Moti <sup>,</sup>	vation 000	Contribution O	Formulation 0000000	Analyses 000000000000	Discussion 000000	Appendix 0000
		nvestment	in health a	across the life	cycle:	
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from the womb to the grave

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#### VERY PRELIMINARY

ASSA meeting, San Francisco January 4, 2016

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#### Disparities widen as children age



Figure: Self-reported health status (1=excellent, 5=poor) for children (LHS) and adults (RHS) from the National Health Interview Survey. College-age adults (ages 18-24) not included due to concerns about representativeness of this sample, and because it is unclear whether these respondents report their current incomes or that of the families in which they were raised. Figure taken from Case, Lubotsky & Paxson (2002).

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# Health disparities originate early in life

- Differences in health become more pronounced as children age (Case, Lubotsky & Paxson, 2002), widening until about age 60 when they appear to narrow (Case & Deaton, 2005)
- Circumstances in womb affect human capital formation
  - intrauterine environment programs fetus to have metabolic characteristics which lead to future disease (Barker et al. 1993; Gluckman & Hanson, 2006)
  - fetal origins effect extends to other domains with sizable effects on educational and labor market outcomes, IQ, adulthood height, marital status, welfare dependency, characteristics of neighborhood (Currie & Hyson 1999; Almond & Currie, 2011)



# Events before age 5 can have large, long lasting impacts

- Adult illnesses are more prevalent and problematic among those who experienced adverse early childhood circumstances (e.g., Galobardes, Lynch & Smith, 2008)
- Heckman and coauthors emphasize the importance of early childhood investments
  - childhood health and human capital formation is a dynamic process (Cunha et al. 2006; Cunha & Heckman, 2007; Heckman, 2007; Heckman, Humphries & Veramendi, 2015)
- This line of research
  - emphasizes the role of childhood cognitive and, particularly, non-cognitive abilities in determining both education and health outcomes in later life
  - illustrates the complexity of early childhood skill formation (abilities are multiple in nature, interact with one another, sensitive and critical periods, etc.)

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# Still unclear what drives effects

- Some claim 40% of early deaths due to behavior, 30% due to genetic predispositions, 15% due to social circumstances, and 10 15% due to shortfalls in medical care (McGinnis et al. 2002)
- Family income is determinant of child health (Case et al. 2002)
  - higher income parents manage chronic health problems better
  - income gradient cannot be explained by child's health at birth, poor child health affecting family income, genetics, differences in health insurance, but cannot rule out health behaviors
- Currie (2009) concludes
  - parental circumstances affect child health
  - child health matters for educational and labor market outcomes
  - but it is too early to tell how important these effects are versus more conventional measures of human capital
  - fetal health appears particularly important
- Environment can become biologically embedded in the body (Gluckman & Hanson, 2006)



### Literature lacks framework to interpret evidence

- Here, emerging empirical evidence is conceptualized and interpreted within a lifecycle framework linking early to late life
- While there are several models of parental investment in children,<sup>1</sup> these
  - model skill formation (under the form of a single skill, with the exception of Cunha, Heckman & Schennach, 2010). See Heckman & Mosso (2014) for a survey
  - do not explicitly model health, a form of human capital with distinct characteristics (Grossman, 1972; Ehrlich & Chuma, 1990; Galama, 2015; Galama & Van Kippersluis, 2015a)
- Grossman (1972) treats childhood health formation as determined outside the model (Heckman, 2007)<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>Caucutt & Lochner (2012); Cunha (2013); Cunha & Heckman (2007); Cunha, Heckman, & Schennach (2010); Del Boca, Flinn & Wiswall (2012, 2014); Gayle, Golan, & Soytas (2013); Lee and Seshadri (2015a,b); Lee, Roys & Seshadri (2015).

<sup>&</sup>lt;sup>2</sup>Jacobson (2000), Bolin et al. (2001, 2002) model production of family (including child) health) but build on degenerate linear investment case (see Galama, 2015) and assume family has single planning horizon. (  $\equiv$  )  $\equiv$ 

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In this p	oaper, we				

- Present an overlapping generations model of early-life parental investment and later-life own investment in health
- Single parent optimizes her education, timing of child birth, consumption, leisure time, time devoted to work and child investment, own and child health, duration of own longevity
- Model captures important relationships between parental characteristics, economic circumstances, childhood skill, health and wealth endowments, biology, and the child's health
- Investment in child's health and saving for inheritance provide distinct means for parent to improve child's future wellbeing
- Upon becoming an adult, child decides to continue her education (and for what duration) or not



# Conceptual framework for health over the life cycle

- Brings together skill, health endowments and their effects on early, late life investments in health, education, and longevity
- In contrast to previous research our theory
  - focusses on life cycle health
  - has a horizon (longevity) that is endogenous
  - is sufficiently tractable that it can be analyzed analytically through comparative dynamic analyses
- We find,
  - parental investments and savings enhanced if additional resources enable child to extend her life
  - model can reproduce early emergence and widening of health disparities, and accelerated aging by those born in poverty



# Schematic of overlapping generations au - 1, au and au + 1



- Generation  $\tau$ 's age measured by  $t_{\tau}$ : born at  $t_{\tau} = -(D_{\tau} B_{\tau})$ , becomes adult at  $t_{\tau} = 0$ , continues her education till  $t_{\tau} = E_{\tau}$ , gives birth to child at  $t_{\tau} = B_{\tau+1}$ , raises child till  $t_{\tau} = D_{\tau+1}$  and dies at  $t_{\tau} = T_{\tau}$ .<sup>3</sup> Endogenous  $E_{\tau}$ ,  $B_{\tau+1}$  and  $T_{\tau}$ . Total lifetime  $= D_{\tau} + T_{\tau}$
- Throughout presentation we measure age by  $t_{\tau}$  and index variables by generation  $\tau 1$ ,  $\tau$ , and  $\tau + 1$

<sup>&</sup>lt;sup>3</sup>Set up similar to OLG model of Lee and Seshadri (2015a,b) but focus is on health, use continuous time, model endogenous longevity, endogenous timing of birth (in addition to endogenous education) and do not include policies.

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### Generation au parent maximizes her life-time utility

$$\mathbb{U}^{\tau} = \max_{Z^{\tau}(t_{\tau})} \left\{ \int_{0}^{T_{\tau}} U[C^{\tau}(t_{\tau}), L^{\tau}(t_{\tau}), H^{\tau}(t_{\tau})] e^{-\rho^{\tau}t_{\tau}} dt_{\tau} \right\}$$
  
+ 
$$\max_{Z^{\tau}(t_{\tau})} \left\{ a_{H}^{\tau} \mathbb{U}^{\tau+1}(D_{\tau+1}) + a_{A}^{\tau} \mathbb{U}^{\tau+1}(T_{\tau}) \right\},$$

 $Z^{\tau}(t_{\tau}) \equiv \{C^{\tau}(t_{\tau}), L^{\tau}(t_{\tau}), X^{\tau}_{H}(t_{\tau}), t^{\tau}_{H}(t_{\tau}), X^{\tau+1}_{H}(t_{\tau}), t^{\tau+1}_{H}(t_{\tau}), E_{\tau}, B_{\tau+1}, T_{\tau}\}$ 

- Utility U(.) (concave) is derived from own consumption  $C^{\tau}(t_{\tau})$ , leisure  $L^{\tau}(t_{\tau})$  and health  $H^{\tau}(t_{\tau})$
- Utility of parent  $a_H^{\tau} \mathbb{U}^{\tau+1}(D_{\tau+1})$  provided by child's health  $H^{\tau+1}(D_{\tau+1})$
- Utility of parent  $a_A^{\tau} \mathbb{U}^{\tau+1}(T_{\tau})$  provided by bequeathing wealth  $A^{\tau}(T_{\tau})$
- Health investment  $I_{H}^{\tau}(t_{\tau})$ ,  $I_{H}^{\tau+1}(t_{\tau})$  (parent, child) consists of goods  $X_{H}^{\tau}(t_{\tau})$ ,  $X_{H}^{\tau+1}(t_{\tau})$  and time inputs  $t_{H}^{\tau}(t_{\tau})$ ,  $t_{H}^{\tau+1}(t_{\tau})$ .<sup>4</sup>
- Endogenous education  $E_{ au}$ , child birth  $B_{ au+1}$ , and length of life  $T_{ au}$

 $<sup>^4</sup>$ Combined according to Cobb-Douglass constant returns process.  $\langle \Box \rangle \rangle \langle \Box \rangle \rangle \langle \Xi \rangle \rangle \langle \Xi \rangle \rangle \langle \Xi \rangle \rangle \langle \Xi \rangle \langle \Box \rangle \langle \Box \rangle \langle \Box \rangle \rangle \langle \Box \rangle \langle \Box$ 

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# Subject to the following dynamic constraints

$$\begin{aligned} \frac{\partial H^{\tau}}{\partial t_{\tau}} &= \mu_{H}^{\tau}(t_{\tau}, E_{\tau}; \theta^{\tau}) I_{H}^{\tau}(t_{\tau})^{\alpha^{\tau}(t_{\tau})} - \sigma_{H}^{\tau}(t_{\tau}) H^{\tau}(t_{\tau}) \\ \frac{\partial H^{\tau+1}}{\partial t_{\tau}} &= \mu_{H}^{\tau+1}(t_{\tau}, E_{\tau}; \theta^{\tau}, \theta^{\tau+1}) I_{H}^{\tau+1}(t_{\tau})^{\alpha^{\tau+1}(t_{\tau})} - \sigma_{H}^{\tau+1}(t_{\tau}) H^{\tau+1}(t_{\tau}) \\ \frac{\partial A^{\tau}}{\partial t_{\tau}} &= r A^{\tau}(t_{\tau}) + Y [H^{\tau}(t_{\tau}), E_{\tau}; \theta^{\tau}] - p_{C}^{\tau}(t_{\tau}) C^{\tau}(t_{\tau}) \\ &- p_{H}^{\tau}(t_{\tau}) X_{H}^{\tau}(t_{\tau}) - p_{H}^{\tau+1}(t_{\tau}) X_{H}^{\tau+1}(t_{\tau}) \end{aligned}$$

and time budget  $\Omega = t_w^\tau(t_\tau) + t_H^\tau(t_\tau) + t_H^{\tau+1}(t_\tau) + L^\tau(t_\tau) + s[H^\tau(t_\tau)]$ 

- Efficiencies  $\mu_H^{\tau}(.)$ ,  $\mu_H^{\tau+1}(.)$  (parent, child) of health production
- Concavity  $\alpha^{\tau}(t_{\tau})$ ,  $\alpha^{\tau+1}(t_{\tau})$  (parent, child) of investment processes
- Depreciation  $\sigma^{ au}_{H}(t_{ au})$ ,  $\sigma^{ au+1}_{H}(t_{ au})$  (parent, child) of health
- Vectors  $\theta^{\tau}$ ,  $\theta^{\tau+1}$  (parent, child) of cognitive and non-cognitive skills
- Time  $t_w^{\tau}(t_{\tau})$  spent working; earnings Y[.]; sick time  $s[H^{\tau}(t_{\tau})]$

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# Initial and end conditions

- A generation  $\tau$  parent starts her adult life with initial health  $H^{\tau}(t_{\tau} = 0) = H_0^{\tau}$  (product of the grandparent's investments)
- Her generation  $\tau + 1$  child is endowed at birth with initial health  $H^{\tau+1}(t_{\tau} = 0) = H_0^{\tau+1}$  and (through parental investments) with  $H^{\tau+1}(t_{\tau} = D_{\tau+1}) = H_{D_{\tau+1}}^{\tau+1}$  at end of childhood
- The parent starts adult life without wealth  $A^{\tau}(0) = A_0^{\tau} = 0$ , and receives a bequest from her parent (the grandparent)  $A^{\tau-1}(T_{\tau-1}) = A_{T_{\tau-1}}^{\tau-1}$  at her parent's death  $T_{\tau-1}$ .<sup>5</sup>
- The parent bequeathes her generation  $\tau + 1$  child with  $A^{\tau}(T_{\tau}) = A^{\tau}_{T_{\tau}}$  when she dies at age  $T_{\tau}$
- Death occurs when health reaches  $H_{\min}$  and life is no longer sustainable (e.g., Grossman, 1972).<sup>6</sup>

<sup>5</sup>Similar to Lee and Seshadri (2015a,b).

<sup>6</sup>Since end state is fixed  $H^{\tau}(T_{\tau}) = H_{\min}$ , it cannot be chosen to have no value  $q_{H}^{\tau}(T_{\tau}) > 0$  (unlike traditional human capital, where the end state is free). As a result, investment in health may grow as death approaches. Condition captures notion that terminal health state is universally low (for natural causes of death physically frail eventually face the great reaper). By contrast, individuals end life with various degrees of cognitive and mental fitness (some of us have the good fortune to stay mentally sharp till death).

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# Optimal control problem

The Hamiltonian of this problem is:

$$\Im(t_{\tau}) = U[.]e^{-\rho^{\tau}t_{\tau}} + q_{H}^{\tau}(t_{\tau})\frac{\partial H^{\tau}}{\partial t_{\tau}} + \lambda_{H}^{\tau+1}(t_{\tau})\frac{\partial H^{\tau+1}}{\partial t_{\tau}} + q_{A}^{\tau}(t_{\tau})\frac{\partial A^{\tau}}{\partial t_{\tau}}$$

The co-states find a natural economic interpretation in a standard result of Pontryagin's maximum principle

$$egin{array}{rll} q^{ au}_A(t_{ au}) &=& rac{\partial \mathbb{U}^{ au}(t_{ au})}{\partial A^{ au}(t_{ au})}, & ext{marginal value of own wealth } A^{ au}(t_{ au}) \ q^{ au}_H(t_{ au}) &=& rac{\partial \mathbb{U}^{ au}(t_{ au})}{\partial H^{ au}(t_{ au})}, & ext{marginal value of own health } H^{ au}(t_{ au}) \ \lambda^{ au+1}_H(t_{ au}) &=& rac{\partial \mathbb{U}^{ au}(t_{ au})}{\partial H^{ au+1}(t_{ au})}, & ext{marginal value to parent of child health } H^{ au+1}(t_{ au}) \end{array}$$

Capture effect (in utils) on remaining (from  $t_{\tau}$  onwards) parental lifetime utility  $\mathbb{U}^{\tau}(t_{\tau})$  as result of additional increment of stock

including utility from child health endowment and child inheritance



# Transversality and other conditions (1 of 2)

Parent seeks to optimize utility she derives from endowing her child with health  $H^{\tau+1}(D_{\tau+1})$  at start of child's adult life  $(t_{\tau} = D_{\tau+1})$ 

$$\lambda_{H}^{ au+1}(D_{ au+1}) \;\; = \;\; a_{H}^{ au} rac{\partial \mathbb{U}^{ au+1}(D_{ au+1})}{\partial H^{ au+1}(D_{ au+1})} = a_{H}^{ au} \, q_{H}^{ au+1}(D_{ au+1})$$

Further, she seeks to optimize utility from bequeathing her child with wealth  $A^{\tau}(T_{\tau})$  at death  $T_{\tau}$ 

$$q_A^{ au}(T_{ au}) \;\; = \;\; a_A^{ au} rac{\partial \mathbb{U}^{ au+1}(T_{ au})}{\partial A^{ au}(T_{ au})}$$

Parent and child may differ in their evaluations of the value of child health and child wealth, by  $a_H^{\tau}$  and  $a_A^{\tau}$  (degree of parental altruism)



### Transversality and other conditions (2 of 2)

Optimal age of death  $T_{\tau}$ , duration of education  $E_{\tau}$ , timing of child birth  $B_{\tau+1}$ , follow from dynamic envelope theorem (Caputo, 2005)

$$\begin{aligned} \frac{\partial}{\partial T_{\tau}} \int_{0}^{T_{\tau}} \Im(t_{\tau}) dt_{\tau} &= \Im(T_{\tau}) = 0\\ \frac{\partial}{\partial E_{\tau}} \int_{0}^{T_{\tau}} \Im(t_{\tau}) dt_{\tau} &= \Im(E_{\tau}^{-}) - \Im(E_{\tau}^{+}) + \int_{0}^{T_{\tau}} \frac{\partial \Im(t_{\tau})}{\partial E_{\tau}} dt_{\tau} = 0\\ \frac{\partial}{\partial B_{\tau+1}} \int_{0}^{T_{\tau}} \Im(t_{\tau}) dt_{\tau} &= \Im(B_{\tau+1}^{-}) - \Im(B_{\tau+1}^{+}) = 0, \end{aligned}$$

where  $E_{\tau}^{-}$ ,  $B_{\tau+1}^{-}$  indicate limit in which  $E_{\tau}$ ,  $B_{\tau+1}$  approached from below, and  $E_{\tau}^{+}$ ,  $B_{\tau+1}^{+}$  when approached from above.<sup>7</sup>

<sup>&</sup>lt;sup>7</sup> Jumps in control variables occur at  $E_{\tau}$  and  $B_{\tau+1}$  as the parent does not work during schooling  $t_{w}^{\tau}(t_{\tau}) = 0$ and she does not spent time on child investment  $t_{H}^{\tau+1}(t_{\tau}) = 0$  before the child is born  $\mathbb{P} \to \mathbb{R} \to \mathbb{R}$ 

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### Comparative dynamic effects depend on co-states

Co-states  $q_A^{\tau}(0)$  and  $\lambda_H^{\tau+1}(D_{\tau+1}) = a_H^{\tau}q_H^{\tau+1}(D_{\tau+1})$  are implicit functions of the initial and end states

$$A_0^{\tau}, A_{T_{\tau-1}}^{\tau-1}, A_{T_{\tau}}^{\tau}, H_0^{\tau+1}, H_0^{\tau}, H_{\min},$$

model parameters

$$r, \rho^{\tau}, a_{H}^{\tau}, a_{A}^{\tau}, D_{\tau+1},$$

model functionals

$$\begin{aligned} &\sigma_{H}^{\tau}(t_{\tau}), \sigma_{H}^{\tau+1}(t_{\tau}), \mu_{H}^{\tau}(.), \mu_{H}^{\tau+1}(.), \alpha^{\tau}(t_{\tau}), \alpha^{\tau+1}(t_{\tau}), \\ &p_{H}^{\tau}(t_{\tau}), p_{H}^{\tau+1}(t_{\tau}), p_{C}^{\tau}(t_{\tau}), \end{aligned}$$

and, as yet unspecified functional forms of the instantaneous utility function U[.], earnings  $Y[H^{\tau}(t_{\tau}), E_{\tau}; \theta^{\tau}]$ , and sick time  $s[H^{\tau}(t_{\tau})]$ 

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Assump	tions				

A natural assumption is that wealth  $A^{\tau}(t_{\tau})$ , health  $H^{\tau}(t_{\tau})$  and longevity  $T_{\tau}$  increase life-time utility  $\mathbb{U}^{\tau}$  at a diminishing rate

$$\frac{\partial q_A^{\tau}(t_{\tau})}{\partial A^{\tau}(t_{\tau})} = \frac{\partial^2 \mathbb{U}^{\tau}(t_{\tau})}{\partial A^{\tau}(t_{\tau})^2} < 0, \quad \frac{\partial q_H^{\tau}(t_{\tau})}{\partial H^{\tau}(t_{\tau})} = \frac{\partial^2 \mathbb{U}^{\tau}(t_{\tau})}{\partial H^{\tau}(t_{\tau})^2} < 0, \quad \frac{\partial^2 \mathbb{U}^{\tau}(t_{\tau})}{\partial T_{\tau}^2} < 0$$

- Thus, for example, wealth  $A^{\tau}(t_{\tau})$  improves utility  $\mathbb{U}^{\tau}(t_{\tau})$  but wealthy parent benefits less from additional wealth  $\delta A^{\tau}(t_{\tau})$  than does a less wealthy parent
- Likewise, for parameters that enter and relax budget constraint, effect is similar to that of wealth  $A^{\tau}(t_{\tau})$
- Direct effects dominate indirect effects<sup>8</sup>

• Reasoning for health  $H^{\tau}(t_{\tau})$  and longevity  $T_{\tau}$  is similar<sup>9</sup>

<sup>8</sup>That is, variations in parameters that enter budget constraint (e.g., wealth  $A^{\tau}(t_{\tau})$ ) result in larger changes in  $q_{A}^{\tau}(t_{\tau})$ , whereas parameters that enter other dynamic constraints (e.g.,  $\mu_{H}^{\tau}(t_{\tau}, E^{\tau}; \theta^{\tau})$ ) result in smaller changes.

<sup>9</sup>As comparative dynamic analyses show, situations may occur where as result of longevity gains diminishing returns no longer apply. Empirical facts suggest wealthy consume more, health behavior improves (Serdula et al. 2004; Pearson et al. 2005) and medical expenditures increase with age (Zweifel, Felder & Meiers, 1999) as health declines, supporting notion of diminishing returns in adulthood. Case for childhood phase and longevity less clear.



# Implicit and explicit functional dependence

Child health and investment in child health are functions of  $q_A^{\tau}(0)$ "wealth effect" and  $\lambda_H^{\tau+1}(D_{\tau+1})$  "endowed health effect" (implicit)

- Parental resources increase investment in child health
- Higher child health endowment  $\lambda_H^{\tau+1}(D_{\tau+1}) = a_H^{\tau} q_H^{\tau+1}(D_{\tau+1})$ reduces value to parent of her child's health (endowed health effect competes with wealth effect)
  - due to diminishing returns, absence of reinforcement and dynamic complementarity in health
  - but higher investment in child health is possible if better child health leads to a longer life to such an extent that the parent invests more in a healthy child than in a less healthy child

In addition, child health and investment in child health depend explicitly on parameters and functionals. Their effects consist of implicit as well as explicit effects

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### When wealth effect dominates endowed health effect

- Parent invests *more* in child health when wealth effect dominates endowed health effect. True for:
  - wealthy  $A_0^{\tau}$ ,  $A_{T_{\tau-1}}^{\tau-1}$  and healthy  $H_0^{\tau}$  (reduces sick time) parent
  - high rates of return to capital r (if positive wealth) and low prices  $p_C^{\tau}(t_{\tau})$ ,  $p_H^{\tau}(t_{\tau})$ ,  $p_H^{\tau+1}(t_{\tau})$
- Additional resources can be used to improve parental utility in various ways and investment in child health is one of them

• Consistent with empirical evidence that children of wealthy parents are healthier



# When endowed health effect dominates wealth effect

- Parent invests *less* in child health when endowed health effect dominates wealth effect. True for:<sup>10</sup>
  - child born healthy  $H_0^{\tau+1}$
- However, while parent invests less in her child's health, health endowment  $H_{D_{\tau+1}}^{\tau+1}$  still higher as result of direct effect
  - for child born healthy, less investment needed to improve later-life health
- Parent devotes freed resources toward child's inheritance, her own consumption, health, and leisure time

<sup>&</sup>lt;sup>10</sup>Again, higher investment in child health is possible if better child health leads to a longer life to such an extent that the parent invests more in a healthy child than in a less healthy child (increasing returns).  $\exists \mapsto \exists \exists \mapsto \exists$ 



# Effect of parental education, skill is ambiguous

- Parental education  $E_{\tau}$ , skills  $\theta^{\tau}$  raise earnings  $Y[H^{\tau}(t_{\tau}), E_{\tau}; \theta^{\tau}]$ increasing investment (permanent income effect)
- But parental education and parental skill also improve efficiency of child health production  $\mu_H^{\tau+1}(t_{\tau}, E_{\tau}; \theta^{\tau}, \theta^{\tau+1})$ (substitution effect)
- Further, opportunity cost of time due to higher wages  $w^{\tau}(t_{\tau}, E_{\tau}; \theta^{\tau})$  reduces investment
- Household income is associated with better child health and more child investment (Case, Lubotsky & Paxson, 2002)
  - permanent income effect appears to dominate opportunity cost of time effect and substitution effect

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### Adult phase identical to decreasing returns in Grossman

- While childhood phase can be solved analytically, for assumed functional forms of the health-production process, solutions for adult phase requires use of phase diagrams (see appendix)
- Solutions of identical form to Grossman model with decreasing returns to investment (Galama, 2015; see appendix)

• Adult phase can thus be successfully analyzed using comparative dynamic methods (as in Galama, 2015)

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#### Wealthy adult invests more, lives longer



**Figure:** The LHS shows the phase diagram of the perturbation due to variation in initial wealth  $\delta A_0^{\tau}$ , for fixed  $T_{\tau}$ . The RHS shows the same phase diagram allowing length of life  $T_{\tau}$  to be free. The four vertical dotted lines represent different values for the end point  $\partial H^{\tau}(T_{\tau})/\partial A_0^{\tau}$ . Endogenous education  $E_{\tau}$  and child birth  $B_{\tau+1}$  do not alter phase diagram as they effect values of co-states but do not change nature of dynamic relations.

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#### Healthy adult invests less, unless she lives relatively long



**Figure:** Phase diagram of the perturbation due to variation in initial health  $\delta H_0^{\tau}$ , for free  $T_{\tau}$ , with starting point,  $\partial H^{\tau}(t_{\tau})/\partial H_0^{\tau} = 1$ , located to the left and right of the steady state. Endogenous education  $E_{\tau}$  and child birth  $B_{\tau+1}$  do not alter phase diagram as they effect values of co-states but do not change nature of dynamic relations.

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### So far we have ignored intergenerational aspect of model

Child health and investment in child health increase in

$$\lambda_{h/a}^{ au+1}(D_{ au+1}) \equiv rac{\lambda_H^{ au+1}(D_{ au+1})}{q_A^{ au}(D_{ au+1})} = rac{a_H^{ au}rac{\partial \mathbb{U}^{ au+1}(D_{ au+1})}{\partial H^{ au+1}(D_{ au+1})}}{a_A^{ au}rac{\partial \mathbb{U}^{ au+1}(T_{ au})}{\partial A^{ au}(T_{ au})}}e^{-(T_{ au}-D_{ au+1})r},$$

where  $\partial \mathbb{U}^{\tau+1}(D_{\tau+1})/\partial H^{\tau+1}(D_{\tau+1}) = q_H^{\tau+1}(D_{\tau+1})$ . Expression:

- captures interactions between parental characteristics, circumstances, childhood endowments, biology, and the child's health
- provides alternative way of explaining parental investment in child health and parental desire to bequeath wealth through their effects on the child's future wellbeing  $\mathbb{U}^{\tau+1}(t_{\tau})$
- is forward looking



### Parent's motivations align with the child's

$$\lambda_{h/a}^{\tau+1}(D_{\tau+1}) \equiv \frac{\lambda_H^{\tau+1}(D_{\tau+1})}{q_A^{\tau}(D_{\tau+1})} = \frac{a_H^{\tau} \frac{\partial \mathbb{U}^{\tau+1}(D_{\tau+1})}{\partial H^{\tau+1}(D_{\tau+1})}}{a_A^{\tau} \frac{\partial \mathbb{U}^{\tau+1}(T_{\tau})}{\partial A^{\tau}(T_{\tau})}} e^{-(T_{\tau} - D_{\tau+1})t}$$

- Parent invests more in child health (and saves less) for a child who benefits more from health [high  $\partial \mathbb{U}^{\tau+1}/\partial H^{\tau+1}(D_{\tau+1})$ ] and less from wealth [low  $\partial \mathbb{U}^{\tau+1}/\partial A^{\tau}(T_{\tau})$ ] (and vice versa)
  - child health raises child's earnings, increases purchases of health investment goods, improving health and reducing sick time, lowering cost of time inputs into health, etc.
- True for child in poor health, but with great opportunities (skilled, educated, wealthy)
- From perspective of parent, returns to investments are high if these resources extend the child's life, amplifying effects

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General	ized Heckn	nan result			

Comparative dynamics can be separated into two components

Total variation = Variation for fixed  $T_{\tau}$  + Variation due to change in  $T_{\tau}$ 

Absent ability to extend life, smaller effect of wealth on education, health (similar to Heckman [1976] for human capital)

- Any additional investment in health, education would have to be compensated by eventual lower investment since life is fixed
- Additional wealth used for additional consumption and leisure mplication: parental investment in health and savings smaller if additional resources do not enable child to extend her life

Motivation	Contribution	Formulation	Analyses	Discussion	Appendix
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Most action due to life extension									

#### Total variation = Variation for fixed $T_{\tau}$ + Variation due to change in $T_{\tau}$

With ability to extend life, wealthy, educated, healthy individuals live longer (similar to Ehrlich & Chuma [1990] for health)

- At high levels of wealth, and hence consumption, only limited marginal utility is gained from extra consumption
- By contrast, investments in health extend life, increasing the period over utility can be enjoyed. These benefits are large.<sup>11</sup>
- Like wealth, education and health are resources

**Implication:** parental investment in health and savings larger if additional resources enable child to extend her life

<sup>&</sup>lt;sup>11</sup>Murphy & Topel (2006) estimate cumulative gains in life expectancy after 1900 were worth over \$1.2 million (per person) to representative American in 2000. イロト イ合ト イミト イミト ミークへぐ

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Most act	tion due to	o life exten	sion		

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# We started to develop a theory of health over the lifecycle

- Based on the canonical theory of the demand for health investment due to Grossman (1972)
- To address issues with commonly employed linear investment models we assume decreasing returns to scale (Galama, 2015)
- In the model, the parent optimizes
  - own education, timing of child birth, consumption, leisure, time devoted to work, duration of her longevity
  - own and child health investment
  - health and wealth she bequeathes to her child

subject to the constraints she faces

- initial own and initial child's health endowment
- resources at her disposition
- own and child environments
- efficiency of own and child health-production processes



- Investment in child's health endowment and saving for her inheritance provides two distinct means by which the parent can improve the future wellbeing of her child U<sup>τ+1</sup>(t<sub>τ</sub>)
- The theory predicts, unsurprisingly, that
  - a wealthy and healthy parent invests more in her child's health
  - also, if the price of consumption and investment is low, parent invests more in her child's health
  - this is consistent with empirical evidence that the children of wealthy parents are healthier
- In addition, parental health reduces sick time, freeing up time that can be devoted to work (wealth effect) and to investment in child health (reduced opportunity cost of time effect)

Motivation	Contribution	Formulation	Analyses	Discussion	Appendix
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Our preli	minary resu	ılts are (2	of 3)		

- Parent invests less in child's health if child is born healthy
- This is because child health reduces the marginal value to the parent of the child's health (endowed health effect)
- However, child health is still higher as less investment is needed to increase the child's health
- Parent devotes the freed resources toward the child's inheritance, her own consumption, health, and leisure time

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# Our preliminary results are (3 of 3)

- Effect of parental education, skill is ambiguous
  - education  $E_{\tau}$ , skills  $\theta^{\tau}$  raise earnings  $Y[H^{\tau}(t_{\tau}), E_{\tau}; \theta^{\tau}]$ increasing investment (permanent income effect)
  - but parental education and skill also improve efficiency of child health production  $\mu_H^{\tau+1}(t_{\tau}, E_{\tau}; \theta^{\tau}, \theta^{\tau+1})$
  - further, opportunity cost of time higher due to higher wages  $w^{\tau}(t_{\tau}, E_{\tau}; \theta^{\tau})$
- Household income associated with better child health and more child investment (Case, Lubotsky & Paxson, 2002)
  - permanent income effect appears to dominate opportunity cost of time effect and substitution effect



# Early emergence and widening of health disparities

- Wealthy, healthy parent invests more in her child's health and saves more for inheritance
- Child starts adult life in better health and with greater wealth
- Wealthy and healthy adult invests more in her health and education, earns higher wages, is wealthier, and lives longer
- Greater investments in health increase health disparities
- Ability to extend life exacerbates investments and disparities (longevity is crucial determinant of return to investments)
- Health of poorer, less educated individuals, decreases faster reducing time devoted to work and thereby earnings, wealth
- Demand for health investment increases faster for low SES as result of declining health, and mortality selection, potentially contributing to subsequent narrowing of gradient in old age

Motivation	Contribution	Formulation	Analyses	Discussion	Appendix
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Next ste	ps				

- Model reinforcement and dynamic complementary in health
- Introduce effect of skills  $\theta^{\tau}$ , education  $E_{\tau}$  on preferences
- Add potential genetic and "stress" pathways
- Include health behaviors of adult and potentially also child

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#### Analytical solutions for adult phase

The first-order condition for investment in parental health is given by

$$q_{h/a}^{\tau}(t_{\tau}) \equiv \frac{q_{H}^{\tau}(t_{\tau})}{q_{A}^{\tau}(t_{\tau})} = \pi_{H}^{\tau}(t_{\tau}), \tag{1}$$

where  $q_{h/a}^{\tau}(t_{\tau})$  is the marginal benefit of investment in the parent's health, and  $\pi_{H}^{\tau}(t_{\tau})$  is it's marginal cost,

$$\pi_{H}^{\tau}(t_{\tau}) \equiv \frac{[p_{H}^{\tau}(t_{\tau})]^{\kappa_{H}} [w^{\tau}(t_{\tau}, \mathcal{E}^{\tau}; \theta^{\tau})]^{1-\kappa_{H}}}{\alpha^{\tau}(t_{\tau}) \mu_{H}^{\tau}(t_{\tau}, \mathcal{E}^{\tau}; \theta^{\tau}) \kappa_{H}^{\kappa_{H}} (1-\kappa_{H})^{1-\kappa_{H}}} [I_{H}^{\tau}(t_{\tau})]^{1-\alpha^{\tau}(t_{\tau})} \equiv \pi_{H,*}^{\tau}(t_{\tau}) [I_{H}^{\tau}(t_{\tau})]^{1-\alpha^{\tau}(t_{\tau})}.$$
(2)

The marginal benefit of investment in the parent's health, evolves according to

$$\frac{q_{h/a}^{\tau}(t_{\tau})}{\partial t_{\tau}} = \left[\sigma_{H}^{\tau}(t_{\tau}) + r\right] q_{h/a}^{\tau}(t_{\tau}) - \left[\frac{1}{q_{A}^{\tau}(0)} \frac{\partial U}{\partial H^{\tau}} e^{-(\rho^{\tau} - r)t_{\tau}} + \frac{\partial Y}{\partial H^{\tau}}\right],\tag{3}$$

and the parent's health evolves according to

$$\frac{\partial H^{\tau}(t_{\tau})}{\partial t_{\tau}} = \mu_{H}^{\tau}(t_{\tau}, E^{\tau}; \theta^{\tau}) \left[ \frac{q_{h/a}^{\tau}(t_{\tau})}{\pi_{H,*}^{\tau}(t_{\tau})} \right]^{\frac{\alpha^{\tau}(t_{\tau})}{1-\alpha^{\tau}(t_{\tau})}} - \sigma_{H}^{\tau}(t_{\tau}) H^{\tau}(t_{\tau}).$$
(4)

Individuals optimally choose longevity  $T_{\tau}$  such that the marginal value of life extension is zero at this age,  $\Im(T_{\tau}) = 0$ ,

$$\Im(T_{\tau}) = U(T_{\tau})e^{-\rho^{\tau}T_{\tau}} + q_{H}^{\tau}(T_{\tau}) \left. \frac{\partial H^{\tau}}{\partial t_{\tau}} \right|_{t_{\tau}=T_{\tau}} + q_{A}^{\tau}(T_{\tau}) \left. \frac{\partial A^{\tau}}{\partial t_{\tau}} \right|_{t_{\tau}=T_{\tau}} = 0.$$
(5)

Relations (3) and (4) and condition (5) identical to Grossman model with decreasing returns in health investment

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#### Analytical solutions for childhood phase (1 of 3)

The first-order condition for investment in child health is given by

$$\lambda_{h/a}^{\tau+1}(t_{\tau}) \equiv \frac{\lambda_{H}^{\tau+1}(t_{\tau})}{q_{A}^{\tau}(t_{\tau})} = \pi_{H}^{\tau+1}(t_{\tau}), \tag{6}$$

where  $\lambda_{h/a}^{\tau+1}(t_{\tau})$  is the marginal benefit of investment in the child's health, and  $\pi_{H}^{\tau+1}(t_{\tau})$  is it's marginal cost,

$$\pi_{H}^{\tau+1}(t_{\tau}) \equiv \frac{\left[\rho_{H}^{\tau+1}(t_{\tau})\right]^{\kappa_{H}} \left[w^{\tau}(t_{\tau}, E^{\tau}; \theta^{\tau})\right]^{1-\kappa_{H}}}{\alpha^{\tau+1}(t_{\tau}) \mu_{H}^{\tau+1}(t_{\tau}, E^{\tau}; \theta^{\tau}, \theta^{\tau+1}) \kappa_{H}^{\kappa_{H}}(1-\kappa_{H})^{1-\kappa_{H}}} \left[l_{H}^{\tau+1}(t_{\tau})\right]^{1-\alpha^{\tau+1}(t_{\tau})}} \equiv \pi_{H,*}^{\tau+1}(t_{\tau}) \left[l_{H}^{\tau+1}(t_{\tau})\right]^{1-\alpha^{\tau+1}(t_{\tau})}.$$
(7)

The marginal benefit consists of the ratio of the marginal value of additional child health  $\lambda_H^{\tau+1}(t_{\tau})$  to the marginal value of additional parental wealth  $q_A^{\tau}(t_{\tau})$ . If the marginal value of additional child health is high, the parent prefers to invest in the child's health, if it is low, the parent prefers to save. The marginal benefit of investment in the child's health evolves according to

$$\frac{\partial \lambda_{h/a}^{\tau+1}(t_{\tau})}{\partial t_{\tau}} = \left[\sigma_{H}^{\tau+1}(t_{\tau}) + r\right] \lambda_{h/a}^{\tau+1}(t_{\tau}), \tag{8}$$

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and the child's health evolves according to

$$\frac{\partial H^{\tau+1}(t_{\tau})}{\partial t_{\tau}} = \mu_{H}^{\tau+1}(t_{\tau}, \mathcal{E}^{\tau}; \theta^{\tau}, \theta^{\tau+1}) \left[ \frac{\lambda_{h/a}^{\tau+1}(t_{\tau})}{\pi_{H,*}^{\tau+1}(t_{\tau})} \right]^{\frac{\alpha^{\tau+1}(t_{\tau})}{1-\alpha^{\tau+1}(t_{\tau})}} - \sigma_{H}^{\tau+1}(t_{\tau}) H^{\tau+1}(t_{\tau}).$$
(9)

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# Analytical solutions for childhood phase (2 of 3)

The analytical solutions of these dynamic relations are:

$$\lambda_{h/a}^{\tau+1}(t_{\tau}) = \lambda_{h/a}^{\tau+1}(D_{\tau+1})e^{-\int_{t_{\tau}}^{D_{\tau}+1}[\sigma_{H}^{\tau+1}(s)+r]ds},$$
(10)

$$I_{H}^{\tau+1}(t_{\tau}) = \left[\frac{\lambda_{h/a}^{\tau+1}(t_{\tau})}{\pi_{H,*}^{\tau+1}(t_{\tau})}\right]^{\frac{1}{1-\alpha^{\tau+1}(t_{\tau})}} = \left[\frac{\lambda_{h/a}^{\tau+1}(D_{\tau+1})}{\pi_{H,*}^{\tau+1}(t_{\tau})}\right]^{\frac{1}{1-\alpha^{\tau+1}(t_{\tau})}} e^{-\frac{1}{1-\alpha^{\tau+1}(t_{\tau})}\int_{t_{\tau}}^{D_{\tau}+1}[\sigma_{H}^{\tau+1}(s)+r]ds}, \quad (11)$$

and

1

$$\begin{aligned} H^{\tau+1}(t_{\tau}) &= H_{0}^{\tau+1}e^{-\int_{0}^{t_{\tau}}\sigma_{H}^{\tau+1}(s)ds} \\ &+ \int_{0}^{t_{\tau}}\mu_{H}^{\tau+1}(x, \mathcal{E}^{\tau}; \theta^{\tau}, \theta^{\tau+1}) \left(\frac{\lambda_{h/a}^{\tau+1}(x)}{\pi_{H,*}^{\tau+1}(x)}\right)^{\frac{\alpha^{\tau+1}(x)}{1-\alpha^{\tau+1}(x)}} e^{-\int_{x}^{t_{\tau}}\sigma_{H}^{\tau+1}(s)ds}dx, \end{aligned}$$
(12)

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# Analytical solutions for childhood phase (3 of 3)

where the endowment of health at the end of childhood equals

$$H^{\tau+1}(D_{\tau+1}) = \int_{0}^{D_{\tau+1}} \mu_{H}^{\tau+1}(x, E^{\tau}; \theta^{\tau}, \theta^{\tau+1}) \left(\frac{\lambda_{h/a}^{\tau+1}(x)}{\pi_{H,*}^{\tau+1}(x)}\right)^{\frac{\alpha^{\tau+1}(x)}{1-\alpha^{\tau+1}(x)}} e^{-\int_{x}^{D_{\tau+1}} \sigma_{H}^{\tau+1}(s)ds} dx + H_{0}^{\tau+1}e^{-\int_{0}^{D_{\tau+1}} \sigma_{H}^{\tau+1}(s)ds},$$
(13)

and

$$\lambda_{h/a}^{\tau+1}(D_{\tau+1}) \equiv \frac{\lambda_{H}^{\tau+1}(D_{\tau+1})}{q_{A}^{\tau}(D_{\tau+1})}.$$
(14)

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