Mandatory Disclosure and Financial Contagion[*]

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Abstract

This paper analyzes the welfare implications of mandatory disclosure of losses at financial institutions when it is common knowledge that some banks have incurred losses but not which ones. We develop a model that features contagion, meaning that banks not hit by shocks may still suffer losses because of their exposure to banks that are. In addition, we assume banks can profitably invest funds provided by outsiders, but will divert these funds if their equity is low. Investors thus value knowing which banks were hit by shocks to assess the equity of the banks they invest in. We find that when the extent of contagion is large, it is possible for no information to be disclosed in equilibrium but for mandatory disclosure to increase welfare by allowing investment that would not have occurred otherwise. Absent contagion, mandatory disclosure cannot raise welfare, even if markets are frozen. Our findings provide insight on when contagion is likely to be a concern, e.g. when banks are highly leveraged against other banks, and thus when mandatory disclosure is likely to be desirable.

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1 Introduction

In trying to explain how the decline in U.S. house prices evolved into a financial crisis and a collapse in trade between financial intermediaries, various analysts have singled out the role of uncertainty as to which entities incurred the bulk of the losses resulting from falling house prices. For instance, Gorton (2008) analyzes the crisis and argues that

> "The ongoing Panic of 2007 is due to a loss of information about the location and size of risks of loss due to default on a number of interlinked securities, special purpose vehicles, and derivatives, all related to subprime mortgages... The introduction of the ABX index revealed that the values of subprime bonds (of the 2006 and 2007 vintage) were falling rapidly in value. But, it was not possible to know where the risk resided and without this information market participants rationally worried about the solvency of their trading counterparties. This led to a general freeze of intra-bank markets, write-downs, and a spiral downwards of the prices of structured products as banks were forced to dump assets."

Market participants emphasized the same phenomenon as the crisis was unfolding. On February 24, 2007, the *Wall Street Journal* attributed the following to former Salomon Brothers vice chairman Lewis Ranieri, the so-called "godfather" of mortgage finance:

> "The problem ... is that in the past few years the business has changed so much that if the U.S. housing market takes another lurch downward, no one will know where all the bodies are buried. 'I don't know how to understand the ripple effects through the system today,' he said during a recent seminar."

In line with this view, some have argued that an important step in eventually stabilizing financial markets was the Fed's decision to release the results of its stress tests on large U.S. banks. These tests required banks to report to Fed examiners how their respective portfolios would fare under various stress scenarios and thus the losses they were vulnerable to. In contrast to the traditional confidentiality accorded to bank examinations, these results were made public. Bernanke (2013) summarizes the view that the public disclosure of the stress-test results played an important role in stabilizing financial markets:

> "In retrospect, the [Supervisory Capital Assessment Program] stands out for me as one of the critical turning points in the financial crisis. It provided anxious investors with something they craved: credible information about prospective losses at banks. Supervisors' public disclosure of the stress test results helped restore confidence in the banking system and enabled its successful recapitalization."

In this paper, we examine whether uncertainty about which banks incurred losses – that is, where the bad apples are located – can lead to market freezes where banks are unable raise to raise outside funds, and can make it desirable for policymakers to force banks to disclose their financial positions. The feature that turns out to be critical for such intervention to be beneficial is contagion. That is, shocks that hit some banks must lead to losses at other banks not directly hit by these shocks, e.g. losses of banks directly exposed to the subprime mortgage market may lead to losses at banks that hold few subprime mortgages.

In what follows, we consider a model of balance-sheet contagion already explored in previous work in which banks hit by shocks default on their obligations to other banks. We modify this model in two ways. First, we allow banks to raise additional funds from outside investors to finance profitable investment projects. However, we introduce an agency problem so that investors only want to invest in banks with sufficient equity. When investors are uncertain about which banks incurred losses, they may refuse to invest in banks altogether, a phenomenon we refer to as a market freeze. Contagion exacerbates this problem, since it raises the possibility that not only the banks hit by shocks have low equity, but that banks not directly hit may as well. Depending on which banks are hit and how they are linked to remaining banks, the impact on the aggregate equity of the banking system may be large. The greater the potential for contagion, then, the more likely market freezes are to occur.

Second, we allow banks to publicly disclose whether they were hit by shocks, i.e. they can choose to release the information examiners would solicit from them under a stress test or hire an external auditor to conduct a stress test. Our model can thus speak to what information banks release and whether banks should be forced to disclose information they chose not to reveal. We show that when the extent of contagion is small, mandatory disclosure cannot improve welfare even if non-disclosure results in a market freeze. But when contagion is large and disclosure costs are low, mandatory disclosure can improve welfare. Intuitively, contagion implies information on the financial health of one bank is relevant for assessing the health of other banks. Since banks fail to internalize such spillovers, too little information is revealed, creating a role for mandatory disclosure. Absent these spillovers, banks internalize the benefits of disclosure. If they choose not to disclose, it must be because the cost of disclosure exceeds the benefits, and forcing them to disclose would be undesirable.

Since our model is somewhat involved, an overview may be helpful. At the heart of our model is a set of banks arranged in a network that reflects what each bank owes other banks. Some banks are hit with shocks that prevent them from fully paying their existing obligations to other banks. Given banks are interconnected, even banks not hit by shocks are vulnerable to losses. All banks, including those hit by a shock, can profitably invest funds raised from outsiders. However, we assume banks can divert the funds they raise. Outside investors

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will then only want to invest in banks with enough equity to forfeit if they divert funds. Banks that want to raise funds can disclose at a cost whether they were hit by a shock. This disclosure must be made before banks know which other banks were hit with shocks, and thus before they know their own equity. Outside investors see all the information that is disclosed and decide which banks if any to invest in and at what terms. If enough banks choose not to disclose their state, investors will be uncertain as to which banks were hit by shocks. Such uncertainty may deter outsiders from investing in any of the banks.

This framework allows us to draw the connection between contagion and the desirability of mandatory disclosure. In particular, we find that network structure only matters for disclosure through its implications for contagion. Our analysis also allows us to show which features give rise to contagion and market freezes, e.g. the degree of leverage banks have against other banks in the network, the magnitude of losses, and the relative and absolute number of banks hit by shocks. Lastly, our approach allows us to derive expressions for contagion probabilities for a particular network with multiple bad banks, a result that may be of interest to researchers working on contagion independently of our results on disclosure.

The paper is structured as follows. In Section 2, we review the related literature. In Section 3, we introduce our model of contagion, focusing on a particular network structure. In Section 4, we allow banks to raise outside funds. We also describe the agency problem that makes investors leery of investing in banks with little equity. In Section 5, we introduce a disclosure decision. We then examine whether non-disclosure can be an equilibrium, and if so whether mandatory disclosure can improve upon this equilibrium. In Section 6, we consider more general network structures. In Section 7, we offer some concluding comments.

2 Literature Review

Our paper is related to several different literatures, specifically work on i) financial contagion and networks, ii) disclosure, iii) market freezes, and iv) stress tests.

Turning first to the literature on contagion, various channels for contagion have been described in the literature. For a survey, see Allen and Babus (2009). For concreteness, we focus on models of contagion based on balance-sheet effects in which a bank hit by a shock is unable to pay its obligations, making it difficult for other banks to meet their obligations. Examples of papers that explore this channel include Kiyotaki and Moore (1997), Allen and Gale (2000), Eisenberg and Noe (2001), Gai and Kapadia (2010), Battiston et al. (2012), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013), and Elliott, Golub, and Jackson (2013). These papers are largely concerned with how the pattern of obligations across banks affects the extent of contagion, and whether certain network structures can reduce the extent of

contagion. Our focus is quite different: We take contagion as given and examine whether policies can be used to mitigate the fallout from contagion once it occurs, e.g. restarting trade in markets that would otherwise remain frozen.

Since our model posits that banks connected via a network communicate information, we should point out that there is a literature on communication and networks, e.g. De-Marzo, Vayanos, and Zwiebel (2003), Calvó-Armengol and de Martí (2007), and Galeotti, Ghiglino, and Squintani (2013). However, these papers study environments in which agents communicate to others *on the network*. By contrast, we study an environment where agents communicate *about* the network, specifically the location of its bad nodes, *to outsiders*.

The other major literature our work relates to concerns disclosure. Verrecchia (2001) and Beyer et al. (2010) provide good surveys of this literature. A key result in this literature, established by Milgrom (1981) and Grossman (1981), is an "unravelling principle" which holds that all private information will be disclosed because agents with favorable information want to avoid being pooled with inferior types and receive worse terms of trade. Beyer et al. (2010) summarize the various conditions subsequent research has established as necessary for this unravelling result to hold: (1) disclosure must be costless; (2) outsiders know agents have private information; (3) all outsiders interpret disclosure identically, i.e. outsiders have no private information; (4) information can be credibly disclosed, i.e. information is verifiable; and (5) agents cannot commit to a disclosure policy ex-ante before observing the relevant information. Violating any one of these conditions can result in equilibria where not all relevant information is conveyed. In our model, non-disclosure can be an equilibrium outcome even when all of these conditions are satisfied. We thus highlight a distinct reason for the failure of the unravelling principle that is due to informational spillovers: In order to know whether a bank in our model is safe to invest in, outside investors need to know not just the bank's own balance sheet, but also the balance sheets of other banks.

Ours is certainly not the first paper to explore disclosure in the presence of informational spillovers. One important predecessor is Admati and Pfleiderer (2000). Their setup also allows for informational spillovers and gives rise to non-disclosure equilibria. However, there are several important differences between the two papers. First, our model features informational complementarities that are not present in their model, whereby information released by one party is essential for determining the state of others. This explains why our model can produce non-disclosure equilibria even when disclosure is costless. By contrast, their model implies full revelation when the cost of disclosure is zero. Another difference between the two papers is that they assume agents commit to disclosing information before learning it, while in our model banks know their losses and then choose to disclose them. Finally, our setup allows us to study contagion and disclosure, something that cannot be deduced from

their setup. That said, Admati and Pfleiderer (2000) do show, as we do, that informational spillovers can make mandatory disclosure welfare-improving.[1]

Our paper is also related to the literature on market freezes. As in our model, this literature emphasizes the role of informational frictions. Some of these papers emphasize private information, where agents are reluctant to trade with others for fear of being exploited by more informed agents. Examples include Rocheteau (2011), Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2012), Camargo and Lester (2011), and Kurlat (2013). Others have focused on uncertainty concerning each agent's own liquidity needs and the needs of others as an impediment to trade. Examples include Caballero and Krishnamurthy (2008) and Gale and Yorulmazer (2013). One difference between our framework and these papers concerns the source of informational frictions. Since in our framework the uncertainty concerns information that can in principle be verified, such as a bank's balance sheet, it naturally focuses attention on the possibility that this information might be revealed. By contrast, previous papers have focused on private information on assets that may be more difficult to verify or information that no agents are privy to and hence cannot be disclosed.

Finally, there is an emerging literature on stress tests. On the empirical front, Peristian, Morgan, and Savino (2010), Bischof and Daske (2012), Ellahie (2012), and Greenlaw et al. (2012) have looked at how the release of stress-test results in the U.S. and Europe affected bank stock prices. These results are complementary to our analysis, which is more concerned with normative questions regarding the desirability of releasing stress-test results. There are also several recent theoretical papers on stress tests, e.g. Goldstein and Sapra (2013), Goldstein and Leitner (2013), Shapiro and Skeie (2012), Spargoli (2012), and Bouvard, Chaigneau, and de Motta (2013). In these papers, banks are not allowed to disclose information on their own. These papers thus sidestep a key question we tackle, namely whether banks might choose not to disclose information even when it is socially desirable for them to do so.

3   A Model of Contagion

We begin with a bare-bones version of our model where banks make no decisions. We use this setup to highlight how contagion works and to propose a summary static to quantify it.

Our approach to modelling contagion follows the balance-sheet contagion models of Eisenberg and Noe (2001), Caballero and Simsek (2012), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013).[2] Formally, there are n banks indexed by $i \in \{0, ..., n-1\}$. Each bank i is

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[1]Earlier work by Foster (1980) and Easterbrook and Fischel (1984) also argues that spillovers may justify mandatory disclosure, although these papers do not develop formal models to analyze this hypothesis.

[2]Other channels for contagion can give rise to similar results. For example, knowing the concentration of losses will be equally important for predicting each bank's equity in fire-sale models of contagion.

endowed with a set of financial obligations $\Lambda_{ij} \geq 0$ to each bank $j \neq i$. Following Eisenberg and Noe (2001), we take these obligations as given without modelling where they come from. Kiyotaki and Moore (1997) and Zawadowski (2013) have shown that banks may enter such obligations without insuring themselves despite the potential for contagion.

For much of our analysis, we follow Caballero and Simsek (2012) in restricting attention to the special case in which

$$\Lambda_{ij} = \begin{cases} \lambda & \text{if } j = (i+1) \pmod{n} \\ 0 & \text{else} \end{cases} \tag{1}$$

This case is known as a ring network or circular network, since the obligations between banks can be depicted graphically as if forming a circle as shown in Figure 1. In Section 6, we show that our analysis can be extended to a larger class of networks. However, since the circular network is expositionally convenient, we begin by focusing on this case.

In addition to the obligations Λ_{ij}, each bank is endowed with some assets. We do not explicitly model the value of these assets, and simply set their value fixed at some $\pi > 0$.

A fixed number of banks b are hit by negative net worth shocks, where $1 \leq b \leq n-1$. We refer to these as "bad" banks. We thus generalize Caballero and Simsek (2012), who assume $b = 1$. Each bad bank incurs a loss ϕ, where ϕ represents a claim on the bank by some outside entity that is not any of the remaining banks in the network. We follow previous work in assuming ϕ is senior to any obligations a bank owes to other banks in the network. Thus, a bank must use its available resources to pay its senior claimant before paying other banks in the network.[3] We shall refer to all remaining banks as "good."

Let $S_j = 1$ if j is a bad bank and 0 otherwise. The vector $S = (S_0, ..., S_{n-1})$ denotes the state of the banking network. By construction, $\sum_{j=0}^{n-1} S_j = b$. We assume shocks are equally likely to hit any bank, i.e. each of the $\binom{n}{b}$ possible locations of the bad banks within the network are equally likely. It follows that $\Pr(S_j = 1) = \frac{b}{n}$ for any bank j.

The purpose of this section is to explain how the location of bad banks, or the realization of S, affects banks in the network. Each bank is either insolvent – unable to pay its obligation λ in full – or solvent, although it may have to liquidate some of its endowment to pay its obligation. As we shall now show, the location of the bad banks determines how many banks are insolvent, which ones, and how much of its endowment each solvent bank retains.

To determine how location matters, let x_j denote bank j's payment to bank $j+1$, and y_j denote bank j's payment to the outside sector. Bank j has $x_{j-1} + \pi$ resources it can draw on to meet its obligations. Our seniority rules imply the outside sector must be paid first.

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[3]We could have alternatively assumed these obligations have equal seniority as obligations to other banks on the network, although this setup is more cumbersome. We thank Fabrice Tourre for pointing this out.

Let $\Phi_j \equiv \phi S_j$ denote bank j's obligation to the outside sector. Then y_j must satisfy

$$y_j = \min\left\{x_{j-1} + \pi, \Phi_j\right\} \tag{2}$$

Bank j can then use any remaining resources to pay bank $j+1$, to which it owes λ, and so

$$x_j = \min\left\{x_{j-1} + \pi - y_j, \lambda\right\} \tag{3}$$

Substituting in for y_j yields a system of equations that defines the payments $\{x_j\}_{j=0}^{n-1}$:

$$x_j = \max\left\{0, \min\left(x_{j-1} + \pi - \Phi_j, \lambda\right)\right\}, \qquad j = 0, ..., n-1 \tag{4}$$

Given a solution $\{x_j\}_{j=0}^{n-1}$ to the system of equations in (4), we can define the equity of bank j as the value of the resources a bank retains after all payments are settled, i.e.

$$e_j = \max\left\{0, \pi - \Phi_j + x_{j-1} - x_j\right\} \tag{5}$$

Although e_j is redundant given the payments x_j, equity will turn out to play an important role in our analysis. Note that both x_j and e_j depend on the state of the network S, i.e. $x_j = x_j(S)$ and $e_j = e_j(S)$, However, below we omit explicit reference to S when this is not essential. Our first result establishes that (4) has a generically unique solution $\{x_j^*\}_{j=0}^{n-1}$.[4]

Proposition 1: For each S, the system (4) has a unique solution $\{x_j^*\}_{j=0}^{n-1}$ if $\phi \neq \frac{n}{b}\pi$.

In the knife-edge case where $n\pi = b\phi$ so total losses of bad banks equal the total endowments of banks, (4) admits multiple solutions for large λ. However, in all of these solutions the outside sector is paid in full, i.e. $y_j = \Phi_j$, and $e_j = 0$ for all j. The only difference across solutions are the notional amounts banks default on to other banks.[5]

In what follows, we will mostly restrict attention to the case of $\phi < \frac{n}{b}\pi$, so the total losses of bad banks $b\phi$ do not exceed the total resources of the banking system, $n\pi$. Although Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) argue that large losses can yield important insights on the nature of contagion, such shocks yields few insights for our purposes. In particular, when $\phi > \frac{n}{b}\pi$, two outcomes are possible, depending on the value of λ. When λ is small, equity $\{e_j\}_{j=0}^{n-1}$ is independent of ϕ, and so this case can be understood even if we restrict $\phi < \frac{n}{b}\pi$. When λ is large, $e_j = 0$ for all j when $\phi > \frac{n}{b}\pi$. Since we are interested in decisions when banks are unsure about their equity, this case offers little insight.

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[4]Our result is a special case of Theorem 2 in Eisenberg and Noe (2001) and Proposition 1 in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013). The latter establishes uniqueness for a generic network Λ_{ij} but does not provide exact conditions for non-uniqueness as we do for the particular network we analyze.

[5]Eisenberg and Noe (2001) also show in their Theorem 1 that $\{e_j\}_{j=0}^{n-1}$ is unique even if $\{x_j\}_{j=0}^{n-1}$ is not.

At the same time, we don't want the loss per bank ϕ to be too small, since as the next proposition shows, $\phi \leq \pi$ implies bad banks are solvent and good banks retain π in full.

Proposition 2: If $\phi \leq \pi$, then $x_j^* = \lambda$ for all j and $e_j = \pi$ for any j for which $S_j = 0$.

The above insights suggest the following restriction on ϕ:

Assumption A1: Losses at bad banks ϕ satisfy $\pi < \phi < \frac{n}{b}\pi$.

When $\phi > \pi$, bad banks will be insolvent: Even if these banks receive the full amount λ owed them, they will have less than λ resources to pay other banks. Assumption A1 thus ensures the equity of bad banks is 0. Since bad banks are insolvent, some good banks will have to liquidate part of their endowment to meet their obligations, and may become insolvent themselves. We use the term contagion to mean a scenario where the equity of some good banks falls below π even though they themselves are not hit by shocks.

To understand how contagion operates, it will help to begin with the case of one bad bank, i.e. $b = 1$, as in Caballero and Simsek (2012). Without loss of generality, let bank 0 be the bad bank. Given that bank 0 receives x_{n-1} from bank $n-1$, the total amount of resources bank 0 can give to bank 1 is $\max\{x_{n-1} + \pi - \phi, 0\}$. We show in Proposition 3 below that under Assumption A1 there is at least one bank that is solvent and can pay its obligation λ in full. From this, it follows that bank $n-1$ must be solvent, since if any bank $j \in \{1, ..., n-2\}$ were solvent, it would pay bank $j+1$ in full, who in turn will pay bank $j+2$ in full, and so on, until we reach bank $n-1$.

Deriving the equity of each bank is straightforward. Bank 0 has $\pi + \lambda$ worth of resources and owes $\phi + \lambda$, so it will fall short on its obligation to bank 1 by

$$\Delta_0 = \min\{\phi - \pi, \lambda\}.$$

Since bank 1 is endowed with $\pi > 0$ resources, it can use them to make up some of the shortfall it inherits when it pays bank 2. If the shortfall $\Delta_0 > \pi$, bank 1 will also be insolvent, although its shortfall will be π less than shortfall it receives. The first bank that inherits a shortfall that is less than or equal to π will be solvent, with an equity position that is at least 0 but strictly less than π. Hence, we can classify banks into three groups: (1) Insolvent banks with zero equity, which includes both the bad bank and possibly several good banks; (2) Solvent banks whose equity is $0 \leq e_j < \pi$, of which there is exactly one when $b = 1$; and (3) Solvent banks that are sufficiently far from the bad bank and have equity equal to π.

Since equity will figure prominently in our analysis below, it will be convenient to work with the case where e_j can take on only two values, 0 or π. For $b = 1$, this requires that $\Delta_0 = \min\{\phi - \pi, \lambda\}$ be an integer multiple of π. For general values of b, we will need to impose that both ϕ and λ are integer multiples of π. Formally, we have

Assumption A2: ϕ and λ are both integer multiples of π.

Assumption A2 ensures that the only group of banks whose equity can differ from 0 or π, namely solvent banks who must still liquidate some of their endowment, have exactly zero equity. The number of good banks with zero equity when $b = 1$ is then

$$k = \frac{\Delta_0}{\pi} = \min\left\{\frac{\phi}{\pi} - 1, \frac{\lambda}{\pi}\right\} \tag{6}$$

Caballero and Simsek (2012) refer to k as the size of the "domino effect" and use it as a measure of contagion from a bad bank to good banks. Two conditions are required for k to be large. First, losses ϕ at each bad bank must be large. When ϕ is small, a bad bank will still be able to pay back a large share of its obligation λ, and so fewer banks will ultimately be affected by the loss. Second, a large k requires the obligation λ be large. Intuitively, when λ is small, banks are not very indebted to one another, and in the limit as $\lambda \to 0$, there will be no effect on good banks regardless of how large losses ϕ at bad banks are. As λ rises, the amount of resources that flow through each bank increases, including at bad banks where they would be grabbed by senior claimants. This starves the banking system of equity, leaving fewer resources for good banks. A higher λ thus shifts resources from banks to senior claimants.[6] As such, contagion merely redistributes resources across agents without imposing a social cost. When we allow banks to invest funds on behalf of outsiders in the next section, however, contagion will matter for welfare.

Armed with this intuition, we can now move to the general case of an arbitrary number of banks, i.e. $1 \leq b \leq n - 1$. We begin with a result that Assumption A1 implies at least one bank will be solvent, regardless of whether Assumption A2 holds.

Proposition 3: If $\phi < \frac{n}{b}\pi$, there exists at least one solvent bank j for which $x_j = \lambda$, and among solvent banks there exists at least one bank j with positive equity, i.e. $e_j > 0$.

As in the case with $b = 1$, there will be three types of banks when $b > 1$: (1) Insolvent banks with zero equity; (2) Solvent banks whose equity is $0 \leq e_j < \pi$; and (3) Solvent banks that are sufficiently far away from a bad bank whose equity $e_j = \pi$. Since we know there is at least one solvent bank j, we can start with this bank and move to bank $j + 1$. If bank $j + 1$ is good, it too will be solvent and its equity will be $e_{j+1} = \pi$. We can continue this way until we eventually reach a bad bank. Without loss of generality, we refer to this bad bank as bank 0. By the same argument as in the case where $b = 1$, Assumption A2 implies that banks $1, ..., k$ will have zero equity, where k is given by (6): Even if all of these banks

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[6]Per Elliott, Golub, and Jackson (2013), increasing λ in our setup implies more integration but not more diversification. However, unlike in their model where greater integration means firms swap their own equity for that of other firms, here greater integration implies greater exposure to shocks at other banks while leaving banks equally vulnerable to their own shocks. Hence, the effect of higher λ is (weakly) monotone.

are good, each will inherit a shortfall of at least π and will have to sell all of its assets. If any of these banks are bad themselves, the shortfall subsequent banks will inherit will be even larger, and so equity at the first k banks will be zero.

If bad banks are sufficiently spread out across the network, so there are at least k banks between any two bad banks, then exactly bk good banks will have zero equity while the rest will have equity π. But if bad banks are located more closely, losses at one bad bank may impact other bad banks, who could in turn default on their senior claimant. The more losses senior claimants absorb, the more equity is preserved within the network. Thus, when $b > 1$, the location of bad banks can matter for the aggregate equity of the network. We now show that for small values of λ, the number of good banks with zero equity will indeed depend on the exact location of bad banks. By contrast, location will not matter for large λ: In that case, exactly bk good banks have zero equity regardless of which banks are bad. Intuitively, large λ imply enough resources flow through each bank that senior claimants can always collect their ϕ in full, and so the residual equity that remains in the network is constant.

We begin with the case where λ is large. Our first result is that this ensures banks will always pay something to other banks. This result does not require Assumption A2 to hold.

Proposition 4: Under Assumption A1, $x_j(S) > 0$ for all j and all S iff $\lambda > b(\phi - \pi)$. When $\lambda \leq b(\phi - \pi)$, there exist realizations of S for which $x_j(S) = 0$ for at least one j.

Given our seniority rules, an implication of Proposition 4 is that for sufficiently large λ, senior claimants will always be fully paid regardless of where bad banks are located. This implies that the total amount of resources left within the banking network also does not depend on where bad banks are located. Since Assumption A2 implies banks can have equity of either 0 or π and total equity is the same for all S, the number of banks with zero equity must be the same for all S. Formally:

Proposition 5: Under Assumptions A1 and A2, if $\lambda > b(\phi - \pi)$, the number of good banks with zero equity is equal to bk regardless of the state of the banking network S.

Next, consider the case where λ is small. Since fewer resources flow through each bank, a bad bank may not be able pay its senior claimant the full amount ϕ. Indeed, when $\lambda < \phi - \pi$, a bad bank will not have enough to pay its senior claimant even if it was paid in full by the bank which owes it λ. In this case, each bad bank j would pay nothing to bank $j + 1$, i.e. $x_j = 0$, regardless of where other bad banks are located. By a similar logic to the case with one bad bank, each bad bank will create a domino effect of wiping out the equity of the next $k = \frac{\lambda}{\pi}$ banks. As can be seen in Figure 2, when all b bad banks are located next to one another, the only good banks whose equity would be wiped out would be the k banks located immediately downstream of the last bad bank. Hence, a total of $b+k$ banks have zero equity.

For $b > 1$, this may be well below the $b + bk$ banks that have zero equity when bad banks are spaced far apart. Similarly, for $\phi - \pi \leq \lambda < b(\phi - \pi)$, the number of banks with zero equity can fall short of bk. The location of the bad banks thus matters for the aggregate equity of the network. That is, uncertainty about which banks are bad matters not just for the equity of any given bank, but for the total equity that remains within the network as a whole.[7]

Since contagion reflects the extent to which good banks are exposed to bad banks, a candidate metric for contagion is the number of good banks with zero equity. When $b = 1$, this is a fixed number k. For $b > 1$, this number can be random. The rest of this section motivates a way to summarize this randomness with a single statistic.

Formally, let ζ denote the number of banks, both good and bad, with zero equity. When $\lambda < \phi - \pi$, we can deduce the distribution of ζ by noting the connection between our model and the discrete version of a well-studied circle-covering problem in applied probability first introduced by Stevens (1939). Consider a fixed number of points drawn at random locations from a circle of length 1. Starting at each respective point, draw an arc going clockwise. Stevens (1939) derived an expression for the probability that the circle will be covered by the arcs given the number of points and length of each arc. In our setting, the number of bad banks is akin to the number of points, while the potential for contagion k, expressed relative to the number of banks in the network, corresponds to the length of each arc. The region of the circle covered by arcs is akin to the fraction of banks with zero equity. The discrete version of this problem was analyzed in Holst (1985), Ivchenko (1994), and Barlevy and Nagaraja (2013). As Holst (1985) notes, the discrete version can be analyzed using results on Bose-Einstein statistics. This allows us to derive an exact distribution for ζ. Our proposed measure of contagion will use the expected value $E[\zeta]$ and can be obtained using results in Ivchenko (1994) and Barlevy and Nagaraja (2013) as summarized in the next lemma.

Lemma 1: Suppose $\lambda < \phi - \pi$. Under Assumptions A1 and A2,

$$E[\zeta] = n - \frac{(n - b)! \, (n - k - 1)!}{(n - 1)! \, (n - b - k - 1)!}$$

where $k = \frac{\lambda}{\pi}$ as defined by (6) for the case where $\lambda < \phi - \pi$.

From our results so far, we can deduce $E[\zeta]$ when $\lambda > b(\phi - \pi)$, since Proposition 5 implies that ζ in this case is deterministically equal to $n - bk - b$, and when $\lambda < (\phi - \pi)$, as reflected in Lemma 1. For intermediate values of λ inbetween, ζ is random, with support

---

[7]When $b = 1$, the location of the bad bank will not matter for aggregates because the ring network is symmetric. If the network were asymmetric, the location of the bad bank can matter, since shocks to more connected banks will have a bigger impact on aggregate equity. For more on aggregation in asymmetric networks, albeit in the context of production, see Acemoglu et al. (2012).

ranging between $b + \frac{\lambda}{\pi} > b + k$ and $b\frac{\phi}{\pi} = bk + b$, where recall k is defined in (6). We could not derive a closed-form expression for $E[\zeta]$ in this case, although in Proposition 6 below we report some comparative statics for $E[\zeta]$ in this case.

We now argue that $E[\zeta]$ can be used to construct a summary statistic for contagion that will be a natural metric for our subsequent analysis. Consider the perspective of a bank that knows it is good. For this bank, contagion means it may be forced to liquidate some of its endowment and end up with an equity below π. Thus, contagion will be reflected in the distribution of equity the bank expects to maintain. Under Assumption A2, e_j can only take on two values. Let p_g denote the probability that a good bank retains its endowment, i.e.,

$$p_g = \Pr\left(e_j = \pi | S_j = 0\right) \tag{7}$$

It is natural to interpret p_g as a measure of contagion: A value of p_g close to 1 implies a good bank will likely avoid liquidating its resources, while a value close to 0 means a good bank will likely have its equity wiped out. The fact that we can reduce the distribution of equity into a single statistic is due to Assumption A2. Without it, or under a more general network structure Λ_{ij}, e_j can assume more than two values. We discuss how to handle this case in Section 6. To obtain an expression for p_g, note that

$$
\begin{aligned}
p_g &= \sum_{z=b+k}^{bk+b} \Pr\left(e_j = \pi | S_j = 0, \zeta = z\right) \Pr\left(\zeta = z\right) \\
&= \sum_{z=b+k}^{bk+b} \frac{n-z}{n-b} \Pr\left(\zeta = z\right) = \frac{n - E[\zeta]}{n-b} .
\end{aligned}
$$

Since $n - b$ banks are good and, on average, $n - E[\zeta]$ banks have positive equity, p_g reflects both the probability a given good bank has zero equity and the average fraction of good banks with zero equity. Using our results for $E[\zeta]$, we can state the following about p_g:

Proposition 6. Under Assumptions A1 and A2,

$$
p_g = \begin{cases}
\prod_{i=1}^{\lambda/\pi} \left(\frac{n-b-i}{n-i}\right) & \text{if } 0 < \lambda < \phi - \pi \\
\Psi\left(b, n, \frac{\phi}{\pi}, \frac{\lambda}{\pi}\right) & \text{if } \phi - \pi \leq \lambda \leq b\left(\phi - \pi\right) \\
1 - \frac{b}{n-b}\left(\frac{\phi}{\pi} - 1\right) & \text{if } b\left(\phi - \pi\right) < \lambda
\end{cases} \tag{8}
$$

where Ψ is weakly decreasing in ϕ/π and in λ/π.

Proposition 6 summarizes how p_g depends on the magnitude of the losses ϕ at bad banks, the depth of financial ties λ, the number of bad banks b, and the total number of banks n.

One feature worth pointing out now is that the effect of bank losses ϕ on p_g depends on λ. For small λ, i.e. for $\lambda < \phi - \pi$, changes in ϕ have no effect on p_g: Increasing ϕ leads to greater losses for senior claimants but has no effect on other banks. For larger λ, increasing ϕ will lower p_g as greater losses at bad banks wipe out equity at a larger number of good banks. For most of our analysis we will take p_g as given, although we will occasionally use these comparative statics to provide an economic interpretation for our results.

Remark 1: We can obtain additional insights on p_g from the limiting case in which the number of banks $n \to \infty$. As we increase n, suppose we keep the fraction of bad banks $\frac{b}{n}$ constant at some θ and hold the domino effect of a single bad bank fixed at k. Let ζ_n denote the (random) number of banks with zero equity when there are n banks in the network. When $\lambda < \phi - \pi$, by Theorem 4.2 in Holst (1985) it follows that the ratio $\frac{\zeta_n}{n}$ converges to a constant as $n \to \infty$. Likewise, the fraction $\frac{n - \zeta_n}{n - b}$ of good banks with equity π converges to a constant. This constant will equal p_g, which recall is just the *expected* fraction of good banks with zero equity. Taking the limit of (8) for the case of $\lambda < \phi - \pi$ as $n \to \infty$ reveals that p_g converges to a simple expression:

$$\lim_{n \to \infty} p_g = (1 - \theta)^k \tag{9}$$

Intuitively, a good bank will only have positive equity if each of the k banks located clockwise to it are good. As $n \to \infty$, the probability any one bank is bad converges to θ independently of what happens to any finite collection of banks around it. Hence, the probability that all of the relevant k neighbor banks are good is $(1 - \theta)^k$. For any given θ, the limiting value of p_g can range between 0 and 1 as k varies from 0 to arbitrarily large integer values. Note that since $k = \min\left\{\frac{\lambda}{\pi}, \frac{\phi}{\pi} - 1\right\}$, values of k that exceed $\frac{1}{\theta} - 1$ will violate the second inequality in Assumption A1, which requires that $\frac{\phi}{\pi}$ be less than $\frac{n}{b} = \frac{1}{\theta}$. However, this restriction can be dispensed with for large n, since the probability that equity is wiped out at all banks becomes exceedingly small even without this assumption. While the limiting case as $n \to \infty$ rules out the empirically interesting case where aggregate bank equity is uncertain, it remains a useful benchmark. For example, it nicely illustrates that the contagion measure p_g in our setup can assume the full range of values, from no contagion ($p_g \to 1$) to full contagion ($p_g \to 0$). ∎

As a final aside, in some of our subsequent analysis we will need the unconditional probability that a bank chosen at random has positive equity. Denote this probability by p_0. Given b bad banks and $n - b$ good banks, and since Assumption A1 implies all bad banks have zero equity, p_0 can be expressed directly in terms of p_g:

$$p_0 = \frac{n - b}{n} p_g + \frac{b}{n} \times 0 = \left(1 - \frac{b}{n}\right) p_g \tag{10}$$

4   Outside Investors and Bank Equity

We now introduce the first modification to our model, allowing banks to raise external funds they can profitably deploy. At the same time, we introduce a moral hazard problem that ensures only banks with enough equity use these funds as intended. Specifically, we let banks divert funds for private gains, a temptation that is mitigated by the equity a bank would have to forfeit. This is meant to capture any action banks with low equity can take that is not in the interest of outside investors, including paying senior claimants. These features can give rise to the possibility of market freezes, i.e. situations in which outsiders refuse to trade with banks. This can occur when outsiders are uncertain as to the location of bad banks. In this section, though, we focus on the full-information case in which the location of the bad banks is common knowledge, leaving the uncertainty case to the next section. The full-information case is of interest for two reasons. First, it reveals what would happen if all banks were forced to disclose whether they were hit by shocks, a policy we eventually explore. Second, it makes clear that in our model, banks known to have low equity will not be able to raise funds to make up for their losses. Even though banks could use such funds to generate income, outside investors are only willing to invest in banks with sufficient equity.

Formally, suppose that outside investors – the same outsiders with senior claims against banks or a new group of outside investors – can choose to invest in any of the n banks in the network. Banks have profitable projects they can undertake, but funding these projects requires outside financing. We assume each bank has a finite number of profitable projects it can undertake. We set the capacity of the bank to 1 unit of resources. On their own, outside investors can earn a gross return of $\underline{r}$ per unit of resources. Banks can earn a gross return of R on the projects they undertake, where $R > \underline{r}$. Thus, there is scope for gains from trade.

We restrict banks and outside investors to transact through debt contracts that are junior to all of the bank's other obligations.[8] Let r_j^* denote the equilibrium gross interest rate bank j offers investors for the funds they invest. We assume the outside sector is large enough that r_j^* is set competitively, i.e. the expected gross return from investing in a bank equals $\underline{r}$. Hence, $r_j^* \geq \underline{r}$, and the most a bank can earn by raising funds is $R - \underline{r}$.

After banks raise funds from outsiders, they can either invest them and earn R, or divert them to a project that accrues a purely private benefit v per unit invested. Private benefits cannot be seized by outsiders, and outsiders cannot monitor banks to prevent diversion. However, they can go after the bank's assets if it fails to pay its obligation r_j^*.

We want v to be large enough so that banks with zero equity will prefer to divert – so the

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[8]As long as the original shareholders can continue to divert the funds they raise, issuing equity instead of debt would not help. In other words, we are implicitly assuming banks are essential for generating the gross return R, so that simply selling the projects to outsiders does not eliminate the underlying agency problem.

moral hazard problem is binding – but not so large that even a bank that keeps its π worth of assets will be tempted to divert funds. The first condition requires $v > R - \underline{r}$, i.e. the private benefit v exceeds the most a bank can earn from undertaking the project. To ensure a bank with equity will not be tempted, we need to make sure that the payoff after undertaking the project, $\pi + R - r_j^*$, exceeds the payoff from diverting funds, $v + \max\{\pi - r_j^*, 0\}$, which reflects the fact that the bank would have to liquidate at least some of its assets to meet its obligation r_j^*. Thus, we need $v < R - \max\{r_j^* - \pi, 0\}$. Since a bank that can be trusted not to divert funds must only offer $\underline{r}$ to outsiders, the condition that ensures banks with assets worth π can credibly promise to invest the funds they raise is if $v < R - \max\{\underline{r} - \pi, 0\}$. The conditions on v we need can be summarized as follows:

Assumption A3: The private benefits v from diverting 1 unit of resources satisfy

$$R - \underline{r} < v < R - \max\{\underline{r} - \pi, 0\} \tag{11}$$

Note that the second inequality in (11) implies $v < R$, so diversion is socially wasteful.

We now show that in the full information benchmark, the same ζ banks that had no equity in the absence of investment will be unable to raise funds and will thus remain with zero equity, while the remaining $n - \zeta$ banks will be able to raise funds and raise their equity to $\pi + R - \underline{r}$. Toward this end, define $I_j \in [0, 1]$ as the amount outsiders invest in bank j. Since (11) involves strict inequalities, banks will either divert the funds they raise or invest. Let $D_j = 1$ if bank j decides to divert the funds and 0 otherwise. Recall that y_j denotes the obligation of bank j to its most senior creditors and x_j its payment to bank $j + 1$. Let w_j denote its payment to outsiders who invest in bank j. Then we have

$$
\begin{aligned}
y_j &= \min\{x_{j-1} + \pi + R(1 - D_j)I_j, \Phi_j\} \\
x_j &= \min\{x_{j-1} + \pi + R(1 - D_j)I_j - y_j, \lambda\} \\
w_j &= \min\{x_{j-1} + \pi + R(1 - D_j)I_j - y_j - x_j, r_j^* I_j\}
\end{aligned}
$$

Finally, the equity at each bank j is given by

$$e_j = \max\{0, x_{j-1} + \pi + R(1 - D_j)I_j - y_j - x_j - w_j\}$$

Let $\{\widehat{y}_j, \widehat{x}_j\}_{j=1}^n$ denote the payments to senior creditors and to banks, respectively, if outside investors could not fund any bank, i.e. if $I_j = 0$ for all j. Likewise, define $\{\widehat{e}_j\}_{j=1}^n$ as the equity positions given $\{\widehat{y}_j, \widehat{x}_j\}_{j=1}^n$, i.e.

$$\widehat{e}_j = \max\{0, \pi - \Phi_j + \widehat{x}_{j-1} - \widehat{x}_j\}$$

The values $\widehat{e}_j$ are just the equity positions we solved for in the previous section. We claim that with full information, $e_j = 0$ whenever $\widehat{e}_j = 0$, and $e_j > 0$ whenever $\widehat{e}_j > 0$.

Proposition 7: Given Assumption A1-A3, with full information, $e_j = 0$ for any bank j for which $\widehat{e}_j = 0$, and $e_j > 0$ if $\widehat{e}_j > 0$. Moreover, $I_j = 0$ if and only if $\widehat{e}_j = 0$.

Proposition 7 shows that even though insolvent banks can try to raise funds to make up their shortfalls, under full information such banks would not be able to do so. Rather, with full information, contagion persists as before. Allowing banks to trade still matters, though, since contagion is now associated with a social cost: When bank balance sheets are linked, shocks that redirect equity from the banking system to senior claimants reduce the scope for banks to create additional surplus. Since we take the network structure as given, we have nothing to say on ways to reduce contagion. Instead, we will focus on the role of information about the location of bad banks. Even if policymakers cannot do anything to alleviate contagion, they may still be able to affect what outside investors know about banks, and thus mitigate the consequences of contagion. For example, revealing which banks are bad may assure outsiders to invest in some banks – those with enough equity – rather than invest in none. To study this possibility, we need to allow banks to choose what information to disclose and to analyze the decisions of investors with incomplete information.

5   Disclosure

We now arrive at the final component into our model – allowing banks to decide whether to disclose their financial position before raising funds. If enough banks decide not to disclose, outsiders must decide whether to invest in banks not knowing exactly where all of the bad banks are located. This allows us to explore the main questions we are after: Under what conditions will market participants be unsure about which banks incurred losses, and in those cases would it be advisable to compel banks to reveal their financial position?

This section is organized as follows. We first describe how we model disclosure. We then provide conditions for the existence of a non-disclosure equilibrium where no bank discloses its S_j. We then examine whether mandatory disclosure can improve welfare relative to this equilibrium. Our essential insight is summarized in Theorem 1, which shows that mandatory disclosure cannot improve welfare when contagion is small but can when contagion is large and disclosure costs are small. Finally, we consider whether there might be other equilibria beyond the non-disclosure equilibrium we focus on. While we provide conditions under which multiple equilibria exist, we argue that our result reflects a tendency for contagion to produce insufficient disclosure rather than a failure by agents to coordinate on a superior equilibrium.

5.1 Modelling Disclosure

To model disclosure, suppose that after nature chooses the location of the b bad banks, each bank j observes S_j but not S_i for $i \neq j$. At this point, all banks simultaneously choose whether to incur a utility cost $c \geq 0$ and disclose S_j. The cost c is meant to capture the effort of conducting and documenting the result of stress-test exercises. In principle, c could reflect the cost of revealing information about trading strategies that rival banks can exploit. But it is not obvious whether we should treat these as costs a social planner would face, so we prefer to interpret c as the costs of producing and communicating information credibly.

Investors observe these announcements and decide what terms to offer each bank, if any. If banks accept such an offer, the outside investor must hand over his funds, giving up the outside option that would have earned $\underline{r}$. After outsiders choose whether to invest, the state of the network S is revealed and banks learn their equity. At this point, banks decide whether to invest the funds they raised or divert them. Finally, profits are realized and obligations are settled. Note that a bad bank with $S_j = 1$ will never want to disclose if $c > 0$. As such, we can describe each bank's decision by $a_j \in \{0, 1\}$, where $a_j = 1$ means bank j announces it is good and $a_j = 0$ means it announces nothing. Outside investors observe $a = (a_1, ..., a_n)$ and choose whether to provide funds to any of the banks. Since we restrict attention to debt contracts, the terms offered to banks can be summarized as an amount of resources each bank j receives, $I_j^*(a)$, and an interest rate $r_j^*(a)$ bank j must repay its investors.

5.2 Existence of a Non-Disclosure Equilibrium

Our first question is under what conditions non-disclosure can be an equilibrium, i.e. where each bank sets $a_j = 0$ expecting $a_i = 0$ for $i \neq j$. This case is of interest because it implies outsiders will be uncertain as to the location of bad banks. For our equilibrium concept, we use the notion of sequential equilibria introduced by Kreps and Wilson (1982). This concept requires off-equilibrium beliefs to coincide with the limit of beliefs from a sequence in which players choose all strategies with positive probability but the weight on suboptimal actions tends to zero. This restriction serves to rule out implausible off-equilibrium path beliefs. For example, without this restriction, off equilibrium outsiders could believe all banks that don't announce are bad, even though there are only b bad banks. Likewise, without this restriction outsiders can maintain particular beliefs about the neighbors of bank j if bank j deviates, even though bank j knows nothing about other banks when it decides on disclosure.

We now show that the existence of a non-disclosure sequential equilibrium depends on two parameters – the cost of disclosure c and the degree of contagion p_g. For non-disclosure to be an equilibrium, each good bank must be willing not to disclose, i.e. set $a_j = 0$, when

it expects no other bank to disclose. To derive what a good bank should do, we need to determine what outsiders would do if no bank discloses and if single good bank discloses, respectively. Assumption A2 implies banks have equity of either 0 or π. If no bank discloses, the probability a randomly chosen bank has equity π is $p_0 = \left(1 - \frac{b}{n}\right) p_g$ as defined in (10). By Assumption A3, we know banks would divert funds if they learn their pre-investment equity is zero. If banks learn their equity is π, whether they invest or divert depends on the r_j^* they are charged. The next lemma summarizes when banks divert funds:

Lemma 2: Under Assumptions A1-A3, any bank j whose pre-investment equity is π will prefer $D_j = 0$ if and only if $r_j^*(a) \leq \bar{r} \equiv \pi + R - v$.

In other words, if outside investors charge a rate above some threshold $\bar{r}$, banks will always prefer to divert funds. In principle, outsiders might still fund banks at a rate above $\bar{r}$, since they can count on grabbing the equity of banks with positive equity. However, it turns out that the equilibrium interest rate charged to any bank never exceeds $\bar{r}$:

Lemma 3: Under Assumptions A1-A3, in any equilibrium, $r_j^*(a) \leq \bar{r}$ for any bank j that receives funding, i.e. for which $I_j^*(a) = 1$.

Assumption A3 ensures that the maximal rate $\bar{r}$ in Lemmas 2 and 3 exceeds the outside option of investors $\underline{r}$.[9] We now argue that if p_0 is small, specifically if $p_0 < \underline{r}/\bar{r} < 1$, then outsiders will not finance any bank in a non-disclosure equilibrium, i.e. $I_j^* = 0$ for all j. Absent any information on S, the rate outside investors must charge to earn as much as their outside option is $\frac{\underline{r}}{p_0}$. From Lemma 3, banks cannot charge above $\bar{r}$ in equilibrium. Hence, the only possible non-disclosure equilibrium when $p_0 < \underline{r}/\bar{r}$ is if $I_j^* = 0$ for all j, or else outsiders must charge banks a rate above $\bar{r}$, which contradicts Lemma 3. Conversely, when $p_0 > \underline{r}/\bar{r}$, a non-disclosure equilibrium requires $I_j^* = 1$ for all j. This is because we can always find a rate $r_j \in \left(\frac{\underline{r}}{p_0}, \bar{r}\right)$ that ensures an expected return above $\underline{r}$ to investors, which both investors and banks would prefer to no trade. Note that since p_0 is proportional to p_g from (10), the cutoff for p_0 can be expressed in terms of p_g, i.e. $I_j^* = 0$ if $p_g < \frac{n}{n-b}\underline{r}/\bar{r}$ and $I_j^* = 1$ if $p_g > \frac{n}{n-b}\underline{r}/\bar{r}$.

Generically, then, when no information is disclosed, outsiders will either invest in all banks or none, depending on p_g. We now use this insight to verify whether a good bank would prefer not to disclose knowing that no other bank will disclose. Consider first the case where $p_g > \frac{n}{n-b}\underline{r}/\bar{r}$, which implies $I_j^* = 1$ for all j if no disclosure is an equilibrium. Since a bank can attract funds even without disclosing, the only benefit to a good bank from disclosing is that it can pay outside investors less than it would have to otherwise. In particular, disclosure will increase the probability outsiders attach to the bank having positive equity from the unconditional probability $p_0 \equiv Pr(e_j = 0)$ to the conditional probability $p_g \equiv Pr(e_j|S_j = 0)$.

---

[9]To see this, consider two cases, $\underline{r} > \pi$ and $\underline{r} \leq \pi$. If $\underline{r} > \pi$, the second inequality in (11) implies $\underline{r} < R + \pi - v \equiv \bar{r}$. If $\underline{r} \leq \pi$, the second inequality in (11) implies $v < R$, and hence $\bar{r} = \pi + R - v > \pi \geq \underline{r}$.

This would allow a bank to borrow at a lower rate than the $\frac{r}{p_0}$ it must pay in equilibrium. More precisely, the payoff to a good bank from not disclosing is given by

$$p_g\left(\pi + R - \frac{r}{p_0}\right) + (1 - p_g)\,v \tag{12}$$

Since a good bank knows it is good, the payoff in (12) is computed using the conditional probability p_g, even though outsiders assign probability p_0 that the bank will have positive equity. If the bank opts to disclose it is good, lenders will compete the rate they would lend to it down to $\frac{r}{p_g} < \frac{r}{p_0}$. Hence, when no other good bank chooses to disclose, good banks will be willing not to disclose their own financial position if and only if the disclosure cost exceeds the maximal gain from lowering the rate they are charged, i.e.

$$c \geq p_g\left(\frac{r}{p_0} - \frac{r}{p_g}\right) = \frac{b\underline{r}}{n - b} \tag{13}$$

Hence, when $p_g > \frac{n}{n-b}\underline{r}/\bar{r}$, a non-disclosure equilibrium exists if and only if $c \geq \frac{b\underline{r}}{n-b}$, i.e. when disclosure costs are large. In this case, the unique non-disclosure equilibrium is one where all banks receive funding. While this is the unique equilibrium without disclosure, other equilibria with partial or full disclosure may exist, an issue we return to below. For now, our only interest is in equilibria with no disclosure.

Next, consider the case where $p_g < \frac{n}{n-b}\underline{r}/\bar{r}$. Recall that in this case, a non-disclosure equilibrium involves no investment in any of the banks, i.e. $I_j^* = 0$ for all j. We need to verify that no good bank would wish to disclose its position given no other bank discloses. Since $I_j^* = 0$ in equilibrium, the only way a bank could benefit from disclosure is if revealing it is good will induce outsiders to fund it. Hence, non-disclosure can be an equilibrium if either unilateral disclosure does not induce outsiders to invest in a bank, or if unilateral disclosure induces investment but the cost of disclosure exceeds the gains from attracting investment.

Given our restriction to sequential equilibria, a good bank that deviates and discloses it is good would expect outside investors to assign probability p_g that it has equity π. Hence, outsiders will demand at least $\frac{r}{p_g}$ from it. From Lemma 2, we know that if $\frac{r}{p_g} > \bar{r}$, a bank will not be able to both pay enough to outsiders and credibly commit not to divert funds. Hence, if $p_g < \underline{r}/\bar{r}$, a good bank will not be able to attract investment if it discloses unilaterally. In this case, non-disclosure is an equilibrium for any $c \geq 0$. The fact that non-disclosure is an equilibrium even when $c = 0$ is of particular interest, since it shows that our model gives rise to non-disclosure equilibria in cases not already encompassed in the survey of Beyer et al. (2010) we discussed above. That is, our model satisfies each of the conditions they identify for non-disclosure to unravel. Our non-disclosure is instead due to an informational

spillover in which information from multiple agents is required to deduce whether a bank has sufficient equity to be worth investing in. This feature has no analog in previous work on disclosure, including work on informational spillovers such as Admati and Pfleiderer (2000), where non-disclosure equilibria only occur when disclosure is costly.[10]

The remaining case is where $\underline{r}/\bar{r} \leq p_g < \frac{n}{n-b}\underline{r}/\bar{r}$. In this case, $p_0 < \underline{r}/\bar{r} \leq p_g$. This means that outsiders will be too worried about default to invest when no bank discloses, but will be willing to invest in a bank if it alone reveals it is good. By disclosing and attracting investment, the bank will achieve an expected gain of

$$p_g \left(R - \underline{r}/p_g \right) + (1 - p_g) v - c$$

Hence, non-disclosure is an equilibrium only when c makes disclosure unprofitable, i.e.

$$c > p_g \left(R - v \right) + v - \underline{r}$$

In short, non-disclosure is an equilibrium if either the probability of contagion p_g is small, enough to render unilateral disclosure ineffective, or if the cost of disclosure c is large. We can collect our findings into the following proposition:

Proposition 8. Assume that Assumptions A2 and A3 hold. Then

1. A non-disclosure equilibrium *with no investment* can only exist if $p_g \leq \min\left(1, \frac{n}{n-b}\underline{r}/\bar{r}\right)$. Such an equilibrium exists if either

   (i) $p_g \leq \underline{r}/\bar{r}$; or
   (ii) $\underline{r}/\bar{r} < p_g \leq \frac{n}{n-b}\underline{r}/\bar{r}$ and $c \geq p_g(R - v) + v - \underline{r}$

2. A non-disclosure equilibrium *with investment* can exist only if $p_g \geq \frac{n}{n-b}(\underline{r}/\bar{r})$. Such an equilibrium exists if

   (i) $\frac{b}{n} \leq 1 - \underline{r}/\bar{r}$ to ensure $\frac{n}{n-b}(\underline{r}/\bar{r}) < 1$; and
   (ii) $c \geq \frac{b}{n-b}\underline{r}$

Figure 3 illustrates these results graphically. The shaded region in the figure corresponds to the region in non-disclosure equilibria exists. Since the thresholds for c are not generally

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[10]Okuno-Fujiwara, Postlewaite, and Suzumura (1990) obtain a result that is closer in spirit to ours. They provide several examples where non-disclosure can be an equilibrium. In one of these (Example 4), a firm can disclose information about another firm. The firm does not benefit from disclosing unfavorable information about its competitor because the competitor is at a corner and acts the same way if information is disclosed or not. This is similar to our result that unilateral disclosure does not induce a change in action by investors.

comparable for $p_g < \frac{n}{n-b} r/\bar{r}$ and $p_g > \frac{n}{n-b} r/\bar{r}$, these two cases are shown separately.

Given that the degree of contagion as reflected in p_g depends on primitives that govern the financial network of banks, we can relate our existence results to features of the underlying network such as the losses ϕ at bad banks and the size of the obligations λ across banks. For example, when ϕ is small, p_g will be close to 1. If there is a non-disclosure equilibrium, then as long as b/n isn't too large, it will be one in which all banks attract funds. Now, suppose news arrives that losses ϕ at bad banks increased, as Gorton (2008) argued occurred after the introduction of the ABX index (see the quote in Section 1). How this affects the non-disclosure equilibrium depends on λ. Recall that we argued in Section 3 that for $\lambda < \phi$, a change in ϕ has no effect on p_g. Thus, for small λ the news of large losses at some banks will have little observable effect: Banks will continue to attract funds. But if λ is large, p_g will fall with ϕ. If p_g falls sufficiently, the only possible non-disclosure equilibrium is one in which no bank attracts funds. Hence, the model suggests that large degrees of leverage against other banks allow shocks to give rise to market freezes that would not occur when λ is smaller. In the next subsection, we show that higher leverage may be related not only to the occurrence of market freezes but to whether mandating disclosure is desirable.

5.3   Mandatory Disclosure and Welfare

We now examine whether mandating disclosure can improve welfare relative to a non-disclosure equilibrium. Forcing all banks to disclose is a natural benchmark given policy-makers are often reluctant to discriminate among banks. This does not mean mandatory disclosure is optimal; it is not in our model. But it can be used to identify when intervention is beneficial. That said, the failure we identify need not require government action, since in principle a mutual association of banks could implement the same policies on their own.

Recall that in a non-disclosure equilibrium, outsiders will generically either invest in all banks or in none. Whether mandatory disclosure can improve upon a non-disclosure equilibrium depends on what investors do in equilibrium. We begin with the case of no investment, i.e. when $p_g < \frac{n}{n-b} r/\bar{r}$. In this case, mandatory disclosure can "unfreeze" markets, allowing those banks with positive equity to attract funds they wouldn't have otherwise. However, the additional surplus this creates comes at the cost of forcing all banks to incur disclosure costs. To determine whether the additional surplus exceeds the cost, note that the expected number of banks with positive equity is $(n-b) p_g$. Each of these banks creates a surplus of $R - \underline{r}$. The cost of forcing all banks to produce information about their losses is cn. Hence, the expected surplus created exceeds the cost of disclosure if and only if

$$(n-b) p_g (R - \underline{r}) - cn > 0. \tag{14}$$

Observe that for $b > 1$, the number of banks with positive equity is random, so (14) represents an *ex-ante* criterion, before the location of bad banks is known. *Ex-post*, the gains from trade can turn out to be too low to justify the cost of disclosure.[11] We will refer to mandatory disclosure as a *welfare improvement over non-disclosure* if (14) holds.

We can now examine whether the conditions that ensure the existence of a non-disclosure equilibrium are compatible with welfare improving mandatory disclosure. From Proposition 8, we know that when $p_g < \underline{r}/\bar{r}$, a non-disclosure equilibrium exists for all $c \geq 0$. By contrast, (14) implies that forcing all firms to disclose will be valuable whenever the cost of disclosure c is not too large. Hence, the region in which no disclosure is an equilibrium but mandatory disclosure is welfare improving is non-empty. Formally,

Proposition 9. Under Assumptions A2 and A3, if $0 < p_g \leq \underline{r}/\bar{r}$ and $c \leq (R - \underline{r})\frac{n-b}{n}p_g$, mandatory disclosure is a welfare improvement over no-disclosure.

Intuitively, at low values of p_g, a good bank that unilaterally discloses its S_j will not be able to attract investment. It can therefore be individually optimal for each bank not to disclose even though all banks would be made better off if they coordinated to disclose.

Next, we turn to the case where $\underline{r}/\bar{r} < p_g < \frac{n}{n-b}\underline{r}/\bar{r}$. For these values of p_g, non-disclosure equilibria exist only for sufficiently large c. By contrast, mandatory disclosure can only improve welfare for sufficiently small c. Thus, it is not obvious that there exists a non-disclosure equilibrium that can be improved upon. However, since the private incentives to disclose need not coincide with the planner's, welfare improvement remains a possibility.

In the next proposition, we provide the conditions under which there exists a non-disclosure equilibrium when $\underline{r}/\bar{r} < p_g < \frac{n}{n-b}\underline{r}/\bar{r}$ that can be improved upon. Two conditions prove to be necessary: v must be below $\underline{r}$, and the fraction of bad banks $\frac{b}{n}$ must not be too large. The first condition implies diversion is costly. To see this, recall that outsiders who invest in banks are assumed to give up the option to earn $\underline{r}$ on their funds. Hence, if a bank learns its equity is zero, the funds it raised will yield a (private) return of v. Since $v < \underline{r}$, identifying the banks with zero equity in advance and avoiding them from taking in funds would yield more total resources that could be used to make both the bank and investor better off. Banks do not take into account these benefits when they choose whether to disclose, and so disclosure may be inefficiently low. As for the role of the fraction of bad banks, recall that only good banks ever contemplate disclosure. The cost for a good bank of disclosure is c, while mandatory disclosure at *all* banks implies a cost of disclosure per

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[11]It is worth noting that one reason stress tests in the U.S. were viewed as successful is that they revealed the U.S. banking system to be relatively well-capitalized. Our model suggests that even if stress tests reveal a different state of affairs, as was arguably the case in Europe, a commitment to run stress-tests may still be a reasonable policy *ex-ante*, i.e. before the results of the tests are known.

good bank of $\frac{n}{n-b}c$. This higher cost may make mandatory disclosure undesirable despite the potential benefits from the release of information. Formally:

Proposition 10. Assume Assumptions A2 and A3 hold. If $\underline{r}/\bar{r} < p_g < \frac{n}{n-b}\underline{r}/\bar{r}$, then

1. If $v \geq \underline{r}$ and there exists a non-disclosure equilibrium, mandatory disclosure cannot be welfare improving over non-disclosure.

2. If $v < \underline{r}$, then

   (a) If $\frac{b}{n} > \left(\frac{\bar{r}}{\underline{r}} - 1\right)\frac{\underline{r}-v}{R-\underline{r}}$, there exists no non-disclosure equilibrium that can be welfare improved via mandatory disclosure.

   (b) If $\frac{b}{n} \leq \left(\frac{\bar{r}}{\underline{r}} - 1\right)\frac{\underline{r}-v}{R-\underline{r}}$, a non-disclosure equilibrium exists upon which mandatory disclosure is welfare improving for pairs (p_g, c) where

   i. $\underline{r}/\bar{r} < p_g < \min\left\{\frac{n}{n-b}\underline{r}/\bar{r}, \frac{\underline{r}-v}{(R-v)-(1-b/n)(R-\underline{r})}\right\}$, and

   ii. $(R-v)p_g + (v-\underline{r}) \leq c \leq \frac{n-b}{n}p_g\left(R-\underline{r}\right)$.

   Moreover, $\min\left\{\frac{n}{n-b}\underline{r}/\bar{r}, \frac{\underline{r}-v}{(R-v)-(1-b/n)(R-\underline{r})}\right\} < 1$, so condition (i) requires $p_g < 1$.

The last part of Proposition 10 implies that a non-disclosure equilibrium can be improved upon only if p_g is strictly below 1. That is, mandatory disclosure to unfreeze markets will only be desirable if there is sufficiently high contagion from bad banks to good banks.

Finally, we turn to the case where $p_g > \frac{n}{n-b}\underline{r}/\bar{r}$. Recall from Proposition 8 that if a non-disclosure equilibrium exists in this case, all banks will be able to raise funds. This does not mean that banks no longer have a reason to disclose: A bank that reveals it is good will be able to offer a lower interest to outside investors. This represents a purely private gain: A bank is able to keep more of the surplus it creates, but disclosure creates no new surplus. As Jovanovic (1982) points out, when disclosure is costly and yields only private gains, mandating disclosure is typically undesirable: It represents a costly activity with no social gains. Fishman and Hagerty (1989) similarly show that when disclosure is driven by rent-seeking, forcing more disclosure than occurs in equilibrium may not be desirable. By contrast, since our model exhibits informational spillovers, mandatory disclosure may be desirable even though each bank's decision to disclose is driven by rent-seeking. To see this, observe that in equilibrium, the available amount of resources is given by

$$(n-b)\,p_g\,(\pi + R) + (n - (n-b)\,p_g)\,v \tag{15}$$

That is, on average $(n-b)\,p_g$ banks have positive equity and invest the funds they raise, while the remainder divert their funds for private gains. By contrast, under mandatory disclosure,

all banks with zero equity will be refused funding and outsiders deploy these funds on their own. Expected available resources after netting out disclosure costs are given by

$$(n - b) p_g (\pi + R) + (n - (n - b) p_g) \underline{r} - cn \tag{16}$$

Although v represents private benefits that cannot be redistributed, comparing (15) and (16) still reveals whether mandatory disclosure can be welfare improving. On the one hand, if mandatory disclosure results in fewer resources, it will be impossible to keep everyone as well off even if redistribution were possible, so mandatory disclosure could not improve welfare if (15) exceeded (16). If mandatory disclosure resulted in more resources, it would be without any diversion, and so the resources created under disclosure can be freely redistributed. The welfare gain from disclosure in this case is not due to generating trade, but to preventing wasteful diversion. When a good bank contemplates the value of disclosure, it ignores the fact that its disclosure may help to identify which other banks have zero equity and prevent diversion. Comparing between (15) and (16) reveals that mandatory disclosure will be welfare improving when c satisfies

$$\frac{b\underline{r}}{n - b} < c < \left(1 - \frac{n - b}{n} p_g\right) (\underline{r} - v) \tag{17}$$

Once again, for this range to be non-empty, two conditions must be satisfied: diversion must be costly, i.e. $v < \underline{r}$, and the fraction of bad banks $\frac{b}{n}$ must not be too large. Formally, for $p_g > \frac{n}{n-b} \underline{r}/\overline{r}$ we have the following proposition:

Proposition 11. Assume Assumptions A2 and A3 hold. Suppose $p_g \geq \frac{n}{n-b} \underline{r}/\overline{r}$. Then

1. If $v \geq \underline{r}$ and there exists a non-disclosure equilibrium, mandatory disclosure cannot be welfare improving over non-disclosure.

2. If $v < \underline{r}$, then

   (a) If $\frac{b}{n} > \frac{\underline{r}-v}{(\underline{r}-v)(1-\underline{r}/\overline{r})+\underline{r}} (1 - \underline{r}/\overline{r})$, there exists no non-disclosure equilibrium that can be welfare improved via mandatory disclosure.

   (b) If $\frac{b}{n} \leq \frac{\underline{r}-v}{(\underline{r}-v)(1-\underline{r}/\overline{r})+\underline{r}} (1 - \underline{r}/\overline{r})$, a non-disclosure equilibrium exists upon which mandatory disclosure is welfare improving whenever

   i. $\frac{n}{n-b} \underline{r}/\overline{r} \leq p_g \leq \frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{\underline{r}}{\underline{r}-v}\right)$, and

   ii. $\frac{b}{n-b} \underline{r} \leq c \leq (1 - \frac{n-b}{n} p_g)(\underline{r} - v)$.

   Moreover, $\frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{\underline{r}}{\underline{r}-v}\right) < 1$, so condition (i) only holds for $p_g < 1$.

Note the parallel with Proposition 10. Once again, mandatory disclosure can only be welfare improving if p_g is strictly below 1, i.e. when there is enough contagion.

We can combine Propositions 9-11 to form a result that captures how the desirability of mandatory disclosure varies with the degree of contagion p_g:

Theorem 1. Assume Assumptions A2 and A3 hold. For p_g close to 1, mandatory disclosure cannot improve upon a non-disclosure equilibrium. Conversely, for p_g close to but not equal to 0, if c is low, the non-disclosure equilibrium can be improved upon.

Remark 2: Note that neither Theorem 1 nor Propositions 8-11 require Assumption A1. In particular, our key results do not depend on being in what Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) describe as the "small shock" regime. This reinforces our observation in Remark 1 that Assumption A1 can be dispensed with, at least for some results. ∎

We close with a few observations. The first concerns comparative statics with respect to p_g. Although mandatory disclosure can be welfare improving when p_g is close to zero, this will not be true in the limit when $p_g = 0$. In that case, there are no banks worth investing in, and so disclosure serves no purpose. More generally, the expected welfare gains from disclosure are not monotonic in p_g: They increase with p_g for p_g below the threshold $\frac{n}{n-b} \underline{r}/\bar{r}$ but decrease with p_g above the threshold. Intuitively, for high degrees of contagion, outsiders will refrain from trade in the absence on information. Mandatory disclosure allows trade, but more contagion implies fewer banks can trade. By contrast, with low degrees of contagion, outsiders will trade with all banks even in the absence of information. Mandatory disclosure then prevents diversion, and more contagion implies more banks divert resources. Note the inherent tensions in the model: When p_g is low, more contagion makes it more likely that mandatory disclosure improves welfare, but it also makes the gains from intervention smaller. When p_g is large, more contagion makes it both more likely that disclosure improves welfare, and it makes such intervention more valuable. But in this case more contagion also makes non-disclosure equilibria where outsiders agree to invest in all firms less likely.

On a related note, the model implies that a decline in p_g can make mandatory disclosure desirable even if markets do not freeze up because there are gains to avoiding wasteful diversion. In fact, a mild decline in p_g may make mandatory disclosure desirable while a larger decline would not, since a larger decline induces a regime shift in which trade is suspended and mandatory disclosure only turns desirable for much larger degrees of contagion.

Finally, we can use our analysis of contagion to relate our results to aspects of the underlying financial network. Recall that a low value of ϕ will imply p_g is close to 1. Thus, when losses at bad banks are small, there will be no need for mandatory disclosure. As losses rise, p_g will be unchanged if λ is small but will fall if λ is large. Higher leverage against

other banks can lead to market freezes and justify intervention that would not be desirable otherwise. This implies that financial regulation that restricts leverage or reduces the degree of interconnectedness among banks may obviate the need for disclosing stress test results. By contrast, the prevalent attitude among policy makers, as captured in Bernanke (2013), views these policies and stress tests as complements that should be adopted simultaneously.

5.4 Multiple Equilibria

Our primary motivation so far was to determine whether it is appropriate to force banks to reveal information they choose not to reveal in equilibrium. As such, we have focused on whether non-disclosure equilibria exist that may necessitate such intervention. However, the existence of non-disclosure equilibria does not rule out other equilibria in which some or even all good banks disclose their status. We now report some results relating to the possibility of multiple equilibria in our model. Under certain conditions, our model suggests that if mandatory disclosure is welfare improving, all banks disclosing will also be an equilibrium. In these cases, mandatory disclosure can be viewed as serving to help banks coordinate on a superior equilibrium. But we also argue that multiple equilibria are not inherent to our setup, so mandatory disclosure is not simply an equilibrium selection device.

We first show that when the number of bad banks b is large, then if mandatory disclosure improves upon a non-disclosure equilibrium with no trade, there exists another equilibrium in which all good banks reveal themselves to be good. Formally, recall that if markets are frozen in the absence of disclosure, forcing disclosure improves welfare if $(n - b)p_g(R - \underline{r}) > cn$, i.e. if the expected surplus created under full revelation exceeds the cost of forcing disclosure. We will now show that this condition ensures that all good banks disclosing must also be an equilibrium for sufficiently large b.

Proposition 12: Suppose $(n - b)p_g(R - \underline{r}) > cn$. Then if $\phi > \pi$ and Assumptions A2 and A3 hold, $a_j = 1$ for all good banks j is an equilibrium if $b > \frac{\bar{r}}{\underline{r}} - 1$.

The reason the number of bad banks b must be large is that this allows outsiders to maintain sufficiently "pessimistic" beliefs about banks that fail to disclose. Recall that our restriction to sequential equilibria limits the beliefs outsiders can entertain about banks off the equilibrium path. In particular, since there are exactly b bad banks, if all other good banks disclose and one good bank deviated and failed it disclose, there would be $b + 1$ banks that announce nothing. Outsiders will then assign equal probability that each bank that doesn't disclose is the good bank, i.e. $\frac{1}{b+1}$. For large b, this probability is close to 0 so beliefs are quite pessimistic. As is well known from previous work on disclosure, pessimistic beliefs can be used to sustain equilibria in which good agents disclose by letting outsiders believe that those who fail to disclose are the worst possible type.

Proposition 12 would seem to suggest that mandatory disclosure is desirable only when it helps agents coordinate on a superior equilibrium. However, this conclusion is not true in general, and it is misleading to read our results as implying that insufficient disclosure arises when agents coordinate on a bad equilibrium. To see this, we make two observations.

First, as our discussion suggests, Proposition 12 requires large values of b. For small values, it need not be the case that there is always another Pareto-superior equilibrium in which good banks can coordinate on whenever intervention is desirable. In Appendix B, we give an example where $b = 1$ for a slightly modified version of the model in which no disclosure is the unique equilibrium yet mandatory disclosure is welfare-improving. In that example, not disclosing is a dominant strategy for each good bank. Thus, mandatory disclosure can make agents better off even without another equilibrium they could coordinate on.[12]

Second, even when b is large, the divergence between private and social incentives admits a role for intervention beyond just addressing a coordination failure. Recall that Proposition 10 shows that mandatory disclosure can be beneficial at intermediate values of p_g when a bank could attract funds if it disclosed its state unilaterally. That is, intervention can be beneficial even when coordination is not a problem. The reason is that banks fail to take into account spillovers from disclosing their information. One way to get at this distinction would be to introduce independent private signals that prevent agents from coordinating their disclosure decisions, and then ask whether the unique equilibrium in that environment is efficient. For example, suppose banks and outsiders receive private signals on ϕ, the loss per bank. Good banks that believe ϕ is small would deduce p_g is close to 1 and prefer to disclose, while good banks that believe ϕ is large would prefer not to disclose if $c > 0$. We conjecture that since banks do not internalize all of the value of the information they disclose, banks would choose not to disclose at a threshold that is lower than the socially optimal one.

6 Alternative Network Structures

So far, our model of financial contagion assumed a particular network structure in which i's obligations to banks $j \neq i$ is given by $\Lambda_{ij} = \lambda$ for $j = i + 1 \pmod{n}$ and 0 otherwise. We now argue that our key results extend to a larger class of networks Λ_{ij}.

A general network corresponds to a specification of liabilities across banks that can be

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[12]Since the existence of multiple equilibria is often related to strategic complementarities, we should note that disclosure in our model is not always a strategic complement: If we restrict other banks to a common disclosure probability, a bank's incentive to disclose can fall with the probability others disclose. This is because there are two offsetting forces in our model: As more banks disclose, each remaining bank is perceived as more likely to be bad, encouraging disclosure. At the same time, if enough of the banks you are exposed to disclose, outsiders may invest in you even if you do not disclose, reducing your incentive to disclose.

summarized by an $n \times n$ matrix Λ with zeros along the diagonal. We continue to restrict attention to networks in which each bank has a zero net position with the remaining banks in the network, i.e. for each $i \in \{0, ..., n-1\}$,

$$\sum_{j \neq i} \Lambda_{ij} = \sum_{j \neq i} \Lambda_{ji} \tag{18}$$

Using network theory terminology, (18) implies Λ is a regular weighted directed network.

As in the case of the circular network, we assume the network is hit by a shock process governed by two parameters: b, the number of bad banks, and ϕ, the losses at each bad bank, where each of the $\binom{n}{b}$ possible locations of the bad banks within the network is equally likely.

Since each bank can now be obligated to any of the other $n-1$ banks, the set of payments is now given by $\{x_{ij}\}_{i \neq j}$ as opposed to just n payments as before. Since each bank can now be obligated to multiple banks, we need a priority rule on how to divide resources when banks fall short of their total obligations. We follow Eisenberg and Noe (2001) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) in assuming that an insolvent bank pays its obligations in a pro-rata basis. That is, define Λ_i as bank i's total obligations to all other banks, i.e.

$$\Lambda_i = \sum_{j=0}^{n-1} \Lambda_{ij} \tag{19}$$

We assume that if bank i's resources fall short of Λ_i, it will pay each bank j to which it is obligated a fraction $\frac{\Lambda_{ij}}{\Lambda_i}$ of the resources it has. The payments x_{ij} thus solve the system

$$x_{ij} = \frac{\Lambda_{ij}}{\Lambda_i} \max \left\{ \min \left\{ \Lambda_i\,,\, \pi - \phi S_i + \sum_{r=0}^{n-1} x_{ri} \right\}\,,\, 0 \right\} \text{ for all } i \neq j \tag{20}$$

where recall $S_i = 1$ if bank i is bad. We can then define the pre-investment equity of bank i, meaning the equity of bank i if it did not raise any outside funds, as

$$e_i = \max \left\{ \pi + \sum_{j=0}^{n-1} x_{ji} - S_j \phi - \sum_{j=0}^{n-1} x_{ij}, 0 \right\} \tag{21}$$

A convenient feature of the circular network we analyzed thus far is that it implies a particular symmetry: Every good bank is equally likely to have its equity wiped out regardless of its identity. This allowed us to summarize contagion with a single statistic p_g. We now argue that similar results on the relationship between contagion and the desirability of mandatory disclosure hold for networks that exhibit a version of this symmetry property, The property we impose involves the distribution of equity e_j:

Definition: A financial network Λ is *symmetrically vulnerable to contagion* given the

shock process $\{b, \phi\}$ if, absent outside funding, the distribution of equity for a good bank is independent of its identity, i.e. if for each $x \in [0, \pi]$, $\Pr(e_j = x | S_j = 0)$ is the same for all j.

One way to ensure that a network is symmetrically vulnerable to contagion is if the network of debt obligations captured by Λ_{ij} is symmetric, a notion we formally define below. To motivate this definition, suppose we have n distinct physical locations. We can assign banks to different physical locations and then trace a directed network across locations based on the obligations between banks. A network is said to be *symmetric* if observing the links across physical locations provides us with no identifying information about the location of *any* individual bank. That is, obligations Λ_{ij} are such that we can put any bank j in any one of the n locations and arrange the remaining banks in such a way that the implied network across physical locations is unchanged. Formally,

Definition: A network Λ is *symmetric* if for any pair k and ℓ in $\{0, ..., n-1\}$ there exists a bijective function $\sigma_{k,\ell} : \{0, ..., n-1\} \rightarrow \{0, ..., n-1\}$ such that (i) $\sigma_{k,\ell}(k) = \ell$ and (ii) for any pair i and j in $\{0, ..., n-1\}$, $\Lambda_{\sigma_{k,\ell}(i), \sigma_{k,\ell}(j)} = \Lambda_{ij}$.

As an example of symmetric networks, consider circulant networks in which banks can be ordered in such a way that the obligation Λ_{ij} between any pair i and j can be expressed solely as a function of the distance $i - j \pmod{n}$. This feature implies that we can arrange banks along a circle in such a way that any rotation of the banks across locations will have no effect on the directed network across locations. Hence, we will not be able to identify any bank after observing the network across locations. The circular network is one example of a circulant network, but other networks that have figured prominently in the literature on financial networks are also circulants, e.g. complete networks with equal liabilities, i.e. $\lambda_{ij} = \lambda$ for all $i \neq j$, partially complete networks where banks have liabilities to multiple banks such as the interconnected ring network in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013), and multiple disconnected symmetric networks, e.g. isolated pairs of banks. While circulant networks are necessarily symmetric, not all symmetric networks are circulant; we give an example of a non-circulant symmetric network in Appendix C. Our restriction thus encompasses a broader class of networks than just circulants. Our next result confirms that symmetry of the network implies symmetric vulnerability to contagion.

Lemma 4: Any regular symmetric network Λ is symmetrically vulnerable to contagion.

Remark 3: It is not necessary for a network to be symmetric to be symmetrically vulnerable to contagion. In Appendix C, we give an example of an asymmetric network that is symmetrically vulnerable to contagion for a particular b and ϕ. That is, we give an example of a network where observing obligations across physical locations fully reveals where each

bank is located, and yet the network is still symmetrically vulnerable to contagion. Thus, our results hold for a broader class of networks than just symmetric networks. ∎

For the circular network we have focused on so far, Assumption A2 ensures that the pre-investment equity e_j for a good bank could only assume two values, 0 and π. Hence, this distribution can be summarized by a single parameter, $p_g = \Pr(e_j = \pi | S_j = 0)$. This will not be true more generally. However, even though the distribution of equity can no longer be summarized by a single statistic p_g, we can still derive analogs to our previous results regarding the determinants of contagion and the connection between the degree of contagion and the desirability of mandatory disclosure.

We start with results on comparative statics on contagion. We index the matrix of obligations across banks Λ by a scale factor λ so that

$$\Lambda(\lambda) = \lambda\,\Lambda(1) \tag{22}$$

That is, the scalar λ multiplies each entry of the baseline matrix $\Lambda(1)$. This is one way to generalize our comparative static in the circular network of simultaneously increasing the obligations between any two banks. Recall that in Proposition 6, we showed that higher λ and higher ϕ led to greater contagion as measured by p_g. In the general case, an analog for lower p_g, and thus more contagion, is a first order stochastically lower distribution for equity. The next proposition establishes that higher λ and ϕ imply more contagion in this sense, even for networks that may not be symmetrically vulnerable to contagion.

Proposition 13. Let Λ be a directed weighted regular network where the matrix Λ is indexed by λ as in (22). Then for each $i \in \{0, ..., n-1\}$ and each $x \in [0, \pi]$ the probability $\Pr\{e_i \leq x \,|\, S_i = 0\}$ is weakly increasing in ϕ and λ.

Finally, we establish an analog to Theorem 1 which shows that the degree of contagion, as reflected in the likelihood of good banks having to liquidate assets and lowering their pre-investment equity to below π, is related to the desirability of mandatory disclosure.

Theorem 2. Suppose Λ is regular and symmetrically vulnerable to contagion. Also, $\phi > \pi$, and Assumption A3 holds. If $\Pr(e_j = \pi | S_j = 0)$ is sufficiently close to 1, mandatory disclosure cannot improve upon non-disclosure. Conversely, there exists an equity level $e^* > 0$ with $0 < e^* < \pi$ such that if $\Pr(e_j \geq e^* | S_j = 0)$ is sufficiently close to but not equal to 0, mandatory disclosure will be welfare improving over non-disclosure for low enough c.

Theorem 2 generalizes Theorem 1 by showing that our result that welfare gains are possible with enough contagion and impossible otherwise holds for a broad class of networks. This does not mean that network structure is irrelevant for the desirability of mandatory

disclosure. Ultimately, the network structure determines the extent of contagion. In other words, although the conditions that ensure mandatory disclosure can be desirable do not depend on the exact network structure, whether these conditions are satisfied does. As an example, consider the complete network in which $\Lambda_{ij} = \lambda$ for all $j \neq 0$. In this case, the exact location of bad banks is irrelevant, since the equity of any good bank will be the same regardless of which banks are bad. Mandatory disclosure can serve no positive role in this case. Consistent with this, note that the distribution of equity at good banks is degenerate in this network, and so $\Pr(e_j \geq x | S_j = 0)$ is equal to either 1 or 0 for all x. Hence, we will never be able to satisfy the requirement of Theorem 2 that $\Pr(e_j \geq e^* | S_j = 0)$ must be close to but strictly above 0 for mandatory disclosure to be desirable.

7 Conclusions and Future Work

This paper shows that when contagion is substantial and disclosure costs are not too high, mandatory disclosure may be welfare-improving. This suggests that using stress tests to force out information on the health of banks can be socially beneficial – provided there is enough potential for contagion across banks. Since contagion depends on the underlying network, stress tests are only beneficial under certain conditions, and other regulatory intervention such as restrictions on leverage with other banks may invalidate the justification for using them. These insights may be relevant for the debate on whether derivatives trading should be shifted from over-the-counter to centralized exchanges.[13] One oft-cited reason for this recommendation is fragility due to chains of indirect exposure to counterparty risk. While we do not model the equivalent to migrating to an exchange, our results suggest mandatory disclosure may offer a partial substitute to migration by addressing some shortcomings of over-the-counter markets, such as the potential for markets to freeze.

Since our model is relatively simple, which makes our arguments, we hope, transparent, it leaves out many features, as we briefly describe here.

The simplicity of our setup relies in part on our restriction to networks that exhibit a convenient symmetry property. This excludes several interesting cases. First, our set up excludes more realistic networks in which some banks are more central than others. One might be able to gain some insights on how this asymmetry matters by looking at sparsely parameterized core-periphery networks as in Babus and Kondor (2013). For example, when is mandatory disclosure only desirable for core banks, in line with the fact that stress tests in practice were limited to core banks, and when is it necessary to force peripheral banks to disclose as well? Second, we might allow the severity of the shocks to vary across banks,

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[13]For a discussion, see Duffie and Zhu (2011) and Duffie, Li, and Lubke (2010) and the references therein.

or the probability of a shock to hit a bank to vary across banks. This type of analysis may suggest better ways of performing stress tests.

More generally, one can use our framework to think about what optimal disclosure policy might be. Mandatory disclosure treats all banks fairly, but it is also inefficient; requiring only $n-1$ banks to disclose in our model is equally informative but less costly. Still more targeted policies do even better, e.g. policies that pay banks as a function of the outcome of their disclosure and then make the outcome public. For example, if less than half of all banks are bad, rewarding banks that disclose they are bad will be preferable to forcing all banks to disclose, as would rewarding banks that disclose they are good if less than half of banks are good. The optimal policy will thus depend on the exact details of the environment.

Another feature of our model that is worth investigating is the importance of our assumption that disclosure is simultaneous. Allowing banks to move sequentially can potentially facilitate coordination. Since we show that our result cannot be entirely attributed to coordination failures, we suspect that some of our results would carry over to dynamic environments. However, sequential disclosure is likely to introduce new issues, such as informational cascades and herding, where information gets "trapped" if banks that are exposed to bad banks choose not to reveal their own state, thereby discouraging the banks exposed to them from disclosing their status.

Another assumption we impose that may be worth relaxing is that banks can provide incontrovertible proof of their state. A more realistic model would allow banks to give an informative yet imperfect signal. This opens new possibilities that may be relevant for the difference between the social and private value of information disclosure. Still another assumption in our model worth relaxing is that banks disclose actual losses. In practice, stress tests ask banks about potential future losses. Whether this matters for our results remains an open question.

Finally, our analysis focuses on stress tests only as a source of information for investors rather a basis for capital injections. The framework we propose here may be a useful start for exploring the role of capital injections beyond issues of disclosure.

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A Proofs

Proof of Proposition 1: We can rewrite the system of equations in (4) as

$$x_j = T_j (x_{j-1}) \equiv \max \{0, \min (x_{j-1} + \pi - \Phi_j, \lambda)\}$$

By repeated substitution, we can reduce this system of equations to a single equation

$$x_0 = T^* (x_0)$$

where

$$T^* (x_0) \equiv T_n \circ T_{n-1} \circ \cdots \circ T_1 (x_0)$$

The mapping T^* is continuous, monotone, bounded. Moreover, for any x and y in $[0, \lambda]$, we have $|T^* (x) - T^* (y)| \leq |x - y|$. Let

$$\begin{aligned}
\underline{x} &= \lim_{m \to \infty} (T^*)^m (0) \\
\overline{x} &= \lim_{m \to \infty} (T^*)^m (\lambda)
\end{aligned}$$

These limits exist given T^* is monotone and bounded. By continuity, $\underline{x}$ and $\overline{x}$ must both be fixed points of T^*, i.e.

$$\underline{x} = T^* (\underline{x}) \ \text{and} \ \overline{x} = T^* (\overline{x})$$

Moreover, by monotonicity, $(T^*)^m (0) \leq (T^*)^m (\lambda)$ for any m. Taking the limit, $\underline{x} \leq \overline{x}$. Hence, the set of fixed points of T^* is nonempty.

Suppose $\underline{x} < \overline{x}$. Then for any $\mu \in (0, 1)$, the value $x_\mu = \mu \underline{x} + (1 - \mu) \overline{x}$ must also be a fixed point of T^*, i.e.

$$x_\mu = T^* (x_\mu)$$

For suppose

$$x_\mu > T^* (x_\mu)$$

In this case, we have

$$\begin{aligned}
x_\mu - \underline{x} &> T^* (x_\mu) - \underline{x} \\
&= T^* (x_\mu) - T^* (\underline{x}) \geq 0
\end{aligned}$$

But this counterfactually implies

$$|x_\mu - \underline{x}| > |T^* (x_\mu) - T^* (\underline{x})|$$

Likewise, if

$$x_\mu < T^* (x_\mu)$$

then we can show that

$$\begin{aligned}\overline{x} - x_\mu &> \overline{x} - T^* (x_\mu) \\ &= T^* (\overline{x}) - T^* (x_\mu) \geq 0\end{aligned}$$

which again counterfactually implies

$$|x_\mu - \underline{x}| > |T^* (x_\mu) - T^* (\underline{x})|$$

We conclude that $T^* (x) = x$ for all $x \in [\overline{x}, \overline{x}]$. Next, we argue that for $x \in [\underline{x}, \overline{x}]$, for all $j \in \{1, ..., n\}$,

$$T_j \circ \cdots \circ T_1 (x) = T_{j-1}(x) + \pi - \Phi_j$$

For suppose not. That is, there exists some j such that either

$$\begin{aligned}&\text{(i)} \quad T_{j-1} (x) + \pi - \Phi_j > \lambda \\ &\text{(ii)} \quad T_{j-1} (x) + \pi - \Phi_j < 0\end{aligned}$$

But then by continuity there must exist at least two values $x' \neq x''$ from $[\underline{x}, \overline{x}]$ such that

$$T_j (x') = T_j (x'')$$

and hence $T^* (x') = T^* (x'')$, which requires $x' = x''$, a contradiction. It follows that

$$T^* (x) = x + \sum_{j=1}^{n} (\pi - \Phi_j)$$

for all $x \in [\underline{x}, \overline{x}]$. But since $T^* (x)$ must equal x in this interval, we must have

$$\sum_{j=1}^{n} (\pi - \Phi_j) = 0$$

This implies that $\underline{x} = \overline{x}$, i.e. the fixed point of T^* is unique, whenever

$$\sum_{j=1}^{n} (\pi - \Phi_j) \neq 0$$

This completes the proof for the case where $n\pi \neq b\phi$. ∎

Proof of Proposition 2: Since $\phi \leq \pi < \dfrac{n}{b}\phi$, we know from Proposition 1 that the (4) has a unique solution. It will suffice to verify that $x_j = \lambda$ is a solution. For any $j \in \{1, ..., n\}$, we have

$$x_j = \max \{0, \min (\lambda + \pi - \Phi_j, \lambda)\}$$

Since $\pi - \Phi_j \geq 0$ whenever $\phi < \pi$, then $x_j = \lambda$ solves the system of equations (4). ∎

Proof of Proposition 3: Suppose $e_j = 0$ for all j. By construction, $e_j \geq \pi - \Phi_j + x_{j-1} - x_j$. Summing up over all j yields

$$\sum_{j=1}^{n} e_j \geq \sum_{j=1}^{n} (\pi - \Phi_j + x_{j-1} - x_j)$$
$$= n\pi - b\phi$$
$$> 0$$

This contradicts the fact that $e_j = 0$ for all j. Hence, there must exist at least one j for which $x_j = \lambda$.

Next, we argue that the fact that $e_j > 0$ for some j implies $x_j = \lambda$ for some j. For suppose not. Since $x_j = \max\{0, \min\{x_{j-1} + \pi - \Phi_j, \lambda\}\}$, it follows that $x_{j-1} + \pi - \Phi_j < \lambda$ for all j. Hence, $x_j = \max\{0, x_{j-1} + \pi - \Phi_j\}$. From this, it follows that $e_j = \max\{\pi - \Phi_j + x_{j-1} - x_j, 0\} = 0$, since either $x_{j-1} + \pi - \Phi_j < 0$ in which case $x_j = 0$ and e_j is the maximum of a negative expression and 0, and thus equal to 0, or else $x_j = x_{j-1} + \pi - \Phi_j$ and so $e_j = \max\{\pi - \Phi_j + x_{j-1} - x_j, 0\} = \max\{0, 0\} = 0$. ∎

Proof of Proposition 4: Define $\hat{S}$ as a state in which all the bad banks are located next to one another. Without loss of generality, we can order banks so that $\hat{S}_j = 1$ for $j = 0$ and $j \in \{n - b + 1, ..., n - 1\}$. We now establish the claim through a sequence of steps. First, we argue that if the state of the network is given by $\hat{S}$, then for λ sufficiently large, all banks will transfer some positive resources to other banks on the network.

Result 1: Suppose $\lambda > b(\phi - \pi)$. Then if $S = \hat{S}$, the fixed point x_j that solves (4) satisfies $x_j > 0$ for all j.

Proof of Result 1: Suppose $x_0 = 0$. Then $x_j = \min\{j\pi, \lambda\}$ for all $j \in \{1, ..., n - b\}$. Since $n\pi > b\phi$ under A1, we can subtract $b\pi$ from both sides to get

$$(n - b)\pi > b(\phi - \pi)$$

Set $\lambda = b(\phi - \pi) + \varepsilon$ where $\varepsilon > 0$. Choose ε sufficiently small so that

$$(n - b)\pi > b(\phi - \pi) + \varepsilon$$

Then $x_{n-b} = \lambda = b(\phi - \pi) + \varepsilon$. Since banks $n - b + 1$ through $n - 1$ are bad, we have

$$x_0 = \max\{0, x_{n-b} - b(\phi - \pi)\} = \varepsilon$$

Therefore, $x_0 > 0$, a contradiction. Since $T^*(x)$ is weakly increasing in λ, then if $T^*(0) > 0$ for $\lambda = b(\phi - \pi) + \varepsilon$, then $T^*(0) > 0$ for any $\lambda' > b(\phi - \pi) + \varepsilon$. ∎

Let $T_j(x; S)$ denote the operator T_j for a particular state of the network S. Likewise, let $T^*(x; S)$ denote the composition of $T_j(x; S)$ for $j = 1, ..., n$ for a particular S. The proof of Result 1 involves showing that for $\lambda > b(\phi - \pi)$, $T^*(0; \hat{S}) > 0$ whenever $\lambda > b(\phi - \pi)$. The next result establishes that as long as $\lambda > b(\phi - \pi)$, then for any vector S that corresponds to the possible location of the b bad banks, $T^*(0; S) > 0$. From this, it follows that as long as $\lambda > b(\phi - \pi)$, the fixed point x_j that solves (4) is positive for all x_j for all S.

Result 2: $T^{*}(0; S) \geq T^{*}(0; \hat{S})$ for all S and all x, including $x = 0$.

Proof: Observe that starting from $\hat{S}$, we can reach any state $S \neq \hat{S}$ with a finite number of steps where each step involves swapping a pair of adjacent banks, one good bank with a lower index and one bad bank with a higher index, so that after swapping them the bad bank has the lower index and the good bank has the higher index. Formally, there exists a sequence of vectors $(S^0, S^1, ..., S^Q)$ where $Q < \infty$ such that $S^0 = \hat{S}$, $S^Q = S$, and for each q

$$S_i^{q+1} = \begin{cases} S_i^q & \text{if } i \notin \{j_q - 1, j_q\} \\ 1 - S_i^q & \text{if } i \in \{j_q - 1, j_q\} \end{cases}$$

for some j_q where $S_{j_q - 1}^q = 1$. Intuitively, we can achieve any desired spacing between the bad banks by first moving bank 0 clockwise, then moving bank $n - 1$, and so on, until finally we move bank $n - b + 2$.

For each q and an initial x_0, define x_j^q as the payment bank j makes if bank 0 pays x_0 to bank 1 and the state of the network is S^q. We can likewise define x_j^{q+1} when the state of the network is S^{q+1}. Formally,

$$\begin{aligned} x_j^q &= T_j \circ \cdots \circ T_1(x_0; S^q) \\ x_j^{q+1} &= T_j \circ \cdots \circ T_1(x_0; S^{q+1}) \end{aligned}$$

By construction, $S_j^q = S_j^{q+1}$ for $j \leq j_q - 2$, which implies $x_{j_q - 2}^q = x_{j_q - 2}^{q+1}$.

Let $G(\xi)$ denote the payment a good bank will make if it receives a payment ξ from its neighboring bank, and let $B(\xi)$ denote the payment a bad bank will make. Then

$$\begin{aligned} G(\xi) &\equiv \max\{0, \min\{\lambda, \xi + \pi\}\} \\ B(\xi) &\equiv \max\{0, \min\{\lambda, \xi + \pi - \phi\}\} \end{aligned}$$

By definition

$$B(\xi) = G(\xi - \phi) \tag{23}$$

Note that $G(\xi)$ is weakly increasing in ξ with slope bounded above by 1. We can now characterize the payment made by bank j_q when $S = S^q$ and $S = S^{q+1}$ using $G(\cdot)$ and $B(\cdot)$ as follows

$$\begin{aligned} x_{j_q}^q &= B\left(G\left(x_{j_q - 2}^q\right)\right) \\ x_{j_q}^{q+1} &= G\left(B\left(x_{j_q - 2}^{q+1}\right)\right) \end{aligned}$$

For any real number ξ, (23) implies

$$\begin{aligned} G(B(\xi)) &= G(G(\xi - \phi)) \\ B(G(\xi)) &= G(G(\xi) - \phi) \end{aligned}$$

Since $G(\cdot)$ has a slope bounded above by 1, then since $\phi > 0$,

$$G(\xi - \phi) \geq G(\xi) - \phi$$

Applying $G(\cdot)$ to both sides and using the fact that $G(\cdot)$ is monotone yields

$$G\left(G\left(\xi - \phi\right)\right) \geq G\left(G\left(\xi\right) - \phi\right)$$

or alternatively

$$G\left(B\left(\xi\right)\right) \geq B\left(G\left(\xi\right)\right)$$

Setting $\xi = x^q_{j_q-2} = x^{q+1}_{j_q-2}$, we have

$$\begin{aligned}
x^q_{j_q} &= B\left(G\left(x^q_{j_q-2}\right)\right) \\
&\leq G\left(B\left(x^q_{j_q-2}\right)\right) \\
&= G\left(B\left(x^{q+1}_{j_q-2}\right)\right) = x^{q+1}_{j_q}
\end{aligned}$$

In other words, the state of the network that minimizes the resources bank 0 has at its disposal is when bank 0 and the $b-1$ banks that come before it are bad.∎

Result 2 implies that for any S, a bank that pays nothing will be left with positive resources with which it can pay. This contradiction proves that if $\lambda > b(\phi - \pi)$, the fixed point of (4) must be strictly positive in all its terms.

Finally, we show that if $\lambda \leq b(\phi - \pi)$, there exists a state a fixed point with $x_j = 0$ for at least one j whenever $S = \hat{S}$.

Result 3: If $\lambda \leq b(\phi - \pi)$, then $x_j = 0$ for some j when $S = \hat{S}$.

Proof of Result 3: The proof is by construction. Suppose $S = \hat{S}$, and consider $x_0 = 0$. Then $x_j = \min\{j\pi, \lambda\}$ for all $j \in \{1, ..., n - b\}$. Since $n\pi > b\phi$, subtracting $b\pi$ from both sides yields

$$(n - b)\pi > b(\phi - \pi)$$

Hence, $x_{n-b} = \lambda \leq b(\phi - \pi)$. Since the next b banks are bad, it follows that

$$x_0 = \min\{0, x_{n-b} - b(\phi - \pi)\} = 0$$

This confirms $x_0 = 0$ is a fixed point of (4). ∎

Proof of Proposition 5: From Proposition 4, we know that $x_j > 0$ for all $j \in \{0, ..., n - 1\}$. Hence,

$$x_j = \min\{\lambda, x_{j-1} + \pi - \Phi_j\}$$

Equity is then given by

$$e_j = \max\{0, x_{j-1} + \pi - \Phi_j - x_j\}$$

We consider each of the two cases for x_j. If $x_j = x_{j-1} + \pi - \Phi_j$, then

$$e_j = x_{j-1} + \pi - \Phi_j - x_j = 0$$

If instead $x_j = \lambda$, then $x_{j-1} + \pi - \Phi_j \geq \lambda$ and so

$$e_j = \max\{0, x_{j-1} + \pi - \Phi_j - \lambda\} = x_{j-1} + \pi - \Phi_j - \lambda$$

Either way, we have

$$e_j = x_{j-1} + \pi - \Phi_j - x_j$$

Summing up the equity values across banks yields

$$\sum_{j=1}^{n} e_j = n\pi - b\phi$$

Hence, the sum of equity values is the same, regardless of S. Assumption A2 implies $e_j \in \{0, \pi\}$. But this implies the cardinality of the set $\{j : e_j = 0\}$ is the same for all S. Let $\zeta \equiv \#\{j : e_j = 0\}$. Then we have

$$\sum_{j=1}^{n} e_j = (n - \zeta)\pi = n\pi - b\phi$$

Since $\lambda > b(\phi - \pi)$, then $\min\{\phi - \pi, \lambda\} = \phi - \pi$. From this, it follows that

$$k \equiv \frac{\min\{\phi - \pi, \lambda\}}{\pi} = \frac{\phi - \pi}{\pi}$$

and so $\phi = (k + 1)\pi$. Hence,

$$(n - \zeta)\pi = n\pi - b(k + 1)\pi$$

which gives

$$\zeta = b(k + 1)$$

as claimed. ∎

Proof of the Proposition 6: For $0 < \lambda < \phi - \pi$, Lemma 1 implies

$$
\begin{aligned}
p_g &= \frac{n - E[\zeta]}{n - b} \\
&= \frac{(n - b)!\,(n - k - 1)!}{(n - b)(n - 1)!\,(n - b - k - 1)!} \\
&= \prod_{i=1}^{k}\left(\frac{n - b - i}{n - i}\right)
\end{aligned}
$$

Since $k = \frac{\min\{\phi - \pi, \lambda\}}{\pi} = \frac{\lambda}{\pi}$, we can rewrite p_g in this case as

$$p_g = \prod_{i=1}^{\lambda/\pi}\left(\frac{n - b - i}{n - i}\right)$$

For $\lambda > b(\phi - \pi)$, Proposition 5 implies $\zeta = bk + b$ with probability 1. Hence,

$$
\begin{aligned}
p_g &= \frac{n - bk - b}{n - b} \\
&= 1 - \frac{bk}{n - b}
\end{aligned}
$$

Since $b \geq 1$, from (6), $\lambda > b(\phi - \pi)$ implies $\lambda > \phi - \pi$, and so $k = \frac{\phi}{\pi} - 1$, and so

$$
p_g = 1 - \frac{b}{n - b}\left(\frac{\phi}{\pi} - 1\right)
$$

Finally, our claim for the case of $\phi - \pi \leq \lambda \leq b(\phi - \pi)$ follows from Proposition 13. $\blacksquare$

Proof of Proposition 7: Our proof is by construction. We know from Proposition 3 that there exists at least one bank for which $\widehat{e}_j > 0$. Start with this bank and move to bank $j + 1$, continuing on until reaching the first bad bank. Without loss of generality, we can refer to this as bank 1. Moreover, we know that $\widehat{x}_0 = \lambda$, i.e. if outsiders did not invest in any of the banks, then bank 0 would be able to pay its obligation to bank 1 in full.

First, we argue that $x_0 = \lambda$, i.e. when banks can raise outside funds, it will still be the case that bank 0 will be able to pay its debt obligation to bank 1 in full. To see this, define

$$
\begin{aligned}
T_j(x) &= \max\{0, \min\{x + \pi + R(1 - D_j)I_j - \Phi_j, \lambda\}\} \\
&\geq \max\{0, \min\{x + \pi - \Phi_j, \lambda\}\} \equiv \widehat{T}_j(x)
\end{aligned}
$$

As before, the payment x_0 must solve the fixed point

$$
x_0 = T^*(x_0) = T_n \circ \cdots \circ T_1(x_0) \tag{24}
$$

Since $T^*(x_0) \geq \widehat{T}^*(x_0)$, then we know that

$$
T^*(\lambda) \geq \widehat{T}^*(\lambda) = \lambda
$$

But $T^*(x) \leq \lambda$ for all x. Hence, $T^*(\lambda) = \lambda$, and so $x_0 = \lambda$ is a fixed point of (24).

Now, suppose bank 1 was able to raise funding, i.e. $I_1 = 1$. Let r_1 denote the rate bank 1 is charged. If bank 1 diverted the funds it obtained, its expected payoff would be v. If it invested the funds, it would get to keep

$$
\max\{\lambda + \pi + R - y_1 - x_1 - w_1, 0\}
$$

where

$$
\begin{aligned}
y_1 &= \min\{\phi, \lambda + \pi + R\} \\
x_1 &= \min\{\lambda, \lambda + \pi + R - y_1\} \\
w_1 &= \min\{r_1^*, \lambda + \pi + R - y_1 - x_1\}
\end{aligned}
$$

If $y_1 = \lambda + \pi + (R - r_1)$, bank 1 would get to keep 0, which is less than v. If $y_1 = \phi$, bank 1 would get to keep

$$\max\{\lambda + \pi + (R - r_1) - \phi - x_1, 0\}$$

which is 0 if $x_1 = \lambda + \pi + R - y_1$ and $\pi + (R - r_1) - \phi$ if $x_1 = \lambda$. Since $\phi > \pi$ under Assumption A1, this is less than $R - r_1$. Moreover, since $r_1 \geq \underline{r}$ in any equilibrium, $R - r_1 \leq R - \underline{r} < v$, where the last inequality follows from Assumption A3. Thus, bank 1 will not be able to raise outside funds, i.e. $I_1 = 0$. From this we can conclude that $e_1 = 0$, since bank 1's resources $\lambda + \pi - \phi$ are less than its obligation of λ to bank 2.

We now proceed by induction. Suppose $e_1 = \cdots = e_{j-1} = 0$ and $I_1 = \cdots = I_{j-1} = 0$. Assumption A2 implies $\widehat{e}_j$ is equal to either 0 or π. We consider each case in turn.

Suppose first that $\widehat{e}_j = 0$. We argue that $I_j = 0$, i.e. if bank j would have zero equity in the absence of investment, then bank j would be unable to raise funds when investment is allowed. For suppose not. Given $x_0 = \widehat{x}_0 = \lambda$ and $I_1 = \cdots = I_{j-1} = 0$, it follows that

$$x_{j-1} = \widehat{x}_{j-1}$$

Since $\widehat{e}_j = 0$, we know that under Assumptions A1 and A2, either $\widehat{x}_{j-1} = \lambda$ or else $\widehat{x}_{j-1} \leq \lambda - \pi$. In the first case, we can apply the same argument we used to establish $I_1 = 0$ to show that $I_j = 1$. In the second case, suppose bank j were able to raise funds. Then if bank j diverts the funds it obtains, its payoff would be v. In particular,

$$
\begin{aligned}
y_j &= \min\{\Phi_j, x_{j-1} + \pi\} = \widehat{y}_j \\
x_j &= \max\{0, \min\{x_{j-1} + \pi - y_j, \lambda\}\} = \widehat{x}_j
\end{aligned}
$$

and since

$$\widehat{e}_j = \max\{0, \widehat{x}_{j-1} + \pi - \widehat{y}_j - \widehat{x}_j\} = 0$$

then even before paying back outside investors w_j, the bank would have no resources left. By contrast, if the bank did not divert, then since $\widehat{x}_{j-1} \leq \lambda - \pi$, its payoff will be at most $R - r_j \leq R - \underline{r} < v$, where $r_j \geq \underline{r}$ is the rate bank j will be charged by outside investors. Hence, $I_j = 0$ as claimed. Since $I_j = 0$ implies $x_j = \widehat{x}_j$, it follows that $e_j = \widehat{e}_j = 0$.

Next, suppose $\widehat{e}_j = \pi$. Note that this implies $S_j = 0$, i.e. j must be a good bank. We want to show that $I_j = 1$ and $x_j = \lambda$. That is, if bank j would have full equity in the absence of investment, then bank j would raise funds when investment is allowed. To see this, observe that $\widehat{e}_j = \pi$ implies $x_{j-1} = \widehat{x}_{j-1} = \lambda$. Hence, we have

$$
\begin{aligned}
y_j &= \min\{\Phi_j, \lambda + \pi\} = 0 \\
x_j &= \max\{0, \min\{\lambda + \pi + R(1 - D_j)I_j, \lambda\}\} = \lambda
\end{aligned}
$$

If the bank obtained funds from outside investors, i.e. $I_j = 1$, and did not divert funds, it would earn $\pi + R - r_j$. If it chose to divert funds, it would receive $v + \min\{\pi - r_j, 0\}$. At $r_j = \underline{r}$, Assumption A3 ensures that the bank would prefer to invest than to divert the funds. Since outsiders can observe the state of each bank, it follows that the unique equilibrium is one where $r_j = \underline{r}$ and $I_j = 1$.

So far, we have established that starting from bank 1, continuing through all the consecutive banks for which $\widehat{e}_j = 0$ implies $I_j = 0$. Once we reach the first bank for which $\widehat{e}_j = \pi$, we know that $x_j = \lambda$, and we can keep going until we reach the next bad bank. Since this bank receives λ, the analysis would be the same as for bank 1. The claim then follows. ∎

Proof of Lemma 2: If bank j has positive equity in equilibrium, it must be that $x_{j-1} = \lambda$, i.e. bank j is paid in full. This is because Assumptions A1 and A2 imply that if $x_{j-1} < \lambda$, then $\widehat{e}_j = 0$, i.e. such a bank would have no equity prior to raising any funds from outside investors. But we know from Assumption A3 that such a bank would divert funds, i.e. $D_j = 1$, and so such a bank would have no equity. Given this, a bank that receives outside funding would choose to invest the funds it raises rather than divert them iff

$$v + \max\left\{\pi - r_j^*, 0\right\} < \pi + R - r_j^* \qquad (25)$$

Suppose $r_j^* < \pi$. In this case, $\max\left\{\pi - r_j^*, 0\right\} = \pi - r_j^*$. But then Assumption A3 tells us that (25) must hold, since it reduces to $v < R$. Next, suppose $r_j^* \geq \pi$. In this case, $\max\left\{\pi - r_j^*, 0\right\} = 0$. In that case, (25) only holds if $\pi \leq r_j^* \leq \pi + R - v$. Since $v < R$, this bound exceeds π. It follows that $D_j = 0$ if and only $r_j^* \leq \pi + R - v$. ∎

Proof of Lemma 3: From Lemma 2, the only scenario we have to explore is whether there exists an equilibrium with $r_j > \overline{r}$ in which a bank with positive equity chooses to divert, i.e. $D_j = 1$. Let p_j denote the probability that bank j has positive equity in equilibrium. Then the expected payoff to bank j is given by $p_j\left(r_j^*(1 - D_j) + \min\left\{\pi, r_j^*\right\} D_j\right)$. When $D_j = 1$, this payoff collapses to $= p_j\pi$. But suppose an outside investor were to charge $r_j = \pi + \varepsilon$ where ε is sufficiently small so ensure that $r_j < \overline{r}$. In that case, the bank would be strictly better off since it is charged a lower rate. Moreover, since $\pi + \varepsilon < \overline{r}$, the bank will invest and pay $r_j = \pi + \varepsilon$ in full, so the investor that charges this amount will be better off. But then the original outcome with $r_j^* > \overline{r}$ could not have been an equilibrium. ∎

Proof of Proposition 10: First, suppose $v \geq \underline{r}$. Then for any $p_g \in (0, 1)$, we have

$$
\begin{aligned}
(R - v)\, p_g + (v - \underline{r}) &= p_g\, (R - \underline{r}) + (1 - p_g)\, (v - \underline{r}) \\
&\geq p_g\, (R - \underline{r}) \\
&> p_g \tfrac{n-b}{n}\, (R - \underline{r})
\end{aligned}
$$

Mandatory disclosure is preferable to no investment if

$$c < (R - \underline{r}) \tfrac{n-b}{n} p_g$$

But from above it follows that

$$c < (R - v) p_g + (v - \underline{r})$$

Since $p_g > \underline{r}/\overline{r}$ implies a good bank that unilaterally discloses will be able to raise funds, while the above inequality implies the benefits from attracting funds exceed the disclosure

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cost, it follows that non-disclosure cannot be an equilibrium whenever mandatory disclosure is preferable to no investment.

Next, suppose $v < \underline{r}$. For any $p_g > \underline{r}/\bar{r}$, a non-disclosure equilibrium with no investment will exist if

$$p_g \leq \frac{n}{n-b} \underline{r}/\bar{r} \text{ and } c \geq (R-v)p_g + (v-\underline{r})$$

and mandatory disclosure will be preferable to no investment if

$$c \leq p_g \frac{n-b}{n} (R - \underline{r})$$

The only way both inequalities involving c can be satisfied is if

$$(R-v)p_g + (v-\underline{r}) \leq p_g \frac{n-b}{n} (R - \underline{r})$$

Rearranging, improvability on a non-disclosure equilibrium with no investment is possible only if

$$p_g \leq \frac{\underline{r} - v}{(R - v) - \frac{n-b}{n} (R - \underline{r})}$$

For this bound to exceed $\underline{r}/\bar{r}$ requires

$$\frac{(\underline{r} - v)}{(R - v) - \left(1 - \frac{b}{n}\right)(R - \underline{r})} > \underline{r}/\bar{r}$$

which, rearranging, implies

$$\frac{b}{n} < \left(\frac{\bar{r}}{\underline{r}} - 1\right) \frac{\underline{r} - v}{R - \underline{r}}$$

Finally, from A3,

$$\frac{\underline{r} - v}{(R - v) - \left(1 - \frac{b}{n}\right)(R - \underline{r})} = \frac{\underline{r} - v}{(\underline{r} - v) + \frac{b}{n}(R - \underline{r})}$$
$$< 1$$

which completes the proof. ∎

Proof of Proposition 11: First, suppose $v \geq \underline{r}$. The expected amount banks pay to investors is $\underline{r}$ both when there is no disclosure and when there is mandatory disclosure. For a good bank, then, the expected payoff under the non-disclosure equilibrium with investment is $p_g R + (1 - p_g) v - \underline{r}$. Under mandatory disclosure, the expected payoff for a good bank is $p_g (R - \underline{r})$, which is strictly lower. This confirms some party will be made worse off with mandatory disclosure, so mandatory disclosure cannot be Pareto improving.

Next, suppose $v < \underline{r}$. A non-disclosure equilibrium with investment can only exist if $c > \frac{b\underline{r}}{n-b}$. At the same time, mandatory disclosure will be Pareto improving relative to an equilibrium where outsiders invest in all banks only if $c < \left(1 - \frac{n-b}{n}p_g\right)(\underline{r} - v)$. For mandatory disclosure to be Pareto improving and for there to exist a non-disclosure equilibrium with

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investment, we need

$$\frac{b\underline{r}}{n-b} < \left(1 - \frac{n-b}{n}p_g\right)(\underline{r}-v)$$

or, rearranging, if

$$p_g \leq \frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right)$$

If this inequality is violated at $p_g = \frac{n}{n-b}\frac{\underline{r}}{\underline{r}}$, then it will be violated for all $p_g \geq \frac{n}{n-b}(\underline{r}/\bar{r})$. Hence, a necessary condition for the existence of a Pareto-improvable non-disclosure equilibrium is for

$$\frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right) \geq \frac{n}{n-b}\underline{r}/\bar{r}$$

Rearranging, we have the condition

$$\frac{b}{n} \leq \frac{\underline{r}-v}{(\underline{r}-v)(1-\underline{r}/\bar{r})+\underline{r}}\left(1 - \frac{\underline{r}}{\bar{r}}\right)$$

Hence, without this condition, there exists no Pareto-improvable non-disclosure equilibrium with investment. With this condition, the interval $\left[\frac{n}{n-b}\underline{r}/\bar{r}, \frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right)\right]$ will be non-empty. For any p_g in this interval, and so the as long as $c \in \left[\frac{b}{n-b}\underline{r}, (1 - \frac{n-b}{n}p_g)(\underline{r}-v)\underline{r}/\bar{r}\right]$, which is necessarily non-empty given the restriction on $\frac{b}{n}$, a non-disclosure equilibrium with investment is Pareto-improvable. Finally, observe that since $v < \underline{r}$, then

$$\frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right) < \frac{n}{n-b}\left(1 - \frac{b}{n-b}\right)$$

But then we have

$$\frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right) < \frac{n}{n-b}\left(\frac{n-2b}{n-b}\right)$$

$$= \frac{n^2 - 2nb}{n^2 - 2nb + b^2} < 1$$

Proof of Proposition 12: Suppose all good banks disclose. To confirm this is an equilibrium, we verify that no good bank wants to deviate. If a good bank discloses, its expected earnings rise by $p_g(R-\underline{r})-c$. Given $(n-b)p_g(R-\underline{r})-cn > 0$, this expected payoff is strictly positive. Next, suppose a good bank deviates and opts not to disclose. Under our refinement, the probability outsiders assign to this bank being good is $\frac{1}{b+1}$. The most optimistic scenario for this bank is that all other bad banks are sufficiently far away that outsiders believe this bank will have positive equity if it is good. In that case, it will be able to attract funding only if $\frac{1}{b+1}\bar{r} > \underline{r}$. If this condition is violated, i.e. if $b > \frac{\bar{r}}{\underline{r}} - 1$, then a bank that fails to disclose will be unable to raise funds in any state of the world, and so its payoff from not disclosing is 0. It follows that all good banks disclosing is an equilibrium. ∎

Proof of Lemma 4. We want to show that for any pair j and k, the distribution of equity e_j for bank j conditional on bank j being good ($S_j = 0$) is the same as the distribution

of equity e_k for bank k conditional on k being good ($S_k = 0$).

Once again, let Ω denote the set of all realizations for S, i.e.

$$\Omega = \left\{ x \in \{0,1\}^n : \sum_{j=1}^n x_j = b \right\}$$

Note that for any two realizations s and s' in Ω, $\Pr(S = s) = \Pr(S = s')$. Suppose we show that there exists a bijective mapping $\varphi : \Omega \to \Omega$ such that (i) $s_j = \varphi_k(s)$, and (ii) $e_j(s) = e_k(\varphi(s))$ for all $s \in \Omega$, i.e. the state and equity of bank j when $S = s$ is the same as the state and equity of bank k when $S = \varphi(s)$. Since all states have the same probability, it follows that $Pr(e_j = x|S_j = 0) = Pr(e_k = x|S_k = 0)$.

Heuristically, we can establish the existence of φ as follows. Suppose we place each bank j at the physical location j. We then construct a directed network across physical locations. Given a vector $s \in \Omega$ that implies which are the b bad banks, we can compute the equity of bank j when $S = s$.

Symmetry implies that for any bank k, we can rearrange banks across locations so that bank k lies in location j and the directed network across physical locations remains unchanged. Suppose we leave the shocks at the same physical locations implied by s. Since we have rearranged banks across locations, this implies the identity of the b bad banks is now generally different. In particular, the state of each bank will be given by $\varphi(s) = (s_{\sigma_{kj}}(0), ..., s_{\sigma_{kj}}(n-1))$ for σ_{kj} as defined in the text.

By construction, $s_k = 1$ when $S = \varphi(s)$ iff $s_j = 1$ when $S = s$. Moreover, by construction, payments across physical locations are the same given payments depend only on flows across locations. This ensures that for any bank i, the equity e_i when $S = s$ is the same as the equity of bank $\sigma_{kj}(i)$ when $S = \varphi(s)$. In particular, the equity of bank k when $S = \varphi(s)$ is the same as the equity of bank j when $S = s$. This completes the proof. ∎

Proof of Proposition 13: We first define the shortfall D_{ij} given the state of the network S as the difference between what bank i owes bank j and what bank i actually pays bank j:

$$D_{ij} = \Lambda_{ij} - x_{ij} \text{ for all } i, j \in \{0, ..., n-1\} \tag{26}$$

We suppress the state of the network from the notation whenever seems clear. We can transform the operator in (20) defined over payments x_{ij} into an operator $F : \mathcal{D} \to \mathcal{D}$, where $\mathcal{D} \subset \mathbb{R}_+^n$ is the space of possible shortfalls given by

$$\mathcal{D} = \{D_{ij} \in [0 , \Lambda_{ij}] , i, j \in \{0, ..., n-1\}\}$$

This operator is defined by

$$(F)(D)_{ij} = \left(\frac{\Lambda_{ij}}{\Lambda_i} \right) \max \left\{ \min \left\{ \Lambda_i , \sum_{m \neq i} D_{mi} - \pi + S_i \phi \right\} , 0 \right\} \tag{27}$$

The set of fixed points of the shortfall operator corresponds to the set of fixed points of the operator defined over payments. Either of these can be used to derive equity, and hence the distribution of equity we wish to characterize.

Our proof now proceeds as follows. First, we show that for each S the shortfall $D(S)$ are weakly increasing in ϕ and in λ. Next we argue that this implies that the distribution of equity is stochastically decreasing with ϕ and in λ for each S. Then the result follows since the distribution of S is not a function of (ϕ, λ).

We use the notation $F_{\phi,\lambda}$ to emphasize the dependence of the operator on the parameters (ϕ, λ). It is easy to show that F is monotone, i.e. $F_{\phi,\lambda}(D') \geq F_{\phi,\lambda}(D)$ if $D' \geq D$, where the comparison is component by component. Thus, by Tarski's fixed point theorem, there exists a smallest fixed point, which is obtained as $D^*(\phi, \lambda) = \lim_{n \to \infty} F^n(0)$. Additionally, F is monotone on (ϕ, λ), i.e. for each $D \in \mathcal{D}$, $F_{\phi',\lambda'}(D) \geq F_{\phi,\lambda}(D)$, whenever $(\phi', \lambda') \geq (\phi, \lambda)$. Then it follows that the smallest fixed point $D^*(\phi, \lambda)$ is increasing in (ϕ, λ).

For any vector of shortfalls D, parameter (ϕ, λ) and state of the network S the implied equity of bank i is:

$$
\begin{aligned}
e_i(S) &= \max \left\{ 0, \ \pi - \phi S_i - \sum_{j=0}^{n-1} \Lambda_{ij} + \sum_{m=0}^{n-1} x_{mi}(S) \right\} \\
&= \max \left\{ 0, \ \pi - \phi S_i - \Lambda_i - \left(\sum_{m=0}^{n-1} D_{mi}(S) + \sum_{m=0}^{n-1} \Lambda_{mi} \right) \right\} \\
&= \max \left\{ 0, \pi - \phi S_i - \sum_{m=0}^{n-1} D_{mi}(S) \right\}
\end{aligned}
\tag{28}
$$

where the last equality follows by regularity of the network, i.e. that $\Lambda_i = \sum_m \Lambda_{mi}$.

Consider the equity corresponding to $D = D^*(\phi, \lambda)$. Equity at bank i is given by

$$
e_i(\phi, \lambda; S) = \max \left\{ 0, \pi - \phi S_i - \sum_{m=0}^{n-1} D^*_{mi}(\phi, \lambda; S) \right\}
\tag{29}
$$

where $D^*_{mi}(\phi, \lambda; S)$ is the amount bank m falls short on bank i for the smallest fixed point for the state S and parameters (ϕ, λ). Using the monotonicity of $D^*(\phi, \lambda)$ it is immediate that $e_i(\phi, \lambda; S)$ is weakly decreasing in (ϕ, λ) for each S. While we have used the smallest fixed point in the definition (29), by Theorem 1 in Eisenberg and Noe (2001) every fixed point of $F_{\phi,\lambda}$ has the same implied equity values for each bank. Hence, the comparative static of equity must be the same for any fixed point.

Finally, the conditional probability of interest is given by

$$
\Pr\{e_j \leq x \,|\, S_j = 0\} = \frac{\sum_{\{S' : S'_j = 0\}} \mathbb{I}\{e_j(\phi, \lambda; S') \leq x\} \Pr\{S'\}}{\sum_{\{S' : S'_j = 0\}} \Pr\{S'\}}
\tag{30}
$$

Since $\Pr\{S'\} = 1/\binom{n}{b}$ for all S', it follows that $\Pr\{e_j \leq x \,|\, S_j = 0\}$ is decreasing in (ϕ, λ). ∎

Proof of Theorem 2: Suppose a bank is able to raise funds from outsiders at a rate r. Once the bank learns its pre-investment equity is e_j, it knows it will earn $e_j + R - r$ if it invests, and $v + \max\{e_j - r, 0\}$ if it diverts. We begin by observing that a bank charged r

will prefer to invest if $e_j > e^*(r)$ and to divert funds if $e_j < e^*(r)$, where

$$e^*(r) = v - R + r \qquad (31)$$

Note that since $v < R$ from Assumption A3, $e^*(r) < r$. Now, suppose $e_j < e^*(r)$. Since $\max\{e_j - r, 0\} = 0$, the fact that $e_j < e^*(r)$ implies

$$e_j + R - r < e_j^* + R - r = v = v + \max\{e_j - r, 0\}$$

and so the bank would prefer to divert. Next, suppose $e^*(r) < e_j \le r$. In that case, $\max\{e_j - r, 0\} = 0$. In this case, $e_j > e^*(r)$ implies

$$e_j + R - r > e_j^* + R - r = v = v + \max\{e_j - r, 0\}$$

and so the bank will prefer to invest. Finally, suppose $e_j > r > e^*(r)$. In that case, $\max\{e_j - r, 0\} = e_j - r$. Since $v < R$ under Assumption A3, we have

$$\begin{aligned} R + e_j - r &> v + e_j - r \\ &= v + \max\{e_j - r, 0\} \end{aligned}$$

and so the bank will prefer to invest in this case as well.

Note that under Assumption A3, $0 < e^*(r) \le \pi$ for $r \in [\underline{r}, \overline{r}]$. In particular, the first inequality in (11) implies that for any $r \ge \underline{r}$,

$$e^*(r) = v + r - R \ge v + \underline{r} - R > 0$$

Since $r \ge \underline{r}$ in equilibrium, the inequality holds in equilibrium. In the other direction, the highest equilibrium rate charged to any bank is $\overline{r} = \pi + R - v$. For $r \le \overline{r}$ we have

$$e^*(r) = v + r - R \le v + \overline{r} - R = \pi$$

Given a network that is symmetrically vulnerable to contagion, $Pr(e_j = x | S_j = 0)$ is the same for all j for any value of x. Hence, we can define

$$p_g^*(r) = \Pr\left(e_j \ge e^*(r) \,|\, S_j = 0\right)$$

That is, $p_g^*(r)$ is the probability that if bank j is good, it will have equity of at least $e^*(r)$, or alternatively the probability that a good bank that raises funds and is charged r will be willing to invest the funds after it learns its equity.

We now derive the analog to Propositions 8-11 to determine when a non-disclosure equilibrium exists and whether mandatory disclosure can improve upon it. The role of $Pr(e_j = \pi | S_j = 0)$ is now replaced with $p_g^*(\tilde{r})$ where $\tilde{r} = \arg\max_r r p_g^*(r)$, i.e. the interest rate that maximizes the expected return to the lender.

First, if a non-disclosure equilibrium exists, we need to determine whether it will involve investment or not. Since $\phi > \pi$, bad banks would divert funds. Hence, outsiders only earn money from the $n - b$ good banks, and then only from those whose equity is high enough that they will choose not to divert the funds they raise. If the maximal expected amount

lenders expect to collect is below $\underline{r}$, a non-disclosure equilibrium must involve no investment. This condition is given by

$$\sup_{r\in[\underline{r},\overline{r}]} \frac{n-b}{n} r p_g^*(r) < \underline{r} \tag{32}$$

If (32) is reversed, then a non-disclosure equilibrium would involve investment; otherwise, a lender and bank could enter a trading relationship that would make both of them better off.

Next, we want to derive conditions for when non-disclosure is an equilibrium. Suppose no other bank disclosed. If a good bank were to deviate and announce it was good, outsiders would expect that if they charged this bank r, the probability they would be paid r is $p_g^*(r)$. Hence, no disclosure is an equilibrium for any $c \geq 0$ if charging the r that maximizes the outside lender's expected return will not yield an expected return to the lender of at least $\underline{r}$, or

$$\sup_{r\in[\underline{r},\overline{r}]} r p_g^*(r) < \underline{r} \tag{33}$$

When (33) is violated, a good bank could raise funds by disclosing it is good. In that case, a non-disclosure equilibrium exists if the cost of disclosure exceed the benefits. In particular, for values of $\sup_{r\in[\underline{r},\overline{r}]} r p_g^*(r)$ such that

$$\underline{r} < \sup_{r\in[\underline{r},\overline{r}]} r p_g^*(r) < \frac{n}{n-b}\underline{r} \tag{34}$$

non-disclosure can be an equilibrium only if

$$p_g^*(r^*)R + (1 - p_g^*(r^*))v - \underline{r} \leq c \tag{35}$$

where r^* is the equilibrium interest rate. The condition above makes use of the fact that in equilibrium, $r^* = \underline{r}/p_g^*(r^*)$. Finally, for

$$\sup_{r\in[\underline{r},\overline{r}]} r p_g^*(r) > \frac{n}{n-b}\underline{r} \tag{36}$$

the only possible non-disclosure is one where outsiders invest in all banks. Let r^* denote the equilibrium rate charged to banks. If a bank were to deviate and reveal itself, it could lower the rate it was charged from r^* to $\frac{n-b}{n}r^*$. Given the cutoff $e^*(r)$ below which a bank would divert is less than r, we know that a bank that diverted would have no equity left. The expected payoff in equilibrium is given by

$$p_g^*(r^*)(R - r^*) + (1 - p_g^*(r^*))v \tag{37}$$

while the expected payoff from deviating is given by

$$p_g^*\left(\frac{n-b}{n}r^*\right)\left(R - \frac{n-b}{n}r^*\right) + \left(1 - p_g^*\left(\frac{n-b}{n}r^*\right)\right)v \tag{38}$$

For non-disclosure to be an equilibrium, the difference between the second and the first expression must be less the cost of disclosure c.

To establish the theorem, define $e^* = e^*(\underline{r})$. Consider the limit as $p_g^*(\underline{r}) \to 0$. Since $p_g^*(r)$ is decreasing in r, it follows that

$$\sup_{r \in [\underline{r}, \overline{r}]} r p_g^*(r) < r p_g^*(\underline{r}) \tag{39}$$

Hence, in the limit, we have $\sup_{r \in [\underline{r}, \overline{r}]} r p_g^*(r) \to 0$ implying the only non-disclosure equilibrium is one where no investment takes place. Moreover, from (33), we know that non-disclosure will be an equilibrium for any $c \geq 0$.

Next, consider the limit as $p_g^*(\overline{r}) \to 1$, i.e. letting $Pr(e_j = \pi | S_j = 0)$ tend to 1. Since $p_g^*(r)$ is decreasing in r, then $p_g^*(r) \to 1$ for all $r \in [\underline{r}, \overline{r}]$, and the argument is identical to the one behind Theorem 1. ∎

B An Example of Unique Equilibrium Dominated by Mandatory Disclosure

In this Appendix, we construct an example where non-disclosure is the *unique* equilibrium but can still be improved upon by forcing banks to disclose. From Proposition 12, we know we need b to be small for this to be true. We therefore set $b = 1$, implying $p_g = \frac{n-1-k}{n-1}$.

We want to construct an example in which $\underline{r} > v$ so that diversion is socially wasteful. We first show that in our model as specified, this condition is incompatible with non-disclosure being the unique equilibrium and mandatory disclosure being preferable to this equilibrium. We then discuss how we can modify the model to allow for $\underline{r} > v$.

Consider first the restriction that mandatory disclosure must be preferable to a non-disclosure equilibrium. We focus on an equilibrium with no investment. Mandatory disclosure will be preferable to this equilibrium if

$$(n - k - 1)(R - \underline{r}) > cn \tag{40}$$

Next, consider the restriction to a unique equilibrium. A necessary condition for the uniqueness of the non-disclosure equilibrium is that all good banks disclosing cannot be an equilibrium. To see the implication of this restriction, suppose bank 0 is good and expects all other good banks to disclose. We need to ensure bank 0 will not want to disclose. If bank 0 discloses, it will receive funding iff it has positive equity, which occurs with probability p_g. Thus, its expected payoff is

$$p_g(R - \underline{r}) - c \tag{41}$$

If bank 0 instead chose not to disclose, it would either receive no funding or funding at different terms. If it received no funding, (40) implies bank 0 would be better off disclosing, an argument we also used in the proof of Proposition 12. Hence, for full disclosure to not be an equilibrium, a good bank must still receive funding if it fails to disclose. If bank 0 failed to disclose, outsiders would exactly two banks who failed to disclose: bank 0 and the one bad bank. Let j denote the location of the bad bank. If $j \in n - k, ..., n - 1$, then outsiders would refuse to invest in bank 0, since bank 0 would have zero equity regardless of which bank was bad. Otherwise, bank 0 could raise funds, but outsiders would assign probability

$\frac{1}{2}$ that bank 0 is bad and charge it $r_0^* = 2\underline{r}$ by outsiders. Bank 0's payoff would then be

$$p_g(R - 2\underline{r}) \tag{42}$$

Comparing (41) and (42) implies that a necessary condition for non-disclosure to be a unique equilibrium is for

$$c > p_g\underline{r} \tag{43}$$

Combining (40) and (43), no disclosure and no investment will be the unique equilibrium and yet dominated by mandatory disclosure only if

$$\underline{r} < \frac{c}{p_g} < R - \underline{r} \tag{44}$$

At the same time, if $v < \underline{r}$ and Assumption A3 hold, then

$$R - \underline{r} < v < \underline{r} \tag{45}$$

But (44) and (45) are in contradiction. Thus, our model as specified does not accommodate a unique no investment equilibrium dominated by full disclosure when $\underline{r} < v$.

We now modify the model to allow $\underline{r} < v$. Suppose that if a *good* bank does not disclose its state S_j, its good status might be revealed to outsiders with some probability, at no cost to the bank. That is, if bank j is good and chooses not to disclose, that fact that $S_j = 0$ might still be revealed with probability ρ, independently of what is revealed about other banks. A bank that contemplates disclosing thus knows that it might be able to communicate the same information without incurring the cost c. The model in the main paper is just a special case in which $\rho = 0$, so this modification generalizes our model. When $\rho > 0$, a good bank may be revealed to be good even it decides not to disclose. Thus, the failure of banks to disclose no longer implies outsiders will have no information. However, it might still be the case that banks inefficiently reduce the odds that their information becomes available.

Our example uses the following parameter values:

$$
\begin{array}{lll}
n = 5 & b = 1 & k = 3 \\
\underline{r} = 1.05 & R = 2 & v = 1 \\
\pi = 1.5 & \rho = .5 & c = 0.137
\end{array}
$$

Since $k = n - b - 1$, this parametrization ensures there will be exactly one good bank with positive equity, i.e. the bank furthest away from the bad bank along the chain of obligations. We now show that for these parameters, non-disclosure is the unique equilibrium, and yet mandatory disclosure improves welfare.

We first verify that no disclosure is an equilibrium. Suppose bank 0 is a good bank, and it expects all remaining good banks to not incur the cost of disclosure. We need to show bank 0 will prefer not to incur the cost of disclosure. For this, we need to determine the payoff to bank 0 from disclosing and not disclosing.

Even though all other good banks are assumed to choose not to disclose, since $\rho > 0$, their state may nevertheless become publicly known. Let m denote the number of good banks other than bank 0 whose state is known. Since one of the bank in $1, ..., n - 1$ is bad, then

m is an integer between 0 and $n - 2$. The probability that exactly m banks will have their state known is

$$\binom{n-2}{m}(1-\rho)^{n-m-2}\rho^m \tag{46}$$

Suppose $0 \le m < n - 3$. Whether bank 0's state is known or not, there will be at least 3 banks with unknown status. Since the one bank with positive equity is located next to the bad bank, there are at least three locations where the one bank with positive equity might be, each of which is equally likely. Since $\bar{r} = \pi + R - v = 2.5 < 3.15 = 3\underline{r}$, investors will not be able to trade with these banks and still cover their outside option. Bank 0 will thus gain nothing from disclosure but incur the cost c if m was within this range.

Next, suppose $m = n - 3$, i.e. there are exactly two other banks whose status is unknown, one good and one bad. Here, it matters whether bank 1 is one of the banks revealed to be good or not. If bank 1 is revealed to be good, then outsiders will not invest in bank 0 given its equity must be 0. If bank 1 is not revealed to be good, then if bank 0 discloses it is good, there will be exactly two banks with unknown status. Outsiders believe each is equally like to be the bad bank. Hence, they would assign probability $\frac{1}{2}$ to bank 0 having positive equity, and so would agree to lend to it at rate $2\underline{r} = 2.1 < 2.5 = \bar{r}$. If bank 0 accepted these terms, it would expect to gain $\frac{1}{2}(R - 2\underline{r}) + \frac{1}{2}v = 0.45 > 0$. Note that bank 0 would gain from borrowing only because it can gain from diverting the funds if it has zero equity; if its equity were π, it would incur a loss given $R > 2\underline{r}$, although it would still go ahead and invest to avoid losing its equity. By contrast, if bank 0 chose not to disclose, there would be 3 banks whose status is unknown, which we just argued implies no banks could raise funds. Given $m = n - 3$, for bank 1 to be revealed as good requires that it be good, which occurs with probability $\frac{n-2}{n-1}$, and that it is not the one good bank whose state isn't revealed, which occurs with probability $\frac{n-3}{n-2}$. Hence, given $m = n - 3$, bank 1 will be revealed as good with probability $\frac{n-3}{n-1}$ and will have its state uncertain with probability $\frac{2}{n-3}$. The expected gain to bank 0 from incurring disclosing is thus

$$\frac{2}{n-1}(1-\rho)\left[\frac{1}{2}(R - 2\underline{r}) + \frac{1}{2}v\right] - c = -0.0245$$

Hence, if bank 0 knew $n - 3$ good banks would be revealed, it would prefer not to disclose.

Finally, suppose $m = n - 2$, i.e. all good banks other than 0 are revealed. Again, it matters what is known about bank 1. If bank 1 is revealed to be good, which occurs with probability $\frac{n-2}{n-1}$, then bank 0 will be unable to raise funds regardless of whether its status is revealed. If bank 1 is not revealed to be good, it must be bad. In that case, bank 0 would be able to raise funds whether its state is known or not, although at different rates. If it chose not to disclose, bank 0 would be charged $2\underline{r}$. However, since bank 0 would know given all other good banks revealed that its equity is π, it would not want to borrow at this rate. If instead bank 0 disclosed, outsiders would know exactly which bank is bad, and bank 0 would be charged $\underline{r}$. The expected gain to bank 0 from incurring the disclosure cost is the gain from being able to borrow at a lower rate when it has positive equity, i.e

$$\frac{1}{n-1}(1-\rho)\left[R - 2\underline{r}\right] - c = -0.01825$$

We have thus shown that when all good banks other than 0 choose not to disclose, then regardless of how many banks will have their state known, bank 0 will be better off not incurring disclosure costs. Knowing only that m will be distributed according to (46), bank 0 will be better off not disclosing. All banks not disclosing is thus an equilibrium.

Next, we argue this equilibrium is dominated by mandatory disclosure. Forcing disclosure reveals the location of the one bank with positive equity and lets outsiders trade with it. This yields $(R - \underline{r}) - cn$ in terms of available net resources. As for the non-disclosure equilibrium above, let m_T denote the total number of banks revealed to be good, so $m_T \in \{0, ..., n - 1\}$. Analogously to (46), the probability that exactly m_T banks disclose is

$$\binom{n-1}{m_T}(1 - \rho)^{n-m_T-2}\rho^{m_T} \tag{47}$$

If $m_T < n - 2$, at least 3 banks fail to disclose, and so no bank will attract funding. The available net resources in this case are 0.

If $m_T = n - 2$, there are exactly 2 banks whose status is not known. Refer to them as bank j and k. There are two cases to consider. First, suppose $1 < |j - k| \pmod{n} < n - 1$. That is, the two banks whose status is revealed are not adjacent. Then the two banks that can have positive equity, $j - 1 \pmod{n}$ and $k - 1 \pmod{n}$, have no private information. In this case, they would both be willing to borrow at the rate $2\underline{r}$ outsiders would charge them. The expected net resources in this case are given by $(R - \underline{r}) + (v - \underline{r})$. If the two banks whose state is unknown are adjacent, then one of the banks that can have positive equity will have private information and one will not. The bank with private information will not receive any funding, since we established above that a bank with private information will only want to borrow if it knows it has zero equity, precisely when outsiders do not want to invest. Thus, only the bank without private information will receive funds. In this case, the expected amount of available net resources are $\frac{1}{2}(R - \underline{r}) + \frac{1}{2}(v - \underline{r})$. The probability that the two banks whose status is unknown are adjacent is $\frac{1}{n-2}$.

If $m_T = n - 1$, the location of all good banks will be revealed. In this case, the amount of available net resources is $(R - \underline{r})$

We can use the probabilities in (47) to compute the expected amount of available resources when no bank chooses to disclose. Subtracting this from the amount of available resources under forced disclosure yields

$$.6875(R - \underline{r}) + .25\left(\frac{n-2}{n-1}(\underline{r} - v) + \frac{1}{n-2}\left[\frac{1}{2}R + \frac{1}{2}v - \underline{r}\right]\right) - cn = 0.007187 \tag{48}$$

This confirms that forcing all firms to disclose is preferable to the non-disclosure equilibrium.

Our last step is to show that non-disclosure is the unique equilibrium. Suppose bank 0 is good. If it discloses its state, then with probability ρ its trade opportunities are unchanged since its state would have been revealed. With probability $1 - \rho$, it might be able to improve its trade opportunities. But this gain is at most $\frac{1}{n-1}(R - \underline{r})$, i.e. being able to raise funds at the lowest possible cost when it has positive equity. The reason is that this is the maximum social surplus created by trade. Since $v < \underline{r}$, the private gain to the bank will be less than the loss of the lender, and so if the lender is kept no worse off the gains of the bank must be

lower. We verified this numerically for our example. But recall we already established that $\frac{1}{n-1}(1-\rho)\left[R - 2\underline{r}\right] - c = -0.01825 < 0$. Hence, disclosure is a dominated strategy for each good bank, implying non-disclosure is the unique equilibrium.

C Examples of Symmetrically-Vulnerable-to-Contagion Networks

In this section, we provide some examples of networks that are symmetrically vulnerable to contagion to highlight the breadth of networks for which our results apply.

Example 1: A symmetric non-circulant network

Our first example demonstrates that the class of symmetric networks is larger than the class of circulant networks, i.e. networks in which we can order banks in such a way that Λ_{ij} is a function of $(i-j) \pmod n$, i.e. the distance between banks. Our example is a weighted directed cuboctahedral network. The financial obligations for this network are given by

$$
\Lambda = \begin{bmatrix}
0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda \\
0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\
\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\
0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda \\
\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0
\end{bmatrix}
$$

The implied network is shown graphically in Figure A1. The distinguishing feature of the cuboctahedral network is that each node belongs to exactly two triangular groups, as evident in Figure A1. None of the circulant networks with 12 nodes possess this feature. Essentially, the obligations Λ_{ij} depend not only on distance but also on whether bank i is even or odd.

**Example 2:** An asymmetric network that is symmetrically vulnerable to contagion

To show that symmetry is not necessary to satisfy symmetric vulnerability to contagion, we construct an example that uses a 4-regular asymmetric undirected graph, i.e. a graph where each node has exactly four vertices (4-regular) and whose automorphism group size is 1 (asymmetric). Gewirtz, Hill, and Quintas (1969) establish that the smallest such network involves 10 nodes. Starting with such a network, which we obtain using the algorithm by Meringer (1999) to compute automorphism group size, we impose equal directional flows of λ so that each node receives 2λ and pays 2λ. The asymmetry of the undirected graph must

carry over to the direct graph. The financial obligations for this network are given by

$$\Lambda = \begin{bmatrix} 0 & \lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & \lambda \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & \lambda \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \end{bmatrix}$$

The implied network is shown graphically in Figure A2.

Consider the case where $b = 1$, $\pi < \phi < 3\pi$, and $\lambda > \phi - \pi$. Although the network is asymmetric, we can easily confirm that e_j can only assume 3 values for each j: 0, $\pi - \frac{\phi - \pi}{2}$, and π with probabilities $\frac{1}{10}$, $\frac{2}{10}$, and $\frac{7}{10}$, respectively.
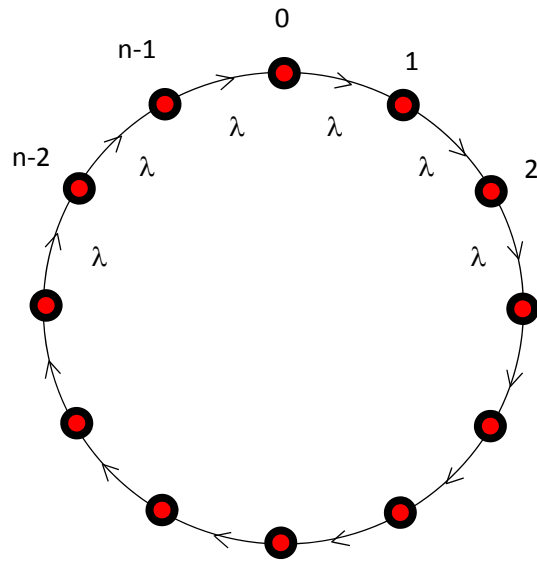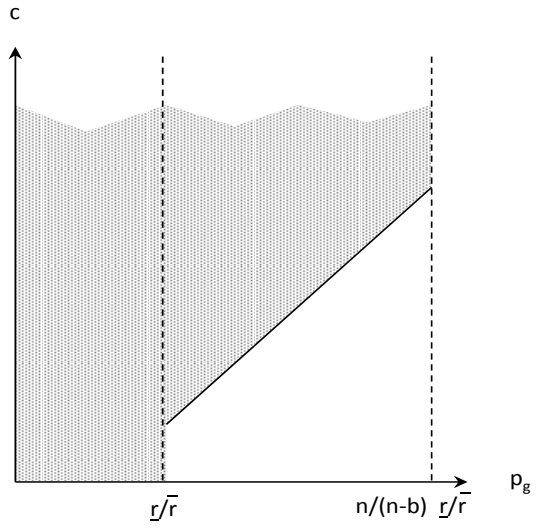
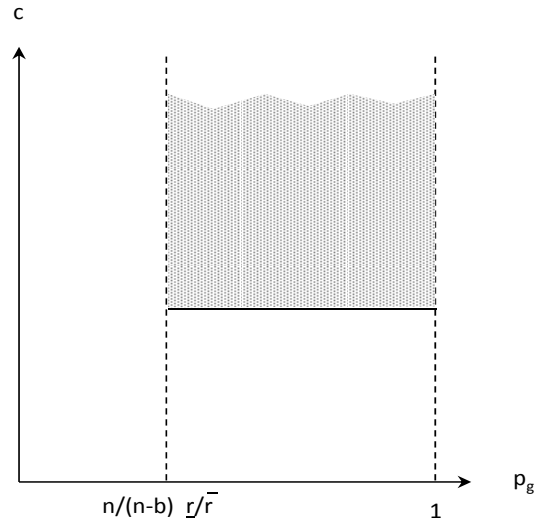**Figure 1: A Circular Network**
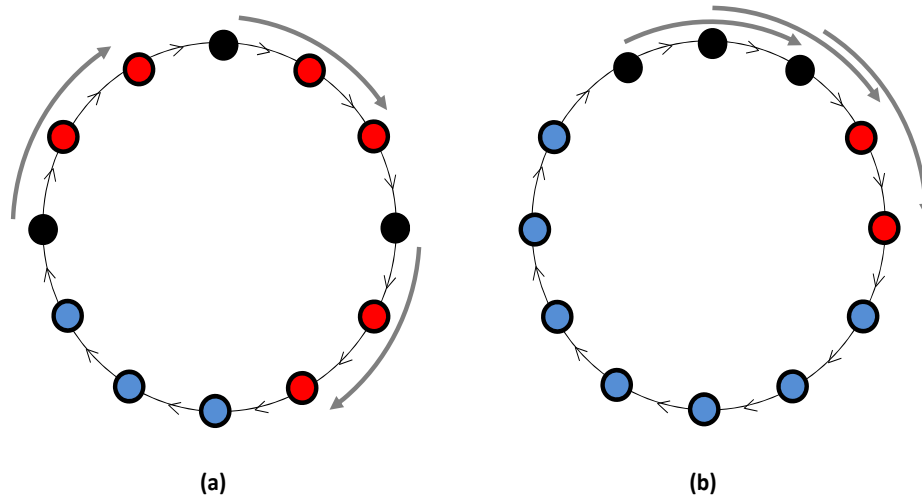
Figure 2: Aggregate Bank Equity Depends on Location of Bad Banks

The figure shows how the location of bad banks can lead to different aggregate bank equity. In the figure, the nodes colored black correspond to bad banks, the nodes colored red are good banks with zero equity, and the nodes colored blue are good banks with equity equal to π. Each circle shows a different realization for the location of the bad banks when n = 12, b = 3, k = 2, and assuming that $\lambda < \phi - \pi$. As seen in the figure, aggregate equity in the bank network is higher when the bad banks are concentrated together, as in part (b), than when bad banks are spaced apart, as in part (a).
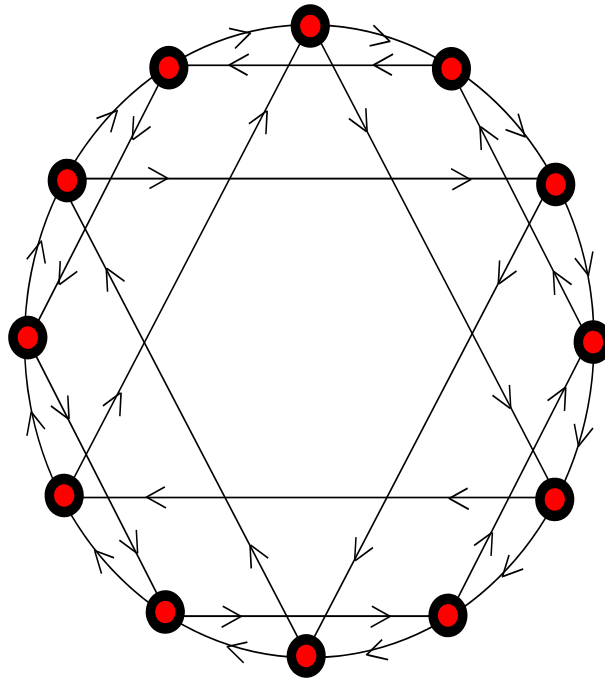
a) $p_g < n/(n-b) \underline{r} / \bar{r}$

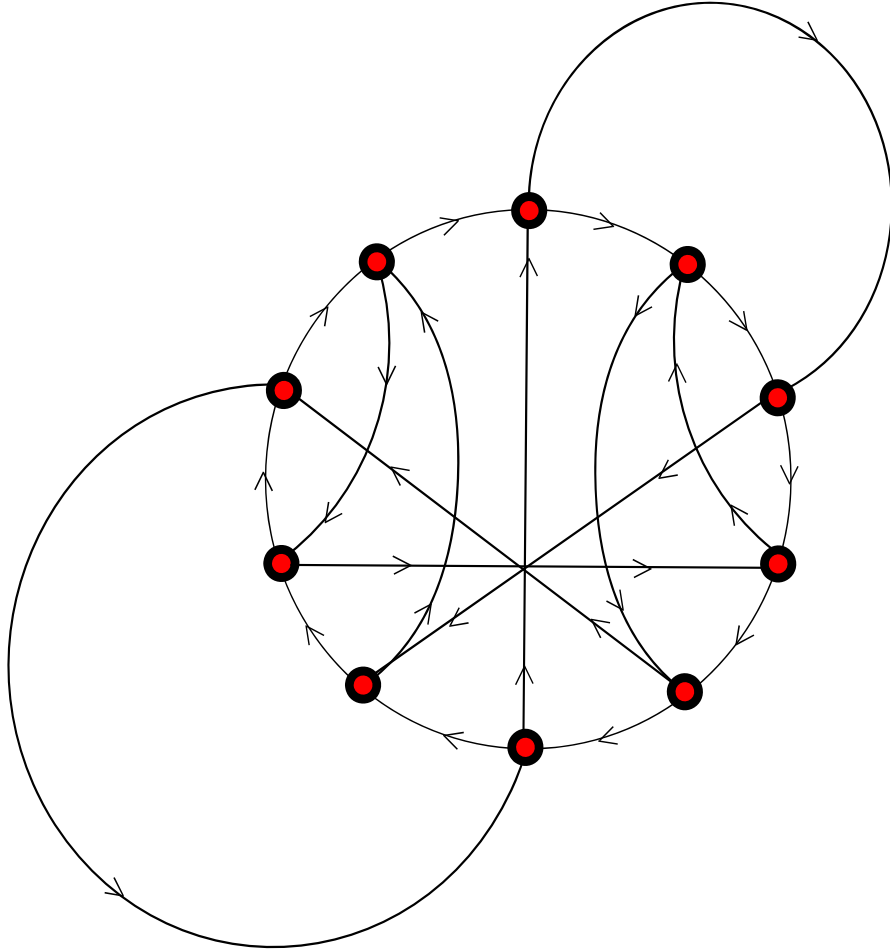b) $p_g > n/(n-b) \underline{r} / \bar{r}$

Figure 3: Region where a non-disclosure equilibrium exists

Figure A1: A Directed Cuboctahedral Network

An example of a symmetric network that cannot be represented as a circulant network

Figure A2: An asymmetric network that can satisfy Symmetric Vulnerability to Contagion