THE BIDDER EXCLUSION EFFECT

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Abstract. We introduce a simple and robust approach to address two key questions in empirical auction analysis: discriminating between models of entry and quantifying the revenue gains from improving auction design. The approach builds on Bulow and Klemperer (1996), connecting their theoretical results to empirical analysis. It applies in a broad range of information settings and auction formats without requiring instruments or estimation of a complex structural model. We demonstrate the approach using US timber and used-car auction data.

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1. Introduction

Empirical work on auctions often requires estimating a complex structural model relying on strong assumptions about the data generating process. For example, computing revenue under counterfactual auction formats typically involves making simplifying assumptions, such as bidders having independent private values (IPV).[1] The researcher may then estimate the underlying distribution of bidder valuations from bid data, and use the estimated distribution to simulate the counterfactuals of interest. While these structural methods are sometimes necessary, we demonstrate that some key questions in the analysis and design of auctions, such as discriminating between models of entry and quantifying the revenue gains from improving auction design (Bulow and Klemperer 1996), can be answered with a simple-to-compute tool we refer to as the *bidder exclusion effect*.

The bidder exclusion effect is the decrease in expected auction revenue when a random bidder is excluded from an auction. The effect is not of intrinsic interest but instead serves as a means to an end, providing a simple and powerful tool for empirical analysis. For the intuition behind how the bidder exclusion effect can be estimated, consider the example of an $n > 2$ bidder ascending auction with private values and no reserve price, where in equilibrium bidders bid their values. If a bidder is excluded at random from the auction, with probability $\frac{n-2}{n}$ he will be one of the $n-2$ lowest bidders, and so his exclusion will not affect revenue. With probability $\frac{2}{n}$, he will be one of the two highest bidders, and revenue will drop from the second-highest to the third-highest bid of the n bidders. The bidder exclusion effect is therefore $\frac{2}{n}$ times the expected difference between the second and third-highest bids.

This estimator is robust to a variety of modeling frameworks which would normally complicate or prohibit identification in auction models. For example, Athey and Haile (2002) prove that the joint distribution of bidder valuations is not identified in ascending auction models with correlated private values because the researcher never

[1] Paarsch and Hong (2006); Hendricks and Porter (2007); Athey and Haile (2007); and Hickman et al. (2012) describe structural econometric methods for auction data and survey the literature.

observes the willingness to pay of the highest bidder. The bidder exclusion effect, however, is identified in this setting—and in a broad class of other auction settings considered in the literature—when the econometrician observes the second and third-highest bids and the number of bidders. With these data, the bidder exclusion effect is point identified in ascending auctions with (possibly asymmetric and correlated) private values and auction-level heterogeneity (observed or unobserved), and can be estimated without requiring instruments, exogenous variation in the number of bidders, or a complex structural model.

While it is particularly straightforward to find the bidder exclusion effect in the example above, we show that it can be bounded above in a wide range of other settings, including common values, bidders bidding below their values (Haile and Tamer 2003), binding reserve prices, and first price auctions. Bounds can also be obtained when the econometrician only observes the second and third-highest bids and not the number of bidders. Incorporating auction-level covariates parametrically or nonparametrically is also straightforward, as the bounds on the bidder exclusion effect reduce to conditional means in these cases.

The bidder exclusion effect sheds light on several important issues in empirical auctions work. First, it is possible to test whether bidders' valuations are independent of the number of bidders who participate in the auction.[2] This type of independence is central to identification in many recent methodological advances.[3] Knowing whether the number of bidders is independent of valuations is also key to modeling bidders' entry decisions. Specifically, a prominent distinction in the auction literature is between entry models which have this property, and models of selective bidder entry, which do not.[4] Choosing an inappropriate entry model might result in misleading estimates of bidders' valuations, or unreliable counterfactual simulations.

[2]This type of independence has a variety of names in the auctions literature: "exogenous participation" (Athey and Haile 2002, 2007), "valuations are independent of N" (Aradillas-López et al. 2013a), or an absence of "selective entry" (Roberts and Sweeting 2013a).

[3]See, for example, Haile and Tamer (2003), Aradillas-López et al. (2013a), and Sections 5.3 and 5.4 of Athey and Haile (2007).

[4]Bajari and Hortacsu (2003); Bajari et al. (2010); Athey et al. (2011); Krasnokutskaya and Seim (2011) and Athey et al. (2013) feature entry models without selection, derived from the theoretical

Although the assumption that bidders' values and the number of bidders are independent is widely used in the empirical auctions literature and is critical for modeling entry, it is rarely tested. We propose a simple test of this assumption, which compares the estimated bidder exclusion effect to the observed difference in average revenue between $n-1$ and and n bidder auctions. If the two quantities are significantly different, then $n-1$ bidder auctions differ from n bidder auctions with one bidder randomly removed, which is evidence against exogeneity of bidder participation. Estimating the bidder exclusion effect can therefore guide the decision of whether to rely on exogenous variation in the number of bidders for identification and, similarly, guide the choice of the most appropriate entry model. In contrast to other methods of discriminating between entry models, this does not require exogenous variation in, or even observing, the number of underlying potential bidders. Nor does it require fully estimating an entry model or other complex structural auction model.[5]

The bidder exclusion effect also allows the analyst to gauge how important it is to set reserve prices optimally. A typical empirical approach to answering this question would rely on assumptions about the distribution of values and the information environment to estimate a detailed model, determine optimal reserve prices using the seller's first-order condition, and finally measure the revenue difference between the optimally designed auction and a no-reserve auction (e.g. Paarsch (1997); Li et al. (2003); Krasnokutskaya (2011)).

We circumvent these steps by using an auction theory result. Under the assumptions of Bulow and Klemperer (1996), sellers raise expected revenue more by adding another bidder than by designing the auction optimally. When revenue is concave in the number of bidders, dropping a bidder has a larger effect on revenue than adding

work of Levin and Smith (1994). Roberts and Sweeting (2013b,a); Bhattacharya et al. (2013) and Gentry and Li (2013), present models of selective entry.

[5]Marmer et al. (2013) and Roberts and Sweeting (2013a) propose tests for selective entry based on observing exogenous variation in the number of potential bidders. Li and Zheng (2009) estimate models with and without selective bidder entry, and select between them on the basis of their predictive power. Aradillas-López et al. (2013b) provide an alternative test of selective entry which is more complex than ours but which has the advantage of not requiring observing multiple order statistics of bids.

a bidder, so the bidder exclusion effect is an upper bound on the revenue gains from improving the auction mechanism, and, in particular, on setting an optimal reserve price. Calculating the bidder exclusion effect allows the researcher to determine whether reserve prices are likely to be important without the effort or assumptions necessary to estimate a more detailed structural model. Additionally, because the bidder exclusion effect is simple to compute, the size of other estimated effects, such as an experimental change in the auction process, can easily be compared to the important benchmark of optimal mechanism design.

We illustrate the uses of the bidder exclusion effect with two applications. First, we use US timber auction data from 1982-1989. We find that removing a single bidder at random would decrease revenue by approximately 13% on average. Under conditions discussed below, this implies that the increase in expected seller revenue from using an optimal reserve price is less than 13%. Timber auctions are a setting in which much of the previous literature has modeled entry as being non-selective.[6] We find evidence of selective entry unconditionally, but after controlling for bidder types (loggers vs. mills), the evidence for selective entry is weaker. This suggests that assuming exogenous variation in the number of bidders may be appropriate in timber auction settings once the bidder type has been accounted for.

Second, we show how to use the bidder exclusion effect to bound the impact of optimal auction design when minimal data is available. We study wholesale used-car auctions in which the number of bidders varies auction by auction but is unobserved to the econometrician. We estimate an upper bound on the bidder exclusion effect, averaged over the unobserved realizations of the number of bidders. From the Bulow-Klemperer theorem, we obtain an upper bound on the revenue increase from an optimal reserve price of approximately \$170. We illustrate how this bound can also be calculated conditional on observable covariates, and we use it as a benchmark for other recently estimated effects from changes in auto auctions (Tadelis and Zettelmeyer 2014; Lacetera et al. 2013; Hortaçsu et al. 2013).

[6]See, for example, Haile and Tamer (2003) and Aradillas-López et al. (2013a).

In the spirit of Haile and Tamer (2003) and Aradillas-López et al. (2013a), our empirical approach does not seek to point identify and estimate the distribution of bidder values. Instead we draw inferences from functions of the value distribution which are point, or partially, identified. To our knowledge, we are the first to point out that a statistic which is straightforward—and in some cases trivial—to compute has implications for modeling entry in auctions or evaluating the revenue impact of optimal mechanism design. More broadly, our approach ties in closely to the recent literature on sufficient statistics for welfare analysis (Chetty 2009; Einav et al. 2010; Jaffe and Weyl 2013), which focuses on obtaining robust welfare or optimality implications from simple empirical objects without estimating detailed structural models.

2. Model and Empirical Strategy

We consider single-unit ascending auctions with risk-neutral bidders throughout, and we assume the auctions analyzed took place without a reserve price.[7] Let N be a random variable denoting the number of auction participants and let n represent realizations of N. Following Athey and Haile (2002), let $W_i = (V_i, S_i)$ denote bidder i's value and private signal, and B_i his bid. With *private values*, $V_i = S_i$ for all i. With *common values*, for all i and j, V_i and S_j are strictly affiliated conditional on any $\chi \subset \{S_k\}_{k \notin j}$, but not perfectly correlated.[8] For the subset of auctions which have exactly n bidders enter, let F^n denote the joint distribution of $W \equiv ((V_i)_{i=1,\dots,n}, (S_i)_{i=1,\dots,n})$. By *bidder symmetry*, we refer to the case where F^n is exchangeable with respect to bidder indices. Let $V^{1:n}, \dots, V^{n:n}$ represent the bidders' valuations ordered from smallest to largest. Similarly, let the random variables $B^{1:n}, \dots, B^{n:n}$ represent their bids ordered from smallest to largest, with realizations of $B^{k:n}$ denoted $b^{k:n}$.

[7]Section 5 discusses counterfactual settings in which reserve prices are used, and Appendix B contains extensions to cases where the auctions analyzed took place with a binding reserve price. Appendix C contains extensions to first price auctions.

[8]The random variables $Y = (V_i, S_j)$ with joint density $f_Y(\cdot)$ are strictly affiliated if for all y, y', $f_Y(y \vee y')f_Y(y \wedge y') > f_Y(y)f_Y(y')$, where $\vee$ denotes component-wise maximum, and $\wedge$ denotes component-wise minimum.

For $k \leq m \leq n$, let $B^{k:m,n}$ represent the k^{th} smallest bid in m bidder auctions, where the m bidders are selected uniformly at random from the n bidders in auctions which had exactly n bidders enter. Some remarks on this quantity are in order. We stress that this is a counterfactual if $m < n$: we assume that it is common knowledge amongst the remaining m bidders that $n - m$ of the original n bidders have been dropped, and that they are competing in an m-bidder auction, not an n-bidder auction. The distribution of $B^{k:m,m}$ and $B^{k:m,n}$ for $m < n$ may be different, as different kinds of goods may attract different numbers of entrants, and bidders may value goods sold in auctions with m entrants differently from those sold in auctions with n entrants. Finally, $B^{k:m,m}$ and $B^{k:m}$ are the same random variable.

Our empirical strategy centers around three key variables. The first is the bidder exclusion effect. We define the bidder exclusion effect in n bidder auctions with no reserve price, $\Delta(n)$, as the expected fall in revenue produced by randomly excluding a bidder from those auctions. In ascending auctions, the bidder exclusion effect is:

$$\Delta(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1,n}),$$

that is, the expected second-highest bid in n bidder auctions, minus the expected second-highest bid in $n-1$ bidder auctions, where those $n-1$ bidder auctions are obtained by publicly dropping a bidder at random from n bidder auctions.

The second variable, $\Delta^{bid}(n)$, is the expected fall in revenue from dropping a *bid* (rather than a bidder) at random, assuming all other bids remain unchanged:

$$\Delta^{bid}(n) \equiv \frac{2}{n} E(B^{n-1:n} - B^{n-2:n}).$$

With probability $\frac{2}{n}$ one of the highest two bids will be dropped, and revenue will drop to the third-highest bid of the original sample, and with probability $\frac{n-2}{n}$, one of the lowest $n-2$ bids will be dropped, and revenue will not change.

The third variable, $\Delta^{obs}(n)$, is the *observed* difference in expected revenue between those auctions in which n bidders choose to enter, and those in which $n-1$ choose to enter:

$$\Delta^{obs}(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1}).$$

Unlike $\Delta(n)$, the quantities $\Delta^{bid}(n)$ and $\Delta^{obs}(n)$ are not counterfactual and can always be estimated using data on the two highest bids and the number of auction entrants. Section 3 gives conditions under which $\Delta(n)$ is equal to, or bounded above by, $\Delta^{bid}(n)$. As Section 4 describes, if entry is selective, then $\Delta(n) \neq \Delta^{obs}(n)$. Combining this with the results of Section 3, we have testable implications of selective entry in terms of the relation between $\Delta^{bid}(n)$ and $\Delta^{obs}(n)$. Section 5 relates the increase in revenue from an optimal reserve to $\Delta(n)$, and, by the results of Section 3, to $\Delta^{bid}(n)$.

3. Identifying and Estimating the Bidder Exclusion Effect

3.1. **The Naïve Approach.** A naïve approach to estimating the bidder exclusion effect is to compare revenue between n bidder and $n-1$ bidder auctions. This approach is appropriate under two strong assumptions, both of which are frequently employed in the structural auctions literature: First, the number of bidders varies exogenously, and, second, the number of bidders is correctly observed. While these assumptions greatly aid in identification and testing, they may not hold in practice.

An appealing alternative is an instrumental variables approach.[9] However, even with valid instruments for bidder participation, instrumental variables estimates would only capture the effect on revenue for those auctions in which the instrument causes more bidders to enter (Angrist and Imbens 1995). Our approach, which we turn to next, does not require instruments or variation in the number of bidders, and applies to all auctions, rather than a subset determined by the choice of instrument.

3.2. **The Basic Model: Point Identification.** In private value ascending auctions with no reserve where bidders bid their values, it follows immediately that the bidder

[9]For example, Haile et al. (2003) use the numbers of nearby sawmills and logging firms as instruments for the number of bidders at US timber auctions, in a test for common values.

exclusion effect is equal to the effect of dropping a bid at random, assuming the other bids remain unchanged. This holds with asymmetric and correlated private values.

Observation 1. *In ascending auctions with private values and no reserve price where bidders bid their values, for all $n > 2$ the bidder exclusion effect $\Delta(n) = \Delta^{bid}(n)$.*

Estimating the bidder exclusion effect in this setting is trivial, as long as the total number of auction participants and the second and third-highest bids are observed. One simply forms the sample analog of $\Delta^{bid}(n)$.[10] Incorporating auction-level observables into estimation is also straightforward. For a vector of auction-level covariates X, one estimates the sample analog of

$$\Delta^{bid}(n|X) \equiv \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}|X), \tag{1}$$

using any parametric or nonparametric approach for estimating conditional means.

3.3. **Extensions to the Basic Model.** We now demonstrate that upper bounds on the bidder exclusion effect are available with private values when losing bidders may drop out below their values (which we refer to throughout as "low bidding"), with symmetric common values, and in settings where the number of entrants is unobserved but it is known be greater than some lower bound. Upper bounds are more important than lower bounds for our applications. Models of selective entry often imply that bidders' valuations are increasing with the total number of entrants (Aradillas-López et al. 2013a,b). An upper bound allows us to detect sufficiently large increases in valuations with the number of entrants, in a sense which will be made precise. Similarly, to bound above the impact of optimal auction design, an upper bound on the bidder exclusion effect is required. Appendices B and C describe further

[10]Alternatively, if all bids are observed, one could estimate the bidder exclusion effect by removing one bid at random from n bidder auctions and computing the average decrease in the second-highest bid. This cannot be applied to ascending auctions, as the highest willingness to pay is unobserved. Moreover, this alternative would be subject to more sampling error, as it effectively involves simulating an indicator variable which is 1 with probability $\frac{2}{n}$ (the indicator for selecting one of the top two bids), rather than directly using the probability $\frac{2}{n}$.

extensions, showing how the bidder exclusion effect can be identified or bounded with binding reserve prices, and in first price auctions.

3.3.1. *Low Bidding.* In the "button auction" model of ascending auctions with private values (Milgrom and Weber (1982)), bidders drop out at their values. As highlighted in Haile and Tamer (2003), in practice bidders' (highest) bids may not equal their values. For example, in English auctions, multiple bidders may attempt to bid at a certain price but only the first bid the auctioneer sees may be recorded. The researcher may be willing to assume that the bidder with the second-highest valuation bids his value, as there are only two bidders remaining at this stage. However she may only be willing to assume that the remaining bids are lower than the bidders' values. The following assumption is equivalent to an argument in Athey and Haile (2002) and similar to an assumption in Haile and Tamer (2003):[11]

Assumption 1. *Bidders have private values. The second-highest bidder bids his value, and lower bidders' bids are less than or equal to their values.*

Our next proposition shows that this assumption can be used to obtain an upper bound on the bidder exclusion effect. The proof, and the proofs of all subsequent results, are in Appendix A.

Proposition 1. *If in ascending auctions with no reserve price, Assumption 1 holds, then for all* $n > 2$ *the bidder exclusion effect* $\Delta(n) \leq \Delta^{bid}(n)$.

3.3.2. *Symmetric Common Values.* Under the assumptions of Observation 1, the change in auction revenue when one bidder is excluded can be computed by removing one bidder's bid, and calculating the fall in revenue assuming the other bids remain unchanged. This is not true with common values, as removing a bidder changes the remaining bidders' equilibrium bidding strategies. It follows from Theorem 9 of Athey

[11]Athey and Haile (2002) argue, "...for many ascending auctions, a plausible alternative hypothesis is that bids $B^{n-2:n}$ and below do not always reflect the full willingness to pay of losing bidders, although $B^{n-1:n}$ does (since only two bidders are active when that bid is placed)." Haile and Tamer (2003) allow for $B^{n-1:n}$ to also not represent the full willingness to pay of the first losing bidder.

and Haile (2002) that with symmetric common values, in the button auction model of ascending auctions, the equality of Observation 1 can be replaced by an inequality.[12] Removing one bidder's bid and assuming other bids remain unchanged overstates the decline in revenue from excluding a random bidder, because it does not account for the increase in bids due to the reduced winner's curse. The following assumption and proposition summarize this result:

Assumption 2. *Bidders have symmetric common values, and bidding follows a button auction format.*

Proposition 2. *If in ascending auctions with no reserve price, Assumption 2 holds, then for all $n > 2$ the bidder exclusion effect $\Delta(n) < \Delta^{bid}(n)$.*

3.3.3. *Unobserved Number of Bidders.* In some settings the number of bidders may not be known to the researcher. In ascending auctions, for example, not all potential bidders may place bids. A lower bound on the number of potential bidders may be known, however, as in our used-car application in Section 7. Let $\underline{n}$ represent this lower bound, such that for all realizations n of the random variable N, $n > \underline{n}$. In this case Propositions 1–2 extend to yield an upper bound on the average bidder exclusion effect, $E(\Delta(N))$, where the expectation is over the unobserved number of bidders N.

Corollary 1. *In ascending auctions with no reserve price, if for all realizations n, $2 < \underline{n} \leq n$ and either Assumption 1 or Assumption 2 holds, then $E(\Delta(N)) \leq \frac{2}{\underline{n}} E(B^{N-1:N} - B^{N-2:N})$.*

Estimation in the case where n is unknown consists of computing the mean gap between second and third order statistics and scaling this quantity by $2/\underline{n}$. The expectation of this gap conditional on covariates can be estimated by standard non-parametric or parametric techniques.

[12]Theorem 9 is used for a different purpose in Athey and Haile (2002). They show that, assuming total bidder participation varies exogenously across auctions, private value models can be tested against common value models.

4. Testing If Valuations and Signals Are Independent of N

If bidders' valuations and signals do not vary systematically with the number of auction participants, then $n-1$ bidder auctions are just like n bidder auctions with one bidder removed at random. This suggests a test of whether bidders' valuations and signals are independent of the number of entrants. If the estimated bidder exclusion effect is significantly different from the observed change in revenue between n and $n-1$ bidder auctions, this is evidence against independence of valuations and signals, and the number of entrants. We develop this test in more detail below. Throughout this section, we assume that the econometrician observes the total number of bidders as well as the second and third-highest bids from all auctions.

Let F_m^n denote the distribution of values and signals of a random subset of m bidders, in auctions which actually had n participating bidders, where the $m \leq n$ bidders are drawn uniformly at random from the n bidders. Following Aradillas-López et al. (2013a), we say that *valuations and signals are independent of* N if $F_m^n = F_m^{n'}$ for any $m \leq n, n'$. With private values, we simply say *valuations are independent of* N, as valuations and signals are equal.

When valuations and signals are independent of N, we have $F_{n-1}^n = F_{n-1}^{n-1}$. Thus if valuations and signals are independent of N, it follows that $E(B^{n-2:n-1,n}) = E(B^{n-2:n-1,n-1}) = E(B^{n-2:n-1})$, and

$$\Delta(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1,n}) \tag{2}$$

$$= E(B^{n-1:n}) - E(B^{n-2:n-1}) \tag{3}$$

$$\equiv \Delta^{obs}(n). \tag{4}$$

If valuations or signals are not independent of N, we refer to entry as being *selective.* Selective entry may occur if bidders observe a signal of their value before entering, so that the entry decision, and consequently the number of entrants, may be correlated with bidder valuations (e.g. Roberts and Sweeting (2013a); Gentry and Li (2013)).[13] By contrast if bidders do not observe signals of their values before entering and play mixed entry strategies, then the variation in the number of entrants is independent of valuations (e.g. Athey et al. (2011); Krasnokutskaya and Seim (2011)).

The previous section discusses estimating the bidder exclusion effect, $\Delta(n)$, when it is point identified, and estimating an upper bound on $\Delta(n)$ when it is not. If the econometrician observes auction revenue and the total number of auction participants, then $\Delta^{obs}(n)$ can be estimated by the observed difference in average revenue between n and $n-1$ bidder auctions. Comparing estimates of $\Delta(n)$ and $\Delta^{obs}(n)$ is the idea behind the test for selective entry.

4.1. **Testing for selective entry with private values.** In private value settings where bidders bid their values, by Observation 1, $\Delta(n) = \Delta^{bid}(n)$ for $n > 2$. If in addition valuations are independent of N, then $\Delta(n) = \Delta^{obs}(n)$, implying that $\Delta^{bid}(n) = \Delta^{obs}(n)$. We define $T(n)$ as

$$\begin{aligned} T(n) &\equiv \Delta^{obs}(n) - \Delta^{bid}(n) \\ &= \left(E(B^{n-1:n}) - E(B^{n-2:n-1})\right) - \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}) \\ &= E\left(\frac{n-2}{n}B^{n-1:n} + \frac{2}{n}B^{n-2:n}\right) - E(B^{n-2:n-1}). \end{aligned} \tag{5}$$

The first term in the final expression is the expected revenue in n bidder auctions when one bidder is dropped at random, and the second term is the expected revenue in $n-1$ bidder auctions.[14]

To assess whether valuations are independent of N (i.e. whether entry is not selective) we test the null hypothesis $T(n) = 0$ for $n > 2$. For any given n this can

[13] Aradillas-López et al. (2013a) prove that in the entry model of Marmer et al. (2013) where potential bidders observe signals of their values when deciding whether to enter, if the distribution of values and signals is symmetric and affiliated, and a symmetric equilibrium exists in cutoff strategies, then valuations are not independent of N (Theorem A3 of Aradillas-López et al. 2013a).

[14] Athey and Haile (2002) propose a test of private vs. common values in which they assume valuations and signals are independent of N and point out that, under this assumption, $T(n) < 0$ in a common values setting and $T(n) = 0$ in a private values setting. We discuss generalizations to common values settings in Section 4.2.

be implemented as a simple t-test.[15] Let A_n represent the set of auctions with n entrants. The test statistic, $\widehat{T}(n)$, for this null is the sample analog of equation (5),

$$\widehat{T}(n) = \frac{1}{|A_n|}\sum_{j\in A_n}\left(\frac{n-2}{n}b_j^{n-1:n} + \frac{2}{n}b_j^{n-2:n}\right) - \frac{1}{|A_{n-1}|}\sum_{j\in A_{n-1}}(b_j^{n-2:n-1}). \qquad (6)$$

A simple regression-based form of this test is as follows. Let $y_j = \frac{n-2}{n}b_j^{n-1:n} + \frac{2}{n}b_j^{n-2:n}$ if $j \in A_n$ and $y_j = b_j^{n-2:n-1}$ if $j \in A_{n-1}$. Regress y_j on a constant and an indicator $\mathbb{1}(j \in A_n)$. The coefficient on the indicator is $\widehat{T}(n)$.

If $\widehat{T}(n)$ is significantly different from 0, the test indicates the presence of selective entry. This test is consistent against all forms of selective entry which affect expected revenue (if $\Delta^{bid}(n) \neq \Delta^{obs}(n)$ then the test rejects with probability approaching 1 as the number of auctions goes to infinity). Appendix D shows Monte Carlo evidence on the power of this test relative to simply comparing mean values in $n-1$ and n bidder auctions in a model of selective entry nesting that of Levin and Smith (1994). The bidder exclusion test is a reasonably powerful alternative to this mean comparison test given that it uses considerably less data. Moreover, the bidder exclusion test is implementable with ascending auction data while the mean comparison test is not.[16]

4.2. **Testing for selective entry with common values or low bidding.** With symmetric common values or low bidding, Propositions 1 and 2 imply that $\Delta(n) \leq \Delta^{bid}(n)$. If in addition valuations and signals are independent of N, then $\Delta(n) = \Delta^{obs}(n)$, implying that $\Delta^{obs}(n) \leq \Delta^{bid}(n)$. We can test this null using the same statistic as before, $\widehat{T}(n)$. Unlike private value settings where bidders bid their values, this test only indicates the presence of selective entry if $\widehat{T}(n)$ is significantly

[15]Standard techniques, like a Wald test or a Bonferroni correction, can be used to test $T(n) = 0$ for all n in some finite set. Note also that this test only uses information on the second and third-highest bids. If more losing bids are available and interpretable as the willingness-to-pay of lower-value bidders, this test could be made more powerful by including information from these losing bids. Intuitively, one could compare the revenue drop which would occur if k out of n bidders were dropped at random to the actual revenue difference between n and $n-k$ bidder auctions.

[16]The bidder exclusion test uses the second and third-highest values in n bidder auctions and the second-highest value in $n-1$ bidder auctions, while the mean comparison test uses *all* n values in n bidder auctions and *all* $n-1$ values in $n-1$ bidder auctions. The mean comparison test cannot be implemented in ascending auctions because the highest valuation is never observed.

greater than zero, and not if it is significantly less than zero. This test is consistent against forms of selective entry in which bidders' values are "sufficiently increasing" with N: precisely, if $E(B^{n-2:n-1,n}) - E(B^{n-2:n-1}) > \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}) - (E(B^{n-1:n}) - E(B^{n-2:n-1,n}))$.[17]

4.3. **Incorporating covariates.** Testing is also possible if valuations and signals are assumed independent of N conditional on a set of observable auction characteristics X rather than unconditionally. The null hypothesis in private values auctions where bidders bid their values is $T(n|X) = 0$, where $T(n|X)$ is defined as

$$T(n|X) \equiv E\left(\frac{n-2}{n}B^{n-1:n} + \frac{2}{n}B^{n-2:n}\middle|X\right) - E(B^{n-2:n-1}|X). \tag{7}$$

With symmetric common values or low bidding, the null is $T(n|X) < 0$. This test can be performed nonparametrically, without assuming any particular form for the conditional means. Chetverikov (2011); Andrews and Shi (2013) and Chernozhukov et al. (2013) develop inference procedures which apply to this setting.

A simple parametric version of this test is as follows. For a fixed n, specify the bidding equation for bidders in auction j as

$$b_j = \alpha_n + \beta X_j + \varepsilon_{nj}, \tag{8}$$

where X_j is a vector of observable characteristics of auction j and $\varepsilon_{nj} \perp\!\!\!\perp X_j$. Then

$$\frac{n-2}{n}b_j^{n-1:n} + \frac{2}{n}b_j^{n-2:n} = a_1 + \beta X_j + e_{1j}, \quad j \in A_n \tag{9}$$

$$b_j^{n-2:n-1} = a_2 + \beta X_j + e_{2j}, \quad j \in A_{n-1} \tag{10}$$

where $a_1 = \alpha_n + E(\frac{n-2}{n}\varepsilon_{nj}^{n-1:n} + \frac{2}{n}\varepsilon_{nj}^{n-2:n})$, $a_2 = \alpha_{n-1} + E(\varepsilon_{(n-1)j}^{n-2:n-1})$, $E(e_{1j}) = E(e_{2j}) = 0$ and $e_{1j}, e_{2j} \perp\!\!\!\perp X_j$.

After controlling for observables, a_1 determines the expected second order statistic (i.e. seller's revenue) when a bidder is removed at random from n bidder auctions and

[17]This feature is shared by the test proposed in Aradillas-López et al. (2013b), which the authors explain, "has power against a fairly wide class of 'typical' violations of [valuations being independent of N]."

a_2 determines the expected second order statistic in $n-1$ bidder auctions when the actual number of bidders is indeed $n-1$. In a private values framework, testing the null hypothesis of equation (7) amounts to testing the null of $a_1 = a_2$. With common values or low bidding, the null is $a_1 < a_2$.

We combine (9) and (10) as follows:

$$y_j = a_2 + (a_1 - a_2)\mathbb{1}(j \in A_n) + \beta X_j + e_{3j}, \quad j \in A_n \cup A_{n-1}, \tag{11}$$

where if $j \in A_n$, then $y_j = \frac{n-2}{n} b_j^{n-1:n} + \frac{2}{n} b_j^{n-2:n}$ and $e_{3j} = e_{1j}$, and if $j \in A_{n-1}$, then $y_j = b_j^{n-2:n-1}$ and $e_{3j} = e_{2j}$. This allows for a convenient regression-based test of the null hypothesis that valuations (or valuations and signals, in the common values case) are independent of N. When $\beta = 0$, this test nests the regression-based test described in Section 4.1.

4.4. **Bidder asymmetries.** The approach to testing for selective entry with private values described above does not require bidder symmetry. If bidders are asymmetric, the assumption of independence between valuations and N is less straightforward, and it less clear what it might mean if a test rejects the assumption. Intuitively, values may fail to be independent of N either because different bidders are more likely to enter depending on N, or because the same bidders enter but the value of the goods sold varies by N. We formalize and prove this statement in Appendix E, following the setup of Coey et al. (2014).

5. Bounding the Impact of Optimal Reserve Prices

The celebrated theorem of Bulow and Klemperer (1996) relates bidder entry to optimal auction design.[18] Under their assumptions, with symmetric bidders and independent signals, an English auction with no reserve price and $n+1$ bidders is more profitable in expectation than *any* mechanism with n bidders. When bidders have

[18] Following Bulow and Klemperer (1996) and most of the auction theory literature, we use "optimal" to mean optimal given a fixed set of participants. If entry is endogenous, then the mechanism's design may affect the number of participants. Optimal reserve prices for fixed and for endogenous entry may be different (McAfee and McMillan (1987); Levin and Smith (1994)).

affiliated signals, Bulow and Klemperer (1996) show that an auction with $n+1$ bidders and no reserve price still outperforms any "standard" mechanism with n bidders.[19] On these grounds, they suggest that sellers may be better off trying to induce more entry than trying to implement a better mechanism. As they acknowledge, this interpretation may be problematic if the new bidders are weaker than the bidders who would have entered anyway (for example, if increased marketing efforts induce lower-value bidders to enter the auction).

We propose an alternative interpretation of their theorem, namely that it can be used in empirical work to easily obtain upper bounds on the effect of improving auction design, without having to estimate a full structural model. Below we use the term "increasing marginal revenue" as it is defined in Bulow and Klemperer (1996).[20] We use the term "optimal reserve price" to refer to a take-it-or-leave-it offer made to the final bidder—the last remaining bidder after the auction ends—which maximizes the expected payment of that bidder.[21] Throughout we assume sellers are risk-neutral.

We make the following assumption before stating our main revenue-bounding result:

Assumption 3. *Expected revenue is concave in the number of bidders.*

Proposition 3. *In ascending auctions with no reserve price, if i) either Assumption 1 or Assumption 2 holds, ii) Assumption 3 holds, and iii) bidders are symmetric and have increasing marginal revenue, then for all $n > 2$ the increase in expected revenue from using the optimal reserve price is less than $\Delta^{bid}(n)$.*

[19] A "standard" mechanism in this context is one in which 1) losers pay nothing, 2) the bidder with the highest signal wins (if anyone) and pays an amount which increases in his own signal given any realization of other bidders' signals. Bulow and Klemperer (1996) highlight a result of Lopomo's (1995), which shows that an optimal mechanism in this class is an English auction followed by a final, take-it-or-leave-it offer to the high bidder (a reserve price). When bidders have correlated values, Crémer and McLean (1988), McAfee et al. (1989), and McAfee and Reny (1992) have provided examples of non-standard mechanisms which extract all bidder surplus and outperform an auction with a reserve price.

[20] See Appendix F for details. Bulow and Klemperer parameterize marginal revenue in terms of bidder "quantity" rather than their private signals, so that their marginal revenue function is decreasing.

[21] As discussed in Bulow and Klemperer (1996), with correlated bidder signals, expected revenue will generally be greater when the reserve is set optimally conditional on the observed bids rather than being set before the auction takes place.

The idea behind the proof is as follows. By the Bulow-Klemperer theorem, the increase in expected revenue from the optimal reserve price is less than the increase in expected revenue from adding another bidder, which by revenue concavity, is in turn less than the bidder exclusion effect. The result follows from Proposition 1, if Assumption 1 holds, and from Proposition 2, if Assumption 2 holds. Assumption 3, revenue being concave in the number of bidders, is natural in many contexts, although it will not hold in all environments.[22] In Appendix F we show that with IPV (or conditionally IPV), symmetry and increasing marginal revenue (or conditionally increasing marginal revenue) imply Assumption 3.[23]

Proposition 3 states that the bidder exclusion effect in n bidder auctions is greater than the revenue increase from an optimal reserve price. As pointed out by Bulow and Klemperer (1996), even when signals are affiliated, an ascending auction with a optimal reserve yields at least as much expected revenue as any standard mechanism (Lopomo (1995)). Thus the increase in expected revenue from using any standard mechanism is also less than $\Delta^{bid}(n)$.[24]

The bidder exclusion effect provides a bound on the impact of optimal reserve prices for a fixed number of bidders, even if the data comes from auctions with selective entry, as estimating the bidder exclusion effect (or its upper bound) does not require exogenous variation in the number of bidders. When the number of bidders is unknown, an upper bound on the average bidder exclusion effect, as described in Section 3.3.3, gives an upper bound on the gain from improving auction design averaged over the (unobserved) realized values of N. Section 7 illustrates this empirically.

[22] Dughmi et al. (2012) provided a counterexample where all bidders have iid values, which are 1 with probability p and 0 otherwise. For sufficiently small p, the revenue increase from one to two bidders is smaller than from two to three bidders. It can be proved that, for any exchangeable value distribution F^{n+1}, a necessary and sufficient condition for the revenue change from $n-1$ to n bidders to be larger than from n to $n+1$ bidders is $3(E(V^{n:n+1})-E(V^{n-1:n+1})) > E(V^{n+1:n+1})-E(V^{n:n+1})$.

[23] We prove a special case of results already established by Dughmi et al. (2012) (Theorem 3.2). The specialization to our current single item auction setting allows us to use only elementary mathematics, in contrast to Dughmi et al. (2012)'s proof which relies on matroid theory.

[24] In later work, Bulow and Klemperer (2002) highlighted that the assumption of marginal revenues increasing in signals may be more stringent in common values settings than in private values settings, and the authors provided examples of common values settings in which the original Bulow and Klemperer (1996) result does not hold.

6. Application 1: Testing for selective entry at timber auctions

Our first application uses US timber auction data to illustrate how the bidder exclusion effect can be used to distinguish between models of entry and bound the impact of optimal auction design. The Forest Service's timber auction data has been used extensively in the empirical auctions literature, and is a natural context to demonstrate the applications of the bidder exclusion effect. For example, Haile and Tamer (2003) and Aradillas-López et al. (2013a) use timber auction data and develop bounds methods relying on the assumption that valuations are independent of N (i.e. that entry is not selective). Below we test the validity of this assumption. Optimal reserve prices have also been a major focus of timber auction work.[25] By estimating the bidder exclusion effect and relating it to the work of Bulow and Klemperer (1996), we are able to side-step the need for a complex structural model and still obtain a simple estimate of the revenue increase from an optimal reserve price.

Our data comes from ascending auctions held in California between 1982 and 1989 in which there were at least three entrants. There are 1,086 such auctions. The data contains all bids, as well as auction level information, such as appraisal variables, measures of local industry activity, and other sale characteristics. By Propositions 1 and 2, with private values and low bidding, or in button auctions with symmetric common values, the bidder exclusion effect is less than $\frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$. We estimate the sample analog of this upper bound conditional on the number of entrants in the auction but unconditional on auction characteristics. Figure 1 shows the results, both as a percentage of revenue, and in absolute terms.[26] The upper bound on the bidder exclusion effect is 12% of auction revenue, averaging over all values of n, with a standard error of 0.4%. Under the conditions of Proposition 3, the average increase in revenue from setting an optimal reserve price is therefore less than around 13%.

[25] For example, Haile and Tamer (2003) and Aradillas-López et al. (2013a) study optimal reserves.

[26] The larger confidence interval for the $n = 7$ auctions in the right panel of Figure 1 is driven by an outlier (an outlier in terms of its realization of $B^{n-1:n} - B^{n-2:n}$ but not as a percentage of revenue, $(B^{n-1:n} - B^{n-2:n})/B^{n-1:n}$, and thus in the left panel the effect is still precisely measured).

We now turn to the test for selective entry. We begin with the simplest version of the test, without controlling for covariates. Table 1 displays the results. In the table, a_1 represents the expected second order statistic when a bidder is removed at random from n bidder auctions, a_2 represents the expected second order statistic in $n-1$ bidder auctions when the actual number of bidders is indeed $n-1$, and the test statistic is given by $\widehat{T}(n) = a_1 - a_2$. For most $n \in \{3, ..., 8\}$, $\widehat{T}(n)$ is insignificant, although at $n = 3$ and $n = 5$, the test statistic is significant and positive, indicating that selective entry may be a concern. Intuitively, a positive $T(n)$ indicates that bidders' values are higher in n than $n-1$ bidder auctions, as might be the case when attractive goods tend to draw many entrants.

Table 2 shows the results of the selective entry test conditional on auction characteristics. The objects a_1, a_2, and $T(n)$ are as in Table 1, but after controlling for covariates (following the parametric procedure described in Section 4.3). We control for appraisal variables (quintiles of the reserve price, selling value, manufacturing costs, logging costs, road construction costs, and dummies for missing road costs and missing appraisals), sale characteristics (species Herfindahl index, density of timber, salvage sale or scale sale dummies, deciles of timber volume, and dummies for forest, year, and primary species), and local industry activity (number of logging companies in the county, sawmills in the county, small firms active in the forest-district in the last year, and big firms active in the forest-district in the last year).

There is stronger evidence for selective entry when controlling for auction characteristics than in the unconditional case. Conditional on auction characteristics, average revenue when a bidder is removed at random from n bidder auctions is higher than average revenue is $n-1$ bidder auctions when $n \in \{3, 4, 5, 7\}$, and this difference is significant at the 95% level. Again, this supports positive selection: bidders' valuations appear to be higher in auctions with more entrants. With $n \in \{6, 8\}$ the difference is negative and insignificant, consistent with a setting where valuations are independent of N. The joint null hypothesis of no selective entry across all $n \in \{3, \ldots, 8\}$ can be rejected at the 99.9% level.

Some bidders in timber auctions may be stronger than others. One common distinction in the literature is between mills, who have the capacity to process the timber, and loggers, who do not. Mills typically have higher valuations than loggers (e.g. Athey et al. 2011, 2013; Roberts and Sweeting 2013a). The evidence of selective entry above may be driven by differences in logger and mill entry patterns. We next turn to the question of whether evidence of selective entry exists, even after restricting attention to a more homogeneous subset of bidders–in this case, loggers.[27]

Table 3 shows the results. The sample size is significantly smaller when restricting to auctions in which all entrants are loggers. At $n = 4$, the test still rejects the null hypothesis that valuations are independent of N. However, the evidence on the whole is much weaker in the loggers-only sample: at $n \in \{3, 5, 6\}$ the difference is much smaller and insignificant, although the smaller sample size may play a role. Together the results from Tables 2 and 3 suggest that selective entry may be a feature of timber auctions but may be reasonably controlled for by accounting for bidder types.

7. Application 2: Bounding the impact of optimal auction design at auto auctions

Our second application uses wholesale used-car auction data to illustrate how the bidder exclusion effect can be used to bound the revenue impact of optimal auction design. Recent studies report causal effects of various interventions at used-car auctions, and our approach allows us to judge the economic relevance of these effects by benchmarking them against the effect of optimal auction design, a question to which much of the auction literature—both theoretical and empirical—is dedicated. This setting also shows how the bidder exclusion effect can be used with minimal assumptions and data requirements. Specifically, records from used-car auctions do not contain the number of bidders and this number can vary from auction to auction, posing challenges for many structural auctions techniques.

[27]There are too few auctions without logger entrants (only 21) to present the same analysis for mills.

Our dataset contains the second and third-highest bids from 6,003 sales of used cars, as well as car characteristics. Summary statistics for these auction sales are shown in Table 4. These cars are mostly late-model (two years old on average), low-mileage cars (33,369 miles on average). The market thickness measure comes from a larger, nationwide sample of auto auction sales from which our final sample is taken, and represents the total number of sales for a given make-by-model-by-age combination.[28] Table 4 shows that, on average, a given make-by-model-by-age combination sold over 1,000 times in the nationwide sample.

Multiple auctions take place in different lanes within the auction house simultaneously, and it is impossible to determine the number of participating bidders (who may be physically present at the auction house or watching online) for any given sale. The data does record the number of online bidders who have logged in to the online console for a given lane at some point during the sale day, which is 35 bidders on average. However, many of these online bidders may not actually be participating bidders for a given sale. Based on personal observations at these auction houses, we choose $\underline{n} = 5$ as a lower bound on the number of bidders present.[29]

Table 4 also displays summary statistics for the second order statistic (the highest observed bid), third order statistic (one bid increment beyond where the third-final bidder dropped out), and the gap between the two. Following Section 3.3.3, when the number of bidders is unobserved, an upper bound on the bidder exclusion effect, averaged over the unobserved values of n, is given by the mean of the gap between second and third order statistics scaled by $2/\underline{n}$. This implies an unconditional estimate

[28]This larger, nationwide sample contains 901,338 auction sales from 27 auction houses nationwide between 2007 and 2010. For each auction in this larger sample, we observe a complete bid log of all bids submitted. However, bid logs at wholesale auto auctions do not contain identities of floor bidders (those who are physically present at the auction); the log simply records the identity as "Floor" for any floor bidder. Therefore, we are only able to identify the third order statistic of bids in cases where at least two of the last three bidders to place bids were online bidders (whose identities are always recorded). This leaves a sample of 8,005 auction sales. We drop recreational vehicles (including boats, motorhomes, motorcycles) or observations lying outside the first or ninety-ninth percentiles of mileage, age, or the number of online bidders. This leaves 6,003 records.

[29]Note that the sales in our data are mainly of cars sold by large fleet/lease sellers rather than small used-car dealers; the latter sales would likely have fewer bidders present. See Larsen (2014).

of the bidder exclusion effect upper bound of $(2/5) \times \$425 = \$170$. This provides an upper bound on the average revenue increase from an optimal reserve price.

Next, following Section 3.3.3, we estimate the upper bound on the average bidder exclusion effect conditional on covariates X, approximating the conditional expectation using a kernel regression.[30] Figure 2 displays the estimates with X being the vehicle's mileage, the vehicle's age, the number of online bidders, or the market thickness measure. Panels A and B suggest that auctions of cars which are lower in mileage or younger in age exhibit a lower bidder exclusion effect. That is, excluding a random bidder from the auction does less damage to revenue in these auctions than in auctions for older, higher mileage cars. This is consistent with sales of newer, lower mileage cars being cases where demand is high and many bidders participate, and hence optimal auction design is less important and a no-reserve auction is likely to perform well. Similarly, Panels C and D suggest that in cases where more bidders have logged in online or where the car is of a frequently sold make-by-model-by-age combination, the bidder exclusion effect is small.

The bidder exclusion effect also leads to bounds on the revenue from bidders which could be expected in any auction, ranging from a no-reserve auction to an auction with an optimal reserve price, as a function of sale characteristics. For each observation j, let lower and upper bounds on revenue conditional on observables X_j be denoted $\underline{\pi}_j(X_j)$ and $\overline{\pi}_j(X_j)$, given by

$$\underline{\pi}_j(X_j) = b_j^{n_j-1:n_j}(X_j) \tag{12}$$

$$\overline{\pi}_j(X_j) = b_j^{n_j-1:n_j}(X_j) + \frac{2}{\underline{n}}\left(b_j^{n_j-1:n_j}(X_j) - b_j^{n_j-2:n_j}(X_j)\right) \tag{13}$$

The lower bound is the observed second order statistic (the no-reserve revenue) and the upper bound is the observed second order statistic plus the scaled gap between second and third order statistics (the expectation of this upper bound is an upper bound on the optimal reserve revenue, under the conditions of Proposition 3). The shaded region in Figure 3 shows the area between the upper pointwise 95% confidence

[30]We use an Epanechnikov kernel and rule-of-thumb bandwidth.

band for $\hat{E}(\overline{\pi}_j|X_j)$ and the lower pointwise 95% confidence band for $\hat{E}(\underline{\pi}_j|X_j)$, where each conditional expectation is estimated using a kernel regression as above.

Panels A and B of Figure 3 demonstrate that the expected revenue is tightly bounded, regardless of the seller's choice of reserve price, for newer, high-mileage cars. Similarly, Panels C and D show that the fraction of expected revenue which could be manipulated through the use of reserve prices is much smaller for auctions of more popular cars or auctions in lanes where more online bidders have logged in.

These estimates are a benchmark for evaluating the economic significance of other effects measured in this industry. Tadelis and Zettelmeyer (2014), through a field experiment at a wholesale auto auction, find that revealing information about the quality of the car through standardized condition reports lead to a difference in revenue of \$643.[31] Lacetera et al. (2013) find that a one-standard-deviation improvement in auctioneer performance raises revenue by \$348.[32] Hortaçsu et al. (2013) find that a 1,000 point increase in credit default swap spreads decrease wholesale auction prices by approximately \$68.[33] The cars studied in these papers have average mileage and age values which correspond to a bidder exclusion effect of approximately \$200 in Figure 3.[34] Therefore, a comparison to the bidder exclusion effect suggests that information-disclosing reports or high-performing auctioneers do more to increase revenues than would the use of an optimal reserve price. Similarly, swings in auction prices resulting from changes in firms' perceived financial stability are at least one-third of the size of price changes which the adoption of optimal reserve prices could generate.

[31]Tables 2 and 3 of Tadelis and Zettelmeyer (2014) show that the probability of sale and expected price conditional on sale for the treatment group, for which condition reports were posted, were 0.455 and \$8,738.90, respectively, vs. 0.392 and \$8,502.20 in the control group.

[32]Table 2, row 8 of Lacetera et al. (2013) shows that one standard deviation of the probability of sale among auctioneers is 0.023. Table 1 shows that the average price on cars conditional on sale is \$15,141. The product of these two numbers is the expected revenue increase from employing an auctioneer who performs one standard deviation above the mean, all else equal.

[33]This number is the price conditional on sale, but Hortaçsu et al. (2013) focus primarily on cars which sell with very high probability (fleet/lease cars). Therefore, the effect on the price conditional on sale is a good approximation for overall effect of financial distress on revenue from bidders.

[34]The average odometer reading and age are 75,959 miles and five years old in Table 16 of Tadelis and Zettelmeyer (2014); 56,237 miles and 4.4. years in Table 1 of Lacetera et al. (2013); and 44,270 miles in Table 4 of Hortaçsu et al. (2013) (which does not report age).

8. Conclusion

We developed a simple procedure for estimating the causal effect of removing a random bidder on auction revenue—the bidder exclusion effect—without requiring instruments, a detailed structural model, or exogenous variation in the number of bidders. Our approach is robust to a wide range of auction settings. The bidder exclusion effect is useful in testing the independence of bidders' valuations and the number of bidders participating, allowing the researcher to distinguish between models of entry. Furthermore, we introduced a new empirical use for the theoretical results of Bulow and Klemperer (1996), demonstrating that the bidder exclusion effect can be used to bound the revenue increase achievable through optimal auction design.

Given that our approach makes only weak assumptions on the auction setting, it cannot identify all objects of interest. For example, while our approach gives bounds on the revenue gain which an optimal reserve price could achieve, stronger assumptions on the structure of bidders' valuations or the underlying entry model would be required in order to recommend a reserve price.

We believe that the bidder exclusion effect is also likely to be useful for other applications. Under certain assumptions, for example, it provides a bound on the revenue loss a seller would face if bidders were to merge or collude. It may also aid in detecting collusion. Appendix G provides a brief discussion of these issues. The bidder exclusion effect also provides a simple specification check of standard assumptions in empirical auctions analysis. For example, under the assumption of independent private values in button auctions, one can invert the second-order statistic distribution to obtain an estimate of the underlying distribution of buyer valuations (Athey and Haile 2007) and simulate the revenue increase under an optimal reserve price. If the simulated revenue increase exceeds the bidder exclusion effect, the validity of either the assumption of independence or the assumption of private values—or both—is in question.

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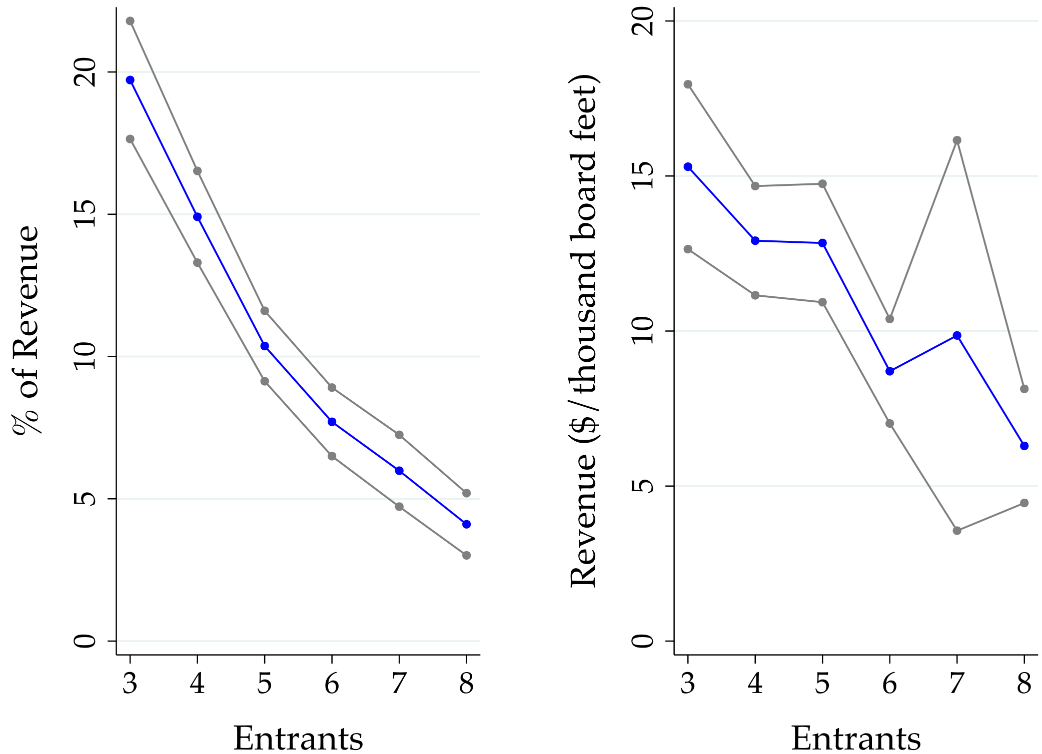
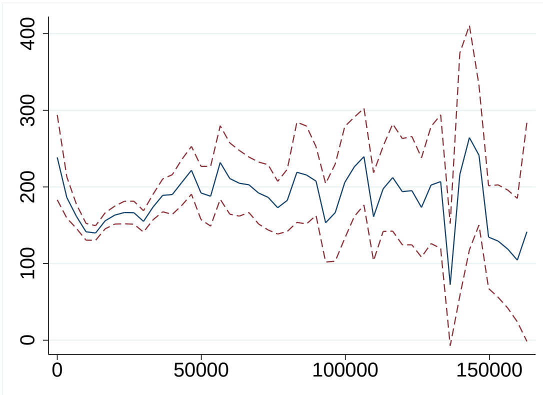
FIGURE 1. Bounding the Bidder Exclusion Effect

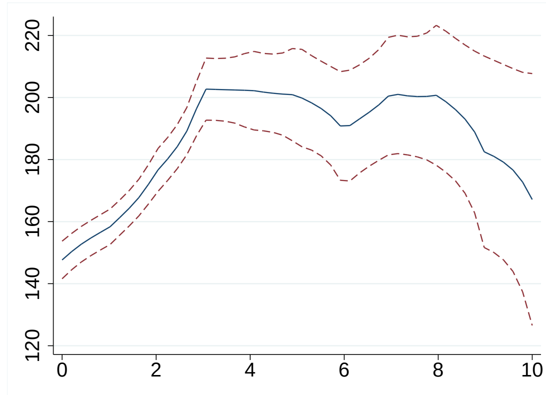


Notes: Graphs show point estimates and 95% pointwise confidence intervals for $\frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$, for various values of n, the total number of entrants in the auction. Estimates in the left graph are expressed as a percentage of auction revenue. Estimates in the right graph are expressed in dollars per thousand board feet.

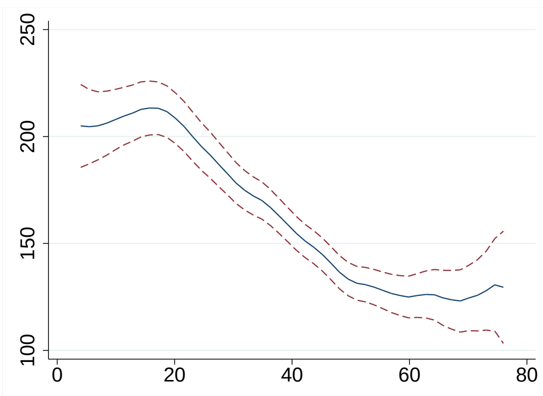
Figure 2. Upper Bounds on the Bidder Exclusion Effect



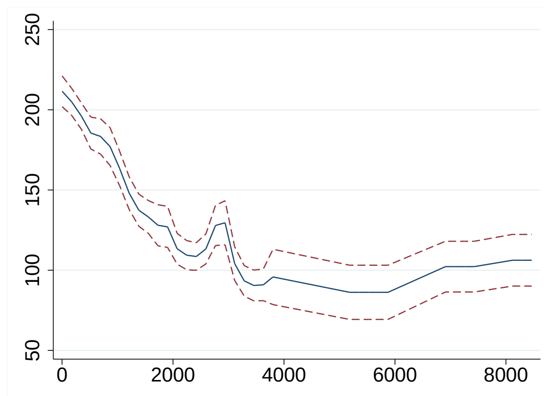
(A) Mileage



(B) Age



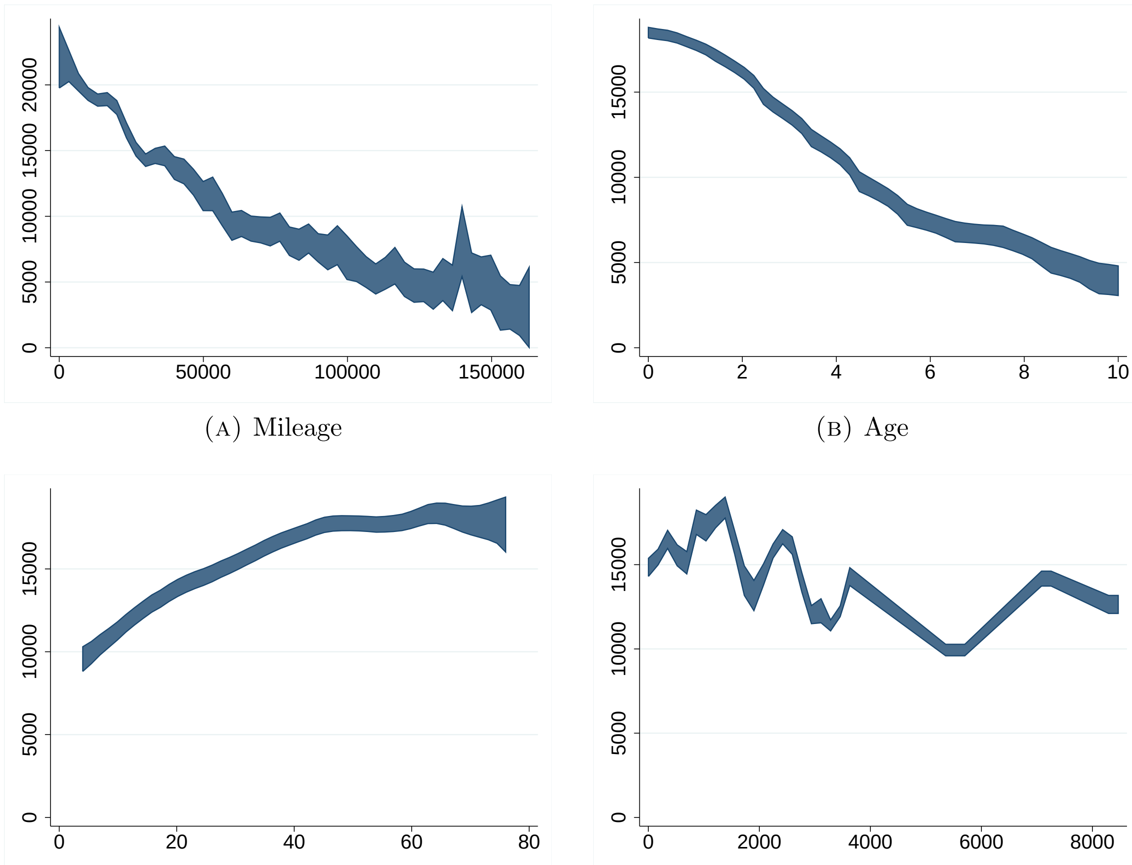
(C) # Online bidders



(D) Market thickness

Notes: Vertical axis in each panel represents upper bounds on the bidder exclusion effect in dollars. Solid line represents bidder exclusion effect conditional on mileage (A), age (B), number of online bidders (C), and market thickness (D). Estimates come from kernel regression with Epanechnikov kernel and rule-of-thumb bandwidth. Dashed lines represent pointwise 95% confidence bands.

FIGURE 3. Upper and Lower Bounds on Expected Revenue



(A) Mileage

(B) Age

(C) # Online bidders

(D) Market thickness

Notes: Vertical axis in each plot represents expected revenue in dollars. Lower boundary of shaded region is given by lower 95% confidence band about the conditional expectation of the second order statistic. Upper boundary of shaded region is given by upper 95% confidence band about the conditional expectation of the sum of the second order statistic and the scaled gap between second and third order statistics (scaled by $2/\underline{n}$). Estimates are conditional on mileage (A), age (B), number of online bidders (C), and market thickness (D), and come from kernel regression with Epanechnikov kernel and rule-of-thumb bandwidth.

Table 1. Unconditional Tests for Selective Entry, All Auctions

Entrants	3	4	5	6	7	8
a_1	78.15***	92.34***	119.75***	125.64***	123.16***	147.24***
	(6.36)	(4.62)	(4.81)	(11.14)	(6.69)	(9.65)
a_2	49.60***	78.37***	89.09***	119.66***	125.02***	130.46***
	(2.62)	(6.43)	(3.50)	(4.85)	(11.15)	(12.04)
$T(n) = a_1 - a_2$	28.55***	13.97	30.66***	5.98	-1.86	16.78
	(6.87)	(7.92)	(5.95)	(12.15)	(13.00)	(15.43)
Sample Size	497	496	456	350	243	164

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Table presents results of test for selective entry unconditional on covariates, for various levels of the number of entrants.

Table 2. Conditional Tests for Selective Entry, All Auctions

Entrants	3	4	5	6	7	8
a_1	-44.52	-120.64	3.34	-95.81**	-111.39	-172.47
	(59.38)	(74.72)	(24.72)	(32.93)	(71.81)	(159.06)
a_2	-64.84	-137.66	-10.47	-89.90**	-124.05	-167.05
	(59.39)	(75.24)	(25.01)	(31.83)	(72.75)	(145.70)
$T(n) = a_1 - a_2$	20.32***	17.02***	13.81***	-5.91	12.66*	-5.42
	(5.33)	(4.21)	(3.63)	(4.75)	(5.78)	(17.94)
Sample Size	497	496	456	350	243	164

Heteroskedasticity-robust standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Table presents results of test for selective entry conditional on covariates described in Section 4.3, for various levels of the number of entrants.

TABLE 3. Testing for Selective Entry, Auctions with Only Loggers

Entrants	3	4	5	6
a_1	-158.28	-183.18	26.03	-285.34**
	(108.96)	(115.53)	(42.50)	(115.06)
a_2	-172.28	-222.73	8.53	-266.30
	(105.59)	(128.23)	(43.13)	(111.34)
$T(n) = a_1 - a_2$	14.00	39.54*	17.51	-19.04
	(16.65)	(19.29)	(9.92)	(25.66)
Sample Size	149	138	109	76

Heteroskedasticity-robust standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Table presents results of test for selective entry conditional on covariates described in Section 4.3, for various levels of the number of entrants, and only for auctions in which all entrants are loggers.

TABLE 4. Wholesale auto auction data descriptive statistics

	Mean	s.d.	Min.	Max.
Mileage	33,369	29,601	13	163,036
Age	1.95	2.18	0	10
# Online bidders	34.71	16.71	4	76
Market thickness	1,196.01	1,648.60	2	8,465
$b^{n-1:n}$	15,429	7,733	300	55,100
$b^{n-2:n}$	15,004	7,648	225	53,400
$b^{n-1:n} - b^{n-2:n}$	425	494	50	6650
Sample size		6,003		

Notes: Age is in years. # Online bidders is the number of bidders who had logged into the lane the car was sold in. Market thickness is the number of cars of the same make-by-model-by-age combination as a given car. $b^{n-1:n}$ is the final auction price (second order statistic), $b^{n-2:n}$ is the highest bid of the third-final bidder (third order statistic).

Appendix A. Proofs

A.1. **Proof of Proposition 1.**

$$\begin{aligned}\Delta(n) &= E(B^{n-1:n}) - E(B^{n-2:n-1,n}) \\ &= E(V^{n-1:n}) - E(V^{n-2:n-1,n}) \\ &= \frac{2}{n}E(V^{n-1:n} - V^{n-2:n}) \\ &\leq \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}).\end{aligned}$$

The first equality is true by definition of $\Delta(n)$. The second follows because by Assumption 1, the second-highest bidder bids his value. The third follows because with probability $\frac{2}{n}$ dropping a value from $(V_1, \ldots, V_n)$ uniformly at random will cause the second-highest value to drop from $V^{n-1:n}$ to $V^{n-2:n}$, and otherwise the second-highest value will be unchanged. The final inequality holds because by Assumption 1, the second-highest bidder bids his value and the third-highest bidder does not bid above his value.

A.2. **Proof of Proposition 2.** Athey and Haile (2002), Theorem 9 proves that for button auction ascending auctions with symmetric common values, regardless of the equilibrium played, $E(B^{n-2:n-1,n}) > \frac{2}{n}E(B^{n-2:n}) + \frac{n-2}{n}E(B^{n-1:n})$. This implies $E(B^{n-1:n}) - E(B^{n-2:n-1,n}) < \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$, as required.

A.3. **Proof of Corollary 1.**

$$\begin{aligned}E(\Delta(N)) &\leq E\left(\frac{2}{N}(B^{N-1:N} - B^{N-2:N})\right) \\ &\leq \frac{2}{\underline{n}}E(B^{N-1:N} - B^{N-2:N}),\end{aligned}$$

where the first inequality follows by Proposition 1 if Assumption 1 holds and from Proposition 2 if Assumption 2 holds, and the second by $n \geq \underline{n}$ for all realizations n of the random variable N.

A.4. **Proof of Proposition 3.** Theorem 1 of Bulow and Klemperer (1996) implies that the increase in expected revenue from using an optimal reserve price is less than the increase in expected revenue from adding another bidder, which by Assumption 3, is less than the bidder exclusion effect. The result follows from Proposition 1 if Assumption 1 holds and from Proposition 2 if Assumption 2 holds.

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Appendix B. Binding Reserve Prices

We consider ascending auctions with private values where bidders bid their values, and where there is a reserve price below which bids are not observed. We modify our notation accordingly: $\Delta(n, r)$ denotes the fall in expected revenue produced by randomly excluding a bidder from n bidder auctions, when the reserve price is r.

Proposition 4. *In ascending auctions with private values and a reserve price of r where bidders bid their value, for all $n > 2$ the bidder exclusion effect* $\Delta(n, r) = \frac{2}{n}E(B^{n-1:n} - \max(B^{n-2:n}, r)|r \le B^{n-1:n}) \Pr(r \le B^{n-1:n}) + \frac{1}{n}r \Pr(B^{n-1:n} < r \le B^{n:n})$.

Proof. If $r \le B^{n-1:n}$, then with probability $\frac{2}{n}$ dropping a bidder at random will cause revenue to fall from $B^{n-1:n}$ to $\max(B^{n-2:n}, r)$, so that in expectation revenue falls by $\frac{2}{n}E(B^{n-1:n} - \max(B^{n-2:n}, r)|r \le B^{n-1:n})$. If $B^{n-1:n} < r \le B^{n:n}$, then with probability $\frac{1}{n}$ dropping a bidder at random will cause revenue to fall from r to 0. If $B^{n:n} < r$, then dropping a bidder at random will not change revenue. These observations imply the result. □

This expression for $\Delta(n, r)$ can be estimated given observed data, as it does not depend on knowing the value of bids lower than the reserve price.

B.1. **Applications of the Bidder Exclusion Effect with Binding Reserve Prices.** When the reserve price equals r in both n and $n-1$ bidder auctions, the expected revenue difference between those auctions is

$$E(\max(B^{n-1:n}, r)|r \le B^{n:n}) \Pr(r \le B^{n:n}) - E(\max(B^{n-2:n-1}, r)|r \le B^{n-1:n-1}) \Pr(r \le B^{n-1:n-1}). \quad (14)$$

If valuations are independent of N, then $F^n_{n-1} = F^{n-1}_{n-1} = F^{n-1}$ and hence expression (14) equals the expression for $\Delta(n, r)$ of Proposition 4. As in Section 4.1, we can test this hypothesis with a t-test, where the test statistic is formed by replacing expectations by sample averages. This test is consistent against forms of selective entry which affect expected revenue, i.e. such that $E(\max(B^{n-2:n-1,n}, r)|r \leq B^{n-1:n-1,n}) \Pr(r \leq B^{n-1:n-1,n}) \neq E(\max(B^{n-2:n-1}, r)|r \leq B^{n-1:n-1}) \Pr(r \leq B^{n-1:n-1})$.

This test can be adapted to incorporate covariates. The null hypothesis is:

$$E\left(\mathbb{1}(r \leq B^{n-1:n})\left(\frac{n-2}{n}B^{n-1:n} + \frac{2}{n}\max\{B^{n-2:n}, r\}\right) + \mathbb{1}(B^{n-1:n} < r \leq B^{n:n})\frac{n-1}{n}r \middle| X\right)$$
$$= E(\mathbb{1}(r \leq B^{n-1:n-1})\max\{B^{n-2:n-1}, r\}|X). \tag{15}$$

This states that, conditional on covariates, revenue in n bidder auctions when one bidder is dropped at random equals revenue in $n-1$ bidder auctions. The regression-based test of Section 6 can be modified to test this restriction.

For the application to optimal mechanism design, we require an upper bound on $\Delta(n, 0)$. Using the fact that bids are non-negative, we have

$$\begin{aligned}
\Delta(n, 0) = &\frac{2}{n}E(B^{n-1:n} - B^{n-2:n}|r \leq B^{n-2:n}) \Pr(r \leq B^{n-2:n}) \\
&+ \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}|B^{n-2:n} < r \leq B^{n-1:n}) \Pr(B^{n-2:n} < r \leq B^{n-1:n}) \\
&+ \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}|B^{n-1:n} < r) \Pr(B^{n-1:n} < r) \qquad (16) \\
\leq &\frac{2}{n}E(B^{n-1:n} - B^{n-2:n}|r \leq B^{n-2:n}) \Pr(r \leq B^{n-2:n}) \\
&+ \frac{2}{n}E(B^{n-1:n}|B^{n-2:n} < r \leq B^{n-1:n}) \Pr(B^{n-2:n} < r \leq B^{n-1:n}) \\
&+ \frac{2}{n}r \Pr(B^{n-1:n} < r) \qquad (17)
\end{aligned}$$

The terms in (17) do not depend on knowing the value of bids lower than the reserve price, and can be estimated given observed data.

Appendix C. First Price Auctions

We now give upper and lower bounds on the bidder exclusion effect in first price auctions with symmetric IPV, and symmetric conditionally independent private values (CIPV). Let $b(V_i, F^n)$ denote bidder i's equilibrium bid, as a function of his value, V_i, and the distribution of bidders' valuations, F^n. We assume V_i is continuously distributed on some interval $[0, u]$. In this section we use subscripts to make explicit the distribution with respect to which expectations are taken, e.g. expected revenue with no reserve price is $E_{F^n}(b(V^{n:n}, F^n))$ in n bidder auctions and is $E_{F^n_{n-1}}(b(V^{n-1:n-1}, F^n_{n-1}))$ when one of the n bidders is randomly excluded. The bidder exclusion effect is $\Delta(n) \equiv E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n_{n-1}}(b(V^{n-1:n-1}, F^n_{n-1}))$.

Proposition 5. *In first price auctions without a reserve price if i) bidders have symmetric independent private values, or ii) there is a random variable U common knowledge to bidders such that bidders have symmetric independent private values conditional on U, then $E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n_{n-1}}(b(V^{n-1:n-1}, F^n)) < \Delta(n) < E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n_{n-1}}(b(V^{n-2:n-1}, F^n))$.*

Proof. We first consider the case of symmetric independent private values. For the lower bound, note that in symmetric independent private values settings, equilibrium bids are strictly increasing in n: $b(v_i, F^n) > b(v_i, F^n_{n-1})$ (see, for example, Krishna (2009)). This implies $E_{F^n_{n-1}} b(V^{n-1:n-1}, F^n) > E_{F^n_{n-1}} b(V^{n-1:n-1}, F^n_{n-1})$, and therefore $E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n_{n-1}} b(V^{n-1:n-1}, F^n) < \Delta(n)$.

For the upper bound, we have

$$E_{F^n_{n-1}}(b(V^{n-2:n-1}, F^n)) < E_{F^n_{n-1}}(V^{n-2:n-1}) \tag{18}$$

$$= E_{F^n_{n-1}}(b(V^{n-1:n-1}, F^n_{n-1})). \tag{19}$$

The inequality holds because equilibrium bids are strictly less than values. The equality holds by revenue equivalence of first and second price auctions with symmetric independent private values. It follows that $\Delta(n) < E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n_{n-1}}(b(V^{n-2:n-1}, F^n))$.

If values are symmetric and CIPV, then because U is common knowledge to bidders these lower and upper bounds hold conditional on every realization of U, and therefore hold unconditionally, taking expectations with respect to U. The bounds thus extend to the conditionally independent private values case. □

The lower bound above is the expected fall in revenue in n bidder auctions when one bid is removed at random, assuming the good will be sold at a price equal to the highest of the remaining bids. The upper bound is the expected fall in revenue in n bidder auctions when one bid is removed at random, assuming the good will be sold at a price equal to the *second* highest of the remaining bids. The following corollary characterizes these bounds more explicitly in terms of the bids from n bidder auctions.

Corollary 2. *In first price auctions without a reserve price if i) bidders have symmetric independent private values, or ii) there is a random variable U common knowledge to bidders such that bidders have symmetric independent private values conditional on U, then*

$$\frac{1}{n}\left(E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n}(b(V^{n-1:n}, F^n))\right) < \Delta(n) \tag{20}$$

and

$$\begin{aligned}\Delta(n) < \frac{n-2}{n}&\left(E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n}(b(V^{n-1:n}, F^n))\right) + \\ &\frac{2}{n}\left(E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n}(b(V^{n-2:n}, F^n))\right).\end{aligned} \tag{21}$$

Proof. For the lower bound, note that with probability $\frac{n-1}{n}$ dropping a bid at random will not change the highest bid, and with probability $\frac{1}{n}$ the highest bid will drop from $b(V^{n:n}, F^n)$ to $b(V^{n-1:n}, F^n)$. For the upper bound, note that with probability $\frac{n-2}{n}$ the difference between the highest bid in the original sample and the second-highest bid after one bid has been dropped at random is $b(V^{n:n}, F^n) - b(V^{n-1:n}, F^n)$, and with probability $\frac{2}{n}$ it is $b(V^{n:n}, F^n) - b(V^{n-2:n}, F^n)$. □

Several remarks on these bounds are in order. The lower bound holds under symmetric correlated private values too, as long as equilibrium bids are strictly increasing in n.[35] The upper bound holds if bidders are risk-averse, as first price auctions raise more revenue than ascending auctions, with symmetric risk-averse bidders in IPV environments (Riley and Samuelson 1981). In the CIPV case, if U is not common knowledge amongst bidders, then bidders' private information is correlated conditional on what they know at the time of bidding. This affects equilibrium bidding behavior and the argument of Proposition 5 does not hold. Finally, the upper bound of Proposition 5 can be replaced by $E_{F^n}(b(V^{n:n}, F^n)) - E_{F^{n'}_{n-1}}(b(V^{n-2:n-1}, F^{n'}))$ for any $n' > n-1$, as bids are below values in n' bidder auctions too. Consequently, $\Delta(n) \leq E_{F^n}(b(V^{n:n}, F^n)) - \sup_{n'} E_{F^{n'}_{n-1}}(b(V^{n-2:n-1}, F^{n'}))$.

C.1. **Applications of the Bidder Exclusion Effect in First Price Auctions.** As with ascending auctions, the bidder exclusion effect can be used to test for selective entry in first price auctions. Under the null hypothesis that entry is not selective, for all $n \geq 2$, $F^n_{n-1} = F^{n-1}_{n-1} = F^{n-1}$. This implies that the bidder exclusion effect $\Delta(n) \equiv E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n_{n-1}}(b(V^{n-1:n-1}, F^n_{n-1}))$ equals $E_{F^n}(b(V^{n:n}, F^n)) - E_{F^{n-1}}(b(V^{n-1:n-1}, F^{n-1}))$. If the sample analog of $E_{F^n}(b(V^{n:n}, F^n)) - E_{F^{n-1}}(b(V^{n-1:n-1}, F^{n-1}))$—which is simply average revenue in n bidder auctions minus average revenue in $n-1$ bidder auctions—lies outside the sample analogs of the lower or upper bounds of Corollary 2, this is evidence against the null hypothesis. This test is consistent against violations of the null when values are "sufficiently" decreasing or increasing with n. Precisely, this is the case if $E_{F^{n-1}}(b(V^{n-1:n-1}, F^{n-1})) > E_{F^n_{n-1}}(b(V^{n-1:n-1}, F^n))$ or $E_{F^{n-1}}(b(V^{n-1:n-1}, F^{n-1})) < E_{F^n_{n-1}}(b(V^{n-2:n-1}, F^n))$. Again, the regression-based test of Section 6 can be modified to test that the null hypothesis holds conditional on observable covariates, rather than unconditionally.

The application to optimal mechanism design also works for first price auctions. The Bulow-Klemperer theorem is stated for ascending auctions, but by revenue equivalence also applies to first price auctions when bidders have symmetric IPV (or symmetric CIPV). Thus Proposition 3 extends to first price auctions, where the upper

[35]Pinkse and Tan (2005) give conditions for this to hold.

bound on the effect on expected revenue of improving mechanism design is given by Corollary 2.

Appendix D. Monte Carlo Power Simulations

For some evidence on how powerful our test of selective entry is, we compare it to another test, which simply compares bidders' mean values in n and $n+1$ bidder auctions. This latter test requires the econometrician to observe all bidders' values. Relative to our test based on the bidder exclusion effect, it requires more data, and does not allow for low bidding. Furthermore, this mean comparison test is not actually feasible in ascending auctions in practice given that the highest bid is never observed.

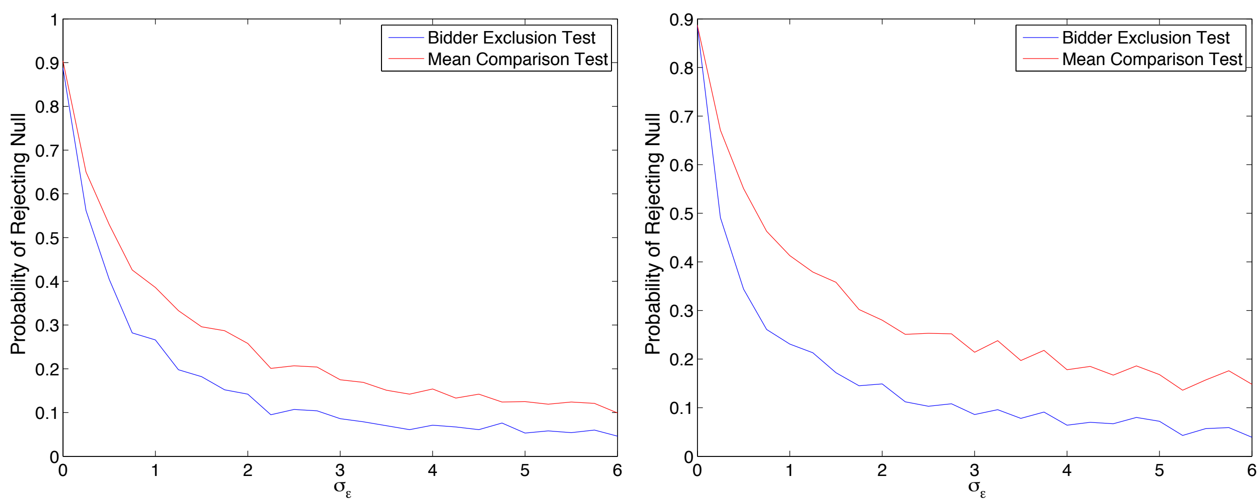
In our simulation, there are 10 potential bidders, who have iid lognormal private values drawn from $\ln N(\theta, 1)$, where θ is itself a random variable. All potential bidders see a common signal $\delta = \theta + \epsilon$, and Bayes update on the value of θ given their observation of δ. The random variables $(\delta, \theta, \epsilon)$ are jointly normally distributed:

$$\begin{pmatrix} \delta \\ \theta \\ \epsilon \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1+\sigma_\epsilon^2, & 1, & \sigma_\epsilon^2 \\ 1, & 1, & 0 \\ \sigma_\epsilon^2, & 0, & \sigma_\epsilon^2 \end{pmatrix}\right). \tag{22}$$

As σ_ϵ increases, the ratio of noise to signal increases, and the variable δ becomes less informative about the variable θ. To learn their value and bid in the ascending auction, potential bidders must pay an entry cost of 0.5. They play mixed entry strategies, entering with a probability p that depends on δ. In the limit as $\sigma_\epsilon \to \infty$, the signal δ is uninformative about θ and the entry probability p no longer varies with δ. This limiting case corresponds to the entry model of Levin and Smith (1994).

For each $\sigma_\epsilon \in \{1, 1.25, 1.5, \ldots, 7\}$, and for $n \in \{3, 4\}$, we generate 1,000 datasets with auctions in which n or $n+1$ bidders choose to enter. Each dataset contains 500 n bidder auctions and 500 $n+1$ bidder auctions. We calculate the probability of rejecting the null hypothesis of no selective entry at the 5% level over the 1,000 datasets, for each value of σ_ϵ and n, and for both bidder exclusion test, and the comparison of means test. Figure 4 shows the rejection probabilities as a function of σ_ϵ. The comparison of means uses more data (in the case of Panel (A), all three bids from $n = 3$ auctions and all four bids from $n = 4$ auctions; and, in the case of Panel (B), all four bids from $n = 4$ auctions and all five bids from $n = 5$ auctions), and is more powerful. The simulation results suggest that when not all bidders' values are observed and the comparison of means test is infeasible (as in ascending auctions), the bidder exclusion based test is a reasonably powerful alternative, especially when entry is more selective (corresponding in this model to low values of σ_ϵ.)

FIGURE 4. Monte Carlo Power Comparison



(A) 3 and 4 Bidder Auctions

(B) 4 and 5 Bidder Auctions

Notes: Figures show the simulated probability of rejecting the null hypothesis of no selective entry for various levels of entry selectiveness (with greater σ_ϵ corresponding to less selective entry), for two tests: one based on a comparison of means between n and $n+1$ bidder auctions, and one based on the bidder exclusion effect computed on n and $n+1$ bidder auctions. The left panel shows the case of $n = 3$, and the right panel shows the case of $n = 4$.

APPENDIX E. ASYMMETRIC BIDDERS

We give sufficient conditions for valuations to be independent of N with asymmetric bidders and private values, following the setup of Coey et al. (2014). Let $\mathbb{N}$ be the full set of potential bidders. Let $\mathcal{P}$ be a random vector representing the identities of bidders participating in an auction, with realizations $P \subset \mathbb{N}$. Let N be a random variable representing the number of bidders participating in an auction, with realizations $n \in \mathbb{N}$. When necessary to clarify the number of bidders in a set of participating bidders, we let P_n denote an arbitrary set of n participating bidders. Define F^P to be the joint distribution of $(V_i)_{i \in P}$ when P is the set of participating bidders.[36] As before, F^n represents the joint distribution of values conditional on there being n entrants, but unconditional on the set of participants. Therefore, $F^n(v_1 \dots v_n) = \sum_{P_n \subset \mathbb{N}} \Pr(\mathcal{P} = P_n | N = n) F^{P_n}(v_1 \dots v_n)$. For $P' \subset P$, let $F^{P'|P}$ denote the joint distribution of $(V_i)_{i \in P'}$ in auctions where P is the set of participants. Finally, let F^P_m denote the joint distribution of values of m bidders drawn uniformly at random without replacement from P, when the set P enters.

Definition 1. ***Valuations are independent of supersets*** *if for all* $P' \subset P$, $F^{P'|P} = F^{P'}$.

Definition 2. ***Bidder identities are independent of*** N *if, for all* P_n, $\Pr(\mathcal{P} = P_n | N = n) = \frac{1}{n+1} \sum_{P_{n+1} \supset P_n} \Pr(\mathcal{P} = P_{n+1} | N = n+1)$.

These definitions describe different kinds of exogeneity. Definition 1 requires that conditional on some set of bidders participating, those bidders' values are independent of which other bidders participate (what Athey and Haile (2002) refer to as exogenous participation). Definition 2 requires that the distribution of participating bidder identities in n bidder auctions is just like the distribution of participating bidder identities in $n+1$ bidder auctions, with one bidder randomly removed.[37] It restricts who participates, but not what their values are. The following proposition shows that together these conditions imply that valuations are independent of N. Consequently,

[36] We adopt the convention that bidders are ordered according to their identities, i.e. if $P = \{2, 5, 12\}$ then F^P is the joint distribution of (V_2, V_5, V_{12}), rather than, for example, the joint distribution of (V_5, V_2, V_{12}).

[37] To see this, fix P_n and note that for each $P_{n+1} \supset P_n$, P_n is obtained by dropping the bidder $P_{n+1} \setminus P_n$ from P_{n+1}. When bidders are dropped uniformly at random, this occurs with probability $\frac{1}{n+1}$.

evidence of selective entry suggests either that valuations are not independent of supersets, or that bidder identities are not independent of N.

Proposition 6. *If valuations are independent of supersets and bidder identities are independent of N, then valuations are independent of N.*

Proof. The proof follows Coey et al. (2014), Lemma 3. It suffices to prove that $F_m^n = F_m^{n+1}$ for any $n \geq m$.

$$
\begin{aligned}
F_m^n(v) &= \sum_{P_n} \Pr(\mathcal{P} = P_n | N = n) F_m^{P_n}(v) \\
&= \sum_{P_n} \sum_{P_{n+1} \supset P_n} \frac{1}{n+1} \Pr(\mathcal{P} = P_{n+1} | N = n+1) F_m^{P_n}(v) \\
&= \sum_{P_{n+1}} \sum_{P_n \subset P_{n+1}} \frac{1}{n+1} \Pr(\mathcal{P} = P_{n+1} | N = n+1) F_m^{P_n}(v) \\
&= \sum_{P_{n+1}} \sum_{P_n \subset P_{n+1}} \frac{1}{n+1} \Pr(\mathcal{P} = P_{n+1} | N = n+1) F_m^{P_n | P_{n+1}}(v) \\
&= \sum_{P_{n+1}} \Pr(\mathcal{P} = P_{n+1} | N = n+1) F_m^{P_{n+1}}(v) \\
&= F_m^{n+1}(v)
\end{aligned}
$$

The second equality follows because bidder identities are independent of N. The fourth equality follows because $F^{P_n} = F^{P_n | P_{n+1}}$, as valuations are independent of supersets. The fifth equality follows because randomly selecting m bidders from $n+1$ bidders is the same as randomly selecting n bidders from $n+1$ bidders, and then randomly selecting m bidders from those n bidders. □

Appendix F. Revenue Concavity

In this section we first provide a precise definition of marginal revenue and then, for symmetric IPV (and CIPV) auctions, give conditions for concavity of expected revenue. Following Bulow and Klemperer (1996), let $(S_1, \ldots, S_n)$ denote the private signals of the n bidders in n bidder auctions. The $n+1^{th}$ bidder, were he to enter, has a private signal denoted S_{n+1}. We denote the marginal distribution of S_j by F_j^n, and the corresponding density by f_j^n. Define $\mathbf{S} \equiv (S_1, \ldots, S_{n+1})$, and $\overline{\mathbf{S}} = \mathbf{S}_{-(n+1)}$, that is, the signals of bidders other than bidder $n+1$. Let $F_j^n(S_j|\mathbf{S}_{-j})$ and $f_j^n(S_j|\mathbf{S}_{-j})$ represent the distribution and density of bidder j's signal conditional on competitors' signals. Let $v_j(\mathbf{S})$ represent the value to bidder j given all signals, and define $\overline{v}_j(\overline{\mathbf{S}}) \equiv E_{S_{n+1}} v_j(\mathbf{S})$. Define $MR_j(\mathbf{S})$ and $\overline{MR}_j(\overline{\mathbf{S}})$ as

$$MR_j(\mathbf{S}) \equiv \frac{-1}{f_j^n(S_j|\mathbf{S}_{-j})} \frac{d}{dS_j} \left(v_j(\mathbf{S})(1 - F_j^n(S_j|\mathbf{S}_{-j})) \right) \tag{23}$$

$$\overline{MR}_j(\overline{\mathbf{S}}) \equiv \frac{-1}{f_j^n(S_j|\overline{\mathbf{S}}_{-j})} \frac{d}{dS_j} \left(\overline{v}_j(\overline{\mathbf{S}})(1 - F_j^n(S_j|\overline{\mathbf{S}}_{-j})) \right) \tag{24}$$

We say bidders have "increasing marginal revenue" (as a function of their private signals) if $S_j > S_i \Rightarrow MR_j(\mathbf{S}) > MR_i(\mathbf{S})$ and $\overline{MR}_j(\overline{\mathbf{S}}) > \overline{MR}_i(\overline{\mathbf{S}})$. Equivalently, bidders have decreasing marginal revenue, when marginal revenue is considered to be a function of bidder "quantity" (i.e. $(1 - F_j^n(S_j|\mathbf{S}_{-j}))$ and $(1 - F_j^n(S_j|\overline{\mathbf{S}}_{-j}))$) rather than of their private signals. In the independent private values case, where this assumption simplifies to the function $MR(S_j) \equiv MR_j(\mathbf{S}) = \overline{MR}_j(\overline{\mathbf{S}}) = S_j - \frac{1-F_j^n(S_j)}{f_j^n(S_j)}$ being increasing in S_j (where as above $S_j = V_j$ for private values).[38]

In the symmetric IPV case, a sufficient condition for concavity of expected revenue is for bidders marginal revenue to be increasing in their values. This is a special case of a result established by Dughmi et al. (2012).

Proposition 7. *In ascending auctions with symmetric independent private values and no reserve price, if bidders' marginal revenue is increasing in their values then expected revenue is concave in the number of bidders.*

Proof. We first prove that if $Z_1, \ldots, Z_{n+1}$ are iid random variables,

$$\begin{aligned} &E(\max\{Z_1, \ldots, Z_n\}) - E(\max\{Z_1, \ldots, Z_{n-1}\}) \geq \\ &\quad E(\max\{Z_1, \ldots, Z_{n+1}\}) - E(\max\{Z_1, \ldots, Z_n\}). \end{aligned} \tag{25}$$

[38]For more on the interpretation of bidders' marginal revenue, see Bulow and Roberts (1989).

For any $z_1, \ldots, z_{n+1} \in \mathbb{R}^{n+1}$,

$$\max\{0, z_{n+1} - \max\{z_1, \ldots, z_{n-1}\}\} \geq \max\{0, z_{n+1} - \max\{z_1, \ldots, z_n\}\}, \tag{26}$$

implying

$$\max\{z_1, \ldots, z_{n-1}, z_{n+1}\} - \max\{z_1, \ldots, z_{n-1}\} \geq \max\{z_1, \ldots, z_{n+1}\} - \max\{z_1, \ldots, z_n\}. \tag{27}$$

Consequently for any random variables $Z_1, \ldots, Z_{n+1}$,

$$E(\max\{Z_1, \ldots, Z_{n-1}, Z_{n+1}\}) - E(\max\{Z_1, \ldots, Z_{n-1}\}) \geq E(\max\{Z_1, \ldots, Z_{n+1}\}) - E(\max\{Z_1, \ldots, Z_n\}), \tag{28}$$

because (27) holds for every realization $z_1, \ldots, z_{n+1}$ of $Z_1, \ldots, Z_{n+1}$. If the Z_i are iid, then $E(\max\{Z_1, \ldots, Z_{n-1}, Z_{n+1}\}) = E(\max\{Z_1, \ldots, Z_n\})$, yielding (25).

The expected revenue from any mechanism is the expected marginal revenue of the winning bidder (Myerson 1981). Ascending auctions assign the good to the bidder with the highest valuation, and therefore highest marginal revenue, because marginal revenue is increasing in valuations. It follows that expected revenue with n bidders is $E(\max\{MR(V_1), \ldots, MR\ (V_n)\})$. As $MR(V_1), \ldots, MR(V_{n+1})$ are iid random variables, we have

$$E(\max\{MR(V_1), \ldots, MR(V_n)\}) - E(\max\{MR(V_1), \ldots, MR(V_{n-1})\}) \geq E(\max\{MR(V_1), \ldots, MR(V_{n+1})\}) - E(\max\{MR(V_1), \ldots, MR(V_n)\}), \tag{29}$$

implying that expected revenue is concave in the number of bidders. □

When there exists a random variable U such that bidder values $V_1, \ldots, V_n$ are iid conditional on U, if marginal revenue is increasing in values conditional on each realization of U, then Proposition 7 applies conditional on each realization of U. Taking expectations over U, it follows that expected revenue is concave in CIPV environments if bidders' marginal revenue curves are increasing in values conditional on each value of U.

Appendix G. Bidder Collusion and Mergers

The bidder exclusion effect may also be used to bound above the expected fall in revenue resulting from mergers or collusion between bidders. Consider first the case of two random bidders forming a bidding ring.[39] This bidding ring excludes one bidder from the auction. If the bidder which is excluded is chosen randomly between the bidders in the ring, and if the bidders were randomly matched when forming a ring, the setting is equivalent to one in which a bidder is randomly—and legally—excluded from the auction. Under the conditions of Observation 1, the seller's losses are given by $\Delta(n) = \Delta^{bid}(n) \equiv \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$. In the case of low bidding or symmetric common values, the seller's losses are bounded above by $\Delta^{bid}(n)$.

The bidder exclusion effect also provides a bound on revenue losses due to mergers between bidders with private values. This argument requires additional steps given that mergers may result in increased production efficiencies and hence an increased willingness to pay of the merged entity. If the merger does not decrease bidders' values, revenue should be at least as great after a merger between two random bidders as after excluding a random bidder. We state the result for the general case of asymmetric bidders. In auctions where $P_n \subset \mathbb{N}$ is the set of participants, let $(V_i)_{i \in P_n}$ denote values before a merger. When bidders i and j merge, let M_k denote the value of bidder $k \neq i, j$, and $M_{i,j}$ denote the value of the merged entity.

Assumption 4. *Mergers do not decrease values: When bidders i and j merge, $M_k \geq V_k$ for all $k \neq i, j$, and $M_{i,j} \geq \max\{V_i, V_j\}$.*

Proposition 8. *In ascending auctions with private values and no reserve price where Assumptions 1 and 4 hold, then for all $n > 2$, $\Delta(n)$ is greater than or equal to the decrease in expected revenue from two randomly selected bidders merging.*

[39] A bidding ring is a group of bidders in an auction who collude in order to keep prices down.

Proof. We follow the notation from Appendix E. In addition, in auctions where P_n is the set of entrants, if $i, j \in P_n$ were to merge we let $F_{n-2:n-1}^{P_n^{i\sim j}}$ denote the distribution of the second-highest value of the unmerged and merged entities, i.e. of $\{M_k\}_{k\neq i,j} \cup \{M_{i,j}\}$.

Consider an n bidder auction with participants P_n. For any $v \in \mathbb{R}$,

$$\begin{aligned} F_{n-2:n-1}^{P_n}(v) &= \frac{1}{n}\sum_{i\in P_n} F_{n-2:n-1}^{P_n\setminus\{i\}|P_n}(v) \\ &\geq \frac{1}{n}\sum_{i\in P_n}\frac{1}{n-1}\sum_{j\in P_n\setminus i} F_{n-2:n-1}^{P_n^{i\sim j}}(v) \end{aligned}$$

The first equality follows from the definition of $F_{n-2:n-1}^{P_n}$. The second equality follows because by the assumption that mergers do not decrease values, for every $j \neq i$, the second-highest term in $\{V_k\}_{k\neq i}$ is less than the second-highest term in $\{M_k\}_{k\neq i,j} \cup \{M_{i,j}\}$. Therefore the distribution of the second-highest value after dropping one of the n bidders in P_n uniformly at random is first order stochastically dominated by the distribution of the second-highest value when two bidders are selected to merge at random. It follows that the bidder exclusion effect is greater than the decrease in expected revenue if two randomly selected bidders merge. This holds conditional on each set of entrants P_n, so it also holds unconditionally, averaging over all P_n, yielding the result. □

In practice it is likely that bidders are not randomly matched to merge or to form bidding rings, and in these cases the loss in expected revenue may exceed than the bidder exclusion effect.[40] However, the expected loss in seller revenue between a no-reserve auction with no collusion or mergers and one with collusion or a merger of two non-random bidders is bounded above by the expectation of the gap between the second and third order statistics of bids, $E(B^{n-1:n} - B^{n-2:n})$. If the number of

[40]Consider the example of bidders 1,2 and 3, with values 0,1, and 1. Assume bidders 2 and 3 merge, forming a bidder with value 1. The drop in revenue from the merger is 1, which is larger than the bidder exclusion effect of $\frac{2}{3}$. Similarly, if bidders know their valuations prior to forming a bidding ring, bidders among the $n-2$ lowest values would have little incentive to collude among themselves. Collusion among the two highest bidders, on the other hand, would lower the auction price to the third order statistic.

bidders is unobserved, the average revenue loss due can be bounded by averaging over all (unobserved) realizations of N.[41]

In the timber auctions example of Section 6—a setting in which Baldwin et al. (1997) point out that accusations of collusion are historically quite common—a bound on the fall in revenue from two random bidders colluding or merging is given by the bidder exclusion effect estimate of 13%. A bound on the loss in seller revenue when instead two non-random bidders form a bidding ring can be recovered simply by scaling the values in Figure 1 by a factor of $n/2$, yielding the expected gap between the second and third order statistics.

In addition to bounding the revenue loss which could occur from mergers or collusion, the bidder exclusion effect may also aid in the detection of collusion. While we do not formally derive such a test, the intuition is as follows. Under the assumptions of Proposition 3 (which include the assumptions of Bulow and Klemperer (1996)), the bidder exclusion effect provides an upper bound on the revenue improvement which a reserve price could achieve. With data from secret reserve price ascending auctions, one could compute what revenue would have been had the secret reserve price not been enforced and compare this quantity to realized revenue.[42] If the observed difference is larger than the bidder exclusion effect, this may be evidence that bids are being artificially depressed through collusion. A similar comparison can be made in ascending or first price auctions without secret reserve prices if the researcher has access to experimental variation in the use of reserve prices.

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[41]This approach to bounding the loss due to collusion or mergers does not allow one to estimate the loss due to collusive actions or mergers which have already occurred, but rather counterfactual impacts which *additional* collusion or mergers would entail. Also, the above approach extends easily to bidding rings or mergers of size larger than two.

[42]Data from public reserve price auctions would not be as useful as data from secret reserve auctions for this test given that when the public reserve price binds it is likely that bids which would have been placed below the reserve price (had the reserve price not been in effect) will not appear in the data.