Pricing Discounts in Forensic Economics

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Introduction

In economics, discounts to future consumption are metered by the preferences for current versus delayed consumption. Future utility, $U(x)$, is priced at the discount $V(x,t)=U(x)e^{-\pi t}$ where π is the rate of utility discount (Samuelson, 1937). In finance, utility is not a concern and discounts are the current price of investments which produce future sequences of money payments where r is the rate of return on the investment. The play between economics and finance is the difference between π and r. The individual (*homo economicus*) will be always saving so long as the interest rate, r, exceeds his rate of utility discount π and he consumes when his rate of utility discount exceeds the rate of interest.

In forensic economics, discounts are made to future dollar damages to avoid overcompensating the plaintiff. Essentially, forensic economists compute expected damages and then calculate the amount of an award as a discounted financial asset to replace those damages. Since the forensic economic literature was created by economists, damage discounts have come to be based on the economist's notion of the interest rate as a spot discount instrument. In this paper, we view damage discounts as they are presented in the finance literature. The first section of the paper rehashes standard discounting theory. The next section works through examples of pricing discounts. We then discuss the importance of knowing how discounts are represented in forensic economics: as inclusive or exclusive of financial risk. We conclude with a simple method of pricing financial risk in forensic economic discounts.

Discounting Theory

The theory of capital asset pricing (Ross, 1977) states that for any asset, W, its *ex ante* expected return at the time instant t is

(1) $$E_W = r_t + b_{Wt}$$

where r_t is the riskless rate of interest and b_{Wt} is some function of expected return of the asset in excess of the riskless rate.[1] If W is a riskless asset, then b_{Wt} is zero and the *ex ante* expected return is simply the riskless rate of interest.

From Brigo and Mercurio (2006), the value of a riskless unitary investment ($\overline{W}(0) = 1$) at any time $t \geq 0$ grows according to the differential equation

(2) $$d\overline{W}(t) = r_t \overline{W}(t)dt$$

where r_t is a positive function of time. At time t before the terminal date s, the value of the investment is

(3) $$\overline{W}(t) = \exp\left(\int_0^t r_s ds\right).$$

A first-order expansion of (3) is

(4) $$\overline{W}(t+\Delta t) = \overline{W}(t)(1 + r(t)\Delta t)\text{, or}$$

(5) $$\frac{\overline{W}(t+\Delta t) - \overline{W}(t)}{\overline{W}(t)} = r(t)\Delta t.$$

In order to determine the discount, we begin at time 0 with A units of money. At time $t > 0$, the investment would be worth $A \times \overline{W}(t)$ units of money. If $A\overline{W}(T) = 1$, then the initial A units of money to fund the investment is determined as

[1] This *ex ante* expected return equation lies at the heart of all pricing models, most notably that of options pricing under the Black and Scholes (1973).

(6) $$A = \frac{1}{\bar{W}(T)}.$$

At any given time t before T, the value of the investment is

(7) $$A\bar{W}(t) = \frac{\bar{W}(t)}{\bar{W}(T)}.$$

Putting (7) together with the deterministic processes (2) and (3), the discount $D(t,T)$ between the two time instants t and T is the money amount at time t that is equivalent to one unit of money payable at time T as given by

(8) $$D(t,T) = \frac{\bar{W}(t)}{\bar{W}(T)} = \exp\left(-\int_t^T r_s ds\right).$$

Suppose an investment begins at \$100 with a riskless rate of 10% per unit of time. From (3), the value of the investment at $t = 1$ is \$110.52 (\$100 × $e^{10\%}$). If we want to produce \$100 at 1 unit of time from now, from (8) the discount or price of the investment is \$90.48 (\$100 × $e^{-10\%}$). In the first example, \$100 is the discount to \$110.52 and \$90.48 is the discount to \$100.

Interest Rates in Bond Markets

The riskless rate r_t depicted above is a spot rate independent of any investment horizon. In the financial market, spot rates are prevalent on demand deposits or as a price to clear a market transaction (e.g., overnight deposits). Beyond spot, there are bond markets which trade claims on specified sums of money delivered at given future dates. Such claims are called pure discount bonds or zero-coupon bonds (or zeros). A T-maturity zero-coupon bond is a contract that guarantees its holder the payment of one unit of currency at time T, with no intermediate payments. The contract value at time $t < T$ is $P(t,T)$ and at maturity

(9) $$P(T,T) = 1, \text{ for all } T.$$

If the interest rate r is deterministic (i.e., non-stochastic), then D is also deterministic and

(10) $$D(t,T) = P(t,T) \text{ for each pair } (t,T).$$

The discount on zeros is determined by the zero rate which is dependent upon the "compounding type" of the bond contract. For continuously compounded constant-rate bonds, the zero rate at time t for the maturity T is denoted by $R(t,T)$ and it is the constant rate at which the investment of $P(t,T)$ units of currency at time accrues continuously to yield an amount of currency at maturity T as

(11) $$R(t,T) := -\frac{\ln P(t,T)}{\tau(t,T)}$$

Where the fraction involved in the continuous compounding is $\tau(t,T) = T - t$, the time difference can be expressed in years. Rearranging, the continuously-compounded interest rate to a constant rate consistent with the zero-coupon bond prices in that

(12) $$\exp\left(R(t,T)\tau(t,T)\right)P(t,T) = 1, \text{ or}$$

(13) $$P(t,T) = \exp(-R(t,T)\tau(t,T))$$

which is the bond price in terms of the continuously compounded zero rate R. If the value of the continuously compounded zero two years from today is \$100 and the annual zero rate is 10%, then the price of the bond at today's present value is \$81.87 (\$100 × $e^{-10\%\times 2}$). Conversely, if a continuously three-year compounded zero has a present day price of \$76.34, then the zero rate is 9% ($-\frac{\ln(\$76.34/\$100)}{3}$).

In forensic economics annual compounding is more prevalent than continuous compounding.[2] The annually-compounded zero-coupon-bond rate at time t for the maturity T is denoted as $Y(t,T)$. The annual zero is the constant rate at which an investment has to be made to

[2] The forensic economic literature has a bad habit of using semi-annually compounded interest rates in annually compounded discount factors (see Foster (2014)).

produce an amount of one unit of currency at maturity, starting from $P(t,T)$ units of currency at time t when reinvesting the obtained amounts once per year. In formulas

(14) $$Y(t,T) := \frac{1}{\left[P(t,T)\right]^{1/\tau(t,T)}} - 1, \text{ or}$$

(15) $$P(t,\mathrm{T})\left(1+Y(t,T)\right)^{\tau(t,T)} = 1, \text{ or}$$

(16) $$P(t,T) = \frac{1}{(1+Y(t,T))^{\tau(t,T)}}.$$

With annual compounding, our example two-year zero-coupon bond with a 10% annual interest rate has a present day price of \$82.64 $\left(\frac{\$100}{(1+10\%)^2}\right)$.

The Zero Curve

Like the investment in equation (1), the value of a bond amount B at the spot rate increases in value at

(17) $$dB = Br(t)dt, \text{ or}$$

at any time t the current value $r(t)$ of the spot rate is the instantaneous rate of increase of the bond value. At time t, equation (17) holds with certainty; however, the subsequent values of the spot rate are not necessarily certain. The accumulations of spot rates across T is called the zero curve. While there have been many variations on the specification of the stochastic nature of the spot rate after time t on the zero curve, the zero curve can be adequately described as

(18) $$P(t,T) = E_t \exp\left(-\int_t^T r(s)ds\right)$$

where E is the stochastic expectation under some set of core rules. For example, the Vasicek (1977) model presents the zero curve under the core rules that the stochastic $r(t)$ is

1. a continuous function of time;

2. that it follows a continuous Markov process meaning that the future development of the spot rate given its present value is independent of the past development that has led to the present value of the spot rate;
3. the price $P(t, s)$ of a zero discount bond is determined by the assessment, at time t, of the segment $\{r(\tau), t \le \tau \le s\}$ of the spot rate process over the term of the bond; and,
4. the market is efficient (no transactions costs, information is available to all investors simultaneously, and every investor acts rationally) (prefers more wealth to less, and uses all available information) with investors having homogeneous expectations so that no profitable *riskless* arbitrage is possible.

Assumptions one to three simply describe a zero curve as the accumulation of market-clearing expectations across time and the purpose of assumption four is largely mathematical. While research has shown that *ex post* arbitrage has occurred, arbitrage is itself stochastic, unpredictable, and short-lived, heeding the phrases "there ain't no such thing as free lunch"[3] or "picking up nickels in front of a steamroller."[4]

Estimating the Zero Curve

By definition, the zero curve represents the financial market's expectation of the riskless rate of interest, r, at every time instant t. In reality, a market for the riskless rate of interest does not exist, chiefly due to the requirements for an efficient market. For example, U.S. Treasury securities are often thought to have the lowest risk and so

> *"it would seem logical that the observed yield on zero-coupon Treasury securities can be used to construct an actual spot-rate curve. However, there are problems with this approach. First, the liquidity of these securities is not as great as that of*

[3] "Economics in Eight Words" El Paso Herald-Post, June 27, 1938

[4] Duarte, Longstaff and Yu (2007). There is also the classic joke: an economist and a normal person go for a walk and the normal person sees what he thinks is a $100 bill lying on ground. When the normal person goes to pick up the bill, the economist says "don't bother bending down, if it were real somebody else would have picked it up before you."

> *the coupon Treasury market. Second, there are maturity sectors of the zero-coupon Treasury market that attract specific investors who may be willing to trade yield in exchange for an attractive feature associated with that particular maturity sector, thereby distorting the term-structure relationship."*[5]

Because of its size, the closest representation to the theoretical riskless rate of interest is found in the market the U.S. Treasury bills and coupon securities. Treasury bills do not pay coupons and have maturities up to 52 weeks; and, Treasury notes and bonds have fixed coupon payments every six months following the issue date. Any Treasury security can be thought of as a collection of zero-coupon investments with each investment having a maturity date equal to the coupon payment date and, in the case of the security's principal, the maturity date. Absent arbitrage, the discounted price of the security is equal to the sum of the discounted price of the collection of zero-coupon investments.

Estimating the zero curve can be accomplished using the bootstrap technique. Suppose a Treasury security with a maturity of 1.5 years has a coupon of $5 and it sold at par. So, the market valuation equation of the cash-flows is:

(19) $$\$100 = \frac{\$5}{(1+z_1)^1} + \frac{\$5}{(1+z_2)^2} + \frac{\$105}{(1+z_3)^3}.$$

where

z_1 = one-half of the annualized six-month theoretical spot rate,

z_2 = one-half of the one-year theoretical spot rate, and

z_3 = one-half of the 1.5-year theoretical spot rate.

If on the same auction date, the 26-week Treasury bill rate was 3% and the 52-week Treasury bill rate was 4%, then the discounted price of the 1.5-year Treasury coupon security was equal to

[5] Fabozzi and Mann (2007), p. 142.

(20) $$\$100 = \frac{\$5}{(1+3\%)^1} + \frac{\$5}{(1+4\%)^2} + \frac{\$105}{(1+z_3)^3}\text{, or}$$

(21) $$\$100 = \$4.8544 + \$4.6228 + \frac{\$105}{(1+z_3)^3}\text{, or}$$

(22) $$\$90.5229 = \frac{\$105}{(1+z_3)^3}\text{, or}$$

(23) $$(1+z_3)^3 = \frac{\$105}{\$90.5229}\text{, or}$$

(24) $$(1+z_3)^3 = \frac{\$105}{\$90.5229}\text{, or}$$

(25) $$z_3 = 5.070\%\,.$$

So, the 1.5-year estimated zero rate is 5.070%.

Given the 1.5-year zero rate, we can calculate the 2-year zero rate, z_4. Suppose again on the same auction date the 2-year Treasury note sold at par with a \$6 coupon. The discounted price of the 2-year Treasury coupon security was equal to

(26) $$\$100 = \frac{\$6}{(1+z_1)^1} + \frac{\$6}{(1+z_2)^2} + \frac{\$6}{(1+z_3)^3} + \frac{\$106}{(1+z_4)^4}\text{, or}$$

(27) $$\$100 = \frac{\$6}{(1+3\%)^1} + \frac{\$6}{(1+4\%)^2} + \frac{\$6}{(1+5.070\%)^2} + \frac{\$106}{(1+z_4)^4}\text{, or}$$

(28) $$(1+z_4)^4 = \frac{\$106}{\$83.4547}\text{, or}$$

(29) $$z_4 = 6.161\%\,.$$

So, the 2-year estimated zero rate is 6.161%. To fill-out the remainder of the zero curve, we can continue the bootstrapping process using the U.S. Treasury note and bond date to the last available maturity date. Currently, there are 300 actively traded U.S. Treasury securities. Some of those securities were issued many years ago. For example, the current price of 30-year bond issued 25 years ago should be approximately the same as the price of a 5-year bond issued today. In order to work towards satisfying theoretical aspects of the zero curve, it is common to use a coupon-bond yield curve constructed from *on-the-run* Treasury securities.[6] On-the-run Treasuries are those that are most recently issued securities by maturity date. Recently issued securities trade close to par and are generally more liquid than older securities which are often held by large institutional investors such as pension funds.

Pricing Discounts

Going back to equation (1), if we want to calculate the dollar value of a riskless asset to replace future damages, then b_{Wt} is zero and the *ex ante* expected return is simply the riskless rate of interest. If any other *ex ante* interest rate is chosen other than the current riskless rate, then interest rate risk is introduced into the expected return and that risk has its own price separate from the risk free discount.

As an example of pricing discounts, to estimate the risk free zero we use the market for U.S. Treasury securities as of the last trading day of November 2014. The U.S. Treasury Department published yield curve by maturity in years is shown in column 2 of Table 1.[7] In column 3 of Table 1, we show the bootstrapped zero coupon yields by maturity. To estimate the zero curve, we first estimated the full bond equivalent yield curve using the Nelson-Siegel cubic spline method. The fourth column of Table 1 shows that under the zero curve the sum of the discounted values of $1 at the end of each maturity year is $20.69. The equivalent constant yield (flat yield curve) discount producing $20.69 is 2.59%. The remainder columns of Table 1 show

[6] The U.S. Treasury Department publishes a daily yield curve constructed from on the run Treasuries. See http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/yieldmethod.aspx.
[7] The Treasury published the daily yield curve on the Internet at http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield.

the discounted value of $1 at the end of each maturity year at various constant zero yields for 30 years. Within Table 1, any constant zero less than 2.59% produces a discounted asset value greater than the current risk free asset value; and, any constant zero greater than 2.59% produces a discounted asset value less than the current risk free asset value. The difference between discounts represents the price of the interest rate risk between yields.

Again, going back to equation (1), if *X* is the discounted asset using the current risk free zero, r_t, and *Y* is a discounted asset at some zero other than r_t, then

(30) $$E_{Xt} = r_t = E_{Yt} - b_{Yt} .$$

Suppose that

(31) $$E_{Xt} = r_t < E_{Yt} - b_{Yt} .$$

If *Y* was represented as a riskless asset, then persons could obtain riskless, costless profits by selling *X* and using the proceeds to buy *Y*. Under our example, suppose *Y* = $15.37 which is the discount of $1 with a constant 5% zero. If *Y* was riskless, then we would always want to sell *X* for $20.69 and then simultaneously buy *Y* for $15.37 in order to earn a riskless, costless arbitrage profit of $5.32. Since that type of arbitrage could never be sustained, *Y* in reality would be a risky asset where the $5.32 is the bonus over *X* that market buyers would demand to purchase *Y*.

Importance to Forensic Economics

As stated in the introduction, forensic economists compute expected damages and then calculate the amount of an award as a discounted financial asset to replace those damages. Because all asset pricing theories and practices require equation (1) to hold, then all riskless payoffs in the future must have the same discounted asset value. If economist *X* and *Y* agree to the future damages, then they must agree to same value of the riskless asset value of those future damages. If an economist computes what he or she represents as a riskless asset value which is different than the riskless zero discount asset value, then that economist is opining an *ex post* riskless arbitrage opportunity, which within the realm of forensic economics, would be seemingly impossible.[8] As described by Brigo and Mercurio (2006 at page 23):

> *"Roughly speaking, the absence of arbitrage is equivalent to the impossibility to invest zero today and receive tomorrow a nonnegative amount that is positive with positive probability. In other words, two portfolios having the same payoff at a given future date must have the same price today."*

Thereby, the importance of discounting to forensic economics is that risk free award amounts to replace risk free future damage amounts must be computed with current risk free zero rates; any other computed award embeds financial risk which has a price to equalize it with the risk free zero discount.

In the practice of forensic economics, discounts of future damages are often misrepresented. For example, consider a plaintiff has a nominal damage of $10,000 one year from today and the expected value of the damage is estimated at $9,900.[9] The value of a riskless financial asset today to replace the $9,900 one year from today is computed with the current yield on one year riskless zeros, say which produces an asset of $9,850. If economist *Z* presents a discounted asset value other than $9,850, then *Z*'s asset contains financial risk. If Z presents an asset with a price of $9,775 that means investors in that asset are demanding a $75 bonus over the current risk free discount. In forensic economics, the $75 is not recognized as a premium necessary to take on the investment, but a supposed guaranteed return. Unfortunately, the Z economists do not prescribe a methodology for how the plaintiff will be able to achieve the $75 excess return with a positive probability (in fact they often represent it as occurring with certainty).

[8] Most forensic economic situations involve no more than a few million dollars in which damage amounts occur annually over the plaintiff's lifetime. At any point in time the plaintiff would have to divest a large amount of their asset award to leverage any potential arbitrage situation.

[9] For example, $10,000 multiplied by one minus the probability of the plaintiff's death over the next year.

Conclusion

This paper began with the standard theory that the *ex ante* expected return on an asset is equal to the riskless rate of interest plus the expected return of the asset in excess of the riskless rate. We showed the theoretical framework for discounting including the riskless zero-coupon bond yield curve. An example of discounting with the derived zero-coupon riskless yield was provided. It is demonstrated that any deviation in the discount rate from the riskless zero measures financial risk which is quantifiable in the discounted asset value. Since the discounted price of all asset values to fund the same future riskless damages must be equal, if an economist opines a discounted asset value other than the riskless value, he or she must explain how that discounted price can occur with a positive probability.

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Table 1. Yields by maturity and the discount of $1 by maturity for 30 years

Maturity in years	Bond equivalent yields	Bootstrapped zero coupon yields	Zero curve discount	2.59%	1%	2%	3%	4%	5%	6%	8%	10%
0.083	0.04%	0.04%										
0.25	0.02%	0.02%										
0.5	0.07%	0.07%										
1	0.13%	0.13%	$0.999	$0.975	$0.990	$0.980	$0.971	$0.962	$0.952	$0.943	$0.926	$0.909
2	0.47%	0.47%	0.991	0.950	0.980	0.961	0.943	0.925	0.907	0.890	0.857	0.826
3	0.88%	0.89%	0.974	0.926	0.971	0.942	0.915	0.889	0.864	0.840	0.794	0.751
4		1.23%	0.952	0.903	0.961	0.924	0.888	0.855	0.823	0.792	0.735	0.683
5	1.49%	1.51%	0.928	0.880	0.951	0.906	0.863	0.822	0.784	0.747	0.681	0.621
6		1.74%	0.902	0.858	0.942	0.888	0.837	0.790	0.746	0.705	0.630	0.564
7	1.89%	1.93%	0.875	0.836	0.933	0.871	0.813	0.760	0.711	0.665	0.583	0.513
8		2.06%	0.849	0.815	0.923	0.853	0.789	0.731	0.677	0.627	0.540	0.467
9		2.16%	0.825	0.795	0.914	0.837	0.766	0.703	0.645	0.592	0.500	0.424
10	2.18%	2.24%	0.802	0.775	0.905	0.820	0.744	0.676	0.614	0.558	0.463	0.386
11		2.30%	0.779	0.755	0.896	0.804	0.722	0.650	0.585	0.527	0.429	0.350
12		2.36%	0.756	0.736	0.887	0.788	0.701	0.625	0.557	0.497	0.397	0.319
13		2.42%	0.733	0.717	0.879	0.773	0.681	0.601	0.530	0.469	0.368	0.290
14		2.47%	0.710	0.699	0.870	0.758	0.661	0.577	0.505	0.442	0.340	0.263
15		2.52%	0.688	0.682	0.861	0.743	0.642	0.555	0.481	0.417	0.315	0.239
16		2.57%	0.666	0.665	0.853	0.728	0.623	0.534	0.458	0.394	0.292	0.218
17		2.61%	0.645	0.648	0.844	0.714	0.605	0.513	0.436	0.371	0.270	0.198
18		2.65%	0.624	0.631	0.836	0.700	0.587	0.494	0.416	0.350	0.250	0.180
19		2.69%	0.604	0.616	0.828	0.686	0.570	0.475	0.396	0.331	0.232	0.164
20	2.62%	2.73%	0.584	0.600	0.820	0.673	0.554	0.456	0.377	0.312	0.215	0.149
21		2.77%	0.564	0.585	0.811	0.660	0.538	0.439	0.359	0.294	0.199	0.135
22		2.80%	0.545	0.570	0.803	0.647	0.522	0.422	0.342	0.278	0.184	0.123
23		2.84%	0.526	0.556	0.795	0.634	0.507	0.406	0.326	0.262	0.170	0.112
24		2.87%	0.507	0.542	0.788	0.622	0.492	0.390	0.310	0.247	0.158	0.102
25		2.91%	0.488	0.528	0.780	0.610	0.478	0.375	0.295	0.233	0.146	0.092
26		2.94%	0.470	0.515	0.772	0.598	0.464	0.361	0.281	0.220	0.135	0.084
27		2.98%	0.453	0.502	0.764	0.586	0.450	0.347	0.268	0.207	0.125	0.076
28		3.01%	0.435	0.489	0.757	0.574	0.437	0.333	0.255	0.196	0.116	0.069
29		3.05%	0.418	0.477	0.749	0.563	0.424	0.321	0.243	0.185	0.107	0.063
30	2.89%	3.09%	0.402	**0.465**	0.742	0.552	0.412	0.308	0.231	0.174	0.099	0.057
			$20.69	$20.69	$25.81	$22.40	$19.60	$17.29	$15.37	$13.76	$11.26	$9.43

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Assumptions

- The present value date is November 28, 2014.
- Damages are $1 per year for 30 years.
- The damage award is to be calculated on a risk free basis.
- U.S. Treasury Securities are assumed to be risk free.
- Economist X computes discounted damages as $20.69.
- Economist Y computes discounted damages as $17.29.

Maturity in years	Bond equivalent yields	Bootstrapped zero coupon yields	Zero curve discount	4% constant yield discount
0.083	0.04%	0.04%		
0.25	0.02%	0.02%		
0.5	0.07%	0.07%		
1	0.13%	0.13%	$0.999	$0.962
2	0.47%	0.47%	0.991	0.925
3	0.88%	0.89%	0.974	0.889
4		1.23%	0.952	0.855
5	1.49%	1.51%	0.928	0.822
6		1.74%	0.902	0.790
7	1.89%	1.93%	0.875	0.760
8		2.06%	0.849	0.731
9		2.16%	0.825	0.703
10	2.18%	2.24%	0.802	0.676
11		2.30%	0.779	0.650
12		2.36%	0.756	0.625
13		2.42%	0.733	0.601
14		2.47%	0.710	0.577
15		2.52%	0.688	0.555
16		2.57%	0.666	0.534
17		2.61%	0.645	0.513
18		2.65%	0.624	0.494
19		2.69%	0.604	0.475
20	2.62%	2.73%	0.584	0.456
21		2.77%	0.564	0.439
22		2.80%	0.545	0.422
23		2.84%	0.526	0.406
24		2.87%	0.507	0.390
25		2.91%	0.488	0.375
26		2.94%	0.470	0.361
27		2.98%	0.453	0.347
28		3.01%	0.435	0.333
29		3.05%	0.418	0.321
30	2.89%	3.09%	0.402	0.308
			$20.69	**$17.29**

Why the $3.40 difference?

- Economist X used the U.S. Treasury yield curve on November 28, 2014 to compute discounted damages.
- Based on historical average yields, economist Y used a constant 4% yield to compute discounted damages.

What does the $3.40 represent?

- Economist Y is opining that $3.40 of riskless financial profit exists.
- In doing so, economist Y violates fundamental economic and financial theory.

Fundamental theory: part one

For any asset, *W*, its *ex ante* expected return at the time instant *t* is

$$E_W = r_t + b_{Wt}$$

where r_t is the riskless rate of interest and b_{Wt} is some function of expected return of the asset in excess of the riskless rate.

Fundamental theory: part two

- The interest rate is a stochastic expectation modeled under strict or loose rules:
  - Continuous and differential
  - Measurable
  - The possibility of profitable riskless arbitrage is non-existent (strict) or minimal (loose).
    - “There ain’t no such thing as free lunch”
    - “Picking up nickels in front of a steamroller.”

Fundamental theory: part three

- “Roughly speaking, the absence of arbitrage is equivalent to the impossibility to invest zero today and receive tomorrow a nonnegative amount that is positive with positive probability. In other words, two portfolios having the same payoff at a given future date must have the same price today.” – *Brigo and Mercurio*

If the payoff damages are the same between experts, so should be the discounted price of the damages between the experts.

- Otherwise, riskless, costless profits can be made.

Example of riskless, costless profits

- Assume that the risk free yield for 30 years is currently a constant 2.59%.
- Economist Y states that the historical average 30 year return risk free investments was 4%, so plaintiff should be able to get that same return over the next 30 years.
- Person Z sells 2.59% bonds and buys economist Y’s bond with a 4% annual yield.
- Person Z collects \$0.465 today from X, uses \$0.308 of those funds to buy economist Y’s bond and pockets \$0.157. At 30 years, Y pays Z \$1 and Z gives \$1 to X.

Conclusion

- Unless future risk-adjusted damages are discounted at the current risk free rate, financial risk is introduced into the discount.
- If financial risk is present in the discount, then that financial risk must be
  1. Recognized,
  2. Measured, and
  3. Priced.