_h, n_S, e_s, e_h, e_S, e_H) + n_S \pi_{Sr}(n_s, n_h, t_s, t_h, n_S, e_s, e_h, e_S, e_H)$

 $e_{H}) + (t_{s} + t_{h} - n_{s})\pi_{Hr}(n_{s}, n_{h}, t_{s}, t_{h}, n_{s}, e_{s}, e_{h}, e_{s}, e_{H}), \text{ and } \pi_{yr}(n_{s}, n_{h}, t_{s}, t_{h}, n_{s}, e_{s}, e_{h}, e_{s}, e_{H})$

$$= \begin{cases} v - (\lambda_{y} + (1 - \lambda_{y})) \frac{\delta}{n-1} (\sum_{y' \in \{s,h\}} (n_{y'} - t_{y'} - 1_{r=y'}) \lambda_{y'} e_{y'} + n_{s} e_{s})) e_{r} d - c(1 - e_{r}), \text{ if } y \in \{s,h\} \\ v - (\lambda_{y} + (1 - \lambda_{y})) \frac{\delta}{n-1} (\sum_{y' \in \{s,h\}} (n_{y'} - t_{y'}) \lambda_{y'} e_{y'} + (n_{s} - 1_{y=s}) e_{s})) e_{y} d - c(1 - e_{y}), \text{ if } y \in \{S,H\} \end{cases}$$

The test allocation maximizes the expected welfare

$$(t_{s}^{*}(n_{s},n_{h}),t_{h}^{*}(n_{s},n_{h})) \in \underset{t_{s} \leq n_{s}, t_{h} \leq n_{h}}{\arg \max} W^{*}(n_{s},n_{h},t_{s},t_{h}),$$
(B2)

where

$$W^{*}(n_{s}, n_{h}, t_{s}, t_{h}) = \sum_{n_{s}=0}^{t_{s}+t_{h}} W(n_{s}, n_{h}, t_{s}, t_{h}, n_{s}, \{e_{y}^{*}(n_{s}, n_{h}, t_{s}, t_{h}, n_{s})\}_{y=s,h,S,H}) \Phi(n_{s}, n_{h}, t_{s}, t_{h}, n_{s}),$$

where $\Phi(n_s, n_h, t_s, t_h, n_s) = \sum_{b=\max[n_s - t_h, 0]}^{\min[n_s, t_s]} \frac{t_s!}{b!(t_s - b)!} \frac{t_h!}{(n_s - b)!(t_h - (n_s - b))!} \lambda_s^b \lambda_h^{(n_s - b)} (1 - \lambda_s)^{t_s - b} (1 - \lambda_h)^{t_h - (n_s - b)}$ is the probability that n_s agents will test positive for infection when t_s agents with suspicions and t_h agents without suspicions are tested.

The agent's expected payoff following a history of signal $y \in \{s,h\}$ and report $r \in \{s,h\}$ is now given

by
$$\Pi_{yr}^{*} = \sum_{n_{s}=0}^{n-1} (p_{r}^{*}(n_{s})(\lambda_{y}f_{s}(r,n_{s}) + (1-\lambda_{y})f_{H}(r,n_{s})) + (1-p_{r}^{*}(n_{s}))f_{y}(r,n_{s}))B_{n_{s},n-1}, \quad \text{where} \quad f_{y}(r,n_{s}) = f_{y}(r,n_{s})$$

$$= \sum_{n_s=0}^{t_s+t_h} \pi_{yr}^* (n_s + 1_{r=s}, n-n_s - 1_{r=s}, t_s^* (n_s + 1_{r=s}, n-n_s - 1_{r=s}), t_h^* (n_s + 1_{r=s}, n-n_s - 1_{r=s}), n_s) \Phi(n_s + 1_{r=s}, n-n_s - 1_{r=s}), t_s^* (n_s + 1_{r=s}, n-n_s - 1_{r=s}), n_s) \Phi(n_s + 1_{r=s}$$

B1. Two Agents

We start with n = 2. First, we establish conditions under which the socially efficient levels of disease control efforts for the untested agents are greater than the privately optimal levels, and demonstrate that the agents with suspicions about being infected are less likely to receive the testing priority when the disease is controlled efficiently.

Lemma 5 Suppose that n = 2, m = 1, and agents reveal their signals to the health authority under the joint disease surveillance and management program. The efficient disease control efforts are given by

$$e_{y}^{M}(n_{s}, n_{h}, t_{s}, t_{h}, n_{s}) = \begin{cases} 1 - y, \text{ for } y \in \{0, 1\} \\ 1_{\lambda_{y}(1 + \delta(1 - n_{s})) < \rho}, \text{ for } y \in \{s, h\} \end{cases}$$
(B3)

and the efficient test allocation (in terms of testing priorities) is given by M(x) = M(x)

$$p_{s}^{M}(0) = 1 - p_{h}^{M}(0) = \frac{1}{2} ,$$

$$p_{s}^{M}(0) = 1 - p_{h}^{M}(1) = \begin{cases} 1, \text{ if } \rho \in [\min[k, \lambda_{s}], \lambda_{s}] \cup (\max[\lambda_{s}, \lambda_{h}(1+\delta)], 1) \\ \frac{1}{2}, \text{ if } \rho \in (\lambda_{s}, \lambda_{h}(1+\delta)) \\ 0, \text{ if } \rho \in (0, \min[k, \lambda_{s}]) \end{cases} , \quad (B4)$$

for any $n_s, n_h \in \{0, 1, 2\}, n_s + n_h = 2, t_s + t_h = 1, n_s \in \{0, 1\}.$

Proof of Lemma 5 By (B1), the expected welfare conditional on n_s, n_h, t_s, t_h, n_s can be written as $W(n_s, n_h, t_s, t_h, n_s, e_s, e_h, e_s, e_H) = 2v - ((n_s + (1 - n_s)\partial\lambda_y e_y)e_{n_s} + (\lambda_y + (1 - \lambda_y)\partial n_s e_{n_s})e_y)d - c(2 - e_{n_s} - e_y)$, where y = s if $(n_s, n_h, t_s, t_h) \in \{(1,1,0,1), (2,0,1,0)\}$ and y = h if $(n_s, n_h, t_s, t_h) \in \{(1,1,0,1), (2,0,1,0)\}$. It is easy to verify that the efficient disease control efforts are given by (B3). Therefore, by (B2), the expected welfare before the test results are known is given by

$$W^{M}(n_{s}, n_{h}, t_{s}, t_{h}) = 2v - \begin{cases} \lambda_{y_{1}}c + \lambda_{y_{2}}\left((1 - \lambda_{y_{1}})\delta + 1\right)d, \text{ if } \rho > \lambda_{y_{2}}\left(1 + \delta\right)\\ c + \lambda_{y_{1}}\lambda_{y_{2}}d, \text{ if } \lambda_{y_{2}} < \rho \le \lambda_{y_{2}}\left(1 + \delta\right)\\ (1 + \lambda_{y_{1}})c, \text{ if } \rho \le \lambda_{y_{2}} \end{cases}$$

where $(y_1, y_2) = (s, s)$ if $(n_s, n_h, t_s, t_h) = (2,0,1,0)$, $(y_1, y_2) = (s,h)$ if $(n_s, n_h, t_s, t_h) = (1,1,1,0)$, $(y_1, y_2) = (h, s)$ if $(n_s, n_h, t_s, t_h) = (1,1,0,1)$, and $(y_1, y_2) = (h,h)$ if $(n_s, n_h, t_s, t_h) = (0,2,0,1)$. The optimal test allocation in (B4) is obtained by comparing $W^{M}(1,1,0,1)$ and $W^{M}(1,1,1,0)$. Note that the full utilization of the diagnostic capacity is always optimal because the health authority can ignore the test results.

From (B3) and (5) for n = 2, m = 1 it follows that an untested agent with signal y exerts an insufficient disease control effort whenever the tested agent is initially healthy and the relative cost of disease control is in an intermediate range, $\rho \in (\lambda_y, \lambda_y(1 + \delta)]$. This happens because the untested agent does not take into account the expected loss incurred by the initially healthy agent from the possible disease transmission, $\delta \lambda_y d$. Under the joint disease surveillance and management program, agents with suspicions tend to be tested less frequently because they are less likely to spread the disease.

Next we will find a transfer scheme that sustains truthful reporting with the smallest expected transfers in equilibrium with efficient testing and disease control policies. For $\rho \in (0, \lambda_h] \cup [\lambda_h(1+\delta), \lambda_s]$ $\cup [\lambda_s(1+\delta), 1)$ the optimal test allocation and transfer scheme are the same as in the regime with voluntary disease control since privately and socially optimal disease control efforts coincide for all $n_s = 0,1$. For $\rho \in (\lambda_s, \lambda_h(1+\delta))$, an untested agent bears the cost of compliance with mandatory disease control that requires controlling the disease whenever the tested agent is healthy. As a result, the final probability that a susceptible agent is infected, $\lambda_{y_1}\lambda_{y_2}$, is independent of which agent is tested. Because *random* testing is efficient and disease control for the untested agent with and without suspicions is mandatory when the tested agent does not carry disease, the need for truthful reporting is obviated, and the efficient outcome can be achieved without transfers.

Now suppose that $\rho \in (\lambda_h, \min[\lambda_s, \lambda_h(1+\delta)])$. Then agents with signal *h* receive the testing priority and the untested agent exerts the disease control effort unless both agents observed signals *h* and the tested agent exerts the full disease control effort. The incremental payoffs from reporting *s* for an agent with signal *y* is now given by

$$\Delta_{y}^{M} = (-\frac{1}{2})(V_{y} - D_{y} - C_{y}), \qquad (B5)$$

where $V_y = (1 - \lambda_y)c$, $D_y = 0$, and $C_y = (1 - \alpha)(1 - \beta)(\lambda_y d - c)$. The new term C_y is the *incremental* expected cost of regulatory compliance with the disease control policy. It consists of the increased exposure to the disease minus the savings from the cost of disease control, $\lambda_y d - c$, multiplied by the probability that the tested agent who does not suspect infection is, in fact, infected, $(1 - \alpha)(1 - \beta)$. Note that reporting *s* decreases or increases the expected cost of compliance depending on whether an agent observed signal

s or *h*, $C_h < 0 < C_s$. For $\rho \le \frac{(1-\alpha)(1-\beta)\lambda_s}{1-\lambda_s+(1-\alpha)(1-\beta)}$, truth-telling is incentive compatible without transfers because $\Delta_h^{\rm M} < 0 \le \Delta_s^{\rm M}$. For $\rho > \frac{(1-\alpha)(1-\beta)\lambda_s}{1-\lambda_s+(1-\alpha)(1-\beta)}$, the truth-telling constraint for agents who suspect infection is binding, $\Delta_h^{\rm M} < \Delta_s^{\rm M} < 0$, and the smallest transfers that sustain truthful reporting are $\tau^{\rm M}(s,S) = \tau^{\rm M}(s,H) = -\Delta_s^{\rm M}/(\frac{1}{2}g_s) > 0 = \tau^{\rm M}(h,S) = \tau^{\rm M}(h,H) = \tau^{\rm M}(.,U)$.

For $\rho \in (\max[\lambda_s, \lambda_h(1+\delta)], \lambda_s(1+\delta)) \cap (0,1)$, agents who suspect infection receive the testing priority and the untested agent does not exert disease control efforts unless he reported *s* and the tested agent is unprotected. Then we have $\Delta_y^M = \frac{1}{2}(V_y - D_y - C_y) \ge (<)0$ depending on whether $x_y \le (>)\rho$, where $V_y = \lambda_y(d-c), D_y = \delta(1-\alpha)(1-\beta)(1-\lambda_y)d$, $C_y = \alpha(1-\beta)(c-\lambda_yd) > 0$, $x_y \equiv (\lambda_y(1+(\alpha(1-\delta)+\delta)(1-\beta)) - \delta(1-\alpha)(1-\beta))/(\lambda_y + \alpha(1-\beta))$. Now the incremental expected cost of compliance is the cost of disease control minus the expected avoided damage from the disease, $c-\lambda_yd$, multiplied by the probability that an initially healthy agent has suspicions about being infected, $\alpha(1-\beta)$. For $x_s < \rho$, we have $\Delta_h < \Delta_s < 0$, and a transfer scheme that pays out the same amount to a tested agent who reported *s* for any outcome of the test, $\tau^M(s, S) = \tau^M(s, H) = -\Delta_s^M/(\frac{1}{2}g_s + g_h) > 0 = \tau^M(h, S) = \tau^M(h, H) = \tau^M(., U)$ makes truthful reporting incentive compatible. Similarly, for $x_h > \rho$, we have $0 < \Delta_h^M < \Delta_s^M$, and a transfer scheme that rewards reporting *h*: $\tau^M(h, S) = \tau^M(h, H) = \Delta_h^M/(\frac{1}{2}g_h) > 0 = \tau^M(s, S) = \tau^M(s, H) = \tau^M(., U)$, sustains credible reporting. However, for $\rho \in [x_h, x_s] \cap (\max[\lambda_s, \lambda_h(1+\delta)], \lambda_s(1+\delta))$ we have $\Delta_h^M \le \Delta_s^M$, and there exists an equilibrium in which reports are credible and the efficient test allocation is implementable without transfers.

Let $\Omega_0^M \equiv (([\lambda_h(1+\delta), \lambda_s] \cup [\lambda_s(1+\delta), 1)) \cap \Omega_0^*) \cup \hat{\Omega}_0$ denote the subset of [0,1] such that for any $\rho \in \Omega_0^M$ there exists an equilibrium in which reports are truthful without transfers under the joint disease surveillance and management policy, where $\hat{\Omega}_0 = (\lambda_s, \lambda_h(1+\delta)) \cup (\lambda_h, \min[\lambda_h(1+\delta), \frac{(1-\alpha)(1-\beta)\lambda_s}{1-\lambda_s+(1-\alpha)(1-\beta)}]] \cup ([x_h, x_s] \cap (\max[\lambda_s, \lambda_h(1+\delta)], \lambda_s(1+\delta)))$. Because $\Omega_0^* \subset \Omega_0^M$, it follows that going from voluntary to efficient disease control may improve welfare through two channels. First, for $\rho \in (\lambda_h, \lambda_h(1+\delta)) \cup (\lambda_s, \lambda_s(1+\delta))$, an untested agent exercises more care to avoid possible disease transmission. Second, for $\rho \in \Omega_0^M \setminus \Omega_0^*$, monetary transfers are no longer needed to efficiently allocate diagnostic tests.

B2. Continuum of Agents

Here we show that under the joint efficient test allocation and disease management policy, the variability in the costs of regulatory compliance across agents can be exploited to sustain credible reporting without monetary transfers even when the number of agents is large as long as the disease occurrence is sufficiently unlikely and private information is precise. Consider a model with a continuum of agents where the size of the population of agents is normalized to one. As in Proposition 2, let $m \in (0,1)$ denote the maximum share of the population that can be tested, t_r denote the share of the tested agents who report $r \in \{s, h\}$, and e_y denote the effort of agents with updated signal $y \in \{s, h, S, H\}$. We assume that the following condition holds:

$$\lambda_{h} \frac{1 + \delta g_{h}(2 - \lambda_{s} - \lambda_{h})}{1 - \lambda_{s} + \lambda_{h}} \leq \rho \leq \lambda_{s} + (1 - \lambda_{s}) \delta g_{h} \lambda_{h},$$
(B6)

Note that (B6) holds whenever private signals are sufficiently precise (β is large) and the disease incursion is sufficiently unlikely (α is large).

Lemma 6 Suppose that condition (B6) holds, the diagnostic capacity, m, is sufficiently small, and private information is revealed to the health authority. Then $e_y^M = 1 - y$ for $y \in \{0,1\} (=\{H,S\}), e_s^M = 0, e_h^M = 1$, and $(t_s^M, t_h^M) = (m, 0)$.

Proof of Lemma 6 The social planner's test allocation and disease management problem can be stated as $\max_{e_s,e_h,e_s,e_H} w(e_s,e_h,e_s,e_H,t_s,t_h) \text{ subject to (A18)}$ (B7)

where $w(e_s, e_h, e_s, e_H, t_s, t_h) = v - d \sum_{y \in \{s,h\}} t_y(\lambda_y(e_s + (1 - e_s)\rho) + (1 - \lambda_y)(R(e_s, e_h, e_s, t_s, t_h)e_H + (1 - e_H)\rho) + (g_y - t_y)$ $((\lambda_y + (1 - \lambda_y)R(e_s, e_h, e_s, t_s, t_h))e_y + \rho(1 - e_y)), \quad R(e_s, e_h, e_s, t_s, t_h) = \delta(e_s(g_s - t_s)\lambda_s + e_h(g_h - t_h) - t_h)\lambda_h + e_s(t_s\lambda_s + t_h\lambda_h)).$ The first-order conditions for the socially optimal disease control efforts by the two types of untested agents evaluated at $(e_s, e_h, e_s, e_h) = (0, 1, 0, 1)$ are given by

$$\frac{\partial w(0,1,0,1,m,0)}{\partial e_s} = -((g_s - m)(\lambda_s + (1 - \lambda_s)\delta g_h \lambda_h - \rho) + (m(1 - \lambda_s) + g_h(1 - \lambda_h))\delta(g_s - m)\lambda_s)d \le 0, \quad (B8a)$$

$$\frac{\partial w(0,1,0,1,m,0)}{\partial e_h} = -(g_h(\lambda_h + (1 - \lambda_h)\delta g_h \lambda_h - \rho) + (m(1 - \lambda_s) + g_h(1 - \lambda_h))\delta g_h \lambda_h)d \ge 0. \quad (B8b)$$

(B8a) holds by the second inequality in (B6). For m = 0, inequality in (B8b) can be rewritten as $\lambda_h + 2(1 - \lambda_h) \delta g_h \lambda_h \le \rho$, which is implied by the first inequality in (B6). Differentiation also establishes that under the joint disease surveillance and management program agents that tested negative do not control the disease, $e_H = 1$, and agents that tested positive control the disease, $e_S = 0$.

Similarly, it is easy to check that for $(t_s, t_h) = (0, m)$ welfare is also maximized at $(e_s, e_h, e_s, e_H) = (0,1,0,1)$ as long as condition (B6) holds, and the maximum average payoff is given by $w(0,1,0,1,0,m) = v - (g_s \rho + m(\lambda_h \rho + (1 - \lambda_h)R(0,1,0,0,m)) + (g_h - m)(\lambda_h + (1 - \lambda_h)R(0,1,0,0,m))d$ (B9) $< w(0,1,0,1,m,0) = v - ((g_s - m)\rho + m(\lambda_s \rho + (1 - \lambda_s)R(0,1,0,m,0)) + g_h(\lambda_h + (1 - \lambda_h)R(0,1,0,m,0))d$. The inequality follows from the second inequality in (B6) when m > 0 is sufficiently close to 0.

Under the joint disease surveillance and management program, given that the diagnostic capacity m is sufficiently small and (B6) holds, agents with suspicions about being infected receive the testing priority and control the disease if they are not tested, and agents without suspicions do not control the disease. Thus, the average initial disease incidence is $R(0,1,0,m,0) = \delta g_h \lambda_h$. The incremental payoff from reporting suspicions for an agent with signal y is now given by

$$\Delta_{y}^{M} = \frac{m}{g_{s}} V_{y} - (1 - \frac{m}{g_{s}}) C_{y}, \qquad (B10)$$

where $V_y = \lambda_y (1-\rho)d$ and $C_y = (\rho - (\lambda_y + (1-\lambda_y)R(0,1,0,m,0)))d$. Agents with suspicions prefer to tell the truth to increase the odds of being selected for testing, $\Delta_s^M > 0$. The efficient disease control policy does not impose any compliance costs on those agents, $C_s \le 0$, because it is privately optimal for them to protect against the disease if they are not selected for testing. However, for agents without suspicions, the increase in the expected cost of compliance with the mandatory disease control policy, $C_h > 0$, offsets the option value of diagnosis when the testing capacity is limited, i.e. $\Delta_h^M \le 0$ for sufficiently small m. Therefore, the truth-telling constraints for both types of agents are satisfied without monetary transfers.

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