Houses, Apartments and Condos: The Governance of Multifamily Housing

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Draft:

December, 2013

In the U.S., homeownership is strongly correlated with the type of building that households occupy. Data from the national American Housing Survey (AHS) indicate that 83 percent of occupied single-family, detached housing in the U.S. is owner-occupied while only 12 percent of units in multifamily buildings are owner-occupied. This paper fills a gap in the homeownership literature by considering the relationship between building size and governance. A portfolio model of investment choice in risky housing assets trades-off a free-rider problem in joint ownership against economies of scale that are obtained in larger buildings with respect to the cost of third-party management. Our results show that the smallest multifamily buildings are particularly disadvantaged for purposes of condo ownership, because free-riding reduces contributions of investor effort while third party management is still relatively expensive. The empirical portion of the paper uses AHS data to estimate the bids of condominium owners with respect to marginal quality, as measured by interior floor space. We confirm that low wealth households are outbid for space in larger condominium buildings by higher wealth households, as predicted by the model.

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1 Introduction

In the U.S., homeownership is strongly correlated with the type of building that households occupy. Data from the national American Housing Survey (AHS) of 2011 indicate that 83 percent of occupied single-family, detached housing in the U.S. is owner-occupied while only 12 percent of units in multifamily buildings are owner-occupied. According to Glaeser and Shapiro (2003), "[t]here are few facts in urban economics as reliable as the fact that people in multi-family units overwhelmingly rent and people in single-family units overwhelmingly own." A stylized fact that emerges from a breakdown of building ownership according to building size, however, is that while condominium ownership of multifamily buildings is quite low among 2-12 unit buildings, it recovers to a national average of over twenty-five percent among larger buildings. This basic pattern of increasing condo ownership with building size persists both within cities, and across cities and regions in the U.S. as depicted in Figures 1 and 2. Unsurprisingly, homeownership rates trend upwards with the increase in condo ownership.

Beyond mere correlations lie important policy implications. The political popularity of homeownership (Gabriel and Rosenthal, 2005), and the willingness of US households to take advantage of the resulting ownership subsidies results in not just more ownership, but more single-family housing than otherwise would be expected. Single family houses occupy more land per household, and the availability of cheap land in the suburbs for those houses causes decentralization, or more pejoratively, sprawl. Decreased density has a number of ramifications including increased (and putatively wasteful) commuting, decreased productivity in the city as a whole, and increased mismatch of workers and vacancies in cities and suburbs.[1] Despite the importance of homeownership, economic theorizing about building size and homeownership has been incomplete or inadequate to explain the basic patterns that we observe in the data.

[1] On commuting and sprawl see Gordon, Kumar and Richardson (1989). On density and productivity, Ciccone and Hall (1996) is the primary reference. Coulson, Laing and Wang (2001) and Ihlandfeldt (1996) discuss spatial mismatch.

This paper presents a portfolio model of investment choice in risky housing assets that can be owned through various alternative forms. A building may be solely owned by a single investor who is the full residual claimant to building value, or by multiple investors who are joint owners of the building. We lump a variety of mechanisms used to coordinate and govern investor relations under this latter heading.[2] Our model emphasizes contributions by both investors and third-party managers to building management, which in turn affects building value. Into this setting, we introduce two sources of frictions related to the number of building co-investors. First, management effort exerted by one investor is assumed to benefit all co-investors in a joint ownership arrangement. The result is that investors bear a personal cost of management effort but only receive a fraction of the benefit produced. Therefore, each investor in a jointly-owned multifamily building under-produces management effort. A sole owner of the same building produces greater total management effort than the sum of effort produced by two or more co-investors.

The second mechanism in the model related to building size is an economy of scale in the cost of effort supplied by third-party managers ("managers"). Thus, in small multifamily buildings, managers are relatively expensive. Adding co-investors increases the building size accessible to a given investor, providing access to lower average costs of third-party management (up to the cost-minimizing building size). Our model trades-off the free-rider problem among investors against economies of scale (and agency costs) associated with hiring third-party management.

Our results show that the smallest multifamily building sizes are particularly disadvantaged for purposes of condo ownership, because free-riding reduces contributions of investor effort while third party management is still relatively expensive. Nonetheless, as the average cost of hiring management falls, bid prices for joint ownership arrangements begin sloping upwards with building size. Investor participation in direct building management falls to

[2] For example, joint ownership is provided by condominium and cooperative arrangements as well as through partnerships and real estate investment trusts (REITs). In any of these arrangements, free-rider problems emerge just as in diffuse stock ownership arrangement for firms.

zero as the number of co-investors increases, such that changes in bid prices with respect to building size are dominated by changes in third-party management costs and become nearly identical for investors among the largest buildings. Therefore, the identity of the marginal investor in larger buildings is heavily influenced by the distribution of wealth. To the extent that the number of sole proprietors of multifamily housing is limited, investors in joint ownership arrangements, which includes condominium ownership, partnership ownership and real estate investment trusts (REITS), may be the marginal investors setting prices. The maintained assumption linking our analysis back to homeownership rates in multifamily buildings is that sole owners of buildings rent the residences therein, while (at least some) joint owners preserve an option to owner-occupy.

The empirical portion of the paper uses the American Housing Survey to estimate the bids of condominium owners with respect to marginal quality, as measured by interior floor space. The bids are modelled as functions of both demographic and unit characteristics, particularly the number of co-investors as measured by the number of units in the building. In order to do this we first account for the selection of buildings into condominium status, as opposed to sole ownership, and then for the selection into owner-occupied condos in order to overcome observability problems discussed below. We then use two-step procedures for recovering the bid functions that were first proposed by Epple (1987) and Bartik (1987). The sample is split into high and low wealth samples. As predicted by the model, the low wealth investors are quite sensitive to the free rider problem, so their gradient with respect to unit count is negative and quite steep. They, therefore, are the winning bidders in small buildings, and are then outbid as condominiums become larger, both in quality and total building size.

The paper proceeds as follows. In the next section we present some of the stylized facts and quandaries to which we have alluded, and describe the literature in this area. Then we present the model, analyze it, and present numerical solutions. The succeeding section estimates the data analogs of the bid functions derived from the theory. This involves several

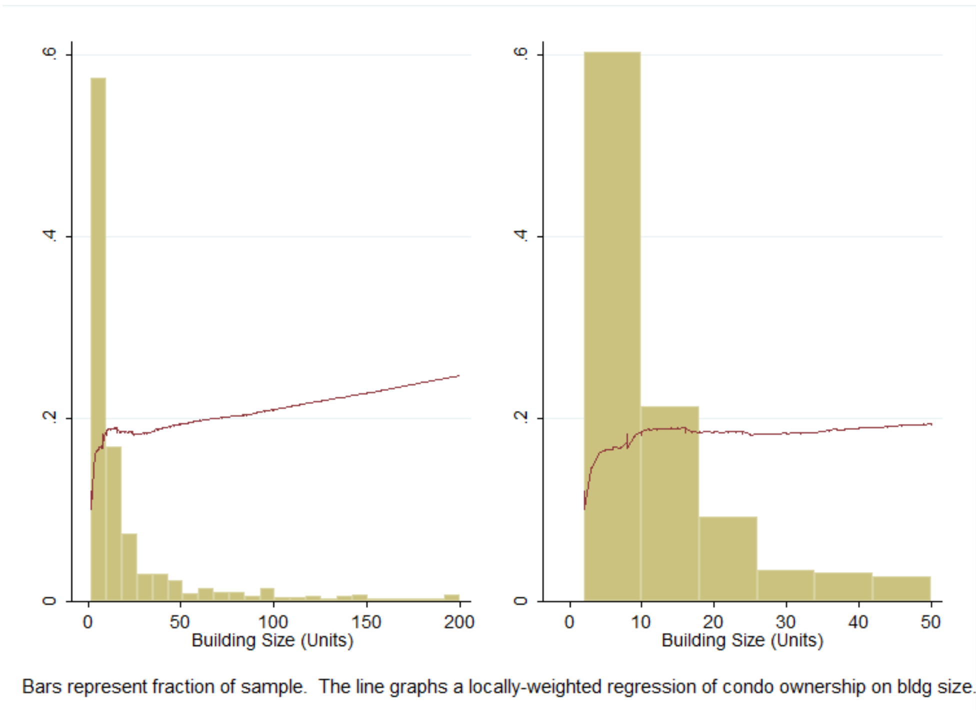

Figure 1: Condo Ownership, All Multifamily (AHS 2011)

econometric issues, which are addressed, and we present empirical evidence consistent with model predictions. We then conclude.

2 Homeownership and Structures

By way of context, about three-fourths of year-round housing in the U.S. is in single family structures. The vertical bars in Figure 1 show that most of the multifamily housing in the U.S. is quite small as well. Over 64% of the multifamily residences surveyed in the 2011 AHS were in buildings of 12 or fewer residences. Confounding simple explanations of the homeownership-structure correlation, condo ownership steeply increases in small buildings (with 12 or fewer residences – see the right panel of Figure 1). Interestingly, the rate of condominium ownership for duplexes in the 2009 AHS data is just over 9%, whereas more than 25% of buildings with more than 50 units are organized as condominiums. This rise in condominium ownership is not sensitive to the location of housing within cities or across

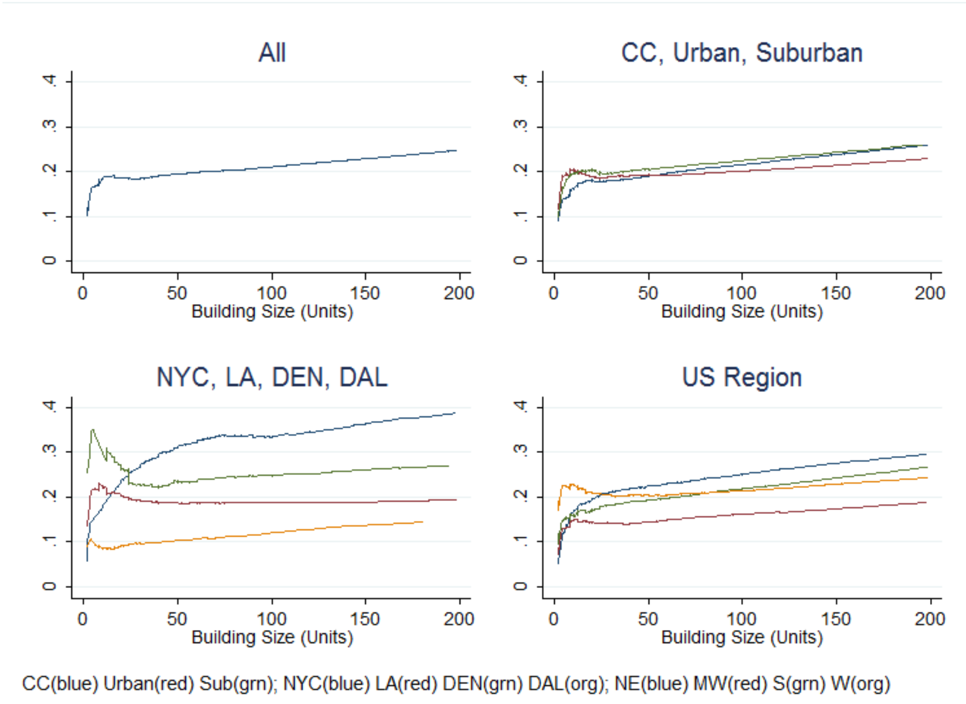

Figure 2: Condo Ownership, Subsamples (AHS 2011)

locations. Figure 2 reproduces Figure 1 for stratifications of the data by different geographic characteristics. We first categorize the data according to where the housing unit is located: within a central city, greater urban area or suburban area of an metropolitan region. We also display these graphs for the cities of New York, Los Angeles, Denver and Dallas. Finally, we graph the rate of condo ownership of housing units within multifamily buildings by region of the U.S. – Northeast, Midwest, South and West. The fact that condominium ownership increases with building size is an important feature of the U.S. housing landscape.

To examine the structure-tenure correlation more carefully than has been done previously, we ask a fundamental question: what is the relationship between building size and its ownership and governance? This is somewhat different than the traditional examination of homeownership (e.g. Coulson and Dalton, 2010; Hilber, 2004; Carrillo and Yezer, 2009) which asks: what characteristics of *households* make them more likely to become owner-occupiers? To introduce this conceptual shift, it is first useful to pin down our vocabulary. Rather obviously, by single-family housing we mean a stand-alone building intended as a residence for a single household. By multifamily, we refer to any arrangement of multiple residences within a building or structure. Multifamily arrangements may also encompass multiple buildings, including multiple single-family houses.

Because one of our main goals is to explain homeownership rates as a function of building size, it is important to recognize that homeownership in multifamily buildings requires a particular brand of governance like condominium or cooperative ownership, which provides investors with a proprietary claim to use and occupy a residence. In contrast, a limited partner in a partnership that owns a multifamily building has no *a priori* right to occupy a residence within the building. We would not consider the investor in this case to be a homeowner. (Nor for that matter would the Census or the IRS.) We do not directly model homeownership. Nonetheless, the ultimate connection between ownership and tenure should be evident. If joint ownership of multifamily buildings is disadvantaged relative to sole ownership for certain types of buildings, we would expect to observe greater rentership in multifamily buildings, all else equal. Thus, our analysis represents a necessary step in establishing the structure-tenure connection.

Literature on housing structures is sparse. Glaeser (2011*b*,*a*) and Glaeser and Shapiro (2003) somewhat informally argue that maintenance and upkeep are building-specific (as opposed to unit-specific) issues and so the problems of decentralized ownership of buildings makes condominiums and cooperatives difficult to manage. These difficulties make it more efficient to put those decisions in the hands of a single owner—a landlord—and make multi-family buildings rentals. However, this casual theorizing fails to explain the observed rise in the rate of condominium ownership among larger multifamily buildings. Linneman (1985) discusses a trade-off between the costs of monitoring a landlord on one hand, and the value of having a landlord solve the free-riding problem, on the other. These trade-offs, in addition to economies of scale for landlords, are assumed to influence the relative productivity of landlord versus owner management in ways that are unmodeled in his investigation of tenure. Williams (1993) proposes a model of multifamily structure ownership which trades off the rental externality described in Henderson and Ioannides (1983) against economies of scale provided by landlords. This model again fails to explain the non-monotonic relationship between homeownership and building size, a fact that the author acknowledges. Ambrose and Goetzmann (1998), Turner (2003) and Hilber (2004) in varying ways find that the locational characteristics of buildings matter; where and when ownership is risky, homeownership by individual households is less likely. None of these papers comes to grips with the basic structure-tenure correlation.

Unlike Linneman (1985) and Williams (1993), we argue that economies of scale are not confined to rental buildings based on suggestive evidence from condominiums. Using data from the state of Massachusetts in 2009, Figure 3 plots the propensity of condo associations to employ professional management services as a function of the number of units in the building.[3] The rapid rise of this probability suggests that smaller buildings do not have sufficient

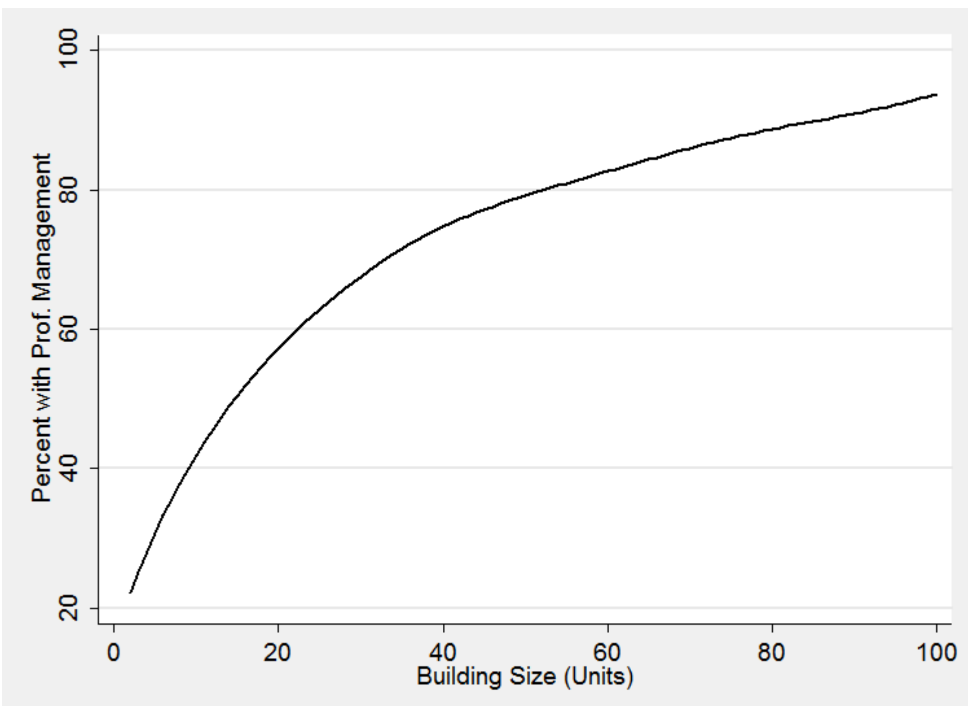

Figure 3: Professional Management in Condominiums

[3]This figure displays the smoothed rate at which buildings of different sizes are professionally managed as revealed by condo listings on MLS PIN in the state of Massachusetts during 2009. N = 53,303 listings with non-missing information about the number of units in the building and whether the building is professionally managed (either by on-site or off-site management).

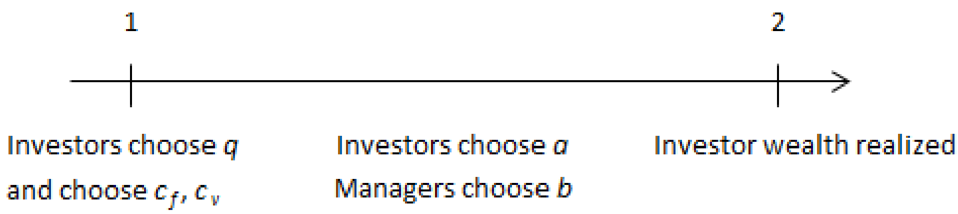

Figure 4: Model Timeline

scale to overcome the fixed costs of bringing in outside management services. Importantly, and even though exact services provided may differ, we expect that both joint owners and sole owners of buildings face similar scale economies in management costs. In the model that follows, we separately incorporate economies of scale for third party management, the agency costs from hiring managers, and the free-rider problem among co-investors.

3 Model

Our model is a basic one period, two date portfolio model in which an investor with initial wealth w_1 chooses between a risk free asset and a risky housing asset at time one in order to maximize expected wealth (realized at time two). In the base model, housing investment is in a single building, or a fraction thereof, so that we may study the implications of building governance when there are multiple investors. Figure 4 sets out the simple timeline for our model. In addition to choosing the level of housing investment, investors choose the terms of a contract with third-party management at time one. Investors also participate in building management. Management effort by both investors and third-party management is exerted after the investment is made, between time one and time two.

Taking the number of co-investors l in a building as given, the investor's indirect utility over wealth is given by

$$U(w_1, l) = \max_{q, c_f, c_v} E\left[v\left(\widetilde{w}\right)\right] \tag{1}$$

where $v(\cdot)$ is the investor's utility function, $\widetilde{w}$ is uncertain time two wealth, and E is the expectations operator. Investors are price-takers who choose an amount of risky housing investment q, and must also choose a fixed and variable component of the manager's contract, c_f and c_v, respectively. The investor's optimization problem is subject to a third-party manager participation constraint, which is described below.

Investors place the balance of their initial wealth endowment w_1 in risk free savings at the rate r. P is the price of a unit of housing services at time one and R is the periodic rent from a unit of housing services that is paid by a tenant at time one for the ensuing period. The net cost of investment in housing is therefore $P - R$. While we initially abstract from consideration of whether the housing investment is owner-occupied, R can be interpreted as imputed rent in the case of owner-occupation. Housing investment and building size are both measured in units of housing services, which comprises both a quantity and quality measure. Ownership shares of multiple investors within a single residential building are equal (although ownership shares can vary between buildings). We characterize building size as $n = ql$ where $l \geq 1$ represents the number of building investors.[4]

We assume that management and maintenance of a building is required in order to produce a flow of housing services to occupants (who are either the condo investors themselves or their tenants). Because we are interested in building-level issues created by different forms of building ownership, we set aside consideration of how a particular residence within a building is maintained. In their model of housing tenure, Henderson and Ioannides (1983) focus on maintenance externalities resulting from the utilization of a particular residence. In contrast, we focus on the administration of building services, and the maintenance of building systems and common areas, like lobbies, hallways and elevators.

Management effort may come from both building investors and managers. Investors choose the terms of a contract with third party management (c_f, c_v) for the building at time one. After time one (but before time two), each investor j in a building non-cooperatively

[4] In the optimization problem that follows, allowing building size to change with changes in investment ensures that $q \leq n$.

chooses effort a_j that is contributed to building management. Managers also choose effort b for a particular building between time one and two. The two sources of effort are perfect substitutes and impact property value independently. Management effort exerted in one period is assumed to influence the flow of housing services in the next period. The benefits of effort in period one for housing services in a future period are capitalized into housing value at time two.

Building value at time two *per unit of housing services* is $P+2\phi_a\sqrt{\sum_i a_i}+\left(2\phi_b\sqrt{b}+\widetilde{\varepsilon}\right)+\widetilde{u}$, where ϕ_a and ϕ_b are the marginal product of investor and manager effort, and $\widetilde{\varepsilon}$ and $\widetilde{u}$ are normally distributed noise with mean 0 and variance σ_{ε}^2 and σ_u^2, respectively. Each investor's property value benefits from the collective effort of all building investors and from manager effort. The cost of a unit of investor effort is linear in both effort and building size, while building value is a concave function of effort.[5]

Manager effort is private information that is not observed by building investors. Rather, investors observe its impact on property value attributed to management, which incorporates the output of management effort along with a random component equal to $2\phi_b\sqrt{b}+\widetilde{\varepsilon}$. Managers incur total cost of effort equal to $b\left(d+\alpha n^2\right)$ which is linear in effort and u-shaped in building size. Therefore, there is an economy of scale in building size with respect to manager costs. Managers have constant absolute risk aversion (CARA) risk preferences represented by a negative exponential utility function with additively separable cost of effort, $V\left(g,n\right)=-e^{-\rho\left(g-b\left(bd+\alpha n^2\right)\right)}$, where ρ is the manager's coefficient of absolute risk aversion and g is her compensation. Investors and a third party manager write a linear contract of the form $g=n\left(c_f+c_v\left(2\phi_b\sqrt{b}+\widetilde{\varepsilon}\right)\right)$. Below, we derive the manager's certainty equivalent value of wealth $\widehat{z}$ and denote the manager's opportunity cost per unit of housing services as $\overline{z}$.

[5] We follow Edmans and Manso (2011) in choosing this functional form. In particular, this set-up ensures a constant technology between sole investors and joint investors in our model.

Second period wealth for investor j is

$$\widetilde{w} = (w_1 - q(P - R))(1 + r) + q\left(P + 2\phi_a\sqrt{\textstyle\sum_i a_i} + (1 - c_v)\left(2\phi_b\sqrt{b} + \widetilde{\varepsilon}\right) + \widetilde{u}\right) - qc_f - a_j en \tag{2}$$

where the summation with respect to a_i is over l co-investors in a particular building. Notice that the variable payment to a manager reduces investor realization of value from third-party management by the fraction c_v. Investors bear a personal cost of management effort equal to $a_j en$ but only benefit from their own effort to the extent of their ownership share. Therefore, investors under-invest in the management effort supplied to the building when $l > 1$.

In the next sections we develop the analytic results of the model, and then numerically solve the model to provide additional insights. We assume that there is an open city with an elastic supply of residential buildings. For an investor with initial wealth w_1, the expected utility from sole ownership of a single family house is $\overline{U}(w_1, 1)$. Requiring the investor to have utility of at least $\overline{U}(w_1, 1)$, we derive investor bids for partial ownership $(q < n)$ of increasing larger buildings. To compare bid prices under joint ownership arrangements to bids from investors who intend to be sole owners, we modify the base model to incorporate sole ownership of multiple buildings. This formulation allows us to derive the bid prices for buildings of different size by letting a sole investor hold a portfolio of buildings in order to satisfy investment demand. The portfolio sequence starts with multiple single family houses, and then the number of buildings owned declines as building size increases, until the portfolio consists of just one building. Using the portfolios, we construct bid prices by holding the sole investor's utility equal to that obtained in the single family portfolio. Under the assumption that sellers sell to the highest bidder, bidding by investors of different wealth, and under alternate ownership structures, determines market prices for multifamily buildings of different size as well as equilibrium ownership arrangements for these buildings.

3.1 Management Effort

In order to find investor bid prices, we first need to derive the optimal effort of investors, conditional on investment level and building size. We also derive manager effort as a function of expected compensation and building size.

3.1.1 Investor Effort

Investor j takes the effort of all other investors within the same building as given and chooses effort to maximize (2). Due to the nature of joint production with other investors in the same building, investor j's choice identifies the optimal level of effort for the entire building:

$$\frac{\phi_a}{el} = \sqrt{\sum_i a_i}. \tag{3}$$

If we assume a symmetric equilibrium (although asymmetric equilibria are feasible), the optimal contribution of the individual investor is:

$$\frac{\phi_a^2}{e^2 l^3} = a_j. \tag{4}$$

Because investors bear the full cost of personal effort, but only obtain benefits for $q < n$ units of housing services, they under-supply effort when $l > 1$. Notice that if an investor owns the whole building, the total effort supplied is $\frac{\phi_a^2}{e^2}$ as compared to total effort of $\frac{\phi_a^2}{e^2 l^2}$ among l co-investors. The difference between these levels widens as the number of building co-investors increases.

3.1.2 Manager Effort

The manager's expected utility is $E\left[-\exp\left(-\rho\left(e_f n + c_v n\left(2\phi_b\sqrt{b} + \widetilde{\varepsilon}\right) - b\left(d + \alpha n^2\right)\right)\right)\right]$. Using the properties of the negative exponential utility function, we re-write the expected utility as

$$V(g,n) = -\exp\left(-\rho\left(c_f n + 2c_v n\phi_b\sqrt{b} - b\left(d+\alpha n^2\right) - \frac{\rho}{2}c_v^2 n^2\sigma^2\right)\right). \tag{5}$$

The manager's optimal choice of effort is given by

$$\frac{c_v^2 q^2 l^2 \phi_b^2}{\left(d+\alpha q^2 l^2\right)^2} = b. \tag{6}$$

Substituting, the utility of the manager can be expressed as $V(g,n) = -\exp(-\rho\widehat{z})$, where

$$\widehat{z} = c_f q l + q^2 l^2 \frac{\left(c_v\phi_b - \alpha q l\right)^2}{4d} - \frac{\rho}{2}c_v^2 q^2 l^2 \phi_b^2 \sigma^2 \tag{7}$$

is the certainty equivalent value of manager wealth.

3.2 Investment Choice

Substituting the definitions of a and b into (2), the investor's objective function in terms of three choice variables is

$$\max_{q,c_v,c_f} E\left[v\left(\widetilde{w}\right)\right] \tag{8}$$

subject to

$$\widehat{z} = \overline{z}n. \tag{9}$$

We obtain four first order conditions from which we are able to simplify the system to two equations in q and c_v. These are:

$$R(1+r) - rP - \overline{z} + \phi_a^2\frac{2l-1}{el^2} + \frac{2ql\phi_b^2}{\left(d+l^2q^2\alpha\right)^2}\left(d+\alpha q^2 l^2\left(1-c_v\right)^2\right) - \rho c_v q l\sigma^2 + \frac{E\left[v'\widetilde{u}\right]}{E\left[v'\right]} = 0, \tag{10}$$

and

$$(1-c_v)\frac{2ql\phi_b^2}{d+\alpha n^2}-\rho c_v q l\sigma^2-\frac{E[v'\tilde{\varepsilon}]}{E[v']}=0, \tag{11}$$

respectively.

4 Investor Prices

To identify the slope of an investor's bid function with respect to the number of building investors, we invoke the envelope theorem and differentiate (1) with respect to l, holding all choice variables at their optimum. In order that investor utility remain unchanged, bid prices (per unit of housing services) are assumed to vary with l. This exercise yields:

$$\frac{\partial P(l)}{\partial l}=\frac{q}{r}\left(c_v\phi_b^2(2-c_v)\frac{d-\alpha q^2l^2}{(d+\alpha q^2l^2)^2}-\frac{2}{eql^3}\phi_a^2(l-1)-\frac{\rho}{2}c_v^2\sigma^2\right) \tag{12}$$

It is difficult to sign this equation. The first term in parentheses on the right hand side of (12) represents a potential benefit of additional co-investors. Notice that by construction, $d-\alpha n^2\geq 0$ at building sizes no larger than the cost-minimizing size. For building sizes less than the cost-minimizing size, additional investors increase building size, thereby lowering the average costs of third party management. The next term in parentheses, which is equal to zero at $l=1$, represents a price discount resulting from the free-rider problem as the number of investors increases. The last term on the on the right hand side of (12) indicates that managers require greater compensation when building size increases (because they bear more total risk, holding c_v constant). Clearly, for large enough buildings the impact of an additional investor on bid prices is negative. Below the cost-minimizing size, utility, and therefore bid prices, may be increasing or decreasing.

In the Appendix, we extend the base model to the case of sole ownership. To uncover bid prices by wealthy investors for small buildings, we must allow these investors to hold a portfolio of small buildings. While the cost of effort is linear in the size of the investor's portfolio, ownership of multiple buildings requires a duplication of effort for each building. Therefore, the costs of effort increases in the number of buildings owned. From this exercise we are able to derive the slope of the sole investor's bid function as building size increases,

$$\frac{\partial P}{\partial n} = \frac{1}{r}\left(c_v \phi_b^2 \left(2 - c_v\right) \frac{d - \alpha n^2}{\left(d + \alpha n^2\right)^2} - \frac{\rho}{2} c_v^2 \sigma^2\right). \tag{13}$$

Compared to (12), we first observe a scaling factor, q, that adjusts for the fact that building size is $n = ql$ in the joint ownership model. This scaling does not affect our interpretation of the two slopes. The remaining difference, once building size is accounted for, is the second term in parentheses in (12). This term quantifies the bid price discount for the free-rider problem. In the limit, however, a goes to zero as the number of co-investors increases. The building size at which a goes to zero is a function of initial investor wealth and the optimal investment level.

One of the main insights of this model is that the impact of the free-rider problem associated with joint ownership arrangements, like condominiums, dissipates with building size. Within small multifamily buildings, the free-rider problem dominates investor bids for joint ownership arrangements, making sole ownership of buildings more valuable than joint ownership. Because the relative gain or loss in utility from changing building size eventually equalizes across investors, the characteristics of the marginal investor depend on supply and demand conditions in the market for large multifamily buildings.

5 Numerical Solutions

Analytically, we have derived a non-linear bid price curve for housing investors as the number of building investors changes. In this section, we parameterize the model in order to arrive at numeric solutions for the model's choice variables. For investor utility, we adopt a constant relative risk aversion form, $E\left[v\left(\widetilde{w}\right)\right] = E\left[\frac{\widetilde{w}^{1-\gamma}}{1-\gamma}\right]$. To characterize equilibrium prices, we solve for constant utility bid prices for multifamily buildings for a variety of initial wealth levels for sole ownership of buildings as well as through joint ownership arrangements. We assume that average wealth investors choose between owning a single family home and a condominium in a multifamily building. We use the average per square foot single family home price in the 2011 American Housing Survey ($94) to anchor solutions for this investor. For wealthier investors, we utilize the extension of the base model found in the Appendix. The modified model identifies the utility of wealthy investors from owning a portfolio of single family houses, and then derive bid prices that hold utility constant from owning fewer, but larger buildings.

5.1 Initial Assumptions

We begin with the following assumptions. First, we interpret units of housing services as square feet of space. Productivity parameters translating investor and manager effort into housing value are set to 0.5. The (real) risk free rate is equal to 2%. Manager opportunity cost ($\overline{z}$) is set to $0.15 per square foot of building size. The owner effort cost parameter is set to $e = .3$ so that the optimal choice of a at $l = 1$ for the average wealth investor produces a gross value of investor effort equal to approximately 3% of single family house value. Manager cost parameters α and d are set so that the cost minimizing building size is 240,000 square feet (approximately, a 200 to 250 unit multifamily building). This size is a current industry norm for efficient operations of a multifamily rental building. Initially, these parameters are $\alpha = 1.32e-8$ and $d = 750$. The total cost of third party management approximates evidence from internet searches about management fees per residence.

Investors' utility shape parameter is $\gamma = 3$. Manager coefficient of absolute risk aversion is $\rho = .5$. Variances of the two noise parameters are initially set relative to the scale of manager output at the cost-minimizing building size and single family house prices, respectively. They are $\sigma_\varepsilon^2 = 0.16$ and $\sigma_u^2 = 12$.

5.2 Results

Figure 5 depicts bid prices for an investor with $95,000 in initial wealth. The investor is assumed to be either a single family owner ($l = 1$) or a condo owner ($l > 1$) due to the size of their investment demand. Given our assumptions about parameters, this group (of about

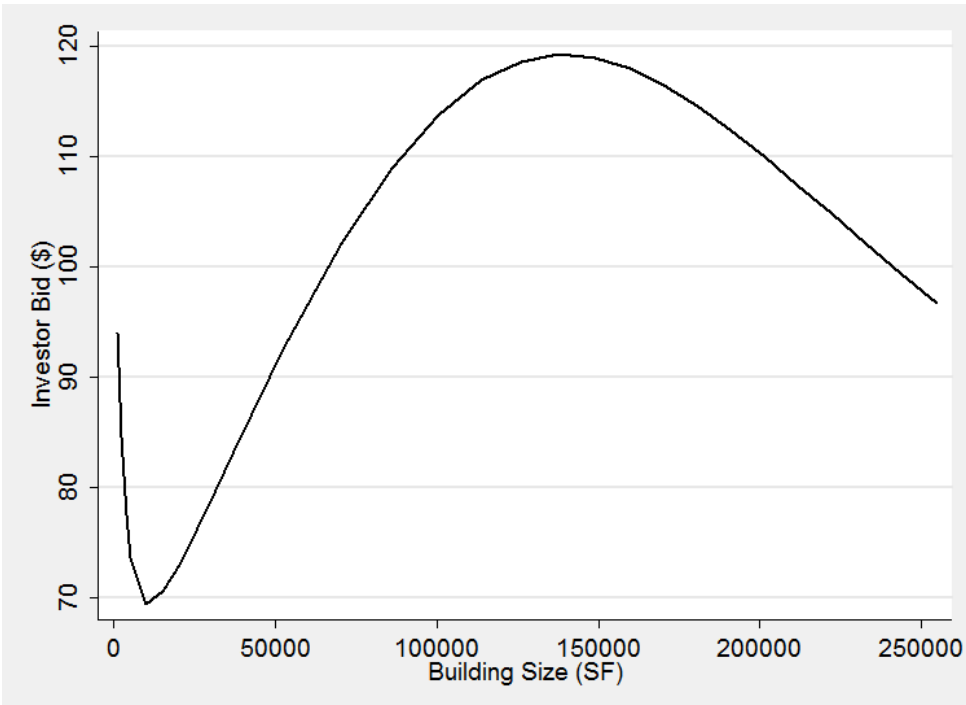

Figure 5: Investor Bids with Initial Wealth of $95,000

average wealth) demands a single family home of approximately 1,200 square feet given a price of $94 per square foot (an average from the 2011 AHS for single family housing).

Underlying these bid prices, investor effort declines dramatically with the addition of co-investors, due to the free rider problem (left panel of Figure 6). Manager effort, on the other hand, increases non-linearly in the right-hand panel of Figure 6. As the number of co-investors increases, investment size also increases and then falls in response to changes total management effort (Figure 7).

For households with initial wealth of more than $95,000, we estimate utility from owning more than one single family house where each house is 1,200 square feet in size with an assumed price of $94 per square foot. We then calculate the bid price for these same wealthy

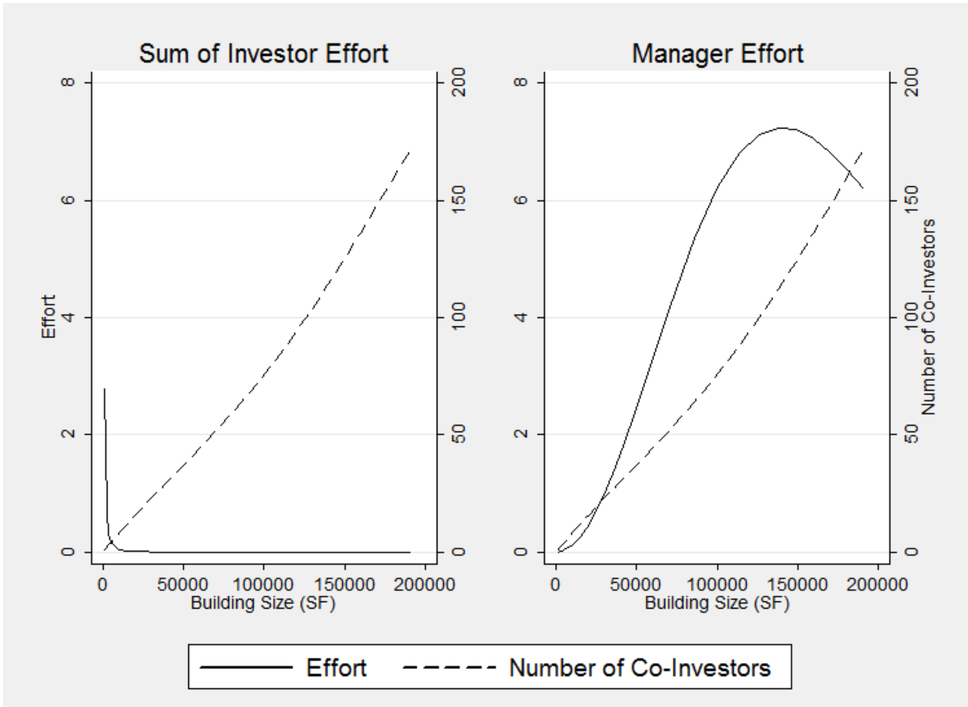

Figure 6: Investor and Manager Effort

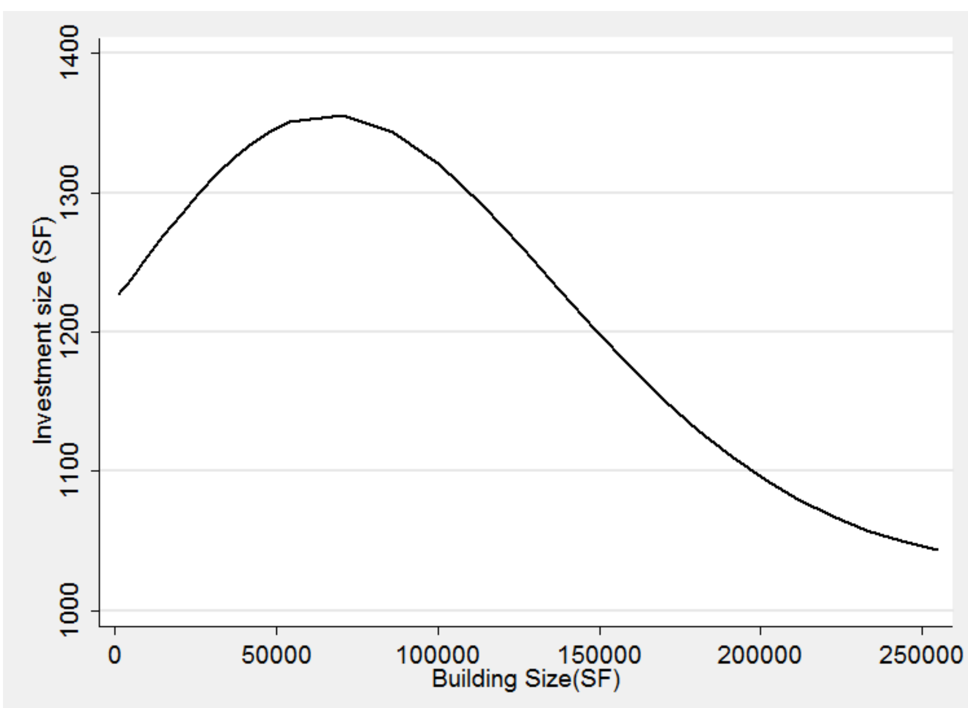

Figure 7: Investment Size for Investor with $95k in Initial Wealth

investors from owning fewer buildings of larger size. For example, assuming that the investor must own four buildings, we solve for the optimal size of these buildings, the terms of the management contract and the price per square foot that yields the same utility level as the single family portfolio. Eventually, we produce a bid price for sole ownership of just one building. The wealthy may co-invest in a larger building by partnering with other investors. In this latter case, ownership transitions to a joint ownership arrangement and the wealthy are subject to the same free-rider problems as less wealthy investors. The main difference is that the size of the building at which wealthier investors require partners is larger.

In Figure 8, we graph bid prices (per square foot) for initial investor wealth of $95,000 and $240,000. At the left of the graph, the investor with $95,000 initial wealth is the sole

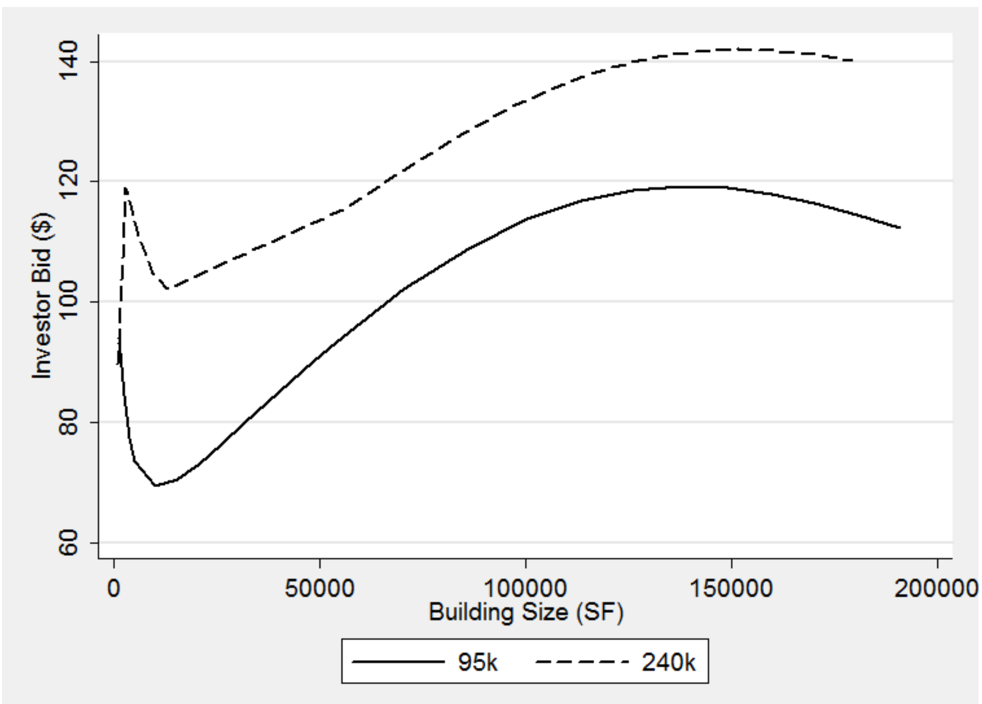

Figure 8: Investor Bids by Building Size

owner of a single family house of about 1,200 square feet. The investor becomes a joint owner in buildings above the size of 1,200 square feet as we move right across the graph. In comparison, the wealthier investor prefers being the sole owner of a building of almost 2,900 square feet as compared to owning two 1,200 square foot houses. To the right of this optimal

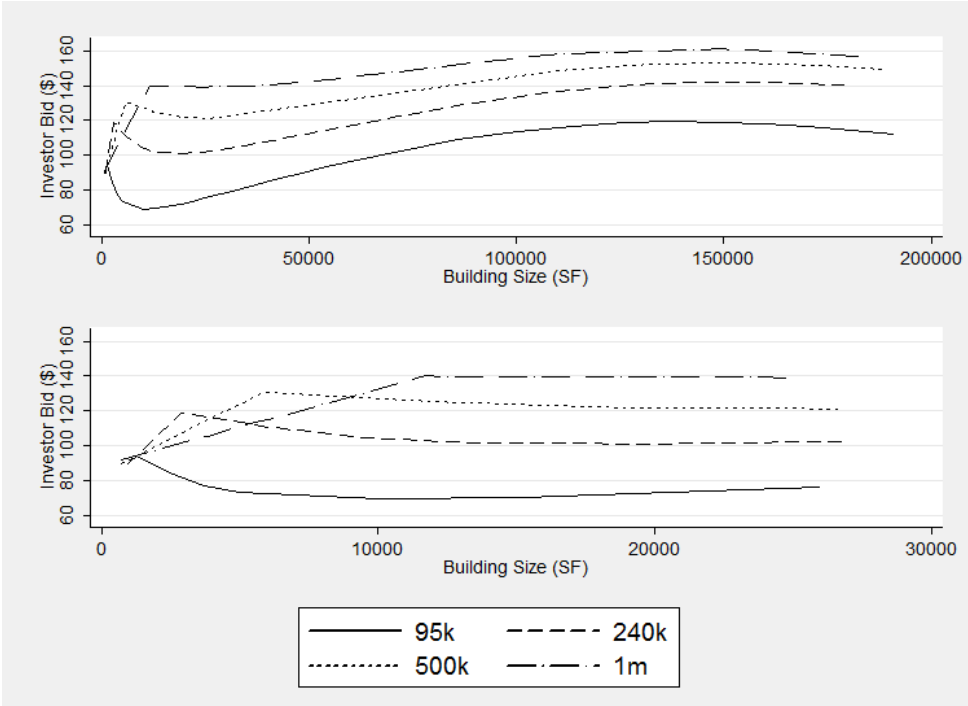

Figure 9: Investor Bids by Initial Wealth and Building Size

building size, the wealthier investor also becomes a joint investor in buildings of increasing size.

In Figure 9, we depict the bid prices for buildings of increasing size for investors with \$94,000, \$240,000, \$500,000, and \$1 million, respectively. Among investors with initial wealth above \$95,000, the less wealthy have the steeper, upward sloping bid curves along the portion of the curve where they bid for sole-ownership of buildings. This implies that in the left-hand portion of either graph in Figure 9, bidding results in a domination of joint ownership arrangements by sole-ownership, and sole-owners sort by wealth into buildings of increasing size. Because sole-owners outbid condo owners in particular, rental tenure should dominate owner-occupation for smaller multifamily buildings.

6 Empirical Models

The theoretical model in the previous section suggests that investor demand for properties in which they perforce share ownership is likely to be affected by the number of co-investors. For small buildings, low-wealth investors are likely to lower bids (per square foot) as the number of co-investors rises because of the increasing level of free-riding that will take place. However, as the scale of the building increases, the ability of the partners to increasingly elicit effort from an outside manager overcomes the free rider problem, and bids begin to rise. The simulations suggested that these bids would flatten out and even decline at large enough scale. The size of the bid is also, naturally enough, sensitive to investor wealth. The simulated bids suggested that for wealthier investors, the effect of the free rider problem is ameliorated by scale due the size of buildings in which the highest wealth investors become co-investors.

In order to do estimate bid functions, we need data on household wealth, household demographic variables, individual condominium prices, and structural attributes of the condominiums, particularly including the floorspace and the number of units in the condo building. All of these, save wealth, are available in the American Housing Survey, and this is our primary source of data. Other surveys calculate household wealth, but are short on the detailed data needed to estimate hedonic price functions for condo prices. Our resolution of this quandary is to use the Panel Survey of Income Dynamics (PSID) to estimate the determinants of wealth as a function of household characteristics, and use this regression function to estimate wealth for the households surveyed in the American Housing Survey. This is the first step in our empirical procedure.

The second step should have been to estimate the hedonic price function for condos – that is, a regression function that maps condo structural and locational attributes into condo prices. However, we are faced with two selectivity issues. The theoretical model implies that for a multifamily structure, the equilibrium ownership of the building is not random. The theoretical model stresses that the number of co-investors is influential in this decision.

Other quality dimensions play a role as well. The empirical literature on homeownership (Hanson, 2012) speaks most directly to this point. It stresses that tax incentives such as the home mortgage interest deduction lead homebuyers to choose higher quality homes, in turn suggesting that buildings of higher quality are more likely to be condos. Quality is only partially observable, therefore the unobserved factors that influence the choice to become condo are correlated with the unobserved attributes that create the price. In other words it is necessary, in the hedonic model, to control for the selective nature of the condo sample. Secondly, as we outline below, the AHS surveys housing units, but of necessity, the interview subjects are the residents of the housing units. For about half of the condominium records in the sample, about half are owner-occupiers, and half are renters. The value of the condo investment (along with condo fees, if any) is reported only in the case that the interviewee is the owner-occupier. But the decision to be an owner-occupier is influenced by the same unobservable factors (albeit with different weights) as was the decision for the building to be organized as a condo in the first place.

Therefore, the estimation of the hedonic price model for condos consists of three stages. The first stage chooses between joint ownership through condominium governance and sole ownership by a landlord who rents the building's units. The building developer sells to the type of buyer that generates the highest bid. We do not observe the bids, but we assume they are summarized by the linear index

$$I_1^* = X_1\beta_1 + e_1 \tag{14}$$

where the index for observation number is suppressed. I_1^* is the net profit from condo organization relative to apartment organization, X_1 is a vector of structural and locational characteristics, and e_1 is the shock to relative profits encountered by the building developer. In the usual way, we do not observe I_1^* but only the decision, so we define $I_1 = 1$ if the building is a condo, and $I_1 = 0$ if not. Then

$$P(I_1 = 1) = P(e_1 > -X_1\beta_1) \tag{15}$$

and on the assumption that e_1 is normally distributed,

$$P(I_1 = 1) = 1 - \Phi(-X_1\beta_1) = \Phi(X_1\beta_1) \tag{16}$$

where Φ is the normal cumulative distribution function. Estimation of stage 1 can proceed as a normal probit.

Stage 2 asks, given condo organization, whether a unit with specified characteristics will be owner-occupied or rented by unit's owner. Again, there are unobserved benefits and costs accruing to each decision, summarized in a linear index:

$$I_2^* = X_2\beta_2 + e_2 \tag{17}$$

where we define the observable decision as $I_2 = 1$ if the unit is rented and $I_2 = 0$ if not. However, the owner-occupation decision is of necessity conditioned on the decision that the building be organized as a condo. Thus there is a selectivity issue if the unobservables that inform the second decision are correlated with those of the first. This suggests the joint estimation of stage 1 and 2 by maximizing the log likelihood function

$$\log L = \sum_{I_1=0} \Phi(X_1\beta_1) + \sum_{I_1=1, I_2=0} \Phi_2(X_1\beta_1 - X_2\beta_2, e_{12}) + \sum_{I_1=1, I_2=1} \Phi_2(X_1\beta_1 X_2\beta_2, \rho_{12}) \tag{18}$$

where Φ_2 is the bivariate normal cumulative distribution function and ρ_{ij} is generically the correlation coefficient of e_i and e_j (Poirier, 1980).

Turning now to the property values equation, we propose a standard hedonic equation of the form

$$\log value = X_3\beta_3 + e_3 \tag{19}$$

we note that there is also a selection issue here, since we only observe values in the case where the building is condo, and the owner elects to owner-occupy. Using identical reasoning as above, we have

$$E(e_3|I_1 = 1, I_2 = 0) = \rho_{13}\xi_1 + \rho_{23}\xi_2 \tag{20}$$

with

$$\xi_1 = \frac{\phi(X_1\beta_1)(\Phi(-X_2\beta_2))}{\Phi_2(X_1\beta_1, -X_2\beta_2, \rho_{12})}, \tag{21}$$

and

$$\xi_2 = \frac{\phi(-X_2\beta_2)(\Phi(X_1\beta_1))}{\Phi_2(X_1\beta_1, -X_2\beta_2, \rho_{12})} \tag{22}$$

as in Lahiri and Song (2005) and Hotchkiss and Pitts (2005) etc. The variables ξ_1, and ξ_2 can be consistently estimated upon obtaining the parameter estimates from (18). Consistent estimates of value in the face of these two selection problems can be obtained through the least squares regression

$$\log value = X_3\beta_3 + \rho_{13}\xi_1 + \rho_{23}\xi_2 + e_3^* \tag{23}$$

on the owner-occupied condos only.

Having obtained consistent estimates of the value function for condominiums, our next step is to derive the bid functions that underlie it. As noted in the hedonic literature (e.g. Rosen (1974) and Epple (1987)) the hedonic function is the upper envelope of bids from different segments of the heterogenous pool of demanders. If the heterogeneity is due to resource constraints – characterized here as wealth, but in Rosen (1974) as income – then normality and concavity of the utility function ensures a single crossing to any pair of bids, and a matching between quality – here characterized as square footage – and wealth. The bid function, the data analogue to the curves calculated in Figures 5 and 6 above, is a function that maps demographic and resource characteristics of the (successful) bidder, along with the structural characteristics, into the marginal price of the characteristic. That is, for some characteristic X^j we write the bid function as

$$\frac{\partial V}{\partial X^j} = Z\omega + X_3\tau + e_4 \tag{24}$$

where the dependent variable is the derivative of the hedonic function with respect to the characteristic – the marginal bid for a unit of that characteristic. Z is a vector of personal characteristics. Rosen (1974) suggested that the estimation of what is in effect a Hicksian demand function is subject to the same kind of endogeneity bias that "ordinary" supply and demand estimation suffers from. In housing market applications, it is reasonable, however to assume (and we do so here) that housing supply is exogenous.

Nevertheless, Bartik (1987) and Epple (1987) note that another kind of simultaneity is present. The hedonic function is by design nonlinear in the characteristics. It must be in order for there to be variation in the dependent variable of (24). The marginal price and quantity of the attribute are simultaneously chosen. If shifts in the error term are caused by unobserved taste differences across consumers, then those shifts which (conditional on Z) cause the choice of bid price, are correlated with the characteristic quantity on the right hand side of the equation. In short, because price and quantity are chosen jointly, quantity is endogenous. To consistently estimate the bid parameters, instruments are needed. Bartik (1987) notes that the instruments must be correlated with X, but uncorrelated with tastes, and variables that shift the budget constraint are therefore valid instruments. The particular implementation that is often used (Bartik, 1987; Coulson and Bond, 1990) is to allow the hedonic function to vary across (geographic) markets. The assumption is that hedonic variation is due to supply constraints and not differences in unobservable taste. Then market-specific variables – market binaries, for instance, but also these binaries interacted with Z – can serve as instruments. As Bartik (1987) notes, nonhousing expenditure (or wealth) is an appropriate member of Z, which implies that total resources (i.e. total wealth) would be an appropriate instrument, when interacted with regional binary variables. We follow this procedure below, and estimate proxies for both total and nonhousing wealth from the PSID data.

7 Empirical Results

In this section we present the three-stage estimation of the hedonic price function for condominium units. The first stage, recall, estimates the probability that the building in which the housing unit is located is jointly owned using condominium governance or solely owned. The second step estimates the probability that the specified unit, conditional on it being a condo, is owner-occupied (such that the value is observed). These two steps are estimated jointly in a maximum likelihood framework. The third step is estimating the hedonic function itself, conditional on the two selection criteria being fulfilled. While fully efficient estimates are realized only if the third step is estimated jointly with the first two, consistent estimates are possible in a two stage procedure, where the second stage merely adds the appropriate Mills ratios to the hedonic model.

Our data source is the 2011 American Housing Survey national sample. The AHS is a biennial survey of housing units and occupants conducting by the US Department of Housing and Urban Affairs. Table 1 outlines some initial facts about the survey. There are 186,448 housing units surveyed. We eliminate those for which some basic information is not available, particularly tenure status, structural status (single or multifamily) or key structural characteristics. We also eliminate mobile homes and public housing. About 75,000 units' records were set aside, primarily because the building was not a permanent structure (i.e. mobile home), not a "typical" housing unit (e.g. group quarters), the unit was vacant, or a household member was not available to interview. Of the remaining 110,132 observations, the table indicates that just over 27,000 (25%) are in multifamily buildings.

Housing Units	
Total Records	186,448
Mobile Homes, etc.	-5,586
Public housing	-4,217
Missing data	-66,513
Sample Housing Units	110,132

	Single family		Multifamily	
Total Housing Units	83,077		27,055	
Owner-Occupied	68,890	(83%)	3,324	(12%)
Units in Solely-Owned Bldg			22,154	
Units in Condo Bldg			4,901	
Owner-Occupied			2,427	(50%)

Table 1: 2011 AHS Sample

Of these 4,900 (18%) are condominium units, of which half are occupied by their owners, and the others rented to other parties. Presumably, the solely-owned multifamily buildings are renter-occupied. The difference between owner-occupied units in multifamily buildings and owner-occupied condo units represents almost 900 owner–occupier–landlords, who solely-own a multifamily building, occupy one unit and rent the remainder. Our interpretation is based on the fact that 65% of these owner-occupied units in non-condo buildings are located in duplexes, and 96% are found in buildings with 12 or fewer units. It seems plausible that owners would also be managers in such small buildings. Overall, the descriptive statistics exhibited here are similar to what has been found in the US Census.

7.1 Wealth

Our first step is to estimate wealth functions using the Panel Survey of Income Dynamics, that can be used to predict wealth for respondents to the AHS. We use the 2011 wave of the PSID; data on persons defined as household heads were downloaded, although total household wealth is the variable of interest. The response coding in the PSID is different from that used in the AHS, which required adjustments to the PSID responses. For instance, in coding education levels, the PSID uses responses 1 through 16 to code actual grad levels completed, while the AHS uses response 31 to code completion of first to fourth grades. For added predictive power, polynomials of age and income are also included in the specification. To additionally aid in the predictability of the sample, we eliminated observations with very large (over \$4,000,000) or very negative (less than -1,000,000) wealth. It is literally impossible to predict wealth that great or that far underwater using demographic variables. There were 47 observations all together in those two categories.

The R-squared of the wealth regression is 31% which, while not large, is respectable for this sort of exercise. The coefficients are sensible; age maps into wealth in a highly nonlinear manner, as would be expected, however wealth seems to be a linear function of income. The schooling and ethnic coefficients coincide with prior expectations, however it is of interest to note that those with between 1 and 10 years of schooling seem to do worse than those with no schooling at all (the omitted category). Figure 10 displays the density of both wealth (solid bars) and predicted wealth (clear). The regression model accurately reproduces the skewed nature of the wealth distribution displayed in the PSID, however it does under-predicted the fraction of participants with wealth near the mode of the actual distribution and under-predicts the number with slightly higher amounts of assets.

7.2 Data and Findings

Table 2 lists, for units in multifamily structures, means and standard deviations, stratified by ownership structure. In the first panel, the data summary is presented for units in solely owned buildings, in the second rentals in condo buildings and in the third, owner-occupied condos. The most obvious takeaway is that there are quality differences, sometimes substantial ones, across these various ownership arrangements. In particular, both types of condo units are larger, and embody more structural amenities, than rental units. Note also that condo units are, on average, newer, although this is partly due to the fact that in most states condominium and cooperative ownership arrangements were not permitted prior

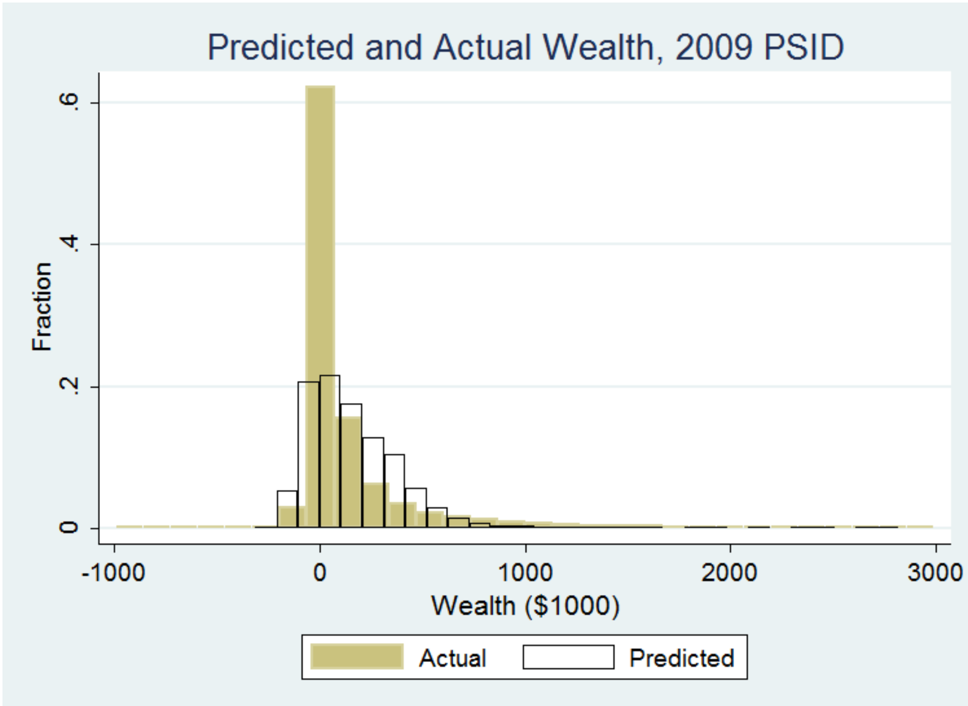

Figure 10: Actual and Predicted Wealth

	Rental Bldg Unit (N = 22,154)		Condo Rental (N = 2467)		Condo Own-Occ. (N = 2434)	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
Number Units	29	64	39	84	43	89
Unit SF	917	841	1000	765	1349	1176
Baths	1.19	0.43	1.35	0.51	1.59	0.58
Half Baths	0.11	0.35	0.19	0.45	0.29	0.51
Fireplace	0.12	0.32	0.21	0.41	0.35	0.48
Porch	0.67	0.47	0.76	0.43	0.84	0.37
A/C	0.52	0.5	0.65	0.48	0.67	0.47
Garage	0.33	0.48	0.47	0.55	0.68	0.47
Yr. Built	1966	25	1974	21	1975	23
Central City	0.46	0.5	0.42	0.49	0.44	0.5
MSA, not CC	0.49	0.5	0.52	0.5	0.52	0.5

Table 2: Sample Statistics for Units in Multifamily Buildings, 2011 AHS

to the early 1960s. There are also notable differences between owner-occupied condos and rental condos, the latter being of lower quality than the former. These quality differences are expected, if only because of the greater tax advantages that higher quality units bring to owner-occupiers, These differences are important, since the observable quality differences may also herald unobservable quality differences which must be accounted for later.

Turning now to the estimation of the three stage model, note that we first cull from the sample observations with unrealistic rents (<$50 per month) or values (<$1000). In both cases these are either properties with extremely low quality, not arms-length transactions, or transcription errors, so that it is appropriate to delete them. In estimating these models it is useful to have "identifying variables" – i.e. variables that influence the choice of condo ownership that do not influence the decision to owner-occupy, and variables that influence the choice to owner-occupy but do not influence value (Lahiri and Song, 2000). This can most easily be seen in the third stage estimation, where the bivariate Mills ratios are entered into the linear regression model of value. If the regressor set in each stage is similar, there can be collinearity issues between the Mills ratios and the determinants of value. There do not appear to be any valid exclusion restrictions that apply, however. Any characteristic that influences the choice to of the owner to occupy the unit (i.e. is of high quality) is likely to have influenced the decision to make the building condo in the first place, and is likely to have an influence on the asset value of the unit. We therefore take X_2, X_3, and X_4 to be identical, and rely on the nonlinear functional form to separately identify the coefficients of the characteristics and the Mills ratios.

The results of the first two (jointly estimated) stages are in Table 3. There are three notable results. The first result is that both condo and owner-occupied probability are strongly associated with unit quality. Almost every observable quality dimension has a coefficient that is both economically and statistically significant, although there are some deviations from this general rule. This confirms an earlier point, that the motives of housing consumers are influenced by the tax incentives to owner-occupation. Higher quality units are

Dependent Variable:	Condo	Hown
No. Units	0.0043***	0.0005
	[0.000]	[0.001]
Units Sq	-1.3E-05***	-2.04E-06
	[0.000]	[0.000]
Units Cu	9.90E-09***	1.26E-09
	[0.000]	[0.000]
Floor		0.0160**
		[0.007]
Baths	0.4059***	0.1702*
	[0.021]	[0.100]
Half Baths	0.3836***	0.0731
	[0.023]	[0.078]
Fireplace	0.3827***	0.0725
	[0.026]	[0.090]
Porch	0.1246***	0.1530**
	[0.024]	[0.065]
A/C	0.1355***	-0.016
	[0.023]	[0.050]
Garage	0.4244***	0.3026**
	[0.021]	[0.125]
Unit SF	0.0001***	0.0001***
	[0.000]	[0.000]
Yr Built	0.0027***	-0.0067***
	[0.000]	[0.001]
Constant	-6.7298***	13.2685***
	[0.952]	[1.857]
Observations	27,055	27,055

Coefficients reported; Standard errors in brackets
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 3: Estimates of the Probability of Condo Ownership, Owner-Occupation

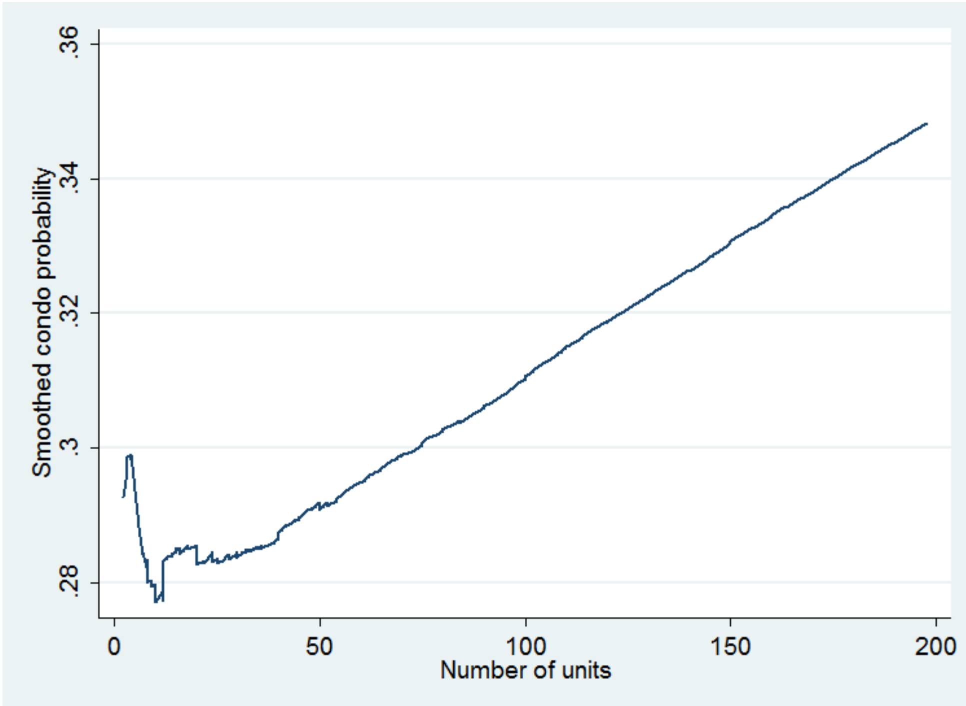

Figure 11: Predicted Probability of Condo Ownership

more likely to be condos, and owner-occupied condos. The second result is, corresponding to Figure 1, that the probability of a building being a condo is strongly, and nonlinearly, associated with the number of units. Figure 11 shows a higher initial rate of condo ownership, followed by a significant drop in and then recovery of the smoothed, predicted probability of condo ownership according to building size. It must be noted, however, that the polynomial cannot imply a flattening of the probability profile over all unit counts, and that beyond 20 units or so, the implied probability rises with the unit count more strongly than that implied by Figure 1. Moving to the probability of owner-occupation, it is of substantial interest to observe that the probability of being owner-occupied is not a function of the number of units. This is highly suggestive: our theoretical model suggests that there are strong investment motives that govern the organization of building ownership. However, we have also noted in this section that the probability of condo ownership might also be influenced by tax incentives of housing consumers. If the unit count were a consumption motive for building organization (i.e. was indicative of higher quality) we would expect it to be a significant determinant of owner occupation, but it is not. Therefore the importance of unit count is strictly as it pertains to investment motives, as suggested by the theory. The third result is that, the test of correlation between the residuals of these two stages indicates that the hypothesis of no correlation can be rejected; accounting for selectivity is important, presumably because omitted quality variables influence both decisions.

We turn now to the estimation of the value functions. As noted previously, we stratify the sample by geographic markets – in this case, Census regions. Note that we still include binaries for metropolitan areas, so that intra-regional variation in the functions is still allowed, but we restrict this variation to intercept terms. The marginal valuations of structural attributes is homogenous within regions, but heterogeneous across. The functional form uses value as the dependent variable (and not, say, its log) and the nonlinearity required to create sufficient variation is created by allowing floorspace to be entered as a cubic polynomial. The number of units is also entered in the regression as a cubic. Table 4 presents these estimates in which the dependent variable is value in thousands of dollars and the coefficients for the cubic polynomials as well geographic variables are suppressed. The parameter estimates vary substantially across regions, especially for the quality binaries such as fireplace, air conditioning, and the like. They also exhibit considerable heterogeneity in the unit count polynomial, but very few of these parameters are significant at standard levels of type I error. The number of bathrooms and the floor of the unit are all large and significant. Importantly, the polynomial factors of square footage are jointly significant, but even the linear term is not estimated particularly precisely. Of equal interest is the fact that the two Mills ratio terms do not have statistically significant coefficients, indicating that the unobserved quality factors that determined condo and ownership probabilities do not seem to have a particularly definable impact on the price of the unit.

	Northeast	Midwest	South	West
Number Units	1.006	-0.022	-0.048	0.811
	[0.827]	[0.289]	[0.452]	[0.518]
Floor	24.811***	12.079***	12.981***	15.431***
	[6.098]	[2.656]	[3.018]	[4.266]
Baths	455.695***	58.269***	71.991***	87.899***
	[44.042]	[16.478]	[18.329]	[26.980]
Half Baths	305.869***	30.751**	31.047**	55.365**
	[51.615]	[15.124]	[13.078]	[24.061]
Fireplace	-24.698	-14.539	-11.614	107.540***
	[54.543]	[15.737]	[18.935]	[22.503]
Porch	-31.375	-6.053	-1.261	12.451
	[51.025]	[20.752]	[30.190]	[35.591]
A/C	-32.544	-16.92	-31.714	-96.282***
	[47.100]	[20.762]	[46.067]	[21.261]
Garage	-118.801**	-16.318	26.3	35.992
	[56.667]	[22.742]	[23.411]	[51.250]
Unit SF	0.249*	0.055	0.063*	0.193*
	[0.138]	[0.042]	[0.033]	[0.100]
Yr. Built	-4.034***	1.359***	-0.29	-5.624***
	[1.130]	[0.491]	[0.636]	[0.780]
$\xi 1$	206.742	-207.594**	-27.694	488.088***
	[270.595]	[99.848]	[91.006]	[143.581]
$\xi 2$	17.424**	2.173	1.477	6.06
	[8.619]	[1.881]	[1.144]	[5.337]
Constant	7,259***	-2,600***	490	10,392***
	[2,072]	[893]	[1,178]	[1,414]
Observations	386	421	508	1,112
Adjusted R-squared	0.486	0.383	0.257	0.306

Standard errors in brackets

*** p<0.01, ** p<0.05, * p<0.1

Table 4: Estimates of Hedonoic Price Functions

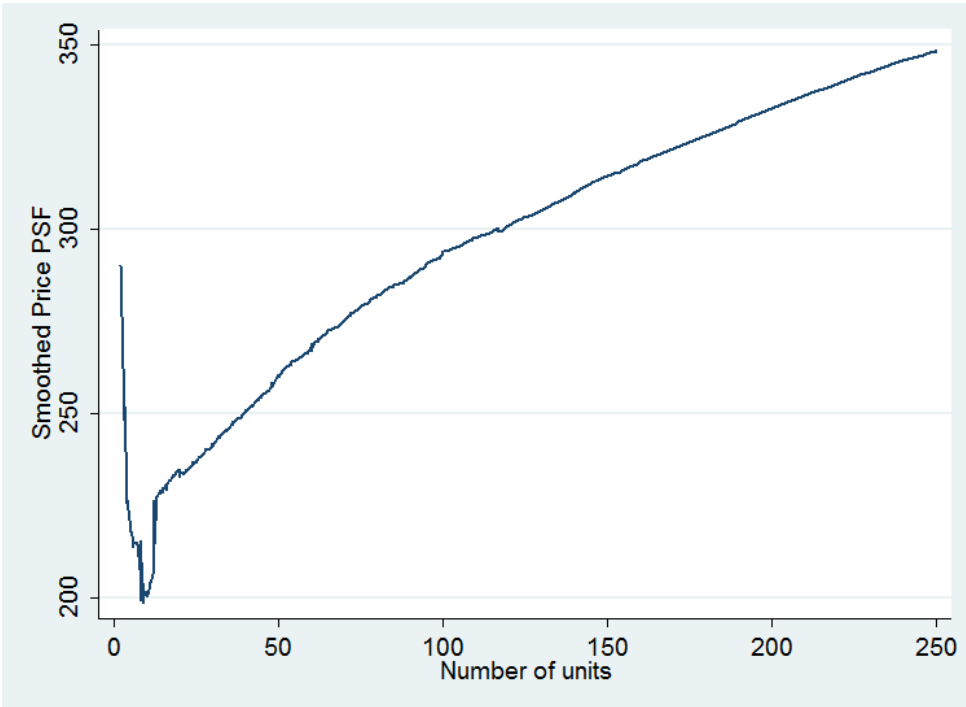

Figure 12: Smoothed Condo Price PSF and Building Size

7.3 Bid Functions

The final stage in this exercise is to estimate bid functions for various wealth levels and unit counts. As a first look at the data from this point of view, we first estimate a bivariate nonparametric relationship between price per square foot and number of units. This is contained in Figure 12. Its resemblance to Figure 5, as derived from the theory model for low wealth investors, is striking. The purpose of the bid function estimation is to map portions of this curve to various wealth groups. In particular, what we observe from Figure 9, which presents theoretical bid curves for various wealth groups, is that if low wealth investors invest in real estate, they will only do so for buildings of small size, i.e. low unit counts. If that occurs, we should also observe a downward (in unit count) sloping bid function for those low wealth investors. At higher unit counts, however, higher wealth investors should be the winning bidders, but at this point, the slope of the bid functions for these wealthier people should be relatively flat. This would be congruent for both the theoretical and empirical graphs of Figures 9 and 12.

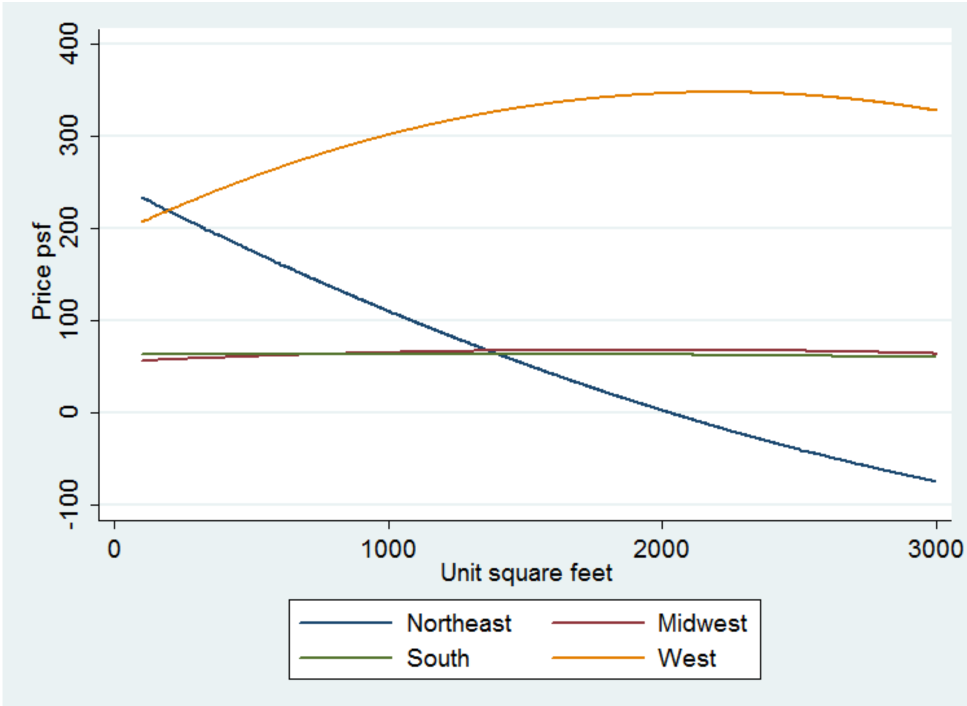

Figure 13: Predicted Price PSF by Region

We use the value functions for the four regions in the previous sections to compute marginal prices-per square foot. These are displayed in Figure 13. Note that there are, as desired, differences across the four regions (although the South and Midwest are fairly similar) with the West having the highest marginal prices. As discussed above, these marginal floorspace prices are calculated for each owner-occupied condo. In order to most clearly see the difference in marginal bid functions for different wealth groups we split the sample into two parts, labeled "Low Wealth" and "High Wealth" with the dividing line at a nonhousing wealth level of $150,000. We separately estimate (24) for these two groups. We include the income of the household and the age of the household head as the demographic variables, Z, and include the housing characteristics from above as well. As noted, the unit square footage is included here, but is clearly endogenous, and so we instrument using regional binaries and these binaries interacted with total wealth. The cubic polynomial of unit count is importantly included as well. The results are displayed in Table 5 (geographic variables are suppressed). The coefficients of importance are first of all, that of unit square

	LowWealth	HighWealth
Unit SF	-0.514***	-0.489***
	[0.114]	[0.099]
Number Units	-2.093***	0.029
	[0.712]	[0.699]
Number Units, sq.	.0066***	0.0007
	[.002]	[0.002]
Number Units, cu.	-4.91E-06**	-3.92E-07
	[1.96E-06]	[0.000]
Income	0.0002	0.0002**
	[0.0005]	[0.0001]
Head HH Age	1.11	2.096**
	[1.229]	[0.893]
Floor	-6.921	1.59
	[4.609]	[7.939]
Baths	210.627***	323.487***
	[47.117]	[62.681]
Half Bath	18.546	206.297***
	[32.642]	[53.141]
Fireplace	-18.385	48.513*
	[27.013]	[26.203]
Porch	35.817	55.654
	[32.213]	[40.774]
A/C	18.693	-109.991***
	[29.211]	[25.071]
Garage	172.500***	84.095**
	[19.222]	[42.562]
Yr. Built	-1.529*	-1.382**
	[0.906]	[0.689]
Constant	3,277*	2,770**
	[1,792]	[1,312]
Observations	903	1,524
Wald Chi2	269.06	229.99
p-value	0.00	0.00

Robust standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Estimates of Bid Functions

feet itself, which is negative for both high and low wealth households, as is quite appropriate for bid functions that display diminishing marginal rates of substitution. Note furthermore that the slope is greater for low wealth households, indicating that high wealth households outbid low wealth ones for larger units. More importantly for our purposes, note the coefficients for the polynomial in the number of units. From the linear terms, we can directly see that, for low unit counts at least, there is a pronounced negative slope for the bids from low wealth households, and that the bids from high wealth households are relatively flat. In Figure 14 we present bid functions for low and high wealth households that are otherwise identical (in both X and Z).[6] Low wealth households are observed to be very sensitive to unit count, and by extension to the free rider problem. High wealth households are not as sensitive to this, and so as the free rider problem worsens, and the expense from condo management rises, they begin to outbid low wealth households at around 60 units, in bigger condo buildings. This is exactly what our theory would predict.

8 Conclusion

In this paper we have generated predictions about bid prices for multifamily buildings of different size and under different forms of ownership. The insights of the model suggest that conclusions about homeownership based on a simple correlation between homeownership rates and buildings size may be misleading. We find that while small multifamily buildings are particularly disadvantaged for condo ownership – and this comprises much of the U.S. multifamily housing stock – economies of scale in the cost of third-party management may make joint ownership valuable in relatively larger buildings. The empirical estimation of bid functions confirms that less wealthy households will tend to outbid wealthier households for ownership of condominiums in smaller buildings. Bid prices for the wealthy may be increasing in the size of buildings to the extent that larger buildings provide for scale economies in

[6] X are set at approximate medians for a condo in the Seattle CMSA. The unit is 800 square feet, with 1 bath and a porch, and was built in 1975. The head's age is set at 49 years, and income is set at $75,000.

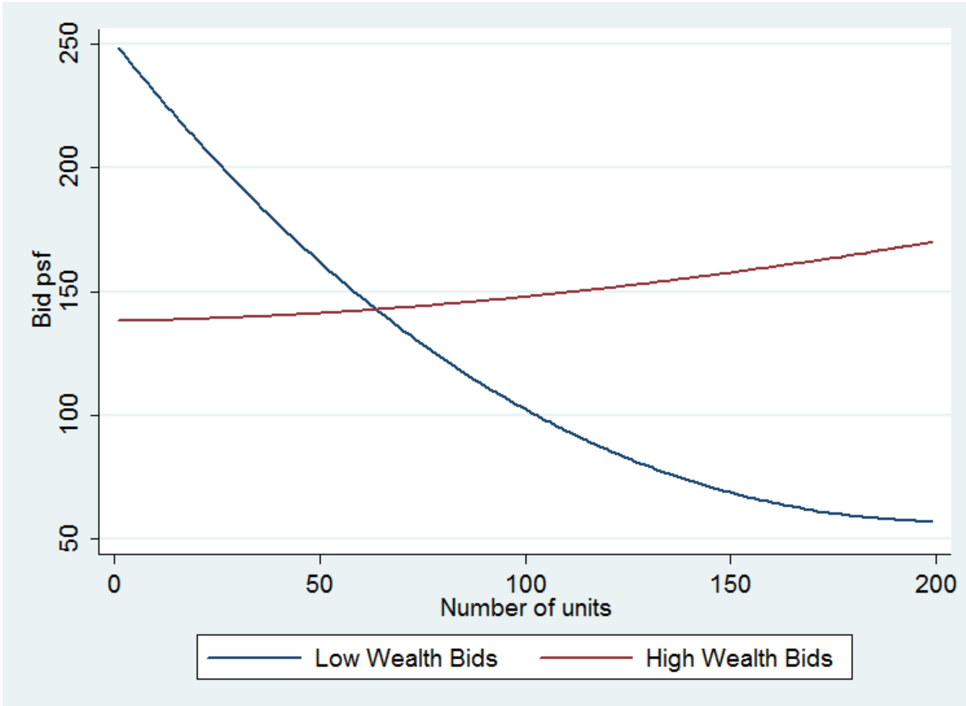

Figure 14: Predicted Bids of Low and High Wealth Households

management and the provision of amenities.

Our results shed light on the fundamental correlation between single family structures and ownership. The "traditional" explanation (Glaeser and Shapiro, 2003) suggests that multifamily units are subject to free rider and coordination problems, and therefore best managed as a solely-owned building. Our results, on the other hand, suggest that these problems can be overcome. The reason that single family homes are owner-occupied is that maintenance in smaller units does not scale up, at least under traditional business models. Unit size aside, it is surely more difficult to manage 500 (possibly dispersed) single family units than one building with 500 units. This issue is of vital importance due to the large amount of newly-vacated homes in the wake of the 2008 housing crisis. The conversion of these properties to rental units by large investors is underway, although it surely remains to be seen whether this is sustainable practice in the long run.[7]

[7] See, for example, Olick (2013), although other reports (Hallman and Berman, 2013) reinforce our point that maintenance is indeed quite costly to scale up in single family portfolios.

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A Sole-Investor Problem

In order to adapt the basic model to allow for ownership of multiple buildings, let $\overline{q}$ represent the number of buildings, while n still denotes total building size. Total investment in risky housing assets is now $\overline{q}n$. The total cost of landlord effort is $a\overline{q}(e\overline{q}n)$. Notice that while the cost of effort is linear in effort within a building, ownership of multiple buildings requires a duplication of effort for each additional building. Using these adjustments, we initially take the number of buildings to be exogenous and define the landlord's indirect utility for a given number of buildings as

$$U^s\left(w_1, \overline{q}\right) = \max_{n, c_f, c_v} E\left[v\left(\widetilde{w}\right)\right].$$

Investor second period realization of wealth is:

$$\widetilde{w}^s = \left(w_1 - \overline{q}n\left(P - R\right)\right)\left(1 + r\right) + \overline{q}n\left(P + 2\phi_a\sqrt{a} + \left(1 - c_v\right)\left(2\phi_b\sqrt{b} + \widetilde{\varepsilon}\right) + \widetilde{u}\right) - \overline{q}ne - ae\overline{q}^2n$$

As before, we solve for the interim choices of effort by the investor:

$$a^s = \frac{\phi_a^2}{e^2\overline{q}^2}.$$

Solving the investor's problem subject to the participation constraint of third-party managers, which remains unchanged relative to the base model, the equation that we use in numeric solutions to identify n is

$$R\left(1 + r\right) - rP - \overline{z} + \left(c_v^2 - 2c_v + 2\right)\frac{n\phi_b^2}{2\left(d + n^2\alpha\right)} - \left(2 - c_v\right)\frac{\rho}{2}c_v n\sigma^2 + \frac{E\left[v'\widetilde{u}\right]}{E\left[v'\right]} = 0.$$

The first order condition for c_v is identical to the base model.

We are use the envelope theorem to derive a slope of the bid function for sole ownership of buildings:

$$\frac{\partial P}{\partial n} = \frac{1}{r}\left(c_v \phi_b^2 \left(2 - c_v\right) \frac{d - \alpha n^2}{\left(d + \alpha n^2\right)^2} - \frac{\rho}{2} c_v^2 \sigma^2\right).$$

The first term on the right hand side is positive for buildings smaller than the cost-minimizing size. Whether or not the slope is positive or negative depends on the particular choice of parameters.

In the numerical solutions for sole investors in multifamily buildings, we initially invert the investor's maximization problem and taking $n = 1200$ as given, solve for the size of the investment q, as well as c_v and c_f at the single family price of \$94 per square foot. Once we obtain the investor's level of utility from owning a portfolio of single family houses, we then solve for n, c_v, c_f, and bid price at which utility is held constant conditional on an integer number of buildings being held in portfolio.

B Symbols

U	indirect investor utility
v	investor utility over wealth
$\widetilde{w}$	investor (uncertain) wealth at time two
P	price per unit of housing services
R	periodic rent per unit of housing services
w_1	endowment at time 1
$\widetilde{\varepsilon}, \widetilde{u}$	normally distributed noise with mean 0 and variance σ_ε^2 and σ_u^2
q	housing investment in units of housing services
r	risk-free rate
c_v	variable component of third party manager compensation
g	total third-party manager compensation
$V(g)$	third party management's utility as a function of compensation
c_f	fixed component of third party manager compensation
ρ	coefficient of manager absolute risk aversion
ϕ_a, ϕ_b	investor and manager production parameter
n	building size in units of housing services
$\widehat{z}$	third party manager's certainty equivalent wealth
$\overline{z}$	third party manager's opportunity cost per unit of housing services
e, d, α	cost of effort parameters