Capital Taxation under Political Constraints[*]

Florian Scheuer
Stanford

Alexander Wolitzky
Stanford

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Abstract

This paper studies optimal dynamic tax policy under the threat of political reform. A policy will be reformed ex post if a large enough political coalition supports reform; thus, sustainable policies are those that will continue to attract enough political support in the future. We find that optimal marginal capital taxes are either progressive or U-shaped, so that savings are subsidized for the poor and/or the middle class but are taxed for the rich. U-shaped capital taxes always emerge when the salient reform threat consists of radically redistributing capital and individuals' political behavior is purely determined by economic motives.

1 Introduction

A fundamental observation in political economy is that high levels of economic inequality may lead to political instability. Most famously, Marx predicted that an increase in the concentration of capital would lead to a revolution, a concern recently revived by Piketty (2014).[1] In this paper, we consider a course of public policy—as well as the resulting economic inequality—to be politically sustainable if and only if it maintains the support of a coalition of citizens large enough to block reform at any point in time. We then ask what is the optimal politically sustainable capital tax policy.

Dynamic tax policy has been intertwined with political coalition formation since the origins of the modern welfare state in 19th century Germany. Following the Paris Commune of 1871, the socialist movement gained momentum until it came to be viewed by the conservative elite as a serious threat (Rimlinger, 1971; Korpi, 1983). According to Esping-Andersen's (1990) seminal account of welfare capitalism, Bismarck responded to this threat by developing corporatist policies, which "institutionalized a middle-class loyalty to the preservation of both occupationally segregated social-insurance programs and, ultimately, to the political forces that brought them into being," (p. 32). Bismarck "deliberately wished to mold the class structure with [his] social-policy initiatives" (p. 59), and some of his most important initiatives resembled capital tax policies. For instance, "early state legislation of pensions was typically undertaken as a means to arrest the growth of labor movements, and to redirect workers' loyalties towards the existing order" (p. 94), and civil servants were granted "extraordinarily lavish welfare provisions" in order to "guarantee proper loyalties and subservience" (p. 59). Thus, to a significant extent the modern welfare state grew out of capital subsidies for the poor and middle class, subsidies which were strategically designed to ensure political stability in the face of the threat of radical redistribution.

This paper studies a simple two-period model that captures these ideas. Individuals consume in both periods but produce in the first period only. Capital accumulation is therefore needed to finance period 2 consumption. The government is concerned about inequality in each period and has access to arbitrary nonlinear labor and capital taxes. The key feature of the model is that the government is able to reform its original policy in each period *if there is sufficient political support for a reform*. Thus, the government's ability to commit to intertemporal tax policy is limited and determined by politics.

A first observation is that the reform threat in period 2 involves full redistribution of capital: the government tolerates inequality only if this provides incentives for production, so it prefers to fully equalize period 2 consumption once production has occurred. Poor voters therefore

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[1]"...there is no ineluctable force standing in the way of a return to extreme concentration of wealth.... If this were to happen, I believe that it would lead to significant political upheaval," (Piketty 2014, p. 422).

tend to support reform, rich voters tend to oppose it, and middle-class voters tend to be close to indifferent. When designing a dynamic tax policy in period 1, the government then takes into account that the period 2 political support of the middle class is particularly sensitive to their period 2 consumption. This gives the government a reason to backload consumption for middle-class voters, that is, to subsidize capital for the middle class: we call this the *political sensitivity effect*. On the other hand, the political support of the poor is also sensitive to their period 2 consumption, simply because they have high marginal utility of consumption, which gives the government a reason to also subsidize capital for the poor: we call this the *utility sensitivity effect*. Finally, the rich are insensitive on both counts, while taxing their wealth has the advantage of reducing consumption for the poor and middle class under an equalizing reform. Thus, the optimal politically sustainable tax schedule subsidizes capital for the poor and/or the middle class, depending on the relative importance of the political and utility sensitivity effects, and taxes capital for the rich. As we argue in more detail in Section 4.3, the novel prediction that optimal marginal capital taxes can be U-shaped resonates well with the Bismarck example, as well as with Director's Law (Stigler, 1970), which observes that public redistribution tends to benefit the middle class rather than the poor.

We also develop several extensions of our model. Most significantly, we consider a variation in the timing of the model that allows the government to commit within each period (though still not across periods) to reforms other than full equalization. For example, in this version of the model, the relevant reform threat in period 2 might involve expropriating the richest 1% of the population to sweeten the reform for the remaining 99%. Here, we show that the government's planning problem may be written as a welfare maximization problem with an additional "no-reform" constraint, where a marginal increase in a given individual's period 2 consumption under the status quo relaxes the no-reform constraint if and only if her period 2 consumption is higher under the reform than under the status quo. Furthermore, the reform is always more egalitarian than the status quo, so it is the poor who have higher period 2 consumption under the reform, and hence the poor face lower capital taxes under the optimal politically sustainable policy. Thus, with within-period commitment, optimal marginal capital taxes are always progressive throughout the income distribution. We also offer an interpretation of the different versions of our model in terms of more direct versus indirect forms of democracy.

This paper lies at the intersection of the public finance literature on dynamic taxation with limited commitment and the political economy literature on endogenous coalition formation. Relative to the public finance literature, we introduce a new model of limited commitment based on political coalition formation: our perspective is that a policy is credible if it retains the support of a coalition large enough to block reform. The most classical branch of the literature on limited commitment assumes a representative agent (Kydland and Prescott, 1977; Fischer, 1980; Klein, Krusell, and Rios-Rull, 2008), which makes the coalition formation

problem degenerate. This remains true in models where "reputation" can mitigate the government's time-inconsistency problem (Kotlikoff, Persson, and Svensson, 1988; Chari and Kehoe, 1990; Benhabib and Rustichini, 1997; Phelan and Stacchetti, 2001). Hassler et al. (2005) and Azzimonti (2011) consider two-type models, which again preclude non-trivial coalitions.[2]

More closely related are the few papers where the extent of commitment is explicitly determined by political economy factors. One such paper is due to Acemoglu, Golosov, and Tsyvinski (2010), who analyze an infinite-horizon Mirrlees model with self-interested politicians and study whether the resulting distortions eventually vanish. A particularly closely related paper that we build on is Farhi et al. (2012). Their model is formally nested as the special case of our model where each individual's political behavior is equally sensitive to marginal changes in her utility, so that, for example, poor voters are not "safer" supporters of an equalizing reform than are middle-class voters. This rules out our political sensitivity effect—and with it the government's need to consider coalition formation—which leads to the prediction of progressive rather than potentially U-shaped capital taxes.

There is also an influential positive political economy literature on redistribution with heterogeneous voters and linear taxes (Bertola, 1993; Perotti, 1993; Alesina and Rodrik, 1994; Persson and Tabellini, 1994). Many papers in this literature incorporate repeated voting with endogenous political preferences (Krusell, Quadrini, and Rios-Rull, 1997; Krusell and Rios-Rull, 1999; Benabou, 2000; Hassler et al., 2003; Bassetto and Benhabib, 2006; Benhabib and Przeworski, 2006; Bassetto, 2008). Restricting to linear taxes leads to a median voter theorem, which again rules out many of the coalition formation issues that underlie our model.

Relative to the coalition formation literature, we sidestep the indeterminacy inherent in most such models by viewing coalition formation as a constraint in a planning problem: that is, we ask what coalition will be formed by a government that needs to maintain a certain level of political support. In most coalition formation models, competition among political parties offering nonlinear tax schedules leads to a Colonel Blotto (or "divide-the-dollar") game, which typically involves mixed strategy equilibria (cf. Myerson, 1993; Lizzeri and Persico, 2001). Our approach is more tractable and is flexible enough to capture in a reduced-form way the outcome of alternative modeling approaches that avoid this feature—such as the probabilistic voting model of Lindbeck and Weibull (1987) or the cooperative model of Aumann and Kurz (1977)—while also remaining closer to the public finance literature on limited commitment.

The result in the coalition formation literature that is most closely related to our approach is Director's Law (Stigler, 1970). The literature on Director's law typically considers static

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[2]Similar commitment problems can also arise in moral hazard models (as opposed to the Mirrleesian adverse selection model considered here). See, for instance, Fudenberg and Tirole (1990) and Netzer and Scheuer (2011) for two-period models where, ex ante, a principal optimally offers incomplete insurance to a risk-averse agent in order to provide incentives, but ex post, once effort is sunk, prefers to provide full insurance. Our approach to modeling limited commitment could also be applied to moral hazard models like these.

settings (Lindbeck and Weibull, 1987; Dixit and Londregan 1996, 1998). To the best of our knowledge, our paper is the first to model how such coalition formation concerns affect optimal dynamic tax policy (though this issue is prominent in the sociology literature on the welfare state, as in Korpi, 1983, and Esping-Andersen, 1990). Another difference is that Director's Law is usually interpreted as predicting a coalition of the poor and middle class against the rich: for example, in Stigler (1970) this happens because ganging up to rob the rich is more profitable than ganging up to rob the poor. In contrast, the problem in our model is how to form a coalition to forestall an equalizing reform, which naturally leads to a coalition of the middle class and the rich. It should also be emphasized that this coalition exists only in terms of opposition to reform; the government remains concerned about inequality, and labor taxes redistribute toward the poor as in standard Mirrlees models.

Finally, our results about the shape of the nonlinear capital tax schedule mirror an extensive literature on the shape of optimal income taxes in static Mirrlees models. In particular, many authors have found U-shaped marginal labor taxes to be optimal (see e.g. Diamond, 1998, and Saez, 2001). However, whereas this property crucially depends on the shape of the underlying skill distribution, our results about U-shaped or progressive marginal capital taxes are completely independent of the form of the skill distribution.

The paper proceeds as follows. Section 2 introduces the model, and Section 3 provides the main results. Section 4 offers a numerical illustration of our results and relates them to capital tax policies in practice today and in past historical episodes. In Section 5, we discuss how our results extend to settings where individuals differ in their savings propensity and to an infinite horizon framework with overlapping generations. Section 6 considers the version of our model where the government can commit to policy within a period. Section 7 concludes. Omitted proofs and further technical details are presented in the appendix.

2   Model

2.1   Preferences and Technology

We consider a standard Mirrlees model with two periods, $t = 1, 2$. There is a government and a continuum of individuals indexed by their ability $\theta \in \Theta$. Assume that Θ is an open subset of $\mathbb{R}$ and that θ has cdf F with positive density f on Θ.

Individuals produce in period 1 only and consume in both periods. A type θ individual has utility function

$$v_1(\theta) = u(c_1(\theta)) + \beta u(c_2(\theta)) - h(y(\theta), \theta),$$

where $c_1(\theta)$ and $c_2(\theta)$ are consumption in periods 1 and 2; u is a strictly increasing, strictly concave, and twice-differentiable consumption utility function satisfying the Inada conditions

$\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$; $\beta > 0$ is the discount factor; $y(\theta)$ is production in period 1; and h is a continuous function that captures disutility from production and is strictly increasing and convex in y with strictly decreasing differences. We also let

$$v_2(\theta) = u(c_2(\theta)),$$

so $v_t(\theta)$ denotes a type θ individual's continuation utility at the beginning of period t.

There is a linear saving technology, so the economy faces aggregate resource constraints in $t = 1, 2$ given by

$$\int c_1(\theta)\, dF + K \leq \int y(\theta)\, dF,$$

$$\int c_2(\theta)\, dF \leq RK,$$

where K is aggregate capital and $R > 0$ is its gross rate of return. These may be combined to form a single intertemporal resource constraint

$$\int \left(c_1(\theta) + \frac{1}{R} c_2(\theta)\right) dF \leq \int y(\theta)\, dF. \tag{1}$$

The government evaluates allocations at the beginning of period t according to

$$\int \psi(v_t(\theta))\, dF,$$

where for now ψ is an arbitrary exogenous function satisfying the following assumption:

Assumption 1 $\psi \circ u$ is strictly increasing and concave.

Assumption 1 ensures that the government has a redistributive motive. In Section 3.4, we endogenize ψ as the outcome of political competition between two parties in a probabilistic voting model.

2.2 Full Commitment Benchmark

As in Mirrlees (1971), the government cannot observe ability. Therefore, the revelation principle implies that the government's problem when it can fully commit to an intertemporal allocation is

$$\max_{c_1,c_2,y} \int \psi\left(u(c_1(\theta)) + \beta u(c_2(\theta)) - h(y(\theta),\theta)\right) dF$$

subject to the intertemporal resource constraint (1) and a standard incentive compatibility constraint

$$u\left(c_{1}\left(\theta\right)\right)+\beta u\left(c_{2}\left(\theta\right)\right)-h\left(y\left(\theta\right),\theta\right)\geq u\left(c_{1}\left(\theta'\right)\right)+\beta u\left(c_{2}\left(\theta'\right)\right)-h\left(y\left(\theta'\right),\theta\right) \text{ for all } \theta,\theta', \quad (2)$$

where c_1, c_2, and y are arbitrary measurable functions from Θ to $\mathbb{R}_+$.

Most of our results will concern the implicit marginal capital tax τ_k, defined by

$$u'\left(c_{1}\left(\theta\right)\right)\equiv\beta R(1-\tau_{k}\left(\theta\right))u'\left(c_{2}\left(\theta\right)\right). \quad (3)$$

This "wedge" is well-defined in any allocation, and can be interpreted as the actual marginal capital tax rate faced by agents of type θ in a nonlinear tax implementation of the optimal allocation, as we discuss in Section 4.2.[3] At a solution to the above full-commitment problem, Atkinson and Stiglitz's (1976) uniform taxation result implies that $\tau_k\left(\theta\right)=0$ for all θ.[4]

2.3 Limited Commitment: Threat of Political Reform

We now introduce our main model, where the government can reform its policy in each period if and only if a large enough coalition of individuals supports reform.

Each period $t=1,2$ begins with a *status quo continuation allocation*, which takes the form $\left(c_1^{SQ}, c_2^{SQ}, y^{SQ}\right)$ in period 1 and $\tilde{c}_2^{SQ}$ in period 2. The period 1 status quo is exogenous and will not play an important role in the results. The timing of the model is as follows:

Period 1:

1. Individuals decide whether to support the period 1 status quo.

2. If at least α individuals support the status quo (where $\alpha \in [0,1]$ is an exogenous parameter), then production and period 1 consumption occur and are given by y^{SQ} and c_1^{SQ}, and the economy moves to period 2 with period 2 status quo given by $\tilde{c}_2^{SQ} = c_2^{SQ}$.

   If fewer than α individuals support the status quo, then the government proposes a reform continuation allocation (c_1, c_2, y). Production y and period 1 consumption c_1 then occur and the economy moves to period 2 with period 2 status quo $\tilde{c}_2^{SQ} = c_2$.

Period 2:

1. Individuals decide whether to support the period 2 status quo.

2. If at least α individuals support the status quo, then period 2 consumption occurs and is given by $\tilde{c}_2^{SQ}$.

   If fewer than α individuals support the status quo, then the government proposes a reform continuation allocation $\hat{c}_2$. Period 2 consumption then occurs and is given by $\hat{c}_2$.

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[3]For results on the intratemporal labor wedge, see also Section 4.1.

[4]Throughout, we omit caveats regarding measure-0 sets when stating results. We address this issue in various proofs where it may cause confusion.

For $t = 1, 2$, let $v_t^{SQ}(\theta)$ be the continuation utility of a type θ individual when at least α individuals support the period t status quo, and let $v_t^R(\theta)$ be the corresponding continuation utility when fewer than α individuals support the period t status quo. A type θ individual supports the period t status quo if and only if

$$v_t^{SQ}(\theta) + \varepsilon^{SQ} \geq v_t^R(\theta) + \varepsilon^R,$$

where ε^{SQ} and ε^R are iid taste shocks that capture an idiosyncratic preference for supporting the status quo or allowing a reform to it, respectively (such as an individual's "conservatism"). Letting H be the cdf of $\varepsilon^{SQ} - \varepsilon^R$, we see that in each period t the status quo remains in place if and only if

$$\int H\left(v_t^{SQ}(\theta) - v_t^R(\theta)\right) dF \geq \alpha. \tag{4}$$

A leading example is the case with zero taste shocks, where political behavior is determined by purely economic considerations. In this case, H is the step function $H(x) = \mathbb{I}\{x \geq 0\}$, where $\mathbb{I}$ is the indicator function. More generally, if H admits a density H', it is natural to assume that H' is single-peaked (or, equivalently, H is "S-shaped"), so that taste shocks can be non-zero but are concentrated around zero, for instance following a normal distribution. For most of the analysis, we assume that H is continuously differentiable, but we also separately consider the important step function case.

The parameter α can result from formal institutional rules (such as super-majority rules in parliament or in a referendum) according to which the government can reform the status quo only if a large enough share of voters are in favor. In this case, the taste shocks ε capture individual voters' inertia or status quo bias. Alternatively, α can result from an informal stability requirement, where the government needs to maintain sufficient support for its policies to prevent political upheaval in the future, as in the Bismarck example. The taste shocks could then include individuals' idiosyncratic preferences for supporting or joining a revolutionary movement.

2.4 Preliminary Observations

We analyze the model by backwards induction. If a share larger than $1 - \alpha$ of the population favor allowing a reform in period 2, then the government will reform the status quo period 2 consumption schedule $\tilde{c}_2^{SQ}$ to the allocation $\hat{c}_2$ that solves

$$\max_{\hat{c}_2} \int \psi\left(u(\hat{c}_2(\theta))\right) dF \ \text{ s.t. } \ \int \hat{c}_2(\theta) dF \leq \int \tilde{c}_2^{SQ}(\theta) dF.$$

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Under Assumption 1, this is the egalitarian consumption schedule given by

$$\hat{c}_2(\theta) = \int \tilde{c}_2^{SQ}(\theta) \, dF \text{ for all } \theta.[5] \tag{5}$$

In the case where (4) holds for $t = 1$, this observation completes the analysis of the model: the exogenous status quo production and period 1 consumption schedules y^{SQ} and c_1^{SQ} are implemented in period 1, and in period 2 the status quo period 2 consumption schedule c_2^{SQ} is either implemented or is reformed to an egalitarian schedule. The non-trivial case therefore occurs when (4) is violated at $t = 1$, where we must consider the government's optimal choice of policy (c_1, c_2, y). We call this the *government's problem*.

In Appendix A, we formulate the model as a formal game between the individuals and the government. The timing of the game is that the government first sets labor and capital tax schedules; individuals then choose how much to produce and consume in period 1; and finally the capital tax schedule may be reformed, and individuals consume their final capital in period 2. We then prove a version of the revelation principle, which says that any implementable allocation (c_1, c_2, y) satisfies (1), (2), and

$$\int H\left(u\left(c_2(\theta)\right) - u\left(\hat{c}_2(\theta)\right)\right) dF \geq \alpha, \tag{6}$$

when viewed as a direct mechanism, where $\hat{c}_2(\theta)$ is given by (5) with $\tilde{c}_2^{SQ} = c_2$. Thus, it is without loss of generality to restrict attention to feasible, incentive compatible direct mechanisms that satisfy the "no-reform" constraint (6). The revelation principle proved in Appendix A does however require the following mild assumption on H, which says that constant period 2 consumption schedules are never reformed:

Assumption 2 $H(0) \geq \alpha$.

3   Optimal Capital Taxes

This section presents our main results on optimal capital taxes, compares them with the most closely related results in the literature, and briefly discusses endogenizing the social welfare functional ψ.

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[5]As will become clear, the fact that this egalitarian reform threat results from optimization by a government with social welfare functional ψ is irrelevant for the analysis. It would be equivalent to exogenously require that the period 2 reform threat is egalitarian. This alternative interpretation, where the reform threat may come from a political force outside the government, can capture the "specter of communism" in the Bismarck example.

3.1 Main Results

Under Assumptions 1 and 2, the revelation principle (proved in Appendix A) implies that the government's problem under the threat of political reform is

$$\max_{c_1,c_2,y} \int \psi \left(u \left(c_1 \left(\theta \right) \right) + \beta u \left(c_2 \left(\theta \right) \right) - h \left(y \left(\theta \right), \theta \right) \right) dF$$

subject to (1), (2), and (6). This section characterizes the resulting optimal capital taxes.

We note at the outset that the government's problem is typically not convex, because $H \left(u \left(c_2 \left(\theta \right) \right) - u \left(\hat{c}_2 \left(\theta \right) \right) \right)$ is typically not concave in $c_2 \left(\theta \right)$, so the constraint set is not convex; for example, step functions are not concave. This is not a "technical" problem (although it will lead to some mathematical complications), but rather a central economic ingredient of the model. In particular, to design a sustainable policy, the government must in effect select a coalition of voters that will support this policy against a potential future reform. This coalition-formation problem is non-convex under the natural assumption that voters who are closer to indifferent between the status quo and the reform are more sensitive to slight changes in these policies.

While the government's problem is not convex, it is still true that it must be solved by any solution to the dual problem

$$\min_{c_1,c_2,y} \int \left(c_1 \left(\theta \right) + \frac{1}{R} c_2 \left(\theta \right) - y \left(\theta \right) \right) dF$$

subject to

$$\int \psi \left(u \left(c_1 \left(\theta \right) \right) + \beta u \left(c_2 \left(\theta \right) \right) - h \left(y \left(\theta \right), \theta \right) \right) dF \geq V, \tag{7}$$

(2), and (6), where V is the value of the primal.[6] Note that in the constraints (2) and (7), $c_1 \left(\theta \right)$ and $c_2 \left(\theta \right)$ only enter through total consumption utility $U \left(\theta \right)$. Hence, any solution must solve the subproblem

$$\min_{c_1,c_2,K} \int \left(c_1 \left(\theta \right) + \frac{1}{R} c_2 \left(\theta \right) \right) dF$$

subject to

$$u \left(c_1 \left(\theta \right) \right) + \beta u \left(c_2 \left(\theta \right) \right) = U \left(\theta \right), \tag{8}$$

$$\int H \left(u \left(c_2 \left(\theta \right) \right) - u \left(RK \right) \right) dF \geq \alpha, \tag{9}$$

and

$$\int c_2 \left(\theta \right) dF \leq RK. \tag{10}$$

We take a Lagrangian approach to this subproblem. This is valid (i.e., constraint qualification is satisfied) for generic values of α, by Theorem 3 of Clarke (1976).[7] The resulting

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[6] If not, then one could take a solution to the dual and vary c_1 so as to increase $u \left(c_1 \left(\theta \right) \right)$ by ε for all θ. This variation would increase the objective while leaving (2) and (6) unaffected, and would not violate (1) for small enough ε.

[7] Clarke's generalized Lagrange multiplier theorem (discussed further in Section 6) coincides with classical

first-order (necessary) conditions deliver the following characterization.

Lemma 1. *In any solution to the government's problem, the intertemporal wedge $\tau_k(\theta)$ satisfies*

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = -R\eta \left[H'(u(c_2(\theta)) - u(RK)) u'(c_2(\theta)) - \bar{H}_K \right] \tag{11}$$

for some multiplier $\eta \geq 0$ on (9), where

$$\bar{H}_K = \int H'(u(c_2(\theta)) - u(RK)) u'(RK) dF. \tag{12}$$

Proof. Substituting out for $c_1(\theta)$ using (8) and letting $\eta \geq 0$ and $\mu \geq 0$ denote multipliers on (9) and (10), respectively, the Lagrangian is

$$\int \left(u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \frac{c_2(\theta)}{R} \right) dF - \eta \int H(u(c_2(\theta)) - u(RK)) dF - \mu \left(RK - \int c_2(\theta) dF \right).$$

If $\eta = 0$, then (9) is slack, and $\tau_k(\theta) = 0$ in any solution as in Atkinson and Stiglitz, so (11) holds. Otherwise, the first-order condition with respect to K gives

$$\frac{\mu}{\eta} = \bar{H}_K. \tag{13}$$

Next, rewrite the Lagrangian as

$$\int \left(u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \frac{1}{R} c_2(\theta) - \eta \left(H(u(c_2(\theta)) - u(RK)) - \frac{\mu}{\eta} c_2(\theta) \right) \right) dF - \mu RK,$$

differentiate under the integral with respect to $c_2(\theta)$, and substitute for μ/η using (13) to obtain the necessary condition

$$\frac{\beta R u'(c_2(\theta))}{u'(c_1(\theta))} = 1 - R\eta \left(H'(u(c_2(\theta)) - u(RK)) u'(c_2(\theta)) - \bar{H}_K \right).$$

Finally, use (3) to rewrite this condition as (11). ∎

The intuition for equation (11) is that, when deciding how much to tax capital for a type θ individual, the government assesses the impact of a marginal unit of her savings on the no-reform constraint (6). Saving by any individual has both a positive and negative effect on this constraint. The negative effect is that saving by a type θ individual uniformly increases everyone's consumption under the reform, which tightens the no-reform constraint: this accounts for the $-\bar{H}_K$ term in (11), which does not depend on θ. The positive effect is that saving by a type θ individual increases her own consumption under the status quo, which relaxes the no-reform constraint: this accounts for the $H'(u(c_2(\theta)) - u(RK)) u'(c_2(\theta))$ term in (11), which depends on θ, and in particular equals the product of the sensitivity of her probability of supporting the reform to her utility (H') and the sensitivity of her utility to her consumption (u').

Our first main result is the following:

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results here, as u and H are continuously differentiable.

Proposition 1. *Optimal marginal capital taxes are inversely related to $H'u'$. Formally, for any two types θ and θ',*

$$\tau_k(\theta) \geq \tau_k(\theta') \Leftrightarrow H'(u(c_2(\theta)) - u(RK)) u'(c_2(\theta)) \leq H'(u(c_2(\theta')) - u(RK)) u'(c_2(\theta')).$$

Proof. Immediate from (11). ∎

Proposition 1 says that when a proposed policy is politically sustainable only if the period 2 allocation is preferred to full redistribution by a large enough share of the population, the shape of the capital tax schedule depends on the product of H' ("political sensitivity") and u' ("utility sensitivity"). Recall that H' is naturally H' is single-peaked, while u' is decreasing. Hence, marginal capital taxes are progressive (in $c_2(\theta)$) for high levels of $c_2(\theta)$, but may be regressive for low levels of $c_2(\theta)$ (and hence U-shaped overall). We record this observation as a corollary. In what follows, a *monotone* solution to the government's problem is one in which $c_2(\theta)$ is non-decreasing in θ. We show in Section 3.2 below that such a solution always exists.

Corollary 1. *If H' is single-peaked at some utility level u^*, then in every monotone solution to the government's problem, optimal marginal capital taxes are progressive on the interval (θ^*, ∞), where $\theta^* = \inf\{\theta : u(c_2(\theta)) \geq u(RK) + u^*\}$*

Proof. If $\theta > \theta'$ and $u(c_2(\theta')) \geq u(RK) + u^*$ in a monotone solution, then $c_2(\theta) \geq c_2(\theta')$ and therefore $H'(u(c_2(\theta)) - u(RK)) u'(c_2(\theta)) \leq H'(u(c_2(\theta')) - u(RK)) u'(c_2(\theta'))$, as $H'(u(x) - u(RK))$ is decreasing in x for $x \geq c_2(\theta') + u^*$ and $u'(x)$ is decreasing in x. The claim then follows from Proposition 1. ∎

An important observation is that the ambiguity in the shape of optimal capital tax schedule below the peak of H' goes away in the leading case where H is a step function, so that the no-reform constraint (6) simply requires that in period 2 mean consumption is below $1 - \alpha$ quantile consumption. In this case, optimal marginal capital taxes are U-shaped in θ.

Proposition 2. *If H is a step function, then in every monotone solution to the government's problem there is an interval of types $[\theta_l, \theta_h]$ such that*

1. *Types $\theta < \theta_l$ and $\theta > \theta_h$ face a common non-negative capital tax τ_k^*.*

2. *Period 2 consumption $c_2(\theta)$ equals RK for all $\theta \in [\theta_l, \theta_h]$. In addition, marginal capital taxes on the interval $[\theta_l, \theta_h]$ are non-decreasing in θ and satisfy $\tau_k(\theta) \leq \tau_k^*$ for all $\theta \in [\theta_l, \theta_h]$.*

Proof. See Appendix C.1. ∎

Figure 1 illustrates the pattern derived in Proposition 2. To sustain support from a large enough fraction α of the population, the government raises the consumption of the "middle class" (types between θ_l and θ_h) to RK, making them just indifferent to a reform. To achieve
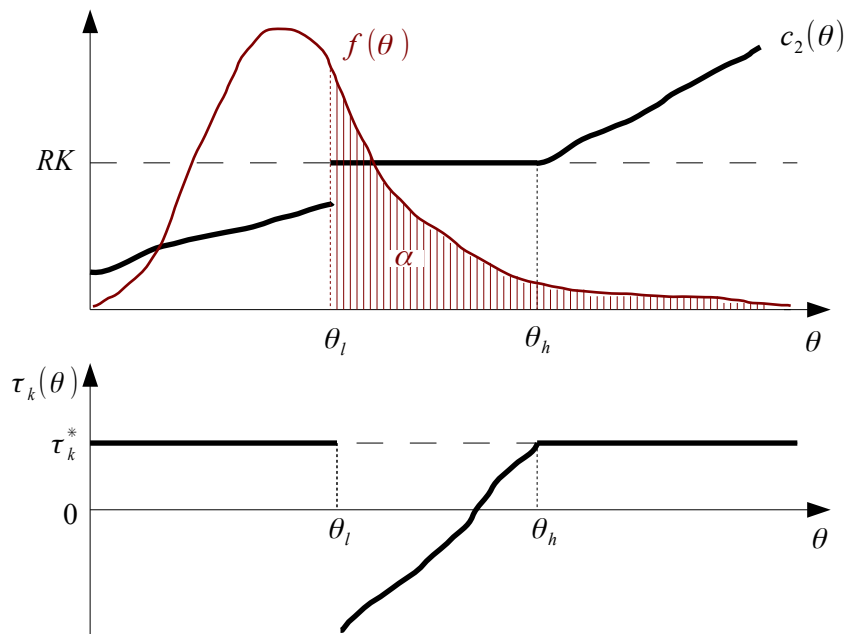
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Figure 1: Capital taxes when H is a step function

this, the government imposes a flat savings tax on the poor and rich (to depress consumption under a reform) and an increasing, lower tax (or a subsidy) on the middle class (to raise their period 2 consumption under the status quo). As in the case where H is smooth but close enough to a step function, this leads to a U-shaped marginal capital tax schedule.

The intuition for these results is simple. Ex post, poor agents tend to support an equalizing reform, rich agents tend to oppose it, and middle-class agents tend to be close to indifferent and thus pivotal; this feature that those in the middle are most sensitive to changes their utility is captured by the assumption that H' is single-peaked. Thus, in order to make the status quo sustainable, the government ensures that middle-class agents' period 2 consumption is high under the status quo and low under the reform. Considering variations in the timing of consumption that leave fixed total consumption utility $U(\theta) = u(c_1(\theta)) + \beta u(c_2(\theta))$ (and thus do not affect incentive compatibility), this may be achieved by subsidizing capital (i.e., back-loading consumption) for the middle class (which increases middle-class period 2 consumption under the status quo) and taxing capital (i.e., frontloading consumption) for the poor and rich (which decreases middle-class period 2 consumption under the reform).

This pattern captures precisely the coalition formation motives underlying the historical examples in the introduction, where securing political support from the middle class was essential under the threat of extreme redistribution. It can also interpreted as a dynamic version of Director's Law, with the difference that, here, the coalition supporting the status quo comprises the middle class and the rich rather than the middle class and the poor (see also Section 4.3).

Outside of the step function case, however, there is also an offsetting effect, which is that

poor agents care more about a marginal dollar (i.e., u' is decreasing), so that all else equal their political support is cheaper.[8] Thus, in order to make the status quo credible, the government subsidizes capital for those agents with high $H'u'$, who may be either poor or middle class (depending on whether the H' or u' effect dominates), and taxes capital for the rich (for whom both H' and u' are low).

It is worth pointing out that none of these results depend on the particular form of the welfare function ψ, as long as it satisfies Assumption 1. For example—and perhaps somewhat surprisingly—optimal marginal capital taxes with step function H remain U-shaped even as ψ approaches the Rawlsian criterion. The shape of labor taxes may change in this case (see Section 4.1), but the shape of capital taxes does not.

3.2   Existence of a Monotone Solution

Lemma 1 and Proposition 1 describe the shape of optimal marginal capital taxes in terms of $c_2(\theta)$, which is itself endogenous to government policy. The economically more important issue is the shape of optimal marginal capital taxes in terms of θ. While it is intuitive that an optimal $c_2(\theta)$ schedule should be monotonically non-decreasing in θ—so that the shape of the tax schedule as a function of θ is the same as its shape as a function of $c_2(\theta)$—this is not immediate in the current context because the government's problem is not convex. In Appendix B, we derive a novel monotone methods result—which we use repeatedly throughout the paper, and which may be useful more broadly—to establish that there does always exist a monotone solution to the government's problem (and that in addition any non-monotonicity in θ can occur only among types that are pooled in terms of production $y(\theta)$ and total consumption utility $U(\theta)$).[9] This existence result underlies Corollary 1 and Proposition 2, which are our main results on the shape of optimal marginal capital taxes in terms of θ.

3.3   Relation to Farhi et al. (2012)

When H is uniform—so that H' is constant—and we allow a resource cost $\kappa \geq 0$ of implementing a reform in period 2, (11) reduces to the optimal capital tax formula of Farhi et al. (2012), which prescribes increasing marginal capital taxes. If H' is single-peaked, we then obtain a U-shaped adjustment to their progressive tax schedule, with taxes being U-shaped overall if and only if $H'u'$ is single-peaked over the range of equilibrium $c_2(\theta)$ levels.

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[8]This effect is a consequence of the assumption that the taste shocks ε^{SQ} and ε^R are additive (or equivalently that H depends on the utility difference $u(c_2(\theta)) - u(\hat{c}_2(\theta))$) and thus goes away in the step function case where the taste shocks always equal zero. See Scheuer and Wolitzky (2014) for how these results extend to alternative specifications of the taste shocks.

[9]Briefly, the result establishes monotonicity of solutions to non-convex constrained optimization problems where the objective has increasing differences in allocation and type and the constraints depend on the allocation only. Mathematically, it is a generalization of the assortative matching theorem of Becker (1973).

Indeed, our model formally nests Farhi et al.'s. To see this, let $\underline{u} = \inf_{c \in C} u(c)$ and $\bar{u} = \sup_{c \in C} u(c)$, where C is some set of relevant consumption levels. Suppose that

$$H\left(u\left(c_2\left(\theta\right)\right) - u\left(\hat{c}_2\left(\theta\right)\right)\right) = \frac{1}{2} + \frac{1}{2}\frac{u\left(c_2\left(\theta\right)\right) - u\left(\hat{c}_2\left(\theta\right)\right)}{\bar{u} - \underline{u}},$$

so that H is uniform and symmetric around zero, and let $\alpha = 1/2$. Then (6) becomes

$$\int u\left(c_2\left(\theta\right)\right) dF \geq u\left(RK - \kappa\right),$$

which is precisely the no-reform constraint in Farhi et al. (2012). To understand this coincidence, recall that the no-reform constraint in their model requires that a utilitarian government does not wish to equalize consumption at resource cost κ, and observe that this is the case if and only if a simple majority of voters does not wish to equalize consumption at resource cost κ in a probabilistic voting model with uniform taste shocks. From this perspective, the results of this section may be viewed as a generalization of Farhi et al.'s analysis to the case where voters' probabilities of supporting reform are not all equally sensitive to marginal policy changes, or equivalently where the problems of maximizing social welfare and maximizing political support do not coincide. This in turn is exactly the case where political coalition formation matters.

3.4 Interpreting and Endogenizing the Social Welfare Functional

We conclude this section by briefly discussing the interpretation of the social welfare functional ψ. The most direct interpretation of ψ is normative: the goal of the model is to locate the optimal tax schedule under social welfare functional ψ, subject to the constraint that the tax schedule must resist future reform. However, an alternative, positive interpretation is that ψ is the outcome of political competition between two office-motivated parties in a probabilistic voting model, as follows:

Suppose that in period $t = 1, 2$ parties $i = A, B$ propose allocations

$$a_t^i\left(\theta\right) = \begin{cases} \left(c_1^i\left(\theta\right), c_2^i\left(\theta\right), y^i\left(\theta\right)\right) & \text{if } t = 1 \\ \hat{c}_2^i\left(\theta\right) & \text{if } t = 2, \end{cases}$$

and an individual of type θ votes for party i with probability $G\left(v_t^i\left(\theta\right) - v_t^j\left(\theta\right)\right)$, where

$$v_t^i\left(\theta\right) = \begin{cases} u\left(c_1^i\left(\theta\right)\right) + \beta u\left(c_2^i\left(\theta\right)\right) - h\left(y^i\left(\theta\right), \theta\right) & \text{if } t = 1 \\ u\left(\hat{c}_2^i\left(\theta\right)\right) & \text{if } t = 2, \end{cases}$$

and G is the cdf of ideological preference shocks for party i over party j, which we assume to be symmetric about 0. Note that in general there is no reason why G should equal H, which

captures "conservative" tastes for maintaining the status quo over allowing a reform rather than ideological preferences for parties.

If a symmetric pure strategy Nash equilibrium in this policy proposal game exists for $t = 2$, each party i's platform must solve

$$\max_{\hat{c}_2^i} \int G\left(u\left(\hat{c}_2^i\left(\theta\right)\right) - u\left(\hat{c}_2^j\left(\theta\right)\right)\right) dF \quad \text{s.t.} \quad \int \hat{c}_2^i\left(\theta\right) dF \leq RK.$$

The standard condition for existence of a symmetric pure strategy Nash equilibrium in this case—due to Lindbeck and Weibull (1987)—is the following:

Assumption 3 $G'\left(u\left(x\right) - y\right) u'\left(x\right)$ is non-increasing in x for all y.

Under this assumption, in the unique symmetric pure strategy Nash equilibrium, both parties propose full equalization of period 2 consumption, as given by (5).

Consequently, if a symmetric pure strategy Nash equilibrium (a_1^*, a_1^*) in the policy proposal game exists for $t = 1$, each party i's platform must solve

$$\max_{a_1^i} \int G\left(v_1^i\left(\theta\right) - v_1^*\left(\theta\right)\right) dF$$

subject to (1), (2), and (6), where these constraints must hold with respect to allocation a_1^i. Letting

$$\psi\left(v_1\left(\theta\right)\right) \equiv G\left(v_1\left(\theta\right) - v_1^*\left(\theta\right)\right) \text{ for all } \theta,$$

we see that each party's platform must solve $\max \int \psi\left(v_1\left(\theta\right)\right) dF$ subject to (1), (2), and (6), which is precisely the government's problem with social welfare functional ψ considered so far.

4 Model Implications

This section considers empirical implications of our results. Section 4.1 provides a numerical illustration. Section 4.2 shows how to implement the optimal allocation with taxes. Section 4.3 compares our results with contemporary and historical wealth redistribution policies.

4.1 Labor Wedge and Numerical Illustration

This paper focuses on qualitative properties of the implicit marginal capital tax (3). We consider it a strength of the model that it allows for sharp results on this margin, despite the complexity of characterizing the entire optimal allocation (which is a standard feature of Mirrlees models). Nonetheless, it is interesting to compute the full optimal allocation numerically in an example. This gives some feel for the quantitative implications of the political economy

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constraints we consider, and also demonstrates that adding these constraints does not make the model intractable numerically.

We first show how to compute the intratemporal labor wedge in the optimal allocation offered by the government in period 1, defined as $\tau_l(\theta) \equiv 1 - h_y(y(\theta), \theta)/u'(c_1(\theta))$, where h_y denotes the partial derivative of the disutility function h with respect to y.[10] In particular, and specializing to $h(y, \theta) = h(y/\theta)$ and $\psi(s) = s$ for simplicity, it is straightforward to show that the implicit labor income tax must satisfy

$$\frac{\tau_l(\theta)}{1 - \tau_l(\theta)} = \left(1 + \frac{1}{\varepsilon(\theta)}\right) \frac{u'(c_1(\theta))}{\theta f(\theta)} \int_\theta^\infty \left(\frac{1}{u'(c_1(s))} - \frac{1}{\lambda}\right) dF \tag{14}$$

whenever there is no bunching, where $\varepsilon(\theta)$ is the Frisch elasticity of labor supply at θ and $\lambda = \left(\int 1/u'(c_1(s))dF\right)^{-1}$ is the multiplier on the resource constraint (1).[11]

The key observation here is that formula (14) is exactly the same as in a static Mirrlees model (see Mirrlees, 1971, or Saez, 2001, for standard interpretations), as well as in our two-period model when there is full commitment (since (14) is completely independent of the form of the political support constraint). In this sense, the labor tax schedule is affected by our political economy constraint only indirectly through the $c_1(\theta)$-schedule on the right-hand side of (14). This justifies our focus on the intertemporal wedge (3) as the key margin of interest, and demonstrates that introducing political constraints does not make the model less tractable along the intratemporal dimension.

Formula (14), together with our characterization of the capital wedge, also allows us to numerically compute the entire optimal allocation $(c_1(\theta), c_2(\theta), y(\theta))$ for a calibrated version of our model. We consider iso-elastic preferences of the form $u(c) = c^{1-\sigma}/(1 - \sigma)$ and $h(l) = \gamma l^{1+1/\varepsilon}/(1 + 1/\varepsilon)$ where $l = y/\theta$, so that the Frisch elasticity of labor supply is constant and given by ε. We set $\varepsilon = 1$, consistent with the evidence in Kimball and Shapiro (2010) and Erosa, Fuster and Kambourov (2011). We interpret a model period as $T = 30$ years and accordingly set $\beta = 0.95^{30}$ and $R = 1/\beta$ (so the optimum under full commitment involves consumption smoothing with $c_1(\theta) = c_2(\theta)$).

For the skill distribution F, we follow Mankiw, Weinzierl and Yagan (2009) who fit a lognormal distribution to the empirical wage distribution from the 2007 Current Population Survey and append a Pareto distribution for the upper tail of wages to obtain asymptotic marginal tax rates as in Saez (2001). We extend their numerical procedure for a static Mirrlees model to our dynamic setting in order to compute both $\tau_l(\theta)$ and $\tau_k(\theta)$ and follow them in

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[10]As before, we focus on the non-trivial case where the initial, exogenous status quo is sufficiently undesirable that a reform in period 1 has political support.

[11]Bunching does not occur whenever the (global) incentive constraints (2) can be replaced by the local incentive constraints $v_1'(\theta) = h_\theta(y(\theta), \theta) \, \forall \theta$, and the monotonicity constraint—requiring that $y(\theta)$ is non-decreasing—is slack. This can be easily checked numerically as in the example we provide below.
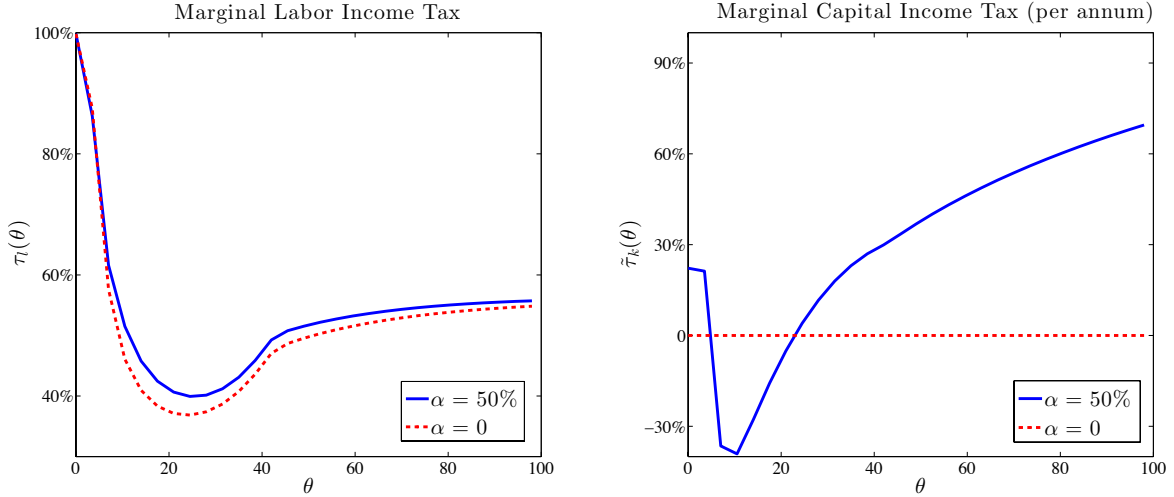
Figure 2: Marginal labor and capital taxes with and without political constraints

setting $\sigma = 1.5$ and $\gamma = 2.55$. We assume that the distribution of taste shocks H is normal with mean 0 and standard deviation 1 (corresponding to just over 10% of mean utility).

The left panel in Figure 2 shows the resulting optimal labor income tax rates (for wages up to \$100/hour) under full commitment ($\alpha = 0$) and when the political constraint (6) binds (in which case we assume majority voting with $\alpha = 50\%$). The right panel shows the optimal annualized marginal capital tax rate $\tilde{\tau}_k(\theta)$ on the net return to saving for both cases.[12] As can be seen from the graphs, the labor tax schedules are very similar under full commitment and limited commitment, confirming that the key effects of the political economy constraint are on the intertemporal margin. Both schedules exhibit the typical U-shaped pattern emphasized in Diamond (1998) and Saez (2001), which is driven by the phase-out of the lump-sum transfer for low wages and by convergence to the asymptotic marginal tax rate due to the Pareto tail for high wages. The right panel demonstrates the U-shaped pattern for the capital tax rate emphasized here and predicted in Section 3.1. In particular, capital tax rates are negative for intermediate wages and positive otherwise (and of sizable absolute amounts).

The underlying consumption and utility schedules are illustrated in Figure 3. The left panel shows consumption in both periods. While c_1 and c_2 coincide under full commitment (as shown by the red line for $\alpha = 0$), they are distorted when the threat of reform is binding. In particular, the capital subsidy increases c_2 (the green line) for intermediate wages in order to increase political support for the status quo to 50% of the population (from 42% under the full commitment solution).[13] Of course, this must lead to an aggregate welfare loss, which here is equivalent to a 2.1% consumption drop for everyone in both periods compared to the full

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[12]Formally, $\tilde{\tau}_k(\theta)$ is defined such that $1 + (1 - \tilde{\tau}_k(\theta)) \left(R^{1/T} - 1 \right) \equiv \left[R(1 - \tau_k(\theta)) \right]^{1/T}$.

[13]Aggregate consumption in periods 1 and 2 remain roughly equal to each other.
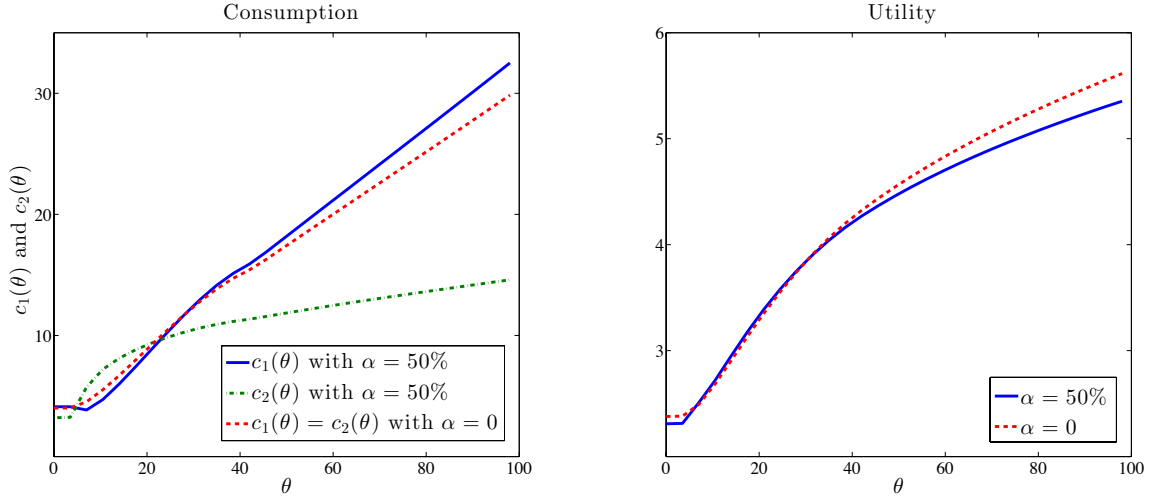
Figure 3: Consumption and utility with and without political constraints

commitment solution. However, there is considerable heterogeneity in how this welfare loss is distributed across the population. In fact, as shown in the right panel, types with intermediate hourly wages benefit from the presence of the political support constraint, whereas all other types lose. This illustrates how our model can generate a pattern of redistribution where the middle class actually benefits from political constraints—rather than just having their consumption backloaded—consistent with Director's Law.

4.2 Tax Implementation

Our analysis so far has implicitly considered direct mechanisms, where the government allocates $c_1(\theta), c_2(\theta)$ and $y(\theta)$ conditional on individual reports about θ, taking into account technological, incentive compatibility and political sustainability constraints. It is straightforward to show that these allocations can alternatively—and more realistically—be implemented through a tax system where each individual is confronted with the same budget set and picks her preferred allocation within this set. In particular, with a non-linear labor income tax T_y and a non-linear capital income tax T_k, individuals are faced with the budget constraint $c_1 + k \leq y - T_y(y)$ in period 1 and $c_2 \leq Rk - T_k(Rk)$ in period 2 and choose c_1, c_2, y, k to maximize $u(c_1) + \beta u(c_2) - h(y, \theta)$ subject to these two constraints.

By Proposition 3 of Farhi et al. (2012), any incentive compatible allocation $(c_1(\theta), c_2(\theta), y(\theta))$ that is non-decreasing in θ can be implemented using such a tax system. Since we show that $c_2(\theta)$ is non-decreasing in an optimal allocation and $y(\theta)$ is non-decreasing by incentive compatibility, their result can be applied to our framework.[14] Moreover, the first-order condi-

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[14]The statement of Proposition 3 in Farhi et al. (2012) also requires $c_1(\theta)$ to be non-decreasing, which holds at the optimum in their model but may or may not hold in ours. However, inspecting their proof reveals that

tions from the above utility-maximization problem imply

$$u'(c_1(\theta)) = \beta R(1 - T_k'(Rk(\theta)))u'(c_2(\theta))$$

for all θ whenever T_k is differentiable, so the wedge $\tau_k(\theta)$ defined in (3) and characterized throughout this paper coincides with the actual marginal capital income tax rate $T_k'(Rk(\theta))$ faced by individuals of type θ in this implementation.[15]

4.3   Relation to Wealth Redistribution Policies in Practice

Our results give clear-cut predictions about the shape of marginal wealth taxes and subsidies. While our model is admittedly stylized, and the level and progressivity of effective capital taxes in advanced economies are hard to measure, it is worth relating our results both to current policy patterns and to historical cases that are consistent with the political dynamics we emphasize.

First, as also noted by Farhi et al. (2012), policies such as income, estate, and wealth taxes, as well as the tax treatment of retirement accounts and subsidies to savings and education by the poor and middle class, contribute to the progressivity of capital taxation. However, policies such as subsidies to college education, many retirement savings programs (where the subsidy is increasing in the marginal income tax rate up to a cap) and the mortgage interest deduction (which subsidizes the accumulation of housing wealth), are likely targeted more directly at middle-class voters than at the very poor. Similarly, Doepke and Schneider (2006) show that the inflation tax effectively redistributes from rich, bond-holding households to middle-class households with fixed-rate mortgage debt, consistent with our version of Director's Law.

Moreover, means-tested government transfers can lead to an overall U-shaped pattern of capital taxes. For example, in the US, only individuals with sufficiently few assets qualify for Medicaid or Federal Student Aid, and only individuals with sufficiently low investment income qualify for the Earned Income Tax Credit. These asset tests can lead to positive marginal savings taxes for the poor. More generally, the contribution to capital tax progressivity of subsidies to savings and education by the poor are at least partially offset by the phase-out of these subsidies, unless eligibility is solely determined by labor income.

Of course, we do not claim that these patterns are necessarily driven by concerns for political stability, especially in the US, where socialist threats have historically been less important than in Europe.[16] However, as alluded to in the introduction, there are important historical cases where the political forces emphasized in our model do appear to have been decisive. In many

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this condition is not actually needed for the result.

[15]If $T_k(Rk(\theta))$ is not differentiable because there is pooling at consumption level $c_2(\theta)$ (so T_k has a convex kink), $\tau_k(\theta)$ is still bounded between the (well-defined) left- and right-derivatives of T_k.

[16]For another explanation of such savings distortions, see e.g. Golosov and Tsyvinsky (2006).

European countries, the insurrection associated with the Paris Commune in 1871, followed by the emergence and growth of socialist parties and labor unions, strongly influenced contemporary elites, who tried to counteract the "specter of communism" by introducing redistributive social insurance programs (Rimlinger, 1971); according to Esping-Andersen (1990), "this was the clear rationale behind the early German, Danish, and Austrian reforms," (p. 94). In a speech to the Reichstag in 1884, Bismarck declared that "had there been no social democracy and had not so many of you feared it, the moderate progress we have made in social reform up to now would on the whole not have existed" (Hentschel, 1983). Indeed, these policies were strongly opposed by social democrats and labor unions at the time, precisely because of the strategic coalition-building motivations underlying them (Esping-Andersen, 1990).

In the twentieth century, this pattern repeated itself when conservative governments in post-World War II Germany and Italy reformed and expanded the welfare state to maintain political support in the face of growing political power of the left. For instance, "Adenauer's great pension reform in 1957 was explicitly designed to resurrect middle-class loyalties" to Germany's conservative-corporatist welfare state (Esping-Andersen, 1990, p. 32). In particular, by tying the benefits of future pensioners to overall economic growth, replacement rates (and hence effective returns to wealth accumulation through the public retirement program) for middle-income earners surged, while basic old-age income protection elements for the poor were abolished (Hinrichs, 2003).

5   Extensions

We now consider two extensions of the model that serve as "robustness checks" on our results. Section 5.1 extends our results to a setting where the full commitment benchmark involves non-zero capital taxes. Section 5.1 extends our two-period model to overlapping generations.

5.1   Heterogeneity in Savings Propensity

We have derived our results in a dynamic Mirrlees model where, in the absence of political constraints, optimal capital taxes are zero. It is natural to ask whether and how they extend to settings where the Atkinson-Stiglitz theorem does not apply to the full commitment benchmark. For example, this is the case when individuals differ in their propensity to save rather than their ability, as emphasized recently by Farhi and Werning (2013) and Piketty and Saez (2013). We briefly demonstrate that, in this case, our results extend in the sense that the pattern of capital taxes found in Section 3.1 can be interpreted as the optimal *addition* to whatever the full commitment capital tax benchmark is.

To this purpose, we consider a modification of our basic two-period model without labor

supply, where individuals simply consume in both periods and all start out with the same initial wealth K_1. Private heterogeneity exclusively enters in the form of savings propensity, so that wealthy individuals in period 2 will be those who are more patient. Formally, preferences are $u(c_1) + \beta u(c_2)$ and β is distributed according to cdf F with support contained in $(0,1)$. An allocation $(c_1(\beta), c_2(\beta))$ is resource feasible if

$$\int c_1(\beta)dF + K_2 \leq RK_1 \ \text{ and } \ \int c_2(\beta)dF \leq RK_2 \tag{15}$$

for some $K_2 \geq 0$ and incentive compatible if

$$u(c_1(\beta)) + \beta u(c_2(\beta)) \geq u(c_1(\beta')) + \beta u(c_2(\beta')) \ \ \forall \beta, \beta'. \tag{16}$$

Considering again the case with $\psi(s) = s$ for simplicity,[17] the government solves in period 1

$$\max_{c_1, c_2, K_2} \int \left[u(c_1(\beta)) + \beta u(c_2(\beta)) \right] dF$$

s.t. (15), (16), and
$$\int H \left(u(c_2(\beta)) - u(RK_2) \right) dF \geq \alpha. \tag{17}$$

It is obvious that implicit capital taxes $\tau_k(\beta)$ (defined as in (3)) are no longer necessarily zero even without the political constraint (17), because these wedges are the only way to achieve redistribution across individuals with different savings propensities β in period 1. However, the following useful decomposition of overall marginal capital taxes in any solution without bunching can be established using standard steps:

$$\frac{\tau_k(\beta)}{1 - \tau_k(\beta)} = \chi(\beta) - \frac{R\eta}{\lambda} \left[H'(u(c_2(\beta)) - u(RK_2)) - \bar{H}_K \right], \tag{18}$$

where $\bar{H}_K$ is defined as in (12) and

$$\chi(\beta) \equiv \frac{Ru'(c_2(\beta))}{f(\beta)} \int_\beta^1 \left(\frac{1}{u'(c_1(\beta))} - \frac{1}{\lambda} \right) dF$$

is the formula for the optimal $\tau_k/(1-\tau_k)$ under full commitment, with $\lambda = \left(\int_0^1 1/u'(c_1(\beta))dF \right)^{-1}$. As can be seen from (18), exactly the same formula for the optimal marginal capital tax as in Lemma 1 appears, with the only difference that it gets added to the (no longer necessarily zero) benchmark marginal capital tax $\chi(\beta)$ that is present even in the absence of political constraints.[18] In this sense, our results extend in a transparent way to situations where the

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[17]It is straightforward to extend the following analysis to general welfare functions or to trace out the set of constrained Pareto efficient allocations in this model.

[18]Of course, this interpretation of an additive adjustment to the capital tax under full commitment holds only

Atkinson-Stiglitz conditions are not met under full commitment.

5.2 Overlapping Generations

There is also a straightforward extension of our two-period model to an infinite-horizon over-lapping generations (OLG) setting. In particular, consider the model from Section 2, except that in every period $t = 1, 2, \ldots$ a new generation is born. As before, individuals live for two periods, produce only when young, and consume in both periods. Each period begins with a capital stock RK_t and a status quo consumption schedule for the old $\tilde{c}_t^O$. The timing in each period t is as follows:

1. Old individuals decide whether to support the status quo.

2. The government chooses a vector $\left(y_t, c_t^Y, \tilde{c}_{t+1}^O\right)$ corresponding to production and consumption for the period t young and status quo consumption for the period $t+1$ old, subject to the resource constraints

$$\int c_t^Y(\theta_t)dF + K_{t+1} = \int y_t(\theta_t)dF \quad \text{and} \quad \int \tilde{c}_{t+1}^O(\theta_t)dF = RK_{t+1}. \tag{19}$$

If fewer than α old individuals support the status quo, the government can also choose a reform consumption schedule $\hat{c}_t^O$ for the period t old, subject to the resource constraint

$$\int \hat{c}_t^O(\theta_{t-1})\, dF = RK_t. \tag{20}$$

Otherwise, consumption for the period t old is given by the status quo $\tilde{c}_t^O$.

The interpretation is that, in each period t, the government sets policies for the currently living generations, i.e. a labor income tax for the young in t and a capital tax for when they will be old in $t+1$, which will become the status quo for the next period. If there is enough support among the currently old, it can also reform their status quo capital tax (which was set in the preceding period) by redistributing their wealth. Note that the resource constraints (19) and (20) rule out intergenerational transfers; we briefly comment on this below.

A *Markov equilibrium* of this model is one where individuals born in period t condition their behavior when young (i.e., production and consumption) only on $\left(y_t, c_t^Y\right)$ and K_t and, when old, condition their support for the status quo only on $\tilde{c}_{t+1}^O$ and K_{t+1}. All of our results for the two-period model of Section 2 immediately extend to the Markov equilibria of this OLG model.

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in terms of the formula (18), since both c_1 and c_2 change when we introduce political constraints.

The model becomes more complicated when intergenerational transfers and non-Markov equilibria are allowed. In non-Markov equilibria, the government could be "punished" for implementing a reform, or for setting particular allocations for the young, which provides an additional source of commitment power. Such history-dependence could also arise with intergenerational transfers, as in this case the period t young would also have to condition their behavior on the status quo $\tilde{c}_t^O$, which was set by the government in period $t-1$. In particular, it is not clear that a suitable definition of Markov equilibrium exists in this case.

6 Within-Period Commitment

In this section, we consider an alternative version of the model where in each period the government proposes a reform *before* individuals decide whether to support the status quo. This change in timing effectively lets the government commit to a policy within each period, though still not across periods. For this section only, we also specialize to the case of a utilitarian government—where ψ is the identity mapping—and also assume that the skill distribution has lower and upper bounds, respectively denoted by $\underline{\theta}$ and $\bar{\theta}$. We establish the unexpected result that in this version of the model optimal capital taxes are progressive throughout the skill distribution, regardless of the shape of H.[19]

Relative to the main model of Section 2, the two versions of the model can be tentatively mapped to more representative and more direct versions of democratic government: in representative democracies, politicians retain the final say on fiscal policy, and are thus unlikely to be able to commit to reforms that they have a strong incentive to modify ex post (as in Section 2); while in direct democracies, politicians may have the "agenda-setting" power to propose a specific policy, while being unable to modify it after it is approved by the voters (as in this section). With this interpretation, our model predicts that capital taxes are more likely to be U-shaped when fiscal policy is ultimately determined by representatives, and more likely to be progressive when determined by direct referendum.[20]

Formally, the timing of the model in this section is as follows:

Period 1:

1. The government proposes a reform continuation allocation (c_1, c_2, y).

2. Individuals decide whether to support the period 1 status quo. If at least α individuals support the status quo, then production and period 1 consumption occur and are given

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[19]We conjecture that this result continues to hold for arbitrary concave ψ.

[20]An alternative interpretation is that the two versions of the model differ in the government's degree of sophistication. In the within-period commitment version, the government is able to design sophisticated vote-buying strategies at the reform stage. In the no within-period commitment version, the government simply pursues its most-preferred (fully equalizing) reform. This can capture naïvety on the part of the government, as well as an inability to commit.

by y^{SQ} and c_1^{SQ}, and the economy moves to period 2 with period 2 status quo given by $\tilde{c}_2^{SQ} = c_2^{SQ}$. If fewer than α individuals support the status quo, then production and period 1 consumption occur and are given by y and c_1, and the economy moves to period 2 with period 2 status quo given by $\tilde{c}_2^{SQ} = c_2$.

Period 2:

1. The government proposes a reform continuation allocation $\hat{c}_2$.

2. Individuals decide whether to support the period 2 status quo. If at least α individuals support the status quo, then period 2 consumption occurs and is given by $\tilde{c}_2^{SQ}$. If fewer than α individuals support the status quo, then period 2 consumption $\hat{c}_2$ occurs.

The model is otherwise the same as in Section 2.

The key implication of this change in timing is that now the no-reform constraint in the resulting period 1 government's planning problem is no longer given by (9), but rather by the condition that there does not exist a reform consumption schedule $\hat{c}_2 : \Theta \to \mathbb{R}$ such that

$$\int u\left(\hat{c}_2\left(\theta\right)\right) dF > \int u\left(c_2\left(\theta\right)\right) dF,$$

$$\int \hat{c}_2\left(\theta\right) dF \leq \int c_2\left(\theta\right) dF - \kappa,$$

$$\int H\left(u\left(c_2\left(\theta\right)\right) - u\left(\hat{c}_2\left(\theta\right)\right)\right) dF \leq \alpha,$$

where κ is an exogenous resource cost of implementing a reform, discussed below. In words, this requires that there is no reform that the government prefers to the status quo, is resource feasible, and would defeat the status quo in terms of popular support. Letting $x\left(\theta\right) \equiv u\left(\hat{c}_2\left(\theta\right)\right) - u\left(c_2\left(\theta\right)\right)$, $\Phi \equiv u^{-1}$, and $u_t(\theta) \equiv u(c_t(\theta))$, note that this constraint is equivalent to the value of the following dual deviation program (DP), which we denote by $R_D\left(u_2\right)$, being greater than $\int \Phi\left(u_2\left(\theta\right)\right) dF - \kappa$:[21]

$$\min_x \int \Phi\left(u_2\left(\theta\right) + x\left(\theta\right)\right) dF \qquad (21)$$

subject to
$$\int x\left(\theta\right) dF \geq 0, \qquad (22)$$

$$\int H\left(-x\left(\theta\right)\right) dF \leq \alpha. \qquad (23)$$

The advantage of this formulation—which we exploit below—is that the status quo utility schedule u_2 enters only through the objective and not through the constraints.

---

[21] The same variation as in footnote 6 shows that a solution to the primal must also solve the dual, both in the deviation program and in the subsequent government's problem.

The dual formulation of the period 1 government's planning problem is therefore

$$\min_{U,u_2,y} \int \left(\Phi\left(U\left(\theta\right) - \beta u_2\left(\theta\right)\right) + \frac{1}{R}\Phi\left(u_2\left(\theta\right)\right) - y\left(\theta\right) \right) dF$$

subject to (2), (7),
$$\int \Phi\left(u_2\left(\theta\right)\right) dF - \kappa \leq R_D\left(u_2\right), \tag{24}$$

and
$$\int H\left(v_1^{SQ}\left(\theta\right) - v_1\left(\theta\right)\right) dF \leq \alpha, \tag{25}$$

where (25) requires that the allocation (c_1, c_2, y) corresponding to (U, u_2, y) receives enough support against the exogenous status quo allocation $\left(c_1^{SQ}, c_2^{SQ}, y^{SQ}\right)$. In Scheuer and Wolitzky (2014), we set up this version of the model as a game between individuals and the government and established a revelation principle that justifies this formulation, as we now do for the main model in Appendix A. We omit this argument here due to space constraints, but note that it relies on the following assumption, which says that the government would rather implement the optimal policy that forestalls reform in period 2 than pay the resource cost κ to implement the full-commitment solution. This is why we allow here for a resource cost of reform.[22]

Assumption 3 The value of the government's dual program is less than the value of the government's full-commitment dual program plus κ.

As with the government's problem in Section 2, non-convexity is an unavoidable feature of the deviation program (DP) under natural specifications of H, and hence its solution $x\left(\theta\right)$ may not be unique. Let

$$X(\theta; u_2) \equiv \{x\left(\theta\right) : x \text{ is a right-continuous solution to (DP) at } u_2\},$$

$$\bar{x}\left(\theta; u_2\right) \equiv \sup X\left(\theta; u_2\right), \quad \text{and} \quad \underline{x}\left(\theta; u_2\right) \equiv \inf X\left(\theta; u_2\right),$$

where we will omit the argument u_2 when its value is unambiguous.[23] Using our general monotone methods lemma, Lemma 5 in Appendix B, we can collect the following results about solutions to (DP):

Lemma 2. *(i) A right-continuous solution to (DP) exists for generic values of α.*
(ii) If $u_2\left(\theta\right) < u_2\left(\theta'\right)$ then $\underline{x}\left(\theta\right) \geq \bar{x}(\theta')$.

---

[22]The resulting asymmetry with the main model is unimportant: one could easily include the resource cost κ in the main model (as we did to facilitate the comparison with Farhi et al. (2012) in Section 3.3), and one could also analyze the current model without the resource cost by restricting attention to equilibria where a reform does not occur in period 2.

[23]Strictly speaking, if u_2 is not monotone, the right-continuity in this definition must be read as being with respect to the following order $\succsim$ on Θ:

$$\theta \succsim \theta' \text{ if either } [u_2\left(\theta\right) > u_2\left(\theta'\right)] \text{ or } [u_2\left(\theta\right) = u_2\left(\theta'\right) \text{ and } \theta \geq \theta'].$$

(iii) $R_D(u_2)$ is locally Lipschitz continuous in u_2 (with the sup metric).

Proof. See Appendix C.2. ∎

The most important part of the lemma is part (ii), which shows that any solution x must be non-increasing in u_2. This formalizes the intuition that, even though reforms are no longer necessarily fully equalizing in this version of the model, they still reduce the inequality implied by the status quo u_2 (since reform utility is $u_2 + x$).

Returning now to the government's period 1 problem, note that only constraint (24) depends on u_2. Hence, at any solution u_2 must solve the subproblem

$$\min_{u_2} \int \left(\Phi\left(U\left(\theta\right) - \beta u_2\left(\theta\right)\right) + \frac{1}{R}\Phi\left(u_2\left(\theta\right)\right) \right) dF \quad \text{subject to (24)}.$$

We begin by again applying Lemma 5, this time to establish the existence of a monotone solution to the government's problem (as in Section 3.2). We restrict attention to these solutions in what follows.

Lemma 3. *There exists a solution to the government's problem in which $u_2\left(\theta\right)$ is non-decreasing.*

Proof. See Appendix C.3. ∎

As in Section 3.1, we take a Lagrangian approach to the subproblem. Constraint qualification is satisfied for generic values of κ, again by Theorem 3 of Clarke (1976). The resulting generalized first order condition characterizes optimal marginal capital taxes, and is the basis of our main results on the progressivity of optimal capital taxes with within-period commitment.

Lemma 4. *In any monotone solution to the government's problem,*

$$\frac{\tau_k\left(\theta\right)}{1 - \tau_k(\theta)} \in \left[R\eta \left(1 - \frac{\Phi'(u_2(\theta) + \bar{x}(\theta))}{\Phi'(u_2(\theta))} \right), R\eta \left(1 - \frac{\Phi'(u_2(\theta) + \underline{x}(\theta))}{\Phi'(u_2(\theta))} \right) \right]. \tag{26}$$

for some multiplier $\eta \geq 0$ on (24).

Proof. By Theorem 1 of Clarke (1976), there exists a multiplier $\eta \geq 0$ on (24) such that the zero vector is an element of the generalized gradient of

$$\int \left(\Phi\left(U\left(\theta\right) - \beta u_2\left(\theta\right)\right) + \frac{1}{R}\Phi\left(u_2\left(\theta\right)\right) \right) dF - \eta \left(R_D\left(u_2\right) - \int \Phi\left(u_2\left(\theta\right)\right) dF + \kappa \right) \tag{27}$$

(we review the definition of the generalized gradient below; see Clarke (1976) for details). Fix such a multiplier, and suppose toward a contradiction that (26) fails on a positive measure set of types for some optimal consumption schedule u_2. Then there exists either a positive measure set Θ' on which $\tau_k(\theta)/(1 - \tau_k\left(\theta\right))$ exceeds $R\eta\left(1 - \Phi'(u_2(\theta) + \underline{x}(\theta))/\Phi'(u_2(\theta))\right)$, or a positive measure set Θ'' on which $\tau_k\left(\theta\right)/(1 - \tau_k\left(\theta\right))$ is less

than $R\eta\left(1 - \Phi'(u_2(\theta) + \bar{x}(\theta))/\Phi'(u_2(\theta)))\right)$. Assume the first case applies; the argument for the second case is symmetric.

Define the vector $\mathbf{1}_{\Theta'} : \Theta \to \mathbb{R}$ by

$$\mathbf{1}_{\Theta'} = \begin{cases} 1 \text{ if } \theta \in \Theta' \\ 0 \text{ if } \theta \notin \Theta'. \end{cases}$$

By definition, the generalized directional derivative of $-R_D(u_2)$ in direction $\mathbf{1}_{\Theta'}$ equals

$$\lim_{\tilde{u}_2 \to u_2, t\downarrow 0} \sup -\frac{R_D\left(\tilde{u}_2 + t\mathbf{1}_{\Theta'}\right) - R_D\left(\tilde{u}_2\right)}{t},$$

where the sup is taken over utility schedules $\tilde{u}_2$ that are monotone on both Θ' and $\Theta \backslash \Theta'$.[24] We claim that this generalized directional derivative is upper-bounded by $-\int_{\Theta'} \Phi'\left(u_2(\theta) + \underline{x}(\theta)\right) dF$. To see this, let $x(\theta; \tilde{u}_2)$ be an arbitrary selection from $X(\theta; \tilde{u}_2)$, let $|\cdot|$ denote the sup norm, and note that

$$\lim_{\tilde{u}_2 \to u_2, t\downarrow 0} \sup -\frac{R_D\left(\tilde{u}_2 + t\mathbf{1}_{\Theta'}\right) - R_D\left(\tilde{u}_2\right)}{t}$$

$$\leq \lim_{\tilde{u}_2 \to u_2, t\downarrow 0} \sup \int_\Theta -\frac{\Phi\left(\tilde{u}_2(\theta) + x(\theta; \tilde{u}_2 + t\mathbf{1}_{\Theta'}) + t\mathbf{1}_{\Theta'}\right) - \Phi\left(\tilde{u}_2(\theta) + x(\theta; \tilde{u}_2 + t\mathbf{1}_{\Theta'})\right)}{t} dF$$

$$= \lim_{\tilde{u}_2 \to u_2, t\downarrow 0} \sup \int_{\Theta'} -\frac{\Phi\left(\tilde{u}_2(\theta) + x(\theta; \tilde{u}_2 + t\mathbf{1}_{\Theta'}) + t\right) - \Phi\left(\tilde{u}_2(\theta) + x(\theta; \tilde{u}_2 + t\mathbf{1}_{\Theta'})\right)}{t} dF$$

$$\leq \lim_{\varepsilon \to 0} \int_{\Theta'} \sup_{|\tilde{u}_2 - u_2| < \varepsilon, t < \varepsilon} -\Phi'\left(\tilde{u}_2(\theta) + \underline{x}(\theta; \tilde{u}_2 + t\mathbf{1}_{\Theta'})\right) dF$$

$$= \int_{\Theta'} \lim_{\varepsilon \to 0} \sup_{|\tilde{u}_2 - u_2| < \varepsilon, t < \varepsilon} -\Phi'\left(\tilde{u}_2(\theta) + \underline{x}(\theta; \tilde{u}_2 + t\mathbf{1}_{\Theta'})\right) dF$$

$$= -\int_{\Theta'} \Phi'\left(u_2(\theta) + \underline{x}(\theta)\right) dF.$$

where the first step follows because $x(\cdot; \tilde{u}_2 + t\mathbf{1}_{\Theta'})$ is feasible in (DP) at $\tilde{u}_2$; the second step is immediate; the third step follows by convexity of Φ and subadditivity of sup; the fourth step follows by the bounded convergence theorem, as by convexity of Φ and monotonicity of the $\tilde{u}_2$'s on Θ' and $\Theta \backslash \Theta'$ the integrand is bounded by

$$\sup_{|\tilde{u}_2 - u_2| < \varepsilon, t < \varepsilon, \theta \in \{\bar{\theta}, \bar{\theta}'\}} -\Phi'\left(\tilde{u}_2(\theta) + \underline{x}(\theta; \tilde{u}_2 + t\mathbf{1}_{\Theta'})\right) < \infty,$$

where $\bar{\theta}' = \sup \Theta'$; and the last step follows by upper hemi-continuity of $X(\theta; \tilde{u}_2)$ in $\tilde{u}_2$.

By definition, the generalized gradient is the set of vectors ζ such that the generalized directional derivative in any direction v exceeds $\zeta(v)$. Hence, if the zero vector is an element of the generalized gradient of (27), then the generalized directional derivative of (27) in direction $\mathbf{1}_{\Theta'}$ is non-negative. As Φ is continuously differentiable, the generalized directional derivatives of all of the terms in (27) except for $R_D(u_2)$ reduce to ordinary (Gâteaux) derivatives, so applying our bound on the generalized directional derivative of $R_D(u_2)$ gives the necessary condition

$$\int_{\Theta'} \left(-\frac{\beta}{u'(c_1(\theta))} + \frac{1}{Ru'(c_2(\theta))} - \eta\left(\Phi'(u_2(\theta) + \underline{x}(\theta)) - \Phi'(u_2(\theta))\right)\right) dF \geq 0.$$

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[24]This allows $\tilde{u}_2(\theta') > \tilde{u}_2(\theta)$ for $\theta' \in \Theta'$ and $\theta \in \Theta \backslash \Theta'$ such that $\theta' < \theta$, so as to include schedules of the form $\tilde{u}_2 + t\mathbf{1}_{\Theta'}$ for monotone $\tilde{u}_2$.

Using the definition (3), this implies

$$\int_{\Theta'} \frac{1}{Ru'(c_2(\theta))} \left(-\frac{1}{1-\tau_k(\theta)} + 1 + R\eta \left(1 - \frac{\Phi'(u_2(\theta) + \underline{x}(\theta))}{\Phi'(u_2(\theta))} \right) \right) dF \geq 0.$$

But, together with the fact that Θ' has positive measure, this contradicts the hypothesis that $\tau_k(\theta)/(1-\tau_k(\theta))$ exceeds $R\eta(1-\Phi'(u_2(\theta) + \underline{x}(\theta))/\Phi'(u_2(\theta)))$ on Θ'. ∎

Lemma 4 implies that all monotone solutions to the government's problem feature progressive marginal capital taxation, in the following sense.

Proposition 3. *In any monotone solution, there exists a threshold type θ^* such that capital is subsidized for types $\theta < \theta^*$ and taxed for types $\theta > \theta^*$.*

Proof. See Appendix C.4. ∎

The intuition for Proposition 3 is that the capital tax, given by equation (26), is designed to make individuals of each type θ internalize the marginal effect of their saving (i.e., backloading their utility) on the no-reform constraint. This involves comparing the effect on required period 2 resources under the most tempting reform, given by $\Phi'(u_2(\theta) + x(\theta))$, with the effect on required resources under the status quo, $\Phi'(u_2(\theta))$.[25] Since by Lemma 2 a reform will equalize period 2 consumption relative to the status quo (i.e., $x(\theta)$ is decreasing), low θ types face $x(\theta) > 0$, so their saving relaxes the no-reform constraint (as $\Phi'(u_2(\theta) + x(\theta)) > \Phi'(u_2(\theta))$ when $x(\theta) > 0$), motivating the capital subsidy. In contrast, high θ types face $x(\theta) < 0$, so their saving tightens the no-reform constraint, which makes it optimal for them to face a capital tax.

This logic is independent of the shape of the function H; notably, it does not depend on whether H' is single-peaked or not. This is in contrast to our results in Section 3, where the shape of H is crucial for the U-shaped pattern of the intertemporal wedge. The reason for this difference is that, in Section 3, the key comparison for determining the capital tax is between $u(c_2(\theta))$ and $u(RK)$, which implies that agents with intermediate $u(c_2(\theta))$ are "pivotal" when H' is single-peaked, and are therefore subsidized. In contrast, in the current section the key comparison is between $\Phi'(u_2(\theta))$ and $\Phi'(u_2(\theta) + x(\theta))$, so that agents with $x(\theta) > 0$ are more sensitive to their period 2 consumption under the status quo than under the reform, which leads them to be subsidized (and $x(\theta)$ is decreasing regardless of the shape of H).

Finally, note that Proposition 3 only shows that capital is subsidized for types below a threshold and taxed for types above it, and not that marginal capital taxes are increasing throughout the entire wealth distribution. We can however establish this stronger result under some additional assumptions.

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[25]This is for the case where $X(\theta) = \{x(\theta)\}$. When $X(\theta)$ is not a singleton, all that can be said in general about the effect of an additional unit of savings on required period 2 resources under the most tempting reform is that it lies in between $\Phi'(u_2(\theta) + \underline{x}(\theta))$ and $\Phi'(u_2(\theta) + \bar{x}(\theta))$.
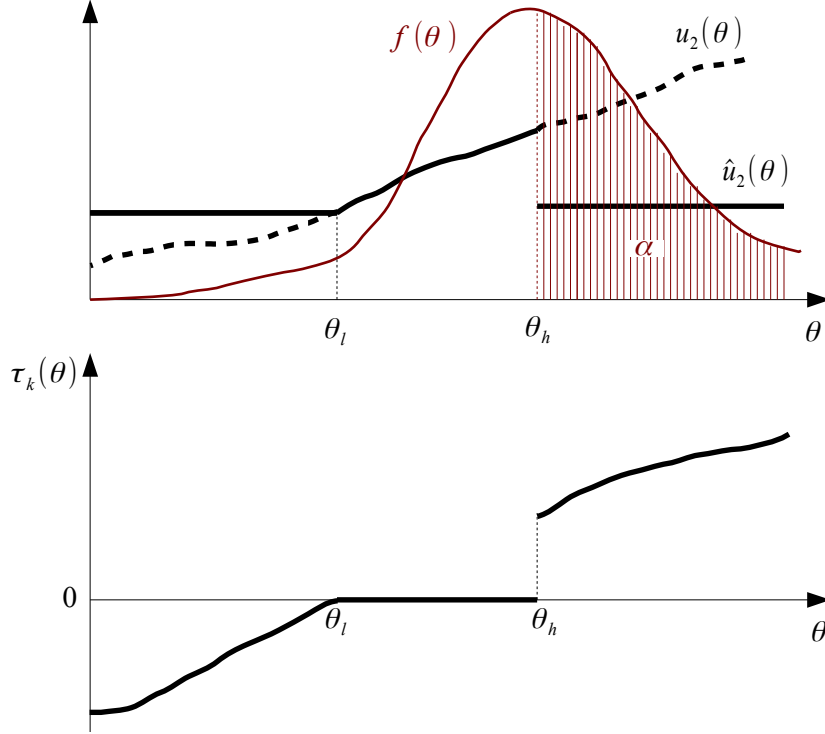
Figure 4: Capital taxes when H is a step function with within-period commitment

Proposition 4. *In any monotone solution, there exists a threshold u_2^* such that*
(i) If H' is single-peaked at 0, then $\tau_k(\theta)$ is non-decreasing on $\{\theta : u_2(\theta) > u_2^*\}$.
(ii) If $-u''(c)/u'(c)^2$ is non-increasing, then $\tau_k(\theta)$ is non-decreasing on $\{\theta : u_2(\theta) < u_2^*\}$.

Proof. See Appendix C.5. ∎

The results of this section can again be particularly easily illustrated for the case where H is a step function. In this case, it can be shown that $X(\theta)$ is single-valued almost everywhere, and in particular that it is flat at $x(\theta) = 0$ for all θ in an interval $[\theta_l, \theta_h]$ (corresponding to types that are indifferent between the status quo and the reform), and is decreasing on the intervals $(\underline{\theta}, \theta_l)$ and $(\theta_h, \overline{\theta})$ (in order to fully equalize reform consumption for types in those intervals). Figure 4 depicts the resulting shape of $\hat{u}_2$ compared to u_2 as well as the intertemporal wedge τ_k, which is weakly increasing in θ.

7 Conclusion

This paper has studied dynamic non-linear taxation under the assumption that a policy is sustainable if it maintains the support of a large enough political coalition over time. Optimal taxes differ starkly from those in settings where the government is free of political constraints.

Rather than predicting zero capital taxes as in the full commitment case, our model predicts progressive or U-shaped capital taxes, so that saving is subsidized for the poor and/or the middle class but taxed for the rich, recalling Director's law of public redistribution (Stigler, 1970). More generally, our analysis suggests that the nature of potential political reforms is an important determinant of the progressivity and middle-class bias of capital taxes, and of redistribution more broadly.

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A   Appendix: Revelation Principle

This section formulates the model as a game between the individuals and the government, and establishes the relevant version of the revelation principle. Individuals' and the government's preferences are as in the text. The game will involve *tax schedules* (T_y, T_k), which importantly are required to satisfy the resource constraint whatever production decisions individuals make. Formally, let $\mathcal{G}$ be the set of probability distributions on $\mathbb{R}_+$, corresponding to possible distributions of output or capital. A labor tax schedule T_y is a map from $\mathbb{R}_+ \times \mathcal{G} \rightarrow \mathbb{R}$ such that $\int_{y \in \mathbb{R}_+} T_y(y, G) \, dG \geq 0$ for all $G \in \mathcal{G}$. A capital tax schedule T_k is a map from $\mathbb{R}_+ \times \mathcal{G} \rightarrow \mathbb{R}$ such that $\int_{k \in \mathbb{R}_+} T_k(Rk, G) \, dG \geq 0$ for all $G \in \mathcal{G}$. The game is as follows:

Period 1:

1. There is an exogenous status quo tax schedule $\left(T_y^{SQ}, T_k^{SQ}\right)$. Individuals decide whether to support the period 1 status quo.

2. If at least α individuals support the status quo, then the labor tax schedule T_y^{SQ} is *implemented* (see below), and the period 2 status quo is set to T_k^{SQ}. If fewer than α individuals support the status quo, then the government proposes a reform tax schedule (T_y, T_k), the labor tax schedule T_y is implemented and the period 2 status quo is set to T_k.

3. Taking as given the implemented labor tax schedule $\tilde{T}_y$ (which may equal either T_y^{SQ} or T_y), individuals produce, pay labor taxes, and consume in period 1, as follows:

   (a) Individuals simultaneously choose production y.

   (b) Given the resulting distribution of output G^y, an individual who produced y pays labor tax $\tilde{T}_y(y, G^y)$.

   (c) An individual with after-tax income $y - T_y(y, G^y)$ chooses period 1 consumption $c_1 \in [0, y - T_y(y, G^y)]$. This leaves her with capital $k = y - T_y(y, G^y) - c_1 \geq 0$. Denote the resulting distribution of capital by G^k.

Period 2:

1. Individuals decide whether to support the period 2 status quo $\tilde{T}_k$ (which may equal either T_k^{SQ} or T_k).

2. If at least α individuals support the status quo, then the capital tax schedule $\tilde{T}_k$ is implemented, meaning that an individual with capital k consumes $Rk - \tilde{T}_k(RK, G^k)$. If fewer than α individuals support the status quo, then the government implements an expropriative reform capital tax schedule, which leads to the period 2 consumption schedule $c_2(\theta) = RK$ for all θ.

Under Assumption 1, it would be equivalent to let the government choose a reform capital tax schedule in the final stage, rather than hard-wiring the choice of an equalizing reform into the definition of the game. The only reason for the current approach is that it simplifies notation in the following solution concept.

A (symmetric, pure strategy, subgame perfect) *equilibrium* consists of a period 1 reform tax schedule (T_y, T_k), and production, consumption, and political support strategies $Y : \mathcal{T}_y \times \mathcal{T}_k \times \Theta \to y$, $C : \mathcal{T}_y \times \mathcal{T}_k \times \Theta \to c_1$, $S_1 : \mathcal{T}_y \times \mathcal{T}_k \times \Theta \to \{0, 1\}$, $S_2 : \mathcal{T}_k \times \mathbb{R}_+ \to \{0, 1\}$ (where $\mathcal{T}_y$ and $\mathcal{T}_k$ are the sets of possible tax schedules $\tilde{T}_y$ and $\tilde{T}_k$, and S_1 and S_2 denote political support strategies in period 1 and 2), such that:

1. (T_y, T_k) maximizes the government's payoff given (Y, C).

2. (Y, C) maximizes the utility of each type θ given $\left(\tilde{T}_y, \tilde{T}_k\right)$ and given that other individuals follow (Y, C, S_1, S_2).

3. S_1 and S_2 are "sincere," in that a type θ individual supports the period t status quo if and only if $v_t^{SQ}(\theta) + \varepsilon^{SQ} \geq v_t^R(\theta) + \varepsilon^R$.

4. $C\left(\tilde{T}_y, \tilde{T}_k, \theta\right) \leq Y\left(\tilde{T}_y, \tilde{T}_k, \theta\right) - \tilde{T}_y\left(Y\left(\tilde{T}_y, \tilde{T}_k, \theta\right), G^{Y(\tilde{T}_y, \tilde{T}_k, \cdot)}\right)$ for all $\tilde{T}_y \in \mathcal{T}_y, \tilde{T}_k \in \mathcal{T}_k, \theta \in \Theta$ (i.e., individuals do not consume more than their after-tax incomes in period 1).

An *allocation* $(c_1 : \Theta \to \mathbb{R}_+, c_2 : \Theta \to \mathbb{R}_+, y : \Theta \to \mathbb{R}_+)$ is a mapping from types to period 1 consumption, period 2 consumption, and production. An allocation is *feasible* if it satisfies the intertemporal resource constraint $\int (c_1(\theta) + c_2(\theta)/R) \, dF \leq \int y(\theta) \, dF$. An allocation is *implementable* if there exists an equilibrium $(T_y, T_k, Y, C, S_1, S_2)$ such that $Y\left(\tilde{T}_y, \tilde{T}_k, \theta\right) = y(\theta) \, \forall \theta \in \Theta$, $C\left(\tilde{T}_y, \tilde{T}_k, \theta\right) = c_1(\theta) \, \forall \theta \in \Theta$ and $D\left(\tilde{T}_y, \tilde{T}_k, Y, C, \theta\right) = c_2(\theta) \, \forall \theta \in \Theta$, where $D\left(\tilde{T}_y, \tilde{T}_k, Y, C, \theta\right)$ is the period 2 consumption of a type θ individual in equilibrium $(T_y, T_k, Y, C, S_1, S_2)$.

In the text, attention is restricted to feasible, incentive-compatible direct mechanisms that satisfy the no-reform constraint $\int H(c_2(\theta), RK) \, dF \geq \alpha$. This approach is justified by the following result.

Proposition 5 (Revelation Principle). *Under Assumption 2, every implementable allocation is feasible, incentive-compatible, and satisfies the no-reform constraint.*

The converse also holds for monotone allocations, as shown in Section 4.2 (building on Proposition 3 of Farhi et al., 2012).

Proof. Showing that any implementable allocation is feasible is a simple accounting exercise. Any implementable allocation is incentive-compatible as a direct mechanism, by the usual revelation principle argument (whether or not it is implemented in an equilibrium in which a reform occurs): a unilateral deviation does not affect the implemented tax schedules or the resulting distributions G^y or G^k, so if $y(\theta)$ and $c_1(\theta)$ are the optimal production and period 1 consumption choices of a type θ individual given others' behavior, then in particular she prefers $(c_1(\theta), c_2(\theta), y(\theta))$ to $(c_1(\theta'), c_2(\theta'), y(\theta'))$ for all $\theta' \in \Theta$. Thus, it suffices to show that every implementable allocation satisfies the no-reform constraint (6) when viewed as a direct mechanism.

To see this, note that if an allocation is implemented in an equilibrium in which no period 2 reform occurs, then it satisfies the no-reform constraint when viewed as a direct mechanism, as the condition for no period 2 reform to occur in equilibrium is precisely the no-reform constraint for the corresponding direct mechanism. In addition, if an allocation (c_1, c_2, y) is implemented in an equilibrium in which a period 2 reform does occur, then c_2 is constant. In this case, Assumption 2 implies that (c_1, c_2, y) satisfies the no-reform constraint when viewed as a direct mechanism. ∎

B Appendix: Monotone Solution to Government Problem

We begin with a novel monotone methods lemma. We will make use of this lemma repeatedly in different contexts throughout the paper, so we state it in general language. Note that the lemma concerns randomized consumption schedules, while the model allows only deterministic consumption schedules. The strategy is thus to show that monotone, deterministic consumption schedules are optimal in the class of all deterministic schedules by showing that they are optimal in the larger class of all randomized schedules.[26] A side benefit of this approach is that it shows that our results would not change if we allowed randomized schedules in the model, as such schedules would not be optimal.

Lemma 5. *Let P be drawn from the set of right-continuous functions from an open set $\Theta \subseteq \mathbb{R}$ to $\Delta(\mathbb{R})$, the set of Borel distributions over real-valued allocations a.[27] Let X be an arbitrary index set, let F be a cdf on Θ with positive density f, and consider the program*

$$W = \sup_P \int \int w(a, t(\theta)) \, dP dF$$

subject to one of the following constraints

$$\int \int y_x(a, A(P)) \, dP dF \leq 0 \text{ for all } x \in X, \text{ where } A(P) = \int \int a \, dP dF, \tag{C}$$

$$or \qquad \int \int y_x(a) \, dP dF \leq 0 \text{ for all } x \in X.[28] \tag{C'}$$

---

[26]Technically, Lemma 5 only implies that monotone schedules are optimal in the class of right-continuous schedules, rather than measurable ones. But it is straightforward to use (11) to show that there do exist right-continuous solutions to the government's problems; the argument is as in the proof of Lemma 2.

[27]Continuity here is with respect to the weak topology. That is, we require that if $\theta' \downarrow \theta$ then $P(\theta')$ converges in distribution to $P(\theta)$.

[28]That is, constraint C is a more general version of constraint C' that allows the functions y_x to depend on the aggregate allocation $A(P)$. Note also that constraint C may equivalently be written as $\sup_{x \in X} \int \int y_x(a, A(P)) \, dP dF \leq 0$, and similarly for constraint C'.

Assume that w is continuous and has strictly increasing differences in a and t, y_x is continuous for all $x \in X$, and t is right-continuous. Then

(i) In any solution P, if $t(\theta') < t(\theta'')$ then $a(\theta') \leq a(\theta'')$ for all $a(\theta') \in \operatorname{supp} P(\theta')$ and $a(\theta'') \in \operatorname{supp} P(\theta'')$.

(i') If the constraint takes the more restrictive form of constraint C', then for any solutions P' and P'', if $t(\theta') < t(\theta'')$ then $a(\theta') \leq a(\theta'')$ for all $a(\theta') \in \operatorname{supp} P'(\theta')$ and $a(\theta'') \in \operatorname{supp} P''(\theta'')$.

(ii) If a solution exists and t is non-decreasing, then there exists a deterministic solution in which a is non-decreasing.

Proof. Part (i). To obtain a contradiction, suppose that there exist $\theta', \theta'' \in \Theta$ such that $t(\theta') < t(\theta'')$ and yet $a(\theta') > a(\theta'')$ for some $a(\theta') \in \operatorname{supp} P(\theta'), a(\theta'') \in \operatorname{supp} P(\theta'')$. Since t and P are right-continuous and Θ is open, there exist disjoint closed intervals of positive length $\Theta' \subseteq \Theta$ and $\Theta'' \subseteq \Theta$ such that $t(\theta') < t(\theta'')$ and $a(\theta') > a(\theta'')$ for some $a(\theta') \in \operatorname{supp} P(\theta'), a(\theta'') \in \operatorname{supp} P(\theta'')$ for all $\theta' \in \Theta', \theta'' \in \Theta''$. Let $\bar{a}(\theta) = \sup\{\operatorname{supp} P(\theta')\}$, $\underline{a}(\theta) = \inf\{\operatorname{supp} P(\theta')\}$, and $\nu \equiv \inf_{\theta' \in \Theta', \theta'' \in \Theta''} \bar{a}(\theta') - \underline{a}(\theta'') > 0$. Without loss of generality, let the lengths of Θ' and Θ'' be equal. Define $\phi: \Theta' \to \Theta''$ by $\phi(\theta) = \theta + \underline{\theta}'' - \underline{\theta}'$, so that in particular ϕ is an invertible bijection. Given a distribution $P(\theta)$, let $\bar{P}(\theta)$ and $\underline{P}(\theta)$ denote the truncation of $P(\theta)$ on $[\bar{a}(\theta) - \nu/4, \bar{a}(\theta)]$ and $[\underline{a}(\theta), \underline{a}(\theta) + \nu/4]$, respectively. Define a new randomized schedule $\hat{P}$ by

$$
\hat{P}(\theta) \equiv \begin{cases} P(\theta) + \frac{\varepsilon}{f(\theta)} \left(\overline{\gamma}(\theta)\underline{P}(\phi(\theta)) - \underline{\gamma}(\phi(\theta))\bar{P}(\theta) \right) & \text{if} & \theta \in \Theta' \\ P(\theta) + \frac{\varepsilon}{f(\theta)} \left(\underline{\gamma}(\theta)\bar{P}(\phi^{-1}(\theta)) - \overline{\gamma}(\phi^{-1}(\theta))\underline{P}(\theta) \right) & \text{if} & \theta \in \Theta'' \\ P(\theta) & \text{if} & \theta \notin \Theta' \cup \Theta'', \end{cases}
$$

where the factors $\overline{\gamma}(\theta) \equiv \int d\bar{P}(\theta)$ and $\underline{\gamma}(\theta) \equiv \int d\underline{P}(\theta)$ ensure that $\hat{P}(\theta)$ integrates to one for each θ, and we fix some $\varepsilon > 0$ such that $\varepsilon < \inf_{\theta \in \Theta' \cup \Theta''} f(\theta)$, which (together with $\overline{\gamma}(\theta), \underline{\gamma}(\theta) \leq 1$ for all θ) ensures that $\hat{P}(\theta)$ is a well-defined probability distribution for all θ.

The variation $\hat{P}$ is constructed such that, for any $x \in X$, we have $\int \int y_x(a) d\hat{P} dF = \int \int y_x(a) dP dF$ because

$$
\int \int y_x(a) d\hat{P} dF - \int \int y_x(a) dP dF
$$
$$
= \varepsilon \int_{\Theta'} \int y_x(a) \left(\overline{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right) d\theta + \varepsilon \int_{\Theta''} \int y_x(a) \left(\underline{\gamma}(\theta) d\overline{P}(\phi^{-1}(\theta)) - \overline{\gamma}(\phi^{-1}(\theta)) d\underline{P}(\theta) \right) d\theta
$$
$$
= \varepsilon \int_{\Theta'} \int y_x(a) \left(\overline{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) + \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) - \overline{\gamma}(\theta) d\underline{P}(\phi(\theta)) \right) d\theta = 0.
$$

In particular, this implies that $A(\hat{P}) = A(P)$ and therefore

$$
\int \int y_x \left(a, A(\hat{P}) \right) d\hat{P} dF = \int \int y_x(a, A(P)) dP dF
$$

for all $x \in X$; that is, if P satisfies constraint C or C', then so does $\hat{P}$. In addition,

$$
\int \int w(a, t(\theta)) d\hat{P} dF - \int \int w(a, t(\theta)) dP dF
$$
$$
= \varepsilon \int_{\Theta'} \left[\int w(a, t(\theta)) \overline{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \int w(a, t(\theta)) \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right] d\theta
$$
$$
+ \varepsilon \int_{\Theta''} \left[\int w(a, t(\theta)) \underline{\gamma}(\theta) d\bar{P}(\phi^{-1}(\theta)) - \int w(a, t(\theta)) \overline{\gamma}(\phi^{-1}(\theta)) d\underline{P}(\theta) \right] d\theta
$$
$$
= \varepsilon \int_{\Theta'} \left[\int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \overline{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right] d\theta.
$$

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For each $\theta \in \Theta'$, $t(\theta) < t(\phi(\theta))$, and because $w(a,t)$ has increasing differences, $w(a,t(\theta)) - w(a,t(\phi(\theta)))$ is decreasing in a. Therefore,

$$\int \left[w(a,t(\theta)) - w(a,t(\phi(\theta))) \right] \overline{\gamma}(\theta) d\underline{P}(\phi(\theta)) > \left[w\left(\underline{a}(\phi(\theta)) + \nu/4, t(\theta)\right) - w\left(\underline{a}(\phi(\theta)) + \nu/4, t(\phi(\theta)))\right) \right] \overline{\gamma}(\theta) \underline{\gamma}(\phi(\theta)),$$

where we used $\int d\underline{P}(\phi(\theta)) = \underline{\gamma}(\phi(\theta))$. Similarly, for each $\theta \in \Theta'$,

$$\int \left[w\left(a,t(\theta)\right) - w\left(a,t(\phi(\theta)))\right) \right] \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) < \left[w\left(\bar{a}(\theta) - \nu/4, t(\theta)\right) - w\left(\bar{a}(\theta) - \nu/4, t(\phi(\theta)))\right) \right] \underline{\gamma}(\phi(\theta)) \overline{\gamma}(\theta).$$

Hence,

$$\varepsilon \int_{\Theta'} \left[\int \left[w(a,t(\theta)) - w(a,t(\phi(\theta))) \right] \overline{\gamma}(\theta) d\underline{P}(\phi(\theta)) - \int \left[w(a,t(\theta)) - w(a,t(\phi(\theta))) \right] \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right] d\theta$$

$$> \varepsilon \int_{\Theta'} \left[\begin{array}{c} w\left(\underline{a}(\phi(\theta)) + \nu/4, t(\theta)\right) - w\left(\underline{a}(\phi(\theta)) + \nu/4, t(\phi(\theta)))\right) \\ - \left[w\left(\bar{a}(\theta) - \nu/4, t(\theta)\right) - w\left(\bar{a}(\theta) - \nu/4, t(\phi(\theta)))\right) \right] \end{array} \right] \underline{\gamma}(\phi(\theta)) \bar{\gamma}(\theta) d\theta$$

$$> 0,$$

where the last inequality follows because $\underline{a}(\phi(\theta)) + \nu/4 < \bar{a}(\theta) - \nu/4$ for all $\theta \in \Theta'$ and $w(a,t(\theta)) - w(a,t(\phi(\theta)))$ is decreasing in a (as $w(a,t)$ has increasing differences). Therefore, $\hat{P}$ achieves a strictly higher value of the objective than P, so P cannot be a solution.

Part (i'). Under Constraint C', if P' and P'' are both solutions then so is the function $\frac{1}{2}P' + \frac{1}{2}P''$ given by $\left(\frac{1}{2}P' + \frac{1}{2}P''\right)(\theta) = \frac{1}{2}P'(\theta) + \frac{1}{2}P''(\theta)$ for all θ. Noting that $\operatorname{supp} P'(\theta') \subseteq \operatorname{supp}\left(\frac{1}{2}P' + \frac{1}{2}P''\right)(\theta')$ and $\operatorname{supp} P''(\theta'') \subseteq \operatorname{supp}\left(\frac{1}{2}P' + \frac{1}{2}P''\right)(\theta'')$, the result follows from applying (i) to $\frac{1}{2}P' + \frac{1}{2}P''$.

Part (ii). Let P be a solution. Taking $\theta'' \downarrow \theta'$ and recalling that $P(\theta)$ is right-continuous, (i) implies that P is already deterministic and monotone over every interval $\Theta' \subseteq \Theta$ on which t is strictly increasing. It remains only to show that P may be replaced by a deterministic and monotone allocation on those intervals Θ' on which t is constant. To see that this is possible, fix such an interval $\Theta' = [\underline{\theta}', \bar{\theta}']$, and let $\underline{a} = \inf\{a_0 : a_0 \in \operatorname{supp} P(\theta), \theta \in \Theta'\}$. Now define the deterministic and monotone allocation $a : \Theta' \to \mathbb{R}$ by

$$a(\theta) = \inf\left\{ a_0 : \int_{\Theta'} \mathbb{I}\{a \in [\underline{a}, a_0]\} dPdF \geq F(\theta) - F(\underline{\theta}') \right\}.$$

It follows that for every interval of allocations $A = [\underline{a}, a_0]$,

$$\int_{\Theta'} \mathbb{I}\{a(\theta) \in A\} dF = \int_{\Theta'} \mathbb{I}\{a \in A\} dPdF,$$

and therefore that the same holds for every measurable set of allocations $A \subseteq \mathbb{R}$. Since t is constant on Θ', this implies that replacing P with a on Θ' does not affect the objective or the constraints of the program. Thus, performing this replacement on all intervals on which t is constant yields a deterministic and monotone solution. ∎

It is straightforward to use Lemma 5 to establish existence of a monotone solution.

Lemma 6. *There exists a solution to the government's problem in which $c_2(\theta)$ is non-decreasing.*

Proof. We relax the government's problem by allowing randomized consumption schedules, and show that there exists a deterministic solution to the relaxed problem with $c_2(\theta)$ non-decreasing. This implies that there exists a solution to the original problem with $c_2(\theta)$ non-decreasing.

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Formally, allow the government to choose, for each $\theta \in \Theta$, a distribution $P(\theta)$ over consumption levels $(c_1(\theta), c_2(\theta))$ such that $P(\theta)$ is right-continuous in the weak topology and $U(\theta) = u(c_1(\theta)) + \beta u(c_2(\theta))$ is constant for all $(c_1(\theta), c_2(\theta)) \in \operatorname{supp} P(\theta)$. Rewrite the dual problem as

$$\min_{U,P,y} \int \int \left(u^{-1}(U(\theta) - \beta u(c_2)) + \frac{1}{R}c_2 - y(\theta) \right) dP dF$$

subject to
$$U(\theta) - h(y(\theta), \theta) \geq U(\theta') - h(y(\theta'), \theta) \text{ for all } \theta, \theta',$$

$$\int (U(\theta) - h(y(\theta), \theta)) dF \geq V,$$

$$\int \int H\left(c_2, \int \int c_2 dP dF\right) dP dF \geq \alpha.$$

Our assumptions ensure that a solution to this problem exists because the objective is continuous and the constraint set is closed and can be bounded using the Inada conditions on u. Moreover, observe that at any solution, P must solve the subproblem

$$\min_{P} \int \int \left(u^{-1}(U(\theta) - \beta u(c_2)) + \frac{1}{R}c_2 \right) dP dF$$

subject to

$$\int \int H\left(c_2, \int \int c_2 dP dF\right) dP dF \geq \alpha.$$

Note that $u^{-1}(U(\theta) - \beta u(c_2(\theta)))$ has strictly decreasing differences in $U(\theta)$ and $c_2(\theta)$ by strict concavity of u, and that $U(\theta)$ is right-continuous and is non-decreasing by the incentive-compatibility constraint (2). The result then follows from Lemma 5 (ii). ∎

C   Appendix: Omitted Proofs

C.1   Proof of Proposition 2

When H is a step function, the government's dual problem is

$$\min_{c_1, c_2, y} \int \left(c_1(\theta) + \frac{1}{R}c_2(\theta) - y(\theta) \right) dF$$

subject to (2), (10), (7), and the no-reform constraint

$$\int \mathbb{I}\{c_2(\theta) \geq RK\} dF \geq \alpha. \tag{28}$$

As in the case where H is differentiable, any solution must solve the subproblem

$$\min_{c_2, K} \int \left(u^{-1}(U(\theta) - \beta u(c_2(\theta))) + \frac{1}{R}c_2(\theta) \right) dF$$

subject to (10) and (28).

In any monotone solution, the set $\{\theta : c_2(\theta) = RK\}$ is an interval, which we take to be a closed interval $[\theta_l, \theta_h]$ without loss of generality (up to consumption at the endpoints).

Claim 1: $\tau_k(\theta) = \tau_k(\theta')$ for almost all $\theta, \theta' \notin [\theta_l, \theta_h]$.

Proof: If not, then there exist a constant $\delta > 0$ and sets of types Θ and Θ' of equal measure such that $\tau_k(\theta) > \tau_k(\theta')$ for all $\theta \in \Theta, \theta' \in \Theta'$ and $|c_2(\theta) - RK| > \delta$ for all $\theta \in \Theta \cup \Theta'$. Vary the solution to the subproblem by increasing $c_2(\theta)$ by $\varepsilon > 0$ for all $\theta \in \Theta$ and decreasing $c_2(\theta)$ by ε for all $\theta \in \Theta'$. This variation does not affect (10) and (28) if $\varepsilon < \delta$, and its first-order effect on the objective is

$$\int_\Theta \left(-\beta \frac{u'(c_2(\theta))}{u'(c_1(\theta))} + \frac{1}{R} \right) dF - \int_{\Theta'} \left(-\beta \frac{u'(c_2(\theta))}{u'(c_1(\theta))} + \frac{1}{R} \right) dF$$
$$= \frac{1}{R} \left(-\int_\Theta \frac{\tau_k(\theta)}{1 - \tau_k(\theta)} dF + \int_{\Theta'} \frac{\tau_k(\theta)}{1 - \tau_k(\theta)} dF \right) < 0.$$

Therefore, the variation is a strict improvement for sufficiently small ε.

Let τ_k^* denote the common capital tax for types $\theta \notin [\theta_l, \theta_h]$.

Claim 2: $\tau_k^* \geq 0$.

Proof: If not, vary the solution to the subproblem by decreasing $c_2(\theta)$ by $\min\{\varepsilon, c_2(\theta) - RK\}$ for all $\theta \notin [\theta_l, \theta_h]$. This variation relaxes (10) and does not affect (28), and its first-order effect on the objective is

$$\int_{\theta \notin [\theta_l, \theta_h]} \frac{\tau_k^*}{1 - \tau_k^*} dF < 0.$$

The variation is a strict improvement for sufficiently small ε.

Claim 3: $\tau_k(\theta)$ *is non-decreasing on* $[\theta_l, \theta_h]$.

Proof: Follows from the fact that $U(\theta)$ is non-decreasing in $[\theta_l, \theta_h]$ (which is a consequence of incentive compatibility) and (3).

Claim 4: $\tau_k(\theta) \leq \tau_k^*$ *for almost all* $\theta \in [\theta_l, \theta_h]$.

Proof: If not, then there exist a constant $\delta > 0$ and sets of types $\Theta \subseteq [\theta_l, \theta_h]$ and Θ' of equal measure such that $\tau_k(\theta) > \tau_k(\theta') = \tau_k^*$ for all $\theta \in \Theta, \theta' \in \Theta'$ and $|c_2(\theta') - RK| > \delta$ for all $\theta \in \Theta'$. Vary the solution to the subproblem by increasing $c_2(\theta)$ by $\varepsilon > 0$ for all $\theta \in \Theta$ and decreasing $c_2(\theta)$ by ε for all $\theta \in \Theta'$. This variation does not affect (10) and relaxes (28) if $\varepsilon < \delta$, and its first-order effect on the objective is

$$\frac{1}{R} \left(-\int_\Theta \frac{\tau_k(\theta)}{1 - \tau_k(\theta)} dF + \int_{\Theta'} \frac{\tau_k^*}{1 - \tau_k^*} dF \right) < 0.$$

The variation is a strict improvement for sufficiently small ε.

C.2   Proof of Lemma 2

(i) A solution to (DP) exists because the objective (21) is continuous and the constraint set defined by (22) and (23) is closed (by continuity of H and F) and can be bounded by the Inada conditions on u. For generic values of α, any solution satisfies the following Lagrange multiplier equation (Clarke, 1976, Theorem 3), where λ is a multiplier on (22) and μ is a multiplier on (23):

$$\Phi'(u_2(\theta) + x(\theta)) + \mu H'(-x(\theta)) = \lambda$$

The set of values of $x(\theta)$ that satisfy this equation is upper hemi-continuous in $u_2(\theta)$, as Φ and H are continuously differentiable. For values $\bar{u}_2$ such that $\{\theta : u_2(\theta) = \bar{u}_2\}$ is a singleton, the corresponding value of $x(\theta)$ may thus be taken to equal its right-hand limit. For the remaining values $\bar{u}_2$, the solution is unaffected by ordering the corresponding values of $x(\theta)$ monotonically in θ, as in the proof of Lemma 5. The resulting allocation is a right-continuous solution.

(ii) Follows from Lemma 5 (i') because $\Phi\left(u_2\left(\theta\right)+x\left(\theta\right)\right)$ has strictly increasing differences in $u_2\left(\theta\right)$ and $x\left(\theta\right)$, as Φ is convex.

(iii) *Lipschitz continuity:* For any consumption schedule $\tilde{u}_2$, let $X\left(\tilde{u}_2\right)$ be the corresponding set of solutions to (DP). Fix a consumption schedule u_2, and for any $\varepsilon > 0$ let

$$X^*\left(\varepsilon\right) = \bigcup_{\tilde{u}_2 : |u_2 - \tilde{u}_2| < \varepsilon} X\left(\tilde{u}_2\right),$$

where $|\cdot|$ is the sup norm. Then

$$R_D\left(\tilde{u}_2\right) = \min_{x \in X^*\left(\varepsilon\right)} \int \Phi\left(\tilde{u}_2\left(\theta\right) + x\left(\theta\right)\right) dF$$

for all $\tilde{u}_2$ such that $|u_2 - \tilde{u}_2| < \varepsilon$. Hence, for all such $\tilde{u}_2$,

$$
\begin{aligned}
\left|R_D\left(u_2\right) - R_D\left(\tilde{u}_2\right)\right| &\leq \max_{x \in X^*\left(\varepsilon\right)} \int \left|\Phi\left(u_2\left(\theta\right) + x\left(\theta\right)\right) - \Phi\left(\tilde{u}_2\left(\theta\right) + x\left(\theta\right)\right)\right| dF \\
&\leq \left|u_2 - \tilde{u}_2\right| \sup_{\substack{\theta \in \Theta, x \in X^*\left(\varepsilon\right), \\ u_2' : \left|u_2 - u_2'\right| < \varepsilon}} \Phi'\left(u_2'\left(\theta\right) + x\left(\theta\right)\right),
\end{aligned}
$$

and so $\quad \dfrac{\left|R_D\left(u_2\right) - R_D\left(\tilde{u}_2\right)\right|}{\left|u_2 - \tilde{u}_2\right|} \leq \sup_{\substack{\theta \in \Theta, x \in X^*\left(\varepsilon\right), \\ u_2' : \left|u_2 - u_2'\right| < \varepsilon}} \Phi'\left(u_2'\left(\theta\right) + x\left(\theta\right)\right) \leq \sup_{\theta \in \Theta, x \in X^*\left(\varepsilon\right)} \Phi'\left(u_2\left(\theta\right) + \varepsilon + \bar{x}\left(\underline{\theta}\right)\right),$

where the second inequality follows from part (ii). The right-hand side of this inequality converges to $\sup_\theta \Phi'\left(u_2\left(\theta\right) + \bar{x}\left(\underline{\theta}\right)\right) < \infty$ as $\varepsilon \to 0$, so $R_D\left(u_2\right)$ is locally Lipschitz continuous.

C.3   Proof of Lemma 3

Constraint $\int \Phi\left(u_2\left(\theta\right)\right) dF - \kappa \leq R_D\left(u_2\right)$ can be written as $\int \Phi\left(u_2\left(\theta\right)\right) dF - \kappa \leq \int \Phi\left(u_2\left(\theta\right) + x\left(\theta\right)\right) dF$ for all $x \in X$, where X is the set of all x-schedules that satisfy (22) and (23). The constraint set therefore takes the same form as (C') in Lemma 5, so the result follows from Lemma 5 (ii) by the same argument as in Lemma 6.

C.4   Proof of Proposition 3

Let $u_2^* = \inf\left\{u_2 : \exists\theta \text{ such that } u_2\left(\theta\right) = u_2 \text{ and } \underline{x}\left(\theta\right) \leq 0\right\}$. Then $u_2\left(\theta\right) < u_2^* \implies \underline{x}\left(\theta\right) > 0$ by definition, and $u_2\left(\theta\right) > u_2^* \implies \bar{x}\left(\theta\right) \leq 0$ by Lemma 2 (ii). Hence, by (26), $u_2\left(\theta\right) < u_2^* \implies \tau_k\left(\theta\right) \leq 0$ and $u_2\left(\theta\right) > u_2^* \implies \tau_k\left(\theta\right) \geq 0$.

The proposition follows immediately if there is no type θ with $u_2\left(\theta\right) = u_2^*$, or if those types θ with $u_2\left(\theta\right) = u_2^*$ either all face positive taxes or all face negative taxes. In the remaining case, let $\theta^* = \inf\{\theta : u_2\left(\theta\right) = u_2^*, \tau_k\left(\theta\right) \geq 0\}$. Note that τ_k is non-decreasing on $\{\theta : u_2\left(\theta\right) = u_2^*\}$ by monotonicity of $U\left(\theta\right)$ (an implication of (2)) and (3). Therefore, $\theta < \theta^*$ implies that either $u_2\left(\theta\right) < u_2^*$ (and hence $\tau_k\left(\theta\right) \leq 0$) or $u_2\left(\theta\right) = u_2^*$ and $\tau_k < 0$. Similarly, $\theta > \theta^*$ implies that either $u_2\left(\theta\right) > u_2^*$ (and hence $\tau_k\left(\theta\right) \geq 0$) or $u_2\left(\theta\right) = u_2^*$ and $\tau_k > 0$. In any case, $\theta < \theta^* \implies \tau_k\left(\theta\right) \leq 0$ and $\theta > \theta^* \implies \tau_k\left(\theta\right) \geq 0$.

C.5   Proof of Proposition 4

Let u_2^* be as in the proof of Proposition 3.
(i) We first claim that if $\theta < \theta'$ and $u_2\left(\theta\right), u_2\left(\theta'\right) > u_2^*$, then $u_2\left(\theta\right) + \underline{x}\left(\theta\right) \geq u_2\left(\theta'\right) + \bar{x}\left(\theta'\right)$. To see this, write

(DP) as

$$\min_{\hat{u}_2} \int H\left(u_2\left(\theta\right) - \hat{u}_2\left(\theta\right)\right) dF$$

subject to
$$\int \Phi\left(\hat{u}_2\left(\theta\right)\right) dF \leq RK - \kappa \quad \text{and} \quad \int \hat{u}_2\left(\theta\right) dF \geq \int u_2\left(\theta\right) dF.^{29}$$

Fix a solution $\hat{u}_2^* : \Theta \to \mathbb{R}$. Let $\Theta' = \{\theta : u_2\left(\theta\right) > u_2^*\}$, $\bar{V} = \int_{\Theta \backslash \Theta'} \hat{u}_2^*\left(\theta\right) dF$, and $\bar{K} = \int_{\Theta \backslash \Theta'} \Phi\left(\hat{u}_2^*\left(\theta\right)\right) dF$. Then a necessary condition for optimality is that the restriction of $\hat{u}_2^*$ to Θ' solves the subproblem

$$\min_{\hat{u}_2 : \Theta' \to \mathbb{R}} \int_{\Theta'} H\left(u_2\left(\theta\right) - \hat{u}_2\left(\theta\right)\right) dF$$

subject to
$$\bar{K} + \int_{\Theta'} \Phi\left(\hat{u}_2\left(\theta\right)\right) dF + \kappa \leq RK \quad \text{and} \quad \bar{V} + \int_{\Theta'} \hat{u}_2 dF \geq \int u_2 dF.$$

If $u_2\left(\theta\right) > u_2^*$ then $u_2\left(\theta\right) \geq \hat{u}_2^*\left(\theta\right)$ for every solution to (DP), by the definition of u_2^* and Lemma 2 (ii). Hence, a necessary condition for optimality is that $\hat{u}_2^*$ still solves the above subproblem when $\hat{u}_2$ is restricted to satisfy $u_2\left(\theta\right) \geq \hat{u}_2\left(\theta\right)$ for all $\theta \in \Theta'$. Now, since H' is single-peaked at 0, the objective in this subproblem has strictly increasing differences in $u_2\left(\theta\right)$ and $\hat{u}_2\left(\theta\right)$ over this range, while $u_2\left(\theta\right)$ does not enter in the constraints except through the constant $\int u_2 dF$, so Lemma 5 (i) implies that at every solution $\hat{u}_2\left(\theta\right)$ is non-increasing in $u_2\left(\theta\right)$, and hence in θ.

We now have $u_2\left(\theta\right) \leq u_2\left(\theta'\right)$ (by definition of monotone solution) and $u_2\left(\theta\right) + \underline{x}\left(\theta\right) \geq u_2\left(\theta'\right) + \bar{x}\left(\theta'\right)$. So in particular

$$\frac{\Phi'\left(u_2\left(\theta\right) + \underline{x}\left(\theta\right)\right)}{\Phi'\left(u_2\left(\theta\right)\right)} \geq \frac{\Phi'\left(u_2\left(\theta'\right) + \bar{x}\left(\theta'\right)\right)}{\Phi'\left(u_2\left(\theta'\right)\right)}. \tag{29}$$

By Lemma 4, this implies that $\tau_k\left(\theta\right) \leq \tau_k\left(\theta'\right)$.

(ii) Note that $-u''\left(c\right)/u'\left(c\right)^2$ is non-increasing $\Leftrightarrow \Phi''\left(u_2\right)/\Phi'\left(u_2\right)$ is non-increasing $\Rightarrow \Phi'\left(u_2 + x\right)/\Phi'\left(u_2\right)$ is non-increasing in $u_2 \, \forall x \geq 0$. Now if $\theta < \theta'$ and $u_2\left(\theta\right), u_2\left(\theta'\right) < u_2^*$, then $u_2\left(\theta\right) \leq u_2\left(\theta'\right)$ by definition of a monotone solution, $\underline{x}\left(\theta\right) \geq \bar{x}\left(\theta'\right)$ by Lemma 2 (ii), and $\underline{x}\left(\theta\right) \geq 0$ by Proposition 3. Therefore, (29) holds by convexity of Φ and the fact that $\Phi'\left(u_2 + \underline{x}\left(\theta\right)\right)/\Phi'\left(u_2\right)$ is non-increasing in u_2. By Lemma 4, this implies that $\tau_k\left(\theta\right) \leq \tau_k\left(\theta'\right)$.

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[29]This dual approach to (DP) is valid whenever the no-reform constraint in the government's problem is binding: if the value of this dual program is less than α, then varying $\hat{u}_2$ toward more equal consumption will decrease $\int \Phi\left(\hat{u}_2\left(\theta\right)\right) dF$ without violating the welfare constraint and will still receive enough support relative to the status quo.