On the joint evolution of culture and institutions*

Alberto Bisin†
New York University and NBER
alberto.bisin@nyu.edu

Thierry Verdier‡
Paris School of Economics and CEPR
tv@pse.ens.fr

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Abstract

What accounts for economic growth and prosperity? What stands at their origin? Recent literature typically searches for single univariate causal explanations: institutions, culture, human capital, geography. In this paper we provide instead a first theoretical modeling of the interaction between different possible explanations for growth and prosperity (in particular, between culture and institutions) and their effects on economic activity. Depending on the economic environment, culture and institutions might complement each other, giving rise to a *multiplier* effect, or on the contrary they can act as substitutes, contrasting each other and limiting their combined ability to spur economic activity. By means of examples we show how the dynamics display non-ergodic behavior, cycles, and other interestingly complex phenomena.

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†Department of Economics, New York University, 19 West Fourth Street, New York, NY, 10012, USA; NBER.
‡Sciences Economiques, Paris School of Economics, 48 Boulevard Jourdan, 75014 Paris, France; CEPR.

1 Introduction

"Era questo un ordine buono, quando i cittadini erano buoni [...] ma diventati i cittadini cattivi, divento' tale ordine pessimo."[1]; Niccolo' Machiavelli, *Discorsi,* I. 16, 1531

The distribution of income across countries in the world is very unequal: according to World Bank data 2011, U.S. GDP per capita in international dollars is 129 times that of the Democratic Republic of Congo, 44 times that of Mali, 13 times that of India and 4 times that of Brazil, for instance. But what makes a poor country poor and a rich country rich? What accounts for economic growth and prosperity? What stands at its *origin*?

The question of *origin* is typically translated, in the economic literature, into one of causation in the language statistics and econometrics. Furthermore, often a single univariate cause is searched for and different possible causes are run one against each other. Acemoglu and Robinson (2012), for instance, argue explicitly against each one of several potential causes (geography, culture, ignorance; Ch. 2, *Theories that don't work*) before laying their argument in favor of institutions in the rest of the volume.

This is not always devoid of problems.[2] More in detail, consider as for instance the arguments for institutions in the literature. They essentially rely on historical natural experiments where institutions are varied in geographical units with common geographical characteristics, culture, and other possible socio-economic determinants of future prosperity.[3]

In many of these examples the assumption that the distinct institutions originated in the natural experiment arise in otherwise common cultural, geographical, environments is disputable. For instance, settlers' mortality rates could be correlated with natives mortality rates and hence pre-colonial development (see e.g., Alsan, 2012, on the habitat for the Tse-Tse fly in Africa).

[1] This was a good institutional order when citizens were good [...] but when citizen became bad, it turned into an horrible order; our translation

[2] Besides the arguments following, there are also methodological reasons to be skeptical about the concept of causation when facing slow-moving non-stationary processes (as, arguably is long-run history). For instance, the origin of the Mafia in Sicily has been reduced with good arguments to a price shock on sulfur an lemon in the 1850's (Buonanno, Durante, Prarolo, and Vanin, 2012); to the lack of city states in the XIV'th century - in turn a consequence of Norman domination (Guiso, Sapienza, and Zingales, 2007); to the Paleolithic split into nomadic pastoralism in 7th millenium B.C. (Alinei, 2007).

We might even suggest ironically that a single origin of economic growth and prosperity is a myth, like the one about the birth of all languages after the Christian God's destruction of the Babel's Tower, an "event" which was indeed "accurately" dated, allegedly on May 5th, 1491 B.C. by James Ussher, in 1650.

[3] Successful examples include: the institutional design of colonial empires, the more extractive the higher settlers' mortality rates (Acemoglu and Robinson, 2001); the spanish colonial policy regarding the forced mining labor system in Peru' (Dell, 2010); the U.S.-Mexico border separating the city of Nogales (Acemoglu and Robinson, 2010); the border separating the island of Hispaniola into two distinct political and institutional systems, Haiti and the Dominican Republic (Diamond, 2010).

Furthermore, even the identification of the historical natural experiment as a change in institution is often debatable, as institutions generally reflect the cultural attitudes of the institution builders. Fischer (1989), for instance, studies institution formation during the early immigration waves in North America, showing how the cultural origins of the different groups of migrants (Puritans, Cavaliers, Quakers, Scots-Irish) affected the institutions they set in place; see also the well-know analysis by Greif (1994) of the institutional set-up of the Genoese and Maghrebi traders.[4] Finally, similar arguments have been produced for culture as the cause of prosperity, historically identifying instances of cultural variation in environments with a common institutional set-up.[5]

Most importantly for our paper, however, the empirical arguments used in this literature, even when not problematic, do not imply the existence of a single relevant cause. Instances where the same institutional change in a single political unit has had differential effects according to the different cultural environments have been extensively documented. The main reference in this respect of course is the work of Putnam on social capital, following the differential effects in the North and in the South of Italy of the institutional decentralization of the 60's and 70's (Putnam, 1993).[6] More generally, instances where institutions and cultural traits have manifestly jointly contributed to the development or the disruption of economic activity are common. This is the case for instance of Italian independent city states in the Renaissance (Guiso, Sapienza, and Zingales, 2007, 2008), industrialization and social capital in Indonesia (Miguel, 2003), the technology of plough, patriarchal institutions and gender attitudes (Alesina and Giuliano, 2011), the authoritarian culture of the sugar plantation regions of Cuba operated with slave labor as opposed to the with liberal culture of the tobacco farms (Ortiz, 1963).[7]

Even the presumption that culture is fundamentally immutable in the relevant time-frame, that is, changing at a much more slower pace than institutions, seems unfounded. Attitudes

[4]Even the institutional changes in Medieval England from the Magna Charta onwards, which arguably stand at the origin of Bristish prosperity and the Industrial Revolution (Acemoglu and Robinson, 2010) could be attributed to a general bourgeois culture as forcefully argued by McCloskey (2006, 2010). The same can be said for the formation of Italian independent city states in the Renaissance (Guiso, Sapienza, and Zingales, 2007 and 2008).

[5]While perhaps the first example of such kind of analysis is Weber's protestant ethic arguments (Weber, 1930), recent examples include the effects of the slave trade on trust within african tribes differently exposed to it but with similar institutional set-up (Nunn and Wantchekon, 2009); individual values about the scope of application of norms of good conduct in Europe (Tabellini, 2008a).

[6]Along similar lines, more recently, see e.g., Durante, Labartino, and Perotti, 2011, on university reform in Italy; Nannicini, Stella, Tabellini, and Troiano, 2010 on voting reform again in Italy; Mauro and Pigliaru (2012) on how culture has different effects when political institutions are centralized or decentralized; Grosjean (2011) on the traditonal (Scottish-Irish) pastoral society honor code in the U.S.; Minasyan (2014) on the effects of development aid institutions depending on donor-recipient cultural differences.

[7]Relatedly, there is also evidence on the complementarity between culture and organization of firms: see e.g., La Porta, Lopez de Silanes, Shleifer, and Vishny (1997) on the effect of trust on firm size; Aghion, Algan, Cajuc, and Shleifer (2010) on the complementarity between distrust and regulation in a model with multiple equilibria; Bloom, Sadun, and Van Reenen (2012) on the organization of firms across countries, and in particular its relationship to culture.

towards redistribution after the institition of welfare states in Europe, for instance, (Alesina and Angeletos, 2005; Alesina and Giuliano, 2010) and in East Germany after unification (Alesina and Fuchs Schuendeln, 2005) also changed very rapidly. So did in various instances the applications of the honor code studied by Appiah (2010). This is also arguably the case for social/civic/human capital after colonization (Glaeser, La Porta, Lopez-de-Silanez, Shleifer, 2004; Easterly and Levine, 2012; and Bisin and Kulkarni, 2012).[8]

Motivated by (this reading of) this literature, therefore, we study socio-economic environments in which culture and institutions jointly evolve and interact. In these environments the *origin*, and hence the causation, question loses most of its interest: culture and institutions are jointly and endogenously determined and they jointly affect economic growth and prosperity, indeed all sorts of economic activity.[9] The focus is moved from the cause (both culture and/or institution can have causal effects) to the process as determined by the interaction.

The objective is to develop an abstract model of culture, institutions, and their joint dynamics. While we aim at an abstract model, we are not after full generality. Rather we aim at a simple model which could help identify conditions under which the interaction of culture and institutions produces specific outcomes of interest.

By means of specific examples, we shall caharacterize conditions under which new institutions set forth cultural and institutional dynamics which reinforce the socio-economic equilibrium pattern the new institutions were designed to achieve and economies in which on the contrary the interaction of culture and institutions ends-up weakening the desired socio-economic equilibrium pattern. In other words, even when studying environments in which the causal effect is indeed from new institutions to economic activity, such effect depends on whether the appropriate culturtal traits develop to support the new institutions. In this context we can define the *cultural multiplier*, a the ratio of the total effect of intitutional change on prosperity divided by the direct effect, that is, the counterfactual effect which would have occurred had the distribution of cultural traits in the population remained constant after the instritutional change. Of couse the *cultural multipler* can be positive or negative - in the first case we say *culture and institutions are complementary*, while in the second case say they are *substitutes*. Of course, were the cause cultural we could similarly define an *institutional multiplier*.

We shall also study example economies displaying non-ergodic behation, in which initial conditions determine important qualitative properties of the evolution of culture and institutions, as well as of the stationary state the process converges to. Finally, we shall indicate how cycles and other interestingly complex behavior can emerge from the interaction of culture and institutions.

[8] An example of a rapid joint change of instututions and culture induce by pro-active policies is the case of the fight against corruption in Hong Kong in the last decades which was driven by institutional change but engendered a deep modification of norms and attitudes towards corruption in the population in just a few years (Clark, 1987 and 1989; see also Hauk and Saez-Marti', 2002).

[9] This view is already clear in Machiavelli's *Principe*, as the quote at the outset demonstrates.

We proceed, in turn, with an abstract model of the dynamics of of institutions (Section 2) and then with an abstract model of cultural evolution (Section 3). We then study the interaction of the two (Section 4). Finally, a series of examples aim at illustrating the analyis and the different forms of interactions (Section 5).

2 A simple model of the dynamics of institutions

We conceptualize institutions as mechanisms through which social choices are delineated and implemented. This is in line with the recent work effort by Acemoglu, Johnson, and Robinson in various pathbreaking contributions (surveyed in Acemoglu, Johnson, and Robinson, 2006) on economic and political institutions. In their view, political institutions are mechanisms for the distribution of political power across different socio-economic groups. It is in turn political power which determines economic institutions which govern (incentivize and constrain) economic activity. In most of their analysis, political institutions represent the mechanism through which the conflict between de jure and de facto political power is resolved into a social choice problem whose manifestation solution are specific economic institutions. More specifically, in Acemoglu (2003) e.g., institutions are represented by an indicator of which political pressure group has the power to control social choice. Institutional change is then the result of voluntary concessions by the controlling group typically under threats of social conflict.[10] This is also the approach taken by Acemoglu and Robinson (2006) to study more specifically the shift between dictatorship and democracy and viceversa. More generally, institutional change can represent an effective commitment mechanism on the part of one political group to extract resources from the others; this is the case, for instance in Besley and Persson (2009a,b, 2010), who study a society with pressure groups alternating in the power to control economic institutions regarding taxation and contractual enforcement.[11] While we share with this literature the view of institutional change as a commitment mechanism, we depart from its notion of political power and control as embedded in one single group. Specifically, we model institutions as Pareto weights associated to the different groups in the social choice problem.[12] This allows us to view institutional change as more incremental (formally, a continuous rather than a discrete change in political control) than just revolutions and regime changes. It also allows us to eschew relying necessarily on social conflict as an explanation of institutional change: institutional change can much more generally

[10] Levine and Modica (2012) and Belloc and Bowles (2012) take a different, explicitly evolutionary, approach to the dynamics of institutions.

[11] Along these lines, Angelucci and Meraglia (2013) study charters to city states in the early Renaissance in Europe as concessions from the king to citizens to check and control the extractive power of fiscal bureaucracies.

[12] See also Guimaraes and Sheedy (2010) who ground the study of institutions in the theory of coalition formation; and Lagunoff (2008) who provides a general study of the theoretical properties of political economy equilibria with dynamic endogenous institutions.

occur as a mechanism to imperfectly and indirectly internalize the lack of commitment and the externalities which plague the social choice problem; social conflict being only one of them and not necessarily the most prevalent in history.[13]

Consider a society with a continuum of agents separated into distinct groups defined in terms of relevant characteristics, i.e., political power and cultural traits.[14] Let $i \in I$ index the political groups and $j \in J$ the cultural groups. Let a^{ij} denote the action of agents of subgroup (i,j) and $a = \{a^{ij}\}_{i,j}$ the vector profile of actions, which we assume lies in some compact set. Let p denote economic policy in society, also in some compact set.[15] Let q^{ij} denote the distribution of the population by cultural group, for each polititical group: the vector profile $q = \{q^{ij}\}_{i,j}$ satisfies $\sum_{j \in J} q^{ij} = 1$, for $i \in I$. [16]

The fraction of agents belonging to subgroup (i,j) have preferences represented by an indirect utility function:

$$u^{ij}\left(a^{ij}, p; a, q\right) \tag{1}$$

The dependence of u^{ij} on a captures indirectly any externality in the economy. The dependence of u^{ij} on q captures instead indirectly the dependence of technologies and resources on the distribution of the population by cultural groups. A natural example would have the externality being represented by the mean action in the population: $A = \sum_{i,j} a_{ij} \lambda_i q^{ij}$, with λ^i denoting the fraction of agents in political group i.

In this society, we identify political institutions with the weights of the groups $i \in I$ in the social choice problem which determines economic policies. Let $\beta^i \geq 0$, denote the weight associated to group i and $\beta = \{\beta^i\}_i$ the vector profile, which satisfies $\sum_{i \in I} \beta^i = 1$.

[13] E.g., Lizzeri and Persico (2004) challenge Acemoglu and Robinson (2000, 2001, 2003) and Conley and Temimi (2001)'s rationalization of the extension of the franchise in early nineteenth century England, as an effect of threats to the stablished order. They argue instead that such institutional change had been motivated by the necessary evolution of public spending which required a commitment to limit particularistic politics in favor of public programs.

[14] Groups can of course be defined also in terms of resources, technologies, and so on. But we shall abstract from these characteristics for simplicity in the paper.

[15] Of course policies might be multi-dimensional, an extension we avoid for simplicity. Also, without loss of generality we could add a parametrization of the component of economic institutions which acts directly on the economic environment. We avoid clogging the notation when not necessary.

[16] Important special cases of this set-up consist of economies in which the political and cultural group coincide. In this cases, the restriction $j = i$ is imposed and, with some abuse of notation, we only use the index i: a^i is the action of group i, q^i its fraction in the population, and so on.

2.1 Societal optimum and equilibria (given institutions and cultural distribution)

"[...] gli assai uomini non si accordano mai ad una legge nuova che riguardi uno nuovo ordine nella citta' se non e' mostro loro da una necessita' che bisogni farlo; e non potendo venire questa necessita' sanza pericolo, e' facil cosa che quella republica rovini, avanti che la si sia condotta a una perfezione d'ordine."[17]; Niccolo' Machiavelli, *Discorsi,* I. 2, 1531.

The *societal optimum given institutions* β *and cultural distribution* q is a tuple $\{a^{eff}, p^{eff}\}$ such that:

$$\{a^{eff}, p^{eff}\} \in \arg\max \sum_i \beta^i \sum_j q^{ij} u^{ij}\left(a^{ij}, p; a, q\right) \tag{2}$$

The social optimum will be generally unattainable in our economy. We introduce instead two distinct equilibrium concept which will play a fundamental role in our analysis. The *societal equilibrium given institutions* β *and cultural distribution* q is a tuple $\{a, p\}$ such that:

$$\begin{array}{l} p \in \arg\max_p \ \sum_i \ \beta^i \sum_j q^{ij} u^{ij}\left(a^{ij}, p; a, q\right) \\ a^{ij} \in \arg\max \ u^{ij}\left(a^{ij}, p; a, q\right) \quad i \in I, \ j \in J. \end{array} \tag{3}$$

That is, the *societal equilibrium* is a Nash equilibrium of the societal game between agents of the two groups and the policy maker operating in an institutional set-up characterized by weights β and cultural distribution q. Note that this simple formulation of the *societal equilibrium* in fact captures lack of commitment on the part of the policy maker, who is not allowed to pick the policy p in advance of the choices of the economic agents.[18]

To model a policy maker with commitment, we define instead the *societal commitment equilibrium given institutions* β *and cultural distribution* q as the Stackelberg Nash equilibrium of the same game, where the policy maker is assumed to be the leader; that is, as a tuple $\{a^{com}, p^{com}\}$ such that:

$$\begin{array}{ll} \{a^{com}, p^{com}\} & \in \arg\max \ \sum_i \ \beta^i \sum_j q^{ij} u^{ij}\left(a^{ij}, p; a, q\right) \\ & s.t. \quad a^{ij} \in \arg\max \ u^{ij}\left(a^{ij}, p; a, q\right), \quad i \in I, \ j \in J \end{array} \tag{4}$$

Under general conditions the *societal optimum,*[19] the *societal equilibrium,* and the *societal commitment equilibrium* are distinct. More precisely,

[17] [...] the majority of people will never agree to a new institutional order for the city unless necessary; and since necessity cannot come without danger, it is easily the case that institutions get into ruins before being perfected in a new order; our translation.

[18] No issues other than notational ones are are involved in modeling a policy maker choosing after the economic agents, thereby strengthening its lack of commitment.

[19] In the interest of lightness, we drop, from now on the qualifier "given institutions β and cultural distribution q" when referring to the equilibrium concepts in the paper.

Proposition 1 *Given any institutions β and cultural distribution q, the* societal equilibrium *and the* societal commitment equilibrium *are both weakly inefficient, that is, they are weakly dominated by the* societal optimum. *On the other hand, the* societal commitment equilibrium *weakly dominates the* societal equilibrium.

Proof. The statement is a straightforward consequence of the fact that, for any (β, q): i) problem (4), which defines a *societal commitment equilibrium* , is a constrained version of problem (2), which in turn defines a *societal optimum*; ii) any *societal equilibrium* satisfying (3) is always contained in the the constrained feasible set of problem (4), which defines a *societal commitment equilibrium*.[20] ■

Importantly, *societal equilibrium* and *societal commitment equilibrium* do not coincide even without the externality, that is, when

$$u^{ij}\left(a^{ij}, p; a, q\right) = u^{ij}\left(a^{ij}, p; q\right) \text{ for any } i \in I,\ j \in J.$$

Even without the externality, it might still be the case in fact that inefficiencies are generated by the policy maker's lack of commitment.

2.2 Institutional design (given cultural distribution)

Future political and economic institutions are designed each generation by the present institutional set-up. We assume that institutional design is myopic, that is, institutions are designed for the future as if they would never be designed anew in the forward future.[21]

In this paper we restrict for simplicity to dychotomous groups, that is $I = \{1, 2\}$ and $J = \{a, b\}$.[22] In this case, and making the dependence on (β, q) explicit, the *societal equilibrium*, the *societal commitment equilibrium*, and the *societal optimum* can be denoted, respectively:

$$[a(\beta, q), p(\beta, q)]\,; \quad [a^{com}(\beta, q), p^{com}(\beta, q)]\,; \quad \left[a^{eff}(\beta, q), p^{eff}(\beta, q)\right]$$

A simple formulation of the design and hence of the dynamics of institutions can be obtained under the following regularity assumption.

Assumption 1 *Utility functions are sufficiently regular so that*
$a(\beta, q)$, $p(\beta, q)$, $a^{com}(\beta, q)$, $p^{com}(\beta, q)$ *are continuous functions.*

[20] Of course, under robust conditions - in particular in all examples we study - domination holds strictly.

[21] We shall relax this assumption later on. We believe the assumption is interesting per se, however. [....more....]

[22] With more than two groups the issue of coalition formation in institutional set-up and change becomes central. We leave this for a subsequent paper. The dynamics of $n \geq 2$ cultural traits has been studied by Bisin, Topa and Verdier (2009) and Montgomery (2009).

Assumption 1 requires obvious but stringent monotone comparative statics requirements for *societal equilibria*.[23]

Adding an index t to denote time, institutions evolve as a solution to the following design problem:

$$\max_{\beta_{t+1}} \sum_{i\in I} \beta_t^i \sum_{j\in J} q_{t+1}^{ij} u^{ij}\left(a^{ij}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1})\right) \tag{5}$$

Since I is dychotomous, and $\sum_i \beta^i = 1$, the dynamics of institutions β is fully determined by the dynamics of a single β^i.

Proposition 2 *Under Assumption 1, and given (q_t, q_{t+1}), the dynamics of institutions β_t^i, $i \in I$, is governed by the following implicit difference equation:*

$$\beta_{t+1}^i = \begin{cases} \beta^i \text{ such that } p^{com}(\beta, q_{t+1}) = p(\beta_t, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{ll} \arg\max p(\beta, q_{t+1}) & \text{if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}),\ \forall 0 \le \beta^i \le 1 \\ \arg\min p(\beta, q_{t+1}) & \text{if } p^{com}(\beta_t, , q_{t+1}) < p(\beta, q_{t+1}),\ \forall 0 \le \beta^i \le 1 \end{array}\right. & \text{else} \end{cases} \tag{6}$$

These dynamics can be intuitively interpreted as follows. At any time t, current institutions β_t induce the choice $p(\beta_t, q_t)$ at equilibrium. But they would rather prefer the choice $p^{com}(\beta_t, q_t)$. Therefore, when designing institutions for time $t+1$, current institutions design ("delegate to") institutions guaranteeing $p^{com}(\beta_t, q_{t+1})$ whenever possible at equilibrium; that is, they design ("delegate to") institutions β_{t+1} such that $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_{t+1})$. Whenever this is not possible, under our assumptions, they will design ("delegate to") institutions guaranteeing at equilibrium a policy choice p as close as possible to $p^{com}(\beta_t, , q_{t+1})$.

The stationary states of the dynamics of institutions and their stability properties can be simply characterized under the following assumption.

Assumption 2 *Utility functions are sufficiently regular so that $p(\beta, q)$ is monotonic in β.*

Assumption 2 implies that the extremal stationary states can only correspond to the corners of the dynamics; that is, $\beta^i = 0, 1$.[24] As a consequence, under Assumption 2, the dynamics of institutions in (6) are then reduced to the following:

$$\beta_{t+1}^i = \begin{cases} \beta^i \text{ such that } p^{com}(\beta, q_{t+1}) = p(\beta_t, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{ll} 1 & \text{if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}),\ \forall 0 \le \beta^i \le 1 \\ 0 & \text{if } p^{com}(\beta_t, q_{t+1}) < p(\beta, q_{t+1}),\ \forall 0 \le \beta^i \le 1 \end{array}\right. & \text{else} \end{cases} \tag{7}$$

It is convenient to define $P(\beta, q) := p(\beta, q) - p^{com}(\beta, q)$.

[23] In the Appendix we spell out the regularity conditions imposed on utility functions to guarantees the desired comparative statics properties.

[24] See the Appendix for a formal discussion. When Assumption 2 is not satisfied the dynamics in (6) might generally be undetermined, as (6) defines a correspondence rather than an equation.

Proposition 3 *Under Assumption 1-2, for any given q, the dynamics of institutions governed by (7) have at least one stationary state. An* interior stationary states β^* *obtains as a solution to $P(\beta, q) = 0$. The* boundary stationary state $\beta^i = 1$ *obtains when $P(\beta, q) \mid_{\beta^i=1} < 0$; while the* boundary stationary state $\beta^i = 0$ *obtains when $P(\beta, q) \mid_{\beta^i=0} > 0$.*[25]

In the following proposition we collect some of the stability properties of (7) which are most relevant in our subsequent analysis. A more complete global stability analysis is not particularly complex but is tedious. We relegate it to the Appendix.

Proposition 4 *Under Assumption 1-2, for any given q, in the continuous time limit, the dynamics governed by (7) satisfies the following properties:*

if $P(\beta, q) < 0$ for any $\beta^i \in [0, 1]$, then $\beta^i = 1$ is a globally stable stationary state.

if $P(\beta, q) > 0$ for any $\beta^i \in [0, 1]$, then $\beta^i = 0$ is a globally stable stationary state;

any boundary stationary state is always locally stable;

if an interior stationary state β^* exists, it is locally stable if $\frac{\partial P(\beta^*, q)}{\partial \beta^i} > 0$.

2.3 Inefficient institutions

It is not generally the case in our set-up that institutions are efficient in a stationary state. By combining the results of Proposition 1 and Proposition 3 we obtain that a stationary *societal equilibrium* at best constitutes a *societal commitment equilibrium* for some institutions. In other words, the institutional dynamics provides a tendency towards efficiency but i) generally not all the way towards a *societal optimal* and ii) for a specific institutional set-up, that is, not necessarily towards a Pareto improvement. Several of the examples we study clearly demonstrate these points.

3 A simple model of the dynamics of cultural traits

We conceptualize culture as preference traits, norms, and attitudes which can be transmitted across generations by means of various socialization practices or can be acquired through socio-economic interactions between peers. Models of the population dynamics of cultural traits along these lines have been extensively studied in the social sciences and in biology.[26]

[25] Note that we arbitrarily define $\beta^i = 1$ (resp. $\beta^i = 0$) as an *interior stationary state* if $P(\beta, q^i) \mid_{\beta^i=1} = 0$ (resp. $P(\beta, q^i) \mid_{\beta^i=0} = 0$).

[26] For an economic approach see a series of papers by Bisin and Verdier (1998, 2000a, 2001a) which build on the work of Cavalli Sforza and Feldman (1973, 1981) in evolutionary biology and of Boyd and Richerson (1985) in anthropology; see Bisin and Verdier (2010) for a recent survey. We briefly introduce them here again for completeness and we refer the reader to the survey for the many details and extensions omitted here.

Cultural transmission is modeled as the result of *direct vertical* (parental) socialization and *horizontal/oblique socialization* in society at large. Fix a political group $i \in I$. Then

i) direct vertical socialization to the parent's trait, say $j \in J$, occurs with probability d^{ij};

ii) if a child from a family with trait j at time t is not directly socialized, which occurs with probability $1 - d^{ij}$, he/she is horizontally/obliquely socialized by picking the trait of a role model chosen randomly in the population inside the political group (i.e., he/she picks trait j with probability q_t^{ij} and trait $j' \neq j$, $j, j' \in J$, with probability $q_t^{ij'} = 1 - q_t^{ij}$).[27]

If we let $P_t^{i,jj}$ (resp. $P_t^{i,jj'}$) denote the probability that a child, in (a family in) political group $i \in I$ with trait j is socialized to trait j (resp. j') at t, we obtain:

$$\begin{aligned} P_t^{i,jj} &= d^{ij} + (1 - d^{ij}) q_t^{ij} \\ P^{i,jj'} &= (1 - d^{ij})(1 - q_t^{ij}) \end{aligned}$$

Let $V^{i,jj}(\beta_{t+1}, q_{t+1})$ (resp. $V^{i,jj'}(\beta_{t+1}, q_{t+1})$) denote the utility to a cultural trait j parent in political group i of a type j (resp. j') child. They depend on the institutional set-up and the cultural distribution the child will face at time $t+1$, when he/she will make his/her economic decision a^{ij}:

$$V^{i,jj}(\beta_{t+1}, q_{t+1}) = u^{ij}\left(a^{ij}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1}), q_{t+1}\right) \quad (8)$$

$$V^{i,j\neq j}(\beta_{t+1}, q_{t+1}) = u^{ij}\left(a^{ij'}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1}), q_{t+1}\right) \quad (9)$$

Let $C(d^{ij})$ denote socialization costs. Direct socialization is then the solution to the following parental socialization problem:

$$\max_{d^{ij} \in [0,1]} -C(d^{ij}) + P_t^{i,jj} V^{i,jj}(\beta_{t+1}, q_{t+1}) + P_t^{i,jj'} V^{i,jj'}(\beta_{t+1}, q_{t+1}), \text{ s. t. } 1) \ .$$

Calling $\Delta V^{ij}(\beta_{t+1}, q_{t+1}) = V^{i,jj}(\beta_{t+1}, q_{t+1}) - V^{i,j\neq j}(\beta_{t+1}, q_{t+1})$, the *cultural intolerance* of trait j in political group i, it follows that the direct socialization, with some notational abuse, has the form:

$$d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1})) = d^{ij}(\beta, q), \ i \in I, \ j \in J \quad (10)$$

Assumption 3 *Utility and socialization cost functions are sufficiently regular so that* $d^{ij} = d^{ij}(\beta, q)$ *is continuous.*

[27] We assume therefore for simplicity that political groups are perfectly segregated, so that the reference population for an agent in subgroup (i, j) is the subgroup itself. This is just for simplicity and extensions are straightforward.

The dynamics of the distribution of the population by cultural trait q is straightforwardly determined. Since J is dychotomous, and $\sum_j q^{ij} = 1$, $i \in I$, the dynamics of institutions q is fully determined by the dynamics of the q^{ij} for each of the political groups $i \in I$.

Proposition 5 *Under Assumption 3, and given β_{t+1}, the dynamics of culture q_t^{ij} is governed by the following difference equation:*

$$q_{t+1}^{ij} - q_t^{ij} = q_t^{ij}(1 - q_t^{ij}) \left(d^{ij} - d^{ij'} \right). \tag{11}$$

evaluated at $d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1}))$ satisfying (10).

It is convenient to define $D^{ij}(\beta, q) := d^{ij}(\beta, q) - d^{ij'}(\beta, q)$.

Proposition 6 *Under Assumption 3, for any given β, the dynamics of institutions governed by (11) have at least the two boundary stationary states, $q^{ij} = 0$ and $q^{ij} = 1$. An* interior stationary states $0 < q^{ij*} < 1$ *obtains as a solution to $D(\beta, q) = 0$.*

In the following proposition we collect some of the stability properties of (11) which are most relevant in our subsequent analysis. As in the case of the dynamics of institutions (7), a more complete global stability analysis is relegated to the Appendix.

Proposition 7 *Under Assumption 3, for any given β, in the continuous time limit, the dynamics governed by (11) satisfies the following properties:*

- *if $D^{ij}(\beta, q) > 0$ for any $q^{ij} \in [0, 1]$, then q_t^{ij} converges to $q^{ij} = 1$ from any initial condition $q_0^{ij} > 0$;*
- *if $D^{ij}(\beta, q) < 0$ for any $q^{ij} \in [0, 1]$, then q_t^{ij} converges to $q^{ij} = 0$ from any initial condition $q_0^{ij} < 1$;*
- *if $D^{ij}(\beta, 1) > 0$, then $q^{ij} = 1$ is locally stable ;*
- *if $D(\beta, 0) < 0$, then $q^{ij} = 0$ is locally stable;*
- *if an interior stationary state q^{ij*} exists, and $\frac{\partial D^{ij}(\beta, q^{ij*})}{\partial q^{ij}} < 0$, it is locally stable.*

It is often convenient to impose the following assumption (we do so in the examples as it simplifies the study of of the dynamics of culture essentially without loss of generality).

Assumption 4 *Socialization costs are quadratic:*

$$C(d^{ij}) = \frac{1}{2} \left(d^{ij} \right)^2.$$

The following corollary characterizes the simplification which can be obtained under quadratic socialization costs:

Corollary 1 *Under Assumption 4,*

$$D^{ij}(\beta, q) = \Delta V^{ij}(\beta, q) q^{ij'} - \Delta V^{ij'}(\beta, q) q^{ij},$$

and hence interior steady states are characterized by solutions to:

$$\frac{\Delta V^{ij}(\beta, q)}{\Delta V^{ij'}(\beta, 1-q)} = \frac{q^{ij}}{q^{ij'}} \tag{12}$$

4 Joint evolution of culture and institutions

Under Assumptions 1-3, the joint dynamics of institutions and culture is governed by the system (7,11), which we report here for convenience:

$$\begin{aligned}
\beta^i_{t+1} &= \begin{cases} \beta^i \text{ such that } p^{com}(\beta, q_{t+1}) - p(\beta_t, q_{t+1}) & \text{if it exists,} \\ \begin{bmatrix} 1 & \text{if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}),\ \forall 0 \leq \beta^i \leq 1 \\ 0 & \text{if } p^{com}(\beta_t, , q_{t+1}) < p(\beta, q_{t+1}),\ \forall 0 \leq \beta^i \leq 1 \end{bmatrix} & \text{else} \end{cases} \\
q^{ij}_{t+1} - q^{ij}_t &= q^{ij}(1 - q^{ij}_t) \left(d^{ij} - d^{ij'} \right), \text{ with } d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1})).
\end{aligned}$$

Even under the strong simplifying assumptions we imposed the non-linear dynamical system (7,11) can display complex dynamics, including cycles and chaos. Very little can be said in general and as a consequence we turn to phase-diagrams in specific examples. We can nonetheless show the following.

Proposition 8 *Under Assumptions 1-3 the dynamical system (7,11) has at least one stationary state. Furthermore, if both the institutional and the cultural dynamics display an interior stationary state, respectively, for all* $0 \leq q^{ij} \leq$ *and all* $0 \leq \beta \leq 1$*, then the dynamical system (7,11) has at least one interior stationary state.*

The proof is detailed in Appendix A, where also formal conditions for the institutional and the cultural dynamics to display at least one interior stationary state are explicitly derived.[28]

4.1 The cultural multiplier

We wish to analyze the comparative dynamics on institutions and culture induced by an exogenous shock to the parameter. More specifically, we wish to uncover conditions under which a shock

[28] Indeed Proposition 8 is re-stated more formally in the Appendix as Proposition A.**??**. To simplify notation, however, Appendix A is restricted to economies where $j = i$, which we introduced in footnote 16.

on institutions could be reinforced (or mitigated) in the long run by the induced dynamics of culture. To this end, we introduce and study the concept of *cultural multiplier*, the ratio of the long run change in institutions due to the shock over the counterfactual long run change that would have happened had the cultural composition of society remained fixed and not co-evolved with the institutions.

We restrict ourselves for notational simplicity to the special economies where political and cultural groups coincide, $j = i$, which we introduced in footnote 16. Furthermore, we take $I = 1, 2$ and, when useful, we adopt the shorthand $q = q^1, \beta = \beta^1$. We consider the case of an interior stable institutional-cultural steady state $(\beta^*, q^*) \in (0,1)^2$.

Let the parameter γ indicate the shock driving the comparative statics. The following assumption simplifies the analysis.

Assumption 5 *Agents' preferences satisfy the following separability condition,*

$$u^i\left(a^i, p; a, q^i\right) = v^i\left(a^i, p\right) + H^i(p; a, q^i), \qquad SP$$

Assumption 5 implies that the policy instrument p affects the optimal private actions, a^i, independently of the economy-level aggregates a and q^i. This in turn implies that the socialization incentives ΔV^i depend only on the equilibrium policy level p. Under 5 the steady state (β^*, q^*) is characterized by the following conditions

$$\begin{aligned} p(\beta, q, \gamma) &= p^{com}(\beta, q, \gamma) \\ \frac{q}{1-q} &= \frac{\Delta V^1(p)}{\Delta V^2(p)} \text{ and } p = p(\beta, q, \gamma) \end{aligned} \tag{13}$$

Let $\beta(q)$ be the steady state institutional manifold associated with the first equation in (13) and $q(\beta, \gamma) = q(p)$ with $p = p(\beta, q, \gamma)$, with some notational abuse, be the steady state interior cultural manifold associated with the second equation $.q(p)$ is actually a well defined function taking values in $[0,1]$.[29] Let as well $p^* = p(\beta^*, q^*, \gamma)$ be the steady state policy choice.

We impose the following, stronger than necessary, regularity conditions:[30]

$$p_\beta(\beta, q, \gamma) > 0 \text{ and } p^{com}_\beta(\beta, q, \gamma) > 0. \qquad ID$$

[29] The local stability of the interior steady state (β^*, q^*) of (13) is obtained under the standard Hessian conditions:

$$\begin{aligned} \left[p_\beta - p^{com}_\beta\right]_{(\beta^*, q^*)} &> 0 \\ 1 - \left[p_q \cdot \widehat{q}_p\right]_{(\beta^*, q^*)} &> 0 \\ \left[(1 - p_q \cdot \widehat{q}_p) \cdot \left[p_\beta - p^{com}_\beta\right] + \widehat{q}_p p_\beta \cdot \left(p_q - p^{com}_q\right)\right]_{(\beta^*, q^*)} &> 0 \end{aligned} \tag{14}$$

[30] Indeed, local conditions are typically sufficient; see Appendix B for details.

$$p_{\gamma}^{com}(\beta^*, q^*, \gamma) > p_{\gamma}(\beta^*, q^*, \gamma) \geq 0 \qquad G$$

Condition ID imposes a monotonic conflict of interest between the two groups in terms of the optimal policy p, designating group 1, without loss of generality, as the group whose members aim at a relatively larger policy level, p. Condition G essentially defines γ in terms of the sign of its effects: an increase in γ, without loss of generality, increases the equilibrium level of the implemented policy p^* around the steady state (β^*, q^*), and when associated to ID, it induces positive institutional dynamics on the weight β when there is no cultural change (i.e., when q remains fixed at its pre-shock value q^*). Indeed, at the steady state (β^*, q^*), the *societal commitment equilibrium* policy $p^{com}(\beta^*, q^*, \gamma)$ is equal to the *societal equilibrium* policy $p(\beta^*, q^*, \gamma)$. Under condition G, a positive shock on γ leads to a positive gap between the two policy outcomes. From (7)), some institutional dynamics away from β^* are triggered. To return to a new instittutional steady state, the local stability conditions of the system (fixing q at its pre-shock value q^*) imply that this policy gap between $p^{com}(.)$ and $p(.)$ should be reduced. This is achieved when the institutional dynamics give more weight to the group that promotes a larger value of the *societal equilibrium* policy $p(.)$. According to condition ID, this is group 1. Hence a larger value of the new steady state weight β of that group.

Definition 1 *The* Cultural multiplier *on institutional change μ, at an interior stable steady state (β^*, q^*) is*

$$\mu = \left(\frac{d\beta^*}{d\gamma}\right) / \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} - 1 \tag{15}$$

Then we have the following proposition characterizing when the *cultural multiplier* is positive or not:

Proposition 9 *Assume that conditions SP, G and ID hold and that (β^*, q^*) is an interior stable institutional-cultural steady state. Then the* cultural multiplier *μ is positive if and only if*

$$\left[p_q^{com}(\beta^*, q^*, \gamma) - p_q(\beta^*, q^*, \gamma)\right] \cdot q_p(p^*) > 0. \tag{16}$$

The *cultural multiplier* is closely connected to how institutional change and cultural change are dynamic complements or substitutes. This is apparent from the main elements of the proof.

Sketch of the proof. First of all, notice that, under local stability of the steady state, $\left[p_q^{com}(\beta^*, q^*, \gamma) - p_q(\beta^*, q^*, \gamma)\right] \geq 0$ (resp. < 0) implies $\beta_q(q) \geq 0$ (resp. < 0).

The comparative statics on (β^*, q^*) are easily(but tediously) obtained:

$$\left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} = \frac{\left(p_{\gamma}^{com} - p_{\gamma}\right)}{\left(p_{\beta} - p_{\beta}^{com}\right)} > 0$$

The last inequality comes from local stability of the steady state and from G. The total effect of $d\gamma$ on institutional change (i.e., taking into account the joint evolution with culture) can be written as:

$$\frac{d\beta^*}{d\gamma} = \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} + \frac{\left(p_q^{com} - p_q\right)}{\left(p_\beta - p_\beta^{com}\right)} \frac{q_p}{1 - p_q q_p} \frac{\frac{\left(p_\gamma^{com} - p_\gamma\right)}{\left(p_\beta - p_\beta^{com}\right)} p_\beta + p_\gamma}{1 - \frac{\left(p_q^{com} - p_q\right)}{\left(p_\beta - p_\beta^{com}\right)} q_p \frac{p_\beta}{1 - p_q q_p}}$$

The result now follows. ■

One can understand the intuition behind condition (16) by disentangling the implications of the two terms $IP = \left[p_q^{com}(\beta^*, q^*, \gamma) - p_q(\beta^*, q^*, \gamma)\right]$ and $q_p(p^*)$. These terms are respectively related to the sign of the slopes of the steady state institutional manifold $\beta(q)$ and the cultural manifold $q(\beta)$ derived from (13). Specifically, coupled with condition ID, a positive (resp. negative sign) sign of IP is associated with a positive (resp. negative) slope of $\beta(q)$. Intuitively, when $IP > 0$ (resp. < 0), there is a positive (resp. negative) association between institutions and culture in the sense that larger cultural groups have more (resp. less) political larger weights in steady state institutions. [31]

Similarly the sign of $q_p(.)$ determines if there is a positive or a negative long term cultural association with respect to a given policy p. When $q_p(.) > 0$ (resp. < 0), a higher level of the policy leads to a larger (resp. smaller) cultural diffusion of the trait that benefits more from that policy (ie. in our case group 1, given condition ID).

The concept of "cultural multiplier" is then related on how institutional change and cultural change are dynamic complements or substitutes. When the slopes of $\beta(q)$ and $q(p)$ have the same signs, institutional and cultural evolutions are dynamic complements and the cultural multiplier is positive. Suppose to fix ideas that both signs are positive. Then any positive shock on γ that triggers a positive impact on the institutional steady state weight β^*, subsequently induces a positive change in the *societal equilibrium* policy p (condition ID). Because of the positive cultural association to the policy (ie. $q_p(.) > 0$), This promotes the cultural diffusion of group 1that benefits from the increase in the policy. Because of the positive association between cultural group size and institutional representation ($ie. \beta_q(q) > 0$), this further increases the institutional steady state weight β^*, leading therefore to a positive cultural multiplier. Any shock that triggers some institutional change in one direction is further enhanced by the associated cultural dynamics that coevolve with the institutions.

Conversely, it is easy to see that institutional change is mitigated by cultural evolution (ie.

[31] A positive association is more likely to be expected in a perfect democratic voting system with large and informed political participation , while a negative association is perhaps more likely to exist when political institutions reflect more the lobbying power of small groups better able to overcome the traditional free rider problems of political organization.

the "cultural multiplier" is negative) when the slopes of $\beta(q)$ and $q(p)$ have opposite signs, [32]

Consider now an economic variable of interest, e.g., per capita income, public good provision, or anything else. The dynamics of this variable would depend entirely on the joint dynamics of institutions and culture. In the context of the model, it is straightforward to isolate the effects of culture from those of institutions and viceversa. Consider formally an aggregate variable: $A(p, q, a^1(p), a^2(p))$. The cultural multiplier on A is now defined as

$$\mu_A = \frac{dA}{d\gamma} \Big/ \left(\frac{dA}{d\gamma}\right)_{q=q^*} - 1.$$

We may decompose the effects of a shock as follows:

$$\frac{dA}{d\gamma} = \left\{ \underbrace{\left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_\beta}_{\text{direct effect}} + \underbrace{\left[A_q + \left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_q\right] \frac{q_p p_\beta}{1 - p_q q_p}}_{\text{indirect effect}} \right\} \frac{d\beta^*}{d\gamma}$$

The effect of γ on institutions will come from a direct effect as well as an indirect one. The direct effect in turn will be composed of two terms: a direct effect of the policy change induced by an institutional change p_β on the aggregate variable A (i.e., the term A_p), and the impact of changes in private actions $a^1(p)$ and $a^2(p)$ as induced also by the policy change p_β), the term $\left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right) p_\beta$). The indirect effect of cultural evolution will come from the compositional effect of changing the cultural group sizes (A_q), plus again the change in policy and private actions $\left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_q$ which such a cultural compositional change induces.

Furthermore,

$$\left(\frac{dA}{d\gamma}\right)_{q=q^*} = \left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_\beta \cdot \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*}$$

recalling the cultural multiplier as $\mu = \left[\left(\frac{d\beta^*}{d\gamma}\right) / \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} - 1\right]$, one obtains easily the cultural multiplier μ_A on the aggregate variable A. It is constituted of two terms:

$$\mu + \frac{\left[A_q + \left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_q\right] \frac{\widehat{q}_p p_\beta}{1 - p_q \widehat{q}_p}}{\left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_\beta} \cdot (1 + \mu) \tag{17}$$

[32] An *institutional multiplier* could be analogously defined as $\left(\frac{dq^*}{d\delta}\right) / \left(\frac{dq^*}{d\delta}\right)_{\beta=\beta^*} - 1$, for some parameter δ affecting only the cultural dynamics. The environment would have to be slightly extended to account for instance that the two groups have different socialization cost functions and δ would reflect the difference between these cost functions. The *institutional multiplier* will be positive under the same condition required for the *cultural multiplier*, (16).

This second term can be decomposed as two further components:

$$\left[\underbrace{\frac{A_q}{\left[A_p+\left(A_{a^1}a_p^1+A_{a^2}a_p^2\right)\right]}}_{\text{compositional effect of cultural change}}+\underbrace{p_q}_{\text{policy effect of cultural change}}\right]\cdot\underbrace{\frac{\widehat{q}_p}{1-p_q\widehat{q}_p}(1+\mu)}_{\text{institutional effect on cultural change}} \tag{18}$$

The intuition for all these terms is the following. Relative to what would be the effect on A of some institutional change when the cultural composition is considered as fixed (at the pre shock value $q=q^*$), there are three additional effects on the aggregate variable A. First there is the direct feedback effect of cultural change on institutions and how consequently this additional institutional change directly affects the variable A through a policy change. This is captured by the cultural multiplier μ as the first term in equation (17). The second term in (17) on the other hand reflects an additional multiplier effect on A that comes in turn from the effect of institutional change on cultural dynamics. Indeed the total effect of institutions on cultural change is captured by the factor term $\frac{\widehat{q}_p}{1-p_q\widehat{q}_p}(1+\mu)$ in (**??**) and can be understood in the following way. The total change in institutions $(1+\mu)$ triggers some policy change in p, which in turn leads to some total change in the cultural composition of the population $\widehat{q}_p/\left(1-p_q\widehat{q}_p\right)$. The induced effect of institutions on cultural diffusion has then two further implications on the aggregate variable A. First, there is the direct compositional effect of changes in cultural groups sizes on the aggregate outcome. Second, there is the effect of cultural change on the *equilibrium societal policy* p, which in turn affects aggregate behavior A. Given that these two effects have to be expressed relative to the impact of institutions per se on A (through the policy p and without cultural dynamics), the first compositional effect is then captured by the term $A_q/\left[A_p+\left(A_{a^1}a_p^1+A_{a^2}a_p^2\right)\right]$ and the second policy effect term is captured is directly by the term p_q in (**??**).

We provide below examples in which, depending on the economic fundamentals, we obtain a positive or a negative cultural multiplier.

5 Examples

In this section we work out several main examples, rich enough to display some interesting cultural and institutional dynamics.[33]

5.1 Elites, workers, and extractive institutions.

Consider an economy populated by workers and members of the elite. Workers are culturally homogeneous, while members of the elite are either work-oriented in their attitudes or not, re-

[33] In all the examples we impose and exploit various regularity conditions without explicit mentioning them. We discuss however all the details in Appendix B.

spectively bourgeois and aristocrats. The elite, as a political group, has the power of taxing workets, but cannot commit ex-ante on the tax rate and cannot observe the workers' production effort. Depending on the political power on the elite, it might tax workers to the point of constraining them at subsistence (a *Malthusian regime*). On the other hand, when this is not the case, the elite might have an incentive to establish less-extractive institutions, to indirectly commit on a lower tax rate, in turn inducing workers to extend an higher production effort. As a consequence, the institutional dynamics of this economy will in general be *non-ergodic*, depending crucially on initial conditions, When initial institutional conditions are not favorable enough to workers, the institutional dynamics will lead to a *Malthusian regime* and institutions remain extractive. On the contrary, when workers are well enough represented, the institutional dynamics lead to a stationary state in which the power is delegated enough to the workers so that they are not taxed. In this case, extractive institutions are undermined by their own inefficiency on labor incentives and the lack of commitment of the policy maker. Interestingly in this example, the process is triggered independently of any technology on the part of the workers to threaten, e.g., by means of a revolution, the power of the aristocrats. In this sense, the mechanism driving the evolution of institutions towards democratization in the example is orthogonal to the one stressed by Acemoglu (2003), Acemoglu and Robinson (2006, 2010), and Acemoglu, Johnson, and Robinson (2006). Furthermore, in this example, extractive institutions are not stable independently of the population distribution of the two political groups, workers and elite. In particular, this is the case even if the relative power of workers is unaffected by their relative size (or even relative income) in society. Once the dynamics of culture is added to the analysis, it turns out that culture and institutions, in a non-Malthusian regime, are complements: bougeois attitudes in the elite become more predominant as institutions become less estractive and taxes decline.

More in detail, workers (group $i = 1$) are in proportion $1 - \lambda$ and members of the elite (group $i = 2$) in proportion λ. All agents have preferences over a consumption good c^{ij} and effort a^{ij}. The technology converting effort in the consumption good is linear in effort (and normalized so that): $c^{ij} = a^{ij}$.

Workers are culturally homogenous[34] and have preferences $U^1\left(c^1, a^1\right) = u(c^1) + v(1 - a^1)$; where $v(.)$ denotes the utility of leisure, normalizing the leisure endowment to 1. Elite members can be of two cultural types. The "bourgeois", in proportion $q^{2b} = q$ of the total elite size λ, have the same preferences as workers, $U^{2b}\left(c^{2b}, a^{2b}\right) = u(c^{2b}) + v(1 - a^{2b})$. The "aristocrat" are instead in proportion $1 - q$ of the elite and have preferences with extreme disutility for work: $U^{2a}\left(c^{2a}, a^{2a}\right) = u(c^{2a}) + \theta v(1 - a^{2a})$; where θ is large enough so as to make aristocrats choose not to work in equilibrium: $a^{2a} = 0$.

Let $\beta^1 = \beta$ denote the institutional weight of the workers. Let in turn p, the policy choice, represent a linear tax on workers' output, a^1. $T = T(p, a^1, \lambda) = pAa^1\frac{1-\lambda}{\lambda}$ then denotes the lump

[34]Hence, for notational simplicity, we drop the apex indexing culture for group 1.

sum transfer received by each member of the elite, by budget balance.

We assume that that there is a survival constraint in the economy: a minimum consumption level needs to be satisfied $c \geq \overline{c}$ with $\overline{c} < A$. Workers only can be at survival as members of the elite have a subsistence endowment large enough to live on without working. [35]

Societal equilibrium. Workers' optimal effort $a^1(p) \geq 0$ is non monotonic in the tax rate p and it typically depends on whether the survival constraint $c^1 \geq \overline{c}$ is binding, as shown in Figure 1: if the survival constraint is not binding (the "non malthusian" regime), $a^1(p)$ is decreasing in p, capturing the disincentive effects of the tax rate on effort; if instead the survival constraint is binding (the "malthusian regime), $a^1(p) = \frac{\overline{c}}{1-p}$ for $p \in [\widehat{p}, 1-\overline{c}]$; that is, increasing in p. Furthermore, the effort choice of bourgeois, $a^{2b}(T) \geq 0$, is decreasing in the transfer level T, while aristocrat do not work for any value of $T \geq 0$.

The *societal equilibrium policy* $p(\beta, q)$, and the *societal commitment policy* $p^{com}(\beta, q)$ are illustrated in Figure 2. Again we can distinguish between the malthusian and non malthusian regime. Typically, when the weight of the workers is low enough, below a threshold $\overline{\beta}_0(q)$, at the *societal equilibrium*, the elite taxes the workers to a level that is larger than $\widehat{p}$, forcing them to a malthusian regime where the survival constraint $\overline{c}$ is binding.[36] On the contrary, if workers are sufficiently powerful, politically, $\beta \geq \overline{\beta}_1(q)$, they are not taxed at all and $p = 0$. At some positive intermediate values of $\beta \in \left(\overline{\beta}_0(q), \overline{\beta}_1(q)\right)$, the societal equilibrium involves an interior positive equilibrium tax rate $p(\beta, q) \in (0, \widehat{p})$ such that the survival constraint is not binding and the economy is in the non malthusian regime. Independently of the regime, however, $p(\beta, q)$ is declining with the workers' weight, β.

The optimal policy at the *societal equilibrium with commitment*, is obviously decreasing in the weight β of workers (and is $= 0$ when β is sufficiently large, $\beta > \overline{\beta}_1(q)$). Furthermwore, when β is small enough, below the threshold $\beta^*(q)$, $p^{com}(\beta, q) > \widehat{p}$ and the societal equilibrium with commitment is in the malthusian regime. When instead $\beta > \beta^*(q)$, $p^{com}(\beta, q) < \widehat{p}$, in the non malthusian regime. Finally, $p^{com}(\beta, q)$ jumps at the threshold $\beta^*(q)$.

Notice importantly that in the non-malthusian regime, $\beta > \beta^*(q)$, it is the case that $p^{com}(\beta, q) < p(\beta, q)$. Indeed, without commitment the elite does not internalize the disincentive effects of taxation on workers' efforts and therefore induces an equilibrium tax that is inefficiently high. The situation however is quite different in the malthusian regime. On the one hand, the elite does not internalize the effect of taxation on workers' effort, which is positive in this regime. This tends

[35] The policy space is assumed bounded in such a way as to always have make survival of the workers feasible: $p \leq 1 - \frac{\overline{c}}{A}$. Assuming that workers have also positive subsistence endowment does not affect significantly our results.

[36] This situation is even more extreme when β even lower, below the threshold $\overline{\beta}_m(q)$). In such a case the workers get taxed to the maximum, $p = 1 - \overline{c}$, and have to supply their full time endowment $a^1 = 1$ to work to maintain their consumption level to the survival limit.

to make the *societal equilibrium policy* $p(\beta, q)$ too low compared to the *societal equilibrium policy with commitment* $p^{com}(\beta, q)$. On the other hand, the elite does not internalize the distortionary effect of taxation, which in a malthusian regime is induced by the constrained effort choice of workers. This tends, on the contrary, to make $p(\beta, q)$ too high compared to $p^{com}(\beta, q)$.

When β is very low, the distortionary effect dominate the positive effect of taxation on labor supply and therefore $p(\beta, q) > p^{com}(\beta, q)$. In this case in fact, the distortionary effect is maximal, as workers are induced to supply their full time endowment to work and have a very large marginal disutility of labor. At the same time, the effect of taxation on labor supply is minimal, as a^1 is already very close to its maximal value.

When instead $\beta < \beta^*(q)$ but it is close enough to $\beta^*(q)$, the distortionary effect tends to be large as the *societal equilibrium policy with commitment* policy jumps from a value smaller than $\widehat{p}$ to a value larger than $\widehat{p}$. Conversely, given that the equilibrium level of effort for the workers is far from its maximal level, the effects of taxation on labor supply are relatively weak. It follows that for β slightly smaller than $\beta^*(q)$, $p(\beta, q) < p^{com}(\beta, q)$. By continuity of the equilibrium policy functions $p(\beta, q)$ and $p^{com}(\beta, q)$ in the range $(0, \beta^*(q))$, there is a point $\beta = \beta^e(q)$ where the two curves cross, as depicted in figure (2). It turns out that this point $\beta^e(q)$ is unique and therefore such that $p(\beta, q) = p^{com}(\beta, q)$.

Institutional dynamics. From the previous discussion, the non-ergodic behavior of the institutional dynamics is apparent. For all initial value $\beta_0 \in [0, \beta^*(q))$, the institutional dynamics converge to the unique steady state $\beta = \beta^e(q)$ and the society ends up in a malthusian regime with low political representation of the workers who are maintained at their survival constraint by exploitative taxation on the part of the elite.[37] Conversely for initial values $\beta_0 \in (\beta^*(q), \overline{\beta}_1(q)]$, the institutional dynamics are very different. The weight of the workers on the institutional setting converge to the unique steady state $\beta = \overline{\beta}_1(q)$, characterized by no taxation, in a non-malthusian regime. Obviously for initial values $\beta_0 > \overline{\beta}_1(q)$, the institutional weight of the workers is already large enough to induce no taxation and therefore no distortions. Institutions do not change and stay at their initial value $\beta_t = \beta_0$ for all $t > 0$. These institutional dynamics are depicted in Figure 2 by the arrows connecting in the usual way the two policy curves $p(\beta, q)$ and $p^{com}(\beta, q)$.

[Figures 1 and 2 about here]

[37] Interestingly in such a malthusian regime, higher taxation may actually increase the efficiency of the rent extraction process as the survival constraint prevents the traditional disincentives on labor supply to kick in. This local effect is arguably instrumental in maintaining of such an exploitative regime for workers. This is reminiscent of an argument in Clark (2009), suggesting that policies that would otherwise appear as having inefficiency costs in a non malthusian world, on the contrary may find some efficiency rationale under malthusian conditions.

Cultural dynamics. Workers are culturally homogenous and hence display no cultural dynamics, they remain workers. Within the elite group, however, bourgeois' children can be socialized to aristocratic values and aristocrats can acquire bourgeois' preferences. This process determines overtime the pattern of preferences inside the elite.

The dynamics of cultural evolution within the elite are determined by the dependence of the relative incentives to socialization $\Delta V^b(p)/\Delta V^a(p)$ to changes in the equilibrium policy instrument p. When the tax rate $\widehat{p}$ at which a malthusian regime is triggered is below the tax rate $p^{\max}$ that maximizes the Laffer curve, $\Delta V^b(p)/\Delta V^a(p)$ is decreasing in p. Indeed, as taxation leads to increased rent extraction on labor, aristocratic preferences are more likely to be transmitted than (work-oriented) bourgeois' preferences inside the elite: the larger the rents of the elite, the larger the socialization advantage of aristocrats. Since equilibrium taxation is a decreasing function of the institutional weight β of workers, the more powerful workers are in society, the larger the diffusion of (work-oriented) bourgeois' preferences inside the elite, and hence inside society.

We depict in figure 3 the cultural steady state manifold $q(\beta) \in (0,1)$ It is an upward sloping curve in the region $\beta \in \left[0, \overline{\beta}_1(q)\right]$ and a vertical line $q = q_0$ in the region $\beta \geq \overline{\beta}_1(q)$ for which there is no redistribution and $p = 0$.[38] The joint evolution of institutions and culture is also depicted in figure 3 that presents the phase diagram of the system. There are two types of steady states: a long run steady state (point M) in the malthusian regime and a full segment $[A, B]$ of steady states in the non malthusian regime. At all points in that segment there is no redistribution from the workers to the elite.

[Figure 3 and 4 about here]

Joint evolution of culture and institutions. The phase diagram in figure 4 indicates how institutions and culture co-evolve after an exogenous shock to society. Suppose for instance that that the economy is at the steady state (β_A, q_A). reflected by point A. Consider for instance an exogenous decrease of the institutional representation of the workers that leads to point C associated to a lower value β_C. Given the initial value q_A of bourgeois members in the elite, this institutional shift induces immediately a positive level of taxes, extracting rents from workers. The relative value to transmit aristocratic preferences increases in the elite, leading first progressively to a reduced fraction of elite members with (work oriented) bourgeois preferences. This makes the commitment problem of rent extraction of the elite more salient. To solve this, institutional change occurs in the direction of readjusting upwards the institutional weight β of the workers. The economy returns to the steady state A with zero taxation and the same long run fraction q_A of elite members with "work-oriented" preferences. Along the transition path,

[38] The slope of the curve $q(\beta)$ changes sign when one moves from the region $\beta \leq \widetilde{\beta}(q)$, to the region $\beta \in \left]\widetilde{\beta}(q), \overline{\beta}(q)\right[$ as the societal equilibrium policy $p(\beta, q)$ moves from being larger than $p^{\max}$ to being smaller than $p^{\max}$.

the time evolution of the fraction of these bourgeois elite members is non monotonic. After an initial decrease associated to the negative institutional shock on β, the institutional readjustment process in favor of workers becomes associated to a positive growth of elite members endowed with such work-oriented preferences, until the economy settles back to point A.

Along the adjustment process, institutional change and culture evolved in a complementary way. Institutional change in favor of individuals endowed with work-oriented preferences stimulates the diffusion of such preferences inside the society, while a larger population of work oriented individuals leads to institutional changes biased in their direction.

5.2 Elites, workers, and the provision of public goods

Consider a version of the previous society in which tax collection contributes to a public good rather than to the elite's revenues. In the spirit of Lizzeri and Persico (2004)'s explanation of the extension of the suffrage in nineteenth century Britain, the elite might have an incentive to establish less-extractive institutions, to indirectly commit on a lower tax rate, in turn inducing workers to extend an higher production effort and hence to contribute more to the public good.In this society as well, culture and institutions are complements, reinforcing each other.

The political and cultural groups are as in the previous society. Contributions to the public good G equal the tax burden on labor, since the government balances its budget: $G = p\left[\lambda a^{11} + (1-\lambda)\left(qa^{2b} + (1-q)a^{2a}\right)\right]$.[39] Abstracting from the issue of a malthusian economy, all individuals are endowed with some subsistence endowment $s > 0$. Utility functions for the different types of agents are as in the previous society with the addition of a separable utility for the public good, for simplicity identical and linear for both political groups: $\Omega \cdot G$ with $\Omega > 0$.Along the lines of the previous analysis, the *societal equilibrium* and the *societal commitment* policies are as in Figure 5. Typically, for β small enough, $\beta \leq \widetilde{\beta}(q)$, all policies $p(\beta, q)$ inducing no labor effort, that is, p larger than the threshold $\geq p_0$, are a *societal equilibrium* policy. In this case, workers have so little power that the natural ex post incentive is to tax them to the extent that they do not provide any labor supply. On the contrary, for $\beta \geq \widetilde{\beta}_M(q)$, labor is not taxed and $p(\beta, q) = 0$. For intermediate values of $\beta \in \left]\widetilde{\beta}(q), \widetilde{\beta}_M(q)\right[$, the *societal equilibrium policy* $p(\beta, q)$ takes interior values and is a decreasing function of β. Indeed the ex-post incentives to finance the public good through labor taxes are lower when the workers' interest are better represented.

As also shown in Figure 5, $p^{com}(\beta, q)$ is a decreasing function of β, always smaller than the tax rate $p^{\max}$ which maximizes tax revenue. Moreover $p^{com}(\beta, q) = 0$ when β is larger than the threshold $\widetilde{\beta}_M(q)$. Most importantly,

$$p^{com}(\beta, q) < p(\beta, q),\ \forall \beta < \widetilde{\beta}_M(q),\ 0 < q < 1.$$

[39] More generally we can think of the goverment choosing the tax rate p and the fraction of the tax collection which is contributed to the public good, but in the interest of simplicity we restrict to a pure public good economy.

In the *societal equilibrium*, the policy maker does not internalize the negative distortion of taxation on the tax base. Hence taxes are systematically (and inefficiently) higher than at the *societal equilibrium with commitment* where such effect is internalized.

[Figure 5 about here]

Institutional dynamics. The institutional dynamics are represented by the arrows in Figure 5. For any value $q \in [0,1]$, the manifold $\beta(q)$ characterizing the steady state institutions is represented by the thick curve $\beta(q) = \min\left(\widetilde{\beta}_M(q), 1\right)$. Furthermore, $\beta_M(q)$ is weakly increasing in q: a larger fraction of work oriented bourgeois individuals in the elite is reflected by an higher institutional weight to workers in the long run. When $\widetilde{\beta}_M(q) \leq 1$, the workers get all the power necessary to prevent them to be taxed, $\beta^e(q) = \widetilde{\beta}_M(q)$, and there is no provision of the public good, $G = 0$. When $\widetilde{\beta}_M(q) > 1$, the workers obtain all the power in the long run, $\beta^e = 1$), but they do finance some positive amount of public good, $G(q) = [\lambda + (1-\lambda)q] \cdot p(1,q) \cdot a(p(1,q)) > 0$. Overall, institutions tend to allocate power to the workers, who are the political group most negatively affected by the policy p and who contribute the most to the provision of the public good.

Cultural dynamics. Similarly to the analysis regarding the previous society, the cultural steady states manifold $q(\beta)$ and the phase diagram of cultural dynamics are shown in Figure 6a) and 6b). In the region $\beta \geq \widetilde{\beta}_M(q)$, the equilibrium policy is $p(\beta, q) = 0$, the terms $\Delta V^{2j}(p)$ take just constant values, $\Delta V^{2j}(0)$,and $q(\beta)$ is just a vertical line $q = q_0$ which does not depend on β. In the region $\beta \in \left(\widetilde{\beta}(q), \widetilde{\beta}_M(q)\right)$, the equilibrium policy $p(\beta, q)$ is interior (ie. $p(\beta, q) \in (0,1)$) and the manifold $q(\beta)$ is an increasing function of β. Indeed the socialization incentives ΔV^{2b} (resp. ΔV^{2a}) to transmit (work-oriented) bourgeois (resp. aristocratic) preferences are decreasing (resp. increasing) in the policy p. Therefore a lower value of the tax policy on labor makes the cultural transmission of bourgeois preferences relatively more advantageous. A higher institutional weight to bourgeois agents, β, leads to a lower equilibrium tax on labor $p = p(\beta, q)$. As this promotes the relative transmission of (work-oriented) bourgeois preferences, the long run fraction $q(\beta)$ of individuals with such preferences rises in the society. Finally, in the region $\beta < \widetilde{\beta}(q)$, workers have so little power that the societal equilibrium outcome induces a large enough tax rate to discourage any production effort and provision of the public good. Given that the bourgeois behave in the same way as the aristocrats, in this case, there is no cultural evolution inside the elite group and q_t remains constant at its initial value q_0.

Joint evolution of culture and institutions. The phase diagrams in Figure 6a) and 6b) imply the following dynamics. In the region $\beta \geq \widetilde{\beta}(q)$, there are no institutional dynamics as

the *societal equilibrium policy* and *societal equilibrium policy with commitment* are identical: $p(\beta, q) = p^{com}(\beta, q) = 0$.

Distinct dynamics depend on where the manifolds $\beta(q)$ and $q(\beta)$ intersect. In case *a)* the manifolds $\beta(q)$ and $q(\beta)$ intersect along the part upward sloping part of $\beta(q) = \widetilde{\beta}(q)$. The full segment of points $[A, B]$ are then stable steady states with the same long run fraction of bourgeois q^A and a range of institutional weights $\beta \in \left[\beta(q^A), 1\right]$ ensuring no taxation and no provision of public goods.

Case *b)* arises when the manifolds $\beta(q)$ and $q(\beta)$ intersect along the flat part $\beta(q) = 1$. In this case there is only one stable steady state C, characterized by all the political power to the bourgeois, $\beta = 1$, and a given interior fraction of such individuals $q(1)$. At such steady state, there is a strictly positive labor tax and some provision of the public good $G > 0$. Case b) is more likely to occur when the valuation of the public good Ω is large enough.

[Figure 6a) and 6b) about here]

It is interesting to illustrate again through this example the importance of the co-evolutionary process between culture and institutions. Consider for instance, in Figure 7, an initial situation as described by point O with initial conditions q_0 and β_0. Consider first a society in which institutions are unchanging: β remains at β_0. Such a society would evolve leading eventually to a larger long run fraction $q(\beta_0) > q_0$ of (work-oriented) bourgeois, as in point W. This would be associated to a positive and larger provision of public goods than initially, $G(q(\beta_0)) > G(q_0) > 0$. Alternatively, consider a society in which culture is unchanging: $q = q_0)>$ Such a society will be driven towards the long run outcome in point Z, at which in the steady state workers have enough power to avoid taxation, $\beta = \widetilde{\beta}_M(q_0)$ and there is no provision of the public good, $G = 0$.

When culture and institutions evolve jointly, however, the society converges to a different configuration, point C. At C, workers have more power than predicted by the institutional dynamics only; that is, $\beta = 1 > \widetilde{\beta}_M(q_0)$. Also, there is a larger predominance of work-oriented bourgeois preferences than what predicted by the cultural dynamics only; that is, $q(1) > q(\beta_0)$. Finally, there is a positive long run provision of the public good, larger than predicted by any of the to independent dynamics separately; that is, $G > G(q(\beta_0)) > 0$.

This example illustrates therefore vividly a case of long run complementarities and "crowding in" between culture and institutions. Indeed taking the terminology of Section 4.1 on the cultural multiplier, there is *positive institutional association* and *positive cultural association*: Indeed a larger fraction of work-oriented bourgeois in the elite leads to institutions giving more weight to labor interests. Conversely, institutions reflecting more labor interests create a policy environment favorable to the long run diffusion of work-oriented bourgeois preferences in the society.

As a consequence the institutional and cultural dynamics magnify each other and the cultural multiplier is positive.

[Figure 7 about here]

5.3 Civic Culture and democratization

The society we consider in this section is one where civic control on the part of workers can limit inefficient redistributive policies enacted by the government in favor of the elite. Civic control requires however the exertion of costly effort on the part of individual workers, inducing a commons problem. In equilibrium only workers characterized by civic-minded attitudes exert civic control. In this society, the institutional dynamics favor the delegation of power to workers (which, abusing language, we call *democratization*), as a commitment device on the part of the elite to restrict, by means of civic control, the inefficient redistributional policies of the government. The most interesting aspect of this society consists in the way institutions and culture interact. Depending on the shape of the elasticity of the cost of exerting effort for the individual workers, in fact, culture and institutiona are either complement or substitute.[40]

More in detail, the society populated by the two political groups in the previous examples: workers, type $i = 1$, and the elite, $i = 2$, in fractions $\lambda^1 = 1 - \lambda^2 = \lambda$. All individuals (workers and elite) are endowed with a fixed amount of resources, ω, taxed at an exogenous tax rate τ. The government uses tax revenues to produce a public good G, but it can also divert some of the revenues as transfers, $T \geq 0$, to the members of the elite. Such transfers are however subject to transaction costs, e.g., to implement secret kick-backs, corruption schemes, hidden accounts, and other "creative" fiscal accounting. Workers can exert effort, a sort of civic society control, to monitor the government, ensuring that as much as possible of the public good is produced. Monitoring by the civil society tends to increase transparency, so that the transaction costs required to transfer resources to the elite on the part of the government are an increasing function of the civic society monitoring. More precisely, $\theta(A)$ denotes the transaction costs: a transfer T produces $\theta(A)T < T$ consumption units available for the elite; where $\theta(A)$ is decreasing in A, the total amount of civic effort exerted by the workers.

The elite has standard preferences over consumption and the public good, $U^2(c^2, G) = c^2 + v(G)$ The workers belong to one of two cultural groups.The first, $j = c$ in proportion q, is composed of *civic-minded* individuals with an intrinsic motivation for exerting civic effort, a^{1c}. More precisely, their preferences are: $U^{1c}\left(c^{1c}, G, a^{1c}, T\right) = c^{1c} + v(G) - \alpha(T)(1 - a^{1c}) - C(a^{1c})$; where

[40] We conjecture that in the substitution case, the joint dynamics might display interesting cyclical behavior.

$c^{1c}+v(G)$ is the direct utility of private consumption and the public good G, while $-\alpha(T)(1-a^{1c})$ is the intrinsic motivation term. Typically, a given level of transfer T to the elite triggers an optimal action of a civic-minded worker, $a^{1c}=a(T)$ increasing in T. The second type of workers, $j=p$, in proportion $1-q$, do not have intrinsic motivations to exert civic monitoring. We call them *passive* members and their preferences are given by $U^{1p}\left(c^{1p},G,a^{1p}\right)=c^{1p}+v(G)-C(a^{1p})$.

The policy choice $p=T/1-\lambda$ depends on the workers' effort a^{11},a^{12} only through the total amount of civic monitoring they exert, $A=\lambda\cdot\left[q\cdot a^{11}+(1-q)\cdot a^{12}\right]$, through its effect on transaction costs $\theta(A)$. As a consequence, the total amount of civic monitoring is a public good, the contribution of each worker effort is negligible, and hence passive workers always choose not to exert any effort, $a^{12}=0$, and civic-minded workers contribute according to their intrinsic motivation.

Given any institutional weight on the workers, $\beta\geq 0$, the *societal equilibrium* and *societal commitment* policies are as in Figure 8. Typically, for large enough workers' weights, $\beta\geq\widehat{\beta}(q)$, there is no transfer diverted to the elite and no civic monitoring at the *societal equilibrium*: $p=0$ and $a^{11}=0$. At some positive low enough values of $\beta\in[0,\widehat{\beta}(q))$, the *societal equilibrium* involves an interior positive equilibrium transfer $p(\beta,q)>0$, decreasing both with the weight β of workers and with the fraction q of civic-minded individuals. Indeed a larger fraction of civic-minded workers leads to more civic monitoring, increasing the transaction costs to divert resources to the elite members. As a consequence, the equilibrium transfer $p(\beta,q)$ is reduced.

At the *societal equilibrium with commitment*, $p^{com}(\beta,q)$ is obviously decreasing in β, vanishing to 0 when β is large enough, $\beta\geq\widehat{\beta}(q)$. Most importantly, $p^{com}(\beta,q)\leq p(\beta,q)$, as long as $q>0$. Indeed, without commitment the policymaker does not internalize the fact that diverting resources out of public good production induces a negative externality on civic-minded workers which in turn leads them to exert civic monitoring.

Institutional dynamics. For all initial value $\beta_0<\widehat{\beta}(q)$ the institutional dynamics converge to a unique steady state $\beta=\widehat{\beta}(q)$. Conversely, for $\beta_0\geq\widehat{\beta}(q)$, there are actually no pressures for institutional change, $\beta_t=\beta_0$ for all $t>0$, as the institutional weight of the workers is already large enough to induce no transfer and therefore no extractive distortion in society.

[Figure 8 about here]

In the steady state, the power is delegated to the workers until no more transfers are diverted to the elite. Furthermore, $\widehat{\beta}(q)$ is decreasing in q. The formal political power of workers in steady state is negatively related to the degree of informal monitoring that can be provided by the fraction of civic-minded individuals. From the institutional perspective, an active civic society acts

as a substitute to formal political power.

Cultural dynamics. The elite is culturally homogenous and hence displays no cultural dynamics, members of the elite remain such. The cultural dynamics within workers are determined by the the relative incentives to socialization $\Delta V^{1c}(p)/\Delta V^{1p}(p)$ as they depend on the equilibrium policy instrument p. In fact, $\Delta V^{1c}(p)/\Delta V^{1p}(p)$ is increasing in p (resp. decreasing in p) when the elasticity $\epsilon(a) = \frac{C'(a))a}{C(a))}$ is increasing (resp. decreasing) in a. Intuitively, the relative advantage of cultural transmission of civic-mindedness depends on the elasticity of the cost of civic effort.

For a fixed institutional weight β, the cultural steady states are characterized by a manifold $q(\beta) \in [0,1]$, and two cases can occur, as presented in Figures (9a) and 9b).

[Figure 9a) and 9b) here]

In the first case, the elasticity $\epsilon(a)$ is decreasing in a. The manifold $q(\beta)$ is then composed of two parts : The part below the curve $\beta = \widehat{\beta}(q)$ has an upward slope (for the stable branches). and the part above the curve $\beta = \widehat{\beta}(q)$ is vertical and such that $q(\beta) = q^*$.

Conversely in the second case, the elasticity $\epsilon(a)$ is increasing in a. The manifold $q(\beta)$ is composed of two parts: the part below the curve $\widehat{\beta}(q)$ has a downward slope; the part above $\widehat{\beta}(q)$ is again given by $q(\beta) = q^*$.

Joint evolution of culture and institutions. The joint evolution of culture and institutions is illustrated in Figures 9a) and 9b). The long run steady states are obtained along segment $[A, B]$ in Figure 9a) and 9b). On any point of such segment, workers have enough power that there is no transfer diverted to the elite. Depending on the sign of variation of $\epsilon(a)$ as a function of a, institutional and cultural dynamics act as complements or substitutes.

To see that suppose that the society settled at the steady state point A and consider for instance an increase in the productive capacity of the economy; i.e., an increase in the endowment ω. Figure 10a) depicts the situation when $\epsilon(a)$ is decreasing in a. In such a case an increase in ω increases the tax base on which to finance the public good G and transfers to the elite T. As a consequence the per capita transfer $p(\beta, q))$ increases and the threshold $\widehat{\beta}(q)$ over which there is no more transfer shifts out (as shown by the red vertical arrow in Figure (10a). Given that $\epsilon(a)$ is decreasing in a, an increase in $p(\beta, q)$ tends to reduce the diffusion of civic-mindedness in society, shifting in the manifold of cultural steady states $q(\beta)$ (as shown by the horizontal red arrow in Figure (10a). At given institutions $\beta = \beta_A$, this leads to a reduction of the fraction of civic-minded workers q. However the higher *societal equilibrium* transfer $p(\beta, q)$ in turn triggers a change in institutions: in order to solve a more salient commitment problem, more power has

to be given to the workers: β increases. This increase in β in turn leads to a reduction of the equilibrium policy $p(\beta, q)$ and therefore to an increase in civic-mindedness in society. At the end of this process the fraction q of civic-minded workers returns to its long run equilibrium value q^* and a higher institutional weight $\beta_{A'}$ is given to the workers to ensure that no transfer is unduly made to the elite. Over the dynamic process, after the initial shock that leads to increased transfers and a reduction of civic-mindedness, the adjustment path towards the new steady state A' involves an increase in the political representation of workers and civic-mindedness moving in the same direction towards A', reinforcing each other.

As shown in Figure (10b), when $\epsilon(a)$ is increasing in a, the opposite adjustment path occurs. The initial positive shock on the endowment ω leads again to an initial increase of transfers to the elite. This however now induces more diffusion of civic-mindedness in society. After that initial shock, the adjustment path towards the new steady state A' now involves political representation of workers and civic-mindedness to move in opposite direction towards A'.

[Figures 10a) and 10b) about here]

5.4 Modernization

The society we study in this section is constructed to study the interaction of culture and institutions in facilitating or impeding a sectoral transition, which we call modernization. While there is no distinction between workers and elite, the political group in society are once again the aristocracy and the bourgeoisie. They are characterized however by different attitudes towards the future, in terms of time-preferences in particular (the more patient group is identified with the bourgeoisie,, along the lines of Doepke and Zilibotti (2006)'s explanation of the industrial revolution in England in the eighteenth century. It will turn out that whether modernization will occur in equilibrium dependson the combination of initial conditions in terms of culture and institutions.

More in detail, in this society agents live two sub-periods t_1 and t_2. In the first sub-period of they have a fixed endowment ω of the consumption good. In order to consume in the second sub-period t_2, however they need to engage in production. They choose between two occupations (or technologies), M, for *modern*, and T, for *traditional*; that is, a^i is discrete, an element of $\{M, T\}$. Production in either of these occupations requires a fixed investment to be undertaken in the first sub-period t_1. More precisely, a required fixed investment c_j, $j \in \{M, T\}$, at t_1 induces production R_j at t_2. Agents are heterogeneous in terms of their skills $s \in [0, 1]$. We assume the distribution of skills is i.i.d. across agents and uniform on $[0, 1]$. The required investment in the modern sector, c_M, depends negatively on the skills s of the agent undertaking it, while c_T is independent of s. Furthermore, the government policy $p \in [0.1]$ reduces the investment c_M,

uniformly for all agents, at a societal cost $C(p)$.[41] Specifically,

$$c_M(s,p) = c_M^0 + 1 - s - p, \quad c_T(p) = c_T^0.$$

We assume that the modern occupation is more skill intensive than the traditional sector and also more productive: $c_M^0 > c_T^0$, $R_M > R_T$. Let $\Delta c = c_M^0 - c_T^0$ and $\Delta R = R_M - R_T$.[42]

The political and the cultural group coincide, the special case $j = i$ introduced in footnote 16. The two groups in the population are distinguished by their time-preference parameter, $0 < \delta^i < 1$. Let $\delta^1 > \delta^2$ (group 1 is the bourgeoisie and group 2 the aristocracy). The time-preference parameters δ^i, $i = 1, 2$ satisfy,

$$\delta^2 < \frac{\Delta c - 1}{\Delta R} < \delta^1 < \frac{\Delta c}{\Delta R} \qquad DD$$

The first inequality implies that all individuals of type 2 are too impatient to produce in the modern sector, whatever their skills s and the level of the policy p. The second inequality ensures that without policy intervention, that is, with $p = 0$, no production will be made in the modern sector. The third inequality ensures on the contrary that some individuals of type 1 can be incentivized to choose to produce in the modern sector when the policy p is large enough. In fact, under these assumptions, an agent of type 1 with skills s will choose occupation M if and only if

$$s \geq s(p) = -\delta^1 \Delta R + \Delta c + 1 - p$$

and hence $S(a,p) = q\,(1 - s(p))$ and production in M is positive if and only if p is greater than a cutoff $p_L = \Delta c - \delta^1 \Delta R$.

At a *societal equilibrium* the government choosing policy p takes as given the skills cutoff at which agents of type 1 choose to produce in sector M, that is, takes as give the fraction of the population in occupation M as well as the total production in the sector. It follows then that a *societal equilibrium* always exists such that $p = 0$ and the modern sector M is inactive. On the other hand, a $p > 0$ reduces the required investment of the agents choosing occupation M, if any such agent exists, and hence their subperiod t_1 consumption. As long as the institutional set-up is such that the government at least in part internalizes these agents preferences, a *societal equilibrium* with $p > 0$ might in principle exist.

In fact it can be shown that there exists a threshold $\underline{\beta} < 1$ such that for $\beta < \underline{\beta}$, the only *societal equilibrium* is the one with $p = 0$ and sector M is inactive; while for $\beta \geq \underline{\beta}$ another *societal equilibrium* arises, with $p(\beta) > 0$, where all agents of type 1 with skills $s > s(p(\beta))$

[41] Detailed assumptions are relegated in the Appendix. Suffices to say here that $C(p)$ is increasing convex and satisfies appropriate boundary conditions.

[42] Note that this example represents formally an extension to the general analysis of Section 2. This is on two dimensions: i) the choices a^i are discrete; ii) agents in the same group i are allowed to be heterogeneous (with respect to skills s). It will appear clear in the following that it can be studied with the same methods.

choose occupation M.[43] At this equilibrium, $p(\beta)$ is increasing.[44] In order to have a well defined dynamics of institutions, for $\beta \geq \underline{\beta}$, we select the *societal equilibrium* that gives the highest payoff to the policy maker, namely the equilibrium with positive production in sector M and associated to $p = p(\beta) > 0$.

The *societal commitment equilibrium* $p^{com}(\beta, q)$ can also be characterized in a similar manner. More precisely one may show that there exists another threshold $\underline{\beta}^{com}(q) < 1$ such that for $\beta < \underline{\beta}^{com}(q)$, the *societal commitment equilibrium* is associated to $p^{com} = 0$ and sector M is inactive; while for $\beta \geq \underline{\beta}^{com}(q)$ the *societal commitment equilibrium* is associated to a policy $p^{com}(\beta, q) > 0$, with all agents of type 1 with skills $s > s(p^{com}(\beta, q))$ choosing occupation M. Furthermore, the policy function $p^{com}(\beta, q)$ is increasing in β and q and satisfies: $p^{com}(\beta, q) \geq p(\beta)$, with equality if and only if $q = 0$. Finally, $\underline{\beta}^{com}(q)$ is decreasing in q and $\underline{\beta}^{com}(q) \leq \underline{\beta}$, with again equality if and only if $q = 0$.

The two equilibrium policy schedules $p(\beta)$ and $p^{com}(\beta, q)$ are drawn in Figure 11).

[Figure 11 about here]

From this , the dynamics of β_t are easily described as follows:
Institutional dynamics. There exists a threshold unique $\widetilde{\beta}^{com}(q) \in \left[\underline{\beta}^{com}(q), \underline{\beta}\right)$ such that, for $\beta_0 < \widetilde{\beta}^{com}(q)$, $\beta_{t+1} = \beta_t = \beta_0$; while for $\beta_0 \geq \widetilde{\beta}^{com}(q)$, $\beta_{t+1} > \beta_t$ and β_t converges to $\beta = 1$.

We can now turn to the dynamics of culture, by studying first the socialization incentives induced by ΔV^1 and ΔV^2.

Cultural dynamics. If $\beta_0 < \widetilde{\beta}^{com}(q)$, $p(\beta) = 0$, $\Delta V^1 = \Delta V^2 = 0$, for any $0 \leq q \leq 1$, and hence $q_{t+1} = q_t = q_0$. If instead $\beta_0 \geq \widetilde{\beta}^{com}(q)$, $p(\beta) > 0$ and ΔV^i can be written as a function of $p(\beta)$ and $\frac{\Delta V^1(p(\beta))}{\Delta V^2(p(\beta))}$ is increasing in β. Therefore the manifold determining the interior stationary states of the dynamics of culture, $q(\beta)$, is also increasing in β, as long as $\beta \geq \underline{\beta}$.

This is reflected in Figure 12 where we also depict the phase diagram of the system. In the region (β, q) below the $\widetilde{\beta}^{com}(q)$ map, there is no dynamics, neither of institutions nor of culture. The economy remains in the traditional sector and population fractions are constant. In this region, that is, the more patient agents in group 1 do not have enough institutional weight to induce a policy that will favor the take-off of the modern sector. On the other hand, in the region above the map $\widetilde{\beta}^{com}(q)$ but below $\underline{\beta}$, institutional dynamics which shifts power towards the more patient group start to occur. When such power becomes large enough, cultural dynamics are set in motion that increase the frequency of group 1's trait in the population. As the transition to the modern sector M strengthens, externalities are larger and further institutional shifts induce

[43] Detailed proofs of all the statements are provided in Appendix C.
[44] $p(\beta)$ satisfies $\beta(p - \underline{p}) = C'(p)$.

policies more favorable to the transition. Institutional and cultural dynamics then complement each other intensifying the transition to the modern sector.

[Figure 12 about here]

5.5 Property rights and conflict

The society studied in this section provides yet another interesting example of non-ergodic behavior, in which the steady state of the joint dynamics of culture and institutions depends on initial conditions. The society is characterized by socio-economic interactions consisting in agents contesting each other's resource endowment under imcomplete protection of property rights. Political and cultural groups coincide as in the previous society and agents are only differentiated by their propensity to act into conflict. Along the lines of the specific groups described by Nisbett (1993), Cohen and Nisbett (1994), or rmore recently Grosjean (2014), as displaying a *culture of honor* one of the cultural gropus in society is more prone to violence than the other individuals, after rituals and practices that individuals partake into in order to be culturally legitimized.

More in detail, in this society people are matched randomly in a contest. Each agent's endowment prior to the contest is $\omega > 0$. Property right protection is the main policy variable, represented by the fraction $p \in [0,1]$ of each agent's endowment which is protected in the contest. After two agents match, their relative effort determines the probability that each of them succeeds in the contest, hence winning the fraction of the endowment of the opponent which is not protected by property rights. More specifically, let a^{hk} denote the effort exerted by an agent h when matching with an agent k. The probability of agent h winning the contest is $\frac{a^{hk}}{a^{hk}+a^{kh}}$.[45] The winner of the contest appropriates of the fraction of the total endowment not protected by property rights, $2(1-p)\omega$.

We assume that there are two political groups $i \in \{1,2\}$ which are fully identified to cultural groups $j \in \{1,2\}$ so that $i = j \in \{1,2\}$. The size of group 1 in society is therefore $\lambda^1 = 1 - \lambda^2 = q^1 = 1 - q^2 = q$.[46]. The two groups are culturally differentiated by their propensity to act into conflict. Specifically group 1 reflects individuals with a *culture of honor*, more prone to violence. Formally, these individuals are endowed with culture in which individuals have to pay a resource cost $F > 0$ allowing them to enjoy afterwards a higher propensity for violent action (ie. a low marginal cost of effort, c^1 in our contest setting). One could see the resource cost F as the typical cost associated to "beating" and "fighting" training sessions, rituals and practices that individuals with such culture go through in order to be legitimized. These violent sessions give them afterwards a "taste" or an ability to engage more easily into violent actions in potential

[45] Formally, this is the case if $a^{hk}, a^{kh} > 0$; while the probability of winning is 1/2 if $a^{hk} = a^{kh} = 0$.

[46] As compared to the general setting in section (?), in this example political groups have some endogenous size. The dynamics however still remain tractable, as poltical groups are fully alligned with cultural groups..

contests with others. On the opposite, group 2 is composed of *conflict-averse,* individuals who do not have such a culture of a ritualized fighting capacity and consequently these individuals are less effective in violent contests. In terms of the model, they do not pay the resource cost F of violence ritualization but have a higher marginal cost of contest effort $c^2 > c^1$ when fighting in contests.

Denote for convenience $\alpha = \left(c^2 - c^1\right)/c^1$. With these notations, q is the fraction of "conflict-prone" individuals in society.

Agents observe the opponent type before choosing their effort.[47] and the Nash equilibrium effort of an agent of type i in his contest with an agent of type j can be solved for straightforwardly, for given property rights p. Denoting by slight abuse of notation such effort as a^{ij}, this is given by:

$$a^{ij} = 2(1-p)\omega \frac{c^j}{\left(c^i + c^j\right)^2},$$

Matching is random, so that an agent in group i will match another agent in the same group with probability q^i and an agent in the other group with probability $1 - q^i$. Let the ex-ante expected payoff for agents of each of the groups at equilibrium be denoted $\Omega_i(p, q)$. It is decreasing in the fraction q of "conflict-prone" indviduals as a larger fraction of "conflict-prone" agents hurts both groups ex-ante. It induces a larger rent dissipation for the conflict-prone agents and a larger probability of extortion (loss of endowment) for the "conflict averse" individuals.

On the other hand, while $\Omega_2(p, q)$ is always increasing in p, $\Omega_1(p, q)$ is increasing in p only for a large enough fraction q of *conflict-prone* agents. Indeed "conflict-averse" individuals always benefit from property right protection, "conflict-prone" agents favor better property rights protection only when their fraction in the population is large enough.

We assume that implementing a level p of property rights protection requires a resource cost $C(p)$ satisfying standard convexity properties (see the Appendix for details).

Denote by $\beta^1 = 1 - \beta^2 = \beta$, the institutional weight of the "conflict prone" group. At the *societal equilibrium* property right protection p is chosen, taking as given the effort choices in contests for agents of the two groups. It can be easily shown that[48] when $\beta \geq q$, *the societal equilibrium* involves no property right protection and $p(\beta, q) = 0$; while for $\beta < q$, a positive level protection of property right is implemented with $p(\beta, q) > 0$. Moreover in such a case $p(\beta, q)$ is a decreasing function of β and is increasing in q. The larger the weight of the "conflict prone" group, the smaller the level fo property right protection, as such group benefits less from this protection. On the other hand, the larger the fraction of the conflict prone individuals in society,

[47] That is, the contest is a complete information game. The expected payoffs of an agent of cultural group j matching with an agent of group l is $W^j(a^{jl}, a^{lj}) = pV + 2(1-p)V\frac{a^{jl}}{a^{ijl}+a^{lj}} - c^j a^{jl}$. This example also represents an extension of the general analysis in Section 2 in that a^j is a multi-dimensional vector; again the same methods apply however.

[48] As usual see Appendix C for details.

the larger the social need for a reduction of conflict efforts dissipated into resource contests and therefore some enhanced degree of protection of property rights.

Similarly one can characterize the *societal equilibrium with commitment,* which now internalizes the impact of property right protection on the effort choices of the two groups in their contests. Specifically, one can show that at a *societal equilibrium with commitment*, there exist a threshold $\widetilde{q}(\alpha) \in]0,1[$ and an increasing function $\beta = \widetilde{\beta}(q)$ with $\widetilde{\beta}(0) < 1$ such that $p^{com}(\beta) = 0$ and there is no protection of property rights if and only if $(\beta, q) \in [0,1]^2$ are such that $q < \widetilde{q}(\alpha)$ and $\beta \geq \widetilde{\beta}(q)$. When conversely β, q do not satisfy such relations, *the societal equilibrium with commitment* involves positive protection of property rights and $p^{com}(\beta, q) > 0$. In such a case, $p^{com}(\beta, q)$ is again decreasing in β and increasing in q. Furthermore, one can show that the boundary $\widetilde{\beta}(q)$ is increasing and equal to 1 for $q < 1$ large enough.

In other words and not surprisingly, the *societal equilibrium with commitment* involves no property right protection when the institutional set-up is favoring the *conflict-prone* group, that is, when β is large enough [49]. More interestingly, however, when the fraction of conflict-prone agents is too high, q is large enough, then such group is always in favor of instituting property rights as a form of self-protection and $p^{com}(\beta, q) > 0$ at any level of β. Finally $p(\beta, q) \leq p^{com}(\beta, q)$, simply reflecting the fact an institutional commitment at property right protection prevent costly efforts to be undertaken into the rentseeking contests.

The two policy schedules $p(\beta, q)$ and $p^{com}(\beta, q)$ are represented in Figure 13, out of which it is then straightforward to study the dynamics of institutions. More specifically, one can immediately see that for any given q, if $\beta_0 > \widetilde{\beta}(q)$, then $\beta_{t+1} = \beta_t = \beta_0$. If instead $\beta_0 < \widetilde{\beta}(q)$, then β_t converges towards $\beta = 0$.

[Figure 13 about here]

We turn now to the dynamics of culture and hence to the socialization incentives of the to cultural traits ΔV^1 and ΔV^2. The "conflict-prone" agents have positive incentives $\Delta V^1(\beta, q)$ to transmit their trait but such incentives decrease with the fraction q of "conflict-prone" individuals in society. Similarly the incentives for the "conflict-averse" agents $\Delta V^2(\beta, q)$ are also positive. But, interestingly, their incentives are increasing with the fraction q of "conflict prone" agents in the population. A larger value of q reduces the expected payoff of "conflict-averse" agents matched with "conflict-prone" agents, thereby reducing the incentives to transmit their own trait; at the same time however, a larger q also increases the cost of effort for "conflict-averse" agents whose children turn out to be "conflict-prone" and undertake the high effort a^{11} when facing

[49] More precisely, a *societal commitment equilibrium* with no property rights also requires that $\frac{c_2}{c_1}$ be large enough; see Appendix C for details.

other "conflict-prone" agents in a contest. This effect tends to increase the value $\Delta V^2(\beta, q)$. It turns out that this second positive effect actually dominates the first negative one and therefore the incentives for "conflict-averse" agents to transmit their trait are positively associated to the fraction of "conflict-prone" agents in the population.

With regards to socialization incentives, property rights protection affects negatively the socialization incentives of the conflict-prone and promotes on the opposite the socialization incentives of conflict-averse agents.[50]

As a consequence, the cultural dynamics has a unique interior stationary state $q(\beta)$ which is increasing in the weight of the "violence prone" group β. Indeed a larger weight of that group implies less protection of property rights and therefore a larger diffusion of a culture of violence in society. Furthermore, $q(\beta) < 1/2$.

With respect to the joint dynamics of culture and institutions, we distinguish two cases (that we represent in Figures 14a) and 14b)).

The first case corresponds to $\alpha = \frac{c_2}{c_1} - 1$ large enough (the culture of violence gives a significant advantage in conflicts) and is represented in Figure (14a). Suppose first the initial conditions (β_0, q_0) are in the stripped region $[1, \beta_A, A, B]$[51]; that is, the "conflict-prone" are well represented in the initial institutions but not as a fraction of the population. The dynamics of culture is not undermining then the institutional set-up serving the interests of the "conflict-prone" group and the system remains with the initial institutional set-up, $\beta_{t+1} = \beta_t = \beta_0$, with no property rights protection. The dynamics of culture converge towards the fraction $\widehat{q}(\alpha)$ of "conflict-prone" individuals.

[Figures 14a) and 14b) about here]

On the other hand when the initial conditions are outside of the stripped region in the figure, "conflict prone" agents are not well represented in the initial institutional set-up and/or they are too numerous. Then the institutional dynamics evolves towards an increased representation of the "conflict-averse" agents, increased property rights protection, and a long run fraction of "conflict-prone" individuals is given by $q(0) < \widehat{q}(\alpha)$. It should be noted that $p(\beta, q) > 0$ along the equilibrium path, the interaction of the dynamics of institutions and culture leads progressively towards a reduction of a culture of violence and also less resources spent in these conficts.

[50] This feature is consistent with the observation by Cohen and Nisbett (1994) and Grosjean (2013) that a "culture of honor" and violence in the US South has persisted and be transmitted because of weak institutions of property rights protection and a need to enforce individually by contest and violence such property rights. It is also consistent with various anthropogical observations that suggest that cultures of violence are more likely to develop in pastoralist and herders' societies where property rights protection on cattle is more difficult to enforce than property rights on land in agrarian societies (see Campbell (1965), Edgerton 1971, Peristiany1965).

[51] where $\beta^*(\alpha)$ is defined as: $\beta^*(\alpha) = \widetilde{\beta}(q^*(\alpha))$.

Interestingly when $\beta_0 \leq \beta_A$, even a smaller fraction of conflict-prone individuals is ultimately self-defeating in terms of institutional dynamics. While for some time the system does not exhibit any institutional change and $\beta_{t+1} = \beta_t = \beta_0$, the underlying cultural dynamics tend to favor the socialization of the conflict-prone agents towards $\widehat{q}(\alpha)$. As soon as q_t passes the threshold of $\widetilde{\beta}^{-1}(\beta_0)$, endogenous institutional dynamics are triggered inducing the implementation of more extensive property rights and institutions biased towards the conflict-averse group. As a consequence of this, the transmission of a culture of violence also regress towards to long run steady state $q(0)$. This example shows therefore cleary the importance of initial institutional and cultural conditions for the long run of society and the non ergodicity properties of this system. Importantly a temporary exogenous institutional shock that gives more formal power to the conflict-averse group may trigger a very different long run trajectory of the institutional and cultural dynamics. Indeed suppose that the society has settled to a point like point A in Figure (14a) with no property rights and a culture of violence at $\widehat{q}(\alpha)$. Then a reduction of β below β_A, leads an endogenous institutional response towards further power to the "conflict averse" individuals. This in turn triggers reinforcing cultural dynamics towards that "conflict averse" group. After a while an inverse institutional shock of similar amplitude will not however bring back the system towards to region without property rights. indeed even if the conflict prone group regains back some formal power for some exogenous reasons, the cultural dynamics have irreversibly driven the system in a region where property rights are protected and there are less individual coflicts for the contest of resources. This suggests that external interventions (colonization, foreign aid, invasions) that changes the balance of poer domestically between groups may have long term effects in terms of institutional and cultural evolution.

The second case corresponds to $\alpha = \frac{c_2}{c_1} - 1$ not too large so that $\widehat{q}(\alpha) > \widetilde{q}(\alpha)$. In this case, represented in Figure (14b), the marginal effort costs c_i are similar across groups and hence "conflict-averse" agents are not much different than "conflict prone" agents. The dynamics of culture and institutions are such that $p(\beta_0, q_0) > 0$, that is property rights are protected for any initial conditions $(\beta_0, q_0) \in [0,1]^2$. The joint dynamics of culture and institutions converge to the stationary state $(0, q(0))$, characterized by institution giving all power to the "conflict-averse" agents and hence a maximal protection of property rights and a small fraction of "conflict-prone" agents in the population.

6 Conclusions

Rather than summarizing our analysis, in these conclusions we just note that in general the joint dynamics of culture and institutions is highly non-linear. In this context, therefore, linear regression methods tend to be at a disadvantage over more structural analyses of the data.

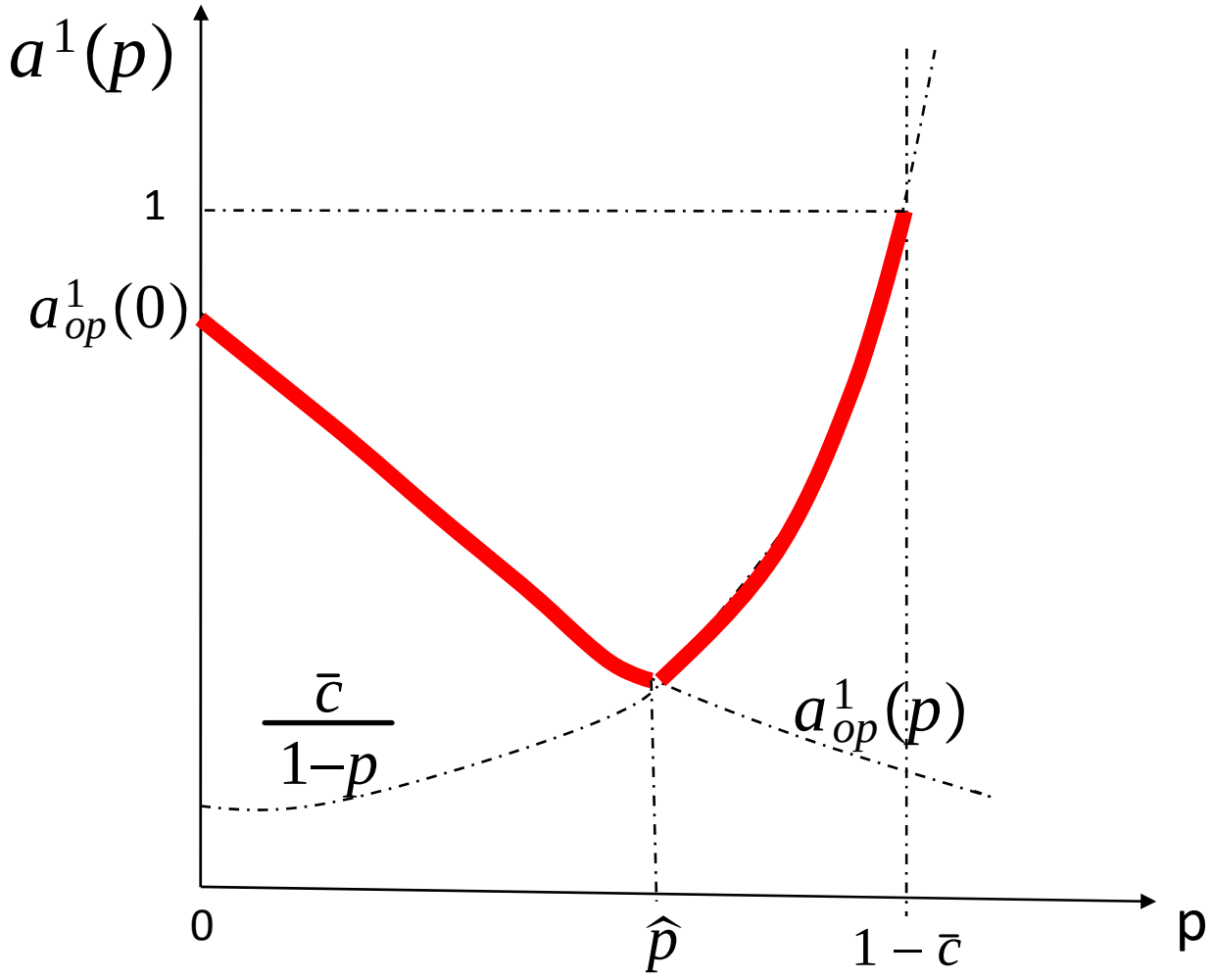


Figure 1: : Elite, Workers and Extractive institutions
Optimal Effort of the Mass workers

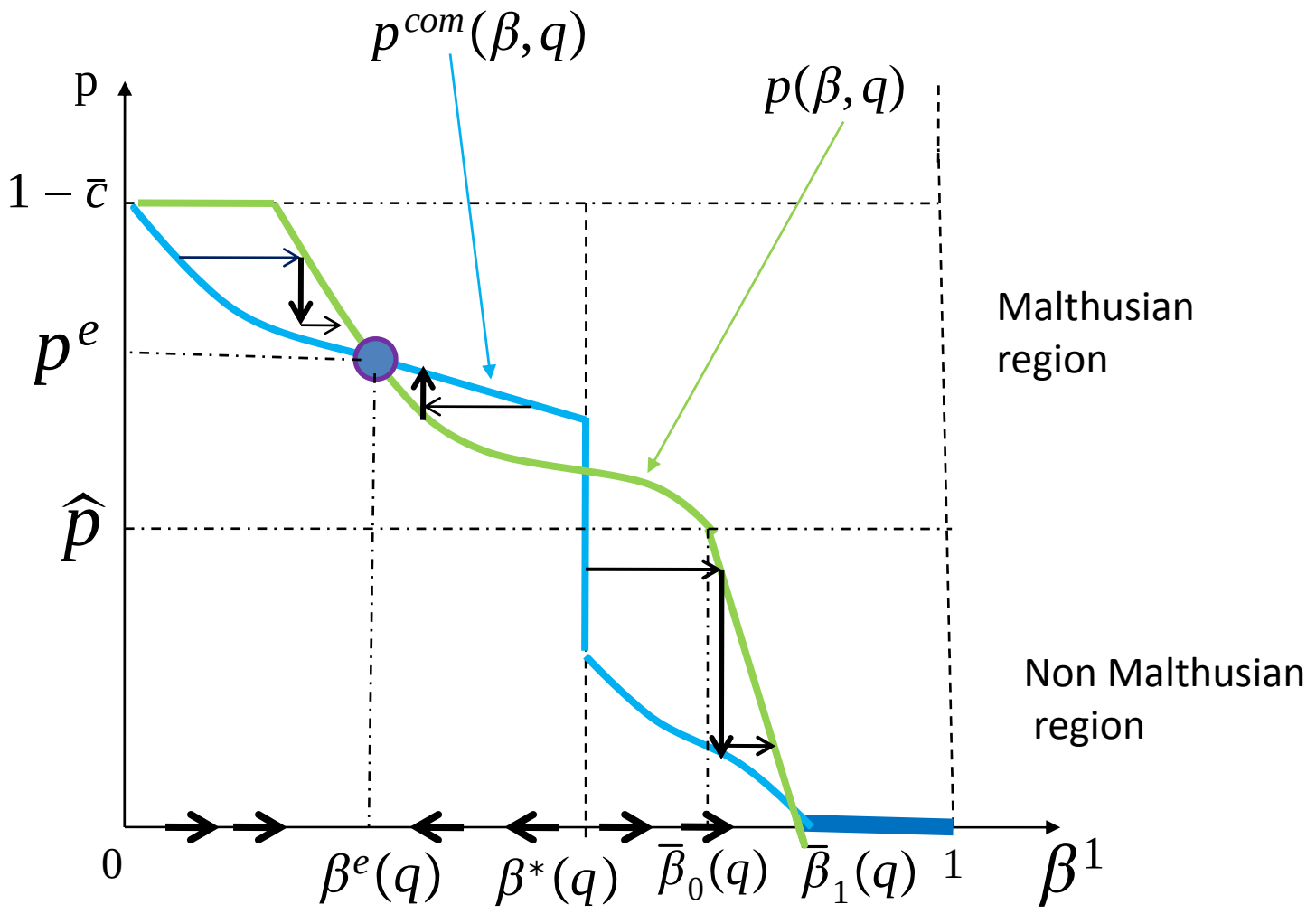


Figure 2: Elite, Workers and Extractive institutions
Equilibrium Policies and Institutional Dynamics

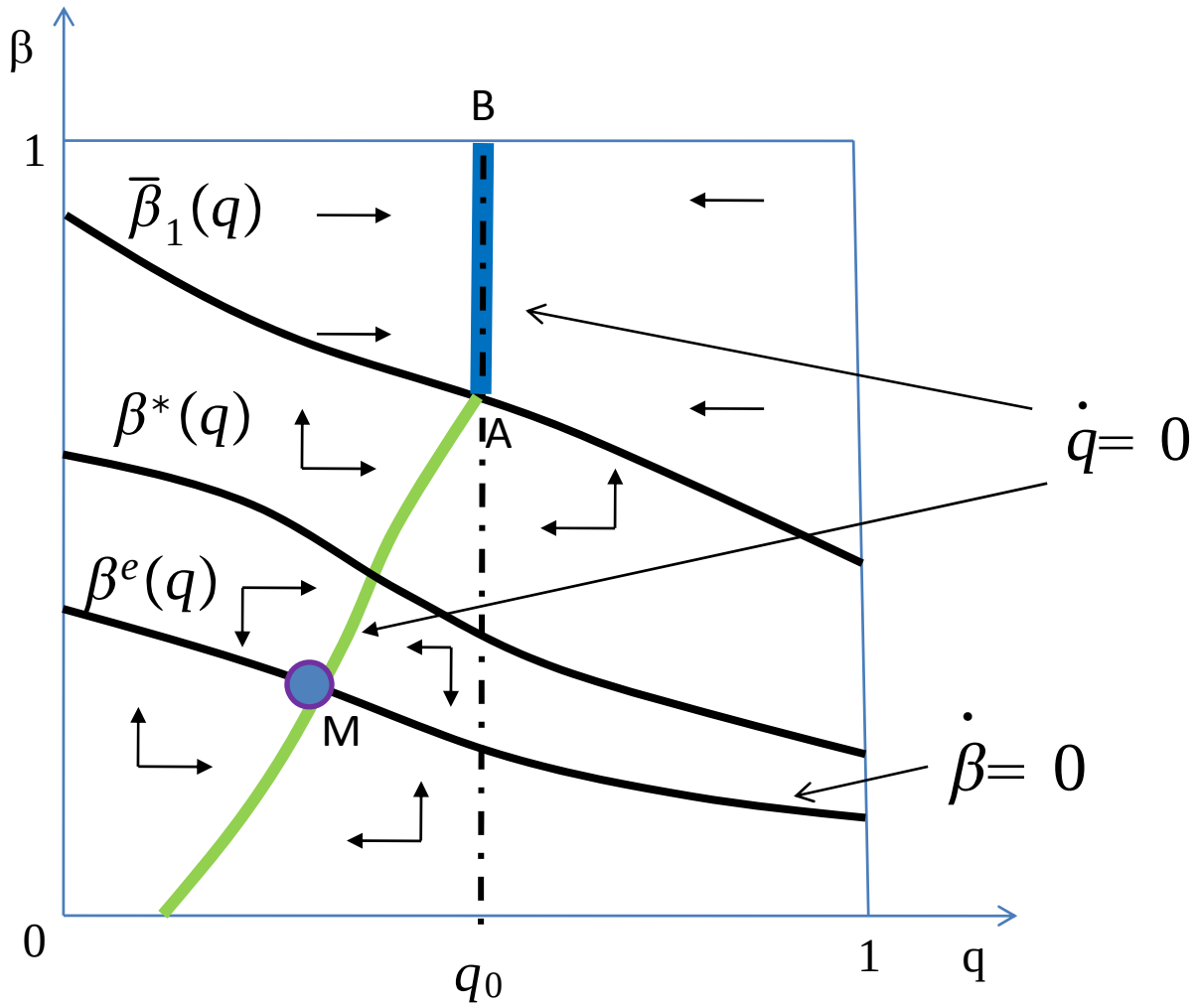


Figure 3: Elite, Workers and Extractive institutions
Phase Diagram Coevolution institutions-culture

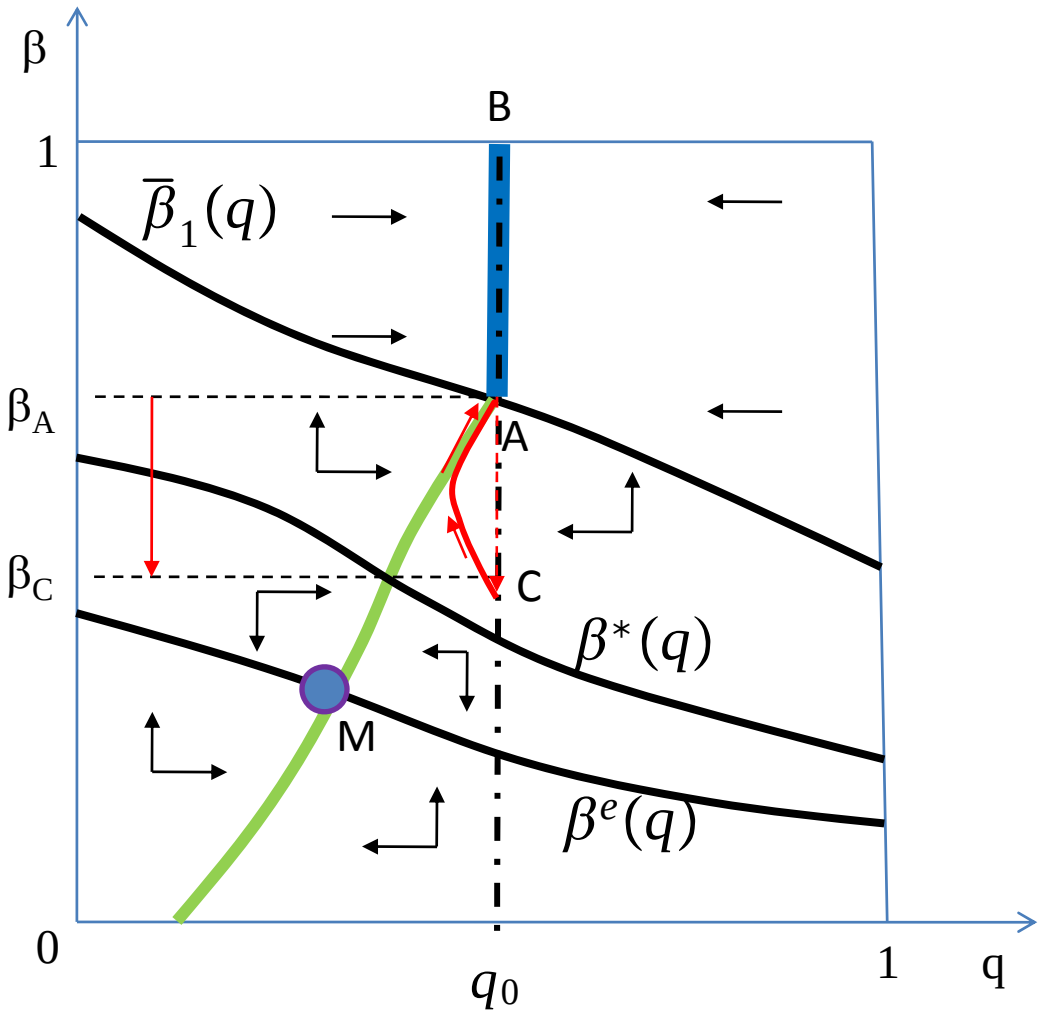


Figure 4: Elite, Workers and Extractive Institutions Shock ,Transition and Co evolution of Institutions and Culture

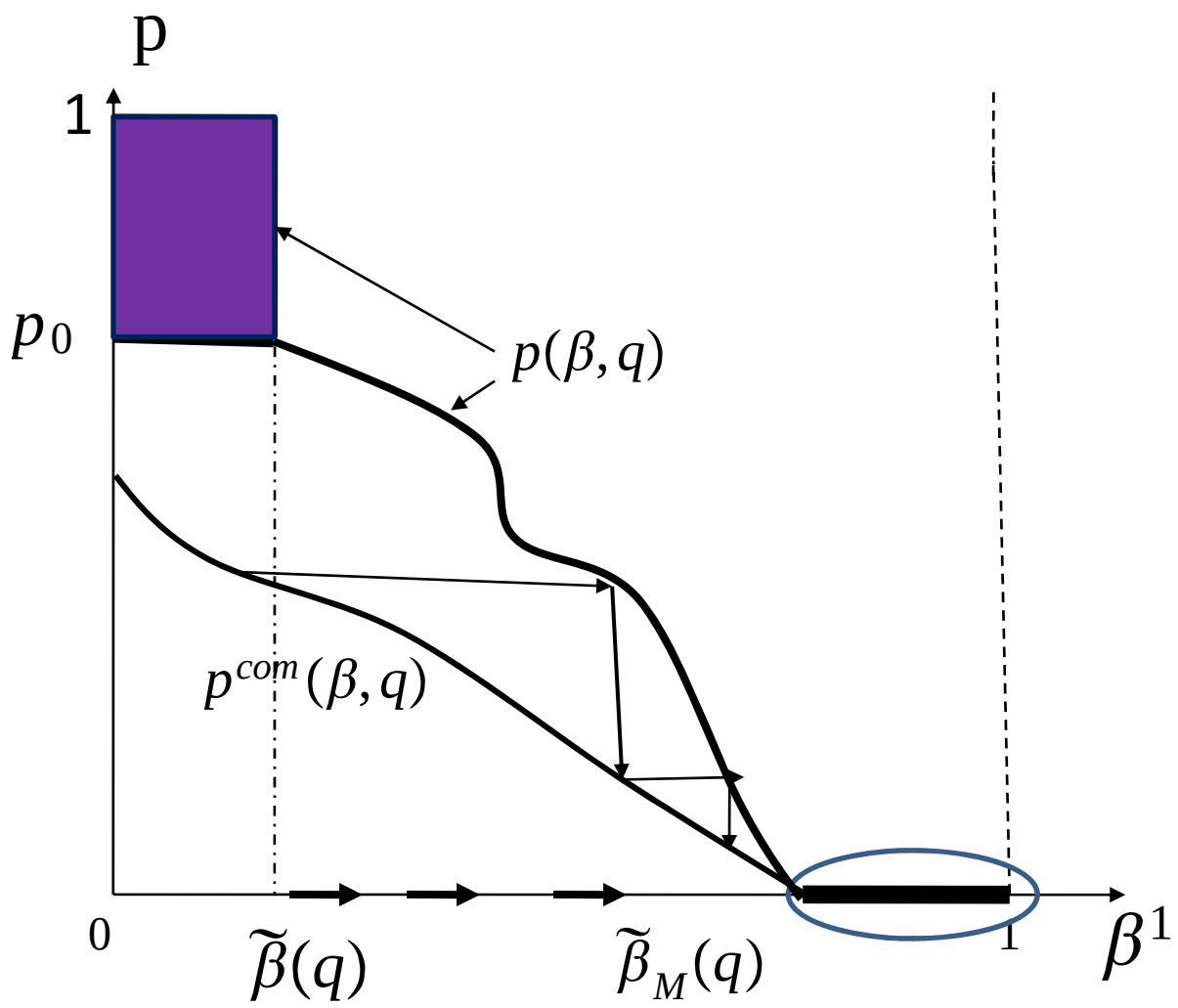


Figure 5: Elites, workers, and the provision of public goods

Equilibrium Policies and Institutional Dynamics

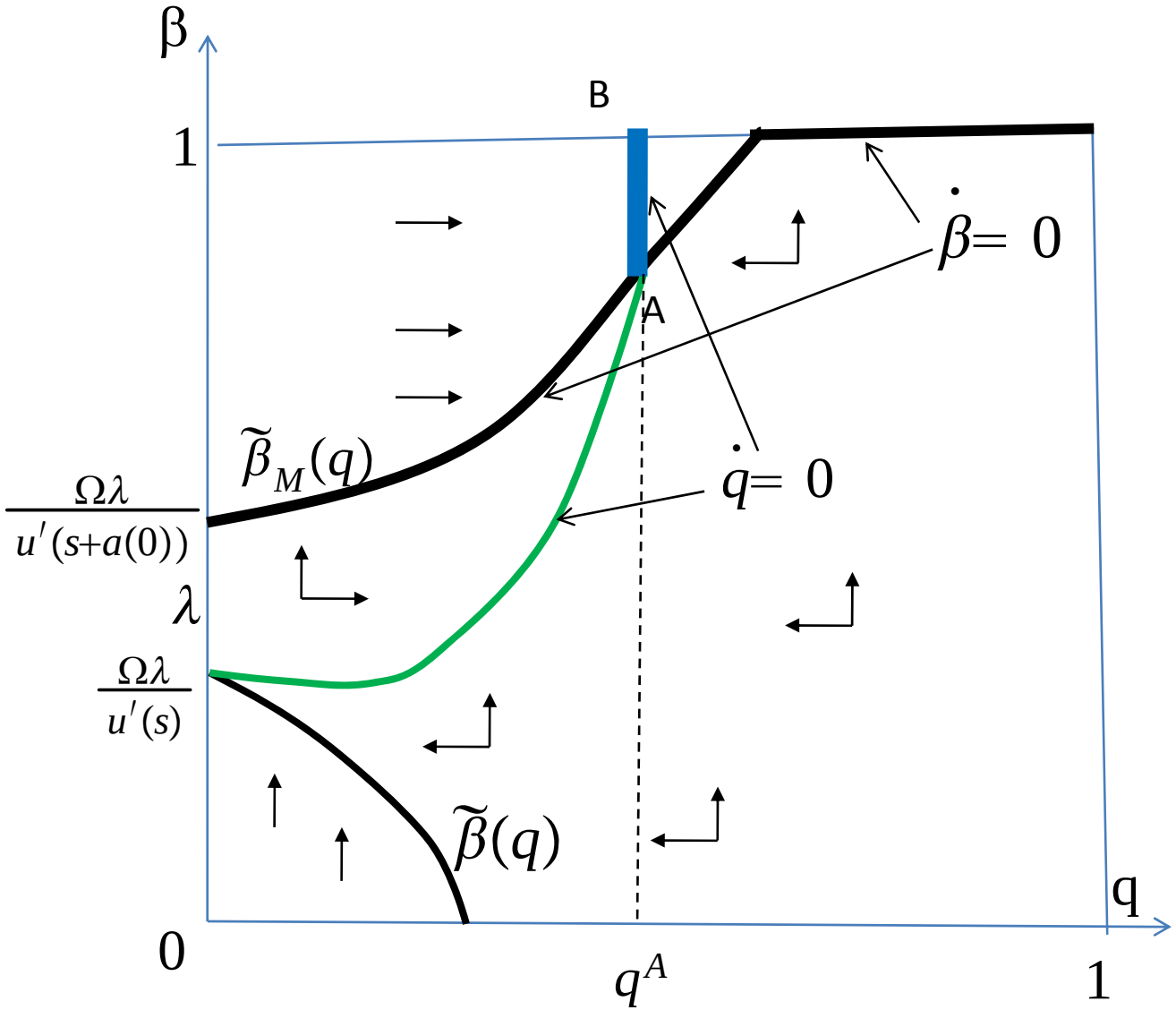


Figure 6a: Elites, workers, and the provision of public goods
Phase diagram public good (case a)

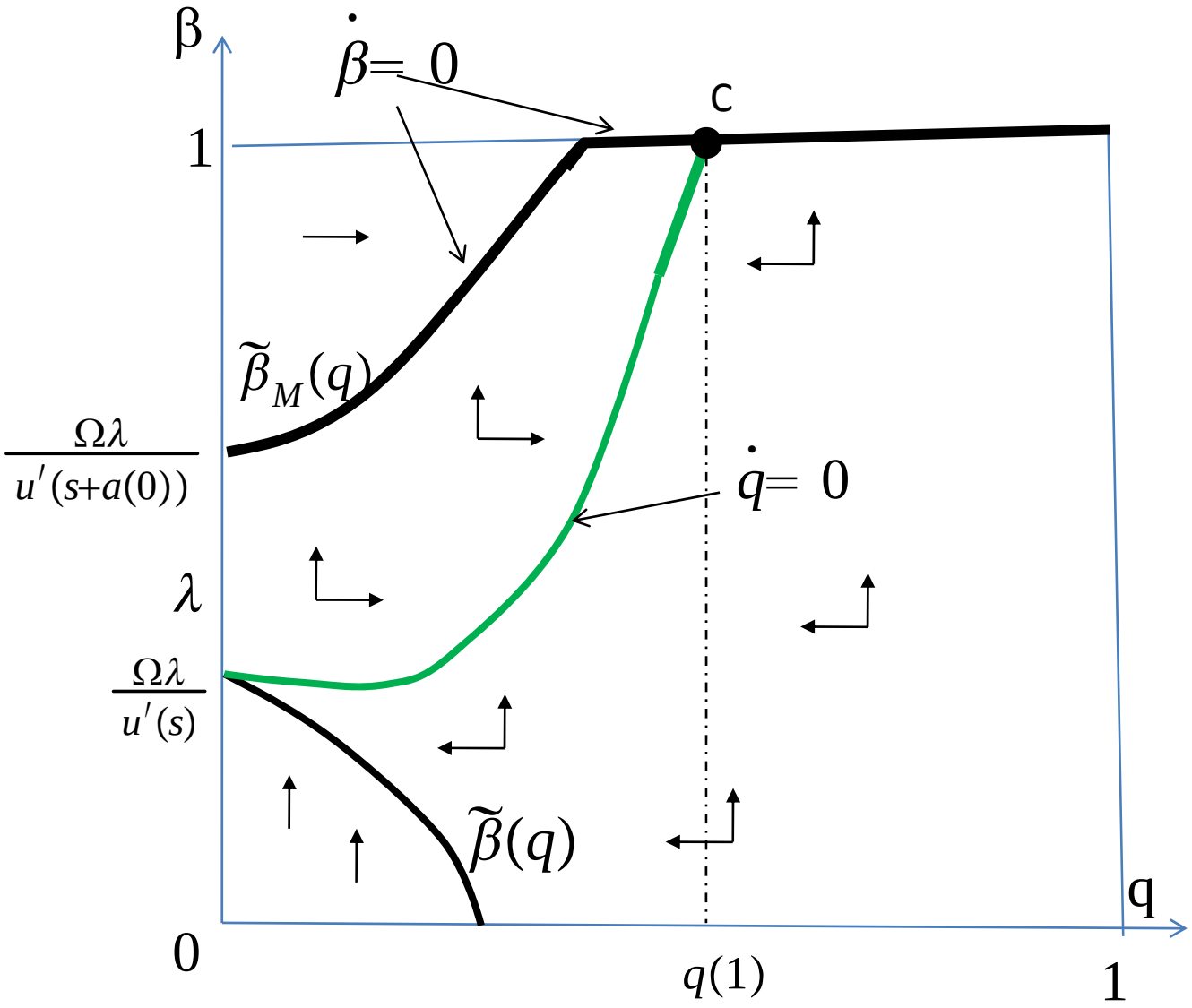


Figure 6b: Elites, workers, and the provision of public goods
Phase diagram public good (case b)

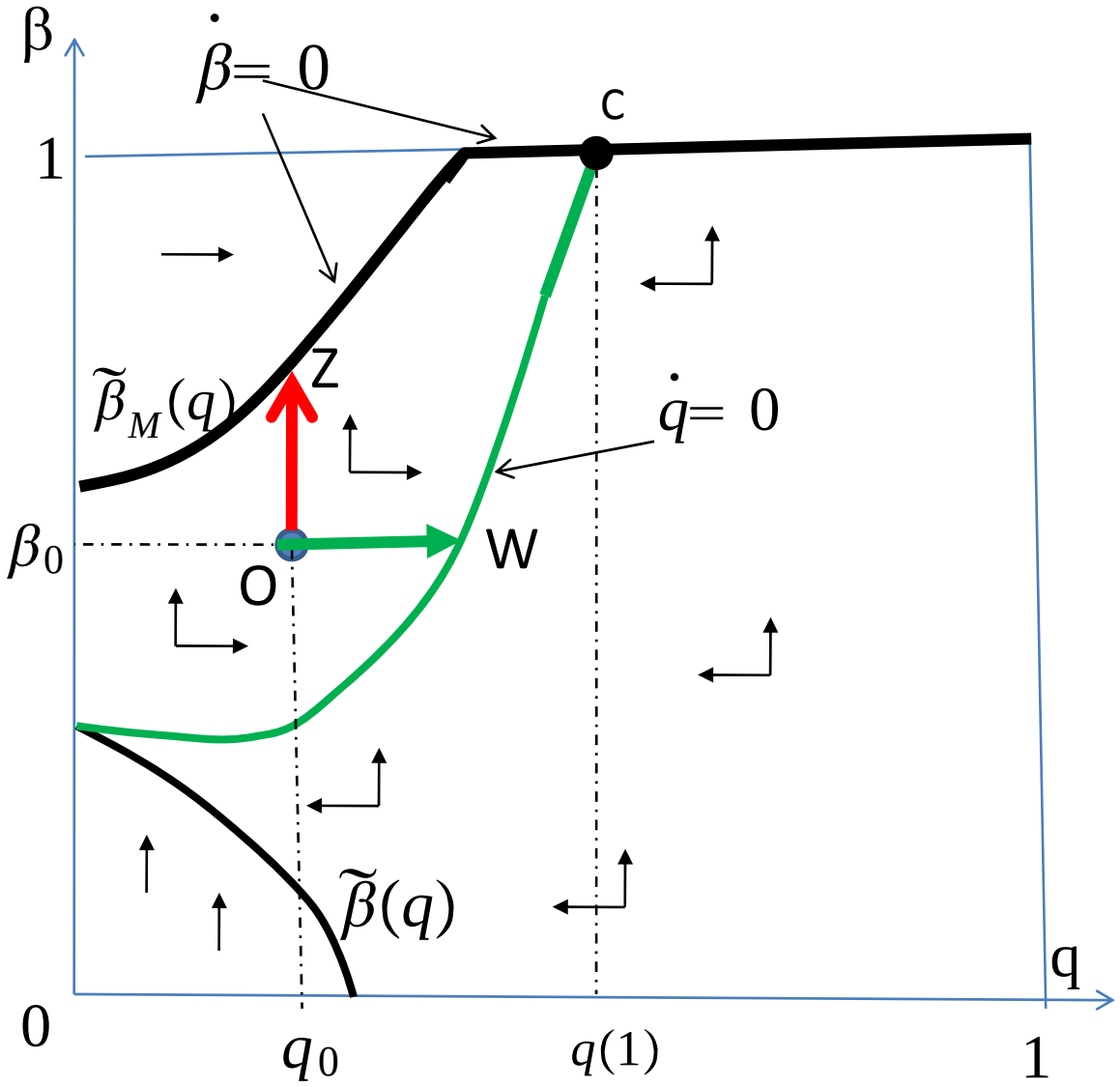


Figure 7: Elites, workers, and the provision of public goods co-evolutionary process between culture and institutions (public good case b)

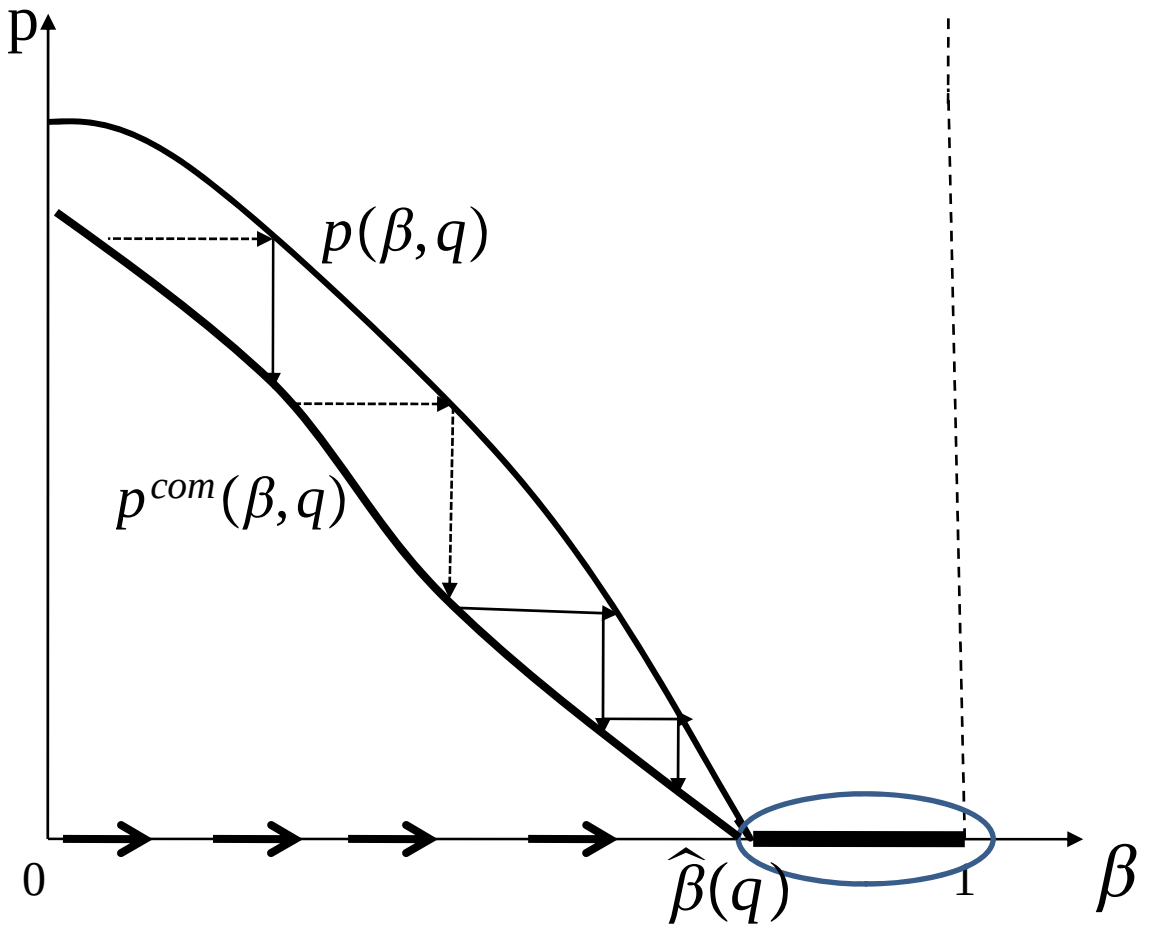
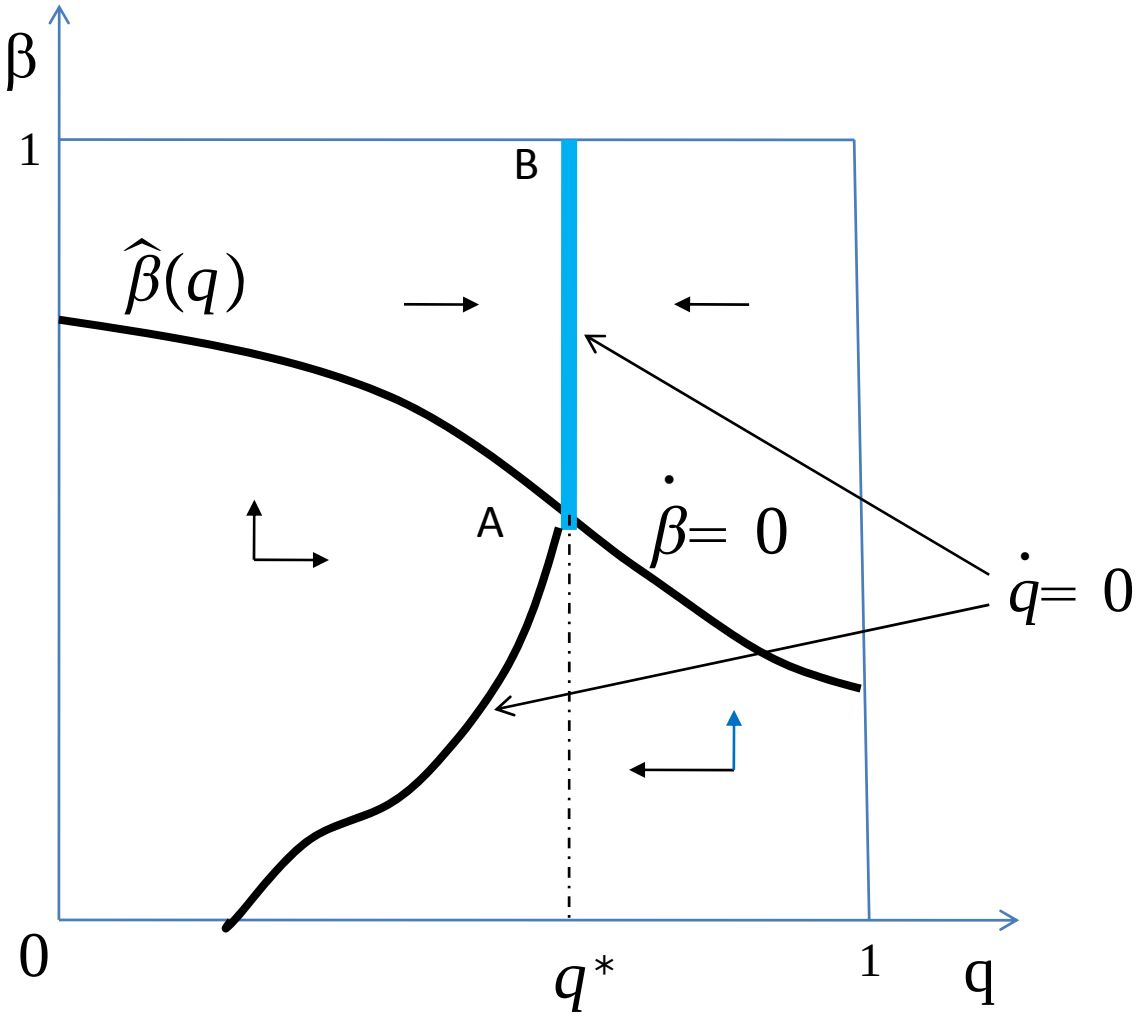
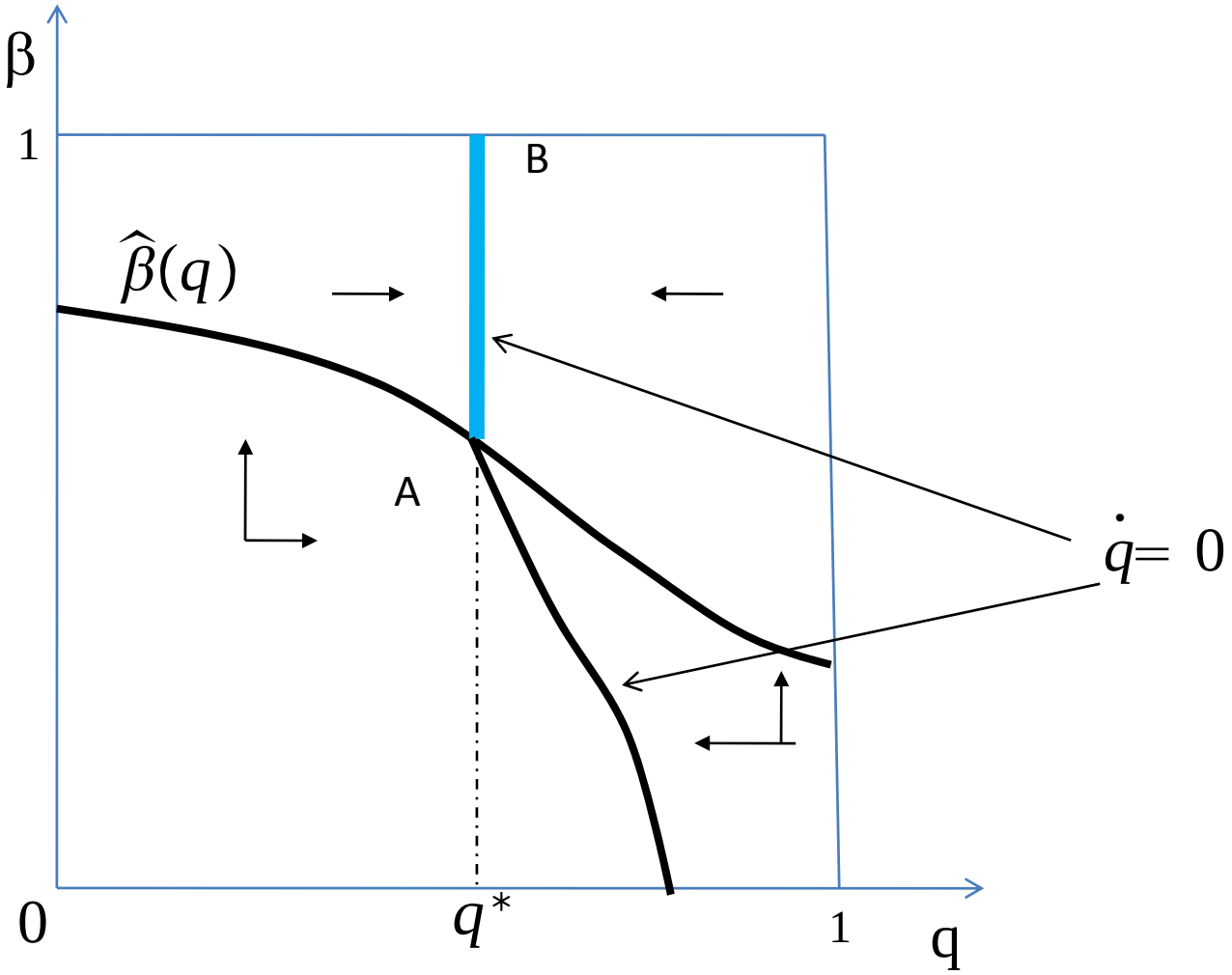


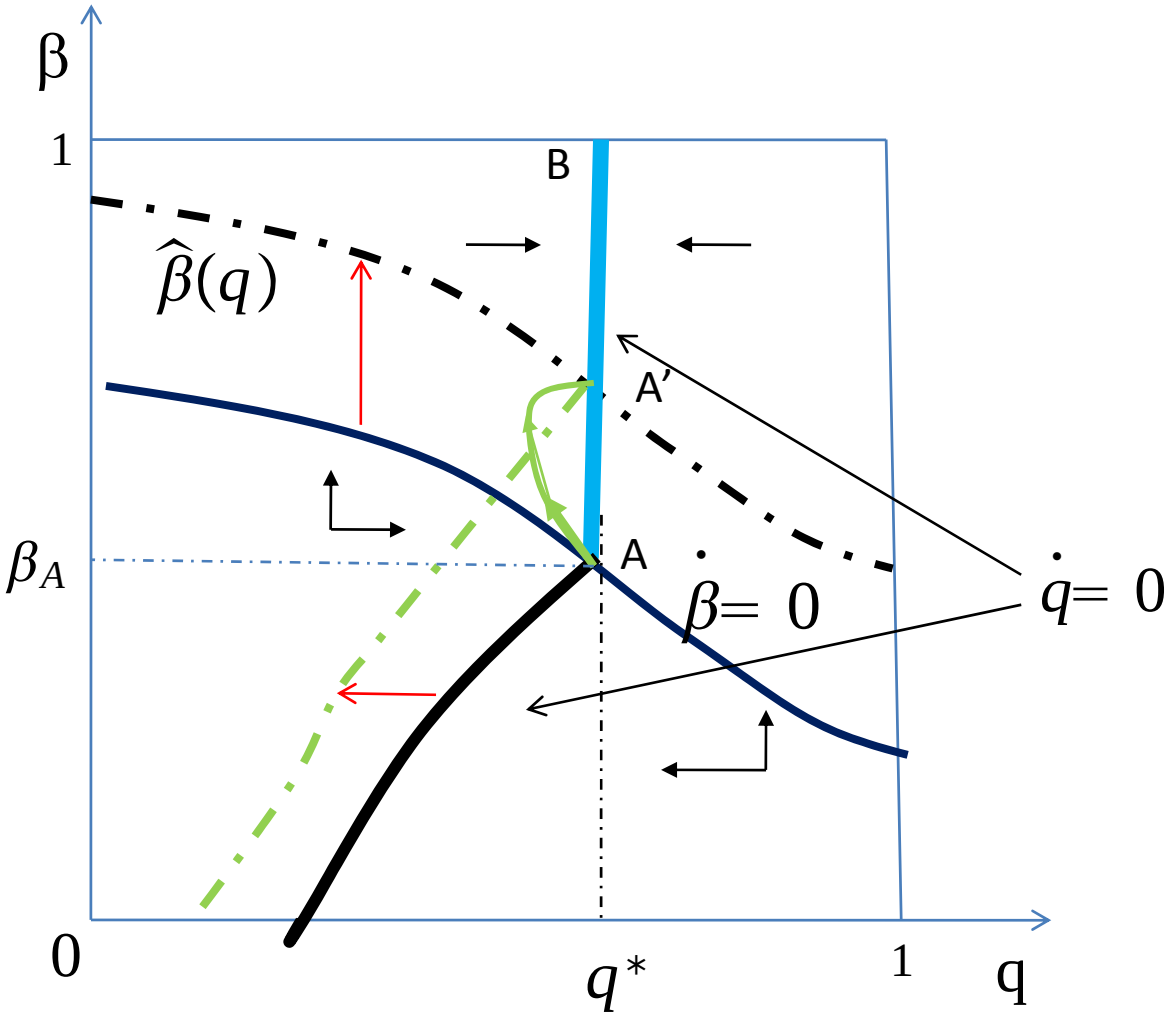
Figure 8: Civic Culture and Democratization
Equilibrium Policies and Institutional Dynamics



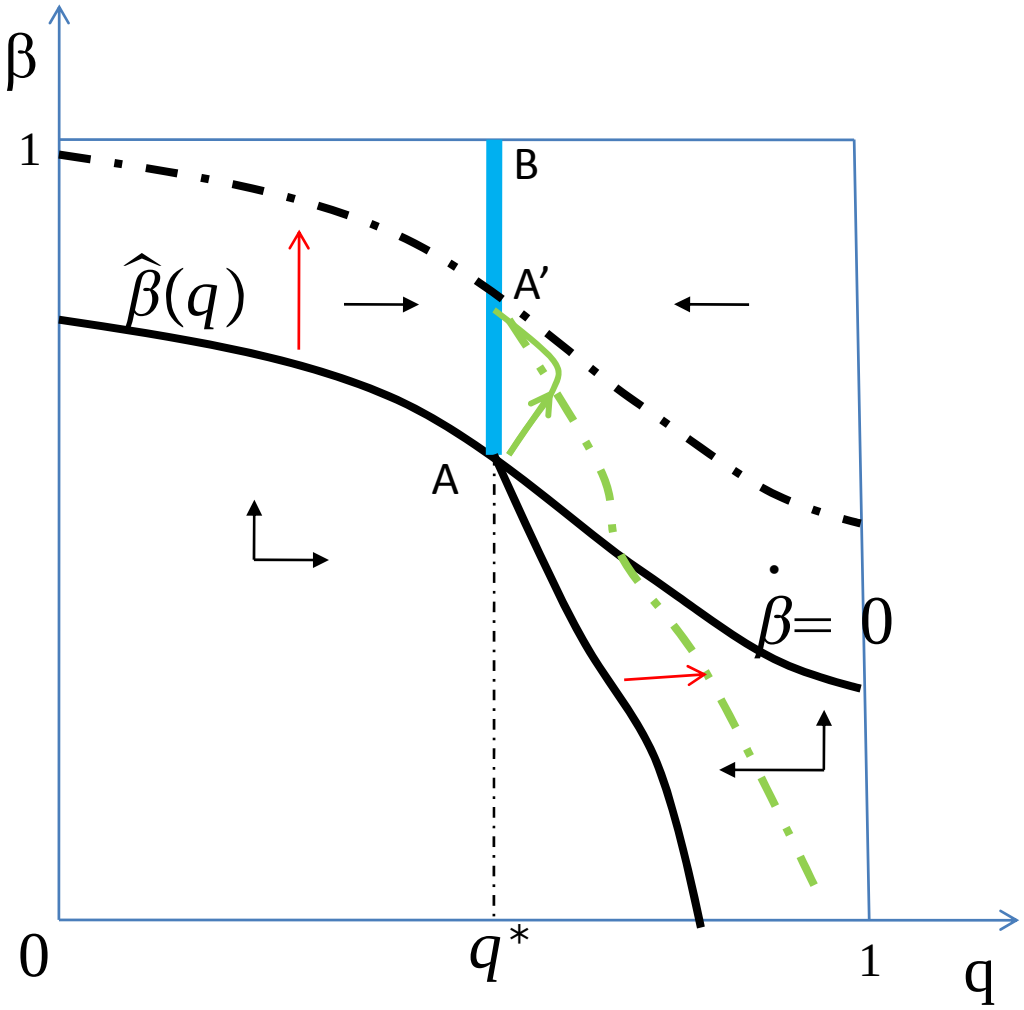
Figures 9a) : Civic Culture and Democratization Phase Diagram



Figures 9b): Civic Culture and Democratization
Phase Diagram



Figures 10a) : Civic Culture and Democratization
Coevolution Institutions and Civic Culture as complements
Positive shock on endowment ω



Figures 10b) : Civic Culture and Democratization
Coevolution Institutions and Civic Culture as substitutes
positive shock on endowment ω

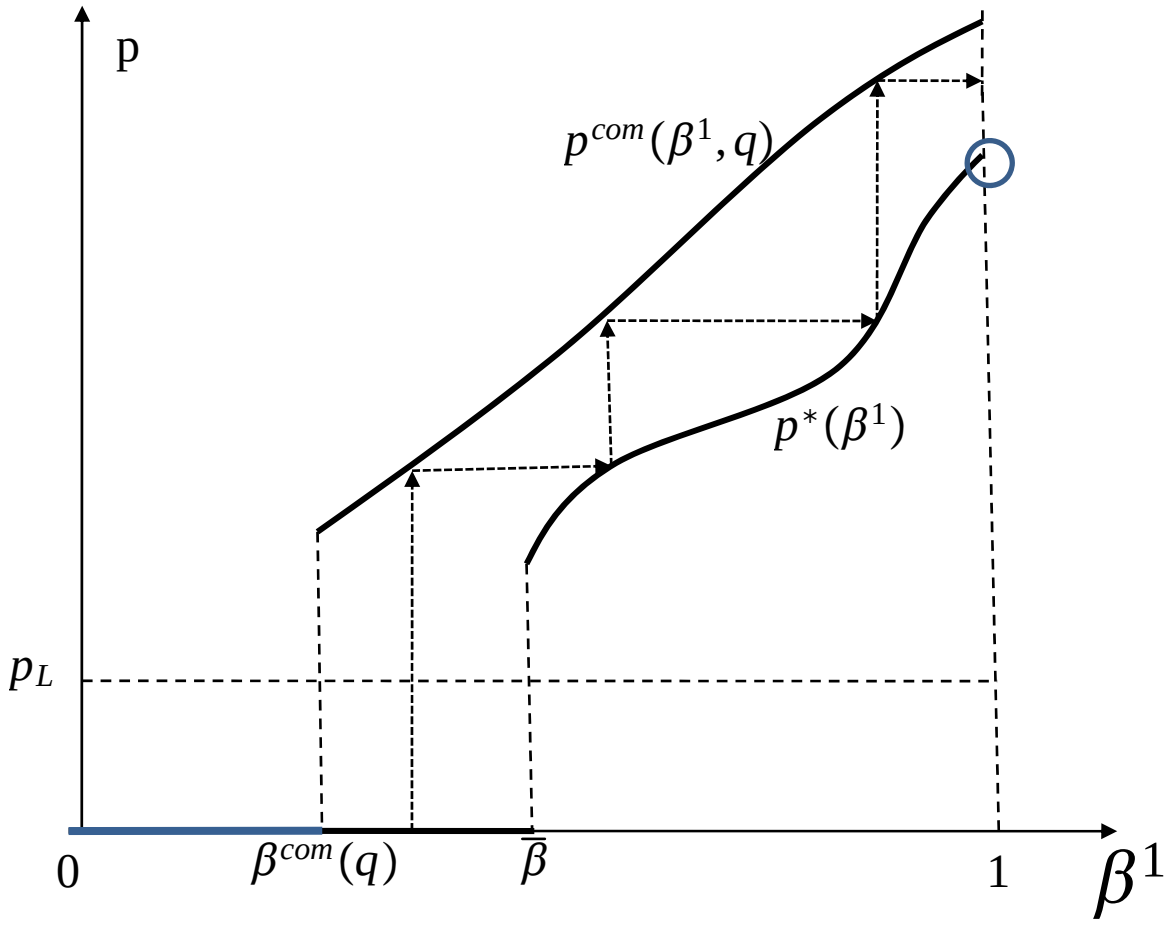


Figure 11: Modernization
Equilibrium Policies and Institutional Dynamics

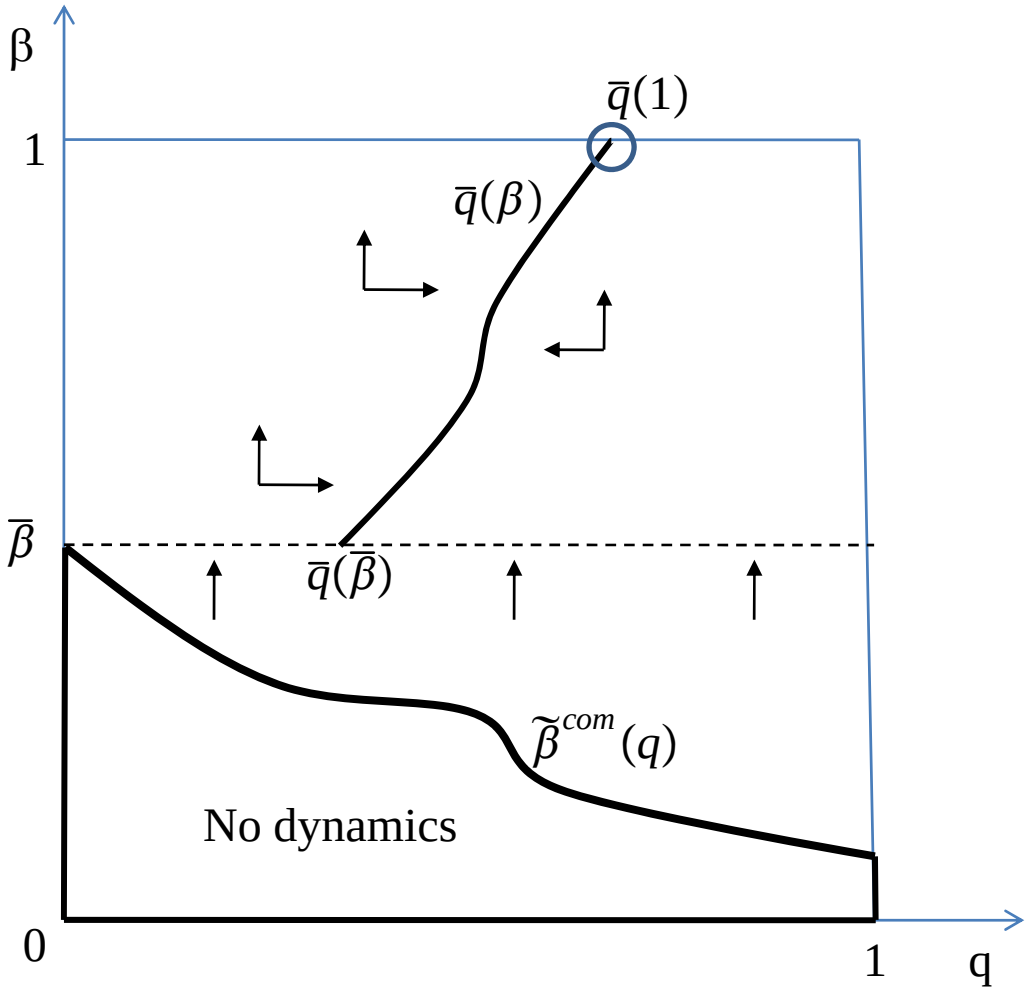


Figure 12) Modernization
Phase Diagram

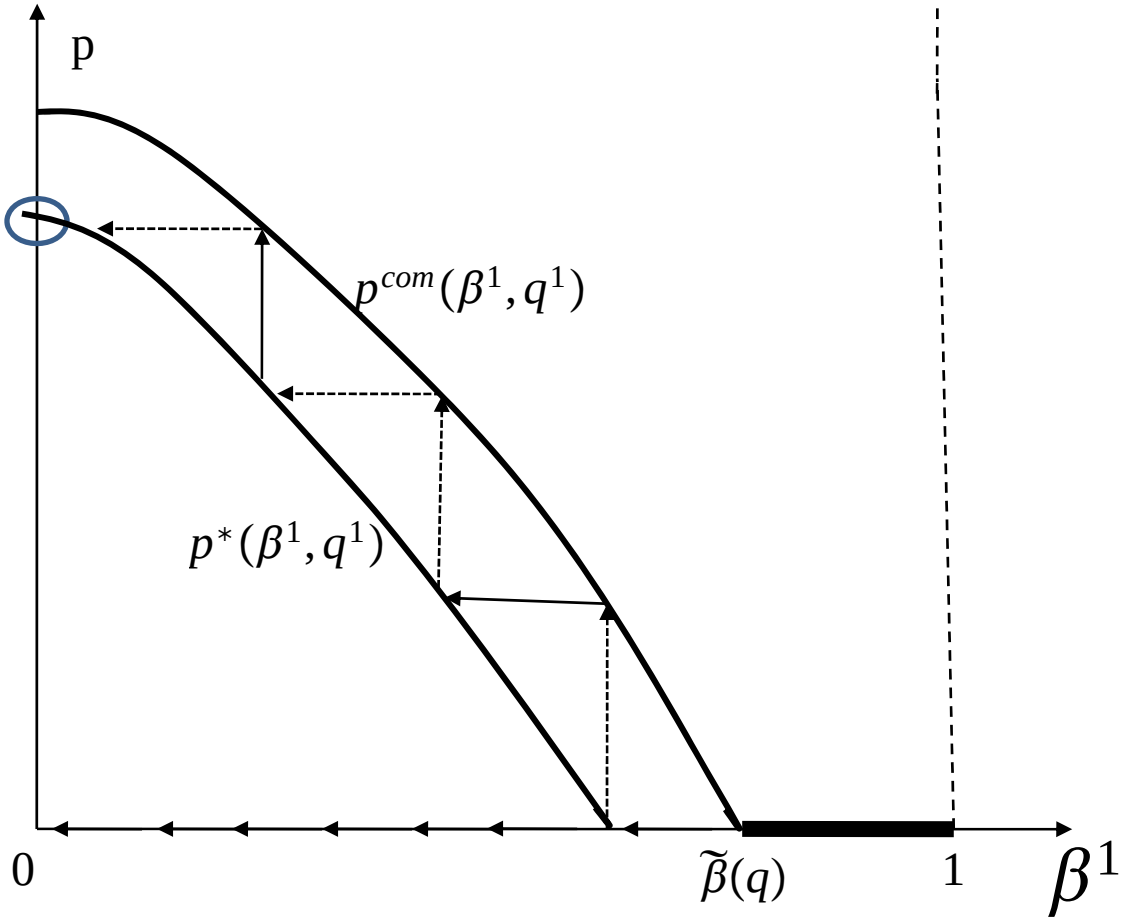
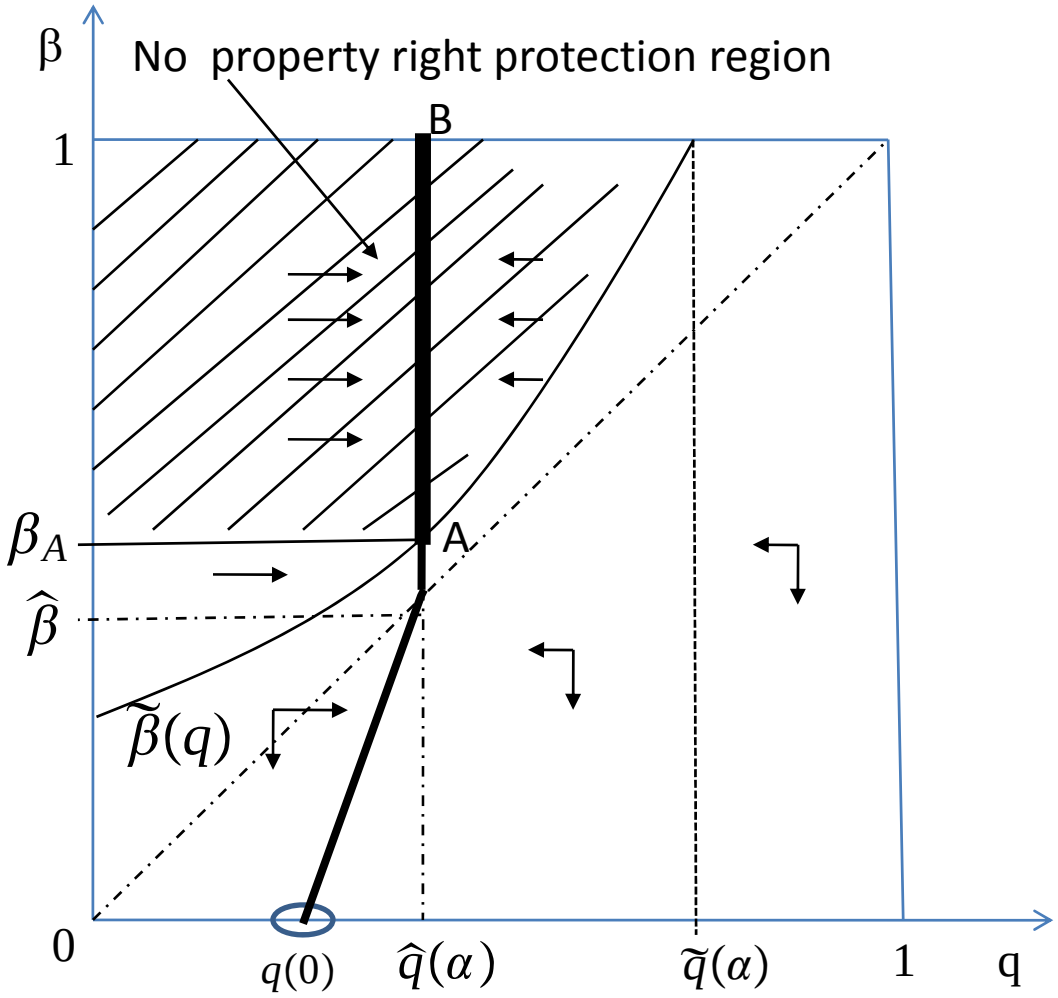
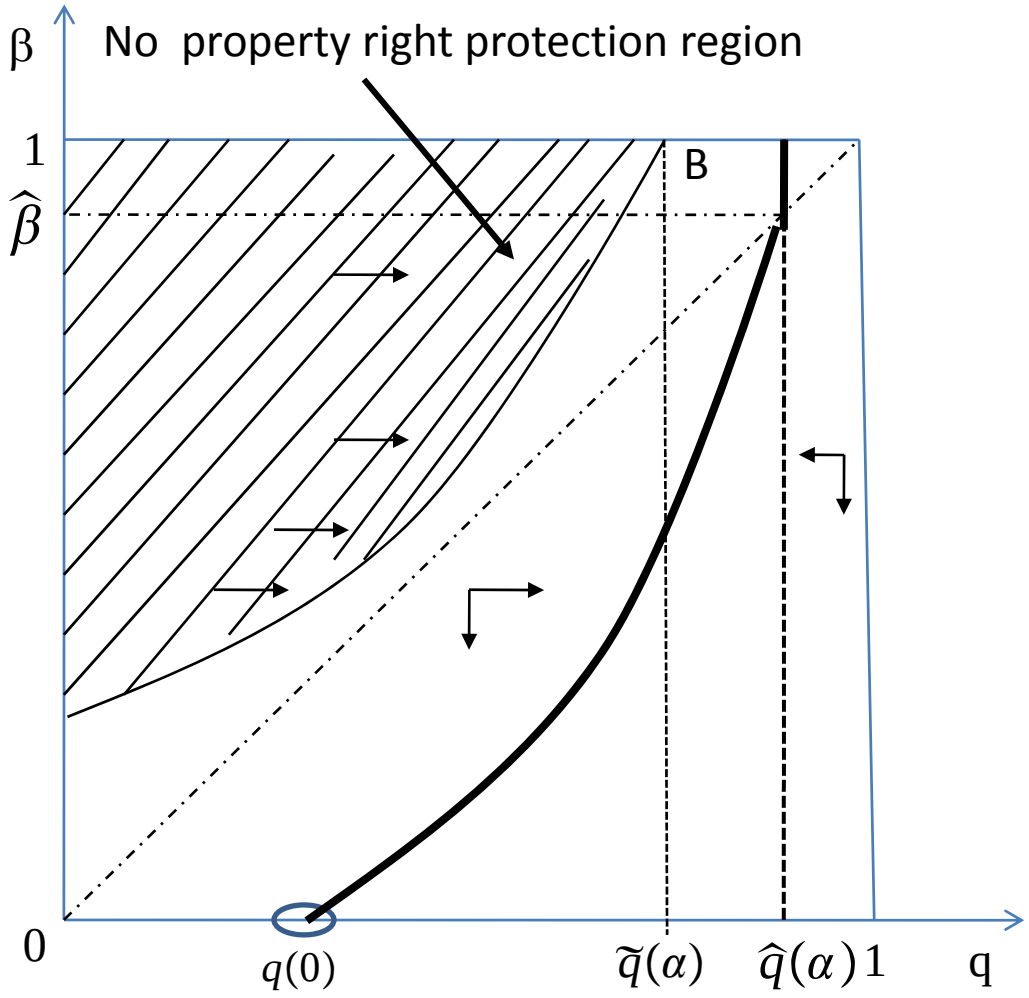


Figure 13) Property Rights and Conflicts
Equilibrium Policies and Institutional Dynamics



Figures 14a) Property Rights and Conflicts
Phase Diagram (α large enough)



Figures 14b) Property rights and conflict
Phase Diagram (α close to α_{min})

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Appendix A: Results on the Dynamical System

In this Appendix we study in some detail the dynamics of our economy. We shall restrict to the case in which $j = i$, as introduced in footnote 16. Furthermore, we keep adopting the convention that, when an apex i is omitted, it refers to $i = 1$.

We study then the dynamics of $(\beta_t, q_t) \in [0,1]^2$. We shall study the dynamical system in continuous time, but it is simpler to describe it in discrete time, as we do in the text. The fundamental dynamics equation, as reported in the text as equations (7,11), are the following:

$$\beta^i_{t+1} = \begin{cases} \beta^i \text{ such that } p^{com}(\beta^i, q^i_{t+1}) = p(\beta^i_t, q^i_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{ll} 1 & \text{if } p^{com}(\beta^i_t, q^i_{t+1}) > p(\beta^i, q^i_{t+1}),\ \forall 0 \le \beta^i \le 1 \\ 0 & \text{if } p^{com}(\beta^i_t, , q^i_{t+1}) < p(\beta^i, , q^i_{t+1}),\ \forall 0 \le \beta^i \le 1 \end{array} \right. & \text{else} \end{cases} \tag{β}$$

$$q^i_{t+1} - q^i_t = q^i(1 - q^i_t)\left(d^i - d^j\right), \text{ with } d^i = d(q^i_t, \Delta V^i(\beta^i_{t+1}, q^i_{t+1})). \tag{q}$$

We impose Assumptions 1-3 and we further assume for regularity that all maps, $p(\beta, q)$, $p^{com}(\beta, q)$, $\Delta V^i(\beta, q)$, $\Delta V^j(\beta, q)$ are smooth.[52]

The dynamics of β_t given q. Let $f : [0,1]^2 \to [0,1]$ denote the map which governs the dynamics of β_{t+1}; that is, which satisfies equation (β) and, in the continuous time limit:

$$\dot{\beta}_t = f(\beta_t, q_t).$$

Lemma A. 1 *Under our assumptions, $f : [0,1]^2 \to [0,1]$ is a continuous function in $(\beta_t, q_t) \in [0,1]^2$.*

Proof. Consider equation (β). First of all note that when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is not satisfied for any β_{t+1}, for some (q_t, q_{t+1}), the assumption that $p(\beta, q)$ is monotonic implies that β_{t+1} is $= 0$ or $= 1$, depending on the sign of $p(\beta_{t+1}, q_{t+1}) - p^{com}(\beta_t, q_t)$. In the continuous time limit $q_{t+1} = q_t$ and hence, in this case, trivially, f maps continuously $(\beta_t, q_t) \in [0,1]^2$ into $\{0\}$.

Consider equation (β), again. We show that β_{t+1} is a continuous function of β_t, q_t, q_{t+1} when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is satisfied. To this end note that the assumed monotonicity in β of $p(\beta, q)$ implies that, when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is satisfied, we can write $\beta_{t+1} = p^{-1}(p, q_t, q_{t+1})$ and hence $\beta_{t+1} = p^{-1}(p^{com}(\beta_t, q_t), q_t, q_{t+1})$, a continuous function. Again, in the continuous time limit $q_{t+1} = q_t$ and hence we can construct a continuous function $f : [0,1]^2 \to \mathbb{R}$ such that $\dot{\beta}_t = f(\beta_t, q_t)$.

Finally, it is straightforward to see that as $p(\beta_{t+1}, q_{t+1}) - p^{com}(\beta_t, q_t)$ crosses 0 $\beta_{t+1} = p^{-1}(p^{com}(\beta_t, q_t), q_t, q_{t+1})$ converges continuously to 0 or 1 depending on the direction of the crossing so as to preserve continuity. ■

[52] As in the text, we continue to drop the index i when convenient, under the convention that $(\beta, q) = (\beta^1, q^1)$.

Let the $\beta : [0,1] \to [0,1]$ map $q \in [0,1]$ into the stationary states of f; that is, $\beta : [0,1] \to [0,1]$ satisfies

$$0 = f(\beta, q), \text{ for any } \beta \in \beta(q)$$

Lemma A. 2 *Under our assumptions, the map $\beta : [0,1] \to [0,1]$ is an non empty and compact valued upper-hemi-continuous correspondence with connected components.*

Proof. The proof is a direct consequence of the continuity of f proved in the Lemma A.1. ■

Let $P(\beta, q) := p(\beta, q) - p^{com}(\beta, q)$. We consider only the regular case in which $P(\beta, q) \neq 0$ at the vertices of $[0,1]^2$, leaving the simple but tedious analysis of the singular cases to the reader. Also, we say that q is a regular point of $\beta \in \beta(q)$ if any stationary stationary state $\beta \in \beta(q)$ satisfies that property that $\frac{\partial P(\beta,q)}{\partial \beta} \neq 0$; that is if $p(\beta, q)$ and $p^{com}(\beta, q)$ intersect transversally. The characterization of $\beta : [0,1] \to [0,1]$ depends crucially on the topological properties of the zeros of $P(\beta, q)$. Let $\pi : [0,1] \to [0,1]$ map q into the stationary states β such that $P(\beta, q) = 0$; that is, the map π satisfies $P(\pi(q), q) = 0$.

Proposition A. 1 *The dynamics of β_t as a function of $q \in [0,1]$ has the following properties,*

1. *$P(0,q) < 0$, $P(1,q) > 0$, for any $q \in [0,1]$, and $p(\beta, q)$ is increasing; or $P(0,q) > 0$, $P(1,q) < 0$, for any $q \in [0,1]$, and $p(\beta, q)$ is decreasing. For any given regular $q \in [0,1]$ there exist an odd number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0, 1$ are also stationary states for given $q \in [0,1]$. The stability properties of the regular stationary states alternate with the smallest and the larger being always locally stable; the boundaries $\beta = 0, 1$ are locally unstable for all $q \in [0,1]$.*

2. *$P(0,q) > 0$, $P(1,q) < 0$, for any $q \in [0,1]$, and $p(\beta, q)$ is increasing; or $P(0,q) < 0$, $P(1,q) > 0$, for any $q \in [0,1]$, and $p(\beta, q)$ is decreasing. For any given $q \in [0,1]$ there exist an odd number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0, 1$ are also stationary states for given $q \in [0,1]$. The stability properties of the regular stationary states alternate with the smallest and the larger being always locally unstable; the boundaries $\beta = 0, 1$ are locally stable.*

3. *$P(0,q) > 0$, $P(1,q) > 0$, for any $q \in [0,1]$. For any given $q \in [0,1]$ there exist either none or an even number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0$ is also a stationary state for given $q \in [0,1]$. The stability properties of the regular stationary states alternate with the smallest always locally unstable; the boundary $\beta = 0$ is locally stable.*

4. *$P(0,q) < 0$, $P(1,q) < 0$, for any $q \in [0,1]$. For any given $q \in [0,1]$ there exist either none or an even number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 1$ is also a*

stationary state for given $q \in [0,1]$. *The stability properties of the regular stationary states alternate with the smallest always locally stable; the boundary* $\beta = 1$ *is locally stable.*

5. $P(0,q)$ *and/or* $P(1,q)$ *change sign with* $q \in [0,1]$. *The characterization obtained above then can be repeated for each sub-interval of* $[0,1]$ *in which the Brouwer degree of the manifold* $\pi(q)$ *is invariant (see the proof). We leave the tedious cathegorization of all possible cases to the reader.*