Bank Capital, Liquid Reserves, and Insolvency Risk*

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Abstract

We develop a dynamic model to assess the effects of liquidity and leverage requirements on banks' insolvency risk. The model features endogenous capital structure, liquid asset holdings, payout, and default decisions. In the model, banks face taxation, flotation costs of securities, and default costs. They are financed with equity, insured deposits, and risky debt. Using the model, we show that mispriced deposit insurance fuels default risk while depositor preference in default decreases it; liquidity requirements have no long-run effects on default risk but may increase it in the short-run; leverage requirements reduce default risk but may significantly reduce bank value.

Keywords: banks; liquidity buffers; capital structure; insolvency risk; regulation

JEL Classification Numbers: G21, G28, G32, G33.

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Banks can impose major risks on the economy. Avoidance of these risks and the associated social costs is the overwhelming concern of prudential regulation. Given the experience in the financial crisis of 2007-2008, in which insufficient liquidity buffers and excessive debt levels made the financial system unable to withstand large negative shocks, the debate on banking regulation has recently evolved around two main ideas. First, equity (or leverage) requirements should be significantly increased, so that if the value of the banks' assets were to decline this would not automatically lead to distress and the resulting losses would be borne by the bank owners (see e.g. Admati and Hellwig (2013)). Second, because most banks assets are illiquid and raising fresh equity is costly, banks should hold a buffer of liquid reserves to be able to cope with short-term losses (see e.g. Gorton (2012)).

While many insightful discussions of leverage and liquidity requirements are available in the literature, financial theory has made little headway in developing models that provide quantitative guidance for bank capital structure decisions or for the effects of regulatory requirements on bank behavior and insolvency risk. Our objective in this paper is therefore twofold. First, we seek to develop a dynamic model of banks' choices of liquid asset holdings, financing, payout, and default policies in the presence of realistic market frictions. Second, we want to use this model to characterize the endogenous response of banks to the imposition of liquidity and leverage requirements and to measure the effects of such requirements on banks' insolvency risk.

We begin our analysis by formulating a dynamic structural model in which banks face taxation, issuance costs of securities, and default costs and may be constrained by a regulator to hold a minimum amount of liquid reserves and/or equity capital. In the model, banks are financed with equity, insured deposits, and risky debt. They hold risky, illiquid assets (e.g. loan portfolios) whose cash flows are subject to both small and frequent shocks as well as large and infrequent negative jumps capturing tail risk. They also have the option to invest in risk-free, liquid assets (such as cash reserves), that can be used to absorb potential losses and save on recapitalization costs. Banks earn revenues from their investments as well as by providing liquidity services to their depositors. Banks maximize shareholder value by choosing their buffers of liquid assets as well as their financing and default policies.

We first consider an hypothetical environment in which banks are unregulated and solve for the bank's optimal capital structure, which involves determining both how much debt to issue and how much liquid reserves to hold. In this environment, we show that financing frictions create a wedge between inside and outside equity and generate a precautionary demand for liquidity as well as an optimal capital structure for banks. Notably, we find that banks facing higher external financing costs have lower debt levels, pay less dividends, hold more liquid reserves, and default earlier.

We also show that banks absorb small and intermediate losses using their liquid reserves or by raising outside funds at a cost when these reserves are insufficient. That is, in our model, small and intermediate losses do not lead to insolvency. We show, however, that large losses due to a realization of tail risk may lead to default. Notably, we find that insolvency risk increases with tail risk, debt levels, and financing costs and decreases with franchise value, defined as the present value of the revenues from the bank's assets. As will become clear, our framework highlights a trade-off between managing insolvency risk ex ante via leverage choices versus ex post via liquidity buffers. That is, in our model, liquidity management, financing decisions, and default decisions are jointly and endogenously determined.

After solving for the policy choices of unregulated banks, we examine the effects of prudential regulation on these policy choices and insolvency risk. We first analyze liquidity requirements that mandate banks to hold a minimum amount of liquid reserves. We show that, when facing such a requirement, banks voluntarily choose to hold liquid buffers in excess of the required minimum in order to reduce the costs associated with breaches of the requirement. Notably, we show that as liquidity requirements rise, banks raise their *target level* of liquid reserves by a like amount, thereby making their liquidity cushion independent of the level chosen by the regulator. We then investigate the implication of this behavior for default risk. We show that, because raising outside funds is costly, an increase in liquidity requirements will generally not lead to an immediate adjustment to the new target buffer. As a result, in the *short-run*, liquidity requirements will lead to a drop in franchise value and to an increase in insolvency risk. We also demonstrate that once there has been an opportunity to build up liquid reserves towards the desired level, changes in liquidity requirements have no impact on insolvency risk. That is, contrary to received wisdom, we find that constraining liquid reserves has no *long-run* impact on the risk of failure of banks.

In addition to liquidity requirements, banks may be subject to leverage requirements. Such requirements do not impose any restriction on what assets banks should hold but impose constraints on the way they fund their operations. As argued by Admati and Hellwig, a tightening of leverage requirements transfers a large fraction of the bank's risks to its shareholders, which otherwise might be passed on to creditors or taxpayers. Leverage requirements generally indicate how much equity capital banks should have relative to their total assets. For example, banks are expected to maintain a Tier 1 capital to asset ratio of 3% under Basel III.[1] Using a calibrated version of the model, we show that such requirements have significant effects on both insolvency risk and bank value. For example, increasing equity capital from its privately optimal level to 20% (as recommended by Admati and Hellwig) decreases the default probability over a one year horizon from 0.1% to 0.0005%. At the same time however, such a requirement reduces total bank value by almost 6%.

The model also allows us to investigate the effects of deposit insurance and depositor preference in default on financing decisions and default risk. We show that mispriced deposit insurance generally allows banks to take on more subordinated risky debt, leading to an increase in insolvency risk. Another way to interpret this result is that the privately optimal level of equity capital increases significantly when deposit insurance is fairly priced. Alternatively, leverage requirements can be seen as a mechanism to reduce both default risk and the mispricing of deposit insurance. Lastly, we show that depositor preference in default increases the cost of subordinated debt and motivates bank shareholders to adopt more conservative financing policies, thereby reducing insolvency risk and contributing to a better alignment of private and social interests.

The literature examining default risk in banks has started with the early contributions of Merton (1977, 1978), in which the objective is to determine the cost of deposit insurance

[1] In July 2013, U.S. banking regulators have also proposed a new restriction on borrowing for U.S. banks, requiring a ratio of (Tier 1) capital to total assets of 6 percent for 8 Systemically Important Financial Institutions and 5 percent for their bank holding companies. Similarly, the Vickers Commission in the U.K. has recommended a 4 percent ratio rather than the 3 percent global standard set by the Basel III regime.

and loan guarantees. Although an important milestone, the analysis of Merton suffers from four limitations. First, it assumes that the bank defaults whenever the assets-to-deposits ratio falls below some *exogenous* barrier. Second, the capital structure of the bank is set exogenously and does not depend on the frictions that it faces such as taxes, bankruptcy costs, or issuance costs of securities. Third, the dynamics of the bank's assets are governed by an exogenous process, implying that there is no connection between the bank's asset and capital structures. Fourth, raising equity is costless, so that liquid reserves are irrelevant.

Most of the recent quantitative banking models examine variants to the first of these assumptions. In these contributions, insolvency is endogenous and triggered by shareholders decision to cease injecting funds in the bank (see e.g. Fries, Mella-Barral, and Perraudin (1997), Bhattacharya, Planck, Strobl, and Zechner (2002), or Décamps, Rochet, and Roger (2004)). While identifying some prime determinants of insolvency risk, these theories assume that asset and liability structures are exogenously given. As a result, they leave open the question of how financing structure and asset structure interact and jointly affect insolvency risk. In addition, these models also maintain the assumption that banks can raise outside funds at no cost, thereby leaving no role for liquid reserves. In that respect, they ignore a number of key determinants of insolvency risk, which are at the centre of most regulatory frameworks, including that of the Basel Committee on Banking Supervision.

Our analysis inherits some of the assumptions of this literature. For example, bank shareholders are protected by limited liability and the bank's objective is to maximize shareholder value. However, it differs from these contributions in three important respects. First, we consider that at least part of a bank's assets are illiquid and that it is costly to issue securities, thereby providing a role for liquid reserves. Second, we incorporate some of the key market imperfections and regulatory requirements that banks face in practice and relate banks' payout, financing, and default policies to these frictions. Third, in our model, financing structure and asset structure interact and jointly affect insolvency risk. We show that these unique features have important implications. For example, while in standard models shareholder value is always increased by making dividend payments to shareholders, this is not the case in our model with frictions, in which shareholders have incentives to protect their franchise value by maintaining adequate liquid reserves.

There exists a large literature analyzing the role of bank capital in regulation (see e.g. Hellman, Murdoch, and Stiglitz (2001), Décamps, Rochet, and Roger (2004), Morrison and White (2005), or Repullo and Suarez (2013)). But it is only recently that the question of bank optimal capital structure has begun to be addressed. The papers that are most closely related to ours in this literature are those by Froot and Stein (1988), Allen, Carletti, and Marquez (2014), Sundaresan and Wang (2014), and Subramanian and Yang (2014). Subramanian and Yang use a dynamic model to examine banks' insolvency risk, focusing on regulators' incentives to step in as banks become distressed. Sundaresan and Wang develop a dynamic model to analyze banks' financing decisions and the effects of deposit insurance and regulatory closure on bank liability structures. In both of these papers, there is no role for liquid reserves as outside financing is costless. In that respect, these papers are complementary to ours. Allen, Carletti, and Marquez show in a static model with bankruptcy costs that banks generally will hold capital to reduce expected bankruptcy costs and the cost of deposit finance. They also show the need for capital regulation when deposits are insured as the cost of deposits does not depend on the risk of the bank's assets and liability structures.

Froot and Stein (1998) build a two-period model in which capital is initially costless but may become costly in the future. In their model, decisions are made by bank shareholders in the first period, in which the future increase in financing costs leads the bank to increase equity capital and to engage in risk management activities to hedge future potential losses. Our model differs from Froot and Stein by considering that banks always face the same set of frictions and choose their payout, financing, and default decisions in response to these frictions as well as regulatory requirements. Another difference is that we consider a dynamic model. Indeed, as argued by Hellwig (1998), a static framework fails to capture important intertemporal effects. For example, in a static model, a regulatory requirement can only affect banks' behavior if it is binding. In practice however, regulatory requirements are binding for a small minority of banks and yet seem to influence the behavior of all banks.

From a methodological perspective, our paper relates to the inventory models of Milne and Whalley (2001), Peura and Keppo (2006), Bolton, Chen, and Wang (2011), Décamps, Mariotti, Rochet, and Villeneuve (2011), and Hugonnier, Malamud, and Morellec (2014) in which financing costs lead banks or corporations to have liquidity buffers. In these models, uncertainty is solely driven by Brownian shocks. As a result, there is no risk of default if firms can raise outside equity. Our paper advances this literature in two important dimensions. First, we incorporate jumps in our analysis to account for tail risk and show that banks may find it optimal to default following large shocks, even if they have access to outside funds. Second, we endogenize not only banks' payout decisions but also their financing and default policies. This allows us to examine the effects of regulation on banks' insolvency risk. As shown in the paper, this problem is difficult to solve because we have two free boundaries (the default and the payout thresholds) instead of one (the payout threshold). Another contribution of this paper is thus to develop a new method based on fixed-point arguments to derive the solution to shareholders' optimization problem.

The paper is organized as follows. Section 1 presents the model. Section 2 derives the value-maximizing payout, financing, and default policies for unregulated banks. Section 3 examines the effects of liquidity and leverage requirements on these policy choices and insolvency risk. Section 4 discusses the model's implications. Section 5 concludes. The proofs are gathered in a separate Appendix.

1 Model

Throughout the paper, agents are risk neutral and discount cash flows at a constant rate $\rho > 0$. Time is continuous and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P; \mathbb{F})$, with the filtration $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions.

The subject of study is a bank held by shareholders that have limited liability. This bank is subject to taxation at rate $\theta \in (0, 1)$. It owns a portfolio of risky assets (e.g. a portfolio of risky loans) as well as liquid reserves (i.e. cash reserves or safe government bonds) and is financed by equity, insured deposits, and risky, subordinated debt. The bank's risky assets generate after-tax cumulative cash flows A_t that evolve according to:

$$dA_t = (1-\theta)\mu dt + \sigma dB_t - Y_{N_t} dN_t.$$

In this equation, B_t is a Brownian motion, N_t is a Poisson process, (μ, σ) are constants, and $(Y_n)_{n=1}^{\infty}$ is a sequence of independently and identically distributed random variables that are drawn from $\mathbb{R}_+$ according to an exponential distribution with mean $\frac{1}{\beta} > 0$. The increments of the Brownian motion represent small and frequent shocks to the bank cash flows. The jumps of the Poisson process represent large losses that may be due, for example, to defaults across the loan portfolio of the bank (see Acharya, Cooley, Richardson, and Walter (2009) for an analysis of the role of tail risk in the 2007-2009 financial crisis). We denote the intensity of the Poisson process by $\lambda > 0$ so that over an infinitesimal time interval there is a probability $\lambda\beta e^{-\beta y} dt$ that the bank makes a loss of size $y \geq 0$.[2] In a non-financial firm, dA_t is the total earnings. In a bank, it only represents the earnings from assets such as loans, not including the income from serving deposit accounts.

The illiquidity of bank assets generally constitutes a major source of banking fragility.[3] Indeed, as discussed in Froot and Stein (1998), "one of the fundamental roles of banks is to invest in assets which, because of their information-sensitive nature, cannot be frictionlessly traded in the capital markets." The standard example of such an illiquid asset is a loan to a small or medium-sized company. To capture this feature, we consider that *risky* assets are illiquid and assume for simplicity that they have zero liquidation value, as in Biais, Mariotti, Rochet, and Villeneuve (2010), DeMarzo, Duffie, and Varas (2013), or Rochet and Sigrist

[2]These jumps may reflect trading losses, such as the \$8.9 billion loss of Morgan Stanley on credit default swaps or the \$7.2 billion loss of Société Générale on European index futures. They may also reflect losses on poorly performing acquisitions, such as the \$10.4 billion loss of Crédit Agricole on Emporiki. Lastly, they may reflect the payments made by a number of banks to authorities in the United States either for having facilitated tax evasion or for having traded with countries under embargo. For example, on June 30, 2014, BNP Paribas has agreed to pay a fine of \$8.9 billion for having violated U.S. sanctions against Cuba, Iran, Sudan, and other countries. For more details, see "Capital Punishment," in *The Economist* July 5, 2014.

[3]This illiquidity problem can be due for example to the special expertise of the bank in evaluating its long-term investments. If the bank tries to sell them before they pay out, other investors might infer that the bank got bad news about the future returns from these investments leading to a decrease of the price at which the bank could sell these assets. That is, liquidity problems may be fundamentally informational. But informational problems are also why we need a banking system in the first place.

(2014). This assumption implies that liquid reserves have the highest possible private value and that liquidity requirements have the highest possible social value. Section 4 examines the effects of positive liquidation values on the predictions of the model.

In addition to risky assets, the bank can (or may be constrained to) hold liquid, risk-free reserves. We denote by S_t the liquid reserves of the bank at any time $t \geq 0$. Holding liquid reserves generally involves deadweight costs. We capture these costs by assuming that the rate of return on liquid reserves is zero.[4] When optimizing its liquid asset holdings, the bank trades-off the lower returns of these assets with the benefits of liquidity.

Banks share some characteristics with non-financial firms: Both have access to the cash flows generated by their assets and both finance their assets with debt and equity. Banks, however, differ from non-financial firms in that they take deposits and provide account services to their depositors. In many countries, deposits are insured by the regulator and the deposit-taking activity comes with heavy regulation. Deposits and the associated account services, deposit insurance, and regulation distinguish the banking business from other businesses and set the capital structure decision of banks apart from that of other firms.

To capture these important differences, we consider that the liability structure of the bank comprises equity, a fixed volume of deposits D, as well as risky, subordinated debt. Deposits are insured against bank failure and require the bank to make a payment $c_D \geq 0$ per unit of time, which includes the interest paid to depositors and a deposit insurance premium.[5] In the model, the rate of return on deposits $\frac{c_D}{D}$ may differ from the fair rate of return, reflecting the income earned by the bank for serving deposit accounts and mispriced deposit insurance. Subordinated debt requires the bank to make a payment $c_L \geq 0$ per unit of time and has a face value L that is endogenized in section 3.2. The rate of return on subordinated debt is thus $\frac{c_L}{L}$ and, as we shall show below, it includes a risk premium to compensate for the risk of default. Summarizing, the bank's debt payments are thus $c = c_D + c_L$ and its earnings

[4] A necessary condition for a well-defined payout policy is that there exists a cost of holding liquidity in that the rate of return on liquid reserves is strictly less than the risk-free rate of return ρ.

[5] As in many recent contributions (see Allen, Carletti, and Marquez (2014), Subramanian and Yang (2014), or Hanson, Shleifer, Stein and Vishny (2014)), we downplay the vulnerability of bank deposits to runs and emphasize the opposite aspect of deposits: Relative to other forms of private-money creation that occur in the shadow-banking sector, bank deposits are highly sticky and not prone to run at the first sign of trouble.

evolve according to

$$dC_t = dA_t - (1-\theta)cdt = (1-\theta)(\mu - c)dt + \sigma dB_t - Y_{N_t}dN_t. \tag{1}$$

Because the distribution of the jump magnitudes has unbounded support, the bank will be unable to withstand large losses with some positive probability, leading to default. In the following, we use a stock based definition of default whereby the bank services debt as long as equity value is positive (as in e.g. Leland (1994), Duffie and Lando (2001), or Sundaresan and Wang (2014)). That is, default is the result of the optimizing behavior of shareholders. In the model, equity capital and liquid reserves serve as buffers against default risk. The bank can increase its liquid reserves either by retaining earnings or by issuing new equity. We consider that when raising outside funds, the bank has to pay a lump-sum cost ϕ. Because of this fixed cost, the bank will raise new funds through lumpy and infrequent issues. In addition, to reduce the costs associated with having to issue fresh equity, the bank will retain earnings and hold liquid reserves.

A payout and financing strategy is a pair $\pi = (P^\pi, R^\pi)$ of adapted, left-continuous, and non-decreasing processes with initial value zero, where P_t^π and R_t^π respectively represent the cumulative payouts to shareholders and the cumulative net financing raised from investors up until time $t \geq 0$. The liquid reserves process associated with a strategy π is defined by:

$$S_t^\pi = s + C_t - P_t^\pi + R_t^\pi,$$

where C_t is defined in equation (1) and s is the initial level of liquid reserves. This equation is a general accounting identity, in which P_t^π and R_t^π are endogenously determined by the bank. It shows that liquid reserves increase with the bank's earnings and with the funds raised from outside investors and decrease with payouts to shareholders. The liquidation time associated with the payout and financing strategy π is then defined by:

$$\tau_\pi = \inf\{t \geq 0 : S_{t+}^\pi = \lim_{u \downarrow t} S_u^\pi \leq 0\}.$$

In the model, bank shareholders make their financing, payout, and default decisions after observing the increment of the cash flow process. As a result, we use left-continuous processes in the definition of strategies and right-hand limits in the definition of liquidation times. Notably, the occurrence of a cash flow jump

$$\Delta C_t \equiv C_t - C_{t-} = C_t - \lim_{s \uparrow t} C_s \leq -S^\pi_{t-}$$

that depletes the liquid reserves of the bank results in default if shareholders do not provide sufficient funds for the right hand limit

$$S^\pi_{t+} = S^\pi_{t-} + \Delta C_t + R^\pi_{t+} - R^\pi_t - P^\pi_{t+} + P^\pi_t$$

to be strictly positive. In the following, we will consider strategies such that $P^\pi_{t+} - P^\pi_t \leq (S^\pi_t + R^\pi_{t+} - R^\pi_t)^+$, where $x^+ \equiv \max(0, x)$, implying that the bank cannot pay out amounts that it does not hold.

Management chooses the payout, financing, and default policies of the bank to maximize the present value of future dividends to existing shareholders, net of the total cost of capital injections. That is, management solves:

$$v(s) = \sup_{\pi \in \Pi(s)} \mathbb{E}_s \left[\int_0^{\tau_\pi} e^{-\rho t} (dP^\pi_t - d\Phi_t(R^\pi)) \right], \tag{2}$$

where $\Pi(s)$ denotes the set of admissible strategies defined in the Appendix and $d\Phi_t(R^\pi)$ represents the contribution of shareholders to the bank, including the cumulative cost of external financing.[6] Because risky assets have no liquidation value, there is no cash flow to

[6]Formally, the cumulative net financing raised from outside investors R^π_t and the total contribution of shareholders to the bank $\Phi_t(R^\pi)$ up until time t are defined by

$$R^\pi_t = \sum_{n=1}^{\infty} 1_{\{t > \xi_n\}} r_n \quad \text{and} \quad \Phi_t(R^\pi) = R^\pi_t + \sum_{n=1}^{\infty} 1_{\{t > \xi_n\}} \phi,$$

for some increasing sequence of stopping times $(\xi_n)_{n=1}^\infty$ that represent the dates at which the bank raises funds from outside investors and some sequence of nonnegative random variables $(r_n)_{n=1}^\infty$ that represent the net financing amounts. See the Appendix for more details.

shareholders in default. That is, we have $(S^{\pi}_{\tau_\pi} - D - L)^+ = 0$. We show in section 3.1 that this is no longer the case in the presence of a regulatory liquidity requirement.

To gain some intuition on the solution to problem (2), note that because of the fixed costs of financing, the bank raises external funds only when cash flow shocks deplete its liquid reserves, i.e. when $s \leq 0$. In such instances, the bank has to either immediately raise new funds to finance the shortfall and continue operating, or default. The bank may choose to default when the shortfall or the cost of refinancing are large. When this is not the case, the bank pays the financing cost ϕ and raises funds so as to maximize equity value. Taking into account these two possibilities, we have that the equity value function satisfies:

$$v(s) = \sup_{b \geq 0} \left(v\left(b\right) - b + s - \phi\right)^+ = \left(\alpha_0^* + s\right)^+, \qquad s \leq 0, \tag{3}$$

where b is the level of the bank's liquid reserves after raising external funds, and

$$\alpha_0^* = v(0) = \sup_{b \geq 0} \left(v\left(b\right) - \left(b + \phi\right)\right)^+$$

is the maximal shortfall that shareholders can accept to refinance. Anticipating, equation (3) shows that two key determinants of default risk in our model are the cost of outside funds ϕ and tail risk, i.e. the possibility for s to become (very) negative.

Because the likelihood of costly refinancing decreases with s, we expect the marginal value of liquid reserves to be decreasing and, therefore, the equity value function to be concave on the positive real line. If this conjecture is verified, then there should exist some barrier level b_0^* such that $v'(s) \geq 1$ if and only if $s \leq b_0^*$ and the optimal payout policy should consist in distributing dividends to maintain liquid reserves at or below the target level b_0^*.

The main difficulty in verifying this conjecture is that one needs to simultaneously determine the values of α_0^* and b_0^*. To circumvent this difficulty, we proceed in two steps. In the first step, we fix the value of the constant α and solve for the bank's payout policy in an auxiliary problem where the bank cannot raise funds but produces the payoff $(\alpha + s)^+$ to shareholders when it runs out of liquid reserves and defaults. In the second step, we show that the constant α can be chosen in such a way that the value of this auxiliary problem coincides with the solution to problem (2) and derive the equity value-maximizing payout, financing, and default policies.

Before moving to these two steps, we first determine the equity value and the bank's policy choices in the benchmark case where there are no refinancing costs. This allows us to derive a first best *franchise value* of equity that will be of repeated use when solving shareholders' optimization problem.

2 Value of an unregulated bank

2.1 First best franchise value

When there are no costs of raising funds, in that $\phi = 0$, any loss can be covered by issuing equity at no cost and there is no need for the bank to hold liquid reserves (as in e.g. Leland (1994) or Duffie and Lando (2001)). As a result, it is optimal to distribute all earnings and the optimization problem of the bank reduces to choosing the default policy that maximizes equity value. That is, management solves

$$v^* = \sup_{\tau \in \mathcal{S}} \mathbb{E}\left[e^{-\rho\tau} |\Delta C_\tau| + \int_0^\tau e^{-\rho t} dC_t \right],$$

where $\mathcal{S}$ denotes the set of all stopping times and $|\Delta C_\tau|$ represents the size of the loss leading to default. Let $a \wedge b = \min(a, b)$. Solving this problem leads to the following result.

Proposition 1 (First best franchise value)**.** *When issuing equity is costless, it is optimal to distribute all positive earnings and the optimal default time is given by*

$$\tau^* = \inf\{t \geq 0 : |\Delta C_t| \geq v^*\},$$

where the first best franchise value of equity $v^* > 0$ *is the unique solution to*

$$\rho v^* = (1 - \theta)(\mu - c) - \lambda \mathbb{E}\left[v^* \wedge Y_1\right], \tag{4}$$

when $c < \mu$ and $v^* = 0$ otherwise. For $c < \mu$, v^* is increasing in the cash flow rate μ, and decreasing in the coupon rate c, the jump intensity λ, and the mean jump size $1/\beta$.

Proposition 1 shows that for any $c < \mu$, it is optimal for bank shareholders to default the first time that the absolute value of a jump of the cash flow process exceeds equity value. In effect, the quantity $|\Delta C_t| = Y_{N_t}$ plays the role of a cost of investment that shareholders have to pay to keep the bank alive following the occurrence of a large loss. To better understand this feature, we can rewrite the Bellman equation (4) as

$$v^* = p(1-\theta)\left(\frac{\mu - c}{\rho}\right) + (1-p)\mathbb{E}\left[(v^* - Y_1)^+\right].$$

with $p = \frac{\rho}{\rho+\lambda} \in (0,1]$. Thus, when issuing equity is costless, the banks' problem can be interpreted as a discrete-time, infinite horizon problem in which shareholders earn $(1-\theta)\,\frac{\mu-c}{\rho}$ each period with probability p and otherwise face a random liquidity shock that they can decide to pay, in which case the bank continues, or not, in which case the bank is liquidated.

To determine the effect of limited liability on bank policy choices, let

$$v_0 = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC_t\right] = \frac{1}{\rho}\left((1-\theta)(\mu - c) - \frac{\lambda}{\beta}\right)$$

stand for the equity value under the assumption that shareholders never default, and denote by $\Omega = v^* - v_0$ the excess value generated by the option of declaring default. Using the Bellman equation (4), it can be shown that this quantity satisfies:[7]

$$\Omega = \left(\frac{\lambda}{\rho}\right)\mathbb{E}\left[(Y_1 - v^*)^+\right]. \tag{5}$$

This equation shows that limited liability only has value if the bank cash flows are subject to jumps, i.e. only if $\lambda > 0$, for otherwise the bank would not be subject to default risk. Importantly, the fact that the first best franchise value of equity v^* is strictly positive for all

[7]While Proposition 1 is valid for any distribution, it is possible to obtain closed-form solutions for equity value and the option to default under specific distributions. For example, when the jumps Y_n are exponentially distributed, the solution to equation (5) is given in closed form by $\Omega = (1/\beta)\mathcal{W}((\lambda/\rho)e^{-v_0})$ where $\mathcal{W}(x)$ is the Lambert$W$ function (see for example Corless, Gonnet, Hare, Jeffrey, and Knuth (1996)).

$c < \mu$, irrespective of the frequency and magnitude of the jumps, implies that it is optimal for the bank to operate even if the present value of cash flows v_0 is negative. That is, limited liability leads shareholders to invest in risky assets that would otherwise have negative net present value. This behavior is reminiscent of the collapse of AIG who was providing insurance by issuing credit default swaps (CDSs). AIG was collecting premia on these CDSs and was expected to make payments only if certain bonds defaulted, which unexpectedly happened in the wake of the subprime crisis. Once the bonds started defaulting, the CDSs had to pay out and AIG was on the hook for billions, which eventually led them to default.

2.2 Equity value and bank policies with no refinancing

Having determined the first best franchise value of equity, we now turn to the solution of the auxiliary problem in which the bank has no access to outside funds. In this case, shareholders cannot refinance, have to default if the bank runs out of liquid reserves, and can only optimize equity value over the bank's payout policy.

Let $\alpha \geq 0$ be a constant and consider an hypothetical bank whose assets produce after-tax net cash flows to shareholders given by dC_t as long as it is in operation, and a cash flow $(\alpha + s)^+$ to shareholders if the bank is liquidated at a point where $S_t = s$. The optimization problem of shareholders in such a bank can be written as

$$w(s; \alpha) = \sup_{\pi \in \Pi_0(s)} \mathbb{E}_s \left[\int_0^{\tau_\pi} e^{-\rho t} dP_t^\pi + e^{-\rho \tau_\pi} (\alpha + S_{\tau_\pi}^\pi)^+ \right], \tag{6}$$

where $\Pi_0(s)$ denotes the subset of admissible strategies such that $R^\pi = 0$ (i.e. no refinancing) and τ_π denotes the first time that liquid reserves are negative. The first term on the right-hand side of (6) captures the present value of all dividend payments until the time of default. The second term gives the present value of the cash flow to shareholders in default.

If the value of α is sufficiently large, it should be optimal for shareholders to liquidate the bank's assets and distribute all the available cash. The following result confirms this intuition and shows that the threshold above which immediate liquidation is optimal coincides with the first best franchise value of equity determined in the previous section.

Lemma 2. *If $\alpha \geq v^*$, it is optimal for shareholders to immediately liquidate.*

Given the above result, we now assume that $\alpha < v^*$. Following the literature on optimal dividend policies (see for example Albrecher and Thonhauser (2009) for a survey), it is natural to conjecture that the optimal payout strategy for shareholders in the auxiliary problem (6) should be of barrier type. Specifically, we conjecture that for any $\alpha < v^*$ there exists a constant barrier $b^*(\alpha)$ such that the optimal policy consists in paying dividends to maintain liquid reserves at or below the level $b^*(\alpha)$. To verify this conjecture, we start by calculating the value of the auxiliary bank's equity under an arbitrary barrier strategy.

Fix a constant $b > 0$ and consider the strategy $\pi_b = (P^b, 0)$ that consists in paying dividends to maintain the liquid reserves of the bank at or below the level b. The cumulative payout associated with this barrier strategy is

$$P_t^b = 1_{\{t>0\}} \max_{0 \leq u < t} (X_u - b)^+$$

where $X_t = s + C_t$ denotes the uncontrolled liquid reserves process (i.e. assuming that there are no dividend payments) and the corresponding value is defined by

$$w(s; \alpha, b) = \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} (\alpha + S_{\tau_{\pi_b}}^{\pi_b})^+ \right].$$

Letting ζ_0 denote the first time that the uncontrolled liquid reserves X_t become negative and using the dividend–penalty identity (see Gerber, Lin, and Yang (2006) and Gerber and Yang (2010)), we have that this value is given by:

$$w(s; \alpha, b) = \begin{cases} (\alpha + s)^+, & \text{for } s \leq 0, \\ \psi(s; \alpha) + \frac{W(s)}{W'(b)} (1 - \psi'(b; \alpha)), & \text{for } 0 < s \leq b, \\ s - b + w(b; \alpha, b), & \text{for } s > b, \end{cases} \tag{7}$$

where the function

$$\psi(s;\alpha) = \mathbb{E}_s\left[e^{-\rho\zeta_0}\left(\alpha + X_{\zeta_0}\right)^+\right]$$

gives the present value of the payment that shareholders receive in liquidation if no dividends are distributed prior to bankruptcy, and $W(s)$ is the $\rho-$scale function of the uncontrolled liquid reserves process. Closed-form expressions for both of these functions as linear combinations of exponentials are provided in the Appendix.

In the payout region (i.e. for $s > b$), the bank pays dividends to maintain its liquid reserves at or below b, so that equity value grows linearly with s. In the earnings retention region (i.e. for $s \leq b$), the term

$$\frac{W(s)}{W'(b)} = w(s;0,b) = \mathbb{E}_s\left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b\right]$$

gives the present value of the dividend payments that shareholders receive until default. The term

$$\psi(s;\alpha) - \psi'(b;\alpha)\frac{W(s)}{W'(b)} = \mathbb{E}_s\left[e^{-\rho\tau_{\pi_b}}(\alpha + S^{\pi_b}_{\tau_{\pi_b}})^+\right]$$

gives the present value of the payoff that shareholders receive in liquidation, with the second term on the left hand side reflecting the effects of payouts on this present value.

Equation (7) shows that in the earnings retention region, the value of a barrier strategy depends on the barrier level only through

$$H(b;\alpha) = \frac{1 - \psi'(b;\alpha)}{W'(b)}.$$

In the Appendix, we show that there exists a unique payout barrier $b^*(\alpha)$ that maximizes this function over $\mathbb{R}_+$ and, relying on a verification theorem for the Bellman equation associated with problem (6), we prove that the corresponding barrier strategy is optimal among all strategies. This leads to the following result.

Proposition 3 (Auxiliary value function). *Consider a bank with no access to outside funds that produces a cash flow $(\alpha+s)^+$ to shareholders in default. The equity value of such a bank is concave and twice continuously differentiable over $(0,\infty)$ and given by*

$$w(s;\alpha) = w(s;\alpha, b^*(\alpha)),$$

where $b^*(\alpha)$ is the unique solution to $H'(b^*(\alpha);\alpha) = 0$. Furthermore, the optimal policy for bank shareholders is to distribute dividends to maintain liquid reserves at or below $b^*(\alpha)$.

As can be seen from equation (7), the value function of problem (6) is linear with slope equal to one in the payout region where $s > b^*(\alpha)$. Combining this property with the smoothness established in Proposition 3 shows that this value function satisfies:

$$0 = w'(b^*(\alpha);\alpha) - 1 = w''(b^*(\alpha);\alpha).$$

The first of these two conditions is known as the smooth-pasting condition and indicates that liquid reserves are reflected down through dividend payments at the constant barrier level $b^*(\alpha)$. The second condition is usually referred to as the high-contact condition and guarantees the optimality of the dividend barrier (see Dumas (1991)).

2.3 Equity value and optimal bank policies

Having solved for the payout policy of the auxiliary bank, we now show how to obtain the optimal policies for problem (2) by endogenizing the value of the constant α that determines the equity value of the auxiliary bank at the point where it runs out of liquid reserves.

Let $b > 0$ be a constant and consider the strategy $\hat{\pi}_b$ that consists in paying dividends to maintain liquid reserves at or below b and in raising funds back to b whenever liquid reserves become negative if that is profitable. Denote by

$$v(s;b) = \mathbb{E}_s\left[\int_0^{\tau_{\hat{\pi}_b}} e^{-\rho t}(dP_t^{\hat{\pi}_b} - d\Phi_t(R^{\hat{\pi}_b}))\right]$$

the corresponding equity value. By definition, this function satisfies

$$v(s;b) = (v(b;b) - b + s - \phi)^+, \qquad \forall s \leq 0.$$

Since the bank does not raise funds before the first time τ_{π_b} that liquid reserves are negative, we have that $R_t^{\hat{\pi}_b} = P_t^{\hat{\pi}_b} - P_t^b = 0$ for all $0 \leq t \leq \tau_{\pi_b}$. Therefore, using the above equations together with the law of iterated expectations, we get that

$$\begin{aligned} v(s;b) &= \mathbb{E}_s\left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho\tau_{\pi_b}} v(S_{\tau_{\pi_b}}^{\pi_b}; b)\right] \\ &= \mathbb{E}_s\left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho\tau_{\pi_b}} (v(b;b) - b + S_{\tau_{\pi_b}}^{\pi_b} - \phi)^+\right] = w(s; v(0;b), b), \end{aligned} \tag{8}$$

where $w(s;\alpha,b)$ is defined as in equation (7). Evaluating the function $v(s;b)$ at the payout barrier b, subtracting $b + \phi$ from both sides of equation (8), and taking the positive part shows that the equity value associated with zero liquid reserves solves

$$\hat{\alpha}(b) \equiv (v(b;b) - b - \phi)^+ = v(0;b) = (w(b;\hat{\alpha}(b), b) - b - \phi)^+ .$$

Given equation (8), the results of the previous section now suggest that to determine the optimal strategy, we need to look for a barrier level b such that $b = b^*(\hat{\alpha}(b))$ or, equivalently, for a fixed point of the function defined by

$$g(\alpha) = (w(b^*(\alpha), \alpha, b^*(\alpha)) - b^*(\alpha) - \phi)^+.$$

We show in the Appendix that this function admits a unique fixed point α_0^* in the interval $[0, v^*)$. If the cost of financing is such that

$$\phi \geq \phi^* \equiv w(b^*(0); 0, b^*(0)) - b^*(0),$$

then this fixed point is given by $\alpha_0^* = 0$ and it is not profitable for shareholders to raise funds when the bank runs out of liquid reserves. If the cost of refinancing is such that $\phi < \phi^*$, then we have that $\alpha_0^* > 0$ and it may be profitable for shareholders to refinance when the bank runs out of liquid reserves, depending on the size of the shortfall. In this case, we have $\alpha_0^* = w(b_0^*; \alpha_0^*, b_0^*) - b_0^* - \phi$ with $b_0^* = b^*(\alpha_0^*)$ and it follows that α_0^* gives both the bank's equity value at the point where it runs out of liquid reserves and the size of the maximum loss that shareholders are willing to refinance.

The identity (8) and the definition of the constants α_0^* and b_0^* imply that the equity value associated with the strategy $\hat{\pi}_{b_0^*}$ is given by

$$v(s; b_0^*) = w(s; \alpha_0^*, b_0^*) = w(s; \alpha_0^*).$$

Using the properties of the auxiliary value function derived in the previous section, we show in the Appendix that this function satisfies

$$v(s; b_0^*) = (\alpha_0^* + s)^+ = \max_{b \geq 0} \left(v(b; b_0^*) - b + s - \phi \right)^+, \qquad s \leq 0,$$

and, relying on a verification theorem for the Bellman equation associated with the bank's optimization problem, we prove that the barrier strategy $\hat{\pi}_{b_0^*}$ is optimal, not only in the class of barrier strategies, but among all strategies. This leads to the following result.

Proposition 4 (Equity value function)**.** *The equity value function is concave and twice continuously differentiable in liquid reserves over* $(0, \infty)$ *and given by*

$$v(s) = v(s; b_0^*) = w(s; \alpha_0^*, b_0^*),$$

where α_0^* *denotes the unique fixed point of* $g(\alpha)$ *in the interval* $[0, v^*)$*. The optimal payout strategy consists in paying dividends to maintain liquid reserves at or below* $b_0^* = b^*(\alpha_0^*)$*. When* $\phi < \phi^*$*, the bank raises funds to move to* b_0^* *whenever liquid reserves become negative with a shortfall smaller than* $\alpha_0^* > 0$*, and liquidates otherwise. When* $\phi \geq \phi^*$*, the bank never raises equity and defaults the first time that liquid reserves reach* $\alpha_0^* = 0$*.*

Proposition 4 shows that when outside funds are costly, the value of equity is a concave function of the bank's liquid reserves over $(0, \infty)$, so that there are *no* incentives for shareholders to increase cash flow volatility even if the bank is levered. In our model, the bank's risk aversion, as measured by the curvature of the value function, is endogenous and depends on the level of liquid reserves. For liquid reserves above b_0^*, the equity value function satisfies

$$v''(s) = w''(s; \alpha_0^*) = 0,$$

so that the bank becomes risk-neutral. For negative values of liquid reserves, the equity value function is linear as long as shareholders find it optimal to refinance and equal to zero in the default region where $s + \alpha_0^* \leq 0$. The value of equity is therefore convex for negative values of liquid reserves. However, since these values can only be reached following the occurrence of a large loss, the bank does not spend any time in this region.[8] The shape and properties of the equity value function are represented in Figure 1.

Under the optimal policy, the bank pays dividends to maintain its liquid reserves at or below b_0^*. This is illustrated by the smooth-pasting and high-contact conditions:

$$0 = v'(b_0^*) - 1 = v''(b_0^*),$$

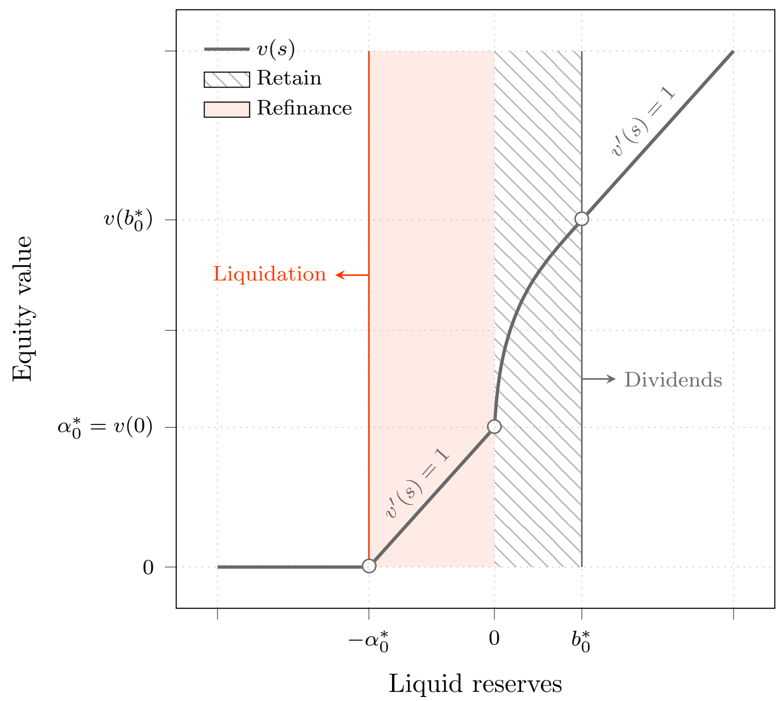
which show that liquid reserves are optimally reflected down at b_0^*. When liquid reserves exceed b_0^*, the bank is fully capitalized and places no premium on internal funds so that it is optimal to make a lump sum payment $s - b_0^*$ to shareholders. The desired level of reserves, b_0^*, results from the trade-off between the cost of raising funds and the cost of holding liquidity.

[8] The model can give the bank the option not to refinance immediately after a negative jump hits (e.g. by introducing a lender of last resort), to allow for counter-cyclical liquidity buffers. A simple way to incorporate this feature is to assume that if a negative jump takes the liquid reserves of the bank below the required minimum, the bank has until the fist arrival time τ_δ of a Poisson process with intensity δ to build liquidity back up, before it is forced to either refinance or liquidate. In this case, the expected duration of this forbearance period is $\Delta \equiv 1/\delta$ and the value of equity for $s \leq 0$ is given by

$$v(s) = \mathbb{E}\left[1_{\tau_0 \leq \tau_\delta} e^{-\rho\tau_0} v(0) + 1_{\tau_\delta \leq \tau_0} e^{-\rho\tau_\delta} \left(v(b^*) - b^* - \phi + S_{\tau_\delta}\right)^+\right],$$

where τ_0 is the first time that the liquid reserves process goes back to 0. See section 4.5.

Figure 1: Equity value for an unregulated bank



Notes. This figure illustrates the shape of the equity value function and the optimal strategy for an unregulated bank. The regions ▭ on each side of the graph correspond respectively to liquidation (left) and dividend payments (right) while the two intermediate regions ▨ and ▭ correspond respectively to earnings retention and refinancing.

Lastly, note that in our model, banks are subject to insolvency risk even though they can raise outside funds. Notably, Proposition 4 shows that it is optimal for shareholders to default following a shock that brings liquid reserves below $-\alpha_0^* = -v(0)$. Our results are thus in sharp contrast with those of Brownian-driven inventory models (see e.g. Milne and Whalley (2001), Peura and Keppo (2006), or Décamps, Mariotti, Rochet, and Villeneuve (2011)) in which there is no default if banks can raise outside equity. Also, while in those models firms always raise the same amount of capital, there exists some time series variation in the amount of funds raised from investors in our model.

3 The effects of liquidity and leverage requirements

This section examines the effects of prudential regulation on the policy choices that maximize equity value and on the risk of insolvency. We introduce prudential regulation in our setup by considering that the regulator can force the bank to maintain its liquid reserves in excess of a given regulatory level and can set a lower bound on equity capital when making leverage choices. That is, our focus is on the instruments of regulation that are being considered in the Third Basel Accord, namely leverage and liquidity requirements.

3.1 Liquidity requirements

To examine the effects of liquidity requirements, we consider a regulatory constraint that mandates the bank to hold a minimum level of liquid reserves $T > 0$ at all times. When $s \leq T$, the bank has the choice of recapitalizing by issuing new equity at a fixed cost ϕ or liquidating. By introducing this liquidity requirement in the model, we are interested in answering the following questions. Can a liquidity requirement affect bank behavior even if it is not binding? If liquidity requirements were to rise, would banks raise their target level of liquid reserves? What are the implications of these behaviors on default risk?

The value of equity in a bank subject to a minimum regulatory level of liquid reserves is defined by the optimization problem

$$v(s;T) = \sup_{\pi \in \Pi(s,T)} \mathbb{E}_s \left[\int_0^{\tau_{\pi,T}} e^{-\rho t} (dP_t^\pi - d\Phi_t(R^\pi)) + e^{-\rho \tau_{\pi,T}} (S_{\tau_{\pi,T}}^\pi - D - L)^+ \right], \tag{9}$$

where $\tau_{\pi,T} = \inf\{t \geq 0 : S_{t+}^\pi \leq T\}$ denotes the default time associated with the strategy π in the presence of a liquidity requirement at the level T, and $\Pi(s,T)$ denotes the set of admissible payout and financing strategies such that $P_{t+}^\pi - P_t^\pi \leq S_t^\pi - T + R_{t+}^\pi - R_t^\pi$. This constraint implies that the regulatory amount of liquid reserves T cannot be distributed. It guarantees that creditors will receive at least this amount in liquidation. The first term on the right-hand side of (9) captures the present value of all dividends to shareholders until default. The second term is the present value of the cash flow to shareholders in default.

Let $\ell(s) = (s - D - L)^+$ denote the payment that shareholders receive if liquidation occurs at point where the bank's liquid reserves are equal to s. Solving shareholders' optimization problem leads to the following result.

Proposition 5 (Liquidity requirements)**.** *Equity value in a bank subject to a minimum regulatory level of liquid reserves $T \geq 0$ is given by:*

$$v(s;T) = \begin{cases} v(s-T;0), & \text{if } \ell(T) \leq \alpha_0^*, \\ w(s-T;\ell(T)), & \text{otherwise,} \end{cases} \tag{10}$$

while the target level of liquid reserves b_T^* satisfies

$$b_T^* = \begin{cases} T + b_0^*, & \text{if } \ell(T) \leq \alpha_0^*, \\ T + b^*(\ell(T) \wedge v^*), & \text{otherwise.} \end{cases} \tag{11}$$

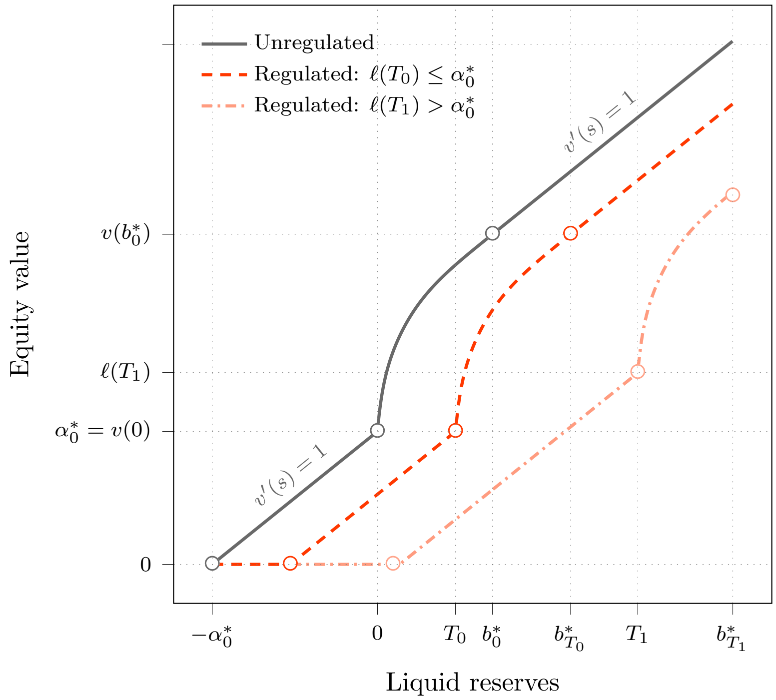
When $\ell(T) \leq \alpha_0^*$, the bank pays dividends to maintain liquid reserves at or below b_T^*, raises funds to move to b_T^* whenever liquid reserves fall below T with a shortfall smaller than α_0^*, and liquidates otherwise. When $\ell(T) > \alpha_0^*$, the bank never raises equity, defaults as soon as $s \leq T$, and pays dividends to maintain liquid reserves at or below b_T^*.

Proposition 5 shows that the effect of liquidity requirements on the bank's policy choices depends on their stringency. When the liquidity requirement is such that $\ell(T) > \alpha_0^* = v(0)$, it is never optimal for shareholders to refinance at T since the cash flow to shareholders in liquidation exceeds the continuation value of equity. In this case, the bank liquidates as soon as liquid reserves fall short of the minimum regulatory level. By contrast, when $\ell(T) \leq \alpha_0^*$, it is optimal for shareholders to refinance at T. In this case, liquidity requirements lead to a drop in equity value as

$$v(s;T) = v(s-T;0) = v(s-T) \leq v(s),$$

and make it optimal for shareholders to default when liquid reserves fall below $T - \alpha_0^*$. Figure 2 illustrates these effects by plotting the value of equity for different liquidity requirements.

Figure 2: Equity value for a regulated bank



Notes. This figure illustrates the change in the equity value when going from an unregulated bank (solid line) to either a regulated bank with T_0 such that $\ell(T_0) \leq \alpha_0^*$ (dashed line) or to a regulated bank with T_1 such that $\ell(T_1) > \alpha_0^*$ (dashed-dotted line).

Given that liquidity requirements reduce equity value, it is immediate to see that in the short-run, i.e. before the bank adjusts to its new target level of liquid reserves, such requirements decrease equity value and increase insolvency risk (see also Figure 5 below). That is, liquidity requirements have a negative transitory effect on insolvency risk. We therefore focus below on the long-run effects of liquidity requirements, i.e. on their effects once the bank has had the opportunity to build up liquid reserves. In addition, given that setting $\ell(T) > \alpha_0^*$ leads to liquidation whenever liquid reserves fall below the regulatory level, we consider that the regulator sets a minimum liquidity requirement satisfying $\ell(T) \leq \alpha_0^*$. In this case, it is optimal for shareholders to refinance when liquid reserves reach the regulatory level T, at which point the bank raises $b_T^* - T = b_0^*$ to move to the target level of liquid reserves. As a result, equity value at the regulatory level T is given by:

$$v(T;T) = (v(b_T^*;T) - (b_T^* - T) - \phi)^+ .$$

This value corresponds to the maximum amount that shareholders are willing to contribute to keep the bank alive following a large loss. Using equations (10) and (11), we then have

$$v(T;T) = (v(b_T^* - T) - (b_T^* - T) - \phi)^+ = (v(b_0^*) - b_0^* - \phi)^+ = \alpha_0^*,$$

so that shareholders default when liquid reserves fall below T with a shortfall greater than α_0^*. This shows that the presence of liquidity requirement increases the default threshold from the unregulated value $-\alpha_0^*$ to the regulated value $T - \alpha_0^*$. Since the target level of liquid reserves satisfies $b_T^* = b_0^* + T$, we immediately get the following result:[9]

Corollary 6 (Liquidity requirements and insolvency risk)**.** *Liquidity requirements have no effect on insolvency risk when banks are optimally capitalized.*

Given that $b_T^* = b_0^* + T$, the cushion of liquid reserves that the bank holds above the required level does not depend on the level chosen by the regulator. In effect, liquidity requirements shifts the support of the distribution of liquid reserves from $(-\alpha_0^*, b_0^*]$ to $(-\alpha_0^* + T, b_0^* + T]$. This is due to the fact that raising the required level of liquid reserves does not change the trade-off between the cost of carrying liquidity inside the bank and the cost of accessing outside liquidity.[10]

Two remarks are in order before we conclude this section. First, while liquidity requirements do not affect the long-run probability of default, they reduce the potential losses of

[9]Section 4.4 provides a detailed analysis of the effects of regulation on insolvency risk. Importantly, introducing counter-cyclical liquidity buffers, as described in footnote 8, would not change our result that liquidity requirements have no long-run impact on default risk for so long as this "forbearance" response is anticipated ex ante by bank shareholders. See section 4.5.

[10]Milne and Whalley (2001) show in a model without default risk that the target level of liquid reserves in banks increases with the reserves requirement so that the buffer of liquid reserves does not depend on regulatory constraints. One important contribution of our paper with endogenous default risk is to show that reserves requirements change not only the target level of liquid reserves in banks but also the selected default threshold, so that such requirements have no long-run effect on default risk in banks.

the bank in default and, hence, the capital injections by the regulator in default. Second, our analysis shows that liquidity requirements increase default risk in the short-run. This may explain why the regulator is implementing these requirements with a long transition period, thereby limiting the effects on bank franchise value and insolvency risk.

3.2 Endogenous leverage and leverage requirements

In this section, we examine the privately optimal mix between equity and debt on the balance sheet of the bank and the effects of regulatory constraints on this mix. To do so, we follow the literature on optimal financing decisions (see e.g. Leland (1994), Duffie and Lando (2001), or Sundaresan and Wang (2014)) and consider a bank that raises an amount D of deposits and issues at par a perpetual debt contract, with face value L and coupon rate c_L.

Assume that the bank is subject to a setup cost $\Psi \geq \phi$ that includes the cost of buying assets and let us determine the set of face values that lead to nonnegative payoffs for both shareholders and debtholders. If creditors agree to the proposed face value L, then the present value to shareholders of setting up the bank is

$$\eta_s(c_L, L, T) = \sup_{e \geq (T+\Psi-D-L)^+} \left(-e + v(D + L + e - \Psi; T|c_L, L)\right),$$

where e represents the amount injected by shareholders and $v(s; T|c_L, L)$, defined as in (9), gives the equity value of a bank that holds s in liquid reserves, is subject to a minimum liquidity requirement $T \geq 0$, and has issued debt with coupon rate c_L and face value L. To derive conditions under which this net present value is nonnegative, consider the function

$$N(c_L, T) = T + b_0^*(c_L) + \Psi - D - v(b_0^*(c_L)|c_L),$$

where $b_0^*(c_L)$ and $v(s|c_L)$ respectively give the optimal payout barrier and the equity value of an unregulated bank with coupon rate c_L. Note that, since shareholders in such a bank do not receive any payment in liquidation, these quantities depend neither on the face value of the debt contract L nor on the amount of deposits D that the bank initially raises. In the Appendix, we establish the following result:

Proposition 7 (Net present value). *The net present value of setting up the bank to shareholders is positive if and only if $L \geq N(c_L, T)$. In this case, this NPV satisfies:*

$$\eta_s(c_L, L, T)^+ = (L - N(c_L, T))^+,$$

and shareholders' strategy $\Theta^*(c_L) = (T, \alpha_0^*(c_L), b_0^*(c_L))$ consists in paying dividends to maintain liquid reserves at or below $T + b_0^*(c_L)$, raising funds to move to $T + b_0^*(c_L)$ whenever liquid reserves fall below T with a shortfall smaller than $\alpha_0^*(c_L)$, and liquidating otherwise.

The first part of Proposition 7 shows that shareholders break even only if they can obtain at least $N(c_L, T)^+$ from creditors. If creditors agree to such a price, then the second part of the proposition shows that shareholders will first adjust liquid reserves to the optimal level

$$b_T^*(c_L) = T + b_0^*(c_L),$$

and will then run the bank according to the barrier strategy $\Theta^*(c_L)$. In such a scenario, the present value to creditors will be

$$\eta_c(c_L, L, T) = d\left(b_T^*(c_L), \Theta^*(c_L) | \, c_L, L\right) - L,$$

where the function

$$d(s, \Theta | c_L, L) = \mathbb{E}\left[\int_0^{\tau(s,\Theta)} e^{-\rho t} c_L dt + e^{-\rho\tau(s,\Theta)} \min\left(S_{\tau(s,\Theta)}(s,\Theta) - D, L\right)^+\right] \tag{12}$$

gives the value of risky debt under the assumption that the bank follows a barrier strategy $\Theta = (T, a, b)$. The first term on the right-hand side of (12) captures the present value of all coupon payments until the default time $\tau(s, \Theta)$ associated with the strategy Θ starting from the initial level s. The second term is the present value of the cash flow to debtholders in default, where we have assumed that deposits were senior to risky debt. (Section 4.3 examines the effects of alternative priority rules on bank leverage and credit spreads.)

In the Appendix, we show that the creditors' present value is strictly decreasing in L and admits a unique root $L^*(c_L, T)$ that lies in $(0, c_L/\rho]$. This result implies that the present value of creditors is positive if and only if the proposed face value lies below $L^*(c_L, T)$. Combining this with Proposition 7 then shows that the set of individually rational face values associated to a coupon rate c_L is given by the interval

$$\left\{L > 0 : N(c_L, T)^+ \leq L \leq L^*(c_L, T)\right\}.$$

If the chosen coupon rate is such that this set is empty, then there is no face value of debt that generates nonnegative payoffs for both parties. If the coupon rate is such that $0 < N(c_L, T) \leq L^*(c_L, T)$, then any face value between these points generates a nonnegative present value for both parties, and it remains to determine how the surplus is shared between shareholders and creditors. To do so, we assume that creditors are competitive and do not have any bargaining power against the bank. In this case, the usual undercutting argument implies that shareholders will be able to sell debt at the highest possible price $L^*(c_L, T)$.

In our model, debt financing reduces the taxes of the bank. At the same time, for any given level of liquid reserves, debt financing increases expected bankruptcy costs and the frequency at which the bank raises costly outside capital. When choosing the coupon payment c_L, the objective of shareholders is to maximize the value of equity after debt has been issued plus the proceeds from the debt issue net of the cost to shareholders of acquiring the bank assets through the provision of required capital. That is, shareholders solve:

$$\sup_{c_L \in \mathcal{C}(T)} \eta_s\left(c_L, L^*(c_L, T), T\right), \tag{13}$$

where the feasible set is defined by

$$\mathcal{C}(T) = \{c_L \geq 0 : N(c_L, T) \leq L^*(c_L, T)\}.$$

One key difference between the optimization problem (13) and leverage choices in dynamic corporate finance models such as Leland (1994), Hackbarth, Miao, and Morellec (2006), or Strebulaev (2007) is that the initial contribution of shareholders to the bank's capital is endogenous and depends on the liquidity constraints faced by the bank. Another important difference is that, in our model, the bank determines its optimal capital structure by balancing tax benefits against both issuance costs and bankruptcy costs.

While problem (13) is not amenable to an explicit solution, we can show that as long as the minimum liquidity requirement does not exceed the amount of deposits—which is the relevant case in practice—the presence of a liquidity requirement will either deter shareholders from setting up the bank or will have no effect on the optimal coupon rate.

Proposition 8 (Optimal leverage and liquidity requirements)**.** *Assume that the optimal coupon rate* $c_L^*(0) \in \mathcal{C}(0)$ *is well-defined. If the liquidity requirement is such that* $T \leq D$ *then either* $\mathcal{C}(T)$ *is empty or* $c_L^*(T) = c_L^*(0)$*.*

In addition to liquidity requirements, the bank may also face leverage ratio requirements. That is, in an attempt to reduce insolvency risk, the regulator may constrain the bank to choose a leverage ratio, defined as equity capital over total asset value, that exceeds some lower bound ℓ^*. Since the bank's leverage ratio is minimized at $s = T$, constraining the bank's leverage ratio to be higher than some value ℓ^* is equivalent to

$$\frac{\alpha_0^*(c_L)}{D + L^*(c_L) + \alpha_0^*(c_L)} \geq \ell^*,$$

or

$$\alpha_0^*(c_L) \geq \frac{\ell^*}{1 - \ell^*}(L^*(c_L) + D).$$

The left-hand side of this equation is decreasing in c_L whereas the right-hand side is increasing in c_L for any $c_L \leq c_L^*(T)$. This shows that constraining the leverage ratio of the bank is equivalent to constraining the bank to choose a coupon level $c_L \in [0, \hat{c})$, where $\hat{c} \geq 0$ is a cap on the coupon payment on risky debt. In our analysis of financing decisions, we will also refer to the debt ratio. This ratio is defined as the market value of debt plus deposits over total bank assets and is equal to one minus the leverage ratio, i.e.

$$\begin{aligned}\text{Debt ratio}(s) &= \frac{D + d(s;T|c_L, L)}{D + d(s;T|c_L, L) + v(s;T|c_L, L)} \\ &= 1 - \frac{v(s;T|c_L, L)}{D + d(s;T|c_L, L) + v(s;T|c_L, L)} = 1 - \text{Leverage ratio}(s).\end{aligned}$$

4 Model analysis

4.1 Parameter values and implied variables

This section provides additional results and illustrates the effects of frictions and regulatory constraints on bank's policy choices using numerical examples. The values of the model parameters used in our base case environment are reported in Table 1. These parameter values imply that large losses represent on average 50% of yearly income and occur every other year in expectation and that the cost of refinancing represents between $\frac{\phi}{b_0^* + \alpha_0^*} = 1.27\%$ (at the default threshold) and $\frac{\phi}{b_0^*} = 6.36\%$ (at the regulatory threshold) of the capital raised. For these parameter values, the optimal coupon on risky debt is $c_L = c_L^*(0) = 11.49\%$ and the corresponding face value of risky debt is $L = 2.25$.

Because liquid reserves fluctuate between their minimum regulatory level and the target buffer set by bank shareholders, the debt ratio of the bank effectively remains between two bands as in the dynamic capital structure models of Fischer, Heinkel, and Zechner (1989), Hackbarth, Miao, and Morellec (2006), Strebulaev (2007), and Morellec, Nikolov, and Schürhoff (2012). Notably, the debt ratio of the bank fluctuates between 91.89% and 93.49% at the optimal coupon level while deposits represent between 61.25% and 62.31% of total asset value, consistent with the figures reported in Gropp and Heider (2010). Lastly, the 1-year default probability for the bank at optimal leverage is 10.06 basis points, consistent with the values reported in Crossen and Zhang (2012) or Hamilton, Munves, and Smith (2010) for 1-year default probabilities of financial firms.

Table 1: Parameter values and implied variables

A. Parameter values		
Parameter	Symbol	Value
Drift rate net of taxes	$(1-\theta)\mu$	0.18
Volatility	σ	0.08
Arrival intensity	λ	0.50
Mean jump size	$1/\beta$	0.09
Tax rate	θ	0.35
Face value of debt	L	2.25
Face value of deposits	D	4.50
Coupon rate	c_L	0.1149
Cost of deposits	c_D	0.045
Discount rate	ρ	0.05
Financing cost	ϕ	0.0075
Liquidity requirement	T	0.00
B. Implied variables		
Variable	Symbol	Value
First best value	v^*	0.6220
Equity value	α_0^*	0.4699
Target level of liquid reserves	b_0^*	0.1179
Share of financing cost		0.0636
1 Year default probability (bps)		10.06
Debt ratio (%)		$91.89 - 93.49$
Deposits over total assets (%)		$61.25 - 62.31$

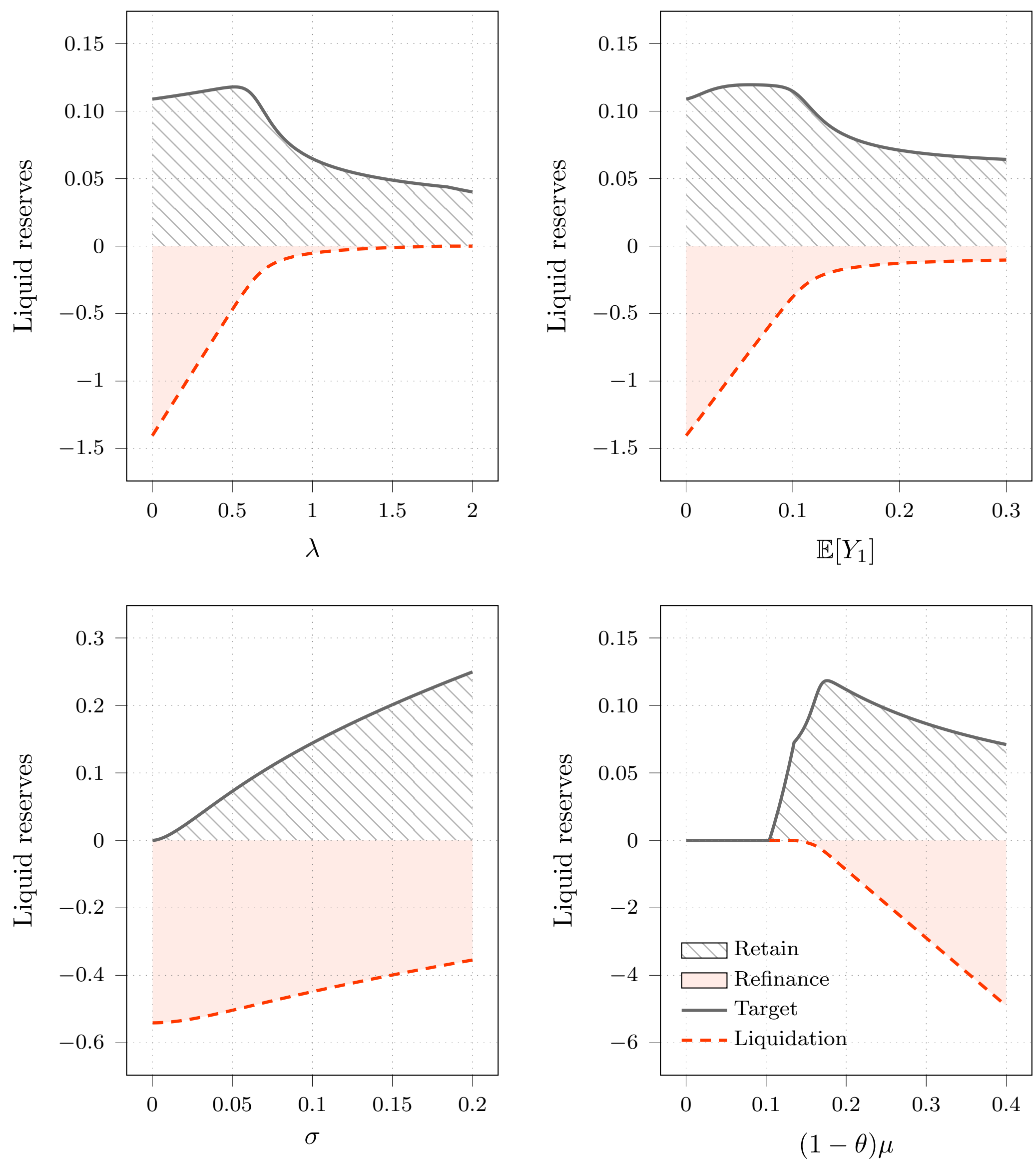
4.2 Default and payout decisions

We start our analysis by examining the determinants of the target level of liquid reserves. To do so, we plot in Figure 3 the unconstrained target buffer b_0^* as a function of the intensity of large losses λ, the expected size of large losses $\frac{1}{\beta}$, the cash flow volatility σ, and the cash flow rate μ. In the model, the target level of liquid reserves maximizes the value of equity and results from a trade-off between the carry cost of liquidity ρ and the cost of raising new capital. Because an increase in volatility increases the likelihood of a costly equity issuance, the target buffer b_0^* increases with σ. An increase in the frequency of large losses λ or in their expected size $\frac{1}{\beta}$ has two opposite effects on b_0^*. First, it increases the likelihood of a costly security issuance and, hence, shareholders' incentives to build up liquid reserves. Second, it reduces the expected cash flow from operating the bank's assets and therefore shareholders' incentives to contribute capital. The first effect dominates for low values of λ and $\frac{1}{\beta}$; the second effect dominates for larger expected losses. The effect of the cash flow rate μ on the target level of liquid reserves also results from two opposite effects. On the one hand, a higher cash flow rate increases the bank's franchise value and shareholders' incentives to contribute capital. On the other hand a higher cash flow rate increases revenues and reduces the role of liquid reserves as a buffer to absorb losses.

Consider next the determinants of the bank's default decision. As shown in Proposition 4, when there is no liquidity requirement in that $T = 0$, default occurs when the bank is hit with a negative shock that takes its liquid reserves below $-\alpha_0^*$. This threshold is given by the value of equity at the target, i.e. $v(b_0^*)$, net of the new provision of capital b_0^* and of the refinancing cost ϕ. By decreasing the value of the claim of incumbent shareholders at refinancing (i.e. by decreasing $v(b_0^*) - b_0^*$), an increase in the intensity of large losses λ, in their expected size $\frac{1}{\beta}$, in the cost of financing ϕ, or in the diffusion coefficient σ lead to an increase in the default threshold. By contrast, an increase in the cash flow rate μ (or, equivalently, a decrease in the coupon rate c) leads to an increase in the value of the claim of incumbent shareholders at refinancing and, therefore, to a decrease in the default threshold. Lastly, when μ falls below $c_D + c_L$, equity becomes worthless and it is optimal to default immediately. These effects are illustrated in Figure 3.

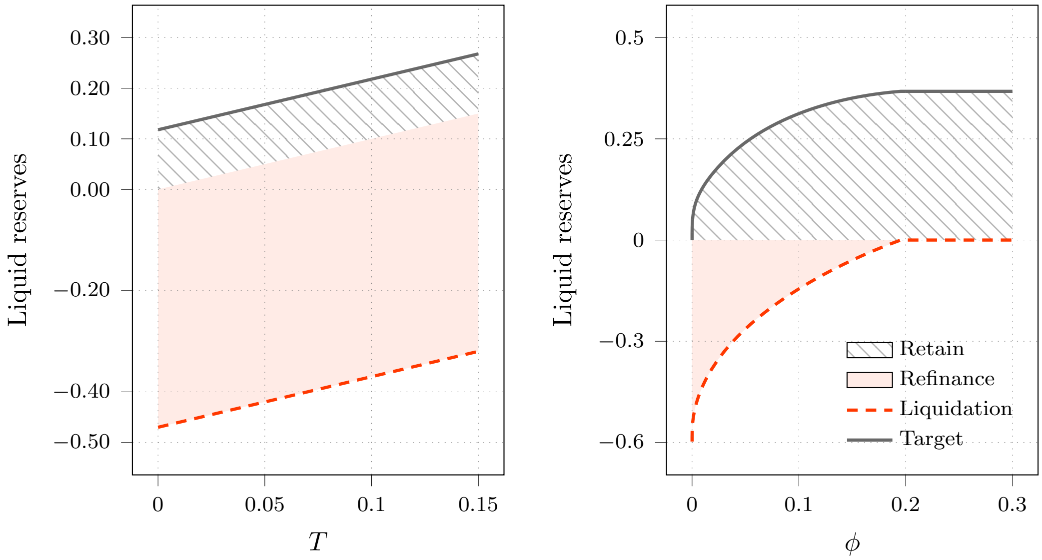
To illustrate the effects of liquidity requirements on bank's policy choices, Figure 4 plots the target level of liquid reserves as well as the default threshold as functions of the minimum requirement T. Consistent with Proposition 5, the figure shows that both thresholds grow linearly with the minimum liquidity requirement T. That is, regulatory requirements affect bank behavior even when they are not binding. The figure also shows that banks will generally hold a significant buffer of liquid reserves in excess of the liquidity requirements in order to reduce refinancing costs. Therefore, our theory predicts that liquidity requirements should be non-binding for most banks. Lastly, Figure 4 shows that both thresholds increase with the cost of external funds ϕ. Indeed, an increase in the cost of raising funds decreases the benefits of refinancing, leading to an increase in the default threshold. In addition,

Figure 3: Effect of the cash-flow parameters



Notes. This figure plots the target level of liquid reserves (solid line) and the liquidation threshold (dashed line) as functions of the jump arrival intensity, the mean jump size, the cash flow volatility and the cash flow drift. In each panel the upper and lower regions correspond respectively to dividend payments and liquidation while the two intermediate regions and correspond respectively to earnings retention and refinancing.

Figure 4: Effect of liquidity requirements and financing costs



Notes. This figure plots the target level of liquid reserves (solid line) and the liquidation threshold (dashed line) as functions of the required level of liquid reserves and the fixed cost of equity financing. In each panel the upper and lower regions ▭ correspond respectively to dividend payments and liquidation while the two intermediate regions ▨ and ▭ correspond respectively to earnings retention and refinancing.

an increase in the cost of external funds raises the value of inside equity and therefore shareholders' incentives to build up liquid reserves.

4.3 Deposit insurance, depositor preference, and insolvency risk

In our model, the terms of deposit insurance and the payments to depositors are fixed exogenously and reflect neither the risk of the bank's assets nor its policy choices. The benefit for bank shareholders of the mispricing of deposit insurance is that it increases the cash flows of the bank by lowering its cost of operation and reduces the present value of bankruptcy costs priced in the debt contracts. This in turn allows the bank to reduce its cost of capital by increasing its debt ratio, leading to a change in insolvency risk.

Table 2: Leverage and default probabilities

			Debt ratio (%)		1 Year default probability (bps.)		
T	$1/\beta$	Base	No deposits	Junior deposits	Base	No deposits	Junior deposits
0	0.09	91.90 – 93.49	83.26 – 86.15	91.90 – 93.49	10.06	7.26	10.06
0	0.18	87.23 – 88.78	58.11 – 61.03	87.23 – 88.78	92.67	40.51	92.67
40%	0.09	91.90 – 93.49	83.61 – 86.51	95.47 – 97.02	10.06	8.34	174.61
40%	0.18	87.23 – 88.78	58.97 – 61.93	99.09 – 100.0	92.67	44.60	8605.73
			Debt ratio (%)		1 Year default probability (bps.)		
T	λ	Base	No deposits	Junior deposits	Base	No deposits	Junior deposits
0	0.5	91.90 – 93.49	83.26 – 86.15	91.90 – 93.49	10.06	7.26	10.06
0	1.0	90.50 – 92.40	72.92 – 77.01	90.50 – 92.50	27.12	13.92	27.12
40%	0.5	91.90 – 93.49	83.61 – 86.51	95.47 – 97.02	10.06	8.34	174.61
40%	1.0	96.44 – 97.71	72.92 – 77.02	99.05 – 100.0	35.63	13.92	9037.13
			Debt ratio (%)		1 Year default probability (bps.)		
T	σ	Base	No deposits	Junior deposits	Base	No deposits	Junior deposits
0	0.08	91.90 – 93.49	83.26 – 86.15	91.90 – 93.49	10.06	7.26	10.06
0	0.16	91.32 – 94.12	82.09 – 87.21	91.32 – 94.12	9.86	7.08	9.86
40%	0.08	91.90 – 93.49	83.61 – 86.51	95.47 – 97.02	10.06	8.34	174.61
40%	0.16	91.32 – 94.12	82.50 – 87.64	95.71 – 98.27	9.86	8.30	308.02

Notes. This table presents the optimal band for debt ratios and the implied one year default probability under different assumptions regarding the level of regulatory capital, the presence and seniority of deposits, the mean jump size and the volatility of cash flows. In the first column the level of regulatory capital is expressed as a fraction of the total face value of debt $L + D$ at the base case.

To investigate the effects of mispriced deposits on banks' financing decisions and insolvency risk, we consider an environment in which $D = c_D = 0$ and all debt is risky and fairly priced. Table II shows that, in such an environment, the bank's debt ratio is significantly reduced and fluctuates between 83.26% and 86.15% at the optimal coupon level. That is, the amount of debt financing is reduced by close to 7.5%. This drop in debt financing leads in turn to a drop in insolvency risk. Notably, the 1-year default probability for the bank at optimal leverage is now 0.0726%. (The Appendix provides closed-form solutions for this default probability.) This suggests that the mispricing of deposit insurance has two opposite effects on default risk. First, it increases cash flows to shareholders and their willingness to absorb losses, for any given coupon rate. Second, it increases the privately optimal amount of debt in the bank's capital structure. In our base case environment, the second effect dominates leading to an increase in insolvency risk. In addition, Table II shows that the drop in insolvency risk is more important in banks with high tail risk, as captured by λ and $1/\beta$. As shown by the table, volatility risk works differently. Notably, an increase in σ leads to an increase in the expected cost of refinancing. This leads the bank to adopt a more conservative financing policy, leading to a drop in the probability of default.

Our analysis so far has assumed that deposits were senior to risky debt. Indeed, in the U.S. for example, Congress has adopted in 1993 a national depositor preference law that elevates the claims of domestic depositors on the assets of a failed bank over the claims of foreign depositors and general creditors. The main objective of this law is to produce cost savings for the Federal Deposit Insurance Company (FDIC) when it resolves failed institutions, thereby limiting injections of public money in the financial sector.

To estimate the effect of this depositor preference on default risk, we now allow the bank to issue risky debt that is secured by the bank's assets. Securing risky debt effectively subordinates depositors, but the depositors do not object to being subordinated as they are fully insured through deposit insurance. In our model, making risky debtholders senior to depositors implies that their cash flow in default is given by: $\min(S^{+}_{\tau(\Theta)}, L)$, where $\tau(\Theta)$ is the default time associated with the use of the strategy Θ. This in turn increases the value of risky debt and leads the bank to take on additional debt.

Table 2 shows that debt ratios and insolvency risk increase when bonds subordinate depositors. That is, high deposit volume combined with secured financing, two factors commonly attributed to a stable banking system, encourage high debt ratios and fuel default risk in banks. Further, the effects of secured debt financing are stronger for banks with high asset volatility and tail risk. This suggests that switching from junior bonds to secured bonds does not necessarily result in less risky bonds. Indeed, while the loss given default decreases by subordinating depositors, increased debt ratios raise the likelihood of default and negatively affect bondholders. That is, we find that depositor preference motivates bank shareholders to adopt more conservative financing policies, thereby reducing insolvency risk and contributing to a better alignment of private and social interests.

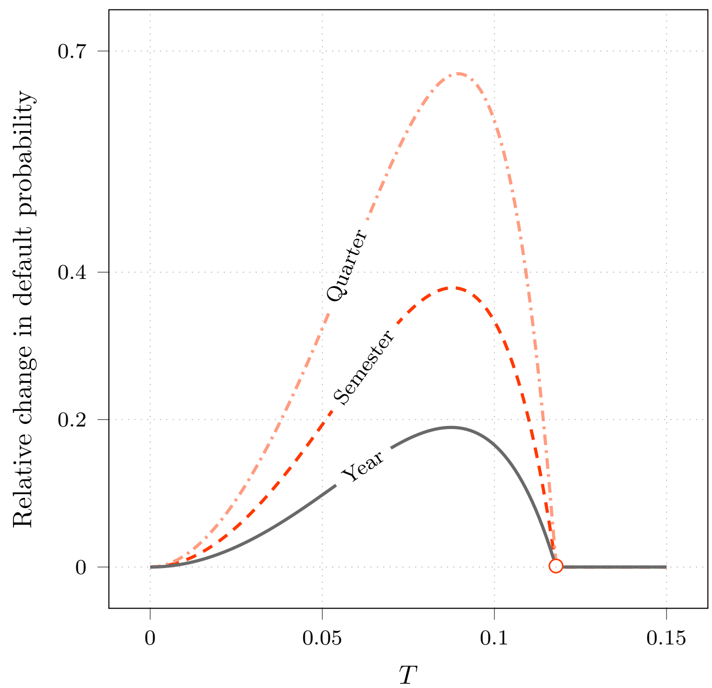
4.4 Regulation, insolvency risk, and bank value

An important question that our model allows to answer is whether regulatory requirements affect insolvency risk. Proposition 5 and Corollary 6 show that altering the liquid reserves of banks has no long-run impact on insolvency risk. To examine the short-run effects of liquidity requirements on insolvency risk, Figure 5 plots the relative change in the probability of default of the bank that results from imposing a minimum liquidity requirement. When the bank follows equity value-maximizing policies, this relative change is given by:

$$\frac{P\left(\tau(b_0^*; T, \alpha_0^*, b_0^*) < t\right)}{P\left(\tau(b_0^*; 0, \alpha_0^*, b_0^*) < t\right)} - 1,$$

where $\tau(b_0^*; T, \alpha_0^*, b_0^*)$ is the time of default for a bank with liquid reserves b_0^*, default trigger at $T - \alpha_0^*$, payout trigger at $T + b_0^*$, and facing a liquidity requirement T. Figure 5 considers three different horizons to compute the change in the probability of default: A three-month horizon ($t = 0.25$), a six-month horizon ($t = 0.5$), and a one-year horizon ($t = 1$). Consistent with the discussion in section 3.1, Figure 5 shows that imposing a minimum liquidity requirement always increases insolvency risk in the short-run. In addition, the figure shows that the increase in the probability of default is larger for shorter horizons, since it is less likely that the bank has built up liquid reserves. The figure also shows that for large liquidity

Figure 5: Insolvency risk and liquidity requirements



Notes. The figure plots the relative change in default probability at a 3 month (dotted line), 6 month (dashed line) and 1 year horizon (solid line) induced by a tightening of the liquidity requirement for a bank with liquid reserves equal to the unregulated target b_0^*.

requirements (i.e. when $T > b_0^*$), it is optimal for the bank to recapitalize immediately and liquidity requirements do not change the probability of default.

One way to restrict the bank's leverage is to constrain its liquid asset holdings by imposing liquidity requirements. However, we have just shown that such constraints lead to a short-run increase in insolvency risk for any given coupon c_L. In addition, as shown by Proposition 8, liquidity requirements have no effects on banks' *ex ante* choice of debt level in that $c_L^*(T) = c_L^*(0)$ for all $T \leq D$. As a result, a more direct mechanism, constraining directly banks' financing decisions, may be necessary. One mechanism advocated by regulators and academics to limit default risk in the banking sector is to impose a minimum leverage ratio for banks (see e.g. Admati and Hellwig (2013)).

Table 3: Leverage requirements, default probabilities, and bank value

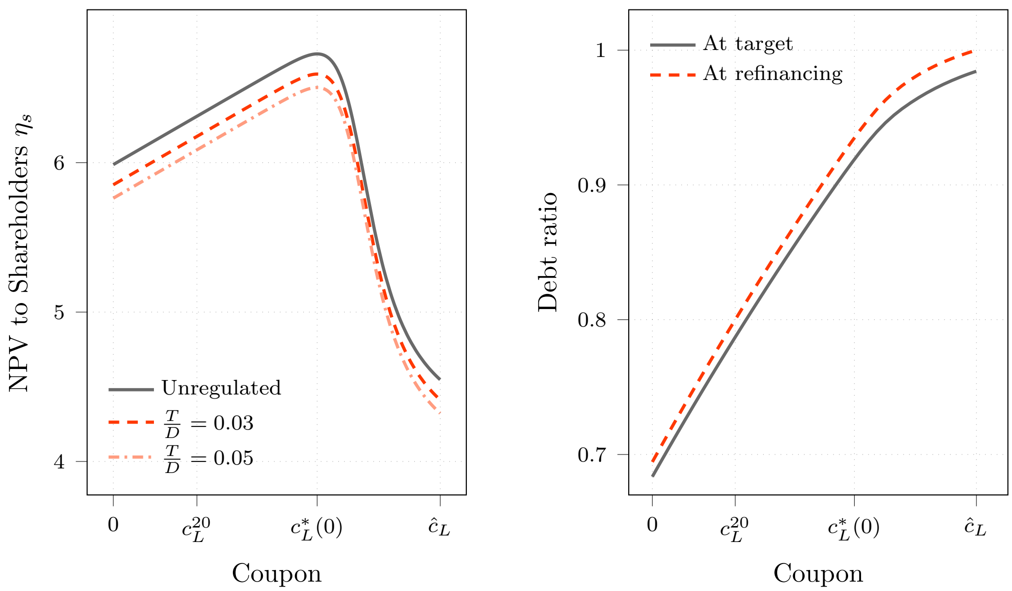
Minimal leverage (%)	Coupon c_L^*	Leverage ratio (%)	Default probability (bps.)	Change in bank value (%)
A. Base case parameters				
0	0.1149	6.508 – 8.104	10.06	0.000
3	0.1149	6.508 – 8.104	10.06	0.000
6	0.1149	6.508 – 8.104	10.06	0.000
10	0.0963	10 – 11.51	0.6835	−1.191
15	0.0711	15 – 16.40	0.01776	−3.568
20	0.0471	20 – 21.29	0.0005488	−5.866
B. High financing cost $\phi = 300$bps.				
0	0.1098	5.82 – 8.729	9.222	0.000
3	0.1098	5.82 – 8.729	9.222	0.000
6	0.1088	6 – 8.901	8.021	−0.009
10	0.0882	10 – 12.73	0.4085	−1.509
15	0.0638	15 – 17.53	0.01194	−3.877
20	0.0404	20 – 22.35	0.0004097	−6.157

Notes. This table presents the optimal coupon, the band for leverage ratios, the default probability over one year and the change in the value of the bank implied by different leverage ratio constraints.

To illustrate the effects of leverage ratio requirements on insolvency risk, Table 3 reports the probability of default over a one-year horizon and the optimal coupon of the bank for different refinancing costs ϕ and different leverage requirements. As shown by the table, leverage ratio requirements have a very strong quantitative effect on insolvency risk. In our base case environment, reducing the coupon rate of the bank from its privately optimal level to $c_L = 4.71\%$ – corresponding to the ratio of equity capital to total assets of 20% recommended by Admati and Hellwig (2013) – reduces the probability of default over a one-year horizon from 0.1006% to 0.0005%. In effect, leverage ratio requirements lead to an increase in the cash flow rate of the bank net of the payments to debtholders and, hence, in an increased willingness of shareholders to absorb losses.[11] The table also shows that

[11] In our model, the bank only has access to one class of risky assets. As a result, the ex-ante constraint on the coupon level that we consider can be easily mapped into a risk-weighted capital ratio. For example, assuming that risk-weighted assets represent 57% of total assets (which is the average number for the financial

Figure 6: Regulation, leverage, and value



Notes. This figure plots the net present value to shareholders as a function of the coupon level for different values of the ratio T/D (left panel) and the leverage at the target level of liquid reserves $b_T^*(c_L)$ and at the refinancing point T as functions of the coupon level (right panel). $c_L^{20} = 0.0471$ indicates the optimal coupon level under a 20% leverage ratio constraint, $c_L^*(0) = 0.1149$ indicates the coupon level that is optimal for shareholders given any liquidity requirement $T \leq D$, and $\hat{c}_L = 0.1843$ indicates the coupon level where where the bank switches to a no-refinancing strategy.

with higher refinancing costs, the bank optimally chooses a lower coupon level. Yet, this lower coupon level translates into a lower leverage ratio due to the drop in equity value (see equation (2)). This in turn implies that the effects of leverage requirement on insolvency risk get larger as refinancing costs increase.

Our analysis so far has shown that while liquidity requirements may increase banks' insolvency risk in the short-run, leverage requirements reduce this risk. Another important aspect of regulation that has been widely discussed in the financial press relates to its effects

industry in North America; see e.g. Le Leslé and Avramova (2012)), our leverage ratio constraint of 6% is equivalent to a risk-weighted capital ratio of 6%/0.57=10.52%. When banks have access to multiple risky assets, risk-based capital charges constrain asset choices to reflect the trade-off between risk and return. See Rochet (1992) for an analysis of the benefits and costs of risk-weights.

on valuations. To measure the value effects of liquidity and leverage requirements, Figure 6 plots the net present value to shareholders (left panel) as well as the bank's debt ratio (right panel) as functions of the selected coupon payment.

As shown by Figure 6, liquidity requirements have significant value effects. In our base case environment, requiring that liquid reserves represent at least 3% of the value of deposits reduces the value of the bank by 1.97%. That is, liquidity requirements lead not only to a short-run increase in default risk but also to a large reduction in the value of the bank. As shown by the figure and Table 3, leverage requirements also reduce total bank value. For example, the total value of the bank is reduced by 5.866% when moving from the privately optimal leverage ratio to a leverage ratio with 20% of equity capital. In our model, this decrease in value is due to an increase in the bank's cost of capital. The table also reveals that the effect of leverage requirements on bank value depends on both their stringency and the level of refinancing costs. Notably, because higher refinancing costs are associated with lower leverage ratios, the (relative) decrease in bank value due to leverage requirements gets more important as refinancing costs increase.

4.5 Model extensions

In this section, we numerically investigate three extensions of our basic setup. First, we introduce a positive liquidation value for risky assets. Notably, we assume that the liquidation value Λ of risky assets is a constant fraction $1-\varphi$ of their first best value $(1-\varphi)\frac{(1-\theta)\mu-\lambda/\beta}{\rho}$. In our calibration, we base the value of φ on the estimates of James (1991) and Flannery (2011) and set $\varphi = 25\%$. Second, we incorporate counter-cyclical liquidity requirements as discussed in footnote 8. Notably, we consider that if a negative jump takes the liquid reserves of the bank below the required minimum, the bank has until the fist arrival time τ_δ of a Poisson process with intensity δ to build liquidity back up, before it is forced to either refinance or liquidate. In our calibration, we assume that the expected duration of this forbearance period is $1/\delta = 1$ year. Third, we assume that the cash held by the bank to meet the liquidity requirement earns a return $\xi > 0$.

Table 4: Model extensions

	Liquidity Requirement T	Deposits D	Leverage ratio band (%)	Franchise value α_0^*	Target $b_T^* - T$	1 Year Default probability (bps.)
A. Liquidation value						
$\varphi = 0$	$\in [0, D]$	4.5	6.50–8.10	0.4699	0.1179	10.05
$\varphi = 0.25$	$\in [0, D - \Lambda]$	4.5	6.50–8.10	0.4699	0.1179	10.05
$\varphi = 0$	0	0	13.84–16.73	0.4992	0.1175	7.26
$\varphi = 0.25$	0	0	12.08–15.02	0.4393	0.1181	14.12
$\varphi = 0.25$	$0.1L_0$	0	11.51–14.46	0.4197	0.1182	17.55
$\varphi = 0.25$	$0.4L_0$	0	8.31–11.31	0.3079	0.1173	61.71
B. Forbearance period						
$\delta = 0$	$\in [0, D]$	4.5	6.50–8.10	0.4699	0.1179	10.05
$\delta = 0.25$	$\in [0, D]$	4.5	2.25–4.15	0.5328	0.0819	7.59
$\delta = 1$	$\in [0, D]$	4.5	2.81–3.76	0.6123	0.0435	2.80
C. Return on regulatory liquid reserves						
$\xi = 0$	$\in [0, D]$	4.5	6.50–8.10	0.4699	0.1179	10.05
$\xi = 0.02$	$0.03D$	4.5	6.48–8.08	0.4720	0.1178	9.83
$\xi = 0.02$	$0.10D$	4.5	6.44–8.00	0.4765	0.1178	9.34
$\xi = 0.02$	$0.4(D + L_0)$	4.5	6.08–7.47	0.5044	0.1175	6.86

Notes. This table presents the optimal band for leverage ratios, the default probability over a year and the change in the value of the optimally capitalised bank implied by different extensions of the model. In the second column $L_0 = 2.25$ gives the face value of the bank's debt at the base case.

Table 4 reports the leverage ratio of the bank, its franchise value, the target level of cash reserves, and the probability of default over a one-year horizon under alternative specifications for $(T, \varphi, \delta, \xi)$. The table also provides information about the level of deposits and, when analyzing the effects of positive liquidation values for risky assets, we also consider that cases in which there are no deposits (as in Table 2), to make sure that bank incentives with respect to financing decisions reflect the change in liquidation payoffs.

Table 4 delivers several results. First, the presence of a liquidation value has no impact on the bank's optimal capital structure as long as we have both

$$\alpha_0^* \geq (T + \Lambda - D - L)^+$$

so that refinancing the bank is optimal, and

$$T + \Lambda - \alpha_0^* - D < 0$$

so that creditors receive nothing in default despite the presence of a liquidation value. When these constraints do not hold (as in the case without deposits in the table), a positive liquidation value increases the value of the bank increases and the privately optimal coupon level, leading to a drop in leverage ratios and to an increase in the default probability.

Second, the table shows that a bank facing counter-cyclical liquidity requirements adopts riskier policies. Indeed, the table shows that the target level of cash reserves of the bank decreases with δ while its debt ratio increases with δ. Despite these adverse effects on private incentives, the introduction of a forbearance period leads to an increase in the bank franchise value α_0^* and to a decrease in default probabilities. Third, total bank value increases as we decrease the cost of holding liquidity, leading to a reduction in the default probability. Lastly, the table shows that our previous results on the effects of liquidity requirements are robust, in that such requirements have no effects on insolvency risk when $\xi = 0$ (that is the default probabilities do not depend on T for any $T \in [0, D - \Lambda]$).

5 Conclusion

We develop a dynamic model of banking in which banks face taxation, flotation costs of securities, and default costs and are financed with equity, insured deposits, and subordinated, risky debt. In this model, liquidity management, financing policies, and default decisions are jointly and endogenously determined. Shareholders have limited liability and banks' policies maximize shareholder value. Using this model, we show that when raising outside funds is costly, inside and outside equity are not perfect substitutes and banks find it optimal to hold buffers of liquid assets even in the absence of regulation. To reduce the risk of default and save on recapitalization costs, banks manage their liquidity buffers dynamically by adjusting their dividend payments to shareholders. Banks facing higher cost of outside financing are less levered, pay less dividends, hold more liquid reserves, and default earlier.

We also show that liquidity requirements constraining banks to hold a minimum amount of liquid reserves have no long-run effects on default risk but may increase it in the short run. By contrast, we find that leverage requirements, which prescribe how much equity capital banks should have relative to their total assets, increase shareholders' willingness to absorb losses, thereby reducing default risk. *Ex ante*, however, such requirements may significantly reduce total bank value by increasing its cost of capital. In our base environment for example, increasing equity capital from the equity value-maximizing level to 20% to total assets – as suggested by Admati and Hellwig (2013) – reduces total bank value by almost 6%.

We also investigate the effects of mispriced deposit insurance and depositor preference in default on banks' financing decisions and insolvency risk. We show that mispriced deposit insurance generally allows the bank to increase its debt ratio, leading to a increase in insolvency risk. By contrast, depositor preference in default motivates bank shareholders to adopt more conservative financing policies, thereby reducing insolvency risk and aligning private and social interests.

While the framework developed in this paper is based on a number of simplifying assumptions, we believe that it is a natural starting point to think about bank equity capital and liquid reserves and to examine the effects of prudential regulation on bank behavior and insolvency risk. In future research, we plan to extend this framework to examine additional issues related to bank optimal capital structures and asset choices.

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