Macroeconomic Risks and Asset Pricing: Evidence from a Dynamic Stochastic General Equilibrium Model

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Abstract

The relation between macroeconomic fundamentals and the cross section of asset returns is studied through the lens of dynamic stochastic general equilibrium (DSGE) models. We provide a full-information Bayesian estimation of the model using seven macroeconomic variables and extract the time series of three fundamental shocks to the economy for the period of 1966Q1-2010Q3: neutral technology (NT) shock, investment-specific technological (IST) shock, and monetary policy (MP) shock. Tests based on the General Method of Moments (GMM) show that the factor model with estimated latent shocks as risk factors performs better than and the model-implied pricing kernel performs as well as the Fama-French three-factor models at the 5% significance level in explaining the cross-sectional returns of a large set of assets including the 25 size/BM, 48 industry, and 8 bond portfolios. Our results show that DSGE models, which have been successful in matching macroeconomic dynamics, have great potential in capturing asset price dynamics as well.

JEL Classification: G31

Keywords: DSGE model, Bayesian MCMC estimation, stock returns, neutral technology shock, investment-specific technology shock, monetary policy shock

1 Introduction

One of the key issues in asset pricing is to understand the economic fundamentals that drive the fluctuations of asset prices. Modern finance theories on asset pricing, however, have mainly focused on the relative pricing of different financial securities. For example, the well-known Black-Scholes-Merton option pricing model considers the relative pricing of option and stock while taking the underlying stock price as given. The celebrated Capital Asset Pricing Model (CAPM) relates individual stock returns to market returns without specifying the economic forces that drive market returns. Modern dynamic term structure models also mainly focus on the relative pricing of bonds across the yield curve. These models tend to assume that the yield curve is driven by some latent state variables without explicitly modeling the economic nature of these variables.

Increasing attention has been paid in the literature to relate asset prices to economic fundamentals as evidenced by the rapid growth of the macro finance literature. For example, the macro term structure literature has been trying to relate term structure dynamics to macro fundamentals. By incorporating the Taylor rule into traditional term structure models, several studies have shown that inflation and output gap can explain a significant portion of the fluctuations of bond yields. The investment based literature has also tried to relate equity returns to firm fundamentals, thus giving economic meaning to empirical based factors (such as HML and SMB) for equity returns. Current attempts to connect macro variables with asset prices, however, are typically based on partial equilibrium analysis. Without a well specified general equilibrium model, it is not clear that the exogenously specified pricing kernels in these "reduced-form" models are consistent with general equilibrium.

The New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models offer such a framework to understand the link between asset prices and economic fundamentals. DSGE models have become a dominant modeling framework in macroeconomics and have been widely used by both academics and central banks around the world for policy analysis, (see, e.g., Clarida, Galí and

Gertler 2000). However, most existing studies on DSGE models in the macroeconomic literature, such as Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007), have mainly focused on the ability of DSGE models in explaining macroeconomic dynamics. If the fundamental fluctuations identified by DSGE models are true economic risks, they should also explain the movements of asset prices because financial assets represent claims on real productive assets. Therefore, financial prices provide an alternative perspective to examine the soundness of DSGE models. Moreover, since financial prices are forward looking and contain market expectations for future economic activities, one can potentially better identify model parameters and latent shocks by incorporating financial prices in the estimation of DSGE models.

In this paper, we study the link between macroeconomic fundamentals and asset pricing through the lens of New Keynesian DSGE models. In particular, we study whether the model-implied stochastic discount factor (pricing kernel) and latent shocks have any explanatory power of the returns of a wide range of financial assets. We focus on three latent shocks: neutral technology (NT) shock, investment-specific technological (IST) shock, and monetary policy (MP) shocks. These three shocks are the most studied risks in the asset pricing literature (see, for example, Balvers and Huang 2007, Kogan and Papanikolaou 2013, and Bernanke and Kuttner 2005). Our analysis is based on the DSGE model considered in Christiano, Trabandt and Walentin (2011) (CTW), which is closely related to Christiano, Eichenbaum and Evans (2005) and includes all the major ingredients of DSGE models. CTW show that this model matches a wide range of macroeconomic variables well. To our knowledge, this paper provides the first study that examines the ability of DSGE models in explaining the cross-sectional stock and bond returns. Our paper makes two important contributions to the macro literature on DSGE models as well as the asset pricing literature.

First, we develop full-information Bayesian Markov Chain Monte Carlo (MCMC) methods for estimating DSGE models using macroeconomic variables. Whereas the Bayesian moment matching methods in CTW essentially match the unconditional moments of the macro variables, our full-information Bayesian MCMC methods fully exploit the conditional information contained in the likelihood function of the macro data. As a result, our methods provide more efficient estimation of model parameters. More important, our Bayesian MCMC methods make it possible to back out the latent shocks, and hence the pricing kernel, from the economy described by the DSGE model. In contrast, the Bayesian moment matching methods cannot back out the latent shocks because they can only match the long-run average features of the data.

Second, we empirically test whether the model-implied pricing kernel and latent shocks price the cross-section of asset returns. Specifically, we evaluate the performance of three pricing kernel models: the model-implied pricing kernel (Model 1), a linear factor model using the estimated latent shocks as risk factors (Model 2), and the Fama-French three-factor model (Model 3) while the last model is used as the benchmark. The pricing kernel in Model 1 is constructed based on the DSGE model parameters and latent shocks estimated from the Bayesian MCMC method and hence is invariant to test assets. In contrast, the linear coefficients in Model 2 and 3 need to be estimated and their values generally vary with different sets of test assets. As suggested by Lowellen, Nagel and Shanken (2010) and Daniel and Titman (2012), we use a large set of test assets including the 25 size/BM and 48 Fama and French (1997) industry portfolios, U.S. government securities portfolios with maturity of 30, 20, 10, 7, 5, 2, and 1 years, and a long-term corporate bond portfolio.

Model estimation and comparison are conducted based on the General Method of Moments (GMM). Two types of weighting matrices, identity matrix and the variance-covariance matrix of the returns on test assets, are used for robustness and they generate largely consistent results. With the identity weighting matrix, linear coefficients in Models 2 and 3 are chosen to minimize the sum of squared pricing errors (SSPE). The variance-covariance matrix of the returns on test assets as a choice of weighting matrix is suggested by Hansen and Jagannathan (1991, 1997), denoted as the Hansen-Jagannathan (HJ) weighting matrix. With the HJ weighting matrix, coefficients are chosen to minimize the Hansen-Jagannathan (HJ) distance, which measures the least squares distances between the proposed pricing kernel and the set of admissible pricing kernels that can correctly price the return space spanned by the test assets. That is, the HJ-distance is a measure of model misspecification errors. Both weighting matrices are invariant to the estimated pricing kernels and hence can be used for performance comparison of different models. Statistical tests based on SSPE (Hall and Pelletier, 2011) and HJ-distance (Li, Xu and Zhang, 2010) are conducted to compare the performance of the three models for the full set of test assets and three of its subgroups: 25 size/BM portfolios, 48 industry portfolios, and 8 bond portfolios, respectively.

Tests based on GMM with the HJ weighting matrix show that Model 2, the linear factor model with estimated latent shocks as risk factors, is the best model among the three. For the full set of test assets, the HJ-distance of Model 2 is the smallest, followed by those of Model 3, the Fama-French three-factor model, and Model 1, the model-implied pricing kernel. The p-values under the null hypotheses that Models 2 and 3 and Models 1 and 3 have equal HJ-distances are 4.97% and 5.24%, respectively. Therefore, at the 5% significance level, the model with estimated latent shocks prices the cross section of asset returns better than the Fama-French three-factor model, while the model-implied pricing kernel performs as well as the Fama-French three-factor model. For the three subgroups of test assets, Model 2 has the smallest HJ-distances for the 48 industry portfolios and the 8 bond portfolios, while the Fama-French three-factor model has the smallest HJ-distance for the 25 size/BM portfolios. However, the differences in HJ-distances among the three models are mostly statistically insignificant for those subgroups.

Tests based on GMM with identity weighting matrix present a similar picture on the comparison of the three models. We use the J statistic, measured as the sum of squared pricing errors scaled by their variance-covariance matrix, to evaluate the overall goodness of fit of the proposed pricing kernel models. For the full set of test assets, Model 2 has the smallest and Model 1 has the largest J values. The p-values of the null hypotheses that Models 2 and 3 and Models 1 and 3 have equal SSPE's are 30.04% and 8.87%. Moreover, Model 2 has the smallest J values for all three subgroups of test assets. However, none of the differences in SSPE's among the three models are statistically significant.

Our result is a testament of the power of the DSGE approach. Note that the latent shocks are extracted from macroeconomic data only and the model-implied pricing kernel is invariant to test assets. It is thus striking that the model-implied pricing kernel explains the cross section of asset returns as well as the Fama-French three-factor model. And the model with estimated latent shocks as risk factors performs even better than the Fama-French three-factor model. Our results highlight the possibility of integrating macroeconomics and asset pricing under a unified modeling framework.

Our paper is most related to Smets and Wouters (2007), who estimate a similar DSGE model with seven latent shocks using Bayesian MCMC method. However, their focus is to match and forecast macroeconomic dynamics. Moreover, Smets and Wouters (2007) estimate a log-linearized model, which is not suitable to study asset pricing questions, while we estimate a model solved to the second-order. Our model setup is the same as the one in Christiano, Trabandt and Walentin (2011) and closely related to Christiano, Eichenbaum and Evans (2005). The focus of these two papers are on examining what economic mechanisms, such as wage rigidities, working capital, and variable capital utilization, are important for capturing the observed macroeconomic dynamics while our focus is on the asset pricing implications of the DSGE model.

Our paper is also related to the literature that aims to explain cross-sectional stock returns using macroeconomic variables, pioneered by Chen, Roll and Ross (1986). The most related ones are Balvers and Huang (2007), Papanikolaou (2011) and Kogan and Papanikolaou (2013), and Bernanke and Kuttner (2005), who study the asset pricing implications of the NT, IST, and MP shocks, respectively. The three shocks in those papers are constructed independent of each other while our three latent shocks are simultaneously constructed from a general equilibrium framework. Moreover, the focus of our paper is not on the asset pricing implications of specific latent shocks, instead is on the methodology that estimates latent shocks, not limited to these three studied in the current paper, and the model-implied stochastic pricing kernels based on macroeconomic variables and Bayesian MCMC methods.

The rest of the paper is organized as follows. Section 2 introduces the DSGE model. Section 3 discusses the full-information Bayesian estimation methods and model implications on asset prices. Section 4 empirically examines the asset pricing implications of the model-implied pricing kernel and risk factors in the cross section. Section 5 concludes.

2 The Model

The DSGE model that we estimate is taken from CTW. The modeled economy contains a perfectly competitive final goods market, a monopolistic competitive intermediate goods market, and households who derive utility from final goods consumption and disutility from supplying labor to production. Nominal price rigidities and wage rigidities in the intermediate goods market are modeled as in Calvo (1983). Government consumes a fixed fraction of GDP every period and the monetary authority sets the nominal interest rate according to a Taylor rule. There are three exogenous shocks in the economy: neutral technology (NT) shocks, investment-specific technological (IST) shocks, and monetary policy (MP) shocks. CTW show that the model matches vey well an important set of macroeconomic variables including: changes in relative prices of investment, real per hour GDP growth rate, unemployment rate, capacity utilization, average weekly hours, consumption-to-GDP ratio, investment-to-GDP ratio, job vacancies, job separation rate, job finding rate, weekly hours per labor force, and Federal Funds Rates. Next, we present the model in details.

2.1 Production sector

There are two industries in the production sector, final goods industry and intermediate goods industry. The production of the final consumption goods uses a continuum of intermediate goods, indexed by $i \in [0,1]$, via the Dixit-Stiglitz aggregator

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}}\, di\right]^{\lambda_f}, \quad \lambda_f > 1\,, \tag{1}$$

where Y_t is the output of final goods, $Y_{i,t}$ is the amount of intermediate goods i used in the final goods production, which in equilibrium equals the output of intermediate goods i, and λ_f measures the substitutability among different intermediate goods. The larger λ is, the more substitutable the intermediate goods are. Since the final goods industry is perfectly competitive, profit maximization leads to the demand function for intermediate goods i:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}}, \tag{2}$$

where P_t is the nominal price of the final consumption goods and P_{it} is the nominal price of intermediate goods i. It can be shown that goods prices satisfy the following relation:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\lambda_f - 1}} di\right)^{-(\lambda_f - 1)}. \tag{3}$$

The production of intermediate goods i employs both capital and labor via the following homogenous production technology

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} K_{i,t}^{\alpha} - z_t^{+} \varphi\,, \tag{4}$$

where z_t is the neutral technology shock, $H_{i,t}$ and $K_{i,t}$ are the labor service and capital service, respectively, employed by firm i, α is the capital share of output, and φ is the fixed production cost. Finally, z_t^+ is defined as

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t\,,$$

where Ψ_t is the investment-specific technology shock, measured as the relative price of consumption goods to investment goods. Assume that z_t and Ψ_t evolve as follows:

$$\mu_{z,t} = \mu_z + \rho_z \mu_{z,t-1} + \sigma_z e_t^z, \quad \text{where } \mu_{z,t} = \Delta \log z_t, \; e_t^z \sim \text{IID}\mathcal{N}(0,1), \tag{5}$$

$$\mu_{\psi,t} = \mu_\psi + \rho_\psi \mu_{\psi,t-1} + \sigma_\psi e_t^\psi, \quad \text{where } \mu_{\psi,t} = \Delta \log \Psi_t, \; e_t^\psi \sim \text{IID}\mathcal{N}(0,1). \tag{6}$$

The intermediate goods industry is assumed to have no entry and exit, which is ensured by choosing a fixed cost ψ that brings zero profits to the intermediate goods producers in the steady state.

Intermediate goods producer i rents capital service K_{it} from households and its net profit at period t is given by $P_{it}Y_{it} - r_t^K K_{it} - W_t H_{it}$. The producer takes the rent of capital service r_t^K and wage rate W_t as given but has market power to set the price of its goods in a Calvo (1983) staggered price setting to maximize its profits. With probability ξ_p, producer i cannot reoptimize its price and has to set its price according to the following rule,

$$P_{i,t} = \pi P_{i,t-1}$$

and with probability $1 - \xi_p$, producer i sets price $P_{i,t}$ to maximize its profits, i.e.,

$$\max_{\{P_{i,t}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\xi_p \beta)^\tau \nu_{t+\tau} \left[P_{i,t} Y_{i,t+\tau\,|\,t} - W_{t+\tau} H_{t+\tau\,|\,t}\right] \tag{7}$$

subject to the demand function in equation (2). In the above objective function, $\nu_{t+\tau}$ is the marginal utility of nominal wage and $Y_{i,t+\tau\,|\,t}$ and $H_{t+\tau\,|\,t}$ refer to the output and labor hiring, respectively, by producer i at time $t+\tau$ if the last time when price P_i is reoptimized is period t.

2.2 Households

Following CTW, we assume that there is a continuum of differentiated labor types indexed with j and uniformly distributed between zero and one. A typical household has infinitely many members covering all the labor types. It is assumed that a household's consumption decision is made based on utilitarian basis. That is, every household member consumes the same amount of consumption goods even though they might have different status of employment. CTW show that a representative household's life-long utility can be written as

$$\sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t - b\, C_{t-1}\right) - A_L \int_0^1 \frac{h_{jt}^{1+\phi}}{1+\phi} \right], \tag{8}$$

subject to the budget constraint

$$P_t \left(C_t + \frac{I_t}{\Psi_t} \right) + B_{t+1} + P_t P_{k',t} \Delta_t \le \int_0^1 W_{jt} h_{jt}\, dj + X_t^K \bar{K}_t + R_{t-1} B_t \tag{9}$$

for $t = 0, 1, \cdots, \infty$. Here, h_{jt} is the number of household members with labor type j who are employed, B_t is the nominal bond holdings purchased by household at $t-1$, $P_{k',t}$ is the market price of one unit capital stock in real term, Δ_t is the amount of capital purchased from the market, X_t^K is the net cash payment to the household by renting out capital $\bar{K}_t$, given by

$$X_t^K = P_t \left[u_t\, r_t^K - \frac{a(u_t)}{\Psi_t} \right].$$

The wage rate of labor type j is determined by a monopoly union who represents all j-type workers and households take the wage rate of each labor type as given.

Households own the economy's physical capital $\bar{K}$. The amount of capital service K_t available for production is given by

$$K_t = u_t\, \bar{K}_t\,,$$

where u_t is the utilization rate of physical capital and utilization incurs a maintenance cost

$$a(u) = b\,\sigma_a\,u^2/2 + b(1-\sigma_a)\,u + b\,(\sigma_a\,/2 - 1)\ , \tag{10}$$

where b and σ_a are constants and chosen such that steady state utilization rate is one and at the steady state $a(u=1)=0$. Note that the maintenance cost $a(u)$ is measured in terms of capital goods, whose relative price to consumption goods is $1/\Phi_t$. A representative household accumulates capital stock according to the following rule:

$$\bar{K}_{t+1} = (1-\delta)\,\bar{K}_t + F(I_t, I_{t-1}) + \Delta_t\,,$$

where Δ_t is the capital stock purchased by the representative household and equals zero in equilibrium because all households are identical. Here, $F(I_t, I_{t-1})$ is the investment adjustment cost, defined as

$$F(I_t, I_{t-1}) = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$$

and

$$S(x_t) = \frac{1}{2}\left\{\exp\left[\sigma_s\left(x_t - \exp(\mu_z^+ + \mu_\psi)\right)\right] + \exp\left[-\sigma_s\left(x_t - \exp(\mu_z^+ + \mu_\psi)\right)\right] - 2\right\}\ ,$$

where $x_t = I_t/I_{t-1}$ and $\exp(\mu_z^+ + \mu_\psi)$ is the steady state growth rate of investment. The parameter σ_s is chosen such that at the steady state $S(\exp(\mu_z^+ + \mu_\psi)) = 0$ and $S'(\exp(\mu_z^+ + \mu_\psi)) = 0$. Note that investment I_t is measured in terms of capital goods. The consumption goods market clearing is then given by

$$Y_t = C_t + G_t + \tilde{I}_t\,,$$

where G_t is government spending and $\tilde{I}$ is investment measured in consumption goods, which also includes the capital maintenance cost $a(u_t)$, i.e.,

$$\tilde{I} = \frac{I_t + u(a_t)}{\Phi_t}.$$

2.3 Labor unions

There are labor contractors who hire all types of labor through labor unions and produce a homogenous labor service H_t, according to the following production function

$$H_t = \left[\int_0^1 h_{jt}^{\frac{1}{\lambda_w}}\, dj\right]^{\lambda_w}, \quad \lambda_w > 1, \tag{11}$$

where λ_w measures the elasticity of substitution among different labor types. The intermediate goods producers employ the homogenous labor service for production. Labor contractors are perfectly competitive, whose profit maximization leads to the demand function for labor type j

$$h_{jt} = H_t \left(\frac{W_{jt}}{W_t}\right)^{\frac{-\lambda_w}{\lambda_w - 1}} \tag{12}$$

It is easy to show that wages satisfy the following relation:

$$W_t = \left(\int_0^1 W_{j,t}^{-\frac{1}{\lambda_w - 1}} dj\right)^{-(\lambda_w - 1)}, \tag{13}$$

where $W_{j,t}$ is the wage of labor type j and W_t is the wage of the homogenous labor service.

Assume that labor unions face the same Calvo (1983) type of wage rigidities. Each period, with probability ξ_w, labor union j cannot reoptimize the wage rate of labor type j and has to set the wage rate according to the following rule

$$W_{jt+1} = \pi_t \mu_{z^+}$$

and with probability $1-\xi_w$, labor union j chooses W_{jt} to maximize households' utility

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\xi_w)^\tau \left[\nu_{t+\tau} W_{jt} h_{t+\tau\,|\,t} - A_L \frac{h_{jt+\tau\,|\,t}^{1+\phi}}{1+\phi}\right] \tag{14}$$

subject to the demand curve for labor type j in equation (12). Here, $h_{jt+\tau\,|\,t}$ is the supply of type j labor at period $t+\tau$ if the last time when labor union j reoptimizes wage rate W_{jt} is period t.

2.4 Fiscal and monetary authorities

Following CTW, fiscal authority in the model simply transfers a fixed fraction g of output as government spending, i.e.,

$$G_t = g\,Y_t\,.$$

Monetary authority sets the level of the short-term nominal interest rate according to the following Taylor rule

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \rho_y \log\left(\frac{Y_t}{Y}\right)\right] + V_t. \tag{15}$$

where R_t is the short-term interest rate, R, π, and Y are steady state values for interest rate, inflation, and output, and V_t is the monetary policy shock, which follows the process

$$V_t = \rho_V V_{t-1} + \sigma_V e_t^V, \tag{16}$$

with $e_t^V \sim \mathrm{IID}\mathcal{N}(0,1)$.

2.5 Model implications on asset prices

Household's utility maximization gives us the stochastic discount factor as follows:

$$M_t = \beta \exp\left(-\Delta c_{t+1} - \Delta q_{t+1}\right) , \tag{17}$$

where q_t is defined as

$$\exp(-q_t) = \frac{1}{1 - b/\Delta c_t} - \mathbb{E}_t\left[\frac{b}{\Delta c_{t+1} - b}\right] .$$

The return on any asset i, R_{it+1}, at $t+1$ satisfies the following Euler equation

$$\mathbb{E}_t\left[R_{it+1} M_{t+1}\right] = 1 .$$

3 Full-Information Bayesian MCMC Estimation

In this section, we develop full-information Bayesian MCMC method for estimating the aforementioned DSGE model based on observed macroeconomic variables. We choose seven macroeconomic variables following Smets and Wouters (2007): per capita output growth (dy), per capita consumption growth (dc), per capita investment growth (di), wage growth (dw), logarithm of inflation (π), logarithm of 3-month T-Bill rate (r), and logarithm of average weekly hours per capita (h) with average normalized as 1. The three fundamental exogenous shocks are neutral technology shocks $\{\mu_{z,t}\}$, investment-specific technology shocks $\{\mu_{\psi,t}\}$ and monetary policy shocks $\{V_t\}$, defined in equations (5), (6), and (16). Given the initial states, the time-series of the aforementioned three exogenous shocks completely determine the outcome of the economy.

3.1 Solution of the model

Our goal is to solve and estimate the economic system described in Section 2 using the actual economic outcomes observed in the data. The model is solved in Dynare [1] to the second order approximation and the details of the solution are provided in the appendix of CTW. Define the vector of state variables at time t as S_t, the vector of endogenous variables that we would like to match with actual data as Υ_t, and the the vector of exogenous shocks as $U_t = \{e_t^z, e_t^\psi, e_t^V\}$. State variables and the variables to be matched with data evolve according the following rules, respectively,

$$\begin{aligned} S_t &= \Gamma^S\left(S_{t-1}, U_t, \mathbf{\Theta}\right), \\ \Upsilon_t &= \Gamma\left(S_{t-1}, U_t, \mathbf{\Theta}\right) \end{aligned} \tag{18}$$

where $\mathbf{\Theta}$ is the vector of model parameters

$$\mathbf{\Theta} = [\beta, \phi, b, \alpha, \delta, \eta_g, \xi_p, \xi_w, K, \lambda_f, \lambda_w, \sigma_a, \sigma_s, \pi_{ss}, \rho_k, \rho_\pi, \rho_y, m_z, \mu_\psi, \sigma_z, \sigma_\psi, \sigma_v, \rho_z, \rho_\psi, \rho_v]$$

For any given initial values of state variables S_0 and exogenous shocks U_0 and the time-series of exogenous shocks, $\{U_s\}_{s=1}^t$, state variables can be obtained by iterating on equation (18), written as

$$S_t = \Gamma^S\left(\Gamma^S\left(\cdots\Gamma^S\left(\Gamma^S\left(S_0, U_0, \mathbf{\Theta}\right), U_1, \mathbf{\Theta}\right)\cdots\right), U_t, \mathbf{\Theta}\right) \equiv \Gamma^{S,(t)}\left(S_0, \{U_s\}_{s=1}^t, \mathbf{\Theta}\right).$$

Thus, the model-implied variables Υ_t can be easily computed as $\Upsilon_t = \Gamma\left(S_{t-1}, U_t, \mathbf{\Theta}\right)$. Our goal is to choose model parameters $\mathbf{\Theta}$ and latent variables $\{U_t\}_{t=1}^T$ such that the model-implied Υ_t is as close to the corresponding values in actual data, Υ_t^{obs}, as possible. The functions Γ and Γ^S are second-order polynomials of S_t and U_t given by Dynare, where the coefficients in these polynomials

[1] Please find detailed information on Dynare at www.dynare.org.

are determined by the values of the parameter set $\mathbf{\Theta}$. [2]

3.2 Full-information Bayesian estimation

Define the time series of observable variables as $\mathbf{\Upsilon}^{obs} = \left\{\Upsilon_t^{obs}\right\}_{t=1}^{T}$ and assume Υ_t^{obs} are observed with independent pricing errors

$$\Upsilon_t^{obs} = \Upsilon_t + \varepsilon_t = \Gamma(X_{t-1}, U_t, \mathbf{\Theta}) + \varepsilon_t$$

where $\Upsilon_t^{obs} = \{dy, dc, di, dw, \pi, r, h\}$, and $\varepsilon_t = \{\varepsilon_{1t}, \cdots, \varepsilon_{7t}\}$ with $\varepsilon_{it} \sim N(0, \sigma_i^2)$ for $i = 1, \cdots, 7$. Υ_t is the model implied values from the $\Gamma(\cdot)$ function that is solved numerically using Dynare package, and the dynamics of U_t is determined through the following evolution equations

$$\begin{cases} \mu_{z,t} = \mu_z(1-\rho_z) + \rho_z\mu_{z,t-1} + \sigma_z e_t^z \\ \mu_{\psi,t-1} = \mu_\psi(1-\rho_\psi) + \rho_\psi\mu_{\psi,t-1} + \sigma_\psi e_t^\psi \\ V_t = \rho_V V_{t-1} + \sigma_V e_t^V \end{cases} .$$

Since U_t $(t = 1, \cdots, T)$ can be uniquely specified by the sequence $(\mu_{z,t}, \mu_{\psi,t}, V_t)$, the main objective of our analysis is to estimate the model parameters, σ_i $(i = 1, \cdots, 7)$ and $\mathbf{\Theta}$, and latent state variables, defined as $\mathbf{X^e} = \{X_t^e\}_{t=1}^T$ where $X_t^e = [\mu_{z,t}, \mu_{\psi,t}, V_t]$, using observation Υ_t^{obs} $(t = 1, \cdots, T)$. The biggest challenge of the analysis is that the marginal likelihood based on parameters has to be obtained by integrating out high dimensional variables (of the order of $3 \times T$ dimension due to latent state variables), creating extremely heavy computing burdens. Bayesian MCMC methods can be used to estimate parameters and latent variables in this situation. In contrast to classical statistical theory, which uses the likelihood $L(\mathbf{\Theta}) \equiv p(\mathbf{\Upsilon}^{obs}|\mathbf{\Theta})$, Bayesian inference adds to the likelihood function the prior distribution for $\mathbf{\Theta}$, called $\pi(\mathbf{\Theta})$. The distribution of $(\mathbf{\Upsilon}^{obs}, \mathbf{X^e})$ and $\pi(\mathbf{\Theta})$ combine to provide a joint distribution for $(\mathbf{\Upsilon}^{obs}, \mathbf{X^e}, \mathbf{\Theta})$ from which the posterior distribution of

[2] The details of the solution method can be found on www.dynare.org.

$(\boldsymbol{\Theta}, \mathbf{X^e})$ given $\boldsymbol{\Upsilon}^{obs}$ is produced

$$p(\boldsymbol{\Theta}, \mathbf{X^e}|\boldsymbol{\Upsilon}^{obs}) = \frac{p(\boldsymbol{\Upsilon}^{obs}, \mathbf{X^e}, \boldsymbol{\Theta})}{\int p(\boldsymbol{\Upsilon}^{obs}, \mathbf{X^e}, \boldsymbol{\Theta}) d\mathbf{X^e} d\boldsymbol{\Theta}} \propto p(\boldsymbol{\Upsilon}^{obs}, \mathbf{X^e}, \boldsymbol{\Theta}) \,.$$

In our context, it is

$$\begin{aligned}
p(\boldsymbol{\Theta}, \mathbf{X^e}|\boldsymbol{\Upsilon}^{obs}) &\propto p(\boldsymbol{\Upsilon}^{obs}|\mathbf{X^e}, \boldsymbol{\Theta}) \times p(\mathbf{X^e}|\boldsymbol{\Theta}) \times \pi(\boldsymbol{\Theta}) \\
&= p(\Upsilon_1^{obs}|\mathbf{X^e}, \boldsymbol{\Theta}) \times p(\Upsilon_2^{obs}|\Upsilon_1^{obs}, \mathbf{X^e}, \boldsymbol{\Theta}) \times \cdots \times p(\Upsilon_T^{obs}|[\Upsilon_1^{obs}, \cdots \Upsilon_{T-1}^{obs}], \mathbf{X^e}, \boldsymbol{\Theta}) \\
&\quad \times p(\mathbf{X^e}|\boldsymbol{\Theta}) \times \pi(\boldsymbol{\Theta}) \\
&\propto \prod_{t=1}^{T} \prod_{i=1}^{7} \frac{1}{\sigma_i} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
&\quad \times \prod_{t=1}^{T} \frac{1}{\sigma_z} \exp\{-\frac{1}{2\sigma_z^2}[\mu_{z,t} - \mu_z(1-\rho_z) - \rho_z \mu_{z,t-1}]^2\} \\
&\quad \times \prod_{t=1}^{T} \frac{1}{\sigma_\psi} \exp\{-\frac{1}{2\sigma_\psi^2}[\mu_{\psi,t} - \mu_\psi(1-\rho_\psi) - \rho_\psi \mu_{\psi,t-1}]^2\} \\
&\quad \times \prod_{t=1}^{T} \frac{1}{\sigma_V} \exp\{-\frac{1}{2\sigma_V^2}[V_t - \rho_V V_{t-1}]^2\} \times \pi(\boldsymbol{\Theta}) \,,
\end{aligned}$$

where $\Upsilon_t^{obs}(i)$ and $\Upsilon_t(i)$ are the observed and model implied values for the i^{th} macroeconomic variable. In general, it is difficult to simulate directly from the above high dimensional posterior distribution. The theory underlying the MCMC algorithms is called Clifford-Hammersley Theorem that can be used to ease computational burden. This theorem states that the joint distribution $p(\boldsymbol{\Theta}, \mathbf{X^e}|\boldsymbol{\Upsilon}^{obs})$ can be represented by the complete conditional distributions $p(\boldsymbol{\Theta}|\mathbf{X^e}, \boldsymbol{\Upsilon}^{obs})$ and $p(\mathbf{X^e}|\boldsymbol{\Theta}, \boldsymbol{\Upsilon}^{obs})$. MCMC algorithm is done iteratively. In each iteration, each parameter is updated based on most recent value of all other parameters and latent variables through sampling from the corresponding complete conditional distribution, and the latent variables at each time t is also updated in the similar fashion. As this is done, the chains converge (theoretically) to the target posterior distribution. Therefore, after a sufficient number of samples, called a burn-in period, the algorithm is then sampling from a converged target posterior distribution.

To find parameter estimates requires some additional machinery. Use of calculus methods will only work nicely if the prior distributions are conjugate priors, leading to tractable solutions. However, in our analysis here, parameters and latent variables are involved into the likelihood through the Dynare package. The solution through the Dynare package is done numerically, resulting in intractable posterior distributions. We therefore turn to Metropolis Hastings Algorithm (MH) for updating both $\mathbf{\Theta}$ and $\mathbf{X^e}$. The MH algorithm is an adaptive rejection sampling method where candidate draw is proposed and then accepted with probability proportional to the ratio of the likelihood of the proposed draw to the current draw. This means that if the new position has a higher likelihood (defined using the posterior distribution), then the parameter values are updated with probability 1. Alternatively, if they are less likely, the parameter values are updated with probability according to the likelihood ratio. Thus the parameter values will tend to stay near the highest probability regions when being sampled and adequately cover the probability space. Given a vector of starting values for the parameters and latent variables, the steps are as follows.

For the j^{th} parameter θ_j in $\mathbf{\Theta}$ $(j = 1, ..., 25)$ in the g^{th} iteration:

- Step 1. Specify a candidate distribution, $h(\theta_j|\theta_j^{(g-1)})$;
- Step 2. Generate a proposed value for parameters, $\theta_j^* \sim h(\theta_j|\theta_j^{(g-1)})$;
- Step 3. Compute the acceptance ratio

$$r_g = \frac{p(\theta_j^*|\theta_{[-j]}, \mathbf{X^e}, \mathbf{\Upsilon}^{obs}) \times h(\theta_j^*|\theta_j^{(g-1)})}{p(\theta_j^{(g-1)}|\theta_{[-j]}, \mathbf{X^e}, \mathbf{\Upsilon}^{obs}) \times h(\theta_j^{(g-1)}|\theta_j^*)}$$

  where $p(\cdot|\theta_{[-j]}, \mathbf{X^e}, \mathbf{\Upsilon}^{obs})$ represents a complete conditional distribution for parameter θ_j and the notation $\theta_{[-j]}$ contains the most updated parameters except for θ_j in $\mathbf{\Theta}$ (see details in the Appendix);

- Step 4. Generate u from an uniform distribution $Uniform[0, 1]$, then set

$$\theta_j^{(g)} = \begin{cases} \theta_j^* & if \quad r_g \geq u \\ \theta_j^{(g-1)} & if \quad r_g < u \end{cases} ;$$

- Step 5. Do this for all θ_j for $j = 1, ..., 25$. Then set g = g + 1 and return to Step 1.

If the candidate distribution is symmetric, the MH algorithm has acceptance ratio equivalent to $p\left(\theta_j^*|\theta_{[-j]}, \mathbf{X^e}, \mathbf{\Upsilon}^{obs}\right) / p\left(\theta_j^{(g-1)}|\theta_{[-j]}, \mathbf{X^e}, \mathbf{\Upsilon}^{obs}\right)$. In implementation, we chose $N(\theta_j^{(g-1)}, c^2)$ with a constant variance c^2 as a candidate distribution for $h(\theta_j|\theta_j^{(g-1)})$. The MH algorithm is conducted iteratively on each parameter in $\mathbf{\Theta}$ and on each latent variable at each time point $t = 1, \cdots, T$. In estimation, we draw posterior samples using the above described MCMC, and use the means of the posterior draws as parameter estimates and the standard deviations of the posterior draws as standard errors of the parameter estimates after a burn-in period. Detailed description about the priors, posterior distributions, and the updating procedures for parameters and latent variables in our model are provided in Appendix A.

3.3 Goodness of model fit

Table 1 presents the estimated posterior means and standard errors of model parameters, which are largely consistent with what CTW find in their estimation. Figure 1 plots the time-series of model-implied and empirical macroeconomic variables. The model-implied output growth (dy), investment growth (di), hours (h), risk-free rate (r), inflation (π) measure very well with the empirical counterparts. However, the model-implied consumption growth (dc) and wage growth (dw) cannot match the volatile high frequency movement in the data, even though both variables capture the low-frequency movements.

To quantify the goodness of our model fit, we regress the empirical variable on its model-implied counterpart, i.e., $X_{i,t} = \alpha_i + \beta_i \hat{X}_{i,t} + \epsilon_{i,t}$. Panel A of Table 2 reports the coefficient estimates and the adjusted R-squared, $\bar{R}^2$. All the beta coefficients are highly significant, indicating a significant correlation between the model-implied seven time-series with their empirical counterparts. The adjusted R-squared presents a similar picture as Figure 1. The model is able to capture 99% of the variations in the three-month T-Bill rate, 93% in hours, 85% in inflation, 63% in output growth and 62% in investment growth, however, can only capture 15% of the variation in consumption growth and 11% in wage growth. Overall, the model-implied variables match the data pretty well, especially given that there are only three latent shocks. We expect the model fit to be significantly improved with additional latent shocks as in Smets and Wouters (2007).

4 Empirical Analysis

With the estimated parameter values and the time series of latent shocks, the stochastic discount factor (pricing kernel) can be computed for our sample period of 1966Q1:2010Q3 based on equation (17). In this section, we test whether the model implied pricing kernel and the latent shocks can explain the cross-section of asset returns. For comparison, we use the Fama and French three-factor model (FF3) as the benchmark. In particular, we examine three asset pricing models:

$$\begin{aligned}
\text{Model 1}: &\quad m_t^1 = m_t^{model}\,, \\
\text{Model 2}: &\quad m_t^2 = b_2 + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V\,, \\
\text{Model 3}: &\quad m_t^3 = b_3 + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}\,,
\end{aligned}$$

where m_t is the empirical pricing kernel and b's are constant to be estimated. The pricing kernels in Model 1 to 3 are the model-implied pricing kernel m_t^{model}, a linear combination of the estimated latent shocks $\{e_t^z, e_t^\psi, e_t^V\}$, and a linear combination of the Fama-French market, size, and value factors $\{r_{mkt,t}, r_{size,t}, r_{hml,t}\}$ (the so-called Fama-French three-factor model), respectively. Since the pricing kernel in Model 1 is constructed based on the DSGE model parameters and latent shocks, it is invariant to test assets. In contrast, the linear coefficients in Models 2 and 3 need to be estimated using moment conditions defined by equation (19), which will be discussed below, and generally vary with the specific test assets used. To estimate the pricing kernels in Models 2 and 3 and compare the performance of the aforementioned three asset pricing models, we take the approach of generalized method of moments (GMM).

Assume that we have n test assets to be priced. The asset pricing models imply

$$\mathbb{E}\left[m_{t+1}^{i}(b)R_{t+1}\right] = \mathbb{E}[P_t]\,, \tag{19}$$

where m_{t+1}^{i} is the stochastic pricing kernel in Model i, $R_{j,t+1}$ is the return vector on test assets, and P_t is the corresponding payoff that equals one if R_{t+1} is gross return and zero if R_{t+1} is excess return. Define a vector of pricing error $u_t \equiv m_{t+1}^{i}(b)R_{t+1} - P_t$ and its sample mean $g_T \equiv \frac{1}{T}\sum_{t=1}^{T} u_t$. The GMM estimate of the parameter vector b minimizes a quadratic form of the sample mean of the pricing errors (Cochrane, 2005)

$$\hat{b} = \text{argmin}_{\{b\}} g_T(b)'\, W\, g_T(b)$$

where W is the weighting matrix.

The issue is how to choose the weighting matrix W. There are three widely used weighting matrices in the empirical asset pricing literature. The so-called optimal weighting matrix is the inverse of the sample variance-covariance matrix of the pricing errors. Hansen (1982) shows that this weighting matrix gives the smallest asymptotic covariances of the estimated parameters. However, this weighting matrix depends on the parameter estimates and varies with the estimated asset pricing models. Thus, it cannot be used to make model comparisons and does not serve our purpose. Cochrane (1996) advocates the use of identity matrix as the weighting matrix, which is equivalent to minimizing the sum of squared pricing errors (SSPE) for the given set of test assets. Besides its implementability for model comparison, this method provides the best graphical representation of predicted returns on the test assets versus their average returns. However, as

Kandel and Stambaugh (1995) point out, as long as the pricing errors are not zero, one can find a linear reformation of the test assets that have arbitrarily large or small pricing errors. In another words, the estimated pricing kernel is not robust to portfolio reformation.

An alternative weighting matrix is proposed by Hansen and Jagannathan (1997) that does not vary with the estimated asset pricing models. Hansen and Jagannathan (1997) develop a distance metric, referred as the HJ-distance, to measure the distance between the proposed pricing kernel and the set of correct pricing kernel. They show that the HJ-distance (δ) can be expressed as [3]

$$\delta = \left\{ \mathbb{E}\left[m^i(b)R - P\right]' \mathbb{E}[RR']^{-1} \mathbb{E}[m^i(b)R - P] \right\}^{1/2} . \tag{20}$$

The sample counterpart of equation (20) is given by $\delta_T = [g_T(b)' \, E_T[RR']^{-1} \, g_T(b)]^{1/2}$ where $E_T[RR'] = RR'/T$ is the sample estimate of $E[RR']$. Hansen and Jagannathan (1997) note that parameter vector b can be chosen to minimize δ. Thus, the model estimation problem becomes

$$\hat{b} = \text{argmin}_{\{b\}} g_T(b)' \, E_T[RR']^{-1} \, g_T(b) \, . \tag{21}$$

It is clear that equation (21) is a standard GMM problem with weighting matrix being $W = E_T[RR']^{-1}$, referred as the Hansen-Jagannathan (HJ) weighting matrix.

The economic interpretation of the HJ-distance is that δ is the maximum pricing error for the asset space expanded by the set of test assets with the norm of individual asset return equal to one. The value of δ is invariant to portfolio reformation of test assets. To see this, form a portfolio of test assets with return $\tilde{R} = \lambda R$ and payoff $\tilde{P} = \lambda P$ with nonsingular matrix λ. The HJ-distance

[3] Please refer to Hansen and Jagannathan (1997) for the detailed description of the HJ-distance and the derivation of equation (20).

associated with the new portfolio is

$$\left\{\mathbb{E}\left[m^i(b)\lambda R - \lambda P\right]' \mathbb{E}[\lambda R R'\lambda']^{-1}\mathbb{E}[m^i(b)\lambda R - \lambda P]\right\}^{1/2}$$
$$= \left\{\mathbb{E}\left[m^i(b)R - P\right]' \mathbb{E}[RR']^{-1}\mathbb{E}[m^i(b)R - P]\right\}^{1/2} .$$

Therefore, the HJ-distance approach does not suffer from the Kandel and Stambaugh (1995) critique. A potential problem with the HJ-distance is that $E[R'R]$ might be near singular in which case inversion is problematic (Cochrane, 2005). In our estimations, we do not encounter inversion problem.

In our empirical analysis, both the identity matrix and the HJ weighting matrix are used in model estimation and evaluation for robustness. To compare the performance of the three models, we conduct two model selection tests. The first one is the the model selection test for strictly non-nested (Models 1 and 3 in our case) and overlapping models (Models 2 and 3) based on HJ-distance, developed by Li, Xu and Zhang (2010). The relative performance of two models are evaluated based on the difference in their HJ-distances. The second test is developed by Hall and Pelletier (2011) for models estimated via GMM with identify weighting matrix. The relative performance of the competing models are evaluated on the differences in their $SSPE$'s. [4]

Our test assets include the commonly used 25 Fama-French size/BM portfolios with additional 48 Fama and French (1997) industry portfolios, 7 portfolios of the U.S. Treasury securities with maturities of 30 years, 20 years, 10 years, 7 years, 5 years, 2 years and 1 year, a portfolio of long-term corporate bonds, and risk-free rate. [5] Lowellen, Nagel and Shanken (2010) and Daniel and Titman (2012) show that returns of the 25 Fama-French size/BM portfolios can be easily explained as long as the risk factors under interest correlate with Fama-French size and book-to-

[4]Details on these two tests can be find in Theorem 2 in Li, Xu and Zhang (2010) and Theorem 3 in Hall and Pelletier (2011).

[5]The industry of Healthcare only starts from 1969 with three firms initially while the rest of our data starts from 1966. In our main results, we discard the return data from Healthcare industry. For robustness, we conduct all the tests based on the sample starting from 1969 with the data from Healthcare industry included. The results are largely consistent.

market factors because of the factor structure in those 25 portfolios returns. Both papers suggest to include other portfolios in the test, such as industry and bond portfolios. We also add risk-free rate to test assets so that the estimated pricing kernels give the average risk-free rate. [6] Returns on 25 size/BM portfolios, 48 industry portfolios and risk-free rates are taken from Kenneth French's website. Returns on portfolios of U.S. Treasury securities with various maturities are from CRSP U.S. Treasury and Inflation Indexes and returns on the portfolio of long-term corporate bonds are from Amit Goyal's website. [7]

Table 3 presents the results of GMM estimation with the HJ weighting matrix. Panel A shows the estimated parameter values in the pricing kernels, the HJ-distances, and the p-values of test statistics based on the full set of test assets. The HJ-distance measures the model misspecification error of the proposed model. Among the three models, the factor model with estimated latent shocks as risk factors (Model 2) has the smallest HJ-distance, $\delta_2 = 1.37$, and the model-implied pricing kernel (Model 1) has the largest HJ-distance, $\delta_2 = 1.44$. The Fama-French three factor model (Model 3) has a HJ-distance of 1.41. The p-values of the hypotheses that the HJ-distances of Models 1 and 3 and Models 2 and 3 are equal, i.e., $\delta_1 = \delta_3$ and $\delta_2 = \delta_3$, is 5.24% and 4.97%, respectively. That is, at the 5% significance level, the model-implied pricing kernel performs as well as the Fama-French three-factor model and the model with estimated latent shocks as risk factors performs better than the Fama-French three-factor model. However, the null hypothesis that the HJ-distance of the proposed pricing kernel is zero is rejected for all three models, indicated by the values of $p(\delta = 0)$ less than 1%.

Panels B to D in Table 3 present the test results based on the three subgroups of test assets, 25 size/BM portfolios, 48 industry portfolios, and 8 bond portfolios, respectively. Model 2 has the smallest HJ-distance for the last two subgroups and Model 3 has the smallest HJ-distance for the

[6] Hansen and Jagannathan (1997) show that linear pricing kernel with a constant parameter that minimizes the HJ-distance will match the mean of the admissible pricing kernel that prices the test assets. Therefore, adding risk-free rate to test assets guarantees that the estimated pricing kernel prices risk-free rate correctly in the GMM estimation with HJ weighting matrix. In the GMM estimation with identity weighting matrix, we normalize the mean of the pricing kernel to the average risk-free rate.

[7] We thank Kenneth French and Amit Goyal for making the data publicly available.

25 size/BM portfolios among the three models. This is not surprising given that the Fama-French three-factor model is designed to explain these 25 portfolio returns. However, even in such a case, the Fama-French three-factor model does not perform significantly better than the model with estimated latent shocks, indicated by $p(\delta 1 = \delta_3)$ being 25.29%. Moreover, the null that the HJ-distance is zero is rejected for Models 1 and 3 based on all three subgroups of test assets, indicated by the values of $p(\delta = 0)$ less than 1% in Panels B to D. On the contrary, Model 2 successfully explains the returns on 48 industry portfolios and the 8 bond portfolios, with $p(\delta_2 = 0)$ being 19.22% and 8.60%, respectively. Therefore, tests based on HJ-distances indicate that Model 2 is the best model among the three for either the full set or subgroups of test assets.

Table 4 presents the results of GMM estimation with identity weighting matrix. The pricing kernels are normalized at the inverse of the average risk-free rate, $\bar{m}$. Panel A shows that estimated parameter values in the pricing kernels, average $SSPE$'s, the overidentification J statistics, and their p-values for all three models based on the full set of test assets. The J statistic is the sum of squared pricing errors weighted by their variance-covariance matrix and is used to evaluate the overall goodness of fit of the proposed models. Similar to the results based on HJ-distances, Model 2 has the best performance among the three with a J statistic of 530.39, compared to 1726.66 from Model 1 and 1653.21 from Model 3. However, Model 3 has the smallest average $SSPE$ ($SSPE$ scaled by the number of test assets), being 0.42, compared to 0.62 from Model 2 and 3.33 from Model 1. The comparison between J and $SSPE$ indicates that Model 3 matches better the average returns of the portfolios that have large return volatilities while Models 1 and 2 match better the ones with low return volatilities. The p-values of the null hypotheses that $SSPE_1 = SSPE_3$ and $SSPE_2 = SSPE_3$ are 8.59% and 29.57%, respectively. Therefore, the three models have statistically indifferent SSPE's. However, all three models are rejected by the overidentification tests based on the J statistics indicated by the values of $p(J)$ less than 1%.

Panels B to D report the test results based the subgroups of test assets: 25 size/BM portfolios, 48 industry portfolios, and the 8 bond portfolios, respectively. Based on the overidentification

J test, Model 2 is not rejected for the subgroups of the 25 size/BM portfolios and the 8 bond portfolios, however is rejected for the 48 industry portfolios with the values of $p(J)$ being 91.58%, 10.85%, and 0.00%, respectively. In contrast, Models 1 and 3 are rejected based on all three subgroups of test assets with the values of $p(J)$ less than 1%. The model selection tests based SSPE indicate that the three models have statistically indifferent SSPE's for all three subgroups of test assets.

In sum, for the full set of test assets, the model selection tests based on HJ-distance indicate that the factor model with estimated latent shocks performs than and the model-implied pricing kernel performs as well as the Fama-French three-factor model at the 5% significance level. The model selection tests based on SSPE indicate that the three models perform equally well. Note that while the test based on SSPE only compares how well the competing pricing kernels price the test assets, the test based on HJ-distance compares how well they price the asset space spanned by the test assets. In that sense, the latter test is a more powerful test on the asset pricing theory presented in equation (19).

The fact that Model 2 performs better than Model 1 implies that the DSGE model used in the estimation is subject to model misspecification errors. However, the sources of risks identified by the DSGE model and the Bayesian MCMC method, i.e., the latent shocks, have great potential in explaining the cross section of asset returns.

5 Conclusion and Future Research

A full-information Bayesian Markov Chain Monte Carlo (MCMC) method is developed for estimating DSGE models using macroeconomic variables only. We implement this method on a standard medium-size DSGE model in CTW to obtain the model parameters and three exogenous latent shocks: neutral technology shock, investment-specific technology shock, and monetary policy shock. Based on the estimations, we construct the model-implied stochastic discount factor and a linear factor model with estimated latent shocks as risk factors for the period of 1966Q1-2010Q3. GMM tests show that at the 5% significance level, the model-implied pricing kernel performs as well as and the factor model with estimated latent shocks performs better than the Fama-French three-factor model in pricing the large set of test assets including the 25 size/BM, 48 industry, 7 government bond and a long-term corporate bond portfolios.

For simplicity, our model incorporates only the three most studied shocks in macroeconomics and asset pricing. Smets and Wouters (2007) show that there are other shocks important for capturing the observed macroeconomic dynamics. Moreover, the recent macroeconomic literature has proposed several new shocks such as risk shock, which is the shock on cross-sectional idiosyncratic uncertainty in Christiano, Motto and Rostagno (2014), and financial shock, which is the shock on firms' borrowing constraint in Jermann and Quadrini (2012). Those shocks are likely to have significant impacts on asset prices. One way to extend the current study is to incorporate those aforementioned shocks in our framework and explore the asset pricing implications of the richer model. Another fruitful research direction is to add financial variables, such as returns on the market portfolio, to the set of variables that the model matches and see whether information from financial markets helps DSGE models to match macroeconomic dynamics.

A Bayesian MCMC Estimation

In this appendix, we provides a brief description about the priors, posterior distributions, and the updating procedures for parameters and latent variables in our model.

- *Posterior of* σ_i $(i = 1, \cdots, 7)$: Set the prior of σ_i as $\sigma_i^2 \sim IG(a, b)$, where a, b are hyper-parameters. The posterior of σ_i^2 is

$$\sigma_i^2 \sim IG(\frac{T}{2} + a, A)$$

where

$$A = \sum_{t=1}^{T} \frac{1}{2}(\Upsilon_t^{obs}(i) - \Upsilon_t(i))^2 + b\,.$$

- *Posterior of* θ_j $(j = 1, \cdots, 25)$: Set the prior of θ_j as $\theta_j \sim N(m, M^2)$ where m, M are hyper-parameters. The posterior of θ_j is

$$\begin{aligned} p\left(\theta_j|\theta_{[-j]}, \mathbf{X^e}, \mathbf{\Upsilon}^{obs}\right) \quad \propto \quad & \prod_{t=1}^{T}\prod_{i=1}^{7} \frac{1}{\sigma_i} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\ & \times \prod_{t=1}^{T} \frac{1}{\sigma_z} \exp\{-\frac{1}{2\sigma_z^2}[\mu_{z,t} - \mu_z(1-\rho_z) - \rho_z\mu_{z,t-1}]^2\} \\ & \times \prod_{t=1}^{T} \frac{1}{\sigma_\psi} \exp\{-\frac{1}{2\sigma_\psi^2}[\mu_{\psi,t} - \mu_\psi(1-\rho_\psi) - \rho_\psi\mu_{\psi,t-1}]^2\} \\ & \times \prod_{t=1}^{T} \frac{1}{\sigma_V} \exp\{-\frac{1}{2\sigma_V^2}[V_t - \rho_V V_{t-1}]^2\} \times \pi(\Theta) \times \exp\{-\frac{(\theta_j - m)^2}{2M^2}\}\,, \end{aligned}$$

where $\theta_{[-j]}$ contains the most updated parameters except for θ_j in $\mathbf{\Theta}$. In implementation, we simplify the above posterior through abandoning terms that do not depend on θ_j, and use MH algorithm to update θ_j.

- *Posterior of* $\{\mu_{z,t}, \mu_{\psi,t}, V_t\}$ $(t = 1, \cdots, T)$: The posterior distribution of $\mu_{z,t}$ $(for\ 1 \leq t < T)$

is

$$
\begin{aligned}
& p\left(\mu_{z,t}|\mathbf{\Theta}, \{\mu_{z,1}, \cdots, \mu_{z,t-1}, \mu_{z,t+1}, \cdots, \mu_{z,T}\}, \{\mu_{\psi,t}\}_{t=1}^{T}, \{V_t\}_{t=1}^{T}, \mathbf{\Upsilon}^{obs}\right) \\
\propto\ & \prod_{s=t}^{T}\prod_{i=1}^{N} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_z^2}[\mu_{z,t} - \mu_z(1-\rho_z) - \rho_z\mu_{z,t-1}]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_z^2}[\mu_{z,t+1} - \mu_z(1-\rho_z) - \rho_z\mu_{z,t}]^2\}\,.
\end{aligned}
$$

For $t = T$, the posterior distribution only involves the first two terms in the above equation. Again, MH algorithm is used to update $\mu_{z,t}$. Updating of $\mu_{\psi,t}$ and V_t $(t = 1, \cdots, T)$ are done in the same way. The analogous posterior distribution for $\mu_{\psi,t}$ is,

$$
\begin{aligned}
& p\left(\mu_{\psi,t}|\mathbf{\Theta}, \{\mu_{\psi,1}, \cdots, \mu_{\psi,t-1}, \mu_{\psi,t+1}, \cdots, \mu_{\psi,T}\}, \{\mu_{z,t}\}_{t=1}^{T}, \{V_t\}_{t=1}^{T}, \mathbf{\Upsilon}^{obs}\right) \\
\propto\ & \prod_{s=t}^{T}\prod_{i=1}^{N} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_\psi^2}[\mu_{\psi,t} - \mu_\psi(1-\rho_\psi) - \rho_\psi\mu_{\psi,t-1}]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_\psi^2}[\mu_{\psi,t+1} - \mu_\psi(1-\rho_\psi) - \rho_\psi\mu_{\psi,t}]^2\}\,.
\end{aligned}
$$

The analogous posterior distribution for V_t is,

$$
\begin{aligned}
& p\left(V_t|\mathbf{\Theta}, \{V_1, \cdots, V_{t-1}, V_{t+1}, \cdots, V_T\}, \{\mu_{z,t}\}_{t=1}^{T}, \{\mu_{\psi,t}\}_{t=1}^{T}, \mathbf{\Upsilon}^{obs}\right) \\
\propto\ & \prod_{s=t}^{T}\prod_{i=1}^{N} \exp\{-\frac{1}{2\sigma_i^2}[\Upsilon_t^{obs}(i) - \Upsilon_t(i)]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_V^2}[V_t - \rho_V V_{t-1}]^2\} \\
& \times \exp\{-\frac{1}{2\sigma_V^2}[V_{t+1} - \rho_V V_t]^2\}\,.
\end{aligned}
$$

References

Balvers, Ronald J. and Dayong Huang. 2007. “Productivity-Based Asset Pricing: Theory and Evidence.” *Journal of Financial Economics* 86(2):405–445.

Bernanke, Ben S. and Kenneth N. Kuttner. 2005. “What Explains the Stock Market’s Reaction to Federal Reserve Policy?” *Journal of Finance* 60(3):1221–1257.

Calvo, Guillermo. 1983. “Staggered Prices in a Utility-Maximizing Framework.” *Journal of Monetary Economics* 12:383–398.

Chen, Nai-Fu, Richard Roll and Stephen A Ross. 1986. “Economic Forces and the Stock Market.” *Journal of Business* 59(3):383–403.

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans. 2005. “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.” *Journal of Political Economy* 113(1):1–45.

Christiano, Lawrence J., Mathias Trabandt and Karl Walentin. 2011. DSGE Models for Monetary Policy Analysis. In *Handbook of Monetary Economics*, ed. Benjamin M. Friedman and Michael Woodford. Vol. 3A Netherlands: North Holland pp. 285–367.

Christiano, Lawrence J., Roberto Motto and Massimo Rostagno. 2014. “Risk Shocks.” *American Economic Review* 104(1):27–65.

Clarida, Richard, Jordi Galí and Mark Gertler. 2000. “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory.” *The Quarterly Journal of Economics* pp. 147–180.

Cochrane, John H. 1996. “A Cross-Sectional Test of an Investment-Based Asset Pricing Model.” *Journal of Political Economy* 104(3):572–621.

Cochrane, John H. 2005. *Asset Pricing*. Revised edition ed. 41 William Street, Princeton, New Jersey 08540: Princeton Univesity Press.

Daniel, Kent and Sheridan Titman. 2012. “Testing factor-model explanations of market anomalies.” *Critical Finance Review* 1:103–139.

Fama, Eugene F. and Kenneth R. French. 1997. “Industry Cost of Equity.” *Journal of Finance* 43:153–193.

Hall, Alastair R. and Denis Pelletier. 2011. “Non-Nested Testing in Models Estimated via Generalized Method of Moments.” *Econometric Theory* 27(02):443–456.

Hansen, Lars Peter. 1982. “Large Sample Properties of Generelized Method of Moments Estimators.” *Econometrica* 50:1029–1054.

Hansen, Lars and Ravi Jagannathan. 1991. “Implications of security market data for models of dynamic economies.” *Journal of Political Economy* 99:225–262.

Hansen, Lars and Ravi Jagannathan. 1997. "Assessing specification errors in stochastic discount factor models." *Journal of Finance* 52:557–590.

Jermann, Urban and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review* 102(1):238–271.

Kandel, Shmuel and Robert F. Stambaugh. 1995. "Portfolio Inefficiency and the Cross-Section of Expected Returns." *Journal of Finance* 50:157–184.

Kogan, Leonid and Dimitris Papanikolaou. 2013. "Firm Characteristics and Stock Returns: The Role of Investment-Specific Shocks." *Review of Financial Studies* 26:2718–2759.

Li, Haitao, Yuewu Xu and Xiaoyan Zhang. 2010. "Evaluating asset pricing models using the second Hansen-Jagannathan distance." *Journal of Financial Economics* 97:279–301.

Lowellen, Jonathan, Stephan Nagel and Jay Shanken. 2010. "A skeptical appraisal of asset-pricing tests." *Journal of Financial Economics* 96(175-194).

Papanikolaou, Dimitris. 2011. "Investment Shocks and Asset Prices." *Journal of Political Economy* 119(4):639–685.

Smets, Frank and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach." *American Economic Review* 97(3):586–606.

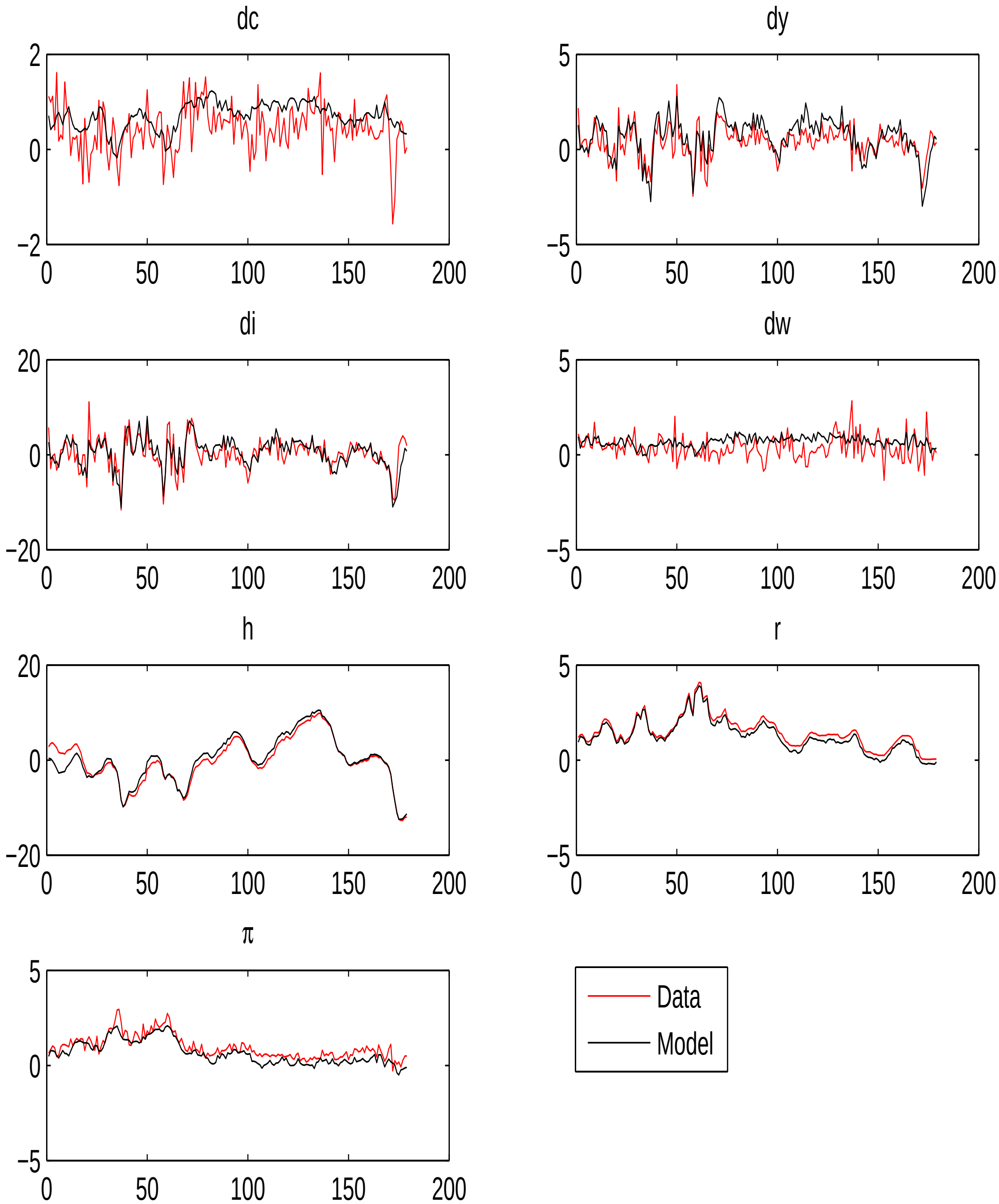

Figure 1: **Observed and Model-Implied Macroeconomic Variables**

This figure plots the estimated the observed (in red) and model-implied (in black) time series of consumption growth (dc), output growth (dy), investment growth (di), wage growth (dw), (normalized) hours (h), 3-month T-Bill rate (r), and inflation (π) during 1966Q1 - 2010Q3.

Table 1: **Estimated Parameter Values**

This table reports the parameters values estimated using the Bayesian Markov Chain Monte Carlo method based on 50,000 Monte Carlo iterations. Observed macroeconomic variables used in the estimation are output growth (dy), consumption growth(dc), investment growth (di), wage growth (dw), logarithm of inflation (π), 3-month T-Bill (r), and employment (h). Sample period is 1966Q1 - 2010Q3.

Parameter	Posterior Mean	Posterior Standard Error
β	0.9980	0.0005
ϕ	1.3838	0.0366
b	0.9598	0.0111
α	0.2308	0.0006
ξ_p	0.6022	0.0064
ξ_w	0.8232	0.0029
λ_f	1.1640	0.0024
λ_w	1.0373	0.0008
σ_a	0.2463	0.0242
σ_s	4.6910	0.2075
π_{ss}	1.0071	0.0023
ρ_R	0.7947	0.0036
ρ_π	1.6597	0.0524
ρ_y	0.1505	0.0079
μ_z	0.0038	0.0001
μ_ψ	0.0025	0.0003
ρ_z	0.1207	0.0664
ρ_ψ	0.7455	0.0500
ρ_v	0.3101	0.0534
σ_z	0.0026	0.0004
σ_ψ	0.0029	0.0003
σ_v	0.0021	0.0001

Table 2: **Summary Statistics**

The table provides the summary statistics on how well the model-implied variables match their observed counterparts and the correlation matrix of the estimated latent shocks. Panel A reports the estimated coefficients and adjusted R^2 of the following regression: $X_t = \alpha + \beta \hat{X}_t + e_t$ for output growth (dy), consumption growth(dc), investment growth (di), wage growth (dw), logarithm of inflation (π), 3-month T-Bill (r), and average weekly hours per capita(h), respectively, where X_t is the observed value and $\hat{X}_t$ is the corresponding model-implied value. Panel B of this table reports the correlation matrix between latent variables, i.e., the neutral technology shock (NT), investment-specific technology shock (IST), and monetary policy shock (MP). All data are sampled quarterly from 1966Q1 to 2010Q3.

Panel A: Goodess of Model Fit			
	α	β	$\bar{R}^2$
dc	-0.08	0.72	0.15
	(-0.82)	(5.74)	
dy	-0.06	0.66	0.63
	(-1.30)	(17.49)	
di	-0.10	0.82	0.62
	(-0.67)	(17.19)	
dw	-0.17	0.79	0.11
	(-1.37)	(4.79)	
h	-0.26	0.94	0.93
	(-2.82)	(49.87)	
r	0.26	0.96	0.99
	(21.10)	(115.84)	
π	0.36	0.92	0.85
	(14.23)	(32.34)	

Panel B: Correlations Matrix of Latent Shocks			
	NT	IST	MP
NT	1.00		
IST	-0.16	1.00	
MP	0.26	-0.33	1.00

Table 3: **GMM Tests with HJ Weighting Matrix**

This table presents the estimations and comparisons of the three asset pricing models based on the Hansen-Jagannathan distance (δ). The stochastic discount factor (pricing kernel) of the first model is computed from the DSGE model. The coefficients in the pricing kernels of Models 2 and 3 are chosen to minimize the corresponding HJ-distances. The HJ-distances and their corresponding p-values are provided for all three models. To compare the performance of the three models, the p-value of the hypothesis that Models 1 and 3 are observationally equivalent, i.e., $p(\delta_1 = \delta_3)$, and the p-value of the hypothesis that Models 2 and 3 are observationally equivalent, i.e., $p(\delta_2 = \delta_3)$, are provided. Details on the test assets can be found in Section 4. The sample is 1966Q1:2010Q3. The t-statistics are in parentheses and Newey-West corrected with lag 5.

Panel A: 25 Size/BM + 48 Industry + 8 Bond Portfolios						
Model 1: $m_t = m_t^{model}$						
				δ_1	$p(\delta_1 = 0)$	$p(\delta_1 = \delta_3)$
1.00				1.44	0.00%	5.24%
Model 2: $m_t = b_2 + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V$						
b_2	b_z	b_ψ	b_V	δ_2	$p(\delta_2 = 0)$	$p(\delta_2 = \delta_3)$
0.87	-0.30	-0.54	-0.46	1.37	0.01%	4.97%
(7.79)	(-1.45)	(-3.04)	(-2.03)			
Model 3: $m_t = b_3 + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}$						
b_3	b_{mkt}	b_{smb}	b_{hml}	δ_3	$p(\delta_3 = 0)$	
1.08	-0.22	-0.06	-0.26	1.41	0.00%	
(9.27)	(-1.71)	(-0.58)	(-2.45)			
Panel B: 25 Size/BM Portfolios						
Model 1: $m_t = m_t^{model}$						
				δ_1	$p(\delta_1 = 0)$	$p(\delta_1 = \delta_3)$
1.00				0.73	0.02%	3.16%
Model 2: $m_t = b_2 + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V$						
b_2	b_z	b_ψ	b_V	δ_2	$p(\delta_2 = 0)$	$p(\delta_2 = \delta_3)$
0.98	-0.46	-0.28	-0.01	0.69	0.14%	25.29%
(5.14)	(-0.99)	(-0.85)	(-0.01)			
Model 3: $m_t = b_3 + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}$						
b_3	b_{mkt}	b_{smb}	b_{hml}	δ_3	$p(\delta_3 = 0)$	
1.08	-0.22	-0.07	-0.28	0.66	0.16%	
(14.78)	(-1.69)	(-0.69)	(-2.66)			

Table 3 Continued

Panel C: 48 Industry Portfolios

Model 1: $m_t = m_t^{model}$

				δ_1	$p(\delta_1 = 0)$	$p(\delta_1 = \delta_3)$
1.00				0.68	2.29%	21.89%

Model 2: $m_t = b_2 + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V$

b_2	b_z	b_ψ	b_V	δ_2	$p(\delta_2 = 0)$	$p(\delta_2 = \delta_3)$
0.96	-0.44	-0.37	-0.07	0.63	19.22%	36.96%
(11.09)	(-1.49)	(-1.48)	(-0.28)			

Model 3: $m_t = b_3 + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}$

b_3	b_{mkt}	b_{smb}	b_{hml}	δ_3	$p(\delta_3 = 0)$
0.99	-0.22	0.10	0.05	0.65	4.38%
(17.16)	(-1.75)	(0.93)	(0.43)		

Panel D: 8 Bond Portfolios

Model 1: $m_t = m_t^{model}$

				δ_1	$p(\delta_1 = 0)$	$p(\delta_1 = \delta_3)$
1.00				0.45	0.04%	16.63%

Model 2: $m_t = b_2 + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V$

b_2	b_z	b_ψ	b_V	δ_2	$p(\delta_2 = 0)$	$p(\delta_2 = \delta_3)$
0.78	0.41	-0.94	-0.72	0.33	8.60%	65.53%
(4.59)	(0.74)	(-2.18)	(-1.07)			

Model 3: $m_t = b_3 + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}$

b_3	b_{mkt}	b_{smb}	b_{hml}	δ_3	$p(\delta_3 = 0)$
1.08	-0.56	-0.14	0.06	0.37	0.98%
(7.16)	(-1.41)	(-0.23)	(0.09)		

Table 4: **GMM Tests with Identity Weighting Matrix**

This table presents the estimations and comparisons of the three asset pricing models based on GMM with identity matrix. The stochastic discount factor (pricing kernel) of the first model is computed from the DSGE model. The coefficients in the pricing kernels of Models 2 and 3 are chosen to minimize the sum of squared pricing errors ($SSPE$). The average $SSPE$, overidentification J statistic and its p-value are provided for each model. Model selection tests in Hall and Pelletier (2011) is conducted to compare the goodness of fit between Models 1 and 3 and between Models 2 and 3, respectively. The p-values of the tests under the null hypothesis that the difference in the average $SSPE$'s of the two competing models is zero are provided under $p(SSPE_1 = SSPE_3)$ and $p(SSPE_2 = SSPE_3)$. Details on the test assets can be found in Section 4. The sample is 1966Q1:2010Q3. The t-statistics are in parentheses and Newey-West corrected with lag 5.

Panel A: 25 Size/BM + 48 Industry + 8 Bond Portfolios						
Model 1: $M_t = m_t^{model}$						
			J	$p(J)$	$SSPE_1$	$p(SSPE_1 = SSPE_3)$
1.00			1726.66	0.00%	3.33	8.59%
Model 2: $M_t = \bar{m} + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V$						
b_z	b_ψ	b_V	J	$p(J)$	$SSPE_2$	$p(SSPE_2 = SSPE_3)$
-0.93	-0.55	-0.63	530.39	0.00%	0.62	29.57%
(-1.73)	(-1.11)	(-0.69)				
Model 3: $M_t = \bar{m} + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}$						
b_{mkt}	b_{smb}	b_{hml}	J	$p(J)$	$SSPE_3$	
-0.27	0.07	-0.18	1654.21	0.00%	0.42	
(-2.41)	(0.67)	(-1.65)				
Panel B: 25 Size/BM Portfolios						
Model 1: $M_t = m_t^{model}$						
			J	$p(J)$	$SSPE_1$	$p(SSPE_1 = SSPE_3)$
1.00			141.45	0.00%	4.76	5.71%
Model 2: $M_t = \bar{m} + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V$						
b_z	b_ψ	b_V	J	$p(J)$	$SSPE_2$	$p(SSPE_2 = SSPE_3)$
-0.49	-1.57	-1.96	13.58	91.58%	0.51	29.52%
(-0.61)	(-1.45)	(-0.70)				
Model 3: $M_t = \bar{m} + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}$						
b_{mkt}	b_{smb}	b_{hml}	J	$p(J)$	$SSPE_3$	
-0.22	-0.06	-0.30	87.99	0.00%	0.15	
(-1.83)	(-0.61)	(-2.97)				

Table 4 Continued

Panel C: 48 Industry Portfolios						
Model 1: $M_t = m_t^{model}$						
			J	$p(J)$	$SSPE_1$	$p(SSPE_1 = SSPE_3)$
1.00			319.72	0.00%	3.09	9.45%
Model 2: $M_t = \bar{m} + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V$						
b_z	b_ψ	b_V	J	$p(J)$	$SSPE_2$	$p(SSPE_2 = SSPE_3)$
-0.92	-0.33	-0.38	105.57	0.00%	0.43	38.49%
(-1.68)	(-0.77)	(-0.54)				
Model 3: $M_t = \bar{m} + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}$						
b_{mkt}	b_{smb}	b_{hml}	J	$p(J)$	$SSPE_3$	
-0.33	0.25	-0.06	194.61	0.00%	0.36	
(-2.89)	(2.03)	(-0.41)				

Panel D: 8 Bond Portfolios						
Model 1: $M_t = m_t^{model}$						
			J	$p(J)$	$SSPE_1$	$p(SSPE_1 = SSPE_3)$
1.00			44.68	0.00%	0.30	17.11%
Model 2: $M_t = \bar{m} + b_z\, e_t^z + b_\psi\, e_t^\psi + b_V\, e_t^V$						
b_z	b_ψ	b_V	J	$p(J)$	$SSPE_2$	$p(SSPE_2 = SSPE_3)$
-0.51	0.04	0.89	9.01	10.85%	0.01	49.94%
(-1.01)	(0.06)	(1.25)				
Model 3: $M_t = \bar{m} + b_{mkt}\, r_{mkt,t} + b_{size}\, r_{size,t} + b_{hml}\, r_{hml,t}$						
b_{mkt}	b_{smb}	b_{hml}	J	$p(J)$	$SSPE_3$	
-0.67	0.47	-0.91	15.95	0.70%	0.01	
(-1.18)	(0.43)	(-1.31)				