

# Measuring Uncertainty\*

Kyle Jurado  
Columbia University

Sydney C. Ludvigson  
NYU and NBER

Serena Ng  
Columbia University

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## Abstract

This paper exploits a data rich environment to provide direct econometric estimates of time-varying macro uncertainty, defined as the common variation in the unforecastable component of a large number of economic indicators. Our estimates display significant independent variations from popular uncertainty proxies, suggesting that much of their variation is not driven by uncertainty. Quantitatively important uncertainty episodes appear far more infrequently than indicated by popular uncertainty proxies, but when they do occur, they have larger and more persistent correlations with real activity. Our estimates provide a benchmark to evaluate theories for which uncertainty shocks play a role in business cycles.

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\*Jurado: Department of Economics, Columbia University, 1019 International Affairs Building, MC 3308, 420 West 118th Street, New York, NY 10027; Email: [kej2108@columbia.edu](mailto:kej2108@columbia.edu).

Ludvigson: Department of Economics, New York University, 19 W.4th Street, 6th Floor, New York, NY 10012; Email: [sydney.ludvigson@nyu.edu](mailto:sydney.ludvigson@nyu.edu); <http://www.econ.nyu.edu/user/ludvigsons/>.

Ng: Department of Economics, Columbia University, 1019 International Affairs Building, MC 3308, 420 West 118th Street, New York, NY 10027; Email: [serena.ng@columbia.edu](mailto:serena.ng@columbia.edu); <http://www.columbia.edu/~sn2294/>.

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Uncertainty estimates and additional results are available from the website [http://www.econ.nyu.edu/user/ludvigsons/jln\\_supp.pdf](http://www.econ.nyu.edu/user/ludvigsons/jln_supp.pdf).

# 1 Introduction

How important is time-varying economic uncertainty and what role does it play in macroeconomic fluctuations? A large and growing body of literature has concerned itself with this question.<sup>1</sup> At a general level, uncertainty is typically defined as the conditional volatility of a disturbance that is unforecastable from the perspective of economic agents. In partial equilibrium settings, increases in uncertainty can depress hiring, investment, or consumption if agents are subject to fixed costs or partial irreversibilities (a “real options” effect), if agents are risk averse (a “precautionary savings” effect), or if financial constraints tighten in response to higher uncertainty (a “financial frictions” effect). In general equilibrium settings, many of these mechanisms continue to imply a role for time-varying uncertainty, although some may also require additional frictions to generate the same effects.

A challenge in empirically examining the behavior of uncertainty, and its relation to macroeconomic activity, is that no objective measure of uncertainty exists. So far, the empirical literature has relied primarily on proxies or indicators of uncertainty, such as the implied or realized volatility of stock market returns, the cross-sectional dispersion of firm profits, stock returns, or productivity, the cross-sectional dispersion of subjective (survey-based) forecasts, or the appearance of certain “uncertainty-related” key words in news publications. While most of these measures have the advantage of being directly observable, their adequacy as proxies for uncertainty depends on how strongly they are correlated with this latent stochastic process.

Unfortunately, the conditions under which common proxies are likely to be tightly linked to the typical theoretical notion of uncertainty defined above may be quite special. For example, stock market volatility can change over time even if there is no change in uncertainty about economic fundamentals, if leverage changes, or if movements in risk aversion or sentiment are important drivers of asset market fluctuations. Cross-sectional dispersion in individual stock returns can fluctuate without any change in uncertainty if there is heterogeneity in the loadings on common risk factors. Similarly, cross-sectional dispersion in firm-level profits, sales, and productivity can fluctuate over the business cycle merely because there is heterogeneity in the cyclicity of firms’ business activity.

This paper provides new measures of uncertainty and relates them to macroeconomic activity. Our goal is to provide superior econometric estimates of uncertainty that are as free as possible both from the structure of specific theoretical models, and from dependencies on any single (or small number) of observable economic indicators. We start from the premise that

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<sup>1</sup>See for example, Bloom (2009); Arellano, Bai, and Kehoe (2011); Bloom, Floetotto, and Jaimovich (2010); Bachmann, Elstner, and Sims (2012); Gilchrist, Sim, and Zakrajsek (2010); Schaal (2012) Bachmann and Bayer (2011); Baker, Bloom, and Davis (2011); Basu and Bundick (2011); Knotek and Khan (2011) Fernández-Villaverde, Pablo Guerrón-Quintana, and Uribe (2011); Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012); Leduc and Liu (2012); Nakamura, Sergeyev, and Steinsson (2012); Orlik and Veldkamp (2013).

what matters for economic decision making is not whether particular economic indicators have become more or less variable or disperse *per se*, but rather whether the economy has become more or less *predictable*; that is, less or more uncertain.

To formalize our notion of uncertainty, let us define  $h$ -period ahead uncertainty in the variable  $y_{jt} \in Y_t = (y_{1t}, \dots, y_{N_y t})'$ , denoted by  $\mathcal{U}_{jt}^y(h)$ , to be the conditional volatility of the purely unforecastable component of the future value of the series. Specifically,

$$\mathcal{U}_{jt}^y(h) \equiv \sqrt{E \left[ (y_{jt+h} - E[y_{jt+h}|I_t])^2 | I_t \right]} \quad (1)$$

where the expectation  $E(\cdot|I_t)$  is taken with respect to information  $I_t$  available to economic agents at time  $t$ . If the expectation today (conditional on all available information) of the squared error in forecasting  $y_{jt+h}$  rises, uncertainty in the variable increases.<sup>2</sup> A measure of *macroeconomic uncertainty* can then be constructed by aggregating individual uncertainty at each date using aggregation weights  $w_j$  :

$$\mathcal{U}_t^y(h) \equiv \text{plim}_{N_y \rightarrow \infty} \sum_{j=1}^{N_y} w_j \mathcal{U}_{jt}^y(h) \equiv E_w[\mathcal{U}_{jt}^y(h)]. \quad (2)$$

We use the terms macro and aggregate uncertainty interchangeably.

We emphasize two features of these definitions. First, we distinguish between uncertainty in a series  $y_{jt}$  and its conditional volatility. The proper measurement of uncertainty requires removing the forecastable component  $E[y_{jt+h}|I_t]$  before computing conditional volatility. Failure to do so will lead to estimates that erroneously categorize forecastable variations as “uncertain.” Thus, uncertainty in a series is *not* the same as the conditional volatility of the raw series: it is important to first remove the forecastable component. While this point may seem fairly straightforward, it is worth noting that almost all measures of stock market volatility (realized or implied) or cross-sectional dispersion currently used in the literature do not take this into account.<sup>3</sup> We show below that this matters empirically for a large number of series, including the stock market.

Second, macroeconomic uncertainty is not equal to the uncertainty in any single series  $y_{jt}$ . Instead, it is a measure of the common variation in uncertainty across many series. This is important because uncertainty-based theories of the business cycle typically require the existence of common (often countercyclical) variation in uncertainty across large numbers of series.

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<sup>2</sup>Forecast error volatility could also be a result of so-called Knightian uncertainty (Knight (1921)), often described as a situation in which agents are uncertain about the probability distribution itself. This definition of uncertainty should also be distinguished from “risk.” In a finance context, risk is often measured by conditional covariance of returns with the stochastic discount factor in equilibrium models. Uncertainty as defined here is distinct from conditional volatility but could be one of several reasons why the conditional variances and covariances of returns vary.

<sup>3</sup>One exception is Gilchrist, Sim, and Zakrajsek (2010), who use the financial factors developed by Fama and French (1992) to control for common forecastable variation in their measure of realized volatility.

Indeed, in many models of the literature cited above, macroeconomic uncertainty is either directly presumed by introducing stochastic volatility into aggregate shocks (e.g., shocks to aggregate technology, representative-agent preferences, monetary or fiscal policy), or indirectly imposed by way of a presumed countercyclical component in the volatilities of individual firm- or household-level disturbances.<sup>4</sup> If these assumptions are correct, we would expect to find evidence of an aggregate uncertainty factor, that is a common component in uncertainty fluctuations that affects many series, sectors, markets, and geographical regions at the same time.

The objective of our paper is therefore to obtain estimates of (1) and (2). To make these measures of uncertainty operational, we require three key ingredients. First, we require an estimate of the forecast  $E[y_{jt+h}|I_t]$ . For this, we form factors from a large set of predictors  $\{X_{it}\}$ ,  $i = 1, 2, \dots, N$ , whose span is as close to  $I_t$  as possible. Using these factors, we then approximate  $E[y_{jt+h}|I_t]$  by a diffusion index forecast, which is ideal for data-rich environments. An important aspect of this data-rich approach is that the diffusion index factors can be treated as known in the subsequent analysis. Second, defining the  $h$ -step-ahead forecast error to be  $V_{jt+h}^y \equiv y_{jt+h} - E[y_{jt+h}|I_t]$ , we require an estimate of the conditional (on time  $t$  information) volatility of this error,  $E[(V_{t+h}^y)^2|I_t]$ . For this, we specify a parametric stochastic volatility model for both the one-step-ahead prediction errors in  $y_{jt}$ , denoted  $\{v_{jt+1}^y\}$ , and the analogous forecast errors for the factors. These volatility estimates are used to recursively compute the values of  $E[(V_{t+h}^y)^2|I_t]$  for  $h > 1$ . As we show below, this procedure takes into account an important property of multistep-ahead forecasts, namely that time-varying volatility in the errors of the predictor variables creates additional unforecastable variation in  $y_{jt+h}$  (above and beyond that created by stochastic volatility in the one-step-ahead prediction error), and contributes to its uncertainty. The third and final ingredient is an estimate of macroeconomic uncertainty  $\mathcal{U}_t^y(h)$  constructed from the individual uncertainty measures  $\mathcal{U}_{jt}^y(h)$ . Our base-case estimate of  $\mathcal{U}_t^y(h)$  is the equally-weighted average of individual uncertainties. It is also possible to let the weights be constructed so that macroeconomic uncertainty is interpreted as the common (latent) factor in the individual measures of uncertainty.

We estimate measures of macroeconomic uncertainty from two post-war datasets of economic activity. The first *macro* dataset is monthly and uses the information in hundreds of primarily macroeconomic and financial indicators. The second *firm level* dataset is quarterly and consists of 155 firm-level observations on profit growth normalized by sales. A measure of macroeconomic uncertainty as defined in (2) may be constructed from either dataset by aggregating the individual uncertainty estimates in the respective dataset. We will refer to estimates

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<sup>4</sup>This common variation is critical for the study of business cycles because if the variability of the idiosyncratic shock were entirely idiosyncratic, it would have no influence on macroeconomic variables. Many models have a role for time-varying firm-level uncertainty driven by a common time-varying variance of some idiosyncratic shock. See, e.g., Bloom (2009), Arellano, Bai, and Kehoe (2011), Bloom, Floetotto, and Jaimovich (2010), Gilchrist, Sim, and Zakrajsek (2010), Schaal (2012), Bachmann and Bayer (2011)).

of macro uncertainty based on the monthly series as *common macro uncertainty* whereas estimates of macro uncertainty based on the quarterly firm-level dataset will be referred to as *common firm-level uncertainty*.

Our main results may be summarized as follows. We find significant independent variation in our estimates of uncertainty as compared to commonly used proxies for uncertainty. An important finding is that our estimates imply far fewer large uncertainty episodes than what is inferred from all of the commonly used proxies we study. For example, consider the 17 uncertainty dates defined in Bloom (2009) as events associated with stock market volatility in excess of 1.65 standard deviations above its trend. By contrast, in a sample extending from 1959:01 to 2011:12, our measure of macro uncertainty exceeds (or come close to exceeding) 1.65 standard deviations from its mean a total of only four (out of 636) months, each of which occur during the three deep recession episodes discussed below. Moreover, an increase in stock market volatility, holding fixed lagged uncertainty, is in fact associated with a slight *decline* in macro uncertainty, according to impulse response functions from a recursively identified bivariate vector autoregression (VAR). Qualitatively, these results are similar for our measures of common firm-level uncertainty in profit growth rates. Taken together, the findings imply that most movements in common uncertainty proxies, such as stock market volatility (the most common proxy), and measures of cross-sectional dispersion, are not associated with a broad-based movement in economic uncertainty as defined in (2). This is important because it suggests that much of the variation in common uncertainty proxies is not being driven by uncertainty.

So how important is time-varying economic uncertainty, and to what extent is it linked to macroeconomic fluctuations? Our estimates of macro uncertainty reveal three big episodes of uncertainty in the post-war period: the months surrounding the 1973-74 and 1981-82 recessions and the Great Recession of 2007-09. Averaged across all uncertainty forecast horizons, the 2007-09 recession represents the most striking episode of heightened uncertainty since 1960, with the 1981-82 recession a close second. Large positive innovations to macro uncertainty lead to a sizable decline in real activity (production, hours, employment), though the decline is more protracted and does not exhibit the “overshooting” behavior found when stock market volatility measures are used to proxy for uncertainty. In an eight variable monthly macro VAR studied by Bloom (2009), common macro uncertainty shocks account for between 3.7 and 13.5% of the forecast error variance in industrial production, depending on the VAR forecast horizon. By contrast, stock market volatility explains between 0 and 6.7%. To form another basis for comparison, shocks to the federal funds rate (a common proxy for unanticipated shocks to monetary policy) account for 2.35% of the 12-month-ahead forecast error variance in this same VAR, while common macro uncertainty shocks account for 13.5%. We also ask how much each series’ time-varying *individual* uncertainty is explained by time-varying *macro* uncertainty and find that the role of the latter is strongly countercyclical, roughly doubling in importance during

recessions.

These results underscore the importance of considering how aggregate uncertainty is measured when assessing its relationship with the macroeconomy. In particular, our estimates imply that quantitatively important uncertainty episodes occur far more infrequently than what is indicated from common uncertainty proxies, but that when they *do* occur, they display larger and more persistent correlations with real activity. Indeed, the deepest, most protracted recessions in our sample are associated with large increases in estimated uncertainty, while more modest reductions in real activity are not. By contrast, common uncertainty proxies are less persistent and spike far more frequently, even in non-recession periods or periods of relative macroeconomic quiescence.

The rest of this paper is organized as follows. Section 2 reviews related empirical literature on uncertainty in more detail. Section 3 outlines the econometric framework employed in our study, and describes how our measures of uncertainty are constructed. Section 4 describes the data and empirical implementation. Section 5 presents our common macro uncertainty estimates, compares our measure to other proxies of uncertainty used in the literature, and considers the dynamic relationship between macro uncertainty and macroeconomic aggregates such as production and employment. Section 6 performs a similar analysis for our estimates of common firm-level uncertainty. Section 7 summarizes and concludes.

To conserve space, a large amount of supplementary material for this paper appears in Jurado, Ludvigson, and Ng (2013). This supplementary document has two parts. The first part provides results from a large number of robustness exercises designed to check the sensitivity of our results to various assumptions (see description below). The second part is a data appendix that contains details on the construction of all data used in this study, including data sources. The complete dataset used in this study, as well as the uncertainty estimates, are available for download from the authors' websites.<sup>5</sup>

## 2 Related Empirical Literature

The measurement of uncertainty is still in its infancy. Existing research has focused on measures of volatility or dispersion as proxies for uncertainty and has often found important relationships between real activity and measures of stock market volatility (e.g., Bloom (2009)) or the volatility in some measure of real activity, or between real activity and dispersion measures such as the cross-sectional spread of firm- and industry-level earnings, productivity or total factor productivity (TFP) growth (e.g. Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012)). An important finding is that these measures of volatility are sharply countercyclical and VAR estimates suggest that they have an impact on output and employment in

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<sup>5</sup><http://www.econ.nyu.edu/user/ludvigsons/data.htm>.

the six months after an innovation in these measures, with a rise in volatility at first depressing real activity and then increasing it, leading to an over-shoot of its long-run level, consistent with the predictions of models with uncertainty as a driving force of macroeconomic fluctuations.

While these analyses are sensible starting places and important cases to understand, we emphasize here that the measures of dispersion and stock market volatility studied may or may not be tightly linked to true economic uncertainty as defined above. Indeed, one of the most popular proxies for uncertainty is closely related to financial market volatility as measured by the VIX, which has a large component that appears driven by factors associated with time-varying risk-aversion rather than economic uncertainty (Bekaert, Hoerova, and Duca (2012)).

A separate strand of the literature focuses on cross-sectional dispersion in  $N_A$  analysts' or firms' subjective expectations as a measure of uncertainty:

$$\mathcal{D}_{jt}^A(h) = \sqrt{\sum_{k=1}^{N_{A_t}} w_k^A \left[ (y_{jt+h} - E(y_{jt+h}|I_{A_k,t}))^2 | I_{A_k,t} \right]^2}$$

where  $I_{A_k,t}$  is the information of agent  $k$  at time  $t$ , and  $w_k^A$  is the weight applied to agent  $k$ . One potential advantage of using  $\mathcal{D}_{jt}^A(h)$  as a proxy for uncertainty is that it treats the conditional forecast of  $y_{jt+h}$  as an observable variable, and therefore does not require estimation of  $E[y_{t+h}|I_{A_k,t}]$ . Bachmann, Elstner, and Sims (2012) follow this approach using a survey of German firms and argue that uncertainty appears to be more an outcome of recessions than a cause, contrary to the predictions of theoretical models such as Bloom (2009) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012). While analysts' forecasts are interesting in their own right, there are several known drawbacks in using them to measure uncertainty. First, subjective expectations are only available for a limited number of series. For example, of the 132 monthly macroeconomic series we will consider in this paper, not even one-fifth have corresponding expectations series. Second, it is not clear that the responses elicited from these surveys accurately capture the conditional expectations of the economy as a whole. The respondents typically sampled are practitioner forecasters; some analysts' forecasts are known to display systematic biases and omit relevant forecasting information (So (2012)), and analysts may have pecuniary incentives to bias their forecasts in a way that economic agents would not. Third, disagreement in survey forecasts could be more reflective of differences in opinion than of uncertainty (e.g., Diether, Malloy, and Scherbina (2002); Mankiw, Reis, and Wolfers (2003)) or, as discussed above, could reflect differences in firm's loadings on aggregate shocks, with constant aggregate and idiosyncratic uncertainty. Fourth, Lahiri and Sheng (2010) show that, even if forecasts are unbiased, disagreement in analysts' point forecasts does not equal (average across analysts) forecast error uncertainty unless the variance of accumulated aggregate shocks over the forecast horizon is zero. They show empirically using the Survey of Professional Forecasters that the variance of the accumulated aggregate shocks can drive a large wedge

between uncertainty and disagreement in times of important economic change, or whenever the forecast horizon is not extremely short. Bachmann, Elstner, and Sims (2012) acknowledge these problems and are careful to address them by using additional proxies for uncertainty, such as an ex-post measure of forecast error variance based on the survey expectations. A similar approach is taken in Scotti (2012) who studies series for which real-time data are available. Whereas these studies focus on variation in outcomes around subjective survey expectations of relatively few variables, we focus on uncertainty around objective statistical forecasts for hundreds of economic series, from which we identify aggregate uncertainty variations to capture uncertainty relevant for the macroeconomy. We view our study as complementary to these.

Our uncertainty measure is also different from proxies based on the unconditional cross-section dispersion of a particular variable:

$$\mathcal{D}_{jt}^B = \sqrt{\frac{1}{N_B} \sum_{k=1}^{N_B} \left[ (y_{jkt} - \frac{1}{N_B} \sum_i y_{jkt})^2 \right]} \quad (3)$$

where  $y_{jkt}$  is a variable indexed by  $j$  (e.g., firm-level profits studied in Bloom (2009)) for firm  $k$ , and  $N_B$  is the sample size of firms reporting profits. Notably, this dispersion has no forward looking component; it is the same for all horizons. This measure suffers from the same drawback as  $\mathcal{D}_{jt}^A(h)$ , namely that it can fluctuate without any change in uncertainty if there is heterogeneity in the cyclical activity of firms business activity.

Carriero, Clark, and Marcellino (2012) consider common sources of variation in the residual volatilities of a Bayesian Vector Autoregression (VAR). This investigation differs from ours in several ways: their focus is on small-order VARs (e.g., 4 or 8 variables) and residual volatility, which corresponds to our definition of uncertainty only when  $h = 1$ ; our interest is in measuring the prevalence of uncertainty across the entire macroeconomy. Their estimation procedure presumes that individual volatilities are not subject to idiosyncratic shocks (only common ones), and it is not possible for some series to have homoskedastic shocks while others have heteroskedastic shocks. We find a large idiosyncratic component in individual volatilities, the magnitude of which varies across series.

An important unresolved issue for empirical analysis of uncertainty concerns the persistence of uncertainty shocks. In models studied by Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), for example, recessions are caused by an increase in uncertainty, which in turn causes a drop in productivity growth. But other researchers who have studied models where uncertainty plays a key role (e.g., Schaal (2012)) have argued that empirical proxies for uncertainty, such as the cross-sectional dispersion in firms' sales growth, are not persistent enough to explain the prolonged levels of unemployment that have occurred during and after some recessions, notably the 2007-2009 recession and its aftermath. Here we provide new measures of uncertainty and its persistence, finding that they are considerably more persistent

than popular proxies such as stock market volatility and measures of dispersion.

### 3 Econometric Framework

We now turn to a description of our econometric framework. A crucial first step in our analysis is to replace the conditional expectation in (1) by a forecast, from which we construct the forecast error that forms the basis of our uncertainty measures. In order to identify a true forecast error, it is important that our predictive model be as rich as possible, so that our measured forecast error is purged of predictive content. A standard approach is to select a set of  $K$  predetermined conditioning variables given by the  $K \times 1$  vector  $W_t$ , and then estimate

$$y_{t+1} = \beta'W_t + \epsilon_{t+1} \tag{4}$$

by least squares. The one period forecast is  $\hat{y}_{t+1|t} = \hat{\beta}'W_t$  where  $\hat{\beta}$  is the least squares estimate of  $\beta$ . A time-varying omitted-information bias may arise if conditioning variables does not span the information sets of economic agents such as financial market participants.<sup>6</sup> This problem is especially important in our exercise since relevant information not used to form forecasts will lead to spurious estimates of uncertainty and its dynamics.

To address this problem, we use the method of diffusion index forecasting whereby a relatively small number of factors estimated from a large number of economic time series are augmented to an otherwise standard forecasting model. The omitted information problem is remedied by including estimated factors, and possibly non-linear functions of these factors, in the forecasting model. This eliminates the arbitrary reliance on a small number of exogenous predictors and enables the use of information in a vast set of economic variables that are more likely to span the unobservable information sets of economic agents. Diffusion index forecasts are increasingly used in data rich environments. Thus we only generically highlight the forecasting step and focus instead on construction of uncertainty, leaving details about estimation of the factors to the on-line supplementary file.

#### 3.1 Construction of Forecast Uncertainty

Let  $X_t = (X_{1t}, \dots, X_{Nt})'$  generically denote the predictors available for analysis. It is assumed that  $X_t$  has been suitably transformed (such as by taking logs and differencing) so as to render the series stationary. We assume that  $X_{it}$  has an approximate factor structure taking the form

$$X_{it} = \Lambda_i^{F'}F_t + e_{it}^X, \tag{5}$$

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<sup>6</sup>Forecasts of both real activity and financial returns are substantially improved by augmenting best-fitting conventional forecasting equations with common factors estimated from large datasets (Stock and Watson (2002), Stock and Watson (2004), Ludvigson and Ng (2007), Ludvigson and Ng (2009)).

where  $F_t$  is an  $r_F \times 1$  vector of latent common factors,  $\Lambda_t^F$  is a corresponding  $r_F \times 1$  vector of latent factor loadings, and  $e_{it}^X$  is a vector of idiosyncratic errors. In an *approximate* dynamic factor structure, the idiosyncratic errors  $e_{it}^X$  are permitted to have a limited amount of cross-sectional correlation.<sup>7</sup> Importantly, the number of factors  $r_F$  is significantly smaller than the number of series,  $N$ .

Let  $y_{jt}$  generically denote a series that we wish to compute uncertainty in and whose value in period  $h \geq 1$  is estimated from a factor augmented forecasting model

$$y_{jt+1} = \phi_j^y(L)y_{jt} + \gamma_j^F(L)\hat{F}_t + \gamma_j^W(L)W_t + v_{jt+1}^y \quad (6)$$

where  $\phi_j^y(L)$ ,  $\gamma_j^F(L)$ , and  $\gamma_j^W(L)$  are finite-order polynomials in the lag operator  $L$  of orders  $p_y$ ,  $p_F$ , and  $p_W$ , respectively,  $\hat{F}_t$  are consistent estimates of a rotation of  $F_t$ , and  $W_t$  are additional predictors of dimension  $r_W$ . These additional predictors could be non-linear functions of the factors  $F_t$ . We use an autoregressive model to obtain forecast of the factors. An important feature of our analysis is that the one-step-ahead prediction error of  $y_{jt+1} = v_{jt+1}^y = \sigma_{jt+1}^y \epsilon_{jt+1}^y$  and of each factor  $v_{kt+1}^F = \sigma_{kt+1}^F \epsilon_{kt+1}^F$  are permitted to have time-varying volatility  $\sigma_{jt+1}^y$  and  $\sigma_{kt+1}^F$ , respectively. This feature generates time-varying uncertainty in the series  $y_{jt}$ .

An alternative (and more compact) representation of the system above is the *factor augmented vector autoregression* (FAVAR). Let  $\mathcal{Z}_t \equiv (\hat{F}_t', W_t')'$  be an  $r = r_F + r_W$  vector which collects the  $r_F$  estimated factors and  $r_W$  additional predictors, and define  $\mathcal{F}_t \equiv (\mathcal{Z}_t, \dots, \mathcal{Z}_{t-q+1})'$ . Also let  $Y_{jt} = (y_{jt}, y_{jt-1}, \dots, y_{jt-q+1})'$ . Then forecasts for any  $h > 1$  can be obtained from the FAVAR system, stacked in first-order companion form:

$$\begin{aligned} \underbrace{\begin{pmatrix} \mathcal{F}_t \\ Y_{jt} \end{pmatrix}}_{(r+1)q \times 1} &= \begin{pmatrix} \underbrace{\Phi_j^F}_{qr \times qr} & \underbrace{\mathbf{0}}_{qr \times q} \\ \underbrace{\Lambda_j^F}_{q \times qr} & \underbrace{\Phi_j^Y}_{q \times q} \end{pmatrix} \begin{pmatrix} \mathcal{F}_{t-1} \\ Y_{jt-1} \end{pmatrix} + \begin{pmatrix} \mathcal{V}_t^F \\ \mathcal{V}_{jt}^Y \end{pmatrix} \\ \mathcal{Y}_{jt} &= \Phi_j^Y \mathcal{Y}_{jt-1} + \mathcal{V}_{jt}^Y, \end{aligned} \quad (7)$$

where  $\Lambda_j^F$  and  $\Phi_j^Y$  are functions of the coefficients in the matrix polynomial lag operators in (6) and  $\Phi^F$  stacks the autoregressive coefficients of the  $F_t$ .<sup>8</sup> By the assumption of stationarity, the largest eigenvalue of  $\Phi_j^Y$  is less than one and, under quadratic loss, the optimal  $h$ -period forecast is the conditional mean:

$$E_t \mathcal{Y}_{jt+h} = (\Phi_j^Y)^h \mathcal{Y}_{jt}.$$

<sup>7</sup>The approximate factor specification requires that the largest eigenvalue of the variance-covariance matrix of idiosyncratic errors be bounded. This limits the contribution of the idiosyncratic covariances to the total variance of  $X$  as  $N$  gets large:  $N^{-1} \sum_{i=1}^N \sum_{j=1}^N |E(e_{it}e_{jt})| \leq M$ , where  $M$  is a constant.

<sup>8</sup>The above specification assumes that the coefficients are time-invariant. An extension to allow for time-varying coefficients as in Cogley and Sargent (2005) would be a significant computational challenge because of the large systems we estimate.

The forecast error variance at  $t$  is

$$\Omega_{jt}^{\mathcal{Y}}(h) \equiv E_t [(\mathcal{Y}_{jt+h} - E_t \mathcal{Y}_{jt+h})(\mathcal{Y}_{jt+h} - E_t \mathcal{Y}_{jt+h})'].$$

Time variation in the mean squared forecast error in general arises from the fact that shocks to *both*  $y_{jt}$  and the predictors  $\mathcal{Z}_t$  may have time-varying variances. We now turn to these implications beginning with  $h = 1$  noting that

$$\Omega_{jt}^{\mathcal{Y}}(1) = E_t(\mathcal{V}_{jt+1}^{\mathcal{Y}} \mathcal{V}_{jt+1}^{\mathcal{Y}'}). \quad (8)$$

For  $h > 1$ , the forecast error variance of  $\mathcal{Y}_{jt+h}$  evolves according to

$$\Omega_{jt}^{\mathcal{Y}}(h) = \Phi_j^{\mathcal{Y}} \Omega_{jt}^{\mathcal{Y}}(h-1) \Phi_j^{\mathcal{Y}'} + E_t(\mathcal{V}_{jt+h}^{\mathcal{Y}} \mathcal{V}_{jt+h}^{\mathcal{Y}'}). \quad (9)$$

As  $h \rightarrow \infty$  the forecast is the unconditional mean and the forecast error variance is the unconditional variance of  $\mathcal{Y}_{jt}$ . This implies that  $\Omega_{jt}^{\mathcal{Y}}(h)$  is less variable as  $h$  increases.

We are interested in the expected (squared) forecast uncertainty of the scalar series  $y_{jt+h}$  given information at time  $t$ , denoted  $\mathcal{U}_{jt}^y(h)$ . This is the square-root of the appropriate entry of the forecast error variance  $\Omega_{jt}^{\mathcal{Y}}(h)$ . More precisely, with  $1_j$  being a selection vector,

$$\mathcal{U}_{jt}^y(h) = \sqrt{1_j' \Omega_{jt}^{\mathcal{Y}}(h) 1_j}. \quad (10)$$

To estimate macro (economy-wide) uncertainty  $\mathcal{U}_t^y(h)$ , we form weighted averages of individual uncertainty estimates:

$$\mathcal{U}_t^y(h) = \sum_{j=1}^{N_y} w_j \mathcal{U}_{jt}^y(h).$$

A simple weighting scheme is to give every series the equal weight of  $1/N_y$ . If individual uncertainty has a factor structure, the weights can be defined by the eigenvector corresponding to the largest eigenvalue of the  $N_y \times N_y$  covariance matrix of the matrix of individual uncertainty.

### 3.2 Time-varying Uncertainty: A Statistical Decomposition

Of interest is whether *time-varying* uncertainty has macroeconomic consequences. Time-varying uncertainty in a series cannot arise when innovations in the model for forecasting the series are entirely homoskedastic. Furthermore, because the future values of the predictors are also unknown,  $h$ -step-ahead uncertainty in  $y_{jt+h}$  is governed by uncertainty in innovations to the predictors  $\hat{F}_t$  and  $W_t$  that have time-varying volatility, as well as time-varying volatility of the unanticipated shock to  $y_{jt+h}$ . And, persistence in uncertainty is driven by autoregressive dynamics in stochastic volatility.

To better understand the time-varying nature of uncertainty, consider first the factor component and its  $h$ -step-ahead error (the argument for  $W_t$  is similar). If the factors  $F_t$  are unforecastable, they can explain in-sample variation of  $y_t$  but is of no use in prediction. Thus suppose that each  $F_t$  is serially correlated and well represented by a univariate AR(1) model:

$$F_t = \Phi^F F_{t-1} + v_t^F.$$

To fix ideas, suppose that there is only one factor. If  $v_t^F$  is a martingale difference with constant variance  $(\sigma^F)^2$ , the forecast error variance  $\Omega^F(h) = \Omega^F(h-1) + (\phi^F)^{2(h-1)}(\sigma^F)^2$  increases with  $h$  but is the same for all  $t$ . We allow the shocks to the common factors  $F$  to exhibit time-varying volatility of the form  $v_t^F = \sigma_t^F \varepsilon_t^F$ , where  $\sigma_t^F$  is the time-varying volatility component and  $\varepsilon_t^F$  is i.i.d. It can be shown that the forecast error variance obeys the recursion

$$\Omega_t^F(h) = (\Phi^F)\Omega_t^F(h-1)\Phi^{F'} + E_t(v_{t+h}^F v_{t+h}^{F'})$$

with  $\Omega_t^F(1) = E_t(v_{t+h}^F)^2$ . The  $h$  period ahead *predictor uncertainty* at time  $t$  is the square root of the  $h$ -step forecast error variance of the predictor. For the common factors  $F$ , predictor uncertainty is

$$\mathcal{U}_t^F(h) = \sqrt{1_F' \Omega_t^F(h) 1_F}$$

where  $1_F$  is an appropriate selection vector.

In addition to time varying volatility in the forecast errors of the predictors,  $y_{jt}$  itself has an innovation  $v_{jt}^y = \sigma_{jt}^y \varepsilon_{jt}^y$  which itself has a time-varying volatility component. The volatilities  $\sigma_{jt}^y$  and  $\sigma_t^F$  are characterized by stochastic volatility models:

$$\begin{aligned} \log(\sigma_t^F)^2 &= \alpha^F + \beta^F \log(\sigma_{t-1}^F)^2 + \tau_F \eta_t^F, & \eta_t^F &\stackrel{iid}{\sim} N(0, 1) \\ \log(\sigma_{jt+1}^y)^2 &= \alpha_j^y + \beta_j^y \log(\sigma_{jt}^y)^2 + \tau_j^y \eta_{jt+1}^y, & \eta_{jt+1}^y &\stackrel{iid}{\sim} N(0, 1) \end{aligned}$$

where  $\varepsilon_{jt+1}^y$  and  $\varepsilon_{t+1}^F \stackrel{iid}{\sim} N(0, 1)$ . The conditional expectation of the squared forecast error is computed to take into account of these time-varying volatilities. In particular, the stochastic volatility model implies

$$E_t(\sigma_{t+h}^F)^2 = \exp \left[ \alpha^F \sum_{s=0}^{h-1} (\beta^F)^{s-1} + \frac{(\tau^F)^2}{2} \sum_{s=0}^{h-1} (\beta^F)^{2(s-1)} + (\beta^F)^h \log(\sigma_t^F)^2 \right]$$

and similarly for  $\sigma_{jt+h}^y$ . Calculations analogous to those for  $F_t$  apply to the additional predictors  $W_t$ . The terms in  $E(\mathcal{V}_{j,t+h}^y \mathcal{V}_{j,t+h}^{y'})$  are computed using the fact that  $E_t(v_{jt+h}^y)^2 = E_t(\sigma_{jt+h}^y)^2$ ,  $E_t(v_{t+h}^F)^2 = E_t(\sigma_{t+h}^F)^2$  and  $E_t(v_{t+h}^W)^2 = E_t(\sigma_{t+h}^W)^2$ . It follows that  $h$ -period-ahead uncertainty of  $y_{jt}$  is affected by a standard (constant) level-effect attributable to homoskedastic variation in  $v_{jt}^y$ , and a second-moment-effect arising from the stochastic volatility in  $v_{jt}^y$ ,  $v_t^F$  and  $v_t^W$ . The level-effect contributes to homoskedastic uncertainty, while the second-moment-effect contributes to *time-varying* uncertainty.

There is, however, a third effect arising from the time-varying covariance between the forecast errors in the predictors and in  $y_{jt}$  that also contributes to time-varying uncertainty. To see this, suppose  $\phi_j^y(L) = \phi_j^y$  (scalar),  $\gamma_j^F(L) = \gamma_j^F$ , and  $W_t$  is empty. The forecasting model

$$y_{jt+1} = \phi_j^y y_{jt} + \gamma_j^F \hat{F}_t + v_{jt+1}^y$$

implies a  $h$ -period-ahead forecasting error  $V_{jt+h}^y$ . When  $h = 1$ ,  $V_{jt+1}^y$  coincides with the innovation  $v_{jt+1}^y$  which is uncorrelated with the one-step-ahead error in forecasting  $F_{t+1}$ , given by  $V_{t+1}^F = v_{t+1}^F$ . When  $h = 2$ , the forecast error for the factor is  $V_{t+2}^F = \Phi^F V_{t+1}^F + v_{t+2}^F$ . The corresponding forecast error for  $y_{jt}$  is:

$$V_{jt+2}^y = v_{jt+2}^y + \phi_j^y V_{jt+1}^y + \gamma_j^F V_{t+1}^F$$

which evidently depends on the one-step-ahead forecasting errors made at time  $t$ , but  $V_{t+1}^y$  and  $V_{t+1}^F$  are uncorrelated. Now consider the  $h = 3$  case. The forecast error is defined by

$$V_{jt+3}^y = v_{jt+3}^y + \phi_j^y V_{jt+2}^y + \gamma_j^F V_{t+2}^F.$$

Evidently,  $V_{jt+3}^y$  will depend on  $V_{jt+2}^y$  and  $V_{t+2}^F$ . But unlike the  $h = 2$  case, the two components  $V_{jt+2}^y$  and  $V_{t+2}^F$  are now correlated because both depend on  $V_{t+1}^F$ .

Therefore, returning to the general case,  $h$ -step-ahead forecast error variance admits the decomposition:

$$\Omega_{jt}^{\mathcal{Y}}(h) = \underbrace{\Phi_j^{\mathcal{Y}} \Omega_{jt}^{\mathcal{Y}} 1}_{\text{autoregressive}} (h-1) + \underbrace{\Omega_{jt}^{\mathcal{Z}}(h-1)}_{\text{Predictor}} + \underbrace{E_t(V_{jt+h}^{\mathcal{Y}} V_{jt+h}^{\mathcal{Y}'})}_{\text{stochastic volatility } \mathcal{Y}} + \underbrace{2\Phi_j^{\mathcal{Y}} \Omega_{jt}^{\mathcal{Y}\mathcal{Z}}(h-1)}_{\text{covariance}} \quad (11)$$

where  $\Omega_{jt}^{\mathcal{Y}\mathcal{Z}}(h) = \text{cov}_t(V_{jt+h}^{\mathcal{Y}}, V_{jt+h}^{\mathcal{Z}})$ . Time variation in uncertainty can thus be mathematically decomposed into four sources: an autoregressive component, a common factor (predictor) component, a stochastic volatility component, and a covariance term. Representation (11), which is equivalent to (10), makes clear that predictor uncertainty plays an important role via the second term  $\Omega_{jt}^{\mathcal{Z}}(h-1)$ . It is time-varying because of stochastic volatility in the innovations to the factors and is in general non-zero for multi-step-ahead forecasts, i.e.,  $h > 1$ . The role of stochastic volatility in the series  $y_j$  comes through the third term, with the role of the covariance between the forecast errors of the series and the predictors comes through the last term. Computing the left-hand-side therefore requires estimates of stochastic volatility in the residuals of every series  $y_j$ , and in every predictor variable  $\mathcal{Z}_j$ .

## 4 Empirical Implementation and Macro Data

Our empirical analysis forms forecasts and common uncertainty from two datasets spanning the period 1959:01-2011:12. The first dataset, denoted  $X^m$ , is an updated version of the of the 132

mostly macroeconomic series used in Ludvigson and Ng (2010). The 132 macro series in  $X^m$  are selected to represent broad categories of macroeconomic time series: real output and income, employment and hours, real retail, manufacturing and trade sales, consumer spending, housing starts, inventories and inventory sales ratios, orders and unfilled orders, compensation and labor costs, capacity utilization measures, price indexes, bond and stock market indexes, and foreign exchange measures. The second dataset, denoted  $X^f$ , is an updated monthly version of the of 147 financial time series used in Ludvigson and Ng (2007). The data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry equity returns. A detailed description of the series is given in the on-line supplementary file.

We combine the macro and financial monthly datasets together into one large “macroeconomic dataset” ( $X$ ) to estimate forecasting factors in these 132+147=279 series. However, we estimate macroeconomic uncertainty  $\mathcal{U}_t^y(h)$  from the individual uncertainties in the 132 macro series only. Uncertainties in the 147 financial series are not computed because  $X^m$  already include some financial indicators, and it is not desirable to over-represent the financial series in the aggregate uncertainty measure.

The stochastic volatility parameters  $\alpha_j, \beta_j, \tau_j$  are estimated from the least square residuals of the forecasting models using Monte Carlo Markov chain (MCMC) methods.<sup>9</sup> In the base-case, the average of these parameters model over the MCMC draws are used to estimate  $\mathcal{U}_{jt}^y(h)$ . Simple averaging is used to obtain an estimate of  $h$  period macro uncertainty denoted

$$\bar{\mathcal{U}}_t^y(h) = \frac{1}{N^y} \sum_{j=1}^{N^y} \hat{\mathcal{U}}_{jt}^y(h). \quad (12)$$

This measure of average uncertainty does not impose any structure on the individual uncertainties. A latent common factor estimate of macro uncertainty, denoted  $\hat{\mathcal{U}}_t^y(h)$ , is also constructed. To ensure that the latent uncertainty factor is positive, the method of principal components is applied to the logarithm of the individual uncertainty estimates and then rescaled. Its construction is detailed in an on-line supplementary file Jurado, Ludvigson, and Ng (2013).

To assess the importance of macro uncertainty in the total uncertainty in series  $j$  over a sample defined by  $\tau$ , we consider the following metric:

$$R_{j\tau}^2(h) = \frac{\text{var}_\tau(\hat{\varphi}_{j\tau}(h)\bar{\mathcal{U}}_t^y(h))}{\text{var}_\tau(\hat{\mathcal{U}}_{jt}^y(h))}. \quad (13)$$

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<sup>9</sup>We use the `STOCHVOL` package in R, which implements the ancillarity-sufficiency interweaving strategy as discussed in Kastner and Fruhwirth-Schnatter (2013) which is less sensitive to whether the mean of the volatility process is in the observation or the state equation. Earlier versions of this paper implements the algorithm of Kim, Shephard, and Chib (1998) using our own MATLAB code.

where  $\hat{\varphi}_{j\tau}(h)$  is the coefficient from a regression of  $\hat{\mathcal{U}}_{jt}^y(h)$  on  $\bar{\mathcal{U}}_t^y(h)$ . Thus  $R_{j\tau}^2(h)$  is the fraction of variation in  $\mathcal{U}_{jt}^y(h)$  explained by macro uncertainty  $\mathcal{U}_t^y(h)$  in the subsample.

Throughout, the factors in the forecasting equation are estimated by the method of static principal components (PCA). Bai and Ng (2006) show that if  $\sqrt{T}/N \rightarrow 0$ , the estimates  $\hat{F}_t$  can be treated as though they were observed in the subsequent forecasting regression. The defining feature of a model with  $r_F$  factors is that the  $r_F$  largest population eigenvalues should increase as  $N$  increases, while the  $N - r_F$  eigenvalues should be bounded. The criterion of Bai and Ng (2002) suggests  $r_F = 12$  forecasting factors  $F_t$  for the combined datasets  $X^m$  and  $X^f$  explaining about 54% of the variation in the 279 series, with the first three factors accounting for 37%, 8%, 3%, respectively. The first factor loads heavily on stock market portfolio returns (such as size and book-market portfolio returns), the excess stock market return, and the log dividend-price ratio. The second factor loads heavily on measures of real activity, such as manufacturing production, employment, total production and employment, and capacity utilization. The third factor loads heavily on risk and term spreads in the bond market.

The potential predictors in the forecasting model are  $\hat{F}_t = (\hat{F}_{1t}, \dots, \hat{F}_{r_F t})'$  and  $W_t$ , where  $W_t$  consists of squares of the first component of  $\hat{F}_t$ , and factors in  $X_{it}^2$  collected into the  $N_G \times 1$  vector  $\hat{G}_t$ . Following Bai and Ng (2008), the predictors ultimately used are selected so as to insure that only those likely to have significant incremental predictive power are included. To do so, we apply a hard thresholding rule using a conservative  $t$  test to retain those  $F_t$  and  $W_t$  that are statistically significant.<sup>10</sup> The most frequently selected predictors are  $\hat{F}_{2t}$ , a “real” factor highly correlated with measures of industrial production and employment,  $\hat{F}_{12t-1}$ , highly correlated with lagged hours,  $\hat{F}_{4t}$ , highly correlated with measures of inflation, and  $\hat{F}_{10t}$ , highly correlated with exchange rates. Four lags of the dependent variable were always included in the predictive regressions.

## 5 Estimates of Macro Uncertainty

We present estimates of macro uncertainty for three uncertainty horizons:  $h = 1, 3$ , and 12 months. Figure 1 plots  $\bar{\mathcal{U}}_t^y(h)$  over time for  $h = 1, 3$ , and 12, along with the NBER recession dates. The matching horizontal bars correspond to 1.65 standard deviations above the mean for each series. Figure 1 shows that macro uncertainty is clearly countercyclical: the correlation of  $\bar{\mathcal{U}}_t^y(h)$  with industrial production growth is -0.62, -0.61, and -0.57 for  $h = 1, 3$ , and 12, respectively. While the level of uncertainty increases with  $h$  (on average), the variability of

<sup>10</sup>Specifically, we begin with a set of candidate predictors that includes all the estimated factors in  $X_{it}$  (the  $\hat{F}_t$ ), the first estimated factor in  $X_{it}^2$  ( $\hat{G}_{1t}$ ), and the square of the first factor in  $X_{it}$  ( $\hat{F}_{1t}^2$ ). We then chose subsets from these by running a regression of  $y_{it+1}$  on a constant, four lags of the dependent variable,  $\hat{F}_t$ ,  $\hat{F}_{1t}^2$ , and  $\hat{G}_{1t}$  (no lags). Regressors are retained if they have a marginal  $t$  statistic greater than 2.575 in the multivariate forecasting regression of  $y_{it+1}$  on the candidate predictors known at time  $t$ .

uncertainty decreases because the forecast tends to the unconditional mean as the forecast horizon tends to infinity. Macro uncertainty exhibits spikes around the 1973-74 and 1981-82 recessions, as well as the Great Recession of 2007-09.

Looking across all uncertainty forecast horizons  $h = 1, 3,$  and  $12,$  the 2007-09 recession clearly represents the most striking episode of heightened uncertainty since 1960. The 1981-82 recession is a close second, especially for forecast horizons  $h = 3$  and  $12.$  Indeed, for these horizons, these are the only two episodes for which macro uncertainty exceeds 1.65 standard deviations above its mean in our sample. Inclusive of  $h = 1,$  the three episodes are the only instances in which  $\bar{\mathcal{U}}_t^y(h)$  exceeds, or comes close to exceeding, 1.65 standard deviation above its mean, implying far fewer uncertainty episodes than other popular proxies for uncertainty, as we show below. Heightened uncertainty is broad-based during these three episodes as the fraction of series with  $\mathcal{U}_{jt}^y(h)$  exceeding their own standard deviation over the full sample are .42, .61, and .51 for 1, 3, and 12 respectively. Further investigation reveals that the three series with the highest uncertainty between 1973:11 and 1975:03 are a producer price index for intermediate materials, a commodity spot price index, and employment in mining. For the 1980:01 and 1982:11 episode, uncertainty is highest for the Fed funds rate, employment in mining, and the 3 months commercial paper rate. Between 2007:12 and 2009:06, uncertainty is highest for the monetary base, non-borrowed reserves and total reserves. These findings are consistent with the historical account of an energy crisis around 1974, a recession of monetary policy origin around 1981, and a financial crisis around 2008 that created challenges for the operation of monetary policy.

Uncertainty in a series is defined above as the volatility of a purely unforecastable error of that series. It is potentially influenced by macro uncertainty shocks and idiosyncratic uncertainty shocks. To assess the relative importance of macro uncertainty  $\bar{\mathcal{U}}_t^y(h)$  in total uncertainty (summed over all series), we compute, for each of the 132 series in the macro dataset, and for  $h = 1, 3,$  and  $12,$   $R_{jt}^2(h)$  as defined in (13). This exercise is performed for the full sample, for recession months, and for non-recession months.<sup>11</sup> The larger is  $R_t^2(h) \equiv \frac{1}{N_y} \sum_{j=1}^{N_y} R_{jt}^2(h),$  the more important is macro uncertainty in explaining total uncertainty.

Table 1 shows that the importance of macro uncertainty grows as the forecast horizon  $h$  increases. On average, uncertainty across all series is much higher for  $h = 3$  and  $h = 12$  than it is for  $h = 1.$  Table 1 also shows that macro uncertainty  $\bar{\mathcal{U}}_t^y(h)$  accounts for a quantitatively large fraction of the variation in total uncertainty in the individual series. For example, when the uncertainty horizon is  $h = 3$  months, the estimated macro uncertainty explains an average (across all series) of 16% of the variation in uncertainty over the non-recession sample. But it explains a much larger 26% in recessions. The results are similar for the  $h = 12$  case. Results

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<sup>11</sup>Recession months are defined by National Bureau of Economic Research dates. Macro uncertainty is estimated over the full sample even when the  $R^2$  statistics are computed over subsamples.

in the right panel of the table based on the common uncertainty factor  $\hat{\mathcal{U}}_t(h)$  reinforce the point that macro uncertainty accounts for a larger fraction of the variation in total uncertainty during recessions.

## 5.1 The Role of the Predictors

We have emphasized the importance of removing the predictable variation in a series before attributing its fluctuations to a movement in uncertainty. How important are these predictable variations in our estimates? Our forecasting regression is

$$y_{jt+1} = \phi_j^y(L)y_{jt} + \gamma_j^F(L)\hat{F}_t + \gamma_j^W(L)W_t + \sigma_{jt+1}^y\varepsilon_{jt+1}.$$

The future values of our predictors  $F$  and  $W$  are unknown and each predictor is forecasted by an AR(4) model. As explained above, time-varying volatility in their forecast errors also contributes to  $h$ -step-ahead uncertainty in the variable  $y_{jt}$  whenever  $h > 1$ . Figure 2 plots predictor uncertainty  $\mathcal{U}_{kt}^F(h)$  for several estimated  $\hat{F}_{kt}$  that display significant stochastic volatility and that are frequently chosen as predictor variables according to the hard thresholding rule. These are,  $\hat{F}_{1t}$  (highly correlated with the stock market),  $\hat{F}_{2t}$  (highly correlated with measures of real activity such as industrial production and employment),  $\hat{F}_{4t}$  (highly correlated with measures of inflation),  $\hat{F}_{5t}$  (highly correlated with the Fama-French risk factors and bond default spreads). This figure also displays estimates of uncertainty for two predictors in  $W$ : the squared value of the first factor  $\hat{F}_{1t}^2$  and for the first factor formed from observations  $X_{it}^2$ , which we denote  $\hat{G}_{1t}$ . These results suggest that uncertainty in the predictor variables is an important contributor to uncertainty in the series  $y_{jt+h}$  to be forecast.

In addition to the stochastic volatility effect, the predictors directly affect the level of the forecast. An important aspect of our uncertainty measure is a forecasting model that exploits as much available information as possible to control for the economic state, so as not to erroneously attributing forecastable variations (as reflected in  $\hat{F}_t$  and  $W_t$ ) to uncertainty in series  $y_{jt+h}$ . Most popular measures of uncertainty do not take these systematic forecasting relationships into account. To examine the role that this information plays in our estimation procedure, we estimate the uncertainty for each series based on the following (potentially misspecified) simple model with constant conditional mean:

$$y_{jt+1} = \mu + \tilde{\sigma}_{jt+1}\tilde{\varepsilon}_{jt+1}. \quad (14)$$

Figure 3 plots the resulting estimates of one-step ahead uncertainty  $\mathcal{U}_{jt}^y(1)$  using this possibly misspecified model and compares it to the corresponding estimates using the full set of chosen predictors (chosen using the hard thresholding rule described above), for several key series in our dataset: total industrial production, employment in manufacturing, non-farm housing

starts, consumer expectations, M2, CPI-inflation, the ten-year/federal funds term spread, and the commercial paper/federal funds rate spread. Figure 3 shows that there is substantial heterogeneity in the time-varying uncertainty estimates across series, suggesting that a good deal of uncertainty is series-specific. But Figure 3 also shows that the estimates of uncertainty in these series are significantly influenced by whether the forecastable variation in these series is removed or not before computing uncertainty: when it is removed, the estimates of uncertainty tend to be lower, much so in some cases. Specifically, uncertainty in each of the eight variables shown in this figure is estimated to be lower during the 2007-09 recession when predictive content is removed than when not, especially for industrial production, employment, and the two interest rate spreads. The difference over time between the two estimates for these variables is quite pronounced in some periods, suggesting that much of the variation in these series is predictable and should not be attributed to uncertainty.<sup>12</sup>

Since stock market volatility is the most commonly used proxy for uncertainty, we further examine in Figure 4 how estimates of stock market uncertainty are affected by whether or not the purely forecastable variation in the stock market is removed before computing uncertainty. This figure compares (i) the estimate of uncertainty in the log difference of the *S&P* 500 index for a case where the conditional mean is assumed constant, implying that no predictable variation is removed, as in (14), with (ii) a case in which only autoregressive terms are included to forecast the stock market, as in

$$y_{jt+1} = \tilde{\phi}_j(L)y_{jt} + \tilde{\sigma}_{jt+1}\tilde{\varepsilon}_{jt+1},$$

with (iii) a case in which all selected factors (using the hard thresholding rule) estimated from the combined macro and financial dataset with 279 indicators are used as predictors. Notice that the first case (constant conditional mean) is most akin to estimates of stock market volatility such as the VXO index studied by Bloom (2009)<sup>13</sup> and discussed further below. We emphasize that stock market volatility measures do not purge movements in the stock market of its predictable piece and are therefore estimates of conditional volatility, not uncertainty. Of course, if there were no predictable component in the stock market, these two estimates would coincide. But Figure 4 shows that there is a substantial predictable component in the log change in the *S&P* price index, which, once removed, makes a quantitatively large difference in the estimated amount of uncertainty over time.<sup>14</sup> Uncertainty in the stock market is substantially

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<sup>12</sup>We have also estimated common macro uncertainty,  $\bar{U}_t^y(h)$  without removing predictable fluctuations. The spikes appear larger than the base case that removes the forecastable component in each series before computing uncertainty. This is especially true for the  $h = 1$  case, where presumably the predictive information is most valuable.

<sup>13</sup>This measure is unavailable before 1986 so Bloom (2009) uses *realized* volatility in the log difference of the S&P 500 Price Index during this period. We still refer to this composite measure as the “VXO Index.”

<sup>14</sup>There is substantial predictive information for excess stock market returns using factors formed from the financial dataset  $X^f$ —see Ludvigson and Ng (2007), as well as in other variables. For more general surveys of the predictable variation in stock market returns, see Cochrane (2005) and Lettau and Ludvigson (2010).

lower in every episode when these forecastable fluctuations are removed compared to when they are not, and is dramatically lower in the recession of 2007-09 compared to what is indicated by ex-post conditional stock market volatility.

We now examine more closely our measure of stock market uncertainty, that corresponding to  $\mathcal{U}_{jt}^y$  where the predictable component is removed, for  $y_{jt}$  equal to the stock market return. If we compare this measure of stock market uncertainty (Figure 4) with macro uncertainty  $\mathcal{U}_t^y$  (Figure 1), we see there are important differences over time in the two series. In particular, there are many (more) large spikes in stock market uncertainty that are not present for macro uncertainty. Unlike macro uncertainty, several of the spikes in financial uncertainty occur outside of recessions. Because stock market volatility is arguably the most common proxy for uncertainty, we further examine the distinction between uncertainty and stock market volatility in the next section.

## 5.2 Uncertainty Versus Stock Market Volatility

In an influential paper, Bloom (2009) emphasizes a measure of stock market volatility as a proxy of uncertainty.<sup>15</sup> This measure is primarily based on the VXO Index, which is constructed by the Chicago Board of Options Exchange from the prices of options contracts written on the S&P 100 Index. In this subsection we compare our macro uncertainty estimates with stock market volatility as a proxy for uncertainty. We update this stock market volatility series to include more recent observations, and plot it along with our estimated macro uncertainty  $\bar{\mathcal{U}}_t^y(h)$  for  $h = 1$  in Figure 5. To construct his benchmark measure of uncertainty “shocks” (plausibly exogenous variation in his proxy of uncertainty), Bloom selects 17 dates (listed in his Table A.1) which are associated with stock market volatility in excess of 1.65 standard deviations above its HP-detrended mean. These 17 dates are marked by vertical lines in the figure. As emphasized above and seen again in Figure 5,  $\bar{\mathcal{U}}_t^y(1)$  exceeds 1.65 standard deviations above its unconditional mean in only three episodes, suggesting far fewer episodes of uncertainty than that indicated by these 17 uncertainty dates.<sup>16</sup>

While  $\bar{\mathcal{U}}_t^y(1)$  is positively correlated with the VXO Index, with a correlation coefficient around 0.5, the VXO Index is itself substantially more volatile than  $\bar{\mathcal{U}}_t^y(1)$ , with many sharp peaks that are not correspondingly reflected by the macro uncertainty measure. For example, the large spike in October 1987 reflects “Black Monday,” which occurred on the 19th of the

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<sup>15</sup>A number of other papers also use stock market volatility to proxy for uncertainty; these include Romer (1990), Leahy and Whited (1996), Hassler (2001), Bloom, Bond, and Van Reenen (2007), Greasley and Madsen (2006), Gilchrist, Sim, and Zakrajsek (2010), and Basu and Bundick (2011)

<sup>16</sup>Bloom (2009) counts uncertainty episodes by the number of times the stock market volatility index exceeds 1.65 standard deviations above its Hodrick-Prescott filtered trend, rather than its unconditional mean. If we do the same for  $\bar{\mathcal{U}}_t^y(1)$ , we find 5 episodes of heightened uncertainty: one in the early mid 1970s (1973:09 and 1974:11), one during the twin recessions in the early 1980s (1980:02 and 1982:02), 1990:01, 2001:10, and 2008:07.

month when stock markets experienced their largest single-day percentage decline in recorded history. While this may accurately reflect the sudden increase in financial market volatility that occurred at that date, our measure of macroeconomic uncertainty barely increases at all. Indeed, it is difficult to imagine that the level of macro uncertainty in the economy in October 1987 (not even a recession year) was on par with the recent financial crisis. Nevertheless, when the VXO index is interpreted as a proxy for uncertainty, this is precisely what is suggested. Other important episodes where the two measures disagree is the recessionary period from 1980-1982, where our measure of uncertainty was high but the VXO index was comparatively low, and the stock market boom and bust of the late 1990s and early 2000s, where the VXO index was high but uncertainty was low.

To more formally investigate the dynamic relationship between the VXO Index and  $\bar{U}_t^y(h)$ , we estimate a bivariate vector autoregression with 12 lags (VAR(12)) in these two variables, with the VXO Index ordered first. Figure 6 reports the impulse responses from a shock to each of the variables, identified using a standard recursive method. For the bivariate VARs that follow, we standardize VXO and  $\bar{U}_t^y(h)$  to have mean zero and unit variance. Figure 6 shows that shocks to macro uncertainty as measured by  $\bar{U}_t^y(h)$  lead to an increase in the VXO Index (third panel) but increases in the VXO Index (second panel) do not lead to increases in  $\bar{U}_t^y(h)$  for any uncertainty horizon  $h$ . We caution that these results are informative only about the dynamic correlation between these two series, and do not identify true causality. Nevertheless, given that stock market volatility is far more variable than uncertainty, these findings suggest that there are many movements in the stock market volatility that do not coincide with movements in uncertainty. If the stock market measure were largely driven by fundamental shocks to uncertainty, we would have expected stock market volatility shocks to be associated with increases in macro uncertainty. The results in Figure 6 do not support that hypothesis. To the contrary, shocks to the VXO Index actually lead to a small *decline* in uncertainty  $\bar{U}_t^y(h)$ , implying that forces other than uncertainty must be behind much of the volatility in the stock market.

From Figure 6 we also see that shocks to  $\bar{U}_t^y(12)$  in this VAR are considerably more persistent than are shocks to VXO: the half-life of VXO to its own shock is 3 months, whereas the half-life of  $\bar{U}_t^y(12)$  to its own shock is 54 months. Moreover, the persistence of shocks to macro uncertainty  $\bar{U}_t^y(h)$  increases with the forecast horizon  $h$ . Thus, macro uncertainty is much more persistent than stock market volatility, a finding relevant for theories where uncertainty is a driving force of economic downturns, including those with more prolonged periods of below-trend economic growth.

### 5.3 Macro Uncertainty and Macroeconomic Dynamics

Existing empirical research on uncertainty has often found important dynamic relationships between real activity and various uncertainty proxies. In particular, these proxies are counter-cyclical and VAR estimates suggest that they have a large impact on output and employment in the months after an innovation in these measures. A key result is that a rise in some proxies (notably stock market volatility) at first depresses real activity and then increases it, leading to an over-shoot of its long-run level, consistent with the predictions of some theoretical models on uncertainty as a driving force of macroeconomic fluctuations.

We estimate impulse responses of industrial production (IP) and employment from eight-variable VAR(12)s following the ordering in Bloom (2009):

$$\begin{bmatrix} \log(\text{S\&P 500 Index}) \\ \textit{uncertainty} \\ \text{federal funds rate} \\ \log(\text{wages}) \\ \log(\text{CPI}) \\ \text{hours} \\ \log(\text{employment}) \\ \log(\text{industrial production}) \end{bmatrix}.$$

Four such VARs are considered with *uncertainty* taken to be either  $\bar{U}_t^y(1)$ ,  $\bar{U}_t^y(3)$ ,  $\bar{U}_t^y(12)$ , or the VXO Index. Following Bloom (2009), we use industrial production, wages, hours and employment for the manufacturing sector only. Bloom (2009) considers a 15-point shock to the error in the VXO equation. This amounts to approximately 4 standard deviations of the identified error. We record responses to 4 standard deviation shocks in  $\bar{U}_t^y(h)$ , so the magnitudes are comparable with those of VXO shocks.

Figure 7 shows that, as in the case of the VXO Index, shocks to  $\bar{U}_t^y(h)$  sharply reduce production and employment. Indeed, the magnitude of the response is larger when the shock is to  $\bar{U}_t^y(h)$  rather than to the VXO Index. An important difference, however, is that shocks to the  $\bar{U}_t^y(h)$  equation do not generate a statistically significant “volatility overshoot,” namely, the rebound in real activity following the initial decline after a positive uncertainty shock found using the VXO Index (last row) and emphasized in Bloom (2009). This finding echoes those in Bachmann, Elstner, and Sims (2012).<sup>17</sup> Unlike the findings in Bachmann, Elstner, and Sims (2012), however, the short-run (within 10 months) responses to our uncertainty shocks are

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<sup>17</sup>The overshoot found by Bloom appears to be sensitive to whether the VXO data are HP (Hodrick and Prescott (1997)) filtered. For these impulse responses, Bloom (2009) HP filters all data in the VAR *except* the VXO index. (This is despite a statement to the contrary in the paper, as confirmed after a careful inspection of the code kindly provided by Bloom.) When the VXO data are not filtered, we find, as reported above, the overshoot also reported in Bloom (2009). However, when we HP filter the VXO index, the overshoot disappears. The lack of overshoot in response to our uncertainty shocks is not, however, sensitive to whether the measure is HP filtered.

sizable. This is especially noteworthy, since uncertainty and VXO shocks are contemporaneously orthogonal to shocks in the level of the stock market. Importantly, the responses to  $\bar{\mathcal{U}}_t^y(h)$  are more protracted than those to the VXO Index, which underscores the greater persistence of these measures as compared to popular uncertainty proxies. We also considered a VAR with uncertainty ordered last; these results are presented in (Jurado, Ludvigson, and Ng (2013)). The effects of uncertainty shocks on real activity remain large and statistically significant.

To study the quantitative importance of uncertainty shocks for macroeconomic fluctuations, Table 2 reports forecast error variance decomposition for production, employment and hours and compares them with the decompositions when VXO is used instead as the proxy for uncertainty in the 8-variable VAR. We use  $k$  here to distinguish the VAR forecast horizon from the uncertainty forecast horizon  $h$ . The table shows the fraction of the VAR forecast error variance that is attributable to common macro uncertainty shocks in  $\bar{\mathcal{U}}_t^y(h)$  and the horizon  $k$  for which shocks to the uncertainty measure  $\bar{\mathcal{U}}_t^y(1)$  or VXO are associated with the greatest fraction of VAR forecast error variance.

From Table 2 we can see that uncertainty shocks with shorter uncertainty horizons  $h$  tend to be associated with a larger fraction of the forecast error variance in these variables than do those with longer uncertainty horizons. When  $h = 1$  ( $h = 12$ ), common macro uncertainty shocks account for between 3.59% (0.52%) and 15.13% (10.46%) of the forecast error variance in industrial production, depending on the VAR forecast horizon. By contrast, stock market volatility explains between 0 and 6.7%. Shocks to  $\bar{\mathcal{U}}_t^y(1)$  and  $\bar{\mathcal{U}}_t^y(3)$  are associated with a maximum of 15% of the forecast error variance in Hours and 15% of the forecast error variance in Production at VAR horizon  $k = 10$  months. By contrast, shocks to VXO are associated with a maximum of 8.24% of the forecast error variance in Hours and 6.7% of the forecast error variance in Production both at VAR horizons  $k = 8$ . Thus, uncertainty is associated with almost double the variation in hours and more than double the variation in production compared to VXO. On the other hand, shocks to  $\bar{\mathcal{U}}_t^y(1)$  and VXO are associated with similar fractions of VAR forecast error variance in Employment, at most VAR forecast horizons  $k$ .

To put the results of Table 2 into perspective, shocks to the federal funds rate (a common proxy for unanticipated shocks to monetary policy) account for just 2.35% of the 12-month VAR forecast error variance of production in this same 8-variable VAR, while common macro uncertainty shocks (to  $\bar{\mathcal{U}}_t^y(1)$ ) account for 14%. Table 2 also shows that the fraction of 12-month ahead VAR forecast error variance in Hours associated with shocks to  $\bar{\mathcal{U}}_t^y(1)$  (equal to 14.30%) is almost twice as large as that associated with VXO (equal to 7.37%). These variance decomposition results are similar if we instead use a 9-variable VAR that includes both VXO and  $\bar{\mathcal{U}}_t^y(1)$ . From this 9-variable VAR, we find that the big driver of VXO are shocks to VXO, not uncertainty. This reinforces the conclusion that stock market volatility is driven largely by shocks other than those to broad-based economic uncertainty, suggesting researchers should be

cautious when using this measure as a proxy for uncertainty.

Due to space constraints, we have reported results only for the base-case estimates described above. A supplementary file available on-line provides additional results designed to check the sensitivity of our results to various assumptions made above. These exercises are based on (i) alternative weights used to aggregate individual uncertainty series; (ii) alternative location statistics of stochastic volatility to construct individual uncertainty series; and (iii) alternative conditioning information based on recursive forecasts to construct diffusion index forecasts. The key findings are qualitatively and quantitatively similar to the ones reported here.

## 5.4 Comparison with Measures of Dispersion

This subsection compares the time-series behavior of  $\bar{U}_t^y(h)$  with four cross-sectional uncertainty proxies studied by Bloom (2009). These are:

1. The cross-sectional dispersion of firm stock returns. This is defined as the within-month cross-sectional standard deviation of stock returns for firms with at least 500 months of data in the Center for Research in Securities Prices (CRSP) stock-returns file. The series is also linearly detrended over our sample period (1960:07-2011:12).
2. The cross-sectional dispersion of firm profit growth. Profit growth rates are normalized by average sales on a monthly basis, so that this measure captures the quarterly cross-sectional standard deviation in  $\frac{\text{profits}_{it} - \text{profits}_{it-1}}{0.5(\text{sales}_{it} + \text{sales}_{it-1})}$ , where  $i = 1, 2, \dots, N_t$  indexes the firms and  $N_t$  denotes the total number of firms observed in month  $t$ . The sample is restricted to firms with at least 150 quarters of data in the Compustat (North America) database.
3. The cross-sectional dispersion of GDP forecasts from the Philadelphia Federal Reserve Bank's biannual Livingston Survey. This is defined as the biannual cross-sectional standard deviation of forecasts of nominal GDP one year ahead. The series is also linearly detrended over our sample period (1960:07-2011:12).
4. The cross-sectional dispersion of industry-level total factor productivity (TFP). This is defined as the annual cross-sectional standard deviation of TFP growth rates within SIC 4-digit manufacturing industries, calculated using the five-factor TFP growth data computed by Bartelsman, Becker, and Gray as a part of the NBER-CES Manufacturing Industry Database (<http://www.nber.org/data/nbprod2005.html>).

These updated series, along with  $\bar{U}_t^y$  (12) are displayed in Figure 8.<sup>18</sup> As was true in the case of stock market volatility in the previous subsection, these measures exhibit quite different

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<sup>18</sup>We update all of these series to include more recent observations, with the exception of the industry-level TFP data because those data after 2005 are not yet available.

behavior from macroeconomic uncertainty. Stock return dispersion tells a story roughly similar to the VXO Index, with a particularly large increase in uncertainty leading up to the 2001 recession that is not present in our measure of macro uncertainty. Firm profit dispersion actually suggests a relatively low level of uncertainty during the 1980-82 recessions when macro uncertainty was high, with a sharp increase towards the end of the 1982 recession, by which time macro uncertainty had declined. GDP forecast dispersion points to a level of uncertainty during each of the 1969-70 and 1990 recessions which is on par with the level of uncertainty during the 2007-09 recession. Again, this contrasts with macro uncertainty which is at a record high in the 2007-09 recession but was not high in the previous episodes. Industry TFP dispersion shows almost no increase in uncertainty during the 1980-82 recessions, and displays the largest increase in January 1997, a time of relative quiescence in macro uncertainty.

To examine the dynamic relationships between these proxies for uncertainty and  $\bar{U}_t^y$  (12), we estimate a series of bivariate VARs analogous to the one from the previous section, but in each case with the VXO Index replaced by one of the four dispersion measures listed above. The lag length for each of these VARs is equal to one year; for monthly data this implies 12 lags, for quarterly data 4 lags, for biannual data 2 lags, and for annual data 1 lag. To match the frequency of the dispersion measure, we aggregate our monthly series  $\bar{U}_t^y(h)$  using averages over the desired period. The resulting impulse responses are graphed in Figure 9.

For the most part, these impulse responses using these proxies for uncertainty paint a similar picture to that obtained using the VXO Index. There is some evidence that increases in  $\bar{U}_t^y(h)$  (for all  $h$ ) lead to increases in measures of dispersion (row 3) but the converse is not true, indicating that there is considerable independent movement in these proxies and our measure of uncertainty. In particular, we see that shocks to macro uncertainty are far more persistent than shocks to any of these proxies: the responses of  $\bar{U}_t^y(h)$  to shocks in  $\bar{U}_t^y(h)$  are far more prolonged (row 4) than are the responses of dispersion proxies to shocks in dispersion (row 1). Moreover, a positive shock to the cross-sectional dispersion in stock returns, profits, and GDP (survey) forecasts leads to a *decline* in macro uncertainty  $\bar{U}_t^y(h)$ , just as a positive shock to the VXO does—see row 2. The exception is that a positive shock to dispersion in TFP does lead to an increase in  $\bar{U}_t^y$  (12). However, this series stops in 2005 and so those estimates do not include the most recent recession for which macro uncertainty shows the largest spike since 1960.

We also consider impulse responses of production and employment for the eight-variable VAR, but using these measures of dispersion as the proxy for uncertainty. These results are reported in Figure 10 and can be summarized as follows. These responses using dispersions to proxy for uncertainty do not always display the intuitive pattern that production and employment should fall as a result of an uncertainty shock. Production falls the most on impact for GDP forecasts, but employment barely responds at all. In the case of stock return dispersion, we see almost no decline immediately after the shock, but then an increase in both production

and employment after about ten quarters, followed by a subsequent decline lasting almost two years. Shocks to the dispersion in firm profits lead to very small, brief declines in both macroeconomic series, followed by a long boom which lasts for over a year. Shocks to GDP forecast dispersion do lead to a drop in production on output, but with no subsequent overshooting dynamics, and almost no response from employment. Finally, an impulse to the dispersion in TFP growth rates, if anything, leads to a slight *increase* in production and employment.

Overall, these results suggest that, like the VXO proxy, increases in measures of cross-sectional dispersion do not necessarily coincide with increases in broad-based macro uncertainty, and indeed they may coincide with a decline in macro uncertainty. These results suggest that, like stock market volatility over time, measures of dispersion vary for many reasons that are unrelated to broad macro uncertainty.

## 6 Results: Firm-Level Common Uncertainty

In this section we turn from our analysis of common macroeconomic uncertainty to examine common variation in uncertainty at the firm level. Specifically, we measure uncertainty in the profit growth of individual firms. For the firm-level dataset, the unit of observation is the change in firm pre-tax profits  $P_{i,t}$ , normalized by a two-period moving average of sales,  $S_{i,t}$ , following Bloom (2009). Given the seasonality in this series, we instead form a year-over-year version of this measure, as detailed in the supplementary on-line file. After converting to a balanced panel, we are left with 155 firms from 1970:Q1-2011:Q2 without missing values.<sup>19</sup> For each firm, the series to be forecast is normalized pretax profits, so again  $y_{it} = X_{it}$ . For the firm-level results, as for the macro results, we form forecasting factors  $F_t$  from the panel  $\{X_{it}\}_{i=1}^{N_{xp}}$ , as well as  $\{X_{it}^2\}_{i=1}^{N_{xp}}$  where  $N_{xp} = 155$ , the number of cross-sectional firm-level observations. We find evidence of two factors in  $\{X_{it}\}_{i=1}^{N_{xp}}$  and one factor in  $\{X_{it}^2\}_{i=1}^{N_{xp}}$ . The  $W_t$  vector of additional predictors includes the macro factors estimated from the macro data set. As before, a conservative  $t$  test is used to include only the predictors that are statistically significant.

One important consideration that is relevant to this microeconomic context is the construction of our panel. Since we need a reasonable number of time series observations to estimate the stochastic volatility process, our base-case cross-sectional average estimate of common firm level uncertainty, we require that the panel be balanced. This leads us to drop about 400 firms per quarter on average. In particular, many of the firms operating towards the beginning of our sample are excluded, because they do not survive until 2011:Q2. This eliminates a large fraction of the cross-sectional variation before 1995. Because of this survivorship bias, it is

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<sup>19</sup>A limitation with Compustat data is that its coverage is restricted to large publicly traded firms. The Census Bureau's ASM data are more comprehensive, but limited to annual observations. Similarly, Total factor productivity may be preferred over profits as the source of uncertainty, but these industry level data eliminates much of the uncertainty at the firm level (Schaal (2012)).

difficult to conclude that our estimated aggregate firm-level uncertainty measure represents a comprehensive measure of the uncertainty facing firms since 1970. Nevertheless, to the extent that the cross-sectional standard deviation of firm profits *within* the balanced panel differs from our estimate of common firm-level uncertainty formed over this same panel, it suggests that simple proxies such as the cross-sectional standard deviation in firm profit growth rates may not be reliable measures of common variation in firm-level uncertainty.

Figure 11 displays the estimated common uncertainty in firm-level profits  $\bar{\mathcal{U}}_t^y(h)$  over time for  $h = 1, 3,$  and  $4$  quarters. Like the measure of macroeconomic uncertainty analyzed above, these estimates point to a rise in uncertainty surrounding the 1973-75, 1980-82 recessions, but not of the same magnitude. Instead, there are larger increases in common firm-level uncertainty surrounding the 2000-01 and 2007-09 recessions. However, this type of aggregate uncertainty is less countercyclical: the correlation of each of these measures with industrial production growth is negative, but smaller in absolute value than is the correlation of the macro uncertainty measures with production growth. This figure also compares our measures of common firm-level uncertainty  $\bar{\mathcal{U}}_t^y(h)$  to the popular proxy for common firm-level uncertainty given by on the cross-sectional dispersion in firm profit growth normalized by sales, denoted  $\mathcal{D}_t^B$  (see equation (3)). with many more spikes in  $\mathcal{D}_t^B$  than in common firm-level uncertainty. Indeed, the dispersion measure exceeds 1.65 standard deviations above its mean dozens of times, while common firm-level uncertainty measures only do so a handful of times. Like the VXO index, there appear to be many movements in the cross-sectional standard deviation of firm profit growth that are not driven by common shocks to uncertainty across firms.

To assess the relative importance of macro uncertainty  $\bar{\mathcal{U}}_t^y(h)$  in total uncertainty, we again compute, for each of the 155 firms in the firm-level dataset, and for  $h = 1$  to  $6$ , the  $R_{jt}^2(h)$  as defined in (13), averaged over  $t$ . As above, this exercise is performed for the full sample, for recession months, and for non-recession months. Table 3 shows that, as was the case for common macroeconomic uncertainty, common firm-level uncertainty comprises a larger fraction of the variation in total uncertainty during recessions than during non-recessions. Indeed, the common firm-level common uncertainty we estimate explains an average of 18% of the variation in total uncertainty for an uncertainty horizon of  $h = 4$  quarters in non-recessions, but it explains double that in recessions. These results echo those using the macro uncertainty measures. Other results (using VARs for example) are qualitatively similar as well, but omitted to conserve space.

## 7 Conclusion

In this paper we have introduced new time series measures of macroeconomic uncertainty. We have strived to ensure that these measures be comprehensive, as free as possible from both the restrictions of theoretical models and/or dependencies on a handful of economic indicators. We

are interested in *macroeconomic* uncertainty, namely uncertainty that may be observed in many economic indicators at the same time, across firms, sectors, markets, and geographic regions. And we are interested in the extent to which this macroeconomic uncertainty is associated with fluctuations in aggregate real activity and financial markets.

Our measures of macroeconomic uncertainty fluctuate in a manner that is often quite distinct from popular proxies for uncertainty, including the volatility of stock market returns (both over time and in the cross-section), the cross-sectional dispersion of firm profits, productivity, or survey-based forecasts. Indeed, our estimates imply far fewer important uncertainty episodes than do popular proxies such as stock market volatility, a measure that forms the basis for the 17 uncertainty dates identified by Bloom (2009). By contrast, we uncover just three big macro uncertainty episodes in the post-war period: the months surrounding the 1973-74 and 1981-82 recessions and the Great Recession of 2007-09, with the 2007-09 recession the most striking episode of heightened uncertainty since 1960. These findings and others reported here suggest that there is much variability in the stock market and in other uncertainty proxies that is not generated by a movement in genuine uncertainty across the broader economy. This occurs both because these proxies over-weight single series in the measurement of macro uncertainty, and because they erroneously attribute forecastable fluctuations to a movement in uncertainty.

Our estimates nevertheless point to a quantitatively important dynamic relationship between uncertainty and real activity. In an eight variable monthly macro VAR, common macro uncertainty shocks are associated with a much larger fraction of the VAR forecast error variance in production and hours worked than are shocks to stock market volatility, or innovations to the federal funds rate. Our estimates also suggest that macro uncertainty is strongly countercyclical, explaining a much larger component of total uncertainty during recessions than in non-recessions, and far more persistent than common uncertainty proxies. We conclude that large macro uncertainty shocks, whether they be common to many aggregate indicators or many firm-level indicators, display quantitatively important linkages with variation in real activity, even more so in recessionary periods.

We caution that our results are silent on the question of causality. (Is uncertainty the cause or effect of recessions?) Our goal, challenging enough, is to develop and estimate a defensible measure of time-varying macro uncertainty that can be tracked over time and related to fluctuations in real activity and asset markets. Nevertheless, our estimates imply that the economy is objectively less predictable in recessions than it is in normal times. This result is not a statement about changing subjective perceptions of uncertainty in recessions as compared to booms. Any theory for which uncertainty is entirely the effect of recessions would need to be consistent with these basic findings.

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Average  $R^2$  From Regressions of Individual Uncertainty on Macro Uncertainty

$h$	$\bar{\mathcal{U}}(h)$			$\widehat{\mathcal{U}}(h)$		
	full sample	recession	non-recession	full sample	recession	non-recession
1	0.18	0.19	0.12	0.17	0.17	0.12
2	0.22	0.24	0.15	0.22	0.24	0.15
3	0.24	0.26	0.16	0.23	0.25	0.16
4	0.25	0.26	0.17	0.23	0.24	0.16
5	0.26	0.27	0.18	0.24	0.25	0.16
6	0.27	0.28	0.19	0.25	0.26	0.16
7	0.28	0.29	0.19	0.25	0.27	0.17
8	0.29	0.30	0.20	0.25	0.27	0.17
9	0.29	0.30	0.20	0.25	0.28	0.17
10	0.29	0.31	0.21	0.25	0.28	0.17
11	0.30	0.31	0.21	0.25	0.29	0.17
12	0.30	0.31	0.21	0.25	0.29	0.17

Table 1: Cross-sectional averages of  $R^2$  values from regressions of  $\mathcal{U}_{jt}^y(h)$  on  $\bar{\mathcal{U}}_t^y(h)$  or  $\widehat{\mathcal{U}}_t^y(h)$  over different subsamples. Uncertainty estimated from the monthly, macro dataset. Recession months are defined according to the NBER Business Cycle Dating Committee.

Variance Decompositions from VAR(12)				
$k$	$\bar{U}(1)$	$\bar{U}(3)$	$\bar{U}(12)$	VXO
Production				
1	3.59	1.10	0.52	0.03
3	6.07	3.16	1.88	0.89
12	14.36	15.13	10.29	5.31
$\infty$	7.16	7.68	5.62	4.25
max	15.13	15.35	10.46	6.67
max $k$	10	11	11	8
Employment				
1	0.50	0.04	0.04	0.00
3	2.53	1.07	0.65	1.21
12	11.85	13.08	9.51	8.93
$\infty$	6.42	7.28	5.49	5.46
max	11.85	13.18	9.52	9.81
max $k$	12	13	13	9
Hours				
1	5.71	2.44	1.59	0.08
3	7.51	3.84	2.09	2.13
12	14.30	13.28	9.02	7.37
$\infty$	9.58	9.08	6.70	6.48
max	15.02	13.67	9.36	8.24
max $k$	10	10	10	8

Table 2: Eight-variable VAR(12) using the VXO Index or  $\bar{U}_t^y(h)$  estimated from the monthly macro dataset for  $h = 1, 3, 12$  as a measure of uncertainty. Each VAR(12) contains, in the following order: log(S&P 500 Index), *uncertainty*, federal funds rate, log(wages), log(CPI), hours, log(employment), and log(industrial production). As in Bloom (2009), all variables are HP filtered, except for the uncertainty measures, which enter in raw levels. The data are monthly and span the period 1960:07-2011:12.

Average  $R^2$  From regressions of Firm-Level Uncertainty on Common Uncertainty

$h$	$\bar{\mathcal{U}}(h)$			$\widehat{\mathcal{U}}(h)$		
	full sample	recession	non-recession	full sample	recession	non-recession
1	0.15	0.29	0.14	0.12	0.27	0.11
2	0.18	0.34	0.16	0.16	0.32	0.14
3	0.19	0.35	0.17	0.17	0.33	0.15
4	0.20	0.36	0.18	0.18	0.33	0.16
5	0.21	0.36	0.18	0.18	0.33	0.16
6	0.21	0.36	0.19	0.18	0.32	0.16

Table 3: Cross-sectional averages of  $R^2$  values from regressions of  $\mathcal{U}_{j_t}^y(h)$  on  $\bar{\mathcal{U}}_t^y(h)$  or  $\widehat{\mathcal{U}}_t^y(h)$  over different subsamples. Uncertainty estimated from the quarterly firm-level dataset with observations on firm profit growth rates normalized by sales. Recession months are defined according to the NBER Business Cycle Dating Committee.

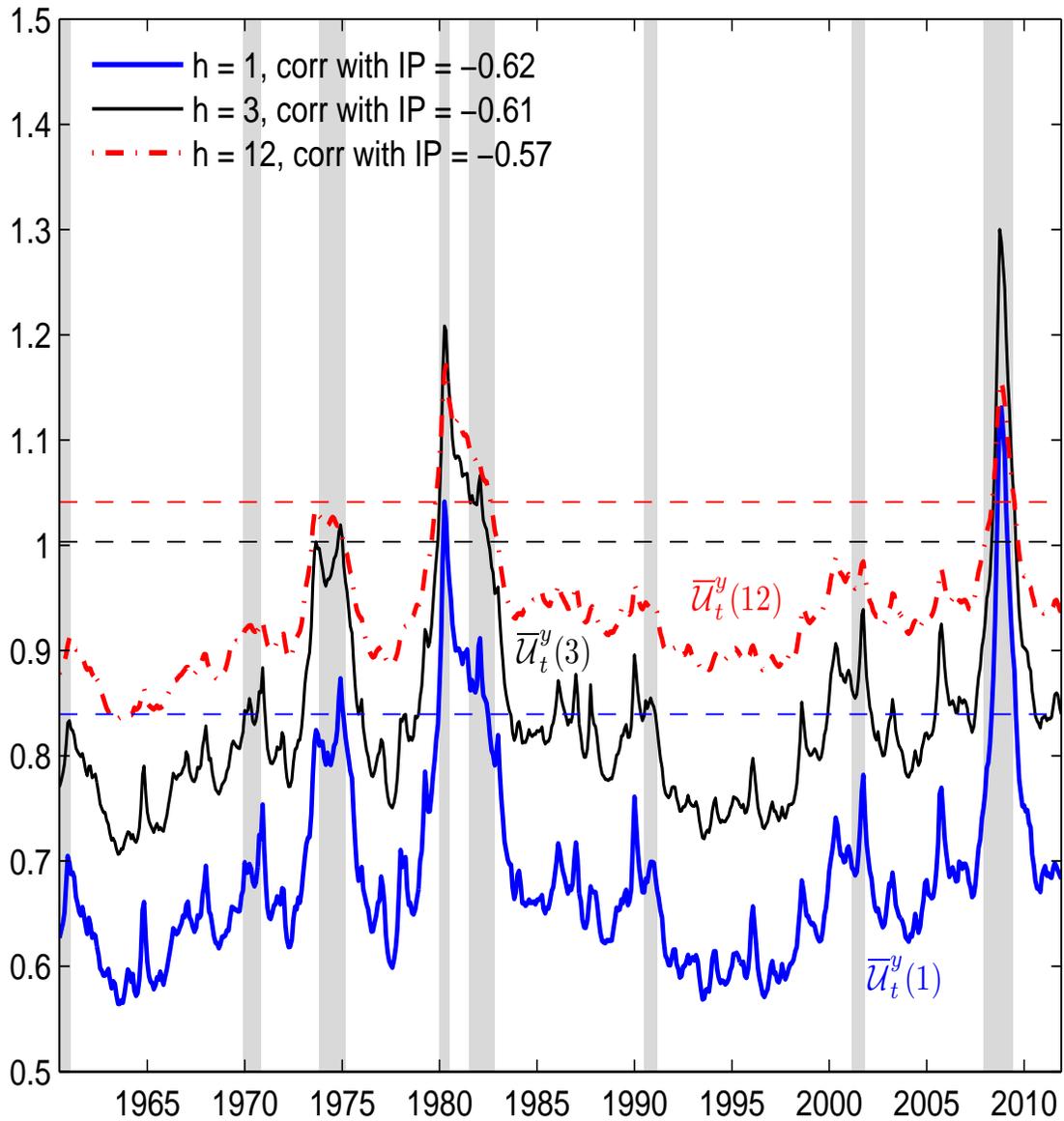


Figure 1: Aggregate Uncertainty:  $\bar{U}_t^y(h)$  for  $h = 1, 3, 12$ . Horizontal lines indicate 1.65 standard deviations above the mean of each series. Industrial Production (IP) growth is computed as the 12-month moving average of monthly growth rates (in percent). The data are monthly and span the period 1960:07-2011:12.

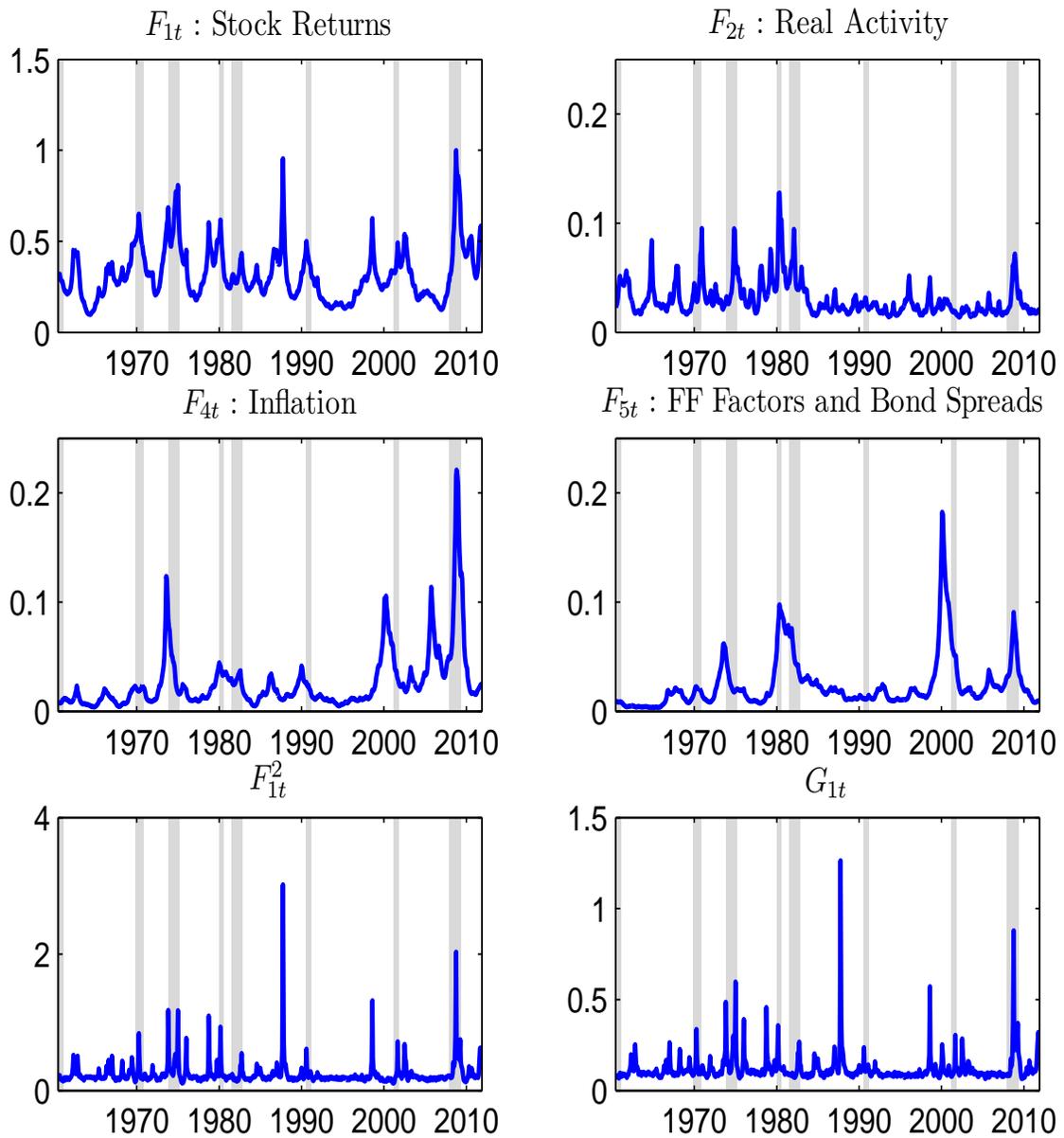


Figure 2: Predictor Uncertainty: This plot displays uncertainty estimates for 6 of the 14 predictors contained in the vector  $\mathcal{Z}_t \equiv (F_t', W_t')'$ .  $F_t$  denotes the 12 factors estimated from  $X_{it}$ , and  $W_t \equiv (F_{1t}^2, G_{1t})'$ , where  $G_{1t}$  is the first factor estimated from  $X_{it}^2$ . Titles represent the types of series which load most heavily on the factor plotted; “FF Factors” means the Fama-French factors (HML, SMB, UMD). The sample period is 1960:01-2011:12.

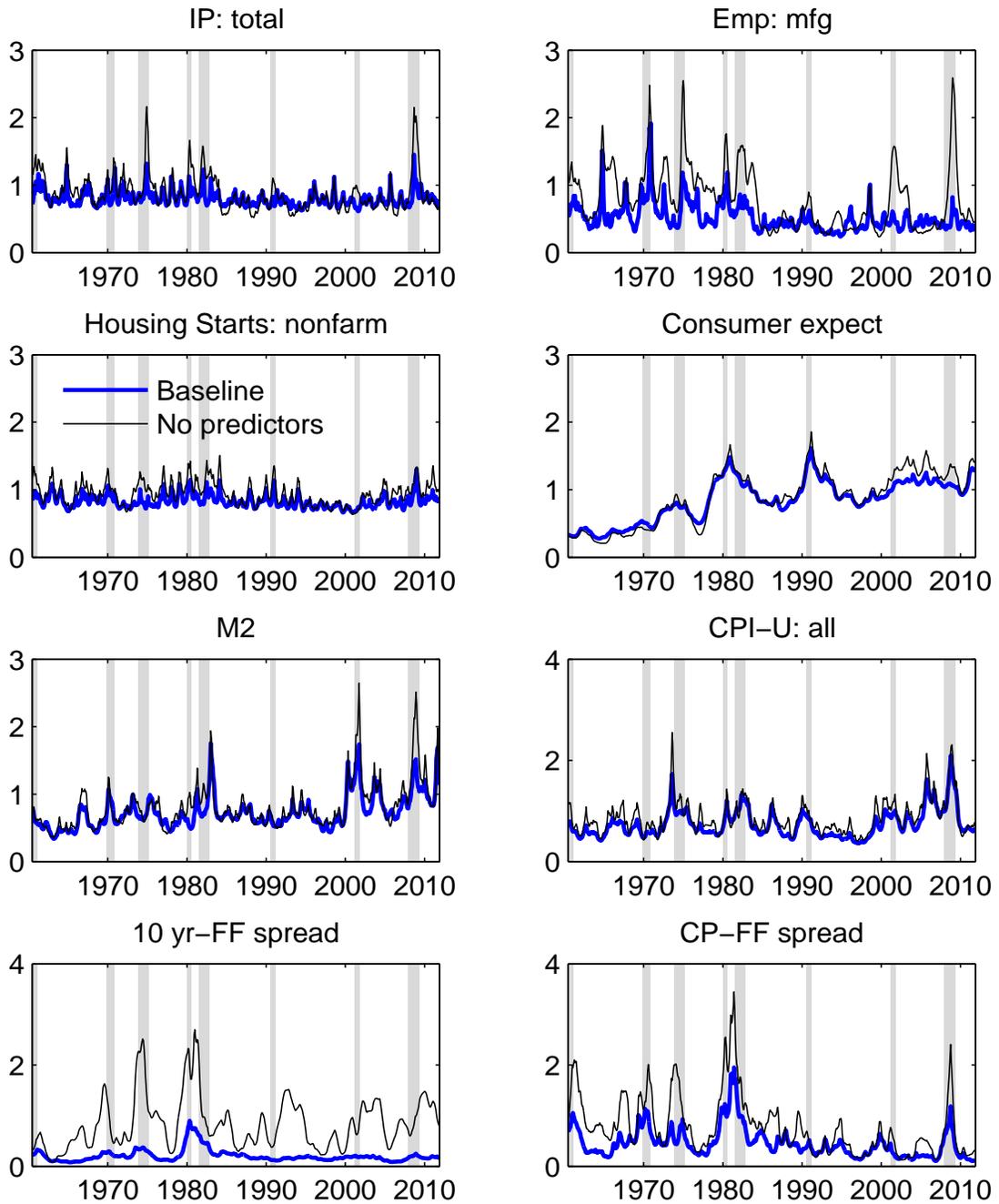


Figure 3: The Role of Predictors: These plots display two estimates of  $U_{jt}^y(1)$  for several key series in our data set. The first is constructed using the full set of predictor variables (“Baseline”); the second is constructed using no predictors (“No predictors”).

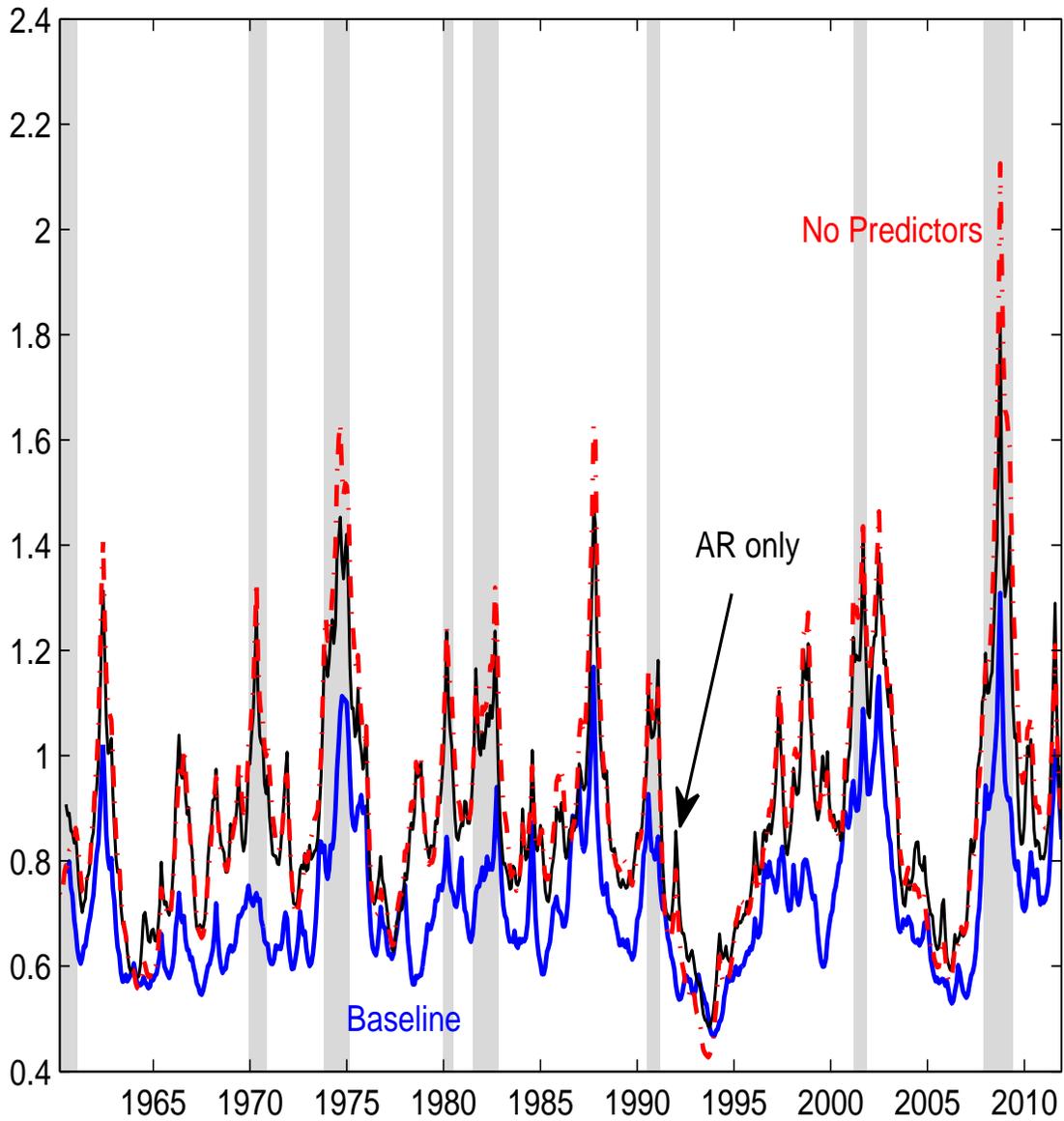


Figure 4: Uncertainty in the S&P 500 Index. These plots show estimates of  $U_{SP500,t}^y(1)$  for the S&P 500 Index based on three different forecasting models. “No Predictors” indicates that no predictors were used, “AR only” indicates that only a fourth-order autoregressive model was used to generate forecast errors, and “Baseline” indicates that the full set of predictor variables was used to generate forecast errors. The sample period is 1960:01-2011:12.

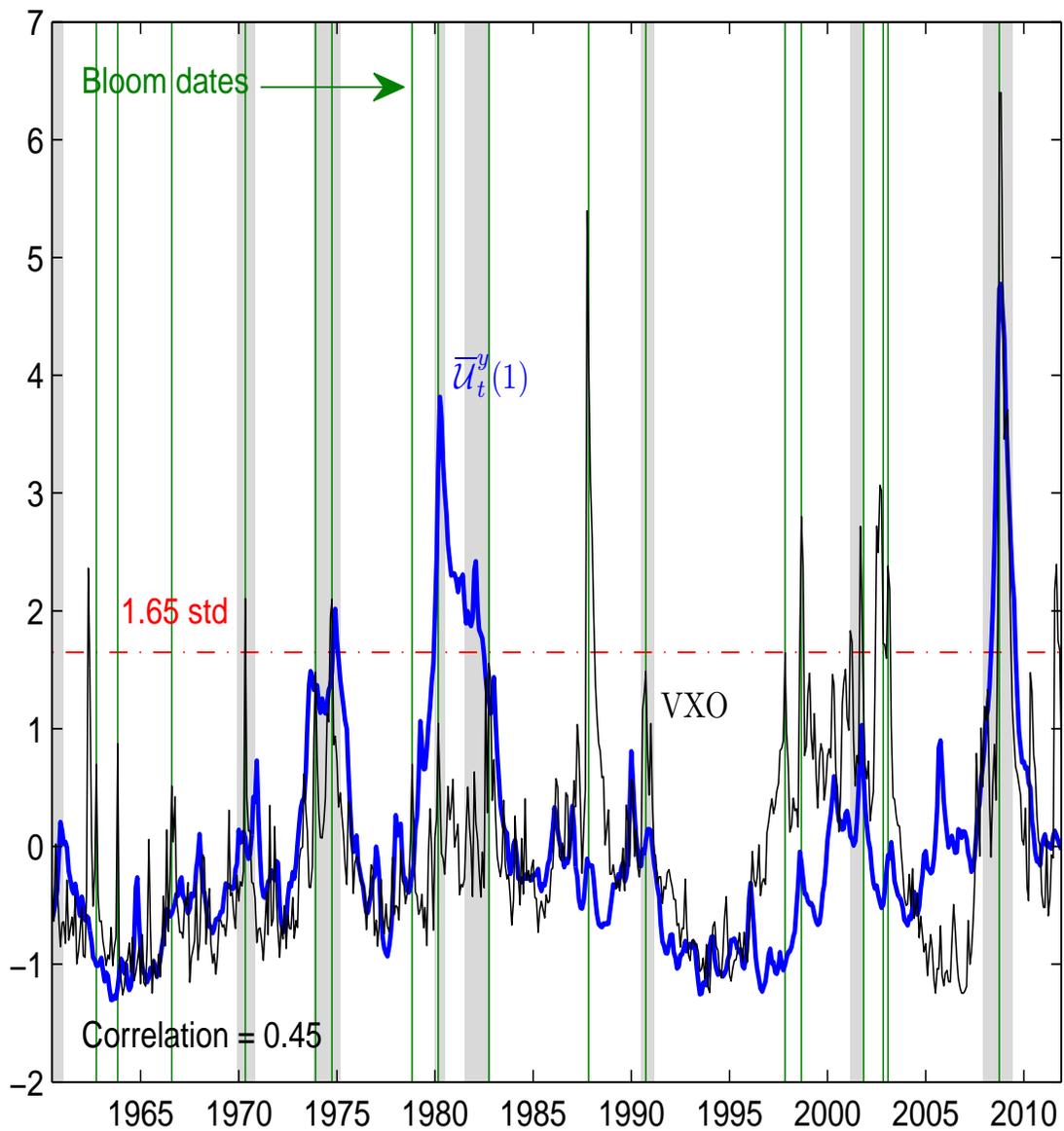


Figure 5: Stock Market Implied Volatility and Uncertainty: This plot shows  $\bar{u}_t^y(1)$  and the VXO index, expressed in standardized units. The vertical lines correspond to the 17 dates in Bloom (2009) Table A.1, which correspond to dates when the VXO index exceeds 1.65 standard deviations above its HP (Hodrick and Prescott, 1997) filtered mean. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). The data are monthly and span 1960:07-2011:12.

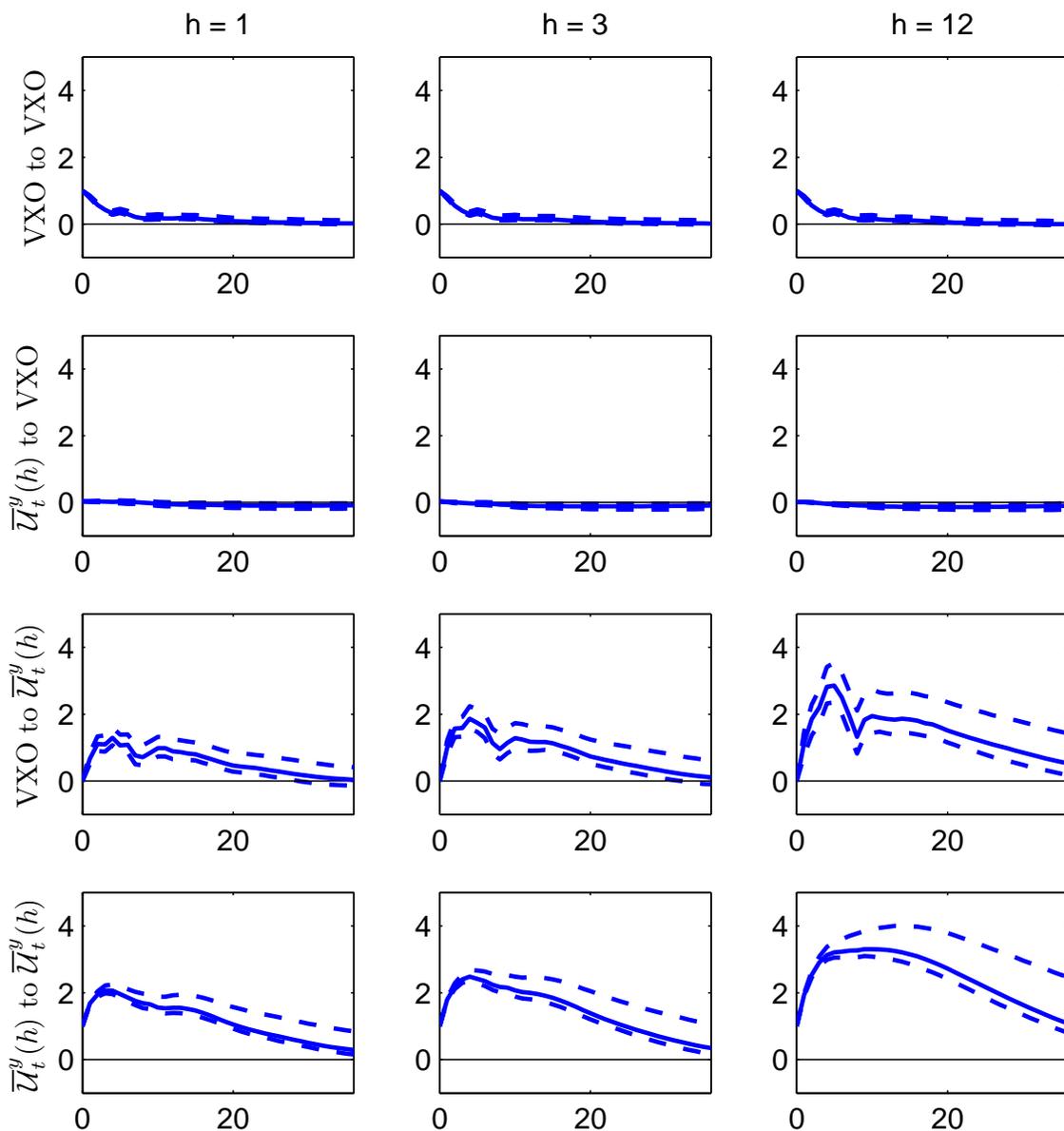


Figure 6: Bivariate VAR(12): VXO index and  $\bar{U}_t^y(h)$  for  $h = 1, 3, 12$ . Each column represents a different bivariate VAR(12), depending on the horizon  $h$  at which aggregate uncertainty is computed. Each row is labeled ‘ $X$  to  $Y$ ’, which indicates the response of variable  $X$  to a unit shock in variable  $Y$ . All variables are standardized before estimation, and periods correspond to one month. The sample is 1960:07-2011:12.

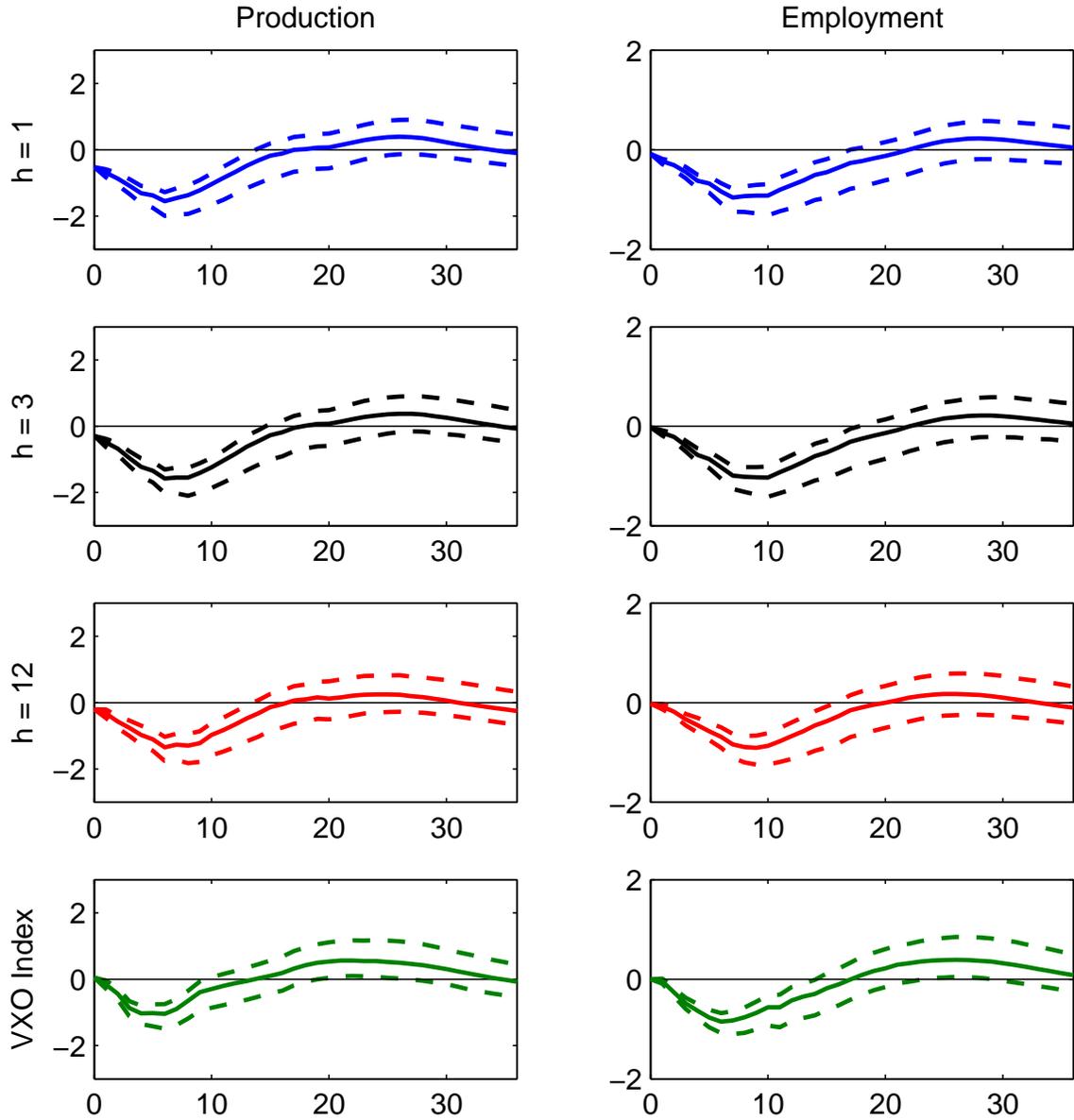


Figure 7: Eight-variable VAR(12) using the VXO Index or  $\bar{U}_t^y(h)$  for  $h = 1, 3, 12$  as a measure of uncertainty. Each VAR(12) contains, in the following order: log(S&P 500 Index), *uncertainty*, federal funds rate, log(wages), log(CPI), hours, log(employment), and log(industrial production). All shocks are a 4 standard deviation impulse, which is the same magnitude considered in Bloom (2009) Figure A.1. As in Bloom (2009), all variables are HP filtered, except for the uncertainty measures, which enter in raw levels. The data are monthly and span the period 1960:07-2011:12.

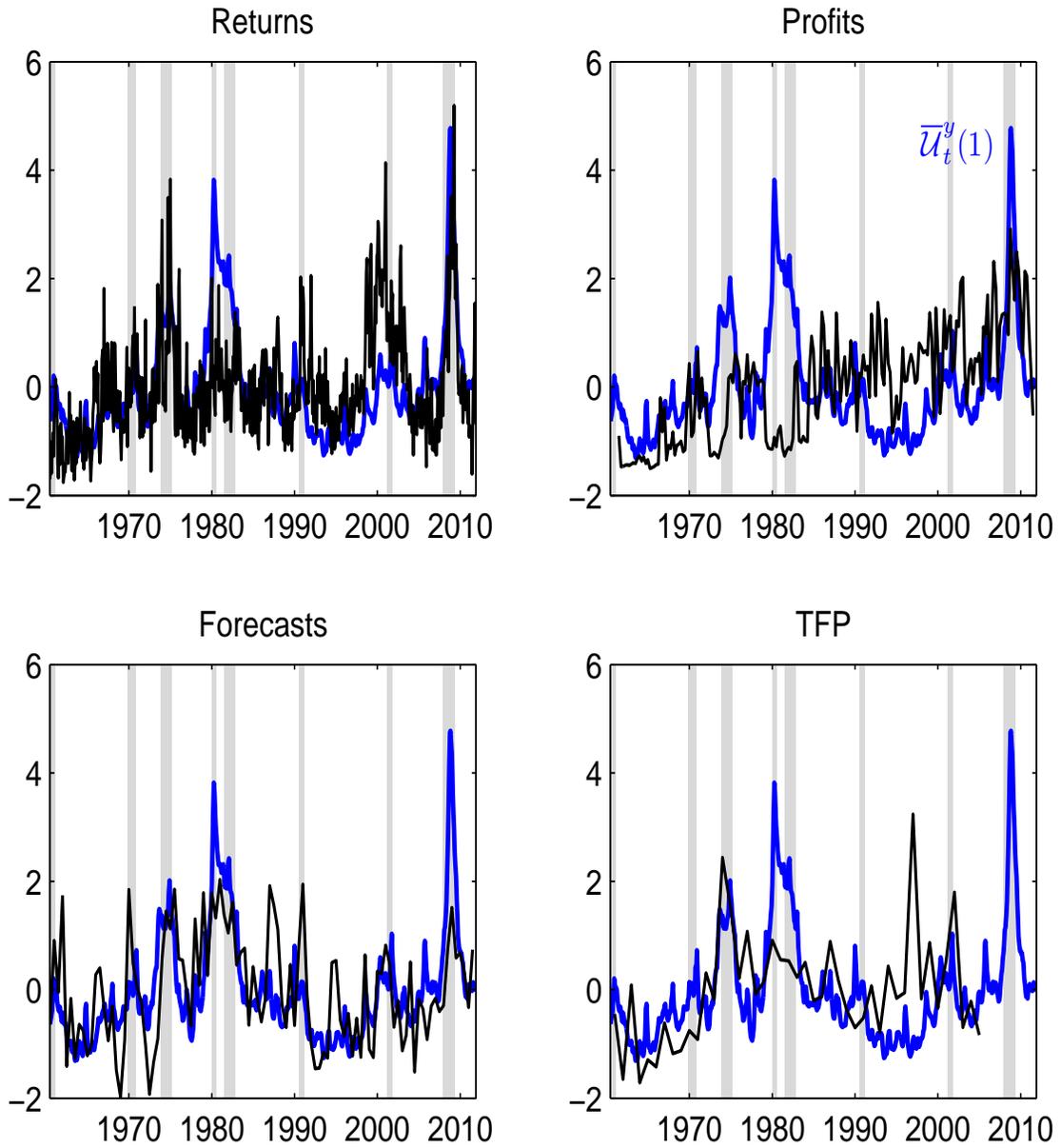


Figure 8: Cross-sectional Dispersion and Uncertainty: This plot shows  $\bar{u}_t^y(1)$  and four dispersion-based proxies, expressed in standardized units. The proxies are (in clockwise order from the northwest panel) the cross-sectional standard deviation of: monthly firm stock returns (CRSP), quarterly firm profit growth (Compustat), yearly SIC 4-digit industry total factor productivity growth (NBER-CES Manufacturing Industry Database), and half-yearly GDP forecasts (Livingston Survey).

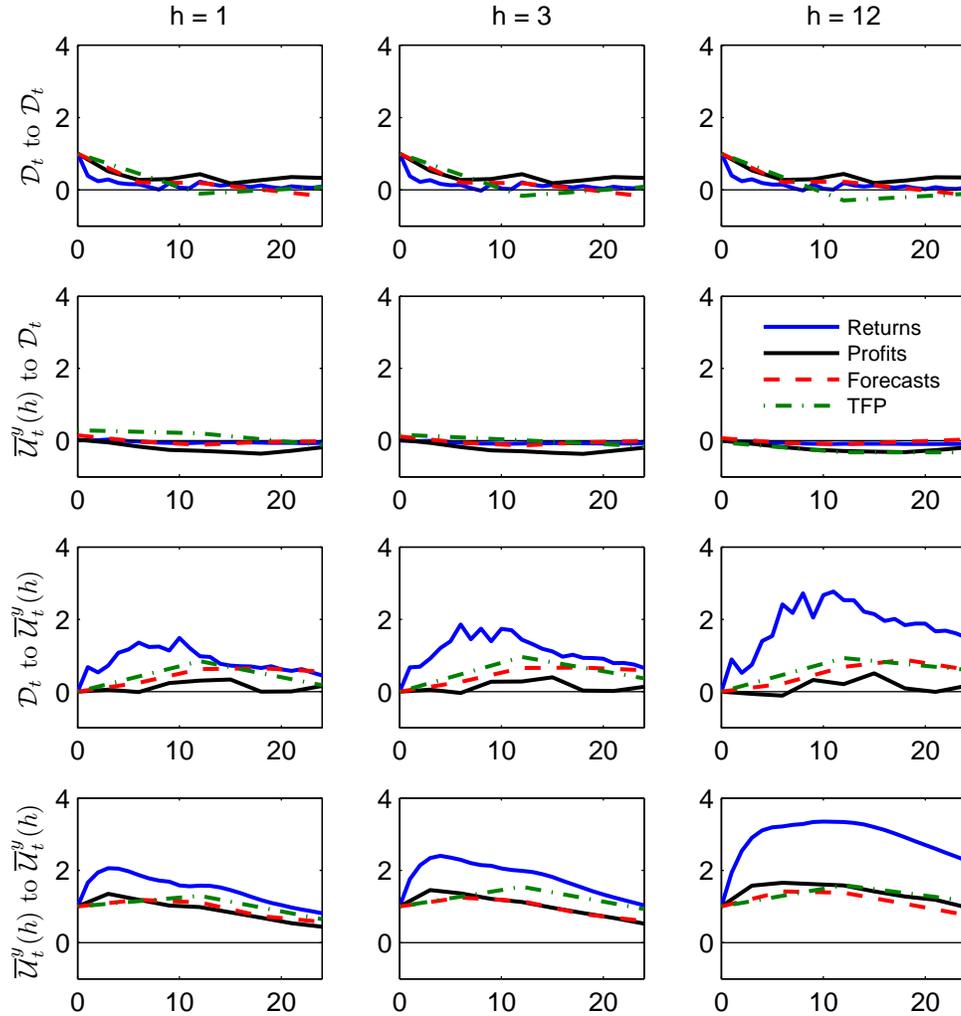


Figure 9: Bivariate VAR(12): Cross-sectional dispersion ( $\mathcal{D}_t$ ) and  $\bar{U}_t^y(h)$  for  $h = 1, 3, 12$ . Each column represents a different bivariate VAR(12), depending on the horizon  $h$  at which aggregate uncertainty is computed. Each row is labeled ‘ $X$  to  $Y$ ’, which indicates the response of variable  $X$  to a unit shock in variable  $Y$ . The different measures of cross-sectional dispersion considered are: monthly firm stock returns (CRSP), quarterly firm profit growth (Compustat), yearly SIC 4-digit industry total factor productivity growth (NBER-CES Manufacturing Industry Database), and half-yearly GDP forecasts (Livingston Survey). No confidence bands have been included, in order to reduce clutter.

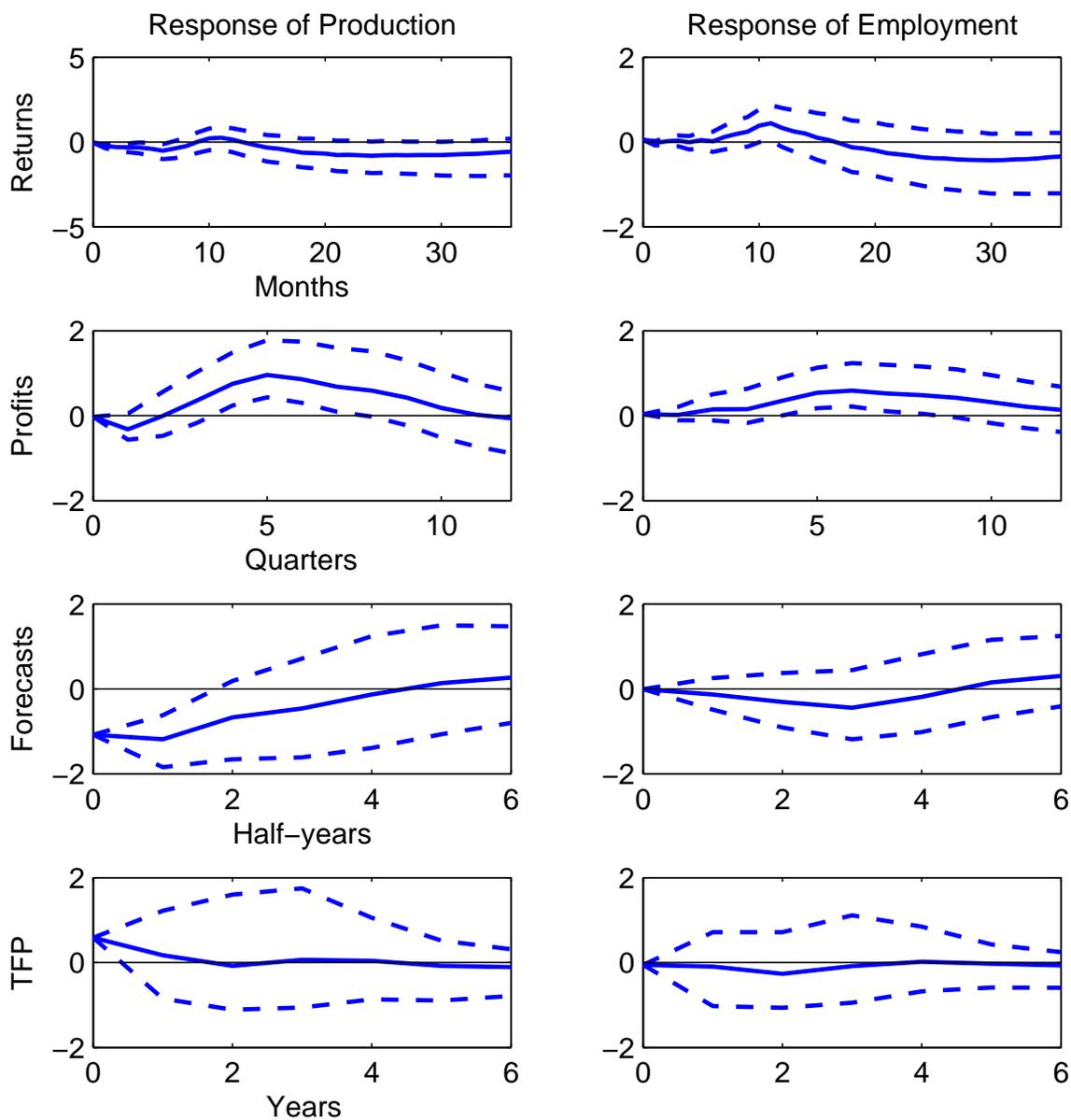


Figure 10: Eight-variable VAR using one of four dispersion-based uncertainty proxies as a measure of uncertainty. Each VAR contains, in the following order:  $\log(\text{S\&P 500 Index})$ , *uncertainty*, federal funds rate,  $\log(\text{wages})$ ,  $\log(\text{CPI})$ , hours,  $\log(\text{employment})$ , and  $\log(\text{industrial production})$ . All shocks are a 4 standard deviation impulse, which is the same magnitude considered in Bloom (2009) Figure A.1. As in Bloom (2009), all variables are HP filtered, except for the uncertainty measures, which enter in raw levels.

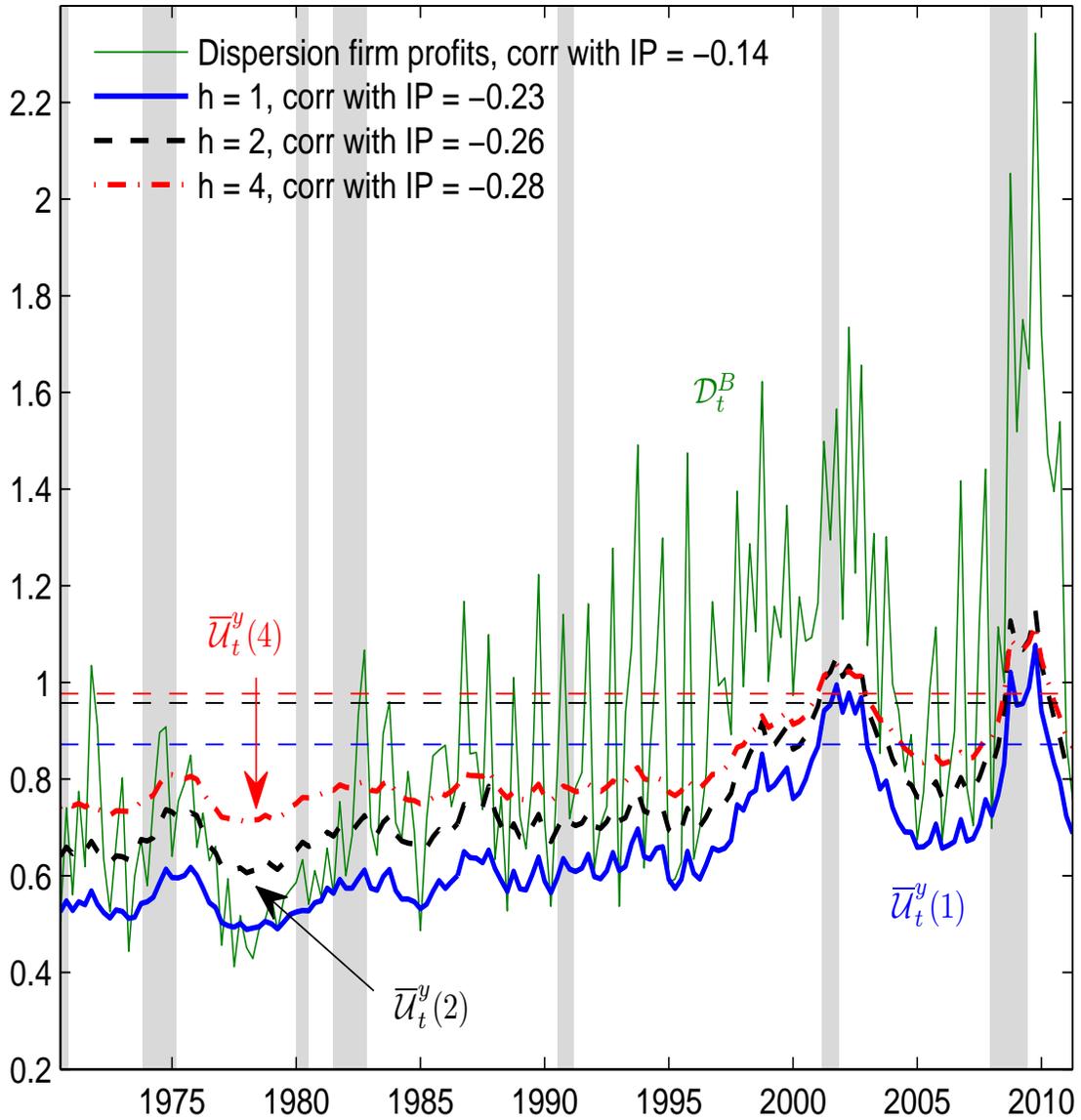


Figure 11: Firm-level Uncertainty:  $\bar{U}_t^y(h)$  for  $h = 1, 2, 4$ . Horizontal lines indicate 1.65 standard deviations above the mean of each series. The thin solid line marked “Dispersion in firm profits” is the cross-sectional standard deviation of firm profit growth, normalized by sales, and denoted  $\mathcal{D}_t^B$ . The dispersion is taken after standardizing the profit growth data. Industrial Production (IP) growth is computed as the 12-month moving average of monthly growth rates (in percent). The data are monthly and span the period 1970:Q1-2011:Q2.