# Self-Fulfilling Debt Crises: Can Monetary Policy Really Help?[1]

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**Abstract**

This paper examines the potential for monetary policy to avoid self-fulfilling sovereign debt crises. We combine a version of the slow-moving debt crisis model proposed by Lorenzoni and Werning (2014) with a standard New Keynesian model. Monetary policy could preclude a debt crisis through raising inflation and output and lowering the real interest rate. These reduce the real value of outstanding debt and the cost of new borrowing, and increase tax revenues and seigniorage. However, the policies needed to avert a crisis generally require excessive inflation for a sustained period of time so that the central bank cannot credibly avoid a self-fulfilling debt crisis.

# 1 Introduction

A popular explanation for sovereign debt crises is self-fulfilling sentiments. If market participants believe that sovereign default of a country is more likely, they demand higher spreads, which raises the debt burden and therefore indeed makes eventual default more likely.[1] This view of self-fulfilling beliefs is consistent with the evidence that the surge in sovereign bond spreads in Europe during 2010-2011 was disconnected from debt ratios and other macroeconomic fundamentals (e.g., de Grauwe and Ji, 2013).[2] It has also been suggested as an explanation for the Argentine crisis of 1998-2002 (Ayres et al., 2015). Recently a debate has developed about what role the central bank may play in avoiding such self-fulfilling debt crises. The central bank has additional tools to support the fiscal authority, either in the form of standard inflation policy or by providing liquidity. Some have argued that the US, Japan, UK and others have avoided such crises altogether because they have their own currency and monetary policy.[3]

The question that we address in this paper is whether central banks can credibly avert self-fulfilling debt crises. We pose this as a general question, without focusing on a specific country or historical episode. To be credible, the costs of central bank policy, particularly inflation, should not outweigh the benefits of avoiding default. For a realistic analysis, we allow for long-term debt, nominal rigidities and dynamics leading to slow-moving debt crises. We do so by combining a standard New Keynesian (NK) model with the "slow moving" debt crisis framework proposed by Lorenzoni and Werning (2014, henceforth LW). The LW model is in the spirit of

[1]For discussions of self-fulfilling crises in sovereign debt models see for example Aguiar et al. (2013), Calvo (1988), Camous and Cooper (2014), Cohen and Villemot (2015), Conesa and Kehoe (2015), Corsetti and Dedola (2014), de Grauwe (2011), de Grauwe and Ji (2013), Gros (2012), Jeanne (2012), Jeanne and Wang (2013), Krugman (2013), Lorenzoni and Werning (2014), and Miller and Zhang (2012). Ayres et al. (2015) show that multiple equilibria arise in sovereign debt models when the government chooses current debt, as opposed to debt at maturity. Even if the government chooses debt at maturity, they show that there are still multiple equilibria when lenders move first (choose an interest rate at which they are willing to lend).

[2]This view was held by the ECB President Draghi himself: "... the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a "bad equilibrium", namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." (press conference, September 6, 2012).

[3]See for example de Grauwe (2011), de Grauwe and Ji (2013), Jeanne (2012) and Krugman (2013).

Calvo (1988), but the latter is a two period model with one-period bonds, while the LW model is dynamic and has long-term bonds. The NK block of the model allows us to consider realistic monetary policy. The central bank can reduce the real value of original debt and of distributed coupons through inflation. It can also reduce the real cost of borrowing by reducing the (riskfree) real interest rate. Finally, expansionary monetary policy delivers seigniorage revenue and can raise the primary surplus by raising output. This all helps to slow down the accumulation of debt.

As is standard in sovereign debt models with self-fulfilling equilibria, there is a region of debt in which there are multiple equilibria. This makes the country exposed to self-fulfilling beliefs that lead to default premia, debt accumulation and possible default. We refer to this debt region as the "multiplicity region". It corresponds to the situation where the central bank remains passive. The objective of the central bank is to conduct monetary policy to move out of this region and therefore guarantee a single no-default equilibrium. While the central bank can in principle do so, there are costs associated with such a policy, particularly inflation.

Our main conclusion is that the central bank policy required to eliminate default equilibria is generally not credible as it implies very high inflation levels. For example, under our benchmark parameterization the price level ultimately increases by a factor 5 and inflation is higher than 20% for 4 years and over 10% for 8 years. This is the case when the debt ratio is in the middle of the multiplicity region, which goes from 80 to 150 percent of GDP. Inflation under optimal policy will of course be modest if debt happens to be near the very bottom of the multiplicity range. Similarly, not much inflation is needed when the multiplicity range itself is very narrow. Neither of these cases are very interesting though as they essentially make the entire problem of self-fulfilling crises irrelevant. Throughout the paper we therefore assume that there is a substantial range of debt in which the country is subject to self-fulfilling debt crises and that debt is generally not at the very bottom of this multiplicity region.

The high level of inflation needed to avoid self-fulfilling crises has an intuitive explanation. Consider the example where the multiplicity region is a debt ratio from 80 to 150 percent of GDP. Assume that current debt is in the middle of that interval, i.e., 115 percent of GDP. If the central bank could raise the price level right away, without delay, the price level would need to rise by 44 percent to reduce debt from 115 to 80 percent of GDP. Assuming quarterly data, this is an annual percentage inflation rate of 176 percent. In practice though, inflation is more gradual, both because of price stickiness and because it is optimal from a welfare point of view to have more gradual inflation. However, this will lead ultimately to a much larger increase in the price level. The reason is that inflation becomes less effective over time because the interest rate for newly issued debt incorporates inflation expectations. For a realistic maturity of government debt, inflation gradually loses power. It is also important to notice, in line with previous literature (e.g., Reis, 2013), that the direct purchase of government debt by the central bank in principle does not affect the main conclusion. The only exception would be the case of a persistent liquidity trap.

We devote significant space to the question of how robust this result is. We consider changes to all the parameters of both the LW and NK components of the model and find that the result is quite robust. We also consider results based exclusively on the LW part of the model. This monetary version of the LW model leads to a condition on inflation and real interest rates over time that needs to be satisfied in order to avoid the multiplicity region. It holds independent of how monetary policy affects inflation and real interest rates and is therefore independent of the NK portion of the model. We do not find a plausible path for real interest rates and inflation that satisfies this default avoidance condition, suggesting that the conclusion does not depend on the specifics of the NK model. Regarding the LW part of the model, the only key parameter is the maturity of the debt. All other parameters of the LW model matter mainly to the extent that they affect the multiplicity region. Given a substantial multiplicity region, the precise value of each parameter does not matter much.

If anything we may be understating how difficult it is for the central bank to avoid self-fulfilling debt crises. We consider a case where in normal times the central bank is able to commit to zero inflation. If initial inflation is large, the additional surprise inflation needed to avoid a self-fulfilling crises is even more costly given the convex cost of inflation. We also abstract from liquidity or rollover crises, such as Cole and Kehoe (2000). As Bocola and Dovis (2015) point out, we often see a substantial shortening of the maturity structure under steep inflation, which leads to additional problems in terms of exposure to rollover crises that we abstract from.

This is not the first paper to analyze the impact of monetary policy in a self-fulfilling debt crisis environment. The main difference is that previous work focuses on more qualitative questions using more stylized frameworks. It typically does not consider standard interest rate policies and considers mainly two-period models with one period bonds, flexible prices, constant real interest rates and a constant output gap. The role of monetary policy was first analyzed by Calvo (1988), who examined the trade-off between outright default and debt deflation. Corsetti and Dedola (2014) extend the Calvo model to allow for both fundamental and self-fulfilling default. They show that with optimal monetary policy debt crises can still happen, but for larger levels of debt. They also show that a crisis can be avoided if government debt can be replaced by risk-free central bank debt that is convertible into cash. Reis (2013) and Jeanne (2012) also develop stylized two-period models with multiple equilibria to illustrate ways in which the central bank can act to avoid the bad equilibrium.

Some papers consider more dynamic models, but still assume flexible prices and one-period bonds.[4] Camous and Cooper (2014) use a dynamic overlapping-generation model with strategic default. They show that the central bank can avoid self-fulfilling default if they commit to a policy where inflation depends on the state (productivity, interest rate, sunspot). Aguiar et al. (2013) consider a dynamic model to analyze the vulnerability to self-fulfilling rollover crises, depending on the aversion of the central bank to inflation. Although a rollover crisis occurs suddenly, it is assumed that there is a grace period to repay the debt, allowing the central bank time to reduce the real value of the debt through inflation. They find that only for intermediate levels of the cost of inflation do debt crises occur under a narrower range of debt values.

The rest of the paper is organized as follows. Section 2 presents the slow-moving debt crisis model based on LW. It starts with a real version of the model and then presents its extension to a monetary environment. Subsequently, it analyzes the various channels of monetary policy in this framework. Section 3 describes the New Keynesian part of the model, discusses results under optimal policy and

[4] Nuño and Thomas (2015) analyze the role of monetary policy with long-term debt in the context of a dynamic Eaton and Gersovitz (1981) model (with a unique equilibrium) and find that debt deflation is not optimal. There are also recent models that examine the impact of monetary policy in the presence of long-term government bonds, but they do not allow for the possibility of sovereign default. For example, Leeper and Zhou (2013) analyze optimal monetary (and fiscal) policy with flexible prices, while Bhattarai et al. (2013) consider a New Keynesian environment at ZLB. Sheedy (2014) and Gomes et al. (2014) examine monetary policy with long-term private sector bonds.

considers sensitivity analysis and extensions. Section 4 provides results that do not rely on the New Keynesian part of the model. Section 5 considers alternative policies. Section 6 concludes. Some of the technical details are left to the Appendix, while additional algebraic details and results can be found in a separate Technical Appendix.

# 2 A Model of Slow-Moving Self-Fulfilling Debt Crisis

In this section we present a dynamic sovereign debt crisis model based on LW. We first describe the basic structure of the model in a real environment. We then extend the model to a monetary environment and discuss the impact of monetary policy on the existence of self-fulfilling debt crises. We focus on the dynamics of asset prices and debt for given interest rates and goods prices. The latter will be determined in a New Keynesian model that we describe in Section 3.

## 2.1 A Real Model

We consider a simplified version of the LW model. As in the applications considered by LW, there is a key date $T$ at which uncertainty about future primary surpluses is resolved and the government makes a decision to default or not.[5] Default occurs at time $T$ if the present value of future primary surpluses is insufficient to repay the debt. We assume that default does not happen prior to date $T$ as there is always a possibility of large primarily surpluses from $T$ onward. In one version of their model LW assume that $T$ is known to all agents, while in another they assume that it is unknown and arrives each period with a certain probability. We mostly adopt the former assumption. In the Technical Appendix we analyze an extension where $T$ is uncertain, which is briefly discussed in Section 3.5.3. While this significantly complicates the analysis, it does not alter the key findings.

The only simplification we adopt relative to LW concerns the process of the primary surplus. For now we assume that the primary surplus $s_t$ is constant at $\overline{s}$

[5] One can for example think of countries that have been hit by a shock that adversely affected their primary surpluses, which is followed by a period of uncertainty about whether and how much the government is able to restore primary surpluses through higher taxation or reduced spending.

between periods 0 and $T-1$. LW assume a fiscal rule whereby the surplus is a function of debt. Not surprisingly, they find that the range of debt where a country is vulnerable to self-fulfilling crises narrows if the fiscal surplus is more responsive to debt. Very responsive fiscal policy could in principle eliminate the concern about self-fulfilling debt crises. In this paper, however, we take vulnerability to self-fulfilling debt crises as given in the absence of monetary policy action. We therefore abstract from such strong stabilizing fiscal policy. However, we will consider an extension where the primary surplus depends on output and is procyclical as this provides an additional avenue through which monetary policy can be effective.

A second assumption concerns the primary surplus value starting at date $T$. Let $\tilde{s}$ denote the maximum potential primary surplus that the government is able to achieve, which becomes known at time $T$ and is constant from thereon. LW assume that it is drawn from a log normal distribution. Instead we assume that it is drawn from a binary distribution, which simplifies the algebra and the presentation. It can take on only two values: $s_{low}$ with probability $\psi$ and $s_{high}$ with probability $1-\psi$. When the present discounted value of $\tilde{s}$ is at least as large as what the government owes on debt, there is no default at time $T$ and the actual surplus is just sufficient to satisfy the budget constraint (generally below $\tilde{s}$). We assume that $s_{high}$ is big enough such that this is always the case when $\tilde{s} = s_{high}$.[6] When $\tilde{s} = s_{low}$ and its present value is insufficient to repay the debt, the government defaults.

A key feature of the model is the presence of long-term debt. As usual in the literature, assume that bonds pay coupons (measured in goods) that depreciate at a rate of $1-\delta$ over time: $\kappa$, $(1-\delta)\kappa$, $(1-\delta)^2\kappa$, and so on.[7] A smaller $\delta$ therefore implies a longer maturity of debt. This facilitates aggregation as a bond issued at $t-s$ corresponds to $(1-\delta)^s$ bonds issued at time $t$. We can then define all outstanding bonds in terms of the equivalent of newly issued bonds. We define $b_t$ as debt measured in terms of the equivalent of newly issued bonds at $t-1$ on which the first coupon is due at time $t$. As in LW, we take $\delta$ as given. It is associated with tradeoffs that are not explicitly modeled, and we do not allow the government to change the maturity to avoid default.

Let $Q_t$ be the price of a government bond. At time $t$ the value of government debt is $Q_t b_{t+1}$. In the absence of default the return on the government bond from

[6] See Technical Appendix for details.

[7] See for example Hatchondo and Martinez (2009).

$t$ to $t+1$ is

$$R_t^g = \frac{(1-\delta)Q_{t+1} + \kappa}{Q_t} \tag{1}$$

If there is default at time $T$, bond holders are able to recover a proportion $\zeta < 1$ of the present discounted value $s^{pdv}$ of the primary surpluses $s_{low}$.[8] In that case the return on the government bond is

$$R_{T-1}^g = \frac{\zeta s^{pdv}}{Q_{T-1} b_T} \tag{2}$$

Government debt evolves according to

$$Q_t b_{t+1} = R_{t-1}^g Q_{t-1} b_t - s_t \tag{3}$$

In the absence of default this may also be written as $Q_t b_{t+1} = ((1-\delta)Q_t + \kappa)b_t - s_t$. The initial stock of debt $b_0$ is given.

We assume that investors also have access to a short-term bond with a gross real interest rate $R_t$. The only shocks in the model occur at time 0 (self-fulfilling shock to expectations) and time $T$ (value of $\tilde{s}$). In other periods the following risk-free arbitrage condition holds (for $t \geq 0$ and $t \neq T-1$):

$$R_t = \frac{(1-\delta)Q_{t+1} + \kappa}{Q_t} \tag{4}$$

For now we assume, as in LW, a constant interest rate, $R_t = R$. In that case $s^{pdv} = Rs_{low}/(R-1)$ is the present discounted value of $s_{low}$. There is no default at time $T$ if $s^{pdv}$ covers current and future debt service at $T$, which is $((1-\delta)Q_T + \kappa)b_T$. Since there is no default after time $T$, $Q_T$ is the risk-free price, equal to the present discounted value of future coupons. For convenience it is assumed that $\kappa = R - 1 + \delta$, so that (4) implies that $Q_T = 1$. This means that there is no default as long as $s^{pdv} \geq Rb_T$, or if

$$b_T \leq \frac{1}{R-1} s_{low} \equiv \tilde{b} \tag{5}$$

When $b_T > \tilde{b}$, the government partially defaults on debt, with investors seizing a fraction $\zeta$ of the present value $s^{pdv}$.

[8] One can think of $\zeta$ as the outcome of a bargaining process between the government (representing taxpayers) and bondholders. Since governments rarely default on all their debt, we assume $\zeta > 0$.

This framework may lead to multiple equilibria and to a slow-moving debt crisis, as described in LW. The existence of multiple equilibria can be seen graphically from the intersection of two schedules, as illustrated in Figure 1. The first schedule, labeled "pricing schedule", is a consistency relationship between price and outstanding debt at $T-1$, in view of the default decision that may be taken at $T$. This is given by:

$$Q_{T-1} = 1 \qquad \text{if } b_T \leq \tilde{b} \tag{6}$$
$$= \psi \frac{\zeta s^{pdv}}{R b_T} + (1-\psi) \qquad \text{if } b_T > \tilde{b} \tag{7}$$

When $b_T \leq \tilde{b}$, the arbitrage condition (4) also applies to $t = T-1$, implying $Q_{T-1} = 1$. When $b_T$ is just above $\tilde{b}$, there is a discrete drop of the price because only a fraction $\zeta$ of primary surpluses can be recovered by bond holders in case of default. For larger values of debt, $Q_{T-1}$ will be even lower as the primary surpluses have to be shared among more bonds.

The second schedule is the "debt accumulation schedule," which expresses the amount of debt that accumulates through time $T-1$ as a function of prices between 0 and $T-1$. Every price $Q_t$ between 0 and $T-1$ can be expressed as a function of $Q_{T-1}$ by integrating (4) backwards from $T-1$ to 0:

$$Q_t - 1 = \left(\frac{1-\delta}{R}\right)^{T-1-t} (Q_{T-1} - 1) \tag{8}$$

Substituting in (3) and integrating the government budget constraint forward from 0 to $T-1$, we get (see Appendix):

$$b_T = (1-\delta)^T b_0 + \frac{\chi^\kappa \kappa b_0 - \chi^s \overline{s}}{Q_{T-1}} \tag{9}$$

where

$$\chi^\kappa = R^{T-1} + (1-\delta)R^{T-2} + (1-\delta)^2 R^{T-3} + ... + (1-\delta)^{T-1}$$
$$\chi^s = 1 + R + R^2 + ... + R^{T-1}$$

The numerator $\chi^\kappa \kappa b_0 - \chi^s \overline{s}$ in (9) corresponds to the accumulated new borrowing between 0 and $T$. We assume that it is positive, which happens when the primary surplus is insufficient to pay the coupons on the initial debt. A sufficient, but not necessary, condition is that the primary surplus itself is negative during this time. The debt accumulation schedule then gives a negative relationship between

$b_T$ and $Q_{T-1}$. When $Q_{T-1}$ is lower, asset prices from 0 to $T-2$ are also lower. This implies a higher yield on newly issued debt, reflecting a premium for possible default at time $T$. These default premia lead to a more rapid accumulation of debt and therefore a higher $b_T$ at $T-1$.

Figure 1 shows these two schedules and illustrates the multiplicity of equilibria. There are two stable equilibria, represented by points A and B. At point A, $Q_{T-1} = 1$. The bond price is then equal to 1 at all times. This is the "good" equilibrium in which there is no default. At point B, $Q_{T-1} < 1$. This is the "bad" equilibrium. Asset prices starting at time 0 are less than 1 in anticipation of possible default at time $T$. Intuitively, when agents believe that default is likely, they demand default premia (implying lower asset prices), leading to a more rapid accumulation of debt, which in a self-fulfilling way indeed makes default more likely.

In the bad equilibrium there is a slow-moving debt crisis. As can be seen from (8), using $Q_{T-1} < 1$, the asset price instantaneously drops at time 0 and then continues to drop all the way to $T-1$. Correspondingly, default premia gradually rise over time. Such a slow-moving crisis occurs only for intermediate levels of debt. When $b_0$ is sufficiently low, the debt accumulation schedule is further to the left, crossing below point C, and only the good equilibrium exists. When $b_0$ is sufficiently high, the debt accumulation schedule is further to the right, crossing above point D, and only a bad equilibrium exists. In that case default is unavoidable when $\tilde{s} = s_{low}$. There is therefore an intermediate region for $b_0$ under which there are multiple equilibria, which we refer to as the multiplicity region.

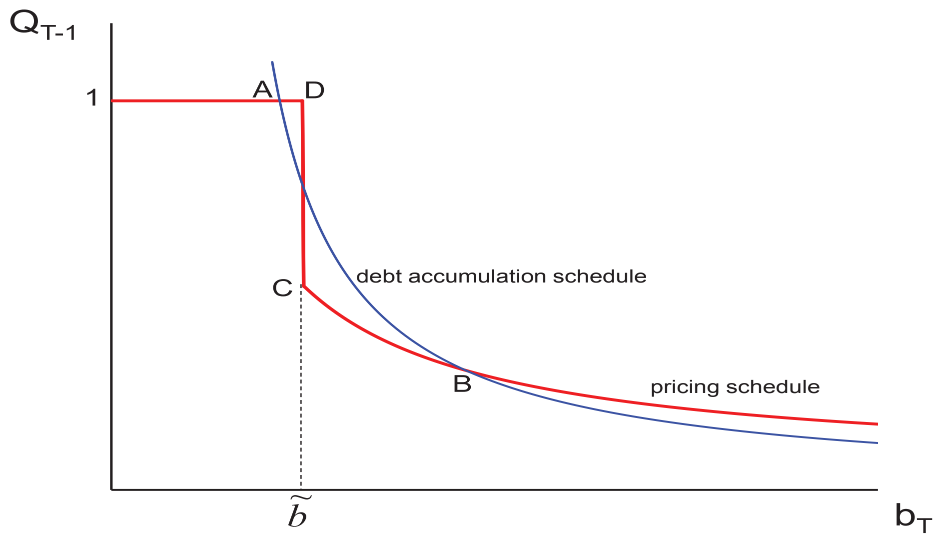
## 2.2 A Monetary Model

We now extend the model to a monetary economy. The goods price level is $P_t$. $R_t$ is now the gross nominal interest rate and $r_t = R_t P_t / P_{t+1}$ the gross real interest rate. The central bank can set the interest rate $R_t$ and affect $P_t$. The coupons on government debt are now nominal. The number of bonds at time $t-1$ is $B_t$ and $B_0$ is given. We define $b_t = B_t / P_t$. The arbitrage equation with no default remains (4), while the government budget constraint for $t \neq T$ becomes

$$Q_t B_{t+1} = ((1-\delta)Q_t + \kappa)B_t - s_t P_t - Z_t \tag{10}$$

where $s_t$ is now the real primary surplus, $s_t P_t$ the nominal surplus, and $Z_t$ is a nominal transfer from the central bank.

Figure 1 Multiple Equilibria Lorenzoni-Werning Model




The central bank budget constraint is :

$$Q_t B^c_{t+1} = ((1-\delta)Q_t + \kappa)B^c_t + [M_t - M_{t-1}] - Z_t \tag{11}$$

where $B^c_t$ are government bonds held by the central bank and are its sole assets. The value of central bank assets decreases with the depreciation of government bonds and payments $Z_t$ to the treasury. It is increased by the coupon payments and an expansion $M_t - M_{t-1}$ of monetary liabilities.

The balance sheets of the central bank and government are interconnected as most central banks pay a measure of net income (including seigniorage) to the Treasury as a dividend.[9] We will therefore consider the consolidated government budget constraint by substituting the central bank constraint into the government budget constraint:

$$Q_t B^p_{t+1} = ((1-\delta)Q_t + \kappa)B^p_t - [M_t - M_{t-1}] - s_t P_t \tag{12}$$

where $B^p_t = B_t - B^c_t$ is government debt held by the general public. The consolidated government can reduce debt to the private sector by issuing monetary liabilities $M_t - M_{t-1}$.

Let $\widetilde{m}$ represent accumulated seigniorage between 0 and $T-1$:

$$\widetilde{m} = \frac{M_{T-1} - M_{T-2}}{P_{T-1}} + r_{T-2}\frac{M_{T-2} - M_{T-3}}{P_{T-2}} + ... + r_0 r_1 ... r_{T-2}\frac{M_0 - M_{-1}}{P_0} \tag{13}$$

[9]See Hall and Reis (2013) for a discussion.

Similarly, let $m^{pdv}$ denote the present discounted value of seigniorage revenues starting at date $T$:

$$m^{pdv} = \frac{M_T - M_{T-1}}{P_T} + \frac{1}{r_T}\frac{M_{T+1} - M_T}{P_{T+1}} + \frac{1}{r_T r_{T+1}}\frac{M_{T+2} - M_{T+1}}{P_{T+2}} + ... \qquad (14)$$

At time $T$ the real obligation of the government to bond holders is $[(1-\delta)Q_T + \kappa]b_T$. The no-default condition is $b_T^p \leq \tilde{b}$, with the latter now defined as

$$\tilde{b} = \frac{s^{pdv} + m^{pdv}}{(1-\delta)Q_T + \kappa} \qquad (15)$$

where

$$s^{pdv} = \left[1 + \frac{1}{r_T} + \frac{1}{r_T r_{T+1}} + ...\right] s_{low} \qquad (16)$$

and $Q_T$ is equal to the present discounted value of coupons:

$$Q_T = \frac{\kappa}{R_T} + \frac{(1-\delta)\kappa}{R_T R_{T+1}} + \frac{(1-\delta)^2\kappa}{R_T R_{T+1} R_{T+2}} + ... \qquad (17)$$

In analogy to the real model, the new pricing schedule becomes

$$\begin{aligned} Q_{T-1} &= \frac{(1-\delta)Q_T + \kappa}{R_{T-1}} && \text{if } b_T^p \leq \tilde{b} \qquad (18) \\ &= \psi \frac{max\{0, \zeta s^{pdv} + m^{pdv}\}}{R_{T-1} b_T^p} + (1-\psi)\frac{(1-\delta)Q_T + \kappa}{R_{T-1}} && \text{if } b_T^p > \tilde{b} \qquad (19) \end{aligned}$$

Since $m^{pdv}$ can potentially be negative, in (19) the minimum return in the bad state is set at 0. The new pricing schedule implies a relationship between $Q_{T-1}$ and $b_T$ that has the same shape as in the real model, but is now impacted by monetary policy through real and nominal interest rates, inflation, and seigniorage.

The debt accumulation schedule now becomes (see Appendix):

$$b_T^p = (1-\delta)^T \frac{B_0^p}{P_T} + \frac{P_{T-1}}{P_T}\frac{\chi^\kappa \kappa B_0^p / P_0 - \chi^s \overline{s} - \widetilde{m}}{Q_{T-1}} \qquad (20)$$

where

$$\begin{aligned} \chi^\kappa &= \left[r_{T-2}...r_1 r_0 + (1-\delta) r_{T-2}...r_1 \frac{P_0}{P_1} + (1-\delta)^2 r_{T-2}...r_2 \frac{P_0}{P_2} + ... + (1-\delta)^{T-1}\frac{P_0}{P_{T-1}}\right] \\ \chi^s &= 1 + r_{T-2} + r_{T-2}r_{T-3} + ... + r_{T-2}...r_1 r_0 \end{aligned}$$

The schedule again implies a negative relationship between $Q_{T-1}$ and $b_T$. Monetary policy shifts the schedule through its impact on interest rates, inflation, and seigniorage.

## 2.3 The Impact of Monetary Policy

Monetary policy affects the paths of interest rates, prices, output and seigniorage, which in turn shifts the two schedules and therefore can affect the existence of self-fulfilling debt crises. The idea is to implement a monetary policy strategy conditional on expectations of sovereign default, which only happens in the crisis equilibrium. If this strategy is successful and credible, the crisis equilibrium is avoided altogether and the policy does not need to be implemented. It is therefore the threat of such a policy that may preclude the crisis equilibrium.

In terms of Figure 1, the crisis equilibrium is avoided when the debt accumulation schedule goes through point C or below. This is the case when

$$\frac{\chi^{\kappa}\kappa B_0^p/P_0 - \chi^s\bar{s} - \widetilde{m}}{s^{pdv} + m^{pdv} - \left((1-\delta)Q_T + \kappa\right)(1-\delta)^T B_0^p/P_T} r_{T-1} \leq \psi \frac{min\{0, \zeta s^{pdv} + m^{pdv}\}}{s^{pdv} + m^{pdv}} + 1 - \psi \tag{21}$$

Note that point C itself is not on the price schedule as its lower section starts for $b_t > \tilde{b}$. It is therefore sufficient that this condition holds as an equality, which corresponds to point C. At point C, $Q_{T-1} < 1$. All prices from 0 to $T-1$ will then be less than one, implying rising default premia that lead to an accumulation of debt. (21) gives a condition for what the central bank needs to do to counteract these rising default premia and avoid default. This condition is key and applies no matter what specific model we assume that relates interest rates, prices and output. We will refer to this as the *default avoidance condition*.

The central bank can impact condition (21) through both *ex ante* policies, taking place between 0 and $T-1$, and *ex post* policies, taking place in period $T$ and afterwards. Ex-ante policies have the effect of shifting the debt accumulation schedule down, while ex-post policies shift the pricing schedule to the right.

Monetary policy can affect the existence of a default equilibrium through inflation, real interest rates, seigniorage and output. Inflation reduces the real value of nominal coupons on the debt outstanding at time 0. Ex-ante policy in the form of inflation prior to time $T$ reduces the real value of coupon payments both before and after $T$. This is captured respectively through $\chi^{\kappa}$ in the numerator of (21) and the term $B_0/P_T$ in the denominator in (21). Inflation after time $T$ only reduces the real value of coupons after $T$, which is reflected in a lower value of $Q_T$ in the denominator.

Reducing real interest rates lowers the cost of new borrowing. For ex-ante policy this is captured through both $\chi^{\kappa}$ and $\chi^s$ in the numerator of (21), which represents the accumulated new borrowing from 0 to $T$. For ex-post policy it shows up through a rise in $s^{pdv}$ in the denominator of (21).[10] Expansionary monetary policy can also lead to a rise in seigniorage. Seigniorage prior to time $T$ reduces the numerator of the left hand side of (21), while seigniorage after time $T$ raises the denominator. Finally, we will also consider an extension where monetary policy can have a favorable effect through output. If we allow the primary surplus to be pro-cyclical, expansionary monetary policy that raises output will raise primary surpluses.

# 3 Illustration with New Keynesian Model

The default avoidance condition (21) depends on interest rates, prices and output. We now consider a specific New Keynesian model that determines prices and output given interest rates that will be controlled by the central bank. The model is used to examine the policies needed to eliminate the default equilibrium. More precisely, we consider the optimal monetary policy that satisfies both the default avoidance condition and the zero lower bound constraint on nominal interest rates.

## 3.1 Model Description

We consider a standard NK model based on Galí (2008, ch. 3), with three extensions suggested by Woodford (2003): i) habit formation; ii) price indexation; iii) lagged response in price adjustment. These extensions are standard in the monetary DSGE literature and are introduced to generate more realistic responses to monetary shocks. The main effect of these extensions is to generate a delayed impact of a monetary policy shock on output and inflation, leading to the humped-shaped response seen in the data.

[10] There is one more subtle real interest rate rate effect, which is specific to the assumption that the central bank knows exactly when the default decision is made. By reducing the real interest rate $r_{T-1}$ the central bank can offset the negative impact of expected default on $Q_{T-1}$. This is captured through the last term on the left hand side of (21).

### 3.1.1 Households

With habit formation, households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t - \eta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} - z\iota_t \right) \tag{22}$$

where total consumption $C_t$ is

$$C_t = \left( \int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{23}$$

and $N_t$ is labor and $z$ is a default cost. We have $\iota_t = 0$ if there is no default at time $t$ and $\iota_t = 1$ if there is default. The default cost does not affect households' decisions, but provides an incentive for authorities to avoid default. Habit persistence, measured by $\eta$, is a common feature in NK models to generate a delayed response of expenditure and output.

The budget constraint is

$$\begin{aligned} &P_t C_t + D_{t+1} + Q_t B_{t+1}^p + M_t = \\ &W_t N_t + \Pi_t - f(M_t, Y_t^n) + R_{t-1} D_t + R_{t-1}^g Q_{t-1} B_t^p + M_{t-1} - T_t \end{aligned} \tag{24}$$

Here $D_{t+1}$ are holdings of one-period bonds that are in zero net supply. $P_t$ is the standard aggregate price level and $W_t$ is the wage level. $\Pi_t$ are firms profits distributed to households and $T_t$ are lump-sum taxes. We will abstract from government consumption, so that the primary surplus is $P_t s_t = T_t$. To generate a money demand, we introduce a transaction cost $f(M_t, Y_t^n)$, where $Y_t^n = P_t Y_t$ is nominal GDP and $\partial f / \partial M \leq 0$. In the benchmark version of the model we assume a cashless economy, with $f(M_t, Y_t^n) = 0$ and $M_t = 0$. We consider an extension with seigniorage in section 3.5.

The first-order conditions with respect to $D_{t+1}$ and $B_{t+1}^p$ are

$$\widetilde{C}_t = \beta E_t R_t \frac{P_t}{P_{t+1}} \widetilde{C}_{t+1} \tag{25}$$

$$\widetilde{C}_t = \beta E_t R_t^g \frac{P_t}{P_{t+1}} \widetilde{C}_{t+1} \tag{26}$$

where

$$\widetilde{C}_t \equiv (C_t - \eta C_{t-1})^{-\sigma} - \eta \beta E_t (C_{t+1} - \eta C_t)^{-\sigma}$$

The combination of (25) and (26) gives the arbitrage equations (4), (18), and (19). This is because government default, which lowers the return on government bonds, does not affect consumption due to Ricardian equivalence.[11]

Let $Y_t$ denote real output and $c_t$, $y_t$ and $y_t^n$ denote logs of consumption, output and the natural rate of output. Using $c_t = y_t$, and defining $x_t = y_t - y_t^n$ as the output gap, log-linearization of the Euler equation (25) gives the dynamic IS equation

$$\tilde{x}_t = E_t\tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}(i_t - E_t\pi_{t+1} - r^n) \tag{27}$$

where

$$\tilde{x}_t = x_t - \eta x_{t-1} - \beta\eta E_t(x_{t+1} - \eta x_t) \tag{28}$$

Here $i_t = ln(R_t)$ will be referred to as the nominal interest rate and $r^n = -ln(\beta)$ is the natural rate of interest. The latter uses our assumption below of constant productivity, which implies a constant natural rate of output.

### 3.1.2 Firms

There is a continuum of firms on the interval $[0,1]$, producing differentiated goods. The production function of firm $i$ is

$$Y_t(i) = AN_t(i)^{1-\alpha} \tag{29}$$

We follow Woodford (2003) by assuming firm-specific labor.

Calvo price setting is assumed, with a fraction $1-\theta$ of firms re-optimizing their price each period. In addition, it is assumed that re-optimization at time $t$ is based on information from date $t-d$. This feature, adopted by Woodford (2003), is in the spirit of the model of information delays of Mankiw and Reis (2002). It has the effect of a delayed impact of a monetary policy shock on inflation, consistent with the data.[12] Analogous to Christiano et al. (2005), Smets and Wouters (2003) and many others, we also adopt an inflation indexation feature in order to generate

[11] When substituting the consolidated government budget constraint $Q_tB_{t+1}^p = R_t^gQ_{t-1}B_t^p - (M_t - M_{t-1}) - T_t$ into the household budget constraint (24), and imposing asset market equilibrium, we get $C_t = Y_t$, which is real GDP and unaffected by default. Here we assume that the transaction cost $f(M_t, Y_t^n)$ is paid to intermediaries that do not require real resources and return their profits to households. It is therefore included in $\Pi_t$.

[12] This feature can also be justified in terms of a delay by which newly chosen prices go into effect.

more persistence of inflation. Firms that do not re-optimize follow the simple indexation rule

$$ln(P_t(i)) = ln(P_{t-1}(i)) + \gamma\pi_{t-1} \tag{30}$$

where $\pi_{t-1} = ln\ P_{t-1} - ln\ P_{t-2}$ is aggregate inflation one period ago.

Leaving the algebra to the Technical Appendix, these features give the following Phillips curve (after linearization):

$$\pi_t = \gamma\pi_{t-1} + \beta E_{t-d}(\pi_{t+1} - \gamma\pi_t) + E_{t-d}(\omega_1 x_t + \omega_2 \tilde{x}_t) \tag{31}$$

where

$$\omega_1 = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{\phi+\alpha}{1-\alpha+(\alpha+\phi)\varepsilon}$$
$$\omega_2 = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1-\alpha}{1-\alpha+(\alpha+\phi)\varepsilon}\frac{\sigma}{(1-\eta\beta)(1-\eta)}$$

### 3.1.3 Monetary Policy

We follow most of the literature by using a quadratic approximation of utility. Conditional on avoiding the default equilibrium, the central bank then minimizes the following objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{\mu_x(x_t - \nu x_{t-1})^2 + \mu_\pi(\pi_t - \gamma\pi_{t-1})^2\right\} \tag{32}$$

where $\nu$, $\mu_x$ and $\mu_\pi$ a function of model parameters (see the Technical Appendix for the derivation). The central bank chooses the optimal path of nominal interest rates over $H > T$ periods. After that, we assume an interest rate rule as in Clarida et al. (1999):

$$i_t - \bar{\imath} = \rho(i_{t-1} - \bar{\imath}) + (1-\rho)(\psi_\pi E_t\pi_{t+1} + \psi_y x_t) \tag{33}$$

where $\bar{\imath} = -ln(\beta)$ is the steady state nominal interest rate. We will choose $H$ to be large. Interest rates between time $T$ and $H$ involve ex-post-policy.[13]

Optimal policy is chosen conditional on two types of constraints. The first is the ZLB constraint that $i_t \geq 0$ for all periods. The second is the default avoidance condition (21) as an equality.[14] Using the NK Phillips curve (31), the dynamic

[13] Since $H$ will be large, the precise policy rule after $H$ does not have much effect on the results.

[14] In the good equilibrium $i_t \geq 0$ is the only constraint and the optimal policy implies $i_t = \bar{\imath}$ each period, delivering zero inflation and a zero output gap. However, conditional on a sunspot that could trigger a default equilibrium condition (21) becomes an additional constraint.

IS equation (27), and the policy rule (33) after time $H$, we solve for the path of inflation and output gap conditional on the set of $H$ interest rates chosen. We then minimize the welfare cost (32) over the $H$ interest rates subject to $i_t \geq 0$ and the default avoidance condition.

## 3.2 Calibration

We consider one period to be a quarter and normalize the constant productivity $A$ such that the natural rate of output is equal to 1 annually (0.25 per quarter). The other parameters are listed in Table 1. The left panel shows the parameters from the LW model, while the right panel lists the parameters that pertain to the New Keynesian part of the model.

A key parameter is $\delta$. In the benchmark parameterization we set it equal to 0.05, which implies a government debt duration of 4.2 years. This is typical in the data. For example, OECD estimates of the Macauley duration in 2010 are 4.0 in the US and 4.4 for the average of the five European countries that experienced a sovereign debt crisis (Greece, Italy, Spain, Portugal and Ireland). The coupon is determined such that $\kappa = 1/\beta - 1 + \delta$.

As we will see, the other LW parameters, $\beta$, $T$ and the fiscal surplus parameters, are not going to be important to the results. They matter mostly by affecting the "multiplicity range" of debt for which there are multiple equilibria. We want this range to be broad as otherwise the entire exercise has little meaning. There is not much need for monetary policy when a country is only subject to self-fulfilling crises for a narrow range of initial debt levels. Although we are by no means aiming to calibrate to a particular historical episode, it is instructive to note that during the Eurozone crisis the range of debt of periphery countries varied substantially in 2010, from 62% in Spain to 148% in Greece.

The range of $B_0$ for which there are multiple equilibria under passive monetary policy ($i_t = \bar{\imath}$) is $[B_{low}, B_{high}]$, where[15]

$$B_{low} = \frac{\beta}{1-\beta} \frac{(\psi\zeta + 1 - \psi)\beta^T s_{low} + (1-\beta^T)\bar{s}}{1 - (1-\zeta)(1-\delta)^T \beta^T \psi} \tag{34}$$

$$B_{high} = \frac{\beta}{1-\beta} \left(\beta^T s_{low} + (1-\beta^T)\bar{s}\right) \tag{35}$$

This range may be wide or narrow, dependent on the chosen parameters. For example, when $\zeta \to 1$, the range narrows to zero. We set $\beta = 0.99$ (4% annual

[15] These values lead to equilibria at points $C$ and $D$ in Figure 1.

natural real rate of interest), $T = 20$ (uncertainty resolved in 5 years), $\bar{s} = -0.01$ (4% annual primary deficit), $s_{low} = 0.02$, $\zeta = 0.5$ and $\psi = 0.95$. This gives a multiplicity range of $[0.79, 1.46]$, so that a country is subject to multiple equilibria when debt is in the range of 79 to 146 percent of GDP, not unlike the variation in European periphery debt levels in 2010. But as we will see, many other combinations of LW parameters deliver the same multiplicity range, with virtually identical results.

The New Keynesian parameters are standard in the literature. The first 5 parameters correspond exactly to those in Gali (2008). The habit formation parameter, the indexation parameter and the parameters in the interest rate rule are all the same as in Christiano et al. (2005). We take $d = 2$ from Woodford (2003, p. 218-219), which also corresponds closely to Rotemberg and Woodford (1997). This set of parameters implies a response to a small monetary policy shock under the Taylor rule that is similar to the empirical VAR results reported by Christiano et al. (2005). The level of output and inflation at their peak correspond exactly to that in the data. Both the output and inflation response is humped shaped like the data, although the peak response (quarter 6 and 3 respectively for inflation and output) occurs a bit earlier than in the data.

Table 1 Calibration

| Lorenzoni-Werning parameters | | New Keynesian parameters | |
|---|---|---|---|
| Parameter | Description | Parameter | Description |
| $\beta = 0.99$ | discount rate | $\sigma = 1$ | elasticity of intertemporal subsitution |
| $\delta = 0.05$ | coupon depreciation rate | $\phi = 1$ | Frisch elasiticity |
| $\kappa = 0.06$ | coupon | $\varepsilon = 6$ | demand elasticity |
| $T = 20$ | quarters before default decision | $\alpha = 0.33$ | capital share |
| $\zeta = 0.5$ | recovery rate | $\theta = 0.66$ | Calvo pricing parameter |
| $\psi = 0.95$ | probability low surplus state | $\eta = 0.65$ | habit parameter |
| $s_{low} = 0.02$ | low state surplus | $\gamma = 1$ | indexation parameter |
| $\bar{s} = -0.01$ | surplus before $T$ | $d = 2$ | lag in price adjustment |
| | | $\rho = 0.8$ | persistence in interest rate rule |
| | | $\psi_\pi = 1.5$ | inflation parameter in interest rule |
| | | $\psi_y = 0.1$ | output parameter in interest rule |

## 3.3 Results under Benchmark Parameterization

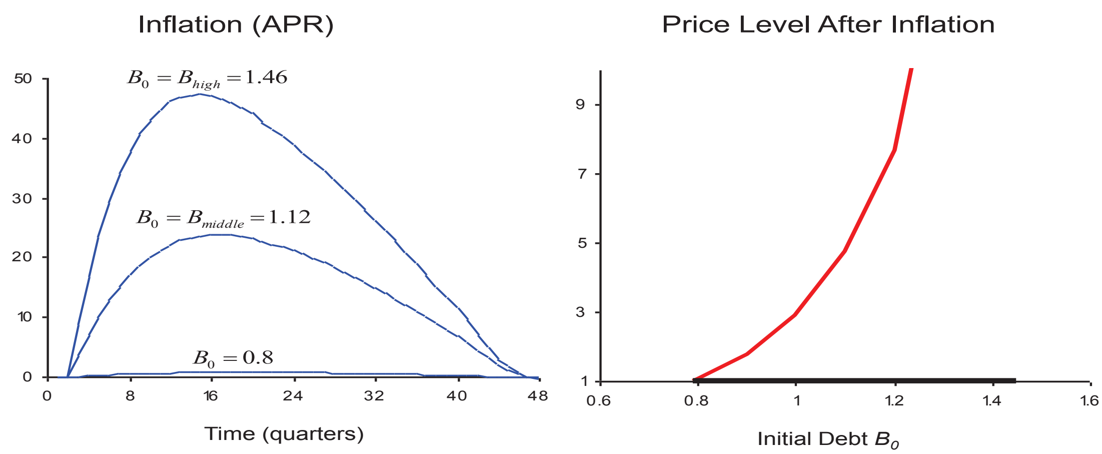
The optimal monetary policy that we have described is credible as long as the welfare cost associated with inflation and non-zero output gaps is less than the cost of default. But rather than comparing the welfare cost to the cost of default, in reporting the results we will mainly focus on the level of inflation needed to satisfy the condition to avoid default under optimal policy. We do so for two reasons. First, the cost of default is hard to measure, including reputational costs, trade exclusion costs, costs through the financial system and political costs. In addition, even within our model the cost of inflation is very sensitive to parameters that otherwise have very little effect on the level of inflation under optimal policy. Second, we will see that the key message that an excessive amount of inflation is needed avoid a self-fulfilling default, is very robust and not affected by parameter assumptions that significantly affect the welfare cost in the model.[16]

Figure 2 shows the dynamics of inflation under optimal policy under the benchmark parameterization for $H = 40$ (which we assume throughout). The results are shown for various levels of $B_0$. The optimal path for inflation is hump shaped. Optimal inflation gradually rises, both due to rigidities and because the welfare cost (32) depends on the change in inflation. Eventually optimal inflation decreases as it becomes less effective over time when the original debt depreciates and is replaced by new debt that incorporates inflation expectations. When $B_0 = B_{middle} = 1.12$, which is in the middle of the range of debt levels giving rise to multiple equilibria, the maximum inflation rate reaches 23.8%. Inflation is over 20% for 4 years, over 10% for 8 years and the price level ultimately increases by a factor 5.3. Inflation needed to avoid default gets even much higher for higher debt levels. When $B_0$ reaches the upper bound $B_{high}$ for multiple equilibria, the maximum inflation rate is close to 47% and ultimately the price level increases by a factor 25! Only when $B_0$ is very close to the lower bound for multiplicity, as illustrated for $B_0 = 0.8$, is little inflation needed.

In order to understand why so much inflation is needed, it is useful to first

[16] At a deeper level, a problem is that there is no consensus on what the exact welfare costs of inflation and output gap are. The welfare costs of inflation depend significantly on the type of price setting (see Ambler (2007) for a discussion of Taylor pricing versus Calvo pricing). The welfare costs of inflation are also broader than the inefficiencies associated with relative price changes that inflation induces. In the model the inflation cost would be zero if all firms raised prices simultaneously. It is also well known that the representative agent nature of the model understates the welfare costs of non-zero output gaps.

Figure 2 Benchmark NK Model: Inflation Needed to Avoid Default




consider a rather extreme experiment where all of the increase in prices happens right away in the first quarter. This cannot happen in the NK model, so assume that prices are perfectly flexible, the real interest rate is constant at $1/\beta$ and the output gap remains zero. When $B_0 = B_{middle} = 1.12$, the price level would need to rise by 42%. This is needed to lower debt so that we are no longer in the multiplicity range. Of course such a policy, even if possible, is not plausible either as it would involve an annualized inflation rate for that quarter of 168%.

In reality inflation will be spread out over a period of time, both because sticky prices imply a gradual change in prices and because it is optimal from a welfare perspective not to have the increase in the price level happen all at once. However, such a delay increases the ultimate increase in the price level that is needed. As the time zero debt depreciates, inflation quickly becomes less effective as it only helps to reduce the real value of coupons on the original time zero debt. The interest rate on new debt incorporates the higher inflation expectations. More inflation is therefore needed to avoid the default equilibrium.

It is important to stress that the economy starts at zero inflation and that the inflation levels mentioned here are unexpected at time 0. If for example an economy already had high expected inflation, the surprise inflation reported here to avoid self-fulfilling debt crises would come on top of that. We deem such high unanticipated inflation rates highly implausible. It is hard to imagine that countries with a central bank truly interested in the general welfare would wish to generate such very large sustained surprise inflation. Indeed, Reinhart and Sbrancia (2015) find that in the post WWII era, particularly 1945-1980, public debt reductions in industrialized countries have been achieved mostly through financial repression as opposed to inflation surprises. To the extent that debt reduction has been partly achieved through inflation surprises, this often has gone hand in hand with financial repression. The extensive use of financial repression tools is a reflection of the difficulty of achieving debt reduction through sustained austerity and inflation surprises alone.

## 3.4 Sensitivity Analysis

We now consider changes to both the LW and NK parameters. Changes in the LW parameters change the multiplicity region $[B_{low}, B_{high}]$. Monetary policy is only of interest if in its absence a country is exposed to self-fulfilling crises for a wide range of debt levels. In other words, we need to assume a substantial multiplicity region. Related to that, the "effort" required from the central bank depends on the extent to which $B_0/B_{low}$ is above 1. When the multiplicity region is narrow, $B_0/B_{low}$ is necessarily close to 1.

Table 2 shows that changes in the LW parameters significantly affect the $[B_{low}, B_{high}]$ region. One way to control for this is to consider combinations of these parameters that lead to exactly the same multiplicity region as under the benchmark. The left panel of Figure 3 shows combinations of $T$, $\bar{s}$ and $s_{low}$ that generate the same $B_{low}$ and $B_{high}$. The panel on the right shows that this has little effect on the path of optimal inflation. Varying $T$ from 10 to 30, while adjusting $\bar{s}$ and $s_{low}$ to keep $B_{low}$ and $B_{high}$ unchanged, gives very similar paths for optimal inflation.

In Figures 4 and 5 and the last two columns of Table 2 we adopt a different approach. We present results when significantly varying one parameter at a time, but keeping $B_0/B_{low} = 1.42$ the same as in the middle of the multiplicity range of the benchmark parameterization. For the LW parameters this implies values of $B_0$ that can be relatively closer to $B_{low}$ or $B_{high}$, dependent on their values for that parameter.[17]

Figure 4 reports optimal inflation for two values of the LW parameters, one

[17] Only for $\zeta = 0.7$ is $B_0$ now slightly above $B_{high}$. For all other parameters the $B_0$ is within the interval for $B_0$ generating multiple equilibria.

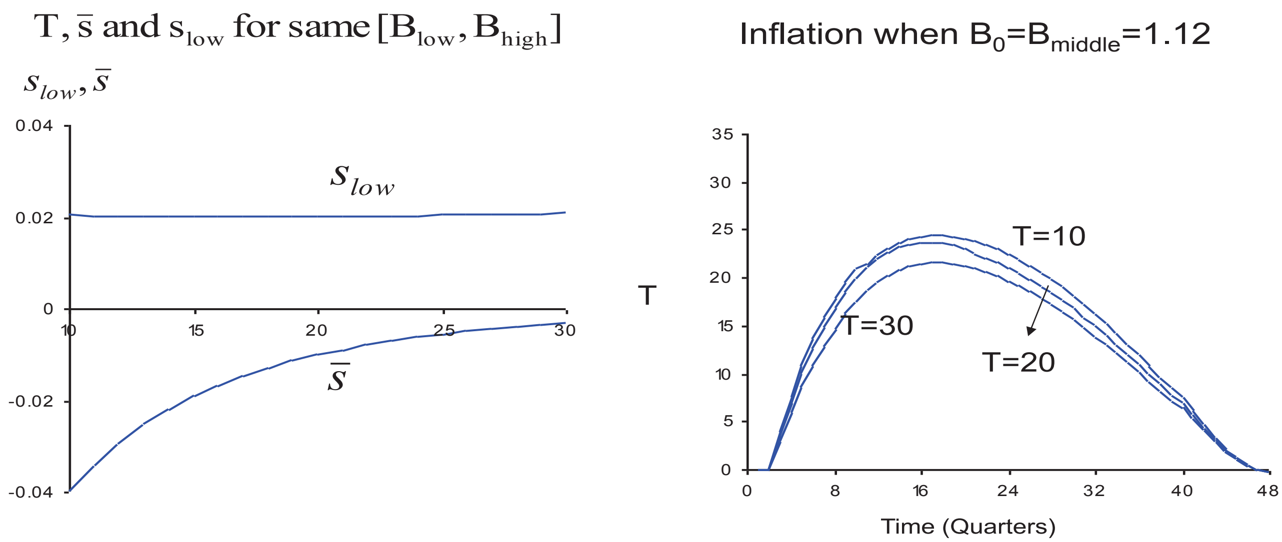
higher and the other lower than in the benchmark. Figure 4 shows that the only LW parameters that substantially affects the results is $\delta$. A lower debt depreciation $\delta$, which implies a longer maturity of debt, implies lower inflation. The reason is that inflation is effective for a longer period of time as the time 0 debt depreciates more slowly. But even when $\delta = 0.025$, so that the duration is 7.2 years, optimal inflation is still above 10% for 6.5 years and the price level ultimately triples.

Figure 5 shows analogous results for the NK parameters. Most parameters again have remarkably little effect. There are two cases where parameter values do matter. One is the lag in price adjustment $d$. With $d = 0$ it is possible to increase inflation from the start, when debt deflation is the most powerful. This reduces the overall inflation needed. But even with $d = 0$, optimal inflation still peaks close to 20% and the price level still more than quadruples as a result of years of inflation (Table 2).

The only other case where parameters matter substantially is the last chart, where we set $\gamma = d = \eta = 0$ as in the Gali (2008) textbook model and compare the results to the benchmark parameterization. In this case all the inflation comes upfront, both because there is no delay in price adjustment and because there is no price indexation. Inflation starts at 23% (APR) in the first quarter, but the ultimate increase in the price level is much less than in the benchmark, only 66%. However, this case is of little practical relevance. The default avoidance condition is met to a large extent by a highly unrealistic steep drop in the real interest rate, from 4% (APR) to -19% in the first quarter, while output rises immediately by 25% in just the first quarter. Even for small monetary shocks it is well known that these parameters lead to a very unrealistic dynamic response of output and inflation to monetary shocks.

A couple of comments are in order about welfare versus inflation. As already pointed out, the welfare cost is very sensitive to NK parameters even when inflation is little affected. For example, the benchmark case gives a welfare cost of 2.8%, measured as a one year percentage drop in consumption or output that generates the same drop in welfare. This seems quite small. But when we increase $\theta$ from 0.66 to 0.8, the welfare cost more than triples to 8.7%, with very little difference in optimal inflation. If we adopt the textbook Gali model, where $\gamma = d = \eta = 0$, the welfare cost is a staggering 85%.

Figure 3 Sensitivity Analysis LW Parameters




## 3.5 Extensions

We consider three extensions: seigniorage, pro-cyclical primary surplus and uncertainty about $T$.

### 3.5.1 Seigniorage

In order to consider the additional role of seigniorage, we use a convenient form of the transaction cost $f(M_t, Y_t^n)$ that gives rise to a standard specification for money demand when $i_t > 0$ ($m_t = ln(M_t)$)[18]:

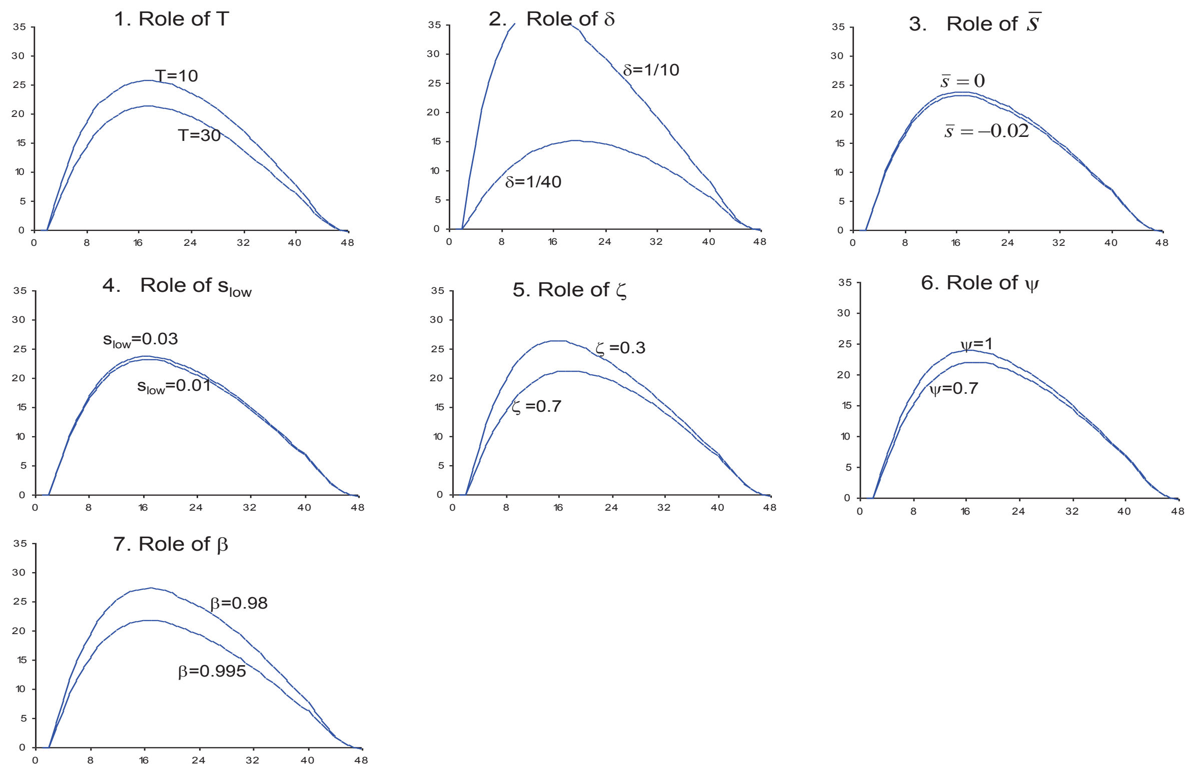
$$m_t = \alpha_m + p_t + y_t - \alpha_i i_t \tag{36}$$

When $i_t$ is close to zero, money demand reaches the satiation level $\alpha_m + p_t + y_t$. We limit ourselves to conventional monetary policy, where the money supply does not go beyond the satiation level. Section 5 makes some comments on unconventional monetary policy.

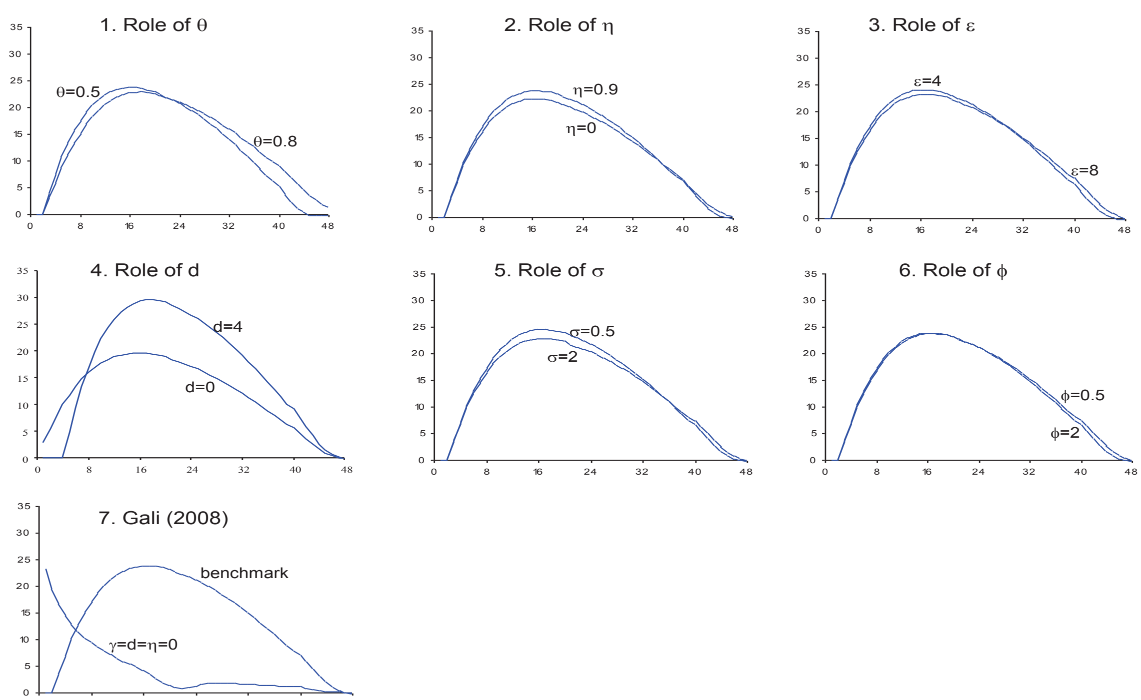
[18]The transaction cost $f(M_t, Y_t^n) = \alpha_0 + M_t \left( ln \left( \frac{M_t}{P_t Y_t} \right) - 1 - \alpha_m \right) / \alpha_i$ gives rise to money demand (36). This function applies for values of $M_t$ where $\partial f / \partial M \leq 0$. Once the derivative becomes zero, we reach a satiation level and we assume that the transaction cost remains constant for larger $M_t$.

| Table 2 Sensitivity Analysis | | | | |
|---|---|---|---|---|
| Parameters | $B_{low}$ | $B_{high}$ | maximum inflation | price level after inflation |
| Benchmark | 0.79 | 1.46 | 23.8 | 5.3 |
| Lorenzoni-Werning Parameters | | | | |
| $T = 10$ | 1.15 | 1.71 | 25.8 | 6.3 |
| $T = 30$ | 0.56 | 1.22 | 21.4 | 4.5 |
| $\delta = 1/40$ | 0.89 | 1.46 | 15.1 | 3.0 |
| $\delta = 1/10$ | 0.71 | 1.46 | 37.5 | 12.0 |
| $\bar{s} = -0.02$ | 0.58 | 1.28 | 23.2 | 5.1 |
| $\bar{s} = 0$ | 1.00 | 1.64 | 23.9 | 5.3 |
| $s_{low} = 0.01$ | 0.28 | 0.63 | 23.2 | 5.1 |
| $s_{low} = 0.03$ | 1.27 | 2.25 | 23.7 | 5.3 |
| $\zeta = 0.3$ | 0.46 | 1.46 | 26.4 | 6.1 |
| $\zeta = 0.7$ | 1.08 | 1.46 | 21.3 | 4.5 |
| $\psi = 0.7$ | 0.98 | 1.46 | 22.0 | 4.7 |
| $\psi = 1$ | 0.75 | 1.46 | 24.0 | 5.4 |
| $\beta = 0.98$ | 0.61 | 1.17 | 26.7 | 6.7 |
| $\beta = 0.995$ | 0.90 | 1.62 | 21.8 | 4.6 |
| New Keynesian Parameters | | | | |
| $\theta = 0.5$ | 0.79 | 1.46 | 23.7 | 5.0 |
| $\theta = 0.8$ | 0.79 | 1.46 | 22.9 | 5.5 |
| $\eta = 0$ | 0.79 | 1.46 | 22.3 | 4.9 |
| $\eta = 0.8$ | 0.79 | 1.46 | 23.8 | 5.3 |
| $\epsilon = 4$ | 0.79 | 1.46 | 24.1 | 5.3 |
| $\epsilon = 8$ | 0.79 | 1.46 | 23.2 | 5.2 |
| $d = 0$ | 0.79 | 1.46 | 19.6 | 4.2 |
| $d = 4$ | 0.79 | 1.46 | 29.6 | 7.2 |
| $\sigma = 2$ | 0.79 | 1.46 | 22.7 | 5.0 |
| $\sigma = 0.5$ | 0.79 | 1.46 | 24.4 | 5.4 |
| $\phi = 2$ | 0.79 | 1.46 | 23.6 | 5.3 |
| $\phi = 0.5$ | 0.79 | 1.46 | 23.7 | 5.2 |
| $\gamma = 0.9$ | 0.79 | 1.46 | 18.0 | 3.2 |
| $\gamma = \eta = d = 0$ | 0.79 | 1.46 | 23.0 | 1.7 |

# Figure 4 Sensitivity Analysis LW Parameters ($B_0/B_{low}=1.42$)




# Figure 5 Sensitivity Analysis NK Parameters ($B_0/B_{low}=1.42$)



Seigniorage revenue depends on the semi-elasticity $\alpha_i$ of money demand. Seigniorage is larger for lower values of $\alpha_i$ since that leads to a smaller drop in real money demand when inflation rises. Engel and West (2005) review many estimates, which fall largely in the range 40-60 for quarterly data. We therefore set $\alpha_i = 40$, which leads to the most seigniorage in this range. The left chart of Figure 6 compares the optimal inflation path with seigniorage to that in the benchmark without seigniorage, assuming $B_0 = B_{middle}$.[19] The effect of seigniorage is clearly negligible. This result is consistent with Reis (2013) and Hilscher et al. (2014). As Reis (2013) puts it, "In spite of the mystique behind the central bank's balance sheet, its resource constraint bounds the dividends it can distribute by the present value of seigniorage, which is a modest share of GDP".

### 3.5.2 Pro-cyclical surplus

In this case, nominal rigidities give the central bank control over the accumulation of debt through the level of output that affects the primary surplus. From 0 through $T-1$ assume that we have

$$s_t = \bar{s} + \lambda(y_t - \bar{y}) \tag{37}$$

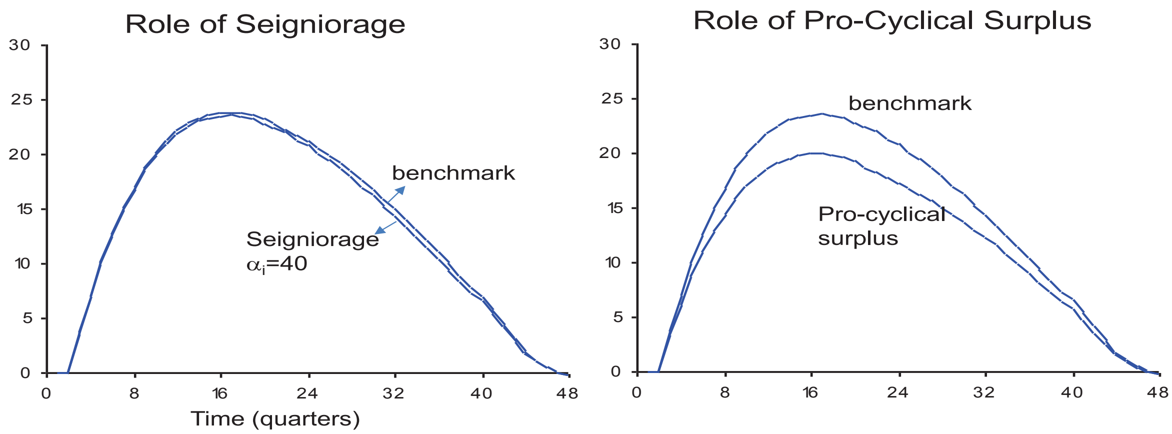
where $\bar{y}$ is steady-state output. We similarly assume that $s_{low}$ is pro-cyclical: $s_{low} = \bar{s}_{low} + \lambda(y_t - \bar{y})$. We set the value of the cyclical parameter of the fiscal surplus to $\lambda = 0.1$, in line with empirical estimates.[20]

With this additional effect from an output increase, the right chart of Figure 6 shows that the optimal inflation decreases slightly, assuming again $B_0 = B_{middle}$. But the effect is again limited. The maximum inflation rate is reduced from 23.8% in the benchmark to 19.9%. The increase in the price level after inflation is reduced from 5.3 under the benchmark to 4.0, which remains excessive. Optimal policy now gives more emphasis to raising output, leading to a first quarter output increase that is 10% (APR), pushing the boundary of what is plausible.

[19] We calibrate $\alpha_m$ to the U.S., such that the satiation level of money corresponds to the monetary base just prior to its sharp rise in the Fall of 2008 when interest rates approached the ZLB. At that time the velocity of the monetary base was 17. This gives $\alpha_m = -1.45$. The velocity is $4P_tY_t/M_t$ as output needs to be annualized, which is equal to $4e^{-\alpha_m}$ at the satiation level.

[20] Note that since $\bar{Y} = 0.25$ for quarterly GDP, the specification implies that $\Delta s = 0.4\Delta Y$. This is consistent for example with estimates by Girouard and André (2005) for the OECD.

Figure 6 Role of Seigniorage and Pro-cyclical Surplus*




*The charts show the inflation rate over time under optimal monetary policy. The left chart compares the benchmark parameterization (cashless economy) to the extension with seigniorage in which $\alpha_i$=40. The right chart compares the benchmark parameterization to the case where the primary surplus is pro-cyclical with $\lambda$=0.1.

### 3.5.3 Uncertainty about T

In the Technical Appendix we discuss one final extension, uncertainty about the date $T$ of the default decision. This significantly complicates the model and we only consider two possible values, $T_1$ and $T_2$, which occur with probabilities $p$ and $1-p$. The key results remain the same. As one might expect, the range for $B_0$ over which there are multiple equilibria is now in between that for the cases where $T = T_1$ and $T = T_2$ without uncertainty. Monetary policy after $T_1$ is now contingent on whether there was a default decision at $T_1$ or not. The key conclusion that an excessive amount of inflation is needed to avoid default (at both $T_1$ and $T_2$) remains unaltered.

# 4 Beyond the NK Model

So far we have cast our analysis in the context of a specific NK model, combined with the LW model. While we have done sensitivity analysis with respect to the various NK parameters, we have not considered alternative versions of the NK block of the model. It is not hard to criticize the specific model we have chosen. We have assumed a particular form of price stickiness (Calvo pricing). One could consider alternatives, such as Taylor price setting or menu costs. We have also abstracted from many features that would complicate the structure, but perhaps make it more realistic, such as investment and wage rigidities. Finally, the dynamic IS equation relies on an intertemporal consumption Euler equation that has recently been criticized in the context of the debate about forward guidance.[21]

In this section we therefore take an alternative approach by considering what paths of real interest rates and inflation are consistent with the default avoidance condition. The advantage of this approach is that we do not need to take any stand on the underlying model that maps monetary policy (interest rate decisions) into inflation and real interest rates. Before presenting the results, we first discuss the constraint imposed by the standard consumption Euler equation in the NK model and how it may have affected the results. In particular, we argue that it limits the extent to which the central bank is capable of lowering real interest rates in a sustained way. After that we consider specific paths of real interest rates and inflation that satisfy the default avoidance condition. This relies only on the monetary version of the LW block of the model.

## 4.1 Euler Equation

The needed inflation may be smaller when lower real interest rates, by lowering the costs of borrowing, help to avoid the default equilibrium. But the consumption Euler equation may constrain the ability of the central bank to reduce real interest rates in a sustained way. In order to see why, abstract from habit formation for a moment ($\eta = 0$). The dynamic IS equation, which comes from the intertemporal consumption Euler, can then be solved as

$$x_0 = -\frac{1}{\sigma} \sum_{t=0}^{\infty} E_0 (r_t - r^n) \tag{38}$$

This precludes a large and sustained drop in the real interest rate as it would imply an enormous and unrealistic immediate change in output at time zero, especially with $\sigma = 1$ as often assumed.

The same point applies when we introduce habit formation, in which case (38) becomes (See Technical Appendix)

$$x_0 = -\frac{1}{\sigma} \sum_{t=0}^{\infty} E_0 \left(1 - (\beta\eta)^{t+1}\right) (r_t - r^n) \tag{39}$$

[21] E.g. see Del Negro et al. (2015) or McKay et al. (2015)

Removing the expectation operator and the $r^n$ for convenience, for the benchmark parameterization ($\sigma = 1$, $\eta = 0.65$), we have

$$x_0 = -0.36r_0 - 0.58r_1 - 0.73r_2 - 0.83r_3 - 0.89r_4 - 0.93r_5 - 0.95r_6 - 0.97r_7 - 0.98r_8 - ... \tag{40}$$

Subsequent coefficients are very close to -1. For the path of real interest rates under optimal policy this implies $x_0 = 0.0157$. This translates into an immediate increase in output of 6.3% on an annualized basis, which is already pushing the boundaries of what is plausible. The real interest rate quickly returns to steady state after dropping from 4% (APR) to 0% for the first two quarters under the benchmark parameterization. This limits the ability of the central bank to satisfy the default avoidance condition.

Related to this, NK models have been found to deliver unrealistically large effects of output and consumption to changes in future interest rates, which Del Negro et al. (2015) have dubbed the forward guidance puzzle. McKay et al. (2015) also argue that it is not realistic that consumption today responds equally to an announced interest rate cut in the far future as to an announced interest rate cut today. Indeed, (38) shows that real interest rate changes at any future date have the same effect on current output (and consumption) as a current real interest rate change. With habit formation future interest rate changes have an even larger effect than current changes. In order to rectify that problem, models have been proposed leading to a reduced effect of real interest rates on consumption and output today when the expected changes occur further into the future. McKay et al. (2015) do so in the context of a model with idiosyncratic risk and borrowing constraints. Del Negro et al. (2015) do so by introducing finite lives through a positive probability of death. In the context of our model, such alternatives allow for a larger drop in future real interest rates without generating unrealistic implications for current output and consumption. We provide an illustration below.

## 4.2 Some Results

Figure 7 considers three scenarios for the real interest rate path, shown in the bottom charts. The top charts show the corresponding inflation needed to satisfy the default avoidance condition. The results are shown as a function of initial debt. We consider both a constant inflation rate over 10 years and a constant inflation rate over 3 years. The latter is represented by the higher line as a higher inflation rate is needed when inflation is limited to three years. The motivation behind this case is that we have seen that inflation is most effective when it occurs soon after the time zero shock, before most of the original debt has to be rolled over and replaced with new debt that incorporates the higher inflation expectations.

The first scenario for real interest rates (scenario A) assumes that the real interest rate simply remains constant and equal to the natural rate. In this case all the burden is on inflation to reach the default avoidance condition. In scenario B the annualized real interest rate immediately drops in period 0 from 4% (the natural rate) to 0. After that we assume that the gap between real interest rate and the natural rate is multiplied by 0.95 each quarter and we close the remaining gap entirely after 40 quarters. As can be seen, this delivers a very large and persistent drop in the real interest rate. Scenario C is similar to scenario B, but the level of the real interest rate is always 2% (APR) lower than in scenario B. So we start from a 2% real interest rate and it drops right away to -2% and then very gradually, over a period of 10 years, returns to 2%. For this case we set the natural rate to 2% by assuming $\beta = 0.995$, while at the same time we change $\bar{s}$ and $s_{low}$ to keep the multiplicity region unchanged, similar to the experiments in Figure 3.[22]

The sharp and persistent drop in real interest rates in scenarios B and C is very large in historical context. Consider for example the 1970s, a decade of significant monetary expansion in many countries leading to steep inflation rates. Italy experienced inflation rates between 10 and 20% for most of the decade. This was the result of extensive money financing of fiscal deficits. The real interest rate in Italy dropped, but no more than in our scenarios B and C.[23]

The drop in real interest rates in scenarios B and C in Figure 5 would lead to an implausible rise in output by more than 30% within a year under the consumption Euler equation of the model. However, output would respond less under alternative modeling approaches mentioned above where future real interest rate changes have a smaller effect on output today. To illustrate this, we have implemented the

[22] $\bar{s}$ is lowered from -0.01 to -0.010787 and $s_{low}$ is lowered from 0.02 to 0.00922. If we did not change these parameters, we would simply be reducing the multiplicity region as the cost of borrowing declines. As discussed many times already, this is like making the problem go away independent of monetary policy.

[23] When subtracting expected inflation from nominal interest rates, using either survey data or econometrics, the real interest rate dropped from slightly above zero to about -4% after the first oil price shock, then returned to zero after a couple of years before dropping somewhat again after the second oil price shock (e.g. see Atkinson and Chouraqui, 1985). Moreover, many other factors, specifically the oil price shocks, obviously played a role as well, leading to a decline in real interest rates also in countries with a more modest monetary policy.

dynamic IS equation from Castelnuovo and Nisticò (2010), who like Del Negro et al. (2015) assume a positive probability of death. They also allow for habit formation as in our model. Their estimated parameters imply

$$x_0 = -0.14 \sum_{t=0}^{\infty} E_0 \Theta^t (r_t - r^n) \tag{41}$$

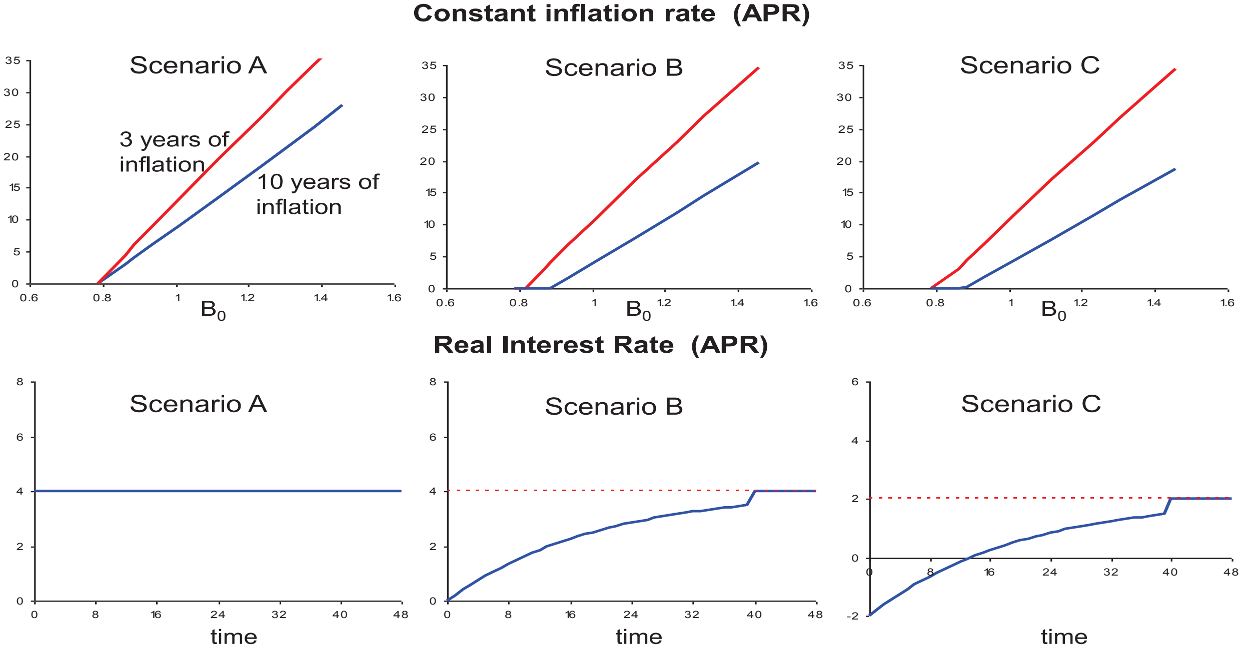
with $\Theta = 0.8$. The weight on the real interest rate 2 years from now is then only a fraction 0.16 of the weight on the current real interest rate. Even with the large drop in real interest rates in scenarios B and C, output then rises only a modest 2.4% (APR) during the first quarter.

However, Figure 7 shows that still large inflation is needed, even with the large sustained drop in real interest rates. Scenarios B and C deliver very similar results for inflation. When inflation is spread over 10 years, annual inflation needs to be anywhere from 0 to 20%, dependent on where we are in the multiplicity region. Correspondingly, the price level needs to increase by a factor between 1 and 6.8. This is a bit less than without the drop in real interest rates, but unless we are near the lower range of debt in the multiplicity region it remains the case very large and sustained inflation is needed to avoid default.

The lowest ultimate increase in the price level is achieved when the real interest rate drops as in scenarios B and C, while at the same time the inflation is limited to the first 3 years, when it is most effective. Annual inflation then varies between 0 to 34% per year. Correspondingly, the price level rises by a factor between 1 and 2.7. Even in this case, unless debt is near the lower end of the multiplicity range, the inflation cost of such policies is generally very substantial.

Of course these results are not entirely model free as we still rely on the LW part of the model. But the LW model matters mainly in generating a certain multiplicity range and in the duration of the government debt. For a realistic assumption about duration and a broad range of multiplicity for debt it is generally difficult to avoid the default equilibrium through monetary policy other than by generating very steep inflation. It is hard to see how this result would change by changing aspects of the LW model as the intuition behind our findings (section 3.3) does not depend on details of the LW framework.

Figure 7 Constant Inflation Needed to Avoid Default*




*Three scenarios for the real interest rate path are shown in the bottom three charts. The corresponding top three charts show the constant inflation that is needed to avoid default. Both a constant inflation rate for 3 years (12 quarters) and a constant inflation rate of 10 years (40 quarters) are reported as a function of the initial debt at time 0.

# 5 Discussion of Alternative Policies

In the NK model, we have examined the role of optimal interest rate policies. But other policies are often mentioned in the context of sovereign debt crises. In this section, we examine three of these policies: i) an interest rate ceiling; ii) quantitative easing at the ZLB; iii) sterilized purchase of debt.

## 5.1 Interest Rate Ceiling

Some, including Calvo (1988), have argued that the bad equilibrium may be avoided if the government commits to an interest rate ceiling. LW counter that if the government refuses to pay more than a certain (real) interest rate, and the market is unwilling to lend at that interest rate, the government would be forced to significantly cut spending or raise taxes without delay. They consider this not to be credible. LW argue that in reality the government will have to do another auction at a price that the market is willing to pay (the bad equilibrium). While they consider a real model without a central bank, alternatively one could imagine the central bank committing to buy government debt at a low (default free) real interest rate. But as we have already seen, this will make little difference. Seigniorage revenue is very small in reality. Ultimately the private sector will need to absorb new debt unless the central bank is able to slow down debt accumulation through the inflationary policies that we have already considered.

## 5.2 Quantitative Easing

When considering the role of seigniorage in section 3.5.1, we assumed that monetary expansions do not go beyond the satiation level. But one can consider much larger monetary expansions that go well beyond the satiation level, where we reach the ZLB. We examine such policies in an earlier draft of this paper, Bacchetta et al. (2015). Such a large monetary expansion can for example result from the central bank buying back a lot of government debt or providing liquidity support to the government that obviates the need for new government borrowing. In both cases government debt to the private sector is reduced and replaced by monetary liabilities. The present discounted value of seigniorage does not change when the money supply increases beyond the satiation level as eventually this expansion needs to be unwound, assuming that the economy cannot be at the ZLB forever.[24] Nonetheless the advantage of this policy is that the consolidated government does not pay default premia on monetary liabilities as it does on government debt in a bad equilibrium.

We find that such policies are only viable if for a substantial period of time, lasting at least through time $T$, we are at a structural ZLB in that the natural real rate is zero or negative. An expansion of money beyond the satiation level that is unwound prior to time $T$ is not helpful as the saving from not paying default premia is offset by a capital loss associated with a gradual drop in the government bond price. For example, if the consolidated government buys back its bonds at time zero and then at time $T-1$ issues new bonds in order to unwind the monetary liabilities, it buys at a higher price then it sells. This problem does not arise if the monetary liabilities are unwound, and new government bonds sold, after time $T$. In that case the policy is effective because no default premia are paid on monetary liabilities. But in practice such a policy is only feasible if the natural rate is close to zero for a long time. If we are not close to a structural ZLB, so that the natural real rate is well above zero, lowering the nominal interest rate to zero for a long period is deflationary. Under flexible prices it implies a negative inflation rate. Under sticky prices deflation tends to come at the cost of a substantial recession.[25]

[24]This will be the case when the natural real interest rate eventually becomes positive.

[25]The consumption Euler equation without habit formation implies that $E_0 \sum_{t=1}^{T} \pi_t + \sigma E_0(y_T -$

## 5.3 Sterilized Debt Purchases

The central bank could potentially sterilize the purchase of government debt by the sale of other securities. There are two ways in which one can imagine the sterilized purchase of government debt by the central bank, which does not increase monetary liabilities and is therefore not inflationary. One case is where the central bank has assets other than government debt on its balance sheet, such as for example gold or foreign exchange reserves. The central bank can purchase government debt, sell gold or reserves, and keep the money supply unchanged. This typically is not a solution though, as central banks' other assets are small compared to government debt.[26] More generally, this illustrates that we should look at government debt as a net concept, subtracting any assets (other than government bonds) that either the central bank or the government itself has on its balance sheet. When this net government debt is in the multiplicity region, the economy becomes subject to self-fulfilling debt crises.

The second case of sterilization applies to a monetary union, where the central bank intervenes in a crisis in its periphery. In the context of the Eurozone it would involve the ECB intervening in a self-fulfilling crisis in its periphery. In that case the central bank can be quite effective. For example, the ECB could buy periphery debt at a low interest rate and sell German debt, without a change in monetary liabilities. The threat alone of doing so is sufficient, which is, in our view, what happened under the OMT policy in the summer of 2012 and the famous Draghi statement "to do whatever it takes". This threat was credible as such an intervention would not overwhelm the ECB.[27] This explains why sovereign spreads quickly fell due to the change in policy. But such a policy applies to a periphery and would not work if the ECB aimed to avoid a self-fulfilling sovereign debt crisis across the entire Eurozone.

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$y_0) = -Tr^n$ when the nominal interest rate is zero, so that in general there will be a combination of falling prices and a drop in output.

[26]There may however be special cases. For example, Switzerland has FX reserves that are larger than government debt.

[27]For example, in 2010 the sum of all the periphery country government deficits together (Greece, Ireland, Portugal, Spain, Italy) amounted to 13% of the ECB balance sheet. And a self-fulfilling default can be avoided even if only a portion of these financing needs are covered by the ECB.

# 6 Conclusion

Several recent contributions have derived analytical conditions under which the central bank can avoid a self-fulfilling sovereign debt crisis. Extreme central bank intervention, generating extraordinary inflation, would surely avoid a sovereign debt crisis. But the cost would be excessive, making such actions not credible. To address the credibility of such policies, we have adopted a dynamic model with many realistic elements that allow for an attempt of quantitative assessment. We introduced a New Keynesian model with nominal rigidities in which monetary policy has realistic effects on output and inflation. We introduced long-term bonds in a slow-moving debt crises model and calibrated the debt maturity to what is observed in many industrialized countries. Overall our conclusion is that, in most cases, the ability of the central bank to avert self-fulfilling crises is limited. Unless debt is close to the bottom of an interval where multiple equilibria occur, monetary policy leads to very high inflation for a sustained period of time.

Several extensions are worthwhile considering for future work. We have focused on a closed economy. In an open economy monetary policy also affects the exchange rate, which affects relative prices and output. Related to that, one can also consider the case where a large share of the debt is held by foreigners or is denominated in a foreign currency. Both might make inflationary central bank policy an even less attractive option. Default is more appealing to the extent that the debt is held by foreigners, while in the case of foreign currency denominated debt a depreciation increases the value of debt denominated in domestic currency. Finally, we have only considered one type of self-fulfilling debt crises, associated with the interaction between sovereign spreads and debt. It would be of interest to also consider rollover crises or even a combination of both types of crises. This also provides an opportunity to consider the endogenous maturity of sovereign debt. As pointed out in the introduction, high inflation can reduce the maturity of government debt, which can lead to exposure to rollover crises.

# Appendix

We derive the debt accumulation schedule in the monetary economy in Section 2.2. It is straightforward to simplify this case to get the schedule for the real economy in Section 2.1. The debt accumulation schedule in the cashless economy (Section 2.2) is a special case of this where $M_t = 0$ at all times. We first derive a relationship between $Q_0$ and $Q_{T-1}$. Integrating forward the one-period arbitrage equation (4) from $t = 1$ to $t = T - 1$, we have:

$$Q_0 = A^{\kappa}\kappa + A^{Q}Q_{T-1} \tag{42}$$

where

$$\begin{aligned} A^{\kappa} &= \frac{1}{R_0} + \frac{1-\delta}{R_0R_1} + \frac{(1-\delta)^2}{R_0R_1R_2} + ... + \frac{(1-\delta)^{T-2}}{R_0R_1R_2...R_{T-2}} && (43) \\ A^{Q} &= \frac{(1-\delta)^{T-1}}{R_0R_1R_2...R_{T-2}} && (44) \end{aligned}$$

Next consider the consolidated budget constraint (12):

$$Q_tB^p_{t+1} = ((1-\delta)Q_t + \kappa)B^p_t - v_t - s_tP_t \tag{45}$$

where $v_t = [M_t - M_{t-1}]$. The government budget constraint at $t = 0$ is:

$$\frac{Q_0B^p_1}{P_0} = ((1-\delta)Q_0 + \kappa)\, b^p_0 - \overline{s} - \frac{v_0}{P_0} \tag{46}$$

For $1 < t < T$

$$\frac{Q_tB^p_{t+1}}{P_t} = r_{t-1}\frac{Q_{t-1}B^p_t}{P_{t-1}} - \overline{s} - \frac{v_t}{P_t} \tag{47}$$

Using equations (47) and (46) and integrating forward, we obtain

$$\begin{aligned} \frac{Q_{T-1}B^p_T}{P_{T-1}} &= r_{T-2}...r_1r_0\frac{Q_0B^p_1}{P_0} - \overline{s}(1 + r_{T-2} + r_{T-2}r_{T-1} + ... + r_{T-2}r_{T-1}...r_1) \\ &- \left[r_1...r_{T-2}\frac{v_1}{P_1} + r_2...r_{T-2}\frac{v_2}{P_2} + ... + \frac{v_{T-1}}{P_{T-1}}\right] && (48) \end{aligned}$$

Combining equation (48) with (46) and (42), we obtain:

$$\begin{aligned} \frac{Q_{T-1}B^p_T}{P_{T-1}} &= r_{T-2}...r_1r_0(1-\delta)b^p_0Q_0 + r_{T-2}...r_1r_0\kappa b^p_0 && (49) \\ &\quad -\overline{s}(1 + r_{T-2} + r_{T-2}r_{T-3} + ... + r_{T-2}...r_1r_0) \\ &\quad - \left[r_0...r_{T-2}\frac{v_0}{P_0} + r_1...r_{T-2}\frac{v_1}{P_1} + ... + \frac{v_{T-1}}{P_{T-1}}\right] \end{aligned}$$

Using equations (42)-(44), we can rewrite equation (49) as

$$\begin{aligned}\frac{Q_{T-1}B_T^p}{P_{T-1}} &= \frac{P_0}{P_{T-1}}(1-\delta)^T b_0^p Q_{T-1} \\ &+r_{T-2}...r_1 r_0\left[1+\frac{1-\delta}{R_0}+\frac{(1-\delta)^2}{R_0R_1}+...+\frac{(1-\delta)^{T-1}}{R_0R_1R_2...R_{T-2}}\right]\kappa b_0^p \\ &-\overline{s}(1+r_{T-2}+r_{T-2}r_{T-3}+...+r_{T-2}...r_1r_0) \\ &-\left[r_0...r_{T-2}\frac{v_0}{P_0}+r_1...r_{T-2}\frac{v_1}{P_1}+...+\frac{v_{T-1}}{P_{T-1}}\right]\end{aligned} \tag{50}$$

Using the expression for $v_t$, the last term in brackets is equal to $\tilde{m}$ as defined in (13). This yields (20).

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