Collective Intertemporal Choice: Time Consistency vs. Time Invariance

Antony Millner*[1] and Geoffrey Heal[2]

[1]London School of Economics and Political Science
[2]Columbia University & NBER

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Abstract

We study collective choice when individuals have heterogeneous discounted utilitarian preferences. Two attractive properties of intertemporal preferences are indistinguishable for individuals, but have dramatically different implications for collective choice. *Time Consistency* requires a plan that is optimal at one evaluation date to be optimal at all later evaluation dates, while *Time Invariance* requires preferences to be unchanged under translations of the time axis. We study the implications of these two properties in a tractable dynamic model that captures both common resource and public goods problems. Utilitarian social planners implement the first best if collective preferences are time consistent, but not if they are time invariant. Decentralized alternatives – property rights (for common resources) and voting (for public goods) – can strictly improve on the planning equilibrium if social preferences are time invariant. We reflect on the implications of these findings for dynamic welfare economics. Revealed preference cannot determine which property we should adopt, but each property is normatively attractive in some contexts.

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*Millner: London School of Economics and Political Science, Houghton St, London, WC2A 2AE, UK (email: a.millner@lse.ac.uk). Heal: Columbia University, 3022 Broadway, New York, NY 10027, USA (email: gmh1@columbia.edu). We are grateful to Ben Groom and co-authors for providing us with the results of their survey on social discounting, and to Elizabeth Baldwin, Simon Dietz, Humberto Llavdador, and especially Marc Fleurbaey for insightful comments.

1 Introduction

Many important decisions in economic life require groups of people with heterogeneous time preferences to implement a collective consumption plan. Examples abound: families must decide on savings and intra-household resource allocation, partners in a firm must decide how to distribute profits between payouts to themselves and investment in the firm's capital, communities with property rights over a natural resource must decide on an extraction plan, and resource rich countries must decide how to consume the proceeds from their sovereign wealth funds. In each of these examples an asset is held in common and is consumed dynamically over time, and the stake-holders in the decision very often have heterogeneous time preferences. How should such decisions be made, given people's different attitudes to time? This is the subject of this paper.

The heterogeneity in people's time preferences is now well documented. Frederick et al. (2002) summarize the empirical literature, which uses experimental and field studies to infer individuals' rates of time preference. Estimates vary from -6%/yr to infinity across the studies they cite, and within study variation in estimates is also large. These studies are positive in nature – they tell us how people behave, and not how they think they, or society, *should* behave for normative purposes. However, the time preferences economists prescribe for normative applications, e.g. in public project appraisal (Arrow et al., 2013), are also highly heterogeneous. This has been highlighted by the long-standing debate about the appropriate rate of time preference for the evaluation of climate change policy (Nordhaus, 2008; Stern, 2007; Weitzman, 2007). A recent survey of economists who are experts on social discounting (Drupp et al., 2014) shows significant variation in their prescriptions for the pure rate of time preference for public project evaluation (see Figure 3 below).[1]

The pervasive heterogeneity in attitudes to time raises conceptual difficulties for intertemporal decision-making that are not present for individual choice. Two attractive properties of preferences – *time consistency* and *time invariance* – are mutually consistent for individuals with discounted utilitarian preferences, but not for social preferences that

[1] The debate over the appropriate pure rate of time preference for public choices stretches back at least as far as the seminal contribution of Ramsey (1928), who argued that 'discounting of future utilities is ethically indefensible and arises purely from a weakness of the imagination.' Nevertheless, Ramsey acknowledged that many might disagree with this ethical prescription (see also Arrow, 1999), and considered positive rates of time preference too. Koopmans (1969) famously observed that very low rates of time preference can lead to the 'paradox of the indefinitely postponed splurge', in which the current decision maker always saves 100% of his income. Subsequent commentators have argued the merits of a variety of discount rates without a clear 'best' value emerging, and different governments have adopted different values for public decision-making.

reflect the heterogeneity in individuals' preferences. Time consistency requires dynamic plans that are optimal today to remain optimal tomorrow. We show that if utilitarian social preferences are time consistent and individuals have heterogeneous discount rates we need to pick a special time τ_0 that serves as the origin of time for welfare computations, now and forever. No matter how much time has elapsed since τ_0, we always evaluate social welfare *as if* we care about agents' lifetime utilities integrated from τ_0 onwards, even though we cannot change the past, and individuals' preferences seem to be forward looking. Time invariance, on the other hand, requires future consumption streams to be evaluated relative to the current evaluation period, i.e. only time differences relative to the present are preference relevant. Time invariant preferences have the attractive feature that there is no 'special' initial time τ_0 that sets the origin of the time axis for the purposes of welfare computations. Each new evaluation period is treated symmetrically, and preferences are solely forward looking.[2] Yet, time invariant utilitarian social preferences are *not* time consistent if discount rates are heterogeneous. Adopting time invariance as a property of social preferences forces us to have a different view of which plans are socially optimal in each new evaluation period, even though each individuals' preferences *seem* to be unchanged from one period to the next.

The conflict between these two properties of intertemporal preferences in a collective choice context is already present in the literature, albeit implicitly. Building on earlier insights by Marglin (1963), Feldstein (1964), Gollier and Zeckhauser (2005) and Zuber (2011), Jackson and Yariv (2015) have shown that Paretian social preferences, such as utilitarianism (in which social welfare is a weighted sum of individuals' welfare), cannot satisfy the stationarity axiom of Koopmans (1960), a cornerstone of the standard discounted utilitarian model of intertemporal choice. The authors take this to mean that utilitarian social welfare functions necessarily violate time consistency when time preferences are heterogeneous. Yet Halevy (2015) has shown that stationarity is only a prerequisite for time consistency if preferences are also time invariant, as implicitly assumed by Jackson and Yariv (2015). Combining the findings of Jackson and Yariv (2015) and Halevy (2015), we show that utilitarian social preferences can satisfy either time consistency, or time

[2] Almost every dynamic preference that is used in the applied literature is time invariant, from the standard discounted utilitarian model (e.g. Samuelson (1937); Strotz (1955)), to models of hyperbolic discounting (e.g. Phelps and Pollak (1968); Akerlof (1991); Laibson (1997)). Caplin and Leahy (2004) consider preferences that only depend on time differences relative to the present, but may be backward looking. We follow Halevy (2015) and define time invariant preferences to be forward looking, so that only *positive* time differences relative to the present are preference relevant. This is the property satisfied by standard discounted utilitarian preferences. We provide formal definitions of time consistency, time invariance, and stationarity below.

invariance, but not both.

This leaves us with a quandary: when should collective choices be governed by the time invariance property, and when by the time consistency property? A natural response to this question might be that social preferences should respect the properties of individuals' preferences – they should be time consistent if individuals are time consistent, and time invariant if individuals are time invariant. But if individuals have discounted utilitarian preferences these two properties are *indistinguishable* by revealed preference.[3] The choice between time consistency and time invariance at the social level is thus inherently normative.

To understand what is at stake when choosing which of these properties to impose on social preferences, we need to investigate their implications in concrete decision problems. To this end, we focus on an analytically tractable model in which a group of agents with heterogeneous time preferences must decide how to manage a productive asset, which may be a common resource or a public good. We begin by deriving the consumption plans that time consistent and time invariant social planners would choose. While time consistent planners implement the first best in all evaluation periods, the plans implemented by rational time invariant planners are inefficient in every evaluation period. This inefficiency suggests that time invariant social planners can improve on their own equilibrium consumption choices by relinquishing some control over decision-making. We demonstrate this formally by deriving a 'second-best' time invariant planner, who has social preferences that obey the same properties as the original planner, but may have different preference parameters. We show that if time invariant planners could delegate decision-making to this alter-ego they would improve on equilibrium outcomes in all evaluation periods, and their consumption plans would be more patient. We relate the 'excess impatience' of the social planner's consumption plan to a measure of the spread of discount rates in the population.

We then consider decentralized decision methods – property rights (for common resource problems), and voting (for public goods). In the case of common resources received economic wisdom suggests that property rights regimes should implement first best allocations. We show that this claim is valid if social preferences are time consistent, but not if

[3]Throughout the paper we assume that individuals have discounted utilitarian preferences, in line with the normative literature on utilitarian preference aggregation (e.g. Zuber, 2011; Jackson and Yariv, 2014, 2015). Halevy (2015) reviews the evidence on individuals' time preferences, noting that many recent studies find little evidence of hyperbolic discounting. In his own study 45-59% of subjects made choices consistent with discounted utilitarianism, substantially more than any of the four other preference categories he considers. While these data give some behavioral credence to this assumption, its normative justification lies in the appeal of Koopmans' stationarity axiom, conjoined with either time consistency or time invariance.

they are time invariant. Nevertheless, we show that property rights can strictly improve on the planning equilibrium in every evaluation period if social preferences are time invariant. Property rights are not feasible if consumption is a public good, so in this case we consider an alternative decentralized approach in which agents vote over public consumption in every period. This procedure can again strictly improve on the planning equilibrium at every evaluation date if social preferences are time invariant, although this result is contingent on the distribution of discount rates in the population, and the Pareto weights that enter the social welfare function. We characterize the set of welfare functions for which voting improves on planning, and show that for empirically plausible distributions of discount rates voting is indeed preferred. Decentralization can thus *strictly* improve on social planning if social welfare is time invariant. These findings demonstrate that standard results on the relationship between planning and decentralization are contingent on an implicit normative choice (i.e. to impose time consistency, rather than time invariance, on social preferences) which cannot be justified by appealing to revealed preference.

Our model allow us to assess the consequences of time consistency and time invariance as properties of social preferences, and demonstrates how traditional economic intuitions depend on which of them we adopt. It also show that the choice between these two properties of social preferences is consequential, in that different decision procedures may be preferred when different properties are adopted. In our final section we argue that the normative appeal of time consistency/invariance is likely to be context dependent: for intra-generational decisions time invariance is often difficult to defend, but it has considerable appeal for inter-generational decisions. Choosing either property nevertheless requires us to make normative judgements that are additional to any information about individuals' preferences that can be revealed through choice.

The paper is structured as follows. Section 2 provides some conceptual background, illustrating how time consistency differs from time invariance, and why this distinction matters for collective intertemporal choice, but not for individual choice. Section 3 sets out our simple model of dynamic choice, analytically derives the equilibrium allocations that social planners with time consistent and time invariant preferences would choose, examines decentralized alternatives to planning (property rights and voting), and analyzes their welfare implications under our two welfare concepts. Section 4 discusses the results, focussing in particular on when time consistency/invariance might be a reasonable property of collective preferences.

1.1 Related literature

At a conceptual level our work is strongly influenced by Halevy (2015), who highlighted the distinction between time consistency, time invariance, and stationarity. While not explicitly concerned with the problem of collective intertemporal choice, the distinction he draws between these properties has clear relevance for this issue, which we investigate here. Halevy however takes a positive approach, using a clever experiment to identify violations of these properties; we are concerned with normative issues, i.e. utilitarian social choice when individuals have heterogeneous discounted utilitarian preferences. While also not concerned with collective choice, Caplin and Leahy's (2004) critique of revealed preference as a guide to dynamic welfare analysis is complementary to ours.

The literature most closely related to our work deals with the aggregation of time preferences via utilitarian social welfare functions. Gollier and Zeckhauser (2005) consider a model in which an exogenous common stream of consumption is divided between heterogeneous agents efficiently, and thus maximizes a weighted sum of agents' individual intertemporal utilities. They derive the representative time preferences of the group, and show that they are generically non-constant, indicating a violation of Koopmans' stationarity axiom. Although time invariance and time consistency are not discussed explicitly, they implicitly impose time consistency on the social preferences they study. Jackson and Yariv (2014, 2015) and Zuber (2011) investigate collective intertemporal choice from an axiomatic perspective, both establishing the conflict between Paretian social welfare functions and the stationarity axiom in greater generality than Gollier and Zeckhauser (2005). In contrast to Gollier and Zeckhauser (2005), Jackson and Yariv (2014, 2015) implicitly adopt a time invariant social welfare function, thus violating time consistency. Zuber (2011) demonstrates that a Paretian social welfare function can only be 'history independent', stationary, and time consistent if all agents share the same discount rate, thus partially anticipating the results of Halevy (2015). We show that the conflicting interpretations of utilitarian social preferences in Gollier and Zeckhauser (2005) and Jackson and Yariv (2014, 2015) are both consistent with revealed preference when individuals have discounted utilitarian preferences. Unlike these authors, we apply both time consistent and time invariant welfare concepts in concrete decision problems, demonstrating how they affect the allocations social planners would choose, and how planning equilibria relate to decentralized decision methods when either property is adopted.

A series of applied papers also implicitly imposes either time consistency or time invariance on utilitarian social preferences. Munro (1979) and Li and Löfgren (2000) implicitly adopt a time consistent social welfare function when studying endogenous resource management decisions when agents have heterogeneous impatience. A literature in finance investigates representative time preferences when agents make private decisions and interact through the market (e.g. Lengwiler, 2005; Jouini et al., 2010; Cvitanić et al., 2012), obtaining results similar to Gollier and Zeckhauser's on the non-constancy of representative discount rates. Market interactions lead to Pareto efficient allocations, yet these can only be considered first best if time invariance of the social welfare function is abandoned. By contrast, e.g. Ekeland and Lazrak (2010) and Iverson et al. (2014) impose time invariance on utilitarian social preferences, thus violating time consistency. None of these papers recognizes that adopting the opposite property to the one they implicitly choose would be consistent with individuals' revealed preferences in their models.

A related literature, stemming from the work of Weitzman (1998, 2001), focuses on aggregation of real (i.e. consumption) discount rates, rather than pure time preferences. Weitzman takes a sample of opinions as to the appropriate (constant) real discount rate for project evaluation, treats these as uncertain estimates of the 'true' underlying rate, and takes expectations of the associated discount factors to derive a declining term structure for the 'certainty equivalent' real discount rate. As Freeman and Groom (2014) observe, opinions about real discount rates conflate ethical views about welfare parameters (e.g. the pure rate of time preference) with empirical estimates of consumption growth rates – they mix tastes and beliefs. This suggests that it is important to pursue approaches that treat preference aggregation as a distinct problem. Our work highlights the normative choices that must be made when we engage with this problem.[4]

Finally, our work relates to a diverse literature on disadvantageous power when intertemporal objectives conflict. If we adopt time invariance as a property of social preferences, a rational social planner who controls all aspects of decision making will choose inefficient plans. We show that social welfare may be strictly improved by delegating decision-making to a decentralized process such as property rights or voting. This finding has analogues in the theory of market power (Salant et al., 1983; Maskin and Newbery, 1990), international cooperation (Rogoff, 1985), and government regulation (Krusell et al., 2002).

[4]See Dasgupta (2001, pp. 187-190) and Gollier (2014) for further discussions of Weitzman's approach.

2 Time consistency, time invariance, and collective intertemporal choice.

Individuals' preferences over infinite consumption streams $\mathbf{c} = (c_t)$ are assumed to have a numerical representation $V_\tau(\mathbf{c})$. The subscript τ indicates that these are preferences *in evaluation period* τ. We thus allow preferences to differ at different times. Throughout the paper we assume that preferences are separable across time.[5] For analytical convenience later on we work with continuous time, but this should be understood as the formal limit of a discrete time finite horizon model as the time step tends to zero, and the time horizon tends to infinity.

Following Halevy (2015), we distinguish three properties of intertemporal preferences: Time consistency, Time invariance, and Stationarity. To define these properties it is useful to establish some notation. We define a 'shift and replace' operator:

$$(\mathbf{x}^{\tau,\Delta}, \mathbf{c}) = \begin{cases} x_t & \text{if } t \in [\tau, \tau+\Delta) \\ c_{t-\Delta} & \text{if } t \in [\tau+\Delta, \infty) \end{cases} \tag{1}$$

where $\tau \in \mathbb{R}, \Delta > 0$. In words, this operator shifts the values of c_t into the future by $\Delta > 0$, and replaces the values of c_t in the time interval $[\tau, \tau+\Delta)$ with the values of x_t in that interval.

Definition 1. (a) *Time consistency*: Optimal plans at any time τ are still optimal at any later time $\tau + \Delta$:

$$\forall \tau \in \mathbb{R}, \Delta > 0 : [\forall \mathbf{c}',\ V_\tau(\mathbf{c}) \geq V_\tau(\mathbf{c}')] \Rightarrow [\forall \mathbf{c}',\ V_{\tau+\Delta}(\mathbf{c}) \geq V_{\tau+\Delta}(\mathbf{c}')]$$

(b) *Time invariance*: Consumption streams are evaluated relative to the current evaluation period:

$$\forall \tau \in \mathbb{R}, \Delta > 0, \mathbf{c}, \mathbf{c}', \mathbf{x}, \mathbf{y} : V_\tau(\mathbf{c}) \geq V_\tau(\mathbf{c}') \iff V_{\tau+\Delta}((\mathbf{x}^{\tau,\Delta}, \mathbf{c})) \geq V_{\tau+\Delta}((\mathbf{y}^{\tau,\Delta}, \mathbf{c}')).$$

(c) *Stationarity*: Preferences at a fixed time τ are independent of common sequences at

[5]This property is ensured by Koopmans' Independence axiom (Koopmans, 1960). For discussions of non-separable intertemporal preferences see e.g. Machina (1989). We return to a discussion of separability in Section 5.

the beginning of consumption streams:

$$\forall \tau \in \mathbb{R}, \Delta > 0, \mathbf{c}, \mathbf{c}', \mathbf{x} : V_\tau(\mathbf{c}) \geq V_\tau(\mathbf{c}') \iff V_\tau((\mathbf{x}^{\tau,\Delta}, \mathbf{c})) \geq V_\tau((\mathbf{x}^{\tau,\Delta}, \mathbf{c}'))$$

Time consistency is a consistency relationship between preferences in *different* evaluation periods. Time invariance also relates preferences at different evaluation periods, but is an altogether different property from time consistency. It requires that preferences be independent of translations of the time axis – shifting both preferences and consumption streams forwards by Δ units of time has no effect on rankings. This implies that preferences at τ depend only on time differences relative to τ, i.e. $t - \tau$. This property also implies a strong form of history independence: preferences at time τ are independent of allocations that preceded τ. Finally, Stationarity is an independence property of preferences in each *fixed* evaluation period. This property is at the heart of Koopmans' (1960) axiomatization of discounted utilitarian time preferences.

Halevy (2015) has observed that, for the purposes of revealed preference, any two of these properties implies the third. Consider the standard Discounted Utilitarian model of intertemporal choice:

$$V_\tau^{DU}(\mathbf{c}) = \int_\tau^\infty U(c_t) e^{-\delta(t-\tau)} dt. \tag{2}$$

These preferences are Stationary, Time Consistent, and Time Invariant.[6] It follows that revealed preference cannot disentangle time consistency and time invariance for discounted utilitarian preferences.

While time consistency and time invariance are inseparable in a single agent context, this is not the case for collective choices that are sensitive to the heterogeneity in agents' preferences, as we now demonstrate.

Suppose that we wish to allocate a collective consumption stream between N agents (indexed by i), each of whom has discounted utilitarian preferences. A vector of time dependent allocations to each of the agents will be written $\vec{\mathbf{c}} = (\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N)$. The notions of time consistency, time invariance, and stationarity are extended to social preferences over

[6] Different axiomatizations of discounted utilitarian preferences employ different properties. Strotz (1955) imposes time invariance and time consistency, but not stationarity, on generalized separable preferences (which may depend on the history of consumption), and arrives at (2). Stationarity is obtained as a consequence of the other two properties. On the other hand, the axiomatization of discounted utilitarianism in Koopmans (1960) employs Stationarity (as well as other independence, sensitivity, and continuity properties), but does not explicitly invoke time invariance or time consistency. His characterization thus does not preclude the utility function and the discount rate from varying with the evaluation date τ. One must add an additional axiom – either time invariance or time consistency, to arrive at (2).

vector-valued consumption streams without modification. Denote agent i's discount rate by $\delta_i \in [0, \infty)$. To focus on the consequences of heterogeneity in attitudes to time we assume away all other sources of heterogeneity between agents. In particular, they are assumed to have a common utility function $U(c_{it})$. In addition we assume that social preferences over consumption allocations are utilitarian. Thus, social welfare in evaluation period τ can be written as:

$$W_\tau(\vec{\mathbf{c}}) = \sum_i w_i(\tau) \int_\tau^\infty U(c_{it}) e^{-\delta_i (t-\tau)} dt, \tag{3}$$
$$w_i(\tau) \geq 0, \sum_i w_i(\tau) = 1.$$

where c_{it} is the consumption allocated to agent i at time t, and $w_i(\tau)$ is the Pareto weight on agent i, which may depend on the evaluation date τ. The case where consumption is a public good corresponds to the restriction that $c_{it} = C_t$ for all i, where C_t is public consumption. We treat this as a special case of our more general setup.

So long as there exist at least two indices i, j with $w_i(\tau) > 0, w_j(\tau) > 0$ and $\delta_i \neq \delta_j$ the preferences $W_\tau(\mathbf{c})$ violate Stationarity. This is demonstrated formally by Jackson and Yariv (2015), and can be easily verified.[7] What of time consistency and time invariance? It is straightforward to achieve either property by choosing the Pareto weights $w_i(\tau)$ in (3) appropriately:

Lemma 1. *1. Social preferences (3) are* ***time consistent*** *if and only if the Pareto weights $w_i(\tau)$ are of the form:*

$$w_i(\tau) = \frac{w_i e^{-\delta_i(\tau - \tau_0)}}{\sum_j w_j e^{-\delta_j(\tau-\tau_0)}}. \tag{4}$$

where $w_i \geq 0$, $\sum_i w_i = 1$, and τ_0 is a constant.

2. Social preferences (3) are ***time invariant*** *if and only if the Pareto weights $w_i(\tau)$ are of the form:*

$$w_i(\tau) = y_i, \tag{5}$$

[7] Jackson and Yariv (2015) entitle their paper 'Collective dynamic choice: The necessity of time inconsistency', claiming that utilitarian social preferences such as (3) necessarily violate time consistency. Their work establishes a clash between stationarity and utilitarianism, but as we have seen, stationarity is only a requirement for time consistency if preferences are also time invariant, an implicit assumption in their analysis. Indeed the axiomatization of utilitarian social preferences due to Hammond (1996) makes explicit use of a time consistency axiom. We are grateful to Marc Fleurbaey for pointing this out to us.

where $y_i \geq 0, \sum_i y_i = 1$.

Proof. See Appendix A. □

Unlike the single agent case, time consistency and time invariance are not interchangeable for social preferences. The time consistent Pareto weights (4) violate time invariance since with this choice the relative weight on each agent's welfare at time t depends on time differences relative to a *fixed* reference time τ_0, and not on differences relative to the current evaluation period, i.e. $t - \tau$. Thus, although individual agents have time invariant preferences, planners act as if time τ_0 is a fixed origin of the time axis. Similarly, time invariant social preferences are *not* time consistent, as the Pareto weights (5) are not of the form (4). In this case social preferences change with the evaluation date τ, even though individuals' preferences do not.

We now make an observation that clarifies the possible interpretations of time consistent and time invariant social preferences.

Lemma 2. *1. If social preferences are time consistent the following interpretations of (3) are equivalent:*

- *Individuals have time invariant preferences, and planners have* τ*-dependent Pareto weights (4).*
- *Individuals have preferences over lifetime utility measured from* τ_0*, and planners' Pareto weights are constant.*

2. If social preferences are time invariant the following interpretations of (3) are equivalent:

- *Individuals have time invariant preferences, and planners have constant Pareto weights (5).*
- *Individuals have preferences over lifetime utility measured from* τ_0*, and planners have* τ*-dependent Pareto weights,* $w_i(\tau) \propto y_i e^{\delta_i(\tau-\tau_0)}$.

Proof. Consider point 1 of the lemma. Suppose that individuals have preferences over lifetime utility measured from τ_0, and planners' Pareto weights are constant. Then social preferences at any time τ are given by:

$$W_\tau(\vec{\mathbf{c}}) = \sum_i w_i \int_{\tau_0}^{\infty} U(c_{it}) e^{-\delta_i(t-\tau_0)} dt \tag{6}$$

Now suppose $\tau > \tau_0$. Let ${}_{\tau_0}\vec{\mathbf{x}}_\tau$ denote the consumption allocation that was realized for $t \in [\tau_0, \tau]$. This allocation cannot be altered, as it is in the past. Thus, for any $\tau > \tau_0$, social preferences are restricted to consumption paths $\vec{\mathbf{c}}_{(\tau)}$ that are continuations of ${}_{\tau_0}\vec{\mathbf{x}}_\tau$ from τ onwards. We can thus write preferences over continuation plans at $\tau > \tau_0$ as:

$$\begin{aligned} W_\tau(\vec{\mathbf{c}}_{(\tau)}) &= \sum_i w_i \int_{\tau_0}^{\tau} U(x_{it}) e^{-\delta_i(t-\tau_0)} dt + \sum_i w_i \int_{\tau}^{\infty} U(c_{it}) e^{-\delta_i(t-\tau_0)} dt \\ &= \underbrace{\sum_i w_i \int_{\tau_0}^{\tau} U(x_{it}) e^{-\delta_i(t-\tau_0)} dt}_{\text{constant}} + \sum_i \left[w_i e^{-\delta_i(\tau-\tau_0)} \right] \int_{\tau}^{\infty} U(c_{it}) e^{-\delta_i(t-\tau)} dt \end{aligned}$$

The first term in this expression is common to all continuation plans, and thus has no effect on preferences. The second term is of the form (2) with a τ-dependent Pareto weight (4). Thus, this planner's preferences over continuation plans at τ coincide with our original interpretation of (3) in which agents are time invariant and planners have Pareto weights (4). The proof of point 2 of the lemma follows analogously.

$\square$

The ambiguity in the interpretation of (3) is a consequence of the fact that we cannot tell whether individuals with discounted utilitarian preferences (2) evaluate consumption streams by looking forward from the evaluation date τ, or whether they have preferences over 'lifetime utility' integrated from τ_0 onwards. Both interpretations give rise to the same individual rankings of continuation consumption streams at each evaluation period τ, and are thus indistinguishable based on revealed preference. This observation is a re-expression of the equivalence between time invariance and time consistency for individuals with discounted utilitarian preferences.

In the remainder of the paper we explore how the choice to impose time consistency or time invariance on social preferences impacts resource allocation by social planners, and the relationship between planning and decentralized allocation methods. For the purposes of our modeling work below we will adopt our original interpretation that emphasizes the time invariance of individual agents' preferences, recognizing that any welfare comparisons we make will also hold for the lifetime utility interpretation of the social welfare function, as indicated by Lemma 3. We return to a discussion of the interpretation of the social welfare function in Section 4, when we discuss the normative implications of imposing time consistency/invariance on social preferences.

3 Collective intertemporal choice in practice

To investigate how time consistency and time invariance of social preferences play out in practice, we focus on an applied problem in which a plan for consuming a resource must be chosen for a group of agents with heterogeneous time preferences. We consider both the case where the resource is common property, and where it is a public good, and characterize the allocations social planners would choose if they are either time consistent, or time invariant. We then examine three alternatives to direct planning – delegation to a 'second best' planner, property rights (for common resources), and voting (for public goods). We show that these alternatives can strictly improve on planning equilibria when social planners have time invariant preferences.

To facilitate the analysis we specialize the social welfare function (3) a little more, by assuming that agents have logarithmic utility functions:

$$U(c) = \ln c \tag{7}$$

Our key conceptual messages do not depend on the choice of utility function, so we opt for the considerable analytical convenience of log utility over generality. Thus if agent i is allocated a consumption path c_{it}, his realized welfare computed at time τ is

$$V_{i\tau} = \int_{\tau}^{\infty} \ln(c_{it}) e^{-\delta_i (t-\tau)} dt. \tag{8}$$

and social welfare at τ is given by

$$W_\tau = \sum_i w_i(\tau) V_{i\tau}. \tag{9}$$

If consumption is a public good, we have that:

$$\forall i,\ c_{it} = C_t. \tag{10}$$

The group of agents derives consumption from a risk-free asset S that yields a constant (net) rate of return $r \geq 0$. S thus evolves according to

$$\dot{S}_t = rS_t - \sum_i c_{it} \quad \text{(Common Resource)} \tag{11}$$

$$\dot{S}_t = rS_t - C_t \quad \text{(Public Good)} \tag{12}$$

and the initial value of S at time τ_0 is S_0. This simple model has many possible interpretations. For example, S could be a household's savings, a firm's capital, the value of a country's sovereign wealth fund, or a stock of environmental quality.

The following observation will be useful. Consider the common resource version of the model, and let the share of aggregate consumption C_t allocated to agent i at time t be $s_i(t)$, with $s_i(t) \geq 0$, $\sum_i s_i(t) = 1$ for all t. Then we have that

$$\begin{aligned} W_\tau &= \sum_i w_i(\tau) \int_\tau^\infty \ln(s_i(t) C_t) e^{-\delta_i(t-\tau)} dt \\ &= \sum_i w_i(\tau) \int_\tau^\infty \ln(C_t) e^{-\delta_i(t-\tau)} dt + \sum_i w_i(\tau) \int_\tau^\infty \ln(s_i(t)) e^{-\delta_i(t-\tau)} dt \end{aligned} \tag{13}$$

This expression shows that when utility is logarithmic the problem of choosing optimal aggregate consumption C_t separates from the problem of choosing optimal consumption shares $s_i(t)$. The state dynamics (11) depend only on $C_t = \sum_i c_{it}$, and not on the shares $s_i(t)$, so the optimal choice of consumption shares in evaluation period τ is independent of the resource dynamics. This further implies that social preferences over *aggregate* consumption streams C_t in a common resource model will be identical to social preferences over *public* consumption streams in a public goods model.[8] This follows since the first term in (13) is identical to the objective function of a utilitarian planner when consumption C_t is a public good. This observation will allow us to treat the public good model as a special case of the common resources model. We thus focus on developing the common resource case in the first instance. The distinction between common and public goods only bites for us when we consider decentralized decision methods, so we defer detailed discussion of public goods until then.

In what follows it will sometimes be useful to use shorthand notation, which we collect here for convenience. We define the weighted average of a quantity x_i ($i = 1..N$) taken with weights y_i ($y_i \geq 0, \sum_j y_j = 1$) as

$$\langle x_i \rangle_{y_i} := \sum_{i=1}^N x_i y_i. \tag{14}$$

If the average operator doesn't have a subscript, this will mean that the weights y_i are

[8]Provided of course that we hold the consumption shares $s_i(t)$ fixed when ranking aggregate consumption streams in the common resource model.

equal:

$$\langle x_i \rangle := \frac{1}{N}\sum_{i=1}^{N} x_i. \tag{15}$$

In addition, the weighted variance will be denoted

$$\mathrm{Var}_{y_i}(x_i) := \langle x_i^2 \rangle_{y_i} - (\langle x_i \rangle_{y_i})^2. \tag{16}$$

3.1 Planning

In this section we describe the choices that rational planners who aim to maximize utilitarian social welfare would make. We first consider the case where planners are time consistent. In this case planners' allocations achieve the first best in all evaluation periods. Next we consider the case of time invariant planners. In this case planners' preferences are not time consistent, and rational planners will anticipate the preferences of their 'future selves', and react optimally to them, giving rise to a dynamic game in the tradition of Phelps and Pollak (1968). We describe the sub-game perfect equilibrium of this game and contrast the two planning equilibria.

3.1.1 Time consistent planning

From Lemma 1, we know that a time consistent utilitarian planner at evaluation period τ will choose agents' consumption c_{it} to maximize

$$W_\tau(\vec{\mathbf{c}}) = \sum_i w_i e^{-\delta_i(\tau-\tau_0)} \int_\tau^\infty \ln(c_{it}) e^{-\delta_i(t-\tau)} dt \;\text{ s.t. }\; \dot{S}_t = rS_t - \sum_i c_{it}, \tag{17}$$

where the initial value of S_t is S_τ. The optimal allocations are summarized by the following proposition:

Proposition 1. *1. The consumption allocated to agent i at time $t \geq \tau_0$ is*

$$c_{it} = \frac{w_i S_0}{\langle \delta_j^{-1} \rangle_{w_j}} e^{-(\delta_i - r)(t-\tau_0)} \tag{18}$$

2. The optimal aggregate consumption plan $C_t = \sum_i c_{it}$ is equivalent to the plan that would be chosen by a representative agent with preferences $\hat{W}_\tau(\mathbf{C}) = \int_{\tau_0}^\infty \ln(C_t) e^{-\hat{\delta}(t)(t-\tau_0)} dt$, *where*

$$\tilde{\delta}(t) = \frac{\sum_i \delta_i w_i e^{-\delta_i(t-\tau_0)}}{\sum_i w_i e^{-\delta_i(t-\tau_0)}}, \tag{19}$$

and

$$\frac{d\tilde{\delta}}{dt} < 0, \ \lim_{t\to\infty} \tilde{\delta}(t) = \delta_L, \tag{20}$$

where L is the index of the agent with the lowest discount rate.

Proof. See Appendix B. □

The consumption allocations in this proposition are Pareto efficient, and achieve the first best in all evaluation periods. This is a consequence of the time consistency of the planner's preferences. By examining the optimal consumption allocations (18) directly we can see exactly how the time invariance assumption is violated in this equilibrium. Let agent L have the lowest discount rate, δ_L. From (18) we see that the ratio of agent $i \neq L$'s consumption to agent L's consumption at time t is proportional to $e^{-(\delta_i-\delta_L)t}$. This ratio is decreasing in time for all $i \neq L$. Thus, regardless of the choice of weights w_i, an increasing share of aggregate consumption is allocated to the agent with the lowest discount rate as time passes. In the limit as $t \to \infty$, the agent with the lowest discount rate receives an infinitely larger consumption share than any other agent. While this is optimal if social preferences are time consistent, it cannot be optimal if social preferences are time invariant. If preferences were time invariant, planners would want to allocate the same share of consumption to agent i in every evaluation period, as their preferences are solely forward looking, and identical in every evaluation period.

3.1.2 Time invariant planning

We now consider the case where social preferences are time invariant, and hence *not* time consistent. In this case a planner at time τ_1 who wants to implement a policy that maximizes (3) with constant Pareto weights knows that at time $\tau_2 > \tau_1$ her 'future self' will implement what she views as the best consumption allocation. However, the allocation that the time τ_2 planner makes will not coincide with what the τ_1 planner would have liked. The planner at τ_1 would like to assign weight $y_i e^{-\delta_i(\tau_2-\tau_1)}$ to agent i's welfare at time τ_2 when deciding on consumption allocations, but from the perspective of the planner at τ_2 the appropriate weight on agent i's welfare at time τ_2 is just y_i. Hence the time inconsistency problem.

The traditional approach to solving models of intertemporal choice with time inconsistent preferences is to treat them as a dynamic game between current and future selves (e.g. Phelps and Pollak, 1968; Laibson, 1997; Barro, 1999). In this approach the planner at τ_1 rationally anticipates the consumption decisions of future planners at all $\tau_2 > \tau_1$. She makes the best decision she can, subject to what these future planners will do. This induces a dynamic game between planners, and we look for sub-game perfect equilibria of this game.

Infinite horizon dynamic games admit a continuum of sub-game perfect equilibria in general (Laibson, 1994; Krusell and Smith, 2003). Recalling however that our model is to be interpreted as the limit of a finite horizon model as the horizon length tends to infinity, the equilibrium of the dynamic game between planners can be shown to be unique, and to correspond to the (linear) Markov Perfect Equilibrium (MPE) (see e.g. Laibson, 1994; Krusell et al., 2002).

We thus focus on characterizing the (linear) MPE of the dynamic game between planners at different evaluation periods. A Markov allocation rule in our context is a profile of state dependent functions $P = \{\sigma_i(S)\}$ such that $c_{it} = \sigma_i(S_t)$ for all $t \geq \tau$. A profile is an MPE if, in the limit as $\epsilon \to 0$, when planners at times $t \in [\tau+\epsilon, \infty)$ in the future use profile P, the best response of the current planner in $t \in [\tau, \tau+\epsilon)$ is also to use P. Although there are N unknown functions $\sigma_i(S)$, it is possible to transform the problem into one that only requires us to solve for a single unknown function. The key observation is that the current planner only influences what happens in the future through her choice of aggregate consumption – how she distributes a given level C_τ of current aggregate consumption across agents has no effect on the actions of future planners, and her stream of future welfare. This is so since future allocations depend only on the state S_t, which is affected only by aggregate consumption decisions. Since the actions of future selves are independent of the distribution of current consumption, the current planner will rationally allocate whatever consumption C_τ she chooses between the agents so as to maximize their current collective welfare. Specifically, given any (as yet unknown) choice of aggregate consumption C_τ, the current planner chooses $c_{i\tau}$ to maximize

$$\sum_i y_i \ln(c_{i\tau}) \text{ s.t. } \sum_i c_{i\tau} = C_\tau.$$

A simple calculation then shows that

$$c_{i\tau} = y_i C_\tau. \tag{21}$$

Thus, in equilibrium, the planner will always allocate a share y_i of aggregate consumption to agent i at each point in time. This implies that the equilibrium allocation to agent i must satisfy $\sigma_i(S) = y_i\sigma(S)$ for some function $\sigma(S_t)$, where aggregate consumption is given by $C_t = \sigma(S_t)$.

Now we need to solve for the equilibrium aggregate consumption rule $\sigma(S)$. To do this, note that the value the planner at $t = \tau$ gets from aggregate consumption C_t consumed by the group at time $t \geq \tau$ is:

$$\sum_i y_i e^{-\delta_i(t-\tau)} \ln(y_i C_t) = \left[\sum_i y_i e^{-\delta_i(t-\tau)}\right] \ln(C_t) + K \tag{22}$$

where K is independent of C_t, and may thus be neglected. Defining

$$\beta(t) = \sum_i y_i e^{-\delta_i(t-\tau)} \tag{23}$$

the problem is to find the MPE of a modified single agent problem with utility function $U(C_t) = \ln(C_t)$ and discount factor $\beta(t)$, and where the resource stock evolves according to $\dot{S}_t = rS_t - C_t$. The next proposition characterizes the linear MPE of this game:

Proposition 2. *1. The linear MPE of the modified planner's problem is given by an aggregate consumption rule $C_t = \sigma(S_t) = AS_t$, where*

$$A = \left[\langle \delta_i^{-1} \rangle_{y_i}\right]^{-1} \tag{24}$$

2. *The aggregate consumption path in the linear MPE is equivalent to the optimal path of a planner with preferences (2), logarithmic utility function, and a (constant) representative discount rate*
$$\hat{\delta} := A. \tag{25}$$

3. *Agent i is assigned a share y_i of aggregate consumption at all times, and achieves equilibrium welfare at time τ given by*

$$V_{i\tau} = \frac{1}{\delta_i} \ln\left(S_\tau y_i \langle \delta_i^{-1} \rangle_{y_i}^{-1}\right) + \frac{1}{\delta_i^2}\left(r - \langle \delta_i^{-1} \rangle_{y_i}^{-1}\right) \tag{26}$$

Proof. See Appendix C. □

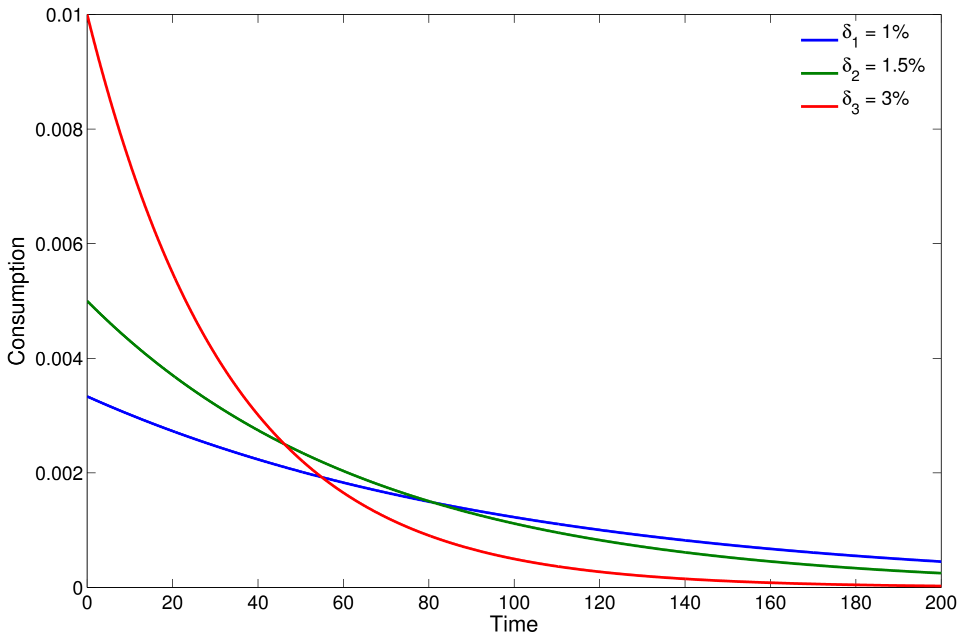
It is clear from the derivation of the effective objective function in (22) that the equilibrium values of aggregate consumption C_t in this common resource problem will once again be identical to the equilibrium consumption path chosen if C_t were a public good.

We make two initial observations about the time invariant planning equilibrium. The first is that, from the perspective of the planner at evaluation period τ, the equilibrium is inefficient. That is, there exist feasible consumption allocations that would increase social welfare, *according to time τ social preferences*. However, owing to the time inconsistency of social preferences, these allocations are not implementable. Any future planner at time $\tau' > \tau$ can increase her social welfare by deviating from the time τ planners' optimal allocation. The time τ planner knows this, anticipates the behavior of all future planners, and reacts rationally to this knowledge. Since all planners behave this way, the resulting equilibrium is inefficient, but fully rational. Second, the time invariant equilibrium is clearly very different from the time consistent equilibrium described in Proposition 1. These differences are summarized by the 'representative' discount rates on aggregate consumption under the two welfare concepts, equations (19) and (25). Whereas the representative discount rate of the time consistent planner is time varying, and declines monotonically to the discount rate of the most patient individual, the representative time invariant discount rate is constant. The constancy of the representative discount rate reflects the time invariance of the game equilibrium consumption path – the game looks the same in each period, except for the current value of the state S_τ.

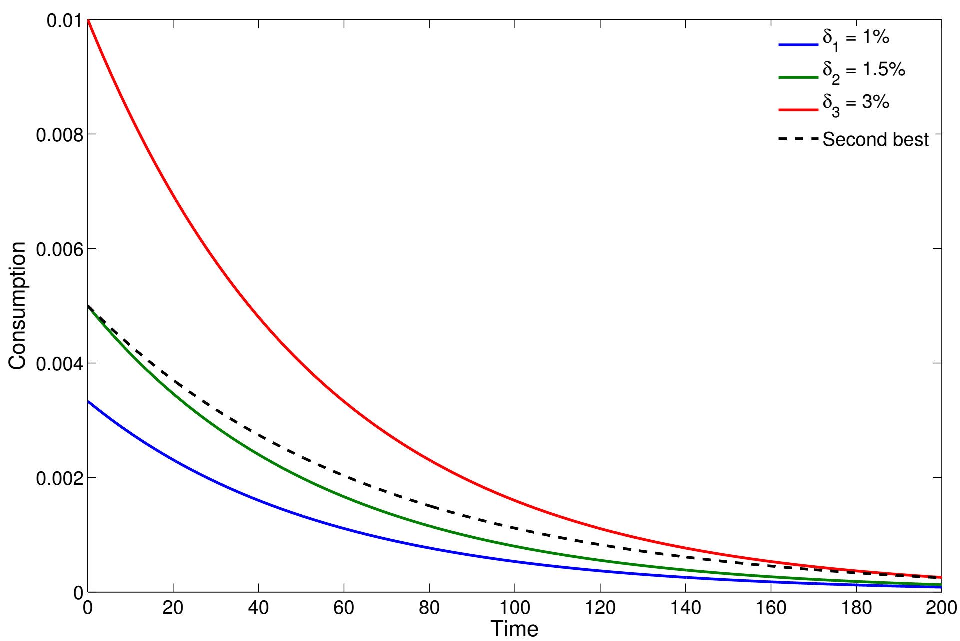
To further illustrate the differences between the time consistent and time invariant equilibria, we plot an example of consumption allocations under these two welfare concepts in Figure 1. If social preferences are time consistent consumption allocations are Pareto efficient, and the consumption shares allocated to agents vary with time. Impatient agents receive a large share at early times, and a small share later on, and vice versa for patient agents. By contrast, if social preferences are time invariant, consumption allocations are not Pareto efficient and agents receive constant consumption shares. This reflects the fact that time invariant preferences are 'memoryless' – the planner in each evaluation period cares only about time differences relative to the present, and not relative to the initial evaluation period τ_0.

3.1.3 Delegation to a 'second best' planner

The fact that equilibrium consumption allocations are inefficient when social preferences are time invariant means that such planners may benefit from delegating decision-making. In the next section we will examine decentralized decision methods and their relationship



(a) Time Consistent social planner



(b) Time Invariant social planner.

Figure 1: Equilibrium consumption allocations under (a) Time Consistent and (b) Time Invariant social welfare functions. In this example we pick $\delta_i \in \{1\%, 1.5\%, 3\%\}, w_i = y_i = \delta_i / \sum_j \delta_j, r = 0, S_0 = 1$. The dashed line in (b) denotes the consumption allocations that a 'second best' time invariant planner would choose – see Proposition 3. The second best planner allocates all agents the same consumption path for this choice of y_i.

to planning equilibria. Here however we ask whether we can identify a 'second best' planner that time invariant planners would like to delegate decision making to.

In general we might be interested in identifying the 'optimal' delegation for time invariant planners. Doing so would require us to evaluate the overall quality of an implemented plan, as conceived by our time invariant social welfare concept. This raises conceptual difficulties. Since time invariant planners at different evaluation periods τ have inconsistent preferences over plans, evaluating an allocation requires us to take a stance on how to trade off social preferences in different evaluation periods. We will not pursue this question too far, as it takes us beyond the concepts of social welfare we have been using, requiring a 'meta preference' over social welfare functions at different evaluation times τ. We do however demonstrate in Appendix D that if meta-preferences over time invariant planners' preferences are Paretian and non-dictatorial, meta-optimal allocations cannot coincide with time consistent utilitarian equilibria.

Instead of selecting the 'best' allocation according to some generalized welfare concept, we can look for the best allocation that can be rationalized by *any* time invariant social preferences. Such a 'second best' planner provides a normative benchmark that retains the properties of our original social preferences, and allows us to quantify how inefficient the original planner's decisions are within this class of preferences.

Let $c_{it}(\vec{z})$ be the game equilibrium consumption path of a time invariant planner with Pareto weights $\vec{z} = (z_i)$. Consider an original planner with Pareto weights y_i. We solve for the second best time invariant planner at time τ by finding the vector of Pareto weights $\vec{z}$ that maximizes

$$\max_{\vec{z}} \sum_i y_i \int_\tau^\infty \ln(c_{it}(\vec{z})) e^{-\delta_i(t-\tau)} dt. \tag{27}$$

Proposition 3. *1. If an original time invariant planner has Pareto weights $\vec{y}$, the second best time invariant planner has Pareto weights $\vec{z}^*$ given by*

$$z_i^* = \frac{y_i \delta_i^{-1}}{\langle \delta_j^{-1} \rangle_{y_j}} \tag{28}$$

in all evaluation periods τ.

2. The game equilibrium consumption path of the second best time invariant planner is also the best consumption path that can be rationalized by social preferences that are *both time invariant and time consistent. Such preferences take the form*

$$W_\tau(\vec{\mathbf{c}}; \vec{z}, \delta_z) = \sum_i z_i \int_\tau^\infty \ln(c_{it}) e^{-\delta_z(t-\tau)} dt. \tag{29}$$

Proof. See Appendix E. □

The first part of this proposition gives an analytical expression for the preferences of the second best time invariant planner. The second part of the proposition shows that the second best consumption plan is in fact the best plan amongst those that would be chosen by *any* time invariant utilitarian planner, including those who are time consistent. This is not obvious, as when preferences are given by (29) equilibria are not determined by solving a dynamic game as in Proposition 2, but rather by straightforward dynamic optimization. In addition, the discount rate δ_z and Pareto weights $\vec{z}$ may be chosen independently when preferences are of the form (29), whereas $\vec{z}$ determines the effective discount rate of the game equilibrium consumption path in Proposition 2 (see eqs. (24–25)). Despite the extra parametric freedom and different solution concept when preferences are of the form (29), the second-best parameter vector in this class of preferences is consistent with the second best parameter vector that emerges from the solution of (27).

Proposition 3 allows us to contrast the second best plan with the original equilibrium that time invariant planners would have implemented. Notice that y_i dominates z_i^* according to the monotone likelihood ratio order. Thus the second best Pareto weights z_i^* place more weight on low discount rates, and less on high discount rates, than the original planner. In a general sense then, the equilibrium that arises from time invariant planning is too impatient relative to the second best. We can make this more explicit by computing the representative discount rate of the second best planner, δ_z^*, using the formulas (25) and (28):

$$\delta_z^* = \langle \delta_i^{-1} \rangle_{z_i^*}^{-1} = \frac{\langle \delta_i^{-1} \rangle_{y_i}}{\langle \delta_i^{-2} \rangle_{y_i}}.$$

Comparing the second best discount rate δ_z^* to the representative discount rate of the original time invariant equilibrium $\hat{\delta}$, we find:

$$\begin{aligned}\delta_z^* - \hat{\delta} &= \frac{-1}{\langle \delta_i^{-1} \rangle_{y_i} \langle \delta_i^{-2} \rangle_{y_i}} \left(\langle \delta_i^{-2} \rangle_{y_i} - \langle \delta_i^{-1} \rangle_{y_i}^2 \right) \\ &\propto -\mathrm{Var}_{y_i}(\delta_i^{-1}) \\ &< 0\end{aligned}$$

The time invariant planner's representative discount rate is thus always too large, relative to the second best, and the excess impatience of the planning equilibrium is proportional to a measure of the dispersion of individuals' discount rates, i.e. the weighted variance $\mathrm{Var}_{y_i}(\delta_i^{-1})$. This result can be seen as a multi-person analogue of excess impatience on game-equilibrium consumption paths when a single agent has quasi-hyperbolic preferences (Phelps and Pollak, 1968).

It is notable that the second best plan can be implemented in a very simple delegation procedure. A planner with weights y_i simply delegates to another time invariant planner with weights z_i^*, who then solves the allocation problem rationally, anticipating the preferences of her future selves. Despite the ease of this procedure, implementing the second best plan requires the planner to *commit* to delegating in every period. In each evaluation period τ, the planner at τ can increase her instantaneous welfare by deviating from the second best equilibrium and implementing her optimal consumption choice today. There is thus always a temptation to 'defect' from the second best equilibrium, even though social welfare would be higher in every evaluation period if the second best is implemented.

3.2 Decentralized alternatives to planning

In this section we investigate decentralized alternatives to planning. As we have observed, planning implements the first best when social preferences are time consistent, but leads to inefficient decisions when social preferences are time invariant. The previous section identified a 'second best' planner when preferences are time invariant; this section investigates decentralized decision methods – property rights (for common resources), and voting (for public goods). These approaches are important because they hand decision-making power back to agents themselves, and are thus more likely to be politically acceptable than relying on a benevolent planner. It may thus be easier for social planners to commit to a decentralized procedure than to the second best plan. The relationship between decentralized decision procedures and social planning is also intrinsically interesting, as it highlights how standard intuitions about the relationship between decentralization and social planning depend on whether our welfare concept is time consistent or time invariant. Property rights can implement the first best in common resource problems if social preferences are time consistent, but not if they are time invariant. Nevertheless, we will show that decentralization is often still desirable – both property rights and voting can improve on time invariant planning equilibria.

3.2.1 Property rights and common resources

In the first instance we consider the case in which the asset S is a common resource, and our social welfare concept is time consistent. In this case there is a one-to-one correspondence between property rights regimes, in which individuals are allocated a share of the resource at $\tau = \tau_0$ which they then manage as they see fit, and the optimal allocations of a time consistent planner:

Proposition 4. *Suppose that social preferences are time consistent with Pareto weights given by (4). Then optimal planning allocations coincide with a property rights regime in which a share $a_i S_0$ of the resource is allocated to agent i at time τ_0, where*

$$a_i = \frac{w_i \delta_i^{-1}}{\langle \delta_j^{-1} \rangle_{w_j}}. \tag{30}$$

Similarly, all property rights regimes with initial resource shares $a_i S_0$ are equivalent to time consistent planning equilibria with

$$w_i = \frac{\delta_i a_i}{\langle \delta_i \rangle_{a_i}}. \tag{31}$$

Proof. See Appendix F. □

Thus any first best allocation can be achieved (for all evaluation periods) by choosing initial endowments appropriately if social preferences are time consistent.

This proposition immediately implies a further result – property rights do *not* implement the first best in all evaluation periods if social preferences are time invariant. At most, property rights can implement the first best *according to the preferences of the time invariant planner at* τ_0. For all $\tau > \tau_0$, the property rights equilibrium will not be socially optimal. This is a simple consequence of the time inconsistency of time invariant preferences.[9]

[9] Of course this observation requires property rights to be enforced at all future dates once established.

Although property rights cannot implement the first best, it is still of interest to know whether they can improve on the time invariant planning equilibrium. In this case our results are more optimistic:

Proposition 5. *Suppose that time invariant planners have Pareto weights y_i, and that a share $\frac{y_i \delta_i^{-1}}{\langle \delta_i^{-1} \rangle_{y_i}} S_0$ of the resource is allocated to agent i at time τ_0.*

1. *This initial property allocation yields the largest value of time invariant welfare out of all possible property rights regimes at τ_0, in every evaluation period $\tau \geq \tau_0$.*
2. *This property rights regime is preferred to the time invariant planning equilibrium in every evaluation period $\tau \geq \tau_0$.*

Proof. See Appendix G. □

Thus, as in our analysis of the second best planner above, if time invariant planners can commit to establishing a system of property rights, they can achieve persistent welfare gains over the equilibrium they would choose for themselves.

3.2.2 Voting and public goods

Property rights can improve on the planning equilibrium when social preferences are time invariant, however they are not available to us when the resource S is a pure public good such as air quality or a stable climate. It is still desirable to ask whether there is a decentralized decision procedure that improves on the planning equilibrium in this case. Voting is a natural candidate for such a procedure.

To be more precise, suppose now that the resource S is a public good, and the group's public consumption in evaluation period τ is C_τ, which is to be decided by ballot. In each period τ each agent may nominate a single value of C_τ for the group to adopt. All agents vote over each pair of nominated consumption values, and the value that gets a majority of votes wins each pairwise contest.[10] A Condorcet winner (if it exists) is a value of C_τ that wins every pairwise contest. If there is a Condorcet winner, it is implemented.

If planners in each evaluation period may seize and redistribute property, the problem of choosing optimal property allocations is formally identical to the equilibrium of the dynamic game between time invariant planners.

[10] We focus on majority rule ballots as they are the most common in practice, and because they satisfy a number of desirable normative properties over a larger domain than any other voting rule (Dasgupta and Maskin, 2008).

Since the current choice of C_τ influences the consumption choices that will be made in future ballots, agents' must anticipate the outcomes of those ballots when forming their preferences over current consumption C_τ. We assume that agents are rational, and thus perfectly anticipate the outcomes of all future ballots when forming their preferences over C_τ. The following proposition characterizes the equilibrium public consumption path that emerges from this sequences of ballots:

Proposition 6. *Agents at time τ who anticipate the outcome of votes over public consumption in future periods have single peaked preferences over current consumption C_τ. Thus, the equilibrium of a voting model with ballots in every period is the optimal consumption plan of the median agent.*

Proof. See Appendix H. □

This result may seem to be in conflict with the analysis of voting over consumption streams in Jackson and Yariv (2015). They show that voting over consumption streams in unrestricted domains is generically intransitive, and thus voting equilibria cannot be represented by the preferences of a single individual such as the median agent. Their analysis assumes however that votes are *once off*, whereas in our result ballots are repeated, so that in each period agents are only voting over a single, unconstrained, value of consumption. The repeated ballot formulation is compelling, as it does not require agents to pre-commit to implementing the outcomes of an initial ballot in all future periods.

We are now interested in comparing the voting equilibrium to the planning equilibrium. Our first result clarifies individual agents' ordinal preferences between the two equilibria

Proposition 7. *A majority of agents always prefers the voting equilibrium to any time invariant planning equilibrium.*

Proof. See Appendix I. □

The heart of the proof of this result is to show that individual agents have single-peaked preferences over consumption streams that are optimal for *some* discounted utilitarian agent. Once we know this the result follows from the fact that any planning equilibrium is observationally equivalent to the optimal plan of an agent with discount rate $\hat{\delta}$ (see Proposition 2), and the fact that the plan corresponding to the median agent's discount rate is a Condorcet winner.

This result is independent of any assumptions about the Pareto weights in the social welfare function, and thus gives a strong indication that voting will often lead to higher social welfare than planning for many time invariant social welfare functions. There is however no analogue of Proposition 5 for public goods – voting will not always dominate planning. To characterize the set of social welfare functions for which voting dominates planning we need to compare social welfare under the two equilibria.

Given that public consumption is determined by the median agent's preferred plan, we can calculate agent i's welfare under voting (see Appendix C.3):

$$V_{i\tau}^{V} = \frac{1}{\delta_i} \ln\left(S_\tau \delta_m\right) + \frac{1}{\delta_i^2}(r - \delta_m) \tag{32}$$

A straightforward adaptation of our results on planning equilibria for common resources to the public goods case allows us to compute agent i's welfare in the planning equilibrium:

$$V_{i\tau}^{I} = \frac{1}{\delta_i} \ln\left(S_\tau \langle \delta_i^{-1} \rangle_{y_i}^{-1}\right) + \frac{1}{\delta_i^2}\left(r - \langle \delta_i^{-1} \rangle_{y_i}^{-1}\right) \tag{33}$$

Using these two expressions, we obtain the following:

Proposition 8. *Define*

$$\alpha := \frac{\langle \delta_i^{-1} \rangle_{y_i}^{-1}}{\delta_m}, \tag{34}$$

$$\phi := \delta_m \langle \delta_i^{-2} \rangle_{y_i} \langle \delta_i^{-1} \rangle_{y_i}^{-1} \tag{35}$$

$$q_i := \delta_i^{-1} \tag{36}$$

1. *Voting gives rise to higher social welfare than the planning equilibrium in each evaluation period τ if and only if*

$$\ln \alpha < \phi(\alpha - 1). \tag{37}$$

2. *If the dependence of the Pareto weights $y_i = y(\delta_i)$ on δ_i is such that*

$$\frac{d}{d\delta}[y(\delta)\delta^{-1}] \geq 0 \tag{38}$$

for all δ, a sufficient condition for voting to dominate planning is:

$$\langle q_i \rangle \leq q_m, \tag{39}$$

where $q_m := \delta_m^{-1}$ is the median value of δ_i^{-1}.

Proof. See Appendix J. □

The first part of the proposition shows that the two dimensionless parameters α and ϕ, which depend only on the vector of Pareto weights $\vec{y}$ and the distribution of discount rates, are sufficient statistics for ranking the voting and planning equilibria. The second part of the proposition shows that if we restrict attention to social welfare functions with Pareto weights that increase fast enough as functions of δ_i, a simple skewness condition on the distribution of the inverses of agents' discount rates is sufficient to ensure that Voting dominates Planning.

We are especially interested in social welfare functions with Pareto weights that satisfy (38), as planners who care about equity will fall in this category. The reason for this is that agents who have higher discount rates will have lower values of welfare, all else being equal, and an equity motivated planner will give these agents more weight to compensate for this. We can see this formally by considering the case of constant consumption paths. Since agents only differ in their attitudes to time, a natural equity requirement is that if the public consumption path C_t does *not* depend on time, all agents should contribute equally to group welfare. There is a unique choice of Pareto weights that ensures this:

$$y_i = \frac{\delta_i}{\sum_j \delta_j}. \tag{40}$$

This follows since agent i's welfare on any constant consumption paths $C_t = C$ is

$$V_{i\tau} = \int_{\tau}^{\infty} \ln C e^{-\delta_i (t-\tau)} dt = \frac{1}{\delta_i} \ln C. \tag{41}$$

The choice of Pareto weights (40) can thus be seen to embody a fairness property. For this choice of weights, $y(\delta_i)\delta_i^{-1}$ is a constant, and thus (38) is satisfied. To further emphasize the equity properties of this choice of weights, note that in the case of common resources, the Pareto weights (40) are the only weights for which planners' preferred consumption allocations are *envy free* at τ_0, i.e. no agent prefers any other agents' allocation.[11] This follows by substituting (40) for w_i in the expression (30), thus showing that a planner at time τ with these Pareto weights would like to choose consumption allocations that are equivalent to a property rights regime in which agents receive equal shares of the resource

[11]The concept of envy free allocations was introduced into economics by Foley (1967) and Varian (1974), and has subsequently been seen as an important fairness property of normative allocation problems (e.g. Fleurbaey and Maniquet, 1997).

at τ.[12] Since agents' welfares are increasing in their resource shares, only an equal division of the resource results in envy free allocations.

While the Pareto weights (40) may embody a compelling fairness concept, Proposition 8 applies to any set of Pareto weights $y_i = y(\delta_i)$ where $y(\delta_i)$ increases at least linearly in δ_i. This clearly admits a much wider set of social preferences, all of which compensate for the intrinsically lower values of welfare that more impatient agents obtain.

The condition (39) requires the distribution of inverse discount rates to be negatively skewed. Intuitively, this seems to imply that the distribution of discount rates itself should be positively skewed. In fact, this is a necessary (but not sufficient) condition for (39) hold. Jensen's inequality implies that $\langle \delta^{-1} \rangle \geq \langle \delta \rangle^{-1}$, and thus a necessary condition for (39) to hold is that $\langle \delta \rangle^{-1} \leq \delta_m^{-1}$, i.e. $\langle \delta \rangle \geq \delta_m$.

We illustrate the implications of (39) in an example in which there are three agents in the population with discount rates $\delta_1 \leq \delta_2 \leq \delta_3$. Define $h_{12} := \delta_1/\delta_2, h_{13} := \delta_1/\delta_3$. Clearly, $0 \leq h_{12} \leq h_{13} \leq 1$. A simple calculation shows that (39) is equivalent to

$$h_{12} \geq \frac{1}{2}(1 + h_{13}). \tag{42}$$

Figure 2 plots the set of three element distributions which satisfy this condition. The figure shows that all these distributions have the property that δ_1 is not 'too small' relative to δ_2, where what counts as too small depends on how large δ_3 is. When δ_3 is very large relative to δ_1 (i.e. $h_{13} \approx 0$), δ_1 needs to be 50% of the value of δ_2, with this ratio climbing as δ_1/δ_3 increases. Thus, the region where (39) is satisfied is intuitively nothing more than a set of distributions that satisfy properties that are qualitatively similar to positive skewness.

For our final illustration of the application of these results, we fix the Pareto weights to be of the equity motivated form (40), and consider an empirical distribution of discount rate prescriptions elicited from economists who are experts in public project evaluation (Drupp et al., 2014). This distribution is illustrated in Figure 3. With our choice of Pareto weights the condition (37), which determines whether voting is preferred to planning, reduces to (defining $d_i = \delta_i/\delta_m$):

$$\ln\langle d_i \rangle < \langle d_i^{-1} \rangle (\langle d_i \rangle - 1). \tag{43}$$

Using the empirical distribution of discount rates, the left hand side of this inequality is equal to 0.78, while the right hand side is equal to 5.26×10^4, and thus the inequality

[12] Of course, in equilibrium time invariant planners would not choose such an allocation, but we are speaking of intrinsic preferences here, i.e. the consumption allocations a planner at τ would like to choose if she were unconstrained by the actions of her future selves.

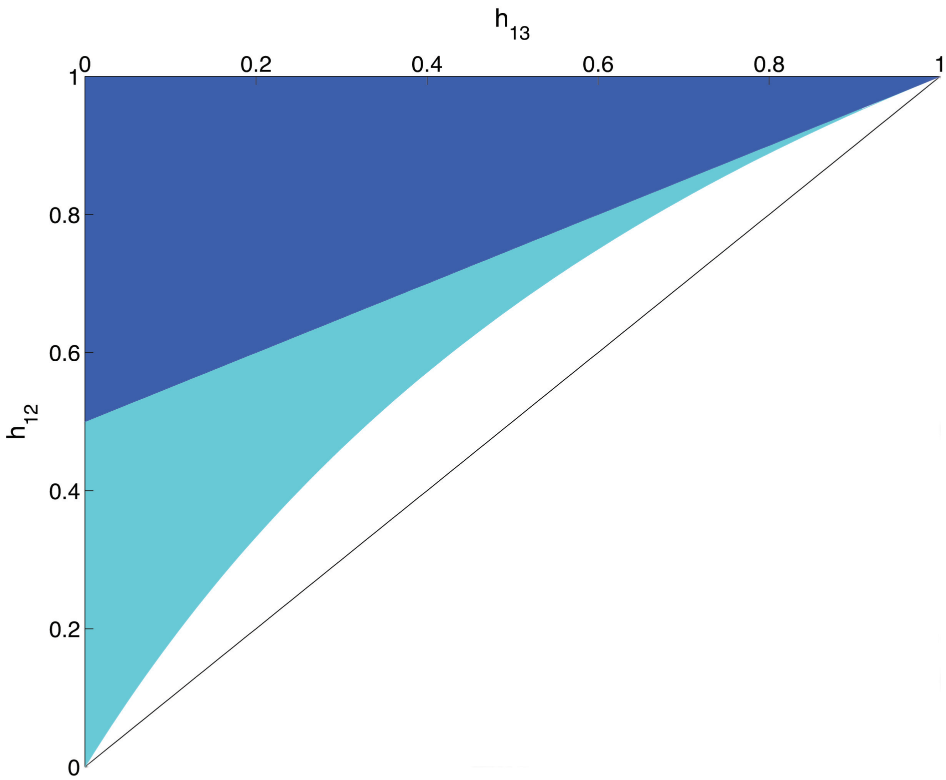


Figure 2: When does voting dominate planning if social preferences are time invariant? The figure represents all distributions with elements $\delta_1 \leq \delta_2 \leq \delta_3$, using the ratios $h_{12} = \delta_1/\delta_2$, and $h_{13} = \delta_1/\delta_3$, where $h_{12} \geq h_{13}$. The shaded dark blue area at the top of the figure denotes the set of all such distributions for which voting dominates planning for *any* social welfare function with Pareto weights y_i such that $y_i\delta_i^{-1}$ is non-decreasing in δ_i. Positively skewed distributions (i.e. $\langle\delta\rangle > \delta_m$), fall in the region $h_{12} > \frac{2h_{13}}{1+h_{13}}$, denoted by the union of the light and dark blue shaded areas.

is satisfied. Thus, for this empirically relevant distribution of time preferences, voting improves on the planning equilibrium. We can quantify just how much voting improves on planning by comparing the differences between these two equilibria and the 'second best' planning equilibrium:

$$\frac{W_\tau(\text{Second Best}) - W_\tau(\text{Planning})}{W_\tau(\text{Second Best}) - W_\tau(\text{Voting})} = 2.17 \tag{44}$$

This ratio is independent of the level of the resource stock S_τ, and thus remains the same in all evaluation periods τ. Thus the voting equilibrium is approximately two times closer to the second best welfare level than the planning equilibrium at all times.

Table 1 summarizes the representative discount rates for planning, voting, and the second best planner for this set of assumptions. Although the decentralized voting approach improves substantially on the consumption path a social planner would choose, it is still less patient than the 'second best' planner's path. Because of the large number of discount rates close to zero in the empirical sample, the second best discount rate $\delta^* = \langle \delta^{-1} \rangle^{-1}$ is only very slightly positive.

4 Discussion

Should we adopt time consistency or time invariance as normative properties of our social welfare function? We have seen that this choice matters, but how should we make it given that revealed preference cannot tell us which property to adopt? We believe that the answer to this question must be context dependent. In order to demonstrate this consider the following stylized scenarios:

1. Ada and Bertha are identical twins; both are childless and single. On their 60th birthday they receive news that a distant cousin they've never met has died, and they will inherit his fortune. Their cousin died intestate, so there is no will to

Table 1: Representative discount rates for a time invariant social welfare function with Pareto weights (40), under planning, voting, and 'second best' planning. Individuals' discount rates are drawn from the distribution depicted in Figure 3.

	Planning ($\hat{\delta} = \langle \delta_i \rangle$)	Voting (δ_m)	Second Best ($\delta^* = \langle \delta^{-1} \rangle^{-1}$)
Representative discount rate	1.15%	0.53%	1.18×10^{-5} %

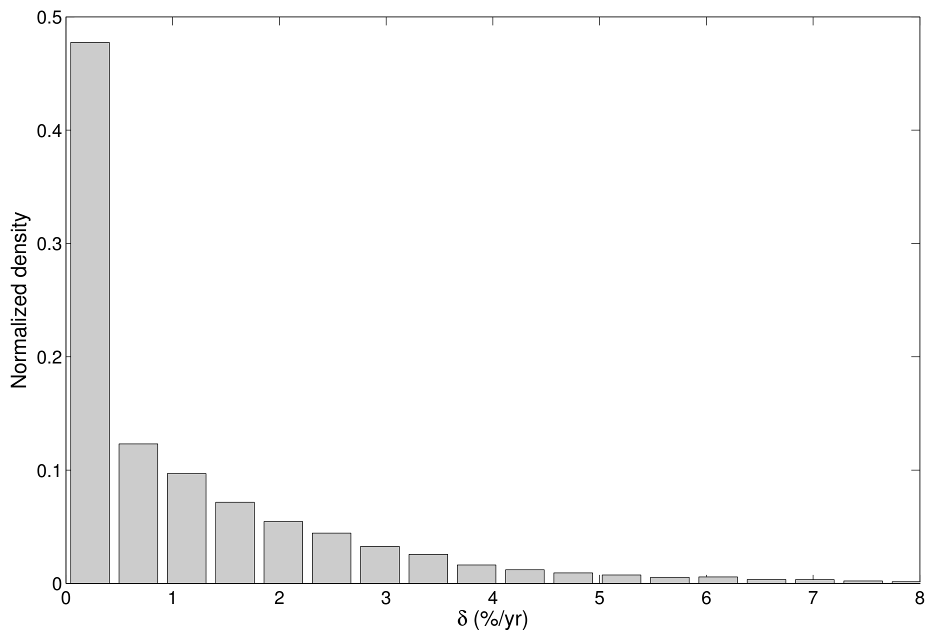


Figure 3: Distribution of the recommended pure rate of time preference δ for public project appraisal, from the Drupp et al. (2014) survey of economists. 180 responses were recorded in the original sample. A kernel density fit has been applied to smooth out the dataset.

specify how the bequest should be divided between them. Ada is naturally impatient, while Bertha is patient. How should a utilitarian social planner allocate the bequest between them?

Suppose that the planner is time consistent. In this case we know that the first best can be obtained by simply allocating each sister a share of the bequest, and letting them spend it as they please. The planner's ethics dictate that an equal share of the bequest be given to each sister. The planner makes this allocation, and walks away happy in the knowledge that she need never revisit this decision.

Now consider a time invariant planner, and suppose that she also initially allocates equal shares to the sisters, before departing on other business, never intending to return. To her surprise however, the planner is asked to revisit the sisters 10 years later and assess their well-being. Ada has led a wild life in the intervening decade, consuming her inheritance much faster than Bertha. But the planner asks herself: what has changed since the sisters were 60? Although Ada has had a better life than Bertha, this is in the past and can't be changed. Since the circumstances are the same, except that the sisters happen to be 10 years older, the planner seizes some of Bertha's carefully saved cash, and reallocates it to Ada, so that their holdings of what remains of the bequest are again equal. Ada immediately spends her new wealth on fast cars and fancy footwear, while Bertha continues to put something away for a rainy day.

2. A nation discovers an enormous oil deposit in its territorial waters. The deposit is large enough to generate rents for generations to come, which will be invested in public goods that benefit all citizens equally. Citizens in the current generation have discounted utilitarian preferences over the wellbeing of future generations. Suppose that in each generation there are equal numbers of citizens who are either Avaricious (i.e. have a high intergenerational discount rate), or Benevolent (i.e. have a low intergenerational discount rate). The citizens must decide on an extraction plan, and agree to adopt a utilitarian approach.

   If society's social welfare function is time consistent a fixed evaluation period τ_0 must be identified to serve as the origin of time for welfare computations in all generations. Given this choice, the optimal plan is equivalent to that of a single agent with a discount rate that declines to the intergenerational discount rate of the Benevolent citizens. After a few generations have past, only Benevolent preferences matter for determining the extraction plan. By Lemma 3, the time consistent planner acts *as if* citizens of the current generation receive utility from the historical consumption of their long deceased forebears.

   If on the other hand the social welfare function is time invariant, both Avaricious and Benevolent citizens' preferences are important determinants of the extraction plan in all generations, and social preferences, like citizens' preferences, depend only on what happens to future generations.

We conjecture that most people would feel that time consistency is a normatively attractive property of social preferences in the first of these scenarios, but not in the second. Conversely, time invariance is likely to be attractive in the second scenario, but not in the first.

What is the difference between these two scenarios? In the first scenario we intuitively feel that Ada should be made to take responsibility for her past excesses, and Bertha should not be penalized for her frugality. A non-paternalistic reason for this could be that individuals experience 'memory utility' from past consumption (e.g. Strotz, 1955; Caplin and Leahy, 2004), as suggested by the lifetime utility interpretation of time consistent social preferences in Lemma 3. Although individuals are not able to express this backward looking aspect of preferences directly through their choices (since the past is fixed), we would be doing them a disservice if we neglected this for the purposes of welfare analysis. A purely normative reason for this could be that there is an evaluation date τ_0 that is ethically salient, even if individuals' preferences are in fact exclusively forward looking. In our scenario, the date of the initial bequest stands out as a natural 'origin' of the time axis. Many people would feel that cumulative utility from the date of the bequest onwards is ethically relevant to the task of apportioning the bequest.

We believe that it is more difficult to substantiate a 'memory utility' component of preferences, or argue for the ethical salience of a special time τ_0, in the second scenario. The unit of analysis in this scenario is intergenerational welfare, rather than an individual's lifetime. It is hard to argue that the wellbeing of those alive today would be higher if history could be altered so that their deceased forebears were better off, keeping the current generation's circumstances unchanged. We may very well feel this way about our own lives, as suggested by Caplin and Leahy (2004), but most peoples' domain of ethical concern begins with their own generation and extends forwards in time. The idea that the wellbeing of a society should accumulate from some 'special' initial generation τ_0 onwards, no matter how long ago τ_0 may have been, is difficult to stomach.[13]

There is no conceptual difficulty with adopting one property of intertemporal preferences for intragenerational decisions and another for intergenerational decisions. We demonstrate this formally in Appendix K, where we solve for the equilibrium consumption path in a hybrid model in which intragenerational social preferences are time consistent, intergenerational social preferences are time invariant, and agents have heterogeneous preferences in both of these dimensions. Some overlapping generations models have in fact adopted a lifetime utility approach to computing intragenerational welfare, and noted the potential for overall social preferences to be time inconsistent if intergenerational prefer-

[13] A possibly counter-argument to this position is that although a time invariant welfare function respects all current citizens' intergenerational preferences, it does not respect the preferences of the dead, which justified the current generation's resource endowment. This would be avoided if we adopted an intergenerational welfare function that is both time invariant and time consistent, and thus respects the preferences of the dead and the living, as well as being independent of any special time τ_0. The difficulty with this, of course, is that such a welfare function must be stationary, and therefore dictatorial with respect to intergenerational discount rates. There is thus a stark tradeoff to be made: We can choose a welfare function that is sensitive to the preferences of all current citizens, but neglects those of the dead. Or we can choose a welfare function that respects the preferences of the living and the dead, but only *one* intergenerational preference can be respected.

ences are time invariant (see e.g. Calvo and Obstfeld, 1988).

5 Conclusions

Contrary to much of the literature on time preference aggregation, this paper makes explicit the normative choices that must be made when making social decisions that reflect the heterogeneity in individuals' time preferences. If individuals have discounted utilitarian preferences (as is commonly assumed in the literature on time preference aggregation), their choices are consistent with both time consistency and time invariance. For the purposes of utilitarian welfare analysis however, we can adopt only one of these properties. The choice to adopt either property has dramatic consequences for the choices rational social planners would make, and the relationship between planning equilibria and decentralized decision methods. If social preferences are time invariant, decentralized methods such as property rights or voting can *strictly* improve on planning equilibria. While the choice between these properties cannot be informed by revealed preference, we have argued that time consistency is normatively attractive for intra-generational choices, while time invariance may be more attractive for inter-generational choice.

The findings in this paper rely on the assumption that individuals have discounted utilitarian preferences, and that social preferences are utilitarian. Both assumptions are routine in the theoretical literature on time preference aggregation, as well as in applications. They are also commonly adopted in the empirical literature that uses revealed preference methodologies to identify how households' collective decisions reflect their members' time preferences (e.g. Mazzocco, 2007; Adams et al., 2014). Aside from satisfying the attractive properties of stationarity, time consistency, and time invariance, discounted utilitarian preferences are highly *tractable*, and thus well suited to applications.

Yet it is clear that such preferences are quite special. In addition to their other properties, they are also additively separable across time. Any additively separable, time invariant, preferences will be difficult to elicit unambiguously by revealed preference, as individuals' choices cannot tell us whether they care about the past (Caplin and Leahy, 2004). This could be seen as an argument for abandoning time-separable preferences (see e.g. Ryder and Heal (1973); Machina (1989); Kreps and Porteus (1978); Epstein and Zin (1989)). Aggregation of non-separable preferences has, to our knowledge, not been explored in the literature, and is a promising subject for future research. Abandoning separability would however come at considerable cost of complexity, and the normative properties of non-separable models are also less well understood. Alternatively, we can keep using separable models, and discounted utilitarianism in particular, but be sensitive to the normative judgements that must be made when applying them to collective choices.

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A Proof of Lemma 1

Part 2 of the lemma is trivial, we prove part 1.

Sufficiency: If the Pareto weights are chosen according to (4) we can write preferences at time τ_1 as

$$\begin{aligned} W_{\tau_1}(\vec{\mathbf{c}}) &\propto \sum_i w_i \int_{\tau_1}^{\tau_2} U(c_{it}) e^{-\delta_i(t-\tau_0)} dt + \sum_i w_i \int_{\tau_2}^{\infty} U(c_{it}) e^{-\delta_i(t-\tau_0)} dt \\ &\propto \sum_i w_i \int_{\tau_1}^{\tau_2} U(c_{it}) e^{-\delta_i(t-\tau_0)} dt + W_{\tau_2}(\vec{\mathbf{c}}). \end{aligned} \tag{45}$$

for any $\tau_2 > \tau_1$. The proportionality factor in these expressions is the denominator of (4), and is preference irrelevant. It is now clear that any optimal plan at τ_1 will be continued at τ_2. If this were not the case it would be possible to find a feasible plan $\vec{\mathbf{c}}_2$ that coincides with the τ_1 optimum $\vec{\mathbf{c}}_1$ on times $[\tau_1, \tau_2]$, but satisfies $W_{\tau_2}(\vec{\mathbf{c}}_2) > W_{\tau_2}(\vec{\mathbf{c}}_1)$. But (45) shows that this contradicts the assumed τ_1 optimality of the plan $\vec{\mathbf{c}}_1$.

Necessity: The welfare function (3) implies that at two evaluation periods τ_1, τ_2 the optimal allocations to agents i and j at a calendar time $t > \tau_1, \tau_2$ must satisfy[14]

$$w_i(\tau_1) U'(c_{it}^1) e^{-\delta_i(t-\tau_1)} = w_j(\tau_1) U'(c_{jt}^1) e^{-\delta_j(t-\tau_1)} \tag{46}$$

$$w_i(\tau_2) U'(c_{it}^2) e^{-\delta_i(t-\tau_2)} = w_j(\tau_2) U'(c_{jt}^2) e^{-\delta_j(t-\tau_2)} \tag{47}$$

If plans are time consistent the solutions to these two equations must coincide: $c_{it}^1 = c_{it}^2, c_{jt}^1 = c_{jt}^2$. So we may divide one of these equations through by the other to find

$$\frac{w_i(\tau_1) e^{\delta_i \tau_1}}{w_j(\tau_1) e^{\delta_j \tau_1}} = \frac{w_i(\tau_2) e^{\delta_i \tau_2}}{w_j(\tau_2) e^{\delta_i \tau_2}}. \tag{48}$$

Since this must be true for any τ_1, τ_2, the ratio $\frac{w_i(\tau) e^{\delta_i \tau}}{w_j(\tau) e^{\delta_j \tau}}$ must be independent of τ for all pairs i, j. This means we can write $w_i(\tau) e^{\delta_i \tau} = Y(\tau) w_i e^{\delta_i \tau_0}$ where the w_i and τ_0 are constants, and factor $Y(\tau)$ does not depend on i. Imposing $\sum_i w_i(\tau) = 1$ for all τ leads to (4).

[14] These conditions follow from the static optimization problem at time t: $\max_{c_{it}} \sum_i w_i(\tau) U(c_{it}) e^{-\delta_i(t-\tau)}$ subject to $\sum_i c_{it} = C_t$, where C_t is some optimal aggregate consumption choice. We do not need to know the value of C_t for the proof.

B Proof of Proposition 1

1. The Hamiltonian of the time consistent planner's problem is:

$$H = \sum_i w_i \ln(c_{it}) e^{-\delta_i(t-\tau_0)} + \lambda_t (rS_t - \sum_i c_{it}) \tag{49}$$

where λ_t is the shadow price of the resource, and S_t evolves according to

$$\dot{S}_t = rS_t - \sum_i c_{it} \tag{50}$$

A standard application of the Maximum principle yields

$$w_i \frac{1}{c_{it}} e^{-\delta_i(t-\tau_0)} = \lambda_t \tag{51}$$

$$\dot{\lambda}_t = -r\lambda_t. \tag{52}$$

Solving the equation for λ_t, we have

$$c_{it} = \frac{w_i}{\lambda_0} e^{-(\delta_i - r)(t-\tau_0)} \tag{53}$$

where λ_0 is the initial shadow price, which we need to solve for. With this solution we can write the evolution equation for the stock in equilibrium as:

$$\dot{S}_t - rS_t = -\sum_i \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)(t-\tau_0)} \right) \tag{54}$$

Multiplying through by an integration factor e^{-rt} and integrating from τ_0 to t:

$$S_t e^{-rt} - S_0 = -\int_{\tau_0}^{t} e^{-rt} \sum_i \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)(t'-\tau_0)} \right) dt' \tag{55}$$

where $S_0 = S(\tau_0)$ is the initial resource stock. The transversality conditions on these solutions require:

$$\lim_{t\to\infty} S_t \lambda_t = \lim_{t\to\infty} S_t \lambda_0 e^{-rt} = 0 \tag{56}$$

Hence, the initial value of the shadow price λ_0 must satisfy:

$$S_0 = \int_{\tau_0}^{\infty} e^{-rt} \sum_i \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)(t-\tau_0)} \right) dt \tag{57}$$

from which we find

$$\lambda_0 = \frac{1}{S_0} \sum_i \frac{w_i}{\delta_i} \tag{58}$$

2. For a proof of this result see Heal and Millner (2013).

C Proof of Proposition 2

1. Let the aggregate consumption policy function be $C_t = \sigma(S_t) = AS_t$. Suppose that planners from $t \in [\tau + \epsilon, \infty)$ follow strategy $\sigma(S)$. The planner at τ's welfare from this is

$$V(C_\tau, \epsilon, A) = \int_{\tau+\epsilon}^{\infty} \ln(AS_t)\beta(t)dt \tag{59}$$

where S_t is the solution of the differential equation

$$\dot{S} = rS - AS;\ S(\tau + \epsilon) = S_\epsilon \tag{60}$$

$$\Rightarrow S_t = S_\epsilon e^{(r-A)t}, \tag{61}$$

and S_ϵ is the stock the current planner bequeaths to his future self at $t = \tau + \epsilon$. Using the state equation, and assuming that ϵ is small, we find

$$S_\epsilon \approx S_\tau(1 + \epsilon(r - C_\tau/S_\tau)). \tag{62}$$

A straightforward calculation then shows that

$$V(C_\tau, \epsilon, A) = \ln(AS_\epsilon)\langle \delta_i^{-1} \rangle_{y_i} + (r - A) \sum_i y_i \left[\frac{e^{-\delta_i \epsilon}(\delta_i \epsilon + 1)}{\delta_i^2} - 1 \right] \tag{63}$$

The planner's total welfare is

$$\begin{aligned}
&\int_{\tau}^{\infty} \ln(C_t)\beta(t)dt \\
&= \int_{\tau}^{\tau+\epsilon} \ln(C_t)\beta(t)dt + \int_{\tau+\epsilon}^{\infty} \ln(C_t)\beta(t)dt \\
&\approx \epsilon \ln(C_\tau) + V(C_\tau, \epsilon, A)
\end{aligned}$$

where the approximation becomes exact as $\epsilon \to 0$. We wish to solve for the optimal C_τ in the limit as $\epsilon \to 0$. We can expand $V(C_\tau, \epsilon, A)$ in powers of ϵ as follows:

$$V(C_\tau, \epsilon, A) = V_0 + \epsilon \left.\frac{\partial V}{\partial \epsilon}\right|_{\epsilon=0} + \mathcal{O}(\epsilon^2)$$

Since the contribution of C_τ to welfare in the current period is first order in ϵ, we care only about the part of $V(C_\tau, \epsilon, A)$ which is also first order in ϵ, and which depends on C_τ. Computing the derivative, evaluating at $\epsilon = 0$, and keeping only the terms that depend on C_τ, we find that

$$\left.\frac{\partial V}{\partial \epsilon}\right|_{\epsilon=0} \sim -\frac{\langle \delta_i^{-1} \rangle_{y_i}}{S_\tau} C_\tau$$

Thus in the limit as $\epsilon \to 0$, C_τ must be chosen such that

$$\begin{aligned}
C_\tau &= \operatorname{argmax} \left[\ln C_\tau - \frac{\langle \delta_i^{-1} \rangle_{y_i}}{S_\tau} C_\tau \right] \\
\Rightarrow C_\tau &= \langle \delta_i^{-1} \rangle_{y_i}^{-1} S_\tau
\end{aligned}$$

In equilibrium, $C_\tau = AS_\tau$, so the equilibrium condition for A is:

$$A = \langle \delta_i^{-1} \rangle_{y_i}^{-1}. \tag{64}$$

2. Straightforward calculations show that the equilibrium aggregate consumption path will be

$$C_t = S_0 A \exp\left[-(A - r)t\right]. \tag{65}$$

Setting $w_i = 1$ for $\delta_i = \delta$, and $w_i = 0$ otherwise, in (18) shows that the optimal consumption path of a single agent with discount rate δ is given by

$$C_t^\delta = S_0 \delta \exp\left(-(\delta - r)t\right). \tag{66}$$

These two consumption paths are equivalent if $\delta = A$.

3. Agent i's welfare in equilibrium in evaluation period τ is given by

$$V_{i\tau} = \int_\tau^\infty \ln(y_i C_t) e^{-\delta_i (t-\tau)} dt.$$

Since the equilibrium consumption path C_t corresponds the optimum of a time consistent and time invariant agent, we know from (65) that at any evaluation period τ we can write the solution as:

$$C_t = S_\tau A \exp\left[-(A - r)(t - \tau)\right]$$

Substituting into the previous expression and performing the integral yields the result.

D Paretian meta-preferences for time invariant social welfare functions

Proposition 9. *Suppose that 'meta-preferences' over social welfare evaluations at different times τ are Paretian. Let $H(\tau - \tau_0)$ be the Pareto weight on social welfare as computed in evaluation period τ, where $H(\tau - \tau_0) \geq 0$ for all $\tau \geq \tau_0$, and*

$$\int_{\tau_0}^\infty H(\tau - \tau_0) d\tau = 1.$$

Then 'meta-optimal' allocations satisfy

$$\begin{aligned} &\max_{c_{it}} \int_{\tau_0}^\infty H(\tau - \tau_0) \left[\int_\tau^\infty \sum_i y_i \ln(c_{it}) e^{-\delta_i (t-\tau)} dt \right] d\tau \\ &\equiv \max_{c_{it}} \sum_i y_i \int_{\tau_0}^\infty \ln(c_{it}) R_i(t - \tau_0) dt \end{aligned} \tag{67}$$

where the functions $R_i(t-\tau_0)$ are defined by

$$R_i(t-\tau_0) = \int_{\tau_0}^{t} H(\tau-\tau_0)e^{-\delta_i(t-\tau)}d\tau. \tag{68}$$

If $H(\tau-\tau_0)$ is non-dictatorial (i.e. non-zero for at least two values of τ), meta-optimal allocations cannot correspond to time consistent utilitarian plans (or property rights regimes) as in Proposition 1. In addition, the representative discount rate corresponding to the meta-optimal plan is always non-constant.

This result shows that adopting any weighting function $H(\tau-\tau_0)$ turns the problem of meta-optimal time invariant allocation into a problem that looks like a standard time consistent allocation problem, but with modified discount factors $R_i(t-\tau_0)$.

To illustrate how the choice of $H(\tau-\tau_0)$ affects $R_i(t-\tau_0)$, consider the following example:

$$H(\tau-\tau_0) = \rho e^{-\rho(\tau-\tau_0)} \Rightarrow R_i(t-\tau_0) = \frac{\rho}{\delta_i-\rho}\left(e^{-\rho(t-\tau_0)} - e^{-\delta_i(t-\tau_0)}\right). \tag{69}$$

With this choice the discount factor on agent i's consumption is a difference between two terms – a discount factor on overall social welfare at time t, and a discount factor on the agents' individual utility at time t. Notice that in this case

$$\begin{aligned}
&\lim_{\rho\to\infty} R_i(t-\tau_0) = e^{-\delta_i(t-\tau_0)} \\
&\lim_{\rho\to 0} R_i(t-\tau_0) \sim \frac{\rho}{\delta_i}(1-e^{-\delta_i(t-\tau_0)})
\end{aligned}$$

The $\rho\to\infty$ limit illustrates convergence to the standard time consistent case when the weighting function places all its weight on $\tau=\tau_0$. On the other hand when ρ is very close to zero, so that the weighting function is almost constant, $R_i(t-\tau_0)$ is an *increasing* function of t, with the weight on $t=\tau_0$ being zero. Assigning equal weight to each time invariant planner thus necessitates an inversion of the standard discounting logic, and gives rise to a *negative* effective discount rate on future utilities. This is intuitive, as only planners at $\tau < t'$ care about outcomes at t'. Thus when planners are weighted equally, the larger is t', the greater the number of planners who care about outcomes at this time, and the greater is the effective discount factor $R_i(t-\tau_0)$.

We do not take a stance on the form of $H(\tau-\tau_0)$ in this paper, as it requires introducing welfare concepts (i.e. a meta-welfare function across planners) that are additional to conventional utilitarian reasoning. Solutions of allocation problems of the form (67) have also been studied by Gollier and Zeckhauser (2005).

E Proof of Proposition 3

The expression (26) tells us the welfare of agent i at time τ in a planning equilibrium:

$$V_{i\tau}^{I} = \frac{1}{\delta_i} \ln \left(S_\tau y_i \langle \delta_i^{-1} \rangle_{y_i}^{-1}\right) + \frac{1}{\delta_i^2} \left(r - \langle \delta_i^{-1} \rangle_{y_i}^{-1}\right) \tag{70}$$

Reviewing the derivation of this expression in Appendix B shows that it may be adapted to derive the welfare agent i obtains for any aggregate consumption path that is an optimum for some agent with discount rate δ_z, and where the consumption share of agent i is constant and equal to z_i:

$$V_{i\tau}(\vec{z}, \delta_z) = \frac{1}{\delta_i} \ln \left(S_\tau z_i \delta_z\right) + \frac{1}{\delta_i^2} \left(r - \delta_z\right) \tag{71}$$

Thus, for any consumption path of this kind, parameterized by $(\vec{z}, \delta_z)$, the welfare a planner with Pareto weights $\vec{y}$ obtains is given by

$$W_\tau(\vec{z}, \delta_z) = \sum_i y_i \left[\frac{1}{\delta_i} \ln \left(S_\tau z_i \delta_z\right) + \frac{1}{\delta_i^2} \left(r - \delta_z\right)\right] \tag{72}$$

From Proposition 2, all planning equilibria give rise to consumption allocations of this form, so if we find the values of $(\vec{z}, \delta_z)$ that maximize the planners welfare, and show that they are consistent with a planning equilibrium, we will be done.

We begin by solving for the optimal values of z_i:

$$\max_{z_i} W_\tau(\vec{z}, \delta_z) \text{ s.t.} \sum_i z_i = 1$$

Solving this constrained optimization problem yields:

$$z_i^* = \frac{y_i \delta_i^{-1}}{\sum_j y_j \delta_j^{-1}} \tag{73}$$

Thus, the optimal choice of z_i is independent of δ_z.

Now we determine the optimal value of δ_z, for arbitrary $\vec{z}$:

$$\begin{aligned}\delta_z^* &= \text{argmax}_{\delta_z} W_\tau(\vec{z}, \delta_z) \\ &= \frac{\langle \delta_i^{-1} \rangle_{y_i}}{\langle \delta_i^{-2} \rangle_{y_i}}\end{aligned}$$

The optimal δ_z is thus also independent of $\vec{z}$.

We now need to show that these optimal parameter values are consistent with a time invariant planning equilibrium. Consider a time invariant planner with Pareto weights z_i^*. From Proposition 2, we know that this planner will allocate a constant share z_i^* of aggregate consumption to agent i. Thus this planner will share out consumption as required to be consistent with (73). We also know from (25) that the equilibrium aggregate consumption path of this planner is equivalent to the optimal path of a time consistent agent with discount rate

$$\begin{aligned}\langle \delta_i^{-1} \rangle_{z_i^*}^{-1} &= \left(\sum_i \delta_i^{-1} \frac{y_i \delta_i^{-1}}{\sum_j y_j \delta_j^{-1}} \right)^{-1} \\ &= \frac{\langle \delta_i^{-1} \rangle_{y_i}}{\langle \delta_i^{-2} \rangle_{y_i}} \\ &= \delta_z^*\end{aligned}$$

Thus, the optimal parameter values $(\vec{z}^*, \delta_z^*)$ correspond to a planning equilibrium, and conversely, this planning equilibrium achieves the highest possible welfare amongst all those consumption paths for which consumption shares are constant, and aggregate consumption is equivalent to the optimal path of some agent. Since by Proposition 2 all planning equilibria have these characteristics, the identified parameter values yield the highest welfare of all possible planning equilibria.

Finally, notice that if social preferences where both time consistent and time invariant, agents would need to share a common discount rate δ', i.e. preferences would take the form:

$$W_{z\tau}(\vec{\mathbf{c}}; \vec{z}, \delta_z) = \sum_i z_i \int_\tau^\infty \ln(c_{it}) e^{-\delta_z (t-\tau)} dt. \tag{74}$$

It is straightforward to see from (18) that optimal consumption allocations with these social preferences will give each agent a constant consumption share z_i, and aggregate consumption will correspond to the optimum of a single agent with discount rate δ_z. Thus, all these allocations fall in the set considered when we solved for the optimal parameter values $(\vec{z}, \delta_z)$, and thus the identified second-best values are optimal for these social preferences too.

It is immediate that this analysis holds for public goods as well, with the minor modification that in that case we no longer need to worry about optimizing agents consumption shares.

F Proof of Proposition 4

From equation (66), we know that if individual i is allocated a share of the resource $a_i S_0$ at time τ_0, and consumes it optimally, his consumption path will be given by:

$$c_{it} = a_i S_0 \delta_i \exp\left(-(\delta_i - r)(t - \tau_0)\right). \tag{75}$$

Comparing this to (18), which describes the consumption allocations that a time consistent planner would choose, we see that the two expressions coincide if

$$a_i \delta_i = \frac{w_i}{\langle \delta_i^{-1} \rangle_{w_i}}. \tag{76}$$

This yields the result.

G Proof of Proposition 5

1. Consider a property rights regime in which a share $a_i S_0$ of the resource is allocated to agent i at time τ_0, where $a_i \geq 0$, $\sum_i a_i = 1$. Using equation (18), the welfare agent i obtains at time τ in this property regime is given by

   $$V_{i\tau} = \int_{\tau}^{\infty} \ln\left(a_i \delta_i S_0 e^{-(\delta_i - r)(t - \tau_0)}\right) e^{-\delta_i (t - \tau)} dt \tag{77}$$

   Writing $t - \tau_0 = (t - \tau) + (\tau - \tau_0)$, and evaluating the resulting integral, yields:

   $$V_{i\tau} = \frac{\ln(a_i \delta_i S_0)}{\delta_i} - \frac{(\tau - \tau_0)(\delta_i - r)}{\delta_i} - \frac{\delta_i - r}{\delta_i^2}. \tag{78}$$

Hence, the welfare the time invariant planner obtains under this property rights regime at time τ is:

$$W_\tau(\text{Property}) = \sum_i y_i \left[\frac{\ln(a_i \delta_i S_0)}{\delta_i} + \frac{r - \delta_i}{\delta_i^2} - \frac{(\tau - \tau_0)(\delta_i - r)}{\delta_i} \right]. \tag{79}$$

To compute the optimal initial property shares a_i we can maximize this expression subject to the constraint $\sum_i a_i = 1$. Computing the solution yields

$$a_i = \frac{y_i \delta_i^{-1}}{\langle \delta_i^{-1} \rangle_{y_i}}. \tag{80}$$

This solution is independent of τ, and therefore these are the optimal initial property shares in all evaluation periods.

2. To compute the value of time invariant welfare given that initial property shares are chosen optimally, simply substitute (80) into (79) to find:

$$W_\tau(\text{Property}^*) = \sum_i y_i \left[\frac{\ln(y_i A S_0)}{\delta_i} + \frac{r - \delta_i}{\delta_i^2} - \frac{(\tau - \tau_0)(\delta_i - r)}{\delta_i} \right]. \tag{81}$$

where $A = \langle \delta_i^{-1} \rangle_{y_i}^{-1}$, and the * reminds us that the initial property shares have been chosen optimally.

From Proposition 2, social welfare in the time invariant planning equilibrium is given by:

$$W_\tau(\text{Plan}) = \sum_i y_i \left[\frac{1}{\delta_i} \ln\left(S_\tau y_i A\right) + \frac{1}{\delta_i^2}\left(r - A\right) \right] \tag{82}$$

In order to compare this value to the welfare level achieved by a property rights regime, we need to know how the stock S_τ evolves in the planning equilibrium. For the time invariant planning equilibrium the group's aggregate consumption rule is $C_\tau = A S_\tau$, and thus we can integrate the state equation for the stock to find

$$S_\tau = S_0 e^{-(A-r)(\tau - \tau_0)}. \tag{83}$$

Thus, we can rewrite social welfare under the time invariant planning equilibrium as:

$$W_\tau(\text{Plan}) = \sum_i y_i \left[\frac{1}{\delta_i} \ln\left(y_i A S_0\right) + \frac{1}{\delta_i^2}\left(r - A\right) - \frac{1}{\delta_i}(\tau - \tau_0)(A - r) \right]. \tag{84}$$

We may now compute the difference in welfare between planning and optimal property rights. The first two terms in equations (84) and (81) are identical, so they cancel out of the welfare difference. Consider the difference between the third terms in these expressions:

$$\begin{aligned}
&-(\tau-\tau_0)\sum_i y_i \frac{1}{\delta_i}(A-r)+(\tau-\tau_0)\sum_i y_i \frac{(\delta_i-r)}{\delta_i} \\
&=(\tau-\tau_0)[-(A-r)A^{-1}+(1-rA^{-1})] \\
&=0.
\end{aligned}$$

Thus, the welfare difference depends only on the difference between the second terms in (84) and (81):

$$\begin{aligned}
W_\tau(\text{Plan})-W_\tau(\text{Property}^*) &= \sum_i y_i \frac{1}{\delta_i^2}(r-A)-\sum_i y_i \frac{1}{\delta_i^2}(r-\delta_i) \\
&= -\langle \delta_i^{-1}\rangle_{y_i}^{-1}\langle \delta_i^{-2}\rangle_{y_i}+\langle \delta_i^{-1}\rangle_{y_i} \\
&= -\langle \delta_i^{-1}\rangle_{y_i}^{-1}[\langle \delta_i^{-2}\rangle_{y_i}-(\langle \delta_i^{-1}\rangle_{y_i})^2] \\
&\propto -\mathrm{Var}_{y_i}(\delta_i^{-1}) \\
&< 0.
\end{aligned}$$

H Proof of Proposition 6

We begin with a useful lemma:

Lemma 3. *Suppose an agent with preference parameter θ receives a payoff of $P(x,\theta)$ from a choice x, where $\frac{\partial^2 P}{\partial x^2}<0$ for all θ. Let $x^*(\theta)$ be the optimal choice of an agent with preferences $P(x,\theta)$, and assume that $x^*(\theta)$ exists and is a smooth function of θ. Define $Q(\theta,\theta')=P(x^*(\theta),\theta')$. If $\frac{\partial^2 P}{\partial x \partial \theta}>0$ then $Q(\theta,\theta')$ is single-peaked in θ.*

Proof.

$$\frac{\partial}{\partial \theta}Q(\theta,\theta')=P_x(x^*(\theta),\theta')\frac{dx(\theta)}{d\theta}. \tag{85}$$

where subscripts denote partial derivatives. Suppose $x(\theta)$ is monotonically increasing in θ. Then $\mathrm{sgn}[P_x(x^*(\theta),\theta')]=\mathrm{sgn}(\theta'-\theta)$, and (85) implies that $Q(\theta,\theta')$ has a global maximum at $\theta=\theta'$, and is a unimodal function of θ. Hence, $Q(\theta,\theta')$ is single peaked in θ. Now $x^*(\theta)$

satisfies

$$P_x(x^*(\theta), \theta) = 0$$

Implicitly differentiating with respect to θ we find

$$P_{xx}\frac{dx^*(\theta)}{d\theta} + P_{x\theta} = 0$$

Since $P_{xx} < 0$, a sufficient condition for $x^*(\theta)$ to be monotonically increasing in θ is for $P_{x\theta} > 0$. $\square$

The proof of the main result is by induction. As in our treatment of the time invariant planners' equilibrium, we consider a finite horizon model in which the game stops at time T, and take the limit as $T \to \infty$. In time period $\tau = T - \Delta\tau$, just before T, an agent with discount rate δ_i has preferences over aggregate consumption C_t given by

$$P_{T-\Delta\tau}(C_{T-\Delta\tau}, \delta_i) = \ln(C_{T-\Delta\tau})\Delta\tau + e^{-\delta_i \Delta\tau}\ln(S_T) \tag{86}$$

where $S_T = (1+r\Delta\tau)S_{T-\Delta\tau} - C_{T-\Delta\tau}\Delta\tau$. Substituting this expression into $P_{T-\Delta\tau}(C_{T-\Delta\tau}, \delta_i)$, and noting that no agent will vote for consumption that makes $S_T \leq 0$ (since $\ln'(0) = \infty$), a simple calculation shows that $\frac{\partial P_{T-\Delta\tau}(C_{T-\Delta\tau},\delta_i)}{\partial\delta_i \partial C_{T-\Delta\tau}} > 0$. Thus preferences at $T - \Delta\tau$ are single peaked, and a vote at $T - \Delta\tau$ results in the median voters' optimal value of current consumption being chosen. Now suppose that the median voters' preferred consumption plan is chosen for all $t \in [\tau + \Delta\tau, T]$, where $\Delta\tau$ is very small. We prove that it will be chosen at τ too. If the median voter's optimal plan is implemented for $t \in [\tau + \Delta\tau, T]$, consumption in this time period will be given by (18) with $w_i = 1$ for $i = m$, and zero otherwise. Hence, by assumption, for this time period consumption is given by

$$C_t = \delta_m S_{\tau+\Delta\tau} e^{-(\delta_m - r)(t-(\tau+\Delta\tau))} \tag{87}$$

where again

$$S_{\tau+\Delta\tau} = (1 + r\Delta\tau)S_\tau - C_\tau \Delta\tau. \tag{88}$$

An agent with discount rate δ_i thus has preferences over current consumption C_τ given by

$$\begin{aligned} P_\tau(C_\tau, \delta_i) &= \ln(C_\tau)\Delta\tau + \int_{\tau+\Delta\tau}^{T} \ln\left[((1+r\Delta\tau)S_\tau - C_\tau\Delta\tau)\delta_m e^{-(\delta_m - r)(t-(\tau+\Delta\tau))}\right] e^{-\delta_i(t-\tau)}dt \\ &\sim \ln(C_\tau)\Delta\tau + \ln((1+r\Delta\tau)S_\tau - C_\tau\Delta\tau) \int_{\tau+\Delta\tau}^{T} e^{-\delta_i(t-\tau)}dt \end{aligned} \tag{89}$$

where in the second line the log has been expanded and terms that do not depend on C_τ or δ_i have been dropped. Using this expression it is now straightforward to show that

$$\frac{\partial P_\tau(C_\tau, \delta_i)}{\partial \delta_i \partial C_\tau} > 0,$$

where once again we use the fact that no voter will want to choose C_τ large enough to make $S_{\tau+\Delta\tau}$ non-positive. By Lemma 3, we have thus proven that preferences over *optimal* values of current consumption, denoted $C^*_\tau(\delta_i)$, are single peaked.

We can now easily extend this result to show that preferences over *arbitrary* values of current consumption C_τ are also single peaked. All we need do is show that $C^*_\tau(\delta_i)$ is a monotonic function of δ_i, and that $C^*_\tau(\delta_i)$ can take any value in the set of feasible values, $[0, \infty)$. We can solve explicitly for $C^*_\tau(\delta_i)$ from (89), taking the limit as $T \to \infty, \Delta\tau \to 0$, to find that

$$C^*_\tau(\delta_i) = \delta_i S_\tau. \tag{90}$$

Clearly $C^*_\tau([0,\infty)) = [0,\infty)$, and $C^*_\tau(\delta_i)$ is increasing in δ_i. Thus, having single peaked preferences over optimal consumption values is equivalent to having single peaked preferences over arbitrary consumption values.

I Proof of Proposition 7

Since the proof of this result is no more difficult with an arbitrary utility function $U(C_t)$, we will do it in general. Let the optimal public consumption plan of an agent with discount rate δ_i be $C(\delta_i) = (C_t^{\delta_i})_{t \geq \tau_0}$. Since individual agents' preferences are both time consistent and time invariant, we can prove the result for e.g. $\tau = 0$, and be sure that it will then hold for all τ. Thus $C(\delta_i)$ is the solution of

$$\max_{C_t} \int_0^\infty U(C_t)\, e^{-\delta_i t} dt \text{ s.t. } \dot{S}_t = rS_t - C_t. \tag{91}$$

We are interested in agents' preferences over the set of optimal plans $\{C(\delta_i)\}$. We begin with a lemma:

Lemma 4. *Suppose that:*

1. *Initial optimal consumption C_0^δ is an increasing function of δ.*

2. *Each pair of consumption paths $\{C(\delta), C(\delta')\}$ has exactly one intersection point, i.e. for any $\delta' > \delta$, there exists a time T such that*

$$\forall t > 0, (T - t)(C_t^{\delta'} - C_t^{\delta}) > 0.$$

Then all agents have single-peaked preferences over optimal consumption paths.

Proof. Let the optimal consumption path for an individual with discount rate δ be $C(\delta) = (c_t^\delta)_{t \geq 0}$. Denote the preferences over aggregate consumption paths of an agent with discount rate δ by $\prec_\delta$.

We first prove that under the conditions of the lemma, given any pair of discount rates $\delta' < \delta''$, for any $\delta < \delta'$ we must have $C(\delta'') \prec_\delta C(\delta')$, and for any $\delta > \delta''$, we must have $C(\delta') \prec_\delta C(\delta'')$. Consider the case $\delta > \delta''$, and let $\delta = \delta'' + \epsilon$, where $\epsilon > 0$. We will evaluate the difference in welfare of agent δ under the two consumption paths $C(\delta')$ and $C(\delta'')$. Let T be the intersection point of the two consumption streams. We have:

$$\begin{aligned}
&\int_0^\infty U(C_t^{\delta''}) e^{-\delta t} dt - \int_0^\infty U(C_t^{\delta'}) e^{-\delta t} dt \\
&= \int_0^T [U(C_t^{\delta''}) - U(C_t^{\delta'})] e^{-\delta t} dt - \int_T^\infty [U(C_t^{\delta'}) - U(C_t^{\delta''})] e^{-\delta t} dt \\
&= \int_0^T [U(C_t^{\delta''}) - U(C_t^{\delta'})] e^{-\delta'' t} e^{-\epsilon t} dt - \int_T^\infty [U(C_t^{\delta'}) - U(C_t^{\delta''})] e^{-\delta'' t} e^{-\epsilon t} dt \\
&\geq e^{-\epsilon T} \int_0^T [U(C_t^{\delta''}) - U(C_t^{\delta'})] e^{-\delta'' t} dt - e^{-\epsilon T} \int_T^\infty [U(C_t^{\delta'}) - U(C_t^{\delta''})] e^{-\delta'' t} dt \\
&= e^{-\epsilon T} \int_0^\infty [U(C_t^{\delta''}) - U(C_t^{\delta'})] e^{-\delta'' t} dt \\
&\geq 0.
\end{aligned}$$

The last inequality follows from the optimality of $C(\delta'')$. Thus we have shown that for any $\delta > \delta''$, $C(\delta') \prec_\delta C(\delta'')$. A similar argument shows that for any $\delta < \delta'$, we must have $C(\delta'') \prec_\delta C(\delta')$.

It is straightforward to see that with these properties in hand, agents' preferences over consumption paths that are optimal for some agent must be single peaked. Consider two paths $C(\delta')$ and $C(\delta'')$, and an agent with discount rate δ, where $\delta < \delta' < \delta''$. From the above properties, we must have $C(\delta'') \prec_\delta C(\delta')$. Similarly, for any paths $C(\delta')$ and $C(\delta'')$ with $\delta'' < \delta' < \delta$, we must have $C(\delta'') \prec_\delta C(\delta')$. Thus all agents' preferences are single peaked. □

We now show that the conditions of this lemma will always be satisfied. Theorem 2 in Becker (1983) shows that optimal initial consumption is an increasing function of δ for any concave production function. As $t \to \infty$, the path $C(\delta)$ tends to zero if $\delta > r$, and $+\infty$ if $\delta < r$. Thus the limiting value of C_t^δ is non-increasing in δ. In addition, all optimal consumption paths are monotonic functions of time (see e.g. Kamien and Schwartz, 1991). All pairs of optimal consumption paths must therefore cross exactly once. The two conditions of Lemma 4 are thus satisfied, and agents have single-peaked preferences over optimal plans.[15]

Since the voting equilibrium is the optimal plan for an agent with discount rate δ_m, and the time invariant planning equilibrium is the optimal plan for an agent with discount rate $\hat{\delta}$, agents have single peaked preferences over these plans. Since agents have single peaked preferences over optimal plans, the classic results of Black (1948) show that the plan corresponding to δ_m is the unique Condorcet winner in the set of optimal plans. Proposition 2 also showed that any time invariant planning equilibrium is observationally equivalent to the optimal plan of a single agent with discount rate $\hat{\delta}$. Thus a majority of agents will always prefer the voting equilibrium to any planning equilibrium.

[15] The proof is no more difficult for an arbitrary concave production function $F(S)$ that admits an interior steady state. In this case the steady state value of consumption on a path $C(\delta)$ is given by $F((F')^{-1}(\delta))$, which by the concavity of F, is again a non-increasing function of δ. The rest of the proof goes through unchanged.

J Proof of Proposition 8

1. By eqs. (32–33), Voting dominates Planning in all evaluation periods iff

$$\begin{aligned}
&\sum_j y_j[\delta_j^{-1}\ln\delta_m - \delta_j^{-2}\delta_m] - \sum_j y_j[\delta_j^{-1}\ln\langle\delta_i^{-1}\rangle_{y_i}^{-1} - \delta_j^{-2}\langle\delta_i^{-1}\rangle_{y_i}^{-1}] > 0 \\
\iff\ & [\langle\delta_i^{-1}\rangle_{y_i}]\ln\frac{\langle\delta_i^{-1}\rangle_{y_i}^{-1}}{\delta_m} < [\langle\delta_i^{-2}\rangle_{y_i}][\langle\delta_i^{-1}\rangle_{y_i}^{-1} - \delta_m] \\
\iff\ & \ln\alpha < [\langle\delta_i^{-2}\rangle_{y_i}\langle\delta_i^{-1}\rangle_{y_i}^{-2}](1-\alpha^{-1}) \\
\iff\ & \ln\alpha < [\delta_m^2\langle\delta_i^{-2}\rangle_{y_i}\alpha^2](1-\alpha^{-1}) \\
\iff\ & \ln\alpha < \phi(\alpha-1).
\end{aligned}$$

2. Since $\ln x < x - 1$ for all $x > 0$, a sufficient condition for (37) to hold is $\phi \leq 1$, i.e.

$$\begin{aligned}
&\delta_m\langle\delta_i^{-2}\rangle_{y_i} < \langle\delta_i^{-1}\rangle_{y_i} \\
\iff\ & \sum_i y_i\delta_i^{-1}\left(\frac{\delta_m}{\delta_i} - 1\right) < 0 \\
\iff\ & \langle\delta_m/\delta_i - 1\rangle_{z_i^*} < 0 \\
\iff\ & \langle\delta_i^{-1}\rangle_{z_i^*} < \delta_m^{-1} \\
\iff\ & \langle q_i\rangle_{z_i^*} < q_m
\end{aligned}$$

where the weights z_i^* are given by (27), and since $f(\delta) = \delta^{-1}$ is a monotonic function, the median of the distribution of q_i is just $q_m = f(\delta_m) = \delta_m^{-1}$. Now,

$$\begin{aligned}
\langle q_i\rangle_{z_i^*} &= N\langle q_i z_i^*\rangle \\
&= N[\langle q_i\rangle\langle z_i^*\rangle + \mathrm{Cov}(q_i, z_i^*)] \\
&= \langle q_i\rangle + N\mathrm{Cov}(q_i, z_i^*)
\end{aligned}$$

where in the last line we've used the fact that $N\langle z_i^*\rangle = \sum_i z_i^* = 1$. Consider the term

$$\mathrm{Cov}(q_i, z_i^*) \propto \mathrm{Cov}(\delta_i^{-1}, y_i\delta_i^{-1}). \tag{92}$$

δ_i^{-1} is strictly decreasing in δ_i, so if $y_i\delta_i^{-1}$ is non-decreasing in δ_i, $\mathrm{Cov}(q_i, z_i^*) < 0$. If this is the case,

$$\langle q_i\rangle < q_m$$

is sufficient to ensure that voting dominates planning.

K A hybrid model of intra- and intergenerational choice.

In this section we sketch out a model of intertemporal choice in which intragenerational social preferences are time consistent, while intergenerational social preferences are time invariant.

Suppose that new generations are born at times $0, \Delta, 2\Delta, \ldots$. Each generation consists of a population of individuals with intragenerational discount rates δ_i and intergenerational discount rates ρ_i. These two distributions of preferences are the same in each generation. Within each generation social preferences are time consistent and Pareto weights are given by (4) with τ_0 chosen to coincide with the birth date of the individuals in that generation. Thus, within a generation, individuals' lifetime utilities are considered relevant for welfare analysis. Intergenerational preferences are however assumed to be time invariant, so that each generation cares only about the welfare of future generations.

We can solve this model in stages. First we solve for optimal consumption decisions within a generation, taking the values of the state variable $S_\tau, S_{\tau+\Delta}$ at the beginning and end of each generation as given. Then we solve the dynamic game between generations – this determines the values of the state variable at the beginning and end of each generation. For simplicity, we assume that consumption is a public good.

The generation beginning at τ, which lives in times $[\tau, \tau+\Delta)$, achieves a welfare level

$$
\begin{aligned}
V_\tau(S_\tau, S_{\tau+\Delta}) &= \max_{C_t} \sum_i w_i \int_\tau^{\tau+\Delta} \ln(C_t) e^{-\delta_i(t-\tau)} dt \ \text{ s.t.} \\
\dot{S}_t &= rS_t - C_t \\
S(\tau) &= S_\tau, \\
S(\tau+\Delta) &= S_{\tau+\Delta}.
\end{aligned}
$$

Straightforward computations with the maximum principle allow us to compute the optimal consumption trajectory on $[\tau, \tau+\Delta]$, and show that

$$
C_t = e^{r(t-(\tau+\Delta))} \frac{e^{r\Delta} S_\tau - S_{\tau+\Delta}}{\sum_j w_j \delta_j^{-1}[1-e^{-\delta_j \Delta}]} \sum_i w_i e^{-\delta_i(t-\tau)}, \ t \in [\tau, \tau+\Delta) \tag{93}
$$

$$
V_\tau(S_\tau, S_{\tau+\Delta}) = K_1 \ln\left(e^{r\Delta} S_\tau - S_{\tau+\Delta}\right) + K_{2\tau} \tag{94}
$$

where $K_1 > 0, K_{2\tau}$ are constants that are independent of S_τ and $S_{\tau+\Delta}$, which we can neglect from now on.

The function $V_\tau(S_\tau, S_{\tau+\Delta})$ is the welfare the generation starting at τ obtains, given that it inherits a resource stock S_τ, and given that it bequeaths a stock $S_{\tau+\Delta}$ to the next generation. Intergenerational preferences at time $\tau \in \{0, \Delta, 2\Delta, \ldots\}$ are thus given by

$$W_\tau = \sum_{n=0}^{\infty} V_{\tau+n\Delta}(S_{\tau+n\Delta}, S_{\tau+(n+1)\Delta}) \left[\sum_i y_i e^{-\rho_i n\Delta} \right]. \tag{95}$$

These preferences are generationally time invariant, i.e. they are unchanged under translations of the time axis of the form $\tau \to \tau + m\Delta$ for any natural number m. They are thus time inconsistent provided that there are at least two agents who disagree on the intergenerational discount rate. To solve for the equilibrium values of the state at the boundaries of the generations we define an 'effective' generational consumption value:

$$\tilde{C}_\tau := S_\tau e^{r\Delta} - S_{\tau+\Delta} \tag{96}$$

The intergenerational dynamics of the state variable are thus described by the difference equation

$$S_{\tau+\Delta} = S_\tau e^{r\Delta} - \tilde{C}_\tau$$

and generation τ's objective function is

$$W_\tau = \sum_{n=0}^{\infty} \ln(\tilde{C}_\tau) \left[\sum_i y_i e^{-\rho_i n\Delta} \right]. \tag{97}$$

The subgame perfect equilibrium of this discrete time dynamic game can be solved for using methods that are analogous to those we used in Proposition 2. We find that in equilibrium

$$\tilde{C}_\tau = \Gamma S_\tau \tag{98}$$

where

$$\Gamma = \frac{e^{r\Delta}}{1 + \sum_i y_i (e^{\rho_i \Delta} - 1)^{-1}}. \tag{99}$$

Hence,

$$S_{n\Delta} = (e^{r\Delta} - \Gamma)^n S_0. \tag{100}$$

From this we find

$$e^{r\Delta}S_{n\Delta} - S_{(n+1)\Delta} = \tilde{C}_{n\Delta} = \Gamma S_{n\Delta} = \Gamma(e^{r\Delta} - \Gamma)^n S_0. \tag{101}$$

Substituting this into (93), we find that in equilibrium consumption will be given by

$$\begin{aligned}
&\text{For } t \in [n\Delta, (n+1)\Delta), \\
&C_t = \frac{\Gamma S_0}{\sum_i w_i \delta_i^{-1}[1 - e^{-\delta_i \Delta}]}(e^{r\Delta} - A)^n e^{-r[(n+1)\Delta - t]} \sum_i w_i e^{-\delta_i(t - n\Delta)}.
\end{aligned}$$

Figure 4 demonstrates the equilibrium consumption path.

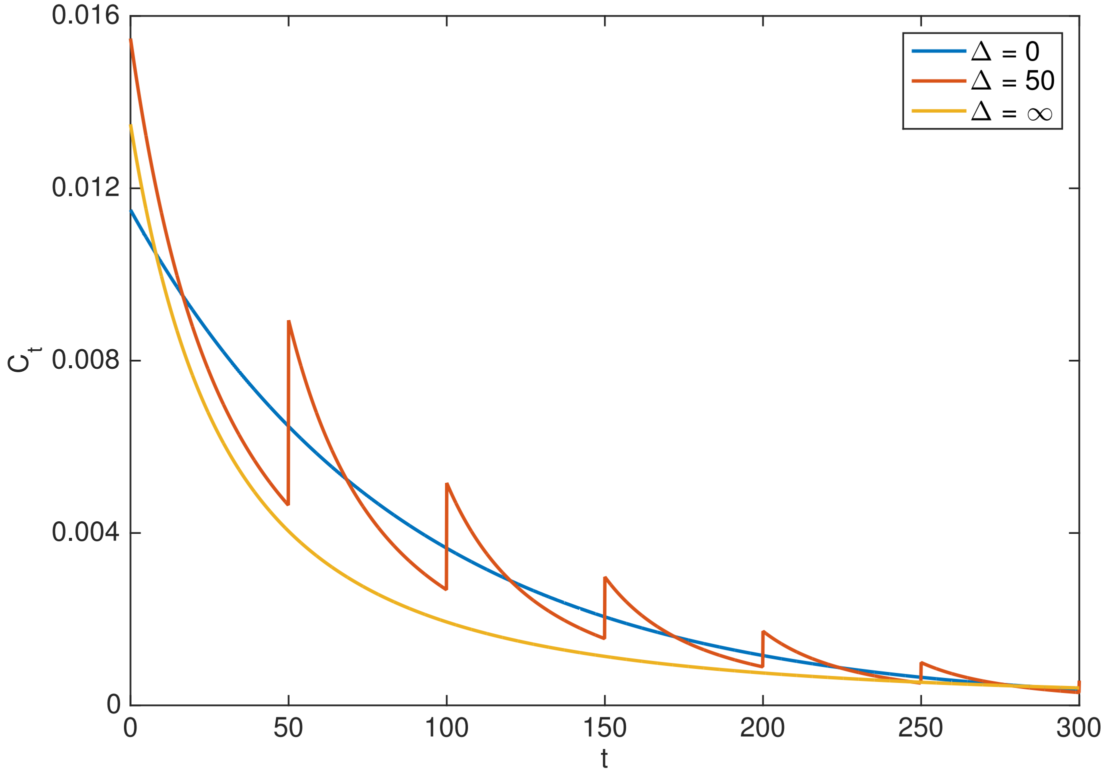

Figure 4: Equilibrium consumption for a hybrid model. Intragenerational social preferences are time consistent, while intergenerational social preferences are time invariant. Both δ_i and ρ_i are distributed according to the distribution in Figure 3, the weights w_i and y_i are chosen in accordance with (40), and $r = 0, S_0 = 1$ in this example. $\Delta \to 0$ corresponds to the case in which social preferences are exclusively time invariant, and $\Delta \to \infty$ to the case in which social preferences are exclusively time consistent, with $\tau_0 = 0$. For intermediate Δ the long run trend of the equilibrium consumption path follows the time invariant consumption path $\Delta = 0$. But within each generation consumption starts high (reflecting the importance of high discount rates early on) and falls rapidly (reflecting the importance of low discount rates towards the end of the generation), thus reflecting the time consistency of the intragenerational welfare function.