Agendas with Priority*

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Draft: December 31, 2014

Abstract

While a wide variety of agendas are used in legislative voting, the literature focuses almost exclusively on Euro-Latin and Anglo-American agendas. These two agendas share three features common to many agendas used in practice: they are *non-repetitive* in that every vote eliminates some alternatives from consideration; *continuous* in that alternatives continue to be contested until they are either eliminated or selected; and, *prioritarian* in that the structure of the agenda discourages the choice of certain alternatives.

In this paper, I characterize the much broader class of social choice rules implemented by *sophisticated voting* on agendas with these features. Not only does the result provide key insights into the kinds of strategic voting outcomes that can arise in the context of legislative voting, it has implications for issues ranging from implementation to tournament solutions and agenda manipulation.

JEL Classification Numbers: C72; D02; D71; D72.

Keywords: Sophisticated Voting; Majority Voting; Agendas; Committees; Axioms; Implementation.

*I would like to thank Michael Gavin and Gail Sullivan for their encouragement; and, Vikram Manjunath and Hervé Moulin for helpful suggestions that have improved the paper considerably. I also thank seminar participants at the *Québec Political Economy Conference*, the *Social Choice and Welfare Conference*, the *COMSOC Conference*, UdeM, and ITAM for their comments—particularly Miguel Ballester, Felix Brandt, Alessandro Lizzeri, Nicola Persico, and Ariel Procaccia.

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I. Introduction

Agenda voting is ubiquitous in legislative decision-making. While a wide variety of agendas are used in practice (see Farquharson [1969]; Miller [1995]; Ordeshook and Schwartz [1987]; and, Riker [1958]), the literature has only studied *sophisticated* (or forward looking) voting (Farquharson [1969]) for a small handful of specific agendas, notably the so-called *Euro-Latin* and *Anglo-American* agendas (see Apesteguia et al. [2014]; Banks [1985]; Miller [1977, 1980, 1995]; and, Sheplse and Weingast [1984]).[1]

In this paper, I introduce a much broader class of agendas, called *priority agendas*, and characterize sophisticated voting outcomes associated with these agendas. For priority agendas, alternatives are added one at a time using a *priority order* and an *amendment rule*. While the former determines *when* a given alternative is to be added to the agenda, the latter determines *how* it is to be added.

The simple recursive structure of these agendas reflects the inherently incremental nature of the legislative process. Since they account for this reality, priority agendas possess two features associated with almost every agenda used in practice (see Ordeshook and Schwartz [1987]; also, Miller [1995]): every vote eliminates some alternatives from consideration; and, alternatives are contested until they are either eliminated or ultimately selected.[2] In other words, priority agendas are *non-repetitive* and *continuous*.

At the same time, priority agendas depart only minimally from the paradigm of Euro-Latin and Anglo-American agendas. Effectively, they only expand the scope of possible proposals. To elaborate, recall that agendas induce "binary" extensive-form games with majority voting in every stage. The figure below illustrates the induced Euro-Latin and Anglo-American games for three alternatives:

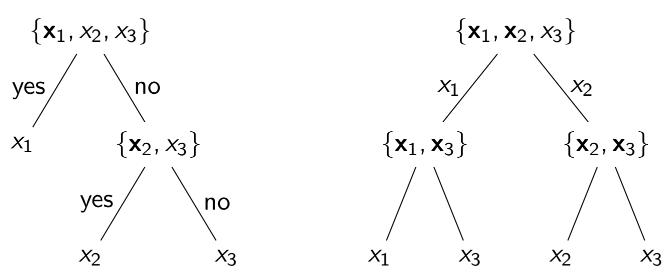

Figure 1: Euro-Latin (*left*) and Anglo-American (*right*) agendas on three alternatives

For the Euro-Latin agenda, voting is by *sequential majority approval*. In every stage, voters consider one alternative for approval (in bold). The selection from the agenda is the first alternative approved by majority. For the *Anglo-American* agenda, voting is by *sequential majority comparison*. In every stage, voters compare two alternatives (in bold) – with the "loser" being eliminated and "winner" moving on to the next stage. The selection is the only alternative not eliminated by the end of this process.

It is straightforward to extend both agendas by proposing new alternatives as "amendments" to items already on the agenda.[3] For a Euro-Latin agenda, a new proposal amends the *last* alternative proposed.

[1] While a variety of names have been used for these two agendas, I follow the nomenclature of Schwartz [2008].

[2] Notably, the two-stage amendment agendas studied by Banks [1989] do not possess the second feature.

[3] I use the term "amendment" only out of convenience. Whether the proposal associated with a given alternative is technically designated an "amendment", a "motion", or a "substitute bill" will depend on how it is added to the agenda.

For an Anglo-American agenda, a new proposal amends *every* alternative proposed before it. To illustrate:

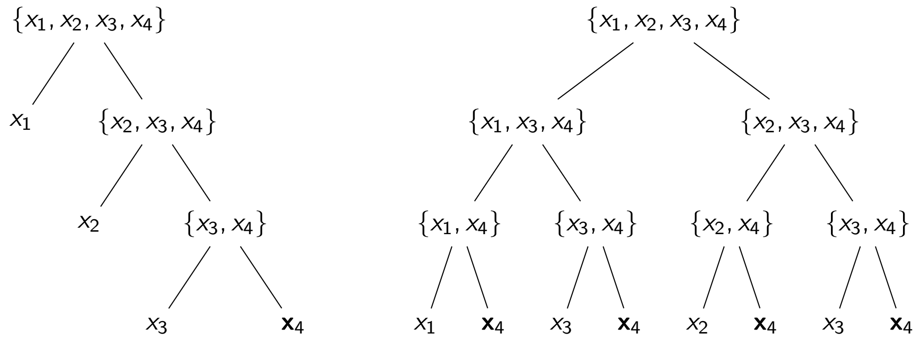

Figure 2: Extending the Euro-Latin (*left*) and Anglo-American (*right*) agendas in Figure 1

Intuitively, the force of an amendment is to confront the voters with an additional decision. For a *Euro-Latin amendment* (*left*), the voters only face such a decision when they would otherwise select the previously last alternative on the agenda (i.e. x_3). For an *Anglo-American amendment* (*right*), the voters face an additional decision regardless of which alternative they would otherwise select.

The more flexible structure of priority agendas allows the agenda-setter to mix and match these two kinds of amendments: a Euro-Latin agenda can be extended by Anglo-American amendment; and, likewise, an Anglo-American agenda can be extended by Euro-Latin amendment. To illustrate:

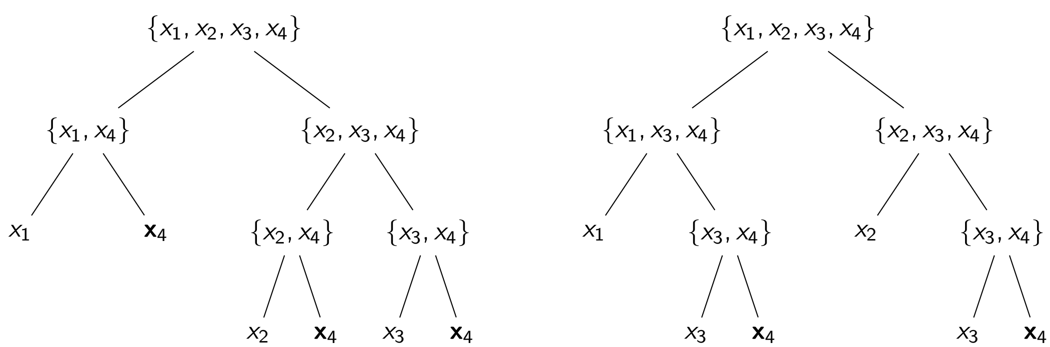

Figure 3: Priority agendas that extend Euro-Latin (*left*) and Anglo-American (*right*) agendas

More broadly, the amendment rules of priority agendas allow a new addition y to amend any alternative x already on the agenda—subject only to the natural restriction that y also amend every alternative added to the agenda after x. Intuitively, an alternative that "takes issue" with a particular alternative on the agenda must also take issue with the other additions that took issue with the same alternative.

Preview of Results: I establish four results related to sophisticated voting—for priority agendas as well as a wider range of agendas used in practice. These results provide key insights into the kinds of strategic voting outcomes that one might expect in the context of legislative decision-making.

Before turning to priority agendas, I first consider the more general case of non-repetitive and continuous agendas in Section III. This broad class of *simple agendas* includes every priority agenda as well as a variety of non-priority agendas. I show that an intuitive *Issue Splitting* condition, which weakens *Path Independence* (Plott [1973]), is effectively sufficient for implementation by sophisticated voting on a simple agenda (Theorem 1). In particular, every social choice function satisfying this condition can be implemented by a *unique* simple agenda. What is more, these agendas have the additional feature that the removal of any alternative results in another simple agenda (Theorem 1*). While not discussed in the literature, many agendas used in practice seem to possess this "recursively simple" structure.

In Section IV, I take up the main question of implementation by priority agenda. I show that every social choice function implementable by priority agenda satisfies a property previously only associated with Euro-Latin and Anglo-American agendas.[4] In particular, there exists a *marginal alternative* which is selected only for preference profiles where it compares favourably with every other alternative (Proposition 1). As with Euro-Latin and Anglo-American agendas, this alternative corresponds with the last addition to the agenda (Miller [1977]; Moulin [1991, Exercise 9.5]). Combined with Issue Splitting, this *Weak Marginalization* property effectively characterizes implementability by priority agenda (Theorem 2).

To conclude, I highlight three ways that priority agendas extend our understanding of legislative voting beyond what is known about sophisticated voting on Euro-Latin and Anglo-American agendas:

(1) Every priority agenda satisfies two normatively appealing *monotonicity* properties previously only known to hold for Euro-Latin and Anglo-American agendas (see Moulin [1986]); Jung [1990]; and, Moulin [1991, Exercise 9.5]). Like these two agendas, the sophisticated voting outcome on *every* priority agenda remains unchanged when it either "improves" in terms of voter preference (Proposition 2) or in terms of priority (Proposition 3). Given Proposition 1, this shows that every property of Euro-Latin and Anglo-American agendas emphasized in the literature holds more generally for priority agendas.

(2) Despite (1), there is a principled way to distinguish Euro-Latin and Anglo-American agendas from every other priority agenda: they are the only priority agendas that, subject to "marginalizing" certain alternatives, otherwise treat the alternatives *neutrally* (Proposition 4). Unlike the separate characterizations of Euro-Latin and Anglo-American voting due to Apesteguia et al. [2014], this common characterization emphasizes a deep *similarity* between the two procedures rather than their *differences*.

(3) By varying how priority is assigned to the alternatives, the Euro-Latin and Anglo-American agendas can be used to define two *tournament solutions* (see Laslier [1997] or Brandt et al. [2015] for an overview of the vast literature on tournament solutions). In particular, the alternatives selected on *some* Euro-Latin agenda (Miller [1977]) define the *Top Cycle*; and, those selected on *some* Anglo-American agenda define the *Banks Set* (Banks [1985]). Since the latter refines the former (see Banks [1985]), the Anglo-American agenda is more discriminating than the Euro-Latin agenda.

This observation generalizes to priority agendas. For a given priority agenda, every alternative in the Banks Set is selected for *some* assignment of priority; and, some alternatives in the Top Cycle may not be selected for *any* assignment of priority. Among priority agendas, this show that the Anglo-American agenda is the most discriminating agenda while the Euro-Latin agenda is the least discriminating.

[4] The recent paper of Iglesias et al. [2014] defines an entire family of "Anglo-American style" agendas with this feature.

Related Literature: At one extreme, the literature on sophisticated voting considers only a narrow class of agendas. Besides the cited work on Euro-Latin and Anglo-American agendas, Banks [1989] studies sophisticated voting for *two-stage amendment agendas*; and, a few papers examine agendas implementing outcomes in the *Iterated Banks Set* (Coughlan and Le Breton [1999]) or outcomes with "high" *Copeland scores* (Fischer et al. [2011]; Horan [2013]; and, Iglesias et al. [2014]).

At the other, the literature focuses on necessary and sufficient conditions for implementation (see McKelvey and Niemi [1978]; Moulin [1986]; Srivastava and Trick [1996]; and, Horan [2013]; see also Brandt et al. [2015] for an overview). While this clearly delimits what can be implemented by sophisticated agenda voting *in general*, it does not help clarify what can be implemented by any *specific* agenda. In part, this lacuna is related to the fact that the results rely on non-constructive proof techniques.

The current paper bridges the gap between these disparate strands of the agenda voting literature. By characterizing sophisticated voting for a wide range of agendas used in practice, it sheds light on the voting outcomes that can be implemented by decision-making procedures used in practice.

II. Basic Definitions

In this section, I briefly review the basic definitions and concepts used in the paper.

The environment consists of an odd number of voters with linear order preferences over the alternatives in a finite set X. A *preference profile* of voters is denoted by P and the collection of all profiles by $\mathbf{P}$. A *decision problem* (P, A) consists of a profile P and a set of alternatives (known as an *issue*) $A \subseteq X$. Where $\mathbf{X}$ denotes the collection of non-empty issues, a *decision rule* defines a mapping $v : \mathbf{P} \times \mathbf{X} \to X$ which selects a single social outcome $v(P, A) \in A$ for every decision problem $(P, A) \in \mathbf{P} \times \mathbf{X}$.

I study the implementation of decision rules by agenda voting. To formalize the notion of an agenda:

Definition 1 An **agenda** $\mathcal{T}_X$ on a set of alternatives X is a rooted binary tree such that:

(1) every terminal node is labeled by (a set consisting of) one alternative in X;[5]

(2) every alternative in X labels one or more terminal nodes; and,

(3) every non-terminal node is labeled by the set of alternatives that label its two successors.[6]

Figures 1-3 of the Introduction clearly illustrate these three features.

For an issue $A \subset X$, one can "prune" the agenda $\mathcal{T}_X$ by deleting the terminal nodes labeled by alternatives in $X \setminus A$. This operation is considered in a number of other papers (Bossert and Sprumont [2013]; Horan [2011]; and, Xu and Zhou [2007]). Like an elimination-style tournament in sports, the idea is that the infeasible alternatives "forfeit" without changing the structure of the agenda.

Definition 2 Given an agenda $\mathcal{T}_X$, the **pruned agenda** $\mathcal{T}_{X|A}$ for an issue $A \subseteq X$ is defined as follows. In sequence, carry out the following three steps:

(1) first delete every terminal node of $\mathcal{T}_X$ labeled by an alternative $x \in X \setminus A$;

[5]Wherever it causes no confusion, I abuse set notation by omitting the brackets for singleton sets.

[6]Property (3) follows the "Farquharson-Miller" definition of agendas rather than the "Ordeshook-Schwartz" definition (see Schwartz [2008]). Since the interest is sophisticated voting rather than *sincere* voting, this is without loss of generality.

(2) then, delete every node with a unique successor, connecting its successor to its predecessor;[7]

(3) and, finally, relabel every non-terminal node of the resulting tree to conform with Definition 1.

To illustrate, consider the agenda $\mathcal{T}_X$ below and the pruned agenda associated with the issue $\{b, c, x\}$:

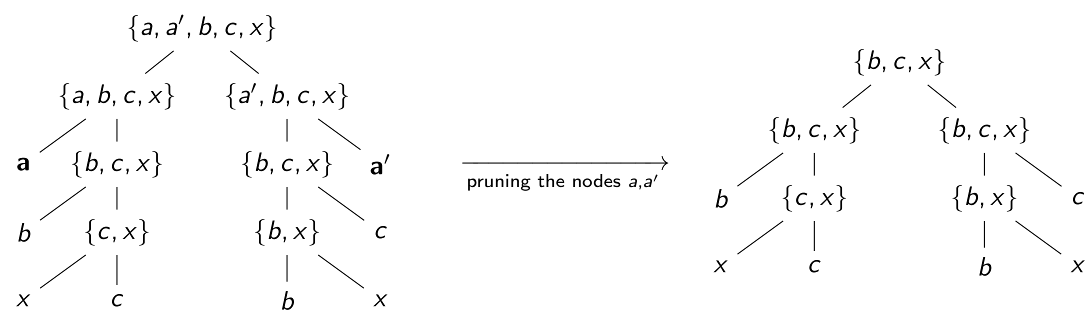

Figure 4: An agenda $\mathcal{T}_X$ (*left*) and its associated pruned agenda $\mathcal{T}_{X|\{b,c,x\}}$ (*right*)

Every agenda $\mathcal{T}_{X|A}$ defines an extensive game form where the outcomes are the terminal nodes and the stage games (or *decision nodes*) consist of majority voting between two subgames. Given a decision problem (P, A), the pair $(\mathcal{T}_{X|A}; P)$ describes a complete information extensive-form game on the pruned agenda $\mathcal{T}_{X|A}$. Every such game is *dominance solvable* (Moulin [1979]). In other words, $(\mathcal{T}_{X|A}; P)$ has a unique *undominated Nash equilibrium* outcome, denoted by $UNE[\mathcal{T}_{X|A}; P]$, for all $A \subseteq X$.

This solution concept corresponds to Farquharson's [1969] notion of *sophisticated voting*. The idea is that sophisticated voters anticipate the outcome of voting in later stages. Since they have a dominant strategy to endorse their preferred candidate in any terminal subgame, voters discount alternatives that lose at this stage. By using this "backward induction" reasoning to roll back the agenda to the root, one obtains the undominated Nash equilibrium outcome (McKelvey and Niemi [1978]).

To formalize the notion of implementation considered in the paper:

Definition 3 *A decision rule v is* ***implementable by agenda*** *if there exists an agenda $\mathcal{T}_X$ such that*

$$v(P, A) = UNE[\mathcal{T}_{X|A}; P]$$

for every decision problem (P, A). In this case, the decision rule v is ***implemented*** *by the agenda $\mathcal{T}_X$.*

Despite superficial appearances, this notion of implementation is no more (and no less) general than the standard notion of implementation where the issue is fixed (i.e. it does not vary from X).

Remark 1 *If a decision rule v is implementable by agenda, then $v(P, A) = v(P^A, X)$ for any profile P^A that coincides with P on A but "demotes" every $x \in X \setminus A$ below all $a \in A$ in every voter preference.*

This shows that the sub-issues $A \subset X$ formally contribute nothing to the difficulty of the implementation problem. Once the agenda-setter has determined what to implement for X, the outcomes for all sub-issues

[7] In case the root node (which has no predecessor) has a unique successor, its successor becomes the new root node.

are determined. Having said this, there are compelling reasons to think about the sub-issues explicitly. For one, some problems of economic interest depend on how the outcomes change when some alternatives become unavailable (as discussed at greater length in Section VI). No less compelling, the sub-issues simplify the statement of the conditions for implementation as well as their interpretation.

III. Implementation by Simple Agenda

After formally defining simple agendas in part (a), I identify two conditions that are sufficient for implementation by simple agenda in part (b). Finally, I provide in part (c) a "recipe" for constructing a simple agenda to implement any decision rule which satisfies these sufficient conditions.

(a) Definition

Given a non-terminal node A of an agenda $\mathcal{T}_X$, the alternative $x \in A$ is said to be *contested* at A if $x \in B \setminus C$ where B and C denote the successors of A. For a simple agenda, every non-terminal node involves a "contest" between two alternatives; and, the "winner" of this contest continues to be contested by other alternatives until it is either eliminated or ultimately selected as the outcome. To formalize:

Definition 4 *A* ***simple agenda*** $\mathcal{S}_X$ *on* X *is an agenda such that*

(i) there exists an alternative $b \in B \setminus C$ *that labels exactly one terminal node below* B *and,*

(ii) there exists an alternative $c \in C \setminus B$ *that labels exactly one terminal node below* C

for every non-terminal node A *of* $\mathcal{S}_X$ *whose successors are labeled* B *and* C.[8]

Equivalently, an agenda is *simple* if it is at once *non-repetitive* and *continuous*.

The non-repetitiveness feature refers to the fact that every stage of voting in the agenda eliminates some alternatives *regardless* of which subgame the voters actually select. Formally:

Definition 5 *An agenda* $\mathcal{T}_X$ *is* ***non-repetitive*** *if*

$$B, C \subset A \text{ for every non-terminal node } A \text{ of } \mathcal{T}_X \text{ whose successors are labeled } B \text{ and } C.$$

Non-repetitive agendas have the appealing feature that the outcome is invariably determined by relatively few votes. Since every subgame contains alternatives unavailable at its "sibling" subgame, the height of a non-repetitive agenda on X is at most $|X| - 1$ so that the number of potential votes is limited by the number of alternatives in X. Clearly, no agenda can guarantee fewer votes in the worst case.

In turn, the continuity feature refers to the fact that some alternatives contested at any stage continue to be contested until they are either eliminated or selected as the outcome. Formally:

Definition 6 *An agenda* $\mathcal{T}_X$ *is* ***continuous*** *if, for every non-terminal node* A *of* $\mathcal{T}_X$ *whose successors are labeled* B *and* C*, some alternative* $x \in B$ *contested at* A *labels exactly one terminal node below* B.[9]

[8] A simple agenda on X will always be denoted by $\mathcal{S}_X$ to help distinguish it from a generic agenda $\mathcal{T}_X$.

[9] Continuity was originally defined only for "Ordeshook-Schwartz" agendas (footnote 6). The stated definition adapts the concept to "Farquharson-Miller" agendas in a way that addresses the concerns of Groseclose and Krebhiel [1993].

For some alternative x contested at the node A of a continuous agenda, there is a unique path starting at A that leads to the selection of x. On this path, the alternative x is contested at every node. Intuitively, this means that every stage of voting on path may be interpreted as a choice between continuing to entertain the possibility of selecting x and rejecting this alternative once and for all.

To help illustrate these two properties, consider the following pair of agendas:

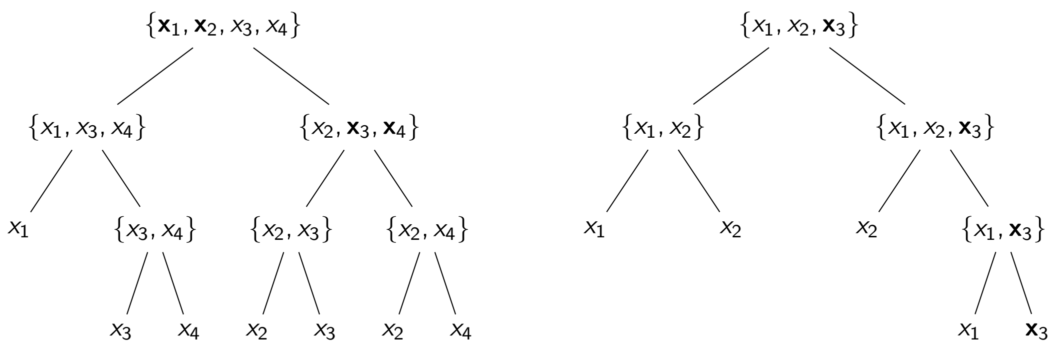

Figure 5: A non-repetitive non-continuous agenda (*left*) and a repetitive continuous agenda (*right*)

The left-hand agenda is non-repetitive: the two successors of each node contain a strict subset of the alternatives. However, it is also non-continuous: while x_2 is contested at the root node, it is not contested at the successor $\{x_2, x_3, x_4\}$.[10] Indeed, it is the *only* alternative not contested at $\{x_2, x_3, x_4\}$.

Conversely, the right-hand agenda is repetitive: the right successor of the root node contains the same alternatives as the root node. At the same time, it is continuous: the only alternative contested at the root node (x_3) appears at a single terminal node (and is thus contested all along this path).

(b) Sufficient Conditions

The sufficient conditions for implementation by simple agenda are natural restrictions related to Plott's [1973] *Path Independence* and Arrow's [1950] *Independence of Irrelevant Alternatives* (IIA).

The first condition, which weakens Path Independence, states that the outcome for every issue can be determined by splitting it into simpler sub-issues. For a decision rule v, an issue $A \subseteq X$ can be *split* if there exists a pair of issues (B, C), called a *splitting*, such that: (i) $B \cap C \neq B, C$ (i.e. B and C are *distinctive*) and $B \cup C = A$ (i.e. B and C *cover* A); and, (ii) $v(P, A) = v(P, \{v(P, B), v(P, C)\})$ for every profile P. To state the splitting condition more formally:

Issue Splitting **(IS)** *For v, every issue can be split into sub-issues.*

By comparison, Path Independence imposes the stronger requirement that the identity in (ii) must hold for *all* pairs of sub-issues (B, C) that cover A, *regardless* of whether these issues are distinctive.[11]

[10] On the other hand, the other alternative contested at the root (x_1) appears at a single terminal node below $\{x_1, x_3, x_4\}$.

[11] This property also weakens the *Division Consistency* (DC) condition of Apesteguia et al. [2014]. For one, it does not require the sub-issues (B, C) of A to be disjoint. More significantly, it does not impose any consistency between the splitting of A and its sub-issues. That is, IS does not require $v(P, D) = v(P, \{v(P, B \cap D), v(P, C \cap D)\})$ for any $D \subset A$.

In the spirit of IIA, the second condition states that outcomes are not affected by alternatives that *never* appeal to a majority.[12] To formalize, an alternative $a \in A$ is the *Condorcet loser* for the decision problem (P, A) if, for all $x \in A \setminus a$, the majority of voters in P prefer x to a. Then, a decision rule v is *independent of the losers* for the issue A if $v(P, A) = v(P, A \setminus a)$ for every profile P where a is the Condorcet loser on (P, A). The condition requires this kind of independence for every issue:

***Independence of the Losing Alternatives* (ILA)** *For every issue, v is independent of the losers.*

Theorem 1 shows that every decision rule v satisfying these two conditions is implementable by a unique simple agenda $\mathcal{S}^v_X$. As discussed in section (c) below, the structure of $\mathcal{S}^v_X$ is straightforward to determine from the outcomes of v for decision problems of three alternatives, called *Condorcet triples*, where the pairwise majority preference forms a cycle:

Theorem 1 *If a decision rule v satisfies IS and ILA, then it is implementable by a unique simple agenda $\mathcal{S}^v_X$ whose structure is determined by the outcomes on Condorcet triples.*

It is worth commenting on the necessity of the two conditions. Clearly, ILA is necessary for implementation by simple agenda. Indeed, it is necessary for implementation by *any* kind of agenda. Since the Condorcet loser cannot win a majority vote in any terminal subgame of an agenda, sophisticated voters disregard it when deciding how to vote in earlier stages. By extending this reasoning, McKelvey and Niemi [1978] identify a more general necessary condition (discussed in Sections V-VI below).

In contrast, IS is not necessary for implementation by simple agenda. To see this, consider the decision rule implemented by the simple agenda $\mathcal{T}_X$ in Figure 4. Let P_{xbc} denote the Condorcet triple with cycle orientation xbc (i.e. x is preferred by majority to b, b to c, and c to x); and, let P_{xcb} denote the triple with the reverse orientation. "Backward induction" shows that $\mathcal{T}_{X|\{b,c,x\}}$ implements a rule that selects b for P_{xbc} and c for P_{xcb}. In other words, $\mathcal{T}_{X|\{b,c,x\}}$ selects the majority preferred alternative between b and c. However, there is no way to do this by splitting $\{b, c, x\}$ (as shown in Table 1 below).[13]

(c) A Recipe

IS and ILA ensure that there is a unique way to split every issue (see Claim 10 of the Appendix).

To get the basic intuition, suppose that (B, C) splits A. Given IS and ILA, it follows that $v(P, D) = v(P, \{v(P, B \cap D), v(P, C \cap D)\})$ on every sub-issue $D \subseteq A$. By way of contradiction, suppose that (B', C') also splits A. If (B, C) and (B', C') are distinct, then there exists a sub-issue $D \subseteq A$ and a profile P that forms a Condorcet triple on D such that

$$v(P, D) = v(P, \{v(P, B \cap D), v(P, C \cap D)\}) \neq v(P, \{v(P, B' \cap D), v(P, C' \cap D)\}) = v(P, D).$$

The unique splitting of every issue makes is straightforward to define the agenda $\mathcal{S}^v_X$ from Theorem 1 recursively. First, define a root node and label it X. Then, for any existing node whose label is a non-

[12] Apesteguia et al. [2014] call this condition *Condorcet Loser Consistency*.

[13] The outcome for P_{xbc} (resp. P_{xcb}) implies that x cannot be paired with b (resp. c). So, the only potential splitting is $(x, \{b, c\})$. Since this requires x as the outcome for both P_{xbc} and P_{xcb} however, there is no way to split $\{b, c, x\}$.

singleton issue A, construct two successors nodes B_A and C_A, labelled according to the unique splitting (B_A, C_A) of A. To illustrate this construction:

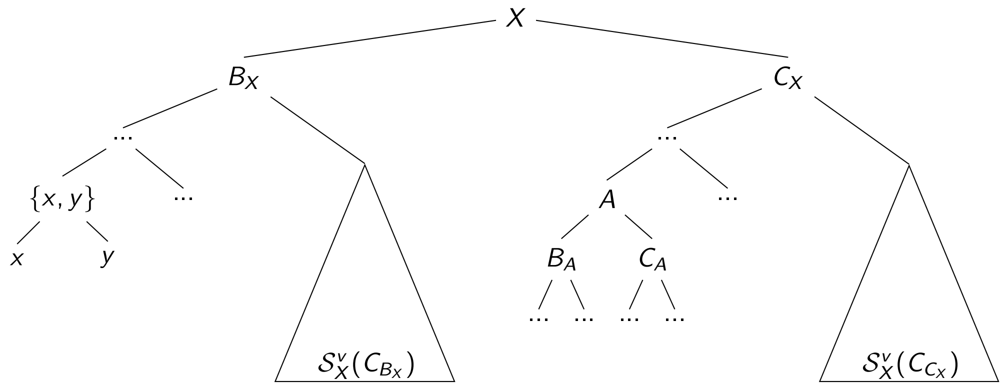

Figure 6: The recursive construction of $\mathcal{S}^v_X$

In Figure 6, the leftmost nodes below B_X illustrate the construction for $|A| = 2$ while the leftmost nodes below C_X illustrate it for $|A| > 2$. In turn, the two triangles represent the subgames starting from the nodes labeled C_{B_X} and C_{C_X} while the ellipses indicate where some details have been omitted.[14]

As Theorem 1 indicates, the outcomes for Condorcet triples may be used to describe $\mathcal{S}^v_X$ more explicitly. For the issue $\{x, b, c\}$, there are three splittings where b and c appear in separate sub-issues:

$$(\{b, x\}, \{c, x\}) \quad (b, \{c, x\}) \quad (\{b, x\}, c)$$

Each of these splittings corresponds to the initial stage game of a different simple agenda on $\{x, b, c\}$:

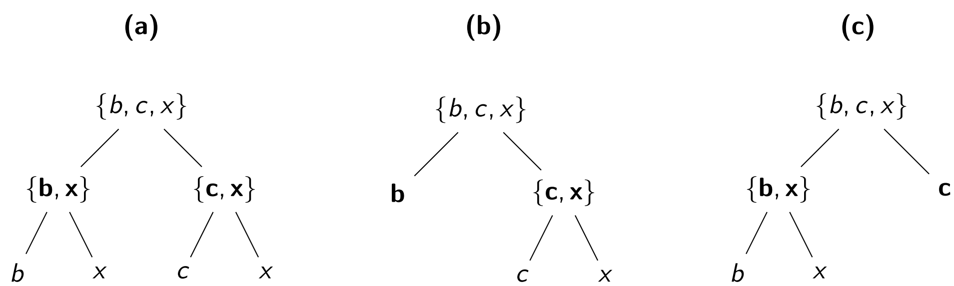

Figure 7: Simple agendas on three alternatives $\{x, b, c\}$

These three agendas implement different combinations of outcomes for the triples P_{xbc} and P_{xcb}:

[14]It is important not to confuse $\mathcal{S}_X(A)$ and $\mathcal{S}_{X|A}$. While the former refers to the subgame at node A in $\mathcal{S}_X$, the latter refers to the agenda obtained from $\mathcal{S}_X$ by pruning away $X \setminus A$. As Figure 4 illustrates, these agendas may be quite different.

Profile\Agenda	(a)	(b)	(c)
P_{xbc}	c	b	c
P_{xcb}	b	b	c
Outcomes	Majority loser between b and c	Outcome b for both triples	Outcome c for both triples

Table 1: Outcomes implemented by agendas (a)-(c) on $\{x, b, c\}$

By construction, there exist alternatives b_A and c_A that appear only on opposite sides of the agenda $\mathcal{S}^v_X(A)$ starting at any node A. For these alternatives, the outcomes on any issue $\{x, b_A, c_A\}$ involving an $x \in A$ must coincide with one of the possibilities in Table 1. After using this observation to locate some $b_A, c_A \in A$, one can use Table 1 to describe the splitting (B_A, C_A) of A in terms of Condorcet triples:

$$B_A \equiv \{b_A\} \cup \{x \in A : \text{type-(a) or type-(c) outcomes on } \{x, b_A, c_A\}\}$$
$$C_A \equiv \{c_A\} \cup \{x \in A : \text{type-(a) or type-(b) outcomes on } \{x, b_A, c_A\}\}$$

Intuitively, type-(a) outcomes reveal that x appears in both sub-issues of (B_A, C_A) while type-(c) outcomes (resp. type-(b) outcomes) reveal that x appears only in the same sub-issue as b_A (resp. c_A).

While the goal was to define a simple agenda for the "grand" issue X, the same approach also defines a simple agenda $\mathcal{S}^v_A$ that implements v for any issue $A \subset X$. This is a straightforward consequence of IS and ILA. This highlights a practical feature of decision rules satisfying these conditions. Instead of using the pruned agenda $\mathcal{S}^v_{X|A}$ to implement the *social choice function* $v(\,\cdot\,, A) : \mathbf{P} \to A$, one can use $\mathcal{S}^v_A$. The advantage is that the latter requires less voting. While every decision node in $\mathcal{S}^v_A$ must affect the outcome for some profile, $\mathcal{S}^v_{X|A}$ may include redundant nodes that cannot affect the outcome for any profile.

To summarize, IS ensures that it is possible to "simplify" the agenda implementing a decision rule on X for every issue $A \subseteq X$. Indeed, it is the defining property of agenda voting rules with this feature:

Theorem 1* *If a decision rule v is implementable by agenda, then it satisfies IS if and only if the social choice function $v(\,\cdot\,, A)$ is implementable by simple agenda for every issue A.*

IV. Implementation by Priority Agenda

I define priority agendas in part (a) and characterize the decision rules that they implement in part (b).

(a) Definition

A priority agenda on X is defined by a pair $(\succsim, \alpha)$ consisting of a *weak priority* $\succsim$ and an *amendment rule* α. Intuitively, $(\succsim, \alpha)$ provides a way to construct the agenda by progressively adding alternatives. Whereas $\succsim$ determines *when* each alternative is added, α determines *how* each is added.

Formally, $\succsim$ is a weak order on X whose indifference classes contain *at most* two alternatives. When $x \succ y$, the idea is that x has higher priority and is added to the agenda before y. When $x \sim y$ however, the two alternatives have *equal* priority and may be added to the agenda in either order.

To formalize the amendment rule α, let X_j denote the j^{th} highest indifference class of $\succsim$ and let $\widetilde{X} \equiv \{\{x\} : x \in X\} \cup \{X_j : |X_j| \neq 1\}$ denote the collection of singletons and equal priority pairs in X. Using this notation, the amendment rule is a mapping $\alpha : X \setminus X_1 \to \widetilde{X}$.

An alternative z is said to *amend* another alternative x if:

$$x \in \alpha(z); \text{ or, } \; y \succ x \succ z \text{ for some } y \in \alpha(z).$$

The interpretation is that, when z is added to the agenda, it amends $y \in \alpha(z)$ and every alternative x already on the agenda with strictly lower priority than y. Consistent with this interpretation, α must satisfy the following restrictions: (i) every new addition to the agenda amends some alternative already on the agenda; (ii) alternatives with the same priority amend the same alternatives; and, (iii) every alternative whose priority is immediately below two alternatives with the same priority amends both.[15]

The Euro-Latin and Anglo-American agendas are easy to describe in terms of this notation:

	$\succsim$	α
Euro-Latin	$x_1 \succ \ldots \succ x_{n-1} \sim x_n$	$\alpha(x_i) = \begin{cases} \{x_{i-1}\} & \text{for } i \neq n \\ \{x_{n-2}\} & \text{for } i = n \end{cases}$
Anglo-American	$x_1 \sim x_2 \succ \ldots \succ x_n$	$\alpha(x_i) = \{x_1, x_2\}$

Table 2: $(\succsim, \alpha)$ for Euro-Latin and Anglo-American agendas of n alternatives

To reconstruct the Euro-Latin and Anglo-American agendas from the $(\succsim, \alpha)$ pairs in this table, one simply adds the alternatives in decreasing order of priority $\succsim$ using the amendment rule α. Indeed, the same type of construction defines an agenda for every priority-amendment pair $(\succsim, \alpha)$. To formalize:

Definition 7 For any pair $(\succsim, \alpha)$ on X, the **priority agenda** $\mathcal{P}_{(\succsim,\alpha)}$ is defined recursively as follows:

(1) Define $\mathcal{P}^1_{(\succsim,\alpha)}$ to be the simple agenda $\mathcal{S}_{X_1}$ on the highest indifference class X_1 of $\succsim$.

(2) Define $\mathcal{P}^{j+1}_{(\succsim,\alpha)}$ by adding the alternatives $x_{j+1} \in X_{j+1}$ to $\mathcal{P}^j_{(\succsim,\alpha)}$ as follows:

(i) Replace every terminal node of $\mathcal{P}^j_{(\succsim,\alpha)}$ labeled by:

- $x_k \in \alpha(x_{j+1})$ with the simple agenda $\mathcal{S}_{\{x_k, X_{j+1}\}}$; and,
- $x_{k'} \in X_{k'}$ for k' s.t. $k < k' \leq j$ with the simple agenda $\mathcal{S}_{\{x_{k'}, X_{j+1}\}}$.

(ii) If $|X_{j+1}| \neq 1$, replace every node X_{j+1} in the agenda resulting from (i) with $\mathcal{S}_{X_{j+1}}$.[16]

(3) Define $\mathcal{P}_{(\succsim,\alpha)}$ to be $\mathcal{P}^K_{(\succsim,\alpha)}$ where K is the number of indifference classes in $\succsim$.

[15] For the interested reader, these three requirements can be formalized as follows: (i) $x \in \alpha(z) \Rightarrow x \succ z$; (ii) $x \sim y \Rightarrow \alpha(x) = \alpha(y)$; and, (iii) $[x \sim y \succ z$ and no $z' \in X$ s.t. $x \sim y \succ z' \succ z] \Rightarrow [x \in \alpha(z)$ or $w \succ x$ for all $w \in \alpha(z)]$.

[16] To be very clear: $\mathcal{S}_{\{x_k, X_{j+1}\}}$ has two terminal nodes x_k and X_{j+1}; and, $\mathcal{S}_{X_{j+1}}$ has two terminal nodes x_{j+1} and x'_{j+1}.

Clearly, this recursive construction defines a simple agenda on X.[17] At any stage $j \leq K$, the simple agenda $\mathcal{P}^{j}_{(\succsim,\alpha)}$ is extended into a longer simple agenda $\mathcal{P}^{j+1}_{(\succsim,\alpha)}$ by appending new simple agendas (of two or three alternatives) to the terminal nodes. The figure below serves to illustrate:

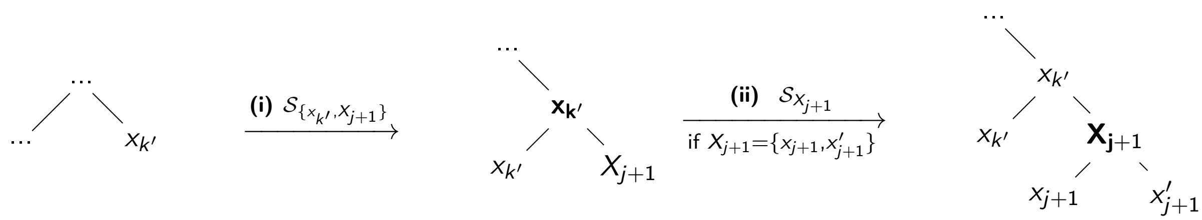

Figure 8: Detail at the terminal node x'_k in stage $j \leq K$ of the construction.

Notice that Definition 7 effectively uses the class of priority agendas on m alternatives to define the class of priority agendas on $m+1$ alternatives. The following example serves to illustrate this point:

Example 1 *There are three consistent ways of proposing a new alternative x_4 to extend the (i) Euro-Latin and (ii) Anglo-American agendas in Figure 1 into priority agendas on $\{x_1, x_2, x_3, x_4\}$.*

(1) By Euro-Latin amendment – *x_4 amends only the last alternative x_3 in $\{x_1, x_2, x_3\}$, which leads to (i) the Euro-Latin agenda in Figure 2 and (ii) the right-hand agenda in Figure 3;*

(2) By Anglo-American amendment – *x_4 amends every alternative in $\{x_1, x_2, x_3\}$, which leads to (i) the left-hand agenda in Figure 3 and (ii) the Anglo-American agenda in Figure 2; and,*

(3) By Intermediate amendment – *x_4 amends x_2 and x_3, which leads to the two agendas below.*

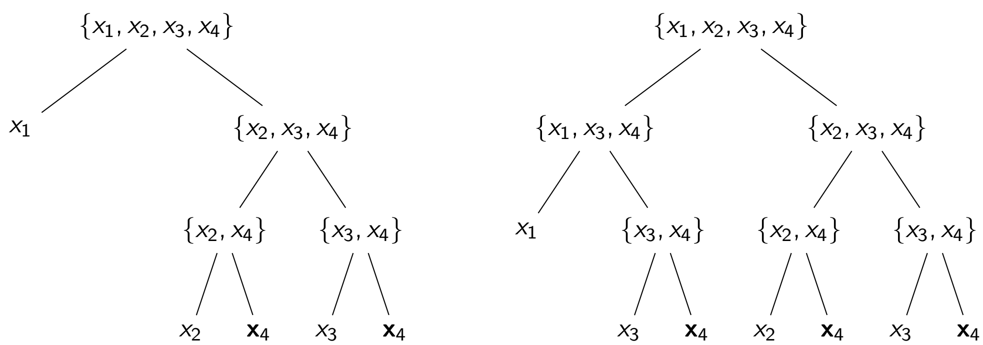

Figure 9: Priority agendas that extend Euro-Latin (*left*) and Anglo-American (*right*) agendas

Since the Euro-Latin and Anglo-American agendas are the only priority agendas on three alternatives, these six agendas constitute the entire class of priority agendas on four alternatives (up to permutation).

Before moving on, it is worth clarifying three issues related to equal priority alternatives:

[17] Technically, one must relabel the non-terminal nodes to conform with Definition 1(3). Since this relabelling is straightforward but cumbersome, it has been omitted to preserve clarity. For the technical details, see Claim 1 of the Appendix.

(1) It is not crucial to add these alternatives at the same time as described in Definition 7. Nothing about the construction would be affected if one added them one at a time—provided that one re-interpreted the amendment rule to mean that the second of the two additions only amends the first.

(2) Alternatives that occupy *symmetric* positions in a priority agenda (like the last two alternatives on a Euro-Latin agenda or the first two on an Anglo-American agenda) may be viewed as having equal priority. The converse is not necessarily true (see e.g. x_1 and x_2 in the right-hand agenda of Figure 9 above).

(3) The previous remarks may give the impression that it is without loss of generality to restrict attention to priority agendas defined by pairs $(\succ, \alpha)$ where $\succ$ is a *strict priority* (i.e. a linear order). To see that this is not the case, consider the following extension of the right-hand agenda in Figure 9:

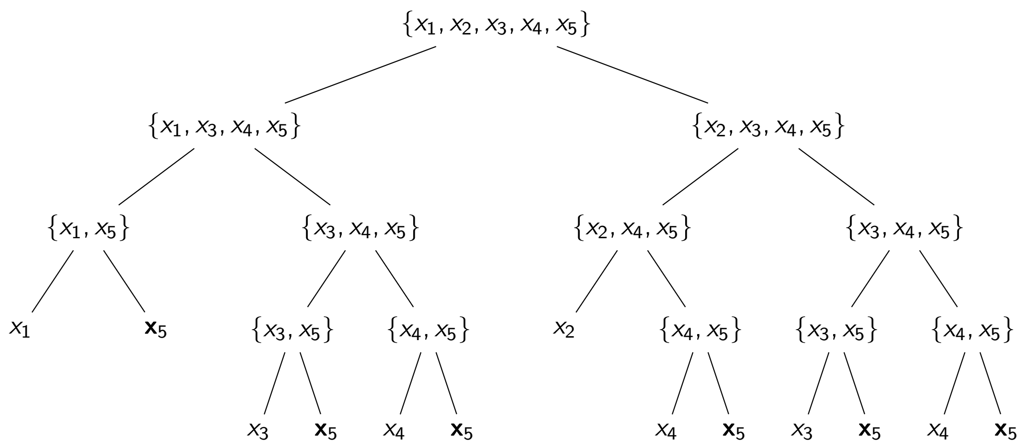

Figure 10: A priority agenda extending the right-hand agenda in Figure 9

For this agenda, assigning a strict priority between x_1 and x_2 leaves no way to define the amendments $\alpha(x_4)$ and $\alpha(x_5)$. The problem is not difficult to see: x_4 does not amend x_1 while x_5 does not amend x_2.

(b) Necessary and Sufficient Conditions

Euro-Latin and Anglo-American agendas are structured so that the lowest priority alternatives are the sophisticated voting outcomes only when they *always* appeal to a majority. To formalize, an alternative $a \in A$ is the *Condorcet winner* for the decision problem (P, A) if, for all $x \in A \setminus a$, the majority of voters in P prefer a to x. Then, an alternative $a^* \in A$ is said to be *marginal* for the issue A when $v(P, A) = a^*$ only if P is a profile where a^* is the Condorcet winner for (P, A). To state the property:

Weak Marginalization **(WM)** *For every issue, v has a marginal alternative.*

It turns out that the same property is satisfied by sophisticated voting on *every* priority agenda:

Proposition 1 *Every decision rule v implementable by priority agenda satisfies Weak Marginalization.*

Indeed, it is the distinguishing feature of decision rules implementable by priority agenda:

Theorem 2 *A decision rule v satisfies IS, ILA, and WM if and only if it is implementable by a priority agenda $\mathcal{P}_X^v$. For any decision rule v satisfying these three conditions, $\mathcal{P}_X^v$ is unique and the pair $(\succsim_v, \alpha_v)$ that defines this agenda is uniquely determined by the outcomes on Condorcet triples.*

To establish the sufficiency of the axioms, the key step is to determine the priority structure imposed by WM. To accomplish this task, the proof relies on the familiar tool of revealed preference:

Definition 8 *Given a decision rule v, define the binary relations $\succ_v$ and $\sim_v$ on X by:*

- $y \succ_v z$ if there exists an issue $A \supseteq \{y, z\}$ where z is marginal but y is not; and,

- $y \sim_v z$ if y is marginal for every issue A where z is marginal and vice versa.

Using $\succ_v$ and $\sim_v$, define the binary relation $\succsim_v$ on X by $y \succsim_v z$ if $y \succ_v z$ or $y \sim_v z$.

For the Anglo-American and Euro-Latin agendas, $\succsim_v$ reflects the underlying weak priority. For every issue A, the first marginalizes the lowest ranked alternative in A according to $\succsim_v$ while the second marginalizes the *two* lowest ranked alternatives.[18] In fact, $\succsim_v$ defines a weak priority with similar features for any decision rule that satisfies IS, ILA, and WM:

Lemma 1 *If v satisfies IS, ILA, and WM, then: **(i)** $\succsim_v$ is a weak priority; and; **(ii)** for every issue A, v marginalizes either the lowest **or** two lowest alternatives in A according to $\succsim_v$.*

As indicated in Theorem 2, it is possible to re-formulate the revealed priority $\succsim_v$ in terms of Condorcet triples. For any two alternatives y and z, Table 1 shows that there are six combinations of outcomes for the triples P_{xyz} and P_{xzy}. Of these, four directly reveal $y \succ_v z$ or $z \succ_v y$ while two combinations are consistent with every possible priority ranking of y and z. By varying the alternative x, it is possible to resolve these ambiguous cases (see Corollary 3 of the Appendix).

Table 1 also shows how to define the amendment rule α_v in terms of Condorcet triples. Intuitively, x is "revealed to amend" a higher priority alternative b if the Condorcet triples for every alternative c with intermediate priority yields type-(a) or type-(c) outcomes—namely the outcomes where x appears in the same sub-issue as b in Figure 7. Then, $\alpha_v(x)$ can be defined as the highest priority alternative(s) that x is revealed to amend (see Definition 11 and Lemma 2 of the Appendix).

In light of Theorem 1, the sufficiency of the axioms in Theorem 2 follows by showing that the simple agenda $\mathcal{S}_X^v$ "branches" in the same way as the priority agenda $\mathcal{P}_X^v$ defined by $(\succsim_v, \alpha_v)$.

V. Discussion

In this section, I examine the relationship of Euro-Latin and Anglo-American agendas to other priority agendas. The goal is to highlight how the latter extend our understanding of legislative voting beyond Euro-Latin and Anglo-American agendas. In part (a), I first describe two distinctive features that every priority agenda shares with Euro-Latin and Anglo-American agendas. In part (b), I then highlight a key difference between these two agendas and other priority agendas. Finally, in part (c), I describe how priority agendas expand the scope of implementation beyond Euro-Latin and Anglo-American agendas.

[18] Eliaz et al. [2011] characterize choice behaviour that is consistent with selecting the "lowest two" alternatives.

(a) Similarities

The literature emphasizes three features of sophisticated voting on Euro-Latin and Anglo-American agendas. One of these features, Weak Marginalization, was used in Section IV to characterize the decision rules implemented by priority agendas. The other two features are monotone comparative statics.

The first is a property of decision rules that relates to changes in voter preferences. Given a profile P, let P^x denote a profile where every preference is identical to P except for *one* voter, whose preference between x and the immediately preferred alternative are reversed.[19] That is, P^x differs from P only by improving x in the eyes of one voter. Using this notation, a decision rule v is *preference monotonic* if, for every decision problem (P, A) such that $v(P, A) = x$, $v(P^x, A) = x$ for every profile P^x.

Like Euro-Latin and Anglo-American agendas, every priority agenda is preference monotonic:

Proposition 2 *Every decision rule v implementable by priority agenda is preference monotonic.*

The second is a property of agendas that relates to changes in priority. Given a priority agenda $\mathcal{P}_X$ defined by $(\succsim, \alpha)$, let $(\succsim_x, \alpha_x)$ denote a pair identical to $(\succsim, \alpha)$ except for *one* alternative x, whose priority and amendment features are swapped with an alternative y such that: (i) $y \succ z \succ x$ for no alternative $z \in X$; or, (ii) $x \sim y$ and x is amended by every alternative that amends y. Let $\mathcal{P}^x_X$ denote the priority agenda defined by $(\succsim_x, \alpha_x)$. Intuitively, x weakly improves in terms of priority in $\mathcal{P}^x_X$ by "swapping places" with y in the agenda.[20] Using this notation, $\mathcal{P}_X$ is *priority monotonic* if, for every decision problem (P, A) such that $UNE[\mathcal{P}_{X|A}, P] = x$, $UNE[\mathcal{P}^x_{X|A}, P] = x$ for $\mathcal{P}^x_X$.

Like Euro-Latin and Anglo-American agendas, every priority agenda is priority monotonic:

Proposition 3 *Every priority agenda $\mathcal{P}_X$ is priority monotonic.*

(b) Differences

Unlike other rules implemented by priority agenda, the Euro-Latin and Anglo-American agendas treat the alternatives neutrally after taking the priorities into account. Intuitively, the outcomes may depend on the "structure" of the profile and the priorities but *not* the "names" of the alternatives. Perhaps the simplest (if not the weakest) implication of this neutrality is that issues of the same size must have the same number of marginal alternatives. Where two issues A, A' such that $|A| = |A'|$ are understood to be *similar*, this neutrality property can be stated more formally as follows:

Neutral Priority **(NP)** *For similar issues, v has the same number of marginal alternatives.*

The next result shows that, besides the Euro-Latin and Anglo-American procedures, no other decision rule implementable by priority agenda satisfies even this weak form of neutrality:

Proposition 4 *The Euro-Latin and Anglo-American procedures both satisfy NP. In fact, they are the only decision rules implementable by priority agenda that satisfy this property.*

[19] For the voter in question, the preference "... $\succ_i y \succ_i x \succ_i$ ..." in P becomes "... $\succ^x_i x \succ^x_i y \succ^x_i$ ..." in P^x.

[20] Formally, $\mathcal{P}^x_X$ can be obtained by permuting the labels of the terminal nodes in $\mathcal{P}_X$ marked x and y.

Whereas the Euro-Latin agenda *always* marginalizes two alternatives, the Anglo-American agenda *always* marginalizes one. No other rule that satisfies IS, ILA, and WM marginalizes the same number of alternatives, even for similar issues. In recent work, Apesteguia et al. [2014] provide a different characterization of the Euro-Latin and Anglo-American procedures using the following properties:

Condorcet Priority **(CP)** *Every issue A has a* ***prioritarian alternative*** *$p^* \in A$ such that $v(P_{p^*xy}, \{p^*, x, y\}) = p^*$ for any Condorcet triple P_{p^*xy} involving alternatives $x, y \in A$.*

Condorcet Anti-Priority **(CA)** *Every issue A has an* ***anti-prioritarian alternative*** *$p_* \in A$ such that $v(P_{p_*xy}, \{p_*, x, y\}) = y$ for any Condorcet triple P_{p_*xy} involving alternatives $x, y \in A$.*

To characterize the Euro-Latin procedure, they use CP along with ILA and *Division Consistency* (DC) (see footnote 11). To characterize the Anglo-American procedure, they use CA along with ILA and a property called *Elimination Consistency* (EC). Theorem 2 shows that one can replace DC and EC in these characterizations by IS:

Corollary 1 *A decision rule v is:*

(i) *a Euro-Latin procedure if and only if it satisfies IS, ILA, and Condorcet Priority.*

(ii) *an Anglo-American procedure if and only if it satisfies IS, ILA, and Condorcet Anti-Priority.*

Using $\succsim_v$ (particularly as characterized in Corollary 3 of the Appendix), it is easy to see that CP marginalizes two alternatives for every issue while CA marginalizes one. This shows that the key difference between the Euro-Latin and Anglo-American procedures is the structure of the amendments associated with each.

(c) Implementation

The majority relation M_P^A associated with a given decision problem (P, A) is defined by

$$xM_P^Ay \text{ if the majority of voters in } P \text{ prefer } x \in A \text{ over } y \in A.$$

Since the number of voters was assumed to be odd, the majority relation M_P^A associated with every decision problem (P, A) defines a *tournament* (a total[21] and asymmetric relation) on A.

Two of the most widely discussed tournament solutions are the *Top Cycle* and the *Banks Set*:

Definition 9 *The* ***Top Cycle*** *$TC(M_P^A)$ is the set of the alternatives that are majority preferred (directly or indirectly) to every other alternative in A. Formally: $TC(M_P^A) = \{a \in A : aM_P^A...M_P^Aa' \text{ for all } a' \in A \setminus a\}$.*

Definition 10 *The* ***Banks Set*** *$BA(M_P^A)$ is the set of alternatives at the top of some maximal M_P^A-transitive chain in A.[22] Formally: $BA(M_P^A) \equiv \{a \in A : a = b_1 \text{ for some maximal } M_P^A\text{-transitive chain } \langle b\rangle\}$.*

[21] Formally, xM_P^Ay or yM_P^Ax for all $x, y \in A$. Thus, M_P^A is incomplete only because it is irreflexive.

[22] Formally: a M_P^A-*transitive chain* is a sequence $\langle b\rangle = b_1, ..., b_m$ of alternatives in A such that $b_iM_P^Ab_j$ for all $j > i$; and, a M_P^A-transitive chain is *maximal* if there is no alternative $a \in A \setminus \langle b\rangle$ such that $a, b_1, ..., b_m$ defines a M_P^A-transitive chain.

While the Top Cycle corresponds to the alternatives that are sophisticated voting outcomes on the Euro-Latin agenda for some priority, the Banks Set corresponds to the alternatives that are sophisticated voting outcomes on the Anglo-American agenda for some priority.

To formalize this observation, consider a priority agenda $\mathcal{P}_A$ defined by $(\succsim, \alpha)$. Given a permutation $\sigma : A \rightarrow A$, let $\mathcal{P}_A^\sigma$ denote the priority agenda defined by permuting the priority $\succsim$ and the amendment rule α according to σ. In other words, $\mathcal{P}_A^\sigma$ is the priority agenda defined by $(\sigma \cdot \succsim, \sigma \cdot \alpha)$. Then:

(i') $a \in TC(M_P^A)$ iff $UNE[\mathcal{P}_A^\sigma; P] = a$ for some permutation σ of a Euro-Latin agenda $\mathcal{P}_A$; and,

(ii') $a \in BA(M_P^A)$ iff $UNE[\mathcal{P}_A^\sigma; P] = a$ for some permutation σ of an Anglo-American agenda $\mathcal{P}_A$.

It turns out that observations (i') and (ii') can be generalized to *all* priority agendas. In particular:

Proposition 5 *Given a priority agenda $\mathcal{P}_X$:*

(i) *$a \notin TC(M_P^A)$ implies $UNE[\mathcal{P}_A^\sigma; P] \neq a$ for every permutation σ; and,*

(ii) *$a \in BA(M_P^A)$ implies $UNE[\mathcal{P}_A^\sigma; P] = a$ for some permutation σ.*[23]

Part (i) follows from a general observation due to McKelvey and Niemi [1978]: *no* agenda ever selects an alternative outside of the Top Cycle. It is included only as a point of contrast to observation (i').

The real novelty is part (ii). Given a decision problem (P, A) and an alternative a in the Banks Set $BA(M_P^A)$, the proof defines a permutation σ such that $\mathcal{P}_A^\sigma$ selects a for *every* priority agenda $\mathcal{P}_A$ on A. The idea generalizes Sheplse and Weingast [1984]. For a given $a \in BA(M_P^A)$, they show that the alternatives of an Anglo-American agenda can be re-prioritizing according to some *sophisticated sequence* so that a is selected.[24] The proof of Proposition 5(ii) shows that this extends to *every* priority agenda.

Proposition 5(ii) has an immediate corollary for implementation by priority agenda:

Corollary 2 *No v implemented by priority agenda selects outside the Banks Set for every (P, A).*

One implication is that priority agendas cannot be used to implement decision rules that select *scoring winners* (according to any of the *Copeland*, *Slater*, or *Markov* scoring rules) for every decision problem.[25]

VI. Conclusion

In this paper, I characterize the sophisticated voting outcomes for a wide range of agendas that possess the kinds of simple features observed in practice. Not only do these results provide key insights into legislative decision-making but they have broader implications for a variety of other issues:

[23] This notation follows the *Restrict-Permute* (R-P) convention that one first restricts $\mathcal{P}_X$ to A (as in Claim 3 of the Appendix) before re-prioritizing the alternatives in $\mathcal{P}_A$ according to $\sigma : A \rightarrow A$. Following an alternative *Permute-Restrict* (P-R) convention, one first re-prioritizes the alternatives in $\mathcal{P}_X$ according to $\sigma : X \rightarrow X$ before restricting $\mathcal{P}_X^\sigma$ to A. For Euro-Latin and Anglo-American agendas, the two conventions lead to identical results. For all other priority agendas, the P-R convention is more flexible. In fact, the R-P convention is just a special case of the P-R convention with the restriction that $\sigma(x) = x$ for all $x \in X \setminus A$. As such, Proposition 5 continues to hold when one instead follows the P-R convention.

[24] Effectively, this establishes the "only if" direction of observation (ii').

[25] For some tournaments, the Banks Set contains no such scoring winners (see Brandt et al. [2015, Figure 3.7]).

(1) Implementation by Agenda: In related work, I give general necessary and sufficient conditions for implementation by agenda (Horan [2013]). *A priori*, these conditions impose no limitations on the size or structure of the implementing agenda. Indeed, some agenda-implementable social choice functions *require* complex agendas, even when the number of alternatives is relatively small (Trick [2006]).

In the interest of identifying "simple" implementing mechanisms (Moore [1992]), it is worth reformulating the three conditions in Theorem 2 using Remark 1. On the conventional implementation domain, ILA states that the outcome cannot be affected by the majority ranking of alternatives outside the *Condorcet Set*. This amounts to the well-known observation of McKelvey and Niemi [1978]. In contrast, the two other conditions (IS and ILA) appear to have no precedent in the literature. Intuitively, both impose strong inter-profile restrictions on what *can* and *cannot* be selected from the Condorcet Set.

(2) Tournament Solutions: By varying the priority of the alternatives, the Euro-Latin and Anglo-American agendas can be used to define the *Top Cycle* and the *Banks Set*. In principle, one could use the same kind of approach to define a tournament solution for *any* priority agenda.[26] This possibility raises an interesting question about another well-known tournament solution: *is there some priority agenda that can be used to define the Uncovered Set (Miller [1980])?*[27]

(3) Agenda Manipulation: The paper has implications for the manipulation of agenda voting. For one, decision rules are well-suited to explore the issue of *strategic candidacy* (Dutta et al. [2002]). Since they range over all decision problems (P, A), the change in the outcome when a candidate (an alternative) drops out of the election becomes part of the primitive. In principle, this makes it possible to extend the analysis of strategic candidacy on Anglo-American agendas to more general classes of agendas.

Equally relevant are the monotonicity features of priority agendas. Since it weakens *strategyproofness* (Sanver and Zwicker [2009]), preference monotonicity has obvious implications for the misrepresentation of preferences by voters.[28] This feature also eliminates the possibility for candidates to gain an advantage by "throwing" majority comparisons with other candidates (see Altman et al. [2009]). In turn, priority monotonicity imposes some mild limitations on the agenda-setter: to prevent an alternative from being selected, the agenda-setter must move it *down* the agenda. Having said this, Proposition 5 shows that the agenda-setter can re-prioritize the alternatives to select any outcome in the Banks Set.

(4) Preference Monotonicity: As the discussion above suggests, this is a desirable property for voting rules (see also Fishburn [1982]). Besides the priority agendas introduced here, it is known that every *knockout agenda* (i.e. each alternative labels exactly one terminal node) implements a preference monotonic decision rule (Altman et al. [2009]; Moulin [1991, Exercise 9.4]). Since the only overlap between knockout agendas and priority agendas are Euro-Latin agendas, this begs the following question: *what features characterize the entire class of agendas implementing preference monotonic decision rules?*

[26] For a given priority agenda $\mathcal{P}_X$, one must also specify *which* restriction of $\mathcal{P}_A$ is to be used for every issue A' of size $|A'| = |A|$. Because of their structure, this is redundant for Euro-Latin and Anglo-American agendas (see also footnote 23).

[27] Given Proposition 5, any such solution must be nested between the Banks Set and the Top Cycle. The Uncovered Set is the only well-known tournament solution in this range (see Brandt et al. [2015, Figure 3.7] or Laslier [1997, Table A.1]).

[28] In the setting of the current paper (i.e. single-valued choice rules on the "universal domain" of strict preference profiles), strategyproofness is equivalent to *Maskin Monotonicity* (see e.g. Muller and Satterthwaite [1977]).

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VIII. Appendix – Proofs

NOTE: Except as indicated, the claims in sections (c)-(g) below suppose that v satisfies IS and ILA.

(a) Proof of Remark 1

To formalize the remark, some notation is required. Given a profile P, let P^A denote the profile that coincides with P on A but, for each voter preference, places all $x \in X \setminus A$ (in a fixed order) below every $a \in A$.

Proof of Remark 1. Suppose v is implemented by $\mathcal{T}_X$. To establish $v(P, A) = v(P^A, X)$, note that "backward induction" determines the UNE on any agenda (McKelvey and Niemi [1978]). In any terminal subgame, it selects the Condorcet winner. One can then delete the Condorcet loser and repeat the argument on the resulting (smaller) agenda. From this observation, it follows that $UNE[\mathcal{T}_X; P^A] = UNE[\mathcal{T}_{X|A}; P^A]$. Since P^A and P coincide on A, $UNE[\mathcal{T}_{X|A}; P^A] = UNE[\mathcal{T}_{X|A}; P]$ as well. So, $UNE[\mathcal{T}_X; P^A] = UNE[\mathcal{T}_{X|A}; P]$ as required. ■

(b) Proof of Propositions 1, 2, and 3

Some additional notation is required. Given a weak order $\succsim$, let $L_{\succsim}(a) \equiv \{x \in X : a \succ x\}$ denote the strict lower contour set of $a \in X$. And, let $\succ^*$ denote any strict order such that $y \succ z$ implies $y \succ^* z$ for all $y, z \in X$. Given an issue $A = \{a_1, ..., a_K\}$ labeled according to $\succ^*$, let A_j^k denote the alternatives in A between a_j and a_k (inclusive). Formally, let $A_j^k \equiv \{a_j, ..., a_k\}$ if $j \leq k \leq K$; and, let $A_j^k \equiv \emptyset$ otherwise.

Claim 1 *Given a priority agenda $\mathcal{P}_{(\succsim,\alpha)}$, the two successor nodes of any non-terminal node A are $a_1 \cup A_j^K$ and A_2^K where $j \in \{3, ..., K+1\}$ and $A = \{a_1, ..., a_K\}$ is labeled according to $\succ^*$. Moreover:*

(i) *$A_2^K = X_{m-K+2}^m$ where $X = \{x_1, ..., x_m\}$ is labeled according to $\succ^*$; and,*

(ii) *$a_2 \succ a_j$ and a_j is a highest priority alternative in A_3^K that amends a_1 (if such an alternative exists).*

Proof. Let B and C denote the labels attached to the two successors of node A. Since $\succsim$ is a weak priority: **(1)** $\max_{\succsim} A = \{a_1, a_2\}$; or, **(2)** $\max_{\succsim} A = \{a_1\}$. Since the claim is trivial if $|A| = 2$, suppose $|A| \geq 3$.

(1) By construction (step (ii) in Definition 7), a_1 and a_2 must have been added at A. Since they must be added to different successors of A, $a_1 \in B \setminus C$ and $a_2 \in C \setminus B$. By definition of α, the next highest priority alternative(s) in X are added under the successors identified with a_1 and a_2. By a straightforward induction argument, it follows that $B = a_1 \cup L_{\succsim}(a_1)$ and $C = a_2 \cup L_{\succsim}(a_2)$. So, $B = a_1 \cup A_3^K$, $C = A_2^K = X_{m-K+2}^m$, and $a_2 \succ a_3$.

(2) By construction (step (i) in Definition 7), a_1 and the next highest priority alternative(s) at A must be added to different successors of A. Defining $A_{-1} \equiv \max_{\succsim}(A \setminus a_1)$, $a_1 \in B \setminus C$ and $A_{-1} \subseteq C \setminus B$. So, $C = A_{-1} \cup L_{\succsim}(a_2) = X_{m-K+2}^m$ by the same argument as (1). To add an $a_j \notin A_{-1}$ under the successor identified with a_1, the construction requires that a_j amends a_1. Then, by the same argument as (1), $B = a_1 \cup A_j^K$ where a_j is a highest priority in $C = X_{m-K+2}^m \equiv A_2^K$ that amends a_1 besides $a_2 \succ a_j$ (if such an alternative exists). ■

Claim 2 *Fix a priority agenda $\mathcal{P}_{(\succsim,\alpha)}$ on $X = \{x_1, ..., x_m\}$ as labeled according to $\succ^*$ and an alternative x_j that amends x_1 and x_2 according to α. Then, every path from the root X to a terminal node in $\mathcal{P}_{(\succsim,\alpha)}$ passes through a non-terminal node $x_i \cup X_j^m$ whose successors are X_j^m and $x_i \cup X_k^m$ for some $i < j < k \leq m+1$.*

Proof. The proof is by strong induction on m. The base case $m = 3$ is straightforward. For the induction step $m = n+1$, consider the two successors of the root X. By Claim 1, these are $x_1 \cup X_k^m$ and X_2^m for $2 < k \leq m+1$.

If $j = 3$, then Claim 1 implies $k = 3$. In this case, $x_1 \cup X_3^m$ and X_2^m are the desired nodes. Every path to a terminal node goes through one of these nodes. And, Claim 1 establishes the following: the successors of $x_1 \cup X_3^m$ are $x_1 \cup X_{k'}^m$ and X_3^m; and, the successors of X_2^m are $x_2 \cup X_{k''}^m$ and X_3^m.

If $j > 3$, then $x_j \in X_3^m$ and $x_j \in X_k^m$ (by Claim 1). Moreover, x_j amends x_3 (by definition of α and the assumption that x_j amends x_1 and x_2). Since the agendas starting at $x_1 \cup X_k^m$ and X_2^m are priority agendas on n or fewer alternatives (where x_j amends the two highest priority alternatives), the induction hypothesis implies that every path to a terminal node starting from $x_1 \cup X_k^m$ or X_2^m passes through a non-terminal node with the desired characteristics. And, since every path from the root X passes through X_2^m or $x_1 \cup X_k^m$, the result follows. ■

The next results require two additional definitions.

- Two agendas $\mathcal{T}_X$ and $\mathcal{T}'_Y$ such that $Y \subset X$ are *outcome-equivalent* on Y if:

$$UNE[\mathcal{T}_{X|A}, P] = UNE[\mathcal{T}'_{Y|A}, P] \text{ for every decision problem } (P, A) \text{ such that } A \subseteq Y.$$

- Given a priority agenda $\mathcal{P}_X$ defined by the pair $(\succsim, \alpha)$, let $\mathcal{P}_{X\setminus x}$ denote the *deleted priority agenda* induced by constructing the agenda without the alternative $x \in X$. Formally, let $\mathcal{P}_{X\setminus x}$ be defined by $(\succsim_{-x}, \alpha_{-x})$ where the priority $\succsim_{-x}$ is the restriction of $\succsim$ to $X \setminus x$ and the amendment rule α_{-x} is defined as follows:

$$\alpha_{-x}(y) \equiv \begin{cases} \alpha(y) & \text{if } y \text{ does not amend } x, \\ \{x'\} & \text{if } \alpha(y) = \{x, x'\}, \\ X_{j+1} & \text{if } \alpha(y) = \{x\} \in X_j \text{ and } |X_j| = 2, \text{ and} \\ \max_{\succsim} \alpha(x) \cup \alpha(y) & \text{otherwise – i.e. } z \succ x \text{ for } z \in \alpha(y). \end{cases}$$

Equivalently, one can obtain $\mathcal{P}_{X\setminus x}$ by "deleting" certain subgames of $\mathcal{P}_X$ where x has highest priority. In particular:

(1) Identify every node C_x of $\mathcal{P}_X$ s.t. $x \in \max_{\succsim} C_x$ and $x \notin \max_{\succsim} C_x^p$ for the predecessor C_x^p of C_x:
- let B_x^s denote the successor of C_x s.t. $x \in B_x^s$; and, let C_x^s denote its sibling.
- let B_x denote the sibling of C_x; and, let $\tilde{B}_x^s$ and $\tilde{C}_x^s$ denote its two successors.

(2) If $C_x^s = \tilde{B}_x^s$ or $C_x^s = \tilde{C}_x^s$, delete the agenda starting at C_x; otherwise, delete the agenda starting at B_x^s.

(3) Delete every non-terminal node with a unique successor, connecting its successor to its predecessor.

(4) To obtain $\mathcal{P}_{X\setminus x}$, relabel every non-terminal node of the resulting tree to conform with Definition 1.

Claim 3 *If $(\succsim, \alpha)$ on X, then $(\succsim_{-x}, \alpha_{-x})$ defines a priority agenda on $X \setminus x$.*

Proof. Clearly, $\succsim_{-x}$ defines a priority on $X \setminus x$. To see that $(\succsim_{-x}, \alpha_{-x})$ defines a priority agenda, one must verify that, in combination with $\succsim_{-x}$, the function α_{-x} satisfies the conditions (i)-(iii) described in Section IV.(a):

(i) To see that $z \in \alpha_{-x}(y) \Rightarrow z \succ_{-x} y$: In the first two cases, $\alpha_{-x}(y)$ inherits (i) from $\alpha(y)$. This follows from the fact that $z \succ y$ implies $z \succ_{-x} y$ for $z \neq x$. In the third case, y cannot have priority immediately lower than x. Otherwise, $\alpha(y) = X_j$ by (iii) on α. Thus, $z \succ_{-x} y$ for $z \in X_{j+1}$. In the final case, fix $z \in \alpha_{-x}(y)$. By construction, $z \succsim z'$ for any $z' \in \alpha(y)$. By (i) on $\alpha(y)$, $z \succsim z' \succ y$ so that $z \succ y$. Thus, $z \succ_{-x} y$.

(ii) To see that $y \sim_{-x} y' \Rightarrow \alpha_{-x}(y) = \alpha_{-x}(y')$: Fix a pair such that $y \sim_{-x} y'$. By definition, $y \sim y'$ and $x \neq y, y'$. In light of this, it is easy to check that $\alpha_{-x}(y)$ inherits (ii) from $\alpha(y)$ in each of the four cases.

(iii) To see that $[z \sim_{-x} z' \succ_{-x} y$ and no $y' \in X \setminus x$ s.t. $z \sim_{-x} z' \succ_{-x} y' \succ_{-x} y] \Rightarrow [z \in \alpha_{-x}(y)$ or $w \succ_{-x} z$ for all $w \in \alpha_{-x}(y)]$: There are four possibilities: (a) $y \succsim x$; (b) $x \succ w \succ y$ for some $w \in X$; (c) $x \in X_j$ immediately precedes y in $\succsim$ with $|X_j| = 2$; and, (d) $x \in X_j$ immediately precedes y in $\succsim$ with $|X_j| = 1$.

- For (a)-(b), it is straightforward to check that $\alpha_{-x}(y)$ inherits (iii) from $\alpha(y)$.

- For (c), the first/third cases cannot arise: y amends x by (i) on α; and, y amends $x, x' \in X_j$ by (iii) on α. In the second/fourth cases, (iii) is vacuously satisfied: x' immediately precedes y but $x' \sim_x z$ for no $z \in X \setminus x$.

- For (d), the first/second/third cases cannot arise: y amends x by (i) on α; y amends x and x' by (iii) on α; and, $|X_j| = 1$. In the fourth case, an issue with (iii) only arises when $|X_{j-1}| = 2$. In that case, x amends $z, z' \in X_{j-1}$ by (iii) on α. By definition, it follows that y must amend $z, z' \in X_{j-1}$ on α_{-x}. ■

It is worth noting that the *deleted* priority agenda $\mathcal{P}_{X \setminus x}$ is formally distinct from the *pruned* priority agenda $\mathcal{P}_{X|X\setminus x}$. Intuitively, $\mathcal{P}_{X \setminus x}$ prunes "higher up" the agenda than $\mathcal{P}_{X|X\setminus x}$ (unless x has lowest priority according to $\succsim$). As a result, it prunes away a larger portion of $\mathcal{P}_X$. However, the two agendas are outcome-equivalent:

Claim 4 *For any priority agenda $\mathcal{P}_X$ and any $x \in X$, $\mathcal{P}_X$ is outcome-equivalent to $\mathcal{P}_{X \setminus x}$ on $X \setminus x$.*

Proof. Suppose $X = \{x_1, ..., x_m\}$ where X is labeled according to $\succ^*$. The proof is by strong induction on m. The base cases $m = 2, 3$ are trivial. For the induction step $m = n + 1$, note that the successors at the root X of $\mathcal{P}_X$ are $(x_1 \cup X_j^m, X_2^m)$ by Claim 1. Consider the three cases: (i) $j = m + 1$; (ii) $j = 3$; and, (iii) $3 < j \leq m$.

The argument is identical in all cases if $x \neq x_1, x_2$. In fact, this argument also works for $x = x_2$ in cases (i) and (iii). Fix any $x = x_i$ s.t. $i \geq 2$ (where $i \neq 2$ if case (ii) is the relevant case). By the induction hypothesis, the priority agenda $\mathcal{P}_{X_2^m} = \mathcal{P}_X(X_2^m)$ starting at the "right" successor X_2^m of the root is outcome-equivalent on $X_2^m \setminus x_i$ to $\mathcal{P}_{X_2^m \setminus x_i}$. Similarly, the priority agenda $\mathcal{P}_{x_1 \cup X_j^m} = \mathcal{P}_X(x_1 \cup X_j^m)$ starting at the "left" successor $x_1 \cup X_j^m$ is outcome-equivalent on $(x_1 \cup X_j^m) \setminus x_i$ to $\mathcal{P}_{(x_1 \cup X_j^m) \setminus x_i}$. Then, "backward induction" shows that the outcome on $\mathcal{P}_X$ for any issue $A \subseteq X \setminus x_i$ does not change if one replaces the agendas at the root by their outcome-equivalents. Since the resulting agenda is $\mathcal{P}_{X \setminus x_i}$ by definition, the claim follows. The following diagram serves to illustrate:

To complete the proof, it suffices to establish the result for $x = x_1$ in cases (i)-(iii) and $x = x_2$ in case (ii).

(i) The claim is trivial since $\mathcal{P}_{X|(X \setminus x_1)} = \mathcal{P}_{X \setminus x_1}$ by definition.

(ii)-(iii) It suffices to show the claim for $x = x_1$. (In case (ii), the same reasoning works for $x = x_2$ as well.) By the induction hypothesis, $\mathcal{P}_{x_1 \cup X_j^m}$ is outcome-equivalent on X_j^m to $\mathcal{P}_{X_j^m}$. Let $\mathcal{T}_{X \setminus x_1}$ denote the agenda obtained from $\mathcal{P}_X$ by replacing the agenda $\mathcal{P}_{x_1 \cup X_j^m}$ at $x_1 \cup X_j^m$ with $\mathcal{P}_{X_j^m}$. To complete the proof, I show that $\mathcal{T}_{X \setminus x_1}$ is outcome-equivalent to the priority agenda $\mathcal{P}_{X_2^m}$ starting at X_2^m.

By way of contradiction, suppose there exists some profile P s.t. $UNE[\mathcal{T}_{X \setminus x_1}, P] \equiv x \neq y \equiv UNE[\mathcal{P}_{X_2^m}, P]$. Since x is the outcome at the root of $\mathcal{T}_{X \setminus x_1}$, "backward induction" establishes that: $UNE[\mathcal{P}_{X_j^m}, P] = x$; and, x is majority preferred to y. By Claims 1 and 2, every path down the agenda from X_2^m reaches a node $x_i \cup X_j^m$ where: (i) the "left" successor is $x_i \cup X_k^m$ for $i < j < k$; and, (ii) the "right" successor is X_j^m.[29] Since $UNE[\mathcal{P}_{X_j^m}, P] = x$, y must be the outcome at one or more "left" successors $x_i \cup X_k^m$. Since $UNE[\mathcal{P}_{X_j^m}, P] = x$ however, y must be eliminated at the predecessor $x_i \cup X_j^m$ of any such "left" successor. So, $UNE[\mathcal{P}_{X_2^m}, P] \neq y$, a contradiction which establishes that $\mathcal{T}_{X \setminus x_1}$ is outcome-equivalent to $\mathcal{P}_{X_2^m}$. ■

[29] Claim 2 is clearly applicable in both cases. In case (ii) (i.e. $j = 3$), x_3 must amend x_2 by definition of α. In case (iii) (i.e. $j > 3$), the fact that x_j amends x_1 implies that x_j must amend x_2 and x_3 (again by definition of α).

Proof of Proposition 1. Suppose v is implementable by a priority agenda $\mathcal{P}_X$ defined by $(\succsim, \alpha)$. I show that x_m is marginal for $X = \{x_1, ..., x_m\}$ as labeled according to $\succ^*$. It then follows that x_m is marginal for any issue A s.t. $x_m \in A$. Since $\mathcal{P}_{X|\{x_1,...,x_{m-1}\}}$ is a priority agenda on $X \setminus x_m$ s.t. $x_j \succsim x_{m-1}$, the same argument shows that x_{m-1} is marginal for any issue $A \subseteq X \setminus x_m$ s.t. $x_{m-1} \in A$. The result follows by extending this reasoning.

The proof that x_m is marginal for X is by strong induction on m. The base cases $m = 2, 3$ are trivial. For the induction step $m = n + 1$, consider the successors of the root X. By Claim 1, these are $x_1 \cup X_k^m$ and X_2^m for $2 < k \leq m + 1$. There are three possibilities: (i) $k = m + 1$; (ii) $k = m$; and, (iii) $2 < k < m$.

(i) In this case, the root splits $\mathcal{P}_X$ into x_1 and a priority agenda $\mathcal{P}_{X_2^m}$ on n alternatives. Since $x_i \succsim x_m$ for all $x_i \in X_2^m$, the induction hypothesis implies that x_m is the outcome on $\mathcal{P}_{X_2^m}$ only if it is the Condorcet winner on X_2^m. Rolling back the agenda to the root node X, x_m is the outcome on $\mathcal{P}_X$ only if it is the Condorcet winner on $\{x_1, x_m\}$ as well. Combining the last two observations gives the result.

(ii) In this case, x_m amends x_1 by Claim 1. Then, by construction of $\mathcal{P}_X$, every terminal subgame pairs x_m against another alternative in X. So, any alternative majority defeated by x_m is eliminated at the last stage (i.e. only x_m and alternatives that beat x_m can be selected as outcomes). Rolling back the agenda to the root node, it then follows that x_m is the outcome on $\mathcal{P}_X$ only if it is the Condorcet winner on X.[30]

(iii) In this case, the agendas $\mathcal{P}_{x_1 \cup X_k^m} = \mathcal{P}_X(x_1 \cup X_k^m)$ and $\mathcal{P}_{X_2^m} = \mathcal{P}_X(X_2^m)$ are priority agendas on n or fewer alternatives. Since $x_i \succsim x_m$ for all $x_i \in X$, the induction hypothesis implies that x_m is the outcome on $\mathcal{P}_X$ only if it is the Condorcet winner on X_k^m (i.e. the intersection of $x_1 \cup X_k^m$ and X_2^m).

In that case, one can prune away X_k^{m-1} from $\mathcal{P}_X$ to obtain $\mathcal{P}_{X|Y}$ where $Y \equiv X_1^{k-1} \cup x_m$. By Claim 4, $\mathcal{P}_X$ is outcome-equivalent on Y to the priority agenda $\mathcal{P}_Y$ (on n or fewer alternatives). Since $x_i \succsim x_m$ for all $x_i \in Y$, the induction hypothesis implies x_m is the outcome on $\mathcal{P}_Y$ (and hence $\mathcal{P}_{X|Y}$) only if it is the Condorcet winner on $X_1^{k-1} \cup x_m$. Combining this with the observation in the last paragraph gives the result. ■

Claim 5 *Every decision rule v implementable by priority agenda satisfies IS.*

Proof. If v is implementable by the priority agenda $\mathcal{P}_X$, then there exists a way to split X. By Claim 4, the same is true for any $X \setminus x$. By applying Claim 4 to the resulting priority agenda $\mathcal{P}_{X \setminus x}$, the same must be true for any $X \setminus \{x, y\}$. Extending this reasoning by induction, it follows that v satisfies IS. ■

Claim 6 *If $\mathcal{P}_X$ is a priority agenda s.t. the successors of the root node X are labeled B and C, then:*

$$UNE[\mathcal{P}_X, P] \in B \cap C \textit{ implies } UNE[\mathcal{P}_X(B), P] = UNE[\mathcal{P}_X(C), P].$$

Proof. The proof is by strong induction on m. The base case $m = 3$ is straightforward. For the induction step $m = n+1$, suppose $(\succsim, \alpha)$ defines $\mathcal{P}_X$ and $X = \{x_1, ..., x_m\}$ is labeled according to $\succ^*$. By Claim 1: $B = x_1 \cup X_j^m$ for some $3 \leq j \leq m + 1$; and, $C = X_2^m$. If $j = m + 1$, then B is a singleton and there is no profile P s.t. $UNE[\mathcal{P}_X, P] \in B \cap C$. So, suppose $j \leq m$ without loss of generality. Now, fix a profile P s.t. $UNE[\mathcal{P}_X, P] = x_k$ for $j \leq k \leq m$. By way of contradiction, suppose the claim is false. Since $UNE[\mathcal{P}_X, P] = x_k$, "backward induction" leads to two possibilities for $UNE[\mathcal{P}_X(B), P] \equiv x_b$ and $UNE[\mathcal{P}_X(C), P] \equiv x_c$: (i) $b = k$ and $c \neq k$; and, (ii) $b \neq k$ and $c = k$. To establish the result, I show that each case leads to a contradiction:

(i) Consider the agenda $\mathcal{T}_{X \setminus x_1}$ (described in Claim 4) where the successors of the root are X_j^m and $C = X_2^m$. Since $UNE[\mathcal{P}_X(B), P] = x_b$, $UNE[\mathcal{P}_{X_j^m}, P] = x_b$. To see this, let B' denote the "left" successor of B in $\mathcal{P}_X$. If $x_b \in B' \cap X_j^m$, then $UNE[\mathcal{P}_{X_j^m}, P] = x_b$ by the induction hypothesis. If $x_b \notin B'$, then $UNE[\mathcal{P}_{X_j^m}, P] = x_b$ as well. (Otherwise, "backward induction" gives $UNE[\mathcal{P}_X(B), P] \neq x_b$.) Since $UNE[\mathcal{P}_X, P] = x_b$, $UNE[\mathcal{P}_{X_j^m}, P] = x_b$

[30] This type of argument is well-known in the literature (see e.g. Theorem 2.2 of Iglesias et al. [2012]).

implies $UNE[\mathcal{T}_{X\setminus x_1}, P] = x_b$ as well. By the argument in Claim 4(ii)-(iii), $\mathcal{T}_{X\setminus x_1}$ must be outcome-equivalent to $\mathcal{P}_{X\setminus x_1} = \mathcal{P}_X(C)$. So, $x_b = UNE[\mathcal{T}_{X\setminus x_1}, P] = UNE[\mathcal{P}_X(C), P] = x_c$, which is a contradiction.

(ii) By Claim 1, X_3^m is the "right" successor of C in $\mathcal{P}_X$. Moreover, $UNE[\mathcal{P}_{X_3^m}, P] = x_c$ by the same reasoning as in (i). Now, consider the priority agenda $\mathcal{P}_{X\setminus x_2}$ on n alternatives. If $j > 3$, then the successors at the root node $X \setminus x_2$ are B and X_3^m. So, "backward induction" gives $UNE[\mathcal{P}_{X\setminus x_2}, P] = x_c$. Since $UNE[\mathcal{P}_X(B), P] = x_b$ however, this contradicts the induction hypothesis. If $j = 3$, then $\mathcal{P}_{X\setminus x_2} = \mathcal{P}_X(B)$. So, $UNE[\mathcal{P}_{X\setminus x_2}, P] = x_b$. Since the "right" successor of B in $\mathcal{P}_{X\setminus x_2}$ is X_3^m and $UNE[\mathcal{P}_{X_3^m}, P] = x_c$ however, this contradicts $UNE[\mathcal{P}_X, P] = x_c$. ■

Proof of Proposition 2. Let $\mathcal{P}_X$ denote the priority agenda that implements v. It suffices to show preference monotonicity for X. Since $UNE[\mathcal{P}_{X|A}, P] = UNE[\mathcal{P}_X, P^A]$ by Remark 1, the result then follows for all $A \subseteq X$.

The proof is by strong induction on m. The base case $m = 3$ is straightforward. For the induction step $m = n+1$, fix a profile P s.t. $UNE[\mathcal{P}_X, P] = x$ and suppose the successors of the root X are B and C. There are two cases: (i) $x \in B \cap C$; and, (ii) $x \in B \setminus C$. **(i)** By Claim 6, $UNE[\mathcal{P}_X(B), P] = UNE[\mathcal{P}_X(C), P] = x$. Since $\mathcal{P}_X(B)$ and $\mathcal{P}_X(C)$ are priority agendas on n or fewer alternatives, $UNE[\mathcal{P}_X(B), P^x] = UNE[\mathcal{P}_X(C), P^x] = x$ by the induction hypothesis. Then, "backward induction" gives $UNE[\mathcal{P}_X, P^x] = x$. **(ii)** Since $\mathcal{P}_X(B)$ is a priority agenda on n or fewer alternatives, $UNE[\mathcal{P}_X(B), P^x] = x$ by the induction hypothesis. Since $x \notin C$, $UNE[\mathcal{P}_X(C), P^x] = UNE[\mathcal{P}_X(C), P]$. Since $UNE[\mathcal{P}_X, P] = x$, "backward induction" gives $UNE[\mathcal{P}_X, P^x] = x$. ■

Proof of Proposition 3. Suppose $\mathcal{P}_X$ is defined by $(\succsim, \alpha)$ where $X = \{x_1, ..., x_m\}$ is labeled according to $\succ^*$. By Remark 1, it suffices to establish the result for X. The proof is by strong induction on m.

The base cases $m = 2, 3$ are straightforward. For the induction step $m = n + 1$, fix a profile P s.t. $UNE[\mathcal{P}_X, P] = x_k$. (Where $x_{k-1} \sim x_k$, suppose that every alternative that amends x_{k-1} also amends x_k.) By Claim 1, the successors of the root X are: $B \equiv x_1 \cup X_j^m$ for some $3 \leq j \leq m+1$; and, $C \equiv X_2^m$. So, there are four cases to consider: (i) $j+1 \leq k \leq m$; (ii) $3 \leq k \leq j-1$; (iii) $k = j$; and, (iv) $k = 2$. To establish the result, I show that $UNE[\mathcal{P}_X^k, P] = x_k$ in each case (where $\mathcal{P}_X^k \equiv \mathcal{P}_X^{x_k}$ to simplify the notation):

(i) Since $x_k \in B \cap C$ and $UNE[\mathcal{P}_X, P] = x_k$, Claim 6 implies $UNE[\mathcal{P}_X(B), P] = UNE[\mathcal{P}_X(C), P] = x_k$. Since $\mathcal{P}_X(B)$ and $\mathcal{P}_X(C)$ are priority agendas on n or fewer alternatives and $k \neq j$, the induction hypothesis implies $UNE[\mathcal{P}_X^k(B), P] = UNE[\mathcal{P}_X^k(C), P] = x_k$. So, $UNE[\mathcal{P}_X^k, P] = x_k$.

(ii) Since $x_k \notin B$ and $UNE[\mathcal{P}_X, P] = x_k$, "backward induction" implies $UNE[\mathcal{P}_X(C), P] = x_k$. Since $\mathcal{P}_X(C)$ is a priority agenda on n or fewer alternatives, the induction hypothesis implies $UNE[\mathcal{P}_X^k(C), P] = UNE[\mathcal{P}_X(C), P] = x_k$. Since $x_k \notin B$, $UNE[\mathcal{P}_X^k, P] = x_k$ by the same kind of reasoning as Proposition 2(ii).

(iii) By the reasoning in case (i), $UNE[\mathcal{P}_X(B), P] = UNE[\mathcal{P}_X(C), P] = x_j$ and $UNE[\mathcal{P}_X^k(C), P] = x_j$. By way of contradiction, suppose $UNE[\mathcal{P}_X^k, P] \neq x_j$. Then, $UNE[\mathcal{P}_X^k, P] = x_1$ by Claim 6. Since $UNE[\mathcal{P}_X^k(C), P] = x_j$, "backward induction" implies that x_1 is majority preferred to x_j.

Since $UNE[\mathcal{P}_X^k, P] = x_1$, "backward induction" implies $UNE[\mathcal{P}_X^k(B_j), P] = x_1$ where $B_j \equiv \{x_1, x_{j-1}\} \cup X_{j+1}^m$ is the "left" successor of the root X in $\mathcal{P}_X^k$. Since the agenda at the "left" successors $B' \equiv x_1 \cup X_{j'}^m$ of B and B_j coincide and $UNE[\mathcal{P}_X^k(B_j), P] = x_1$, "backward induction" implies $UNE[\mathcal{P}_X^k(B'), P] = UNE[\mathcal{P}_X(B'), P] = x_1$. Since $UNE[\mathcal{P}_X(B), P] = x_j$ however, "backward induction" implies that x_j is majority preferred to x_1. Since this contradicts the inference drawn in the last paragraph, it follows that $UNE[\mathcal{P}_X^k, P] = x_j$.

(iv) While more involved than case (iii), the basic proof technique in this case is similar. Since $UNE[\mathcal{P}_X, P] = x_2$ and $x_2 \notin B$, "backward induction" implies $UNE[\mathcal{P}_X(C), P] = x_2$. Let $B' \equiv x_2 \cup X_{j'}^m$ and $C' \equiv X_3^m$ denote the two successors of C in $\mathcal{P}_X$. Since x_2 only appears at one terminal node below B' and $UNE[\mathcal{P}_X(C), P] = x_2$, x_2 is majority preferred to $UNE[\mathcal{P}_X(C'), P]$ and every alternative it meets on the "backward induction" path in $\mathcal{P}_X(B')$. Now, consider $\mathcal{P}_X^2$, letting $B_2 \equiv x_2 \cup X_j^m$ and $C_2 \equiv x_1 \cup X_3^m$ denote its two successors. By construction,

$B_2' \equiv x_1 \cup X_{j'}^m$ and $C_2' \equiv C'$ are the two successors of C_2 in $\mathcal{P}_X^2$.

Since every alternative that amends x_1 in $\mathcal{P}_X$ also amends x_2, everything that x_2 meets on the "backward induction path" in $\mathcal{P}_X(B_2)$ is something that it meets in $\mathcal{P}_X(B')$. Since x_2 is majority preferred to all of these alternatives by the first observation in the last paragraph, $UNE[\mathcal{P}_X^2(B_2), P] = x_2$. Moreover, $UNE[\mathcal{P}_X^2(C_2'), P] = UNE[\mathcal{P}_X(C'), P]$ by the second observation in the last paragraph.

By way of contradiction, suppose $UNE[\mathcal{P}_X^2, P] \neq x_2$. Since $UNE[\mathcal{P}_X^2(B_2), P] = x_2$ and x_2 is majority preferred to $UNE[\mathcal{P}_X^2(C_2'), P] = UNE[\mathcal{P}_X(C'), P]$, "backward induction" implies $UNE[\mathcal{P}_X^2, P] = UNE[\mathcal{P}_X^2(B_2'), P]$. Since x_2 is majority preferred to everything that x_1 meets along the "backward induction path" in $\mathcal{P}_X(B_2')$, $UNE[\mathcal{P}_X^2, P] = UNE[\mathcal{P}_X^2(B_2'), P] = x_1$. Since $UNE[\mathcal{P}_X^2(B_2), P] = x_2$, x_1 is majority preferred to x_2.

Since $UNE[\mathcal{P}_X^2(B_2'), P] = x_1$, the same kind of reasoning as in the previous paragraphs establishes that $UNE[\mathcal{P}_X^2(B), P] = x_1$. Since $UNE[\mathcal{P}_X^2, P] = x_2$ however, x_2 is majority preferred to x_1. Since this contradicts the inference drawn in the last paragraph, it follows that $UNE[\mathcal{P}_X^2, P] = x_2$. ■

(c) Proof of Theorems 1 and 1*

Claim 7 *For any two profiles P, P' that coincide on A, $v(P, A) = v(P', A)$.*

Proof. The proof is by induction on $|A|$. The base case $|A| = 2$ follows from ILA. For the induction step, $v(P, A) = v(P, \{v(P, B), v(P, C)\}) = v(P, \{v(P', B), v(P', C)\}) = v(P', \{v(P', B), v(P', C)\}) = v(P', A)$ follows from IS, the induction hypothesis, and the base case. ■

Using the definition of P^A from section (a) above, one can establish an analog of Remark 1:

Claim 8 *For any decision problem (P, A), $v(P, A) = v(P^A, X)$.*

Proof. By ILA, $v(P^A, X) = ... = v(P^A, A)$. Since $v(P^A, A) = v(P, A)$ by Claim 7, $v(P^A, X) = v(P, A)$. ■

Claim 9 *Suppose (B, C) splits A for v. Then, for all $D \subseteq A$:*

(i) *$v(P, D) = v(P, \{v(P, B \cap D), v(P, C \cap D)\})$; and,*

(ii) *$(B \cap D, C \cap D)$ splits D if $D \neq B \cap D, C \cap D$.*

Proof. Fix some $x \in A$ and let P_x coincide with P except x is demoted to Condorcet loser on A. Then, $v(P, A \setminus x) = v(P_x, A \setminus x) = v(P_x, A) = v(P_x, \{v(P_x, B), v(P_x, C)\}) = ... = v(P, \{v(P, B \setminus x), v(P, C \setminus x)\})$ by Claim 7, ILA, and IS. Part (i) follows by repeated application of this reasoning. For part (ii), observe that $D \neq B \cap D, C \cap D$ implies $B \cap C \cap D \neq B \cap D, C \cap D$. Then, given part (i), $(B \cap D, C \cap D)$ splits D. ■

Claim 10 *For v, there is a unique way to split every issue.*

Proof. The proof is by induction on $|A|$. The claim holds trivially for $|A| = 2$ and is straightforward to show for $|A| = 3$. While there are a number of cases to check, the idea is relatively simple. Contrary to IS, $v(P, \{v(P, B), v(P, C)\}) \neq v(P, \{v(P, B'), v(P, C')\})$ for some cyclic profile P.

To complete the induction, suppose the claim holds for $|A| = n$ and consider the case $|A| = n + 1$. By way of contradiction, suppose (B, C) and (B', C') are distinct splittings of A. First, suppose (B, C) and (B', C') both partition A. Then, there exists some $x \in A$ such that $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$ are distinct partitions of $A \setminus x$. But, this contradicts the induction hypothesis. Next, suppose $x \in B \cap C$. If $x \in B' \cap C'$, then $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$ are distinct splittings of $A \setminus x$, which again contradicts the induction hypothesis. Finally, suppose $x \in B' \setminus C'$. There are two possibilities:

- $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$ coincide: Without loss of generality, $B \setminus x = B' \setminus x$ and $C \setminus x = C'$ (since the only other possibility is symmetric). Since (B, C) splits A, there exist alternatives $b \in B \setminus C$ and $c \in C \setminus B$. Since $B \setminus x = B' \setminus x$ and $C \setminus x = C'$, $b \in B' \setminus C'$ and $c \in C' \setminus B'$. Now, consider $D = \{b, c, x\}$. Collecting the observations above: $(B \cap D, C \cap D) = (\{x, b\}, \{x, c\})$ and $(B' \cap D, C' \cap D) = (\{x, b\}, \{c\})$. For the profile P_{bxc} whose Condorcet set is the cyclic triple bxc: $v(P_{bxc}, \{v(P_{bxc}, B \cap D), v(P_{bxc}, C \cap D)\}) = v(P_{bxc}, \{b, x\}) = b$; and, $v(P_{bxc}, \{v(P_{bxc}, B' \cap D), v(P_{bxc}, C' \cap D)\}) = v(P_{bxc}, \{b, c\}) = c$. But, this contradicts Claim 9(i).

- $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$ are distinct: First, suppose $B' \setminus C' \neq x$. Then $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$ are distinct splittings of $A \setminus x$, which contradicts the induction hypothesis. Next, suppose $B' \setminus C' = x$. Since (B, C) splits A, there exist alternatives $b \in B \setminus C$ and $c \in C \setminus B$. Now, consider $D = \{b, c, x\}$. Collecting the observations above: $(B \cap D, C \cap D) = (\{x, b\}, \{x, c\})$, $x \in B' \setminus C'$, and $b, c \in C'$. The last two set inclusions leave three possibilities: (i) $(B' \cap D, C' \cap D) = (\{x\}, \{b, c\})$; (ii) $(B' \cap D, C' \cap D) = (\{x, b\}, \{b, c\})$; and, (iii) $(B' \cap D, C' \cap D) = (\{x, b, c\}, \{b, c\})$. In cases (i)-(ii): $v(P_{bcx}, \{v(P_{bcx}, B \cap D), v(P_{bcx}, C \cap D)\}) = v(P_{bcx}, \{x, c\}) = c$; and, $v(P_{bcx}, \{v(P_{bcx}, B' \cap D), v(P_{bcx}, C' \cap D)\}) = v(P_{bcx}, \{x, b\}) = x$. But, this contradicts Claim 9(i). In case (iii): $v(P_{bcx}, D) = c$ by Claim 9(i); and, $v(P_{bcx}, D) = v(P_{bcx}, \{v(P_{bcx}, B' \cap D), v(P_{bcx}, C' \cap D)\}) = v(P_{bcx}, \{v(P_{bcx}, D), v(P_{bcx}, \{b, c\})\}) = v(P_{bcx}, \{c, b\}) = b$. But, this is a contradiction. ■

Claim 11 *$\mathcal{S}_X^v$ is continuous.*

Proof. The proof is by strong induction on $m \equiv |X|$. The base cases $m = 2, 3$ follow directly from the definition of $\mathcal{S}_X^v$ and IS. For the induction step $m = n + 1$, consider the root node X of $\mathcal{S}_X^v$. Let B and C denote its two successors. By IS and the induction hypothesis, the agendas $\mathcal{S}_X^v(B)$ and $\mathcal{S}_X^v(C)$ are simple.

Let B' and C' denote the two successors of B. And, let $b \in B' \setminus C'$ (resp. $c \in C' \setminus B'$) denote some alternative that labels one terminal node below B' (resp. C'). To complete the proof, it suffices to show that $b \notin C$ or $c \notin C$. (The argument for C is similar.) By way of contradiction, suppose $b, c \in C$. By IS, there exists some $x \in B \setminus C$. By Claim 9, it follows that $v(P, \{x, b, c\}) = v(P, \{v(P, \{x, b, c\}), v(P, \{b, c\})\}$.

By Claim 9 and the assumption about B', the only possible splittings of $\{x, b, c\}$ are: (i) $(\{b\}, \{c, x\})$; (ii) $(\{b, x\}, \{c\})$; or, (iii) $(\{b, x\}, \{c, x\})$. By the formula in the last paragraph, each of these cases entails a contradiction: (i) $b = v(P_{xcb}, \{x, b, c\}) \neq v(P_{xcb}, \{v(P_{xcb}, \{x, b, c\}), v(P_{xcb}, \{b, c\})\} = c$; (ii) $c = v(P_{xbc}, \{x, b, c\}) \neq v(P_{xbc}, \{v(P_{xbc}, \{x, b, c\}), v(P_{xbc}, \{b, c\})\} = b$; or, (iii) both of the contradictions obtained in cases (i)-(ii). ■

Proof of Theorem 1. Using the approach described in the text, the structure of the agenda S_X^v can be determined from outcomes on Condorcet triples. By construction, S_X^v is non-repetitive. By Claim 11, S_X^v is continuous.

To show that $\mathcal{S}_X^v$ implements v, I show that $UNE[\mathcal{S}_X^v; P] = v(P, X)$ for any profile P. Since $UNE[\mathcal{S}_{X|A}^v; P] = UNE[\mathcal{S}_X^v; P^A]$ (by Remark 1) and $v(P^A, X) = v(P, A)$ (by Claim 8), $UNE[\mathcal{S}_{X|A}^v; P] = v(P, A)$ for any $A \subset X$. To see that $UNE[\mathcal{S}_X^v; P] = v(P, X)$, use "backward induction" on $\mathcal{S}_X^v$ (see the proof of Remark 1). In any terminal subgame, the UNE selects the Condorcet winner. By ILA, so does v. By deleting the Condorcet loser and continuing in this fashion, $UNE[\mathcal{S}_X^v; P] = v(P, X)$ follows immediately by IS and the construction of $\mathcal{S}_X^v$. Finally, Claim 10 ensures that $\mathcal{S}_X^v$ is the unique simple agenda implementing v. For any simple agenda $\mathcal{S}_X$ implementing v, the subgames at any node A must induce the unique splitting of A. So, $\mathcal{S}_X$ must coincide with $\mathcal{S}_X^v$. ■

Proof of Theorem 1*. (using the assumptions about v in the statement of the Theorem) ($\Rightarrow$) Since ILA is necessary for v to be implementable by agenda and v satisfies IS by assumption, the result follows from the discussion in the text. ($\Leftarrow$) Fix an issue A. Since $v(\cdot, A)$ is implementable by simple agenda, "backward induction" establishes that A can be split. Since this is true for every $A \subseteq X$, v satisfies IS. ■

(d) Proof of Theorem 2

Sub-sections (i) and (ii) establish Lemmas 1 and 2. The proof of Theorem 2 is given in sub-section (iii).

(i) Proof of Lemma 1

Claim 12 *If a^* is marginal in A for v, then it is marginal in $A \setminus x$ for all $x \in A \setminus a^*$.*

Proof. Fix any $x \in A \setminus a^*$. By way of contradiction, suppose $v(P, A \setminus x) = a^*$ for some profile P where a^* is not the Condorcet winner in $A \setminus x$. By Claim 7, $v(P_x, A \setminus x) = v(P, A \setminus x)$ for any profile P_x that coincides with P except x is demoted to Condorcet loser in A. Moreover, $v(P_x, A) = v(P_x, A \setminus x)$ by ILA. So, $v(P_x, A) = a^*$, which contradicts the assumption that a^* is marginal in A. ■

Claim 13 *For v, every issue A has at most two marginal alternatives.*

Proof. Suppose otherwise. Denote any three marginal alternatives by x, y, and z and consider the triple P_{xyz} (as defined in the text). Then, $v(P_{xyz}, \{x, y, z\}) \notin \{x, y, z\}$ by Claim 12, which is a contradiction. ■

Claim 14 *Suppose (B, C) splits A for v and a^* is marginal in A. Then:*

(i) *if $a^* \in C \setminus B$, then $(B, C) = (b, A \setminus b)$; and,*

(ii) *if $a^* \in B \cap C$ and $a^{**} \in C \setminus a^*$ is also marginal in A, then $a^{**} \in B \cap C$.*

Proof. (i) By way of contradiction, suppose $|B \setminus C| \geq 2$. Fix $b, b' \in B \setminus C$ and consider the triple $P_{a^*bb'}$. By Claim 9, $v(P_{a^*bb'}, \{a^*, b, b'\}) = v(P_{a^*bb'}, \{v(P_{a^*bb'}, \{b, b'\}), a^*\}) = a^*$. By Claim 12, this contradicts the assumption that a^* is marginal in A. **(ii)** By way of contradiction, suppose $a^{**} \notin B$. Fix some $b \in B \setminus C$ and consider the triple $P_{a^*ba^{**}}$. Then, a contradiction obtains along the same lines as (i). ■

Claim 15 *If v satisfies WM, then $\succ_v$ is asymmetric and $\succsim_v$ is complete.*

Proof. The completeness of $\succsim_v$ is a direct consequence of the asymmetry of $\succ_v$. To see that $\succ_v$ is asymmetric, suppose $y \succ_v z$ and $z \succ_v y$ for some $y, z \in X$. Let Y and Z denote the issues leading to the inferences $y \succ_v z$ and $z \succ_v y$. The proof that this amounts to a contradiction is by induction on $|Y \cup Z|$.

For $|Y \cup Z| = 4$: suppose $Y = \{a^*, y, z\}$ and $Z = \{x, y, z\}$. (Every other case is ruled out by ILA or Claim 12.) By WM and Claims 12-13, the only possible marginal alternatives in $Y \cup Z$ are: a^* and x; or, one of the two, say a^*. Now, consider the unique splitting (B, C) of $\{a^*, x, y, z\}$. There are two possibilities: (i) $a^* \in C \setminus B$; and, (ii) $a^* \in B \cap C$. **(i)** By Claim 14(i), $(B, C) = (b, \{a^*, x, y, z\} \setminus b)$ with $b \neq a^*$. By Claim 9, every possibility for b leads to a contradiction: if $b = y$, then y is not marginal in $\{a^*, y, z\}$; if $b = z$, then z is not marginal in $\{x, y, z\}$; and, if $b = x$, then y is marginal in $\{x, y, z\}$.[31] **(ii)** By Claims 9 and 14(ii), $y \in B \cap C$. So, $(B, C) = (\{a^*, y, z\}, \{a^*, x, y\})$. But, then z is not marginal in $\{x, y, z\}$.

For $|Y \cup Z| = n + 1$: Y and Z have one or two marginal alternatives (by WM and Claim 13). If both have two, then this reduces to the case $|Y \cup Z| = n$ by Claim 12. If both have one, then y or z is marginal in $Y \cup Z$ by WM and Claims 12-13. So, either y is marginal in Z or z is marginal in Y by Claim 12, both contradictions. So, suppose y and a^* are marginal in Y while z is marginal in Z. By WM and Claim 12, a^* is the only marginal alternative in $Y \cup Z$ and $a^* \notin Z$. By Claim 12, it also follows that: a^* and y are marginal in $\{a^*, y, z\}$; and, z is marginal in $\{x, y, z\}$ for any $x \in Z$.

Now, consider the splitting (B, C) of $Z^* = Z \cup a^*$. As in the base case, there are two possibilities: (i) $(B, C) = (\{b\}, Z^* \setminus b)$ with $b \neq a^*$; and, (ii) $a^* \in B \cap C$. For both, I claim that y and z must appear in the

[31] This last case cannot occur if x is marginal in $\{a^*, x, y, z\}$.

same sub-issues as a^*. **(i)** As in the base case, $b \neq y, z$. So, $\{y, z\} \subseteq C$ as claimed. **(ii)** As in the base case, $y \in B \cap C$. This, in turn, implies $z \in B \cap C$. To see why, suppose $z \in B \setminus C$ and fix some $x \in C \setminus B$. Then, as in the base case, z cannot be marginal in $\{x, y, z\}$. So, $\{y, z\} \subseteq B \cap C$ as claimed.

Continuing in the same vein on the sub-issues B and C, it follows that y and z always appear in the same sub-issues (up to the splitting of $\{a^*, y, z\}$). Now, construct the agenda $\mathcal{S}^v_{Z \cup a^*}$. By the last observation, y and z appear in exactly the same subgames of $\mathcal{S}^v_{Z \cup a^* | Z}$ (i.e. after a^* is deleted). By assumption, $v(P, Z) = y$ for some profile P where y is not the Condorcet winner in Z. Since $\mathcal{S}^v_Z$ implements v on Z by Theorem 1, $v(P, Z) = UNE[\mathcal{S}^v_Z; P] = y$. To show a contradiction, consider the related profile P_σ that permutes z and y in every voter's preference. From the symmetry of $\mathcal{S}^v_Z$, $v(P_\sigma, Z) = UNE[\mathcal{S}^v_Z; P_\sigma] = z$. But, this contradicts the assumption that z is marginal in Z and establishes that $\succ_v$ is asymmetric. ■

Claim 16 *If v satisfies WM, $\succsim_v$ is a weak order whose indifference classes contain one or two alternatives.*

Proof. Since $\succsim_v$ is complete by Claim 15, showing transitivity proves $\succsim_v$ is a weak order. Fix $x \succsim_v y \succsim_v z$. By way of contradiction, suppose $z \succsim_v x$. By WM, some alternative is marginal in $A = \{x, y, z\}$. By definition of $\succsim_v$, it then follows that A has three marginal alternatives. But, this contradicts Claim 13. This rules out the possibility that $z \succsim_v x$. Since $\succsim_v$ is complete by Claim 15, it follows that $x \succ_v z$, which shows that the indifference classes of $\succsim_v$ may contain at most two alternatives. ■

Proof of Lemma 1. Claim 16 establishes (i). To establish (ii), fix an issue A. By WM, some $x \in A$ must be marginal. Let $z \in \min_{\succsim_v} A$ and $y \equiv \min_{\succsim_v} A \setminus z$. If $x \neq y, z$, one obtains a contradiction along the lines of Claim 16. So, suppose $x = y$. Since $\succsim_v$ is complete by Claim 15, there are two possibilities. If $x \succsim_v z$, then z is marginal as well. If $z \succ_v x$, then x may be the only marginal alternative. ■

Corollary 3 *If v satisfies WM, then:*

(i) *$y \succ_v z$ if and only if there exists an $x \in X$ such that $v(P_{xyz}, \{x, y, z\}) = y$; and,*

(ii) *$y \sim_v z$ if and only if*
$$\left\{ \begin{array}{c} v(P_{xyz}, \{x,y,z\}) = z \;\; \text{and} \;\; v(P_{xzy}, \{x,y,z\}) = y \\ \textbf{\textit{or}} \\ v(P_{xyz}, \{x,y,z\}) = x \;\; \text{and} \;\; v(P_{xzy}, \{x,y,z\}) = x \end{array} \right\} \text{ for all } x \in X.$$

Proof. (i) ($\Leftarrow$) From the six possible splittings of $\{x, y, z\}$, $v(P_{xyz}, \{x, y, z\}) = y$ implies $v(P_{xzy}, \{x, y, z\}) \in \{x, y\}$. So, $y \succ_v z$. ($\Rightarrow$) By way of contradiction, suppose $v(P_{xyz}, \{x, y, z\}) \neq y$ for all $x \in X$. Since $y \succ_v z$, asymmetry implies $v(P_{xyz}, \{x, y, z\}) \neq z$ for all $x \in X$. So, $v(P_{xyz}, \{x, y, z\}) = x$ for all $x \in X$. By the argument in ($\Leftarrow$), $v(P_{xzy}, \{x, y, z\}) \in \{x, z\}$ for all $x \in X$. Since $y \succ_v z$, asymmetry implies $v(P_{xzy}, \{x, y, z\}) = x$ for all $x \in X$. Now, fix an issue A s.t $|A| \geq 3$ with splitting (B, C). First, observe that $y, z \in B \cap C$ or $y, z \in C \setminus B$. Otherwise, $y \in C \setminus B$ and $z \in B$ without loss of generality. If $z \in B \setminus C$, Claim 9 shows that $v(P_{xyz}, \{x, y, z\}), v(P_{xzy}, \{x, y, z\}) \neq x$ for any $x \in A$. If $z \in B \cap C$, there is a similar contradiction for $x \in B \setminus C$. Since $y, z \in B \cap C$ or $y, z \in C \setminus B$ for the splitting (B, C) of any issue A, y and z appear in the same subgames of $\mathcal{S}^v_X$. Since $y \succ_v z$, a contradiction obtains by the argument in Claim 15. **(ii)** Fix any $x \in X$. Given the six possible splittings of $\{x, y, z\}$, the result follows from (i) and the fact that $\succsim_v$ is a weak order (by Claim 15). ■

(ii) Proof of Lemma 2

Claim 17 *If v satisfies WM and (B, C) is the unique splitting of A, then $|C \setminus B| \geq 2$ implies $|B \setminus C| = 1$.*

Proof. By way of contradiction, suppose $|C \setminus B|, |B \setminus C| \geq 2$. Fix any $b, b' \in B \setminus C$ and $c, c' \in C \setminus B$. By Claim 9, $(\{b, b'\}, \{c, c'\})$ is the unique splitting of $A' = \{b, b', c, c'\}$. For all $a \in A'$, it follows that $v(P(a), A') = a$ for some profile $P(a)$ where a is not the Condorcet winner, which contradicts WM. ■

Using the definitions in section (b) above:

Claim 18 *If v satisfies WM and $A = \{a_1, ..., a_K\}$ is labeled according to $\succ_v^*$ for $K \geq 2$, then:*

$$(a_1 \cup A_j^K, A_2^K) \text{ splits } A \text{ for some } j \text{ s.t. } j \in \{3, ..., K+1\}.$$

Proof. By Claim 14(i), there are two possibilities for the unique splitting (B, C) of A: (i) either $(B, C) = (b, A \setminus b)$ with $b \neq a_K$; or, (ii) $a_K \in B \cap C$. In either case, I show that (B, C) has the form required.

(i) In this case, it suffices to show $b = a_1$. By way of contradiction, suppose $b = a_k$ for some $k \neq 1, K$. Then, $v(P_{a_K a_k a_1}, \{a_1, a_k, a_K\}) = a_k$ by Claim 9 so that $a_k \succ_v a_1$ by Lemma 3. Since $a_1 \succsim_v a_k$ by assumption, $a_k \succ_v a_1$ contradicts the fact that $\succsim_v$ is a weak order (by Claim 15). So, $b = a_1$ as required.

(ii) By Claim 17, there are two possibilities: (1) $(B, C) = (A \setminus c, A \setminus b)$; and, (2) $(B, C) = (b \cup B', A \setminus b)$ for $B' \subset A \setminus b$ and $|A \setminus B'| \geq 3$. **(1)** It suffices to show $b = a_1$ and $c = a_2$. If $a_1 \neq b, c$, then the outcomes on $\{a_1, b, c\}$ lead to the contradictions $b, c \succ_v a_1$ following the same kind of reasoning as in case (i). So, $b = a_1$ without loss of generality. If $a_2 \neq c$, then the outcomes on $\{a_1, a_2, c\}$ lead to the contradiction $c \succ_v a_2$. So, $c = a_2$. **(2)** It suffices to show: (a) $b = a_1$; (b) $a_2 \notin B'$; and, (c) $a_k \in B'$ implies $a_{k+1} \in B'$. **(a)** By the same reasoning as (1), $a_1 \notin B \cap C = B'$. If $a_1 \neq b$, then $\{a_1, b, c\}$ leads to the contradiction $b \succ_v a_1$ for $c \notin b \cup B'$. So, $b = a_1$. **(b)** If $a_2 \in B'$, then $\{a_1, a_2, c\}$ leads to the contradiction $c \succ_v a_2$ for $c \notin a_1 \cup B'$ given (a). So, $a_2 \notin B'$. **(c)** If $a_k \in B'$ and $a_{k+1} \notin B'$, then $\{a_1, a_k, a_{k+1}\}$ leads to the contradiction $a_{k+1} \succ_v a_k$ given (a). ■

Definition 11 *Given a decision rule v with revealed priority $\succsim_v$, x is* ***revealed to amend*** *$b \succ_v x$ if:*

$$v(P_{xbc}, \{b, c, x\}) = c \text{ for all } c \in X \text{ such that } b \succsim_v c \succ_v x.$$

Define α_v as follows: $b \in \alpha_v(x)$ if x is revealed to amend b and x is <u>not</u> revealed to amend any $a \succ_v b$.

Lemma 2 *If v satisfies WM, then α_v is an amendment rule.*

Proof. Definition 11 and Corollary 3 ensure the following: (i) $x \in \alpha_v(z) \Rightarrow x \succ_v z$; and, (iii) $[x \sim_v y \succ_v z$ and no $z' \in X$ s.t. $x \sim_v y \succ_v z' \succ_v z] \Rightarrow [x \in \alpha_v(z)$ or $w \succ_v x$ for all $w \in \alpha_v(z)]$. I show: (ii) $x \sim_v y \Rightarrow \alpha_v(x) = \alpha_v(y)$.

(ii) It suffices to show that y is revealed to amend b if x is revealed to amend b. By way of contradiction, suppose y is not revealed to amend b. By Definition 11, there exists some c s.t. $b \succsim_v c \succ_v y \sim_v x$ and, moreover, $v(P_{ybc}, \{b, c, y\}) \neq c$ for all such c. By Corollary 3, $v(P_{ybc}, \{b, c, y\}) \neq y$. Otherwise, $y \succ_v b$ which contradicts the fact that $\succsim_v$ is a weak order (by Claim 15). So, $v(P_{ybc}, \{b, c, y\}) = b$ which, by Corollary 3, implies $b \succ_v c$. Finally, Definition 11(i) implies $v(P_{xbc}, \{b, c, x\}) = c$ for all c s.t. $b \succsim_v c \succ_v x$ (since x is revealed to amend b). To summarize, $v(P_{xbc}, \{b, c, x\}) = c$ and $v(P_{ybc}, \{b, c, y\}) = b$ for some c s.t. $b \succ_v c \succ_v y \sim_v x$.

By Claim 18, the splitting of $\{b, c, x, y\}$ is $(b \cup B', \{c, x, y\})$ for $B' \subseteq \{x, y\}$. By Claim 9, $v(P_{xbc}, \{b, c, x\}) = c$ implies $x \in B'$ and $v(P_{ybc}, \{b, c, y\}) = b$ implies $y \notin B'$. So, the splitting of $\{b, c, x, y\}$ is $(\{b, x\}, \{c, x, y\})$. By Claim 9, this implies $v(P_{byx}, \{b, x, y\}) = y$ so that $y \succ_v x$ by Corollary 3, which is a contradiction. ■

(iii) Proof of Theorem 2

Proof of Theorem 2. (**using the assumptions about v in the statement of the Theorem**) ($\Leftarrow$) The discussion in the text following Theorem 1 shows that v satisfies ILA (i.e. any decision rule implemented by an agenda satisfies ILA). In turn, Proposition 1 and Claim 5 show that v satisfies WM and IS.

($\Rightarrow$) This follows from the fact that the simple agenda $\mathcal{S}_X^v$ from Theorem 1 coincides with the priority agenda $\mathcal{P}_X^v$ defined by $(\succsim_v, \alpha_v)$. To establish this fact, it suffices to show that the successors at the root node of $\mathcal{P}_X^v$ and $\mathcal{S}_X^v$ coincide. Extending this reasoning by induction, it follows that $\mathcal{S}_X^v$ coincides with $\mathcal{P}_X^v$.

Consider the root node $X = \{x_1, ..., x_m\}$ of $\mathcal{S}_X^v$ as labeled according to $\succ_v^*$. By Claim 18, the successors of X are $x_1 \cup X_j^m$ and X_2^m for some j s.t. $j \in \{3, ..., m+1\}$. There are two cases: **(i)** X_j^m is empty (i.e. $j = m+1$); or, **(i)** X_j^m is non-empty (i.e. $j \leq m$). The fact that the successors of X on $\mathcal{S}_X^v$ coincide with the successors on $\mathcal{P}_X^v$ follows by Claim 1 in both cases: **(i)** Claim 9 applied to (x_1, X_2^m) gives $v(P_{x_j x_1 c}, \{x_1, x_j, x\}) = x_1$ for all x_j, c s.t. $x_1 \succsim_v c \succ_v x_j$. So, no x_j s.t. $x_2 \succ_v x_j$ is revealed to amend x_1 (by Definition 11). **(ii)** Claim 9 applied to $(x_1 \cup X_j^m, X_2^m)$ gives $v(P_{x_j x_k x_1}, \{x_1, x_k, x_j\}) = x_1$ and $v(P_{x_j x_1 x_k}, \{x_1, x_k, x_j\}) = x_k$ for $x_k \in X_2^{j-1}$. This shows that $x_k \succ_v x_j$ (by Corollary 3) and x_j is revealed to amend x_1. To see that no x_k s.t. $x_2 \succ_v x_k \succ_v x_j$ is revealed to amend x_1, it is enough to observe that $v(P_{x_k x_1 x_2}, \{x_1, x_2, x_k\}) = x_1$ for all $x_k \in X_3^{j-1}$. ■

(e) Proof of Proposition 4

Claim 19 *If every issue A s.t. $|A| \neq 1$ has two marginal alternatives, then $\mathcal{S}_X^v$ is a Euro-Latin agenda.*

Proof. Consider the splitting (B_1, X_1) of X and let a^* denote a marginal alternative in X. By Claim 14(i), there are two possibilities: (1) $a^* \in B_1 \cap X_1$ with $b_1 \in B_1 \setminus X_1$ and $x_1 \in X_1 \setminus B_1$; and, (2) $(B_1, X_1) = (b_1, X \setminus b_1)$. For (1), Claim 9 implies that $\{a^*, b_1, x_1\}$ has one marginal alternative a^*, a contradiction. So, the splitting must be (2). Continuing in the same vein on X_1 establishes that $\mathcal{S}_X^v$ is a Euro-Latin agenda. ■

Claim 20 *If every issue A s.t. $|A| \neq 2$ has a unique marginal alternative, then $\mathcal{S}_X^v$ is an Anglo-American agenda.*

Proof. Consider the splitting (B, C) of X. If $|C \setminus B| \geq 1$ (with $b \in B \setminus C$ and $c, c' \in C \setminus B$), then Claim 9 implies that $\{b, c, c'\}$ has two marginal alternatives c and c', a contradiction. This shows that $|C \setminus B| = |B \setminus C| = 1$. In other words, $(B, C) = (X \setminus c_1, X \setminus b_1)$ for some $b_1 \in B$ and $c_1 \in C$. Continuing in the same vein on $X \setminus c_1$ and $X \setminus b_1$ establishes that $\mathcal{S}_X^v$ is an Anglo-American agenda. ■

Proof of Proposition 4. (**using the assumptions about v in the statement of the Theorem**) Regarding the first part of the claim: the Euro-Latin procedure has two marginal alternatives for all A s.t. $|A| \neq 1$; and, the Anglo-American procedure has a unique marginal alternative for all A s.t. $|A| \neq 2$.

Regarding the second part of the claim, suppose $|X| \geq 3$. (If $|X| = 2$, the claim is trivial.) By Claim 13, there are two cases: (i) X has two marginal alternatives a_1^* and a_2^*; or, (ii) X has a unique marginal alternative.

(i) By Theorem 1 and Claim 19, it suffices to show that all A s.t. $|A| \neq 1$ have two marginal alternatives. By Claim 12, a_1^* and a_2^* are marginal in $X \setminus x$ for all $x \neq a_1^*, a_2^*$. By NP, every $X \setminus x$ has two marginal alternatives. Continuing in the same vein, the result follows by a simple inductive argument.

(ii) By Theorem 1 and Claim 20, it suffices to show that all A s.t. $|A| \neq 2$ have one marginal alternative. By way of contradiction, suppose $|X| \geq 4$ and some $X \setminus x$ has two marginal alternatives. Then, by the argument in case (i), every $A \neq X$ s.t. $|A| \neq 1$ has two marginal alternatives. To establish the contradiction, consider the splitting (B_1, X_1) of X. By the argument in Claim 19, $(B_1, X_1) = (b_1, X \setminus b_1)$. Since $\mathcal{S}_{X_1}^v$ is Euro-Latin by Claim 19, this shows that $\mathcal{S}_X^v$ is as well. It follows that X has two marginal alternatives, which is a contradiction. ■

(f) Proof of Corollary 1

For part (i), Apesteguia et al. [2014] show CP is necessary. Sufficiency follows from Claim 19 and:

Claim 21 *If v satisfies CP, then every issue A such that $|A| \neq 1$ has two marginal alternatives.*

For part (ii), Apesteguia et al. [2014] show CA is necessary. Sufficiency follows from Claim 20 and:

Claim 22 *If v satisfies CP, then every issue A such that $|A| \neq 2$ has a unique marginal alternative. What is more, this alternative coincides with the unique anti-prioritarian alternative in A when $|A| \geq 3$.*

Since the proofs of these claims are quite similar, I point out only where the differences arise:

Proof. The proof of Claim 21 (22) is by induction on $|X|$. For the base cases $|X| = 2, 3$, the claim follows from ILA and CP (CA). For the induction step, note that all $A \subset X$ satisfy the claim by the induction hypothesis. To see that X also satisfies the claim, consider the splitting (B, C) of X. There are two possibilities for the prioritarian (anti-prioritarian) alternative p in X: (i) $p \in B \cap C$; and, (ii) $p \in B \setminus C$.

For **Claim 21**: Consider $b \in B \setminus C$ and $c \in C \setminus B$. Using Claim 9, (i) leads to the contradiction that p is not prioritarian in $\{b, c, p\}$ (let alone X). So, (ii) must hold. Using the same kind of reasoning, it can be shown that $B = \{p\}$. (The idea is to consider an issue $\{b', c, p\}$ s.t. $b' \in B$, $c \in C \setminus B$. While there are several cases, a contradiction obtains for each.) By the induction hypothesis, $X \setminus p$ has two marginal alternatives. Since the splitting of X is $(\{p\}, X \setminus p)$, IS implies that these alternatives are marginal in X as well.

For **Claim 22**: Consider $b \in B$ and $c \in C \setminus B$. Using Claim 9, (ii) leads to the contradiction that p is not anti-prioritarian in $\{b, c, p\}$ (let alone X). So, (i) must hold. By the induction hypothesis, p is marginal in B and C (since it is anti-prioritarian for these issues). By IS, it then follows that p is marginal in X. Finally, by Claim 12 and the induction hypothesis, there can be no other marginal alternative in X. ∎

(g) Proof of Proposition 5

For a given decision problem (P, A), a sequence $\langle a \rangle \equiv a_1, ..., a_n$ is said to be *universal* for $a \in A$ if:

(1) every alternative in A appears exactly once in the sequence $\langle a \rangle$;

(2) $\langle a \rangle$ contains a M_P^A-transitive subsequence $\langle b \rangle \equiv b_1, ..., b_m$ such that $a = b_1$; and,

(3) every alternative $a_i \in \langle a \rangle \setminus \langle b \rangle$ satisfies the following two conditions:

  (i) a_i appears before some alternative $b \in \langle b \rangle$ in the sequence $\langle a \rangle$; and,

  (ii) $b_j M_P^A a_i$ for the first $b_j \in \langle b \rangle$ after a_i in the sequence $\langle a \rangle$.

Two observations about universal sequences will be critical for the result:

Claim 23 *A sequence $\langle a \rangle$ in A is universal for at most one alternative in A.*

Proof. By way of contradiction, suppose $\langle a \rangle = a_1, ..., a_n$ is universal for distinct alternatives a_i and a_j. Without loss of generality, suppose $i < j$. Let $\langle b^i \rangle$ and $\langle b^j \rangle$ denote the transitive subsequences associated with a_i and a_j, respectively. By condition (3.i), it follows that $b_1^i = a_i$ and $b_1^j = a_j$. So, $a_i \notin \langle b^j \rangle$. Hence, $a_j M_P^A a_i$ by condition (3.ii). Since $\langle b^i \rangle$ is a transitive sequence with a_i maximal by condition (2), $a_j M_P^A a_i$ implies $a_j \notin \langle b^i \rangle$.

By condition (3.ii), $a_j \notin \langle b^i \rangle$ implies $\tilde{b}^i M_P^A a_j$ for some $\tilde{b}^i \in \langle b^i \rangle$ such that $\tilde{b}^i = a_{k(i)}$ and $k(i) > j$. Since $\langle b^j \rangle$ is a transitive subsequence by condition (2), $\tilde{b}^i M_P^A a_j$ implies $\tilde{b}^i \notin \langle b^j \rangle$. By condition (3.ii), $\tilde{b}^i \notin \langle b^j \rangle$ implies $\tilde{b}^j M_P^A \tilde{b}^i$ for some $\tilde{b}^j \in \langle b^j \rangle$ such that $\tilde{b}^j = a_{k(j)}$ and $k(j) > k(i)$. Since $\langle b^i \rangle$ is a transitive subsequence by condition (2), $\tilde{b}^j M_P^A \tilde{b}^i$ implies $\tilde{b}^j \notin \langle b^i \rangle$. Continuing in this vein leads to the contradiction that $\langle a \rangle$ does not terminate. ∎

Claim 24 *There exists a universal sequence for a on (P, A) iff $a \in BA(M_P^A)$.*

Proof. ($\Rightarrow$) Fix some $b \in A \setminus BA(M_P^A)$. By way of contradiction, suppose that $\langle a \rangle$ is universal for b on (P, A). Consider the transitive subsequence $\langle b \rangle = b_1, ..., b_m$ with $b_1 = b$. Since $b \in A \setminus BA(M_P^A)$, $\langle b \rangle$ is not a maximal transitive sequence. In other words, there exists some $a \in A \setminus \langle b \rangle$ such that $a, b_1, ..., b_m$ is a transitive sequence. So, there exists no $b_j \in \langle b \rangle$ such that $b_j M_P^A a$. Since a must appear before some $b_j \in \langle b \rangle$ by condition (3.i), this contradicts condition (3.ii). In particular, no $b_j \in \langle b \rangle$ may comes after a in $\langle a \rangle$.

($\Leftarrow$) Since $a \in BA(M_P^A)$, there exists a maximal transitive sequence $\langle b \rangle$ in A with $b_1 = a$. For every $a \in A \setminus \langle b \rangle$, there exists some $b \in \langle b \rangle$ such that $bM_P^A a$. Otherwise, $a, b_1, ..., b_m$ is a transitive sequence, which contradicts the maximality of $\langle b \rangle$. Let $b_{i(a)}$ denote the last alternative in $\langle b \rangle$ such that $b_{i(a)} M_P^A a$. Extend $\langle b \rangle$ into a sequence $\langle a \rangle$ on A by inserting each $a \in A \setminus \langle b \rangle$ between the alternatives $b_{i(a)-1}$ and $b_{i(a)}$. By construction, $\langle a \rangle$ satisfies conditions (1)-(3). So, $\langle a \rangle$ is a universal sequence for a on (P, A). ■

Given a priority agenda $\mathcal{P}_A$ on A defined by $(\succsim, \alpha)$, define a strict ordering $\succ^*$ from $\succsim$. Let $\langle a \rangle^* = a_1^*, ..., a_n^*$ denote the sequence defined by taking the alternatives in A in ascending order according to $\succ^*$. Given another sequence $\langle a \rangle = a_1, ..., a_n$ of the alternatives in A, define the permutation $\sigma^*_{\langle a \rangle} : A \to A$ by $\sigma^*_{\langle a \rangle}(a_i) = a_i^*$. To simplify the notation, let $\mathcal{P}_A^{\langle a \rangle} \equiv \mathcal{P}_A^{\sigma^*_{\langle a \rangle}}$ denote the agenda where the i^{th} alternative in the sequence $\langle a \rangle$ occupies the i^{th} highest priority position in the agenda.

Claim 25 *Given a priority agenda $\mathcal{P}_A$ on A and a universal sequence $\langle a \rangle$ for a on (P, A), $UNE[\mathcal{P}_A^{\langle a \rangle}, P] = a$.*

Proof. Since $\langle a \rangle$ in A is universal for at most one alternative in A by Claim 23, the claim is well-defined.

The proof is by strong induction on $|A|$. The base case $|A| = 2$ is trivial. So, suppose the claim holds for $|A| = n$. For $|A| = n + 1$, fix a universal sequence $\langle a \rangle$ for a on (P, A) with transitive subsequence $\langle b \rangle$ satisfying conditions (2)-(3). Given Claim 1, let $C \equiv a_1 \cup A_k^{n+1}$ and $C' \equiv A_2^{n+1}$ for $2 < k < n + 2$ denote the successors of the root node in $\mathcal{P}_A^{\langle a \rangle}$; and, let $\langle c \rangle$ and $\langle c' \rangle$ denote the subsequences associated with C and C'.

There are three cases to consider: (i) $a \in C \setminus C'$; (ii) $a \in C \cap C'$; and, (iii) $a \in C' \setminus C$.

(i) Since $|C \setminus C'| = 1$, it follows that $a = a_1$. Then, it is easy to check that $\langle c \rangle = a, a_k, ..., a_{n+1}$ is a universal sequence for a on (P, C). Since the last alternative in $\langle a \rangle$ must be in $\langle b \rangle$ by condition (3.i), it is also easy to verify that $\langle c' \rangle = a_2, ..., b_2, ..., a_{n+1}$ is a universal sequence for b_2 on (P, C'). Since $\mathcal{P}_A^{\langle a \rangle}(C)$ and $\mathcal{P}_A^{\langle a \rangle}(C')$ are both priority agendas, $UNE[\mathcal{P}_A^{\langle a \rangle}(C); P] = a$ and $UNE[\mathcal{P}_A^{\langle a \rangle}(C); P] = b_2$ by the induction hypothesis. Since $aM_P^A b_2$ by construction, "backward induction" implies $UNE[\mathcal{P}_A^{\langle a \rangle}; P] = a$.

(ii) Since $|C \setminus C'| = 1$, it follows that $a \neq a_1$. So, $a \in C$ implies $a = a_j$ for some $j \geq k$. As in case (i), it is easy to check that $\langle c \rangle = a_1, a_k, ..., a, ..., a_{n+1}$ is a universal sequence for a on (P, C). Likewise, it is easy to verify that $\langle c' \rangle = a_2, ..., a, ..., a_{n+1}$ is a universal sequence for a on (P, C'). So, $UNE[\mathcal{P}_A^{\langle a \rangle}(C); P] = a = UNE[\mathcal{P}_A^{\langle a \rangle}(C'); P]$ by the induction hypothesis. Using "backward induction", it then follows that $UNE[\mathcal{P}_A^{\langle a \rangle}; P] = a$.

(iii) Since $|C \setminus C'| = 1$, $a \neq a_1$. So, $a \in C$ implies $a = a_j$ for some $j \geq k$. As in case (ii), $\langle c' \rangle = a_2, ..., a, ..., a_{n+1}$ is a universal sequence for a on (P, C'). So, $UNE[\mathcal{P}_A^{\langle a \rangle}(C'); P] = a$ by the induction hypothesis. By way of contradiction, suppose $UNE[\mathcal{P}_A^{\langle a \rangle}; P] \neq a$. Since $C \setminus C' = \{c\}$ and $UNE[\mathcal{P}_A^{\langle a \rangle}(C'); P] = a$, Claim 6 implies $UNE[\mathcal{P}_A^{\langle a \rangle}; P] = c$. Since $c \notin C'$, it follows that $UNE[\mathcal{P}_A^{\langle a \rangle}(C); P] = c$. Given $UNE[\mathcal{P}_A^{\langle a \rangle}; P] = c$, "backward induction" implies $cM_P^A a$, which contradicts condition (3.ii). ■

Proof of Proposition 5. **(i)** This is a consequence of McKelvey and Niemi's [1978] observation that *no* agenda ever selects an alternative outside of the Top Cycle. **(ii)** Fix a priority agenda $\mathcal{P}_X$ on X and consider the deleted agenda $\mathcal{P}_A$ on A. Given an alternative $a \in BA(M_P^A)$, Claim 24 ensures that there exists a universal sequence $\langle a \rangle$ for a on (P, A). Then, Claim 25 establishes that a is selected from $\mathcal{P}_A^{\langle a \rangle}$. ■

IX. Appendix – Independence of the Axioms

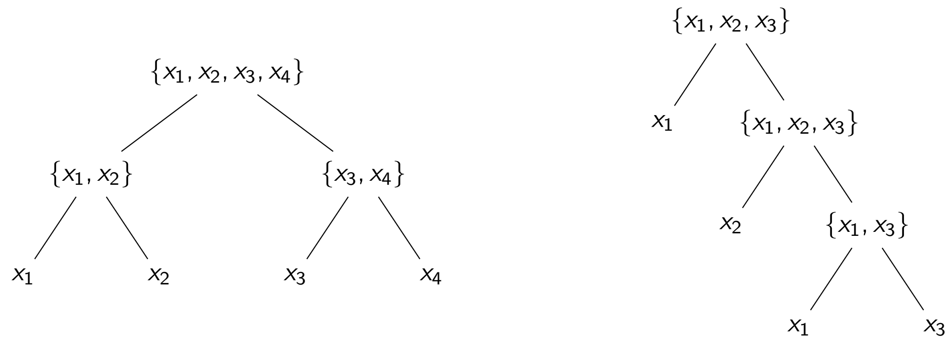

Figure 11: Agenda $\mathcal{T}_1$ (*left*) and Agenda $\mathcal{T}_2$ (*right*)

It is easy to see that the decision rule v_1 induced by $\mathcal{T}_1$ satisfies IS and ILA. To see that it violates WM, note that $v_1(P_1, \{x_1, x_2, x_3, x_4\}) = x_1$ for the profile P_1 where: x_1 is majority preferred to x_2 and x_3 but not x_4; and, x_3 is majority preferred to x_4. So, x_1 is selected without being the Condorcet winner. Since the other alternatives are symmetrically placed, there are also profiles where they are selected without being the Condorcet winner.

It is easy to see that the decision rule v_2 induced by $\mathcal{T}_2$ satisfies ILA. To see that it satisfies WM, note that $v(P, \{x_1, x_2, x_3\}) = x_3$ only if x_3 is the Condorcet winner on $\{x_1, x_2, x_3\}$. To see that it violates IS, note that $v_2(P_{123}, \{x_1, x_2, x_3\}) = x_1$ and $v_2(P_{132}, \{x_1, x_2, x_3\}) = x_2$. As such, the more preferred between x_1 and x_2 is selected for both Condorcet triples. By Table 1, this cannot be achieved with any simple agenda.

Finally, consider the decision rule v_3 on $\{x_1, x_2, x_3\}$ that selects: the majority preferred alternative between x_1 and x_2 when both are available; x_i on $\{x_i, x_3\}$; and, x_i on $\{x_i\}$. Since $(\{x_1, x_3\}, \{x_2, x_3\})$ splits $\{x_1, x_2, x_3\}$, v_3 satisfies IS. Since x_3 is trivially marginal, v_3 also satisfies WM. To see that it violates ILA, consider a profile P_3 where x_3 is the Condorcet winner on $\{x_1, x_2, x_3\}$. If v_3 satisfies ILA, then $v_3(P_3, \{x_1, x_2, x_3\}) = ... = x_3$. But, this contradicts the assumption that $v_3(P_3, \{x_1, x_2, x_3\}) = v_3(P_3, \{x_1, x_2\}) \neq x_3$.