Phasing out the GSEs *

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Abstract

We develop a new model of the mortgage market that emphasizes the role of the financial sector and the government. Risk tolerant savers act as intermediaries between risk averse depositors and impatient borrowers. Both borrowers and intermediaries can default. The government provides both mortgage guarantees and deposit insurance. Underpriced government mortgage guarantees lead to more and riskier mortgage originations and higher financial sector leverage. Mortgage crises occasionally turn into financial crises and government bailouts due to the fragility of the intermediaries' balance sheets. Foreclosure crises beget fiscal uncertainty, further disrupting the optimal allocation of risk in the economy. Increasing the price of the mortgage guarantee "crowds in" the private sector, reduces financial fragility, leads to fewer but safer mortgages, lowers house prices, and raises mortgage and risk-free interest rates. Due to a more robust financial sector and less fiscal uncertainty, consumption smoothing improves and foreclosure rates fall. While borrowers are nearly indifferent to a world with or without mortgage guarantees, savers are substantially better off. While aggregate welfare increases, so does wealth inequality.

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1 Introduction

Government and quasi-government entities dominate mortgage finance in the U.S. Over the past five years, the government-sponsored enterprises, Fannie Mae and Freddie Mac, and the Federal Housing Administration have stood behind 80% of the newly originated mortgages.[1] Ever since the collapse of the GSEs in September of 2008 and the conservatorship which socialized housing finance, there have been many proposals to bring back private capital.[2] The main idea of these policy proposals is to dramatically reduce the size and scope of the government guarantee on standard (conforming) mortgages. Because the proposed reform would turn a largely public into a largely private housing finance market, there is both uncertainty and concern about its impact on house prices, the stable provision of mortgage credit, financial sector stability, and ultimately welfare.[3]

Understanding the economic impact of wholesale mortgage finance reform of the kind currently debated requires a general equilibrium model. Such a model must recognize the important role that residential real estate and mortgage markets have come to play in the financial system and the macro-economy of rich countries (Jorda, Schularick, and Taylor (2014)). It must also recognize the large footprint of the government. This paper proposes a new general equilibrium model of the housing and mortgage markets where the interaction of the financial sector and the government plays a central role.

In our benchmark model, the government provides mortgage default insurance at low cost. The financial sector issues mortgages to borrowers and decides for how many of those mortgages to buy the government guarantee. Guaranteed mortgages dominate banks' portfolios due to

[1]Currently, of the \$9.85 trillion stock of residential mortgages, 57% are Agency Mortgage-backed Securities guaranteed by Fannie Mae, Freddie Mac, and Ginnie Mae. Private-label mortgage backed securities make up less than 8% of the stock. The rest is unsecuritized first liens held by the GSEs and the banking sector (28%) and second liens (7%). Acharya, Richardson, Van Nieuwerburgh, and White (2011) provide an in-depth discussion of the history of the GSEs, their growth, and collapse.

[2]The Obama Administration released a first report along these lines in February 2011. The bills proposed -but not passed- by Corker-Warner in 2013 and Johnson-Crapo in 2014 provide the most recent attempts at legislative reform.

[3]The financial and real estate industries, the Mortgage Bankers Association, and consumer advocate groups have all vehemently argued to keep a form of government guarantee in place. They favor a solution with a public mortgage guarantor that would succeed Fannie and Freddie, and fear that there may "not be enough private capital" in the mortgage market in a fully private system. They argue a private market solution would jeopardize the stable provision of mortgage credit for a broad cross-section of households. In contrast, the Congressional Budget Office recently argued that a privatization of housing finance would have minimal impact (CBO 2014).

the low insurance premium (guarantee fee or g-fee) that the government charges banks, as well as due to the small amount of regulatory capital that banks must hold against guaranteed mortgage bonds. As in the real world, mortgages are long-term, prepayable, and defaultable. The financial sector enjoys a government bailout guarantee, which is equivalent to deposit insurance in our setting. Deposit insurance is an important feature of any financial system that the literature on mortgage finance has not considered hitherto.[4]

A main result is that underpriced mortgage guarantees beget financial sector risk taking. As the least risk averse among the agents in the model, bankers desire a high return-high risk portfolio. But taking advantage of the underpriced mortgage guarantee lowers the risk of banks' assets. This prompts banks to increase leverage. The favorable regulatory capital treatment of guaranteed mortgage bonds enables such high leverage. Deposit insurance further propels leverage, since it makes banks' lenders, the depositors, insensitive to the risk of a banking collapse. A second way in which banks increase risk is through their mortgage origination decisions. They grow the size of the mortgage portfolio. And they increase its riskiness, by raising mortgage debt-to-income and loan-to-value ratios. Low equilibrium mortgage rates make borrowers willing to take on higher mortgage debt. In sum, the government's underpriced mortgage guarantee distorts financial sector leverage and leads to a larger financial sector and lower underwriting standards as banks pursue their desired risk-reward ratio. Deposit insurance and low bank equity requirements enable high financial sector leverage. House prices are inflated.

Ex-post, a larger and riskier mortgage portfolio produces higher mortgage default and loss rates. These losses produce deadweight costs of foreclosures, a first source of welfare losses for the economy. Given banks' low net worth and high leverage, housing crises occasionally turn into financial crises, defined as bank insolvencies. Thus, the economy with underpriced mortgage guarantees generates financial sector fragility. It also displays high house price volatility.

The government absorbs most of the default-induced losses as part of the mortgage default insurance contract and bails out the banks if they are insolvent. It issues debt to absorb the

[4]Deposit insurance can be thought of more broadly as encompassing implicit government guarantees for short-term financial sector liabilities such as money market funds, asset-backed commercial paper, repurchase agreements, etc. Indeed, the government stepped in to bail out these markets in the Fall of 2008 and in the Spring of 2009.

costs of the guarantee payouts. Government debt is both high and volatile as a result. This dual government intervention confers two advantages. First, the government's ability to issue debt in bad times allows society to spread out the fiscal costs of mortgage defaults over time, preventing households from having to cut consumption in bad states of the world. Second, the mortgage guarantees (mostly) protect banks' balance sheets from mortgage losses and enable banks to continue their intermediation function. However, in equilibrium someone must be buying the additional government debt issued to finance the mortgage insurance payouts and bank bailouts. Risk averse depositors are the marginal agents who buy the additional government debt. Since they dislike volatility in their consumption, the fiscal volatility caused by government guarantees is an important welfare cost to the economy. Through the mortgage guarantee program, the government shifts some of the burden of bearing mortgage losses from the intermediaries to the depositors whose preferences make them less suitable to bear this risk.

Confronted with this fiscal uncertainty, depositors increase precautionary savings, resulting in low equilibrium interest rates ex-ante. Low rates are good news for borrowers because they help keep mortgage rates down (despite the high default rates), but bad news for savers.

Capturing the spirit of the proposed mortgage finance reforms, our main policy experiment is to increase the cost of government mortgage insurance from the low level observed until recently. Naturally, higher guarantee fees "crowd in" the private sector: They induce a shift in the composition of banks' assets from guaranteed to private mortgage bonds. With banks now bearing more of the mortgage credit risk, their portfolio risk increases. Banks choose to reduce leverage. Intermediary net worth is higher on average so that banks have more "skin in the game." Because of sufficient intermediary capital, banks are able to continue lending even during housing crises. The provision of mortgage credit is as stable in the "private-sector" model as in the benchmark economy with severely priced government guarantees. This result dispels the notion that the GSEs are needed to guarantee stable access to mortgage finance. Banks also choose to reduce the size and riskiness of their mortgage portfolio. Thus, mortgage default and loss rates are lower in the private sector economy. Mortgage crises are less likely to turn into financial crises because of the sturdier bank balance sheets. The key insight is that abolition of mortgage guarantees leads banks to take less risk, moving them farther from their leverage constraints. Their higher capital ratios enable them to better perform their function of absorbing aggregate risk from both borrowers and depositors.

With higher g-fees, mortgage guarantees become less attractive. The lower quantity of mortgage insurance bought and the lower loss rates on mortgages combine to dramatically reduce fiscal uncertainty (the volatility of government debt issuance). This allows depositors to achieve a less volatile consumption stream. Ex-ante, they reduce precautionary savings so that the private sector equilibrium features higher risk-free interest rates. The higher risk-free interest rate more than offsets the decline in the mortgage spread (due to lower mortgage defaults and a lower mortgage risk premium due to better risk sharing) so that mortgage interest rates are higher in the private sector economy.

At g-fees that are high enough to crowd out the mortgage guarantee completely, we find that risk-free interest rates are higher by 70 basis points, mortgage interest rates are higher by 22 basis points, house prices are lower by 6.3%, the mortgage market is smaller by 8.7%, and intermediary leverage is lower by 7.3 percentage points. Mortgages are safer: debt-to-income ratios are 6.2% lower. Mortgage default and loss rates are 40% lower, as are deadweight losses from foreclosure. Bank defaults (financial sector bailouts) are nearly eliminated. The overall effect of phasing out the GSEs is an increase in social welfare of 0.63% in consumption equivalence units. Borrowers welfare increases only marginally (+0.04%) while depositors (+1.32%) and intermediaries (+1.69%) gain substantially. Borrowers benefit from the improved risk sharing, but they lose their mortgage subsidy, face higher mortgage rates, tighter lending standards, and lower house prices. Depositors benefit from higher interest rates and less fiscal uncertainty. Thus, while abolishing the GSEs is a Pareto improvement, it increases wealth inequality.

These results are steady-state comparisons. We also compute a transition from the low to the high g-fee economy and find that borrowers suffer in the short-run from the reform. Prices adjust quickly but stocks of debt and wealth take longer to adjust so that the losses from higher mortgage rates and lower house prices hit immediately while the benefits from improved risk sharing only accrue gradually.

We study an intermediate economy where the guarantee fee is similar to the value charged by the GSEs anno 2015. At even higher levels for the g-fee, we find that the government guarantee is only taken up in bad times. This dovetails with the "mortgage insurer of last resort" option in the Obama Administration proposal which envisions activating the government only in crises

(Scharfstein and Sunderam (2011)). These economies improve on the benchmark but have lower welfare than the fully private sector economy where guarantee fees are set so high that they become unattractive in good and in bad states of the world. The benefit of mortgage guarantees, to support the financial sector in bad times (ex-post) when its capacity to intermediate is in jeopardy, is outweighed by its cost, which is to distort the optimal allocation of aggregate risk in the economy (as reflected in the ex-ante risk-free interest rate).

We use our model to quantitatively evaluate 2014 the Johnson-Crapo bill which proposes to put 10% private capital in front of a catastrophic government guarantee. The first 10% of losses in the event of a mortgage crisis would be born by the private sector. The government would step in only when the loss rate exceeds 10% and bear the losses in excess of 10%. Assuming a guarantee fee of 20 basis points for the catastrophic guarantee, we find that the Johnson-Crapo proposal generates a substantial welfare gain of 0.67%. This gain is slightly larger than the one we obtained in our main policy experiment of phasing out the standard full credit guarantee (+0.63%). The large private sector loss absorption of 10% eliminates most moral hazard and results in similarly low financial sector leverage, mortgage market size and mortgage risk (loss rates) as in the private sector economy. But now intermediaries enjoy the benefit of the government insurance during very severe mortgage crisis. This improves risk sharing compared to the no-guarantee economy. This experiment demonstrates that a well-designed mortgage insurance system can improve welfare. It also demonstrates that our model does not generate a private sector solution as its automatic optimal outcome.

2 Related Literature

Our paper contributes to several strands of the literature on housing, finance, and macroeconomics. Unlike recent quantitative work that explores the causes and consequences of the housing boom,[5] this paper focuses on the current and future state of the housing finance system and the role the government plays in this system. It shares with these models a focus on quantitative implications and on general equilibrium considerations. In particular, house prices

[5]Examples include Ortalo-Magné and Rady (2006), Kiyotaki, Michaelides, and Nikolov (2011), Favilukis, Ludvigson, and Van Nieuwerburgh (2015), Landvoigt, Piazzesi, and Schneider (2015), Landvoigt (2012), Chu (2014). See Davis and Van Nieuwerburgh (2015) for a recent review of the literature.

and interest rates are determined in equilibrium rather than exogenously specified. We simplify by working in an endowment economy with a constant housing stock.[6]

Like another strand of the literature, our model features borrowers defaulting optimally on their mortgages.[7] Unlike most of that literature, our lenders are not risk-neutral but risk averse. A default risk premium is priced into the mortgage contract. It is time-varying and depends on the covariance of the risk taker's intertemporal marginal rate of substitution with the payoff on the mortgage loan. We assume that lenders impose maximum LTV ratios on borrowers, chosen to match observed mortgage debt/income ratios and default rates in normal and mortgage crisis times. Unlike most of the literature, our mortgage contract is a long-term contract. This is important because of the centrality of the 30-year fixed-rate mortgage in the debate on U.S. housing finance reform. We calibrate our mortgage contract to exhibit the same amount of interest rate risk as the outstanding pool of agency mortgage-backed securities. Our setting is ideally suited to study the interaction of default and prepayment risk. Government policy affects the quantity and price of both of these risks.

The main difference between our housing finance model and the literature is our focus on the interplay between the financial sector and the government. A recent literature in asset pricing has emphasized the central role of financial intermediaries in the crisis. It models these intermediaries as a separate set of agents.[8] Usually, intermediaries have access to a different technology from other agents. In our model, as in Drechsler, Savov, and Schnabl (2014), intermediaries arise from differences in preferences (risk aversion) instead. The least risk averse savers choose to issue short-term debt ("deposits") to the more risk averse savers. Like in the part of the intermediation literature that emphasizes debt constraints, our intermediaries face borrowing (margin) constraints which link the amount of short-term liabilities to the collateral value of their assets. In this class of models, the net worth of the financial sector is the key state variable which governs risk sharing and asset prices. In our model, intermediary wealth is

[6] The role of housing supply and construction are studied in Favilukis, Ludvigson, and Van Nieuwerburgh (2015), Chatterjee and Eyigungor (2009), Hedlund (2014), and Boldrin, Garriga, Peralta-Alva, and Sanchez (2013).

[7] Recent examples of equilibrium models with default are Corbae and Quintin (2014), Garriga and Schlagenhauf (2009), Chatterjee and Eyigungor (2009), Jeske, Krueger, and Mitman (2013), Landvoigt (2012), Arslan, Guler, and Taskin (2013), and Hedlund (2014).

[8] Recent examples include Brunnermeier and Sannikov (2014), He and Krishnamurty (2013), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), and Maggiori (2013). Brunnermeier, Eisenbach, and Sannikov (2013) provides a review of this literature.

also an important state variable, but it is not the only one. The wealth of the depositors, the wealth of the borrowers, and the outstanding amount of government debt all have important effects on equilibrium allocations and prices.

Unlike most of the intermediary asset pricing literature, we explicitly model the intermediary's decision to default. When intermediary net worth threatens to go negative, intermediaries can choose to offload all assets and liabilities onto the government. The government bailout option is equivalent to deposit insurance in the model. By studying the role of the financial sector in the provision of mortgages we capture the stylized fact of Jorda, Schularick, and Taylor (2014), that "over a 5-year window run ups in mortgage lending and run ups in house prices raise the likelihood of a subsequent financial crises. Mortgage and house price booms are predictive of future financial crises, and this effect has also become much more dramatic since WW2." We ask how government intervention in the form of asset (mortgage) guarantees or liability (deposit) guarantees affects this nexus.

Our result that the economy without a government guarantee for mortgages features a better capitalized and more stable financial system is of interest to the literature on capital regulation of financial institutions.[9] Our work contributes a quantitative general equilibrium model to that discussion, with an emphasis on the role of mortgages and mortgage guarantees. Our framework should be useful to investigate the quantitative implications of deposit insurance and higher capital requirements. See Begenau (2015) for a similar approach.

Finally, our paper contributes to the literature that quantifies the effect of government policies in the housing market. Most work focuses on studying the effect of abolishing the mortgage interest rate tax deductibility and the tax exemption of imputed rental income of owner-occupied housing.[10] When house and rental prices are determined endogenously, such policy changes tend to lower house prices and price-rent ratios and cause an increase in home ownership rates. Our model has no renters and misses the home ownership channel. However, the results from this literature suggest that the welfare gains from a GSE phaseout would be further amplified in a model like ours with renters. Lower house price-rent ratios would benefit

[9]See for example, Kashyap, R., and Stein (2008), Hart and Zingales (2011), Maddaloni and Peydro (2011), Admati and Hellwig (2013), and Admati, DeMarzo, Hellwig, and Pfleiderer (2013).

[10]For example, Gervais (2002), Chambers, Garriga, and Schlagenhauf (2009), Floetotto, Kirker, and Stroebel (2012), and Sommer and Sullivan (2013).

renters who want to become home owners.

This literature also finds that abolishing the fiscal benefits from home ownership would not only increase overall welfare but also reduce inequality. Studying the GSE subsidies, Jeske, Krueger, and Mitman (2013) reach a similar conclusion regarding welfare and inequality. In a model similar to Jeske, Krueger, and Mitman (2013), Gete and Zecchetto (2015) argue that the poor, high-credit risk households suffer a disproportionate increase in the cost of mortgage credit from an abolition of GSEs, offsetting the reduction in inequality emphasized by Jeske et al. Our work identifies a force towards raising inequality upon GSE reform. Jeske et al. (2013) emphasize heterogeneity among borrowers who face (partially) uninsurable idiosyncratic income risk. Our work focuses on the role of the financial sector and the interaction between the mortgage guarantee and the bailout guarantee. We emphasize heterogeneity between borrowers, banks, and savers. We assume that our borrowers are able to perfectly insure idiosyncratic risk with one another, and we focus instead on how aggregate risk is spread between borrowers, banks, and depositors. We do not investigate whether GSEs provide insurance against idiosyncratic risk. However, we find that underpriced guarantees result in a large increase in foreclosures, and so whatever missing insurance role the GSEs play would have to be strong enough to offset this effect.[11]

3 The Model

3.1 Endowments, Preferences, Technology, Timing

Endowments The model is a two-good endowment economy with a non-housing and a housing Lucas tree. The fruit of the non-housing tree, *output* Y_t, grows and its growth rate is subject to aggregate shocks. The different households are endowed with a fixed and non-tradeable share of this tree. This endowment can be interpreted as labor income. The size of the housing tree

[11]Our model differs from Jeske et al. (2013) in various other respects. They model mortgage guarantees as a tax-financed mortgage interest rate subsidy. In our model, the government sells mortgage insurance to the private sector. Second, our mortgages are long-term in nature while theirs are one-period contracts. Third, our lenders are risk-averse, while theirs are risk-neutral. Fourth, their model has a simple construction sector and constant house prices, while our model has a fixed housing supply and endogenous house prices. Fifth, their model has a rental sector while ours does not. Sixth, our government can issue debt while theirs has to balance the budget every period. Thus, the two papers complement each other nicely.

(housing stock) grows at the same stochastic trend as output. The total quantity of housing shares is fixed and normalized to $\bar{K}$. The housing stock yields fruit (housing services) proportional to the stock.

Preferences The model features a government and three groups of households. Impatient households will play the role of borrowers in equilibrium (denoted by superscript B), while patient households will turn out to be savers. There are two type of savers, differentiated by their risk aversion coefficient; we refer to the less risk averse savers as "risk takers" (denoted by superscript R) and the more risk averse as "depositors" (denoted by D). Thus, for the rate of impatience we assume that $\beta_R = \beta_D > \beta_B$, and for the coefficient of relative risk aversion we assume that $\sigma_R < \sigma_B \leq \sigma_D$. All agents have Epstein-Zin preferences over the joint consumption bundle which is a Cobb-Douglas aggregate of housing and non-housing consumption with aggregation parameter θ.

$$U_t^j = \left\{ (1-\beta_j) \left(u_t^j\right)^{1-1/\nu} + \beta_j \left(\mathrm{E}_t \left[(U_{t+1}^j)^{1-\sigma_j} \right] \right)^{\frac{1-1/\nu}{1-\sigma_j}} \right\}^{\frac{1}{1-1/\nu}} \tag{1}$$

$$u_t^j = \left(C_t^j\right)^{1-\theta} \left(A_K K_{t-1}^j\right)^{\theta} \tag{2}$$

C_j^t is numeraire non-housing consumption and the constant A_K specifies the housing services from owning the housing stock, expressed in units of the numeraire. All agents share the same elasticity of intertemporal substitution ν.

Figure 1 depicts the balance sheets of the different agents in the economy and the flows of funds between them.

Technology In addition to housing, there are three assets in the economy. The first is a one-period short-term bond. The second is a mortgage bond, which aggregates the mortgage loans made to all borrower households. The third is mortgage insurance which the government sells to the private market. The guarantee turns a defaultable long-term mortgage bond into a default-free government-guaranteed mortgage bond. The government exogenously sets the price of the guarantee.

Borrowers experience housing depreciation shocks and may choose to default on their mortgage. There is no recourse; savers and possibly the government (ultimately the tax payers) bear the loss depending on whether mortgage loans are held in the form of private or government-guaranteed mortgage bonds, respectively. A novel model ingredient is that risk takers may also choose to default and declare bankruptcy. Default wipes clean their negative wealth position with no further consequences; the losses are absorbed by the government in a "financial sector bailout."

Timing The timing of agents' decisions at the beginning of period t is as follows:

1. Income shocks for all types of agents and housing depreciation shocks for borrower households are realized.

2. Risk takers (financial intermediaries) decide on a bankruptcy policy. In case of a bankruptcy, their financial wealth is set to zero and they incur a utility penalty. At the time of the decision, the magnitude of the penalty is unknown.[12] Risk takers know its probability distribution and maximize expected utility by specifying a binding decision rule for each possible realization of the penalty.[13]

3. Borrowers decide on mortgage default.

4. Risk takers' utility penalty shock is realized and they follow their bankruptcy decision rule from step 2. In case of bankruptcy, the government picks up the shortfall in repayments to debt holders (depositors).

5. Borrowers choose how much of the remaining mortgage balance to prepay (refinance). All agents solve their consumption and portfolio choice problems. Markets clear. All agents consume.

Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves and predict the

[12]Introducing a random utility penalty is a technical assumption we make for tractability. It makes the value function differentiable and allows us to use our numerical methods which rely on this differentiability. This randomization assumption is common in labor market models (Hansen (1985)). Additionally, uncertainty about the consequences of a systematic banking crisis and insolvency may seem quite reasonable.

[13]The assumption of making a binding default decision is necessitated in the presence of Epstein-Zin preferences.

bankruptcy decisions of borrowers and risk takers. We now describe each of the three types of household problems and the government problem in detail.

3.2 Borrower's Problem

Mortgages As in reality, mortgage contracts are long-term, defaultable, and prepayable. The mortgage is a long-term contract, modeled as a perpetuity. Bond coupon (mortgage) payments decline geometrically, $\{1, \delta, \delta^2, \ldots\}$, where δ captures the duration of the mortgage. A mortgage can default, in which case the lenders have recourse to the housing collateral. We introduce a "face value" $F = \frac{\alpha}{1-\delta}$, a fixed fraction α of the mortgage payments (per unit of mortgage bond), at which the mortgage can be prepaid. Prepayment incurs a cost detailed below. Mortgage payments can be deducted from income for tax purposes at a rate $\tau_t^m = (1-\alpha)\tau_t$, where τ_t is the income tax rate and the fraction $(1-\alpha)$ reflects the interest component of mortgage payments .

Borrower Default There is a representative family of borrowers, consisting of a measure one of members. Each member receives the same stochastic labor income $Y_t^B \propto Y_t$, chooses the same quantity of housing k_{t-1}^B s.t. $\int_0^1 k_{t-1}^B di = K_{t-1}^B$, and the same quantity of outstanding mortgage bonds a_t^B s.t. $\int_0^1 a_t^B di = A_t^B$.

After having received income and having chosen house and mortgage size, each family member draws an idiosyncratic housing depreciation shock $\omega_{i,t} \sim F_\omega(\cdot)$ which proportionally lowers the value of the house by $(1-\omega_{i,t})p_t k_{t-1}^B$. The value of the house after stochastic depreciation is $\omega_{i,t} p_t k_{t-1}^B$. We denote the cross-sectional mean and standard deviation by $\mu_\omega = \mathrm{E}_i[\omega_{i,t}]$ and $\sigma_{t,\omega} = (\mathrm{Var}_i[\omega_{i,t}])^{0.5}$, where the latter varies over time. The variable $\sigma_{t,\omega}$ governs the mortgage credit risk in the economy; it is the second exogenous aggregate state variable (in addition to aggregate income growth).

Each borrower household member then optimally decides whether or not to default on the mortgages. The houses that the borrower family defaults on are turned over to (foreclosed by) the lender. Let the function $\iota(\omega) : [0, \infty) \to \{0, 1\}$ indicate the borrower's decision to default on a house of quality ω. We conjecture and later verify that the optimal default decision is characterized by a threshold level ω_t^*, such that borrowers default on all houses with $\omega_{i,t} \leq \omega_t^*$

and repay the debt for all other houses. Using the threshold level ω_t^*, we define $Z_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $Z_K(\omega_t^*)p_t K_{t-1}^B$ to be the value to the borrowers of the residual (non-defaulted) housing stock after default decisions have been made. We have:

$$Z_A(\omega_t^*) = \int_0^\infty (1-\iota(\omega))\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \geq \omega_t^*], \tag{3}$$

$$Z_K(\omega_t^*) = \int_0^\infty (1-\iota(\omega))\omega\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \geq \omega_t^*]\ \mathrm{E}[\omega_{i,t} \,|\, \omega_{i,t} \geq \omega_t^*] \tag{4}$$

After making a coupon payment of 1 per unit of remaining outstanding mortgage, the amount of outstanding mortgages declines to $\delta Z_A(\omega_t^*)\, A_t^B$.

Prepayment Next, the households can choose to prepay a quantity of the outstanding mortgages R_t^B by paying the face value F per unit to the lender. We denote by $Z_t^R \equiv R_t^B/A_t^B$ the ratio of prepaid mortgages to beginning-of-period mortgages. Prepayment incurs a monetary cost Ψ. We use an adjustment cost function $\Psi(R_t^B, A_t^B)$ that is convex in the fraction prepaid Z_t^R, capturing bottlenecks in the mortgage refinance infrastructure when too large a share of mortgages are prepaid at once.

Borrower Problem Statement The borrower family's problem is to choose consumption C_t^B, housing K_t^B, default threshold ω_t^*, prepayment quantity R_t^B, and new mortgage debt B_t^B to maximize life-time utility U_t^B in (1), subject to the budget constraint:

$$C_t^B + (1-\tau_t^m) Z_A(\omega_t^*) A_t^B + p_t K_t^B + F R_t^B + \Psi(R_t^B, A_t^B) \leq (1-\tau_t) Y_t^B + G_t^{T,B} + Z_K(\omega_t^*) p_t K_{t-1}^B + q_t^m B_t^B, \tag{5}$$

an evolution equation for outstanding mortgage debt:

$$A_{t+1}^B = \delta Z_A(\omega_t^*)\, A_t^B + B_t^B - R_t^B, \tag{6}$$

a maximum loan-to-value constraint:

$$F A_{t+1}^B \leq \phi p_t K_t^B. \tag{7}$$

and a double constraint on the amount of mortgages that can be refinanced:

$$0 \leq R_t^B \leq \delta Z_A(\omega_t^*) A_t^B. \tag{8}$$

Outstanding mortgage debt at the end of the period (equation 6) is the sum of the remaining mortgage debt after default and new borrowing B_t^B minus prepayments. The borrower household uses after-tax labor income, net transfer income from the government ($G_t^{T,B}$), residual housing wealth, and new mortgage debt raised to pay for consumption, mortgage debt service net of mortgage interest deductibility, new home purchases, prepayments FR_t^B and associated prepayment costs $\Psi(R_t^B, A_t^B)$. New mortgage debt raised is $q_t^m B_t^B$, where q_t^m is the price of one unit of mortgage bonds in terms of the numeraire good.

The borrowing constraint in (7) caps the face value of mortgage debt at the end of the period, FA_{t+1}^B, to a fraction of the market value of the underlying housing, $p_t K_t^B$, where ϕ is the maximum loan-to-value ratio. With such a constraint, declines in house prices (in bad times) tighten borrowing constraints. It is the first of two occasionally binding borrowing constraints in the model.

The refinancing constraints in equation (8) ensure that the amount prepaid is between 0 and the outstanding balance after the default decision was made. Equivalently, the share prepaid, Z_t^R, must be between 0 and $\delta Z_A(\omega_t^*)$.

3.3 Risk Takers

Next we study the problem of the risk taker households, who lend to borrower households and borrow from depositor households. Hence, we refer to this household type as intermediaries.[14]

Risk-taker Default After shocks to income and housing depreciation have been realized, the risk taker (financial intermediary) chooses whether or not to declare bankruptcy. Risk takers who declare bankruptcy have all their assets and liabilities liquidated.[15] They also incur

[14]Note that we could separately model risk taker households as the shareholders of the banks and the banks they own. For simplicity we combine the two balance sheets.

[15]The mortgages are bonds that trade in a competitive market. They are sold during the liquidation and bought by the banks that start off the following period with zero financial wealth and the exogenous income

a stochastic utility penalty ρ_t, with $\rho_t \sim F_\rho$, i.i.d. over time and independent of all other shocks. At the time of the bankruptcy decision, risk takers do not yet know the realization of the bankruptcy penalty. Rather, they have to commit to a bankruptcy decision rule $D(\rho)$: $\mathbb{R} \to \{0, 1\}$, that specifies the optimal decision for every possible realization of ρ_t. Risk takers choose $D(\rho)$ to maximize expected utility at the beginning of the period. We conjecture and later verify that the optimal default decision is characterized by a threshold level ρ_t^*, such that risk takers default for all realizations for which the utility cost exceeds the threshold. As we explain below, risk taker default leads to a government bailout.

After the realization of the penalty, risk takers execute their bankruptcy choice according to the decision rule. They then face a consumption and portfolio choice problem, where they allocate their wealth between a short-term risk-free bond, a private mortgage bond, and a government-guaranteed mortgage bond.

Private Mortgage Bond A private mortgage bond is a simple pass-through vehicle, aggregating the mortgages of the borrowers. The coupon payment on performing mortgages in the current period is $A_t^B Z_A(\omega_t^*)$, which is the number of mortgage bonds times the fraction that is performing times the coupon payment of 1 per unit of performing bond. For mortgages that go in foreclosure, the risk taker repossesses the homes. These homes are worth $(1-\zeta)\left(\mu_\omega - Z_K(\omega_t^*)\right) p_t K_{t-1}^B$, where ζ is the fraction of home value destroyed in a foreclosure. It represents a deadweight loss to the economy. Thus, the total payoff per unit of private mortgage bond is:

$$M_{t,P} = Z_A(\omega_t^*) + \frac{(1-\zeta)(\mu_\omega - Z_K(\omega_t^*))p_t K_{t-1}^B}{A_t^B}.$$

The price of the bond is q_t^m per unit.

Government-guaranteed Mortgage Bond A government-guaranteed bond is a security with the same duration (maturity and cash-flow structure) as a private mortgage bond. The only difference is that it carries no mortgage default risk because of the government guarantee. To prevent having to keep track of an additional state variable, we model guarantees as one-

stream.

period default insurance.[16] Combining one unit of a private mortgage bond with one unit of default insurance creates a mortgage bond that is government-guaranteed for one period. One unit of a government-guaranteed mortgage bond has the following payoff:

$$M_{t,G} = 1 + (1 - Z_A(\omega_t^*))\delta F$$

The first term is the coupon of 1 on all loans in the pool, performing and non-performing. The second term is compensation for the loss in principal of defaulted loans. Owners of guaranteed loans receive a principal repayment F. The price of the bond is $q_t^m + \gamma_t$ per unit. The government sets the price of insurance γ_t per unit of bond to the government. The choice between guaranteed and private mortgage bonds is the main choice of interest in the paper.

Risk-taker wealth Denote risk-taker financial wealth at the start of period t by W_t^R. This financial intermediary net worth is a key state variable.

$$\begin{aligned} W_t^R = {} & (M_{t,P} + \delta Z_A(\omega_t^*) q_t^m - Z_t^R[q_t^m - F]) A_{t,P}^R \\ & + (M_{t,G} + \delta Z_A(\omega_t^*) q_t^m - Z_t^R[q_t^m - F]) A_{t,G}^R + B_{t-1}^R. \end{aligned}$$

It consists of the market value of the portfolio of private and guaranteed bonds bought last period, as well as the short-term bonds from last period which mature this period. When the holdings of short-term bonds are negative, the last term is short-term debt which must be repaid this period. Since the mortgage guarantee is valid for only one period, both private and government-guaranteed bonds bought last period trade for the same price q_t^m. Voluntary (i.e., rate-induced rather than default-induced) mortgage prepayments "come in at par" F. Since such prepayments only happen when $q_t^m > F$, they represent a loss to the intermediary. If the portfolio consists entirely of guaranteed bonds, prepayments constitute an important source of risk driving fluctuations in risk taker net worth.

[16] Rolling over default insurance every period for the life of the loan is the equivalent to the real-world long-term guarantees provided by Fannie Mae and Freddie Mac. Having the choice of renewal each period makes our guarantees more flexible, and hence more valuable, than those in the real world.

Consumption-Portfolio Choice Problem While intertemporal preferences are still specified by equation (1), intraperiod utility u_t^j depends on the bankruptcy decision and penalty:

$$u_t^R = \frac{\left(C_t^R\right)^{1-\theta}\left(A_K K_{t-1}^R\right)^{\theta}}{\exp\left(D(\rho_t)\rho_t\right)}.$$

Entering with wealth W_t^R, the risk taker's problem is to choose consumption C_t^R, holdings of private mortgage bonds $A_{t+1,P}^R$, holdings of government-guaranteed mortgage bonds $A_{t+1,G}^R$, and short-term bonds B_t^R to maximize life-time utility U_t^R in (1), subject to the budget constraint:

$$C_t^R + q_t^m A_{t+1,P}^R + (q_t^m + \gamma_t)A_{t+1,G}^R + q_t^f B_t^R + (1-\mu_\omega)p_t K_{t-1}^R \leq (1-\tau_t)Y_t^R + G_t^{T,R} + W_t^R, \tag{9}$$

and the following constraints:

$$A_{t+1,P}^R \geq 0, \tag{10}$$

$$A_{t+1,G}^R \geq 0, \tag{11}$$

$$-q_t^f B_t^R \leq q_t^m \varkappa(\xi_P A_{t+1,P}^R + \xi_G A_{t+1,G}^R). \tag{12}$$

The budget constraint (9) shows that the risk-taker uses after-tax labor income, net transfer income, and beginning-of-period wealth to pay for consumption, purchases of private and government-guaranteed mortgage bonds and short-term bonds, and for housing repairs which undo the effects of depreciation. We do not allow for negative positions in either long-term mortgage bond (equations 10 and 11). A key constraint in the model is (12). A negative position in the short-term bond is akin to the risk taker issuing short-term bonds, or equivalently deposits. The negative position in the short-term bond must be collateralized by the market value of the risk taker's holdings of long-term mortgage bonds. The parameters ξ_P and ξ_G together with $\varkappa$ determine how useful private and government-guaranteed mortgage bonds are as collateral. In the calibration, we will assume that guaranteed mortgages are better collateral: $\xi_G > \xi_P$. The constraint captures the reality of Basel II/III-type risk weights that restrict intermediary leverage.[17]

[17]The short-term borrowing is akin to a repo contract. It allows the intermediary to buy a mortgage bond by borrowing a fraction ξ of the purchase price while only using a fraction $1-\xi$ of the purchase price, the margin requirement, of her own capital. One can think of the guaranteed bond as a private mortgage bond

3.4 Depositors

The second type of savers, depositors, receive labor income, $Y_t^D \propto Y_t$, own a fixed share of the housing stock K_t^D, and solve a standard consumption-savings problem. Entering with wealth $W_t^D = B_{t-1}^D$, the depositor's problem is to choose consumption C_t^D and holdings of short-term bonds B_t^D to maximize life-time utility U_t^D in (1), subject to the budget constraint:

$$C_t^D + q_t^f B_t^D + (1-\mu_\omega)p_t K_{t-1}^D \leq (1-\tau_t)Y_t^D + G_t^{T,D} + W_t^D, \tag{13}$$

and short-sales constraints on bond holdings:

$$B_t^D \quad \geq 0,. \tag{14}$$

The budget constraint (13) is similar to that of the risk taker. We also do not allow depositor's to take a negative position in the short-term bond (14), consistent with our assumption that the depositor must not declare bankruptcy.

3.5 Government

We model the government as set of exogenously specified tax, spending, bailout, and debt issuance policies.[18] Government tax revenues, T_t, are labor income tax receipts minus mortgage interest deduction tax expenditures plus mortgage guarantee fee income:

$$T_t = \tau_t Y_t - \tau_t^m Z_A(\omega_t^*) A_t^B + \gamma_t (A_{t,G}^R + A_{t,G}^D)$$

Government expenditures, G_t are the sum of payoffs on mortgage guarantees, financial sector bailouts, other exogenous government spending, G_t^o, and government transfer spending G_t^T:

$$G_t = (M_{t,G} - M_{t,P})A_{t,G}^R - D(\rho_t)W_t^R + G_t^o + G_t^T$$

plus a government guarantee (a credit default swap or mortgage insurance). Implicit in constraint (12) is the assumption that the government guarantee itself is an off-balance sheet item that cannot be collateralized.

[18] We consolidate the role of the GSEs and that of the Treasury department into one government, reflecting the reality as of September 2008.

The mortgage guarantee pays to the risk takers the difference in cash-flow between a guaranteed and a private mortgage bond, for each unit of guaranteed bond they purchase. The bailout to the financial sector equals the negative of the financial wealth of the risk taker, W_t^R, in the event of a bankruptcy ($D(\rho_t) = 1$). By bailing out the intermediaries, the government renders intermediaries' liabilities, deposits, risk-free. In the model, risk taker bankruptcies, limited liability for risk takers, and deposit insurance are equivalent.

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \tag{15}$$

We impose a transversality condition on government debt:

$$\lim_{u \to \infty} \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+u}^D B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^D$ is the SDF of the depositor.[19] Because of its unique ability to tax and repay its debt, the government can spread out the cost of mortgage default waves and financial sector rescue operations over time. We are interested in understanding whether the government's ability to tax and issue debt leads to an increased stability of mortgage credit provision in the world with government guarantees.

Government policy parameters are $\Theta_t = (\tau_t, \gamma_t, G_t^o, \phi, \xi_G, \xi_P, \mu_\rho)$. The parameters ϕ in equation (7) and (ξ_G, ξ_P) in equation (12) can be thought of as macro-prudential policy tools which govern household and intermediary leverage. We added the parameter μ_ρ that governs the mean utility cost of bankruptcy to risk takers to the set of policy levers, since the government may have some ability to control the fortunes of the financial sector in the event of a bankruptcy. This cost directly affects the strength of deposit insurance but could also include reputational costs of bank defaults.

[19]We show below that the risk averse saver is the marginal agent for short-term risk-free debt.

3.6 Equilibrium

Given a sequence of aggregate income and house valuation shocks $\{Y_t, \sigma_{\omega,t}\}$ and utility costs of default shocks $\{\rho_t\}$, and given a government policy $\{\Theta_t\}$, a competitive equilibrium is an allocation $\{C_t^B, K_t^B, B_t^B, R_t^B\}$ for borrowers, $\{C_t^R, A_{t,P}^R, A_{t,G}^R, B_t^R\}$ for risk takers, $\{C_t^D, B_t^D\}$ for depositors, default policies $\iota(\omega_{it})$ and $D(\rho_t)$, and a price vector $\{p_t, q_t^m, q_t^f\}$, such that given the prices, borrowers, depositors, and risk-takers maximize life-time utility subject to their constraints, the government satisfies its budget constraint, and markets clear.

The market clearing conditions are:

$$\text{Risk-free bonds: } B_t^G = B_t^D + B_t^R \tag{16}$$

$$\text{Mortgages: } A_t^B = A_{t,G}^R + A_{t,P}^R \tag{17}$$

$$\text{Housing tree shares: } K_t^B + K_t^R + K_t^D = \bar{K} \tag{18}$$

$$\begin{aligned} \text{Consumption: } Y_t = {} & C_t^B + C_t^R + C_t^D + (1 - \mu_{t,\omega}) p_t \bar{K} + G_t^o + \\ & \zeta(\mu_{t,\omega} - Z_K(\omega_t^*)) \frac{p_t K_{t-1}^B}{A_t^B} + \Psi(R_t^B, A_t^B) \end{aligned}$$

The last equation states that total non-housing resources equal the sum of non-housing consumption expenditures and home renovations by the households, discretionary spending by the government, and lost resources due to the deadweight costs of foreclosure and mortgage refinancing.

3.7 Welfare

In order to compare economies that differ in the policy parameter vector Θ_t, we must take a stance on how to weigh the different agents. We propose a utilitarian social welfare function summing value functions of the agents according to their population weights ℓ:

$$\mathcal{W}_t(\cdot; \Theta_t) = \ell^B V_t^B + \ell^D V_t^D + \ell^R V_t^R,$$

where the $V^i(\cdot)$ functions are the value functions defined in the appendix. A nice feature of value functions under Epstein-Zin preferences is that they are homogeneous of degree one in consumption. Thus, a $\lambda\%$ increase in the value function from a policy change is also a $\lambda\%$ change in consumption units.

4 Model Solution and Calibration

Appendix A presents the Bellman equations for each of the three household types and derives first-order conditions for optimality. We highlight some key features of the solution here by inspecting these FOC. We then turn to the calibration.

4.1 First Order Conditions

Borrower FOCs First, since borrowers are the only households freely choosing their housing position, their choice pins down the price of housing in the economy. Let $\tilde{\mathcal{M}}^i_{t,t+1}$ be the intertemporal marginal rate of substitution (or stochastic discount factor) for agent $i \in \{B, D, R\}$, with expressions provided in the Appendix. At the optimum, house prices satisfy the recursion:

$$p_t \left[1 - \tilde{\lambda}^B_t \phi\right] = \mathrm{E}_t \left[\tilde{\mathcal{M}}^B_{t,t+1} e^{g_{t+1}} \left\{p_{t+1} Z_K(\omega^*_{t+1}) + \frac{\theta C^B_{t+1}}{(1-\theta) K^B_t}\right\}\right]. \tag{19}$$

The marginal cost of housing on the left-hand side consists of the house price p_t minus a term which reflects the collateral benefit of housing; an extra unit of housing relaxes the maximum LTV constraint (7). The right hand side captures the expected discounted future marginal benefits which depends on the resale value of the non-defaulted housing stock as well as on the dividend from housing, which is the intratemporal marginal rate of substitution between housing and non-housing goods.

Second, we analyze the borrower's optimal foreclosure decision. In the appendix, we show that the optimal default threshold is given by:

$$\omega^*_t = \frac{(1 - \tau^m_t + \delta q^m_t - \delta \tilde{\lambda}^{RB}_t) A^B_t}{p_t K^B_{t-1}}. \tag{20}$$

At the threshold level ω^*_t, the cost from foreclosure (and mortgage debt relief), which is the loss of a house valued at $\omega^*_t p_t K^B_{t-1}$, exactly equals the expected cost from continuing the service the mortgage (including the option to default in the future which is encoded in q_t^m) and keeping the house. The cutoff has an intuitive interpretation. It is the aggregate loan-to-value ratio of the borrowers, with both mortgage debt and housing valued at market prices. When the *market* leverage of the borrower increases, the house value threshold ω_t^* rises and default becomes more likely. Note that when the borrower exercises her prepayment option to its maximum extent, $\tilde{\lambda}_t^{RB} > 0$ and default becomes less likely. Hence the default option and the prepayment option interact. A valuable refinancing option gives the borrower incentives to postpone a default decision as in Deng, Quigley, and Van Order (2000).

Third, the optimal share of outstanding mortgages that the borrower chooses to prepay, $Z_t^R = R_t^B / A_t^B$ is given by:

$$\psi Z_t^R = q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB}. \tag{21}$$

This balances the marginal cost of refinancing on the left-hand side with the marginal benefit on the right-hand side. For an internal prepayment choice, the marginal benefit is to increase the value of mortgage debt raised by $q_t^m - F$. Intuitively, when current mortgage rates are lower than when the mortgage was originated, the mortgage is a premium bond and trades at a price q^m above par value F. By refinancing a marginal unit of debt, the borrower gains $q^m - F$. If $q^m - F$ is large enough, the borrower will want to refinance all outstanding debt ($Z_t^R = \delta Z_A(\omega^*)$). The multiplier on the refinancing upper bound activates ($\tilde{\lambda}_t^{RB} > 0$). Conversely, when $q_t^m < F$, refinancing is not useful and the multiplier on the lower refinancing bound, $\tilde{\mu}_t^{RB} > 0$, turns positive to keep $Z_t^R = 0$.

Fourth, from the borrower's first order condition for A^B, we can read off the demand for mortgage debt.

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*) \left(1 - \tau^m - \frac{\psi \left(Z_{t+1}^R \right)^2}{2 Z_A(\omega_{t+1}^*)} - \delta \tilde{\lambda}_{t+1}^{RB} + \delta q_{t+1}^m \right) \right]. \tag{22}$$

A unit of mortgage debt obtained generates an amount q_t^m today but uses up some borrowing capacity, which is costly when the borrower's loan-to-value constraint binds ($\tilde{\lambda}_t^B > 0$). The non-defaulted part of the debt must be serviced in future periods, modulo a mortgage interest tax deduction, as long as it is not prepaid.

Depositor FOC The risk averse saver buys short-term debt issued by the risk taker. This debt is equivalent to government debt by virtue of the deposit insurance. The depositor's first-order condition for the short-term bond, assuming the short-sales constraint is not binding, is:

$$q_t^f \quad = \quad \mathrm{E}_t \left[\tilde{\mathcal{M}}^D_{t,t+1} \right].$$

The depositor's precautionary savings incentives are a crucial force determining equilibrium risk-free interest rates.

Risk Taker FOCs Next, we turn to the risk taker's default decision. The risk taker will optimally default whenever the utility costs of doing so is sufficiently small: $\rho_t < \rho_t^*$. The threshold depends on her wealth W_t^R and the state variables $\mathcal{S}_t^R$ that are exogenous to the risk taker, including the wealth of the borrower and of the depositor, and the outstanding amount of government debt. At the threshold, she is indifferent between defaulting and offloading her (negative) wealth onto the government or carrying on:

$$V^R(0, \rho_t^*, \mathcal{S}_t^R) = V^R(W_t^S, 0, \mathcal{S}_t^R),$$

where the value function is defined in Appendix A.

Second, the risk taker can invest in both government guaranteed and private MBS. The respective first-order conditions are:

$$\begin{aligned} q_t^m + \gamma_t \quad &= \quad \mathrm{E}_t \left[\tilde{\mathcal{M}}^R_{t,t+1} \left(M_{G,t+1} + \delta Z_A(\omega^*_{t+1}) q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F] \right) \right] + q_t^m \varkappa \xi_G \tilde{\lambda}_t^R \\ q_t^m \quad &= \quad \mathrm{E}_t \left[\tilde{\mathcal{M}}^R_{t,t+1} \left(M_{P,t+1} + \delta Z_A(\omega^*_{t+1}) q^m_{t+1} - Z^R_{t+1}[q^m_{t+1} - F] \right) \right] + q_t^m \varkappa \xi_P \tilde{\lambda}_t^R. \end{aligned}$$

Absent binding risk taker borrowing constraints ($\tilde{\lambda}_t^R = 0$), the marginal cost of a guaranteed mortgage bond is the price q_t^m plus the guarantee fee γ_t (expressed as a price) while the benefit is the expected discounted value of the bond tomorrow, which consists of the coupon payment and the repayment of principal in case of default (both are in M_G) plus the resale value of the non-defaulted portion of the mortgage bond. When there are prepayments, the market value of the bond is adjusted for the difference between the market value and the face value, on the share of mortgages that gets prepaid. If the collateral constraint is binding, the benefit is increased by the relaxation of the borrowing constraint, and depends on the haircut ξ_G for guaranteed mortgages. The first-order condition for private mortgages is similar, without the guarantee fee term, with a different collateral requirement term (ξ_P), and a different mortgage payoff M_P.

An equivalent way of restating the risk taker's choice is in terms of how many units of mortgages to originate to borrowers, and for how much of these holdings to buy default insurance from the government. The optimal amount of default insurance to buy solves:

$$\gamma_t = \mathrm{E}_t \left[\tilde{\mathcal{M}}^R_{t,t+1} \left(M_{G,t+1} - M_{P,t+1} \right) \right] + \tilde{\lambda}^R_t \varkappa q^m_t \left(\xi_G - \xi_P \right).$$

Risk takers will buy insurance until the marginal cost of insurance on the left equals the marginal benefit. An extra unit of default insurance increases the payoff of the mortgage and it increases the collateralizability of a mortgage, a benefit which only matters when the borrowing constraint binds. A binding risk taker leverage constraint increases demand for mortgage bonds, and especially for guaranteed bonds given their low risk weight (high ξ_G).

4.2 Calibration

The parameters of the model and their targets are summarized in Table 1.

Aggregate Income The model is calibrated at annual frequency. Aggregate endowment or labor income Y_t follows:

$$Y_t = Y_{t-1} \exp(g_t)$$
$$g_t = \rho_g g_{t-1} + (1 - \rho_g)\bar{g} + \epsilon_t, \qquad \epsilon_t \sim iid\, \mathcal{N}(0, \sigma_g)$$

We scale all variables by permanent income in order render the problem stationary. Given the persistence of income growth, g_t becomes a state variable. We discretize the g_t process into a 5-state Markov chain using the method of Rouwenhorst (1995). The procedure matches the mean, volatility, and persistence of GDP growth by choosing both the grid points and the transition probabilities between them. We use annual data on real per capita GDP growth from the BEA NIPA tables from 1929-2014 excluding the war years 1940-1945. The resulting mean is 1.9%, the standard deviation is 3.9%, and the persistence is 0.42. The states, the transition probability matrix, and the stationary distribution are listed in Appendix B.1.

Foreclosure crises The stochastic depreciation shocks or idiosyncratic house value shocks, $\omega_{i,t}$, are drawn from a Gamma distribution characterized by shape and a scale parameters $(\chi_{t,0}, \chi_{t,1})$. $F_\omega(\cdot; \chi_{t,0}, \chi_{t,1})$ is the corresponding CDF. We choose $\{\chi_{t,0}, \chi_{t,1}\}$ to keep the mean μ_ω constant at 0.975, implying annual depreciation of housing of 2.5%, a standard value, and to let the cross-sectional standard deviation $\sigma_{t,\omega}$ follow a 2-state Markov chain. Fluctuations in $\sigma_{t,\omega}$ govern the aggregate mortgage credit risk and represent the second source of exogenous aggregate risk. We refer to states with the high value for $\sigma_{t,\omega}$ as mortgage crises or foreclosure crises.

We set the two values $(\sigma_{H,\omega}, \sigma_{L,\omega}) = (0.10, 0.14)$ and the deadweight losses of foreclosure $(\zeta_H, \zeta_L) = (0.25, 0.425)$ in order to match the mortgage default rates and severities (losses given default) in normal times and in mortgage crises. In the benchmark model with low guarantee fees, and given all other parameter choices, these parameters imply equilibrium mortgage default rates of 1.6% in normal times and 12.7% in mortgage crises. The unconditional default rate is 2.7%. They imply equilibrium severities of 28.2% in normal times and 47.0% in crises. Mortgage default and severity rates combine to produce unconditional mortgage loss rates of 1.0% per year; 0.46% in normal times and 6.1% in crises. Appendix B.2 discusses the empirical evidence and argues that these numbers are a good match for the data. We note that the values for σ_ω are in line with standard values for individual house price shocks (Landvoigt, Piazzesi, and Schneider (2015)). Our unconditional severities are 30%, in line with typical values in the literature (Campbell, Giglio, and Pathak (2011)).

To pin down the transition probabilities of the 2-state Markov chain for $\sigma_{t,\omega}$, we assume that when the aggregate income growth rate in the current period is high (g is in one of the top three income states), there is a zero chance of transitioning from the $\sigma_{L,\omega}$ to the $\sigma_{H,\omega}$ state and a 100% chance of transitioning from the $\sigma_{H,\omega}$ to the $\sigma_{L,\omega}$ state. Conditional on low growth (g is in one of the bottom two income states) we calibrate the two transition probability parameters (rows have to sum to 1), p^{ω}_{LL} and p^{ω}_{HH}, to match the frequency and length of mortgage crises. Based on the argument by Jorda et al. (2014) that most financial crises after WW-II are related to the mortgage market and the historical frequency of financial crises in Reinhart and Rogoff (2009), we target a 10% probability of mortgage crises. Conditional on a crisis, we set the expected length to 2 years, based on evidence in Jorda et al. and Reinhart and Rogoff. Thus, the model implies that not all recessions are mortgage crises, but all mortgage crises are recessions.[20]

Population and wealth shares To pin down the labor income and housing shares for borrowers, depositors, and risk takers, we calculate a net fixed-income position for each household in the Survey of Consumer Finance (SCF).[21] Net fixed income equals total bond and bond-equivalent holdings minus total debt. If this position is positive, we consider a household to be a saver, otherwise it is a borrower. For savers, we calculate the amount of risky assets, defined as their holdings of stocks, business wealth, and real estate wealth, as well as the share of these risky assets in total wealth. We define risk takes as households that are within the top 5% of risky asset holdings and have a risky asset share of at least 75%. This delivers population shares of $\ell^B = 47\%$, $\ell^D = 51\%$, and $\ell^R = 2\%$. Based on this classification and the same SCF data, borrowers receive 38% of aggregate income and own 39% of residential real estate. Depositors receive 52% of income and 49% of housing wealth. Finally, risk takers receive 10% of income and 12% of housing wealth. By virtue of the calibration, the model thus matches basic aspects of the observed income and wealth inequality.

Mortgages In our model, a government-guaranteed MBS is a geometric bond. The issuer of one bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. The same is true if the borrower refinances the mortgage. We estimate values for δ and F such that

[20]In a long simulation, 33% of recessions are mortgage crises. This compares to a fraction of 6/22 (≈27%) in Jorda et al. (2014). The correlation between $\sigma_{t,\omega}$ and g_t is -0.42. The model generates persistence in the mortgage default rate of 0.02 in the low g-fee economy and 0.08 in the high g-fee economy. The persistence depends on, among other things, the persistence of $\sigma_{t,\omega}$ which is 0.478.

[21]We use all survey waves from 1995 until 2013 and average across them.

the duration of the geometric mortgage in the model matches the duration of the portfolio of outstanding mortgage-backed securities, as measured by the Barclays MBS Index, across a range of historically observed mortgage rates. This novel procedure, detailed in Appendix B.3, recognizes that the mortgage in the model represents the pool of all outstanding mortgages of all vintages. We find that values of $\delta = 0.95$ and $\alpha = 0.52$ imply a relationship between price and mortgage rate for the geometric mortgage that closely matches the price-rate relationship for a real-life MBS pool consisting of fixed-rate mortgages issues across a range of vintages. The average duration in model and data of the mortgage (pool) is about 4 years. Like the real-life MBS pool, the geometric mortgage price is convex in rates when rates are high (the prepayment option is out-of-the-money) and concave when rates are low ("negative convexity" when the prepayment option is in-the-money). Thus, the geometric mortgage has the same interest rate risk (duration) of real-life mortgages for different interest rate scenarios. Despite its simplicity, the perpetual mortgage with prepayment captures the key features of real-life guaranteed MBS pools.

Borrowers can obtain a mortgage with face value up to a fraction ϕ of the market value of their house. We set the LTV ratio parameter $\phi = 0.65$ to match the average mortgage debt-to-income ratio for borrowers in the SCF of 130%. The calibration produces an unconditional mortgage debt-to-income ratio among borrowers of 148% in the benchmark model, somewhat overshooting the target. In the benchmark model, borrowers' mean loan-to-value ratios are 63.8% in book value and 76.2% in market value terms.

We set the marginal prepayment cost parameter ψ so that the benchmark model generates reasonable conditional prepayment speeds. We target an average speed in the range of 15-20% annually.[22]

Government parameters Government fiscal policy consists of mortgage guarantee policy, a financial sector bailout policy, and general taxation and spending policies. In our desire to have a quantitatively meaningful model, we believe it is important to also capture non-housing realted fiscal policy. After the conservatorship of Fannie Mae and Freddie Mac in September

[22]CPR rates depend strongly on the interest rate and mortgage vintage, even among conventional 30-year mortgages. For example, in December 2009, CPRs ranged from 6% to 34%. In October 2013, they ranged from 14% to 24% (SIFMA prepayment tables, benchmark scenario). In 2003, CPRs were as high as 80%.

2008, the merger of the GSEs and the Treasury Department became a reality.

Starting with the guarantee policy, our parameter γ specifies the cost of a guarantee, expressed in the same units as the price of the mortgage. Real-world guarantee fees are expressed as a surcharge to the interest rate. We consider several values for γ with implied g-fees ranging from 20 to 300 basis points. Freddie Mac's management and g-fee rate was stable at around 20bps from 2000 to 2012. Similarly, Fannie Mae's single-family effective g-fee was also right around 20bps between 2000 and 2009. Thus, our benchmark model is the 20 basis point g-fee economy. The main policy experiment in the paper is to raise γ and investigate the effects from higher guarantee fees. Interestingly, Freddie Mac has increased its g-fee gradually from 20bps at the start of 2012 to 32 bps at the end of 2014, while Fannie Mae has increased its g-fee from 20bps at the start of 2009 to 41 bps at the end of 2014 (Urban Institute Housing Finance Policy Center, December 2014 update). Fannie's g-fees on new single-family originations currently average 63bps.

We set the proportional income tax rate equal to $\tau = 20.4\%$ in order to match average discretionary tax revenue to trend GDP in U.S. data. The discretionary tax revenue in the 1946-2013 data of 19.97% is after mortgage interest deductions, which is about 0.43% of trend GDP. Hence, we set a tax rate before MID of 20.4%.[23] As explained before, the model features mortgage interest rate deductibility at the income tax rate. Because our geometric mortgages do not distinguish between interest and principal payments, we assume that the entire mortgage payment is deductible but at a lower rate, $\tau^m = (1 - \alpha) \times \tau$.[24] Tax revenues are pro-cyclical, as in the data. Every dollar of income is taxed at the same tax rate. Risk takers are only 2% of the population but pay 10% of the income taxes since they earn 10% of the income.

We set exogenous government spending equal to $G^o = 0.163$ (times trend GDP of 1) in order to match average exogenous government spending to trend GDP in the 1946-2013 U.S.

[23]In our numerical work, we keep the ratio of government debt to GDP stationary by decreasing tax rates τ_t when debt-to-GDP threatens falls below $\underline{b^G} = 0$ and by increasing tax rates when debt-to-GDP exceedd $\overline{b^G} = 1.2$. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -30%. Tax rates are gradually and convexly increased until they hit 50% ay a debt-to-GDP ratio of 160%. Our simulations never reach the -30% and +160% debt/GDP states. These profligacy and austerity tax policies do not affect the amount of resources that are available for private consumption in the economy.

[24]As discussed in Appendix B.3, the sum of all mortgage payments is $1/(1 - \delta)$ and $F = \alpha/(1 - \delta)$ is the payment of "principal." Hence, the fraction of "interest payments" is the fraction $(1 - \alpha)$. In the equilibrium with low g-fees, the mortgage interest deductibility expense is 0.46% of trend GDP, very close to the target.

data of 16.3%.[25] This exogenous spending is wasted. We also allow for transfer spending of 3.18% of GDP, which equals the net transfer spending in the 1946-2013 data. This spending is distributed lump-sum to the agents in proportion to their population share. As a fraction of *realized* GDP, expenditures fluctuate, mimicking their counter-cyclicality in the data.

We can interpret the risk-taker borrowing constraint parameters, $\varkappa$, ξ_G and ξ_P as regulatory capital constraints set by the government. Under Basel II and III, "first liens on a single-family home that are prudently underwritten and performing" enjoy a 50% risk weight and all others a 100% risk weight. Agency MBS receive a 20% risk weight. Given that we think of the non-guaranteed mortgage market as the subprime and Alt-A market, a capital charge of 8% (100% risk weight) seems most appropriate for ξ_P. Given that the government guaranteed mortgages are the counterpart to agency MBS, we set a capital charge of 1.6% (20% risk weight) for ξ_G. We set the additional margin $\varkappa$ to match average leverage ratios of the financial sector, given all other parameters. Since mortgage assets are predominantly held by leveraged financial institutions, we calculate leverage for those kinds of institutions. The average ratio of total debt to total assets for 1985-2014 is 95.6%.[26] Since the non-mortgage portfolio of these institutions have higher risk weights than their mortgage portfolio, we find that $\varkappa < 1$.

Utility cost of risk-taker bankruptcy The model features a random utility penalty that risk takers suffer when they default. Because random default is mostly a technical assumption, it is sufficient to have a small penalty. We assume ρ_t is normally distributed with a mean of μ_ρ =1, i.e., a zero utility penalty on average, and a small standard deviation of $\sigma_\rho = 0.05$.

[25]The data are from Table 3.1 from the BEA. Exogenous government spending is defined as consumption expenditures (line 18) plus subsidies (line 27) minus the surplus of government enterprises (line 16). It excludes interest service on the debt and net spending on social security and other entitlement programs. Government revenues are defined as current receipts (line 1), which excludes social security tax receipts. Trend GDP is calculated with the Hodrick Prescott Filter.

[26]Specifically, we include U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations (Fed Bailout entities e.g. Maiden Lanes), GSEs, Agency- and GSE-backed Mortgage pools (before consolidation), Issuers of ABS, REITs, and Life and Property-Casualty Insurance Companies. Krisnamurthy and Vissing-Jorgensen (2011) identify a group of financial institutions as net suppliers of safe, liquid assets. This group is the same as ours except that we add insurance companies and take out money market mutual funds, since we are interested in leveraged financial firms. For comparison, leverage for the Krisnamurthy and Vissing-Jorgensen institutions is 90.7% for the 1985-2014 sample. The group of excluded, non-levered financial institutions are Money Market Mutual Funds, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds. Total financial sector leverage, including these non-levered institutions, is 60.6%.

The mean size of the penalty affects the frequency of financial sector defaults (and government bailouts). The lower μ_ρ, the lower the resistance to declare bankruptcy, and the higher the frequency of bank defaults. The standard deviation affects the correlation between negative financial intermediary wealth and bank defaults. Given those parameters, the frequency of financial crises (government bailouts of the risk-taker) depends on the frequency of foreclosure crises, and the endogenous choices (asset composition and liability choice) of the risk taker.

Preference parameters Preference parameters are harder to pin down directly by data since they affect many equilibrium quantities and prices simultaneously. However, the discussion of the first-order conditions above helps us connect the various parameters to specific equilibrium objects they have a disproportionate effect on.

The coefficients of risk aversion are $\sigma_R = 1$, $\sigma_B = 8$, and $\sigma_D = 20$. The annual subjective time discount factors are $\beta_R = \beta_D = 0.98$ and $\beta_B = 0.88$. Risk aversion and the time discount factor of the depositor disproportionately affect the short-term interest rate and its volatility. The benchmark model generates a mean one-year real risk-free interest rate of 1.1% with a standard deviation of 3.0%. In the data, the mean real interest rate is 1.20% with a volatility of 1.97% over the period 1985-2014.[27] The borrower's discount factor governs mortgage debt and ultimately house prices. In the model, housing wealth to trend GDP is 2.24, while in the Flow of Funds data (1985-2014) it is 2.41.[28] Borrower risk aversion is set to target the volatility of the annual change in household mortgage debt to GDP (Flow of Funds and NIPA), which is 4.2% in the 1985-2014 data. Our low g-fee economy produces a volatility of 1.8%. The risk takers have log period utility and their subjective discount factor is set equal to that of the depositors. We set the elasticity of inter-temporal substitution equal to 1 for all agents, a common value in the asset pricing literature.

[27]To calculate the real rate, we take the nominal one year constant maturity Treasury yield (FRED) and subtract expected inflation over the next 12 months from the Survey of professional Forecasters. The mean interest rate is sensitive to the sample period. Over the period 1990-2014, the mean is 0.72% and over the period 1998-2014, it is only 0.21%.

[28]The number in the data includes the real estate owned by the corporate sector since our model is a model of the entire economy but does not include real estate-owning firms. Real estate owned by the household and non-corporate sector is 1.51 times GDP on average over this period.

4.3 Computation

This is a complicated model to solve given the presence of occasionally binding constraints for both borrowers and risk takers. We provide a non-linear global solution method, policy time iteration, which is a variant of the parameterized expectations approach. As explained in more detail in computational Appendix C, policy functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of non-linear equations including the Euler equations and the Kuhn-Tucker conditions expressed as equalities. Kubler and Schmedders (2003) show that there exist stationary equilibria in this class of models when all exogenous state variables follow Markov chains, as is the case here. Our solution method is a variant of theirs.

5 Main Results: Phasing out the GSEs

The main experiment in the paper is to compare an economy with and without government-guaranteed mortgages. Specifically, we compute a sequence of economies that only differ by the mortgage guarantee fee γ_t set by the government. We compare equilibrium prices, quantities, and ultimately welfare across economies. All economies feature a government bailout guarantee to the financial sector, or equivalently, deposit insurance. We simulate each economy for 10,000 periods and report unconditional means and standard deviations across the simulations in Tables 2 and 3.

Our benchmark model is one where the government provides the mortgage guarantee relatively cheaply. We set γ_t to a value that implies an annual rate spread of 20bps.[29] The benchmark "low g-fee" case represents the period between the late 1990s and the late 2000s when g-fees were around 20bps. In the interest of space, we only report detailed results for two intermediate economies: the 55bps and 100bps g-fee cases. The former is of particular interest since it reflects the level of g-fees observed today. The latter is of interest because, at a 100bp g-fee, guaranteed mortgages turn out to dominate during mortgage crises while private bonds

[29]The interest rate on private bonds can be calculated as $r_{P,t} = \log\left(\frac{1}{q_t^m} + \delta\right)$, and the rate on guaranteed bonds is $r_{G,t} = \log\left(\frac{1}{q^m+\gamma_t} + \delta\right)$. The effective g-fee, quoted as a difference in rates, is therefore given by $r_{P,t} - r_{G,t}$.

are dominant in normal times. This outcome is reminiscent of Option B in the Obama Administration's policy document of February 2010 which envisions setting the g-fee high enough so that it is only taken up in crises. Finally, we report on a "high g-fee" economy, the 275bps g-fee economy, where guarantees are expensive enough that they are never bought. In this last economy, the GSEs are "phased out" and the mortgage market functions as a private market without government intervention (except for deposit insurance).

5.1 Prices

The first panel of Table 2 shows that risk-free and mortgage interest rates are low and house prices are high in the benchmark low g-fee economy. As g-fees rise, risk-free and mortgage interest rates rise while house prices fall. The cheap mortgage guarantees lead to mortgage interest rates of 3.5%, 22 basis points lower than in the high g-fee economy. This magnitude of subsidy to mortgage rates is similar to what the empirical research has inferred form the spread between conforming and jumbo mortgage loans.

To show that the mortgage guarantee is indeed underpriced, we compute the actuarially fair guarantee fee. It is the fee that a hypothetical risk-neutral agent with the same degree of patience as the savers would charge for the mortgage guarantee payment upon a default ($M_G - M_P$). The actuarially fair g-fee depends on the model in which it is computed. In the 20bp g-fee economy, whose equilibrium displays financial fragility, the actuarially fair g-fee is 77 basis points.

Short-term interest rates vary more substantially across economies, with 1.1% annual real interest rates in the low g-fee economy, 70 basis points lower than in the high g-fee economy. The key reason for low real interest rates is that the low g-fee economy is riskier, as explained further below. Depositors, who often are the marginal agents in the risk-free bond market and who are the most risk averse of all agents, have strong precautionary savings motives which push down interest rates.

The entire rise in mortgage rates is due to the rise in risk-free interest rates. Mortgage spreads, the difference between mortgage and risk-free interest rates, go down as the g-fee rises, a reflection of declining mortgage default rates.

Faced with low mortgage rates, borrowers who are the marginal agents in the housing sector demand more housing. Given a fixed housing supply (relative to trend growth), increased housing demand results in higher house prices. The low g-fee economy's house prices are 6.3% higher than in the high g-fee economy. Thus, phasing out the GSEs would lead to a non-trivial decline in house prices.

House prices are also more volatile in the low g-fee economy: 14% annual standard deviation compared to 12% in the high g-fee economy. This is a consequence of the higher volatility in the demand for mortgage debt in the low g-fee economy, as discussed further below.

5.2 Borrowers

Faced with high house prices and low mortgage rates, borrowers demand more mortgage debt. The steady state stock of mortgages outstanding is high in the low g-fee equilibrium (0.053 units A^B or 63.4% of GDP in market value terms). The average borrower LTV ratio is 63.8% and borrowers' mortgage debt-to-income is 1.49 on average, both are close to the averages for borrowers in SCF data. When g-fees and mortgage rates rise, the size of the mortgage market shrinks. The mortgage market also becomes safer: the mortgage debt-to-income ratio drops by 9% points.

We recall that the optimal mortgage default policy for the borrower family depends on the mark-to-market LTV ratio (equation 20). That ratio is the highest and thus mortgages are the riskiest in the low g-fee economy. The average mortgage default rate is 2.7% while the average severity rate (loss given default) is 30.2%. Both match the data. They deliver a mortgage loss rate which is 1.0% on average. Both default and loss rates fluctuate substantially across aggregate states of the world (output growth and mortgage crisis vs. normal times). Loss rates from mortgage defaults are 6.1% in housing crises but less than 0.5% in normal times.

In the high g-fee economy, the mortgage default rate is 1.8%, a reduction by almost 40% compared to the low g-fee economy. Since the severity rate is the same across economies, this translates in a loss rate of 0.7% unconditionally. The lower mark-to-market LTV ratio implies a lower mortgage default rate. It is itself the result of lower mortgage prices q^m a and smaller amount of mortgage debt and occurs despite lower house prices. The first sense in which the private sector economy is safer is that borrowers have more home equity and mortgages default less frequently. Fewer foreclosures lead to less deadweight losses from foreclose, a reduction in deadweight losses. The reduction in mortgage loss rates results in lower mortgage spreads in the high g-fee economy.

In the benchmark economy, mortgage debt is not only higher, it is also more volatile across aggregate states of the world. Specifically, there is a larger drop in mortgage credit during crises episodes (high σ_ω states). An oft-invoked rationale for government guarantees is to ensure the stable provision of mortgage credit at all times.[30] Our measure of the stability of the provision of mortgage credit is the standard deviation of mortgage debt to income growth. This volatility is 2.9% in the low g-fee economy. Surprisingly, this volatility initially *decreases* to 2.7% as the g-fee increases to 55bp. As we will see below, banks are better capitalized in the 55bp economy. The volatility inches back up as g-fees increase further and banks take on more credit risk. As we approach the private market economy, the volatility reaches a level equal to that in the low g-fee economy. Even in crisis periods, the decline in mortgage credit relative to income is smaller in absolute value in the high g-fee economy than in the low g-fee economy. The popular fear that a private mortgage system would lead to large swings in the availability of mortgage credit, especially in bad times, is unwarranted in our model.

In terms of their prepayment decisions, borrowers prepay 15.8% of non-defaulted mortgages on average, matching historical data. As the g-fee rises, prepayment rates go down slightly. Higher equilibrium mortgage rates reduce the benefit from refinancing. Financial intermediaries face more mortgage credit risk but less prepayment risk in the private sector economy.

5.3 Risk Takers

The third panel of Table 2 reports on the risk takers. As financial intermediaries, they make long-term mortgage loans to impatient borrower households and borrow short-term from patient depositor households. They play the traditional role of maturity transformation. Given their low risk aversion, they are the most willing to bear fluctuations in their net worth among all

[30] Indeed, Fannie Mae was founded in the Great Depression when a massive default wave of banks threatened the supply of mortgage credit. By guaranteeing mortgages, it is widely believed to make mortgage markets more liquid thereby ensuring that banks are willing to lend even in bad times.

agents. Given sufficient intermediary capital, they can absorb a disproportionate amount of aggregate risk.

Low g-fees In the low g-fee economy, risk takers hold nearly all of their assets in the form of government-guaranteed bonds. They buy mortgage guarantees both in normal times and in mortgage crises (high σ_ω) states, taking advantage of the cheap mortgage guarantees provided by the government. As a result, the asset side of their balance sheet is largely shielded from mortgage default risk. Bearing little default risk on their assets and facing a low interest rate on safe deposits, banks use substantial leverage in order to achieve their desired risk-return combination. The intermediary leverage constraint allows banks who exclusively hold guaranteed mortgage bonds to have a maximum leverage ratio of 96.4%. Banks hit that constraint in 32.7% of the periods. The average bank leverage (market value of debt to market value of assets) ratio is 95.6%, matching the data. Average risk taker wealth is modest, at 2.9% of trend GDP. Banks have little "skin in the game." The constraint binds more frequently in normal times (34.8%) than in crises (14.7%) because of precautionary deleveraging in crises. Leverage is pro-cyclical in the low g-fee economy.

In addition to taking risk through leverage, banks in the low g-fee economy have a larger balance sheet of mortgages. Due to the combination of low risk taker wealth, high leverage, and a large and risky mortgage portfolio, the banking system is fragile. When adverse income or mortgage credit shocks hit, risk taker net worth falls. The reason this happens despite the prevalence of guaranteed mortgages on intermediaries' balance sheets is mark-to-market losses on guaranteed bonds. Mortgage defaults act as prepayments for holders of guaranteed bonds. Since agency bonds trade above par prior to prepayment, but prepayments come in at par, prepayments constitutes a loss for the holder or agency MBS.[31] The average mark-to-market loss rate given a prepayment is 10.8% (12.6% in crises). The overall loss rate on banks' mortgage portfolio is 0.38% (0.23% in normal times and 1.68% in crises). Losses are nearly entirely due to prepayment-related (both default- and rate-induced) losses rather than due to credit losses/mortgage arrears.

When intermediary net worth turns negative, which occurs in 0.27% of the simulation peri-

[31]Put differently, prepayments happen when interest rates are low and reinvestment opportunities are poor.

ods, the government steps in to bail out the financial sector. Thus mortgage crises can trigger financial crises, a relationship documented in the work of Jorda et al. (2014). The table reports the return on risk-taker wealth. It is 3.6% excluding the bankruptcy events, but 3.3% including such events (unreported). This difference illustrates the option value introduced by the possibility to go bankrupt. The return on risk-taker wealth is 12% in a crisis, consistent with the result in intermediary-based asset pricing literature that risk premia increase when intermediary capital is scarce.

Higher g-fees As g-fees rise, the composition of the risk taker portfolio shifts towards private bonds. In the 55bp economy, guaranteed bonds still make up 98.4% of the portfolio. This is consistent with the situation today, where g-fees have risen to about 55bp and yet guaranteed bonds continue to dominate.

When g-fees go up to 100 basis points, banks guarantee only 11% of their portfolio. This dramatic reversal occurs because risk takers buy the guarantee only in crises, when guaranteed bonds constitute 93% of their portfolio. In good times they prefer an all-private portfolio. The state uncontingent g-fee is too cheap to forego in bad times but too expensive in normal times. This 100 basis point g-fee economy is reminiscent of Option B of the Obama Administration housing reform plan, which envisions a g-fee level that is high enough so that it would only be attractive in bad times (Scharfstein and Sunderam, 2011).

In the high g-fee economy, risk takers shift exclusively towards holding private MBS. They do not buy default insurance from the government, neither in good nor in bad times. The 275 basis point g-fee is high enough to "crowd-in" the private sector at all times. The 275bp g-fee economy implements Option A in the Obama plan which envisions an entirely private mortgage market.

Our main result is that increasing the g-fee lowers the riskiness of the financial sector. Risk taker leverage falls from 95.6% in the 20bp and 95.2% in the 55bp economy to 89.0% in the 100bp economy, and 88.3% in the high g-fee economy. As the portfolio shifts towards private mortgages, the risk taker's collateral constraint becomes tighter because private mortgages carry higher regulatory capital requirements ($\xi_P > \xi_G$). But this is not the main driver of the lower leverage. Rather, banks choose to stay away from their leverage constraint in most periods;

their leverage constraint binds in less than 20% of periods compared to 33% in the low g-fee economy. Having to bear mortgage credit risk, banks attain their desired high risk-high return portfolio without the need of having to lever up as much.

The financial sector is well-enough capitalized (6.9% equity) and has low enough leverage that it can guarantee the stable provision of mortgage credit in good and bad times, relative to a system with a government backstop where banks are poorly capitalized and prone to occasional collapses. Higher g-fees reduce the incidence of bank insolvencies and concomitant bank bailouts. Interestingly, risk taker leverage becomes counter-cyclical for higher g-fees as the intermediaries are sufficiently strong to desire an increase in lending in bad times.[32] In sum, low g-fees create moral hazard: faced with cheap mortgage guarantees which offload mortgage credit risk onto the taxpayer, banks endogenously increase leverage, make more and riskier mortgages. The incidence of foreclosure increases and ultimately increases the fragility of the financial system.

The model endogenously generate a negative relationship between the amount of default and prepayment risk. In the high g-fee economy, banks face less prepayment risk (CPRs are 4% points lower) but more of the credit risk. However, there is less credit risk to be born because the mortgage default rate is lower and the mortgage portfolio smaller. In the low g-fee economy there is endogenously more credit risk and more prepayment risk created, but banks only bear the latter.

5.4 Depositors and Government

Depositors have high risk aversion and thus loathe large fluctuations in consumption across states of the world. Their strong precautionary savings demand makes them willing to lend to intermediaries and the government at low interest rates. Deposit insurance is important because it makes depositors' claims on the banks risk-free, irrespective on the riskiness of the bank's balance sheet.

[32]Our result is a cousin of the volatility paradox in Brunnermeier and Sannikov (2014) where an increase in the fundamental volatility of the asset endogenously reduces the risk appetite of the intermediaries.

Low G-fees The high mortgage loss rates are mostly absorbed by the government, as are the occasional bank insolvencies. Both lead to a surge in government expenditures, financed with government debt. Government debt to trend GDP is 15.0% on average with a high standard deviation of 21.8%. Tax revenues only slightly exceed government discretionary and transfer spending, and in the aftermath of a severe mortgage crisis it takes many years of small surpluses as well as the absence of a new mortgage crisis to reduce government debt back to steady state levels.

Depositors must hold not only the debt issued by the banks (deposits) but also the debt issued by the government. High risk taker leverage and high government debt in the low g-fee economy lead to high equilibrium holdings of short-term debt by depositors. All else equal, the large supply should result in a low price of short-term debt, or equivalently, a high interest rate to induce the depositor to hold all that debt.[33] Yet on average the low g-fee economy exhibits a *low* average equilibrium interest rate. The reason is that the precautionary savings demand more than offsets the supply effect.

During mortgage crises government debt shoots up as the government pays out on mortgage guarantees and occasionally on bank bailouts. This increase in the supply of short-term debt by the government is only somewhat offset by lower risk taker leverage, so that on net the supply of bonds grows during crises. The ultra low real interest rates in crises make debt issuance attractive for the risk taker and government alike. The depositor absorbs this debt increase in equilibrium by increasing savings and reducing consumption.

In sum, by protecting the financial sector from mortgage defaults, the government shifts more of the consumption fluctuations across states of the world onto the depositor. By virtue of the depositor's high risk aversion, she is more unwilling to bear such consumption fluctuations than the risk taker. The depositor responds to the "fiscal uncertainty" by saving a lot more at all times to absorb at least some of the fluctuations in government debt with existing savings. The result is a low equilibrium interest rate and low average financial income for the depositor. The low interest rate is a signature of the large amount of risk in the low g-fee economy.

[33]Indeed, the interest rate during income contractions that coincide with mortgage crises (-1.7%) is more than a percentage point higher than the interest rate during contractions that do not coincide with crises (-2.8%).

High G-fees The high g-fee economy witnesses the same fraction of mortgage crises. But these crises result in much lower mortgage loss rates. Furthermore, the mortgage losses are no longer borne by the government but rather absorbed by the intermediaries' balance sheet. Average government debt and its volatility fall precipitously. The lower equilibrium supply of both government debt and risk taker debt (deposits) would result in a lower interest rate if savers were risk neutral.[34] But overall interest rates are *higher* in the high g-fee economy because the precautionary savings effect again dominates given the high risk aversion of the depositors. A safe economy without financial fragility and therefore with low and predictable government debt induces depositors to scale back their precautionary saving demand. The fall in demand for safe assets exceeds the decline in the supply, explaining higher equilibrium real interest rates.

In summary, when the g-fee is high enough, risk takers are well enough capitalized and their intermediation capacity is rarely impaired. They bear and hence internalize all mortgage default risk. In contrast, in the low g-fee economy, mortgage crisis episodes frequently disrupt risk takers' intermediation function. During these crises, the risk free rate drops sharply and government debt increases sharply, effectively making depositors bear a greater part of the mortgage default risk.

5.5 Welfare

We measure aggregate welfare as the population-weighted average of the value functions of the three types of agents.[35] The first row of Table 3 shows that it is 0.63% higher in the high g-fee economy than in the benchmark low-g-fee economy. In unreported results for a series of intermediate g-fee economies, we find that aggregate welfare increases monotonically in the g-fee. We consider the 0.63 percent improvement in consumption equivalence terms from GSE reform to be a substantial effect.

There are two effects that help understand the aggregate welfare gain: an improvement in risk sharing and a reduction in deadweight losses. First, risk sharing between the different types

[34] Indeed, the large difference between crisis and non-crisis interest rates attributable to additional supply of risk-free debt, holding aggregate income constant, entirely disappears.

[35] With unit EIS, the value functions are in units of composite consumption $C^{1-\rho}K^{\rho}$. Therefore, increases in aggregate welfare can be directly interpreted as consumption-equivalence gains.

of agents generally improves as g-fees increase. To measure the extent of the improvement, we compute the ratios of (log) marginal utilities between the different types. If markets were complete, agents would be able to achieve perfect risk sharing by forming portfolios that keep these ratios constant. Hence, larger volatilities of these marginal utility (MU) ratios indicate worse risk sharing between the different types of agents. Table 3 lists the average MU ratios and their volatilities for borrowers/risk takers, and risk takers/depositors, as these are the pairs of agents that directly trade with each other. The volatilities of both ratios is lower in the high g-fee economy than in the low g-fee economy. The MU ratio volatility between borrower and risk taker falls by 22.6%. The decline in the MU ratio volatility between risk taker and depositor is 7.7%. We see similarly large improvements in risk sharing if we look at consumption volatility for the three types of agents: -9.3% for the borrower, -20.7% for the depositor, and -8.9% for the risk taker.[36] Intermediary wealth is a crucial driver of the overall degree of risk sharing between the agents in the economy. In the private economy, banks are better able to provide consumption smoothing services to both borrowers and depositors because they are better capitalized and less fragile. Improved risk sharing also increases the risk-free rate and therefore mean consumption for savers.

The second source of the welfare gain is a reduction in deadweight losses. The first deadweight loss is the one associated with mortgage foreclosures. Lower deadweight costs leave more resources for private consumption each period. This deadweight loss is 0.59% of GDP in the 20bps economy and 0.36% of GDP in the 275bp economy. While the deadweight loss falls by 39%, it remains modest. The economy also benefits from lower deadweight losses from prepayment costs in high g-fee economies since prepayment rates are lower. These losses are of the same magnitude as those from foreclosures and fall by about the same percentage. Finally, there is a reduction in housing maintenance costs going from the low to the high g-fee economy because maintenance expenses are proportional to the value of the house. A reduction in housing consumption leaves more resources for non-housing consumption. Combined, the increases in

[36]While the risk sharing between borrower and risk taker improves monotonically as g-fees rise, the risk sharing between the risk taker and the depositor is highest at an intermediate g-fee closest to the actuarially fair g-fee (around 65bp). Similarly, the volatility of consumption for the risk taker is the lowest at that g-fee. At low g-fee levels, the high leverage and risk-taking of intermediaries makes their consumption volatile. As the g-fee rises from 20bps to about 65bps, leverage falls sharply but the risk taker's portfolio is still largely protected by government guarantees. However, as g-fees rise further above 65bps, the risk taker portfolio tilts towards private bonds, and this makes consumption volatility rise again, despite further reductions in leverage.

resources benefits the depositor and risk taker, both of which increase mean consumption. The borrower's consumption declines because she faces higher mortgage interest rates in the high g-fee economy.

What are the distributional consequences of a mortgage market privatization? Close inspection of the value function of each of the three household types shows that the borrower's welfare stays almost constant between the low and the high g-fee economies (+0.04%), while depositor welfare (+1.32%) and risk taker welfare (+1.69%) both increase substantially.[37] The near-absence of a welfare loss for borrowers is surprising since taking away underpriced mortgage guarantees increases mortgage rates and lowers property values. The important offset to a decline in her consumption comes from the improvement in risk sharing. In conclusion, while GSE reform is a Pareto improvement, it redistributes wealth from borrowers to savers thereby raising inequality.

In Figure 2, we study a transitional experiment rather than a steady-state comparison. We assume that the economy starts in the 20-bp g-fee equilibrium at typical values of the state variables. The economy then undergoes a once-and-for-all change to the 275bps g-fee economy. We find that prices adjust rapidly, while state variables such as the wealth distribution adjust gradually. As a result of the sudden rise in interest rates and decline in house prices, borrowers' value function falls upon impact. It only slowly recovers as the intermediary sector's wealth accumulation gradually facilitates better risk sharing. Depositors and risk takers gain upon impact as well as in the long-run. Aggregate welfare rapidly stabilizes.

6 Alternative Policy Experiments

6.1 State-contingent Guarantees

Figure 3 shows the actuarially fair g-fee for the low and high g-fee economies as well as several intermediate economies. The solid line shows the unconditional average, the dashed line the actuarially fair g-fee in crises, and the dotted line the fair g-fee in normal times. The figure also draws in the 45-degree line along which actual and actuarially fair g-fees are equal. We

[37]Given the homogeneity properties of the value function, log changes in value functions are directly interpretable as consumption equivalence changes and hence directly comparable across agents.

note that the actuarially fair g-fee is decreasing in the exogenous g-fee (solid line with circles). The higher the g-fee, the safer the mortgages are, and the more stable the financial sector. To break even, a risk neutral insurer could charge a lower average rate. The actuarially fair g-fee declines from 77 basis points in the 20bp economy to 54 basis points in the 275bp economy. The fixed point where the actual and fair g-fees equate is around 60 basis points. Relative to the risk-neutral benchmark, mortgage guarantees are overpriced *on average* in the economies with a g-fee above 60 basis points and unconditionally underpriced in economies with g-fees below 60bp.

The figure also makes clear that the actuarially fair g-fee is state contingent. During mortgage crises (high σ_ω states), the mortgage loss rate is a lot higher and a much higher g-fee must be charged to break even (dashed line with squares). For g-fees below 150bp, risk takers would always want to buy guarantees in crisis times since risk takers are not risk neutral but risk averse. It turns out, we must go to 275 basis points to make guarantees expensive enough so that they are almost never bought in any of the states of the world. The 100bp economy is an interesting one. Risk takers overwhelmingly buy the mortgage guarantee in crisis periods but overwhelmingly hold private mortgage bonds in normal times. The actuarially fair g-fee in that economy is 158bp in crisis times while it is 47bp in normal times. Thus, the 100bp non-state contingent guarantee fee is attractively priced only in crises. Appendix B.4 discusses whether there is there scope for welfare-enhancing private mortgage insurance in the model.

We study a policy experiment where the government charges a high g-fee of 100bps in good times (expansions and normal mortgage credit risk states) and a low g-fee of 55bps in bad times (mortgage crises and recessions). This policy has been proposed by Scharfstein and Sunderam (2011) as well as in the Obama Administration's policy paper of 2010, known as long-run Option B. The results are reported in columns 5 and 6 of Table 4. There is not much to be gained by making g-fees time-varying. Welfare is higher than in the fixed 55bps economy, and lower to than in the fixed 100bps economy. Comparing the riskiness of the fixed 55bps economy and the counter-cyclical g-fee economy, there are two effects. On one hand, higher g-fees in booms reduce risk, consistent with our previous experiments. On the other hand, time variation in g-fees introduces a new source of risk, because next period's g-fee affects next period's prices. The offset from the latter reduces the appeal of a counter-cyclical g-fee.

6.2 Catastrophic Insurance

Our framework is well-suited to quantitatively evaluate a recent legislative proposal in the U.S. Senate Banking Committee, the Johnson-Crapo or JC proposal.[38] The proposal envisions changing the nature of the government-provided mortgage guarantee by mandating mortgage lenders to hold a substantial buffer of private capital, with a view towards better protecting the taxpayer. Having more private capital at risk, mortgage underwriting would be more prudent and intermediary moral hazard would be diminished. Specifically, the government guarantee would only kick in after the private sector has borne a 10% mortgage loss rate. Losses above 10% would be absorbed by the government. The industry has retorted that a 10% private loss rate is too high and proposed 5% instead. We evaluate both proposals. Our paper is the first to provide a detailed quantitative analysis of Johnson-Crapo, including all the general equilibrium effects on risk taking, interest rates, house prices, and the distributional effects for the various types of tax payers.

In the model, we change the definition of the government guarantee. When risk takers buy a mortgage guarantee, it pays out only if the loss rate on the mortgage pool exceeds 10%. If the loss is less than 10%, the guarantee is worthless and the guaranteed bond has the same payoff as a private mortgage bond. If the loss is higher than the threshold, the guaranteed bond pays out an amount equal to the losses above the threshold. For ease of comparison, we keep the regulatory capital advantages of guaranteed bonds from the benchmark economy. We assume that the government offers the catastrophic insurance at 20 basis points.[39] We compute the actuarially fair cost of the catastrophic guarantee, at the new equilibrium. The last two columns of Table 4 present the results.

The JC economy is similar to the high g-fee economy in several aspects. It has lower house prices, higher mortgage rates, and a smaller mortgage market. In market value terms, risk taker mortgage assets are 8.7% lower in the JC 10% economy than in the benchmark low g-fee

[38] The "Housing Finance Reform and Taxpayer Protection Act of 2014" introduced by senators Corker and Warner preceded the draft bill introduced by Senators Johnson and Crapo and voted in the Senate Committee on Banking, Housing, and Urban Affairs on May 15, 2014. The 13-9 vote was not strong enough to force a full senate floor vote.

[39] This assumption does not significantly affect results. Appendix D reports results from two additional experiments. In the first one, the catastrophic insurance is priced at 5bp. The second one keeps the g-fee at 20 bp but provides insurance for losses in excess of 5%, rather than 10% percent. Results are qualitatively similar.

economy. House prices fall by 6.2%. There is a substantial reduction in mortgage default rates, just like in the high g-fee economy.

In the JC economy, the guarantee's actuarially fair cost is 2 basis points. Insurance is quite *over*priced at 20bp. As a result, risk takers hold fewer guaranteed bonds (29% of their portfolio). While the guarantee is less generous, the insured mortgages are also endogenously less risky, so that the loss rate on *guaranteed* bonds is lower than in the low g-fee case. Risk takers' portfolio loss rate is 0.56%, which is 47% higher than in the benchmark economy but 17% lower than in the high g-fee economy when only private mortgages are held, as the guaranteed bonds are still substantially safer than uninsured bonds. Absent the severely underpriced guarantee, borrower leverage is lower, and lower default and loss rates on mortgages require smaller and less frequent guarantee payouts. As a result, government debt is much lower in the JC economies.

Risk taker leverage is lower at 91.6%, suggesting that the increased losses they must bear reduce their appetite for high leverage. The protection offered by the catastrophic guarantee increases their appetite to resume lending after a mortgage crisis and they run more often into binding constraints (77% on average, 88% in crises). The return on risk takers' wealth is now lower (2.9% excluding bankruptcy periods) but also less volatile, with volatility dropping by 26%.

Interestingly, aggregate welfare in the JC 10% case is slightly higher than in the high g-fee economy (+0.67% versus +0.63%). Borrowers' welfare changes in the almost identical way as from a phase-out of the guarantee (+0.05%). Depositors gain more than in the main experiment (+1.38% vs. +1.32%), while risk takers gain slightly less (+1.23% versus +1.69%). The gain for depositors comes from the much lower and less volatile government debt, just as in the main experiment. In addition, risk takers are better able to help depositors smooth consumption. This ability is higher in the JC economy than in the high g-fee economy because the catastrophic guarantee protects the banks in very adverse states of the world. We can see the effect of the catastrophic insurance by inspecting the consumption distribution of risk takers: while in the high g-fee economy the 0.1-percentile of risk taker consumption is 0.045, the same percentile is higher at 0.051 in the JC 10% economy (mean risk taker consumption is approximately 0.075 in both economies). The volatility of risk taker consumption falls substantially, besting the change in the main experiment. The benefits of this insurance for banks accrue primarily to depositors: equilibrium interest rates reflect the safe environment and are even (6bp) higher than in the high g-fee economy. As a result, the mean consumption gain for the risk taker is not as high as in the main experiment, because the return on bank equity and the mortgage spread are not as high in the JC 10% economy as in the high g-fee economy, explaining the slight smaller overall gain for risk takers.

7 Great Recession Experiment

This sections explores implications of the model for period of low aggregate economic growth and high credit risk, a situation akin to the Great Recession in 2008-10 in the United States. Specifically, we are interested in its implications for foreclosures and house prices.

To capture the foreclosure delays which were rampant in judiciary states, we modify our model slightly.[40] In the benchmark model, all borrowers with negative equity default (recall equation 20). In this extension, we still assume that borrowers with negative equity miss their mortgage payment, but now only a fraction default if the economy is in a mortgage crisis. We set this fraction to half. If a borrower is still underwater the following period and the the economy is still in the high σ_ω state, there is again a 50% probability of default. This mechanism spreads out foreclosures over multiple years during a prolonged crisis. It also reduces the total foreclosure rate relative to the benchmark model.

Once the extended low g-fee model is solved, we feed in a particular sequence of aggregate shocks mimicking the conditions during the period 2001–2013.[41] The left panel of Figure 4 plots GDP (Y) resulting from the exogenous growth dynamics we fed into the model. GDP peaks in 2007 and bottoms out in 2010.

The middle panel shows the (endogenous) loss rate on mortgages. As in the data, there are almost no mortgage losses until 2007. In 2008, the foreclosure crisis starts and mortgage

[40]The average length of a foreclosure processes increased dramatically during and after the financial crisis from about 200 days nationwide in 2007 to 630 days in 2015. Foreclosure processes lasting 2.5-3 years are not uncommon in judiciary foreclosure states such as New Jersey, Florida, and New York according to Realty Trac.

[41]Specifically, we assume the state variable σ_ω takes on its normal-times value in 2001–2007 and 2011–2013, but its higher crisis value in 2008-2010. For the second aggregate state variable, economic growth g, we assume the following sequence. In 2005–2007, the economy is in a strong expansion (highest of 5 grid points). In 2002–04 and 2011–13, the economy is in a mild expansion. In 2001 and 2010, the economy is in a mild recession, and in 2008 and 2009, it is in a severe recession.

loss rates spike. Because of the foreclosure delay mechanism, foreclosures and mortgage losses remain elevated in 2009 and 2010. The right panel plots the house prices. It shows that the model is able to generate a 22% drop in house prices between 2007 and 2008. This accounts for about 2/3rds of the observed house price drop in the data and shows that the economy has enough risk to generate meaningful house price dynamics. As in the data, house prices are back to 2003 levels by 2013.

8 Conclusion

Underpriced, government-provided mortgage default insurance leads to moral hazard in the financial sector. Banks become too levered, make too many mortgages, and make riskier mortgages. House prices are too high, mortgage and risk-free interest rates are too low. Mortgage default and loss rates are too high and mortgage crises may lead to the insolvency of the financial sector. Even though the government can mitigate the fallout from such crises by spreading out the costs over time by issuing bonds, savers must buy these bonds at inopportune times. The allocation of risk is suboptimal.

We document a substantial welfare gain from moving to a private mortgage system, a transition which can be effectuated by raising the cost of the government mortgage guarantees. The private market provides a safer financial sector with fewer mortgage foreclosures and better intermediation between borrowers and savers. It features lower fiscal volatility. While the policy change is a Pareto improvement, it benefits depositors and bankers more and raises wealth inequality. We find that recent policy proposals in which the government only provides catastrophic loss insurance behind private loss-bearing capacity strike a good balance between keeping banks' moral hazard at bay while providing some backstop for the financial system in very bad states of the world.

There are several other promising avenues for further exploration. First, the framework is well suited to study macro-prudential regulation. How good a substitute for GSE reform are tighter bank capital regulations or maximum loan-to-value rules on mortgages? We explore these questions in a companion paper (Elenev, Landvoigt, and Van Nieuwerburgh, 2015).

Second, the model currently abstracts from the choice between owning and renting. Abolishing the mortgage guarantees may well affect the home ownership rate. If house price-to-rent ratios fall in the aftermath of the policy reform, as they do in recent models that study the abolition of mortgage interest rate deductibility, phasing out the GSEs may well boost the home ownership rate.

Third, because a GSE phaseout reduces government debt, it reduces the need for taxation. If taxation were distortionary, it would further amplify the welfare benefits form a GSE phaseout.

Fourth, we abstract from the feedback effect from the mortgage lending complex to the rest of the financial sector. Mortgage guarantees only apply to conforming mortgages. Subsidies in this segment of the market may affect mortgage credit provision, mortgage rates, and risk taking in the non-conforming mortgage market. Introducing two types of mortgages and studying the interaction between the GSEs and the private-label mortgage lenders would be interesting.

Fifth, we abstract from the feedback effect from the mortgage lending complex to the real economy. In a world with subsidized mortgage lending, lending to capital-constrained entrepreneurs may get crowded out. To the extent that entrepreneurs have productive investment opportunities which drive economic growth, a GSE phaseout would have additional welfare benefits we currently do not capture. Elenev, Landvoigt, and Van Nieuwerburgh (2016) take a step in this direction and study a model with production. In addition, there may be interesting real effects on the construction sector.

Sixth, the model is a natural laboratory to study innovation in mortgage contract design. What mortgage contract implements the best allocation of aggregate risk? Are shared appreciation mortgages which make mortgage payments contingent on house prices a good idea from this perspective? Would overall welfare be higher if more housing equity products were available to borrowers?

Finally, our model is also a natural laboratory to explore the effects of government purchases of mortgages. The GSEs were a large buyer of guaranteed and non-guaranteed mortgages, accumulating a combined portfolio of $1.7 trillion dollars by 2007. Since then, they have reduced their holdings by 50%. The Federal Reserve was a large buyer of guaranteed mortgage bonds, accumulating $1.8 trillion as part of its QE1 and QE3 programs during and in the aftermath of the financial crisis. It is merely a matter of time before the Fed will start to shrink the size of these holdings. Thus, over the next several years, the U.S. is likely to see a major change from governmental to private ownership of at least 25% of the secondary mortgage market, one of the largest fixed income markets in the world. A complete understanding of such dramatic shift on the mortgage market, house prices, bond yields, the macro-economy, and the financial sector remains an important challenge for future research.

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Table 1: Calibration

This table reports the parameter values of our model.

Parameter	Description	Value	Target
		Exogenous Shocks	
$\bar{g}$	mean income growth	1.9%	Mean rpc GDP gr 29-13
σ_g	vol. income growth	3.9%	Vol rpc GDP gr 29-13
ρ_g	persistence income growth	0.41	AC(1) rpc GDP gr 29-13
μ_ω	mean idio. depr. shock	2.5%	Housing depreciation Census
σ_ω	vol. idio. depr. shock	{0.10,0.14}	Mortgage default rates (Appendix B.2)
p^ω_{LL}, p^ω_{HH}	transition prob	0.2,0.99	Frequency and duration of mortgage crises
		Population, Income, and Housing Shares	
ℓ^i	pop. shares $i \in \{B, D, R\}$	{47,51,2}%	Population shares SCF 95-13
Y^i	inc. shares $i \in \{B, D, R\}$	{38,52,10}%	Income shares SCF 95-13
K^i	housing shares $i \in \{B, D, R\}$	{39,49,12}%	Housing wealth shares SCF 95-13
		Mortgages	
ζ	DWL of foreclosure	{0.25,0.425}	Mortgage severities (Appendix B.2)
δ	average life mortgage pool	0.95	Duration Fcn. (Appendix B.3)
α	guarantee payout fraction	0.52	Duration Fcn. (Appendix B.3)
ϕ	maximum LTV ratio	0.65	Borrowers' mortg. debt-to-inc. SCF 95-13
ψ	refinancing cost parameter	8	Mean Conditional Prepayment Rate
		Preferences	
σ^B	risk aversion B	8	Vol househ. mortgage debt to GDP 85-14
β^B	time discount factor B	0.88	Mean housing wealth to GDP 85-14
θ^B	housing expenditure share	0.20	Housing expend. share NIPA
σ^D	risk aversion D	20	Vol. risk-free rate 85-14
$\beta^D = \beta^R$	time discount factor D, R	0.98	Mean risk-free rate 85-14
σ^R	risk aversion R	1	Standard Value
ν	intertemp. elasticity of subst.	1	Standard Value
		Government Policy	
τ	income tax rate	19.83%	BEA govt. revenues to trend GDP 46-13
G^o	exogenous govmt spending	15.8%	BEA govt. spending to trend GDP 46-13
G^T	govmt transfers to agents	3.41%	BEA govt. net transfers to trend GDP 46-13
$\varkappa$	margin	98%	Fin. sector leverage Flow of Funds 85-14
ξ_G	margin guaranteed MBS	1.6%	Basel reg. capital charge agency MBS
ξ_P	margin private MBS	8%	Basel reg. capital charge non-agency mortg.

Table 2: Phasing Out the GSEs: Main Results

	20 bp g-fee		**55 bp g-fee**		**100 bp g-fee**		**275 bp g-fee**	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
				Prices				
Risk free rate	1.13%	3.00%	1.20%	2.91%	1.50%	3.00%	1.83%	3.14%
Mortgage rate	3.52%	0.24%	3.59%	0.25%	3.67%	0.26%	3.74%	0.26%
House price	2.240	0.142	2.199	0.131	2.152	0.122	2.100	0.120
Actuarially Fair g-fee	0.77%	0.43%	0.67%	0.40%	0.59%	0.38%	0.54%	0.31%
				Borrower				
Mortgage debt	0.053	0.001	0.052	0.001	0.051	0.001	0.050	0.001
Borrower LTV	63.79%	3.93%	63.80%	3.67%	63.81%	3.46%	63.79%	3.50%
Market value of debt LTV	75.72%	6.47%	75.12%	6.21%	74.49%	6.04%	73.91%	6.02%
Borrower debt to income	1.489	0.040	1.462	0.030	1.432	0.024	1.397	0.025
Debt/income growth	0.04%	2.86%	0.04%	2.73%	0.04%	2.79%	0.04%	2.87%
Mortgage default rate	2.74%	6.20%	2.32%	5.15%	1.97%	4.21%	1.76%	3.92%
Severity rate	30.15%	5.76%	30.10%	5.74%	30.06%	5.73%	30.02%	5.72%
Mortgage loss rate	1.04%	2.75%	0.88%	2.25%	0.75%	1.82%	0.68%	1.72%
Rate-induced prepayment rate	15.83%	4.24%	14.49%	4.27%	13.11%	4.42%	11.92%	4.38%
				Risk-Taker				
Market value of bank assets	0.634	0.018	0.617	0.014	0.599	0.013	0.579	0.013
Fraction guaranteed	99.96%	0.55%	98.41%	5.48%	10.88%	28.77%	0.58%	5.12%
Risk taker leverage	95.59%	0.92%	95.22%	1.21%	88.97%	1.62%	88.27%	1.77%
Risk taker wealth	0.029	0.012	0.031	0.013	0.068	0.016	0.069	0.016
Fraction $\lambda^R > 0$	32.66%	46.90%	35.07%	47.72%	28.73%	45.25%	19.95%	39.97%
MTM Loss Given Prepayment	10.78%	2.59%	9.95%	2.65%	9.08%	2.80%	8.31%	2.81%
Loss rate guaranteed	0.38%	0.88%	0.31%	0.72%	0.25%	0.57%	0.21%	0.50%
Loss rate portfolio	0.38%	0.88%	0.31%	0.73%	0.61%	1.56%	0.67%	1.72%
Bankruptcy frequency	0.27%	5.19%	0.06%	2.45%	0.16%	4.00%	0.22%	4.69%
Return on RT wealth[a]	3.56%	35.74%	3.53%	36.11%	3.87%	17.75%	3.85%	18.07%
				Government				
Government debt / GDP	14.96%	21.81%	−0.57%	9.95%	−4.70%	4.83%	−6.15%	3.26%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The model in the first 2 columns has a mortgage guarantee fee of 20 basis points (20 bp g-fee), the model in columns 3 and 4 has an average g-fee of 55 basis points, the model in columns 5 and 6 has an average g-fee of 100 basis points, and the model in the last two columns has an average g-fee of 275 basis points.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.

Table 3: Phasing Out the GSEs: Welfare and Risk Sharing

	50 bp g-fee		**100 bp g-fee**		**275 bp g-fee**	
	mean	stdev	mean	stdev	mean	stdev
Aggregate Welfare[a]	+0.14%	+0.31%	+0.20%	+0.16%	+0.63%	+0.23%
Value function borrower[a]	+0.05%	−0.24%	+0.04%	−0.95%	+0.04%	−1.37%
Value Function depositor[a]	+0.23%	−0.27%	+0.37%	+0.12%	+1.32%	+0.94%
Value function risk taker[a]	+0.57%	+4.32%	+1.10%	+4.36%	+1.69%	+6.86%
Consumption borrower	−0.20%	−3.88%	−0.47%	−8.06%	−0.56%	−9.28%
Consumption depositor	+0.81%	−4.09%	+1.52%	−13.80%	+2.15%	−20.68%
Consumption risk taker	+0.42%	−11.35%	+1.47%	−16.20%	+1.79%	−8.94%
MU ratio borrower/risk taker[b]	−2.02%	−5.19%	−19.61%	−18.08%	−13.96%	−22.61%
MU ratio risk taker/depositor[b]	+1.16%	−11.11%	+1.22%	−20.90%	+1.21%	−7.69%
DWL from Foreclosure	−16.55%	−17.99%	−30.28%	−34.34%	−39.39%	−39.95%
DWL from Prepayment	−16.47%	−9.18%	−31.19%	−16.71%	−43.36%	−25.74%
Maintenance Costs	−1.83%	−22.53%	−3.92%	−34.98%	−6.23%	−34.00%

The table reports percent changes in unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of three different models relative to the 20 bp g-fee benchmark. The model in the first 2 columns has a mortgage guarantee fee of 55 basis points (50 bp g-fee). The model in columns 3 and 4 has an average g-fee of 100 basis points, and the model in the last two columns has an average g-fee of 275 basis points.

[a]: With unit EIS the value functions are in units of composite consumption $C^{1-\rho}K^{\rho}$. Therefore differences in values have a direct interpretation as consumption-equivalent welfare differences.

[b]: Marginal utility ratios are calculated as the difference of the logarithm of marginal utilities.

Table 4: The Role of Countercyclical Charges and Catastrophic Insurance

	20 bp g-fee		275 bp g-fee		CC g-fees		JC 10%	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
Prices								
Risk free rate	1.13%	3.00%	1.83%	3.14%	1.34%	2.93%	1.89%	3.24%
Mortgage rate	3.52%	0.24%	3.74%	0.26%	3.63%	0.26%	3.75%	0.27%
House price	2.240	0.142	2.100	0.120	2.185	0.123	2.101	0.119
Risk-Taker								
Market value of bank assets	0.634	0.018	0.579	0.013	0.611	0.013	0.579	0.014
Fraction guaranteed	99.96%	0.55%	0.58%	5.12%	11.70%	30.29%	28.81%	30.00%
Risk taker leverage	95.59%	0.92%	88.27%	1.77%	89.13%	1.51%	91.60%	2.40%
Risk taker wealth	0.029	0.012	0.069	0.016	0.068	0.015	0.050	0.019
Fraction $\lambda^R > 0$	32.66%	46.90%	19.95%	39.97%	29.76%	45.72%	76.90%	42.15%
Bankruptcy frequency	0.27%	5.19%	0.22%	4.69%	0.17%	4.12%	0.36%	5.99%
Return on RT wealth[a]	3.56%	35.74%	3.85%	18.07%	3.95%	17.58%	2.88%	26.42%
Borrower								
Mortgage debt	0.053	0.001	0.050	0.001	0.052	0.001	0.050	0.001
Borrower LTV	63.79%	3.93%	63.79%	3.50%	63.80%	3.44%	63.81%	3.46%
Market value of debt LTV	75.72%	6.47%	73.91%	6.02%	74.80%	6.04%	73.91%	6.03%
Borrower debt to income	1.489	0.040	1.397	0.025	1.453	0.024	1.398	0.024
Debt/income growth	0.04%	2.86%	0.04%	2.87%	0.04%	2.79%	0.04%	2.96%
Default rate	2.74%	6.20%	1.76%	3.92%	2.09%	4.38%	1.76%	3.87%
Rate-induced prepayment rate	15.83%	4.24%	11.92%	4.38%	13.79%	4.45%	11.88%	4.48%
Loss Given Default	30.15%	5.76%	30.02%	5.72%	30.08%	5.73%	30.02%	5.72%
MTM Loss Given Prepayment	10.78%	2.59%	8.31%	2.81%	9.50%	2.79%	8.28%	2.88%
Loss rate private	1.04%	2.75%	0.68%	1.72%	0.80%	1.89%	0.68%	1.69%
Loss rate guaranteed	0.38%	0.88%	0.21%	0.50%	0.28%	0.62%	0.21%	0.50%
Loss rate portfolio	0.38%	0.88%	0.67%	1.72%	0.63%	1.56%	0.56%	1.45%
Government								
Government debt / GDP	14.96%	21.81%	−6.15%	3.26%	−3.28%	6.43%	−6.38%	3.27%
Actuarially Fair g-fee	0.77%	0.43%	0.54%	0.31%	0.61%	0.43%	0.02%	0.01%
Welfare								
Aggregate Welfare	0.279	0.008	+0.63%	+0.23%	+0.15%	+0.21%	+0.67%	+0.48%
Value Function borrower	0.319	0.010	+0.04%	−1.37%	+0.05%	−0.76%	+0.05%	−1.17%
Value Function depositor	0.249	0.006	+1.32%	+0.94%	+0.26%	−0.02%	+1.38%	+1.27%
Value function risk taker	0.083	0.000	+1.69%	+6.86%	+1.22%	+2.14%	+1.23%	+42.92%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The first two models (first 4 columns) are the benchmark and high g-fee models from Table 2. The model in columns 6 and 7 two columns has a capital charge for guaranteed bonds set to 8% (same as for private bonds). The last 2 columns report results for an economy where the government guarantees only losses in excess of 10%. Like in the benchmark economy, guarantees in the last two models are both prices at 20 bp.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.

Figure 1: Balance sheets of agents in model economy

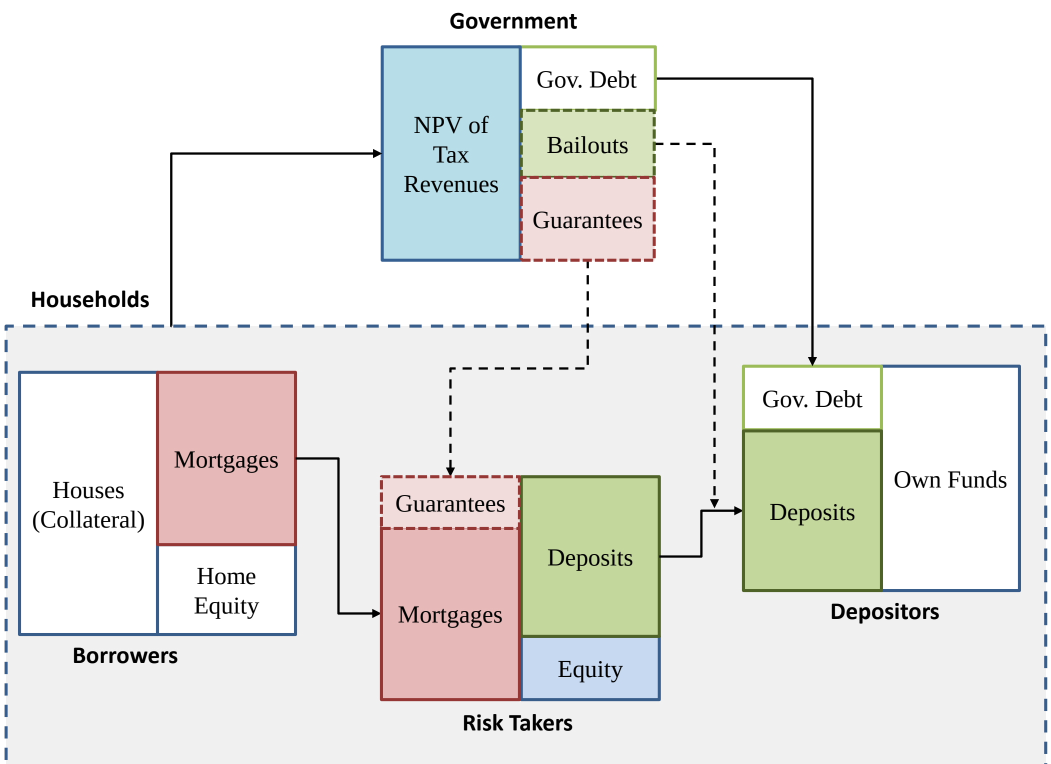
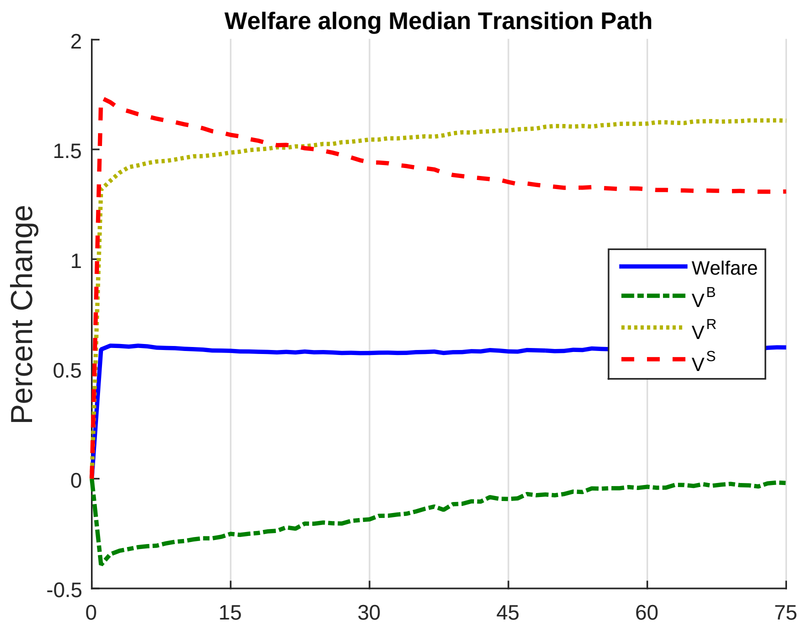
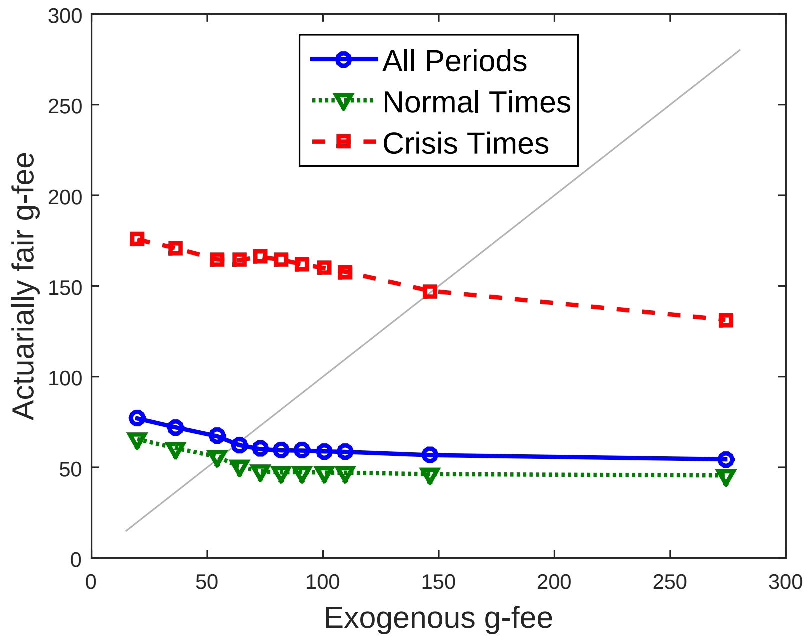


Figure 2: Transition



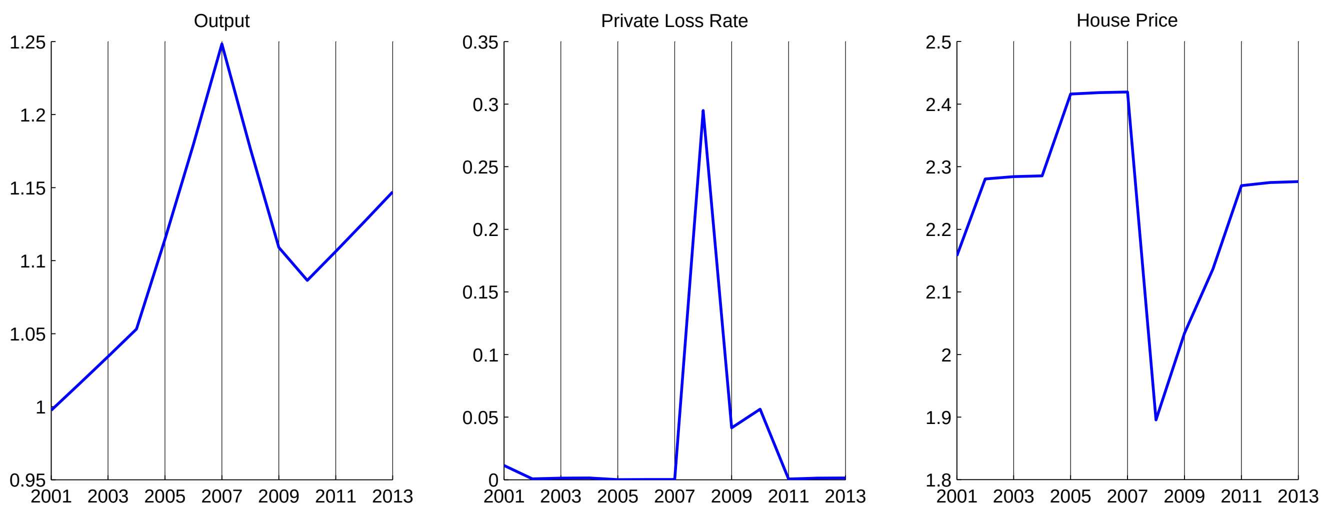
The graph plots aggregate welfare as well as the value functions of the three types of agents along the median transition path from the low g-fee economy to the high g-fee economy. The economy switches from the 20bps to the 275bps g-fee at time 0. The median transition path is computed based on 10,000 simulations.

Figure 3: Actuarially Fair G-Fees



The graphs show the actuarially fair g-fee (y-axis) for seven economies that differ by their exogenously given g-fee (x-axis). The solid line with circles denotes the average g-fee across all periods in a long simulation. The dotted line with triangles denotes the average g-fee during normal times whereas the dotted line with squares denotes the average g-fee during mortgage crises (high σ_ω) times.

Figure 4: Great Recession in the Model



The left panel plots the exogenously assumed path for GDP. We start the low g-fee economy in the year 2000 at typical values for the state variable and in a mild expansion. The exact shock sequence we feed in for the years from 2001 until 2013 is given in the main text. The middle plots the resulting loss rate on mortgages. The right panel shows the house prices.

Online Appendix "Phasing Out the GSEs" Vadim Elenev, Tim Landvoigt, and Stijn Van Nieuwerburgh

A Model Appendix

We reformulate the problem of risk taker, depositor, and borrower to ensure stationarity of the problem. We do so by scaling all variables by permanent income.

A.1 Borrower problem

A.1.1 Preliminaries

We start by defining some preliminaries.

$$\begin{aligned} Z_A(\omega_t^*) &= [1 - F_\omega(\omega_t^*; \chi)] \\ Z_K(\omega_t^*) &= [1 - F_\omega(\omega_t^*; \chi)] \mathrm{E}\left[w_{i,t} \mid \omega_{i,t} \geq \omega_t^*; \chi\right] \end{aligned}$$

and $F_\omega(\cdot; \chi)$ is the CDF of $\omega_{i,t}$ with parameters χ. Assume $\omega_{i,t}$ are drawn from a Gamma distribution with shape and scale parameters $\chi = (\chi_0, \chi_1)$ such that

$$\begin{aligned} \mu_\omega &= \mathrm{E}_i[\omega_{i,t}; \chi_0, \chi_1] = \chi_0 \chi_1 \\ \sigma_{t,\omega}^2 &= \mathrm{Var}_i[\omega_{i,t}; \chi_0, \chi_1] = \chi_0 \chi_1^2 \end{aligned}$$

From Landsman and Valdez (2004, equation 22), we know that

$$\mathrm{E}[\omega | \omega \geq \bar{\omega}] = \mu_\omega \frac{1 - F_\omega(\bar{\omega}; \chi_0 + 1, \chi_1)}{1 - F_\omega(\bar{\omega}; \chi_0, \chi_1)}$$

so the closed form expression for Z_K is

$$Z_K(\omega_t^*) = \mu_\omega \left[1 - F_\omega(\omega_t^*; \chi_0 + 1, \chi_1)\right]$$

It is useful to compute the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$:

$$\begin{aligned} \frac{\partial Z_K(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} \omega f_\omega(\omega) d\omega = -\omega_t^* f_\omega(\omega_t^*), \\ \frac{\partial Z_A(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} f_\omega(\omega) d\omega = -f_\omega(\omega_t^*), \end{aligned}$$

where $f_\omega(\cdot)$ is the p.d.f. of a Gamma distribution with parameters (χ_0, χ_1).

Prepayment Cost Let

$$\Psi(R_t^B, A_t^B) = \frac{\psi}{2} \left(\frac{R_t^B}{A_t^B} \right)^2 A_t^B$$

Then partial derivatives are

$$\Psi_R(R_t^B, A_t^B) = \psi \frac{R_t^B}{A_t^B} \tag{23}$$

$$\Psi_A(R_t^B, A_t^B) = -\frac{\psi}{2}\left(\frac{R_t^B}{A_t^B}\right)^2 \tag{24}$$

A.1.2 Statement of stationary problem

Let $\mathcal{S}_t^B = \left(g_t, \sigma_{\omega,t}, W_t^R, W_t^D, B_{t-1}^G\right)$ represent state variables exogenous to the borrower's decision. We consider the borrower's problem in the current period after income and house depreciation shocks have been realized, after the risk taker has chosen a default policy, and after the risk taker's random utility penalty is realized. Then the borrower's value function, transformed to ensure stationarity, is:

$$\begin{aligned} V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B) = \max_{\{C_t^B, K_t^B, \omega_t^*, R_t^B, B_t^B\}} & \Bigg\{(1-\beta_B)\left[\left(C_t^B\right)^{1-\theta}\left(A_K K_{t-1}^B\right)^{\theta}\right]^{1-1/\nu} + \\ & + \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K_t^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\Bigg\}^{\frac{1}{1-1/\nu}} \end{aligned}$$

subject to

$$C_t^B = (1-\tau_t)Y_t^B + G_t^{T,B} + Z_K(\omega_t^*)p_t K_{t-1}^B + q_t^m B_t^B - (1-\tau_t^m)Z_A(\omega_t^*)A_t^B - p_t K_t^B - F R_t^B - \Psi(R_t^B, A_t^B) \tag{25}$$

$$A_{t+1}^B = e^{-g_{t+1}}\left[\delta Z_A\left(\omega_t^*\right)A_t^B - R_t^B + B_t^B\right] \tag{26}$$

$$\phi p_t K_t^B \geq F\left[\delta Z_A\left(\omega_t^*\right)A_t^B - R_t^B + B_t^B\right] \tag{27}$$

$$0 \leq R_t^B \leq \delta Z_A\left(\omega_t^*\right)A_t^B \tag{28}$$

$$\mathcal{S}_{t+1}^B = h(\mathcal{S}_t^B) \tag{29}$$

where the functions Z_K and Z_A are defined in the preliminaries above.

The continuation value $\tilde{V}^B(\cdot)$ must take into account the default decision of the risk taker at the beginning of next period. We anticipate here and show below that that default decision takes the form of a cutoff rule:

$$\begin{aligned} \tilde{V}^B\left(K_{t-1}^B, A_t^B, \mathcal{S}_t^B\right) = & F_\rho(\rho_t^*)\mathrm{E}_\rho\left[V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)\,|\,\rho < \rho_t^*\right] + (1-F_\rho(\rho_t^*))\,\mathrm{E}_\rho\left[V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)\,|\,\rho > \rho_t^*\right] \\ = & F_\rho(\rho_t^*)V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^S(\rho_t < \rho_t^*)) + (1-F_\rho(\rho_t^*))\,V^B(K_{t-1}^B, A_t^B, \mathcal{S}_t^S(\rho_t > \rho_t^*)), \end{aligned} \tag{30}$$

where (30) obtains because the expectation terms conditional on realizations of ρ_t and ρ_t^* only differ in the values of the aggregate state variables.

Denote the value function and the partial derivatives of the value function as:

$$\begin{aligned} V_t^B &\equiv V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B), \\ V_{A,t}^B &\equiv \frac{\partial V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)}{\partial A_t^B}, \\ V_{K,t}^B &\equiv \frac{\partial V(K_{t-1}^B, A_t^B, \mathcal{S}_t^B)}{\partial K_{t-1}^B}. \end{aligned}$$

Therefore the marginal values of borrowing and of housing of $\tilde{V}^B(\cdot)$ are:

$$\tilde{V}^B_{A,t} = F_\rho(\rho^*_t)\frac{\partial V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t(\rho_t < \rho^*_t))}{\partial A^B_t} + (1 - F_\rho(\rho^*_t))\,\frac{\partial V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t(\rho_t > \rho^*_t)}{\partial A^B_t}$$
$$\tilde{V}^B_{K,t} = F_\rho(\rho^*_t)\frac{\partial V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t(\rho_t < \rho^*_t))}{\partial K^B_{t-1}} + (1 - F_\rho(\rho^*_t))\,\frac{\partial V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t(\rho_t > \rho^*_t)}{\partial K^B_{t-1}}$$

Denote the certainty equivalent of future utility as:

$$CE^B_t = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}$$

Recall that

$$u^B_t = \left(C^B_t\right)^{1-\theta}\left(A_K K^B_{t-1}\right)^\theta$$

A.1.3 First-order conditions

New mortgages The FOC for new mortgage loans B^B_t is:

$$\begin{aligned}
0 = \frac{1}{1-1/\nu}&\left\{(1-\beta_B)\left[\left(C^B_t\right)^{1-\theta}\left(A_K K^B_{t-1}\right)^\theta\right]^{1-1/\nu} + \right.\\
&\left. + \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\right\}^{\frac{1}{1-1/\nu}-1} \times \\
\times &\left\{(1-1/\nu)(1-\beta_B)\left[\left(C^B_t\right)^{1-\theta}\left(A_K K^B_{t-1}\right)^\theta\right]^{-1/\nu}(1-\theta)(A_K K^B_{t-1})^\theta (C^B_t)^{-\theta} q^m_t + \right.\\
&+ \beta_B\frac{1-1/\nu}{1-\sigma_B}\mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}-1}\times\\
&\left.\times\; E_t\left[(1-\sigma_B)\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{-\sigma_B} e^{g_{t+1}}\tilde{V}^B_{A,t+1}e^{-g_{t+1}}\right]\right\} - \lambda^B_t F
\end{aligned}$$

where λ^B_t is the Lagrange multiplier on the borrowing constraint.

Simplifying, we get:

$$\begin{aligned}
&q^m_t \frac{1-\theta}{C^B_t}(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu} = \\
&\lambda^B_t F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}^B_{t+1})^{-\sigma_B}\tilde{V}^B_{A,t+1}](CE^B_t)^{\sigma_B - 1/\nu}(V^B_t)^{1/\nu}
\end{aligned} \tag{31}$$

Observe that we can rewrite equation (31) as:

$$q^m_t = \frac{C^B_t}{(1-\theta)(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}}\left\{\lambda^B_t F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}^B_{t+1})^{-\sigma_B}\tilde{V}^B_{A,t+1}](CE^B_t)^{\sigma_B-1/\nu}(V^B_t)^{1/\nu}\right\}.$$

We define the rescaled Lagrange multiplier of the borrower as the original multiplier divided by marginal utility of current consumption:

$$\tilde{\lambda}^B_t = \lambda^B_t \frac{C^B_t}{(1-\theta)(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}}.$$

Then we can solve for the mortgage price as:

$$q_t^m = \tilde{\lambda}_t^B F - \beta_B \frac{C_t^B \left\{ \mathrm{E}_t[(e^{g_{t+1}} \tilde{V}_{t+1}^B)^{-\sigma_B} \tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu} (V_t^B)^{1/\nu} \right\}}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}. \tag{32}$$

Houses The FOC for new purchases of houses K_t^B is:

$$\begin{aligned} 0 = \frac{1}{1-1/\nu}(V_t^B)^{1/\nu} \times \Big\{ &-(1-1/\nu)(1-\beta_B)(u_t^B)^{-1/\nu}(1-\theta)(A_K K_{t-1}^B)^\theta (C_t^B)^{-\theta} p_t + \\ &+ \frac{1-1/\nu}{1-\sigma_B}\beta_B (CE_t^B)^{\sigma_B - 1/\nu} \mathrm{E}_t[(1-\sigma_B)(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B} e^{g_{t+1}} \tilde{V}_{K,t+1}^B] \Big\} + \lambda_t^B \phi p_t. \end{aligned}$$

Simplifying, we get:

$$\begin{aligned} &p_t \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ &\quad \lambda_t^B \phi p_t + \beta_B \mathrm{E}_t[e^{(1-\sigma_B)g_{t+1}}(\tilde{V}_{t+1}^B)^{-\sigma_B} \tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu} \end{aligned} \tag{33}$$

Default Threshold Taking the first-order condition with respect to $\omega_t^\star$ and using the expressions for the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$ in the preliminaries above yields:

$$\begin{aligned} &f_\omega(\omega_t^*) \left[\omega_t^* p_t K_{t-1}^B - (1-\tau_t^m) A_t^B \right] \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ &\quad \delta A_t^B f_\omega(\omega_t^*) \left\{ \lambda_t^B F - \lambda_t^{RB} - \beta_B \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{V}_{t+1}^B \right)^{-\sigma_B} \tilde{V}_{A,t+1}^B \right] \times (CE_t^B)^{\sigma_B - 1/\nu} (V_t^B)^{1/\nu} \right\}. \end{aligned}$$

This can be simplified by replacing the term in braces on the right-hand side using the FOC for new loans (32) and solving for ω_t^* to give:

$$\omega_t^* = \frac{A_t^B(1-\tau_t^m + \delta q_t^m - \delta \tilde{\lambda}_t^{RB})}{p_t K_{t-1}^B}, \tag{34}$$

where the rescaled Lagrange multiplier on the upper refinancing bound is:

$$\tilde{\lambda}_t^{RB} = \lambda_t^{RB} \frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Prepayment The FOC for repayments R_t^B is:

$$\begin{aligned} &[F + \Psi_R(R_t^B, A_t^B)] \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ &\quad \mu_t^{RB} - \lambda_t^{RB} + \lambda_t^B F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B} \tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}, \end{aligned} \tag{35}$$

where λ_t^{RB} is the Lagrange multiplier on the upper bound on R_t^B and μ_t^{RB} is the Lagrange multiplier on the lower bound. Combining with (32), we obtain:

$$\Psi_R(R_t^B, A_t^B) = q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB},$$

where we defined the lower bound Lagrange multiplier on refinancing as the original multiplier divided by marginal utility of consumption:

$$\tilde{\mu}_t^{RB} = \mu_t^B \frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Recall the definition $Z_t^R = R_t^B/A_t^B$. Using the functional form of Ψ_R from (23), the optimal prepayment fraction is:

$$Z_t^R = \frac{1}{\psi}\left(q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB}\right) \tag{36}$$

A.1.4 Marginal Values of State Variables and SDF

Mortgages Taking the derivative of the value function with respect to A_t^B gives:

$$\begin{aligned}V_{A,t}^B = &-\left(1-\tau_t^m + \frac{\Psi_A(R_t^B, A_t^B)}{Z_A(\omega_t^*)}\right) Z_A(\omega_t^*)\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} \\ &-\delta Z_A(\omega_t^*)\{\lambda_t^B F - \lambda_t^{RB} - \beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B] \times (CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}\}.\end{aligned}$$

Note that we can substitute for the term in braces using equation (31) and for Ψ_A using (24):

$$V_{A,t}^B = -Z_A(\omega_t^*)\left(1-\tau_t^m - \frac{\psi\left(Z_t^R\right)^2}{2Z_A(\omega_t^*)} + \delta q_t^m - \delta\tilde{\lambda}_t^{RB}\right)\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}. \tag{37}$$

Houses Taking the derivative of the value function with respect to K_{t-1}^B gives:

$$V_{K,t}^B = \left[p_t Z_K(\omega_t^*) + \frac{\theta C_t^B}{(1-\theta)K_{t-1}^B}\right]\frac{1-\theta}{C_t^B}(V_t^B)^{1/\nu}(1-\beta_B)(u_t^B)^{1-1/\nu}. \tag{38}$$

SDF Define the borrower's intertemporal marginal rate of substitution between t and $t+1$, conditional on a particular realization of ρ_{t+1} as:

$$\begin{aligned}\mathcal{M}_{t,t+1}^B(\rho_{t+1}) &= \frac{\partial V_t^B/\partial C_{t+1}^B}{\partial V_t^B/\partial C_t^B} = \frac{\partial V_t^B}{\partial V_{t+1}^B}e^{-g_{t+1}}\frac{\partial V_{t+1}^B/\partial C_{t+1}^B}{\partial V_t^B/\partial C_t^B} \\ &= (V_t^B)^{1/\nu}\beta_B(CE_t^B)^{\sigma_B-1/\nu}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\frac{\frac{1-\theta}{C_{t+1}^B}(1-\beta_B)(V_{t+1}^B)^{1/\nu}(u_{t+1}^B)^{1-1/\nu}}{\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}} \\ &= \beta_B e^{-\sigma_B g_{t+1}}\left(\frac{C_{t+1}^B}{C_t^B}\right)^{-1}\left(\frac{u_{t+1}^B}{u_t^B}\right)^{1-1/\nu}\left(\frac{V_{t+1}^B}{CE_t^B}\right)^{-(\sigma_B-1/\nu)}\end{aligned}$$

We can then define the stochastic discount factor (SDF) of borrowers as:

$$\tilde{\mathcal{M}}_{t,t+1}^B = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^B(\rho_{t+1} < \rho_{t+1}^*) + (1-F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^B(\rho_{t+1} > \rho_{t+1}^*),$$

where $\mathcal{M}_{t,t+1}^B(\rho_{t+1} < \rho_{t+1}^*)$ and $\mathcal{M}_{t,t+1}^B(\rho_{t+1} > \rho_{t+1}^*)$ are the IMRSs, conditional on the two possible realizations of state variables.

A.1.5 Euler Equations

Mortgages Recall that $\tilde{V}_{A,t+1}^B$ is a linear combination of $V_{A,t+1}^B$ conditional on ρ_t being below and above the threshold, and with each $V_{A,t+1}^B$ given by equation (37). Substituting in for $\tilde{V}_{A,t+1}^B$ in (32) and using the SDF expression, we get the recursion:

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*)\left(1-\tau^m - \frac{\psi\left(Z_{t+1}^R\right)^2}{2Z_A(\omega_{t+1}^*)} - \delta\tilde{\lambda}_{t+1}^{RB} + \delta q_{t+1}^m\right)\right]. \tag{39}$$

Houses Likewise, observe that we can write (33) as:

$$p_t\left[1-\tilde{\lambda}_t^B\phi\right]=\frac{\beta_B\mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}}{\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}$$

Recall that $\tilde{V}_{K,t+1}^B$ is a linear combination of $V_{K,t+1}^B$ conditional on ρ_t being below and above the threshold, and with each $V_{K,t+1}^B$ given by equation (38). Substituting in for $\tilde{V}_{K,t+1}^B$ and using the SDF expression, we get the recursion:

$$p_t\left[1-\tilde{\lambda}_t^B\phi\right]=\mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^B e^{g_{t+1}}\left\{p_{t+1}Z_K(\omega_{t+1}^*)+\frac{\theta C_{t+1}^B}{(1-\theta)K_t^B}\right\}\right] \tag{40}$$

A.2 Risk Takers

A.2.1 Statement of stationary problem

Denote by W_t^R risk taker wealth at the beginning of the period, before their bankruptcy decision. Then wealth after realization of the penalty ρ_t is:

$$\tilde{W}_t^R=(1-D(\rho_t))W_t^R,$$

and the effective utility penalty is:

$$\tilde{\rho}_t=D(\rho_t)\rho_t.$$

Let $\mathcal{S}_t^R=\left(g_t,\sigma_{\omega,t},W_t^D,A_t^B,B_{t-1}^G\right)$ denote all other aggregate state variables exogenous to risk takers.

After the default decision, risk takers face the following optimization problem over consumption and portfolio composition, formulated to ensure stationarity:

$$\begin{aligned}V^R(\tilde{W}_t^R,\tilde{\rho}_t,\mathcal{S}_t^R)=\max_{C_t^R,A_{t+1,P}^R,A_{t+1,P}^R,B_t^R}&\Bigg\{(1-\beta_R)\left[\frac{(C_t^R)^{1-\theta}(K_{t-1}^R)^\theta}{e^{\tilde{\rho}_t}}\right]^{1-1/\nu}\\&+\beta_R\mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^R\left(W_{t+1}^R,\mathcal{S}_{t+1}^R\right)\right)^{1-\sigma_R}\right]^{\frac{1-1/\nu}{1-\sigma_R}}\Bigg\}^{\frac{1}{1-1/\nu}}\end{aligned} \tag{41}$$

subject to:

$$(1-\tau^S)Y_t^R+\tilde{W}_t^R+G_t^{T,R}=C_t^R+(1-\mu_\omega)p_tK_{t-1}^R+q_t^mA_{t+1,P}^R+(q_t^m+\gamma_t)A_{t+1,G}^R+q_t^fB_t^R, \tag{42}$$

$$\begin{aligned}W_{t+1}^R=e^{-g_{t+1}}&\left[(M_{t+1,P}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m-Z_{t+1}^R[q_{t+1}^m-F])A_{t+1,P}^R\right.\\&\left.+(M_{t+1,G}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m-Z_{t+1}^R[q_{t+1}^m-F])A_{t+1,G}^R+B_t^R\right],\end{aligned} \tag{43}$$

$$q_t^fB_t^R\geq-q_t^m(\xi_PA_{t+1,P}^R+\xi_GA_{t+1,G}^R), \tag{44}$$

$$A_{t+1,G}^R\geq0, \tag{45}$$

$$A_{t+1,P}^R\geq0, \tag{46}$$

$$\mathcal{S}_{t+1}^R=h(\mathcal{S}_t^R). \tag{47}$$

The continuation value $\tilde{V}^R\left(W_{t+1}^R,\mathcal{S}_{t+1}^R\right)$ is the outcome of the optimization problem risk takers face at the beginning of the following period, i.e., before the decision over the optimal bankruptcy rule. This continuation value function is given by:

$$\tilde{V}^R(W_t^R,\mathcal{S}_t^R)=\max_{D(\rho)}\mathrm{E}_\rho\left[D(\rho)V^R(0,\rho,\mathcal{S}_t^R)+(1-D(\rho))V^R(W_t^R,0,\mathcal{S}_t^R)\right] \tag{48}$$

Define the certainty equivalent of future utility as:

$$CE_t^R = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^R\left(W_{t+1}^R, \mathcal{S}_{t+1}^R\right)\right)^{1-\sigma_R}\right]^{\frac{1}{1-\sigma_R}}. \tag{49}$$

and the composite within-period utility (evaluated at $\rho = 0$) as:

$$u_t^R = (C_t^R)^{1-\theta}(A_K K_{t-1}^R)^{\theta}.$$

A.2.2 First-order conditions

Optimal Default Decision The optimization consists of choosing a function $D(\rho) : \mathbb{R} \to \{0, 1\}$ that specifies for each possible realization of the penalty ρ whether or not to default.

Since the value function $V^R(W, \rho, \mathcal{S}_t^R)$ defined in (41) is increasing in wealth W and decreasing in the penalty ρ, there will generally exist an optimal threshold penalty ρ^* such that for a given W_t^R, risk-takers optimally default for all realizations $\rho < \rho^*$. Hence we can equivalently write the optimization problem in (48) as

$$\begin{aligned}\tilde{V}^R(W_t^R, \mathcal{S}_t^R) =& \max_{\rho^*} \mathrm{E}_\rho\left[\mathbb{1}[\rho < \rho^*]\, V^R(0, \rho, \mathcal{S}_t^R) + (1 - \mathbb{1}[\rho < \rho^*])\, V^R(W_t^R, 0, \mathcal{S}_t^R)\right] \\ =& \max_{\rho^*} F_\rho(\rho^*)\, \mathrm{E}_\rho\left[V^R(0, \rho, \mathcal{S}_t^R)\,|\,\rho < \rho^*\right] + (1 - F_\rho(\rho^*))\, V^R(W_t^R, 0, \mathcal{S}_t^R).\end{aligned}$$

The solution ρ_t^* is characterized by the first-order condition:

$$V^R(0, \rho_t^*, \mathcal{S}_t^R) = V^R(W_t^S, 0, \mathcal{S}_t^R).$$

By defining the partial inverse $\mathcal{F} : (0, \infty) \to (-\infty, \infty)$ of $V^S(\cdot)$ in its second argument as

$$\left\{(x, y) : y = \mathcal{F}(x) \Leftrightarrow x = V^R(0, y)\right\},$$

we get that

$$\rho_t^* = \mathcal{F}(V^R(W_t^R, 0, \mathcal{S}_t^R)), \tag{50}$$

and by substituting the solution into (48), we obtain

$$\tilde{V}^R(W_t^R, \mathcal{S}_t^R) = F_\rho(\rho_t^*)\mathrm{E}_\rho\left[V^R(0, \rho, \mathcal{S}_t^R)\,|\,\rho < \rho_t^*\right] + (1 - F_\rho(\rho_t^*))\, V^R(W_t^R, 0, \mathcal{S}_t^R). \tag{51}$$

Equations (41), (50), and (51) completely characterize the optimization problem of risk-takers.

To compute the optimal bankruptcy threshold ρ_t^*, note that the inverse value function defined in equation (50) is given by:

$$\mathcal{F}(x) = \begin{cases}\log((1-\beta_R)u_t^R) - \frac{1}{1-1/\nu}\log\left(x^{1-1/\nu} - \beta_R(CE_t^R)^{1-1/\nu}\right) & \text{for } \nu > 1 \\ (1-\beta_R)\log(u_t^R) + \beta_R\log(CE_t^R) - \log(x) - (1-\beta_R) & \text{if } \nu = 1.\end{cases}$$

Optimal Portfolio Choice The first-order condition for the short-term bond position is:

$$\begin{aligned}&q_t^f \frac{1-\theta}{C_t^R}(1-\beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \\ &\lambda_t^R q_t^f + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}\end{aligned} \tag{52}$$

where λ_t^R is the Lagrange multiplier on the borrowing constraint (44).

The first order condition for the government-guaranteed mortgage bond position is:

$$(q_t^m + \gamma_t)\frac{1-\theta}{C_t^R}(1-\beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \lambda_t^R \xi_G q_t^m + \mu_{G,t}^R$$
$$+ \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R \left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}, \quad (53)$$

where $\mu_{t,G}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (45).

The first order condition for the private mortgage bond position is:

$$q_t^m \frac{1-\theta}{C_t^R}(1-\beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \lambda_t^R \xi_P q_t^m + \mu_{P,t}^R$$
$$+ \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R \left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}, \quad (54)$$

where $\mu_{t,P}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (46).

A.2.3 Marginal value of wealth and SDF

Differentiating (51) gives the marginal value of wealth

$$\tilde{V}_{W,t}^R = (1 - F_\rho(\rho_t^*))\,\frac{\partial V^R(W_t^R, 0, \mathcal{S}_t^R)}{\partial W_t^R},$$

where

$$\frac{\partial V^R(W_t^R, 0, \mathcal{S}_t^R)}{\partial W_t^R} = \frac{1-\theta}{C_t^R}(1-\beta_R)(V^R(W_t^R, 0, \mathcal{S}_t^R))^{1/\nu}(u_t^R)^{1-1/\nu},$$

The stochastic discount factor of risk-takers is therefore

$$\mathcal{M}_{t,t+1}^R = \beta_R e^{-\sigma_R g_{t+1}}\left(\frac{V^R(W_{t+1}^R, 0, \mathcal{S}_{t+1}^R)}{CE_t^R}\right)^{-(\sigma_R - 1/\nu)}\left(\frac{C_{t+1}^R}{C_t^R}\right)^{-1}\left(\frac{u_{t+1}^R}{u_t^R}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}_{t,t+1}^R = (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^R$$

A.2.4 Euler Equations

It is then possible to show that the FOC with respect to B_t^R, $A_{t+1,G}^R$, and $A_{t+1,P}^R$ respectively, are:

$$q_t^f = q_t^f \tilde{\lambda}_t^R + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\right], \quad (55)$$

$$q_t^m + \gamma_t = q_t^m \xi_G \tilde{\lambda}_t^R + \tilde{\mu}_{t,G} + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right], \quad (56)$$

$$q_t^m = q_t^m \xi_P \tilde{\lambda}_t^R + \tilde{\mu}_{t,P} + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right]. \quad (57)$$

A.3 Depositor

We state here a slightly more general problem than in the main text whereby we allow the depositor to also invest in government-guaranteed mortgage bonds in addition to short-term government bonds. The problem in the main text then arises as a special case where we impose the additional constraint that the guaranteed mortgage bond holdings must be non-positive. The Lagrange multiplier on this constraint tells us whether the depositor in the restricted problem would want to hold guaranteed bonds, evaluated at the equilibrium allocation of the restricted model.

A.3.1 Statement of stationary problem

Let $\mathcal{S}_t^D = \left(g_t, \sigma_{\omega,t}, W_t^R, A_t^B, B_{t-1}^G\right)$ be the depositor's state vector capturing all exogenous state variables. Scaling by permanent income, the stationary problem of the depositor -after the risk taker has made default her decision and the utility cost of default is realized- is:

$$V^D(W_t^D, \mathcal{S}_t^D) = \max_{\{C_t^D, B_t^D, A_{t+1,G}^D\}} \Bigg\{(1-\beta_D)\left[\left(C_t^D\right)^{1-\theta}\left(A_K K_{t-1}^D\right)^{\theta}\right]^{1-1/\nu} + \\ + \beta_D \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W_{t+1}^D, \mathcal{S}_{t+1}^D)\right)^{1-\sigma_D}\right]^{\frac{1-1/\nu}{1-\sigma_D}}\Bigg\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C_t^D = (1-\tau_t^S)Y_t^D + G_t^{T,D} + W_t^D - (q_t^m + \gamma_t)A_{t+1,G}^D - q_t^f B_t^D - (1-\mu_{t,\omega})p_t K_{t-1}^D \tag{58}$$
$$W_{t+1}^D = e^{-g_{t+1}}\left[(M_{t+1,G} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F])A_{t+1,G}^D + B_t^D\right] \tag{59}$$
$$B_t^D \geq 0 \tag{60}$$
$$A_{t+1,G}^D \geq 0 \tag{61}$$
$$\mathcal{S}_{t+1}^D = h(\mathcal{S}_t^D) \tag{62}$$

As before, we will drop the arguments of the value function and denote marginal values of wealth and mortgages as:

$$V_t^D \equiv V_t^D(W_t^D, \mathcal{S}_t^D),$$
$$V_{W,t}^D \equiv \frac{\partial V_t^D(W_t^D, \mathcal{S}_t^D)}{\partial W_t^D},$$

Denote the certainty equivalent of future utility as:

$$CE_t^D = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W_t^D, \mathcal{S}_t^D)\right)^{1-\sigma_D}\right],$$

and the composite within-period utility as:

$$u_t^D = (C_t^D)^{1-\theta}(A_K K_{t-1}^D)^{\theta}.$$

Like the borrower, the depositor must take into account the risk-taker's default decisions and the realization of the utility penalty of default. Therefore the marginal value of wealth is:

$$\tilde{V}_{W,t}^D = F_\rho(\rho_t^*)\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t < \rho_t^*))}{\partial W_t^D} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t > \rho_t^*)}{\partial W_t^D}.$$

A.3.2 First-order conditions

The first-order condition for the short-term bond position is:

$$q_t^f \frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu} = \\ \lambda_t^D + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^D)^{-\sigma_D}\tilde{V}_{W,t+1}^D](CE_t^D)^{\sigma_D - 1/\nu}(V_t^D)^{1/\nu} \tag{63}$$

where λ_t^D is the Lagrange multiplier on the no-borrowing constraint (60).

The first order condition for the government-guaranteed mortgage bond position is:

$$(q_t^m + \gamma_t)\frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu} =$$

$$\mu_{G,t}^D + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^D)^{-\sigma_D}\tilde{V}_{W,t+1}^D \left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^D)^{\sigma_D - 1/\nu}(V_t^D)^{1/\nu}, \quad (64)$$

where $\mu_{t,G}^D$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (61).

A.3.3 Marginal Values of State Variables and SDF

Marginal value of wealth is:

$$V_{W,t}^D = \frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu}, \tag{65}$$

and for the continuation value function:

$$\tilde{V}_{W,t}^D = F_\rho(\rho_t^*)\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t < \rho_t^*))}{\partial W_t^D} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t > \rho_t^*)}{\partial W_t^D}.$$

Defining the SDF in the same fashion as we did for the borrower, we get:

$$\mathcal{M}_{t,t+1}^D(\rho_t) = \beta_D e^{-\sigma_D g_{t+1}}\left(\frac{V_{t+1}^D}{CE_t^D}\right)^{-(\sigma_D - 1/\nu)}\left(\frac{C_{t+1}^D}{C_t^D}\right)^{-1}\left(\frac{u_{t+1}^D}{u_t^D}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}_{t,t+1}^D = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^D(\rho_{t+1} < \rho_{t+1}^*) + (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^D(\rho_{t+1} > \rho_{t+1}^*).$$

A.3.4 Euler Equations

Combining the first-order condition for short-term bonds (63) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q_t^f = \tilde{\lambda}_t^D + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^D\right] \tag{66}$$

where $\tilde{\lambda}_t^D$ is the original multiplier λ_t^D divided by the marginal value of wealth.

Similarly, from (64) we get the Euler Equation for guaranteed mortgages:

$$q_t^m + \gamma_t = \tilde{\mu}_{G,t}^D + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^D\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right] \tag{67}$$

A.4 Equilibrium

The optimality conditions describing the problem are (25), (34), (36), (39) and (40) for borrowers, (42), (55), (56), and (57) for risk takers, and (58), (66), and (67) for depositors. We add complementary slackness conditions for the constraints (27) and (28) for borrowers, (44), (45), and (46) for risk-takers, and (60) and (61) for depositors. Together with the market clearing conditions (16), (17), and (18), these equations fully characterize the economy.

B Calibration Appendix

B.1 States and Transition Probabilities

After discretizing the aggregate real per capita income growth process as a Markov chain using the Rouwenhorst method, we obtain the following five states for g:

$$[0.943, 0.980, 1.018, 1.058, 1.101]$$

with 5×5 transition probability matrix:

$$\begin{bmatrix} 0.254 & 0.415 & 0.254 & 0.069 & 0.007 \\ 0.103 & 0.381 & 0.363 & 0.134 & 0.017 \\ 0.042 & 0.242 & 0.430 & 0.242 & 0.042 \\ 0.017 & 0.134 & 0.363 & 0.381 & 0.103 \\ 0.007 & 0.069 & 0.254 & 0.415 & 0.254. \end{bmatrix}$$

We discretize the process for σ_ω^2 into a two-state Markov chain that is correlated with income growth g. The two states are:

$$[.078, .203]$$

The transition probability matrix, conditional on being in one of the bottom two g states is:

$$\begin{bmatrix} 0.80 & 0.20 \\ 0.01 & 0.99 \end{bmatrix}$$

The transition probability matrix, conditional on being in one of the top three g states is:

$$\begin{bmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \end{bmatrix}$$

The stationary distribution for the joint Markov chain of g and σ_ω^2 is

State	1	2	3	4	5	6	7
g	0.943	0.943	0.980	0.980	1.018	1.058	1.101
σ_ω^2	0.078	0.203	0.078	0.203	0.078	0.078	0.078
Prob.	0.039	0.023	0.167	0.081	0.372	0.255	0.063

From a long simulation, we obtain the following mean, standard deviation, and persistence for g: 1.019, .039, and .42, respectively. We obtain the following mean, standard deviation, and persistence for σ_ω^2: .092, .039, and .46, respectively. We obtain a correlation between g and σ_ω of -0.42.

B.2 Evidence on default rates and mortgage severities

Since not all mortgage delinquencies result in foreclosures (loans can cure or get modified), we use the fraction of loans that 90-day or more delinquent or in foreclosure as the real world counterpart to our model's default rate. Some loans that were 90-day delinquent or more received a loan modification, but many of these modifications resulted in a redefault 12 to24 months later. Given that our model abstracts from modifications, using a somewhat broader criterion of delinquency than foreclosures-only seems warranted.

The observed 90-day plus (including foreclosures) default rate rose from 2% at the start of 2007 to just under 10% in 2010.Q1. Since then, the default rate has been gradually falling back, to 4.7% by 2014.Q3 (Mortgage Bankers Association and Urban Institute). The slow decline in foreclosure rates in the data is partly due to legal delays in the foreclosure process, especially in judicial states like New York and Florida where the average foreclosure process takes up to 1000 days. In other part it is due to re-defaults on modified loans. Since, neither is a feature of the model, it seems reasonable to interpret the abnormally high default rates of the post-2013 period as due to such delays, and to reassign them to the 2010-2012 period. If we assume that the foreclosure rate will return to its normal 2% level by the end of 2016, then such reassignment delivers an average foreclosure rate of 8.5% during the 2007-2012 foreclosure crisis. Absent reassignment, the average default rate would be 5.9% over the 2007-2016 period. Jeske et al. (2014) target only a 0.5% foreclosure rate, but their calibration is to the pre-2006 sample. The evidence from the post-2006 period dramatically raises the long-term mean default rate.

Fannie Mae's 10K filings for 2007 to 2013 show that severities, or losses-given-default, on conventional single-family loans were 4% in 2006, 11% in 2007, 26% in 2008, 37% in 2009, 34% in 2010, 35% in 2011, 31% in 2012, and 24% in 2013. Severities on Fannie's non-conforming (mostly Alt-A and subprime) portfolio holdings exceed 60% in all these years. If anything, the severity rate on Fannie's *non-conforming* holdings is lower than that of the overall non-conforming market due to advantageous selection (Adelino et al 2014). Given that the non-conforming market accounted for half of all mortgage originations in 2004-2007, the severities on conventional loans are too low to accurately reflect the market-wide severities. To take account of this composition effect, we target a market-wide severity rate of 40% in the crisis (2007-2012). We target a severity rate of 15% in non-crisis years (pre-2007 and post-2012), based on Fannie's experience in that period and the much smaller size of the non-conforming mortgage market in those years.

Combining a default rate of 2% in normal times with a severity of 15%, we obtain a loss rate of 0.3% in normal times. Combining the default rate of 8.5% during a foreclosure crisis with the severity of 40% in crises, we obtain a 3.4% loss rate.

To obtain mortgage debt to GDP in normal times and in crisis times, we calculate a time series of household mortgage debt (including debt on multi-family real estate owned by the household sector) and divide by GDP. Since mortgage debt-GDP saw a gradual decrease for reasons related to new technology, such as automated underwriting and securitization, we focus attention on the post-1985 period. Mortgage debt-GDP averages to 54% in the 1985-1999 period. We target this for our normal times value. Mortgage debt-GDP averages to 78% in the 2000-2014 period. We target that number for our crisis number.

B.3 Long-term mortgages

Our model's mortgages are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. Real life mortgages have a finite maturity (usually 30 years) and a principal payment. They also have a vintage (year of origination), whereas our mortgages combine all vintages in one variable. This appendix explains how to map the geometric mortgages in our model into real-world mortgages.

Our model's mortgage refers to the entire pool of all outstanding mortgages. In reality, this pool not only consists of newly issued 30-year fixed-rate mortgages (FRMs), but also of newly issued 15-year mortgages, other mortgage types such as hybrid adjustable-rate mortgages (ARMs), as well as all prior vintages of all mortgage types. This includes, for example, 30 year FRMs issued 29 years ago. The Barclays U.S. Mortgage Backed Securities (MBS) Index is the best available measure of the overall pool of outstanding government-guaranteed mortgages. It tracks agency mortgage backed pass-through securities (both fixed-rate and hybrid ARM) guaranteed by Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC). The index is constructed by grouping individual TBA-deliverable MBS pools into aggregates or generics based on program, coupon and vintage. For this MBS index we obtain a time series of monthly price, duration (the sensitivity of prices to interest rates), weighted-average life (WAL), and weighted-average coupon (WAC) for January 1989 until December 2014.

Our calibration strategy is to choose values for δ and F so that the relationship between price and interest rate (duration) is the same for the observed Barclays MBS Index and for the model's geometric bond. We proceed in two steps. In the first step, we construct a simple model to price a pool of MBS bonds and calibrate it to match the observed time series of MBS durations. With this auxiliary model in hand, we then choose the two parameters to match the price-rate curve in the auxiliary model and the geometric mortgage model.

B.3.1 Step 1: A simple MBS pricing model

Changes in duration of the Barclays MBS index are often driven by changes in the index composition. As mortgages are prepaid and new ones are issued with different coupons, both the weighted-average-life and weighted-average-coupon of the Index change significantly. Any model that wants to have a chance at matching the observed durations must account of these compositional changes.

For simplicity, we assume that all mortgages are 30-year fixed-rate mortgages. We construct a portfolio of MBS with *remaining* maturities ranging from 1 to 360 months. Each month, a fraction of each MBS prepays. We assume that the prepayment rate is given by a function $CPR(c-r)$ which depends on the "prepayment incentive" of that particular MBS, defined as the difference between the original coupon rate of that mortgage and the current mortgage rate. We assume that every prepayment is a refinancing: a dollar of mortgage balance prepaid result in a dollar of new mortgage balance originated at the new mortgage rate. In addition, each period an exogenously given amount of new mortgages are originated with a coupon equal to that month's mortgage rate to reflect purchase originations (as opposed to refinancing originations).

In a given month t, each mortgage i has starting balance bal_t^i, pays a monthly mortgage pmt_t^i of which int_t^i is interest and $prin_t^i$ is scheduled principal, where i is the remaining maturity of the mortgage, i.e., the mortgage was originated at time $t-(360-i)-1$. Denote the unscheduled principal payments, or prepayments, by prp_t^i. Let SMM_t^i be the prepayment rate in month t on that mortgage. The evolution equations for actual mortgage cash flows are:

$$\begin{aligned}
int_t^i &= \frac{c_{t-(360-i)-1}}{12} \times bal_t^i \\
prin_t^i &= pmt_t^i - int_t^i \\
prp_t^i &= SMM_t^i(bal_t^i - prin_t^i) \\
bal_{t+1}^{i-1} &= (1-SMM_t^i)(bal_t^i - prin_t^i) \\
pmt_{t+1}^{i-1} &= (1-SMM_t^i)pmt_t^i
\end{aligned}$$

The initial payment is given by the standard annuity formula, normalizing the amount borrowed to 1.

$$\begin{aligned}
pmt_t^{360} &= \frac{\frac{c_{t-1}}{12}}{1-(1+c_{t-1}/12)^{-360}} \\
bal_t^{360} &= 1 + \sum_{i=1}^{360} prp_{t-1}^i
\end{aligned}$$

The last equation says that the initial balance of new 30-year FRMs is comprised on 1 unit of purchase originations, an exogenously given flow of originations each period, plus refinancing originations which equal all prepayments from the previous period.

Furthermore, at every month t we compute *projected* cash flows on each mortgage assuming mortgage rates stay constant from t until maturity i. These projected cash flows follow the same evolution equations as presented above. Denote these projected cash flows with a tilde over the variable.

We can then compute the price P_t, (modified) duration Dur_t, and weighted-average-life WAL_t of the MBS *portfolio* comprised of all vintages:

$$P_t = \sum_{i=1}^{360} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1 + r_t/12)^s}$$

$$Dur_t = \frac{1}{1 + \frac{r_t}{12}} \sum_{i=1}^{360} \frac{1}{P_t^i} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1 + r_t/12)^s} s$$

$$WAL_t^i = \sum_{i=1}^{360} \frac{\sum_{s=0}^{i} (\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}) s}{\sum_{i=1}^{360} \sum_{s=0}^{i} (\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s})}$$

What remains to be specified is our prepayment model delivering the single-month mortality SMM_t^i used above. Following practice, we assume an annual constant prepayment rate (CPR) which is a S-shaped function of the rate incentive: $CPR_t^i = CPR(r_t - c_{t-(360-i)-1})$:

$$CPR(x) = \underline{CPR} + (\overline{CPR} - \underline{CPR}) \left(1 - \frac{\exp(\psi(x - \bar{x})}{1 + \exp(\psi(x - \bar{x}))} \right)$$

The annual CPR implies a monthly SMM $SMM_t^i = factor_i \times (1 - (1 - (CPR_t^i)^{1/12})$. The multiplicative $factor_i$ allows us to deal with slow prepayments early in the life of the mortgage (the "ramp-up" phase) and late in the life of the mortgage (the "burn-out" phase). For simplicity, we make $factor_i$ linearly increasing from 0 in month 1 (when $i = 360$) to 1 in month 30, flat at 1 between month 30 and month 180 and linearly decreasing back to 0 between months 180 and month 360. We choose the CPR curve parameters $\left\{ \underline{CPR}, \overline{CPR}, \psi, \bar{x} \right\}$ to minimize the sum of squared errors between the time series of model-implied duration $\{Dur_t\}$ and observed duration on the Barclays index.

To produce the time-series of model-implied duration $\{Dur_t\}$, we feed in the observed 30-year conventional fixed rate mortgage rate (MORTGAGE30US in FRED), $\{r_t\}$. We initialize the portfolio many years before the start of our time series data to ensure that the model is in steady state by the time our time series for the Barclays index starts. Specifically, we start the computation in April 1903 by issuing 1 MBS. By March 1933, we have a complete portfolio of 360 fixed-rate amortizing mortgages, maturing any month from April 1933 to March 1963.

The left panel of Figure 1 shows the observed time series of duration on the Barclays MBS index plotted against the model-implied duration on the MBS pool. The two time series track each other quite closely despite several strong modeling assumptions. The resulting CPR curve looks close to historical average prepayment behavior on agency MBS, as prepayment data from SIFMA indicate. CPR is slightly above 40% when the rate incentive is 200 basis points or more, about 15% when the rate incentive is zero, and slightly above 5% when the rate incentive is below -200 basis points.

B.3.2 Step 2: Matching MBS pool to perpetual mortgage in our model

With a well-calibrated auxiliary model for a MBS pool, we now proceed to match key features of that auxiliary model's MBS pool to the mortgage in our model, which is a geometrically declining perpetuity.

We start by computing the price $P(r)$ of a fixed-rate MBS with maturity T and coupon c as a function of the current real MBS rate r, using the constant prepayment rate function $\hat{CPR}(r) = CPR(r - c)$ obtained from step 1. For T and c we use the time-series average of the weighted-average maturity and weighted-average real coupon, respectively, from the model-implied MBS pool obtained in step 1.[42]

We can write the steady-state price of a guaranteed geometric mortgage with parameters (δ, F) and a per-

[42] To get real mortgage rates from nominal mortgage rates, we subtract realized inflation over the following year. To get real coupons and MBS rates from real mortgage rates, we subtract 50 bps to account for servicing and guarantee fees.

period fee γ paid for the life of the loan recursively as:

$$Q(r,\gamma)+\gamma = \frac{1}{1+r}\left(1 + C\hat{P}R(r)\delta F + (1 - C\hat{P}R(r))\delta(Q(r,\gamma)+\gamma)\right)$$

Solving for $Q(r,\gamma)$, we get

$$Q(r,\gamma) = \frac{1 + C\hat{P}R(r)\delta F}{1 + r - \delta(1 - C\hat{P}R(r))} - \gamma. \tag{68}$$

Note that the fee γ in equation 68 is quoted in units of the guaranteed bond's price. However, in the data MBS pool we observe a guarantee and servicing fee of approximately 50 bp on average that is charged as a spread on top of a bond's yield. During the calibration, we thus need to use the net-of-fees rate for the MBS pool and the gross-of-fees rate for the geometric bond.

The stage 2 calibration determines how many units X of the geometric mortgage with parameters (δ, F) one needs to sell to hedge one unit of the MBS against parallel shifts in interest rates, across the range of historical mortgage rates:

$$\min_{\delta,F,X,\gamma} \int [P(r) - XQ(r + 0.005, \gamma)]^2 dr,$$

subject to

$$\log\left(\frac{1}{Q} + \delta\right) = \log\left(\frac{1}{Q+\gamma} + \delta\right) + 0.005. \tag{69}$$

The equality constraint 69 determines the price-fee γ that corresponds to the 50 bps rate-fee. The LHS is the gross-of-fees mortgage rate and the RHS is the equivalent net-of-fees mortgage rate plus the 50 bps fee[43]. Generally the equivalent price-fee will depend on the level of the price, which is endogenous to the minimization problem. Thus the constraint determines γ as the equivalent price-fee when the MBS trades at par (with price 1) so that $Q = 1/X$.

We estimate values of $\delta = 0.948$, $F = 9.910$, which implies $\alpha = 0.520$, and $X = 0.1080$. For the model calibration, we only need δ and α. The right panel of Figure 1 shows that the fit is excellent. The average error is only 0.34% of the MBS pool price.

In conclusion, despite its simplicity, the perpetual mortgage in the model captures all important features of real life mortgages (or MBS pools). The relationship between price and interest rate is convex when rates are high and concave ("negative convexity") when rates are low, which is when the prepayment option is in the money. It matches the interest rate risk (duration) of real-life mortgages, for different interest rate scenarios.

B.4 Private Mortgage Insurance

An interesting question is whether there is there scope for welfare-enhancing private mortgage insurance. In the low g-fee equilibrium, private mortgage insurance could not compete with the government's underpriced guarantees. In the high g-fee equilibrium, risk takers would buy a mortgage guarantee in crisis periods if it were available at all times for a state-uncontingent 54bps. This logic is incomplete, however. The presence of the guarantee would change the equilibrium of the economy, reintroducing moral hazard, and leading to larger and riskier intermediary balance sheets. The increase in risk would increase the actuarially fair g-fee. Second, the provider of private mortgage insurance would go bankrupt since she would need to charge the much higher crisis-only guarantee fee of 131 basis points if banks only bought the guarantee in crisis times. We would have to take a stance on the particulars of the private mortgage insurance sector. Should it not have the same preferences as the risk takers (being also part of the leveraged financial sector)? Should it not enjoy the same bailout guarantees as the banks (cfr. AIG)? Clearly a thorough analysis of this question would require adding a fifth balance sheet for the PMI sector. Such an extension adds substantial numerical complexity and is beyond the scope of this paper. Suffice to say that if the PMI sector has the same preferences as the intermediaries and enjoys the same treatment by the government, we can simply think of the current intermediary sector as the consolidated balance sheet of both banks and mortgage insurers. Since the quantity of mortgage insurance

[43] The yield of a geometric bond with price Q and duration parameter δ is $r = \log\left(\frac{1}{Q} + \delta\right)$.

would both be an asset and a liability of the consolidated sector, its equilibrium size would be indeterminate. The equilibrium allocations and prices (including the actuarially fair g-fee) would be the same as the economy we currently compute.

C Computational Solution

The computational solution of the model is implemented using what Judd (1998) calls "time iteration" on the system of equations that characterizes the equilibrium of the economy defined in appendix section A.4. The general solution approach for heterogeneous agent models with incomplete markets and portfolio constraints that we employ in this paper is well described by Kubler and Schmedders (2003).

The procedure consists of the following steps

1. **Define approximating basis for the unknown functions.** The unknown functions of the state variables that need to be computed are the set of endogenous objects specified in the equilibrium definition. These are the prices, agents' choice variables, and the Lagrange multipliers on the portfolio constraints. There is an equal number of unknown functions and nonlinear functional equations. To approximate the unknown functions in the space of the two exogenous state variables $[Y_t, \sigma_{\omega_t}]$ and four endogenous state variables $[A_t^B, W_t^R, W_t^S, G_t]$, we discretize the state space and use multivariate linear interpolation (splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation). One endogenous state variable can be eliminated for computational purposes since its value is implied by the agents' budget constraints, conditional on any three other state variables. As pointed out by several previous studies such as Kubler and Schmedders (2003), portfolio constraints lead to additional computational challenges since portfolio policies may not be smooth functions of state variables due to occasionally binding constraints. Hence we cluster grid points in areas of the state space where constraints transition from slack to binding, and we test the accuracy of the approximation by computing relative Euler equation errors.

2. **Iteratively solve for the unknown functions.** Given an initial guess $\mathcal{C}^0(\mathcal{S})$ to compute tomorrow's optimal policies as functions of tomorrow's states, solve the system of nonlinear equations for the current optimal policies at each point in the discretized state space. Expectations are computed using quadrature methods. Using the solution vector for current policies, compute the next iterate of the approximation $\mathcal{C}^1(\mathcal{S})$ and repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Judd, Kubler, and Schmedders (2002) show how Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose.

3. **Simulate the model for many periods using approximated policy functions.** To obtain the quantitative results, we simulate the model for 10,000 periods after a "burn-in" phase of 500 periods. We verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed.

In a long simulation, errors in the nonlinear equations are low. Table 1 reports the median error, the 95^{th} percentile of the error distribution, the 99^{th}, and 99.5^{th} percentiles.

D Additional Experiments

Our main policy experiment consisted of raising the g-fees. In the main body of the paper, we also reviewed two alternative experiments. The first changed capital requirements on guaranteed mortgages. The second considered a legislative proposal to to "put private capital in front of a government guarantee" by limiting guarantees to losses in excess of 10%. We now study three more policy experiments and compare their welfare consequences to those in the main policy experiment. The first experiment explores the effect of limited liability. The next two experiments study alternate ways to make guarantees operative only when losses are catastrophic. These exercises help to further illuminate the interaction of government guarantees, deposit insurance, and risk taker leverage.

D.1 Limited Liability

The second alternative policy we consider is one that weakens deposit insurance. The knowledge that they (and their depositors) will be bailed out by the government if their net worth turns negative leads banks to take on more risk. We weaken deposit insurance, or equivalently weaken limited liability for banks, by increasing the mean (μ_ρ) of the utility penalty that banks incur for insolvency. The third and fourth columns of Table 2 labeled "high μ_ρ" report the results.

Many of the effects are similar as for the tighter leverage constraint. Guarantees remain very valuable and dominate the portfolio, even more so than in the previous experiment. Portfolio delinquency and loss rates are close to the benchmark low g-fee economy. One big difference to the main experiment is that weaker limited liability does not lead banks to reduce leverage. Risk taker net worth only increases marginally. Still, this small increase in net worth, combined with the higher utility cost of bankruptcy is enough to eliminate all bankruptcies. As a result of the high leverage and government debt, depositors must hold substantial amounts of safe assets and interest rates are a bit higher than the benchmark model as a result. The real short rate increases by 2 basis points to 1.15%.

We find no significant effect on aggregate welfare from this policy. It has an aggregate welfare loss of 0.02%. Borrowers' welfare is unaffected and both risk takers and depositors lose slightly. In sum, while increasing the costs of bank bankruptcy is successful at eliminating bank bankruptcies, it has a small negative aggregate welfare effect. This demonstrates that intermediary bankruptcies are not the driving force behind our welfare results. The key issue rather is the underpricing of the guarantee, which is as paramount in this economy as in the low g-fee benchmark.

D.2 Catastrophic Insurance

Columns 6 and 7 of Table 2 report results for a catastrophic insurance policy which "kicks in" at losses of 5%. Welfare increases are smaller than when the private sector loss is capped at 10% (+0.56% vs +0.67%). This increase in welfare is smaller than that from a complete phase-out. Thus the largest welfare gains are obtained when the private sector bears enough losses to reduce its risk taking, but not so much as to debilitate its intermediation function which is important to achieve the best distribution of aggregate risk in the economy.

The last two columns of Table 2 report results for a catastrophic insurance experiment in which losses are capped at 10% but where the insurance is offered at a much lower price of 5 bp instead of the 20bp discussed in the main text. This policy has higher welfare gains of +0.69%, compared to +0.67% for the 20bp catastrophic guarantee and +0.63% for the full phase-out. The main difference with the more expensive catastrophic guarantee is that because the guarantee is cheaper (and closer to the actuarially fair cost of 2bp), risk takers are much more likely to purchase it. This protects them better to unexpected catastrophic shocks than in the 20bp JC economy. It further improves risk sharing and raises interest rates.

Table 1: Computational Errors

	Percentile				
	50th	75th	95th	99th	Max
(39)	0.0010	0.0020	0.0085	0.0130	0.0239
(40)	0.0007	0.0014	0.0058	0.0088	0.0164
(57)	0.0008	0.0050	0.0157	0.0266	0.0691
(55)	0.0008	0.0055	0.0167	0.0294	0.0746
(66)	0.0002	0.0004	0.0009	0.0016	0.0083
(27)	0.0002	0.0007	0.0033	0.0058	0.0313
(44)	0.0003	0.0006	0.0025	0.0034	0.0086
(60)	0.0030	0.0042	0.0084	0.0092	0.0230
(16)	0.0002	0.0003	0.0010	0.0022	0.0088
(46)	0.0000	0.0000	0.0000	0.0008	0.0108
(28)	0.0000	0.0001	0.0003	0.0014	0.0103
(56)	0.0008	0.0051	0.0160	0.0280	0.0736
(45)	0.0000	0.0000	0.0001	0.0003	0.0029
(43)	0.0001	0.0001	0.0003	0.0013	0.0279
(15)	0.0003	0.0005	0.0020	0.0023	0.0487

The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 10,000 period simulation of the 20 bps g-fee model. The first 13 equations define policy functions. They are a subset of the 22 equations that define the equilibrium. The last two equations define evolutions of risk-taker wealth and government debt, respectively.

Table 2: The Role of Limited Liability and Catastrophic Insurance

	20 bp g-fee		**High μ_ρ**		**JC 5%**		**JC 10%, 5 bp**	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
				Prices				
Risk free rate	1.13%	3.00%	1.15%	3.07%	1.82%	3.22%	1.92%	3.21%
Mortgage rate	3.52%	0.24%	3.52%	0.24%	3.73%	0.26%	3.74%	0.25%
House price	2.240	0.142	2.240	0.142	2.113	0.121	2.106	0.123
				Risk Taker				
Market value of bank assets	0.634	0.018	0.634	0.018	0.584	0.013	0.581	0.012
Fraction guaranteed	99.96%	0.55%	99.92%	0.89%	54.53%	34.10%	75.37%	24.19%
Risk taker leverage	95.59%	0.92%	95.28%	1.05%	93.31%	2.33%	94.90%	1.56%
Risk taker wealth	0.029	0.012	0.031	0.013	0.040	0.018	0.030	0.014
Fraction $\lambda^R > 0$	32.66%	46.90%	27.17%	44.49%	85.20%	35.51%	98.82%	10.80%
Bankruptcy frequency	0.27%	5.19%	0.00%	0.00%	0.02%	1.41%	0.44%	6.62%
Return on RT wealth[a]	3.56%	35.74%	2.84%	34.69%	2.71%	30.95%	3.50%	37.00%
				Borrower				
Mortgage debt	0.053	0.001	0.053	0.001	0.050	0.001	0.050	0.001
Borrower LTV	63.79%	3.93%	63.79%	3.90%	63.81%	3.48%	63.78%	3.50%
Market value of debt LTV	75.72%	6.47%	75.72%	6.47%	74.07%	6.04%	73.96%	6.00%
Borrower debt to income	1.489	0.040	1.488	0.040	1.406	0.025	1.400	0.024
Debt/income growth	0.04%	2.86%	0.04%	2.90%	0.04%	2.89%	0.04%	2.75%
Default rate	2.74%	6.20%	2.75%	6.18%	1.82%	4.01%	1.76%	3.79%
Rate-induced prepayment rate	15.83%	4.24%	15.81%	4.30%	12.22%	4.43%	12.07%	4.28%
Loss Given Default	30.15%	5.76%	30.15%	5.76%	30.03%	5.72%	30.03%	5.72%
MTM Loss Given Prepayment	10.78%	2.59%	10.77%	2.62%	8.51%	2.83%	8.42%	2.74%
Loss rate private	1.04%	2.75%	1.05%	2.73%	0.70%	1.75%	0.67%	1.64%
Loss rate guaranteed	0.38%	0.88%	0.38%	0.88%	0.22%	0.52%	0.21%	0.49%
Loss rate portfolio	0.38%	0.88%	0.38%	0.88%	0.42%	1.12%	0.34%	0.83%
				Government				
Government debt / GDP	14.96%	21.81%	15.37%	22.05%	−6.03%	3.76%	−6.06%	3.95%
Actuarially Fair g-fee	0.77%	0.43%	0.77%	0.43%	0.11%	0.08%	0.02%	0.01%
				Welfare				
Aggregate Welfare	0.279	0.008	−0.02%	+0.05%	+0.56%	+0.49%	+0.69%	+0.41%
Value Function borrower	0.319	0.010	0.00%	+0.02%	+0.05%	−1.02%	+0.04%	−1.14%
Value Function depositor	0.249	0.006	−0.04%	+0.12%	+1.16%	+1.07%	+1.46%	+1.06%
Value function risk taker	0.083	0.000	−0.24%	+6.44%	+0.96%	+40.15%	+1.02%	+8.94%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The first two columns are the benchmark model from Table 2. The next two columns have a higher mean utility cost of default μ_ρ. The model in columns 6 and 7 report results for an economy where the government offers catastrophic insurance i.e. guarantees only losses in excess of 5%. The last 2 columns report results for a catastrophic insurance economy where the attachment point is 10%, like in Table 4, but at a lower price of 5 bp.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.

Figure 1: Matching Mortgages in Model to Data

The left panel plots the observed time series of duration on the Barclays MBS index (solid line) plotted against the duration on the model-implied MBS pool (dashed-line). The right panel plots the mortgage price-interest rate relationship for the model-implied MBS pool (solid line) and the model-implied geometrically declining perpetual mortgage (dashed line). Prices on a $100 face value mortgage are on the vertical axis, while interest rates are on the horizontal axis. The Barclays MBS index data are from Bloomberg for the period 1989 until 2014 (daily frequency). The calculations also use the 30-year fixed-rate mortgage rate from FRED.