Conflict, Evolution, Hegemony, and the Power of the State☆

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Abstract

In a model of evolution driven by conflict between societies more powerful states have an advantage. When the influence of outsiders is small we show that this results in a tendency to hegemony. In a simple example in which institutions differ in their exclusiveness we find that these hegemonies will be inefficiently extractive in the sense of having inefficiently high taxes, high compensation for state officials, and low welfare. The theory also predicts that they are most likely overthrown by fanatic bands who maximize power ignoring incentive constraints.
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1. Introduction

A robust finding of the game theory literature is that with the possibility of future punishment and rewards there are many possible equilibria. One interpretation is that these represent alternative social norms or institutions. Indeed, we do observe a wide array of different institutions both across space and time: political systems in particular range from relatively autocratic ("exclusive" in our terminology) to quite democratic ("inclusive" in the terminology of Acemoglu and Robinson (2012)). A natural question is the positive one: among these different institutions are we more likely to observe some than others? Are efficient institutions likely to be more successful than inefficient ones? A natural setting for this question is an evolutionary one - and one possible answer is that of Ely (2002)[3] who shows how voluntary migration leads to efficiency. But we do not believe that historically people have moved from one location to another through a kind of voluntary immigration into the arms of welcoming neighbors. Rather people and institutions have more often spread through invasion and conflict.[4] Moreover, significant institutional change has most often arisen in the aftermath of the disruption caused by warfare and other conflict between societies. This leads us to address the question of which institutions are likely to be long-lived in the context of evolution driven by conflict between societies.

We develop a theory of the evolution of institutions driven by conflict where the state power generated by institutions determines their ability to prevail in a conflict.

1. We show that there are only two types of enduring environments: environments in which institutions are diverse, including both efficient and extractive institutions; and environments in which a single state controls the entire area within well defined and hard to penetrate geographical boundaries. Such a state we call a hegemony. We find that hegemonies should be commonplace, and that hegemonic institutions are extractive and state power maximizing.

2. We give specific criteria on economic fundamentals that determine which the environment will be. Generally speaking strong outside influence, weak tax collecting technology, and poor ability to aggregate state power favors diverse institutions while conversely weak outside influence, strong tax collecting technology and effective aggregation of state power favor hegemony of the strongest.

3. We show that even in environments that favor long-lived hegemonies they are periodically broken with shorter periods of conflict between competing - and possibly more efficient - states. While the bulk of the evolutionary literature is limited to describing long-run ergodic states, we

[3] Ely uses a model similar to the one used here, but similar results using more biologically oriented models have been around for some time. For example Aoki (1982) uses a migration model to study efficiency, while more recently Rogers, Deshpande and Feldman (2011) use a migration model to show how unequal resources can lead to long-run inequality.

[4] This was apparently true even in the earliest of times. Bowles and Choi (2013) argue that farming was initially an inferior technology to foraging and became widespread not because it was eagerly adopted by imitators, but rather because farmers had "formidable military technology" that enabled them to successfully encroach on foragers.

are able to describe the transition process. In particular, we show that fanatic bands - groups who are temporarily out of equilibrium - play a key role in the downfall of hegemonies.

4. Unlike the bulk of the evolutionary literature, which is limited to very special classes of games, by combining long-run evolution with short-run learning, we are able to give broad results for arbitrary games. This method has potentially other applications, for example, if societies compete through trade, innovation, or migration rather than conflict and warfare.

5. To relate the abstract concept of state power to more concrete economic fundamentals and economic institutions we develop a simple static economic model of taxation in the spirit of Besley and Persson (2010) that incorporates Acemoglu and Robinson (2012)'s concept of inclusiveness.

The following stylized facts about the historical evidence support the predictions of the theory and illustrate its empirical content.

Stylized Fact 1: Hegemonies are common. The idea of history being dominated by hegemonic states may seem a strange one, but with some important exceptions it is borne out by historical facts.[5] Take, for example, the largely geographically isolated region of China: bounded by jungles in the South, deserts on the West, cold arid wasteland in the North and the Pacific Ocean in the East. We find that during the 2,234 years beginning from when we have decent historical records in 221 BCE the area was ruled by a hegemonic state roughly 72% of the time, with five interregna. Less reliable records exist for the area of Egypt, but in the 1,617 years from 2686 BCE to the end of the new Kingdom in 1069 BCE we see hegemonic rule 87% of the time with two interregna. In Persia during the 1,201 years from 550 BCE to 651 CE we see hegemony 84% of the time with two interregna. England has been largely hegemonic within the geographically confined area of the island of Britain for 947 years from 1066 CE to the present. The Roman Empire ruled the Mediterranean area as a hegemony for 422 years from the advent of Augustus in 27 BCE to the permanent division into Eastern and Western Empires in 395 CE and the Eastern Roman Empire lasted an additional 429 years until the advent of the Caliphate in 814 CE. The Caliphate itself lasted 444 years until the Mongol invasion in 1258. After a 259 gap, the Ottoman Empire established a hegemony over the same general area for 304 years from the conquest of Egypt in 1517 CE to the Greek revolution in 1821 CE.

Stylized Fact 2: Hegemonies are prevalent where outsiders are weak. While hegemonies are common in history, there are two glaring exceptions: except for brief periods neither the subcontinent of India nor, following the fall of the Western Roman Empire, the area of continental Europe were subject to a hegemonic state. According to our theory hegemonies will not persist when there are strong outsiders protected by geographical barriers. In the case of both continental Europe and India this is the case. In the case of Europe following the fall of Rome and up to around 1066 we have the continued interference of northerners - the Vikings especially were well protected by their own geography. Following 1066 we have the constant interference of England - also safe behind a water barrier: during this period we observe that England constantly intervened

[5] Sources and calculations of historical data on hegemonies and population is in Appendix 1.

in continental conflicts but always to support the weaker side, and eventually this policy of balance of power became explicit.[6] India also was subject to repeated invasion from central Asia - protected not by water but by difficult desert and mountain terrain.[7] Of course China too was subject to outside influence - particularly that of the Mongols. However, the relative size of the Mongolia is quite small relative to China - less than half a percent of the population - while the population of Scandinavia was about 5% that of continental Europe, that in central Asia about 5% that of India, while England was about 8% of continental Europe. These exceptions are in fact exactly what is predicted by our evolutionary theory: we show that as outside influence grows the fraction of time hegemony will reign decreases.[8]

Stylized Fact 3: Nomadic foragers - who unlike farmers lack the most primitive food storage technology to build and accumulate power - do not form hegemonies. A discussion of this can be found in Bowles and Choi (2013).

To proceed to our final stylized fact we wish to bring to the reader's attention that from the narrow perspective of establishing long run results on hegemony it is sufficient to assume that there is equilibrium at each moment of time. While we find this an unsatisfying approach, our finding that replacing "equilibrium at each moment of time" with "learning that converges rapidly to equilibrium" leads to similar results about hegemony is not surprising. However, a closer look at the transition dynamics uncovers the fact that the model of rapid convergence to equilibrium does not have the same implications for the transition between hegemonies: in particular it highlights the relevance of "fanatic bands" - groups of individuals who are not bound by incentive constraints because they have not yet learned their way to equilibrium. According to the theory, these should play an important role in the downfall of hegemonies, and in addition the presence of hegemonies and fanatic bands should also lead to the phenomenon of short-lived empires.

Stylized Fact 4: Short-lived empires are common and fanatic bands bring down hegemonies. The prominent role of charismatic leaders and their armies in the collapse of empires is apparent in any history book. With respect to the collapse of hegemonies, the most obvious examples are the collapse of the Achaemenid Empire in Persia brought about by the invasion of Alexander the Great

[6] It is not completely correct to view England and Scandinavia as "outsiders" as at various time they had continental interests and conversely, but the key point is that they had a core area relatively safe from invasion. In a different direction Hoffman (2013) argues a role also for the Western Catholic church which in Europe acted as a balancing force much akin to to the outsiders of our model.

[7] The exact nature of the asymmetry in the physical geographical barrier is uncertain, but it is a fact that India has been invaded numerous times successfully from Central Asia, but there have been no successful conquests of Central Asia from India. Phil Hoffman in a private communication suggests that part of the answer may lie in the fact that the area of Central Asia is well suited for raising horses and India is not, and that horses play a central military role in conflict between Central Asia and India.

[8] Note that geographical factors matter in our argument only in so far as they give rise to outsiders who influence the evolution of the relationships between the other groups. An existing literature, including Diamond (1998), gives physical geography a direct role, arguing for example that the terrain of Western Europe is more defensible than that of China, hence less susceptible to hegemony. Besides this particular claim being challenged on physical grounds (Hoffman (2013)), such considerations have no bite in the Indian case. Incidentally: while this discussion includes only the area of Europe, Asia and North Africa, it should be borne in mind that until modern times 90% of the world population lived in this area.

and the collapse of the Caliphate brought about the the invasion of the Mongols. Note that both resulted in short-lived successor empires, and this is a common destiny of this type of conquest. Indeed, in the annals of short-lived empires we may also count those of Ashoka the Great, Attila the Hun, Charlemagne, Tamerlane, Napoleon and Hitler. A common characteristic of the successful conquerors is that the lifestyle of the roving and fierce warrior was quickly dropped - which in our interpretation indicates that the usual incentive constraints were absent during the expansion. Alexander's empire collapsed because his successors preferred the settled life of luxury over that of the nomadic conqueror, and much the same was true in the other cases. Moreover, while in many instances hegemonies ended by splintering into states or civil war, in those cases for which we have records, this was proceeded by a series of civil insurrections weakening the central state and strengthening the periphery. Finally - again where we have records - the insurrections involved rather fanatical bands: for example, the yellow turbans whose rebellion began the process that ended the hegemony of the later Han period in China believed that an apocalypse would engulf the government and at the same time turn the sky yellow. The Boxers whose rebellion brought about the weakening of the Qing dynasty which fell about a decade later were noted also for their fanaticism.

With respect to institutions, as we indicated, states with greater power will have an advantage over less powerful states. To focus thinking we examine a simple model in which institutions and incentives determine state power. A key question is why individuals in a society will contribute to state power? First, there is a significant public goods problem. Worse, given that outside conquest and disruption is relatively rare, it is hard to believe that military spending levels would stand much of a cost-benefit analysis, so that the "good" of state power is perhaps not very good at all. Hence while the public goods problem may be solved by a state run by officials who collect taxes, the question remains why these officials do not collude to consume tax revenue rather than use it to augment the power of the state. The answer we propose to this question is that the incentive of state officials to acquire state power is not so much in defense of the state from intruders but rather to collect taxes from which they can consume and to maintain themselves in office. Roughly speaking we view state officials as preferring to consume "jewelry" rather than "swords," but they need the swords to collect the tax revenue to pay for the jewelry - swords which then come in useful in conflict. We model this in a simple way by assuming that greater state power increases the ability of the state to collect taxes. Of course a more powerful state enables the collection of greater revenues for officials only if the army and other forces of state power respond to the wishes of those officials. Whether this is true depends on inclusivity of institutions. In an inclusive democracy, for instance, this is not so much the case - officials may try to send the army into the streets to collect revenues for their own benefit, but in a meaningful democracy the army will be loyal to institutions not persons and will not follow such orders.

Putting the paper in context, the idea that evolution can lead to both cooperation and inefficiency is scarcely new, nor is the idea that evolutionary pressure may be driven by conflict. There is a long literature on group selection in evolution: there may be positive assortative matching as discussed by Bergstrom (2003). Or there can be noise that leads to a trade-off between incentive constraints and group welfare as in the work of Price (1970, 1972). Yet another approach is through differential extinction as in Boorman and Levitt (1973). Conflict, as opposed to migration, as a source of evolutionary pressure is examined in Bowles (2006), who shows how intergroup competition can lead to the evolution of altruism. Bowles, Choi and Hopfensitz (2003) and Choi and Bowles (2007) study in group altruism versus out group hostility in a model driven by conflict. Rowthorn and Seabright (2010) explain a drop in welfare during the neolithic transition as arising from the greater difficulty of defending agricultural resources. More broadly, there is a great deal of work on the evolution of preferences as well as of institutions: for example Blume and Easley (1992), Dekel, Ely and Yilankaya (2007), Alger and Weibull (2010), Levine et al (2011) or Bottazzi and Dindo (2011). Some of this work is focused more on biological evolution than social evolution. As Bisin (2001) and Bisin and Topa (2004) point out the two are not the same. This paper is driven by somewhat different goals than earlier work. We are interested in an environment where individual incentives matter; and in an environment where the selection between the resulting equilibria are driven by conflict over resources ("land"). By combining the idea of the conflict resolution function introduced by Hirshleifer (2001) and subsequently studied in the economic literature on conflict [9] with the stochastic tools of Kandori, Mailath and Rob (1993), Young (1993) and Ellison (2000) we are able with relatively weak assumptions to show when state power maximizing hegemonies do and do not arise.

The present line of research complements that of Besley, Persson and others on the dynamics of state capacity, for example Besley and Persson (2010). Broadly speaking, they study the determinants of state capacity within a state facing potential conflict with an exogenous opponent, or facing internal conflict; we analyze the dynamics of actual conflict among different states with endogenously changing opponents and look at state power in the resulting long run configuration. We return to discuss their work at the end of section 2.

2. A Static Example

We start with a simple static model which illustrates how state power may be determined by institutions and incentives within a society. The model yields novel insights on the relation between state power and extractiveness of institutions and welfare, and motivates the introduction of a map from actions to state power in the general dynamic model of section 4. The model of this section is in the spirit of Besley and Persson (2010), but differs in some respects that we discuss below.

The "size" of a society, in terms of land resources controlled, will play a crucial role in in the evolutionary dynamics, but land plays no role in the statics, so here we conduct the analysis on a representative unit of land. As summarized in the introduction, the idea for this model is that state power determines strength in conflict, but officials determine state power mainly to collect taxes

[9] See, for example, Garfinkel and Skaperdas (2007) or Hausken (2005). An important focus of this literature has been in figuring out how shares of resources are determined by the conflict resolution function.

in their own interest. The results of the section relate state power and welfare to the exclusivity of institutions and their extractiveness.

There are two types of players: producers $i = P$ and state officials $i = O$. The choice variable for producers is the effort level $a^P \in [0,1]$ and for officials the level of state power $a^O \in [0,1]$. State officials move first and choose the level of state power; producers move second and choose the effort level.

Internally state power serves the purpose of collecting taxes as well as providing public goods. The tax rate is determined by the level of state power a^O together with a parameter $\chi \in [0,1]$ describing social institutions. Higher state power enables to collect more taxes. The institutions that give rise to χ we view as fixed in the short-run model described here although in the dynamic model studied below they will change over time in response to evolutionary pressures.[10] The relevant aspect of institutions summarized by the parameter χ describes the extend to which institutions enable the use of state power to collect taxes.[11] It represents the "exclusivity" of those institutions: relatively inclusive institutions, such as democratic one, use a variety of checks and balances to limit the application of state power - courts, appeals processes and so forth. In the extreme when $\chi = 0$ we imagine that it is essentially impossible to collect taxes because individuals who fail to pay taxes may engage in endless appeal to the courts. At the opposite extreme when $\chi = 1$ tax collectors can simply seize resources from producers at gunpoint without any institutional constraint. Hence we define *fiscal capacity* $b = \chi a^O$ as the product of the exclusivity parameter χ and state power. When $\chi = 0$ there is no fiscal capacity; when $\chi = 1$ fiscal capacity is the same as state power. The actual tax rate is given by $\overline{\tau} \equiv \min\{1, \tau b\}$ where τ is *tax effectiveness,* a technological parameter specifying how effective fiscal capacity is in collecting taxes. We assume $\tau > 1$ so that tax system is efficient enough to pay the cost of collecting taxes.

Producers are modeled as a single representative player. Effort translates into output one for one. Producer's utility is output net of taxes and the quadratic cost of providing effort, plus a benefit from public goods provided through state power:

$$u^P = (1-\overline{\tau})a^P - [ca^P + (1/2)(1-c)\left(a^P\right)^2] + \xi^P a^O \quad 0 < c < 1$$

Note that the cost function has been normalized so that the marginal cost of a unit of effort is 1, and that ξ^P measures the extent to which state power affects producers utility by providing public goods.

State officials act collusively as the residual claimants of tax revenue net of the resources devoted

[10] We do not explicitly model the decisions to adhere to social norms that underly institutions: we refer the reader to the literature on repeated games such as Fudenberg and Maskin (1986) or Fudenberg Levine and Maskin (1994) and especially Kandori (1992)'s work on social norms. In an earlier version of this paper Levine and Modica (2012) these decisions were explicitly analyzed - without however leading to different conclusions.

[11] For computational simplicity in analyzing statics χ and a^i will be treated as continuous, but in the analysis of evolutionary dynamics they will be treated as discrete.

to building state power: their utility is

$$u^O = \overline{\tau} a^P - a^O + \xi^O a^O$$

where $\xi^O a^O$ is the officials utility from the public goods provided by a^O. We assume $\xi^O \le 1$. [12]Notice that we allow negative utility for state officials - implicitly they have resources so that the state can operate with a deficit. Our results are not sensitive to this modeling simplification as in equilibrium officials never choose to do this.

An action profile (a^P, a^O) for a society is an *equilibrium* if it is subgame perfect when the officials move first, or equivalently, a Stackelberg equilibrium. Results for the quadratic case are worked out in Appendix 2 with a complete analysis in Web Appendix 2. The economy can be summarized by means of the tax-revenue and profit functions given respectively by

$$G(b) = \overline{\tau} a^P \quad \text{and} \quad \Pi(b) = G(b) + u^P - \xi^P a^O$$

Note that the utility of the representative producer and state officials are measured in compatible units in the sense that a unit of utility lost by the producer in taxes is a unit of utility gained by the state officials so that welfare $W(b) = u^P + u^O$ is given by $\Pi(b) - (1 - \xi^P - \xi^O)a^O$. The utility of state officials is $G(b) - (1 - \xi^O)a^O$.

In Appendix 2 we show that in the quadratic case the following salient facts about these functions hold: $G(b) = 0$ at $b = 0$ and for $b \ge \overline{b} \equiv (1 - c)/\tau$ while for $0 \le b \le \overline{b}$ $G(b)$ is twice continuously differentiable with $G''(b) < 0$; $G'(0) > 1$, and $G'(b) + bG''(b)$ is decreasing. For $0 \le b \le \overline{b}$ we have profits $\Pi(b)$ twice continuously differentiable, decreasing and $\Pi''(b) < 0$ with $\Pi(b) = 0$ for $b \ge \overline{b}$. Finally, $\Pi(b) - G(b)$ is decreasing. Stepping beyond the linear/quadratic case, when these properties are satisfied for some value of $\overline{b}$ we will refer to the economy as *proper.*

We are interested in which institutions achieve specific benchmarks. Specifically in proper economies, we can describe institutions χ for which the equilibrium maximizes state power and for which it maximizes welfare. In Appendix 2 we show that:

Theorem 1. *In a proper economy there is a unique equilibrium level of state power $a^O(\chi)$, and it is single peaked in χ; so it has a unique argmax $\chi_2 > 0$. There is a unique welfare maximizing level of exclusivity χ_1, and $\chi_1 \le \chi_2$. There is a $\overline{\xi} \ge 1$ such that if $\xi^P + \xi^O \le \overline{\xi}$ then $\chi_1 < \chi_2$.*

Thus state power maximization leads to greater exclusiveness than welfare maximization.

We also have a relationship between exclusivity and what Acemoglu and Robinson (2000) call extractiveness. In Appendix 2 we show that:

Theorem 2. *In a proper economy profits $\Pi(\chi a^O(\chi))$ are decreasing in χ, while tax revenues $G(\chi a^O(\chi))$, fiscal capacity $\chi a^O(\chi)$, and the utility of state officials $u^O(\chi, a^O(\chi))$ are all increasing in χ.* [13] *For $\chi \ge \chi_1$ producer utility is decreasing in χ and if $\xi^P + \xi^O < 1$ so is welfare. If*

[12]As detailed in Web Appendix 2 this rules out cases where officials choose positive state power even if producers do not produce at all.

[13]We have not specified a relationship between fiscal capacity and the tax rate, but expect that as in the linear quadratic case the tax rate is increasing in fiscal capacity.

$\xi^P + \xi^O \geq 1$ *welfare is decreasing for* $\chi_1 \leq \chi \leq \chi_2$.

Theorems 1 and 2 imply that institutions that maximize state power have greater extractiveness than those that maximize welfare.

Comments on the model. As we indicated, the model of this section is in the spirit of Besley and Persson (2010). A formal difference is that in their model investment in fiscal capacity - the ability to raise taxes - lowers the level of the public good which gives the ability to fight external enemies in the future. Hence they model investment in fiscal capacity and state power as substitutes, while we view them as complements. Our justification for a simpler model - which can be applied to a changing configuration of a multitude of interacting states - is that historically a strong military has played a key role in maintaining internal order - a prerequisite for the ability to collect taxes - and the same military serves in conflict. The other difference - dictated again by the different goals of the models - is that in their model the value of land is implicitly heterogeneous across countries: they explain how higher state power may be due to high valuation of the defense public good, which is ultimately determined by high land value which in turn raises the risk of potential conflict; [14] we consider homogeneous land and concentrate on differences in military spending due to institutional conditions (the χ parameter) which we next determine endogenously through a model of evolution.

3. Dynamics with Two Societies

We now wish to consider how institutions χ are determined by evolutionary pressure. To begin, we analyze a greatly simplified evolutionary dynamic. Subsequently we show that the qualitative properties of this example hold under much broader conditions.

In our preliminary analysis we assume that there are two societies, that both are proper economies, and that equilibrium action profiles are always chosen. These societies share the same technology and differ only in inclusiveness χ. To focus thoughts, it is useful to think of one value χ_1 maximizing welfare, the other χ_2 maximizing state power. These two societies will compete over land, with their chances of winning or losing land governed by a conflict resolution function that depends upon relative state power. We shall posit a positive relation between state power and strength in conflict, so it will come as no surprise that evolutionary forces will favor the society with greater state power. On the other hand our main questions concerns not “does more state power do better?” but rather, “how much better does greater state power enable a society to do?” and “how often we are likely to see a hegemony?”

Assume then that two societies $j = 1, 2$ compete over an integral number $L > 1$ units of land. [15] Both societies are assumed to implement their unique equilibrium profile, and consequently generate $a^O(\chi_j) > 0$ units of state power per unit of land, $j = 1, 2$. Observe that $a^O(\chi_1) \leq a^O(\chi_2)$, that is,

[14] In this context some of the highest military expenditures in the world are in the Gulf states. One may explain this by saying that they have dangerous neighbors - but the more fundamental economic fact is that they have valuable oil that their neighbors might like to seize, as for example when Iraq invaded Kuwait in 1990. Systematic research on differential land value as a potential source of conflict is now under way, as in Caselli, Morelli and Rohner (2013) who relate the frequency of conflict to the location of natural resources.

[15] If there is a single unit of land then there must always be a hegemony.

the efficient society is the weaker society. At time t society j controls an integral number $L_{jt} \geq 0$ units of land where $L_{1t} + L_{2t} = L$. If $L_{jt} = L$ for $j = 1$ or $j = 2$ we have a *hegemony.* [16]

Control over land follows a Markov process with state variable L_{1t}, the amount of land belonging to the weaker society. The transition probabilities are determined by a conflict resolution function, in which each period there may be conflict resulting in one of the two societies losing a unit of land to the other: that is $|L_{j,t+1} - L_{jt}| \leq 1$. The conflict resolution probabilities will depend on the power of the two societies.

We refer to the loss of a unit of land on the part of society j as a *disruption* and write π_{jt} for the corresponding probability. We assume that if the opponent has some land, so $L_{-j} > 0$ then the probability of disruption is a fixed constant $1/2 > p > 0$. Notice that this probability must be less than $1/2$ since if each society holds a unit of land, then each society has a chance p of losing a unit of land, and they cannot both do so. For a hegemonic society, we assume that the probability of losing a unit of land depends on three parameters $a_0, \alpha > 0$ and $1 > \epsilon > 0$ and is given by

$$p\epsilon^{\max\{0, a^O(\chi_j)L^\alpha - a_0\}}$$

Here $a^0(\chi_j)L^\alpha$ represents the aggregate state power of a hegemonic state that has $L_j = L$ units of land and we refer to $\rho(\chi_j) = \max\{0, a^O(\chi_j)L^\alpha - a_0\}$ as the *hegemonic resistance.*

The conflict resolution function is governed by four parameters: $p, a_0, \alpha\epsilon$. The first parameter p determines how often land is won or lost - we may think of this as a measure of the period length - if the period is very short, then there is little chance a unit of land is lost in a single period. For a fixed period length p controls how quickly change takes place - a higher value of p effectively speeds up the time scale. This can be significant for applications, as p can be determined by technology and geography that will generally differ across time and space. For example, if we accept Diamond (1998)'s thesis that the terrain of Europe is more difficult than that of China (see, however, Hoffman (2013)), this could have the effect of slowing down warfare, leading to a lower value of p in Western Europe, which would in turn mean longer "interregna" relative to China.

The parameter a_0 is a threshold: if state power falls below this level then the hegemony has little resistance to disruption. As state power rises above this level then resistance to disruption increases. The underlying idea is that if outside forces - whose interests are against emergence of a hegemony - are strong, then a great deal of state power is needed to prevent disruption from the outsiders: so we think of a_0 as a measure of the strength of outside forces. This interpretation arises from the global picture of competition we are modeling, in which conflict is not confined to the forces directly engaged in it because it also indirectly affects other states whose interests are sensitive to its outcome.

The parameter α describes how well state power aggregates. If $\alpha = 1$ state power grows linearly with land. If $\alpha = 0$ more land does not increase the ability to prevail in a conflict. For example, with high population density and food storage technology it is possible to concentrate large masses

[16] The definition of hegemony in the more general model of section 4 is on page 15.

of troops from many locations so that state power will grow more or less linearly with land - twice the land, twice the troops - so $\alpha = 1$. With low population density and no food storage technology, concentrating a large number of troops in one location dooms them to starvations: hence twice the land does not imply that armies can be twice as large, meaning that α is small. The aggregation technology can also depend on geographical factors: for example, for a "circular" state the perimeter that must be defended expands as the square root as the amount of land, while for a "linear" state with flanks protected by geographically difficult terrains, the perimeter to be defended may be independent of the amount of land, and so forth.

Finally, ϵ measures how sensitive is the outcome of conflict to institutions and to state power. If ϵ is large then increasing state power has little effect on the chance of disruption. In the extreme case where $\epsilon = 1$, for example, increasing state power has no effect on the chance of disruption.

To understand the dynamics, observe first that when $\epsilon = 0$ the hegemonic states are absorbing and the non-hegemonic states ($0 < L_1 < L$) are transient, so in the long-run there is a hegemony, and if the initial condition is uniform over L_1, each society has an equal chance of having the long-run hegemony since the Markov process is symmetric in this case.

When $\epsilon > 0$ the situation is quite different: all states are positively recurrent and there is a unique stationary probability distribution representing the frequency with which each state occurs. Since this is a simple birth-death chain, the stationary probabilities can be explicitly computed. In particular the stationary probability of society j having a hegemony is

$$\sigma_j = \frac{1}{1 + (L-2)\epsilon^{\rho(\chi_{-j})} + \epsilon^{\rho(\chi_{-j}) - \rho(\chi_j)}}.$$

We can manipulate this expression to characterize the average frequency of time the system spends in hegemony:

Theorem 3. *If $a_0 \geq a^O(\chi_2)$ the stationary distribution over states is uniform regardless of ϵ. If $a_0 < a^O(\chi_2)$ then as $\epsilon \to 0$ we have $\sigma_1 + \sigma_2 \to 1$. If $a^O(\chi_2) > a^O(\chi_1)$ then in addition $\sigma_2 \to 1$ and $\sigma_1 \to 0$. For fixed $\epsilon > 0$ time spent in hegemony $\sigma_1 + \sigma_2$ declines with outside influence a_0 and as $\rho(\chi_1) \to 0$ it approaches $2/L$.*

Notice that $1/L$ represents the size of a unit of land relative to the total amount of land. In this model it represents the amount of land that an invader must successfully conquer to get a "toehold" enough to have an appreciable chance of success. As this grows smaller, the fraction of time there is a hegemony falls to zero. In other words, strong outside forces, and a small "toehold" needed for success means little hegemony.

We can summarize the proposition by saying that with strong outsiders there is no tendency towards hegemony, while with weak outsiders the tendency is towards a hegemony of the stronger state. Notice that the circumstances that favor hegemony are exactly the same that favor the society with greater state power. Hegemony and high state power go hand in hand. The circumstances under which we might expect to see institutions with less than maximum state power for appreciable amounts of time are circumstances where hegemony is uncommon and competing states more likely.

From an evolutionary perspective what matter is fitness - in this case state power. From an economic perspective we are not so interested in what level of state power is favored by evolution, but rather what it implies about institutions. Without significant outside influence, we will generally see a hegemony of the institutions χ_2 that maximize state power. In comparison to efficient institutions χ_1 we see from Theorems 1 and 2 that absent significant outside influence there will be a hegemony that will be inefficient and excessively extractive. By contrast, when there is significant outside influence, hegemony will be much less common, and efficient institutions will persist more frequently.

We see also that better ability to aggregate state power makes hegemonies longer-lived. We expect agricultural societies with their higher population densities and food storage technology to be able to better aggregate state power, which contributes to explaining why (as mentioned in the introduction) hegemonies started to form in agricultural societies and not in the pre-existing nomadic forager societies.

Finally, it is of interest to consider tax effectiveness τ. Note that $a^O(\chi) \leq \tau$ so that hegemonic resistance is at most $\rho(\chi_j) = \max\{0, \tau - a_0\}$. Hence for low values of τ the hegemonic resistance is zero, meaning that there is no particular tendency to hegemony. This provides an additional reason that nomadic foragers do not form hegemonies while sedentary farmers do: since nomads carry their wealth with them, it is difficult to tax them.

Notice that we have assumed away technological differences. Exogenous technological differences can easily be introduced - with the obvious conclusion that given the same institutions and absent significant outside influence we expect a hegemony of the superior technology. We do not think, however, that exogenous technological differences are that interesting: more interesting are endogenous technological differences that arise from difference in institutions. Unlike the example, the general model of the next section is broad enough to allow for models in which different institutions lead to different levels of technology.[17] We do not examine such models here, but it provides an interesting area for future research.

4. General Evolutionary Dynamics

We are now going to generalize the static and dynamic models of the previous sections to seek a broader theorem about the emergence and nature of hegemony. Specifically: we consider an arbitrary finite number of diverse societies which may compete with one another; we relax the assumption that equilibrium profiles are always chosen; and we work with a general functional form of the conflict resolution function.

To generalize the static model we allow for an arbitrary finite list of societies $j = 1, \ldots, M$. Each society j has a set of players $i = 1, 2, \ldots, N_j$, although as in the example we understand these to be player roles that may involve representative individuals or collusive groups (in the example $N_j = 2$). Each player has a finite set of actions $a_{ij} \in A_{ij}$ and we denote by $a_j \in A_j$

[17]As the model does not allow for the possibility of continuing growth, it is not broad enough to study technologies that lead to different long-run growth rates.

the corresponding action profiles. We do not explicitly model utility and incentive constraints, but assume rather that for each society there is a set of equilibrium profiles $E_j \subseteq A_j$. These are the profiles for which incentive constraints are satisfied - although the solution concept might depend on the context - in the example E_j is a singleton containing the unique Stackelberg equilibrium in which state officials are the leader. We allow the possibility that E_j is empty.

Finally, as anticipated in the motivating section 2, the relation between actions and state power is given as a map from profiles to state power: $\gamma_j : A_j \to \Re_+$. As in section 3, we will assume that state power and size are positively related to strength in conflict. Note the "size" part of the assumption, which is a main point of this paper: we are asserting that in a competing world it is not income per capita that matters most - what matters is the aggregation of allied forces.

Next we consider evolutionary dynamics. Each society at a moment of time $t = 1, 2, \ldots$ plays an action profile a_{jt} and controls an integral amount of land L_{jt} where $\sum_{j=1}^{M} L_{jt} = L$. If $L_{jt} > 0$ we refer to a society as active, otherwise it is inactive.

In the dynamic example of the previous section we constrained action profiles to lie in E_j. We now drop the requirement that incentive constraints are satisfied at every moment of time, and instead make assumptions about a learning process by which individuals modify their actions and expectations over time. Note that this is consistent with requiring equilibrium at each moment of time as in the example: if we wish to impose this requirement, we simply take $E_j = A_j$ to be a singleton.

We start by considering (for active societies) what a steady state of a learning process should be like. Two things should be true: first, players should expect that today will be the same as yesterday; second, given that expectation, it should be optimal to play the same way as yesterday. In other words, what happened yesterday should be an equilibrium, and in addition that equilibrium should be expected to recur today. In a learning process, the expectation that today should be the same as yesterday will be based on having observed that in the past this has indeed been true. Suppose that we are not yet in a steady state but in fact yesterday was an equilibrium so that $a_{jt-1} \in E_j$ and today is the same as yesterday so that $a_{jt} = a_{jt-1}$. A simple model of learning is to assert that in this case that there is a chance $1 > \psi_j > 0$ that expectations of tomorrow are that it will be the same as today - that we enter a steady state. To indicate this, we introduce a state variable b_{jt} that can take on two values, 1 for steady state expectations and 0 otherwise. When $b_{jt} = 1$ we say that society j is *stable.* If $a_{jt} \notin E_j$ then necessarily $b_{jt} = 0$. If $L_{j,t+1} > 0$, $a_{jt} \in E_j$ and $b_{jt} = 1$ then $a_{j,t+1} = a_{jt}$ and $b_{j,t+1} = b_{jt} = 1$ that is, once an active society achieves a steady state it stays there as long as it remains active.

For active yet unstable societies in which $b_{jt} = 0$ we assume that there is a transition function $P(a_{j,t+1}|a_{jt}) > 0$ that puts positive weight on all profiles. In other words, when people are unsure about the future there is a degree of randomness in their behavior - charismatic leaders may arise, populist nonsense may be believed and so forth.

The stochastic process we are going to study will have a conflict resolution function parametrized by a variable ϵ measuring the sensitivity of conflict to state power and the stable equilibria will be obtained as $\epsilon \to 0$. The key property of the model is that - if there is an equilibrium - the society arrives with probability one to equilibrium in finite time and that the probability of doing so in any given time is appreciable in the sense of being positive and independent of ϵ. That is relative to evolutionary forces, which operate at a speed diminishing in ϵ, learning is relatively fast. This particular model was chosen for concreteness and notational simplicity. Only the property of arrival at equilibrium at with probability one in finite time at a rate independent of ϵ is used in the proof. There are many learning processes which have this property: In Levine and Modica (2013) a slightly more elaborate procedure is given which is decentralized and depends on longer histories of play. Both the dynamic here and the Levine and Modica (2013) dynamic are simplified version of the stochastic individual learning procedure that Foster and Young (2003) introduce and for which they show Nash equilibrium are stochastically stable. It is known from the work of Hart and Mas-Colell (2003) that the only decentralized learning procedures that have this property are stochastic, and they give a deeper discussion of the types of stochastic learning processes that do have this property in Hart and Mas-Colell (2013).

From an economic perspective we think the assumption of relatively rapid learning is the relevant one: we observe that even in highly unexpected and disrupted situations - such as refugee camps - people seem to quickly find modes of behavior that are sensible for the new environment. From the perspective of the model it may not be a terrible surprise that results are robust to replacing the assumption of "always in equilibrium" with the assumption of "quickly in equilibrium." However, as we shall see, this can be misleading, because while the long-run results about hegemony are indeed robust, the particular paths by which hegemony is reached can be quite different when we do not assume equilibrium at every moment of time.

Turning to inactive societies in which $L_j = 0$ we assume that they play a null action profile $a_j = 0$ where $\gamma_j(0) = 0$ - that is inactive societies have no state power. If $E_j = A_j$ then $b_j = 1$,[18] and the society will be stable when it enters, otherwise, as one would not expect that people in a newly created society would ordinarily be instantly in equilibrium, we assume $b_j = 0$ and the society will be unstable when it becomes active. When a society first become active the initial profile is chosen randomly according to $P(a_{j,t+1}|0) > 0$. As inactive societies becoming active represent an experiment with new institutions it makes sense in the context to suppose that new action profiles are experimented with at the same time.[19]

Stepping back, the overall state vector at time t is $s_t = \{a_{jt}, L_{jt,} b_{jt}\}_{j=1}^{J} \in S$ and it evolves according to a Markov process $M(\epsilon)$ that depends on a parameter $\epsilon \geq 0$. We will study the process for small ϵ. To fully specify this process on S we must indicate how land is gained and lost.

As in the example the movement of land between societies is governed by a conflict resolution

[18] If all profiles are equilibrium profiles then there is nothing to learn.

[19] We also need to assume, at least in the case of non-trivial societies (A_j not a singleton), that the fact that a society is active does not preempt the use of the same institutions by another group - something that would not make very much sense. Define two societies j, j' to *use identical institutions* if $A_j = A_{j'}, E_j = E_{j'}$ and $\gamma_j(\cdot) = \gamma_{j'}(\cdot)$. Formally, we assume that for every society j which is non-trivial there exists a society $j' \neq j$ with identical institutions.

function - except that now we have to contend with the possibility of many societies. As before we continue to assume that at most one unit of land changes hands in any given period. We assume that the probability that society j is disrupted and loses a unit of land $\pi_{jt} = \pi(b_{jt}, \gamma_{jt}, L_{jt}, \gamma_{-j,t}, L_{-j,t})[\epsilon]$ depends on the stability of the society and on the level of state power and land holdings of the society and that of rival societies. Notice that since at most one unit of land can change hands each period $\sum_{j=1}^{M} \pi_{jt} \leq 1$ and the shocks must necessarily be correlated. We assume, in fact, that $\sum_{j=1}^{M} \pi_{jt} < \overline{\pi} < 1$ so that there is always a chance, regardless of ϵ that no land changes hands. If a unit of land is lost it is gained by a society chosen randomly according to the function $\lambda(k|j, \gamma_t, L_t) > 0$ for $k \neq j$ and $\lambda(j|j, \gamma_t, L_t) = 0$.

With respect to the conflict resolution function $\pi(b_j, \gamma_j, L_j, \gamma_{-j}, L_{-j})[\epsilon]$ we first assume that for $\epsilon > 0$ we have $\pi(b_j, \gamma_j, L_j, \gamma_{-j}, L_{-j})[\epsilon] > 0$. Second we assume that it is symmetric in γ_{-j}, L_{-j} so that the names of the societies do not matter, only their state power and land holding. Third, we assume that it is monotone: non-increasing in γ_j, L_j and non-decreasing in γ_{-j}, L_j.

Fourth, we assume that an unstable society always has an *appreciable* chance of losing land meaning that $\pi(0, \gamma_j, L_j, \gamma_{-j}, L_{-j})[\epsilon] > 0$ independent of ϵ. The idea is that when expectations of the future are uncertain players have a choice between experimenting with different actions - or with different institutions. Under our assumption that institutions can change on at most a single unit of land in a single period, experimentation means that a single unit of land is lost to the new institution. Notice that the stability of opposing societies does not matter: how disruptive opponents are depends upon their strength and not upon whether or not they are stable. Of course if they are not stable, the actions taken by that society are likely to change in the future, and as a consequence their future ability to be disruptive may be greater or less than their current ability.

Our final and key assumptions concern chance of a stable society losing a unit of land. Define the resistance

$$r(\gamma_j, L_j, \gamma_{-j}, L_{-j}) \equiv \lim_{\epsilon \to 0} \frac{\log \pi(1, \gamma_j, L_j, \gamma_{-j}, L_{-j})[\epsilon]}{\log \epsilon}.$$

This is assumed to exist and satisfy the regularity condition that

$$\text{if } r(\gamma_j, L_j, \gamma_{-j}, L_{-j}) = 0 \text{ then } \lim_{\epsilon \to 0} \pi(1, \gamma_j, L_j, \gamma_{-j}, L_{-j})[\epsilon] > 0.$$

If resistance is non-zero it is assumed to be strictly monotone: strictly increasing in γ_j, L_j and strictly decreasing in γ_{-j}, L_j.

To rule out the possibility of a stalemate where societies are effectively unable to disrupt each other we assume that the weakest active society always has an appreciable chance of losing land: for any profile $\gamma_j, L_j, \gamma_{-j}, L_{-j}$

$$\min_{L_{j'} > 0} r(\gamma_{j'}, L_{j'}, \gamma_{-j'}, L_{-j'}) = 0.$$

Finally, for any profile $\gamma_j, L_j, \gamma_{-j}, L_{-j}$for which j is active and in which at least two opponents are active define the profile $\tilde{\gamma}_{-j}, \tilde{L}_{-j}$ in which all enemy land belongs to the strongest opponent j'. We assume that is better to face divided opponents than a unified one: $r(\gamma_j, L_j, \gamma_{-j}, L_{-j}) \geq$

$r(\gamma_j, L_j, \tilde{\gamma}_{-j}, \tilde{L}_{-j})$ with strict inequality if $r(\gamma_j, L_j, \tilde{\gamma}_{-j}, \tilde{L}_{-j}) > 0$. Notice that the example in section 3 satisfies these assumptions.

We have now defined a Markov process $M(\epsilon)$ on the state space S. Within the state space S we identify certain states as *hegemonic*. A hegemony s_t at (j, a_j) means that $a_{jt} = a_j \in E_j$, that society j is stable $b_{jt} = 1$, and that society j has all the land $L_{jt} = L$. We assume that there is at least one hegemonic state - that is, that the set E_j is nonempty for at least one j. For any hegemonic class (j, a_j) we define the *hegemonic resistance* $\rho(a_j) = r(\gamma(a_j), L, 0, 0)$ to be the resistance to disruption when a hegemony is obtained. Since this is the resistance when no other society has any land, we expect it to depend not only on the strength of the hegemony $\gamma(a_j)$, but also on the strength and ability of outside forces to disrupt the hegemony. Hence the expectation that - as in the example of section 3 - stronger outside forces will decrease hegemonic resistance.

We can now describe the stationary distribution $\mu(\epsilon)$ of the Markov processes $M(\epsilon)$ - this tells us how frequently different states will occur.

Theorem 4. *[Main Theorem] For $\epsilon > 0$ there is a unique $\mu(\epsilon)$ that places positive weight on all states. As $\epsilon \to 0$ there is a unique limit μ. If $\max_{j, a_j \in E_j} \rho(a_j) = 0$ then μ places positive weight on all states (hegemonic or not). If $\max_{j, a_j \in E_j} \rho(a_j) > 0$ then μ places weight only on hegemonic states j, a_j that have maximal equilibrium state power $\gamma_j(a_j) = \max_{j', a_{j'} \in E_{j'}} \gamma_{j'}(a_{j'})$.*

For economies which are proper in the sense of section 2, this says that if hegemonic resistance is positive then inefficiently extractive hegemonies will be observed most of the time; if on the other hand hegemonic resistance is zero in any hegemony, there will be a diversity of institutions, including efficient ones. Notice that the general criterion - hegemonic resistance positive or zero - is exactly the same as the example of section 3. In this sense we find that our earlier results do not depend on the specific assumptions concerning equilibrium, functional form, or the specific structure of society. In particular, economic factors such as the strength of outside forces, the ability to aggregate state power and the efficiency of the technology for taxation will have the same impact as in the example.

We now review here some of the important elements of the proof - the remaining details can be found in Web Appendix 1. If all hegemonies have no resistance $\max_{j, a_j \in E_j} \rho(a_j) = 0$, then due to monotonicity, they can have no resistance if they have less land. This means that all transitions have no resistance, so by the regularity condition have probabilities bounded away from zero independent of ϵ. In this case taking the limit as $\epsilon \to 0$ is irrelevant, and all states have positive probability regardless of ϵ.

Next observe that when $\epsilon = 0$ absorbing states correspond to hegemonies j, a_j with $\rho(a_j) > 0$. Weaker hegemonies are transient, and because of the assumption that at least one active society has no resistance, non-hegemonies are also transient. Within the absorbing hegemonies, we notices that maximizing state power and maximizing hegemonic resistance is the same thing. Specifically, $r(\gamma_j, L, 0, 0)$ is weakly increasing in γ_j and strictly so if it is positive, so when $\max_{j, a_j \in E_j} \rho(a_j) = \max_{j, a_j \in E_j} r(\gamma(a_j), L, 0, 0) > 0$ maximizing $\gamma_j(a_j)$ is the same thing as maximizing $r(\gamma(a_j), L, 0, 0)$.

To show that as $\epsilon \to 0$ only these maximal strength hegemonies have positive weight we use a method of Ellison.

The method of Ellison requires us to compute the least resistance path from one absorbing hegemonic state (j, a_j) to another. It turns out this is relatively easy to compute. The key is that the least resistance to losing a unit of land occurs when there is a single opponent who is as strong as possible. Since this strongest opponent may not be stable and may not satisfy the incentive constraints, we refer to it as a *fanatic band*. Let $3, a_3$ denote the fanatic band. Let $2, a_2$ denote a strongest hegemonic state. Note that this may be weaker than the nightmare society, since the fanatic band need not satisfy incentive constraints - we will return to this in the next section. Finally, let $1, a_1$ denote a second strongest hegemonic state. The total resistance by the strongest hegemonic state to such a fanatic band taking over is called by Ellison the radius and is the sum of resistances to the invader

$$R(a_2) = \sum_{L_3=0}^{L-1} r(\gamma(a_2), L - L_3, \gamma(a_3), L_3, 0, 0).$$

From monotonicity this is strictly increasing in γ_j, that is, the stronger the defender, the greater the resistance to being taken over by the fanatic band.

On the other hand, Ellison asks us to consider the co-radius, which measures how quickly the system returns to the hegemony $2, a_2$. We can show that this is the resistance of the second strongest hegemony to the fanatic band

$$CR(a_2) = \sum_{L_3=0}^{L-1} r(\gamma(a_1), L - L_3, \gamma(a_3), L_3, 0, 0).$$

By monotonicity we see that $R(a_2) > CR(a_2)$. Ellison shows that as $\epsilon \to 0$ the ratio of time during which $(2, a_2)$ has a hegemony to the time at which it does not is approximately

$$1/\epsilon^{R(a_2) - CR(a_2)}.$$

and in the limit this ratio goes to infinity: most of the time there is a hegemonic class that maximizes state power.

Remark. (Relation to Literature on Group Evolution) The novelty of our approach lies in our treatment of incentive compatibility. Existing literature in the area mainly focuses on the interplay between individual and group evolutionary selection: individual behavior which increases fitness of a group, typically some form of "generosity", may be harmful for individual fitness. This is the case both in the Haystack Model as in Maynard Smith (1982) or Richerson and Boyd (2001) and in Bowles' model of conflict and evolution (Bowles (2009)). The equilibrium dimension in the group selection literature is generally missing. One exception is Boyd and Richerson (1990) who consider a setting with multiple Evolutionary Stable Strategies and show that group selection can be operative at the level of the equilibrium.

5. Fanatic Bands and the Decline of Hegemonies

The theoretical literature on evolution has focused on the long-run stationary probability distribution - and existing results do not provide information about the transition paths between stochastically stable states, although there must be continued movement between these states. In the current context, existing results tell us that - when conditions are right - we will see hegemonies most of the time, and also that they will fall for brief periods of time - but not how they will fall. To this question - the nature of the fall - we now turn our attention. Our goal is to highlight both the significance of "quick convergence to equilibrium through learning" rather than "always in equilibrium" and in the case of quick convergence to equilibrium to demonstrate the key role played by what we call fanatic bands.

To avoid triviality, we assume $\max_{j,a_j \in E_j} \rho(a_j) > 0$ and that ϵ is small so that in fact by Theorem 4 hegemonies are commonplace. Second, we make the not unnatural assumption that $\max \gamma_j(a_j) > \max_{j,a_j \in E_j} \gamma_j(a_j)$, that is, that the greatest state power is generated when incentive constraints are violated - note that this rules out "always in equilibrium." In the static example, for this amounts to the state officials choosing state power $a^O = 1$. We refer to a k, a_k that achieves the free maximum of state power as a *fanatic band.* We have assumed that such a band does not satisfy the incentive constraints - and on account of the learning process this means that they are necessarily short-lived - eventually they will learn their way back to equilibrium. However, this does not mean that they do not play a significant role in the dynamics. Empirically, we see that such fanatic bands have played a significant role in the decline of hegemonies in the sense that, for example, Alexander the Great conquered the Persian empire although his followers quickly found that they preferred the sedentary life of great rulers to the ascetic life of soldiers on the march. The same can be said of the descendants of Genghis Kahn, and of various bands of revolutionaries who - after taking power - generally discovered, or their descendants discovered - that they preferred secular luxury to the rude life of a revolutionary - in our terms, they were absorbed by a stable society.

In our interpretation these fanatic bands violate the incentive constraints but eventually learn that they would really prefer secular luxury. An alternative interpretation is that, as in Harsanyi (1973), they suffer from short-run preference shocks making it optimal for a period of time to live a rude and ascetic life. The particular interpretation has no economic consequence as, in either case, the fanatic behavior is short-lived.

To talk about the decline of a hegemony, we must first say what it means for a hegemony to fall. A natural interpretation is that the hegemony faces a "significant" level of competition, or what amounts to the same thing, that it has lost a certain amount of land L_-. As hegemonies suffer small rebellions all the time and quickly stamp them out, we insist that this amount of land be significant in the sense that we assume $L_- > 1$. Of course some hegemonies may so weak that they fall pretty much the first moment they are touched by rebellion of any sort - in this case fanatic bands are scarcely needed to topple them. So we will focus on *non-weak* hegemonies, by which we mean $r(\gamma_j(a_j), L-1, \gamma_k(a_k), 1, 0, 0) > 0$, that is, they have some resistance even to a fanatic band after losing only a single unit of land.

In this setting we wish to ask how a hegemony at j, a_j falls, that is, loses L_- units of land, without first returning to hegemony. In particular we would like to know how likely it is that a fanatic band plays an essential role in this transition. Let us say that a *fanatic band has an essential role* if during the transition there is a period of time and a fanatic band k, a_k such all the land not held by the hegemony during that period is held by the fanatic band. Let Q_f be the probability that the transition takes place and a fanatic band has an essential role during the transition. We want to contrast this with a hegemony's fall where on the contrary no fanatic band has an essential role during the transition. Let Q_{-f} be the probability of this other event.

Historical evidence shows that empires are most often overthrown by what we have described as fanatic bands. In our model this is confirmed rather sharply: as the next theorem shows, for small ϵ the probability Q_f becomes infinitely greater than Q_{-f}:

Theorem 5. *For a non-weak hegemony and any* $L \geq L_- > 1$ *we have* $\lim_{\epsilon \to 0} Q_f / Q_{-f} \to \infty$.

The proof, which uses the method of hitting times can be found in Appendix 3. The intuition is that in an evolutionary model dynamics are driven by "luck." To overcome a large powerful hegemonic society requires a considerable amount of luck. The best kind of luck to have is to have a great deal of power: a strong organization, good technology - and a charismatic and brilliant leader concerned with conquest over personal pleasure. Even better luck is to have that leader convince his followers to set aside their incentive constraints as well. Such luck will not last long - eventually we know that warriors or revolutionaries or their descendants will prefer to follow their incentives and consume "jewelry" rather than "swords" - but the luck can last long enough - as it did for Alexander the Great or Genghis Kahn - to conquer the relevant world. Since a fanatic band is in a race with time - to overcome the hegemony before collapsing of its own internal contradictions - it is not obvious that a slower steadier attack by a stable but weaker opponent might not have a better chance of success. Never-the-less, the theorem shows that when hegemonies are likely to arise - when ϵ is small - this is not the case.

6. Conclusion

Readers of grand theories of history such as those of McNeil (1963), Cipolla (1965), Diamond (1998) or Acemoglu and Robinson (2012) will not find surprising the idea that ideas are spread by the conquest of the less advanced by the more advanced - indeed it seems almost ubiquitous in their anecdotes and discussions. Missing from these accounts, however, is a model of dynamic competition between conflicting societies. Here we introduce such a model and find that there is a tendency towards hegemony when outside forces are weak - but less so when they are strong.[20] We also find that these hegemonies tend to maximize state power and that this results in inefficiently high exclusiveness which in turn determines inefficiently high extractiveness, that is high taxes,

[20] A recent empirical paper on the relation between warfare and institutions in the Italian *Risorgimento* is Dincecco, Federico and Vindigni (2011).

high income for state officials, low income for producers, and low welfare. Moreover, the theory sheds new light on the role - so prominent in all history books - of charismatic leaders and their armies in the fall of empires.

The theory has several implications. In the introduction we gave a broad view of hegemony and the connection to the strength of outside forces. Here we give some more speculative thoughts about institutions and history as seen through the lens of hegemonic state power and outside influence.

Democracy and military spending. In the range between welfare maximization and state power maximization the theory predicts a positive relationship between exclusiveness and state power. If one takes military spending as a measure of state power this suggests that more democratic societies would generally spend less on the military than less democratic societies. This is a robust finding in the empirical political science literature: see for example Dunne and Perlo-Freeman (2003) or Dunne et al (2008).

Decreasing exclusiveness. Analyses such as those of Hoffman and Rosenthal (2000) argue that the transition from absolute to constitutional monarchy in Europe was determined by the higher tax revenue to be employed for military purposes which a parliament could generate. This can occur in our model if technological change increases the efficiency of tax collection τ in which case it will reduce the optimal degree of exclusiveness. For example improved military technology - the development of firearms, for example - can improve the efficiency of tax collection (raise τ) resulting in both state power maximizing and welfare maximizing levels of exclusiveness declining.

Technology and state power. We have modeled the effect of exclusiveness on output as taking place through the tax system. There can also be a direct effect of exclusiveness lowering productivity as suggested for example in Acemoglu and Robinson (2012).[21] As indicated above, it is possible to study models of endogenous technology driven by institutional differences in our general evolutionary model. We do not expect that it will change the general nature of the conclusions from the simple example, but may have additional interesting implications. In particular the nature of technology may interact with institutions. For example, at the beginning of the cold war, technology favored assembly line manufacturing which is relatively amenable to central planning, and so the Soviet Union, a particularly exclusive and extractive system, was able to compete successfully with the United States. By contrast as technology changed to favor greater decentralization and inclusiveness, it is likely that the enormous growth of GDP in the United States relative to the Soviet Union made it impossible for the Soviet Union to continue to compete.

Nationalism. We have characterized institutions by exclusiveness - the extent to which state power is unchecked in collecting taxes. Another dimension in which institutions may differ is in the extent to which tax revenue is checked in being used as external state power. It is simple to modify the model to include another multiplier which we might think of as "nationalism" which converts the portion of tax revenue devoted to state power to actual (external) state power. At the extreme

[21] There can also be a direct benefit to government officials of state power, for example, they may have a taste for warfare as in Hoffman (2013) - this would not change the qualitative nature of our results.

we can think of this as being zero in the case of Japan where the constitution prohibits the use of military force externally.[22] Such a multiplier has no implication for welfare, but obviously state power is maximized when the coefficient of nationalism is one. In other words: nationalism is a necessary characteristic of long-lived societies - which may help to explain its prevalence.[23]

An evolutionary model is a model of the very long-run and historically hegemonic societies have lasted many centuries. Moreover, the decline and fall of societies according to the model is driven by infrequent bad luck. Applying the model to current affairs is particularly speculative, more so given that modern institutions are quite recent with the oldest ones being those of the U.S.[24] However, consistent with the theory the U.S. - which has a very high level of military expenditure - has had an effective hegemony over the North American continent for 237 years. We notice also that despite modern technology the large oceans appear to still provide a formidable military obstacle - it seems unlikely that either the United States or China will bypass these barriers to establish a world hegemony. With five major "rivals" in Eurasia - Europe, Russia, China, India and the Islamic block - we may anticipate that the U.S. may play the role in the future of the Eurasian continent that we believe England did in continental Europe - that of preventing hegemony and preserving competition.[25] Indeed: the United States took over the role from England of outside power in Europe during the 20th Century - without the intervention of the United States during the Second World War, it is likely that either Germany or Russia would have established a hegemony over Europe.

Appendix 1: Historical Data and Computations

Summary of Chinese dynastic history taken from Table 1.1 of Maddison (1998).

- 221 BCE - 206 BCE: Ch'in (hegemonic); 206 BCE - 8 CE: early Han (hegemonic); 8 CE - 23 CE: interregnum
- 23 CE - 220 CE: later Han (hegemonic); 220 CE - 589 CE: Empire disintegrated
- 589 CE - 617 CE: Sui (hegemonic); 618 CE - 906 CE: T'ang (hegemonic); 906 CE - 960 CE: empire disintegrated
- 960 CE-1127 CE: Sung(hegemonic); 1127 CE - 1279 CE: interregnum (Jurchen/Yuan in North, Southern Sung)
- 1279 CE - 1368 CE: Yuan (hegemonic); 1368 CE - 1644 CE: Ming (hegemonic)

[22]In an earlier version of this paper Levine and Modica (2012) we considered "expansionism" which allowed the use of state power for defense but not for offense. However, we showed that under mild conditions such societies will have little evolutionary success.

[23]Notice that just as exclusiveness may have a direct effect on technology, so may nationalism: for example, the same desire to protect against outsiders and to conquer them may also inhibit the peaceful arrival of productive immigrants and so lower output. Never-the-less unless this force is very strong, maximizing state power will involve a substantial amount of nationalism.

[24]The European Union - which includes England - as well as current institutions in China and India are all post World War II.

[25]Note that the ratio of U.S. to Eurasian population is similar to that of England to continental Europe.

- 1644 CE - 1911 CE: Chi'ng (hegemonic); 1911 CE - 1949 CE: interregnum
- 1949 CE - 2013 CE: Communist (hegemonic)

The total number of years covered is 2234. There are five interregna totally 630 years, so 72% of the period is hegemonic.

Summary of ancient Egyptian history taken from Shaw (2000).

- 2686 BCE - 2160 BCE: Old Kingdom (hegemony); 2160 BCE - 2055 BCE: first intermediate period
- 2055 BCE - 1650 BCE: Middle Kingdom (hegemonic); 1650 BCE - 1550 BCE: second intermediate period
- 1550 BCE - 1069 BCE: New Kingdom (hegemonic)

The total number of years covered is 1617. There are two interregna totally 205 years, so 87% of the period is hegemonic.

Summary of ancient Persian history taken from Daryaee (2012)[26]

- 550 BCE - 330 BCE: Achaemenid Persian Empire (hegemonic); 330 BCE - 250 BCE: interregnum
- 250 BCE - 114 CE: Parthian Empire (hegemonic); 114 CE - 224 CE: interregnum
- 224 CE - 651 CE: Sassanian Empire (hegemonic)

The total number of years covered is 1201. There are two interregna totally 190 years, so 84% of the period is hegemonic.

Additional history

We count the hegemony of Rome from Augustus in 27 BCE to the permanent division into the Eastern and Western Empires in 395 CE, a period of 422 years. The Western Empire did not maintain a hegemony for a significant period after this. We date the end of the hegemony of the Eastern Empire to the expansion of the Caliphate in 814 CE. - 395 a period of 429 years.

In England we date the beginning of the hegemony from the Norman conquest in 1066, a period of 947 years.

We date the hegemony of the Ottoman Empire from the conquest of Egypt in 1517 to the Greek revolution in 1821, a period of 304 years.

[26] Additional information about the Parthian Empire from Wright (2006). Note that the conventional dates of the Parthian Empire conclude with the Sassanian Empire, but the hegemony of the Parthian Empire appears to have ended following the war with Rome in 114 AD, so we take that as the end date.

Demographic data taken from Maddison (2013)

Ratio of population of Mongolia to China in 1820 CE (the earliest date for which there is an estimate of the Mongolian population): .2%.

Ratio of population of Scandinavia to Western Europe excluding Greece and the British Isles in 1000 CE 5%.

Ratio of population of United Kingdom to all of Western Europe: In 1000 CE it is 8% and remains relatively stable until it rises in the early late 1800s, rising to 19% in 1820 CE and remaining relatively stable since then.

Estimate of ratio of population of Central Asia to India in 1820 CE. The ratio of the population of Afghanistan to India in 1820 is 1.6%. Data for the rest of Central Asia is not available until 1950 CE. We computed the ratio of the population of the rest of Central Asia (the Soviet "stans": Kazakhstan, Kyrgyzstan, Tajikstan, Turkmenistan, and Uzbekistan) to Afghanistan in 1950 CE when data is available as being about double that of Afghanistan. Assuming that ratio is about the same as in 1820 CE we estimate the overall ratio of the population of Central Asia to India in 1820 CE as about 5%.

Appendix 2: Analysis of the Static Example

Recall the requirements for an economy to be proper: $G(b) = 0$ at $b = 0$ and for $b \geq \overline{b} \equiv (1-c)/\tau$ while for $0 \leq b \leq \overline{b}$ $G(b)$ is twice continuously differentiable with $G''(b) < 0$. Moreover, $G'(0) > 1$ and $G'(b) + bG''(b)$ is decreasing. For $0 \leq b \leq \overline{b}$ $\Pi(b)$ is twice continuously differentiable, decreasing and $\Pi''(b) < 0$ with $\Pi(b) = 0$ for $b \geq \overline{b}$. Finally, $\Pi(b) - G(b)$ is decreasing.

Lemma 1. *In the linear quadratic economy optimal effort is given by*

$$a^P = \max\{0, 1 - \frac{\overline{\tau}}{1-c}\}$$

Proof. The problem of the producer is to maximize, with respect to a^P utility

$$u^P = (1-\overline{\tau})a^P - [ca^P + (1/2)(1-c)\left(a^P\right)^2] + \xi^P a^O$$

where $\overline{\tau} = \min\{1, \tau\chi a^O\}$. The derivative of u^P is $(1-\overline{\tau}) - [c + (1-c)a^P]$. This is negative for all a^P if $\overline{\tau} > 1-c$, otherwise it hits zero at $1 - \overline{\tau}/(1-c)$. Solving gives the desired result. □

Proposition 1. *The linear quadratic economy is proper.*

Proof. Suppose $b < \overline{b} \equiv (1-c)/\tau$. Recall that $\overline{\tau} = \min\{1, \tau b\}$. Since $b < \overline{b}$ we have $\tau b < 1 - c \leq 1$ so $\overline{\tau} = \tau b$. Hence in this case by Lemma 1

$$a^P = 1 - \frac{\overline{\tau}}{1-c}$$

Suppose $b \geq \overline{b} \equiv (1-c)/\tau$. If $\tau b > 1$ then $\overline{\tau} = 1 \geq 1 - c$. If $\tau b \leq 1$ then $\overline{\tau} = \tau b \geq 1 - c$. Hence in this case by Lemma 1 $a^P = 0$.

Tax revenue is $G(b) = \overline{\tau}a^P$. If $b \geq \overline{b}$ this is zero. Otherwise

$$G(b) = \tau b \left[1 - \frac{\tau b}{1-c}\right]$$

which is obviously concave. From $G'(b) = \tau[1 - 2\tau b/(1-c)]$ we see that $G'(0) = \tau > 1$. Moreover

$$G'(b) + bG''(b) = \tau - b\frac{4\tau^2}{1-c}$$

is obviously decreasing in b.

Profit is $\Pi(b) = G(b) + u^P - \xi^P a^O$. If $b \geq \overline{b}$ this is zero. Otherwise we have

$$\Pi(b) = \frac{1-c}{2}\left[1 - \frac{\tau b}{1-c}\right]^2 + \tau b\left[1 - \frac{\tau b}{1-c}\right] \qquad \Pi'(b) = -\frac{\tau^2 b}{1-c}$$

Thus profit is decreasing in b and $\Pi''(b) < 0$.

Finally, for $b \leq \overline{b}$ we have

$$\Pi(b) - G(b) = (1-\overline{\tau})a^P - \left[ca^P + (1/2)(1-c)\left(a^P\right)^2\right]$$

$$= (1-\overline{\tau})\left[1 - \frac{\overline{\tau}}{1-c}\right] - c\left[1 - \frac{\overline{\tau}}{1-c}\right] - \frac{1}{2}(1-c)\left(1 - \frac{\overline{\tau}}{1-c}\right)^2$$

$$= \frac{1}{2}(1-c)\left(1 - \frac{\overline{\tau}}{1-c}\right)^2$$

which since $\overline{\tau} = \min\{1, \tau b\}$ and $\overline{\tau} < 1-c$ is decreasing in b. □

Theorem. *[1 from the text] In a proper economy there is a unique equilibrium level of state power $a^O(\chi)$, and it is single peaked in χ; so it has a unique argmax $\chi_2 > 0$. There is a unique welfare maximizing level of exclusivity χ_1, and $\chi_1 \leq \chi_2$. There is a $\overline{\xi} \geq 1$ such that if $\xi^P + \xi^O \leq \overline{\xi}$ then $\chi_1 < \chi_2$.*

Proof. The official's utility is given by $u^O = G(\chi a^O) - (1-\xi^O)a^O$, where $G(b)$ has $G''(b) < 0$ on $[0, \overline{b}]$ and $G(b) = 0$ for $b > \overline{b}$. Hence the maximum with respect to a^O is unique and is either 0 if $\chi G'(0) < 1 - \xi^O$ or the unique to the first order condition $\chi G'(\chi a^O) = 1 - \xi^O$ otherwise. In the latter case, we have from the implicit function theorem

$$\frac{da^O}{d\chi} = -\frac{G'(b) + bG''(b)}{\chi^2 G''(b)}$$

and $G'(b) + bG''(b)$ decreasing implies that this derivative can be zero at most once, so the function $a^O(\chi)$ is continuous and single peaked. Note that since $G'(0) > 1 - \xi^O$ from the first order condition the solution $a^O(1) > 0$ implying a positive level of state power is feasible, and hence that the argmax $\chi_2 > 0$.

Welfare is $W(a^O) = \Pi(\chi a^O) - (1 - \xi^P - \xi^O)a^O$ and since $\Pi''(b) < 0$ in $[0, \overline{b}]$ there is a unique maximum at some $\hat{a}^O$. Suppose $\chi > \chi_2$. Then there is a $\chi' < \chi_2$ with $a^O(\chi') = a^O(\chi)$. Since $\Pi(b)$ is decreasing, this implies that $W(a^O(\chi')) > W(a^O(\chi))$. Hence $\chi_1 \leq \chi_2$, and exact equality is possible only if either $\chi_2 = 0$ or $\hat{a}^O > \max_\chi a^O(\chi)$. But $\chi_2 > 0$ and $a^O(\chi_2) > 0$; and if $\xi^P + \xi^O < 1$ then $W(a^O)$ is decreasing, so $\hat{a}^O = 0 < a^O(\chi_2)$. □

Theorem. *[2 from the text] In a proper economy profits $\Pi(\chi a^O(\chi))$ is decreasing in χ, while tax revenues $G(\chi a^O(\chi))$, fiscal capacity $\chi a^O(\chi)$, and the utility of state officials $u^O(\chi, a^O(\chi))$ are all increasing in χ. For $\chi \geq \chi_1$ producer utility is decreasing in χ and if $\xi^P + \xi^O < 1$ so is welfare. If $\xi^P + \xi^O \geq 1$ welfare is decreasing for $\chi_1 \leq \chi \leq \chi_2$.*

Proof. The first order condition for maximizing u^O is $\chi G'(b) = 1 - \xi^O$ so by the envelope theorem we have $du^O/d\chi = a^O(\chi)G'(\chi a^O(\chi)) = a^O(\chi)/\chi > 0$. So the utility of state officials is increasing in χ; and from $db/d\chi = -1/\chi^2 G''(b) > 0$ fiscal capacity is also increasing in χ. For tax revenues $G(\chi a^O(\chi))$ since fiscal capacity is increasing in χ it is sufficient that $G'(\chi a^O(\chi)) > 0$, which follows from the first order condition for maximizing u^O. Profits decrease with χ because it is assumed to be decreasing in fiscal capacity which increases with χ.

For $\chi_1 \leq \chi \leq \chi_2$ we have $a^O(\chi)$ increasing, while $W(a^O)$ is concave in a^O so welfare is decreasing in χ. Since state official utility is increasing, it follows that producer utility must be decreasing. For $\chi > \chi_2$ we have fiscal capacity b increasing and state power a^O decreasing. Since producer utility is welfare minus state official utility it is $u^P = \Pi(b) - G(b) + \xi^P a^O$, and $\Pi(b) - G(b)$ is assumed to be decreasing. When $\xi^P + \xi^O < 1$ we also have welfare $W = \Pi(b) - (1 - \xi^P - \xi^O)a^O$ decreasing. $\square$

Appendix 3: Fanatic Bands

Let L_k^k be least amount of land such that $r(\gamma_j(a_j), L - L_k^k, \gamma_k(a_k), L_k^k, 0, 0) = 0$; let $L_{k-}^k = \min\{L_-, L_k^k\}$ - by assumption $L_{k-}^k > 1$. Let $\rho = r(\gamma_j(a_j), L, 0, 0)$: this is the resistance to the hegemony losing the first unit of land. Let $R_f = \sum_{L_k=1}^{L_{k-}^k - 1} r(\gamma_j(a_j), L - L_k, \gamma_k(a_k), L_k, 0, 0)$: this is the resistance of the hegemony to the fanatic band of losing additional land until either the hegemony falls or its resistance to the fanatic band falls to zero. Let $R_{-f} = \sum_{L_k=1}^{L_{k-}^k - 1} \min_{L_{-j}} r(\gamma_j(a_j), L - L_k, \gamma_{-j}(a_{-j}), L_{-j})$: this is greatest possible resistance to the amount of land falling to L_{k-}^k when no fanatic band has an essential role. By monotonicity $R_{-f} > R_f$ (for if no fanatic band has an essential role then some society during the transition must be playing actions with less than maximum state power). Then Theorem 5 in the text follows directly from the bounds

Lemma 2. *For some constants C_f, C_{-f} we have $Q_f \geq C_f \epsilon^{\rho + R_f}$,$Q_{-f} \leq C_{-f} \epsilon^{\rho + R_{-f}}$*

Proof. To establish the first inequality observe that one way to get from $L_j = L$ to $L_j = L - L_-$ without hitting $L_j = L$ is by means of the path $L_k = 1, 2, \ldots, L_-$ in which the fanatic band grabs one piece of land each period. This has resistance $\rho + R_f$. The probability bound then follows from the definition of resistance and the fact that the probability that some fanatic band has a role sometime during the transition is at least as great as the probability this particular fanatic band has this role at this particular point in the transition.

To establish the second inequality, observe that the first thing that must happen is that the hegemony must lose a unit of land. The resistance to this is ρ so by the properties of resistances, the probability has the form $D\epsilon^\rho$. Following this the hegemony must lose a second unit of land without first falling back into hegemony. The probability this happens depends upon the current state: the particular society and actions that have taken over the first unit of land. However by assumption this is not a fanatic band. Let $Q(L-2, t_2)$ be the greatest probability over all states consistent with the hegemony holding $L-1$ units of land that exclude a fanatic band of first hitting $L_j = L - 2$ without reverting to hegemony for the first time after t_2 periods. Following this, we must get to $L - 3$ without first reverting to hegemony. Again this depends on the current state. Again, we know that a fanatic band does not occupy all the land not dominated by the hegemony.

Let $Q(L-3, t_3)$ be the largest probability over all these states of first hitting $L_j = L-3$ without reverting to hegemony for the first time after t_3 periods following t_2. Continuing in this way, we may define $Q(\ell, t_{L-\ell})$ for going from $\ell+1$ to ℓ for the first time without reverting to hegemony. This gives us the inequality

$$\begin{aligned} Q_{-f} &\leq D\epsilon^{\rho} \sum_{t_2=1}^{\infty} \sum_{t_3=1}^{\infty} \cdots \sum_{t_{L-L_-}}^{\infty} Q(L-2, t_2) Q(L-3, t_3) \ldots Q(L_-, t_{L-L_-}) \\ &= D\epsilon^{\rho} \prod_{\ell=L_-}^{L-2} \sum_{t=1}^{\infty} Q(\ell, t) \leq D\epsilon^{\rho} \prod_{\ell=L_{k-}^{k}}^{L-2} \sum_{t=1}^{\infty} Q(\ell, t) \end{aligned}$$

The key idea is that $Q(\ell, t)$ has a bound that on the one hand is proportional to the chance of going from $\ell+1$ to ℓ in one step, and on the other declines exponentially in t so that the sums converge. In a certain sense each period there is a minimal chance that the hegemony is restored: so to hit ℓ for the first time in period t it must be that the coin is flipped successfully t times (hegemony is not restored), then the transition from $\ell+1$ to ℓ must take place.

To make this precise, define

$$\underline{\pi} = \min\{ \min_{r(\gamma_{jt}, L_{jt}, \gamma_{-j,t}, L_{-j,t})=0 | L_{jt}>0} \pi(1, \gamma_{jt}, L_{jt}, \gamma_{-j,t}, L_{-j,t})[\epsilon], \min_{L_{jt}>0} \pi(0, \gamma_{jt}, L_{jt}, \gamma_{-j,t}, L_{-j,t})[\epsilon] \}$$

and $\underline{\lambda} = \min_{k \neq j} \lambda(k|j, \gamma_t, L_t)$ and observe by assumption both of these are strictly positive. Next, break t into blocks of length $L-\ell$ - the minimum number of transitions needed to get back to hegemony. Notice that there are $\lfloor t/(L-\ell) \rfloor \geq t/(L-\ell) - 1$ such blocks (where $\lfloor . \rfloor$ denotes the greatest integer less than or equal to). Notice that by monotonicity and the definition of L_{k-}^{k} we have $r(\gamma_j(a_j), \ell, \gamma_k(a_k), L-\ell, \gamma_{-j-k}(a_{-j-k}), 0) > 0$, hence by monotonicity and the divided opponents assumption $r(\gamma_j(a_j), \ell, \gamma_{-j}(a_{-j}), L_{-j}) > 0$. It follows from the absence of stalemate that there is some $j' \neq j$ with $L_{j'} > 0$ that has no resistance, hence at least a $\underline{\pi}$ chance of losing a unit of land, and there is a chance of at least $\underline{\lambda}$ that this land it taken over by j. Hence in each block there is a chance of at least $p = (\underline{\pi}\underline{\lambda})^{L-\ell}$ of returning to $L_j = L$. In other words, each $L-\ell$ periods you must flip a coin with probability $1-p$ in order not to have returned to hegemony. Using the definition of resistance we therefore have the inequality

$$\begin{aligned} Q(\ell, t) &\leq (1-p)^{[t/(L-\ell)]-1} F \epsilon^{\min_{L_{-j}} r(\gamma_j(a_j), \ell, \gamma_{-j}(a_{-j}), L_{-j})} \\ &\leq F \frac{\left[(1-p)^{1/(L-1)}\right]^t}{1-p} \epsilon^{\min_{L_{-j}} r(\gamma_j(a_j), \ell, \gamma_{-j}(a_{-j}), L_{-j})}, \end{aligned}$$

Summing we find the desired bound

$$Q_{-f} \leq DF \frac{(1-p)^{1/(L-1)}}{(1-p)\left[1-(1-p)^{1/(L-1)}\right]} \epsilon^{\rho} \epsilon^{R_{-f}}.$$

□

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Web Appendix 1: Proof of the Main Theorem

Theorem. *[Main Theorem from the text] For $\epsilon > 0$ there is a unique $\mu(\epsilon)$ that places positive weight on all states. As $\epsilon \to 0$ there is a unique limit μ. If $\max_{a_j \in E_j} \rho(a_j) = 0$ then μ places positive weight on all states (hegemonic or not). If $\max_{a_j \in E_j} \rho(a_j) > 0$ then μ places weight only on hegemonic states j, a_j that have maximal equilibrium state power $\gamma_j(a_j) = \max_{a_{j'} \in E_{j'}} \gamma_{j'}(a_{j'})$.*

Proof. If $\max_{a_j \in E_j} \rho(a_j) = 0$ then by monotonicity, the resistance of all societies is zero under all circumstances. If this is the case, or if $\epsilon > 0$ then all feasible transitions have positive probability. It is then just a matter of checking that any state can be reached from any other by a finite sequence of feasible transitions. Hence the process $M(\epsilon)$ is positively recurrent, implying that $\mu(\epsilon)$ is unique and places positive weight on all states. Next, observe that the stationary distributions are the solutions of $\mu(\epsilon) = M(\epsilon)\mu(\epsilon)$. Under our assumptions in the limit as $\epsilon \to 0$ we have $M(\epsilon) \to M(0)$ from which it follows that if μ is any limit point of $\mu(\epsilon)$ then μ is a stationary distribution for $M(0)$. As $\mu(0)$ is unique it follows that $\mu(\epsilon) \to \mu(0)$.

Now suppose $\max_{j, a_j \in E_j} \rho(a_j) > 0$. A theorem of Young (1993) shows that from the assumption that $M(\epsilon)$ is regular it is still the case that $\mu(\epsilon)$ has a unique limit μ. Now however $M(0)$ can have many stationary distributions, so the question is: which one is μ? Notice that any hegemonic state for which $\rho(a_j) > 0$ is absorbing in $M(0)$ since when $\epsilon = 0$ the probability of disruption is zero. On the other hand from any other type of state, the argument of the previous paragraph shows that there is a positive probability of reaching one of these absorbing states, so all other states are transient. Hence μ can place weight only on such hegemonic states. If the second highest value of $\rho(a_j) = 0$, then the result holds trivially. Otherwise we use a theorem of Ellison (2000) to show that these classes have probability zero in μ.

Let H denote the set of hegemonic absorbing states, that is, the states $s_t \in H$ are hegemonies (j, a_j) for which $\rho(a_j) > 0$. In addition to writing $s_t \in H$ we write $(j, a_j) \in H$. Also write H^* for those hegemonic absorbing states that maximize state power.

To apply Ellison's method we must determine for each $\hat{s}_t \in H$ the basin consisting of states s_t for which when $\epsilon = 0$ the probability of reaching $\hat{s}_t$ is one. Suppose $\hat{s}_t$ is a hegemony for (j, a_j). In any state s_t with $\pi(1, \gamma_{jt}, L_{jt}, \gamma_{-jt}, L_{-jt})[0] = 0$, by assumption it must be the case that for some k and $b_k = 1$ we have $\pi(b_k, \gamma_{kt}, L_{kt}, \gamma_{-kt}, L_{-kt})[0] > 0$, while this is always true for $b_k = 0$ by assumption. Hence in this case there is no chance of j losing land, and a positive probability of gaining it (from society k), meaning with probability one we return to a hegemonic j. That is, s_t is in the basin of $\hat{s}_t$. On the other hand if $\pi(1, \gamma_{jt}, L_{jt}, \gamma_{-jt}, L_{-jt})[0] > 0$ so that there is a positive chance j loses land, since that will only increase the subsequent chance of losing land, there is a positive probability it will lose all its land and become absorbed in some different hegemonic state. Hence the basin are exactly those states for which $\pi(1, \gamma_{jt}, L_{jt}, \gamma_{-jt}, L_{-jt})[0] = 0$.

The radius of j, a_j is defined as the least resistance path out of the basin of $\hat{s}_t$. The initial resistance to losing a unit of land is $\rho(a_j)$. Subsequently the greatest chance of losing another unit of land occurs when there is a single opponent who has the highest possible state power. Consider then a society k and profile a_k such that $\gamma_k(a_k)$ is maximal over all societies and profiles. There are two cases: either some such a_k is an equilibrium - the equilibrium case - in which case it is part of the hegemonic states we are trying to establish have positive probability in μ, or every such a_k fails to be an equilibrium, in which case we refer to a_k as a fanatic band.[27]

[27] We assume below that $k \neq j$. It is possible to select such a k except in one case: in the equilibrium case when there is a single j, a_j in $H*$ and that society is always in equilibrium and has no duplicate. This is the case in the example. However: the only change needed in subsequent computations is for the radius of j, a_j itself - and assuming that the society can invade itself gives a valid lower bound on the radius, so the result is not affected.

We can now show two things: first that the radius of any hegemony $\hat{s}_t$ at j, a_j is strictly increasing in γ_j. Second, in this model all roads lead to Rome: if $\hat{s}_t \in H \backslash H^*$ and $s_t^* \in H^*$ there is a least resistance route out of the basin of $\hat{s}_t$ which also leads to s_t^*. In the equilibrium case, we may assume that hegemony of s_t^* is the invader k, a_k. In both the equilibrium and fanatic band case we assume each period one unit of land is lost to k, a_k - until further resistance is futile (reaches zero). Since this is the edge of the basin the radius $R_j(a_j)$ is the sum of resistances until the threshold is reached, and notice that since each individual resistance is strictly increasing in γ_j so is the radius. In the equilibrium case there is from that point a zero resistance path in which k, a_k continues to acquire land until s_t^* is reached. In the fanatic band case, there is a zero resistance path to every other hegemonic absorbing state: the band remains unstable and takes over all the land with no resistance. Since it is unstable, it than can lose units of land without any resistance, and there is a positive probability independent of ϵ so that the land is lost to any hegemony whatever including s_t^*.

The *co-radius* of s_t^* is the greatest over $\hat{s}_t \in H \backslash H^*$ of the least resistance of any path from $\hat{s}_t$ to s_t^*. In general the least resistance path from $\hat{s}_t$ to s_t^* may not include a least resistance path out of the basin of $\hat{s}_t$, but in this model it does because all roads lead to Rome. This means that the least resistance from $\hat{s}_t$ to s_t^* is $R_j(a_j)$. Hence the co-radius of (k, a_k) is just

$$CR_k(a_k) = \max_{(j,a_j) \in H \backslash H^*} R_j(a_j).$$

Hence if $k, a_k \in H^*$ then $R_k(a_k) > CR_k(a_k)$ which is Ellison's criterion for μ to place weight only on H^*. $\square$

Web Appendix 2: Detailed Analysis of the Linear/Quadratic Case

Recall that $\xi^O \leq 1$ by assumption. We will point out in the course of the analysis that otherwise officials may want to set $a^O > 0$ even if it makes producers lay idle. Also recall from Lemma 1 of Appendix 2 the solution to the problem of the producer:

$$a^P = \max\{0, 1 - \frac{\overline{\tau}}{1-c}\}.$$

The problem of the state official is to maximize

$$u^O = \overline{\tau} a^P - (1 - \xi^O) a^O = \min\{1, \tau\chi a^O\} \cdot \max\Big\{0, 1 - \frac{\min\{1, \tau\chi a^O\}}{1-c}\Big\} - (1 - \xi^O) a^O$$

Suppose $\tau\chi < 1$. Then for all $a^O \leq 1$ also $\tau\chi a^O < 1$ so

$$u^O = \tau\chi a^O \cdot \max\{0, 1 - \frac{\tau\chi a^O}{1-c}\} - (1 - \xi^O) a^O$$

Now if a^O is such that $\tau\chi a^O \geq 1 - c$ then $u^O(a^O) < 0 = u^O(0)$[28] hence such a value cannot be

[28] This would not be true if $\xi^O > 1$, and it can be checked that equilibrium would have $a^P = 0$ and $a^O = 1$ for ξ^O large enough.

optimal; so we may take $\tau\chi a^O < 1-c$, in which case

$$\begin{aligned} u^O &= a^O[\tau\chi - (1-\xi^O) - \frac{(\tau\chi)^2}{1-c}a^O] \\ &= -\frac{(\tau\chi)^2}{1-c}a^O[a^O - \frac{(1-c)(\tau\chi-(1-\xi^O))}{(\tau\chi)^2}] \end{aligned}$$

This is a concave parabola. If $\tau\chi - (1-\xi^O) \le 0$ i.e. $\xi^O \le 1-\tau\chi$ then max $a^O = 0$. Otherwise argmax is

$$a^O = \frac{(1-c)(\tau\chi-(1-\xi^O))}{2(\tau\chi)^2}$$

positive and smaller than $(1-c)/\tau\chi$ in the current parameter range.

Now suppose $\tau\chi \ge 1$. Then $\frac{1-c}{\tau\chi} \le 1-c < 1$. For $a^O \ge \frac{1-c}{\tau\chi}$ one has $\bar\tau \ge 1-c$ so inserting $a^P = 0$ into u^O we get $u^O = -(1-\xi^O)a^O \le 0 = u^O(0)$,[29] whence optimal choice may be again searched in the range $a^O < (1-c)/(\tau\chi)$. In this range $\bar\tau = \tau\chi a^O < 1-c$ so

$$\begin{aligned} u^O &= \tau\chi a^O(1-\frac{\tau\chi a^O}{1-c}) - (1-\xi^O)a^O \\ &= \tau\chi a^O - \frac{\left[\tau\chi a^O\right]^2}{1-c} - (1-\xi^O)a^O \\ &= a^O[-\frac{(\tau\chi)^2}{1-c}a^O + (\tau\chi-(1-\xi^O))] \\ &= -\frac{(\tau\chi)^2}{1-c}a^O[a^O - \frac{(1-c)(\tau\chi-(1-\xi^O))}{(\tau\chi)^2}] \end{aligned}$$

Again this is a concave parabola, but now we know $\tau\chi - (1-\xi^O) > 0$ so argmax is the positive one above.

Thus

$$a^O(\chi) = \max\Big\{0, \frac{1-c}{2}\frac{\tau\chi-(1-\xi^O)}{(\tau\chi)^2}\Big\}$$

Note that $\tau\chi a^O(\chi) < 1-c$ for all χ. Thus the equilibrium tax rate is the following, increasing in χ:

$$\bar\tau = \tau\chi a^O(\chi) = \max\Big\{0, \frac{1-c}{2}(1-\frac{1-\xi^O}{\tau\chi})\Big\}$$

And inserting this into optimal producer's choice we get (since $\bar\tau < 1-c$)

$$a^P(\chi) = 1 - \frac{\bar\tau}{1-c} = 1 - \max\Big\{0, \frac{1}{2}(1-\frac{1-\xi^O}{\tau\chi})\Big\}$$

Plugging back optimal choices we can compute utilities. If $\tau\chi < 1$ and $\xi^O \le 1-\tau\chi$ then $a^O = u^O = 0$, $a^P = 1$ and $u^P = (1-c)/2$. Note that this is a "libertarian" equilibrium: officials

[29] Again this would fail if $\xi^O > 1$, and the pathological equilibrium of the previous note could again arise.

impose no taxes because it would be unprofitable to do so and producers exert maximum effort. But state power, crucial in interactions with other societies, is zero.

For $\tau\chi < 1$ and $\xi^O > 1 - \tau\chi$ or $\tau\chi \geq 1$ we have

$$a^P = \frac{1}{2}(1 + \frac{1-\xi^O}{\tau\chi}) \qquad a^O(\chi) = \frac{1-c}{2}\frac{\tau\chi - (1-\xi^O)}{(\tau\chi)^2}$$

Utility of producers can be computed to be

$$u^P = \frac{1-c}{8(\tau\chi)^2}\{[\tau\chi + 1 - \xi^O]^2 + 4\xi^P[\tau\chi - (1-\xi^O)]\}$$

Utility of state officials is

$$u^O = \frac{1-c}{4}\Big[1 - \frac{1-\xi^O}{\tau\chi}\Big]^2.$$