# Intangibles and Real Business Cycle

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> > December 31, 2014

# Abstract

The recent financial crisis has created a significant deviation between real world and existing business cycle theories. This paper follows the spirit of McGrattan and Prescott (2012) and reassesses this labor productivity puzzle by considering a real business cycle (RBC) model with R&D assets and organizational capital. It contributes to the literature in two ways. First, R&D assets and organizational capital are assumed to be produced using different technologies. Second, in addition to utilize BEA's recently published R&D data, we construct our own data on organizational capital, which are often constructed by relying on a lot of guesswork in previous studies. Our accounting exercises confirm the finding of McGrattan and Prescott (2012) that the labor productivity puzzle is much less of an issue when intangible capitals are incorporated into real business cycle models.

# 1. Introduction

Low correlation between labor productivity and GDP that has occurred in the United States since mid-1980's (often referred to as "labor productivity puzzle") has led some researchers to question the usefulness of real business cycle (RBC) theories — theories that assume cyclical fluctuations are driven in large part by shocks to total factor productivity (TFP) and that predict labor productivity is procyclical. In response to this questioning, economists, such as McGrattan and Prescott (2012), have argued that this low correlation is much less of an issue when intangibles are incorporated into real business cycle models. This argument is well-founded because business investments in intangibles have increased significantly in the United States over the past few decades and the estimated spending scale reached to 13.1% of GDP by 2000 (Corrado *et al.*, 2009). However, to incorporate intangibles into real business cycle models, researchers are faced with an inevitable problem: there is no arms-length market for most intangibles and the majority of them are developed for a firm's own use. As a result, previous studies on intangibles and real business cycle have to rely on a lot of guesswork to measure intangible capitals (McGrattan and Prescott, 2012).

This paper aims to fill in this gap by constructing our own data on intangible capitals and reassessing the labor productivity puzzle. In doing so, we follow the spirit of McGratten and Prescott (2012) and consider a RBC model where the two most important components of intangible capital — R&D assets and organizational capital — are produced using different technologies. The idea behind the distinguishment between R&D and organizational capital is that the two intangible capitals are not perfect substitutes for each other and their natures of production are different. On the data side, the Bureau of Economic Analysis (BEA) has developed methodologies to measure R&D assets and software capital (Li, 2012; Robbins *et al.*, 2012). In 2013, BEA started publishing R&D assets. In addition, we apply Li (2012)'s methodology to estimate the depreciation rates of organizational capital for all BEA's major 2-digit sectors, including 3-digit manufacturing sectors, and use the perpetual inventory method to construct the annual stock of organizational capital from 1990 to 2014.

Our RBC model with R&D and organizational capital does very well in accounting for the recent downturn. Particularly, our model predicts that both measured aggregate and business labor productivity rise between 2008 and 2010 while GDP was falling. This result confirms the finding of McGrattan and Prescott (2012) that the labor productivity puzzle is much less of an issue when intangible capitals are incorporated into real business cycle models.

The rest of the paper is organized as follows. Section 2 lays out the RBC theory with R&D assets and organizational capital. Section 3 describes how intangible capital stocks are constructed and how model parameters are estimated. Section 4 presents model outputs. Section 5 concludes.

#### 2. Theory

#### 2.1. A RBC Model with R&D and Organizational Capital

Following the spirit of McGrattan and Prescott (2012), we consider the following three technologies available to business, which are given by

$$y_{bt} = A_t^1 \left( k_{Tt}^1 \right)^{\theta} k_{It}^{\phi} \left( h_t^1 \right)^{1-\theta-\phi} = A_t^1 \left( k_{Tt}^1 \right)^{\theta} \left[ a \left( k_{It}^1 \right)^{\gamma} + (1-a) \left( k_{It}^2 \right)^{\gamma} \right]^{\phi/\gamma} \left( h_t^1 \right)^{1-\theta-\phi}$$
(1)

$$x_{It}^{1} = A_{t}^{2} \left(k_{Tt}^{2}\right)^{\theta} k_{It}^{\phi} \left(h_{t}^{2}\right)^{1-\theta-\phi} = A_{t}^{2} \left(k_{Tt}^{2}\right)^{\theta} \left[a \left(k_{It}^{1}\right)^{\gamma} + (1-a) \left(k_{It}^{2}\right)^{\gamma}\right]^{\phi/\gamma} \left(h_{t}^{2}\right)^{1-\theta-\phi}$$
(2)

$$x_{It}^{2} = A_{t}^{3} \left(k_{Tt}^{3}\right)^{\theta} k_{It}^{\phi} \left(h_{t}^{3}\right)^{1-\theta-\phi} = A_{t}^{3} \left(k_{Tt}^{3}\right)^{\theta} \left[a \left(k_{It}^{1}\right)^{\gamma} + (1-a) \left(k_{It}^{2}\right)^{\gamma}\right]^{\phi/\gamma} \left(h_{t}^{3}\right)^{1-\theta-\phi}$$
(3)

Firms produce business output  $y_b$  using their tangible capital  $k_T^1$ , labor  $h^1$ , and intangible capital  $k_I$ . Note that  $k_I$  is an aggregate of two intangible capitals — R&D  $(k_I^1)$  and organizational capital  $(k_I^2)$ . More specifically,  $k_I = \left[a \left(k_{It}^1\right)^{\gamma} + (1-a) \left(k_{It}^2\right)^{\gamma}\right]^{1/\gamma}$  is obtained by aggregating  $k_I^1$  and  $k_I^2$  using a constant elasticity substitution (CES) aggregator function, where a is the share parameter and  $\gamma$  determines the degree of substitutability of two intangible capitals. To distinguish  $k_{It}$ ,  $k_{It}^1$ , and  $k_{It}^2$ , we refer to the former as "aggregate intangible capital". Firms produce R&D using tangible capital  $k_T^2$ , labor  $h^2$ , and intangible capital  $k_I$ . Firms produce organizational capital using tangible capital  $k_T^3$ , labor  $h^3$ , and intangible capital  $k_I$ . As in the first sector,  $k_I$  is a CES aggregate of the two intangible capitals  $(k_I^1$  and  $k_I^2)$  in the latter two sectors. In addition,  $A^i$  (i = 1, 2, 3) denote TFP for the three sectors (i.e., business, R&D, and organizational capital) respectively. As noted by McGrattan and Prescott (2012), the two intangible capitals are not split among the three sectors, because they are used both to sell final goods and services and to design and develop new intangible capitals.

The household maximizes the following intertemporal utility function for given  $(k_{T0}^1, k_{T0}^2, k_{T0}^3, k_{I0}^1, k_{I0}^2)$ ,

$$E\sum_{t=0}^{\infty}\beta^{t}\left[\log c_{t}+\psi\log\left(1-h_{t}\right)\right]N_{t},$$
(4)

subject to

$$c_{t} + x_{Tt} + q_{t}^{1} x_{It}^{1} + q_{t}^{2} x_{It}^{2} = r_{Tt} k_{Tt} + r_{It}^{1} k_{It}^{1} + r_{It}^{2} k_{It}^{2} + w_{t} h_{t} + \varsigma_{t}$$

$$-\tau_{ct} c_{t} - \tau_{ht} (w_{t} h_{t} - (1 - \chi^{1}) q_{t}^{1} x_{It}^{1} - (1 - \chi^{2}) q_{t}^{2} x_{It}^{2}) - \tau_{k} k_{Tt}$$

$$-\tau_{p} \{ r_{Tt} k_{Tt} + r_{It}^{1} k_{It}^{1} + r_{It}^{2} k_{It}^{2} - \delta_{T} k_{Tt} - \chi_{qt}^{1} x_{It}^{1} - \chi_{qt}^{2} x_{It}^{2} - \tau_{k} k_{Tt} \}$$

$$-\tau_{d} \{ r_{Tt} k_{Tt} + r_{It}^{1} k_{It}^{1} + r_{It}^{2} k_{It}^{2} - x_{Tt} - \chi^{1} q_{t}^{1} x_{It}^{1} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} \}$$

$$-\tau_{p} (r_{Tt} k_{Tt} + r_{It}^{1} k_{It}^{1} + r_{It}^{2} k_{It}^{2} - \delta_{T} k_{Tt} - \chi^{1} q_{t}^{1} x_{It}^{1} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{1} k_{Tt} + r_{It}^{2} k_{It}^{2} - \delta_{T} k_{Tt} - \chi^{1} q_{t}^{1} x_{It}^{1} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{1} k_{Tt} + r_{It}^{2} k_{It}^{2} - \delta_{T} k_{Tt} - \chi^{1} q_{t}^{1} x_{It}^{1} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{1} k_{Tt} + r_{It}^{2} k_{It}^{2} - \delta_{T} k_{Tt} - \chi^{1} q_{t}^{1} x_{It}^{1} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{2} k_{Tt} + r_{It}^{2} k_{It}^{2} - \delta_{T} k_{Tt} - \chi^{1} q_{t}^{1} x_{It}^{1} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{2} k_{Tt} + r_{p}^{2} k_{Tt}^{2} - \delta_{T} k_{Tt} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{2} k_{Tt} + r_{p}^{2} k_{Tt}^{2} - \delta_{T} k_{Tt} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{2} k_{Tt} + r_{p}^{2} k_{Tt}^{2} - \delta_{T} k_{Tt} - \chi^{2} q_{t}^{2} x_{It}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{2} k_{Tt} + r_{p}^{2} k_{Tt}^{2} - \delta_{T} k_{Tt} - \chi^{2} q_{t}^{2} x_{Tt}^{2} - \tau_{k} k_{Tt} + \tau_{p}^{2} k_{Tt} + \tau_{p}^{2} k_{Tt} + \tau_{p}^{2} k_{Tt} + \tau_{p}^{2} k_{Tt}^{2} - \delta_{T} k_{Tt} - \chi^{2} q_{t}^{2} k_{Tt}^{2} - \tau_{p}^{2} k_{Tt} + \tau_{p}^{2} k_{Tt} + \tau_{p}^{2} k_{Tt} + \tau_{p}^{2} k_{T} + \tau_{p}^{2} k_{Tt} + \tau_{p}^{2} k_{T} + \tau_{p}^{2} k$$

$$k_{T,t+1} = [(1 - \delta_T)k_{Tt} + x_{Tt}]/(1 + \eta),$$
(6)

$$k_{I,t+1}^{1} = [(1 - \delta_{I}^{1})k_{It}^{1} + x_{It}^{1}]/(1 + \eta),$$
(7)

$$k_{I,t+1}^2 = [(1 - \delta_I^2)k_{It}^2 + x_{It}^2]/(1 + \eta),$$
(8)

where all variables are expressed in per capita terms,  $N_t = N_0(1 + \eta)^t$  is the population in period t, and  $\beta$  is the utility discount rate. c denotes consumption including both private and public consumption.  $x_T$  denotes tangible investment including both private and public tangible investment. Note that  $x_I^1$  denotes investment in R&D and  $q^1$  denotes its price in terms of consumption goods, and that  $x_I^2$  denotes investment in organizational capital and  $q^2$  denotes its price in terms of consumption goods.  $r_T$ ,  $r_I^1$ , and  $r_I^2$  denote the rental rates for business tangible capital, R&D, and organizational capital products.  $\delta_T$ ,  $\delta_I^1$  and  $\delta_I^2$  denote depreciation rates for tangible capital, R&D, and organizational products.

zational capital, respectively.  $\zeta$  denotes other income, and the remaining terms in the household budget constraint are tax payments.

Following the spirit of McGrattan and Prescott (2012), taxes are levied on consumption, labor income, tangible capital (that is, property), profits, and capital distributions at rates  $\tau_c$ ,  $\tau_h$ ,  $\tau_k$ ,  $\tau_p$ , and  $\tau_d$  respectively. It is worth noting that taxable income for the tax on profits is net of depreciation and property tax, and taxable income for the tax on distributions is net of property tax and profits tax. In addition, we have assumed tax rates for consumption and labor varies over time.

Moreover, we assume that other income  $\zeta$  is exogenous in the household's decision problem and that nonbusiness labor income is included in wh. In addition, hours, investment, and output in the nonbusiness sector are treated as exogenous, because this sector is not important for the issues being addressed. In other words, in our simulations of the model, we set the paths of nonbusiness hours  $\{\overline{h}_{nt}\}$ , investment  $\{\overline{x}_{nt}\}$ , and output  $\{\overline{y}_{nt}\}$  in the model's nonbusiness sector equal to U.S. paths. Measured output, which corresponds to GDP, is the sum of  $y_b$  and  $\overline{y}_n$ . Measured tangible investment is the sum of business tangible investment  $x_T$  and nonbusiness tangible investment  $\overline{x}_n$ . Measured hours h is the sum of business hours  $h_1 + h_2 + h_3$  and nonbusiness hours  $\overline{h}_n$ .

Let  $\chi^1$  ( $\chi^2$ ) denotes the fraction of investment on R&D (organizational capital) financed by capital owners. The amount  $\chi^1 q^1 x_I^1 + \chi^2 q^2 x_I^2$  then represents expensed investment financed by the capital owners who have lower accounting profits the greater this type of investment. The amount  $(1 - \chi^1)q^1 x_I^1 + (1 - \chi^2)q^2 x_I^2$  is what McGrattan and Prescott (2010, 2012) refer to as "sweat investment", which is financed by workers who have lower compensation if these two types of investment are greater.

Gross domestic product in the economy is the sum of total consumption (public plus private) and tangible investment (public plus private) for business and nonbusiness; in per capita terms GDP is  $c + x_T + \overline{x}_n$ . Gross domestic income (GDI) is the sum of all labor income less sweat investment  $wh - (1 - \chi^1)q^1x_I^1 - (1 - \chi^2)q^2x_I^2$ , business capital income less expensed investment,  $r_Tk_T + r_{It}^1k_{It}^1 + r_{It}^2k_{It}^2 - \chi^1q^1x_I^1 - \chi^2q^2x_I^2$ , and nonbusiness capital income (which is found residually as the difference between GDP and the other components of GDI). Summing terms gives us GDI equal to  $y_b + y_n$ . Total output and income — which is not what is measured by national accountants — includes the value of intangible capital and is therefore equal to GDP (or GDI) plus  $q^1x_I^1 + q^2x_I^2$ .

#### 2.2. Measured Labor Productivity and True Labor Productivity

In this subsection we show that the model outlined in Subsection 2.1 has the potential to resolve the labor productivity puzzle. As noted by McGrattan and Prescott (2012), the introduction of intangible capital(s) and nonneutral TFP (that is, the three TFPs do not change by the same factor) means that the positive correlation between output and measured labor productivity may disappear. There are two reasons for this result. First, measured output of the business sector in (1) does not depend on total business hours  $h^1 + h^2 + h^3$ , only on business hours allocated to the production of final goods and services. Second, true output of the business sector is  $y_b + q^1x_I^1 + q^2x_I^2$ , not  $y_b$ . Therefore, there is a difference between measured labor productivity and true labor productivity.

More specifically, for the aggregate economy, measured labor productivity is the ratio of GDP to total hours (i.e.,  $(y_b + y_n)/h$ ), whereas true labor productivity is the ratio of total output to total hours (i.e.,  $(y_b + y_n + q^1x_I^1 + q^2x_I^2)/h$ ). For the business sector, measured labor productivity is the ratio of business value added to total business hours (i.e.,  $y_b/(h^1 + h^2 + h^3)$ , whereas true labor productivity is the ratio of total business output to total business hours (i.e.,  $(y_b + q^1x_I^1 + q^2x_I^2)/(h^1 + h^2 + h^3)$ , whereas true labor productivity is the ratio of total business output to total business hours (i.e.,  $(y_b + q^1x_I^1 + q^2x_I^2)/(h^1 + h^2 + h^3)$  (or, equivalently, the ratio of output of final goods and services in the business sector to total hours allocated to production of final goods and services, i.e.,  $y_b/(h^1)$ .

What does the difference between measured labor productivity and true labor productivity imply for the labor productivity puzzle? If shocks to the sectoral TFPs move in opposite directions or change at different rates, the model predicts a shift in hours from one activity to another. Suppose that true output in the business sector  $y_b + q^1 x_I^1 + q^2 x_I^2$  and true labor productivity  $(y_b + q^1 x_I^1 + q^2 x_I^2) / (h^1 + h^2 + h^3)$  both fall in a downturn. What that means for measured labor productivity  $(y_b/(h^1 + h^2 + h^3))$  depends on the change in  $q^1 x_I^1 + q^2 x_I^2$  relative to output  $y_b$ . If investment falls by more than output, which is typical in recession periods, then it is possible that measured labor productivity would rise.

To give an artificial numerical example, suppose that before the downturn,  $y_b=200,\,q^1x_I^1+$ 

 $q^2x_I^2 = 100$  and  $h^1 + h^2 + h^3 = 50$ . After the downturn  $y_b = 110$ ,  $q^1x_I^1 + q^2x_I^2 = 30$  and  $h^1 + h^2 + h^3 = 25$ . The true labor productivity before and after the downturn are 6 (i.e., (200 + 100)/50 = 4) and 5.6 (i.e., (110 + 30)/25 = 5.6) respectively, indicating a decline in true labor productivity. In contrast, the measured labor productivity before the downturn and that after the downturn are 6 (i.e., 200/50 = 4) and 5.5 (i.e., 110/25 = 5.5), indicating an increase in measured labor productivity. This example suggests that measured labor productivity can rise even GDP and true labor productivity declines!

# 3. Construction of Intangible Capital Stocks and Estimation of Parameters Related to Intangible Capitals

As discussed in the Introduction, a contribution of this study is that we construct our own intangible capital stocks and then estimate parameters related to these stocks. Thus, in this section we focus on how intangible capital stocks are constructed and how these parameters are estimated.

#### 3.1. Construction of Intangible Capital Stocks

For R&D, methods for estimating its depreciation rates and constructing its stock have been widely documented. For example, Li (2012) developed a forward-looking profit model and used BEA data to estimate R&D depreciation rates for the ten R&D intensive industries defined in BEA's R&D satellite account, which has been released in late 2013 (Li, 2012). However, little effort has been made to estimate the depreciation rate of organizational capital and construct the stock of organizational capital. Thus, in this subsection we focus on the construction of stock of organizational capital.

## 3.1.1. Data Source

The data on organizational capital used in this study are obtained from Compustat and cover the period 1987 — 2013. We use the list of NAICS codes for all Bureau of Economic Analysis (BEA)'s 2-digit sectors, including 3-digit manufacturing sectors, to match the corresponding SIC codes in the Compustat dataset. Following previous studies, we use sales, general, and administrative (SG&A) expense as a proxy for a firm's investment in organizational capital (Lev and Radhakrishnan, 2005; Eisfeldt and Papanikolaou, 2013). Firms report this expense in their annual income statements<sup>1</sup>. It includes most of the expenditures that generate organizational capital, such as employee training costs, brand enhancement activities, consulting fees, and the installation and management costs of supply chains. One may question whether it is a valid measure of firms' investment in organizational capital on the ground that SG&A expenditures may include some items that are unrelated to the improvement of firms' organizational efficiency. Interested readers are referred to Eisfeldt and Papanikolaou (2013) for rigorous justifications for the use of this measure. *3.1.2. Methodology for Estimation of the Depreciation of Organizational Capital* 

Depreciation rates are needed to construct organizational capital. Eisfeldt and Papanikolaou (2013) used 15% as the depreciation rate of organizational capital, a number estimated by Griliches (1981) for the depreciation rate of R&D assets for major manufacturing industries during the 1970s. However, since each industry has a different competition environment, business practices, and technological progress, depreciation rates for organizational capital and R&D assets vary across industries. Furthermore, although both organizational capital and R&D assets are intangibles, the nature of their productions and their relationships with market competition should be different. That is, we should expect that even within the same industry, the depreciation rate for R&D assets differ from that of organizational capital.

In this study we use Li's (2012) forward-looking profit model to estimate the depreciation rates of organizational capital ( $\delta_{OC}$ ). To estimate the industry-specific depreciation rates for organizational capital for all BEA 2-digit, including 3-digit manufacturing sector, we use the non-linear least squares method to estimate the model. The results are summarized in Table 1.

<sup>&</sup>lt;sup>1</sup>Note that the SG&A expenditures in the Compustat dataset contain the number of R&D expenditures. We need to deduct the number of R&D expenditures from the SG&A number.

Table 1: Summary of the Depreciation

BEA Sector	$\delta_{OC}$	BEA Sector	$\delta_{OC}$
11	11%	332	19%
21	48%	333 & 334	12%
22	5%	335	8%
23	34%	3361 & 3364	8%
311	7%	337	22%
313	17%	339	5%
315	4%	42	38%
321	27%	44	12%
322	21%	48	6%
323	9%	51	17%
324	56%	FIRE	19%
325	1%	Professional	2%
326	11%	Public	15%
327	17%	Retail	18%
331	48%		

Rates of Organizational Capital

# 3.1.3. Construction of Organizational Capital Stock

To construct the stock of organizational capital at industry level, we use the method for constructing R&D stocks for U.S. manufacturing industries employed by Hall (1993). First, we deflate each industry's annual SG&A expenditures by using the GDP deflator with 2005 as the base year. Second, we apply the perpetual inventory method, together with our estimated depreciation rates, to construct the annual stock of organizational capital. Lastly, we multiply the real stock of organizational capital for each year by the GDP deflator to obtain nominal stock of organizational capital for that year. The industry-specific annual stocks of organizational capital are aggregated to derive the economy-wide annual stock of organizational capital from 1990 to 2014. We apply the same procedure to construct the annual stock of R&D assets for each industry.

Since the Compustat dataset only covers public firms, to construct the annual stock of organizational capital for the whole economy, we calculate the annual ratio of R&D assets to organizational capital based on the Compustat dataset. Moreover, utilizing the published BEA's annual data on R&D assets and the calculated annual ratio of R&D assets to organizational capital based on the Compustat dataset, we derive the annual stock of organizational capital for the whole economy from 1994 to 2014<sup>2</sup>.

#### 3.2 Estimation of Parameters Related to Intangible Capitals

For the parameters that are unrelated to intangible capitals, we use the values used by Mc-Grattan and Prescott (2012), namely,  $\eta = 0.010$  (growth in population), g = 0.019 (growth in technology),  $\beta = 0.979$  (discount factor),  $\psi = 1.186$  (utility parameter),  $\delta_T = 0.039$  (tangible capital),  $\tau_k = 0.014$  (tax rate on property),  $\tau_p = 0.296$  (tax rate on profits),  $\tau_d = 0.078$  (tax rate on distributions), and  $\chi^1 = \chi^2 = 0.5$  (fractions of R&D and organizational capital financed by workers). For the values of the parameters that vary over time, such as  $\tau_{ct}$  (tax rate on consumption),  $\tau_{ht}$  (tax rate on labor income),  $A_t^1$  (TFP for the business sector), nonbusiness series ( $\overline{y}_{nt}$ ,  $\overline{x}_{nt}$ , and  $\overline{h}_{nt}$ ), we also use the values used by McGrattan and Prescott (2012) (see Table 2 of McGrattan and Prescott (2012) for more details).

However, for the parameters that are related to intangible capitals, we estimate them based on our estimates of intangible capital stocks. We start by describing how  $\theta$ ,  $\phi$ , and  $(1 - \theta - \phi)$  are estimated. Taking log of both sides of (1) yields

$$\log(y_t) = \log A_t^1 + \theta \log(k_{Tt}^1) + \phi \log\left[a(k_{It}^1)^{\gamma} + (1-a)(k_{It}^2)^{\gamma}\right]^{1/\gamma} + (1-\theta - \phi) \log(h_t^1),$$
(9)

where  $y_t$  is total output instead of GDP. There are two ways to obtain the estimates of these three parameters. One is to add a statistical noise to (9) and then empirically estimate the resulting regression equation. The other is to theoretically show that these three parameters are equal to the

 $<sup>^{2}</sup>$ This method assumes that the annual ratio of R&D stocks to organizational capital in U.S. public firms is the same as the ratio for the whole economy.

cost shares of tangible capital, aggregate intangible capital, and labor respectively.

In this study we adopt the second approach and show that  $\theta$ ,  $\phi$ , and  $(1 - \theta - \phi)$  are equal to the cost shares of tangible capital, aggregate intangible capital, and labor respectively. From (9), it is straightforward to show that

$$\theta = \frac{\partial \log (y_{bt})}{\partial \log (k_{Tt}^1)} = \frac{\partial y_{bt}}{\partial k_{Tt}^1} \times \frac{k_{Tt}^1}{y_{bt}}.$$
(10)

It is straightforward to show that under the assumptions of perfect competition and constant returns to scale,  $\partial y_{bt} / \partial k_{Tt}^1$  is equal to the price of tangible capital. Thus, (9) can be further written as

$$\theta = \frac{\partial y_{bt}}{\partial k_{Tt}^1} \times \frac{k_{Tt}^1}{y_{bt}} = \frac{p_{k_t} \times k_{Tt}^1}{y_{bt}} = \frac{p_{k_t} \times k_{Tt}^1}{c_{bt}},\tag{11}$$

where  $p_{k_t}$  is the price of tangible capital and  $c_{bt}$  is total cost. The last equality of (11) is obtained by noting that  $y_{bt} = c_{bt}$  under perfect competition. According to (11), is the actual share of tangible capital. Similarly,  $\phi$  and  $(1 - \theta - \phi)$  are the shares of aggregate intangible capital, and labor respectively.

However, to calculate the share of aggregate intangible capital, we still need the values of a and  $\gamma$ . As is well known, a is the share parameter and thus is calculated as the ratio of nominal R&D stock to the sum of nominal R&D stock and nominal organizational capital stock. With regard to  $\gamma$ ,  $s = 1/(1 - \gamma)$  is the elasticity of substitution between. We experiment with different values for  $\gamma$ , ranging from 0 to 1, and find that our model predictions are quite robust to the changes in  $\gamma$  (though we do note that there is small quantitative changes in the model predications). Thus, it is safe to set  $\gamma = 0.5$ . In addition, since we don't have information on the price of the aggregate intangible capital, we assume that tangible capital and the aggregate intangible capital are paid the same rental price. Under this assumption, the ratio of the share of tangible capital to the stock of the aggregate intangible capital is equal to the ratio of the stock of tangible capital to the stock of the aggregate intangible capital. With the values for a and  $\gamma$  and this latter assumption, we obtain the following values for  $\theta$ ,  $\phi$ , and  $(1 - \theta - \phi)$ :  $\theta = 26.7\%$ ,  $\phi = 7.8\%$ , and  $(1 - \theta - \phi) = 65.5\%$ .

# 4. Model Predictions

Figures 1–7 show the equilibrium paths for GDP, hours worked, aggregate labor productivity, business labor productivity, per capita consumption, per capita investment, per capita GDP and output, tangible and intangible capitals in the model along with their counterparts from actual U.S. time series. All series, with the exception of hours, are detrended by the growth in labor-augmenting technical change (that is,  $(1 + g)^t$ ). The U.S. data are detrended in the same way. The series are then indexed so that the values in the starting year equal 100. Overall, our results support McGrattan and Prescott (2012)'s argument that the inclusion of intangible capital and nonneutral technology to the model was crucial in accounting for high productivity and low GDP during the period.

Specifically, Figure 1 plots actual and predicted GDP. As can be seen from the figure, predicted GDP closely follows actual GDP . Figures 2 and 3 show predicted and U.S. total (tangible) investment and consumption. As can seen from Figure 2, the decline in total tangible investment between 2004 and 2008 is underpredicted, while the decline in total tangible investment between 2008 and 2011 is overpredicted. For example, predicted investment is below trend by about 2 percent in 2008, whereas the model predicts that it is below by 0.8 percent. Predicted investment is below trend by about 3 percent in 2009, whereas the model predicts that it is below by 0.7 percent. Correspondingly, the underprediction of the fall in investment over the period 2004 - 2008 implies an overprediction of the fall in consumption over the same period. And the overprediction of the fall in investment over the same period. Figure 4 shows predicted and U.S. per capita hours of work, which lines up rather well too.

Figure 5 plots predicted and U.S. measured aggregate labor productivity. We note that measured aggregate labor productivity rises between 2008 and 2010, a period when GDP falls (see Figure 1). This result is because measured aggregate labor productivity does not take into account intangible capitals. Figure 6 plots predicted and U.S. measured business labor productivity, which also rises between 2009 and 2010. These two results show that labor productivity rises for both the aggregate economy and the business sector during the Great Recession.

Figure 7 plots predicted and U.S. true aggregate labor productivity. It is very interesting to note that true aggregate labor productivity declines between 2008 and 2010. This result is because true aggregate labor productivity accounts for intangible capitals. Figure 8 plots predicted and U.S. true business labor productivity, which also falls between 2009 and 2010. These two results show that true labor productivity declines for both the aggregate economy and the business sector during the Great Recession, confirming the importance of accounting for intangible capitals.

Figure 9 compares the path for model GDP and the path for model total output. As can be seen from this figure, total output falls by more between 2008 and 2010 because intangible investment falls by more than the value added of final goods and services. Figure 10 compares model predictions of tangible investment and investment on aggregate intangible capital in the business sector. Compared with tangible investment, intangible investment increases by more before the Great Recession, but falls by less after the Great Recession. Specifically, in 2008, intangible investment is roughly 41 percent above trend, while tangible investment is 13 percent above the trend. In 2010, intangible investment is roughly 47 percent below trend, while tangible investment is 72 percent below the trend. Both investments start to recover in 2011.

## 5. Conclusion

"Labor productivity puzzle" that occurred in the U.S. over the past three decades has led some researchers to question the usefulness of real business cycle (RBC) theories. Economists, such as McGrattan and Prescott (2012), have argued that this low correlation is much less of an issue when intangibles are incorporated into real business cycle models. Following this spirit, we consider a RBC model with R&D assets and organizational capital treated separately. More importantly, we construct our own data on the two intangible capitals, which are often constructed by relying on a lot of guesswork in previous studies. Our model predicts that both measured aggregate and business labor productivity rise between 2008 and 2010 while GDP was falling, thus supporting McGrattan and Prescott (2012)'s argument that the labor productivity puzzle is much less of an issue when intangible capitals are incorporated into real business cycle models.

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