Dissecting Factors*

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December 2013

Abstract

Firm size and book-to-market ratio can each be split into two components: one correlated with changes in the market value of equity and the other with everything else. We construct factors from these four components and show that although each component explains variation in returns, only the components correlated with changes in the market value of equity have positive prices of risk. Portfolios based on the unpriced parts generate statistically and economically significant three-factor alphas. Even though average returns are flat across these portfolios, their loadings on SMB and HML differ significantly. Hence, the three-factor model assigns significantly negative alphas to "high" portfolios and positive alphas to "low" portfolios. These results are relevant for practice. Applying the Fama and French (2010) bootstrapping methodology, we find that the estimated fraction of skilled mutual fund managers increases from 4% to 27% when we control for the unpriced parts of size and value. Also, the negative correlation between gross profitability and value is entirely due to the unpriced part of value.

*Joseph Gerakos is with the University of Chicago Booth School of Business and Juhani Linnainmaa is with the University of Chicago Booth School of Business and NBER. We thank Laurent Barras (discussant), John Campbell, John Cochrane, Kent Daniel (discussant), Peter Easton, Gene Fama, Wayne Ferson (discussant), Mark Grinblatt, Tarek Hassan, John Heaton, Ravi Jagannathan, Matti Keloharju, Ralph Koijen, Martin Lettau, Jon Lewellen (discussant), Laura Liu (discussant), Toby Moskowitz, Robert Novy-Marx (discussant), Ľuboš Pástor, Christopher Polk (discussant), Josh Pollet, Pietro Veronesi, John Wei, Feixue Xie, and seminar participants at the University of Chicago Booth School of Business, the University of Illinois at Urbana-Champaign, the Chicago Quantitative Alliance Fall 2012 Conference, the University of New South Wales, the University of Sydney, the University of Technology, Sydney, the 2012 HKUST Finance Symposium, the DePaul-Chicago Fed Finance Seminar, the ASU Sonoran Winter Finance Conference 2013, UCLA, the 2013 SFS Finance Cavalcade, the 2013 Western Finance Association conference, the 2013 Oxford Asset Pricing Retreat, the NBER 2013 Summer Institute Asset Pricing Meetings, the 2013 Inquire UK Seminar, and the 2013 European Finance Association Annual Meetings for helpful discussions and Jan Schneemeier for research assistance. We thank Ľuboš Pástor, Robert Stambaugh, and Luke Taylor for providing the merged and reconciled CRSP-Morningstar database.

1 Introduction

Fama and French (1996) reject the three-factor model due to its inability to price corner portfolios when stocks are sorted by size and book-to-market ratios. Nonetheless, it remains the most widely used asset pricing model nearly twenty years later. It is used either as-is or augmented with a momentum factor. But in either case the model fails to capture a large swath of patterns in average returns.[1] Rather than further map the model's boundaries by finding new anomalies, we dissect the factors already in the model.

We show that the SMB and HML factors, which are constructed by sorting stocks into portfolios by size and book-to-market ratios, combine multiple risk factors with different risk premia. The HML factor combines two factors, one with a risk premium of 6.5% per year between 1963 and 2012, and the other with a zero risk premium. The SMB factor in turn combines two factors with risk premia of 7.6% and 0.2% per year. SMB and HML have these properties because value and size come in different flavors. Even though small stocks comove with other small stocks, and value stocks comove with other value stocks, not all flavors of size and value are compensated in the form of higher average returns. Because of this mismatch between covariances and average returns, the three-factor model incorrectly penalizes some small stocks and value stocks.

We can illustrate the problems that result from the "baby" factors that comprise SMB and HML by using an example based on Ross's (1976) arbitrage pricing theory. Suppose the true model describing asset returns is

$$r_{it}^e = \beta_{1,i} F_{1t} + \beta_{2,i} F_{2t} + \varepsilon_{it}, \tag{1}$$

in which the risk premia are $\lambda_1 = 5\%$ and $\lambda_2 = 0\%$, and the factors are orthogonal with equal variances.

[1] See, for example, Stambaugh, Yu, and Yuan (2012) and Novy-Marx (2013) for lists of strategies that deliver anomalous returns vis-a-vis the three-factor model.

If stocks A and B have betas of $\beta_{1,\mathrm{A}} = 2$, $\beta_{2,\mathrm{A}} = 0$ and $\beta_{1,\mathrm{B}} = 0$, $\beta_{2,\mathrm{B}} = 2$, then these stocks' risk premia are 10% and 0%. But if we develop an asset pricing model that uses $F_t = F_{1t} + F_{2t}$ as a single factor, both stocks have the same beta of one against F_t:

$$\frac{\mathrm{cov}(r_{it}^e, F_{1t} + F_{2t})}{\mathrm{var}(F_{1t} + F_{2t})} = \frac{1}{2}(\beta_{1,i} + \beta_{2,i}) = 1 \text{ for } i = \mathrm{A}, \mathrm{B}. \tag{2}$$

A model that includes just F_t predicts a risk premium of 5% for each stock. Hence, we would underestimate stock A's risk premium by 5% and overestimate stock B's by an equal amount.

Our main result is that both SMB and HML behave similarly to the composite F_t in this example. The three-factor model assigns too high or too low alphas to strategies that mix unequal amounts of the different flavors of size or value. The economic magnitude of this problem is large. A long-short strategy that trades only the unpriced (or residual) part of value has a three-factor model alpha of 5.5% per year with a t-value of 3.7. A strategy based on the unpriced part of size has a three-factor model alpha of 2.6% per year with a t-value of 2.4. These limitations of the three-factor model differ from those emphasized by Fama and French (1996). They show that the three-factor model cannot price some corner portfolios after double-sorting stocks by size and book-to-market—we show that the problems with SMB and HML are pervasive.

The different flavors of size and value matter in practice. Performance evaluation relies on asset pricing models—abnormal performance determines future compensation and influences fund flows.[2] We show that the majority of mutual funds that trade value and small stocks trade disproportionately the types of value and size that are not compensated in the form of high average returns. Because the three-factor model does not distinguish between different flavors of value and size, it lowers the alphas

[2]See, for example, Berk and van Binsbergen (2012).

of all value and small stock managers alike. When we re-estimate alphas using a multi-factor model that includes the traditional three factors along with factors based on the unpriced parts of size and value, the t-values for the alphas of over 65% of mutual funds increase. Moreover, when we apply the Fama and French (2010) bootstrapping methodology, we find that the estimated fraction of skilled managers increases from 4% to 27% when we control for the unpriced parts of size and value. That such a seemingly small perturbation to the model greatly alters inferences is discomforting. A great deal of attention has been devoted to correctly distinguishing luck from skill.[3] But such concerns may be overwhelmed by those arising from using risk factors that combine "baby" factors with different risk premia.

The unpriced flavors of size and value also explain the negative correlation between gross profitability and value documented by Novy-Marx (2013). He shows that a strategy based on gross profitability (sales minus costs of goods sold divided by total assets) earns average excess returns and is negatively correlated with value. Hence, such a strategy allows an investor to invest in a growth strategy while earning value-like returns. When we evaluate this strategy using a multi-factor model that includes the traditional three factors along with control factors based on the unpriced parts of size and value, we find that the negative correlation between gross profitability and value disappears.

That size and value come in different flavors is important for research that seeks to uncover the nature of risks for which the premia associated with the HML and SMB factors compensate. One explanation, for example, is that the size and value premia compensate for the risk of financial distress—but the empirical evidence is at best mixed.[4] Our results suggest why so little consensus has emerged in this area. Because both SMB and HML consist of "baby" factors, each with its own risks

[3] See, for example, Kosowski, Timmermann, Wermers, and White (2006), Barras, Scaillet, and Wermers (2010), Fama and French (2010), and Linnainmaa (2013).

[4] See, for example, Lakonishok, Shleifer, and Vishny (1994), Fama and French (1995), and Campbell, Hilscher, and Szilagyi (2008).

and rewards, it is difficult to detect these risks and rewards in the bundles that comprise SMB and HML.

Our results inform attempts to extend the three-factor model's pricing abilities. Some current work in this area constructs new factors that are kept approximately orthogonal to the SMB and HML factors.[5] This goal of orthogonality is difficult to implement because it requires sorting stocks into increasingly less-diversified portfolios. Moreover, such an orthogonality constraint *guarantees* that the new factors cannot partial out the impurities embedded in the SMB and HML factors. A model augmented by new orthogonal factors can capture new sources of risk but it cannot offset the limitations of the existing factors. Trying to create additional orthogonal factors is misguided *if* the existing factors already confound multiple primitive factors. To overcome the limitations of the factors already in the model, we either need to reconstruct SMB and HML to consist just of the priced flavors of size and value, or include new factors that embed linearly independent spans of these flavors.

The priced and unpriced parts of size and value have clean economic interpretations. In both cases, the priced part relates to changes in the market value of equity and is distinct from the momentum and net issuance anomalies. The reason changes in the market value of equity are informative about expected returns can be seen through the lens of the dividend discount valuation model,

$$M_t = \sum_{\tau=1}^{\infty} \mathrm{E}_t \left[\frac{D_{t+\tau}}{(1+r)^\tau} \right]. \tag{3}$$

This model implies that firm size is always informative about expected returns if the asset pricing model does not fully account for these differences through other factors (Berk 1995). The reason why the *level* of market value of equity does not fully reveal expected returns, r, is that expected cash

[5] See, for example, Fama and French (2013).

flows vary across firms. This variation in cash flows motivates the use of book-to-market ratios. Fama and French (2008) note that under clean surplus accounting, the dividend D_t is earnings, Y_t, minus the change in retained earnings, dB_t. The market-to-book ratio then is

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} \mathrm{E}_t \left[\frac{Y_{t+\tau} - dB_{t+\tau}}{(1+r)^\tau} \right]}{B_t}. \tag{4}$$

Controlling for expected cash flows, a higher book-to-market ratio implies a higher expected stock return.

The better are the proxies for expected cash flows, the more information about expected returns we can recover from book-to-market ratios. However, *if* expected cash flows are sufficiently stable over time whereas expected returns vary, we can recover information about expected returns using an alternative method: differencing market values of equity over time. Suppose that cash flows grow deterministically at a rate g per year and that a stock's expected return is r. Then a stock's value today is

$$M_t = \frac{D_{t+1}}{r-g}. \tag{5}$$

If a one-off shock changes the stock's expected return from r to $r + \Delta r$ (without affecting its cash flows) then, to the first-order approximation, the log-change in the market value of equity equals:

$$\log\left(\frac{M_{t+1}}{M_t}\right) \approx -\frac{1}{r-g}\Delta r. \tag{6}$$

That is, a firm that experiences an increase in the market value of equity has a lower expected return than a firm whose market value of equity increases. This argument mirrors Berk's (1995) observation about the *levels* of market value of equity: firms with low market values of equity must have on average higher expected returns than large firms because their future cash flows are discounted back at higher rates. We suggest that this line of reasoning extends to *changes* in market values of equity if expected returns are time-varying.

Even without holding expected cash flows fixed, changes in market values of equity are likely informative about changes in expected returns. Properties of expected return and cash flow shocks determine how much information the changes in market values of equity recover relative to using firm sizes and book-to-market ratios. Our result is that the information in changes in the market value of equity subsumes the additional information in sizes and book-to-market ratios. This result does not imply that there is no more useful information in book-to-market ratios. By definition, book-to-market ratios contain *all* information about expected returns—it is only a question of finding good enough variables to hold expected cash flows fixed in order to make use of that information.

Daniel and Titman (2006) and Fama and French (2008) also examine the components of the book-to-market ratio. Fama and French (2008) show that the evolution of book-to-market ratios contains additional information about expected returns. They do not, however, examine the implication of this result on the HML factor's ability to explain average returns.

Daniel and Titman (2006) use "brute force" to decompose changes in book-to-market ratios into stock returns and a proxy for tangible information based on accounting performance ("book returns"). They find that only stock returns orthogonal to "book returns" reverse. However, Gerakos and Linnainmaa (2013) show that this decomposition is technically infeasible—it generates a "book-return" component contaminated by past book-to-market ratios, net issuances, dividends, and stock returns. Importantly, the effects of these contaminants vary with whether a firm is classified as either value or growth. For example, if a value firm issues equity, this action gets recorded as poor tangible performance. In contrast, if a growth firm issues equity, this action gets recorded as positive tangible performance. The effect of dividends also splits based on whether the firm is either value or growth. Instead of reflecting the firm's accounting performance, one-third of the variation in "book returns" is due to the correlations induced by using "brute force" to split the book-to-market ratio. Hence, when stock returns are decomposed into "tangible" and "intangible" parts, it is perhaps not surprising that the "tangible"-information part is uncorrelated with future returns. The Daniel and Titman (2006) result therefore provides no insight into whether baby factors comprise HML.

Our work also differs from Daniel and Titman (2006) and Fama and French (2008) in that neither study examines whether different components of firm size have different prices of risk, nor do they examine the connection between the existence of these disparate baby factors and performance evaluation and gross profitability.

In the following section we describe the data used in our analysis. Section 3 decomposes book-to-market ratios and market values of equity into priced and unpriced parts. Section 4 explores the asset pricing implications of the priced and unpriced parts of book-to-market ratios and market values of equity. Sections 5 and 6 show how the unpriced parts affect the evaluation of mutual fund skill and drive the negative relation between value and gross profitability. Section 7 concludes.

2 Data

2.1 CRSP and Compustat data on individual stocks

We take stock returns from CRSP and accounting data from Compustat. Our sample starts with all firms traded on NYSE, Amex, and Nasdaq. For these firms, we calculate the book value of equity (shareholder equity, plus balance sheet deferred taxes, plus balance sheet investment tax credits, minus preferred stock). We set missing values of balance sheet deferred taxes and investment tax credit equal to zero. To calculate the value of preferred stock, we set it equal to the redemption value if available, or else the liquidation value, or the carrying value. If shareholders' equity is missing, we set it equal to the value of common equity if available, or total assets minus total liabilities. We then use the Davis, Fama, and French (2000) book values of equity from Ken French's website to fill in missing values of the book value of equity. Because we require gross profitability, we start our main sample in July 1963. We end it in December 2012. In robustness tests we extend the sample back to July 1932.

For the book-to-market ratio, we use the market value of equity as per the fiscal year end and calculate it as the CRSP month-end share price times the Compustat shares outstanding if available, or else the CRSP shares outstanding. We lag book-to-market ratios by at least six months so that companies have released their annual financial statements. For example, if a firm's fiscal year ends in December, we begin using the December information at the end of June. When we calculate book-to-market ratios, we align the numerator, the denominator, and all the components of the decomposition at the same point in time. Therefore, the five-year changes in the book and market values of equity are the five-year changes up to the date when the book-to-market ratio is computed. For example, for firms with December 2008 fiscal year ends, the book and market value of equity terms are measured as of December 2008 and the five-year changes in the book and market values of equity are from December 2003 through December 2008. We winsorize explanatory variables in Fama-MacBeth regressions at 0.5% and 99.5% levels. The sample consists of firms that have non-missing market values of equity and book-to-market ratios both today and five years prior, and that have non-missing returns for month $t + 1$.

2.2 Merged and reconciled CRSP-Morningstar data on mutual funds

We take mutual fund returns from the CRSP survivorship bias free database and Morningstar. We use the merging procedure of Pástor, Stambaugh, and Taylor (2013) to reconcile the two data sources, identify possibly erroneous entries, and correct mistakes in returns, expenses, and assets under management.[6] Because our methodology (described later) follows that used by Fama and French (2010), we impose their sample restrictions. We exclude index funds, sector funds, funds of funds, and international funds and keep only actively managed U.S. equity funds. We restrict the sample to funds that start in or after 1984 to avoid biases that may arise from poorly performing funds that only report annual returns (Fama and French 2010). We do include funds that start in or after 2007 to avoid having a large number of funds with short return histories. We require a minimum of eight months of returns, and let the funds enter our sample when their assets under management reach a minimum of $5 million in December 2000 dollars. This filter removes the incubation bias documented by Evans (2010). Overall, these filters lead to final sample of 2,350 U.S. equity mutual funds.

3 Decompositions

Book-to-market ratio. Following Fama and French (2008), we decompose the time series evolution of book-to-market ratios using the following identity:

$$bm_t \equiv bm_{t-k} + \sum_{s=0}^{k-1} dbe_{t-s} - \sum_{s=0}^{k-1} dme_{t-s}, \tag{7}$$

[6]The procedure of Pástor, Stambaugh, and Taylor (2013) builds on earlier clean-up efforts of Berk and van Binsbergen (2012).

where bm_t is the log of the book-to-market ratio at time t, dbe_t is the annual change in the log of the book value of equity, and dme_t is the annual change in the log of the market value of equity. This identity implies that in a regression of bm_t against the components on the right-hand side of equation (7), the slopes on bm_{t-k} and dbe_{t-s}s would equal one, and those on dme_{t-s}s would equal negative one.

Components of bm_t can, however, differ significantly from each other in their contribution to the *variation* of current book-to-market ratios. This variation is what ultimately matters when sorting stocks by book-to-market ratios to measure value and growth or to construct the HML factor. Because stock returns are more volatile than accounting variables, changes in the market value of equity likely drive more of the cross-sectional variation in book-to-market ratios. Changes in the market and book values of equity are also not independent of each other: a change in the book value of equity usually reflects in market valuations, either contemporaneously or at lead or lag.

Market value of equity. The SMB factor, motivated by Banz's (1981) empirical results, is defined as the return difference between small and large stocks, holding the book-to-market ratio constant. To decompose the market value of equity, we split firm i's market value at date t into its market value at the time it first appears on the CRSP tape (denoted by $t_{i,0}$) plus the subsequent cumulative change in the market value:

$$me_{i,t} = me_{i,t_{i,0}} + \sum_{s=0}^{t-t_{i,0}-1} dme_{i,t-s}. \tag{8}$$

Similar to bm_t, these components can significantly differ from each other in their contribution to variation in current market values of equity. Such differences matter when we sort stocks by their market value of equity to measure the size premium or construct the SMB factor.

3.1 Which components explain cross-sectional variation?

Our decomposition of the cross-sectional variation in book-to-market ratios starts from the identity that a variable's covariance with itself equals its variance:

$$\begin{aligned}\mathrm{var}(bm_t) &= \mathrm{cov}(bm_t, bm_{t-k} + \sum_{s=0}^{k-1} dbe_{t-s} - \sum_{s=0}^{k-1} dme_{t-s}) \\ &= \mathrm{cov}(bm_t, bm_{t-k}) + \sum_{s=0}^{k-1} \mathrm{cov}(bm_t, dbe_{t-s}) + \sum_{s=0}^{k-1} \mathrm{cov}(bm_t, -dme_{t-s}). \qquad (9)\end{aligned}$$

Dividing both sides of this equation through by $\mathrm{var}(bm_t)$ gives each term's percentage contribution to the variance of today's book-to-market ratios.

The cross-sectional variation in market values of equity can be similarly decomposed:

$$\begin{aligned}\mathrm{var}(me_t) &\equiv \mathrm{cov}(me_t, me_{t-k} + \sum_{s=0}^{k-1} dme_{t-s}) \\ &= \mathrm{cov}(me_t, me_{t-k}) + \mathrm{cov}(me_t, \sum_{s=0}^{k-1} dme_{t-s}). \qquad (10)\end{aligned}$$

Once again, dividing both sides through by $\mathrm{var}(me_t)$ gives each term's percentage contribution to the variance of today's market value of equity.

Our use of the term "variance decomposition" is consistent with its usage in prior research.[7] Our decompositions, however, measure the covariation between today's book-to-market ratios and market values of equity and their components, and can therefore be negative. For book-to-market ratios, these covariances have the same interpretation as the analysis of Fama and French (1995), who show that value firms experienced low profitability for the prior five years. These estimates tell us what type of firms end up in different portfolios when sorted by their book-to-market ratios or market values of

[7] See Cochrane (1992).

equity. In particular, if a component's covariance with today's book-to-market ratios or market value of equity is zero, then any information contained in this component is lost in univariate portfolio sorts, because this component does not vary across portfolios.[8]

Table 1 presents the variance-decomposition estimates. We estimate the covariances of equations (9) and (10) with year fixed effects. Most of the variation in today's book-to-market ratios arises from lagged book-to-market ratios and changes in the market value of equity. In the one-year decomposition, 79% of the variation is due to the prior year's book-to-market ratio, 21% is due to (minus) the change in the market value of equity, and basically none of the variation is due to the change in the book value of equity. At the five year horizon, almost half of the variation is due to old book-to-market ratios, 56% is due to changes in market value of equity, and the difference (−5%) is due to changes in the book value of equity. The negative sign on the change in the book value indicates that when the book value of equity increases, the market value of equity generally increases even more, thereby resulting in lower book-to-market ratios in the cross-section.[9]

Cross-sectional differences in market values of equity are persistent. At the one year horizon, less than 10% of the variation in current market value of equity is due to the change in the market value of equity; the remainder is due to old market values of equity. Even at the five year horizon, 82% of the variation is due to old market values of equity. Hence, current market values of equity observed in the cross-section are primarily due to past market values.

These decomposition results address the question of what information size and book-to-market

[8]This argument relates to Lewellen, Nagel, and Shanken's (2010) critique of using 25 size- and book-to-market-sorted portfolios to test asset pricing models. They argue that a sort of stocks in these two dimensions imposes a rigid factor structure—whatever asset pricing factors were in stock returns prior to sorting are largely gone after the sort.

[9]The covariance term $\text{cov}(bm_t, dbe_{t-s})$ can be written as the variance of dbe_{t-s} and its covariances with all other terms of the decomposition. The results here suggest that the sum of these other covariances is large enough to more than offset dbe_{t-s}'s own variance. These results are similar to that observed in the price-dividend ratio decompositions where future returns appear to account for more than 100% of the variation in the price-dividend ratios (Cochrane 2005, p. 400).

Table 1: Cross-sectional variation of the book-to-market ratio and the market value of equity

This table decomposes the book-to-market ratio in year t (bm_t) into the book-to-market ratio in year $t-k$ (bm_{t-k}) plus the changes in the book (dbe_{t-k}) and market ($-dme_{t-k}$) values of equity from year $t-k$ to t and the market value of equity in year t (me_t) into the market value of equity in year $t-k$ (me_{t-k}) plus the change in change in the market value of equity from year $t-k$ to t ($\sum dme_{t-k}$). The first row presents a one-year decomposition, the second row a two-year decomposition, and so forth. Each estimate is the ratio of variation in today's explained by the component indicated by the column. A firm is used for estimating year $t-k$ covariances if the necessary information for it is available in years $t-k$ and t. The sample is from 1963 through 2012 for horizon = 1, from 1964 through 2012 for horizon = 2, and so forth. The covariances used to measure these ratios (see equations (9) and (10)) are estimated with year fixed effects. Standard errors, reported in square brackets, are block bootstrapped by year.

Horizon,	bm_t			me_t	
years	bm_{t-k}	$\sum dbe_\tau$	$-\sum dme_\tau$	me_{t-k}	$\sum dme_\tau$
1	0.79	0.00	0.21	0.91	0.09
	[0.01]	[0.01]	[0.02]	[0.01]	[0.01]
2	0.67	−0.02	0.34	0.88	0.12
	[0.02]	[0.01]	[0.02]	[0.01]	[0.01]
3	0.60	−0.03	0.43	0.86	0.14
	[0.02]	[0.01]	[0.02]	[0.01]	[0.01]
4	0.54	−0.04	0.50	0.84	0.16
	[0.02]	[0.01]	[0.02]	[0.01]	[0.01]
5	0.49	−0.05	0.56	0.82	0.18
	[0.02]	[0.01]	[0.02]	[0.01]	[0.01]
6	0.47	−0.06	0.59	0.81	0.19
	[0.02]	[0.02]	[0.02]	[0.01]	[0.01]
7	0.44	−0.07	0.63	0.80	0.20
	[0.01]	[0.02]	[0.02]	[0.01]	[0.01]
8	0.42	−0.07	0.65	0.79	0.21
	[0.01]	[0.02]	[0.02]	[0.01]	[0.01]
9	0.41	−0.08	0.67	0.79	0.21
	[0.01]	[0.01]	[0.02]	[0.01]	[0.01]
10	0.40	−0.09	0.68	0.78	0.22
	[0.01]	[0.01]	[0.02]	[0.01]	[0.01]

ratio-sorted portfolios pick up from the data. The algebraic decomposition could lead one to believe that, because bm_t is the cumulation of past changes in the book and market values, every part of the history plays as prominent a role. Table 1 rejects this view. The book-to-market ratios observed in the cross-section today are primarily due to what these ratios were in the past plus an adjustment for the changes in the market value of equity during the intermittent years. Thus, a sort on today's book-to-market ratios mostly sorts stocks by their old book-to-market ratios and changes in the market value of equity. Similarly, a sort on today's market value of equity mostly sorts stocks by their past market values.

3.2 Average returns, book-to-market ratio, firm size, and changes in the market value of equity

Book-to-market ratios and firm sizes correlate with future returns because some or all of their components correlate with those returns. Table 2 evaluates the extent to which changes in market values of equity *subsume* book-to-market ratio and firm size in Fama-MacBeth regressions of returns on firm characteristics. The starting point is the baseline regression in column (1):

$$r_{j,t} = b_0 + b_1 r_{j,t}^{1,1} + b_2 r_{j,t}^{2,12} + b_3 me_{j,t} + b_4 bm_{j,t} + e_{j,t+1}, \tag{11}$$

which models returns as a function of prior one-month returns ("short-term reversal"), prior one-year returns skipping a month ("momentum"), firm size, and book-to-market ratio. In the baseline regression presented in column (1), the average slope on firm size is significantly negative and that on book-to-market ratio is significantly positive.

Regressions (2) through (6) augment the baseline regression by adding one-year log-changes in the

Table 2: Average returns, the book-to-market ratio, firm size, and changes in the market value of equity

This table shows average Fama-MacBeth regression slopes and their t-values from cross-sectional regressions to predict monthly returns. The regressions, estimated monthly using data from July 1963 through December 2012, include the following variables: prior one-month returns, $r_{j,t}^{1,1}$; prior one-year returns skipping a month, $r_{j,t}^{2,12}$; the natural logarithm of the market value of equity, me_t; the natural logarithm of the book-to-market ratio, bm_t; the change in the log market value of equity over fiscal year $t-s$, dme_{t-s}; the log-change in the percentile rank in the firm size distribution between initiation of coverage on CRSP and time t, rank $\text{chg}_t = \ln\left({}^{me\ \text{rank}_t}/_{me\ \text{rank}_0}\right)$. Regression (7) adds one-year changes in the market value of equity up to year $t-10$. Because we only require firms to have existed for up to five years, some firms do not have changes in market value of equity beyond year $t-5$. For these firms we set the change in the market value of equity equal to that month's cross-sectional mean computed over firms with non-missing values.

	Model									
Regressor	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$r_t^{1,1}$	−5.91	−6.01	−6.12	−6.18	−6.24	−6.27	−6.39	−6.17	−6.10	−5.77
	(−14.88)	(−15.30)	(−15.73)	(−15.94)	(−16.09)	(−16.22)	(−16.65)	(−16.08)	(−16.02)	(−14.68)
$r_t^{2,12}$	0.64	0.64	0.62	0.61	0.58	0.57	0.54	0.65	0.68	0.74
	(3.62)	(3.72)	(3.57)	(3.54)	(3.40)	(3.35)	(3.20)	(3.87)	(4.04)	(4.26)
me_t	−0.10	−0.10	−0.10	−0.09	−0.09	−0.09	−0.08	−0.05	−0.02	
	(−2.84)	(−2.83)	(−2.77)	(−2.78)	(−2.75)	(−2.69)	(−2.37)	(−1.70)	(−0.58)	
bm_t	0.27	0.19	0.14	0.11	0.10	0.07	0.07	0.07	0.08	
	(4.56)	(3.25)	(2.36)	(1.94)	(1.64)	(1.25)	(1.23)	(1.27)	(1.40)	
dme_t		0.24	0.27	0.30	0.32	0.33	0.36	−0.25	−0.23	−0.28
		(2.58)	(2.74)	(3.06)	(3.27)	(3.39)	(3.64)	(−2.68)	(−2.43)	(−2.93)
dme_{t-1}			0.25	0.25	0.28	0.31	0.33	−0.22	−0.20	−0.24
			(3.04)	(3.03)	(3.30)	(3.58)	(3.90)	(−2.64)	(−2.40)	(−2.80)
dme_{t-2}				0.25	0.27	0.29	0.29	−0.21	−0.19	−0.24
				(3.87)	(4.02)	(4.39)	(4.34)	(−3.24)	(−3.02)	(−3.12)
dme_{t-3}					0.24	0.25	0.26	−0.18	−0.17	−0.24
					(3.80)	(3.94)	(4.02)	(−2.86)	(−2.75)	(−3.36)
dme_{t-4}						0.21	0.23	−0.13	−0.12	−0.17
						(3.83)	(4.20)	(−2.50)	(−2.22)	(−2.68)
Older *dmes*							Yes			
rank chg_t								−0.30	−0.34	−0.37
								(−6.61)	(−6.37)	(−5.77)
rank chg_t^2									0.20	0.23
									(6.55)	(6.47)
Avg. R^2	4.21%	4.53%	4.84%	5.02%	5.18%	5.29%	5.67%	5.45%	5.62%	4.60%

market value of equity. Regression (2) includes the one-year change from the end of fiscal year $t-2$ to the end of fiscal year $t-1$; regression (3) adds to that the one-year change from the end of fiscal year $t-3$ to the end of fiscal year $t-2$; and so forth. The slope on the book-to-market ratio changes significant as we move to the right. It decreases from 0.27 to 0.19 when we control for the one-year change in the market value of equity; to 0.14 when we control for two years of changes; and to 0.07 in regression (6) when we control for five years of changes. At the same time as the slope decreases by three-quarters, book-to-market ratio loses its statistical significance. In regression (6) the t-value associated with this variable is just 1.25, down from 4.56 in the baseline regression.

The addition of changes in the market value of equity also modifies the slope on the market value of equity albeit at a much lower pace. This difference between firm size and book-to-market ratio is consistent with Table 1's variance decomposition. Whereas the five years of changes in the market value of equity account for 56% of the variation in book-to-market ratio, they account for just 18% of the variation in firm sizes. Regression (7) adds one-year changes in the market value of equity up to year $t-10$. Because we only require firms to have existed for up to five years, some firms do not have changes in market value of equity beyond year 5. For these firms we set the change in the market value of equity equal to that month's cross-sectional mean computed over firms with non-missing values. In this specification, the slope on the market value of equity nudges down just a little bit, suggesting that older changes in the market value of equity contribute toward me_t's ability to explain average returns.

Regression (8) replaces the year $t-5$ to year $t-10$ changes in market values of equity with one variable, rank chg_t, that captures total changes in the market value of equity since initiation of coverage on CRSP. Because firms differ in age, the total changes in the market value of equity—$\sum_{s=0}^{t-t_{i,0}-1} dme_{i,t-s}$—are not directly comparable. We therefore measure firms' *relative* movements in the firm size distribution between the initial date and month t. We calculate each firm's percentile rank in the distribution of firm size as of the end of the first month that it appears on CRSP (me rank_0). We then calculate the log-change in this rank between this initial date and time t, $\text{rank chg}_t = \ln(me\ \text{rank}_t / me\ \text{rank}_0)$. This variable measures the long-term change in firm size relative to all other public firms. To illustrate, suppose a firm listed in June 1950 and its initial percentile rank in the firm size distribution at that date was 10%. If the firm still exists in June 2000 and its percentile rank is 40%, we compute $\text{rank chg}_t = \ln(0.4/0.1) = 1.39$.

Both firm size and book-to-market ratio are statistically insignificant in regression (8) that includes five years of changes in market value of equity and rank chg_t. The reason for why firm size still lingers near significance (t-value $= -1.70$) is either that differences in firms' initial sizes explain variation in average returns or that rank chg_t, given its percentile-rank construction, does not get the functional form right. Regression (9) suggests it is the latter: when the regression includes the second power of this variable, firm size attenuates further (t-value $= -0.58$).

The results Table 2's columns (1) through (9) show that in the baseline regression without changes in the market value of equity, book-to-market ratio and firm size predict future returns because they correlate with changes in the market value of equity. Book-to-market ratio is about relatively short-term changes in the market value of equity—adding changes beyond year five does not affect the slope estimate. Firm size, by contrast, is largely about long-term changes in the market value of equity.

Although it is the changes in market values of equity that forecast returns, the residual parts of book-to-market ratio and firm size explain variation in returns. When in regression (10) we drop book-to-market ratio and firm size from the list of regressors, the average R^2 decreases from 5.62% to just 4.60%—which is close to the value in the baseline regression (4.21%). Even though stocks with high or low "residual" book-to-market ratios or firm sizes do not earn consistently higher or lower returns, they comove with each other.

4 Asset pricing

4.1 Portfolio sorts based on the priced and unpriced components of book-to-market ratios and firm sizes

The Fama-MacBeth regressions in Table 2 show that at least part of book-to-market ratio's and firm size's ability to explain variation in average returns stems from these variables' correlations with changes in market values of equity. Moreover, the R^2 estimates suggest that if we were to sort stocks into portfolios by those components of book-to-market ratios and firm sizes that are orthogonal to changes in market values of equity, these portfolios would have different loadings on the HML and SMB factors. Because the HML and SMB factors sort stocks into portfolios based on the *overall* book-to-market ratios and firm sizes—and not just the parts that explain differences in average returns—these factors carry within them not only variation tied to changes-in-the-market-value-of-equity component but also that that tied to the residual book-to-market ratios and firm sizes.

Table 3 explores this issue by sorting stocks first into deciles based on firms' book-to-market ratios and sizes, and then by the two parts of these variables. We follow Table 2's results and break the cross-sectional variation in book-to-market ratios into two components. The first component is that part of book-to-market ratio that is correlated with changes in the market value of equity and the other is everything else. We extract these parts by estimating monthly cross-sectional regressions of log-book-to-market ratios on the variables in Table 2 that capture represent changes in the market value of equity: dme_t, dme_{t-1}, dme_{t-2}, dme_{t-3}, dme_{t-4}, rank chg_t, and rank chg_t^2. We call the fitted value, $\widehat{bm}_t$, the priced component and the residual part, bm_t^e, the unpriced component. We similarly break the variation in firm size into priced, $\widehat{me}_t$, and unpriced, me_t^e, components by estimating cross-sectional regressions of firm size against the same regressors.

Table 3: The priced and unpriced parts of book-to-market ratios and market values of equity

This table reports excess returns, CAPM alphas, and three-factor model alphas and factor loadings for portfolios sorted by book-to-market ratio and firm size (Panel A), the priced components of book-to-market ratio and firm size (Panel B), and the unpriced components of book-to-market ratio and firm size (Panel C). We sort stocks into deciles based on NYSE breakpoints at the end of each June and hold the portfolio for the next year. The adjusted R^2s in the last column are from time-series regressions of the high-minus-low portfolio. The last column reports the Gibbons, Ross, and Shanken (1989) test statistic and its associated p-value. The loadings-rows report the estimated loadings from the three-factor model. Test statistics for market betas (MKTRF) compare the estimated slopes to 1.0. The estimated slopes on SMB and HML are compared to zero. t-values are in parentheses.

Panel A: Sorts on book-to-market ratio

Sorting variable	$\hat{\alpha}$	Portfolio decile 1	2	9	10	GRS	High − Low $\hat{\alpha}$	R^2
Current	$\bar{r}^e$	0.40	0.51	0.69	0.87		0.48	
BE/ME:		(1.98)	(2.63)	(3.49)	(3.86)		(2.58)	
bm_t	CAPM	−0.06	0.05	0.28	0.43	1.68	0.50	0.0%
		(−0.83)	(0.91)	(2.62)	(3.09)	(0.08)	(2.67)	
	FF	0.15	0.11	−0.07	−0.06	2.25	−0.21	70.0%
		(2.56)	(2.00)	(−1.06)	(−0.62)	(0.01)	(−2.01)	
	$\hat{b}_{\text{mktrf}}$	0.97	1.00	1.02	1.07		0.10	
		(−1.99)	(−0.12)	(1.39)	(3.20)		(3.95)	
	$\hat{b}_{\text{smb}}$	−0.20	−0.12	0.14	0.36		0.55	
		(−10.01)	(−6.27)	(5.94)	(11.75)		(16.05)	
	$\hat{b}_{\text{hml}}$	−0.39	−0.09	0.70	0.93		1.32	
		(−18.40)	(−4.44)	(27.75)	(28.42)		(35.52)	
Priced comp.	$\bar{r}^e$	0.28	0.43	0.83	0.83		0.54	
of BE/ME:		(1.13)	(2.08)	(3.98)	(3.31)		(2.82)	
$\widehat{bm}_t$	CAPM	−0.29	−0.04	0.40	0.32	2.42	0.61	1.4%
		(−3.05)	(−0.57)	(3.51)	(2.25)	(0.01)	(3.14)	
	FF	−0.09	0.04	0.06	−0.04	1.07	0.06	40.9%
		(−1.10)	(0.54)	(0.72)	(−0.32)	(0.38)	(0.36)	
	$\hat{b}_{\text{mktrf}}$	1.17	1.05	1.04	1.12		−0.05	
		(8.66)	(2.79)	(1.90)	(4.51)		(−1.35)	
	$\hat{b}_{\text{smb}}$	0.01	−0.14	0.19	0.52		0.52	
		(0.23)	(−5.59)	(6.52)	(13.79)		(10.24)	
	$\hat{b}_{\text{hml}}$	−0.42	−0.14	0.65	0.58		1.00	
		(−14.19)	(−5.00)	(20.95)	(14.22)		(18.45)	

Panel A: Sorts on book-to-market ratio (continued)

Sorting variable	$\hat{\alpha}$	Portfolio decile 1	2	9	10	GRS	High − Low $\hat{\alpha}$	R^2
Unpriced comp.	$\bar{r}^e$	0.46	0.48	0.55	0.46		0.00	
of BE/ME:		(2.47)	(2.42)	(2.74)	(2.08)		(−0.01)	
bm_t^e	CAPM	0.04	0.01	0.12	−0.02	1.50	−0.05	1.8%
		(0.51)	(0.21)	(1.19)	(−0.16)	(0.14)	(−0.36)	
	FF	0.20	0.08	−0.19	−0.27	2.46	−0.46	35.5%
		(2.99)	(1.49)	(−2.66)	(−2.84)	(0.01)	(−3.65)	
	$\hat{b}_{\text{mktrf}}$	0.92	1.03	1.04	1.11		0.19	
		(−5.07)	(2.32)	(2.37)	(5.14)		(6.44)	
	$\hat{b}_{\text{smb}}$	−0.21	−0.12	0.13	0.13		0.33	
		(−9.50)	(−6.41)	(5.61)	(4.11)		(7.96)	
	$\hat{b}_{\text{hml}}$	−0.27	−0.12	0.62	0.49		0.76	
		(−11.48)	(−5.66)	(23.85)	(14.55)		(16.73)	

Panel B: Sorts on firm size

Sorting variable	$\hat{\alpha}$	Portfolio decile 1	2	9	10	GRS	High − Low $\hat{\alpha}$	R^2
Current	$\bar{r}^e$	0.92	0.81	0.50	0.40		−0.52	
ME:		(3.62)	(3.21)	(2.69)	(2.29)		(−2.71)	
me_t	CAPM	0.43	0.29	0.06	−0.02	3.71	−0.45	2.2%
		(2.69)	(2.09)	(1.16)	(−0.44)	(0.00)	(−2.34)	
	FF	0.08	−0.05	−0.03	0.05	1.86	−0.03	86.5%
		(1.07)	(−1.04)	(−0.60)	(1.94)	(0.05)	(−0.41)	
	$\hat{b}_{\text{mktrf}}$	0.90	1.00	1.00	0.97		0.07	
		(−5.92)	(0.12)	(0.45)	(−5.33)		(4.08)	
	$\hat{b}_{\text{smb}}$	1.17	1.03	−0.01	−0.29		−1.46	
		(48.26)	(59.11)	(−0.75)	(−34.81)		(−60.28)	
	$\hat{b}_{\text{hml}}$	0.37	0.39	0.19	−0.05		−0.42	
		(14.27)	(20.89)	(11.30)	(−5.42)		(−16.14)	
Priced comp.	$\bar{r}^e$	0.94	0.66	0.40	0.31		−0.63	
of ME:		(3.36)	(2.89)	(2.01)	(1.43)		(−3.49)	
$\widehat{me}_t$	CAPM	0.36	0.18	−0.07	−0.20	3.12	−0.55	2.5%
		(2.38)	(1.49)	(−1.08)	(−2.82)	(0.00)	(−3.11)	
	FF	−0.01	−0.19	0.00	−0.06	2.23	−0.05	58.2%
		(−0.07)	(−2.10)	(0.03)	(−0.91)	(0.01)	(−0.43)	
	$\hat{b}_{\text{mktrf}}$	1.18	1.12	1.02	1.07		−0.10	
		(7.69)	(5.68)	(1.53)	(4.91)		(−3.69)	
	$\hat{b}_{\text{smb}}$	0.89	0.33	−0.12	−0.06		−0.95	
		(27.27)	(11.14)	(−6.01)	(−2.85)		(−24.08)	
	$\hat{b}_{\text{hml}}$	0.48	0.68	−0.11	−0.28		−0.76	
		(13.76)	(20.99)	(−4.93)	(−12.11)		(−17.93)	

Panel B: Sorts on firm size (continued)

Sorting variable	$\hat{\alpha}$	Portfolio decile 1	2	9	10	GRS	High − Low $\hat{\alpha}$	R^2
Unpriced comp.	$\bar{r}^e$	0.46	0.56	0.49	0.44		−0.02	
of ME:		(1.81)	(2.38)	(2.65)	(2.58)		(−0.11)	
me_t^e	CAPM	−0.10	0.03	0.05	0.04	2.28	0.13	13.8%
		(−0.84)	(0.34)	(1.04)	(0.75)	(0.01)	(0.87)	
	FF	−0.16	−0.03	0.04	0.05	1.81	0.22	69.0%
		(−2.21)	(−0.46)	(0.80)	(1.53)	(0.06)	(2.35)	
	$\hat{b}_{\text{mktrf}}$	1.05	1.02	1.02	0.95		−0.10	
		(2.76)	(1.25)	(1.52)	(−5.62)		(−4.35)	
	$\hat{b}_{\text{smb}}$	0.72	0.58	−0.12	−0.25		−0.97	
		(28.89)	(24.66)	(−8.32)	(−21.50)		(−31.31)	
	$\hat{b}_{\text{hml}}$	−0.09	−0.05	0.07	0.04		0.13	
		(−3.36)	(−1.92)	(4.38)	(3.36)		(3.97)	

Table 3 Panel A shows average excess returns, CAPM alphas, and Fama and French (1993) three-factor model alphas for portfolios sorted by book-to-market ratios and the priced and unpriced components of book-to-market ratios. We use NYSE breakpoints and follow the Fama and French (1996) portfolio formation and rebalancing conventions. The sample is restricted to those firms for which we have the necessary data to decompose book-to-market ratios and firm sizes. (We use the standard Fama and French (1996) size- and book-to-market ratio-sorted portfolios as test assets in Section 4.2.)

The portfolios sorted on $\widehat{bm}_t$ spread out returns more than bm_t. The return spread between the top and bottom decile is 48 basis points (t-value = 2.58) per month for bm_t but 54 basis points (t-value = 2.82) for the priced part. Investors can earn on average 72 basis points more per year by trading the priced component of book-to-market ratios. A CAPM adjustment increases the high-minus-low spreads to 50 and 61 basis points, and the annualized return difference between the two strategies to 1.3% per year. In contrast, the unpriced part (bm_t^e) does not spread excess returns between the top and bottom deciles. Similarly, the CAPM alpha for the spread between the top and bottom deciles is insignificant for the portfolios sorted on the unpriced part.

But the three-factor alphas differ markedly across deciles because of how these strategies load on the HML factor. The loadings of the high-minus-low strategies based on book-to-market ratio and the two components of it are 1.32 (t = 35.5), 1.00 (t = 18.5), and 0.76 (t = 16.7). The three-factor model alpha on a strategy that purchases "value" stocks and sells "growth" stocks, as determined by the residual book-to-market ratio, bm_t^e, is −46 basis points with a t-value = −3.65! The reason for this large negative alpha is that from the three-factor model's viewpoint the "high" portfolio should earn higher returns than the "low" portfolio. But because the excess returns are, in fact, flat across these residual book-to-market ratio portfolios, all of the three-factor model's adjustment (that is, the $\hat{\beta}_{\text{hml}}\hat{\lambda}_{\text{hml}}$ term) becomes alpha.

A manager could use the unpriced part to enhance his three-factor model alphas. No matter what strategy a manager pursues, mixing in at least a fraction of the unpriced-part strategy would increase alphas. More interestingly, a manager who trades on value could earn the value premium while masking the source of these profits from the three-factor model by buying the $\widehat{bm}_t$-based strategy and shorting the bm_t^e-based strategy. A one dollar long-one dollar short strategy based on the two parts earns an excess return of $(54 - 0)/2 = 27$ basis points per month (t-value = 2.52). Because this strategy correlates only weakly with the HML factor, it keeps almost all of its excess returns as alphas—the three-factor model alpha is 26 basis points per month (t-value = 2.4).

An important result in Panel A is that the explanatory power of the three-factor model is significantly higher for the high-minus-low portfolio based on the current book-to-market ratio (70.0%) than for that based on the priced component (40.9%). Moreover, the explanatory power for the high-minus-low portfolio based on the unpriced component is 35.5%. The implication here, together with the R^2s presented in the last two columns Table 2's Fama-MacBeth regressions, is that HML captures systematic variation in returns that is not priced, or that at least has a lower price of risk than the rest of HML. That is, value stocks comove with other value stocks even when the "value" is of the flavor that does not earn a risk premium.

Table 3 Panel B sorts stocks into portfolios by market values of equity (me_t), the priced part of market values of equity ($\widehat{me}_t$), and the unpriced part of market values of equity (me^e_t). These estimates tell a similar story to Panel A's results. The average excess return on the high-minus-low portfolio is -49 basis points with a t-value of -3.49. The priced part generates a similar return and t-value (-63 basis points with t-value $= -2.83$). Average returns do not vary across portfolios sorted by the unpriced part of size—the excess return on the high-minus-low portfolio is -2 basis points (t-value $=$ -0.11). But the three-factor alpha is significant for the portfolios sorted by the unpriced part of size (22 basis points; t-value $= 2.35$).

The high and low portfolios of the priced and unpriced parts are also similar in terms of their characteristics. When stocks are sorted by their actual book-to-market ratio, the average book-to-market ratios of the top and bottom deciles are 0.21 and 2.51. Similarly, average book-to-market ratios are 0.36 and 1.26 when the sort is based on the priced part, and 0.23 and 1.87 when it is based on the unpriced part. Thus, a traditional value strategy mixes stocks that have always been value stocks or that have seen an increase in the book value of equity (the unpriced part) with those that became value stocks through a decrease in the market value of equity (the priced part). When stocks are sorted by the priced and unpriced parts of size, the bottom deciles have similar average market capitalizations: \$36, \$86, and \$144 million for actual size and the priced and unpriced parts of size. Hence, a small-cap strategy mixes stocks that have always been small (the unpriced part) with those that have become small (the priced part). The top deciles, however, differ in terms market capitalizations of \$23.7, \$2.2, and \$12.4 billion, because many large-cap stocks are stocks that have always been large-cap stocks (i.e., firms rarely make extreme movements up the size distribution).

4.2 Asset pricing model perturbation

We next compare the ability of the traditional three-factor model and three alternative models to price size and book-to-market sorted portfolios. The first alternative model augments the traditional three-factor model with control factors based on high-minus-low portfolios of the unpriced parts of size and book-to-market. We call the factors based on the unpriced parts of size and book-to-market "control factors" because they have zero means, and thus affect alphas only to the extent that they correlate with the other factors. The book-to-market ratio control factor, for example, is the return on the $10-1$ strategy in which stocks are sorted into portfolios by bm_t^e. The second alternative model includes just the control factors based on the unpriced parts of size and book-to-market along with the market return. The third alternative model includes factors based on high-minus-low portfolios of the priced parts of size and book-to-market along with the market return.

Table 4 uses the standard Fama and French (1996) size- and book-to-market ratio-sorted portfolios as test assets, and examines how well alternative models price the portfolios. For each model we present the estimated alphas for the two high and two low portfolios, and the estimated alphas and adjusted R^2s for the high-minus-low portfolios. The rightmost column reports the Gibbons, Ross, and Shanken (1989) test statistic and its associated p-value.

The traditional three-factor model does not price the BE/ME sorted portfolios. The estimated alpha for the high-minus-low portfolio is -26 basis points with a t-value of -2.72, and the joint pricing errors are significant with a p-value of 0.022. The high-minus-low alpha is negative because, from the three-factor model's viewpoint, the spread in excess returns should be larger than what it is in the data. The reason this spread is not high enough is that when stocks are sorted into portfolios by bm_ts, the high and low portfolios also contain stocks that have high or low residual book-to-market ratios.

Table 4: Asset pricing model perturbation

This table sorts stocks into deciles based on book-to-market ratio and firm size, and prices these portfolios using the standard Fama and French (1996) three-factor model ("FF3") and three alternative asset pricing models: "FF3 with control factors" adds high-minus-low portfolios based on the unpriced parts of the book-to-market ratio and size to the three-factor model; "Control factors only" replaces SMB and HML factors of the three-factor model with the high-minus-low portfolios based on the unpriced parts of the book-to-market ratio and size; "Priced components only" replaces the SMB and HML factors with high-minus-low portfolios based on the priced parts of the book-to-market ratio and size. The table reports alphas, t(alpha)s, adjusted R^2s, and Gibbons, Ross, and Shanken (1989) test statistics and the associated p-values from the time-series regressions.

AP	Portfolio decile					High − Low	
Model	1	2	9	10	GRS	$\hat{\alpha}$	R^2
			Test assets: BE/ME deciles				
FF3	0.15	0.06	−0.02	−0.11	2.11	−0.26	76.2%
	(2.65)	(1.12)	(−0.36)	(−1.28)	(0.022)	(−2.72)	
FF3 with control factors	0.06	0.04	0.02	−0.04	1.10	−0.11	81.3%
	(1.23)	(0.72)	(0.38)	(−0.51)	(0.359)	(−1.24)	
Control factors only	−0.12	0.00	0.34	0.42	2.43	0.54	48.3%
	(−1.99)	(−0.04)	(4.16)	(3.61)	(0.008)	(3.87)	
Priced components only	0.03	0.03	0.14	0.09	0.82	0.06	42.6%
	(0.47)	(0.60)	(1.66)	(0.81)	(0.606)	(0.41)	
			Test assets: ME deciles				
FF3	−0.09	−0.12	0.01	0.04	2.59	0.13	86.7%
	(−1.14)	(−2.74)	(0.21)	(2.09)	(0.004)	(1.72)	
FF3 with control factors	−0.08	−0.12	0.06	0.01	2.13	0.10	87.0%
	(−1.09)	(−2.70)	(1.41)	(0.66)	(0.020)	(1.27)	
Control factors only	0.32	0.21	0.08	−0.06	3.49	−0.37	52.9%
	(2.54)	(2.22)	(2.11)	(−2.30)	(0.000)	(−2.70)	
Priced components only	−0.09	−0.14	0.04	0.03	2.03	0.12	45.0%
	(−0.67)	(−1.30)	(0.96)	(1.06)	(0.029)	(0.80)	

These stocks flatten the pattern in average return because average returns do not vary across residual book-to-market ratio portfolios. The alternative model that augments the three-factor model with the control factors demonstrates more success in pricing the BE/ME sorted portfolios. Its high-minus-low alpha is insignificant (−11 basis points; t-value = −1.24) as are the joint pricing errors.

The control factors alone do not price book-to-market ratio-sorted portfolios. The high-minus-low alpha is positive and highly significant (54 basis points with a t-value of 3.87) as are the joint pricing errors (p-value = 0.008) in an alternative model that includes only the market factor and the control factors. This rejection is not surprising given that neither control factor earns a risk premium, and so no matter how the portfolios load on these factors, the factors cannot match the return differences between high and low portfolios.

To demonstrate that SMB and HML derive their pricing ability from the priced parts of size and book-to-market, we also evaluate an alternative model that includes just the factors based on the priced components along with the market return. These "priced factors" are the high-minus-low portfolio returns for $\widehat{bm}_t$ and $\widehat{me}_t$ from Table 3. For this alternative model, the high-minus-low alpha is close to zero (6 basis points) with a t-value of 0.41. Moreover, in contrast with the three-factor model, the joint pricing errors are insignificant (p-value = 0.606).

The results for the size-sorted portfolios are similar, even though the control factors do not completely remedy the model. Although the alphas on the high-minus-low portfolios are insignificant in every specification, the pricing errors are jointly large enough that the GRS test rejects all four models. Nevertheless, when we augment the three-factor model with the control factors, the p-value associated with the GRS test increases from 0.004 to 0.02. For the model that includes just the market return and the control factors, the pricing errors are jointly significant with a p-value less than 0.001. Hence, the control factors alone cannot price the size deciles. In contrast, the alternative three-factor model that includes the two priced components along with the market return performs as well as the three-factor model augmented with the control factors.

4.3 Robustness

Our results on the priced and unpriced parts of size and value are distinct from momentum (Jegadeesh and Titman (1993)). When we use Carhart's (1997) four-factor model to evaluate the priced and unpriced parts of size and value, the alphas are similar to those reported in Table 3: the alphas on long-short portfolios built on the priced parts remain insignificant, and the alphas on portfolios built on the unpriced parts are -44 basis points per month (t-value $= -3.47$) and 21 basis points per month (t-value $= 2.24$) for value and size. The reason the results parts are distinct from momentum is that momentum lasts only up to one year (Novy-Marx 2012). In contrast, as Tables 1 and 2 show, book-to-market ratio and firm size capture much longer term returns. The Fama-MacBeth regressions in Table 2 that motivate the decomposition also control for prior one-year returns.

Our results are also distinct from the net issuance anomaly. We construct a net-issuance factor by sorting stocks into deciles based on five-year net issuances (= log-change in the market value of equity − log-returns excluding dividends) and computing the value-weighted return on the high-minus-low strategy. This construction, as Daniel and Titman (2006) note, picks up the changes in the market value of equity that are due to either repurchases or issuances. Our results are not affected by the addition of this net-issuance factor to either the three- or four-factor model. The alphas on high-minus-low strategies built on the unpriced parts of value and size are -40 basis points (t-value $= -3.07$) and 23 basis points (t-value $= 2.38$) per month for value and size. The strategies built on the priced components remain statistically insignificant.

Our results on the priced and unpriced parts of value complement studies that split the variation in book-to-market ratios into intra- and inter-industry components. Cohen and Polk (1998), Asness, Porter, and Stevens (2000), and Novy-Marx (2013) show that strategies that trade on intra-industry variation in book-to-market ratios are as profitable but less volatile than those that trade on canonical book-to-market ratios. Repeating Table 1's decomposition for industry-adjusted book-to-market ratios, we find that the cross-sectional variation in industry-adjusted book-to-market ratios is also largely due to past changes in the market value of equity. In these decompositions the importance of changes in the market value of equity increases at the expense of changes in the book value of equity. Therefore, a sort on industry-adjusted book-to-market ratios is also largely a sort on the changes in the market value of equity.

In Fama-MacBeth regressions of returns on industry-adjusted size and book-to-market ratio, the slopes are -0.105 (t-value $= -3.14$) and 0.324 (t-value $= 7.38$), higher than those reported in Table 2 for the unadjusted variables and consistent with Novy-Marx's (2013) estimates. However, controlling for the changes in the market value of equity as in regression (9) of Table 2, industry-adjusted firm size becomes insignificant and the slope on the industry-adjusted book-to-market ratio decreases to 0.136 (t-value $= 3.22$). Thus, all of the industry-adjusted size variable's ability, and a large portion of the industry-adjusted book-to-market ratio's ability, to predict returns comes from the changes in the market value of equity. But the significance of the industry-adjusted book-to-market ratio suggests that industry-adjustment increases this variable's predictive ability for reasons unrelated to changes in the market value of equity.

Our main tests use data from July 1963 through December 2012. The downside of extending the sample back from 1963 to the beginning of CRSP is that left-truncation becomes an issue—we do not observe pre-1927 changes in market values of equity even for firms that had been publicly traded companies long before January 1927. Nevertheless, results are similar if we use the full sample starting in the July 1932. (By starting in 1932, we observe the minimum required five years of historical data.) For Table 2's baseline specification (1), the slope estimates on firm size and book-to-market ratio are

-0.128 ($t = -4.00$) and 0.252 ($t = 4.91$). When we control in specification (6) for five years of changes in market values equity, these slopes attenuate to -0.114 ($t = -3.93$) and 0.084 ($t = 1.84$), similar to what happens in the main sample, and the changes in the market value of equity are negative and significant. Finally, in specification (9) that also controls for long-term changes in market values of equity, both slopes are statistically insignificant at -0.049 ($t = -1.87$) and 0.070 ($t = 1.57$) and the long-term changes in the market value of equity are negative and significant.

The portfolio sort results of Table 3 remain largely unchanged as well. The return spread between the top and bottom decile is 62 basis points (t-value = 2.80) for book-to-market ratio-sorted portfolios, and 74 basis points (t-value = 3.21) and 19 basis points (t-value = 1.17) for the priced and unpriced components. The high-minus-low size portfolio earns -71 basis points (t-value = -3.02) per month, and the priced and unpriced components earn -79 basis points (t-value = -3.68) and 6 basis points (t-value = 0.50). The three-factor model alphas are insignificant for the priced parts of value and size (-14 and 4 basis points per month with t-values of -1.08 and 0.40). The three-factor model alphas are -37 basis points (t-value = -3.15) and 10 basis points (t-value = 0.97) for the unpriced parts of book-to-market ratio and firm size.

5 Measuring skill among mutual fund managers

Researchers and practitioners alike use multi-factor models extensively to evaluate money managers' abilities.[10] If managers' strategies covary with the unpriced parts of firm size and book-to-market ratio, then estimated three-factor alphas do not properly mirror their abilities. Suppose, for example, that value fund managers do not distinguish between the two components of value. The three-factor

[10] See, for example, Kosowski, Timmermann, Wermers, and White (2006), Barras, Scaillet, and Wermers (2010), Fama and French (2010), and Linnainmaa (2013).

alpha estimates will be too low for those managers who mostly purchase stocks with high residual book-to-market ratios, and too high for those managers who mostly own stocks that have become value stocks as their market values have fallen. We examine the extent to which inferences about mutual fund managers abilities are influenced by the fact that SMB and HML clump together baby factors with different prices of risk.

We first measure the shifts in factor loadings and the $t(\alpha)$ distribution when we augment the traditional three-factor model with control factors based on the unpriced parts of firm size and book-to-market ratio. We then use the Fama and French (2010) methodology to evaluate how the sensitive the estimate of the number of skilled fund managers is to perturbations in the asset pricing model. Similar to Fama and French (2010), we base our inferences throughout this section on the $t(\alpha)$ distributions to account for differences in the precision at which the alphas are estimated.

5.1 Asset pricing model perturbation and shifts in mutual fund's loadings and alphas

We estimate the traditional three-factor model and the three-factor model augmented with control factors for each fund j:

$$r_{j,t} - r_{f,t} = \alpha_j + b_j \text{MKTRF}_t + s_j \text{SMB}_t + h_j \text{HML}_t + e_{j,t} \quad (12)$$

$$r_{j,t} - r_{f,t} = \alpha_j + b_j \text{MKTRF}_t + s_j \text{SMB}_t + h_j \text{HML}_t + u_j me^e_{t,\text{control}} + w_j bm^e_{t,\text{control}} + e_{j,t}, \quad (13)$$

where $r_{j,t}$ is fund j's month-t return net of fees, $r_{f,t}$ is the one-month Treasury-bill rate, MKTRF_t is the return on the value-weighted market portfolio in excess of the Treasury-bill rate, SMB_t and HML_t are the size and value factors of the three-factor model, $bm^e_{t,\text{control}}$ is the month-t return a on long-short

Table 5: Asset pricing model perturbation and skill among mutual fund managers

This table reports the distributions of estimated factor loadings and $t(\alpha)$s for actively managed U.S. equity mutual funds. The first three columns present these estimates from the traditional three-factor model. The final five columns present estimates from a three-factor model augmented with control factors constructed from the residual components firm size and book-to-market ratio. The regression specifications are:

$$\begin{aligned} r_{j,t} - r_{f,t} &= \alpha_j + b_j \mathrm{MKTRF}_t + s_j \mathrm{SMB}_t + h_j \mathrm{HML}_t + e_{j,t} \\ r_{j,t} - r_{f,t} &= \alpha_j + b_j \mathrm{MKTRF}_t + s_j \mathrm{SMB}_t + h_j \mathrm{HML}_t + u_j me^e_{t,\text{control}} + w_j bm^e_{t,\text{control}} + e_{j,t}. \end{aligned}$$

Row "% > 0" reports the fraction of positive estimates. The t-value underneath this row is from a test that this fraction equals one-half. We estimate these models using monthly returns for U.S. equity mutual funds obtained from a merged-and-reconciled CRSP/Morningstar database. We restrict our sample to funds that start in or after 1984 and before 2007, and require a minimum of eight months. Funds enter our sample after their assets under management reach a minimum of $5 million in December 2000 dollars. The sample consists of 2,350 funds.

	FF3 model			FF3 model augmented with me and bm control factors				
Statistic	$\hat{b}_{\text{smb}}$	$\hat{b}_{\text{hml}}$	$t(\hat{a})$	$\Delta\hat{b}_{\text{smb}}$	$\Delta\hat{b}_{\text{hml}}$	$\Delta t(\hat{a})$	$\hat{b}_{\text{me control}}$	$\hat{b}_{\text{bm control}}$
p_5	−0.19	−0.49	−2.52	−0.54	−0.12	−0.65	−0.20	−0.14
p_{10}	−0.16	−0.39	−2.04	−0.37	−0.08	−0.43	−0.12	−0.09
p_{25}	−0.08	−0.18	−1.30	−0.18	−0.04	−0.12	−0.03	−0.04
p_{50}	0.09	0.03	−0.46	−0.06	0.00	0.21	0.06	0.02
p_{75}	0.46	0.25	0.41	0.03	0.03	0.56	0.18	0.09
p_{90}	0.72	0.43	1.15	0.12	0.07	0.92	0.35	0.19
p_{95}	0.84	0.54	1.62	0.20	0.11	1.17	0.52	0.25
Mean	0.20	0.20	0.20	−0.09	−0.01	0.22	0.09	0.04
	(28.53)	(4.43)	(−17.05)	(−16.22)	(−3.12)	(10.97)	(16.25)	(10.28)
% > 0	61.4%	54.6%	36.6%	33.4%	46.6%	65.7%	66.9%	59.1%
	(11.36)	(4.52)	(−13.54)	(−17.06)	(−3.31)	(16.08)	(17.40)	(9.02)

portfolio based on the unpriced part of book-to-market ratio, and $me^e_{t,\text{control}}$ is the month-t return on a long-short portfolio based on the unpriced part of size.

Table 5 shows the distributions of $\hat{b}_{\text{smb}}$, $\hat{b}_{\text{hml}}$, and $t(\alpha)$ in the three-factor model, the distributions of the changes in these quantities as we augment the model with the control factors, and the loadings on the control factors in the augmented model. The first two columns show that many funds tilt their portfolios toward small value stocks—the SMB loading is positive for 61% of the funds and the HML loadings is positive for 55% of the funds. Over 36% of funds have positive alphas.

Both the loadings and $t(\alpha)$s shift significantly when we perturb the model. The SMB loadings decrease for two-thirds of funds and the HML loads decrease for 53% of funds. The reason for these shifts lies in how the funds load on the control factors. The loading on $me^e_{t,\text{control}}$ shows that two-thirds of funds invest disproportionately in "always-small" small firms; the remaining 33% overweight their investments in small firms that have become small during their existence as public companies. The loadings on $\hat{b}_{\text{bm control}}$ show a similar pattern. Almost 60% of funds have portfolios weighted toward the types of value firms that do not earn above-average returns. The test statistics on the $\% > 0$ row test whether these estimated fractions equal one-half. We reject this null with t-values of 9.02 and 17.40. The consequence of these shifts in factor loadings—away from the SMB and HML factors and toward the control factors—is that the alpha distribution shifts to the right. The $\Delta t(\alpha)$ column shows that alphas of almost two-thirds of funds increase when we augment the three-factor model with the control factors.

Figure 1 presents a histogram of these fund-specific changes in $t(\alpha)$s when we move from the traditional three-factor model to the augmented model. The histogram shows the pronounced rightward shift, and it also suggests that the shift is slightly right-skewed, that is, large positive changes are more frequent than large negative changes. In Table 5 the 5th and 95th percentiles of the $\Delta t(\alpha)$-distribution are -0.65 and 1.17. These marked shifts in the alpha distribution affect inferences about the number of skilled fund managers.

5.2 Fama and French (2010) analysis of luck versus skill

Fama and French (2010) use a novel bootstrapping technique to estimate the fraction of managers who have enough skill to consistently outperform passive benchmarks after fees. Because fund returns

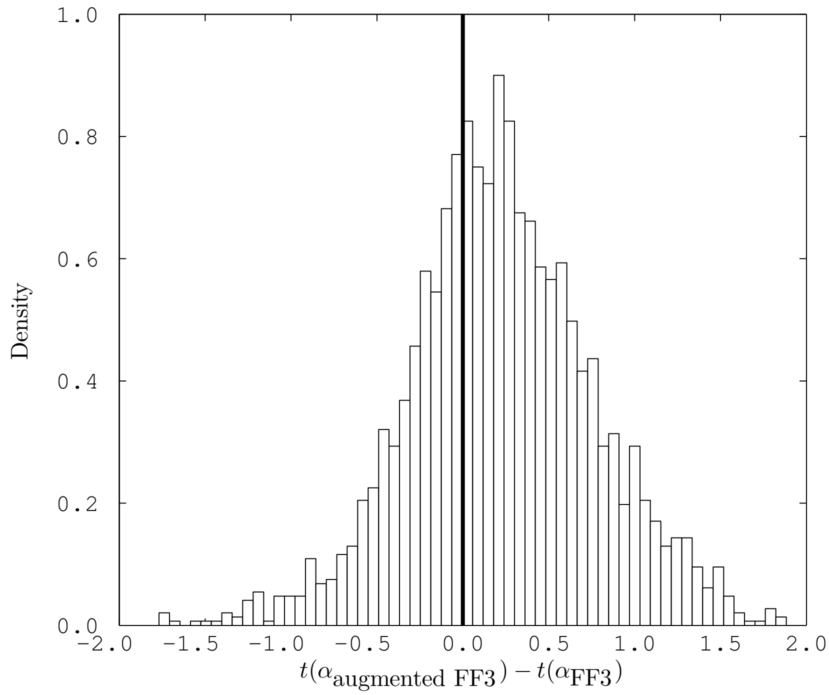

Figure 1: **Histogram of differences in funds' $t(\alpha)$s estimated using a three-factor model and a three-factor model augmented with control factors.** This figure plots the distribution of fund-specific differences in $t(\alpha)$s estimated using the standard three-factor model and a three-factor model augmented with two control factors based on the unpriced parts of firm size and book-to-market ratios. We estimate these models using monthly returns for U.S. equity mutual funds obtained from a merged-and-reconciled CRSP/Morningstar database. We restrict our sample to funds that start in or after 1984 and before 2007, and require a minimum of eight months. Funds enter our sample after their assets under management reach a minimum of $5 million in December 2000 dollars. The sample consists of 2,350 funds.

are very noisy, funds can have high or low alphas (or $t(\alpha)$s) just by luck. The empirical difficulty then is disentangling luck from skill. Fama and French assess skill using the following procedure:

1. Estimate each fund's alpha using all available data;
2. Set funds' full-sample alphas to zero by subtracting estimated alphas from monthly funds returns;
3. Resample *months* from the panel with replacement to preserve the covariance structure of fund returns and factors.

4. Re-estimate alphas of all funds using the resampled data; and

5. Go back to step 3 and repeat the simulation procedure 10,000 times.

By setting funds' full-sample alphas to zero, the variation in the re-estimated alphas (and $t(\alpha)$s) is due to noise. Fama and French (2010) then examine how the true distribution of $t(\alpha)$s differs from the simulated distributions. The benefit of the by-month sampling scheme is that it retains the covariance structure of fund returns and factors, so the bootstrapping procedure properly accounts for correlated observations.[11]

Fama and French's (2010) main analysis is based on the analysis of likelihoods. They compute the percentiles of the actual $t(\alpha)$-distribution and then report the fraction of simulations in which the corresponding percentile is lower. If, for example, the simulated 90th percentile of the $t(\alpha)$ distribution is often lower than the corresponding percentile in the actual $t(\alpha)$ distribution, then fund managers at this percentile appear to have skill—that is, their $t(\alpha)$s are higher than what we would expect them to be by luck alone. Fama and French (2010) conclude that only a handful of managers have skill. In the three-factor model only at the top-2% percentiles of the actual $t(\alpha)$ distribution the t-values dominate the simulated t-values more than 50% of the time.

Figure 2 repeats the Fama and French (2010) analysis using the combined-and-reconciled CRSP-Morningstar database. The solid line corresponds to the main specification in Fama and French (2010), reported in Table 3 of that paper. Our estimates of likelihoods are very close to Fama and French's estimates despite the differences in the sample. The fraction of simulations in which the actual $t(\alpha)$ percentile dominates the simulated percentile is stuck at zero up to around the 40th percentile, after which it begins its slow drift upwards. However, it is only at the very right tail of the distribution

[11] The test statistics in Table 5 assume that all observations across funds are independent of each other. But this is not a reasonable assumption. Funds may hold similar portfolios and, as Fama and French (2010) note, the regressors—that is, the factor realizations—are the same for all funds. The bootstrapping procedure addresses these issues.

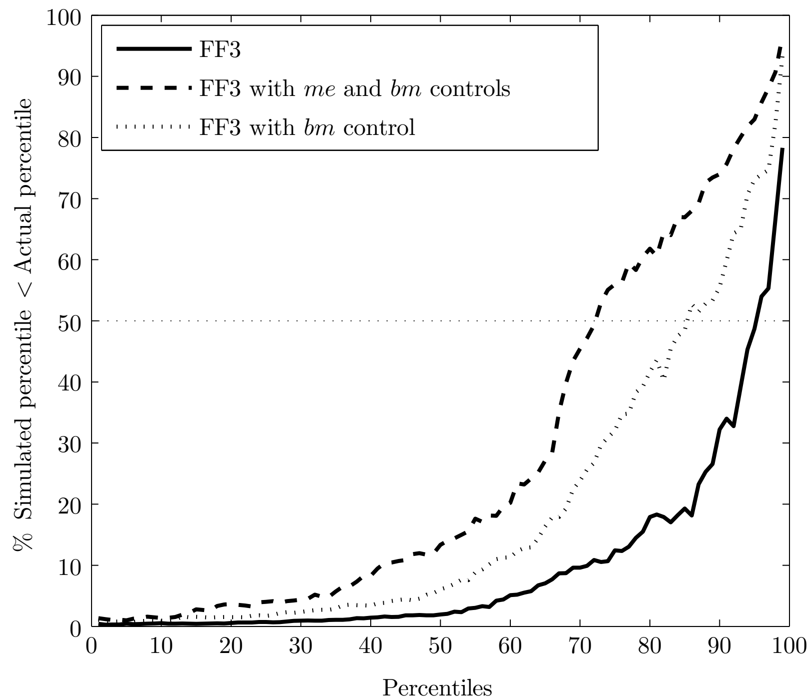

Figure 2: **Fama and French (2010) luck-versus-skill analysis of mutual fund returns.** We estimate alphas from the traditional three-factor model (solid line), this model augmented with just the book-to-market ratio control factor (dotted line), and this model augmented with both the book-to-market ratio and firm size control factors (dashed line). We record the actual distributions of $t(\alpha)$s from these models and then subtract estimated alphas from monthly fund returns. We then resample months 10,000 times. In each simulation we re-estimate the alphas, construct the $t(\alpha)$ distribution, and compare the percentiles of each simulated distribution against the actual $t(\alpha)$ distribution. y-axis reports the fraction of simulations in which the percentile indicated on the x-axis is *lower* than the actual percentile. We estimate the models using monthly returns for actively managed U.S. equity mutual funds obtained from a merged-and-reconciled CRSP/Morningstar database. We restrict our sample to funds that start in or after 1984 and before 2007, and require a minimum of eight months of data. Funds enter our sample after their assets under management reach a minimum of \$5 million in December 2000 dollars. The sample consists of 2,350 funds.

where this fraction increases above one-half. That is, similar to Fama and French (2010), only 4% fund managers have higher $t(\alpha)$s than what we would expect to observe if all alphas were due to luck alone. This is the estimated fraction of managers who can consistently deliver performance that covers their expenses under the three-factor model.

The dashed line in Figure 2 repeats the Fama-French simulations using the three-factor model augmented with the control factors. It shows that inferences about the fraction of skilled managers are very sensitivity to perturbations in the asset pricing model. The dashed line crosses the one-half threshold between the 72nd and 73rd percentiles, so the estimated fraction of skilled managers increases from 4% in the three-factor model to 27% as we augment the model with the control factors. That is, up to a quarter of fund managers may have enough skill to pick stocks well enough to cover the costs they impose on their investors. The dotted line in Figure 2 augments the traditional three-factor model using just the book-to-market-ratio control factor. The estimated fraction of skilled fund managers in this model, at 14%, is between that in the three-factor model and the full augmented model.

6 Gross profitability

Novy-Marx (2013) shows that a trading strategy based on gross profitability (sales minus cost of goods sold divided by total assets) earns a significant average excess return. More importantly, he shows that such a growth strategy is negatively correlated with the traditional value strategy. Therefore, the strategy's three-factor model alpha is greater than its excess return. We next examine whether the unpriced parts of firm size or book-to-market ratio drive the gross profitability strategy's negative correlation with value.

Following Novy-Marx (2013), we create gross profitability deciles based on NYSE breakpoints. For these deciles, Table 6 presents average excess returns, three-factor loadings and alphas, and factor loadings and alphas for a three-factor model augmented with factors based on the unpriced parts of size and value, $bm^e_{t,\text{control}}$ and $me^e_{t,\text{control}}$. As per Novy-Marx (2013), the high-minus-low portfolio of excess

Table 6: Gross profitability, size, and value

This table sorts stocks into deciles based on gross profitability (= sales − cost of goods sold divided by total assets) and reports average excess returns, three-factor alphas and factor loadings, and alphas and factor loadings for a model that augments the three-factor model with control factors constructed from the unpriced parts of firm size and book-to-market for the deciles and high-minus-low hedge portfolios. The deciles are based on NYSE breakpoints.

	Portfolio decile										High − Low	
Model	Low	2	3	4	5	6	7	8	9	High	$\hat{\alpha}$	R^2
					Excess returns							
r^e	0.31	0.35	0.41	0.40	0.57	0.51	0.43	0.45	0.57	0.69	0.39	
	(1.56)	(1.93)	(2.06)	(2.03)	(2.84)	(2.52)	(2.01)	(2.16)	(2.92)	(3.53)	(2.80)	
					Fama-French three-factor model							
b	0.95	0.96	1.03	1.03	1.02	1.00	1.03	0.98	0.93	0.90	−0.05	9.2%
	(−2.33)	(−2.35)	(1.89)	(1.46)	(0.98)	(0.27)	(1.83)	(−0.88)	(−4.18)	(−4.94)	(−1.56)	
s	0.08	−0.03	−0.10	−0.03	0.01	0.12	0.06	0.00	0.00	−0.05	−0.13	
	(2.63)	(−1.39)	(−3.86)	(−1.01)	(0.41)	(5.02)	(2.27)	(−0.16)	(−0.16)	(−1.73)	(−2.84)	
h	0.09	0.35	0.21	0.17	0.10	0.03	−0.20	−0.25	−0.27	−0.28	−0.37	
	(2.91)	(12.97)	(7.64)	(6.08)	(3.53)	(1.11)	(−7.21)	(−8.87)	(−10.56)	(−9.23)	(−7.75)	
a	−0.18	−0.21	−0.12	−0.12	0.06	0.01	0.02	0.10	0.25	0.40	0.58	
	(−2.06)	(−2.78)	(−1.60)	(−1.56)	(0.82)	(0.14)	(0.31)	(1.22)	(3.56)	(4.71)	(4.34)	
					Fama-French three-factor model augmented with size and value control factors							
b	0.92	0.92	0.99	0.99	0.99	0.99	1.02	1.00	0.97	0.98	0.05	30.0%
	(−3.64)	(−4.47)	(−0.65)	(−0.34)	(−0.66)	(−0.66)	(1.00)	(−0.08)	(−1.86)	(−1.43)	(1.86)	
s	−0.04	−0.22	−0.23	−0.04	−0.04	0.11	0.00	0.05	0.09	0.13	0.17	
	(−0.93)	(−5.68)	(−6.04)	(−0.94)	(−1.04)	(2.88)	(0.00)	(1.21)	(2.60)	(3.39)	(2.71)	
h	0.02	0.28	0.07	0.01	−0.02	−0.05	−0.24	−0.21	−0.13	−0.02	−0.04	
	(0.59)	(8.56)	(2.02)	(0.31)	(−0.59)	(−1.45)	(−6.63)	(−5.83)	(−4.25)	(−0.59)	(−0.79)	
$b_{\text{me control}}$	0.09	0.15	0.07	−0.06	0.00	−0.02	0.04	−0.04	−0.04	−0.06	−0.15	
	(2.16)	(4.52)	(2.10)	(−1.62)	(0.02)	(−0.73)	(1.15)	(−1.01)	(−1.14)	(−1.86)	(−2.72)	
$b_{\text{bm control}}$	0.11	0.11	0.20	0.20	0.16	0.10	0.05	−0.06	−0.19	−0.35	−0.46	
	(3.52)	(4.63)	(8.15)	(7.77)	(5.99)	(3.89)	(2.04)	(−2.16)	(−8.18)	(−14.52)	(−11.28)	
a	−0.11	−0.12	−0.01	−0.04	0.14	0.05	0.06	0.06	0.16	0.22	0.34	
	(−1.30)	(−1.70)	(−0.19)	(−0.55)	(1.79)	(0.68)	(0.75)	(0.77)	(2.34)	(3.11)	(2.82)	

returns generates a significant monthly return of 39 basis points (t-value of 2.80) and a significant three-factor alpha of 58 basis points (t-value of 4.34). Also consistent with Novy-Marx (2013), the high-minus-low portfolio has a significantly negative loading on HML (-0.37 with a t-value of -7.75) in the three-factor model.

The negative correlation with value, however, is entirely absent when we augment three-factor model with the control factors based on the unpriced parts of book-to-market and size. The high-minus-low strategy's loading on HML *attenuates* to -0.04 (t-value of -0.79) because this strategy correlates significantly with the unpriced parts of both firm size and book-to-market ratio. The coefficient on the book-to-market ratio control factor is negative and highly significant (-0.46 with a t-value of -11.28) and the coefficient on the firm size control factor is negative and significant (-0.15 with a t-value of -2.72). In addition, the adjusted R^2 for the high-minus-low strategy increases from 9.2% to 30.0% when we augment the three-factor model with the control factors.

The consequence of the shift in HML loadings is that the gross profitability strategy's alpha in the augmented three-factor model attenuates back to the level of excess returns (34 basis points with a t-value of 2.82). Hence, gross profitability is negatively correlated with the unpriced part of the book-to-market ratio and it is this negative correlation that boosts the strategy's alpha when evaluated using the traditional three-factor model. In spite of this result, it is worthwhile to emphasize that the remaining part of the gross-profitability strategy *is* profitable and clearly separate from value and size no matter how they are defined.

7 Conclusions

We show that when firm size and book-to-market ratio are decomposed into parts correlated with changes in the market value of equity and into residual parts, only the parts correlated with changes in the market value of equity predict returns. At the same time, firms with high or low *residual* book-to-market ratios or firm sizes comove with each other. As a consequence, portfolios based on the residual parts of firm size and book-to-market ratio generate significant three-factor alphas: 22 basis points (t-value = 2.35) and -46 basis points (t-value = -3.65) per month.

Two applications illustrate that these results are relevant for practice. First, many mutual funds hold portfolios positively correlated with the unpriced parts of firm size and book-to-market ratio. The estimated fraction of skilled mutual fund managers increases from 4% to 27% when we move from the three-factor model to a model that controls for the unpriced components. Second, the unpriced part of book-to-market ratio is responsible for the negative correlation between gross profitability and value. This negative correlation is entirely absent in a model that controls for the unpriced components of firm size and book-to-market ratio.

Our analyses relate to recent work on the three- and four-factor models. Cremers, Petäjistö, and Zitzewitz (2013) show that some stock indices have statistically significant nonzero alphas in the four-factor model. They trace these nonzero alphas to the model's inability to price the size/book-to-market corner portfolios (Fama and French 1993), and to the fact that the model's market index includes securities other than stocks. In contrast, our results on the SMB and HML factors are not about the corner portfolios, but about these factors combining multiple "baby" factors with different risk premia.

Asness and Frazzini (2011) find that adjusting book-to-market ratios to use more timely price data increases the value premium. Our Fama-MacBeth regressions are consistent with their findings. They find that the value premium increases because more recent market values of equity reduce the role that recent changes in the book value of equity play in the book-to-market ratios. In fact, one obtains a similar result not only by using more timely records of market values of equity, but also by using *older* records of book value of equity.

Fama and French (2008) show that the changes in the market and book values of equity are approximately equally important among large stocks in Fama-MacBeth regressions of returns on net issuances and these changes. These results are also consistent with our findings. Changes in book value of equity are informative about expected returns, but only when conditioning on additional information.[12] Our results on the HML factor show that when the book-to-market ratio is taken in isolation, it produces an asset pricing factor with a non-uniform price of risk.

Although we focus on the SMB and HML factors, our message is more general. If empirical risk factors combine primitive risk factors with different prices of risk, we misestimate expected returns. This issue applies to any empirical risk factor. How can researchers address this problem? First, they can construct better empirical factors by paying attention to the uniformity of factors' risk premia. This approach is, however, difficult given that we do not observe the primitive economic risks to which empirical risk factors map. Second, they can condition on more information by adding new factors—it does not matter how the factors are confounded as long as they jointly span all primitive factors. But it is difficult to evaluate ex ante whether the factors completely span all mean-variance-efficient portfolios. And additional factors come at the cost of lost degrees of freedom. At a minimum, it is worth demonstrating how sensitive estimates (such as mutual fund manager skill and trading strategy

[12] Gerakos and Linnainmaa (2013) note that a firm's book value of equity can increase because it is profitable (which predicts high future returns) or because it issues equity (which predicts low future returns). If an empirical model does not condition on net issuances to disentangle these two channels, changes in book value of equity are approximately uncorrelated with future returns.

alphas) are to small perturbations in the asset pricing model.

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