Wage Rigidity and Labor Market Dynamics with Sorting*

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Abstract

This paper adds two-sided ex ante heterogeneity and a production technology inducing sorting to the canonical Diamond-Mortensen-Pissarides (DMP) search and matching model. This modification considerably improves the model's empirical performance. Due to assortative matching, match-specific wages are non-monotonous in firm type, as illustrated by Eeckhout and Kircher (2011); that is, an optimal, wage-maximizing employer exists for every worker. In a frictional labor market, however, wage-maximizing matches are unlikely to materialize, resulting in mismatch and output loss compared to the first-best allocation of workers to jobs. A lower expected value of match-specific wages, in turn, provides additional incentives for firms to create vacancies in response to a favorable productivity shock. Thus, ex ante heterogeneity and sorting have important implications not only for the equilibrium but also for the dynamic properties of the model. Specifically, the modifications solve the problem that standard DMP models do not generate enough volatility in response to shocks, also known as the "Shimer Puzzle" (Shimer, 2005). The necessary amplification to overcome the volatility puzzle stems from an endogenously generated wage rigidity, which is of reasonable magnitude given empirical evidence from the U.S. labor market.

Keywords: Sorting, Mismatch, Wage Rigidity, Heterogeneity, Unemployment, Search and Matching, Aggregate Fluctuations

JEL Classifications: E24, E32, J63, J64

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1 Introduction

A large literature in labor economics shows how frictions and information imperfections prevent labor markets from clearing. This explains the existence of equilibrium unemployment. Job seekers and employers need to spend time and money to locate a suitable counterpart and negotiate before eventually forming a match to start production. Intuitively, this labor market imperfection can be understood as an outcome of heterogeneity. Workers and firms differ, for instance, with respect to their location or the supply and demand of specific skills, making finding a suitable partner and forming a match costly. The Diamond-Mortensen-Pissarides (henceforth DMP) search and matching framework, as summarized in Pissarides (2000), owes a large part of its success to its mathematically elegant way of incorporating this coordination friction without making the heterogeneity across workers and firms explicit.[1]

The downside of this approach is that the individual-specific dimension of match formation is cloaked. To show that relaxing this assumption potentially has important implications for the model's empirical performance, this paper takes explicit heterogeneity across firms and workers as a starting point. Shimer and Smith (2000), Teulings and Gautier (2004), Atakan (2006), Gautier et al. (2010), Eeckhout and Kircher (2011), and Hagedorn et al. (2012) all show the potential importance of complementarity in production and assortative matching[2]—henceforth sorting in short—in the context of a search model with heterogenous workers and firms. I assume positive sorting based on comparative advantage and show that this modification has important implications both for the equilibrium and the dynamic properties of a search and matching model. My primary finding is that the augmented model creates a wage rigidity of reasonable magnitude given empirical evidence from U.S. labor market data. Wage rigidity is defined, as it is common in this literature, as an elasticity of wages with respect to labor productivity of less than 1; that is, wages do not fully adjust and do not follow labor productivity one-to-one. Haefke et al. (2013) report an elasticity of 0.8 for the U.S. labor market (with a standard error of 0.4). I compute the same elasticity using simulated data from the search and matching model with sorting and get a result of 0.861.

In contrast to much of the literature, I obtain this result even though I keep the simple, well-known structure of the standard DMP model intact, including the Nash bargaining

[1] In the baseline DMP model, the arrival of suitable match opportunities is governed by a Poisson process which depends on the aggregate state of the labor market. Representative workers and firms base their optimal, forward-looking decisions solely on the expected law of motion of this aggregate state, which is the tightness of the labor market, defined as job-openings per unemployed worker.

[2] Details and technical conditions follow in sections 2 and 5.

solution. I emphasize this because Shimer (2005) finds that it is due to the simple Nash bargaining solution that the model largely fails to generate sufficient volatility in response to shocks. He argues that the latter problem, also known as “Shimer Puzzle”, has a distinct connection to the sharing rule used in the model. In turn, many papers indeed show that the volatility puzzle can be solved by making wages less responsive to shocks, either by simply assuming fully inflexible wages (Hall (2005)) or by changing the calibration of the model such that the outside option of the worker is higher, what also translates to lower and less flexible wages (Hagedorn and Manovskii (2008)).[3]

The model presented in this paper endogenously generates a wage rigidity using a standard sharing rule. The rigidity arises solely out of equilibrium properties of the search and matching model with sorting: unequal bargaining powers lead to an asymmetry between the reservation values of workers and firms in equilibrium. In the wage equation, this appears as a relative, i.e. match-specific labor market tightness component which is strictly lower than the aggregate value. Firms' matching sets are thus wider in equilibrium and more volatile in response to shocks. The equilibrium framework is inspired by Hagedorn et al. (2012). Sorting makes the allocation of workers to jobs meaningful, both in terms of match-specific output and overall welfare. It implies that an optimal counterpart exists for every market participant. In the absence of labor market frictions, a firm (worker) would match instantaneously with the optimal employee (employer)—and only with the optimal one—leading to a Walrasian first-best allocation like in Gary Becker's neoclassical marriage market model (Becker (1973)). Due to frictions and costly search, however, this optimal, welfare-maximizing allocation of workers to jobs can never be realized. An equilibrium feature of this class of models is that workers and firms are picky, that is, they are willing to match only with a subset of types in the vicinity of their optimal allocation. The cardinality of these individual-specific subsets, which are known as matching sets, is dependent on the degree of complementarity in production. They are endogenously determined in the model and have important implications for match-specific wages because they influence the outside options in the Nash bargaining solution, that is, the values of being unemployed or maintaining an unfilled vacancy, respectively. This is the key mechanism through which sorting influences wage formation and the dynamics of the model. To generate results that are comparable to previous literature, I calibrate the model along the lines of Shimer (2005) for the U.S. labor market and conduct numerical simulations in the presence of aggregate uncertainty.

3 These approaches to solve the volatility puzzle are highly debated in the literature, mainly for reasons of empirical plausibility, see Hornstein et al. (2005)

I show that in a simple dynamic DMP model with heterogeneity and sorting no additional assumptions are necessary to generate rigid wages. Also, I note that the DMP model's volatility problem vanishes with sorting. In the presence of aggregate uncertainty, two channels contribute to this amplification: both the endogenously generated wage rigidity and the effect of sorting on the forward-looking job creation decision boost firms' incentives to post vacancies in response to positive aggregate productivity shocks. Since the wage rigidity generated by the model is both empirically reasonable and rather moderate, it accounts only for roughly half of the additional volatility needed to align the model with moments of empirical labor market time series.[4] Thus, a second channel is needed to generate the additional amplification necessary to fully overcome the Shimer puzzle: once a positive shock hits the economy, the non-monotonous surplus function shifts upwards. Firms re-optimize their forward-looking job creation decision: they choose wider matching sets because the surplus from an increased number of potential matches is larger than zero. Therefore, a higher number of vacancy postings is the optimal response to a positive shock. This generates additional amplification besides the wage-rigidity effect. The firms' expected surplus changes along two margins: In response to a positive shock, firstly, the expected potential wage payments to every worker in the equilibrium matching set are lower due to the rigidity. Secondly, additional workers with potentially positive surplus enter the equation due to wider matching sets. Via both channels, the DMP model with sorting creates a large amount of amplification in response to shocks and solves the Shimer puzzle without changing the well-understood basic structure of the workhorse model in macro-labor economics.

The contribution of this paper goes beyond the labor market. Explaining the market imperfections that one can observe in the data in form of sluggish and imperfect price adjustments is a key challenge of modern macroeconomics. Only taking these rigidities into account enables macroeconomic models to generate dynamics which are consistent with empirical facts. Thus, many papers rely on complex model extensions to generate rigid prices and wages to match the data.[5] Incorporating a labor market model with sorting, which is capable of endogenously generating a rigidity, into larger macroeconomic models with heterogeneous agents is therefore a highly promising avenue for future research. This is one of the first papers that analyzes two-sided heterogeneity and assortative matching in a fully dynamic equilibrium labor search model. What

[4] As documented in the literature, amplification is a result of less responsive wages because they lead to a larger share of the additional surplus that the firm can absorb in response to a positive shock.

[5] Blanchard and Galí (2010) provide a nice overview of this literature, which is discussed in section 5.

distinguishes it from other recent papers that develop comparable models (Bagger and Lentz (2014), Gautier and Teulings (2014), Lise and Robin (2013)) is the focus on wage formation, rigidity, and the consequences for labor market dynamics. Furthermore, as compared to richer models, I stick to most of the standard model's assumptions for two reasons: to present the mechanism behind the endogenously generated wage rigidity as transparent as possible and to generate results which are directly comparable and applicable to both previous work on the dynamics of DMP models and the wider field of macroeconomic models with frictional labor markets. In fact, once the type space of the model with heterogeneity is collapsed into a single point, it is straightforward to reduce the model to the well-known textbook equations of the basic DMP model. I show, however, that the simplifying assumptions made to ensure clarity and comparability do not induce any loss of generality with respect to the key mechanism at the core of the augmented model.

The remainder of this paper is structured as follows: section 2 introduces the model and derives the wage-formation mechanism in the presence of heterogeneous agents and sorting as a bargaining outcome in the recursive framework. The equilibrium is characterized and the computational strategy is briefly outlined. Section 3 presents numerical simulations under aggregate uncertainty and compares the results of the augmented model to the baseline model in the light of empirical moments from the U.S. labor market data. Section 4 analyzes the interplay between sorting, wage formation, and the model's dynamics in more detail and presents robustness checks. Section 5 discusses the results in the light of related theoretical and empirical literature. Section 6 concludes.

2 The Model

2.1 Setup

I construct a fully dynamic two-sided equilibrium search and matching model with discounting and transferable utility. Workers and firms are heterogeneous. Uncertainty with respect to worker and firm type is not considered, that is, agents know their own type and the types of all possible counterparts. Time is discrete but time indices are omitted where not necessary for clarity of exposition. Workers and firms are risk neutral, infinitely lived, and seek to maximize their expected discounted future income streams. β represents the common discount factor. As in the canonical search and matching model, only unemployed individuals search randomly in this model; employed workers do not search.[6] In the event of unemployment, workers receive constant unemployment benefits b every period, which can also be interpreted as the value of non-market activity. Vacant firms have to pay per-period vacancy posting costs κ, representing expenses for posting vacancies, screening applications, and so forth. Productive activity commences when a firm and a worker who meet in the labor market are able to jointly produce a non-negative surplus given their types. If this is the case, the surplus is shared according to the standard Nash bargaining solution with workers having bargaining power α. Every period, matches between firms and workers may terminate for two possible reasons: Firstly, they are subject to idiosyncratic separation shocks, which lead to immediate dissolution of the employment relationship. These shocks hit a match with an exogenous per-period probability δ, the separation rate. Additionally, the model allows for endogenous separations in the presence of aggregate uncertainty. A separation may occur at the margins of the agents' matching sets after a negative productivity shock hits the economy and renders employment relationships which previously generated positive surpluses unprofitable.

The recursive setup of the model mainly builds on Hagedorn et al. (2012). However, I simplify the timing structure to be in line with the baseline setup as in Pissarides (2000).[7] Search models with two-sided ex ante heterogeneity date back to Shimer and Smith (2000) who were the first to add search frictions to the classical Beckerian assignment model (Becker (1973)), thereby inducing departure from the Walrasian equilibrium. In the Shimer and Smith (2000) model agents are picky in equilibrium, that is, they are not willing to form a match with every possible counterpart since for some pairings the surplus is negative. This feature severely complicates the analysis since search equilibrium cannot be pinned down analytically. The Shimer and Smith (2000) paper forms a bridge between the literature on matching of heterogeneous agents and models of frictional markets, which usually rely on representative agents (Diamond (1982), Mortensen (1982), Pissarides (2000)), although there are notable exceptions (Acemoglu (2001), Bertola and Caballero (1994), Davis (2001)).

[6] The assumption of no search on-the-job is discussed in sections 3 and 4.

[7] I assume that matches cannot break up in the period they are formed. Hagedorn et al. (2012) assume a more complex timing structure with two subperiods: Matches are formed in the first subperiod. In the second subperiod, before production takes place, separation can occur for all matches, including newly formed ones. This assumption is inconsequential for the properties of the model.

Table 1: Labor market states, density functions, and relation to aggregate labor market variables

Description	Density/Distribution Function	Aggregate Value
Active matches	$g_m(s,c)$	$M = \iint g_m(s,c)\ \mathrm{d}s\mathrm{d}c$
Employed workers	$g_e(s) = \int g_m(s,c)\ \mathrm{d}c$	$E = \int g_e(s)\ \mathrm{d}s$
Unemployed workers	$g_u(s) = g_w(s) - g_e(s)$	$U = \int g_u(s)\ \mathrm{d}s$
Producing firms	$g_p(c) = \int g_m(s,c)\ \mathrm{d}s$	$P = \int g_p(c)\ \mathrm{d}c$
Vacant firms	$g_v(c) = g_f(c) - g_p(c)$	$V = \int g_v(c)\ \mathrm{d}c$

2.2 The Type Space

I assume symmetric, atomless distributions of both types. Workers are endowed with heterogeneous skills, indexed by s, and firms differ from each other in terms of job complexity, indexed by c. Both skills and job complexity can be viewed as a one-dimensional representation of a larger, multi-dimensional set of worker and firm characteristics. I assume that both firms and workers can be unambiguously ranked based on skills and job complexity and this global ranking is known to the researcher.[8] The indices, s and c, are distributed uniformly on the open interval. This is a stationary model, so the overall probability density function for workers and firms are time constant and denoted by, $g_w(s)$ and $g_f(c)$.[9] Table 1 shows how these densities relate to the distribution of unemployed workers of type s, $g_u(s)$, and the distribution of vacant firms of type c, $g_v(c)$, which are both equilibrium objects. They entail the probability of meeting a specific worker or firm type. The aggregate number of vacancies and unemployed workers, V and U, in turn, are computed by taking the integral over the distribution of vacant firms or unemployed workers, respectively. $g_m(s,c)$ is the two-dimensional distribution of active matches. The integral over either dimension yields the probability distributions of employed workers, $g_e(s)$, and producing firms, $g_p(c)$. The sum of both distributions of matched agents ($g_e(s)$ and $g_p(c)$) and unmatched agents ($g_u(s)$ and $g_v(c)$) has to equal the overall density functions for workers and firms, $g_w(s)$ and $g_f(c)$.

8 In an empirical setting, constructing such a ranking given the available matched employer-employee data is indeed the most difficult part of identifying the sign and the strength of sorting, see also the discussions in Eeckhout and Kircher (2011) and Hagedorn et al. (2012).

9 Since the type space is symmetric and both densities relate to the same uniform distribution, one overall density function would be sufficient to set up the model. However, I keep $g_w(s)$ and $g_f(c)$ for clarity of exposition.

2.3 Production

In the model economy, a productive relationship can be imagined as a single worker operating a single machine. Thus, I assume that every firm has exactly one job to fill. Translated to an environment with multi-worker firms, this simplistic structure implicitly assumes that the output of a given match is independent of all other matches of the firm. Hence, the output of a multi-worker firm is simply the sum of all single-match outputs with individual workers.[10] The worker's labor is the only input to the production process, that is, I ignore capital.

This paper's central assumption is positive assortative matching (PAM). Output is match specific due to two-sided heterogeneity. PAM requires that the production technology features complementarities, i.e., it has to be supermodular.[11] The arguments of the twice differentiable and strictly concave non-negative production function are the worker's skill s and the firm's job complexity c. It is increasing in both arguments and has strictly positive cross-derivatives. Thus, the more similar a firm and a worker are in terms of their type, the higher is the output they can jointly produce. A high-skilled worker will produce more in a complex job compared to a low-skilled worker. To fulfill these demands, I use the circular model proposed in Gautier et al. (2010) to calculate match output as a function of the "distance" between worker and firm, defined along the circumference of a circle:

$$F(s,c) = F(x) = 1 - \frac{1}{2}\gamma x^2, \;\; x \equiv |s - c|. \tag{1}$$

x, the distance, can be seen as a measure of mismatch between the worker and the firm. Thus, the function is maximized for $x = 0$, i.e., for the case of an optimal match. The parameter γ governs the degree of complementarity of worker types and, thus, the cost of mismatch. Conceptually, the parameter γ is related to the "complexity dispersion parameter" discussed in Teulings and Gautier (2004) and Teulings (2005). The lower γ, the more substitutable are different types in production. I assume a small γ, i.e., a small degree of complementarity. It turns out that small gains from sorting are sufficient to

[10] See also Lentz and Mortensen (2010). While this simplifying assumption is common in the current literature, a theoretical approach to heterogeneity in models with multi-worker firms is an exciting avenue for future research.

[11] Generally, supermodularity necessitates positive cross-derivatives of the production function, that is, the higher the first argument of the function, the higher is the marginal value of an increase in the second argument. In the presence of costly search, Shimer and Smith (2000) show that for for any search frictions and type distribution, log-supermodularity is a necessary condition for the existence of a search equilibrium. It ensures convexity of the matching sets. Topkis (1998) provides more details on the application of closely related mathematical concepts (lattice theory).

achieve the desired amplification effect (see Section 3). However, a minimum constraint on γ ensures that workers do not accept every job in equilibrium. For $\gamma = 0$, the output of a match would be independent of worker and firm types, just like in the baseline case with type-independent output $F = 1$ in a model of homogenous workers and firms. Note that this production function has an interior maximum that distinguishes it from early analyses of search equilibria in models with two-sided heterogeneity. Shimer and Smith (2000) and Teulings and Gautier (2004) rely on a hierarchical production structure with high-skilled workers and high-productivity firms having an absolute advantage. Intuitively, absolute advantage implies that high-skilled workers will always produce more than low-skilled workers, at any firm. Gautier et al. (2006) discuss the relation of the hierarchical and circular models. The circular model is particularly suitable for the analysis of models with on-the-job search as it avoids a corner problem.[12] However, its empirical applicability is limited since it is hard to imagine data on worker and firm characteristics that can be represented meaningfully on a circle. Since this paper is theoretically motivated, however, I employ the circular model nevertheless. The interior maximum of the production function delivers a nice intuition for the model-generated non-monotonicity because it represents the optimal match that maximizes output and wage. The critical property of search equilibrium that leads to the model-generated wage rigidity—the pickiness of workers and firms—does not hinge on this assumption. It also arises in models with a hierarchical structure, see Shimer and Smith (2000).

To assess the dynamic properties of a search and matching model with sorting in section 3, I run stochastic simulations. I model aggregate uncertainty using a stochastic process for labor productivity that is calibrated to match the data. Aggregate labor productivity is a general, type-independent component of production technology that affects the output of every match in equal measure. Formally, the aggregate labor productivity process and its fluctuations enter the model in a simple multiplicative way:

$$F_t(s, c) = F(s, c) \times Y_t. \qquad (2)$$

The output of a match between worker s and firm c at time t is thus simply the product of the time-constant and match-specific part, $F(s, c)$, and the time-varying and type-

[12] See Gautier et al. (2006), p. 120. The corner problem refers to the fact that the matching sets of the extreme types in the hierarchical model "cannot have both an interior upper and lower bound." This problem becomes especially apparent in models with on-the-job search since workers and firms move up the type space over time. Without on-the-job search, however, the mass of workers and firms at the tails of the type distribution is low, which justifies the use of Taylor approximations of search equilibrium in this class of models, as proposed by Teulings and Gautier (2004).

independent aggregate labor productivity process, Y_t. In section 3, Y_t is shocked. In steady state, however, Y_t is unity. Thus, it can be ignored for the characterization of equilibrium.

2.4 Search and Matching with Sorting

I add ex ante heterogeneity and a production technology as defined above to the canonical Diamond-Mortensen-Pissarides (DMP) search and matching model. The model economy's labor market is characterized by random search of unemployed agents only. Labor market frictions prevent meeting the optimal counterpart. Thus, workers and firms rationally form suboptimal matches as long as the surplus to share is weakly positive. This holds true even though I ignore uncertainty with respect to types. In the event of an encounter that would yield a negative surplus, no match is formed and both parties continue to search. The search technology is linear, that is, meetings are governed by the common Cobb-Douglas type matching function with constant returns to scale:

$$M(U,V) = \vartheta \; U^{\xi} \; V^{1-\xi}. \tag{3}$$

The parameter ξ $(1-\xi)$ represents the elasticity of new matches with respect to unemployment (vacancies). ϑ is a scale parameter. Since the matching function is homogenous of degree one, the Poisson arrival rates are functions of labor market tightness $\theta = {}^{V}\!/_{U}$:

$$q_v(\theta) = \frac{M(U,V)}{V} = M\left(\frac{U}{V},1\right) = \vartheta\theta^{-\xi}, \tag{4}$$

$$q_u(\theta) = \frac{M(U,V)}{U} = M\left(1,\frac{V}{U}\right) = \vartheta\theta^{1-\xi}. \tag{5}$$

In the frictional labor market, $q_v(\theta)$ is the rate at which vacant firms meet unemployed workers and, correspondingly, $q_u(\theta)$ represents the probability that unemployed workers meet vacant firms. $q_v(\theta)$ is decreasing and $q_u(\theta)$ is increasing in labor market tightness θ. In the search and matching model with heterogeneity, not all meetings will result in a productive employment relationship. If a firm and a worker meet and find that they are too different in terms of their types, i.e., the surplus of the match would be negative, both parties prefer to continue search. Thus, for every firm (worker) only a subset of the type space of workers (firms) qualifies for a match. Formally, two matching sets, $M_s(c)$ and $M_c(s)$, delimit the types of suitable counterparts for a firm c or worker s. Thus, the set $M_s(c)$ $(M_c(s))$ contains all worker (firm) types with which a given firm (worker) of type c (s) is willing to match. The surplus, $S(s,c)$, is weakly positive for those matches.[13]

$$S(s,c) \geq 0 \leftrightarrow s \in M_s(c) \leftrightarrow c \in M_c(s). \tag{6}$$

As in Hagedorn et al. (2012), a joint, two-dimensional matching set $\mathcal{M}$ contains all matches (s,c) that form in equilibrium:

$$\mathcal{M} \equiv \{(s,c) : s \in M_s(c) \wedge c \in M_c(s)\}. \tag{7}$$

The possibility of meeting an unsuitable counterpart, i.e., $S(s,c) < 0$, has to be incorporated in the set of recursive equations that pin down the option values for both workers and firms in both states (matched and unmatched). Following Hagedorn et al. (2012), albeit with small modifications, these four value functions are written as follows:[14]

$$V_e(s,c) = W(s,c) + \beta \mathbb{E}\left[\underbrace{\delta V_u(s)}_{\text{separation}} + \underbrace{(1-\delta)\max\{V_e(s,c), V_u(s)\}}_{\text{continued employment}}\right] \tag{8}$$

$V_e(s,c)$ is the value of an employed worker of type s at firm c. It is determined by the match-specific wage, $W(s,c)$, plus the future expected value of two contingencies, discounted by β. First, the value of unemployment in case of an exogenous separation shock (probability δ) and, second the value of non-separation and continued employment at firm c (probability $(1-\delta)$. Note that the max operator entails the possibility of an endogenous separation: the match will also terminate once the value of continued employment falls short of the value of unemployment. The asset value of an unemployed worker, $V_u(s)$, has to incorporate three possibilities:

$$V_u(s) = b + \beta \mathbb{E}\left[\underbrace{(1-q_u(\theta))V_u(s)}_{\text{no meeting}} + \underbrace{q_u(\theta)\int_{M_c(s)} \frac{g_v(c)}{V} V_e(s,c)\ \mathrm{d}c}_{\text{successful match}} + \underbrace{q_u(\theta)V_u(s)\int_{\overline{M_c(s)}} \frac{g_v(c)}{V}\ \mathrm{d}c}_{\text{meet unacceptable firm}}\right] \tag{9}$$

[13] I assume that a match is formed in case of indifference ($S(s,c) = 0$).

[14] Recall that I do not consider the possibility of an immediate separation before production starts and allow for endogenous separations in the face of aggregate uncertainty (see Section 2.1). Hagedorn et al.'s (2012) notation has been slightly altered. For the exposition of the equilibrium value functions, time-varying stochastic labor productivity can be excluded since it is assumed to be unity in steady state.

In case of unemployment all workers receive the same payment b. With probability $(1 - q_u(\theta))$ the worker will not meet any firm in the current period, stay unemployed, and continue to receive the value of unemployment in the next period. With probability $q_u(\theta)$, a meeting occurs. In the event the firm's type is an element of the worker's matching set and vice versa, a match is formed and production starts. Note that $g_v(c)/V$ represents the probability of meeting a single firm type in the labor market. The type-specific value of this fraction works like a weight on the surplus. The firm could also be an unsuitable match, i.e., be of a type that is not an element of the worker's matching set, $M_c(s)$, but of its complementary set, $\overline{M_c(s)}$. The complementary set contains all firms with which the worker is not willing to match. The value of this eventuality is captured by the third term. The firms' asset value equations are constructed in a similar way:

$$V_p(s,c) = F(s,c) - W(s,c) + \beta \mathbb{E}\left[\underbrace{\delta V_v(c)}_{\text{separation}} + \underbrace{(1-\delta)\max\{V_p(s,c), V_v(c)\}}_{\text{continued employment}}\right] \tag{10}$$

A productive employment relationship generates match-specific output $F(s,c)$ for the firm, determined by the production technology defined in Section 2.3. The firm has to pay the bargained match-specific wage to the employee. In the following period, the match breaks up with probability δ, leading to the option value of a vacancy, or is sustained with probability $(1-\delta)$, yielding the same value also in the next period, but discounted with β. Again, the max operator allows for an endogenous termination of the employment relationship. Finally, the asset value of a vacant firm is as follows:

$$V_v(c) = -\kappa + \beta \mathbb{E}\left[\underbrace{(1-q_v(\theta))V_v(c)}_{\text{no meeting}} + \underbrace{q_v(\theta)\int_{M_s(c)} \frac{g_u(s)}{U} V_p(s,c)\,\mathrm{d}s}_{\text{successful match}} + \underbrace{q_v(\theta)V_v(c)\int_{\overline{M_s(c)}} \frac{g_u(s)}{U}\,\mathrm{d}s}_{\text{meet unacceptable worker}}\right] \tag{11}$$

The constant cost of maintaining an open vacancy, κ, which must be paid every period, enters the value function with a negative sign. The contingencies with respect to the next period consist of the possibility of not meeting a worker, the possibility of meeting a suitable worker and filling the vacant job, and the value of meeting an unsuitable worker and continuing search, constructed similarly to equation (9).

To determine how the surplus is divided in case of a suitable match, I apply the standard Nash bargaining solution of the baseline search and matching model (Pissarides (2000)). For both the worker and the firm, the respective share of surplus equals the additional value of being matched compared to the value of continued search, which serves as threat point in the bargaining game. The split of the surplus from a match between worker s and firm c is governed by bargaining power parameter $\alpha \in [0,1]$:

$$S(s,c) = V_p(s,c) - V_v(c) + V_e(s,c) - V_u(s) \tag{12}$$

$$\alpha S(s,c) = V_e(s,c) - V_u(s) \tag{13}$$

$$(1-\alpha)S(s,c) = V_p(s,c) - V_v(c) \tag{14}$$

To simplify Equations (8) to (11) I apply the Nash bargaining solution and plug in the surplus sharing rules of Equations (13) and (14). This leads to a simplified system of steady-state value functions with the equilibrium surplus under the integrals:

$$V_e(s,c) = W(s,c) + \beta\mathbb{E}\left[V_u(s) + \alpha(1-\delta)\max\{S(s,c),0\}\right] \tag{15}$$

$$V_u(s) = b + \beta\mathbb{E}\left[V_u(s) + \alpha q_u(\theta)\int\limits_{M_c(s)} \frac{g_v(c)}{V} S(s,c)\ \mathrm{d}c\right] \tag{16}$$

$$V_p(s,c) = F(s,c) - W(s,c) + \beta\mathbb{E}\left[V_v(c) + (1-\alpha)(1-\delta)\max\{S(s,c),0\}\right] \tag{17}$$

$$V_v(c) = -\kappa + \beta\mathbb{E}\left[V_v(c) + (1-\alpha)q_v(\theta)\int\limits_{M_s(c)} \frac{g_u(s)}{U} S(s,c)\ \mathrm{d}s\right] \tag{18}$$

As is commonly assumed in search and matching models, firms will post vacancies as long as there is an opportunity to realize additional profits. Thus, the value of vacant jobs, as represented by Equation (18), has to be zero in equilibrium. The following free-entry condition holds:

$$\frac{\kappa}{q_v(\theta)} = \beta(1-\alpha)\mathbb{E}\left[\int\limits_{M_s(c)} \frac{g_u(s)}{U} S(s,c)\ \mathrm{d}s\right]. \tag{19}$$

This is simply a modification of equation (18) with $V_v(c) \stackrel{!}{=} 0$. As in the baseline model, the expected cost of hiring a worker, $\kappa/q_v(\theta)$ has to equal the future discounted profits of a job.[15] This equation is also known as the job-creation condition, because the firm

[15] Note that $1/q_v(\theta)$ is the expected duration of a vacancy.

creates additional jobs until another vacancy posting would not yield additional profits. In the model, this value is a function of the integral over the firm-specific matching set, taking into account the surplus of every worker type the firm would be willing to match with, appropriately weighted by the probability of meeting every specific type, ${}^{g_u(s)}/_{U}$. To derive the equation for match-specific wages, the Nash bargaining solution is used again. Using Equations (13) and (14) and equating them via the surplus the following must hold true:

$$V_e(s,c) - V_u(s) = \frac{\alpha}{1-\alpha}(V_p(s,c) - V_v(c)). \tag{20}$$

$V_v(c)$ drops out in equilibrium due to the imposed free-entry condition. Plugging in the value functions of equations (15), (16), and (17), maximizing the Nash product, and performing some algebra yields an expression that determines the match specific-wage, $W(s,c)$:

$$W(s,c) = \alpha F(s,c) + (1-\alpha)\beta\alpha\mathbb{E}\left[q_u(\theta)\int\limits_{M_c(s)} \frac{g_v(c)}{V} S(s,c)\ \mathrm{d}c\right] + (1-\alpha)b. \tag{21}$$

The wage of a given worker is thus a convex combination of the output produced with the present firm and the worker's outside option, that is, the value of being unemployed, which depends on the expected value of the surplus with all potential employers in his matching set. Depending on the distribution of vacant firms, $g_v(c)$, the surplus is weighted with the probability of actually meeting a specific firm type. The wider the matching set, the higher is the negotiated wage because the worker has more valuable outside options. Note that for all workers in the matching set, the surplus is weakly positive by definition. After factoring out α, the free-entry condition (Equation (19)) can be plugged into this wage equation to arrive at the following expression:

$$W(s,c) = \alpha\left(F(s,c) + \kappa\mathbb{E}\left[\theta\frac{\int_{M_c(s)} \frac{g_v(c)}{V} S(s,c)\ \mathrm{d}c}{\int_{M_s(c)} \frac{g_u(s)}{U} S(s,c)\ \mathrm{d}s}\right]\right) + (1-\alpha)b. \tag{22}$$

Using the free-entry condition, the same logic regarding the integral over the matching set applies also from the firm's perspective. The integral over all workers in the matching set $M_s(c)$ of a firm c influences the negotiated wage negatively through the denominator. The wider the firm's matching set the better is the firm's outside option of continued search and thus the lower the match-specific wage of worker s employed at firm c. Note that the aggregate labor market tightness θ in front of the quotient cancels out with ${}^{1}/_{V}$

in the numerator and $1/U$ in the denominator:

$$W(s,c) = \alpha \left(F(s,c) + \kappa \mathbb{E} \left[\frac{\int_{M_c(s)} g_v(c) S(s,c) \text{ d}c}{\int_{M_s(c)} g_u(s) S(s,c) \text{ d}s} \right] \right) + (1-\alpha) b. \tag{23}$$

The final expression defining the match-specific wage $W(s,c)$, (23), does not depend on the aggregate numbers of unemployed workers and vacancies. Instead, the quotient, call it "relative" or "match-specific" labor market tightness, depends on the expectation of the distributions of vacancies and unemployed workers and surpluses for the types within the respective matching set. For clarity of exposition in the following, I assign the following functional symbols to the integral terms:

$$\Phi_w(s) = \int_{M_c(s)} g_v(c) S(s,c) \text{ d}c, \tag{24}$$

$$\Phi_f(c) = \int_{M_s(c)} g_u(s) S(s,c) \text{ d}s. \tag{25}$$

Note that these integral expressions are nothing but the workers' and firms' reservation values, $R_w(s)$ and $R_f(c)$, without the parameters and aggregate values that drop out when deriving the wage eqauation. They are easily derived from equations (16) and (18):

$$R_w(s) = (1-\beta) V_u(s) = b + \beta \alpha \mathbb{E} \left[\frac{q_u(\theta)}{V} \int_{M_c(s)} g_v(c) S(s,c) \text{ d}c \right], \tag{26}$$

$$R_f(c) = (1-\beta) V_v(c) = -\kappa + \beta (1-\alpha) \mathbb{E} \left[\frac{q_v(\theta)}{U} \int_{M_s(c)} g_u(s) S(s,c) \text{ d}s \right]. \tag{27}$$

Using this additional notation, the wage equation can be written like this:

$$W(s,c) = \alpha \left(F(s,c) + \kappa \mathbb{E} \left[\frac{\Phi_w(s)}{\Phi_f(c)} \right] \right) + (1-\alpha) b. \tag{28}$$

The close relation to the baseline DMP now becomes very clear. Compare equation (28) with its textbook counterpart for a DMP model with constant labor productivity and homogenous firms and workers:[16]

$$W = \alpha(F + \kappa\theta) + (1 - \alpha)b. \tag{29}$$

In the baseline version of the model, the wage positively responds to changes in aggregate labor market tightness. The higher the labor market tightness ($\theta = {}^{V}\!/_{U}$), the more difficult it is for firms to hire a worker. If there is fierce competition between many firms for relatively few unemployed workers, wages are higher. Equation (28) generalizes this notion for a framework with heterogeneous agents. Note that equation (28) collapses to the textbook equation (29) in case of constant labor productivity and homogenous workers and firms. This underlines the close connection between this model and the baseline model and is a key to understanding the augmented model's dynamics. The simulation results presented in section 3 include a baseline case that confirms that the model without modifications exactly reproduces the results presented in Shimer (2005).

The key difference between the baseline DMP model and the augmented version with sorting presented in this paper is that the match-specific wage does not depend on aggregate labor market tightness, but on the match-specific ratio of the negotiating firm's and worker's reservation values. This object, relative labor market tightness, is nothing butthe ratio of all outside options, properly weighted for each point in the type space, both from the perspective of the firm and the worker. The intuition behind this modification of the baseline model is straightforward: aggregate tightness may be high, but if there are a great many unemployed workers in the segment of the type space that matters for a given firm, it has no incentive to pay a high wage and the worker extracts less. Thus, intuitively, the worker's bargaining position does not depend on scarcity or abundance of other unemployed workers outside the matching set of the potential employer. The worker's bargaining position is worsened, however, in case there are many unemployed of a very similar type, that is, who also appear in the same firm's matching set. The quotient, relative labor market tightness, is a key object since it generates the endogenous wage rigidity that drives the results of this paper. Note that in the baseline calibration of the search and matching model, the worker has a higher bargaining power than the firm.[17] It turns out that the fixed point of the surplus flow

[16] Note that this equation corresponds to Shimer's Equation (7) (Shimer (2005), p. 41), which is a slightly generalized version of Equation (1.20) in Pissarides (2000), p.17.

[17] Following the Hosios (1990) condition for socially efficient vacancy posting, the bargaining power of the worker is set equal to the matching function's elasticity parameter ξ. Based on empirical evidence, this parameter value is usually set to 0.72 in the literature, implying a higher bargaining power and, thus, a larger share of surplus for the worker.

equation, which is the primary equilibrium object presented in the next section, critically depends on this assumption. With unequal bargaining powers, the firm's and worker's reservation values, i.e. the integrals over their respective matching sets, are asymmetric.

Using the standard calibration of the DMP model, firms' matching sets are wider in equilibrium. Their reservation value is higher and also more volatile in response to shocks. In equation (23), therefore, relative labor market tightness has to be smaller than one, given that the firms' reservation value is the denominator. This finding is the key property of the DMP model with sorting as it generates the endogenous wage rigidity. As compared to aggregate labor market tightness in the baseline model, which commonly equals 1 in steady state, relative labor market tightness is smaller than one and leads to a lower equilibrium wage for every point in the type space. In response to a shock, this property dampens the impact of stochastic labor productivity on the wage.Moreover, the negotiated wage that worker s would receive while being employed at firm c obviously depends on match-specific output, $F(s, c)$. Thus, match-specific wages also depend directly on the production function and the degree of sorting in the labor market. This is the second difference with respect to equation (29) with constant and type-independent match output F. Section 4 shows in detail how the effect on wage dynamics can be decomposed and, thus, to what extent both modifications are responsible for the amplification effect we find.

2.5 The Steady State

Due the integrals in the Bellman equations (Equations (16) and (18)), in which both the integrand and the domain of integration are endogenous, this model cannot be solved analytically. This is a well-documented property of this class of models. For example, Teulings and Gautier (2004) note that "it is precisely this complexity which has prohibited progress in this type of models."[18] Teulings and Gautier (2004) propose a second-order Taylor series approximation of the steady state around the frictionless, Walrasian equilibrium to circumvent this complication. It has been also shown in the literature, however, that iterative fixed point procedures can be applied to numerically approximate the steady state of this class of models. The solution of Shimer and Smith's model (2000) is computed iteratively on a discrete grid. The iterative procedures used to define the equilibrium in Lise et al. (2013) and Hagedorn et al. (2012) are in principle similar to this approach. I make use of the same computational strategy, that is, value

[18] See Teulings and Gautier (2004), p. 559.

function iteration on a discrete grid, to find the steady state of the model. Using this technique, the equilibrium can be computed reliably with high precision in a relatively short amount of time even for a relatively finely spaced grid.[19] Simulations under uncertainty, however, as they are carried out to analyze model dynamics, are much more computationally costly. Computing the results presented in this paper, which use 100 workers and firms only, takes more than a week on a standard personal computer.

The steady state of the search and matching model with ex ante heterogeneity and sorting is a quadruplet $(S(s,c), \mathcal{M}, g_u(s), g_v(c))$. Equilibrium surpluses, $S(s,c)$, determine the optimal matching sets $\mathcal{M}$ via the simple non-negativity condition. The following functional equation for match-specific equilibrium surpluses is the result of simply summing up equations (15) to (18):

$$S(s,c) = F(s,c) + \beta\mathbb{E}\left[(1-\delta)\max\{S(s,c),0\}\right] - \left(b + \beta\mathbb{E}\left[\alpha q_u(\theta)\int\limits_{M_c(s)} \frac{g_v(c)}{V} S(s,c)\ \mathrm{d}c\right]\right) - \left(-\kappa + \beta\mathbb{E}\left[(1-\alpha)q_v(\theta)\int\limits_{M_s(c)} \frac{g_u(s)}{U} S(s,c)\ \mathrm{d}s\right]\right). \tag{30}$$

This is the equilibrium surplus flow equation. The endogenous wage rigidity, the key feature of this model, arises due to the way in which the bargaining powers enter equation (30). Note that α acts like a weight on the firms and workers reservation value, respectively. In case of $\alpha \neq 0.5$, an asymmetry arises in equilibrium, which is illustrated graphically below in figure... With a higher bargaining power on the side of the worker, i.e. $\alpha > 0.5$, the arising asymmetry implies that the surplus falls more steeply in the worker type than in the firm type offside the optimal allocation. The firms compensate this by optimally choosing a wider matching set. The joint distribution of active matches in equilibrium, which pins down the type distributions of unemployed workers and vacant firms, $g_u(s)$ and $g_v(c)$, is computed using the steady-state flow condition of a stationary search and matching model. Computing the integrals of the latter two probability distribution functions yields the aggregate flows of unemployed workers and vacant firms, which in turn determine aggregate labor market tightness, arrival rates,

[19] Finding the equilibrium for 1,000 workers and firms takes only a couple of seconds on a standard personal computer.

and the flow of matches.

$$\delta g_m(s,c) = g_u(s)\ q_u(\theta)\ \frac{g_v(c)}{V}\ \mathbb{1}\{S(s,c) \geq 0\}. \tag{31}$$

The LHS represents the number of dissolved matches, which has to equal new match formation for all pairs (s,c) that form in equilibrium, i.e., $\forall(s,c) \in \mathcal{M}$. Recall that the necessary condition for a match is a weakly positive surplus, represented by the indicator function in Equation (31). If this condition is satisfied, new match formation arises for every possible pair as the product of the density of an unemployed worker of type s, the aggregate job-finding rate $q_u(\theta)$, and the probability of meeting any specific firm type c in the labor market, which is nothing but the density of this type of vacant firm divided by the overall number of vacancies.

To approximate search equilibrium, I alternate between computing the fixed point of the surplus flow equation (Equation (30)) and updating the distribution of active matches using the steady-state flows (Equation (31)). This procedure is repeated until the decision rule converges as described in Hagedorn et al. (2012).[20]

3 Numerical Simulations and Empirical Analysis

The Shimer Puzzle revolves around the search and matching model's ability (or lack thereof) to explain stylized facts of labor market data. Methodologically, the empirical performance of this class of structural models is tested using numerical simulations in a stochastic environment. The focus lies on labor productivity shocks to add a stochastic dimension to the model. While the dynamics of the baseline model (Pissarides (2000)) are consistent with a number of important stylized facts, for example, a strong and persistent negative correlation between vacancies and unemployment (the Beveridge curve), Shimer (2005) shows that the model fails to generate a sufficient degree of volatility in response to a productivity shock.[21] Shimer (2005) emphasizes that his approach "is not an attack on the search approach to labor markets, but rather a critique of the commonly-used Nash bargaining assumption for wage determination."[22] My results show, however, that it is indeed possible to solve the volatility puzzle without departing

[20] This procedure is proposed in Hagedorn et al. (2012), Appendix II. Convergence is achieved once the absolute difference of the surplus between two iterative steps is less than 10^{-12}.

[21] Shimer (2005) also considers separation rate shocks, and comes to essentially the same conclusions. Stochastic labor productivity is far more common in the literature, which is why concentrate on this concept.

[22] Shimer (2005), p. 45.

from the common Nash bargaining framework by generalizing the model to account for explicit heterogeneity and complementarity.

I find that second moments of simulated data from the augmented model are considerably closer to the data than are those generated using the baseline model. Thus, the search and matching model with heterogeneity and sorting is able to solve the Shimer puzzle. The amplification is both a result of the complementarity in production, i.e. positive sorting, and the endogenous wage rigidity.[23] The degree of model-generated wage rigidity, as calculated below, turns out to be supported by empirical evidence from the U.S. labor market as reported by Haefke et al. (2013).

In the following, a numerical simulation exercise similar to that of Shimer (2005) is conducted. I simulate the model in log-linearized form using Dynare (Adjemian et al. (2014)) and compare the dynamics of a baseline model, which reproduces Shimer's results, and the equilibrium search and matching model with sorting presented in section 2. Since the models are log-linearized around the steady state, numerical results must be understood as elasticities, i.e., percentage deviations from the steady state. As in Shimer (2005), one period of time in the discrete model is set to be one quarter.

3.1 Calibration Based on Business Cycle Properties

Table 2 shows the calibration of the model based on quarterly U.S. labor market data used for the simulation exercise. To ensure direct comparability of the dynamics of the augmented model with the results in Shimer (2005), identical parameter values are used. As Kydland and Prescott (1996) state, parameter values for model calibration must not be chosen so as to bring the model as close to the data as possible. In fact, Shimer's main point is that the baseline model is *not* close to the data in terms of volatility. Instead, parameter values for calibration should be chosen to be in line with long-run time series observations and micro-evidence from cross-sectional data. Therefore, Shimer (2005) computed the values for b, κ, and δ directly from U.S. labor market data for the period of 1951—2003. The value of non-market activity b is computed using U.S. replacement rates in relation to mean labor income under the assumption that this value is entirely determined by unemployment benefits. Vacancy posting costs κ are set to resemble average U.S. labor market tightness. A value of 0.1 for the separation rate represents an average employment spell of about 2.5 years in the United States in the relevant

[23] For a decomposition of the amplification into the effect of complementarity and wage rigidity see section 4.

Table 2: Parameter values for the quarterly calibration of the search and matching model for the U.S. labor market (1951—2003)

Parameter	Symbol	Value	Source
Substitutability	γ	2	Gautier and Teulings (2014)
Discount factor	β	0.99	Shimer (2005) (discount rate 0.012)
Separation rate	δ	0.1	Shimer (2005)
Workers' bargaining power	α	0.72	Hosios (1990) condition: $\alpha = \xi$,
Matching function elasticity	ξ	0.72	Shimer (2005)
Value of non-market activity	b	0.4	Shimer (2005)
Vacancy posting costs	κ	0.213	Shimer (2005)
First order autocorrelation	ρ	0.765	Hagedorn and Manovskii (2008)
Standard deviation	σ_ϵ	0.013	Hagedorn and Manovskii (2008)

period (1951—2003).[24] In the period relevant for the model, annual interest rates were roughly 5% in the United States. Thus, the quarterly discount rate is set to 0.012, which translates to a discount factor (as it appears in the model equations) of $\beta = {}^{1}/_{1.012} \approx 0.99$. The Hosios (1990) condition for socially efficient vacancy posting in the decentralized equilibrium leads to the equalization of workers' bargaining power and the elasticity parameter of the matching function. The matching function elasticity, in turn, is set by Shimer (2005) at 0.72, which is within the empirically-supported range reported by Petrongolo and Pissarides (2001) and resembles the average job-finding rate in U.S. labor market data.The only parameter that is not part of the baseline search and matching model and its calibration proposed in Shimer (2005) is the degree of complementarity γ of the production function (Equation (1)). I use a value of 2 as proposed in Gautier and Teulings (2014) and provide a robustness check in section 4. Given the production function (Equation (1)), a relatively low degree of complementarity suffices to achieve the desired amplification effect. The value of γ has to be high enough, though, to induce the agents to be picky in equilibrium.[25] As introduced in section 2, the labor productivity Y_t can be imagined as an underlying technology that enables labor to be used productively.

[24] δ is the arrival rate of a Poisson process. Thus, average waiting until the modeled event occurs is simply $\frac{1}{\delta}$. Thus $\frac{1}{0.1} = 10$ quarters $= 2.5$ years.

[25] A low degree of complementarity leads to an equilibrium with matching sets that comprise the whole type space.

It is a stochastic process, that is, it varies over time but is type-independent and affects all matches symmetrically. As in Shimer (2005), it is normalized to 1 in steady state and a calibrated to resemble empirical labor productivity in the U.S. over the relevant period of time. Regarding the functional form, I follow Hagedorn and Manovskii (2008) and set up stochastic labor productivity as a first-order autoregressive process:

$$Y_t = Y_{t-1}^{\rho}\, e^{\epsilon_t} \leftrightarrow y_t = \rho\, y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2). \tag{32}$$

Lower-case letters represent labor productivity in logs. $\rho \in (0, 1)$ captures the degree of first-order autocorrelation of this AR(1) process. Innovations are drawn from a Gaussian distribution with mean 0 and standard deviation σ_ϵ. Both parameters are set to match quarterly U.S. labor productivity.[26] All values in Table 2 are based on quarterly data. Shimer's (2005) simulation results are reported as deviations from an HP trend and this can be seen as a convention in the literature.[27]

3.2 The Amplification Effect of Sorting

Once the simulations have been conducted, I find that second moments of simulated time series data from the search and matching model with ex ante heterogeneity and positive sorting are very close to the observations for U.S. labor market data. In particular, the volatility of unemployment, vacancies, labor market tightness, and the job-finding rate are much closer to the empirical second moments than the results reported by Shimer (2005) using the baseline search and matching model. Table 3 compares the main results of this analysis to those of Shimer (2005) and the data. The first two rows of Table 3 show the well-known result reported in Shimer (2005). The original Shimer Puzzle is immediately apparent. The standard deviations of unemployment, vacancies, labor market tightness, and the job-finding rate in simulated time series data miss the standard deviations in the data by a factor of about 10—20. This severe lack of volatility—the

[26] Shimer (2005), Hornstein et al. (2005) and Hagedorn and Manovskii (2008) report the parameter values necessary to represent U.S. labor productivity "as seasonally adjusted quarterly real average output per person in the non-farm business sector constructed by the BLS" (Hagedorn and Manovskii (2008), p. 1694).

[27] The Hodrick-Prescott (HP) filter is a technique for decomposing the trend and the cyclical component of a time series (see Hodrick and Prescott (1997)). Shimer (2005) sets the smoothing parameter of the filter to $\lambda = 10^5$ instead of to the more common value of $\lambda = 1600$ for quarterly data. This makes the cyclical component more volatile and more persistent. I use the same value as Shimer to generate comparable moments. Hornstein et al. (2005) point out that a more volatile trend, using the common smoothing parameter $\lambda = 1600$ for quarterly data, "has almost no effect on the relative volatilities" (p. 23).

Table 3: Standard deviations of labor market variables

	Standard deviations	U	V	θ	$q_u(\theta)$
1.	U.S. data	0.190	0.202	0.382	0.118
2.	Results of Shimer (2005)	0.009	0.027	0.035	0.010
3.	This paper (baseline)	0.009	0.026	0.035	0.010
4.	This paper (sorting)	0.215	0.617	0.831	0.234

Rows 1 & 2: Based on Tables 1 and 3 in Shimer (2005), pp. 28/39. Calculated based on quarterly U.S. data, 1951—2003. Rows 3 & 4: Standard deviations of simulated data from the author's model with and without sorting. All moments come from HP-filtered data with $\lambda = 10^5$.

tune of several orders of magnitude—is what began the discussion on how to fix the dynamics of the model to make it appropriate for policy analysis. As mentioned before, a central argument in this literature is that rigid wages can amplify the model's response to a productivity shock.

The model with sorting developed in this paper endogenously generates rigid wages and a serious amplification effect without additional assumptions and without changing Shimer's (2005) calibration strategy.[28] The key figures are reported in the fourth row of Table 3. The second moments of time series data for the main labor market aggregates from the augmented model with sorting are much closer to the data than the corresponding numbers generated using the model without sorting. The standard deviations reported in the third row are for comparison. They show that the model developed in this paper reproduces the results of Shimer (2005) when heterogeneity and complementarity are switched off. The model reduces to the baseline model, as illustrated by the wage equation in Section 2.4. This shows that the amplification of the model's dynamics using the augmented version is indeed a consequence of the modifications I introduced. Additionally, it is ensured, that the results in row 4 are directly comparable to Shimer (2005).

The standard deviation of the HP-filtered unemployment time series (0.215) is very close to what we see in the data (0.190). This shows that the search and matching model with sorting is able to generate realistic employment dynamics via the channels that have been emphasized in the light of empirical evidence. The simulated standard

[28] Rigid wages in this dynamic context mean that the elasticity of wages with respect to labor productivity is less than 1. In the baseline model, wages follow the labor productivity process one-to-one, i.e., they are fully flexible. The endogenously generated wage rigidity and its magnitude in light of empirical evidence are discussed in Section 4.1.

deviations of vacancies (V), labor market tightness (θ), and the job-finding rate ($q_u(\theta)$) are also much higher than in the baseline model. They even overshoot their empirical counterparts by a factor of roughly two. Recall that both the tightness and the job-finding rate are simple functions of V. Thus, the excess volatility must stem from firm's vacancy posting decisions which are obviously amplified too much using the simple functional form assumptions for the production structure with complementarity and the standard parametrization from the literature explained above. While it is interesting to think about ways that avoid this discrepancy between the volatility of unemployment and vacancies, it is important to note that the aim of the exercise presented in this paper is not to exactly match the empirical moments; rather, the main finding is that all standard deviations of simulated time series data generated using the model with sorting are reasonably close to the data, whereas the baseline model misses empirical second moments by several orders of magnitude.[29]

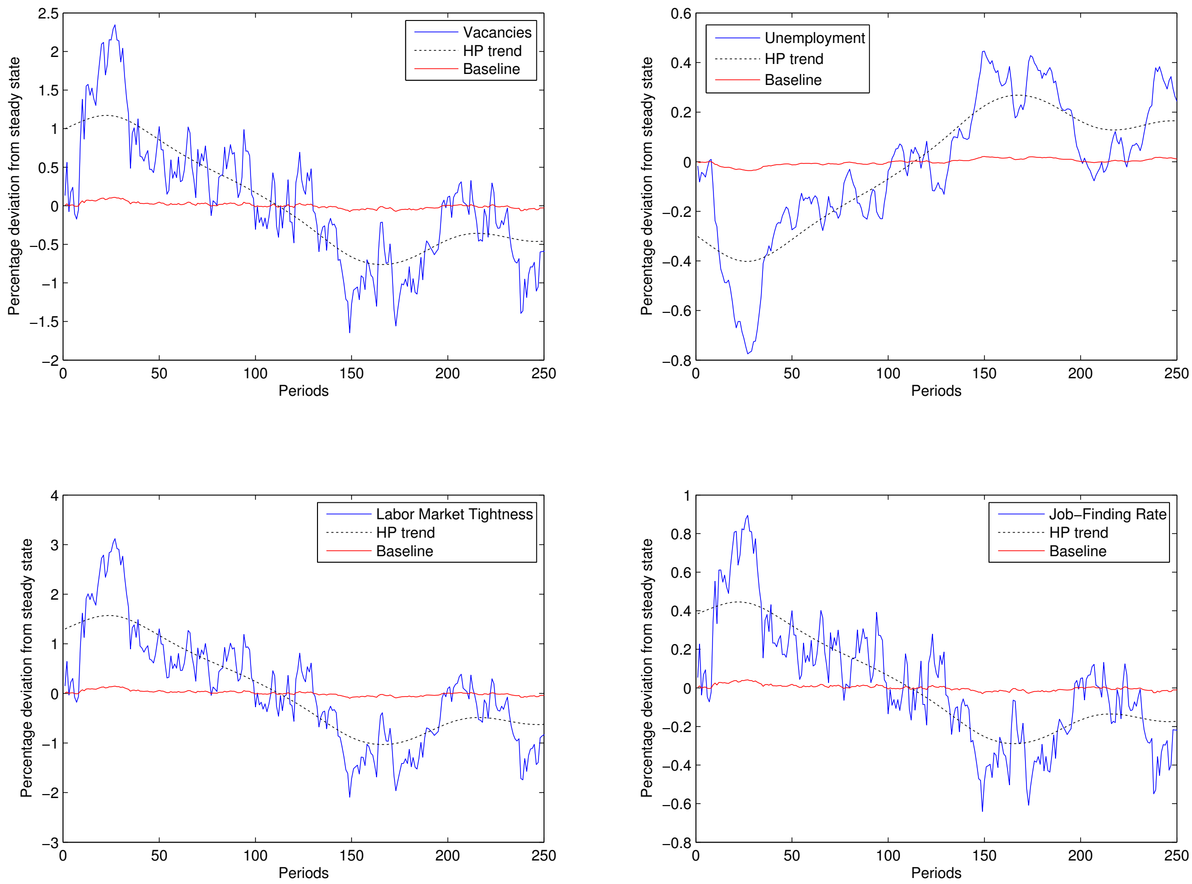
Figure 1 shows simulated time series, in terms of percentage deviations from steady state, for the main labor market variables over 250 quarters: unemployment, vacancies, labor market tightness, and the job-finding rate. The red series come from the baseline model without sorting; the blue series are simulations of the model with sorting. The huge amplification effect in response to stochastic labor productivity is obvious for all four variables. The four series also illustrate a number of features of the search and matching environment. While the number of vacancies, labor market tightness, and the job-finding rate are obviously positively correlated and go up in response to a favorable productivity shock, unemployment moves in the other direction and falls in response to the shock. This negative correlation between unemployment and vacancies resembles the well-known Beveridge curve, a feature of the data that both the model with sorting and the baseline model generate.

4 Robustness Checks

This section takes a deeper look at the simulation results presented in the previous section, i.e. at the documented amplification of the dynamics of the presented search and matching model with two-sided heterogeneity and sorting. To learn more about the channels that lead to the improved empirical performance, the results shall be illuminated

[29] Note that due to the complexity of the model, results reported here come from a single simulation on the whole type space, approximated by a discrete grid with dimensions 100^2. The Shimer (2005) results are bootstrapped standard errors from 10,000 simulations of the baseline model. Thus, some caution is warranted when interpreting the results.

Figure 1: Simulated time series of labor market variables for the sorting model (blue) and the baseline model (red)



from three perspectives:

1. Is the endogenously generated wage rigidity of reasonable magnitude given existing empirical evidence for the U.S. labor market?

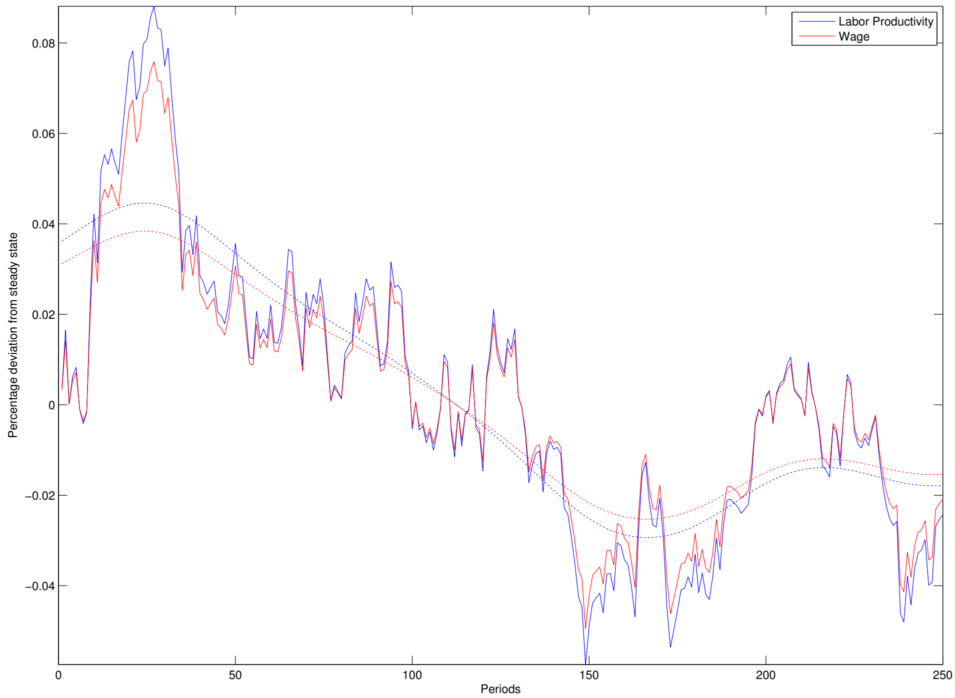
2. What channels contribute to the generated amplification of the model's dynamics?

3. How do the results respond to changes in the degree of complementarity γ?

4.1 Rigid Wages

Shimer (2005) does not analyze wage data for the U.S. labor market and does not use the wage-formation mechanism to characterize the equilibrium of his model. Therefore, I analyze the degree of model-generated wage rigidity separately. Perfectly flexible wages, as in the baseline model, respond directly to stochastic changes of labor productivity in the form of a one-to-one co-movement. The elasticity of wages with respect to labor productivity is therefore 1. Totally rigid wages, on the other hand, would not react at all to changes in labor productivity, implying an elasticity of 0 (as in Hall (2005)). Figure 2 shows simulated time series of wages and labor productivity for both models,

Figure 2: Simulated time series of wages and labor productivity



with and without sorting. The blue series represents stochastic labor productivity. In the baseline mode with fully flexible wages, the wage time series looks exactly like the time series of labor productivity since wages adjust instantaneously. Wage dynamics generated using the model with sorting, illustrated by the red time series, turn out to be less volatile. They do not fully adjust even though the model is simulated using a the very same calibrated labor productivity process. Since wages do not adjust one-to-one to labor productivity, the elasticity of wages with respect to labor productivity must be somewhat smaller than 1. This is exactly the result one would expect. In the baseline model, wages are too responsive. After a favorable shock, they soak up most of the extra productivity since they adjust perfectly flexibly and this leads to the insufficient responsiveness of other model variables. This is the essence of the Shimer Puzzle. In the model with sorting, however, the one-to-one link between wages and labor productivity is dampened. The arising wage rigidity, which is visible in Figure 2, limits the extent to which wages rise in response to the shock. Therefore, wages are less volatile, which gives rise to the amplification effect documented above.

To check whether the model-generated rigidity is of a reasonable magnitude, I refer to the empirical literature on this issue. I rely on Haefke et al. (2013).[30] The authors find an an elasticity of wages with respect to labor productivity of 0.8 with a relatively large standard errot of 0.4. This elasticity can be computed simply as the coefficient η_1 using a simple linear regression of wages on labor productivity in logs and first differences:

$$\Delta \log W_t(s,c) = \eta_0 + \eta_1 \Delta \log Y_t + \varepsilon_t \tag{33}$$

Running this simple regression using the simulated time series data of the model with sorting yields a wage elasticity of 0.861. Given the relatively large standard error of the estimate reported by Haefke et al. (2013) (0.4), the elasticity of wages with respect to labor productivity generated by the model with sorting lies well in the empirically supported range. Hagedorn and Manovskii (2008) also compute the relevant elasticity from U.S. wage and productivity data and report a coefficient of 0.449, which is within the range supported by Haefke et al. (2013) as well, albeit at the lower end. It is reassuring that the rigidity implied by the model with sorting lies close to the empirical benchmarks. Furthermore, Hall and Milgrom (2008) provide an analysis of the degree of wage flexibility needed to enable their adapted DMP model to reproduce empirical unemployment fluctuations. They propose using an alternating offer bargaining game instead of the common but not undisputed Nash bargaining approach.[32] Their model implies an elasticity/derivative of about 0.7, which is again in line both with the finding of this paper and the empirical evidence.

Given the strong amplification effect of the augmented model documented above, the degree of wage rigidity I find appears to be small at first sight. Thus, it is important to note that the empirically backed elasticity of 0.861, and thus the endogenous wage rigidity, is not the only source of amplification in the model with sorting. The next section will decompose the amplification in the effect of rigid wages and a second effect of sorting on firms' job creation decision.

[30] Haefke et al. (2013) focus primarily on the different degrees of wage rigidity for newly hired workers as compared established employment relationships. The length of an employment spell plays no role for wage determination in a search and matching model. Thus, I can compare the reported elasticity of wages with respect to labor productivity for new hires to my results. Haefke et al. (2013) note that ths value "is an appropriate and informative calibration target for search and matching models."[31]

[32] Recall the quote at the beginning of this section: Shimer himself concludes from his analysis that the Nash bargaining approach lies at the heart of the volatility puzzle.

4.2 Decomposition

Besides the direct effect that the wage rigidity has on firms' expected wage payments, sorting unfolds a second effect on forward-looking vacancy posting decisions. Recall the firms' job creation condition with $^M/_V$ plugged in for $q_v(\theta)$ and time indices added:

$$\frac{\kappa V_t}{M_t} = \beta(1-\alpha)\mathbb{E}_t \left[\int_{M_s(c)} \frac{g_u(s)}{U} S(s,c) \, \mathrm{d}s \right]_{t+1} . \qquad (34)$$

The value that the firm evaluates on the RHS in a forward-looking manner depends on the non-monotonous surplus function that has an interior maximum. After a positive productivity shock hits the economy, the non-monotonous surplus function shifts upwards. Firms re-optimize their forward-looking job creation decision: they choose wider matching sets because the surplus from an increased number of potential matches is larger than zero. Therefore, a higher number of vacancy postings is the optimal response to a positive shock. This generates additional amplification besides the wage-rigidity effect. The firms' expected surplus changes along two margins: In response to a positive shock, firstly, the expected potential wage payments to every worker in the equilibrium matching set are lower due to the rigidity. Secondly, additional workers with potentially positive surplus enter the equation due to wider matching sets. This leads to a higher expected value of future surpluses on the side of the firm and, consequentially, amplification. As can easily be seen from equation (34), a higher RHS has to correspond to a larger number of vacancy postings V, which is the firm's control variable. Thus, in this model wages do not need to be extremely rigid (as in Hall (2005)) to generate sufficient volatility. To find out to what degree both the wage rigidity and the complementarity effect contribute to the documented amplification, I take the calculated wage elasticity of 0.861 and impose it on a standard model without sorting and heterogeneity. The gap in volatility, which cannot be explained by the effect of rigid wages, must then be the direct amplification effect of sorting. The last row of table 4 shows the results of this exercise. The volatility of labor market variables from a simulation of the baseline model with imposed wage rigidity remain too small as compared to the data. As one would expect from the model's well-known structure, the volatility of all variables is higher by a factor of 3-5 due to the rigidity. However, this is not sufficient to generate the degree of volatility we see in the data. Especially the standard deviation of unemployment remains tiny in the model with wage rigidity only. I conclude that the

Table 4: Decomposition of the amplification effect

	Standard deviations	U	V	θ	$q_u(\theta)$
1.	U.S. data	0.190	0.202	0.382	0.118
2.	Results of Shimer (2005)	0.009	0.027	0.035	0.010
3.	This paper (baseline)	0.009	0.026	0.035	0.010
4.	This paper (sorting)	0.215	0.617	0.831	0.234
5.	This paper (rigidity only)	0.038	0.109	0.146	0.041

Rows 1 & 2: Based on Tables 1 and 3 in Shimer (2005), pp. 28/39. Calculated based on quarterly U.S. data, 1951—2003. Rows 3, 4 & 5: Standard deviations of simulated data from the author's model. All moments come from HP-filtered data with $\lambda = 10^5$.

direct effect of sorting on the model dynamics is important. The small but empirically reasonable wage rigidity alone does not sufficiently amplify the model. An additional amplification factor of 2-5, depending on the variable, is necessary to align the model with the data. This additional amplification arises from the dynamic adjustment of the matching sets in response to shocks.

4.3 The Choice of γ

As a final robustness check, I examine to what extent the results I document depend on the chosen of the degree of complementarity in the production function. As mentioned before, γ is the only parameter of the model with sorting that has not been thoroughly discussed in the literature, primarily because only very few other papers analyze equilibrium labor search models with sorting.[33] Recall that the higher is γ, the stronger is the degree of complementarity in production and the more picky the agents will be in equilibrium. I compare the value of $\gamma = 2$, as it is proposed in the literature (Gautier and Teulings (2014), Hagedorn et al. (2012)) and used to compute the results presented in section 3, with a value of $\gamma = 1$, that is, a lower degree of complementarity.[34] As table 5 shows, a lower degree of complementarity and, thus, wider matching sets than in the baseline calibration of the search and matching model with sorting further amplify the model to an empirically implausible degree. This confirms that the choice of γ is

[33] The most closely related work is discusses in section 5.

[34] I chose a lower degree of complementarity first to check how the model behaves as the production structure gets closer to the baseline case without complementarity. Robustness tests with more extreme degrees of complementarity are work in progress.

Table 5: The influence of γ on amplification

	Standard deviations	U	V	θ	$q_u(\theta)$
1.	U.S. data	0.190	0.202	0.382	0.118
2.	Results of Shimer (2005)	0.009	0.027	0.035	0.010
3.	This paper (baseline)	0.009	0.026	0.035	0.010
4.	This paper ($\gamma = 2$)	0.215	0.617	0.831	0.234
5.	This paper ($\gamma = 1$)	0.260	0.746	1.006	0.283

Rows 1 & 2: Based on Tables 1 and 3 in Shimer (2005), pp. 28/39. Calculated based on quarterly U.S. data, 1951—2003. Rows 3, 4 & 5: Standard deviations of simulated data from the author's model. All moments come from HP-filtered data with $\lambda = 10^5$.

highly important for the performance of the augmented model. The results indicate that a slightly higher value of γ might be conductive to aligning the model with the data. However, sound empirical evidence for the importance of sorting in the U.S. labor market is necessary to make an informed calibration choice with respect to this parameter value in this class of models. This is an interesting avenue for future research.

5 Discussion in Relation to the Literature

This paper constructs an equilibrium search and matching model with two-sided heterogeneity and sorting. Gautier et al. (2006) provide a good and compact overview of the development of this class of labor search models. The relation to previous literature is best understood from two reference points. The first reference point is the assignment game among heterogeneous agents in a frictionless market following Becker (1973). Becker characterizes the equilibrium of a marriage market, that is, a market with non-transferable utility. In this frictionless, neoclassical model, all types are only willing to match with their one optimal counterparts to form matches which jointly maximize payoffs. Thus, matching sets consist of only a single element. Shimer and Smith (2000) take this model out of its Walrasian equilibrium by adding frictions and costly search. Additionally, they consider transferable utility, thereby building a bridge to labor market applications of the theory. In the paper, technical conditions for positive assortative matching in equilibrium are derived and existence is proven: in a nutshell, types need to be complements in production. Thus, models featuring positive sorting rely on a supermodular production function which is increasing in its two arguments, the worker's skill and the firm's productivity, and twice continuously differentiable.

The second reference point is the baseline search and matching model of the labor market as outlined in the textbook by Pissarides (2000) and used in—among many other papers—Shimer (2005). The DMP model has been very successful due to its ability to explain equilibrium unemployment and a number of important stylized facts of labor market data (e.g. the Beveridge curve). Due to the fact that it abstracts from heterogeneity and thus considers only one representative type of workers and firms, the matching sets of those types embrace the whole type space.[35] In other words, every agent is willing to match with every other agent, but frictions limit the number of encounters in the labor market and, therefore, unemployment exists. Wages are totally flexible in this model. In a dynamic framework with aggregate uncertainty, they adjust instantaneously and follow labor productivity one-to-one. This point is central to the discussion started by Shimer (2005) and Hall (2005). Shimer (2005) points out that even though the standard search and matching framework appealingly predicts the long-run equilibrium of the labor market, the volatility of data from stochastic simulations of the baseline model is too small by a factor of up to 20 compared to empirical second moments of U.S. labor market data. This obstacle has become known as the Shimer Puzzle. Departure from totally flexible wages, that is, incorporating rigid wages into the baseline model, has been used to solve this problem since rigid wages increase firms' incentives to create jobs in a cyclical upswing, that is, in response to a positive productivity shock.[36] Due to the rigidity, wages no longer follow labor productivity one-to-one. Thus, wages become less volatile and the dynamics of the model are amplified since more vacancies will be posted until the firms' free entry condition is satisfied. However, the assumptions used in the literature to justify rigid wages are highly debated. Hall (2005) shows that the volatility puzzle vanishes once wages are made fully inflexible. This, however, implies a counterfactual wage volatility of zero. Hagedorn and Manovskii (2008) show that the dynamics can be amplified by increasing the value of the workers' outside option of non-market activity. A higher calibrated value of the respective model parameter leads to lower wage outcomes in the Nash bargaining game, however, with rather unrealistic consequences.[37]

35 Which is simply a point.

36 The baseline model is calibrated to ensure that vacancy posting is socially efficient according to the Hosios (1990) condition. This leads to a too-high share of the additional surplus for the worker and diminishes firms' incentives to post additional jobs given free entry, which leads to the well-known problem that labor market variables react insufficiently to a shock.

37 Hornstein et al. (2005) and Costain and Reiter (2008) conclude that the calibration strategy proposed by Hagedorn and Manovskii (2008) is implausible because a 15% increase in the value of

I construct a model in between the two reference points: in a labor market with search frictions and assortativeness through complementarity, suboptimal matches between heterogeneous jobs and workers arise in equilibrium and persist through time. This setup generates sufficient volatility in a search and matching model without the additional assumptions that earlier approaches make.[38]

Frictions rationalize the formation of matches beyond the optimal allocation of workers to jobs because heterogeneity and match-specific output give rise to the possibility that the option value of continued search for the optimal partner could be lower than the option value of starting production immediately. The key object of this optimal stopping problem is the production function and the degree of complementarity between its inputs, worker skills and firm productivity. It governs the acceptable degree of suboptimality, that is, the acceptable divergence from the optimal allocation. The higher the degree of complementarity between the inputs of the production function is, the higher is the penalty from a suboptimal match in form of foregone output leading to narrower matching sets and a higher aggregate welfare-loss relative to the optimal allocation. The production function's degree of complementarity is thus a parameter of supreme empirical interest, also because the degree of sorting contains important implications for policies which are concerned with the optimal allocation of scarce resources in the labor market. A large literature attempts to identify the sign and strength of sorting using matched employer-employee data and reduced form empirical models with worker and firm fixed effects representing unobservable worker and firm characteristics. (see Abowd et al. (1999) and follow-up papers). While adding fixed effects considerably improves these models' explanatory power regarding wage dispersion, many authors inferred that sorting does not systematically affect wage formation because the correlation between the two fixed effects is typically statistically indistinguishable from zero. If positive sorting played a significant role in explaining wage formation, one would expect to find a strong positive correlation between the two fixed effects.

An empirical and intuitive explanation for this small correlation of worker and firm fixed effects is the so-called limited mobility bias (Andrews et al. (2008), Andrews et al.

non-market activity implies that the unemployment rate doubles.

38 The impact of worker heterogeneity on the dynamics of a search and matching model is also in the focus of an interesting article by Pries (2008). He analyzes how worker heterogeneity can impact the dynamics of a search and matching model, however, without analyzing wage formation in particular. He introduces heterogeneity, two types of workers and homogenous firms, in a much simpler way than this paper. Pries (2008) finds some amplification, which he ascribes to a changing composition of the pool of unemployed workers over the business cycle. However, this modification does not to sufficiently amplify the model.

(2012)), which implies that estimated correlations are biased downward due to estimation error. A conclusion regarding the importance of sorting for wage determination would thus be misleading. The fewer there are observations of movers between firms in matched employer-employee data, the larger the bias and the smaller is the estimated correlation, hence the name.[39]

Eeckhout and Kircher (2011) present a theoretical argument why one cannot conclude that sorting is irrelevant for match formation and wage determination in the labor market based on inference from reduced form empirical models. The key argument is that in theory, assortative matching leads to type-specific wages that, for a given worker, do not monotonically increase in the type of the firm. The fixed effects methodology used in studies that build on Abowd et al. (1999) cannot account such non-linear relations between the unobserved time-invariant determinants of wages as it necessitates additive separability. A theoretical model with sorting, however, implies an inverted U-shape pattern of a given worker's wages in the firm type. This is also a feature of the model presented in this paper. Beyond the extremum of this non-monotonous function, which represents the first best allocation of this worker and his highest achievable wage, remuneration is lower because the worker has to compensate the firm for hiring him and not waiting to match with a more suitable candidate. Due to search frictions, the probability of matching with the optimal type on a continuous type space is zero for both workers and firms. They have to decide about the acceptable degree of mismatch by optimally choosing the width of their matching sets. The condition for a type to be an element of the matching set is a high enough surplus to compensate both parties for the foregone option value of continued search. While it is impossible to identify the sign of sorting in the simple two-period model used by Eeckhout and Kircher (2011) with wage data alone, Hagedorn et al. (2012) show that the production function—and thus the sign and strength of sorting—is non-parametrically identifiable using standard matched employer-employee data and a fully-dynamic search model with discounting. In a different paper that focuses on workers' search intensity as driving force of sorting, Bagger and Lentz (2014) also solve this identification problem and document significant positive sorting for the Danish labor market using structural econometric techniques. Due to a renewed and broad interest in the empirical identification of labor market sort-

[39] Using German data, Andrews et al. (2012) find positive correlations between worker and firm fixed effects for establishments with a large number of movers and thus conclude that the true correlation should be positive for Germany. Mendes et al. (2010) document comparable evidence in support of positive sorting. Using very detailed Portuguese data, the authors construct a measure of firm-specific productivity and thus do not have to rely on firm fixed effects. They find strong evidence for PAM in Portugal.

ing more evidence for different countries can be expected in the near future. For this paper, these new insights are critically important. Since positive sorting is the central assumption of the model, the key parameter of the production function, which governs sorting via the degree of complementarity, has to be calibrated in an informed way using sound empirical evidence, that has also been emphasized in section 4.3.

The contribution of this paper focuses on wage formation and the aggregate dynamics of a search and matching model with two-sided heterogeneity and positive sorting, which is the central assumption. Two classes of production functions are commonly used in models with two-sided heterogeneity and sorting. Shimer and Smith (2000) propose a hierarchical model with global absolute advantage of high types whereas Marimon and Zilibotti (1999) introduce a circular model of production based on comparative advantage.[40] I use the circular model, without loss of generality, because the interior maximum of the production function based on comparative advantage, as it is used in Gautier et al. (2010) and Gautier and Teulings (2014), avoids the corner problem that hierarchical models face. The results presented in this paper do not hinge on this assumption.

Once the model is build an its equilibrium characterized, a common way to assess the empirical plausibility of macroeconomic models is to perform numerical simulations and compare the model-generated dynamics to empirical evidence for the respective variables of interest. Thus, I use a dynamic discrete-time version of the model and calibrated it for U.S. labor market data (1951—2003) as in Shimer (2005). Therefore, comparability to the baseline model is ensured and the wage rigidity and amplification effects I find are a direct result of complementarity and sorting. Furthermore, the model-generated wage rigidity is of a reasonable magnitude given U.S. wage data for new hires. Haefke et al. (2013) quantitatively analyze wage rigidity for the U.S. labor market both for existing employment relationships and new hires. Since the baseline search and matching model with Nash bargaining is essentially a model of new hires, the elasticity of wages with respect to labor productivity for this group, as reported by Haefke et al. (2013), is a reasonable metric to assess the plausibility of the theoretical model and the simulation results.[41]

Rigidities of prices and wages play a key role in modern macroeconomics. Blanchard

[40] Gautier et al. (2006) discuss both approaches in detail and highlight their respective theoretical and empirical implications.

[41] Note that wages do not play any allocational role in a random search model. The Nash bargaining solution simply determines how the surplus is shared in every time-period given the state of the model in the same period. Thus, the length of an employment spell does not influence wages and there is no meaningful distinction between new hires and existing employment relationships.

and Galí (2010) show how nominal rigidities influence aggregate fluctuations in a DSGE model with a search and matching labor market component. The key insight is that the central bank faces a trade-off between inflation and unemployment fluctuations if and only if inefficient unemployment fluctuations arise in the presence of productivity shocks. This can only be the case, however, if the labor market bloc of the DSGE model generates a sufficient degree of amplification in response to shocks, otherwise unemployment would simply not be responsive on a meaningful scale, contrary to the data. Thus, the Shimer puzzle plays a key role also in enabling larger macroeconomic models to generate dynamics which are consistent with the data and, thus, allow for model-based policy advice in the presence of macroeconomic fluctuations. Blanchard and Galí (2010) also survey a number of earlier papers that explored the implications of real and nominal rigidities in different variants of real business cycle (RBC) or New-Keynesian macro (NKM) models and also link them to the Shimer puzzle literature. The models developed by Merz (1995), Andolfatto (1996), Christoffel and Linzert (2005), Krause and Lubik (2007), Faia (2008), and Gertler and Trigari (2009), among many others, are all examples of complex DSGE models, studied by calibration and simulation, that integrate variants of price and wage staggering to create persistence and sufficient amplification for the analysis of different kinds of shocks. Gertler and Trigari (2009), for example, adapt the well-known Calvo (1983) pricing structure for wage formation in the labor market. Rigid wages are generated by only allowing a calibrated fraction of matches to renegotiate wages in every period. This complex and empirically reasonable modification brings the model's dynamics considerably closer to the data. The key contribution of this paper, in the light of the literature surveyed by Blanchard and Galí (2010), is that the search and matching model with two-sided heterogeneity and sorting is able to generate rigid wages as well as large and inefficient unemployment fluctuations, in line with the data, without additional model extensions. Intuitively, the channels of transmission are similar to those explored in the literature. This shows that sufficient amplification in the presence of shocks and the simple Nash bargaining solution are not necessarily mutually exclusive as concluded by Shimer (2005) and Hall (2005).

The model developed in this paper is closely related to the standard DMP models that have been extensively analyzed in the literature mentioned above. It can even be shown that it nests the textbook model. Once the continuous type-space is collapsed into a single point, the case of a representative worker and firm, the equations simplify to their textbook versions. Therefore, the mechanism of transmission in the presence of productivity shocks are transparent and the results are directly applicable to larger macroeconomic models. Thus, incorporating a frictional labor market with sorting into a large macroeconomic model with heterogeneous agents is a fascinating topic for future research. One should, however, keep in mind the well-known limitations of the standard search and matching model. First, the model uses a linear search technology in the form of an aggregate matching function with constant returns to scale. While this approach has not been rejected by the data in numerous empirical studies (for a summary, see Petrongolo and Pissarides (2001)), it is by no means undisputed.[42] Second, the assumption that only unemployed individuals search might be less convincing in a model with heterogeneity. The literature contains some empirical evidence on job-to-job flows (for the United States, see, e.g., Fallick and Fleischman (2004) and Nagypál (2005)). A recent series of papers by Jeremy Lise, Jean-Marc Robin, and co-authors (Lise et al. (2013), Lise and Robin (2013), Lamadon et al. (2013)) apply a sequential bargaining framework in the spirit of Postel-Vinay and Robin (2002) to allow for on-the-job search and poaching. Even though these models generate reasonable dynamics, they cannot be applied directly to the literature on wage rigidity and the Shimer Puzzle, which is the focus and central contribution of this paper. In the context of the model presented in this paper, I argue that the impact of on-the-job search on model dynamics can to a large extent be understood by examining varying degrees of complementarity as in on model dynamics that . Recall that the key property of the model is that workers and firms are picky in equilibrium. The lower the degree of complementarity, the lower is pickiness in equilibrium, that is, the wider are the agents matching sets. On-the-job search has a similar effect on the model's equilibrium. To to the possibility of continued search while being employed, workers save themselves the opportunity to switch to a better match. Therefore, the initial choice which job to accept out of unemployment less binding and, thus, the range of marginally acceptable matches on the worker side is wider. Section 4.3 shows, however, that wider matching sets might amplify the model's dynamics up to an empirically implausible degree.

42 Kohlbrecher et al. (2014) show in a recent working paper how the characteristic co-movement of matches and unemployment/vacancies can also arise in a model without a matching function due to idiosyncratic labor productivity and free entry. Thus, matching function estimations might be seriously biased. Additionally, Borowczyk-Martins et al. (2013) emphasize that empirical models that estimate the elasticities of a standard matching function can be severely biased due to endogeneity. They propose an empirical strategy that eliminates this bias by using lagged values of the key variables as instruments.

6 Conclusions

A large class of macro models uses complex model extensions to generate sufficient dynamics in response to shocks. Often, the labor market is in the center of attention because rigid wages can be source of inefficient unemployment fluctuations over the business cycle, what is a key labor market policy concern. Search and matching models are the workhorse environment in this kind of application. However, the most widely applied version features fully flexible wages and cannot create less-responsive wages by itself.

This paper shows how two-sided ex ante heterogeneity and a production function with complementarity, i.e., positive assortative matching, can help to overcome this shortcoming and improve the canonical search and matching model's ability to match empirical moments of U.S. labor market data. The model endogenously generates a wage rigidity and does not rely on any additional assumptions besides heterogeneity and sorting. This is the key distinction of this paper as compared to earlier approaches to fix the canonical search and matching model's dynamics. The endogenous wage rigidity is of empirically reasonable magnitude. Alone, however, it does not suffice to bring the model's volatility close enough to the data. However, the direct effect of complementarity on firms vacancy posting decisions provides an additional source of amplification, with the results that the model is capable of overcoming Shimer's volatility puzzle. Both effects provides additional incentives for firms to create jobs in response to a favorable productivity shock. This mechanism significantly amplifies the simulated standard deviations of unemployment, vacancies, labor market tightness, and the job-finding rate compared to the baseline model of Shimer (2005), who shows that the canonical search and matching model with homogeneous firms and workers misses empirical business cycle statistics by a factor of 10—20. Aside from the added heterogeneity, our model's elements are standard in the literature and it collapses to the textbook search and matching model (Pissarides (2000)) for the case of homogenous workers and firms. Thus, I also rely on the empirical results of Shimer (2005) to calibrate the model and to ensure comparability of results.

From a theoretical perspective, the augmented wage-formation mechanism in the model with sorting is particularly interesting. It takes into account the fact that heterogeneous firms and workers are picky in search equilibrium, i.e., not every possible match will be formed in the labor market because some pairs are not able to generate a positive surplus. Aggregate labor market tightness in the wage equation is replaced by a measure of relative labor market tightness which takes into account all type-specific outside options for every worker and firm given their endogenous matching sets. Furthermore, I show that the wage becomes non-monotonous in firm type for every worker in a model with sorting, as theorized by Eeckhout and Kircher (2011). Since the optimal, wage-maximizing match never occurs in a frictional labor market, this non-monotonicity delivers the main intuition for the wage rigidity I find.

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