A Bayesian DSGE Model of
Stock Market Bubbles and Business Cycles[*]

Jianjun Miao[†], Pengfei Wang[‡] and Zhiwei Xu[§]

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Abstract

We present an estimated DSGE model of stock market bubbles and business cycles using Bayesian methods. Bubbles emerge through a positive feedback loop mechanism supported by self-fulfilling beliefs. We identify a sentiment shock which drives the movements of bubbles and is transmitted to the real economy through endogenous credit constraints. This shock explains most of the stock market fluctuations and sizable fractions of the variations in real quantities. It generates the comovement between stock prices and the real economy and is the dominant force behind the internet bubbles and the Great Recession.

Keywords: Stock Market Bubbles, Bayesian Estimation, DSGE, Credit Constraints, Business Cycles, Sentiment Shock

JEL codes: E22, E32, E44

[†]Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Email: miaoj@bu.edu. Tel.: 617-353-6675.

[‡]Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Email: pfwang@ust.hk. Tel: (+852) 2358 7612.

[§]Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China. Email: xuzhiwei09@gmail.com. Tel.: (+86) 021-52301587.

1. Introduction

The U.S. stock market is volatile relative to fundamentals as is evident from Figure 1, which presents the monthly data of the real Standard and Poor's Composite Stock Price Index from January 1871 to January 2011, and the corresponding series of real earnings. Two recent boom-bust episodes are remarkable. Starting from January 1995, the stock market rose persistently and peaked in August 2000. During this period, the stock market rose by about 1.8 times. This boom is often attributed to the internet bubble. Following the peak in August 2000, the stock market crashed, reaching the bottom in February 2003. The stock market lost about 47 percent. Then the stock market went up again and reached the peak in October 2007. This stock market runup is often attributed to the housing market bubble. Following the burst of the bubble, the U.S. economy entered the Great Recession, with the stock market dropping 52 percent from October 2007 through March 2009.
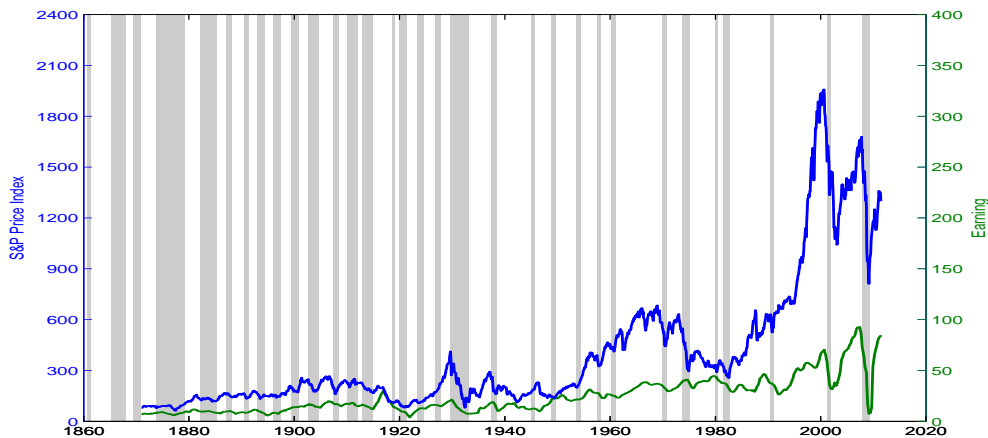

Figure 1: Real stock price indexes and real earnings. Source: Robert Shiller's website: http://www.econ.yale.edu/shiller/data.htm.

The U.S. stock market comoves with macroeconomic quantities. The boom phase is often associated with strong output, consumption, investment, and hours worked, while the bust phase is often associated with economic downturns. Stock prices, consumption, investment, and hours worked are procyclical, i.e., they exhibit a positive contemporaneous correlation with output (see Table 3 presented later).

The preceding observations raise several questions. What are the key forces driving the boom-bust episodes? Are they driven by economic fundamentals, or are they bubbles? What explains the comovement between the stock market and the macroeconomic quantities? These questions are challenging for macroeconomists. Standard macroeconomic models treat the stock market as a sideshow. In particular, after solving for macroeconomic quantities in a social planner's problem, one can derive the stock price that supports these quantities in a competitive equilibrium. Much attention has been devoted to the equity premium puzzle (Hansen and Singleton (1983) and Mehra and Prescott (1988)). However, the preceding questions have remained underexplored.

The goal of this paper is to provide an estimated dynamic stochastic general equilibrium (DSGE) model to address these questions. To the best of our knowledge, this paper provides the first estimated DSGE model of stock market bubbles using Bayesian methods. Our model-based, full-information econometric methodology has several advantages over the single-equation or the vector autoregression (VAR) approach used in the early literature to identify bubbles.[1] First, because neither bubbles nor fundamentals are observable, the literature fails to differentiate between misspecified fundamentals and bubbles (see Gurkaynak (2008) for a recent survey). By contrast, we treat bubbles as a latent variable in a DSGE model. The state space representation of the DSGE model allows us to conduct Bayesian inference of the latent variables by using observable data. We can answer the question of whether bubbles are important by comparing the marginal likelihoods of a DSGE model with bubbles and an alternative DSGE model without bubbles. Second, the single-equation or the VAR approach does not produce a time series of the bubble component and the shock behind the variation in bubbles. Thus, it is difficult to evaluate whether the properties of bubbles are in line with our daily-life experience. By contrast, we can simulate our model based on the estimated parameters and shocks to generate a time series of bubbles. Third, because our model is structural, we can do counterfactual analysis to examine the role of bubbles in generating fluctuations in macroeconomic quantities.

We set up a real business cycle (RBC) model with three standard elements: habit formation, investment adjustment costs, and variable capacity utilization. The novel element of our model is the assumption that firms are subject to idiosyncratic investment efficiency shocks and face endogenous credit constraints as in Miao and Wang (2011a,b, 2012a,b), and Miao, Wang, and Xu (2012). Under this assumption, a stock market bubble can exist through a positive feedback loop mechanism supported by self-fulfilling beliefs. The intuition is as follows. Suppose that households have optimistic beliefs about the stock market value of a firm. The firm uses its assets as collateral to borrow from the lender. If both the lender and the firm believe that firm assets have high value, then the firm can borrow more and make more investment. This makes firm value indeed high, supporting people's initial optimistic beliefs. Bubbles can burst if people believe they can. By no arbitrage, if a bubble in an asset bursts, a new one in the same asset cannot emerge. To facilitate recurrent bubbles in the model, we introduce exogenous entry and exit. New entrants bring new bubbles to the economy, making the total bubble in the economy stationary. We show that the aggregate stock market value is equal to the capital value (Tobin's marginal Q times the capital stock) plus a bubble (or speculative) component.

We introduce a sentiment shock which drives the fluctuations in the bubble and hence the stock price. This shock reflects households' beliefs about the relative size of the old bubble to the new bubble. This shock is transmitted to the real economy through credit constraints. Its movements affect the tightness of the credit constraints and hence a firm's borrowing capacity. This in turn affects a firm's investment decisions and hence output.[2] In addition to this shock, we incorporate five

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[1]See Phillpis and Yu (2011) for a recent econometric test for bubbles.

[2]Chirinko and Schaller (2001), Goyal and Yamada (2004), and Gilchrist, Himmelberg, and Huberman (2005) find

other shocks often studied in the literature: permanent and transitory labor-augmenting technology (or TFP) shocks, the permanent investment-specific technology (IST) shock, the labor supply shock, and the financial shock (a shock to the external financing constraint). We estimate our model using Bayesian methods to fit six U.S. time series data of consumption, investment, hours, the relative price of investment goods, stock prices, and the Chicago Fed's National Financial Conditions Index (NFCI). Our full-information, model-based empirical strategy for identifying the sentiment shock exploits the fact that in the theoretical model the observable variables react differently to different types of shocks. We then use our estimated model to address the questions raised earlier. We also use our model to shed light on two major bubble and crash episodes: (i) the internet bubble during the late 1990s and its subsequent crash, and (ii) the recent stock market bubble caused by the housing bubble and the subsequent Great Recession.

Our baseline estimation results show that the sentiment shock explains most of the fluctuations in the stock price at the business cycle frequency. It also explains a sizable fraction of the variations in investment, consumption, and output. Consistent with the RBC literature, the two TFP shocks together explain most of the variations in these quantities. Historical decomposition of shocks shows that the sentiment shock explains almost all of the stock market booms and busts. In addition, it is the dominant driving force behind the movements in investment during the internet bubble and crash and the recent stock market bubble and the subsequent Great Recession. The sentiment shock also accounts for a large share of the fall in consumption during the Great Recession. But it is not a dominant driver behind the consumption movements during the internet bubble and crash. For both boom-bust episodes, the labor supply shock, instead of the sentiment shock, is the major driving force behind the movements in labor hours.

To examine the robustness of our findings, we study two model variations. First, we incorporate the consumer sentiment index (CSI) data from the University of Michigan in the estimation since this index is highly correlated with the smoothed sentiment shock. We introduce measurement errors into the measurement equation for this data. We also allow SCI to be correlated with business cycles and allow the sentiment shock to be correlated with other shocks in the model. Second, we follow Ireland (2004) and estimate a hybrid model that combines the DSGE framework with the VAR model. We remove all shocks from the baseline model except for the sentiment shock. We then formulate the measurement equations into a VAR system. We find that our results in the baseline model are robust to the two model variations, although the impact of the sentiment shock is weakened. As a conservative estimate, the sentiment shock explains about 73, 17, 10, and 20 percent of the fluctuations in the stock market, output, investment, and consumption, respectively.

The transmission mechanism for the comovement between the stock market and the real economy is as follows. In response to a positive sentiment shock, the bubble and the stock price rise. This relaxes firms' credit constraints and raises their investments. Importantly, the rise in the

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empirical evidence that investment responds to the stock market value beyond the fundamentals. See Gan (2007) and Chaney, Sraer, and Thesmar (2009) for empirical evidence on the relation between collateral constraints and investment.

bubble has a capital reallocation effect, making resources move to more productive firms. This makes investment more efficient. Tobin's marginal Q falls as the capital stock rises, causing the capacity utilization rate to rise. This induces the labor demand to rise. The wealth effect due to the rise in stock prices causes consumption to rise and the labor supply to fall. It turns out that the rise in the labor demand dominates the fall in the labor supply, and hence labor hours increase. The increased hours and capacity utilization together raise output.

The sentiment shock in our model is similar to the financial shock in that the impact of both shocks is transmitted to the real economy through the credit constraints. One difference is that, unlike the sentiment shock, the financial shock cannot generate the comovement among stock prices, investment, and consumption as well as the excessive volatility in the stock market. Another difference is that the sentiment shock directly affects stock prices. Without using the stock price data in the estimation, the financial shock is important, while the sentiment shock is not. However, when the stock price data is included in the estimation, the sentiment shock displaces the financial shock, making the impact of the latter much smaller.

We emphasize that the sentiment shock is not simply a residual used to explain the stock market volatility. When we shut down this shock and introduce measurement errors into the measurement equation for the stock price data, we find that the measurement errors explain most of the variation in the stock prices. But this model cannot explain the comovement between the stock market and the real economy.

It is challenging for standard DSGE models to explain this comovement and the stock market booms and busts. In these models, the stock market value is equal to Tobin's marginal Q times the capital stock. One often needs a large investment adjustment cost parameter to make Tobin's marginal Q highly volatile. In addition, one also has to introduce other sources of shocks to drive the comovement between the marginal Q and real quantities because many shocks studied in the literature fail to generate either the right comovement or the right relative volatility in the data. For example, the TFP shock cannot generate large volatility of the stock price, while the IST shock generates counterfactual comovements of the marginal Q (hence stock prices) and the relative price of investment goods if both series are used as observable data. The financial shock typically makes investment and consumption move in opposite directions and causes stock prices to move countercyclically.

Our finding that the usual macroeconomic risks such as the TFP and IST shocks do not explain much the variations in the stock market is consistent with that in Li, Li, and Yu (2013). Without incorporating the stock price data, Li, Li, and Yu (2013) estimate the DSGE model of Christiano, Trabandt and Walentin (2011) using Bayesian methods and extract the TFP, IST, and monetary policy shocks from this model. They find that these shocks predict the future stock returns with the adjusted R-squares ranging from 0.02 to 0.03 for one-month horizon, from 0.02 to 0.04 for one-quarter horizon, and from 0.07 to 0.13 for one-year horizon. They also show that this predictability outperforms other studies in the literature.

Recently, two types of shocks have drawn wide attention: the news shock and the risk (or uncertainty) shock. The news shock cannot generate the comovement in a standard RBC model (Barro and King (1984) and Wang (2012)). To generate the comovements, Beaudry and Portier (2004) incorporate multisectoral adjustment costs, Christiano et al. (2008) introduce nominal rigidities and inflation-targeting monetary policy, and Jaimovich and Rebelo (2009) consider preferences that exhibit a weak short-run wealth effect on the labor supply. These three papers study calibrated DSGE models and do not examine the empirical importance of the news shock.[3] Fujiwara, Hirose, and Shintani (2011) and Schmitt-Grohe and Uribe (2012) study this issue using the Bayesian DSGE approach. Most Bayesian DSGE models do not incorporate stock prices as observable data for estimation. As Schmitt-Grohe and Uribe (2012) point out, "as is well known, the neoclassical model does not provide a fully adequate explanation of asset price movements."[4]

By incorporating the stock price data, Christiano, Motto, Rostagno (2010, 2012) argue that the risk shock, related to that in Bloom (2009), displaces the marginal efficiency of investment shock and is the most important shock driving business cycles.[5] They also introduce a news shock to the risk shock, instead of TFP. Their models are based on Bernanke, Gertler and Gilchrist (1999) and identify the credit constrained entrepreneurs' net worth as the stock market value in the data. By contrast, we use the aggregate market value of the firms in the model as the stock price index in the data, which is more consistent with the conventional measurement.

As in Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Jermann and Quadrini (2012), and Liu, Wang and Zha (2013), financial frictions play an important role in our model. Unlike in these papers and in Christiano, Motto, Rostagno (2010, 2012), firms in our model are not financially constrained in the aggregate. Our model features firm heterogeneity. Some firms are financially constrained, while others are not. In the aggregate, firms can be self-financing. This feature is consistent with the empirical evidence documented by Chari, Christiano, and Kehoe (2008) and Ohanian (2010). Unlike the representative firm setup, there is a capital reallocation channel for the financial frictions to impact the real economy.

Our paper is closely related to the literature on rational bubbles (Tirole (1982), Weil (1987), and Santos and Woodford (1997)). The recent Great Recession has generated renewed interest in this literature. Recent important contributions include Kocherlakota (2009), Farhi and Tirole (2010), Hirano and Yanagawa (2010), Martin and Ventura (2011a,b), Wang and Wen (2011), Miao and Wang (2011a,b, 2012a,b), and Miao, Wang and Xu (2012). Most papers in this literature are theoretical, while Wang and Wen (2011) provide some calibration exercises. Except for Miao and Wang (2011a,b, 2012a,b) and Miao, Wang, and Xu (2012), all other papers study bubbles in intrinsically useless assets or assets with exogenously given payoffs.

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[3]Beaudry and Portier (2006) study the empirical implications of the news shock using the VAR approach.

[4]In Section 6.8 of their paper, Schmitt-Grohe and Uribe (2012) discuss briefly how the share of unconditional variance explained by anticipated shocks will change when stock prices are included as observable data. But they do not include stock prices in their baseline estimation.

[5]It is difficult for shocks to the TFP shock's variance (uncertainty shocks) to generate comovements among investment, consumption, hours, and stock prices in standard DSGE models (see, e.g., Basu and Bundick (2011)).

Our paper is also related to Farmer (2012a,b), which argue that multiple equilibria supported by self-fulfilling beliefs can help understand the recent Great Recession. Farmer provides a search model and replaces the Nash bargaining equation for the wage determination with an equation to determine the expected stock future price. In particular, he assumes that the expected future stock price relative to the price level or the real wage is determined by an exogenously given variable representing beliefs. The evolution of this variable is determined by a belief function. Unlike Farmer's approach, we model beliefs as a sentiment shock to the relative size of the old bubble to the new bubble. We then derive a no-arbitrage equation for the bubble in equilibrium. No extra equation is imposed exogenously.

The remainder of the paper proceeds as follows. Section 2 presents the baseline model. Section 3 estimates model parameters using Bayesian methods. Section 4 analyzes the estimated model's economic implications. Section 5 conducts a sensitivity analysis by estimating four alternative models. Section 6 concludes. Technical details are relegated to appendices.

2. The Baseline Model

We consider an infinite-horizon economy that consists of households, firms, capital goods producers, and financial intermediaries. Households supply labor to firms, deposit funds in competitive financial intermediaries, and trade firm shares in a stock market. Firms produce final goods that are used for consumption and investment. Capital goods producers produce investment goods subject to adjustment costs. Firms purchase investment goods from capital goods producers subject to credit constraints. Firms finance investment using internal funds, new equity issuance, and external borrowing. Firms and households can save in competitive financial intermediaries (or banks), which make one-period loans to borrowers. As a starting point, we assume that there is no friction in financial intermediaries so that we treat them as a veil. In addition, we do not consider money or monetary policy and study a real model of business cycles.

2.1. Households

There is a continuum of identical households of measure unity. Each household derives utility from consumption and leisure according to the following expected utility function:

$$E \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - hC_{t-1}) - \psi_t N_t \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, $h \in (0,1)$ is the habit persistence parameter, C_t denotes consumption, N_t denotes labor, and ψ_t represents a labor supply shock. This shock accounts for the labor wedge and may proxy for a variety of labor market frictions that could be important in the real world. Assume that $\ln \psi_t$ follows an AR(1) process. The specification of linear disutility of labor reflects indivisible labor in the RBC literature and helps generate large

fluctuations in hours worked relative to productivity.

The representative household's budget constraint is given by

$$C_t + P_t^s s_{t+1} + \frac{d_{t+1}}{R_{dt}} = W_t N_t + \Pi_t + (D_t + P_t^s)s_t + d_t, \ s_0 = 1, \ d_0 = 0, \tag{2}$$

where s_t, P_t^s, d_t, R_{dt}, W_t, Π_t, and D_t denote share holdings, the aggregate stock price of all final goods firms, deposits in the financial intermediaries, the deposit rate, the wage rate, the profit from capital goods producers, and the aggregate dividend, respectively. The household is subject to a borrowing constraint, $d_{t+1} \geq 0$. Without a borrowing constraint, a bubble cannot exist (e.g., Kocherlakota (1992, 2009)). In equilibrium, $s_t = 1$. The household's first-order conditions are given by

$$\Lambda_t W_t = \psi_t, \tag{3}$$

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta E_t \frac{h}{C_{t+1} - hC_t}, \tag{4}$$

$$\frac{1}{R_{dt}} \geq \beta(1 - \delta_e)E_t \frac{\Lambda_{t+1}}{\Lambda_t}, \ \text{with equality when } d_{t+1} > 0, \tag{5}$$

where Λ_t represents the marginal utility of consumption.

2.2. Firms

There is a continuum of final goods firms of measure unity. Suppose that households believe that each firm's stock may contain a bubble. They also believe that the bubble may burst with some probability. By rational expectations, a bubble cannot reemerge in the same firm after bursting. Otherwise there would be an arbitrage opportunity. This means that none of the firms would contain any bubble once all bubbles have bursted if no new firms enter the economy. As a result, we follow Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), and Gertler and Kiyotaki (2011), and assume exogenous entry and exit, for simplicity. A firm may die with an exogenously given probability δ_e each period. After death, its value is zero and a new firm enters the economy without costs so that the total measure of firms is fixed at unity in each period. A new firm entering at date t starts with an initial capital stock K_{0t} and then operates in the same way as an incumbent firm. The new firm may bring a new bubble into the economy.[6]

An incumbent firm $j \in [0,1]$ combines capital K_t^j and labor N_t^j to produce final goods Y_t^j using the following production function:[7]

$$Y_t^j = (u_t^j K_t^j)^\alpha \left(A_t N_t^j \right)^{1-\alpha}, \tag{6}$$

where $\alpha \in (0,1)$, u_t^j denotes the capacity utilization rate, and A_t denotes the labor-augmenting

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[6]See Martin and Ventura (2011b) for a related overlapping generations model with recurrent bubbles.

[7]A firm can be identified by its age. Hence, we may use the notation $K_{t,\tau}$ to denote firm j's capital stock K_t^j if its age is τ. Because we want to emphasize the special role of bubbles, we only use such a notation for the bubble.

technology shock. Given the Cobb-Douglas production function, we may also refer to A_t as a total factor productivity (TFP) shock. For a new firm entering at date t, we set $K_t^j = K_{0t}$. Assume that A_t is composed of a permanent component A_t^p and a transitory (mean-reverting) component A_t^m such that $A_t = A_t^p A_t^m$, where $\ln \lambda_{at} \equiv \ln \left(A_t^p / A_{t-1}^p \right)$ and $\ln A_t^m$ follow independent AR(1) processes.

Assume that the capital depreciation rate between period t and period $t+1$ is given by $\delta_t^j = \delta(u_t^j)$, where δ is a twice continuously differentiable convex function that maps a positive number into $[0, 1]$. We do not need to parameterize the function δ since we use the log-linearization solution method. We only need it to be such that the steady-state capacity utilization rate is normalized to 1. The capital stock evolves according to

$$K_{t+1}^j = (1 - \delta_t^j)K_t^j + \varepsilon_t^j I_t^j, \tag{7}$$

where I_t^j denotes investment and ε_t^j measures the efficiency of the investment. Assume that investment is irreversible at the firm level so that $I_t^j \geq 0$. Assume that ε_t^j is IID across firms and over time and is drawn from the fixed cumulative distribution Φ over $[\varepsilon_{\min}, \varepsilon_{\max}] \subset (0, \infty)$ with mean 1 and the probability density function ϕ. This shock induces firm heterogeneity in the model. For tractability, assume that the capacity utilization decision is made before the observation of investment efficiency shock ε_t^j. Consequently, the optimal capacity utilization does not depend on the idiosyncratic shock ε_t^j.

Given the wage rate w_t and the capacity utilization rate u_t^j, the firm chooses labor demand N_t^j to solve the following problem:

$$R_t u_t^j K_t^j = \max_{N_t^j} \ (u_t^j K_t^j)^\alpha (A_t N_t^j)^{1-\alpha} - W_t N_t^j, \tag{8}$$

where

$$R_t \equiv \alpha \left[\frac{(1-\alpha) A_t}{W_t} \right]^{\frac{1-\alpha}{\alpha}}. \tag{9}$$

In each period t, firm j can make investment I_t^j by purchasing investment goods from capital producers at the price P_t. Its flow-of-funds constraint is given by

$$D_t^j + L_t^j + P_t I_t^j = u_t^j R_t K_t^j + \frac{L_{t+1}^j}{R_{ft}}, \tag{10}$$

where $L_{t+1}^j > 0 (< 0)$ represents borrowing (savings), R_{ft} represents the interest rate, and $D_t^j > 0 (< 0)$ represents dividends (new equity issuance). Assume that external financial markets are imperfect so that firms are subject to the following constraint on new equity issuance:

$$D_t^j \geq -\eta_t K_t^j, \tag{11}$$

where η_t is an exogenous stochastic shock to equity issuance. In addition, external borrowing is

subject to a credit constraint:

$$E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, L_{t+1}^j) \geq E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, 0) - E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(\xi_t K_t^j, 0), \quad (12)$$

where $V_{t,\tau}(k, l, \varepsilon)$ represents the cum-dividends stock market value of the firm with assets k, debt l, and idiosyncratic investment efficiency shock ε at time t with age τ, and $\bar{V}_{t,\tau}(k_t, l_t) \equiv \int V_{t,\tau}(k_t, l_t, \varepsilon) d\Phi(\varepsilon)$ represents the ex ante value after integrating out ε. Here, ξ_t represents a collateral shock that reflects the friction in the credit market as in Jermann and Quadrini (2011) and Liu, Wang, Zha (LWZ for short) (2013). Note that τ represents the age of firm j. We will show below that equity value depends on the age because it contains a bubble component which is age dependent.

Following Miao and Wang (2011a), we can interpret (12) as an incentive constraint in a contracting problem between the firm and the lender when the firm has limited commitment.[8] In any period t, firm j chooses to borrow L_{t+1}^j / R_{ft}. It may default on debt L_{t+1}^j at the beginning of period $t+1$ before the realization of the idiosyncratic investment efficiency shock and conditional on its surviving in period $t+1$. If it does not default, it obtains continuation value $\beta(1 - \delta_e) E_t \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, L_{t+1}^j)$. If it defaults, debt is renegotiated and the repayment is relieved. The lender can seize the collateralized asset $\xi_t K_t^j$ and keep the firm running with these assets by reorganizing the firm.[9] Thus the threat value to the lender is $\beta(1 - \delta_e) E_t \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(\xi_t K_t^j, 0)$. Following Jermann and Quadrini (2012), assume that the firm has a full bargaining power. Then the expression on the right-hand side of (12) is the value of the firm if it chooses to default. Thus, constraint (12) ensures firm j has no incentive to default in equilibrium.[10]

2.3. Decision Problem

We describe firm j's decision problem by dynamic programming:

$$
\begin{aligned}
V_{t,\tau}\left(K_t^j, L_t^j, \varepsilon_t^j\right) = \max_{I_t^j, u_t^j, L_{t+1}^j} \quad & R_t u_t^j K_t^j - P_t I_t^j - L_t^j + \frac{L_{t+1}^j}{R_{ft}} \\
& + (1 - \delta_e) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{t+1,\tau+1}\left(K_{t+1}^j, L_{t+1}^j, \varepsilon_{t+1}^j\right),
\end{aligned}
$$

subject to (7), (12), and

$$0 \leq P_t I_t^j \leq u_t^j R_t K_t^j + \eta_t K_t^j - L_t^j + \frac{L_{t+1}^j}{R_{ft}}, \quad (13)$$

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[8]Miao and Wang (2011a) show that other types of credit constraints such as self-enforcing debt constraints can also generate bubbles.

[9]Using $\xi_t K_{t+1}^j$ as collateral does not change our key insight, but makes the analysis slightly more complicated (see Miao and Wang (2011a)).

[10]Miao and Wang (2011a) discuss other forms of credit constraints under which a bubble can exist. The key idea is that a bubble helps relax credit constraints.

where we have used equations (10) and (11). We conjecture that the value function takes the following form:

$$V_{t,\tau}(K_t^j, L_t^j, \varepsilon_t^j) = v_t(\varepsilon_t^j)K_t^j + b_{t,\tau}(\varepsilon_t^j) - v_{Lt}(\varepsilon_t^j)L_t^j, \qquad (14)$$

where $v_t(\varepsilon_t^j)$, $b_{t,\tau}(\varepsilon_t^j) \geq 0$, and $v_{Lt}(\varepsilon_t^j)$ depend only on idiosyncratic shock ε_t^j and aggregate state variables. The form in (14) is intuitive following Hayashi (1982). Since we assume competitive markets with constant-returns-to-scale technology, it is natural that firm value takes a linear functional form. However, in the presence of credit constraints (12), firm value may contain a speculative component, $b_{t,\tau}(\varepsilon_t^j)$. Either $b_{t,\tau}(\varepsilon_t^j) = 0$ or $b_{t,\tau}(\varepsilon_t^j) > 0$ can be an equilibrium solution depending on agents' beliefs (note that the preceding dynamic programming problem does not give a contraction mapping). As in Miao and Wang (2011a), we may interpret this component as a bubble.

Define the date-t ex-dividend stock price of the firm of age τ as

$$P_{t,\tau}^{s,j} = (1 - \delta_e)E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,\tau+1}(K_{t+1}^j, L_{t+1}^j).$$

Given the above conjectured form in (14), we have

$$P_{t,\tau}^{s,j} = Q_t K_{t+1}^j + B_{t,\tau} - \frac{1}{R_{ft}} L_{t+1}^j, \qquad (15)$$

where we define

$$Q_t = (1 - \delta_e)E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} v_{t+1}(\varepsilon_{t+1}^j), \ \ B_{t,\tau} = (1 - \delta_e)E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} b_{t+1,\tau+1}\left(\varepsilon_{t+1}^j\right). \qquad (16)$$

Note that Q_t and $B_{t,\tau}$ do not depend on idiosyncratic shocks because they are integrated out. We interpret Q_t and $B_{t,\tau}$ as the (shadow) price of installed capital (Tobin's marginal Q) and the average bubble of the firm, respectively. Note that marginal Q and the investment goods price P_t are different in our model due to financial frictions and idiosyncratic investment efficiency shocks. In addition, marginal Q is not equal to average Q in our model because of the existence of a bubble. Given the conjectured value function (15), the credit constraint (12) becomes

$$\frac{1}{R_{ft}} L_{t+1}^j \leq Q_t \xi_t K_t^j + B_{t,\tau}. \qquad (17)$$

We then have the following proposition:

Proposition 1 *(i) The optimal investment level I_t^j of firm j with a bubble satisfies*

$$P_t I_t^j = \begin{cases} u_t R_t K_t^j + \eta_t K_t^j + Q_t \xi_t K_t^j + B_{t,\tau} - L_t^j & \text{if } \varepsilon_t^j \geq \frac{P_t}{Q_t} \\ 0 & \text{otherwise} \end{cases}. \qquad (18)$$

10

(ii) Each firm chooses the same capacity utilization rate u_t satisfying

$$R_t(1 + G_t) = Q_t\delta'(u_t), \tag{19}$$

where

$$G_t = \int_{\varepsilon \geq P_t/Q_t} (Q_t/P_t\varepsilon - 1)d\Phi(\varepsilon). \tag{20}$$

(iii) The bubble, the price of installed capital, and the lending rate satisfy

$$B_{t,\tau} = \beta(1 - \delta_e)E_t\frac{\Lambda_{t+1}}{\Lambda_t}B_{t+1,\tau+1}(1 + G_{t+1}), \tag{21}$$

$$Q_t = \beta(1 - \delta_e)E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[u_{t+1}R_{t+1} + Q_{t+1}(1 - \delta_{t+1}) + (u_{t+1}R_{t+1} + \xi_{t+1}Q_{t+1} + \eta_{t+1})G_{t+1}\right], \tag{22}$$

$$\frac{1}{R_{ft}} = \beta(1 - \delta_e)E_t\frac{\Lambda_{t+1}}{\Lambda_t}(1 + G_{t+1}), \tag{23}$$

where $\delta_t = \delta(u_t)$.

The intuition behind the investment rule given in equation (18) is the following. The cost of one unit of investment is the purchasing price P_t. The associated benefit is the marginal Q multiplied by the investment efficiency ε_t^j. If the benefit exceeds the cost $Q_t\varepsilon_t^j \geq P_t$, the firm will invest. Otherwise, the firm makes zero investment. This investment rule implies that firm-level investment is lumpy, which is similar to the case with fixed adjustment costs. Equation (18) shows that the investment rate increases with cash flows R_t, marginal Q, Q_t, and the bubble, $B_{t,\tau}$.

Equation (17) shows that the existence of a bubble $B_{t,\tau}$ relaxes the credit constraint, and hence allows the firm to make more investment. Thus, the bubble term $B_{t,\tau}$ enters the investment rule in (18). In addition, the existence of a bubble in the aggregate economy affects the equilibrium Q_t and P_t and hence the investment threshold $\varepsilon_t^* \equiv P_t/Q_t$. This also implies that the bubble has an extensive margin effect by affecting the number of investing firms. We call this effect of the bubble the capital reallocation effect.

The bubble must satisfy the no-arbitrage condition given in (21). Having a bubble at time t costs $B_{t,\tau}$ dollars. The benefit consists of two components: (i) The bubble has the value $B_{t+1,\tau+1}$ at $t + 1$. (ii) The bubble can help the firm generate dividends $B_{t+1,\tau+1}G_{t+1}$. The intuition is that a dollar of the bubble increases the borrowing capacity by one dollar as revealed by (17). This allows the firm to make more investment, generating additional dividends $(\varepsilon Q_t/P_t - 1)$ for the efficiency shock $\varepsilon \geq P_t/Q_t$. The expected investment benefit is given by (20). Thus, $B_{t+1,\tau+1}(1 + G_{t+1})$ represents the sum of "dividends" and the reselling value of the bubble. Using the stochastic discount factor $\beta\Lambda_{t+1}/\Lambda_t$ and considering the possibility of firm death, equation (21) says that the cost of having the bubble is equal to the expected benefit.

Note that the bubble $B_{t,\tau}$ is non-predetermined. Clearly, $B_{t,\tau} = B_{t+1,\tau+1} = 0$ is a solution to (21). If no one believes in a bubble, then a bubble cannot exist. We shall show below an equilibrium

with bubble $B_{t,\tau} > 0$ exists. Both types of equilibria are self-fulfilling. Note that the transversality condition cannot rule out a bubble because of the additional benefit G_{t+1} generated by the bubble.

The right-hand side of equation (19) gives the tradeoff between the cost and the benefit of a unit increase in the capacity utilization rate for a unit of capital. A high utilization rate makes capital depreciate faster. But it can generate additional profits and also additional investment benefits.

Equation (22) is an asset pricing equation of marginal Q. The dividends from capital consist of the rental rate $u_{t+1}R_{t+1}$ in efficiency units and the investment benefit $(u_{t+1}R_{t+1}+\xi_{t+1}Q_{t+1})G_{t+1}$ of an additional unit increase in capital. The reselling value of undepreciated capital is $Q_{t+1}(1 - \delta_{t+1})$.

Equation (23) is an asset pricing equation for the interest rate. For firms that decide not to invest and save (buying the bonds issued by other firms). For every one dollar saved today, the firm will earn R_{ft} in the next period. The firm may receive a favorable investment shock in the next period and invest R_{ft} to generate additional dividends $(\varepsilon Q_{t+1}/P_{t+1} - 1)$ in the next period. Hence the total return on saving will be $R_{ft}(1 + G_{t+1})$.

2.4.   Sentiment Shock

To model households' beliefs about the movements of the bubble, we introduce a sentiment shock. Suppose that households believe that the new firm in period t may contain a bubble of size $B_{t,0} = b_t^* > 0$ with probability ω. Then the total new bubble is given by $\omega\delta_e b_t^*$.

Suppose that households believe that the relative size of the bubbles at date $t + \tau$ for any two firms born at date t and $t + 1$ is given by θ_t, i.e.,

$$\frac{B_{t+\tau,\tau}}{B_{t+\tau,\tau-1}} = \theta_t, \quad t \geq 0, \ \tau \geq 1, \tag{24}$$

where θ_t follows an exogenously given process:

$$\ln \theta_t = (1 - \rho_\theta)\bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta,t}, \tag{25}$$

where $\bar{\theta}$ is the mean, $\rho_\theta \in (-1, 1)$ is the persistence parameter, and $\varepsilon_{\theta,t}$ is an IID normal random variable with mean zero and variance σ_θ^2. We interpret this process as a *sentiment shock*, which reflects household beliefs about the fluctuations in bubbles.[11] These beliefs may change randomly over time. It follows from (24) that

$$B_{t,0} = b_t^*, \ B_{t,1} = \theta_{t-1}b_t^*, \ B_{t,2} = \theta_{t-1}\theta_{t-2}b_t^*, ..., \ t \geq 0. \tag{26}$$

This equation implies that the sizes of new bubbles and old bubbles are linked by the sentiment shock. The sentiment shock changes the relative sizes. Note that the growth rate $B_{t+1,\tau+1}/B_{t,\tau}$ of

---

[11]In a different formulation available upon request, we may interpret θ_t as the probability that the bubble survives in the next period. This formulation is isomorphic to the present model. In particular, m_t in equation (32) can be interpreted as the mass of firms having bubbles. Equation (34) is the asset pricing equation for the bubble B_t^a/m_t. The advantage of the present setup is that we allow θ_t to be greater than 1.

the bubble in the same firm born at any given date $t - \tau$ must satisfy the equilibrium restriction derived in equation (21).

2.5. Capital Producers

Capital goods producers create new investment goods using input of final output subject to adjustment costs, as in Gertler and Kiyotaki (2011). They sell new investment goods to firms with investing opportunities at the price P_t. The objective function of a capital producer is to choose $\{I_t\}$ to solve:

$$
\max_{\{I_t\}} \ E \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left\{ P_t I_t - \left[1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 \right] \frac{I_t}{Z_t} \right\},
$$

where $\bar{\lambda}_I$ is the steady-state growth rate of aggregate investment, $\Omega > 0$ is the adjustment cost parameter, and Z_t represents an IST shock as in Greenwood, Hercowitz and Krusell (1997). The growth rate $\bar{\lambda}_I$ will be determined in Section 3. Following Justiniano, Primiceri, and Tambalotti (2011), we assume that $Z_t = Z_{t-1}\lambda_{zt}$, where $\ln \lambda_{zt}$ follows an AR(1) process. The optimal level of investment goods satisfies the first-order condition:

$$
\begin{aligned}
Z_t P_t \ &= \ 1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 + \Omega \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right) \frac{I_t}{I_{t-1}} \\
&\quad - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \Omega \left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I \right) \frac{Z_t}{Z_{t+1}} \left(\frac{I_{t+1}}{I_t} \right)^2.
\end{aligned}
\tag{27}
$$

2.6. Aggregation and Equilibrium

Let $K_t = \int K_t^j dj$ denote the aggregate capital stock of all firms in the end of period $t - 1$ before the realization of the death shock. Let X_t denote the aggregate capital stock after the realization of the death shock, but before new investment and depreciation take place. Then

$$
X_t = (1 - \delta_e) K_t + \delta_e K_{0t},
\tag{28}
$$

where we have included the capital stock brought by new entrants.

Define aggregate output and aggregate labor as $Y_t = \int_0^1 Y_t^j dj$ and $N_t = \int_0^1 Y_t^j dj$. By Proposition 1, all firms choose the same capacity utilization rate. Thus, all firms have the same capital-labor ratio. By the linear homogeneity property of the production function, we can then show that

$$
Y_t = (u_t X_t)^\alpha (A_t N_t)^{1-\alpha}.
\tag{29}
$$

As a result, the wage rate is given by

$$
W_t = \frac{(1 - \alpha) Y_t}{N_t},
\tag{30}
$$

Let B_t^a denote the total bubble in period t. Adding up the bubble of the firms of all ages and using (26) yield:

$$B_t^a = \sum_{\tau=0}^{t} (1 - \delta_e)^\tau \delta_e \omega B_{t,\tau} \equiv m_t b_t^*,$$

(31)

where m_t satisfies the recursion,

$$m_t = m_{t-1}(1 - \delta_e)\theta_{t-1} + \delta_e \omega, \ m_0 = \delta_e \omega.$$

(32)

The process $\{m_t\}$ is stationary in the neighborhood of the steady state as long as $(1 - \delta_e)\bar{\theta} < 1$.

By equations (26) and (21),

$$b_t^* = \beta(1 - \delta_e)\theta_t E_t \frac{\Lambda_{t+1}}{\Lambda_t} b_{t+1}^* (1 + G_{t+1}).$$

(33)

This equation gives an equilibrium restriction on the size of the new bubble. Substituting (31) into the above equation yields:

$$B_t^a = \beta(1 - \delta_e)\theta_t E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{m_t}{m_{t+1}} B_{t+1}^a (1 + G_{t+1}).$$

(34)

This equation gives an equilibrium restriction on the value of the total bubble in the economy. The above two equations prevent any arbitrage opportunities for old and new bubbles. Equations (32) and (34) reveal that a sentiment shock affects the relative size m_t and hence the aggregate bubble B_t^a.

Aggregating all firm value in (15), we obtain the aggregate stock market value of the firm:

$$P_t^s = Q_t K_{t+1} + B_t^a.$$

This equation reveals that the aggregate stock price consists of two components: the fundamental $Q_t K_{t+1}$ and the bubble B_t^a.

Competitive financial intermediaries implies that the deposit rate is equal to the lending rate so that $R_{dt} = R_{ft}(1 - \delta_e)$, where we have taken into account that firms die with probability δ_e. It follows from (23) and $G_{t+1} > 0$ that

$$\frac{1}{R_{dt}} = \frac{1}{(1 - \delta_e)R_{ft}} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 + G_{t+1}) > \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}.$$

(35)

Thus, households prefer to borrow until their borrowing constraints bind, i.e., $d_{t+1} = 0$. Without borrowing constraints, no arbitrage implies that $G_{t+1} = 0$. Equation (21) and the transversality condition will rule out bubbles.

By the market-clearing conditions for bank loans, $L_t = \int L_t^j dj = d_t = 0$ for all $t \geq 0$. This means that firms with high investment efficiency shocks borrow and invest, while all other firms

save and lend.

Let $I_t = \int I_t^j dj$ denote aggregate investment. Using Proposition 1 and adding up (18) for firms of all ages, we can use a law of large numbers to drive aggregate investment as

$$
\begin{aligned}
P_t I_t &= [(u_t R_t + \xi_t Q_t + \eta_t)X_t + B_t^a - L_t] \int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon) \\
&= [(u_t R_t + \xi_t Q_t + \eta_t)X_t + B_t^a] \int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon),
\end{aligned}
\tag{36}
$$

where in the second line we have used the fact that $L_t = 0$. Similarly, the aggregate capital stock evolves according to

$$
\begin{aligned}
K_{t+1} &= (1 - \delta_t)X_t + \int I_t^j \varepsilon_t^j dj \\
&= (1 - \delta_t)X_t + I_t \frac{\int_{\varepsilon > \frac{P_t}{Q_t}} \varepsilon d\Phi(\varepsilon)}{\int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon)},
\end{aligned}
\tag{37}
$$

where we have used a law of large numbers and the fact that I_t^j and ε_t^j are independent by Proposition 1.

The total capacity of external financing is given by

$$
\underbrace{\eta_t K_t}_{\text{new equity}} + \underbrace{\xi_t Q_t K_t + B_t^a}_{\text{debt}},
\tag{38}
$$

where we have used equations (11) and (17) to conduct aggregation. Then the fluctuation in this capacity reflects the overall financial market conditions. We can use a single shock defined as

$$
\zeta_t \equiv \eta_t / Q_t + \xi_t,
\tag{39}
$$

to capture the disturbance to the degree of the overall financial constraints and rewrite the total capacity of external financing as $\zeta_t Q_t K_t + B_t^a$. Assume that $\ln \zeta_t$ follows an AR(1) process. Using (39), equations (22) and (36) become

$$
Q_t = \beta(1 - \delta_e)E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[u_{t+1}R_{t+1} + Q_{t+1}(1 - \delta_{t+1}) + (u_{t+1}R_{t+1} + \zeta_{t+1}Q_{t+1})G_{t+1} \right],
\tag{40}
$$

$$
P_t I_t = [(u_t R_t + \zeta_t Q_t)X_t + B_t^a] \int_{\varepsilon > \frac{P_t}{Q_t}} d\Phi(\varepsilon).
\tag{41}
$$

In Section 4, we shall estimate the shock ζ_t instead of its two components η_t and ξ_t.

The resource constraint is given by

$$C_t + \left[1 + \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2\right]\frac{I_t}{Z_t} = Y_t. \tag{42}$$

A competitive equilibrium consists of stochastic processes of 15 aggregate endogenous variables, $\{C_t, I_t, Y_t, N_t, K_t, u_t, Q_t, X_t, W_t, R_t, P_t, m_t, B_t^a, R_{ft}, \Lambda_t\}$ such that 15 equations (42), (41), (29), (3), (37), (19), (40), (28), (30), (9), (27), (32), (34), (23) and (4) hold, where G_t satisfies (20) and $\delta_t = \delta(u_t)$.

There may exist two types of equilibrium: bubbly equilibrium in which $B_t^a > 0$ for all t and bubbleless equilibrium in which $B_t^a = 0$ for all t. A bubbly equilibrium can be supported by the belief that a new firm may bring a new bubble with a positive probability $\omega > 0$. A sentiment shock θ_t can generate fluctuations in the aggregate bubble B_t^a because households believe that the size of the old bubble relative to that of the new bubble fluctuates randomly over time. A bubbleless equilibrium can be supported by the belief that either old or new firms do not contain any bubble ($\omega = \theta_t = m_t = 0$). In the next section, we characterize the steady-state existence conditions for these two types of equilibria.

3. Bayesian Estimation

Since the model has two unit roots, one in the investment-specific technology shock and the other in the TFP shock, we have to appropriately transform the equilibrium system into a stationary one. In Appendix B, we present the transformed equilibrium system and in Appendix C we show that the transformed equilibrium system has a nonstochastic steady state in which all the above transformed variables are constant over time. We solve the transformed system numerically by log-linearizing around the nonstochastic steady state. We seek saddle-path stable solutions. We shall focus on the bubbly equilibrium as our benchmark.

3.1. Shocks and Data

We use Bayesian methods to fit the log-linearized model to the U.S. data. Our model has six orthogonal shocks: persistent and transitory TFP shocks (λ_{at}, A_t^m), the investment-specific technology shock Z_t, the labor supply shock ψ_t, the financial shock ζ_t, and the sentiment shock θ_t. We need six data series to identify these shocks. We choose the following five quarterly U.S. time series data: the relative price of investment (P_t), real per capita consumption (C_t), real per capita investment in consumption units (I_t/Z_t), per capita hours (N_t), and real per capita stock price index (defined as $P_t^s = Q_t K_{t+1} + B_t^a$ in the model). The first four series are taken from LWZ (2013), and the stock price data is the S&P composite index downloaded from Robert Shiller's website. We normalize it by the price index for non-durable goods and population. The sample period covers the first quarter of 1975 through the fourth quarter of 2010. More details about the data construction can

be found in Appendix A in LWZ (2013).

The sixth data series is the Chicago Fed's National Financial Conditions Index (NFCI), which is used to identify the financial shock ζ_t. In Section 4.2 we will show that without including the NFCI data, the estimation would produce a counter-intuitive smoothed financial shock series. The NFCI is a comprehensive index on U.S. financial conditions in money markets, debt and equity markets, as well as the traditional and shadow banking systems. The NFCI is normalized to have mean zero and standard deviation of one over a sample period extending back to 1973. A positive (negative) number means tight (loose) financial conditions. The data extends back to 1973 and is available at quarterly frequency.[12] We have also tried several subindices of NFCI (other variation of the NFCI index) and the results are similar.

Besides the standard measurement equations, we include the following measurement equation:

$$NFCI_t = -f_1\hat{\zeta}_t - f_2\hat{Q}_t - f_3\left(\hat{B}_t^a - \hat{K}_t\right), \tag{43}$$

where $f_1 > 0$, $f_2 > 0$, $f_3 > 0$, $\hat{\zeta}_t$ denotes log-deviation from the steady state, and $\hat{Q}_t$, $\hat{B}_t^a$, and $\hat{K}_t$ denote the log-deviations from the steady state for the corresponding detrended variables. This equation is motivated from (38) and (39). The total capacity of external financing is $\zeta_t Q_t K_t + B_t^a$ and its fluctuation depends on financial market conditions, represented by the NFCI. The preceding measurement equation relates the NFCI to the log-linearized expression of finacing capacity normalized by capital K_t. The intuition is that an increase in either one of $\hat{\zeta}_t$, $\hat{Q}_t$, or $\hat{B}_t^a - \hat{K}_t$ will reduce the NFCI and hence reduce the tightness in the overall financial market as revealed in equation (38).

In principle, one could use the credit market data such as total debt to identify the credit shock ξ_t and use the equity market data such as aggregate new equity issuance to identify the equity issuance shock η_t. We have not followed this approach because aggregate debt is zero in our model, but firms can borrow and save among themselves. Our model is consistent with the empirical evidence documented by Chari, Christiano, and Kehoe (2008) and Ohanian (2010). They find that the corporate sector typically has substantial cash reserves and thus can be largely self-financing. In addition, our modeling of using one shock to describe the financial market conditions is parsimonious. Our purpose is not to identify all shocks that drive the financial market conditions, but to study how the sentiment shock and a single reduced-form financial shock to the financial market conditions affect the real economy.

3.2. Parameter Estimates

As in Section 3, we focus on the steady state for the stationary equilibrium in which the capacity utilization rate and the investment goods price are both equal to 1. Due to the log-linearization

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[12]See Brave and Butter (2011) for a detailed description on the construction of the NFCI index at http://qa.chicagofed.org/digital_assets/publications/economic_perspectives/2011/1qtr2011_part2_brave_butters.pdf

solution method, we do not need to parameterize the depreciation function $\delta\left(\cdot\right)$ and the distribution function $\Phi\left(\cdot\right)$. As shown in Appendices C and D, we only need to know the steady-state values of $\delta\left(1\right)$, $\delta'\left(1\right)$, $\delta''\left(1\right)$, $\Phi\left(\varepsilon^*\right)$, and $\mu \equiv \frac{\phi(\varepsilon^*)\varepsilon^*}{1-\Phi(\varepsilon^*)}$, where ε^* is the steady-state investment threshold for the shock ε_t. We treat these values as parameters to be either estimated or calibrated.

We partition the model parameters into three subsets. The first subset of parameters includes the structural parameters, which are calibrated using the steady-state relations. This set of parameters is collected in $\Psi_1 = \{\beta,\, \alpha,\, \delta\left(1\right),\, \delta'\left(1\right),\, \delta_e,\, \bar{\psi},\, \Phi\left(\varepsilon^*\right),\, g_\gamma,\, \bar{\lambda}_z,\, K_0/\tilde{K},\, \bar{\theta},\, \omega\}$, where $\bar{\psi}$ is the mean labor supply shock, g_γ is the steady-state gross growth rate of output, $\bar{\lambda}_z$ is the steady-state gross growth rate of IST, K_0 is the detrended capital stock endowed by the new entrants, and $\tilde{K}$ is the detrended steady-state aggregate capital stock. Note that the parameter ω does not affect the steady-state bubble-output ratio by Proposition 2 in Appendix C. In addition, as Appendix D shows, it does not affect the log-linearized dynamic system. Thus, it can take any positive value, say, $\omega = 0.5$.

As is standard in the literature, we fix the discount factor β at 0.99, the capital share parameter α at 0.3, and the steady-state depreciation rate $\delta\left(1\right)$ at 0.025. Using (C.20), we can pin down $\delta'\left(1\right)$ to ensure that the steady-state capacity utilization rate is equal to one. We choose $\bar{\psi}$ such that the steady-state average hours are 0.25 as in the data. Using data from the U.S. Bureau of the Census, we compute the exit rate as the ratio of the number of closed original establishments with non-zero employment to the number of total establishments with non-zero employment. The average annual exit rate from 1990 to 2007 is 7.8 percent, implying about 2 percent of quarterly exit rate. Thus, we set the exit rate δ_e at 0.02.[13] This number is consistent with the literature. For instance, Bilbiie, Ghironi and Melitz (2012) set the quarterly firm exit rate to be 0.025, Bernard, Redding and Schott (2010) find an quarterly 2.2 percent minimum production destruction rate. Using (C.27) in the appendix, we can pin down $\Phi\left(\varepsilon^*\right)$ by targeting the steady-state investment-output ratio $(\tilde{I}/\tilde{Y})$ at 0.20 as in the data, given that we know the other parameter values. We set the growth rate of per capita output $g_\gamma = 1.0042$ and the growth rate of the investment-specific technology $\bar{\lambda}_z = 1.0121$ as in the data reported by LWZ (2013). Using equation (B.6), we can then pin down the average growth rate of TFP, $\bar{\lambda}_a$. Dunne, Roberts and Samuelson (1988) document that the average relative size of entrants to all firms in the period 1972-1982 is about 0.20. We thus set the ratio of the initial capital stock of new entry firms to the average capital stock $K_0/\tilde{K}$ to 0.20. By (23) and (34), the growth rate of bubbles of the surviving firms in the steady state is given by $\bar{\theta} = R_f/g_\gamma$. We use this equation to pin down $\bar{\theta}$, the calibrated value is 0.9975.[14] In summary, Table 1 presents the values assigned to the calibrated parameters in Ψ_1.

The second subset of parameters $\Psi_2 = \left\{h,\, \Omega,\, \delta''/\delta'\left(1\right),\, \bar{\zeta},\, \mu\right\}$ includes the habit formation parameter h, the investment-adjustment cost parameter Ω, the capacity utilization parameter $\delta''/\delta'\left(1\right)$, the mean value of the financial shock $\bar{\zeta}$, and the elasticity of the probability of undertaking

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[13]Our results are not sensitive to this number.

[14]In particular, we use the 3-month treasury bill rates from 1975Q2-2010Q4 adjusted by the expected inflation rate (from the University of Michigan's survey of consumer) and take the average to obtain the steady state R_f of 1.0017.

Table 1. Calibrated Parameters

Parameter	Value	Description
β	0.99	Subjective discounting factor
α	0.3	Capital share in production
$\delta(1)$	0.025	Steady-state depreciation rate
δ_e	0.020	Exit rate
N	0.25	Steady-state hours
g_γ	1.0042	Steady-state gross growth rate of output
$\bar{\lambda}_z$	1.0121	Steady-state gross growth rate of investment-specific technology
u	1	Steady-state capacity utilization rate
$\tilde{I}/\tilde{Y}$	0.2	Steady-state investment-output ratio
$K_0/\tilde{K}$	0.20	Ratio of capital endowment for an entrant to total capital stock
$\bar{\theta}$	0.9975	Relative size of the old bubble to the new bubble
ω	0.5	Fraction of entrants with bubbles

investment at the steady-state cut-off $\mu \equiv \frac{\phi(\varepsilon^*)\varepsilon^*}{1-\Phi(\varepsilon^*)}$. These parameter values are estimated by the Bayesian method.

Following LWZ (2013), we assume that the prior of h follows the beta distribution with mean 0.3333 and standard deviation 0.235. This prior implies that the two shape parameters in the Beta distribution are given by 1 and 2. The prior density declines linearly as h increases from 0 to 1. The 90 percent interval of this prior density covers most calibrated values for the habit formation parameter used in the literature (e.g., Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum and Evans (2005)).

Following LWZ (2013), we assume that the prior for Ω follows the gamma distribution with mean 2 and standard deviation 2. The 90 percent interval of this prior ranges from 0.1 to 6, which covers most values used in the DSGE literature (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), Liu, Waggoner, and Zha (2012), and LWZ (2013)).

For $\delta''/\delta'(1)$, we assume that the prior follows the gamma distribution with mean 1 and standard deviation 1. The 90 percent interval of this prior covers the range from 0.05 to 3, which covers most calibrated values for $\delta''/\delta'(1)$ (e.g., Jaimovich and Rebelo (2009)).

For $\bar{\zeta}$, we assume that the prior follows the beta distribution with mean 0.3 and standard deviation 0.1. The 95 percent interval of this prior density ranges roughly from 0.1 to 0.5. Covas and den Hann (2011) document that $\bar{\zeta}$ ranges from 0.1 to 0.4 for various sizes of firms. Our prior covers their empirical estimates. We find that our estimate of $\bar{\zeta}$ is quite robust and not sensitive to the prior distribution.

For μ, we assume that the prior follows the gamma distribution with mean 2 and standard deviation 2. The 90 percent interval of this prior ranges from 0.1 to 6, which is wide enough to cover low to high elasticity used in the literature. For example, if we assume that ε follows the Pareto distribution $1 - \varepsilon^{-\varsigma}$, then $\mu = \varsigma$. Wang and Wen (2012) estimate that ς is equal to 2.4,

Table 2. Prior and Posterior Distributions

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	St.Dev.	Mode	Mean	5%	95%
h	Beta	0.333	0.235	0.5365	0.5443	0.4864	0.6046
Ω	Gamma	2	2	0.0297	0.0337	0.0110	0.0557
δ''/δ'	Gamma	1	1	11.7869	11.9229	8.2706	15.1467
$\bar{\zeta}$	Beta	0.3	0.1	0.2975	0.2981	0.2337	0.3566
μ	Gamma	2	2	2.5336	2.5844	2.1143	3.0557
f_1	Gamma	1	1	0.0521	0.0444	0.0125	0.0718
f_2	Gamma	1	1	4.7264	4.7855	2.7891	6.6858
f_3	Gamma	1	1	0.4068	0.3291	0.0101	0.5749
ρ_a	Beta	0.5	0.2	0.9695	0.9643	0.9431	0.9868
ρ_{a^m}	Beta	0.5	0.2	0.9629	0.9635	0.9471	0.9808
ρ_z	Beta	0.5	0.2	0.3595	0.3427	0.2233	0.4609
ρ_θ	Beta	0.5	0.2	0.9285	0.9242	0.8990	0.9495
ρ_ψ	Beta	0.5	0.2	0.9879	0.9802	0.9650	0.9968
ρ_ζ	Beta	0.5	0.2	0.8770	0.8735	0.8104	0.9390
σ_a	Inv-Gamma	0.01	Inf	0.0022	0.0024	0.0018	0.0029
σ_{a^m}	Inv-Gamma	0.01	Inf	0.0101	0.0105	0.0093	0.0116
σ_z	Inv-Gamma	0.01	Inf	0.0059	0.0060	0.0053	0.0066
σ_θ	Inv-Gamma	0.1	Inf	0.1839	0.1925	0.1226	0.2603
σ_ψ	Inv-Gamma	0.01	Inf	0.0080	0.0082	0.0072	0.0092
σ_ζ	Inv-Gamma	0.01	Inf	0.0077	0.0083	0.0048	0.0118

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

which lies in our range.

For the coefficients in the measurement equation of the financial condition index, f_1, f_2, f_3, we assume that the priors follow the gamma distribution with mean 1 and standard deviation 1. The 90 percent interval of this prior covers fairly large range from 0.05 to 3. We find that our estimates of these parameters are quite robust and not sensitive to the prior distribution.

The third subset of parameters is $\Psi_3 = \{\rho_i, \sigma_i\}$ for $i \in \{a, z, a^m, \theta, \zeta, \psi\}$, where ρ_i and σ_i denote the persistence parameters and the standard deviations of the six structural shocks, respectively. Following Smets and Wouters (2007) and LWZ (2013), we assume that ρ_i follows a beta distribution with mean 0.5 and standard deviation 0.2. The prior for σ_i follows inverse gamma distribution with mean 0.01 and standard deviation ∞, except for σ_θ. For the sentiment shock θ_t, we assume that the prior mean of σ_θ is equal to 0.1. The choice of this high prior volatility is based on the fact that the stock price is the main data used to identify the sentiment shock. Since we know that the stock market is very volatile, it is natural to specify a large prior volatility for the sentiment shock. As a robustness check, we also consider the prior mean 0.01 of σ_θ and find similar results (see Appendix E).

Table 2 presents the prior distributions of the parameters in groups two Ψ_2 and three Ψ_3. It also presents the modes, means, and 5th and 95th percentiles of the posterior distributions for

those parameters obtained using the Metropolis-Hastings algorithm with 200,000 draws.[15] In later analysis, we choose the posterior modes as the parameter values for all simulations.

Table 2 reveals that our estimates of most parameters are consistent with those in the literature (e.g., LWZ (2013)). We shall highlight some of the estimates. First, the sentiment shock is highly persistent and volatile. The posterior mode and mean of the AR(1) coefficient are equal to 0.9285 and 0.9242, respectively. The posterior mode and mean of the standard error are equal to 0.1839 and 0.1925, respectively. Second, our estimated investment adjustment cost parameter is small. The posterior mode and mean of this parameter are equal to 0.0297 and 0.0337, respectively. This result is important because a large adjustment cost parameter is needed for most DSGE models in the literature to explain the variations in stock market prices or returns. But a large value is inconsistent with micro-level evidence (Cooper and Haltiwanger (2006)). For example, the estimate in Christiano, Motto, and Rostagno (2009) is 29.22. The intuition is that a large investment adjustment cost parameter makes Tobin's marginal Q very volatile, which helps explain the volatility of the stock market value. By contrast, in our model the aggregate stock market value contains a separate bubble component. The movement of the stock market value is largely determined by the bubble component which is driven largely by the sentiment shock. According to our estimated parameter values, the bubble component accounts for about 14 percent of the stock market value in the steady state. We will show below that this small component plays a dominant role in explaining fluctuations in the stock market as well as macroeconomic quantities.

3.3. Model Fit

To evaluate our model performance, we present in Table 3 the baseline model's predictions regarding standard deviations, correlations with output, and serial correlations of output, consumption, investment, hours, and stock prices. This table also presents results for four re-estimated comparison models that will be discussed later. The model moments are computed using the simulated data from the estimated model when all shocks are turned on. We take the posterior modes as parameter values. Both simulated and actual data are in logs and HP filtered.

From Table 3, we observe that the estimated model fits the empirical moments from the actual data quite well. We highlight two results. First, our model matches closely the stock market volatility in the data (0.1088 versus 0.1082). This result is remarkable because most neoclassical models in finance or macroeconomics have difficulty in explaining the stock market volatility (Shiller (1981)). Second, our model matches the persistence of macroeconomic quantities and stock prices as well as their comovements. As is well known, many real business cycle models have difficulty in generating the persistence of output because they lack an endogenous amplification and propagation mechanism. Our estimated model with bubbles identifies a new shock, the sentiment shock, and provides a powerful amplification and propagation mechanism for this shock.

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[15]We have checked that our estimates pass Iskrev's (2010) test of identification.

Table 3. Business Cycles Statistics

	Y	C	I	N	SP	P
		Standard Deviations				
U.S. Data	0.0170	0.0093	0.0419	0.0179	0.1082	0.0111
Baseline Model	0.0184	0.0146	0.0429	0.0130	0.1058	0.0106
No Stock Price	0.0115	0.0095	0.0304	0.0111	0.0132	0.0122
No Sentiment	0.0150	0.0140	0.0332	0.0157	0.1020	0.0249
No Bubble	0.0177	0.0165	0.0410	0.0187	0.1028	0.0261
Extended	0.0246	0.0194	0.0535	0.0125	0.1220	0.0112
		Standard Deviations Relative to Y				
U.S. Data	1.0000	0.5452	2.4672	1.0539	6.3714	0.6540
Baseline Model	1.0000	0.7898	2.3236	0.7029	5.7366	0.5753
No Stock Price	1.0000	0.8252	2.6335	0.9570	1.1460	1.0568
No Sentiment	1.0000	0.9293	2.2072	1.0414	6.7769	1.6519
No Bubble	1.0000	0.9328	2.3152	1.0559	5.8066	1.4768
Extended	1.0000	0.7885	2.1738	0.5084	4.9564	0.4535
		First Order Autocorrelations				
U.S. Data	0.9003	0.9023	0.8671	0.9255	0.7652	0.8620
Baseline Model	0.8903	0.9258	0.7944	0.7834	0.7607	0.8474
No Stock Price	0.8303	0.8872	0.7348	0.7699	0.7219	0.8827
No Sentiment	0.9063	0.9142	0.8295	0.7361	0.7219	0.8112
No Bubble	0.9372	0.9417	0.8710	0.7817	0.7220	0.7548
Extended	0.9117	0.9400	0.8365	0.8368	0.7637	0.8574
		Correlation with Y				
U.S. Data	1.0000	0.9296	0.9705	0.8208	0.4217	-0.1287
Baseline Model	1.0000	0.9364	0.8788	0.6133	0.3943	-0.0707
No Stock Price	1.0000	0.8774	0.7990	0.6759	0.4546	-0.0779
No Sentiment	1.0000	0.8514	0.7364	0.5554	0.0596	-0.1443
No Bubble	1.0000	0.8995	0.7101	0.5237	0.0784	0.0703
Extended	1.0000	0.9582	0.9098	0.6407	0.4978	-0.0797

Note: The model moments are computed using the simulated data (20,000 periods) from the estimated model at the posterior mode. All series are logged and detrended with the HP filter. The columns labeled Y, C, I, N, SP, and P refer, respectively, to output, consumption, investment, hours worked, the stock price, and the relative price of investment goods. "No Bubble" corresponds to the model without bubbles. "No Sentiment" corresponds to the baseline model without sentiment shocks. "No Stock Price" corresponds to the baseline model without using the stock price data in estimation. "Extended" corresponds to the model in Section 5.2.

4. Economic Implications

In this section, we discuss the model's empirical implications based on the estimated parameters. We address the following questions: How much does each shock contribute to the variations in the stock market, output, investment, consumption, and hours? What explains the stock market booms and busts? Does the stock market affect the real economy? We then use our model to shed light on two major bubble and crash episodes in the U.S. economy: (i) the internet bubble during the late 1990s and its subsequent crash, and (ii) the recent stock market bubble in tandem with the housing bubble and the subsequent Great Recession.

4.1. Relative Importance of the Shocks

Our estimated model helps us evaluate the relative importance of the shocks in driving fluctuations in the growth rates of stock prices and macroeconomic quantities. We do this through the variance decomposition. Table 4 reports this decomposition across the six structural shocks at the business cycle frequency.[16]

Table 4 shows that the sentiment shock accounts for about 98 percent of the stock market fluctuations. The contributions of the other shocks are negligible. The sentiment shock is transmitted from the stock market to the real economy through the credit constraints. A sentiment shock causes the fluctuations in the credit limit and hence affects a firm's investment decisions. This in turn affects aggregate investment and aggregate output. Table 4 reveals that the sentiment shock explains about 20 and 31 percent of the fluctuations in investment and output, respectively. The sentiment shock is the dominating force driving the fluctuations in consumption, accounting for about 32 percent of its variation. This is due to the large wealth effect caused by the fluctuations in the stock market value.

The two TFP shocks are important in explaining variations in macroeconomic quantities as in the RBC literature, but they barely affect the stock market fluctuations.

The labor supply shock accounts for most of the fluctuations in hours (about 72 percent). It also contributes to a sizable fraction of fluctuations in output, investment and consumption. This shock is a reduced-form shock capturing the labor wedge. A similar finding is reported in LWZ (2013) and Justiniano, Primiceri, and Tambalotti (2011).

The permanent IST shock does not explain much of the fluctuations in investment, output, consumption, and hours. This is because our model is designed to fit the data of the relative price of the investment goods and the IST shock is tied to the fluctuations in the relative price of investment goods. This result is consistent with the findings reported in Justiniano, Primiceri, and Tambalotti (2011), LWZ (2013), Christiano, Motto and Rostagno (2010), and Liu, Waggoner, and Zha (2012).

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[16]We compute variance decomposition using the spectrum of the linearized models and an inverse first difference filter for stock prices, output, consumption, investment to reconstruct the levels. The spectral density is computed from the state space representation of the model with 2000 bins for frequencies covering that range of periodicities.

Table 4. Variance Decomposition at Business Cycle Frequencies

	Sentiment	Financial	IST	Agrowth	Atrans	Labor	MeaErr
				Stock Price			
Baseline	98.45	0.00	0.22	0.53	0.51	0.29	–
No Stock Price	0.70	27.61	2.43	23.32	5.94	39.99	–
No Bubble	–	0.01	1.70	3.11	0.42	1.21	93.55
No Sentiment	–	0.00	1.14	4.49	0.54	0.90	92.93
Extended	73.37	11.26	0.04	1.78	6.44	7.09	–
				Output			
Baseline	30.56	0.11	0.93	18.09	31.17	19.13	–
No Stock Price	14.29	14.48	1.21	35.48	2.34	32.21	–
No Bubble	–	0.46	21.45	31.00	15.03	32.05	0.00
No Sentiment	–	0.09	22.49	12.97	31.22	33.23	0.00
Extended	17.22	2.45	1.18	53.59	13.04	12.53	–
				Investment			
Baseline	19.54	0.72	4.17	36.22	25.31	14.04	–
No Stock Price	23.94	60.36	0.40	6.75	0.92	7.63	–
No Bubble	–	1.85	53.78	20.68	6.82	16.88	0.00
No Sentiment	–	0.49	55.95	20.73	10.27	12.56	0.00
Extended	10.49	1.16	6.27	37.47	36.18	8.43	–
				Consumption			
Baseline	31.68	0.03	0.38	18.68	29.78	19.45	–
No Stock Price	3.97	0.64	1.45	49.10	2.58	42.26	–
No Bubble	–	0.02	6.11	58.10	11.98	23.79	0.00
No Sentiment	–	0.00	7.70	36.37	27.80	28.13	0.00
Extended	19.92	3.67	0.52	57.25	3.62	15.02	–
				Hours			
Baseline	4.20	0.41	2.10	19.00	2.67	71.62	–
No Stock Price	4.42	19.30	0.04	1.65	1.68	72.91	–
No Bubble	–	0.30	21.07	21.24	10.36	47.04	0.00
No Sentiment	–	0.09	20.70	15.79	12.62	50.81	0.00
Extended	2.50	0.77	5.42	10.08	15.03	66.19	–

Note: "No Bubble" corresponds to the model without bubbles. "No Sentiment" corresponds to the baseline model without sentiment shocks. "No Stock Price" corresponds to the baseline model without using the stock price data in estimation. "Extended" corresponds to the model in Section 5.2. "MeaErr" denotes the measurement error in the measurement equation for the stock prices.

Our estimated financial shock is highly persistent, but accounts for a negligible fraction of fluctuations in stock prices, investment, consumption, output, and hours. The intuition is that the sentiment shock is similar to the financial shock since both shocks affect the credit constraints. However, the sentiment shock displaces the financial shock once the stock price data is included in the estimation because only this shock can generate comovement between stock prices and macro quantities, as well as the excessive volatility of stock prices. Table 4 shows that when the stock price data is not included in the estimation, the re-estimated financial shock becomes much more important, explaining about 28, 14, 60 percent of the variations in stock prices, output, and investment, respectively. However, this re-estimated model cannot explain the stock market volatility. As Table 3 reveals, the stock price volatility generated by the re-estimated model is about 0.01, while the volatility in the data is about 0.11.

4.2. What Explains Stock Market Booms and Busts?

From the variance decomposition, we find that the sentiment shock is the most important driving force behind the fluctuation in the stock market.[17] Why are other shocks not important? To address this question, we derive the log-linearized detrended stock price as

$$\hat{P}_t^s = \frac{\tilde{K}\tilde{Q}}{\tilde{P}^s}(\hat{Q}_t + \hat{K}_{t+1}) + \frac{\tilde{B}^a}{\tilde{P}^s}\hat{B}_t^a, \tag{44}$$

where a variable with a tilde denotes its steady state detrended value and a variable with a hat denotes the relative deviation from the steady state. We also use equation (D.13) in the appendix to derive

$$\hat{B}_t^a = -\hat{\Lambda}_t + \left[1 - \beta(1 - \delta_e)\bar{\theta}\right]\varphi_G \sum_{j=1}^{\infty} E_t(\hat{P}_{t+j} - \hat{Q}_{t+j}) + \frac{1 - (1 - \delta_e)\bar{\theta}}{(1 - \delta_e)\bar{\theta}} \sum_{j=1}^{\infty} E_t\hat{m}_{t+j}. \tag{45}$$

In the preceding equation, φ_G is a negative number given in (D.7) in the appendix. Equation (44) shows that the variations in the stock price are determined by the variations in marginal Q, $\hat{Q}_t$, the capital stock, $\hat{K}_{t+1}$, and the bubble, $\hat{B}_t^a$. As is well known in the literature, the capital stock is a slow-moving variable and cannot generate large fluctuations in the stock price. The variation in marginal Q can be large if the capital adjustment cost parameter is large. But according to our estimation, this parameter is small and hence movements in marginal Q cannot generate large fluctuations in the stock price. Equation (45) reveals that the variation in the bubble is largely determined by the variation in the expected future relative size of the aggregate bubble to the new bubble, $\hat{m}_{t+j}$, because the variations in $\hat{\Lambda}_t$, $\hat{P}_{t+j}$ and $\hat{Q}_{t+j}$ are small. The variation in $\hat{m}_{t+j}$ is determined by the sentiment shock $\hat{\theta}_{t+j}$ as shown in equation (32). According to our estimation, the sentiment shock is the dominant driver of the stock market fluctuations, even though the bubble component accounts for a small share of the stock price ($\tilde{B}^a/\tilde{P}^s = 0.14$) in the deterministic steady

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[17]We use the Campbell-Shiller approximation (Campbell (1999)) to compute the stock return volatility and find that the sentiment shock explains more than 90 percent of the stock return volatility.

state.

Why are the other shocks not important drivers of the stock market fluctuations? We first note that the IST shock cannot be the primary driver when we allow the model to fit both the stock price data and the relative price of investment goods data. This is because the price of the investment goods is countercyclical, but the stock market value is procyclical. A positive IST shock can reduce the price of the investment goods, but it also reduces the marginal Q and hence the stock market value.

The labor supply shock cannot be the primary driver either. Since it affects the marginal utility of leisure directly, it is an important shock to explain the variation in hours. However, it cannot generate large movements in the stock price because its impact on the marginal Q is small.

We next turn to the two TFP shocks, which are considered to be the main driver of the fluctuations in real quantities in the RBC literature. Figure 2 shows that a permanent TFP shock cannot be an important driver of the stock market movements. A permanent TFP shock reduces marginal Q because it reduces the future marginal utility of consumption due to the wealth effect. Though it raises the bubble in the stock price, the net impact on the stock price is negative and small. As Figure 2 shows, the impulse responses of output are similar to those of the stock price. This implies that the volatility of the stock market would be counterfactually similar to that of output growth if the permanent TFP shock were the driving force.

As illustrated in Figure 2, although a positive transitory TFP shock raises both marginal Q and the bubble, its impact on the stock price is small, compared to that on consumption, investment, and output. Thus, it cannot explain the high relative volatility of the stock market.[18]

Recently, Jermann and Quadrini (2012) show that the financial shock is important for business cycles. LWZ (2013) find that the housing demand shock displaces the financial shock when the housing price data is included in estimation. Figure 2 shows that once the stock market data is incorporated, the role of the financial shock is significantly weakened. The intuition is that an increase in the financial shock causes the credit constraints to be relaxed, thereby raising investment. Since it does not affect output directly, consumption falls on impact. Thus, the financial shock cannot generate comovement between consumption and investment. As capital accumulation rises, marginal Q falls, causing the fundamental value of the stock market to fall. In addition, the bubble component also falls on impact because there is no room for a bubble as the credit constraints are already relaxed. As a result, the net impact of an increase in the financial shock is to reduce the stock price, implying that the financial shock cannot drive the stock market cyclicality.

Now consider the impact of a sentiment shock presented in Figure 3. A positive sentiment shock raises the size of the bubble, causing the credit constraints to be relaxed. Thus, firms make more investment. As capital accumulation rises, marginal Q falls so that the fundamental value of the stock market also falls. But this fall is dominated by the rise in the bubble component, causing

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[18]Note that both a permanent and a transitory TFP shocks can generate a fall in hours on impact. This is due to the presence of habit formation utility and investment adjustment costs (see, Fransis and Ramey (1998) and Smets and Wouters (2007)).
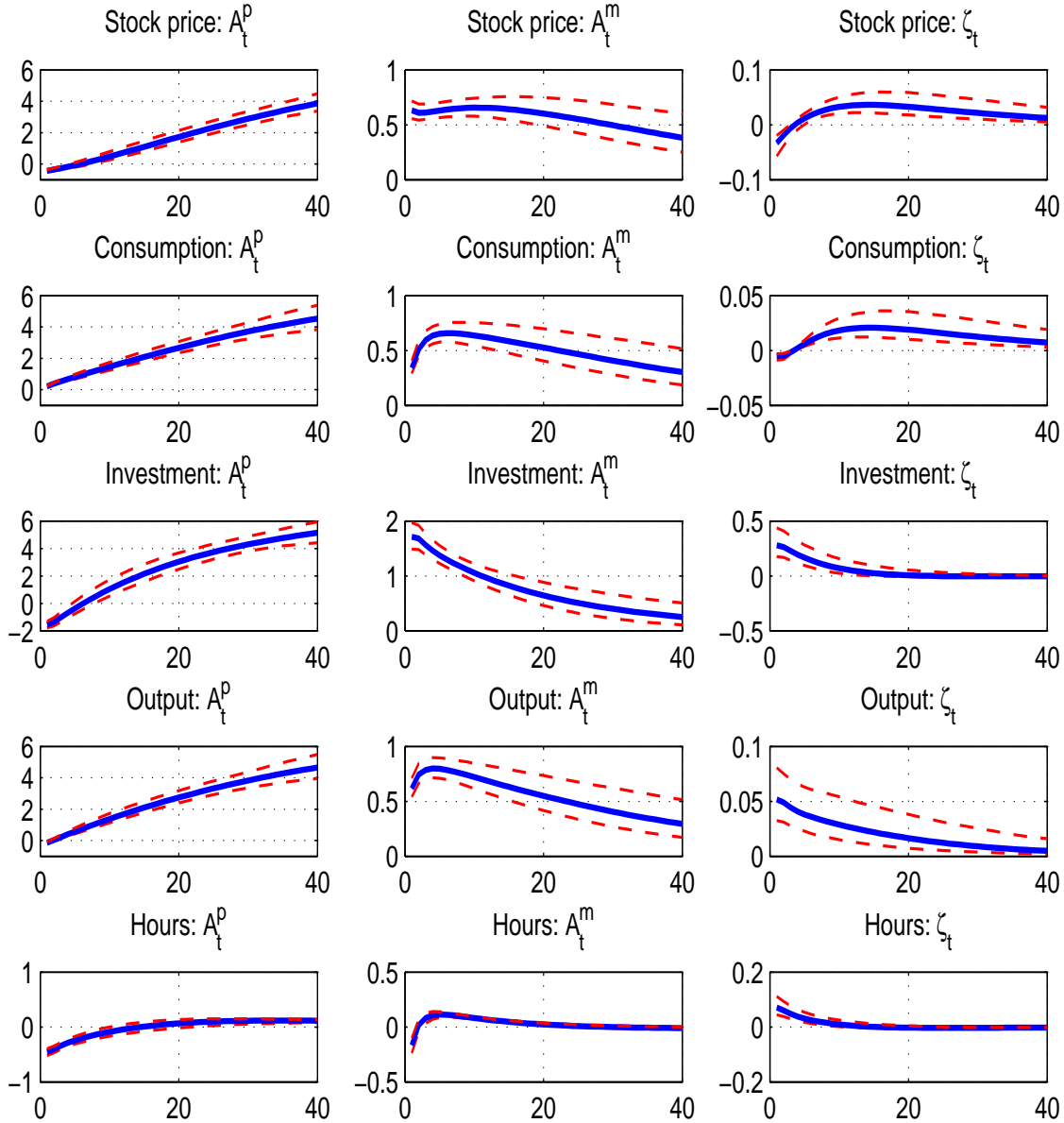
Figure 2: Impulse responses to a one-standard-deviation permanent TFP shock (A_t^p), transitory TFP shock (A_t^m), and financial shock (ζ_t) in the baseline model. All vertical axes are in percentage. We compute the responses for 20,000 draws from the posterior distributions. The solid line is the median value, the dashed lines indicate the 90 percent confidence interval.
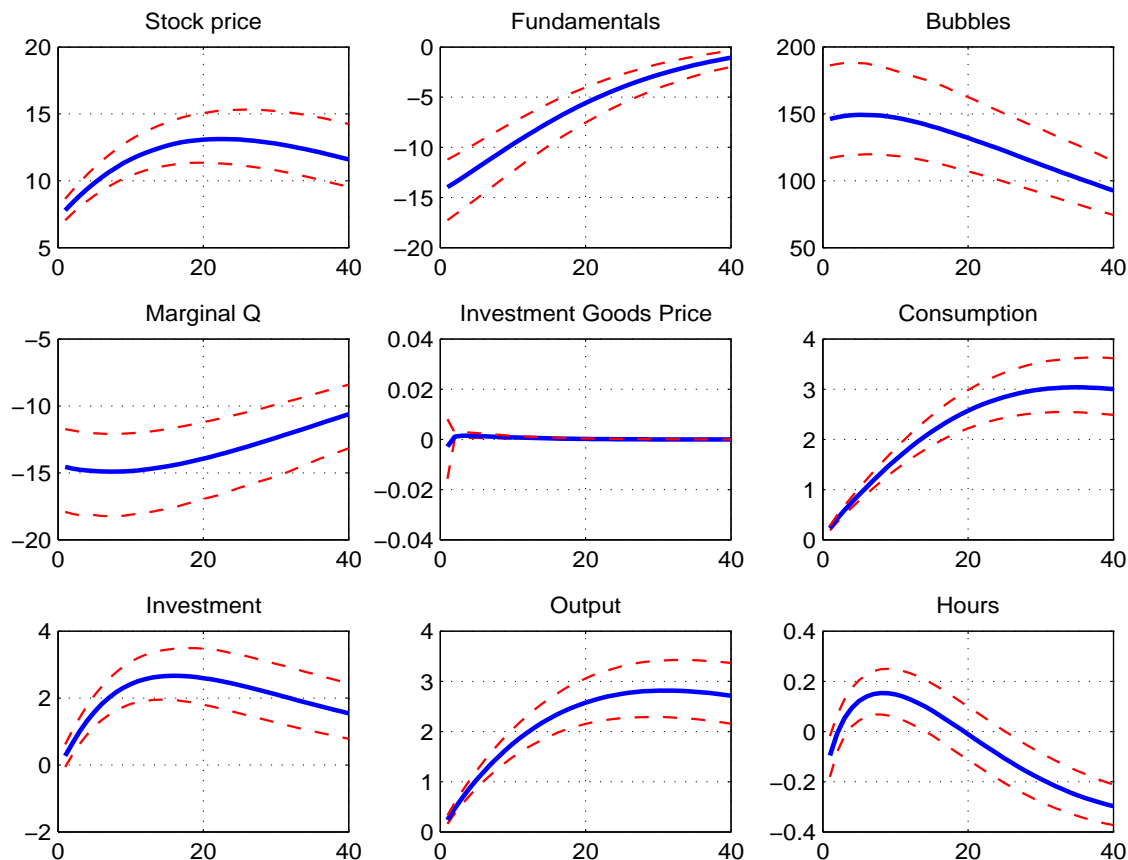
Figure 3: Impulse responses to a one-standard-deviation sentiment shock in the baseline model. All vertical axes are in percentage. We compute the responses for 20,000 draws from the posterior distributions. The solid line is the median value, the dashed lines indicate the 90 percent confidence interval.

the stock price to rise on impact. This in turn causes consumption to rise due to the wealth effect. The capacity utilization rate also rises due to the fall of marginal Q, causing the labor demand to rise. The rise in the labor demand is dominated by the fall in the labor supply due to the wealth effect, and hence labor hours fall on the impact period, but rises afterward. The increased capacity utilization raises output.

Notice that on impact the stock price rises by about 8 percent, which is much larger than the impact effects on output (0.2 percent), consumption (0.2 percent) and investment (0.3 percent). This result indicates that the sentiment shock can generate a large volatility of the stock market relative to that of consumption, investment, and output. The sentiment shock has a small impact on the price of investment goods. This allows the movements of the price of investment goods to be explained by the IST shock.

The top panel of Figure 4 presents the smoothed estimate of the sentiment shock $\hat{\theta}_t = \ln\left(\theta_t/\bar{\theta}\right)$. The middle panel plots the historical demeaned logged stock price growth and the fitted demeaned logged stock price growth from the model when all shocks are turned on and when only the sentiment
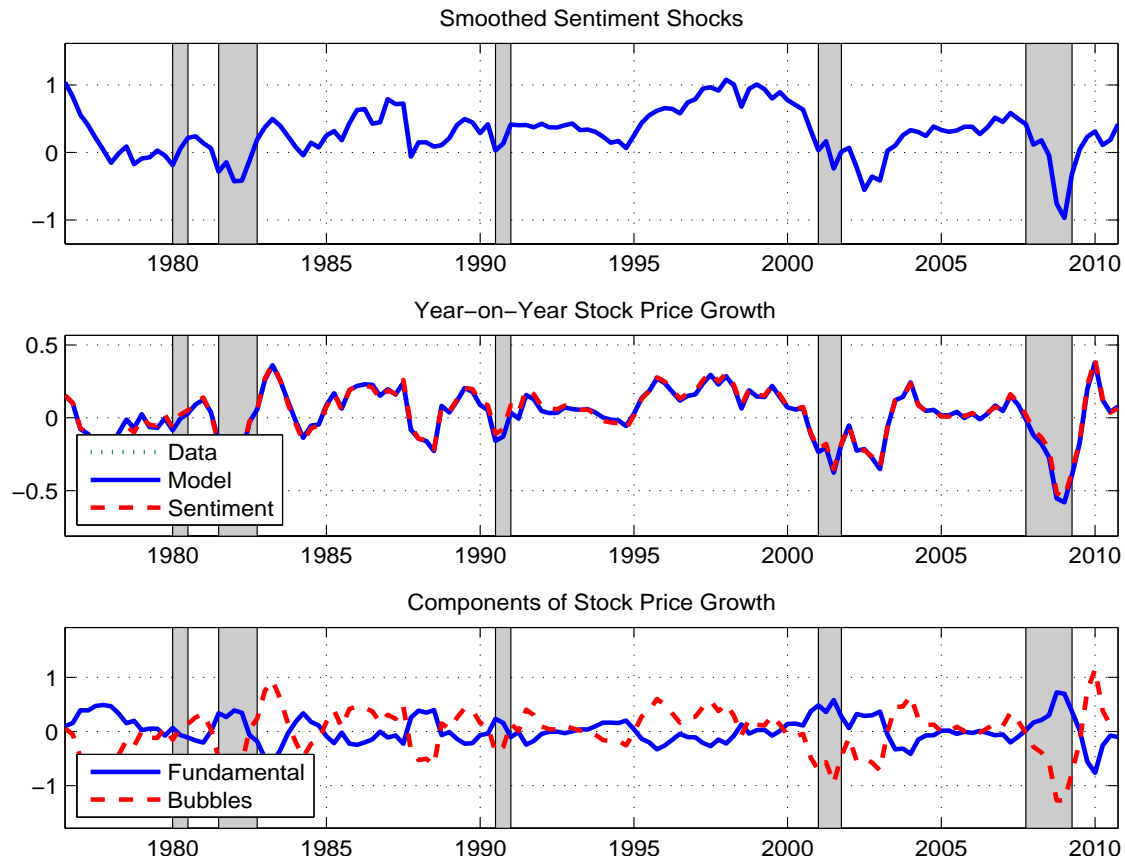
Figure 4: The top panel plots the smoothed sentiment shocks estimated from the baseline model. The middle panel plots the year-on-year growth data of the actual stock prices (labeled "Data") and the smoothed estimates of the stock prices based on all seven shocks (labeled "Model") and on the sentiment shock only (labeled "Sentiment"). The bottom panel plots the smoothed estimates of the bubble and the fundamental components of stock prices.

shock is turned on. We cannot find visual differences among the three lines, indicating that the sentiment shock drives almost all of the stock market fluctuations. Comparing these two panels reveals that the fluctuations in the sentiment shock and in the stock market follow an almost identical pattern. This implies that the boom of the stock market is fed by the optimistic sentiment of growing bubbles and the bust is fueled by the pessimistic sentiment of shrinking bubbles or the collapse of bubbles. The bottom panel of Figure 4 presents the two components of the stock price when all shocks are turned on: demeaned growth of logged bubble values and demeaned growth of logged fundamental values. This panel reveals that the movements of the bubble component and the stock price follow an almost identical pattern. But the movements of the fundamental component and the stock price follow almost opposite patterns, indicating that the stock market fluctuations cannot be explained by fundamentals.

4.3. Understanding Major Bubble and Crash Episodes

The U.S. economy has experienced two major bubble and crash episodes: (i) the internet bubble during the late 1990s and its subsequent crash, and (ii) the recent stock market bubble in tandem with the housing bubble and the subsequent Great Recession. Can our model help understand these two episodes? To address this question, we compute the paths of stock prices, business investment, consumption, and labor hours implied by our estimated model when all shocks are turned on and when the sentiment shock alone is at work. We then compare these paths with the actual data during these two episodes.

Figure 5 shows that our estimated DSGE model fits the actual data almost exactly. In addition, the sentiment shock plays the most important role. In particular, the sentiment shock is the dominant driving force behind the fluctuations in stock prices and investment. We also find that there are sizable gaps between the actual consumption and labor data and the simulated data when the sentiment shock alone is turned on. This suggests that other shocks are also important in driving the variations in consumption and hours. In particular, the permanent TFP shock accounts for a large share of the variation in consumption and the labor supply shock accounts for most of the variation in labor hours, as suggested by the variance decomposition reported in Table 4. The labor supply shock captures the labor wedge and may be interpreted as a reduced form representation of the labor market friction. Our result suggests that labor market frictions played a significant role in accounting for drops in hours growth especially during the Great Recession. Modeling such frictions is an interesting future research topic, which is beyond the scope of this paper.

What is the role of the financial shock? The top panel of Figure 6 plots the smoothed financial shock, indicating that there was a large negative financial shock that tightened credit constraints during the Great Recession. This shock helps explain the gap between the actual investment data and the estimated data when only the sentiment shock is turned on (see the top right panel of Figure 5). Figure 6 also shows that the IT bubble in 1990s and the subsequent crash were not quite related to financial shocks because the smoothed financial shock moved countercyclically during
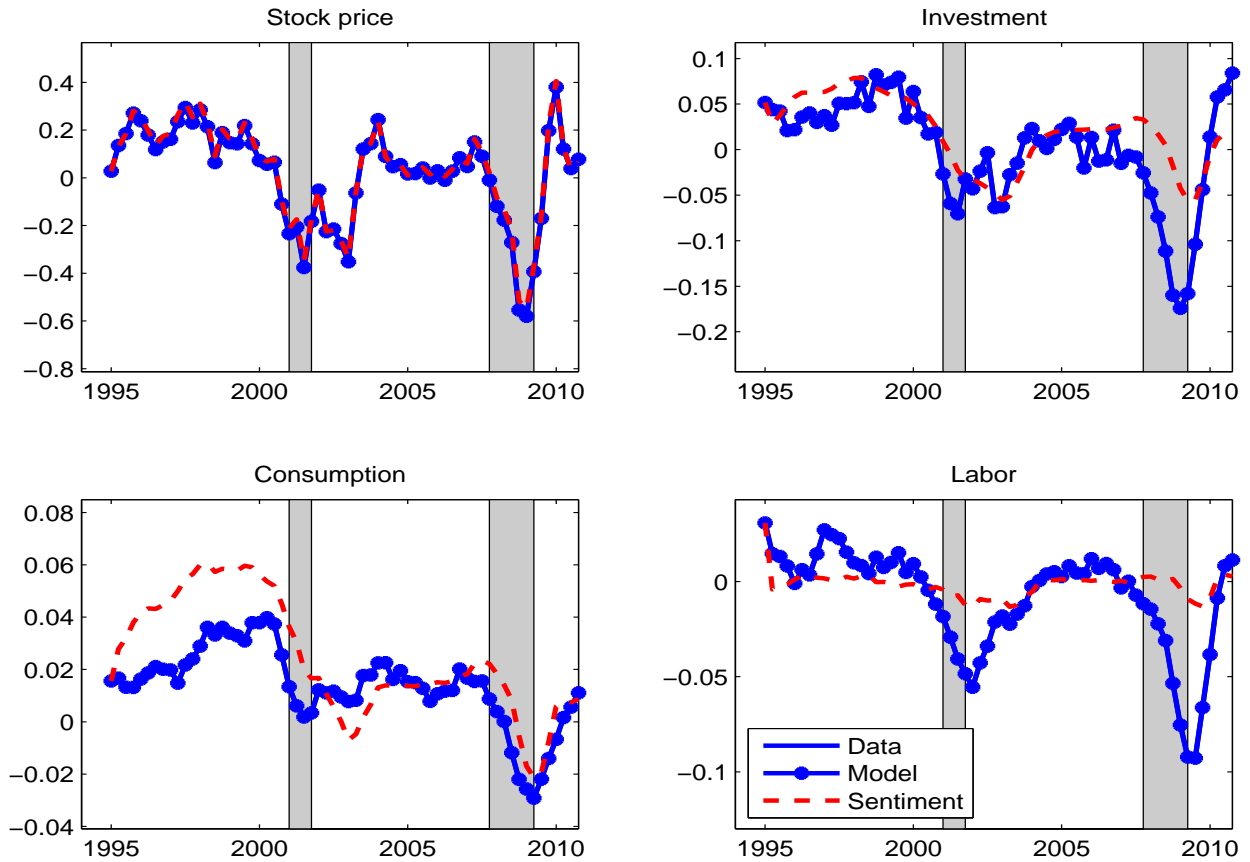
Figure 5: The internet bubble and Great Recession episodes. This figure plots the year-on-year growth rates of stock prices, investment, consumption, and labor hours. The shaded area is the NBER recession bar. Data: actual data. Model: model fitted data when all shocks are turned on. Sentiment: model fitted data when only the sentiment shock is turned on.

that period. Instead, the sentiment shock is the most important shock to explain that episode.
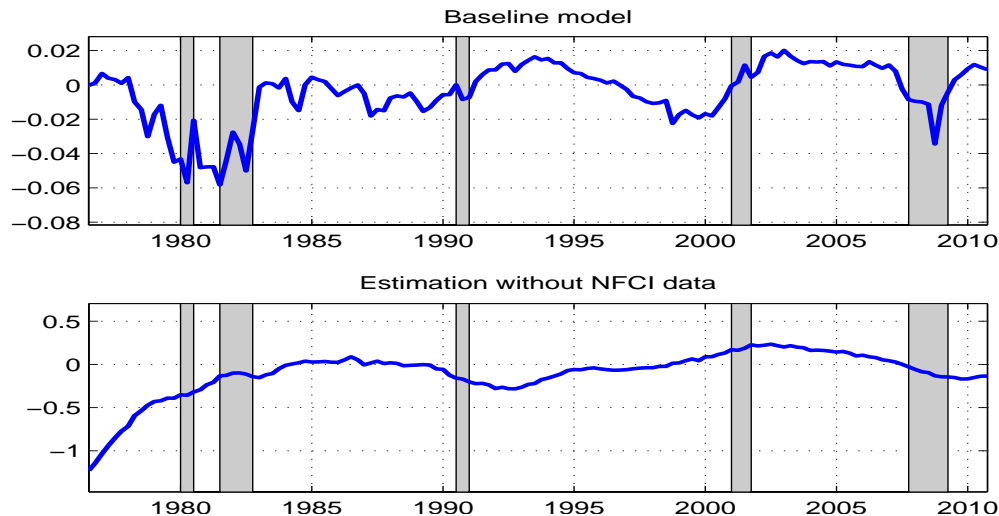

Figure 6: Smoothed financial shocks $\hat{\zeta}_t$. The shaded areas represent NBER recession bars.

To see why the NFCI data is important to identify the financial shock, we estimate the model without the NFCI data and obtain the smoothed financial shock on the bottom panel of Figure 6. We find that the correlation between the smoothed financial shock and NFCI is only -0.03, whereas the correlation in the baseline estimation is around -0.88, indicating that the NFCI data is informative for identification. In addition, the bottom panel of Figure 6 shows that there was an increasing sequence of financial shocks during the IT bubble period. The shock series is relatively smooth and there was no significant drop during the Great Recession. All these counterintuitive features are in contrast to those shown on the top panel of Figure 6 and are inconsistent with the common view.

5.   Understanding the Sentiment Shock

In this section, we conduct various sensitivity analyses and robustness checks to understand the nature of the sentiment shock.

5.1.   Two Alternative Models

To further understand the role of the sentiment shock in economic fluctuations, we estimate two alternative models without this shock. The first alternative model is derived from our baseline model presented in Section 2 after removing the sentiment shock in equation (25) and setting $\theta_t = \bar{\theta} = 0.9975$. In the second alternative model, we replace the credit constraint (12) with the

Kiyotaki-Moore type constraint:

$$\frac{L_{t+1}^j}{R_{ft}} \le (1 - \delta_e)\xi_t Q_t K_{t+1}^j. \tag{46}$$

The resulting equilibrium is identical to the bubbleless equilibrium in our baseline model. In addition, in order to make the above two models flexible enough to match the stock prices and to avoid the stochastic singularity problem, we add measurement errors in the observation equation for stock prices. Table 5 presents the variance decompositions for the two estimated alternative models.

We find that the measurement errors explain almost all of the stock market volatility in the two alternative models. In particular, they explain about 93 percent of the fluctuations in the stock prices in the alternative model without sentiment shocks. The IST shock and the two TFP shocks together explain about 87 percent of the investment fluctuation. The impact of the financial shock is still negligible as in our baseline model. The large impact of the measurement errors indicate that these models are misspecified.

Similar patterns emerge in the alternative model without bubbles as Table 4 reveals. In particular, the measurement errors now explain 94 percent of the fluctuations in the stock prices, and the IST shock and the two TFP shocks together explain about 80 of the fluctuations in the investment. Again, the financial shock plays a negligible role.

To compare the performance of our baseline model with that of the alternative models, we first compute the marginal likelihoods based on the Laplace approximation. We find that the log marginal likelihoods for our baseline model, the model without sentiment shocks, and the model without bubbles are equal to 2226.9, 2098.4, and 2092.0, respectively. This suggests that the data favor our baseline model.

We then report the business cycle moments based on the simulated data from the two alternative models in Table 3. Compared to the baseline model, the two alternative models perform much worse in the following two dimensions. First, the model without sentiment shocks and the model without bubbles counterfactually predict that the stock market and output are almost uncorrelated, though they can fit the stock price volatilities quite well due to the measurement errors. Thus, the sentiment shock not only helps explain the stock market volatility, but also plays an important role in driving the comovement between the stock market and the real economy. Second, the two alternative models overpredict the volatility of the relative price of investment goods by about twice as much.

5.2. Consumer Sentiment Index

In our model, the sentiment shock is an unobserved latent variable. We infer its properties from our six time series of the U.S. data using an estimated model. We find that the consumer sentiment index (CSI) published monthly by the University of Michigan and Thomson Reuters is highly

correlated with our sentiment shock as illustrated in Figure 7.[19] The correlation is 0.61. We now incorporate this data in the estimation and consider the following measurement equation:

$$CSI_t = CSI + b_1\hat{\theta}_t + b_2\Delta\hat{Y}_t + b_3\Delta\hat{Y}_{t-1} + b_4\Delta\hat{Y}_{t-2} + b_5\Delta\hat{Y}_{t-3} + \varepsilon_{cci,t}^{err},$$

where Δ denotes the first difference operator. In this equation, we allow for measurement errors $\varepsilon_{cci,t}^{err}$ and the correlation between CSI and business cycles (i.e., output growth in the past four quarters). This specification captures the fact that CSI may be influenced by current and past GDP growth. We also allow the sentiment shock to be correlated with other shocks in the model such that

$$\begin{aligned}
\hat{\theta}_t &= \hat{\theta}_{1t} + \hat{\theta}_{2t}, \quad \hat{\theta}_{1t} = \rho_\theta\hat{\theta}_{1t-1} + \hat{\varepsilon}_{\theta,t}, \\
\hat{\theta}_{2t} &= a_1\hat{\zeta}_t + a_2\hat{A}_t^m + a_3\hat{\lambda}_{a,t} + a_4\hat{\lambda}_{z,t} + a_5\hat{\psi}_t,
\end{aligned}$$

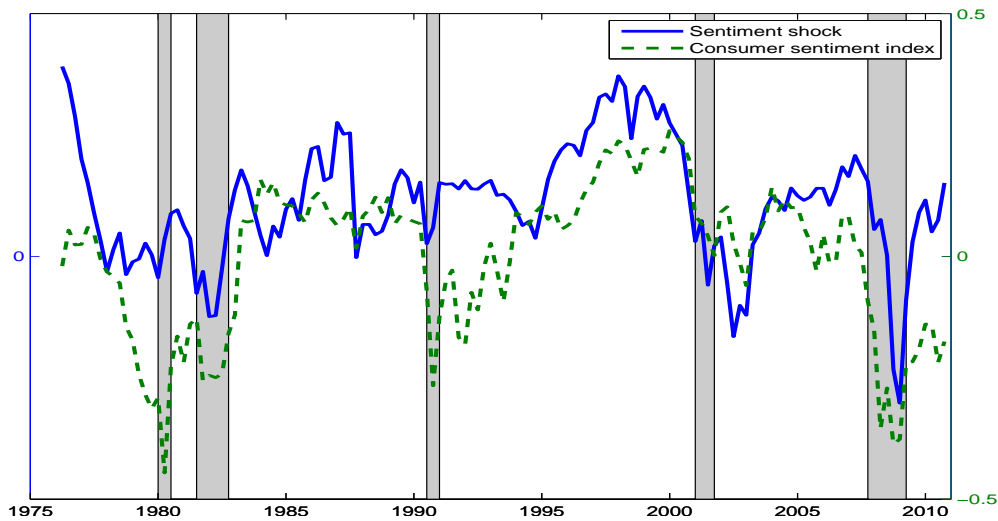where $\lambda_{a,t} = A_t^p/A_{t-1}^p$ and $\lambda_{z,t} = Z_t/Z_{t-1}$.

Figure 7: This figure plots the sentiment shock estimated from the baseline model and the consumer sentiment index downloaded from the University of Michigan. Both series are measured as the deviation from the mean divided by the mean. The shaded areas represent NBER recession bars.

Tables 3 and 4 present results based on the estimated parameter values.[20] Table 4 shows that the impact of the sentiment shock is weakened compared to the baseline model. But it is still the dominant force driving the stock market fluctuations, explaining about 73 percent of the variation. It also explains a sizable fraction of the variations in real quantities. In particular, it explains about 17, 10 and 20 percent of the variations in output, investment and consumption, respectively. The

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[19]This index is normalized to have a value of 100 in December 1964. At least 500 telephone interviews are conducted each month of a continental United States sample (Alaska and Hawaii are excluded). Five core questions are asked. An important objective of this index is to judge the consumer's level of optimism/pessimism.

[20]The parameter estimates are available upon request.

two TFP shocks are the most important force in explaining these quantities. But they are still not important in explaining the stock market fluctuations. Table 3 shows that the extended model and the baseline model perform almost equally well in explaining business cycle statistics.

5.3. A Hybrid Model

Our baseline model has abstracted away from many other potentially important shocks such as news shocks or uncertainty shocks. Thus, it is possible that the sentiment shock is not important at all in explaining stock prices and real variables if other shocks are taken into account. To examine this possibility, we follow the methodology of Ireland (2004) and combine the DSGE model with the VAR model. We then estimate this hybrid model using Bayesian methods.[21] Following Ireland (2004), we now shut down all the shocks in the baseline model except the sentiment shock, and introduce four measurement errors into the measurement equations for the data $\left\{\Delta P_t^{s\text{Data}}, \Delta C_t^{\text{Data}}, \Delta I_t^{\text{Data}}, \ln N^{\text{Data}}\right\}$. Specifically, let

$$
\begin{bmatrix} \Delta P_t^{s\text{Data}} \\ \Delta C_t^{\text{Data}} \\ \Delta I_t^{\text{Data}} \\ \ln N^{\text{Data}} \end{bmatrix} = \begin{bmatrix} \Delta \hat{P}_t^s \\ \Delta \hat{C}_t \\ \Delta \hat{I}_t \\ \hat{N}_t \end{bmatrix} + \begin{bmatrix} \ln(g_\gamma) \\ \ln(g_\gamma) \\ \ln(g_\gamma) \\ \ln(\bar{N}) \end{bmatrix} + \overrightarrow{\nu}_t,
\tag{47}
$$

where $\overrightarrow{\nu}_t$ is the vector contains four measurement errors, g_γ is the gross growth rate of output, and $\bar{N}$ is the average hours in the data. Following Ireland (2004), we assume that the measurement errors $\overrightarrow{\nu}_t$ follow a VAR(1) process:

$$
\overrightarrow{\nu}_t = \mathcal{A} \overrightarrow{\nu}_{t-1} + \mathcal{B}\hat{\varepsilon}_{\nu,t},
\tag{48}
$$

where $\mathcal{A}$ is the coefficient matrix and $\mathcal{B}$ is assumed to be lower-triangular such that the innovations in $\hat{\varepsilon}_{\nu,t}$ are orthogonal to each other.

The measurement errors in equation (48) can be considered as a combination of all omitted structural shocks in our baseline model and allow for potential model misspecifications. We allow the measurement errors to be flexible enough so that the data are not necessarily driven by the sentiment shock. The idea is that, if the sentiment shock is not the driving force, then equations (47) and (48) form a first-order Bayesian VAR system and the measurement errors should be important in explaining fluctuations in the data of $\left\{\Delta P_t^{s\text{Data}}, \Delta C_t^{\text{Data}}, \Delta I_t^{\text{Data}}, \ln N^{\text{Data}}\right\}$. On the other hand, if the baseline model is correctly specified and the sentiment shock is the main source of fluctuations, then the estimated measurement errors will be unimportant.

The variance decomposition shows that the sentiment shock remains the single most important factor accounting for the stock price variation although its importance is somewhat reduced. It explains about 82 percent of the variation in the stock prices. It still accounts for significant frac-

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[21] We thank Tao Zha for suggesting us to conduct this analysis.

tions of fluctuations in investment, consumption and output, explaining about 26, 38, 35 percent, respectively. As in the baseline model, the sentiment shock is not important in explaining the fluctuation in hours. We also find that the estimates of the common parameters in the hybrid model are very similar to those in the baseline model. The smoothed sentiment shock is still highly correlated with the consumer sentiment data, the correlation is about 0.73. These results suggest that the importance of the sentiment shock is robust to the model variation and specification of different shocks.

6. Conclusion

Stock markets are highly volatile and it is challenging to explain their movements entirely by fundamentals. Many people believe that bubbles, fads or irrationality may play an important role in determining stock prices. This idea has been developed extensively in the theoretical literature. However, the development of the empirical literature is hindered by the lack of identification of bubbles using the VAR approach or other reduced-form regression analysis. As a result, the empirical importance of bubbles for the stock market and for the real economy is unclear.

The main contribution of this paper is that it provides a Bayesian DSGE model of stock market bubbles and business cycles. Stock market bubbles emerge endogenously through a positive feedback loop mechanism supported by self-fulfilling beliefs. Using Bayesian methods, we identify a sentiment shock that drives the movements of bubbles and hence stock prices. Unlike many other demand side shocks such as news shocks and uncertainty shocks, the sentiment shock can generate comovements among consumption, investment, hours, output and stock prices. Our Bayesian estimation shows that the sentiment shock explains most of the stock market volatility and sizable fractions of the variations in investment, consumption, and output. It is the driving force behind the comovements between stock prices and macroeconomic quantities. In addition to the empirical contribution, our paper also makes a theoretical contribution to the literature on rational bubbles by modeling recurrent bubbles in an infinite-horizon DSGE framework. Our theoretical model is useful to address many other quantitative or empirical questions. For example, our model focuses on the real side and does not consider inflation and monetary policy. Should monetary policy respond to asset price bubbles? Miao, Wang and Xu (2012) study this question by embedding the present model in a dynamic new Keynesian framework.

References

Barro, Robert, and Robert G. King, 1984, Time Separable Preferences and International Substitution Models of Business Cycles, *Quarterly Journal of Economics* 99, 817-839.

Basu, Susanto and Brent Bundick, 2011, Uncertainty Shocks in a Model of Effective Demand, working paper, Boston College.

Beaudry, Paul, and Franck Portier, 2004 An Exploration into Pigou's Theory of Cycles, *Journal of Monetary Economics* 51, 1183–216.

Beaudry, Paul, and Franck Portier, 2006, Stock Prices, News, and Economic Fluctuations, *American Economic Review* 96, 1293–307.

Bernard, Andrew, Stephen Redding and Peter K. Schott, 2010, Multiple-Product Firms and Product Switching, *American Economic Review*, 100(1), 70-97.

Bernanke, Ben, Mark Gertler and Simon Gilchrist, 1999, Financial Accelerator in a Quantitative Business Cycle Framework, In: J. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, Elsevier Science, Amsterdam.

Bilbiie, Florin, Fabio Ghironi and Marc J. Melitz, 2012, Endogenous Entry, Product Variety, and Business Cycles, *Journal of Political Economy*, 120(2), 304-345.

Bloom, Nicholas, 2009, The Impact of Uncertainty Shocks, Econometrica 77, 623-685.

Boldrin, Michele, Lawrence J. Christiano and Jonas D. M. Fisher, 2001. Habit Persistence, Asset Returns, and the Business Cycle, *American Economic Review* 91, 149-166.

Brave, Scott and R. Andrew Butters, 2011, Monitoring Financial Stability: A Financial Conditions Index Approach, Economic Perspectives of the Federal Reserve Bank of Chicago, 1Q.

Campbell, John Y., 1999, Asset Prices, Consumption and the Business Cycle, in John B. Taylor, and Michael Woodford, eds.: *Handbook of Macroeconomics*, vol. 1 (Elsevier Science, North-Holland, Amsterdam).

Carlstrom, Charles T. and Timothy S. Fuerst, 1997, Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis, *American Economic Review* 87, 893-910.

Chaney, Thomas, David Sraer, and David Thesmar, 2009, The Collateral Channel: How Real Estate Shocks affect Corporate Investment, working paper, Princeton University.

Chari, V. V., Lawrence Christiano, and Patrick J. Kehoe, 2008, Facts and Myths about the Financial Crisis of 2008, Working Paper 666, Federal Reserve Bank of Minneapolis.

Chirinko, Robert S. and Huntley Schaller, 2001. Business Fixed Investment and Bubbles: The Japanese Case, *American Economic Review* 91, 663-680.

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans, 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* 113, 1-45.

Christiano, Lawrence, Cosmin Ilut, Roberto Motto, and Massimo Rostagno, 2008, Monetary Policy and Stock Market Boom-Bust Cycles, Working Paper Series 955, European Central Bank.

Christiano, Lawrence, Roberto Motto and Massimo Rostagno, 2010, Financial Factors in Economic Fluctuations, Working Paper Series 1192, European Central Bank.

Christiano, Lawrence, Roberto Motto and Massimo Rostagno, 2012, Risk Shocks, working paper, Northwestern University.

Christiano, Lawrence J., Mathias Trabandt and Karl Walentin. 2011. DSGE Models for Monetary Policy Analysis. In Handbook of Monetary Economics, ed. Benjamin M. Friedman and Michael Woodford. Vol. 3A Netherlands: North Holland, pp. 285-367.

Cooper, Russell and John Haltiwanger, 2006, On the Nature of Capital Adjustment Costs, *Review of Economic Studies* 73, 611-634.

Covas, Francisco and Wouter J. Den Haan, 2011. The Cyclical Behavior of Debt and Equity Finance, *American Economic Review* 101, 877-899.

Dunne, Timothy, Mark J. Roberts and Larry Samuelson, 1988. Patterns of Firm Entry and Exit in U.S. Manufacturing Industries, *Rand Journal of Economics* 19, 495-515.

Farhi, Emmanuel and Jean Tirole, 2010, Bubbly Liquidity, forthcoming in *Review of Economic Studies*.

Farmer, Roger E. A., 2012a, Confidence, Crashes and Animal Spirits, *Economic Journal* 122, 155-172.

Farmer, Roger E. A., 2012b, The Stock Market Crash of 2008 Caused the Great Recession: Theory and Evidence, forthcoming in *Journal of Economic Dynamics and Control*.

Fujiwara Ippei, Yasuo Hirose and Mototsugu Shintani, 2011, Can News Be a Major Source of Aggregate Fluctuations? A Bayesian DSGE Approach, *Journal of Money, Credit and Banking* 43, 1-29.

Gan, Jie, 2007, Collateral, Debt Capacity, and Corporate Investment: Evidence from a Natural Experiment, *Journal of Financial Economics* 85, 709-734.

Gertler, Mark and Nobuhiro Kiyotaki, 2010, Financial Intermediation and Credit Policy in Business Cycle Analysis, working paper, NYU.

Gilchrist, Simon, Charles Himmelberg and Gur Huberman, 2005, Do Stock Price Bubbles Influence Corporate Investment?, *Journal of Monetary Economics* 52, 805-827.

Goyal, Vidhan K. and Takeshi Yamada, 2004, Asset Price Shocks, Financial Constraints, and Investment: Evidence from Japan, *Journal of Business* 77, 175-200.

Greenwood, Jeremy, Zvi Hercowitz and Per Krusell, 1997. Long-Run Implications of Investment-Specific Technological Change, *American Economic Review* 87, 342-62.

Gurkaynak, Refet S., 2008, Econometric Test of Asset Price Bubbles: Taking Stock, *Journal of Economic Surveys* 22, 166-186.

Hansen, Lars Peter and Kenneth Singleton, 1983, Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns, *Journal of Political Economy* 91, 249-265.

Hayashi, Fumio, 1982, Tobin's Marginal q and Average q: A Neoclassical Interpretation, *Econometrica* 50, 213-224.

Hirano, Tomohiro and Noriyuki Yanagawa, 2010, Asset Bubbles, Endogenous Growth and Financial Frictions, working paper, University of Tokyo.

Ireland, Perter N., 2004, A method for Taking Models to the Data, *Journal of Economic Dynamics and Control* 28, 1205-1226.

Iskrev, Nicolay, 2010, Local Identification in DSGE Models, *Journal of Monetary Economics* 57, 189-210.

Jaimovich, Nir and Sergio Rebelo, 2009. Can News about the Future Drive the Business Cycle?, *American Economic Review* 99, 1097-1118.

Jermann, Urban, and Vincenzo Quadrini, 2011, Macroeconomic Effects of Financial Shocks, forthcoming in *American Economic Review*.

Justiniano, Alejandro, Giorgio Primiceri and Andrea Tambalotti, 2011. Investment Shocks and the Relative Price of Investment, *Review of Economic Dynamics* 14, 101-121.

Kiyotaki, Nobuhiro, and John Moore, 1997, Credit Cycles, *Journal of Political Economy* 105, 211-248.

Kocherlakota, Narayana, 2009, Bursting Bubbles: Consequences and Cures, working paper, University of Minnesota.

Li, Haito, Xuenan Li, and Cindy Yu, 2013, Macroeconomic Risks and Asset Pricing: Evidence from a Dynamic Stochastic General Equilibrium Model, working paper, CKGSB.

Liu, Zheng, Daniel F. Waggoner and Tao Zha, 2011. Sources of Macroeconomic Fluctuations: A Regime-Switching DSGE Approach, *Quantitative Economics* 2, 251-301.

Liu, Zheng, Pengfei Wang and Tao Zha, 2013, Land-Price Dynamics and Macroeconomic Fluctuations, *Econometrica* 81, 1147-1184.

Martin, Alberto and Jaume Ventura, 2011a, Theoretical Notes on Bubbles and the Current Crisis, *IMF Economic Review* 59, 6-40.

Martin, Alberto and Jaume Ventura, 2011b, Economic Growth with Bubbles, forthcoming in *American Economic Review.*

Mehra, Rajnish and Edward C. Prescott, 1985, The Equity Premium: A Puzzle, *Journal of Monetary Economics* 15, 145-161.

Miao, Jianjun and Pengfei Wang, 2011a, Bubbles and Credit Constraints, working paper, Boston University and HKUST.

Miao, Jianjun and Pengfei Wang, 2011b, Sectoral Bubbles and Endogenous Growth, working paper, Boston University and HKUST.

Miao, Jianjun and Pengfei Wang, 2012a, Banking Bubbles and Financial Crisis, working paper, Boston University and HKUST.

Miao, Jianjun and Pengfei Wang, 2012b, Bubbles and Total Factor Productivity, forthcoming in *American Economic Review Papers and Proceedings.*

Miao, Jianjun, Pengfei Wang, and Lifang Xu, 2012, Stock Market Bubbles and Unemployment, working paper, Boston University and HKUST.

Miao, Jianjun, Pengfei Wang, and Zhiwei Xu, 2012, Stock Market Bubbles and Monetary Policy, Boston University and HKUST.

Ohanian, Lee E., 2010, The Economic Crisis from a Neoclassical Perspective, *Journal of Economic Perspectives* 24, 45-66.

Phillips, Peter C.B. and Jun Yu, Dating the Timeline of Financial Bubbles during the Subprime Crisis, *Quantitative Economics* 2, 455-491.

Santos, Manuel S. and Michael Woodford, 1997, Rational Asset Pricing Bubbles, *Econometrica* 65, 19-58.

Schmitt-Grohe, Stephanie and Martin Uribe, 2012, What's News In Business Cycles, forthcoming in *Econometrica.*

Shiller, Robert J, 1981, Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?, *American Economic Review* 71, 421-36.

Sims, Christopher A. and Zha, Tao, 1998. Bayesian Methods for Dynamic Multivariate Models, *International Economic Review* 39(4), 949-968.

Smets, Frank and Rafael Wouters, 2007, Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, *American Economic Review* 97, 586-606.

Tirole, Jean, 1985, Asset Bubbles and Overlapping Generations, *Econometrica* 53, 1499-1528.

Wang, Pengfei, 2012, Understanding Expectation-Driven Fluctuations: A Labor-Market Approach, *Journal of Money, Credit, and Banking* 44, 487-506.

Wang, Pengfei and Yi Wen, 2011, Speculative Bubbles and Financial Crisis, forthcoming in *American Economic Journal: Macroeconomics.*

Wang, Pengfei and Yi Wen, 2012. Hayashi Meets Kiyotaki and Moore: A Theory of Capital Adjustment, *Review of Economic Dynamics* 15, 207-225.

Weil, Philippe, 1987, Confidence and the Real Value of Money in an Overlapping Generations Economy, *Quarterly Journal of Economics* 102, 1-22.

Appendix

A   Proof of Proposition 1:

We use a conjecture and verification strategy to find the decision rules at the firm level. We first study the optimal investment problem by fixing the capacity utilization rate u_t^j. Using (14) and (16), we can write firm j's dynamic programming problem as

$$v_t(\varepsilon_t^j)K_t^j + b_{t,\tau}(\varepsilon_t^j) - v_{Lt}(\varepsilon_t^j)L_t^j = \max_{I_t^j, L_{t+1}^j} u_t^j R_t K_t^j - P_t I_t^j - L_t^j + \frac{L_{t+1}^j}{R_{ft}} \tag{A.1}$$

$$+ Q_t[(1-\delta_t^j)K_t^j + \varepsilon_t^j I_t^j] + B_{t,\tau} - Q_{Lt}L_{t+1}^j,$$

subject to the investment constraint:

$$0 \le P_t I_t^j \le u_t^j R_t K_t^j - L_t^j + \frac{L_{t+1}^j}{R_{ft}} + \eta_t K_t^j. \tag{A.2}$$

For $\varepsilon_t^j \le P_t/Q_t$, $I_t^j = 0$. Optimizing over L_{t+1}^j yields $Q_{Lt} = 1/R_{ft}$. For $\varepsilon_{t+1}^j \ge P_t/Q_t$, the optimal investment level must reach the upper bound in the above investment constraint. We can then immediately derive the optimal investment rule in (18). In addition, the credit constraint (17) must bind so that

$$\frac{1}{R_{ft}}L_{t+1}^j = Q_t \xi_t K_t^j + B_{t,\tau}. \tag{A.3}$$

Substituting the optimal investment rule and $Q_{Lt} = 1/R_{ft}$ into (A.1) yields:

$$v_t(\varepsilon_t^j)K_t^j + b_{t,\tau}(\varepsilon_t^j) - v_{Lt}(\varepsilon_t^j)L_t^j$$
$$= u_t^j R_t K_t^j + Q_t(1-\delta_t^j)K_t^j + B_{t,\tau} - L_t^j$$
$$+ \max\{Q_t \varepsilon_t^j/P_t - 1, 0\} \times \left(u_t^j R_t K_t^j + \eta_t K_t^j - L_t^j + \frac{L_{t+1}^j}{R_{ft}} \right). \tag{A.4}$$

Since u_t^j is determined before observing ε_t^j, it solves the following problem:

$$\max_{u_t^j} u_t^j R_t K_t^j + Q_t(1-\delta_t^j)K_t^j + G_t u_t^j R_t K_t^j, \tag{A.5}$$

where G_t is defined by (20). We then obtain the first order condition

$$R_t(1+G_t) = Q_t \delta'(u_t^j). \tag{A.6}$$

Since $\delta_t^j = \delta(u_t^j)$ is convex, this condition is also sufficient for optimality. From this condition, we can immediately deduce that optimal u_t^j does not depend on firm identity so that we can remove the superscript j.

By defining $\delta_t \equiv \delta(u_t)$, (A.4) becomes

$$
\begin{aligned}
&v_t(\varepsilon_t^j)K_t^j + b_{t,\tau}(\varepsilon_t^j) - v_{Lt}(\varepsilon_t^j)L_t^j \\
&= \quad u_t R_t K_t^j + Q_t(1 - \delta_t)K_t^j + B_{t,\tau} - L_t^j \\
&\quad + \max\{Q_t\varepsilon_t^j/P_t - 1, 0\} \times \left(u_t R_t K_t^j + \eta_t K_t^j - L_t^j + \frac{L_{t+1}^j}{R_{ft}} \right),
\end{aligned}
$$

where L_{t+1}^j/R_{ft} is given by (A.3). Matching coefficients yields:

$$
v_t(\varepsilon_t^j) = \begin{cases} u_t R_t + Q_t(1 - \delta_t) + (Q_t\varepsilon_t^j/P_t - 1)(u_t R_t + \eta_t + \xi_t Q_t) & \text{if } \varepsilon_t^j \geq \frac{P_t}{Q_t} \\ u_t R_t + Q_t(1 - \delta_t) & \text{otherwise} \end{cases}, \tag{A.7}
$$

$$
b_{t,\tau}(\varepsilon_t^j) = \begin{cases} \left(Q_t\varepsilon_t^j/P_t - 1 \right) B_{t,\tau} & \text{if } \varepsilon_t^j \geq \frac{P_t}{Q_t} \\ B_{t,\tau} & \text{otherwise} \end{cases}, \tag{A.8}
$$

and

$$
v_{Lt}(\varepsilon_t^j) = \begin{cases} Q_t\varepsilon_t^j/P_t - 1 & \text{if } \varepsilon_t^j \geq \frac{P_t}{Q_t} \\ 1 & \text{otherwise} \end{cases}.
$$

Using equation (16), we then obtain (21) and (22) and (23).   Q.E.D.

B   Stationary Equilibrium

We define the following transformed variables:

$$
\begin{aligned}
\tilde{C}_t &\equiv \frac{C_t}{\Gamma_t}, \ \tilde{I}_t \equiv \frac{I_t}{Z_t\Gamma_t}, \ \tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t}, \ \tilde{K}_t \equiv \frac{K_t}{\Gamma_{t-1}Z_{t-1}}, \\
\tilde{P}_t^s &\equiv \frac{P_t^s}{\Gamma_t}, \ \tilde{B}_t^a \equiv \frac{B_t^a}{\Gamma_t}, \ \tilde{X}_t \equiv \frac{X_t}{\Gamma_t Z_t}, \ \tilde{W}_t \equiv \frac{W_t}{\Gamma_t}, \\
\tilde{Q}_t &\equiv Q_t Z_t, \ \tilde{P}_t = P_t Z_t, \ \tilde{R}_t = R_t Z_t, \ \tilde{\Lambda}_t \equiv \Lambda_t \Gamma_t,
\end{aligned}
$$

where $\Gamma_t = Z_t^{\frac{\alpha}{1-\alpha}} A_t$. The other variables are stationary and there is no need to scale them. To be consistent with a balanced growth path, we also assume that $K_{0t} = \Gamma_{t-1}Z_{t-1}K_0$, where K_0 is a constant.

The six shocks in the model are given by

1. The permanent TFP shock,

$$
A_t^p = A_{t-1}^p \lambda_{at}, \ \ln \lambda_{at} = (1 - \rho_a) \ln \bar{\lambda}_a + \rho_a \ln \lambda_{a,t-1} + \varepsilon_{at}. \tag{B.1}
$$

2. The transitory TFP shock,

$$
\ln A_t^m = \rho_{a^m} \ln A_{t-1}^m + \varepsilon_{a^m,t}. \tag{B.2}
$$

3. The IST shock,

$$
Z_t = Z_{t-1}\lambda_{zt}, \ \ln \lambda_{zt} = (1 - \rho_z) \ln \bar{\lambda}_z + \rho_z \ln \lambda_{z,t-1} + \varepsilon_{zt}. \tag{B.3}
$$

4. The sentiment shock,
$$\ln \theta_t = (1 - \rho_\theta) \bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta,t}. \tag{B.4}$$

5. The labor shock,
$$\ln \psi_t = (1 - \rho_\psi) \ln \bar{\psi} + \rho_\psi \ln \psi_{t-1} + \varepsilon_{\psi t}. \tag{B.5}$$

6. The financial shock,
$$\ln \zeta_t = (1 - \rho_\zeta) \ln \bar{\zeta} + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}.$$

Here, all innovations are mutually independent and are independently and identically distributed normal random variables.

Denote by $g_{\gamma t} \equiv \Gamma_t / \Gamma_{t-1}$ the growth rate of Γ_t. Denote by g_γ the nonstochastic steady-state of $g_{\gamma t}$, satisfying
$$\ln g_\gamma \equiv \frac{\alpha}{1 - \alpha} \ln \bar{\lambda}_z + \ln \bar{\lambda}_a. \tag{B.6}$$

On the nonstochastic balanced growth path, investment and capital grow at the rate of $\bar{\lambda}_I \equiv g_\gamma \bar{\lambda}_z$; consumption, output, wages, and bubbles grow at the rate of g_γ; and the rental rate of capital, Tobin's marginal Q, and the relative price of investment goods decrease at the rate $\bar{\lambda}_z$.

After the transformation described in Section 3, we can derive a system of 15 equations for 15 transformed variables: $\{\tilde{C}_t, \tilde{I}_t, \tilde{Y}_t, N_t, \tilde{K}_t, u_t, \tilde{Q}_t, \tilde{X}_t, \tilde{P}_t, \tilde{W}_t, \tilde{R}_t, m_t, \tilde{B}_t^a, R_{ft}, \tilde{\Lambda}_t\}$.

1. Resource constraint:
$$\tilde{C}_t + \left[1 + \frac{\Omega}{2} \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{zt} g_{\gamma t} - \bar{\lambda}_I \right)^2 \right] \tilde{I}_t = \tilde{Y}_t, \tag{B.7}$$

where $g_{zt} = Z_t / Z_{t-1}$.

2. Aggregate Investment:
$$\tilde{I}_t = \left(\alpha \tilde{Y}_t + \zeta_t \tilde{Q}_t \tilde{X}_t + \tilde{B}_t^a \right) \frac{1 - \Phi(\varepsilon_t^*)}{\tilde{P}_t}, \tag{B.8}$$

where $\varepsilon_t^* = \tilde{P}_t / \tilde{Q}_t$.

3. Aggregate output:
$$\tilde{Y}_t = \left(u_t \tilde{X}_t \right)^\alpha N_t^{1-\alpha}. \tag{B.9}$$

4. Labor supply:
$$(1 - \alpha) \frac{\tilde{Y}_t}{N_t} \tilde{\Lambda}_t = \psi_t. \tag{B.10}$$

5. The law of motion for capital:
$$\tilde{K}_{t+1} = (1 - \delta_t) \tilde{X}_t + \tilde{I}_t \frac{\Sigma(\varepsilon_t^*)}{1 - \Phi(\varepsilon_t^*)}, \tag{B.11}$$

where
$$\Sigma(\varepsilon_t^*) \equiv \int_{\varepsilon > \varepsilon_t^*} \varepsilon d\Phi(\varepsilon).$$

6. Capacity utilization:
$$\alpha \frac{\tilde{Y}_t}{u_t \tilde{X}_t} (1 + G_t) = \tilde{Q}_t \delta'(u_t), \tag{B.12}$$

where
$$G_t = \int_{\varepsilon > \varepsilon_t^*} (\varepsilon / \varepsilon_t^* - 1) \, d\Phi(\varepsilon) = \frac{\Sigma(\varepsilon_t^*)}{\varepsilon_t^*} + \Phi(\varepsilon_t^*) - 1.$$

7. Marginal Q:

$$\tilde{Q}_t = \beta(1 - \delta_e) E_t \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \frac{\tilde{Q}_{t+1}}{g_{zt+1} g_{\gamma t+1}} \left[u_{t+1} \delta'(u_{t+1}) + (1 - \delta_{t+1}) + \zeta_{t+1} G_{t+1} \right]. \tag{B.13}$$

8. Effective capital stock used in production:

$$\tilde{X}_t = \frac{1 - \delta_e}{g_{zt} g_{\gamma t}} \tilde{K}_t + \delta_e K_0. \tag{B.14}$$

9. Euler equation for investment goods producers:

$$
\begin{aligned}
\tilde{P}_t &= 1 + \frac{\Omega}{2} \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{zt} g_{\gamma t} - \bar{\lambda}_I \right)^2 + \Omega \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{zt} g_{\gamma t} - \bar{\lambda}_I \right) \frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{zt} g_{\gamma t} \\
&\quad - \beta E_t \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \Omega \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t} g_{zt+1} g_{\gamma t+1} - \bar{\lambda}_I \right) \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t} \right)^2 g_{zt+1} g_{\gamma t+1}. \tag{B.15}
\end{aligned}
$$

10. The wage rate:

$$\tilde{W}_t = (1 - \alpha) \frac{\tilde{Y}_t}{N_t}. \tag{B.16}$$

11. The rental rate of capital:

$$\tilde{R}_t = \frac{\alpha \tilde{Y}_t}{u_t \tilde{X}_t}. \tag{B.17}$$

12. Evolution of the number of bubbly firms:

$$m_t = m_{t-1}(1 - \delta_e)\theta_{t-1} + \delta_e \omega. \tag{B.18}$$

13. Evolution of the total value of the bubble:

$$\tilde{B}_t^a = \beta E_t \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \tilde{B}_{t+1}^a (1 + G_{t+1}) (1 - \delta_e)\theta_t \frac{m_t}{m_{t+1}}. \tag{B.19}$$

14. The risk-free rate:

$$\frac{1}{R_{ft}} = \beta E_t \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \frac{1}{g_{\gamma t+1}} (1 + G_{t+1}) (1 - \delta_e). \tag{B.20}$$

15. Marginal utility for consumption:

$$\tilde{\Lambda}_t = \frac{1}{\tilde{C}_t - h\tilde{C}_{t-1}/g_{\gamma t}} - \beta E_t \frac{h}{\tilde{C}_{t+1} g_{\gamma t+1} - h\tilde{C}_t}. \tag{B.21}$$

C Steady State

The transformed system presented in Appendix B has a nonstochastic steady state. We eliminate $\tilde{W}_t$ and $\tilde{R}_t$ and then obtain a system of 15 equations for 15 steady-state values: $\{\tilde{C}, \tilde{I}, \tilde{Y}, N, \tilde{K},$ $u, \tilde{Q}, \tilde{X}, \tilde{P}, \tilde{W}, \tilde{R}, m, \tilde{B}^a, R_f, \tilde{\Lambda}\}$, where we have removed time subscripts. We assume that the function $\delta(\cdot)$ is such that the steady-state capacity utilization rate is equal to 1. In addition, we set $\tilde{Q} = 1$ which pins down G.

1. Resource constraint:
$$\tilde{C} + \tilde{I} = \tilde{Y}, \tag{C.1}$$

    where we have used the fact that $\bar{\lambda}_I = \bar{\lambda}_z g_\gamma$.

2. Aggregate investment:
$$\tilde{I} = \left(\alpha \tilde{Y} + \bar{\zeta}\tilde{Q}\tilde{X} + \tilde{B}^a \right) \frac{1 - \Phi(\varepsilon^*)}{\tilde{P}}, \tag{C.2}$$

    where $1 - \Phi(\varepsilon^*) = \int_{\varepsilon > \varepsilon^*} d\Phi(\varepsilon)$, and $\varepsilon^* = \tilde{P}/\tilde{Q}$.

3. Aggregate output:
$$\tilde{Y} = \tilde{X}^\alpha N^{1-\alpha}. \tag{C.3}$$

4. Labor supply:
$$(1 - \alpha)\frac{\tilde{Y}}{N}\tilde{\Lambda} = \bar{\psi}. \tag{C.4}$$

5. End-of-period capital stock:
$$\tilde{K} = (1 - \delta(1))\tilde{X} + \tilde{I}\frac{\Sigma(\varepsilon^*)}{1 - \Phi(\varepsilon^*)}, \tag{C.5}$$

    where
$$\Sigma(\varepsilon^*) \equiv \int_{\varepsilon > \varepsilon^*} \varepsilon d\Phi(\varepsilon).$$

6. Capacity utilization:
$$\alpha\frac{\tilde{Y}}{\tilde{X}}(1 + G) = \tilde{Q}\delta'(1), \tag{C.6}$$

    where
$$G = \int_{\varepsilon > \varepsilon^*} (\varepsilon/\varepsilon^* - 1)\, d\Phi(\varepsilon) = \frac{\Sigma(\varepsilon^*)}{\varepsilon^*} + \Phi(\varepsilon^*) - 1.$$

7. Marginal Q:
$$1 = \beta(1 - \delta_e)\frac{1}{\bar{\lambda}_z g_\gamma}\left[\delta'(1) + 1 - \delta(1) + \bar{\zeta}G \right]. \tag{C.7}$$

8. Effective capital stock used in production:
$$\tilde{X} = \frac{1 - \delta_e}{\bar{\lambda}_z g_\gamma}\tilde{K} + \delta_e K_0. \tag{C.8}$$

9. Euler equation for investment goods producers:

$$\tilde{P} = 1. \tag{C.9}$$

10. The wage rate:

$$\tilde{W} = (1 - \alpha)\frac{\tilde{Y}}{N}. \tag{C.10}$$

11. The rental rate of capital:

$$\tilde{R} = \frac{\alpha\tilde{Y}}{\tilde{X}}. \tag{C.11}$$

12. Evolution of the number of bubbly firms:

$$m = m(1 - \delta_e)\bar{\theta} + \delta_e\omega. \tag{C.12}$$

13. Evolution of the total value of the bubble:

$$\tilde{B}^a = \beta\tilde{B}^a\,(1 + G)\,(1 - \delta_e)\bar{\theta}. \tag{C.13}$$

14. The risk-free rate:

$$\frac{1}{R_f} = \beta\frac{1}{g_\gamma}\,(1 + G)\,(1 - \delta_e). \tag{C.14}$$

15. Marginal utility for consumption:

$$\tilde{\Lambda} = \frac{1}{\tilde{C} - h\tilde{C}/g_\gamma} - \frac{\beta h}{\tilde{C}g_\gamma - h\tilde{C}}. \tag{C.15}$$

For convenience, define $\varepsilon_t^* = P_t/Q_t = \tilde{P}_t/\tilde{Q}_t$ as the investment threshold. We use a variable without the time subscript to denote its steady-state value in the transformed stationary system. The following proposition characterizes the bubbly steady state.[22]

Proposition 2 *Suppose that $\omega > 0$ and $0 < \varepsilon_{\min} < \beta(1 - \delta_e)\bar{\theta} < \beta$. Then there exists a unique steady-state threshold $\varepsilon^* \in (\varepsilon_{\min}, \varepsilon_{\max})$ satisfying*

$$\int_{\varepsilon > \varepsilon^*} (\varepsilon/\varepsilon^* - 1)\, d\Phi\,(\varepsilon) = \frac{1}{\beta(1 - \delta_e)\bar{\theta}} - 1. \tag{C.16}$$

If the parameter values are such that

$$\frac{\tilde{B}^a}{\tilde{Y}} = \frac{[\varphi_k - (1 - \delta(1))]\,\varphi_x}{1/\left[\beta(1 - \delta_e)\bar{\theta}\right] - \Phi\,(\varepsilon^*)} - \alpha - \bar{\zeta}\varphi_x > 0, \tag{C.17}$$

where we define

$$\varphi_k \equiv \left(\frac{1 - \delta_e}{\bar{\lambda}_z g_\gamma} + \delta_e\frac{K_0}{\tilde{K}}\right)^{-1}, \tag{C.18}$$

---

[22]The bubbleless steady state can be obtained by setting $\tilde{B}^a = 0$ and $m = \omega = 0$. Thus, we can remove equations (C.13) and (C.12).

$$\varphi_x \equiv \frac{\alpha}{\bar{\lambda}_z g_\gamma \bar{\theta} - (1 - \delta(1))\beta(1 - \delta_e)\bar{\theta} - \bar{\zeta}\left[1 - \beta(1 - \delta_e)\bar{\theta}\right]}, \tag{C.19}$$

then there exists a unique bubbly steady-state equilibrium with the bubble-output ratio given in (C.17). The steady-state growth rate of the bubble is given by $\bar{\theta} = R_f/g_\gamma$, where R_f is the steady-state interest rate. In addition, if

$$\delta'(1) = \frac{\alpha}{\beta(1 - \delta_e)\bar{\theta}} \frac{1}{\varphi_x}, \tag{C.20}$$

then the capacity utilization rate in this steady state is equal to 1.

Proof: In the steady state, equation (B.15) implies that $\tilde{P} = 1$. Hence by definition we have $\varepsilon^* = 1/\tilde{Q}$. Then by the evolution equation (B.19) of the total bubble, we obtain the steady-state relation:

$$\frac{1}{\beta(1 - \delta_e)\bar{\theta}} - 1 = G = \int_{\varepsilon > \varepsilon^*} (\varepsilon/\varepsilon^* - 1)\, d\Phi(\varepsilon). \tag{C.21}$$

Define the expression on the right-hand side of the last equality as a function of ε^*, $G(\varepsilon^*)$. Then we have $G(\varepsilon_{\min}) = \frac{1}{\varepsilon_{\min}} - 1$ and $G(\varepsilon_{\max}) = 0$. Given the assumption that $\varepsilon_{\min} < \beta(1 - \delta_e)\bar{\theta}$, there is a unique solution ε^* to equation (C.21) by the intermediate value theorem. In addition, by the definition of G, we have

$$G = \frac{\Sigma(\varepsilon^*)}{\varepsilon^*} - [1 - \Phi(\varepsilon^*)],$$

where $\Sigma(\varepsilon^*) = \int_{\varepsilon > \varepsilon^*} \varepsilon\, d\Phi(\varepsilon)$. Thus $\Sigma(\varepsilon^*)$ can be expressed as

$$\Sigma(\varepsilon^*) = [G + 1 - \Phi(\varepsilon^*)]\varepsilon^*. \tag{C.22}$$

Suppose that the steady-state capacity utilization rate is equal to 1. The steady-state version of (B.13) gives (C.7) and the steady-state version of (B.12) gives (C.6). Using these two equations, we can derive

$$\alpha\frac{\tilde{Y}}{\tilde{X}} = \frac{\tilde{Q}}{1 + G}\left[\frac{g_z g_\gamma}{\beta(1 - \delta_e)} - (1 - \delta(1)) - \bar{\zeta}G\right]. \tag{C.23}$$

Substituting equation (C.21) into the above equation yields:

$$\frac{\tilde{Q}\tilde{X}}{\tilde{Y}} = \varphi_x, \tag{C.24}$$

where φ_x is given by (C.19). In order to support the steady-state $u = 1$, we use equation (B.12) and (C.24) to show that condition (C.20) must be satisfied.

From (B.14), the end-of-period capital stock to the output ratio in the steady state satisfies

$$\frac{\tilde{K}}{\tilde{Y}} = \varphi_k\frac{\tilde{X}}{\tilde{Y}}, \tag{C.25}$$

where φ_k is given by (C.18). Then from equation (B.11), we can derive the steady-state relation:

$$
\begin{aligned}
\frac{\tilde{I}}{\tilde{Y}} &= \frac{1 - \Phi\left(\varepsilon^*\right)}{\Sigma\left(\varepsilon^*\right)}\left[\varphi_k - \left(1 - \delta\left(1\right)\right)\right]\frac{\tilde{X}}{\tilde{Y}} \\
&= \frac{1 - \Phi\left(\varepsilon^*\right)}{\left[G + 1 - \Phi\left(\varepsilon^*\right)\right]}\left[\varphi_k - \left(1 - \delta\left(1\right)\right)\right]\frac{\tilde{Q}\tilde{X}}{\tilde{Y}} \\
&= \frac{\left[1 - \Phi\left(\varepsilon^*\right)\right]\left[\varphi_k - \left(1 - \delta\left(1\right)\right)\right]\varphi_x}{G + 1 - \Phi\left(\varepsilon^*\right)},
\end{aligned}
\tag{C.26}
$$

where the second line follows from (C.22) and $\varepsilon^* = 1/\tilde{Q}$ and the last line follows from (C.24). After substituting (C.21) into the above equation, we solve for $1 - \Phi\left(\varepsilon^*\right)$:

$$
1 - \Phi\left(\varepsilon^*\right) = \frac{1/\left[\beta(1 - \delta_e)\bar{\theta}\right] - 1}{\left(\tilde{I}/\tilde{Y}\right)^{-1}\left[\varphi_k - \left(1 - \delta\left(1\right)\right)\right]\varphi_x - 1},
\tag{C.27}
$$

From (B.8), the steady-state total value of bubble to GDP ratio is given by

$$
\frac{\tilde{B}^a}{\tilde{Y}} = \frac{\tilde{I}}{\tilde{Y}}\frac{1}{1 - \Phi\left(\varepsilon^*\right)} - \alpha - \bar{\zeta}\frac{\tilde{Q}\tilde{X}}{\tilde{Y}}.
$$

Substituting (C.21), (C.26) and (C.24) into the above equation yields (C.17). We require $\tilde{B}^a/\tilde{Y} > 0$. By (23) and (34), the growth rate of bubbles of the surviving firms in the steady state is given by $\bar{\theta} = R_f/g_\gamma$. Q.E.D.

D   Log-linearized System

We eliminate equations for $\tilde{W}_t$ and $\tilde{R}_t$. The log-linearized system for 13 variables $\{\tilde{C}_t, \tilde{I}_t, \tilde{Y}_t, N_t,$ $\tilde{K}_t, u_t, \tilde{Q}_t, \tilde{X}_t, \tilde{P}_t, m_t, \tilde{B}_t^a, R_{ft}, \tilde{\Lambda}_t\}$ including two growth rates are summarized as follows:

1. Resource constraint:

$$
\hat{Y}_t = \frac{\tilde{C}}{\tilde{Y}}\hat{C}_t + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t.
\tag{D.1}
$$

2. Aggregate investment:

$$
\begin{aligned}
\hat{I}_t = &\ \frac{\alpha}{\alpha + \bar{\zeta}\varphi_x + \tilde{B}^a/\tilde{Y}}\hat{Y}_t + \frac{\bar{\zeta}\varphi_x}{\alpha + \bar{\zeta}\varphi_x + \tilde{B}^a/Y}\left(\hat{\zeta}_t + \hat{Q}_t + \hat{X}_t\right) \\
&+ \frac{\tilde{B}^a/\tilde{Y}}{\alpha + \bar{\zeta}\varphi_x + \tilde{B}^a/\tilde{Y}}\hat{B}_t^a - \mu\hat{\varepsilon}_t^* - \hat{P}_t,
\end{aligned}
\tag{D.2}
$$

where

$$
\mu = \frac{\phi\left(\varepsilon^*\right)\varepsilon^*}{1 - \Phi\left(\varepsilon^*\right)}, \quad \hat{\varepsilon}_t^* = \hat{P}_t - \hat{Q}_t.
\tag{D.3}
$$

3. Aggregate output:

$$
\tilde{Y}_t = \alpha\left(\hat{u}_t + \hat{X}_t\right) + \left(1 - \alpha\right)\hat{N}_t.
\tag{D.4}
$$

4. Labor supply:
$$\hat{\Lambda}_t + \hat{Y}_t - \hat{N}_t = \hat{\psi}_t. \tag{D.5}$$

5. End of period the capital stock:
$$\hat{K}_{t+1} = -\frac{\delta'(1)}{\varphi_k}\hat{u}_t + \frac{1-\delta(1)}{\varphi_k}\hat{X}_t + \left(1 - \frac{1-\delta(1)}{\varphi_k}\right)\left(\hat{I}_t - \frac{\mu}{\varphi_G}\hat{\varepsilon}_t^*\right), \tag{D.6}$$

where
$$\varphi_G \equiv -\frac{1-\Phi(\varepsilon^*)}{G} - 1. \tag{D.7}$$

6. Capacity utilization:
$$\hat{Y}_t - \hat{X}_t + \left[1 - \beta(1-\delta_e)\bar{\theta}\right]\varphi_G\hat{\varepsilon}_t^* = \hat{Q}_t + \left(1 + \frac{\delta''(1)}{\delta'(1)}\right)\hat{u}_t. \tag{D.8}$$

7. Marginal Q:
$$
\begin{aligned}
\hat{Q}_t &= E_t\left(\hat{\Lambda}_{t+1} - \hat{\Lambda}_t\right) + E_t\left(\hat{Q}_{t+1} - \hat{g}_{zt+1} - \hat{g}_{\gamma t+1}\right) \\
&+ \frac{\beta(1-\delta_e)\delta'(1)}{\bar{\lambda}_z g_\gamma}\frac{\delta''(1)}{\delta'(1)}E_t\hat{u}_{t+1} \\
&+ \frac{\bar{\zeta}\beta(1-\delta_e)G}{\bar{\lambda}_z g_\gamma}E_t\left(\hat{\zeta}_{t+1} + \varphi_G\hat{\varepsilon}_{t+1}^*\right).
\end{aligned} \tag{D.9}
$$

8. Effective capital stock
$$\hat{X}_t = \frac{1-\delta_e}{\bar{\lambda}_z g_\gamma}\varphi_k\left(\hat{K}_t - \hat{g}_{zt} - \hat{g}_{\gamma t}\right). \tag{D.10}$$

9. Euler equation for investment goods producers:
$$
\begin{aligned}
\hat{P}_t &= E_t[(1+\beta)\,\Omega g_\gamma^2\bar{\lambda}_z^2\hat{I}_t + \Omega\bar{\lambda}_z^2 g_\gamma^2\left(\hat{g}_{\gamma t} + \hat{g}_{zt}\right) - \Omega\bar{\lambda}_z^2 g_\gamma^2\hat{I}_{t-1} \\
&\quad - \beta\Omega\bar{\lambda}_z^2 g_\gamma^2\left(\hat{I}_{t+1} + \hat{g}_{zt+1} + \hat{g}_{\gamma t+1}\right)].
\end{aligned} \tag{D.11}
$$

10. Evolution of the number of bubbly firms:
$$\hat{m}_t = (1-\delta_e)\bar{\theta}\hat{m}_{t-1} + (1-\delta_e)\bar{\theta}\hat{\theta}_{t-1}. \tag{D.12}$$

11. Evolution of the total value of the bubble:
$$
\begin{aligned}
\hat{B}_t^a &= E_t\left(\hat{\Lambda}_{t+1} - \hat{\Lambda}_t + \hat{B}_{t+1}^a\right) + \left[1 - \beta(1-\delta_e)\bar{\theta}\right]\varphi_G E_t\hat{\varepsilon}_{t+1}^* \\
&+ \frac{1 - (1-\delta_e)\bar{\theta}}{(1-\delta_e)\bar{\theta}}E_t\hat{m}_{t+1}.
\end{aligned} \tag{D.13}
$$

12. The risk-free rate

$$-\hat{R}_{ft} = E_t\left(\hat{\Lambda}_{t+1} - \hat{\Lambda}_t - \hat{g}_{\gamma t+1}\right) + [1 - \beta(1 - \delta_e)R_f/g_\gamma]\,\varphi_G E_t\hat{\varepsilon}^*_{t+1}. \qquad \text{(D.14)}$$

13. Marginal utility for consumption:

$$
\begin{aligned}
\hat{\Lambda}_t \;=\;& \frac{g_\gamma}{g_\gamma - \beta h}\left[-\frac{g_\gamma}{g_\gamma - h}\hat{C}_t + \frac{h}{g_\gamma - h}\left(\hat{C}_{-1} - \hat{g}_{\gamma t}\right)\right] \\
& -\frac{\beta h}{g_\gamma - \beta h}E_t\left[-\frac{g_\gamma}{g_\gamma - h}\left(\hat{C}_{t+1} + \hat{g}_{\gamma t+1}\right) + \frac{h}{g_\gamma - h}\hat{C}_t\right]. \qquad \text{(D.15)}
\end{aligned}
$$

14. The growth rate of consumption goods

$$\hat{g}_{\gamma t} = \frac{\alpha}{1 - \alpha}\hat{\lambda}_{zt} + \left(\hat{\lambda}_{at} + \hat{A}^m_t - \hat{A}^m_{t-1}\right). \qquad \text{(D.16)}$$

15. The growth rate of the investment goods price:

$$\hat{g}_{zt} = \hat{\lambda}_{zt}. \qquad \text{(D.17)}$$

In the above system G is determined by (C.13),

$$G = \frac{1}{\beta\left(1 - \delta_e\right)\bar{\theta}} - 1, \qquad \text{(D.18)}$$

$(1 - \Phi\left(\varepsilon^*\right))$ is given by (C.27), and $\delta'\left(1\right)$ satisfies (C.20). The log-linearized shock processes are listed below.

1. The permanent technology shock:

$$\hat{\lambda}_{at} = \rho_a\hat{\lambda}_{at-1} + \varepsilon_{at}. \qquad \text{(D.19)}$$

2. The transitory technology shock:

$$\hat{A}^m_t = \rho_{am}\hat{A}^m_{t-1} + \varepsilon_{a^m,t}. \qquad \text{(D.20)}$$

3. The permanent investment-specific technology shock:

$$\hat{\lambda}_{zt} = \rho_z\hat{\lambda}_{zt-1} + \varepsilon_{zt}. \qquad \text{(D.21)}$$

4. The labor supply shock:

$$\hat{\psi}_t = \rho_\psi\hat{\psi}_{t-1} + \varepsilon_{\psi t}. \qquad \text{(D.22)}$$

5. The financial shock:

$$\hat{\zeta}_t = \rho_\zeta\hat{\zeta}_{t-1} + \varepsilon_{\zeta t}. \qquad \text{(D.23)}$$

6. The sentiment shock:

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \varepsilon_{\theta t}. \qquad \text{(D.24)}$$

Table E.1 Prior and posterior distributions of shock parameters

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	St.Dev.	Mode	Mean	5%	95%
h	Beta	0.333	0.235	0.5438	0.5434	0.4871	0.6052
Ω	Gamma	2	2	0.0290	0.0344	0.0112	0.0573
δ''/δ'	Gamma	1	1	11.4409	11.9169	8.3284	15.4937
$\bar{\zeta}$	Beta	0.3	0.1	0.2918	0.2969	0.2246	0.3595
μ	Gamma	2	2	2.5714	2.6029	2.1241	3.1928
f_1	Gamma	1	1	0.0535	0.0440	0.0132	0.0718
f_2	Gamma	1	1	4.7268	4.8177	2.5445	7.0784
f_3	Gamma	1	1	0.4164	0.3232	0.0047	0.5628
ρ_a	Beta	0.5	0.2	0.9615	0.9650	0.9432	0.9866
ρ_{a^m}	Beta	0.5	0.2	0.9672	0.9636	0.9456	0.9806
ρ_z	Beta	0.5	0.2	0.3592	0.3411	0.2229	0.4568
ρ_θ	Beta	0.5	0.2	0.9304	0.9238	0.8970	0.9521
ρ_ψ	Beta	0.5	0.2	0.9874	0.9799	0.9638	0.9965
ρ_ζ	Beta	0.5	0.2	0.8777	0.8735	0.8089	0.9411
σ_a	Inv-Gamma	0.01	Inf	0.0023	0.0023	0.0018	0.0029
σ_{a^m}	Inv-Gamma	0.01	Inf	0.0103	0.0104	0.0093	0.0116
σ_z	Inv-Gamma	0.01	Inf	0.0059	0.0060	0.0054	0.0067
σ_θ	Inv-Gamma	0.01	Inf	*0.1785*	*0.1946*	*0.1165*	*0.2638*
σ_ψ	Inv-Gamma	0.01	Inf	0.0081	0.0082	0.0072	0.0093
σ_ζ	Inv-Gamma	0.01	Inf	0.0077	0.0084	0.0042	0.0121

E   Robustness

To see if our result is robust for a smaller prior mean of σ_θ, we set the prior as Inv-Gamma with mean 0.01 and standard deviation infinite. We re-do Bayesian estimation and report estimation results in Table E.1. We find that these results are very similar to those in the baseline estimation.