Riding the Bubble with Convex Incentives*

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ABSTRACT

We show that the convex incentives of institutional investors like hedge funds can explain their failure to trade against mispricing. The standard mean-variance and hedging components that make up the portfolio often imply opposite stances against mispricing. The first component represents the manager's bets against overpriced securities. By contrast, the manager's hedge against the risk of forfeiting an end-of-period performance fee can result in substantial over-investment in overpriced securities. This "bubble-riding" component is more likely to drive the manager's portfolio as overpricing increases. We show that this rationale holds in equilibrium and can substantially exacerbate mispricing.

Keywords: Money management, convex incentives, incomplete information, mispricing.

JEL Classification: D81, D82, D83, G11, G23.

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1 Introduction

The role of institutional investors in enhancing financial markets efficiency has come under increased scrutiny following recent episodes of prolonged—perceived—mispricing. Indeed, several empirical studies document that mutual funds, and especially hedge funds, were heavily invested in technology stocks during the "tech bubble" of the late 1990s.[1] Such over-investment in overvalued securities, or "bubble-riding", has often been associated with herd behavior resulting from career concerns. Yet, some authors argue that career concerns should be substantially lessened by the use of short-term incentive contracts.[2] Bubble-riding by institutional investors thus seems particularly puzzling in light of the widespread use of pay-for-performance and the high power of the incentives commonly observed in the asset management industry.

In this paper, we show the opposite effect: high-power incentives of money managers can lead to the bubble-riding behavior observed in the literature. Using a dynamic investment model with no career concerns, we show that short-term contracts can induce bubble-riding when they include "convex" incentives, i.e. those that reward good performance more than they penalize poor outcomes. In the model, convex incentives induce large changes in effective risk aversion as a function of the manager's performance relative to the benchmark specified in the fee contract. Excess investment—with respect to the case of no convex incentives—in an overpriced asset arises as the manager either increases the portfolio exposure to the asset ("risk shifting") or mimics the benchmark ("indexing") in response to the changes in effective risk aversion. Whereas risk-shifting and indexing have been previously studied in the literature, we argue that under mispricing both behaviors can precisely imply investing less aggressively against mispriced securities than in the absence of convex incentives. We further argue that this bubble-riding behavior can worsen with the level of mispricing. In turn, we show that this behavior exacerbates the equilibrium mispricing with respect to the case in which there are no convex incentives.

Convex incentives are ubiquitous in the money management industry. Among hedge funds, an explicit convexity arises from their typical fee structure. This structure includes a flat management fee plus a "bonus" performance fee—typically, several times larger than the management fee—over profits in excess of a hurdle performance rate or a high-water mark. In the mutual fund industry, an implicit convexity in manager's incentives results from the asymmetric relation between a fund's performance and its clients' share purchases and redemptions, as documented by Chevalier and Ellison (1997) and Sirri and Tufano (1998)).[3]

We explore the consequences of this type of non-linear incentives in a dynamic equilibrium

[1] See, e.g., Brunnermeier and Nagel (2004), Greenwood and Nagel (2009), and Griffin, Harris, Shu, and Topaloglu (2011).

[2] See, e.g., Scharfstein and Stein (1990) and Dass, Massa, and Patgiri (2008).

[3] A further source of convexity in mutual fund managers' incentives is created by the prevalence of bonus payments in their end-of-year compensation packages (see, e.g., Farnsworth and Taylor (2006), and Ma, Tang, and Gomez (2015)).

setting. Our model features risk averse money managers and retail investors. The representative money manager is subject to convex incentives. This convexity follows from a "bonus" component, contingent on good performance relative to a given benchmark. Whereas the manager has complete information about asset fundamentals, retail investors are "uninformed", in the sense that they do not observe fundamentals but learn about them from the realization of asset dividends over time. The informational asymmetry between the money manager and retail investors in our setup implies that the manager has superior information, an assumption consistent with the empirical findings in Brunnermeier and Nagel (2004) and Hendershott, Livdan, and Schurhoff (2014). In addition, superior information can be interpreted as a short-cut for the higher ability of at least some money managers, as argued in a strand of the literature (e.g. Berk and Green (2004)).

Since the optimal asset allocation in equilibrium does not have an analytic expression, we first study the interaction among the different components of the manager's allocation in a partial equilibrium setting. Rather than adopting arbitrary dynamics for security prices we assume that mispricing arises from the trading of the retail investors, whose aggregate asset holdings represent the market portfolio. In this simplified setting, we solve for the informed manager's trading strategies in closed-form. Depending on the uninformed traders' up-to-date inference of the underlying asset parameters, prices can be higher or lower than "fundamental value," i.e. the corresponding prices in a full-information economy—as observed by the informed manager. Thus, time-varying learning by the uninformed traders leads to time-variation in the level of asset mispricing, potentially resulting in periods of large overpricing (as well as underpricing) of securities. As a particular example of mispricing that has received plenty of attention in the literature, we associate episodes of large overpricing with bubbles.[4]

Under these price dynamics, we first solve for how much an informed direct trader—one who has the same information and risk aversion as the manager but faces no convex incentives—optimally invests in mispriced assets. This case provides us with a standard or, following Basak, Pavlova, and Shapiro (2007), "normal" policy against which we can assess the effects of convex incentives. For the short investment horizon we consider in this paper, we show that (i) the normal portfolio overweights underpriced assets and underweights overpriced assets relative to the market portfolio, (ii) the size of these positions increases with the extent of mispricing and (iii) the normal policy can result in substantial short-sale positions for largely overvalued securities.

Next, we examine the extent to which convex incentives can make the money manager trade more or less aggressively against mispricing than in the absence of these incentives—i.e., under the normal policy. The manager's optimal dynamic trading strategy includes a mean-variance component that summarizes the manager's bets against mispricing, but also a hedging component against the risk of underperforming or, equivalently, of forfeiting the performance-

[4] A detailed characterization of the emergence and dynamics of bubbles is beyond the scope of this paper. See, for example, Brunnermeier and Oehmke (2013) for such a characterization.

linked bonus payment. This hedging component features well-known risk-shifting and indexing behaviors when the manager is under- or outperforming the benchmark, respectively. Our main contribution is to show that, under mispricing, both behaviors (i) can lead the manager to over-invest in overvalued assets relative to the case of no convex incentives, and (ii) can distort the manager's investment policy further as mispricing worsens.

Although convex incentives induce similar distortions on the trading against mispricing across the different types of money managers (e.g., hedge fund and mutual fund managers), these distortions are particularly pronounced in the case of hedge fund managers. The high degree of convexity in hedge fund incentive fees leads managers to overweight the overpriced stock *more than the uninformed investors* with high probability. This outcome is due to the risk-shifting component, and results from the combination of three factors. First, a hedge fund "underperforms" until it meets the hurdle that triggers the performance fee. But meeting this hurdle can take an extended period of time, and the stock overvaluation can persist or even worsen during this time. The small to moderate underperformance that prevails during these time "activates" the risk-shifting component in the manager's portfolio. Second, the high power of the incentive fees drastically reduces the risk aversion of the manager and magnifies the absolute value of the risk-shifting component. Third, the benchmark associated with the incentive fees in the hedge fund industry drives risk-shifting towards *overweighting*, rather than underweighting, the overpriced asset in the manager's portfolio. This last result is not trivial, as this factor needs not work in the direction that exacerbates overpricing for all types of money managers *even under convex incentives*. In particular, the manager of a mutual fund shifts risk by underweighting instead the same overpriced stock, even under a positive excess expected return.

As the price of the overvalued stock keeps rising, the long position in the stock pays off and the prospects to secure the performance fees improve. To lock-in the interim outperformance that warrants the performance fee payment, the optimal policy becomes more conservative and tilts the portfolio towards the indexing component. This results in substitution of the risk-free security for the overpriced stock as overpricing worsens. When the overpricing is so severe that the stock expected excess return turns negative, the normal policy sells the overpriced stock short, whereas the indexing component limits the extent of short selling below the normal policy. The resulting difference drives the manager's over-investment in the overpriced stock with negative risk premium. This endogenous constraint on short selling limits the manager's bets against overvalued assets even in the absence of explicit portfolio constraints. The resulting conservative behavior contrasts with the common view of hedge funds as absolute-return investment vehicles.

Next, we consider the equilibrium setting in which both hedge funds and retail investors have positive weights in the market and their investment decisions have price impact. We show that the existence of convex incentives for the fund manager exacerbates the mispricing of the asset with respect to the normal case—in the terminology of Basak, Pavlova, and Shapiro (2007)—without convex incentives. Moreover, we show that with high probability hedge funds can exacerbate mispricing even beyond the levels prevailing in an economy populated by uninformed investors only. Finally, we find a limited role for sophisticated investors in stabilizing financial markets in situations of large overpricing. These findings lead us to conclude that, when managers face convex incentives, their optimal investment strategy can often be inconsistent with the common wisdom that bets against mispricing should increase as mispricing worsens, or that they would help prices correct back to "fundamental values."

An interesting aspect of our analysis is that we can justify some "puzzles" regarding the trading strategy of presumably sophisticated investors without recurring to behavioral arguments, and only using incentives documented in the literature—although not standard in financial models. In particular, we argue that informed hedge funds may find optimal to invest in overpriced stocks in a proportion even higher than the market portfolio, as documented by Brunnermeier and Nagel (2004). This result is also consistent with evidence from the experimental work in Holmen, Kirchler, and Kleinlercher (2014), who find that trading at inflated prices is rational for subjects with convex incentives. Also, we provide an incentive-based—as opposed to financial constraint-based—explanation for the low short interest during overpricing periods as documented by, e.g., Stein and Lamont (2004).

From a methodological perspective, our paper is closest to the literature on money managers' risk taking in response to incentives. In particular, we build on Basak, Pavlova, and Shapiro (2007) and extend their analysis to a setup in which risky assets can be potentially mispriced due to an information wedge between managers and other investors in the economy. This allows us to interpret the risk-shifting and indexing effects in Basak, Pavlova, and Shapiro (2007) in terms of trading either against or in the direction of mispricing under different levels of over- and undervaluation. Our equilibrium model with hedge funds is based on the setting of Cuoco and Kaniel (2011). In a career concerns model with risk-neutral agents, Makarov and Plantin (2014) show that managers chasing investors' flows can invest in securities with negative expected returns and tail risk. In general equilibrium models with symmetric information about asset fundamentals, Vayanos and Woolley (2013) and Buffa, Vayanos, and Woolley (2014) find that money managers subject to time-varying investors' flows, or perceiving fees that depend linearly on relative performance, can push prices away from fundamental value. Malamud and Petrov (2014) further study the effects of convex incentives on price informativeness and volatility in a general equilibrium model with asymmetrically informed and risk-neutral managers.

Our paper also contributes to the theoretical literature on rational explanations of limits to arbitrage. Allen and Gorton (1993) argue that unskilled fund managers with limited liability buy overvalued assets in order to appear skilled. Our analysis shows that also skilled managers may choose to buy overvalued assets. Shleifer and Vishny (1997) show that managers trade less aggressively than expected in presence of an arbitrage opportunity when they face the risk of investors' capital withdrawals. Liu and Longstaff (2004) prove that capital-constrained risk-averse arbitrageurs can trade conservatively in the presence of arbitrage opportunities and even lose money in the process. Stein (2009) suggests that sophisticated investors can buy an overvalued asset due to an unawareness of the aggregate capital involved in eliminating the mispricing.

Sato (2009) shows that the synchronization problem identified by Abreu and Brunnermeier (2003) can exacerbate the persistence of bubbles when traders are portfolio managers subject to relative performance concerns. Investors' strategies in our model are not limited by financial constraints, and our simple asset pricing setup leaves out synchronization and "crowded-trade" risks. Therefore, our explanation complements the existing rationalizations of bubble-riding behavior using only the type of compensation arrangements for money managers commonly observed in practice.

The paper is structured as follows. In Section 2 we describe the economic setting. We derive the optimal investment strategies of the informed money manager in partial equilibrium in Section 3. In Section 4 we examine the price impact of these strategies in general equilibrium. We close the paper with some conclusions in Section 5.

2 Economic Setting

We are interested in the effects of convex compensation on the trading of informed—i.e., sophisticated—institutional investors against security mispricing and, in turn, in how these decisions affect security mispricing.[5] We concentrate our analysis on the behavior of money managers such as hedge funds for which explicit or implicit option-like compensation structures have been extensively reported in the literature.[6] We introduce the financial markets, the agents, the information structure and the agents' problems in our economy in the next subsections.

2.1 Financial Markets

We consider a pure exchange economy over the finite period $t \in [0, T]$. Financial markets consist of one risk-less asset with price β and one risky asset (henceforth, a "stock") with price S. The risk-less asset pays a constant interest rate r per unit of time, whereas the stock represents a claim to the dividend D_T at $t = T . D_T$ is the terminal value of a dividend process with initial value d_0 and dynamics given by:

$$dD_t = D_t(\rho dt + \delta dB_t). \tag{1}$$

The dividend mean growth rate ρ and volatility δ are positive constants, and B is a standard Brownian motion process under the probability measure P that explains the dynamics of this economy. Everyone observes δ; however, as we describe later, only a money manager with superior information observes ρ. The constant ρ is the unobserved realization at $t = 0$ of a random variable with normal distribution $\mathbf{N}(\rho_0, v_0)$, for given constant prior ρ_0 and variance $v_0 \geq 0$.

[5] See Brunnermeier and Nagel (2004) and Hendershott, Livdan, and Schurhoff (2014) for evidence of superior information on the part of institutional investors.

[6] See references in the introduction.

We assume that the risk-less asset is in zero net supply, while the stock is in unit supply. The stock price satisfies the following dynamics:

$$dS_t = S_t(\mu_t dt + \sigma_t dB_t), \tag{2}$$

with mean rate of return μ_t and volatility $\sigma_t > 0$ to be determined in equilibrium.

2.2 Investors

There are two types of investors in the financial markets: money managers and retail investors. Investors of each type can be seen as belonging to a large pool (a continuum) of identical investors with no individual price impact. Following standard aggregation results and our competitive assumption, we refer indistinctly to the investors and the representative investors within each type. All investors have constant relative risk aversion (CRRA) preferences with the same coefficient $\gamma > 1$ and maximize expected utility from final wealth only.[7] As in Basak and Pavlova (2013), the representative money manager and retail investor in our economy are initially endowed with fractions $\theta \in [0, 1]$ and $1 - \theta$ of the stock.

What differentiates money managers from retail investors in our model is (i) their access to information about fundamentals, and (ii) their incentives.

2.2.1 Money Managers

Money managers have superior information over other investors in the economy because they observe the realization of the dividend drift ρ at $t = 0$. The representative money manager has no initial wealth but receives an end-of-period compensation W_T^M, which is the product of a fee rate f and assets under management (AUM) W at T. In principle, the amount of AUM and the managers' compensation could be determined by delegating investors, but we leave this decision outside of the model.[8] The manager dynamically chooses the time-t fraction ϕ_t of W_t ($t \in [0, T]$) that is allocated to the stock. Given $W_0 = w_0$, the value of the portfolio (AUM) follows:

$$dW_t = W_t\left(r + \phi_t(\mu_t - r)\right) dt + W_t \phi_t \sigma_t dB_t. \tag{3}$$

The compensation fee f_T is a function of the manager's performance relative to a benchmark index Y (henceforth just "benchmark"). For an arbitrary initial value Y_0, this benchmark represents a long-only fixed-weight portfolio investing a fraction $\phi^Y \in [0, 1]$ of its value in the stock and the remaining fraction in the risk-free asset:

$$dY_t = Y_t\left(r + \phi^Y(\mu_t - r)\right) dt + Y_t \phi^Y \sigma_t dB_t. \tag{4}$$

[7] We also discuss the case $\gamma = 1$ (log preferences) in Section 3.

[8] This is in the spirit of Basak and Pavlova (2013). For an equilibrium model endogenizing the contract between institutions and their investors, or the amount of AUM, see Buffa, Vayanos, and Woolley (2014) and Vayanos and Woolley (2013).

The fee rate f_T is specified as the following 2-piece function of performance relative to the benchmark:

$$f_T = k\left(\frac{R_T}{\bar{\zeta} R_T^Y}\right)^{\alpha_1} \mathbb{1}_{\{R_T < \bar{\zeta} R_T^Y\}} + k\left(\frac{R_T}{\bar{\zeta} R_T^Y}\right)^{\alpha_2} \mathbb{1}_{\{R_T \geq \bar{\zeta} R_T^Y\}}, \tag{5}$$

where $k, \bar{\zeta} > 0$, $0 \leq \alpha_1 \leq \alpha_2$, $R_T \equiv W_T/W_0$, and $R_T^Y \equiv Y_T/Y_0$. This specification is a generalization of the function used to represent mutual fund investors' flow-performance relationship by Basak and Makarov (2014), and is similar to the incentive function in Kaniel and Kondor (2013).

The fee rate (5) is positive and (weakly) increasing in the manager's relative performance R_T/R_T^Y. It implies an asymmetric relation between relative performance and perceived fees whenever the slope α_1 in the *underperformance* region, $\{R_T < \bar{\zeta} R_T^Y\}$, is smaller than the slope α_2 in the *outperformance* region, $\{R_T \geq \bar{\zeta} R_T^Y\}$. In particular, $\alpha_1 < \alpha_2$ implies that the fees perceived by the manager increase with performance at a faster rate when relative performance R_T/R_T^Y exceeds a threshold $\bar{\zeta}$. This asymmetric relation implies an option-like, or *convex* compensation scheme, according to which the manager receives a "bonus" payment for relatively good performance. This bonus is given by the difference in fee rates between the outperformance and underperformance regions in (5).

The specification (5) is general enough to capture the incentives of different types of money managers, most notably hedge fund and mutual fund managers. The case of mutual fund managers, for whom the benchmark (4) is a broad stock market index, has been examined extensively in the literature (see, e.g., Basak and Pavlova (2013), Kaniel and Kondor (2013), Vayanos and Woolley (2013), Buffa, Vayanos, and Woolley (2014)). Therefore, we focus on the case of hedge fund managers following so-called "absolute return" strategies, whose incentive fees depend on their fund performance against a money market rate (e.g., 3-month LIBOR) plus a spread. However, to the possible extent we keep the analysis general enough to include the cases of mutual funds and other money managers with performance concerns relative to the stock index or other portfolios of the stock index and the risk-free asset.

2.2.2 Retail Investors

Retail investors are *uninformed*, in the sense that they do not observe the realized value of the dividend growth rate ρ at $t = 0$. We refer to these as U-investors.[9] At each time $t \in [0, T]$, U-investors allocate a fraction ϕ_t^U of their wealth W_t^U to the stock and the remaining fraction to the risk-less asset to maximize utility from final wealth at time T. Given initial wealth $w_0 = S_0$ their wealth process evolves according to:

$$dW_t^U = W_t^U \left(r + \phi_t^U (\mu_t - r)\right) dt + W_t^U \phi_t^U \sigma_t dB_t. \tag{6}$$

[9] Assuming that all retail investors are uninformed makes the model tractable but is not key for our results. Qualitatively, we just need that prices do not reflect all the information available to the money manager.

Since they do not observe ρ at $t = 0$, U-investors have to infer it from the observation of dividends during the investment period. We explain the dynamics of the posterior of the expected dividend growth rate in subsection 2.3.

2.3 Information Structure

Money managers have an information advantage over U-investors, in the sense that managers observe the realization of the dividend growth rate ρ at $t = 0$ whereas U-investors have to infer its value over time. We assume that U-investors are dogmatic about their beliefs and disregard managers' information. This may occur because U-investors fail to recognize that managers *do* observe ρ and believe instead that they only have a (potentially) different prior, but we abstract from the specific reasons behind this behavior and take the difference in information as exogenously given. This assumption implies that retail investors learn from the observation of the dividend process D_t over the period $t \in [0, T]$ but ignore the information about ρ contained in the prices, via the price impact of money managers. While irrational given that managers have superior information, this limited learning by retail investors is enough to create a rich enough mispricing dynamics in our model. Moreover, it allows us to avoid the intractability of embedding our setup in a noisy rational expectation equilibrium (REE) framework featuring irrational noise traders alongside managers and U-investors.[10] As such, the information structure in our model is a variation of Wang (1993) and Kogan, Ross, Wang, and Westerfield (2006).

Because U-investors face incomplete information while managers face complete information about economic fundamentals, each type of investor forms expectations under a different probability measure. U-investors start the investment period with a prior distribution for ρ and update it over time according to Bayes rule, based on the arrival of information D_t. We denote by $\tilde{P}$ the probability measure that describes the dynamics of the dividend process according to U-investors' priors. Letting $\tilde{E}$ denote the expectation under $\tilde{P}$, the distribution of ρ conditional on D_t is Gaussian, with mean $\tilde{\rho}_t \equiv \tilde{E}\left[\rho|D_t\right]$ and variance $v_t \equiv \tilde{E}\left[(\rho - \tilde{\rho}_t)^2|D_t\right]$ satisfying:[11]

$$\begin{cases} d\tilde{\rho}_t = \frac{v_t}{\delta}\frac{1}{\delta}\left(\frac{dD_t}{D_t} - \tilde{\rho}_t dt\right) \equiv \frac{v_t}{\delta} d\tilde{B}_t, \\ dv_t = -\frac{v_t^2}{\delta^2} dt, \end{cases} \tag{7}$$

for the initial values $\tilde{\rho}_0 = \rho_0$ and v_0. $\tilde{B}_t$ is a standard Brownian motion with respect to the filtration $\mathcal{F}_t^D$ generated by the dividend process D under $\tilde{P}$, with dynamics $d\tilde{B}_t \equiv 1/\delta\,(dD_t/D_t - \tilde{\rho}_t dt) = dB_t + 1/\delta(\rho - \tilde{\rho}_t)dt$. We let $\tilde{\mu}_t \equiv \tilde{E}\left[\mu_t|D_t\right]$ and $\tilde{\eta}_t \equiv (\tilde{\mu}_t - r)/\sigma_t$ denote the time-t inferred stock mean rate of return and market price of risk under $\tilde{P}$. From U-investors' perspective, markets are complete with respect to the observable states of the economy (a single risky asset S driven by

[10] Our results in Sections 3 and 4 draw on wealth effects stemming from the CRRA preferences of the investors in our model. These wealth effects and the high non-linearity of managers' optimal policies as a function of the state variables would make the filtering problem of inferring ρ from prices intractable for the U-investors in our model. This, in turn, would render a characterization of the equilibrium unfeasible.

[11] See, e.g., Liptser and Shirayayev (2001).

a single Brownian motion $\tilde{B}$). Absent arbitrage opportunities, U-investors see financial markets as driven by a unique state-price deflator (SPD) $\tilde{\pi}$ with dynamics $d\tilde{\pi}_t = -r\tilde{\pi}_t dt - \tilde{\pi}_t \tilde{\eta}_t d\tilde{B}_t$.

The representative manager observes the true value of the dividend growth rate ρ, and sees the stock price dynamics under the true probability P as given by:

$$\begin{aligned} dS_t =& S_t\Big(\tilde{\mu}_t dt + \sigma_t(\frac{\rho - \tilde{\rho}_t}{\delta}dt + dB_t)\Big) \\ =& S_t\Big((\tilde{\mu}_t + \sigma_t \frac{\rho - \tilde{\rho}_t}{\delta})dt + \sigma_t dB_t\Big), \end{aligned} \tag{8}$$

with the following dynamics for U-investors' estimated growth rate $\tilde{\rho}_t$

$$d\tilde{\rho}_t = \frac{v_t}{\delta^2}(\rho - \tilde{\rho}_t)dt + \frac{v_t}{\delta}dB_t. \tag{9}$$

Similarly, the market price of risk under P consists of the sum of the market price of risk under $\tilde{P}$ and a term proportional to the uninformed investors' estimation error $\rho - \tilde{\rho}_t$: $\eta_t = \tilde{\eta}_t + 1/\delta(\rho - \tilde{\rho}_t)$.

Markets are complete for the money manager, who sees financial markets as driven by a unique SPD π with dynamics $d\pi_t = -r\pi_t dt - \pi_t \eta_t dB_t$, i.e.:

$$\begin{aligned} \pi_t =& \exp\left\{-rt - \frac{1}{2}\int_0^t \eta_s^2 ds - \int_0^t \eta_s dB_s\right\} \\ =& \tilde{\pi}_t \xi_t, \end{aligned} \tag{10}$$

where

$$\xi_t \equiv \exp\left\{-\frac{1}{2}\int_0^t \left(\frac{\rho - \tilde{\rho}_s}{\delta}\right)^2 ds - \int_0^t \frac{\rho - \tilde{\rho}_s}{\delta} dB_s\right\} \tag{11}$$

is the likelihood process (a P-martingale) for the measure transformation from P to $\tilde{P}$: $\xi_t = d\tilde{P}/dP$ on $\mathcal{F}_t$. Therefore, the extent to which the manager's SPD, π_t, differs from the SPD of retail investors, $\tilde{\pi}_t$, depends on the size and sign of the estimation error $\rho - \tilde{\rho}_t$.

2.4 Investors' Problems

Managers allocate a fraction $\hat{\phi}_t$ of AUM W_t to the stock market to maximize expected utility over their terminal wealth:

$$E_0\left[\frac{(W_T^M)^{1-\gamma}}{1-\gamma}\right], \tag{12}$$

subject to initial AUM $w_0 = \theta S_0$ and the self-financing constraint (3).

Similarly, U-investors allocate a fraction $\hat{\phi}_t^U$ of their wealth W_t^U to the stock market to maximize expected utility over terminal wealth:

$$\tilde{E}_0\left[\frac{(W_T^U)^{1-\gamma}}{1-\gamma}\right], \tag{13}$$

subject to initial wealth $w_0^U = (1-\theta)S_0$ and the self-financing constraint (6), re-expressed in terms of observable variables as:

$$dW_t^U = W_t^U \left(r + \phi_t^U (\tilde{\mu}_t - r)\right) dt + W_t^U \phi_t^U \sigma_t d\tilde{B}_t. \tag{14}$$

A solution to our model consists of a set of investment policies and asset prices such that: (i) the individual investment policies of the managers and the U-investors are optimal, and (ii) bond and stock markets clear. We first examine the managers' trading against mispricing in partial equilibrium in the next Section. We analyze the price impact of these trading policies to Section 4. All proofs are given in Appendix A.

3 Optimal Investment Strategy

In this section we derive the optimal investment policy of the investors in our economy for the particular case of $\theta = 0$, i.e. prices are determined exclusively by the retail investors. Although unrealistic, this case allows us to characterize managers' trading against mispricing in closed form.[12] Our main goal is to build intuition for the more realistic case in which informed managers can affect equilibrium prices in Section 4, for which analytic expressions cannot be derived. We solve for the dynamics of prices in a first stage, and for the optimal portfolio of the money manager under these prices in a second stage.

3.1 Price Dynamics

When $\theta = 0$, market clearing along with the requirement of no-arbitrage implies:[13]

$$W_T^U = S_T = D_T. \tag{15}$$

The optimal investment strategy of retail investors in this case determines the following price dynamics:

Proposition 1. *Let $\tau = T - t > 0$. Equilibrium stock prices and the SPD of the uninformed investors are given by:*

$$S_t = D_t \exp\left\{ \left(\tilde{\rho}_t - r - \gamma\delta^2 - \left(\gamma - \frac{1}{2}\right) \upsilon_t \tau \right) \tau \right\}, \tag{16}$$

$$\tilde{\pi}_t = \lambda^{-1} D_t^{-\gamma} \exp\left\{ \left(r - \gamma\tilde{\rho}_t + \frac{1+\gamma}{2}\gamma\delta^2 + \frac{\gamma^2}{2}\upsilon_t \tau \right) \tau \right\}, \tag{17}$$

where $\lambda > 0$ is the Lagrange multiplier of the equivalent static problem and its solution is given in Appendix A. Under the probability $\tilde{P}$, equilibrium stock mean return, volatility and market

[12] Our approach in this Section is in the spirit of DeLong, Shleifer, Summers, and Waldman (1990), who analyze the survival of irrational traders in a model in which noise traders do not affect prices. By contrast, we focus on the trading of the informed manager in partial equilibrium when less informed traders determine prices.

[13] The condition of absence of arbitrage in the stock market requires that the stock price equals the liquidation dividend at the terminal date: $S_T = D_T$.

price of risk are time-varying and deterministic, as given by:

$$\begin{aligned}\tilde{\mu}_t &= r + \gamma\sigma_t^2,\\ \sigma_t &= \delta + \frac{v_t}{\delta}\tau,\\ \tilde{\eta}_t &= \gamma\sigma_t.\end{aligned} \tag{18}$$

The stock price dynamics in equations (16) are affected by investors' estimate of the dividend growth rate $\tilde{\rho}_t$ and by their uncertainty v_t. U-investors' incomplete information can then introduce a wedge between the stock market price in this economy and its "fundamental value," understood as the stock price S^{CI} that would prevail if all traders in the economy had complete and symmetric information about the dividend growth rate ρ (i.e., $\rho_0 = \rho$ and $v_0 = 0$).

Arguably, any situation in which $S \neq S^{CI}$ would be perceived as stock *mispricing* by fully informed investors such as the money manager in our setup.[14] Hence, we measure the extent of stock overvaluation as of time $t < T$ by the quantity $OV_t \equiv \left(S_t/S_t^{CI}\right)^{1/\tau} - 1$, and the extent of mispricing by the quantity $MP_t \equiv |OV_t|$.[15] We say that stock mispricing reflects *overvaluation* or *overpricing* (respectively, *undervaluation* or *underpricing*) whenever $OV_t > 0$ ($OV_t < 0$). Since by no-arbitrage $S_T = D_T = S_T^{CI}$, stock mispricing equals zero at the terminal date T. The following Lemma characterizes the stock fundamental value and the extent of overvaluation as perceived by the informed manager at any interim period:

Corollary 1. *Under complete information for the retail investors in the economy, stock prices are:*

$$S_t^{CI} = D_t \exp\left\{\left(\rho - r - \gamma\delta^2\right)\tau\right\}. \tag{19}$$

The stock mean return, volatility and market price of risk are:

$$\begin{aligned}\mu^{CI} &= r + \gamma\left(\sigma^{CI}\right)^2,\\ \sigma^{CI} &= \delta,\\ \eta^{CI} &= \gamma\sigma^{CI}.\end{aligned} \tag{20}$$

The time-t stock overvaluation OV_t, as perceived by a fully-informed agent, is:

$$OV_t = \exp\left\{\left(\tilde{\rho}_t - \rho - \left(\gamma - \frac{1}{2}\right) v_t\tau\right)\right\} - 1. \tag{21}$$

As expected, an over-estimation of the mean dividend growth rate by U-investors, $\tilde{\rho}_t > \rho$, will typically lead to stock overvaluation.[16] Moreover, the extent of overvaluation OV_t is increasing

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[14] Our definition of over- and undervaluation in this paper is in the spirit of the equivalent definitions in Grossman and Stiglitz (1980).

[15] As can be seen from the proof of Corollary 1 below, the ratio S_t/S_t^{CI} depends on the time to maturity τ. The definitions of OV_t and MP_t then make the extent of mispricing at different dates comparable.

[16] The stock will be overvalued as long as U-investors over-estimate the dividend growth rate by a large

in the over-estimation margin $\tilde{\rho}_t - \rho$, consistent with the intuition that better perceived dividend growth prospects leads U-investors to push prices further up.

We note that there is no uncertainty about the date at which prices converge to the fundamental value, since by construction $S_T = S_T^{CI}$ with probability one, even though the path to convergence is random. This rules out bubble-riding behavior due to "synchronization" risk as originally studied by Abreu and Brunnermeier (2003) and generalized to a delegated portfolio management context by Sato (2009).

We further note that, due to the assumption in this Section of no price impact by money managers, U-investors are fully invested in the stock at all times, i.e. $\phi^U \equiv \hat{\phi}_t^U = 1$ for all $t \in [0, T]$. In this particular setting, the stock price S (equivalently, U-investors' wealth W^U) can be interpreted as the value of the *market portfolio*. We draw on this interpretation when we assess the trading policies of an informed investor in the next subsection.

3.2 Optimal Investment Strategy of the Money Manager

According to equation (21), the manager observes that time-varying learning by U-investors induces time-variation in the level of asset mispricing, potentially resulting in sustained periods of stock overpricing or underpricing. An important particular case of stock overpricing is the price appreciation observed during financial bubbles. Although fully characterizing the emergence and dynamics of a bubble is beyond the scope of this paper, a main goal of this Section is to analyze the behavior of money managers under the asset overvaluation typical of bubbles. Whenever possible, we relate our results to the observed behavior of money managers as documented by prior empirical literature.

3.2.1 Benchmark Case: Investment Policy without Convex Incentives

In order to single out the effects of convex incentives on the manager's trading strategy, we first examine the dynamic policy of a hypothetical retail investor with superior information—but without this type of incentives. We follow Basak, Pavlova, and Shapiro (2007) in referring to this standard (default) policy as the *normal* (N) policy.

For an arbitrary coefficient of relative risk aversion $\tilde{\gamma} > 1$, we define $\phi_{\tilde{\gamma},t}^N$ as the time-t ($t \in [0, T]$) normal trading in the stock of an investor with RRA coefficient $\tilde{\gamma}$. Proposition 2 characterizes $\phi_{\tilde{\gamma},\cdot}^N$ along with the associated portfolio value process $W_{\tilde{\gamma},\cdot}^N$:

Proposition 2. *For $t \in [0, T]$, the normal trading strategy $\phi_{\tilde{\gamma},t}^N$ and associated portfolio value process $W_{\tilde{\gamma},t}^N$ are:*

$$\phi_{\tilde{\gamma},t}^N \equiv \frac{\delta^2 + v_t \tau}{\delta^2 + \frac{v_t}{\tilde{\gamma}} \tau} \frac{\eta_t}{\tilde{\gamma} \sigma_t}, \tag{22}$$

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enough margin: $\tilde{\rho}_t - \rho > (\gamma - .5) v_t \tau > 0$. This implies that a low enough over-estimation of fundamentals, $(\gamma - .5) v_t \tau > \tilde{\rho}_t - \rho > 0$, is still consistent with an undervalued stock in our economy with sufficiently risk-averse U-traders.

$$\hat{W}^N_{\tilde{\gamma},t} = (\lambda_N \pi_t)^{-\frac{1}{\tilde{\gamma}}} Z_{1-\frac{1}{\tilde{\gamma}},t,T}, \tag{23}$$

where for any $\psi \in (0,1)$ *and* $0 \le t \le t' \le T$

$$Z_{\psi,t,t'} \equiv \delta^{\psi} \sqrt{\frac{(\delta^2 + v_t(t'-t))^{1-\psi}}{\delta^2 + (1-\psi)v_t(t'-t)}} \exp\left\{-\psi r(t'-t) - \frac{\psi(1-\psi)\delta^2(t'-t)}{\delta^2 + (1-\psi)v_t(t'-t)} \frac{\eta_t^2}{2}\right\}, \tag{24}$$

and $\lambda_N = \left(\frac{Z_{1-\frac{1}{\tilde{\gamma}},0,T}}{w_0}\right)^{\tilde{\gamma}}$.

Comparing the portfolio weight in the stock of the U-investors (the market portfolio), ϕ^U, to the normal policy of an equally risk-averse ($\tilde{\gamma} = \gamma$) investor, $\phi^N_{\gamma,t}$, we obtain the following:

Corollary 2. *For* $t \in [0,T]$*, the normal excess holding of the stock relative to the market is:*

$$\phi^N_{\gamma,t} - \phi^U = -\frac{1}{\gamma} \frac{\tilde{\rho}_t - \rho - (\gamma-1)v_t\tau}{\delta^2 + \frac{v_t}{\gamma}\tau}. \tag{25}$$

Thus, the normal portfolio implies a lower stock holding than the market, $\phi^N_{\gamma,t} < \phi^U$*, iff:*

$$\begin{aligned} \tilde{\rho}_t >& \rho + (\gamma-1)v_t\tau \\ \Leftrightarrow OV_t >& \exp\left\{-\frac{1}{2}v_t\tau\right\} - 1. \end{aligned} \tag{26}$$

The normal portfolio implies a higher stock holding than the market if the converse of (26) *holds; both holdings are the same when* (26) *holds as an equality.*

Table 1 summarizes the relationship between over-estimation of fundamentals, stock overpricing and the normal policy. Except for a typically low-probability range of underpricing, $OV_t \in \left(\exp\left\{-\frac{1}{2}v_t\tau\right\} - 1, 0\right)$ (corresponding to an estimation error $(\tilde{\rho}_t - \rho) \in ((\gamma-1)v_t\tau, (\gamma-0.5)v_t\tau)$),[17] we see that for an *overpriced* stock, $S_t > S_t^{CI}$, the normal portfolio *underweights* the stock relative to the market ($\phi^N_{\gamma,t} < \phi^U$), and conversely for an *underpriced* security ($\phi^N_{\gamma,t} > \phi^U$ for $S_t < S_t^{CI}$). Moreover, rewriting equation (25) as:

$$\phi^N_{\gamma,t} - \phi^U = -\frac{1}{\gamma} \frac{\ln(1 + OV_t) + \frac{1}{2}v_t\tau}{\delta^2 + \frac{v_t}{\gamma}\tau}, \tag{27}$$

we see that a higher overvaluation leads to larger stock underweighting in the normal portfolio relative to the market, potentially resulting in sizable short positions in the stock for high levels of overpricing.

We emphasize that even though the informed agent underweights the stock relative to the market, the allocation to the stock can still be positive in our setting. This is because the local

[17] In these states, the normal portfolio underweights slightly underpriced securities. This occurs because a positive but small enough over-estimation of the dividend growth rate by U-investors, $0 \le (\gamma-1)v_t\tau < \tilde{\rho}_t - \rho < (\gamma - .5)v_t\tau$, does not translate into stock overpricing (see Section 3.1) but still leads to below-normal stock holdings. However, these states occur with low probability for short enough investment horizons T.

expected return μ can be greater than the risk-free rate r despite the fact that the stock is overvalued. The local expected return is determined by retail investors, who eventually learn the true dynamics of the dividend process. However, they learn only gradually over a horizon that largely exceeds the evaluation period of the money manager. In this case, a risk-averse informed investor will optimally hold a long position in the stock for diversification purposes. Therefore a puzzle in practice should be not that money managers invest in overvalued stocks, but rather that they invest more than the market portfolio, as documented by Brunnermeier and Nagel (2004).

	$\tilde{\rho}_t - \rho$			
	$(-\infty, 0]$	$(0, (\gamma-1)v_t\tau)$	$((\gamma-1)v_t\tau, (\gamma-0.5)v_t\tau)$	$((\gamma-0.5)v_t\tau, +\infty)$
$sgn(OV_t)$	$-$	$-$	$-$	$+$
$sgn(\phi^N_{\gamma,t} - \phi^U)$	$+$	$+$	$-$	$-$

Table 1: Relation Between Fundamentals, Overpricing and the Normal Portfolio

Summing up, for the short investment horizon we consider in this Section we have shown that the normal policy is consistent with the commonly expected behavior of an informed trader under efficient financial markets. In particular, (i) the informed trader overweights underpriced assets and underweights overpriced assets relative to the market portfolio, (ii) the size of the informed trader's bets against mispricing increases with the extent of mispricing and (iii) can even result in substantial short-sale positions for largely overvalued securities. In the next subsection we contrast this with the behavior of an informed trader under convex incentives—the money manager.

3.2.2 The Informed Money Manager

Previous authors have suggested that high-powered incentives can alleviate the bubble-riding behavior associated with career concerns of money managers (see, e.g., Scharfstein and Stein (1990), Dass, Massa, and Patgiri (2008)). A high enough weight on managers' short-term performance, the argument goes, should offset the negative longer-term effects of a loss of reputation and make managers more willing to deviate from the herd and away from overvalued assets. In this subsection and the next we assess the validity of this argument under the convex managerial incentives that we describe in section 2.

Within our setup, a simple low-powered compensation arrangement consists in setting the fee rate equal to a (possibly small) positive constant c, i.e. $f_T = c$. Under this arrangement, it is straightforward to show that the manager's optimal trading strategy equals the normal policy at all times. For high-powered compensation arrangements such as convex contracts to offset incentives to over-invest in overpriced assets, the resulting trading against mispricing should be at least as aggressive as under the normal policy.

In what follows, we relate the manager's optimal policy to the allocations in the stock ϕ^Y, ϕ^U and $\phi^N_{\tilde{\gamma},\cdot}$ of the benchmark portfolio, the market (U-investors') portfolio and the normal policy (for a given RRA coefficient $\tilde{\gamma}$) introduced in previous sections. As before, we let $\tau = T - t$ denote the time remaining to horizon, and we introduce the following additional notation: $\zeta \equiv \bar{\zeta} W_0 / Y_0$, and $\gamma_i \equiv \gamma + \alpha_i(\gamma - 1)$, for $i = 1, 2$. ζ is the normalized performance threshold. γ_1 and γ_2 represent the manager's *effective* RRA in the *underperformance* and *outperformance* regions, respectively, of section 2.2.1. The interpretation of γ_i follows from computing the RRA coefficient corresponding to the manager's utility function when wealth is augmented by the fee rate (5) and implies that, for $\alpha_2 > \alpha_1 > 0$, the manager's effective RRA in the outperformance region increases by $\gamma_2 - \gamma_1 > 0$ relative to the underperformance region. This is the consequence of the higher sensitivity of expected fees to relative performance when the manager's funds do better than the benchmark.[18] Our main result of this Section is stated in the following:

Proposition 3. *The informed manager's optimal holdings of the stock is:*

$$\hat{\phi}_t = \omega_t \phi^N_{\gamma_1,t} + (1-\omega_t)\phi^N_{\gamma_2,t} + \left[\omega_t \left(1 - \frac{\gamma}{\gamma_1}\right) \frac{\delta}{\underline{\sigma}_{\gamma_1,t}} + (1-\omega_t)\left(1 - \frac{\gamma}{\gamma_2}\right)\frac{\delta}{\underline{\sigma}_{\gamma_2,t}}\right]\phi^Y$$
$$+ \frac{1}{\sqrt{\sigma_t \tau}}\left[\omega_t \Phi_{1,t} + (1-\omega_t)\Phi_{2,t}\right] \tag{28}$$

where the weight $\omega_t \in [0,1]$ is increasing in the time-t probability $\Pi_{1,t}$ of underperforming at T as given below,

$$\Phi_{i,t} \equiv \frac{1}{\underline{\sigma}_{\gamma_i,t}} \frac{\mathcal{N}'(\underline{d}_{i,t}) - \mathcal{N}'(\bar{d}_{i,t})}{\Pi_{i,t}}, \tag{29}$$

for

$$\underline{d}_{i,t} \equiv \tfrac{\gamma\delta^2 \underline{\sigma}_{\gamma,t}}{v_t\sqrt{\tau}} \sqrt{\tfrac{\delta^2 + v_t\tau}{\delta^2 + \frac{v_t}{\gamma_i}\tau}}\left(\phi^N_{\gamma,t} - \phi^Y\right) - \tfrac{\sigma_t}{v_t\sqrt{\tau}}\sqrt{\tfrac{\delta^2 + \frac{v_t}{\gamma_i}\tau}{\delta^2 + v_t\tau}}\Gamma, \quad \bar{d}_{i,t} \equiv \underline{d}_{i,t} + 2\tfrac{\sigma_t}{v_t\sqrt{\tau}}\sqrt{\tfrac{\delta^2 + \frac{v_t}{\gamma_i}\tau}{\delta^2 + v_t\tau}}\Gamma, \tag{30}$$

$\Pi_{1,t} \equiv \mathcal{N}(\underline{d}_{1,t}) + 1 - \mathcal{N}(\bar{d}_{1,t})$, $\Pi_{2,t} \equiv \mathcal{N}(\bar{d}_{2,t}) - \mathcal{N}(\underline{d}_{2,t})$, $\mathcal{N}(\cdot)$ *is the standard normal cumulative distribution function,* $\underline{\sigma}_{\tilde{\gamma},t} \equiv \delta + \frac{v_t}{\tilde{\gamma}\delta}\tau$, *and* $\Gamma \geq 0$ *is as given in Appendix A.*

The manager's optimal portfolio (28) consists of the sum of three components, each of which is a weighted average with weights ω_t and $1 - \omega_t$ of a specific portfolio in the underperformance region and its counterpart in the outperformance region:

(1) A *mean-variance* component $\omega_t \phi^N_{\gamma_1,t} + (1-\omega_t)\phi^N_{\gamma_2,t}$. The normal portfolios $\phi^N_{\gamma_1,t}$ and $\phi^N_{\gamma_2,t}$ reflect the different relative risk aversion of the manager in the underperformance (RRA coefficient γ_1) and outperformance (RRA coefficient $\gamma_2 > \gamma_1$) regions. This component has the same sign as the normal policy $\phi^N_{\gamma,t}$ but a smaller absolute value, implying a smaller position in the stock. Moreover, $|\phi^N_{\gamma_2,t}| \leq |\phi^N_{\gamma_1,t}| \leq |\phi^N_{\gamma,t}|$, implying a more conservative mean-variance policy in the outperformance relative to the underperformance regions.

[18] To see this, note that changes in *actual* wealth are augmented by a fee rate $k\left(W_T/\zeta W^Y_T\right)^{\alpha_2}$ in the outperformance region, but only by a flow rate $k\left(W_T/\zeta W^Y_T\right)^{\alpha_1} < k\left(W_T/\zeta W^Y_T\right)^{\alpha_2}$ for $\alpha_2 > \alpha_1$ outside of it. The fee charged by a top performer is increasing in wealth at a higher rate. Therefore, effective wealth fluctuates more in response to the same change in actual wealth in this region than in the underperformance region, raising manager's effective risk aversion.

(2) An *indexing* component, scaling down the benchmark weight in the stock ϕ^Y by the factor $\left[\omega_t \left(1 - \gamma/\gamma_1\right) \delta/\underline{\sigma}_{\gamma_1,t} + \left(1 - \omega_t\right) \left(1 - \gamma/\gamma_2\right) \delta/\underline{\sigma}_{\gamma_2,t}\right] \in (0, 1)$. Since we examine long-only benchmarks ($0 \leq \phi^Y \leq 1$), and the scaling factor takes values in the interval $(0, 1)$, this component represents either long or zero positions in the stock. We discuss the weight in the stock of this component relative to the normal policy in Corollary 3 below.

(3) An additional component, proportional to the sum $\omega_t \Phi_{1,t} + (1 - \omega_t)\Phi_{2,t}$. Portfolios $\Phi_{1,t}$ and $\Phi_{2,t}$ are non-linear functions of the difference between the normal and benchmark portfolios $\phi^N_{\gamma,t} - \phi^Y$ and can reflect large long or short positions in the stock. We refer to this as the *risk-shifting* component, and assess the direction in which this component deviates the manager's trading from the normal policy in in Corollary 3 below.

As usual, we can interpret the difference between the manager's portfolio (28) and the mean-variance component (1) as the manager's *hedging demand*. The manager in our model hedges against the risk of underperforming or, equivalently, of not receiving the performance fee in the outperformance region. Given the decomposition above, this hedging demand is captured by the indexing and risk-shifting components. We analyze these components in the following:

Corollary 3. *At any interim state of the economy as of time $t \in [0, T)$, the sign of the risk-shifting component in the manager's portfolio (28) equals the sign of $(\phi^N_{\gamma,t} - \phi^Y)$:*

$$\operatorname{sgn}\left(\frac{1}{\sqrt{\sigma_t \tau}}(\omega_t \Phi_{1,t} + (1 - \omega_t)\Phi_{2,t})\right) = \operatorname{sgn}(\phi^N_{\gamma,t} - \phi^Y). \tag{31}$$

The indexing component overweights the stock whenever $\phi^N_{\gamma,t} < 0$.

Whether the risk-shifting component represents a long or a short position in the stock depends on the sign of the difference in the allocations to the stock of the normal and the benchmark portfolios, as first pointed out by Basak, Pavlova, and Shapiro (2007). Unlike in their model, the normal policy in our setup is not constant but varies with the (state-dependent) extent of stock mispricing. Thus, the same manager may over- or under-invest in mispriced securities with respect to the normal portfolio at different points in time. Notably, risk-shifting in our setup can lead to over-investment in overpriced securities, even with respect to the market portfolio, when $\phi^Y < \phi^N_{\gamma,t} < \phi^U$.[19] We show later with a numerical example that the risk-shifting component dominates the portfolio when the manager underperforms the benchmark by a small to moderate margin. In practice, the typical benchmark in hedge fund performance fees tend to differ substantially from the market portfolio, with $\phi^Y \ll \phi^U$. Therefore, we expect the risk-shifting portfolio to especially distort an informed hedge fund manager's trading against mispricing. We verify this intuition with our numerical example below.

[19] This can be seen by reference to equations (27) and (31). According to (27), an overvalued stock ($OV_t > 0$) calls for a reduction in the normal stock holding, $\phi^N_{\gamma,t}$, relative to the stock holding of the market, ϕ^U. According to equation (31), though, the risk-shifting component represents a long position in the stock. Under standard parameterizations of the model, this long position can result in a long overall position $\hat{\phi}_t$ that exceeds ϕ^U (and thus $\phi^N_{\gamma,t}$) by a large margin.

Within the indexing component, the manager tilts the portfolio towards the outperformance component $(1-\gamma/\gamma_2)\delta/\underline{\sigma}_{\gamma_2,t}\phi^Y$ as the probability of outperforming increases (ω_t falls). Since $0 \leq (1-\gamma/\gamma_1)\delta/\underline{\sigma}_{\gamma_1,t}\phi^Y \leq (1-\gamma/\gamma_2)\delta/\underline{\sigma}_{\gamma_2,t}\phi^Y \leq \phi^Y$, the outperforming indexing component is closer to the benchmark than its underperforming counterpart. This reflects the well-known *lock-in* effect according to which an outperformer ("winner") and risk-averse manager prefers to secure an interim relative gain by investing like the benchmark. In our setup, this effect intensifies with the fee-performance sensitivity α_2. Whenever the benchmark overweights the stock with respect to the normal policy $\phi^N_{\gamma,t}$, the indexing component tilts the portfolio towards overweighting the stock. By the lock-in effect highlighted above, this effect is more severe in the outperformance region. Given that a benchmark weight in the stock is non-negative, the indexing component always over-invests when the normal policy represents a short position in the stock, as claimed in Corollary 3. A stock market bubble characterized by a negative risk premium then leads to an indexing component of the manager's portfolio that overweights the overpriced stock.

3.2.3 Numerical Example: the Case of Hedge Funds

The typical fee contract in the hedge fund industry ("two-twenty") stipulates a 2% management fee along with an incentive fee equal to 20% of investment profits beyond a designated benchmark performance. The preassigned benchmark is usually a money market rate such as LIBOR plus a spread instead of a market index, and is consistent with the goal of most hedge funds of delivering "absolute returns" in all market conditions.[20]

To capture these incentives, we set the benchmark weight in the stock to zero ($\phi^Y = 0$), in which case $Y_T = \beta_0 e^{rT}$. Defining the continuously-compounded rates $r_T \equiv \ln(R_T)/T, r^Y_T \equiv \ln\left(R^Y_T\right)/T = r$, setting the threshold $\bar{\zeta} = e^{hT}$ for the spread (hurdle rate) $h \geq 0$, and $\alpha_1 = 0$, we show in Appendix B.1 that up to a first-order approximation our fee rate (5) is:

$$f_T \approx kT + kT\alpha_2(r_T - (r+h))^+, \tag{32}$$

where $x^+ \equiv \max(0,x)$. Using equation (32), it is easy to see how we can calibrate the two-twenty contract with a positive hurdle rate within our setup by setting, e.g., $kT = 1, kT\alpha_2 = 5$, and $(r+h) = 1.5\% + 5\% = 6.5\%$.[21]

We next look into how the asset price dynamics described by Proposition 1 affect the trading

[20] The fee structure often includes a "high-water mark" stipulating that the fund has to recover losses before any incentive fee can be charged following a year in which the fund declines in value. Such provisions may reduce the long-term risk-seeking incentives of a hedge fund manager, as analyzed by Hodder and Jackwerth (2007), Panageas and Westerfield (2009), and Drechsler (2013). Since we focus on the trading behavior of the manager over short-horizons, we follow Buraschi, Kosowski, and Sritrakul (2011) in assuming that the high-water mark is prespecified at the beginning of the period, and allow for differences in high-water marks by varying our spread parameter $\bar{\zeta}$ in (5).

[21] Given the exponential form of the performance fee (5) in the outperformance region, a value of $\alpha_2 = 5$ suffices to reflect a performance fee 10 times larger than the base management fee for most of the relevant domain of relative performance.

of a hedge fund managers according to Proposition 3. Figure 1 illustrates the manager's allocation in the risky asset (solid blue line) at a given point in time ($t = 3/4T$) across different states of overvaluation OV_t. The figures assume an initial stock overvaluation of 4%, corresponding to a realized dividend growth rate ρ one standard deviation lower than U-investors' prior belief ρ_0.

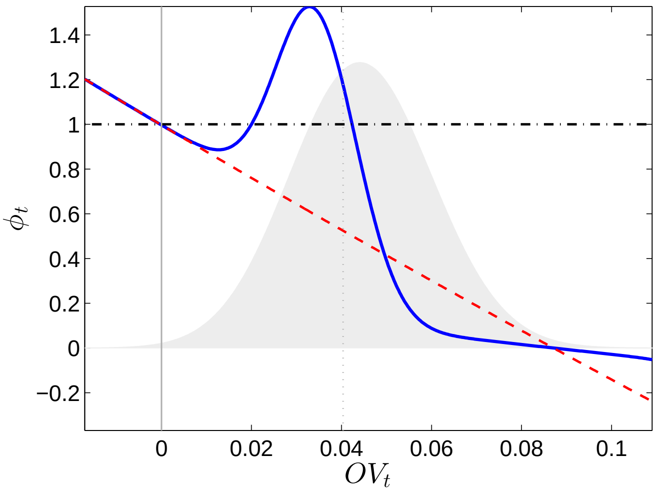


Figure 1: **Interim Portfolio Weight of a Hedge Fund Manager in a Mispriced Stock**
The solid blue, dashed red and dash-and-dot black lines represent the portfolio weights of a hedge fund manager ($\hat{\phi}_t$), the normal policy ($\phi^N_{\gamma,t}$) and the market (ϕ^U), respectively, in the mispriced stock for different degrees of overvaluation OV_t as of $t = 3/4T$. The grey area depicts the time-0 actual probability of the corresponding overvaluation state at $t = 3/4T$. Results obtain from a time-0 overvaluation of 4% (following a realized dividend growth rate 1 std. dev. below the prior ρ_0), as marked by the vertical dotted line. We set: $\alpha_1 = 0, \alpha_2 = 5, \bar{\zeta} = 1.05, \phi^Y = 0$. The rest of the parameters are as follows: $T = 1, r = 1.5\%, \delta = .0129, v_0 = 0.05^2, \gamma = 5$.

Both small and large stock overvaluations can lead the managers to trade *less aggressively against the mispriced security than the normal policy.* The greater probability mass (grey shaded area) over states in which the manager over-invests shows that this is the most likely behavior even though situations of both above- and below-normal exposure to the stock can occur.

Notably, the high degree of convexity in hedge fund incentive fees leads managers to overweight the overpriced stock *more than the uninformed investors* under a high-probability subset of overvaluation states. For a similar level of overpricing as at $t = 0$, the stock holdings of the manager at $t = 3/4T$ can exceed the proportion in the market portfolio by more than 50%. This outcome is due exclusively to the risk-shifting component, and results from the combination of three factors. First, a hedge fund is an underperformer until it meets the hurdle that triggers the performance fee. But meeting this hurdle can take an extended period of time, even if the stock overvaluation persists after three quarters of the year have elapsed. As Figure 2 illustrates, a worsening—relative to the initial values—of the stock overpricing can still be consistent with the small to moderate underperformance that "activates" the risk-shifting component in the manager's portfolio. Second, the high power of the incentive fees drastically reduces the risk aversion of the manager and magnifies the absolute value of the risk-shifting component. Third, the benchmark associated with the incentive fees in the hedge fund industry drives risk-shifting towards *overweighting*, rather than underweighting, the overpriced asset in the manager's portfolio. This last result is not trivial, as this factor needs not work in the direction that exacerbates overpricing—once managers' price impact is considered in the next Section—for all types of money managers *even under convex incentives*. In particular, we show below that the manager of a mutual fund shifts risk by *underweighting* instead the same overpriced stock, despite its positive excess expected return.

We explain the intuition behind hedge funds' over-investment in overpriced assets with Figure 2, which illustrates the manager's portfolio for a hypothetical trajectory of stock overvaluation under the same parameterization as in Fig. 1, along with the manager's performance relative to the benchmark. This trajectory is meant to resemble the dynamics of security overpricing in the initial phase of a bubble.[22]

At the beginning of the period the option implied by the manager's incentive fee is "out of the money." In other words, the manager starts off below the performance threshold necessary to receive the incentive fee. In order to reach this threshold, the risk-shifting component increases the weight in the overpriced stock (solid blue line) over the weights in both the normal (dashed red line) and market (dash-and-dot black line) portfolios. This happens as long as the manager *has not attained a sufficient outperformance*. The overinvestment in the stock follows from Corollary 3. As noted above, hedge funds' absolute performance condition is equivalent to a (scaled) money market benchmark for which $\phi^Y = 0$. This risk-free asset-only benchmark implies a lower position in the stock than the weight $\phi^N_{\gamma,\cdot}$ preferred by a manager with finite risk aversion.[23] Following Corollary 3, the manager shifts risk by aggressively overweighting the overpriced stock in the portfolio, as can be seen in both Fig. 1 and Fig. 2.[24]

This excess investment in the overpriced stock is consistent with the bubble-riding behavior documented empirically by Brunnermeier and Nagel (2004) for hedge funds during the build-up of the tech bubble in the late 1990s. These authors show that several hedge funds overweighted, relative to the market, highly overpriced technology stocks in their portfolios before the bubble burst.

As prices keep rising in Fig. 2, this strategy eventually pays off and the fund performance exceeds the threshold. As an *outperformer* ($R_t > \bar{\zeta} R^Y_t$), the manager's effective risk aversion increases. To lock-in the interim outperformance that warrants the performance fee payment, the optimal policy becomes more conservative and tilts the portfolio towards the indexing component. This results in a substitution of the risk-free security for the overpriced stock as overpricing

[22] More precisely, we examine a particular sequence of shocks that results in the (unlikely) smooth path in the figure. At each point in time, the manager's portfolio allocation depends on the same ex ante uncertainty about the future values of state variables as in Fig. 1. We choose this path to facilitate the analysis only, as our conclusions do not depend on that particular realization.

[23] More precisely, this occurs as long as $\ln(1 + OV_t) < \gamma\delta^2 + .5 v_t \tau$ according to equation (27).

[24] We expect this intuition to survive under multiple risky assets. The reason is that, as long as the overvalued asset has a positive risk premium and provides some diversification value, the normal portfolio will include a positive holding in this asset. Then, a manager levering up the normal portfolio component could indirectly lever up the overpriced asset as well.

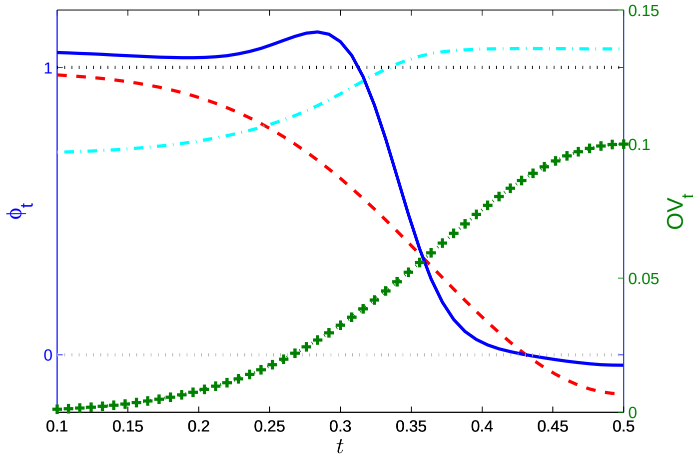


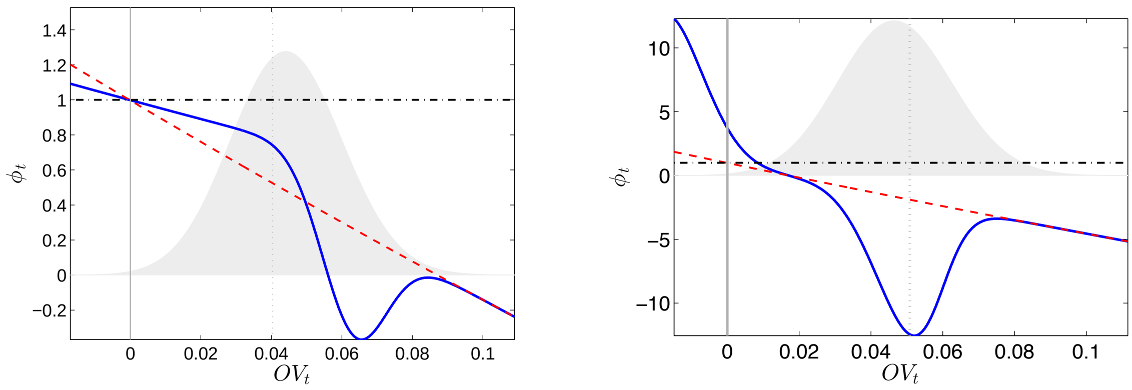
Figure 2: **Hedge Fund's Investment in the Stock during an Overvaluation Path**

The solid blue, dashed red and dotted black lines on the left axis represent the informed manager's ($\hat{\phi}_t$), normal ($\phi^N_{\gamma,t}$) and market (ϕ^U) portfolio weights, respectively, in the stock for a hypothetical overvaluation path OV_t (green crossed line, right axis). The cyan dash-and-dot line represents the manager's relative performance $R_t/(\bar{\zeta}R^Y_t)$ on the left axis. Results obtain from a realized dividend growth rate 1 std. dev. below the prior ρ_0. We assume: $\alpha_1 = 0, \alpha_2 = 5, \bar{\zeta} = 1.05, \phi^Y = 0, T = 1, r = 1.5\%, \delta = .0129, v_0 = 0.05^2, \gamma = 5$.

worsens. While the risk premium on the overpriced stock remains positive, the manager's tilt towards the indexing component results in more aggressive trading than the normal portfolio against the mispricing. When the overpricing is so severe that the stock expected excess return turns negative, the normal policy sells the overpriced stock short, whereas the indexing component limits the extent of short selling below the normal policy. The resulting difference drives the manager's over-investment in the overpriced stock with negative risk premium. This benchmark-induced conservative behavior contrasts with the common view of hedge funds as absolute-return investment vehicles.

As can be seen in Fig. 1, benchmarking concerns can induce the hedge fund manager to short-sell substantially less of the mispriced stock than the normal policy for large levels of overvaluation OV_t. Absent explicit portfolio constraints, the incentives in our model then lead to endogenous *short-sale restrictions*. This behavior agrees with the decline in short interest in NASDAQ stocks during the tech bubble documented by Stein and Lamont (2004). We show in Section 4 that these incentive-based limits to short-selling can hamper the role of sophisticated investors in stabilizing the stock market in the same fashion that explicit short-sale constraints limit pessimistic investors' trading against overvaluation in models of disagreement (see, e.g., Hong and Stein (2007)).

We note that for money managers following benchmarks different from those of hedge funds, like mutual funds, the risk-shifting component needs not overweight an asset with positive expected excess return. In the mutual fund industry, an implicit convexity results from the relation between a fund's performance and its clients' share purchases and redemptions. Indeed, an extensive literature (see, e.g., Chevalier and Ellison (1997), and Sirri and Tufano (1998)) documents that mutual fund inflows after good performance relative to a stock market index largely exceed outflows following poor excess returns. Since mutual funds' revenue is commonly proportional to their assets under management (AUM), such a convex flow-performance relationship suggests an implicit option-like relation between mutual fund performance in excess of the stock market and managerial compensation.



(a) MF Manager

(b) HF Log Preferences

Figure 3: **Interim Portfolio Weight in a Mispriced Stock**

The solid blue, dashed red and dash-and-dot black lines represent the portfolio weights of a manager ($\hat{\phi}_t$), the normal policy ($\phi^N_{\gamma,t}$) and the market (ϕ^U), respectively, in the mispriced stock for different degrees of overvaluation OV_t as of $t = 3/4T$. Panel 3(a) shows the case of a mutual fund (MF) manager subject to a convex flow-performance relationship ($\alpha_1 = 0, \alpha_2 = 1.5, \bar{\zeta} = 0.94, \phi^Y = 1, \gamma = 1$, see Appendix B.2). Panel 3(b) shows the case of a hedge fund (HF) manager with log preferences ($\alpha_1 = 0, \alpha_2 = 5, \bar{\zeta} = 1.05, \phi^Y = 0, \gamma = 1$). The grey area depicts the time-0 actual probability of the corresponding overvaluation state at $t = 3/4T$. Results obtain from a time-0 overvaluation of 4% (following a realized dividend growth rate 1 std. dev. below the prior ρ_0), as marked by the vertical dotted line. The rest of the parameters are as follows: $T = 1, r = 1.5\%, \delta = .0129, v_0 = 0.05^2$.

Figure 3(a) plots the optimal investment strategy of a mutual fund manager under the same scenario as in Fig. 1. We explain how we parameterize eq. (5) to reflect the convex incentives in the mutual fund industry in Appendix B.2. Unlike the hedge fund manager, a mutual fund manager can shift risk by *underweighting* the overpriced stock in the portfolio relative to the normal policy. In particular, the manager can sell the stock short even when it has a positive expected excess return and is held long in the normal portfolio. This is because the risk-shifting component of the mutual fund manager represents a *more aggressive stance against mispricing* than the normal policy. As can be seen in the Fig. 1 for low levels of overvaluation, a mutual fund manager can also overweight the overpriced stock relative to the normal policy. This is driven exclusively by the indexing component, which makes the manager mimic partially the stock market index when outperforming. This effect is similar to the price distortion arising from managers' unwillingness to deviate from their benchmark in Buffa, Vayanos, and Woolley (2014).

We also note that the over-investment in overpriced assets in our model is not just the result of convex incentives reducing the manager's risk aversion. On the contrary, the effects arise and become more severe for managers that are more risk-averse than implied by log preferences.

Figure 3(b) shows the case of a manager with log preferences, and thus lower risk aversion than the manager in our analysis. If the over-investment in overpriced assets were driven by a decrease in risk aversion only, the manager with log preferences should overweight the overpriced asset more. By contrast, the portfolio under log preferences sells the overpriced security even faster than the normal policy in almost all overpricing states. Only for a very small range of overpricing does the log manager overweight the overpriced asset relative to the normal policy. This case agrees with the intuition in Dass, Massa, and Patgiri (2008), who hypothesize that convex incentives should lead managers to trade aggressively against security overpricing.[25] Log preferences also eliminate the need to hedge against underperformance when the manager is outperforming, shutting down the indexing component that drives the overinvestment for large levels of overpricing in our setup.

We highlight that in our analysis so far the manager perceives the fees at the same time T as the stock price converges to its fundamental value. Therefore, our results do not hinge on the manager's conservatism in anticipation of potential losses triggered by further widening in mispricing, as originally suggested by Shleifer and Vishny (1997).

We conclude this Section by examining the effects of the manager's information advantage on its aggressiveness to trade against mispricing. Figure 4 plots the hedge fund manager's time-t excess holdings of the stock, relative to the normal portfolio, for different values of the manager's information advantage v_0. To remove the dependence on the realization of ρ, the policies are plotted as a function of the current overvaluation OV_t by averaging the manager's trading across all paths of the inferred growth rate $\tilde{\rho}_t$ that lead to that particular value of overvaluation as of $t = 0.5$. If the manager traded more aggressively against the mispricing than the normal policy, the policies should plot in the top-left and bottom-right quadrants of the figure.

Generalizing our results above, on average the hedge fund manager fails to trade against any levels of undervaluation, as well as against high levels of overvaluation. Crucially, the problem becomes worse in general as (i) up-to-date overvaluation OV_t worsens, and (ii) the manager's initial information advantage v_0 improves. The economic intuition behind (i) is as follows: a larger initial mispricing improves the odds that the manager will outperform early on and lock in the outperformance thereafter. The hedge fund manager locks in the current performance by (partially) mimicking a risk-free asset, which results in either too few holdings of an underpriced stock, or too little trading against a severely overpriced stock. The intuition underlying (ii) is similar: as the manager's advantage over retail traders widens, the probability of outperforming the benchmark increases and the indexing component tends to dominate the manager's portfolio. This behavior leads to an overly conservative stance against mispricing and worsens precisely when the manager's potential profits from trading against the mispricing are greatest.

[25] This aggressive trading against mispricing is explained by the normal policy under low risk aversion. Unlike a more risk-averse investor, the normal portfolio of a log investor will bet against an overpriced security in the presence of the slightest mispricing. Since by Corollary 3 the risk-shifting component of a manager's portfolio invests in the same direction as the normal policy, the manager with convex incentives and log preferences will invest even more aggressively against the overpricing than in the absence of these incentives.

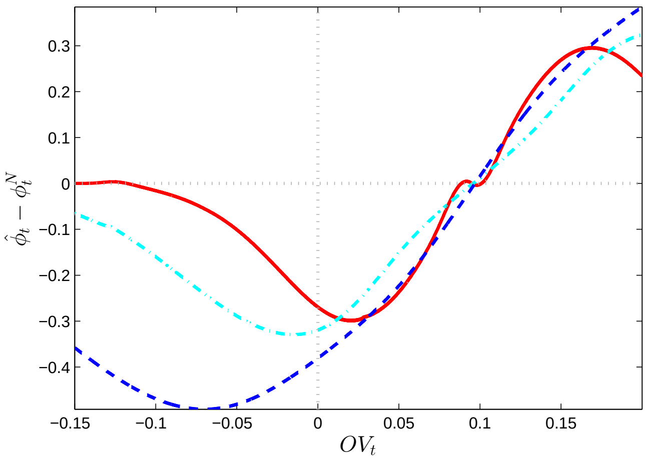


Figure 4: **Hedge Fund Manager's Average Over-Investment in the Mispriced Stock**
Average time-t optimal weight in the stock of the hedge fund manager in excess of the normal portfolio, $\hat{\phi}_t - \phi_t^N$, as of mid-year ($t = 0.5$). For each level of overvaluation OV_t, the average is computed over all the paths of the inferred dividend growth rate $\tilde{\rho}_t$ that result in that particular value of OV_t. The red solid, cyan dash-and-dot and blue dashed lines correspond, respectively, to levels of U-investors' initial uncertainty $\sqrt{v_0}$ equal to 3.7%, 7.4% and 11.1%. We assume: $\alpha_1 = 0, \alpha_2 = 5, \bar{\zeta} = 1.05, \phi^Y = 0, \tilde{\rho}_0 = 0.0238, T = 1, r = 1.5\%, \delta = .0129, \gamma = 5$.

Summing up, informed money managers responding to convex incentives can trade less aggressively against mispricing than would be expected absent such incentives. This problem is particularly severe for sophisticated investors like hedge fund managers, and in situations of very high over- and underpricing. In the next Section we show how this behavior can exacerbate mispricing under the general equilibrium in which money managers have positive price impact.

4 Convex Incentives and Equilibrium Mispricing

In this section we assess the price impact of the policies of money managers with superior information but convex incentives under general equilibrium. More precisely, we abandon our assumption that $\theta = 0$ of Section 3 to focus on the effectiveness of these policies in correcting asset mispricing when $\theta \in (0, 1)$. Although our setup lends itself to examine the price impact of money managers following any type of benchmark of the type (4), for brevity of exposition we concentrate our results on the relatively under-explored case of hedge funds for which the benchmark is, effectively, a money market rate.

To solve for the equilibrium state-price density in closed form, we examine the piece-wise linear approximation (32) to the fee rate (5) directly, and assume that the manager's compensation W^M is the product of this fee rate times initial AUM W_0:

$$W_T^M = \big(kT + kT\alpha_2(r_T - (r + h))^+\big)W_0 \tag{33}$$

Importantly, this variant retains all the features of our previous specification of the hedge fund manager's incentives and induces optimal portfolio policies that are almost indistinguishable from the policies in Section 3.2.[26] The only (slight) difference is observed in the strength of the indexing effect in the outperformance region: by applying the fee rate to a *fixed* instead of to a *stochastic* level of AUM (W_0 instead of W_T) the manager becomes less risk averse in this region than in the case examined in Section 3.2, while still remaining more risk averse than in the underperformance region. Given the importance of the indexing component in the manager's portfolio in limiting the trading against mispricing highlighted above (particularly for severe mispricing), we expect this alternative compensation specification to exacerbate the equilibrium mispricing in this section to a *lesser* extent than would the incentives analyzed so far. In this sense, the impact of managers' policies on severe levels of mispricing that we derive below can be seen as a lower bound on the results that can be expected under slightly different fee arrangements.

When $\theta \in (0,1)$, market clearing in the risk-free and risky assets,

$$W_T + W_T^U = S_T = D_T, \tag{34}$$

implies the following:

Proposition 4. *For a given realization of the dividend growth rate ρ and for $t \in [0,T]$, if an equilibrium exists the (manager's) SPD and the risky asset price are given by:*

$$\pi_t = e^{r(T-t)} \int_{-\infty}^{+\infty} \pi_T \varphi(B_T|B_t) dB_T, \tag{35}$$

$$S_t = \pi_t^{-1} \int_{-\infty}^{+\infty} \pi_T D_T \varphi(B_T|B_t) dB_T, \tag{36}$$

where $\varphi(\cdot|D_t)$ is the normal density with conditional mean and variance B_t and $T-t$,

$$\pi_T = \lambda_M^{-1} \begin{cases} \left[\dfrac{D_T}{(kT)^{\frac{1}{\gamma}-1} + \left(\frac{\lambda_M}{\lambda_U}\xi_T\right)^{\frac{1}{\gamma}}}\right]^{-\gamma}, & \text{if } D_T < \underline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right), \\ (kT(1+\alpha_2))^{1-\gamma}\left[\frac{\gamma-1}{\gamma}\frac{1}{\kappa-1}\frac{\alpha_2}{1+\alpha_2}\zeta\beta_T\right]^{-\gamma}, & \text{if } \underline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right) \le D_T < \overline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right), \\ \left[\dfrac{D_T - \frac{\alpha_2}{1+\alpha_2}\zeta\beta_T}{(kT(1+\alpha_2))^{\frac{1}{\gamma}-1} + \left(\frac{\lambda_M}{\lambda_U}\xi_T\right)^{\frac{1}{\gamma}}}\right]^{-\gamma}, & \text{if } \overline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right) \le D_T, \end{cases} \tag{37}$$

$\kappa \equiv (1+\alpha_2)^{1-1/\gamma} > 1, \zeta \equiv \bar{\zeta}W_0/Y_0$, the functions $\underline{D}(\cdot), \overline{D}(\cdot)$ are as given in Appendix A, and the constants λ_M, λ_U are the Lagrange multipliers for the manager and the retail investor that solve the following system of (algebraic) equations:

$$\begin{cases} e^{rT}\int_{-\infty}^{+\infty} \pi_T \varphi(B_T) dB_T & = 1, \\ \int_{-\infty}^{+\infty} \pi_T^{1-\frac{1}{\gamma}} (\xi_T/\lambda_U)^{\frac{1}{\gamma}} \varphi(B_T) dB_T & = (1-\theta)S_0. \end{cases} \tag{38}$$

[26] Illustrations of the similarity of the manager's policies in response to the two types of incentives are available from the authors upon request.

An equilibrium exists if and only if a solution to (38) exists.

The first equation in (38) is a normalization of the initial value of the SPD, whereas the second equation is the retail investor's budget constraint. The non-linearity of the optimal policy of the manager (28) results in a three-piece equilibrium SPD. This SPD adopts different functional forms in the "bad", "intermediate" and "good" states of the economy according to (38). Given this SPD, direct integration against a normal density allows for a relatively straightforward computation of a solution (in case it exists) to (38) and, in turn, of the equilibrium prices (36).

We next follow this procedure to find the price impact of the manager's policies of Section 3.2.3, assuming that the informed manager is endowed with half the share of the stock: $\theta = 0.5$. For comparability, we examine an identical parameterization of our model as in Section 3.[27] We note however that our results are robust to alternative values of the model parameters.

4.1 Benchmark case: Equilibrium Mispricing without Convex Incentives

We first examine the equilibrium mispricing in an economy in which the informed investor (the manager) has no convex incentives. This case corresponds to the *normal* policy defined in Section 3.2. For consistency, we refer to this as the *normal* case. It represents the relevant comparison benchmark to assess the effects of convex incentives on the price impact of an informed investor: differences between the price impact of the hedge fund manager and the normal policy can be attributed solely to the specific incentives that hedge fund managers face.

Figure 5 plots the equilibrium mispricing $((S_t/SCI_t)^{1/\tau} - 1)$ prevailing under the normal economy for different states as of $t = 3/4T$. This states correspond to the different values of stock overvaluation OV_t defined in Section 3.2, i.e. the equilibrium mispricing that would prevail absent any informed investor in the economy ($\theta = 0$). For comparison, a 45-degree line is also included in the graph.

Absent convex incentives, an informed investor always corrects prices towards their "fundamental values," reducing the extent of mispricing across all economic states. Indeed, when the uninformed investors underestimate the growth rate of dividend and underweight the stock in their portfolios, they push the stock price below its fundamental value. The informed investor expects the uninformed traders to revise their estimates up in the future as better news (higher realizations of D_t) arrive in the market. In anticipation of the upward pressure that uninformed investors will exert on the stock price once they learn the better prospects, the normal policy overweights the stock. The resulting increased demand for the stock, relative to the all-uninformed-investors economy, pushes prices up before the news arrive and corrects part of the mispricing (given that the informed investors are endowed with only half the supply of shares of the stock) in the process.

[27] In particular, we assume that the manager receives the compensation fees at the end of the year, i.e. $T = 1$, and that the initial estimation error of the uniformed investors is 4% (following a realized dividend growth rate 1 std. dev. below the prior ρ_0). The rest of the parameters are as follows: $\alpha_1 = 0, \alpha_2 = 5, \bar{\zeta} = 1.05, \phi^Y = 0, r = 1.5\%, \delta = .0129, v_0 = 0.05^2, \gamma = 5$.

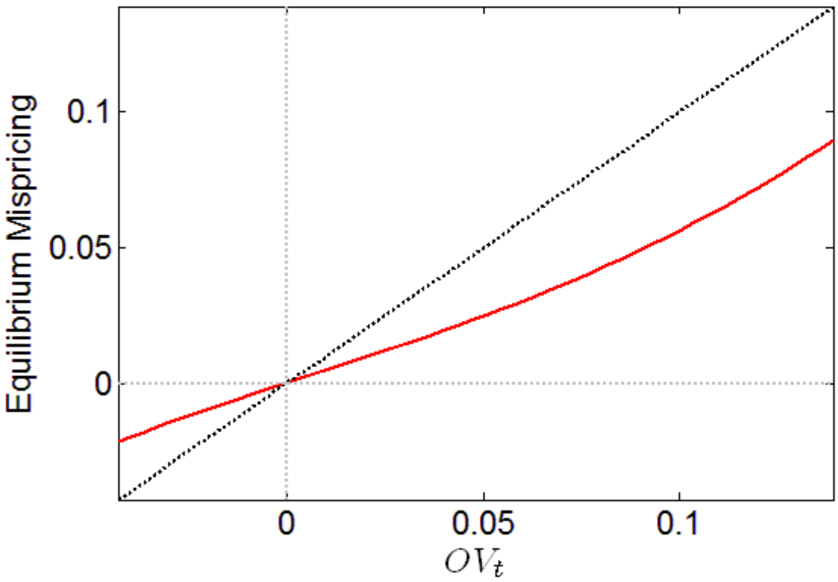


Figure 5: **Equilibrium Mispricing as of $t = 3/4T$ in the Normal Case**

The solid red and black dotted lines represent the equilibrium mispricing $(S_t/SCI_t)^{1/\tau} - 1$ under the normal case (no convex incentives) and the all-uninformed ($\theta = 0$) case. Results obtain from a realized dividend growth rate 1 std. dev. below the prior ρ_0. We assume: $r = 1.5\%, \delta = .0129, \upsilon_0 = 0.05^2, \gamma = 5$.

Similarly, the normal policy has a stabilizing role in situations of stock overpricing. A streak of good news can make uninformed investors too optimistic about the future prospects of the economy. They will demand more shares of the stock and push the stock price up and above its fundamental value in the process. The informed normal investor expect uninformed trader to revise down their estimates to more realistic as worse news arrive in the future. Anticipating the future price drop the normal investor will demand more shares of the stock and bring part of the correction in the stock mispricing ahead of time.

This correcting effect on prices of the normal policy reduces the initial stock mispricing substantially. As mentioned above, the initial stock overvaluation depicted in the graph amounts to 4%, corresponding to the realization of ρ falling one standard deviation below the uninformed prior ρ_0. The normal policy shrinks that initial overpricing by 53.4% even though the informed investor is endowed with only half of the stock in our example.

4.2 Impact of Convex Incentives

Our partial equilibrium analysis of the manager's investment policy in Section 3.2 suggests that convex incentives could exacerbate mispricing in many situations. In particular, the risk-shifting and indexing components of the manager's portfolio can overweight an overpriced asset relative to the normal policy. However, for intermediate ranges of stock overvaluation the indexing component in the manager's portfolio can also underweight the stock as long as the stock risk premium remains positive. Given that the manager outperforms the benchmark in these situations, fund AUM may exceed the wealth of a normal investor in the same circumstance. The expected net effect on current and future prices then remains uncertain unless examined within the general equilibrium framework in this Section.

We plot the equilibrium mispricing under convex incentives in Figure 6. For both the hedge fund manager and normal case, results are plotted as a fraction of the mispricing that would prevail absent any informed investor in the economy ($\theta = 0$), and against this level of mispricing on the x-axis.

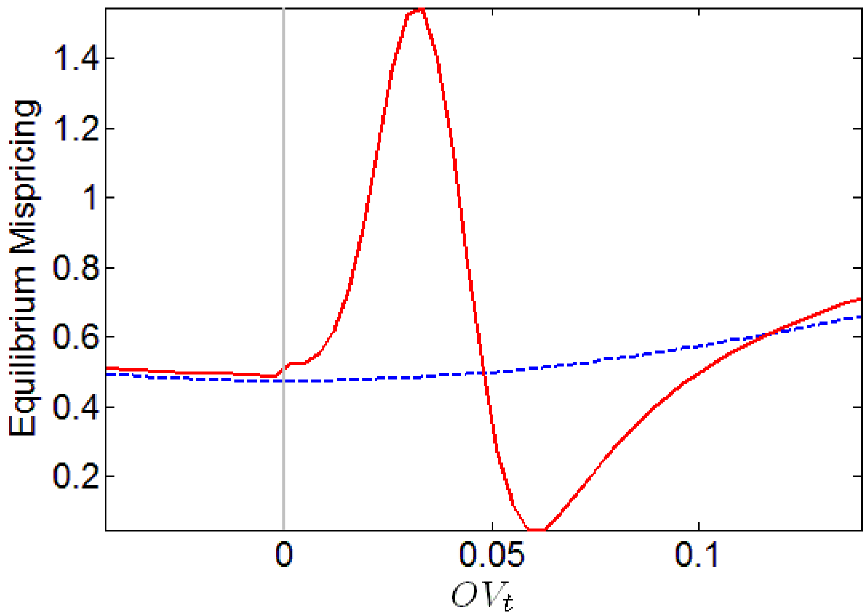


Figure 6: **Equilibrium Mispricing as of** $t = 3/4T$

The solid red and blue dashed lines represent the equilibrium mispricing $(S_t/SCI_t)^{1/\tau} - 1$ under the hedge fund manager and normal (no convex incentives) cases, as a fraction of the mispricing prevailing when all investors are uninformed ($\theta = 0$). Results obtain from a realized dividend growth rate 1 std. dev. below the prior ρ_0. We assume: $\alpha_1 = 0, \alpha_2 = 5, \bar{\zeta} = 1.05, \phi^Y = 0, T = 1, r = 1.5\%, \delta = .0129, v_0 = 0.05^2, \gamma = 5, \theta = 0.5$.

Convex incentives can exacerbate overpricing not only relative to the normal case, but even *beyond the overpricing prevailing in a fully-uninformed economy.* Indeed, the overpricing under convex incentives can be up to 55% higher than if all investors are over-optimistic about the prospects of the economy. This is in spite of the fact that, unlike these investors, the manager is actually aware that the asset is overpriced. However, the convex incentives induce the manager to shift risk until she secures a large enough margin over the risk-free benchmark in the performance fee. Following Corollary 3, given the low (zero) risk of the benchmark the manager optimally risk-shifts by taking a large long position in the stock rather than trading against its overpricing. The true, lower ρ is revealed slowly over time to the uninformed investors. Thus, the manager expects to make a large enough excess profit from this risk-shifting position before stock prices revert back to fundamental values.

Once a sufficient outperformance is secured, the indexing component in the manager's portfolio can help correct overpricing faster than in the normal case for intermediate but not for severe levels of overvaluation. In the latter situation, the endogenous constraint on short-selling that we highlighted in Section 3.2 severely limits, relative to the normal policy, the trading of the manager against the overvaluation. The resulting overvaluation can be 8.1% higher than in the normal case, compared to a 51.8% higher when all investors are uninformed.

Crucially, the manager's investment policy has a positive net impact on the initial stock overpricing. Indeed, the trading of the manager corrects only 28.2% of the overpricing that would prevail if all investors were uninformed. Compared to the 53.4% correction that would be attained in the absence of convex incentives, we conclude that the overweight in overpriced assets of the risk-shifting and indexing component in the manager's portfolio more than offset her trading against mispricing under intermediate levels of overpricing. The net effect of convex incentives is an upward bias on the price of already overvalued stocks.

5 Conclusions

In this paper we consider the effects of convex incentives on the trading against mispricing of a money manager with superior information. According to the standard paradigm, the trading of an agent with superior information should vary depending on the level of mispricing or deviation of the security price from the fundamentals. In particular, the investor should underweight an overpriced security and overweight an underpriced security. Even in the presence of career concerns, a recent line of research suggests that short-term incentive contracts should induce trading against overpricing and offset the bubble-riding behavior resulting from these concerns.

We find that convex incentives alter these conclusions. In particular, it can be optimal for an informed money manager to over-invest—relative to the standard level or even to the market portfolio—in overpriced securities, so as if "riding the bubble." We further show that this behavior worsens as expected overpricing increases. Our model is able to reconcile some puzzling empirical findings without recurring to behavioral arguments, and only using incentives documented in the literature—although not standard in financial models. We study the problem in detail in partial equilibrium, where we get analytic expressions, but we show that our conclusions hold in general equilibrium and result in a significant exacerbation of security mispricing.

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Appendix

A Proofs and Auxiliary Results

We start by stating two auxiliary lemmas that are used throughout the remaining proofs.

Lemma A1. *Let $\tau = T - t$. For $0 \leq t \leq T$, $\alpha \in \mathbb{R}$:*

$$\tilde{E}_t\left[D_T^\alpha\right] = D_t^\alpha \exp\left\{\left(\alpha\tilde{\rho}_t - \frac{1-\alpha}{2}\alpha\delta^2 + \frac{\alpha^2}{2}v_t\tau\right)\tau\right\} \tag{39}$$

Proof. The dynamics of D_t under the filtered probability are $dD_t = D_t(\tilde{\rho}_t dt + \delta d\tilde{B}_t)$, or, for $0 \leq t \leq t' \leq T$:

$$\begin{aligned} D_{t'} &= D_t e^{\int_t^{t'}\left(\tilde{\rho}_s - \frac{\delta^2}{2}\right)ds + \delta(\tilde{B}_{t'} - \tilde{B}_t)} \\ &= D_t e^{-\frac{\delta^2}{2}(t'-t) + \int_t^{t'}\tilde{\rho}_s ds + \delta(\tilde{B}_{t'} - \tilde{B}_t)}. \end{aligned} \tag{40}$$

From (7), and using the solution to v as: $v_t = \frac{\delta^2 v_0}{\delta^2 + v_0 t}$,

$$\begin{aligned} \frac{d\tilde{\rho}_t}{v_t} = \delta d\tilde{B}_t &\Rightarrow \int_t^{t'}\frac{d\tilde{\rho}_t}{v_t} = \delta(\tilde{B}_{t'} - \tilde{B}_t) \\ &\Rightarrow \frac{\tilde{\rho}_{t'}}{v_{t'}} - \frac{\tilde{\rho}_t}{v_t} - \int_t^{t'}\tilde{\rho}_s ds = \delta(\tilde{B}_{t'} - \tilde{B}_t) \\ &\Rightarrow \frac{\tilde{\rho}_{t'}}{v_{t'}} - \frac{\tilde{\rho}_t}{v_t} = \int_t^{t'}\tilde{\rho}_s ds + \delta(\tilde{B}_{t'} - \tilde{B}_t), \end{aligned} \tag{41}$$

which allows us to re-express (40) as:

$$D_t e^{-\frac{\delta^2}{2}(t'-t) + \int_t^{t'}\tilde{\rho}_s ds + \delta(\tilde{B}_{t'} - \tilde{B}_t)} = D_t e^{-\frac{\delta^2}{2}(t'-t) + \frac{\tilde{\rho}_{t'}}{v_{t'}} - \frac{\tilde{\rho}_t}{v_t}}. \tag{42}$$

Note that, conditioning on $\mathcal{F}_t^D$, the only random variable in the former expression is $\tilde{\rho}_{t'}$. Moreover, from (7) we know that $\tilde{\rho}_t$ is a linear diffusion with deterministic volatility, so:

$$\tilde{\rho}_{t'}|\tilde{\rho}_t = \tilde{\rho}_t + \frac{1}{\delta}\int_t^{t'} v_s d\tilde{B}_s|\tilde{\rho}_t \sim \mathbf{N}\left(\tilde{\rho}_t, \sigma^2_{\tilde{\rho},t,t'}\right), \tag{43}$$

with $\sigma^2_{\tilde{\rho},t,t'} = \frac{1}{\delta^2}\int_t^{t'}(v_s)^2 ds = v_t - v_{t'}$. This implies that $D_T|D_t$ is log-normally distributed with deterministic mean and variance, so:

$$\begin{aligned} \tilde{E}_t\left[D_T^\alpha\right] =& D_t^\alpha e^{-\alpha\frac{\delta^2}{2}\tau - \alpha\frac{\delta^2}{v_t}\tilde{\rho}_t} E_t\left[e^{\alpha\frac{\delta^2}{v_T}\tilde{\rho}_T}\right] \\ =& D_t^\alpha \exp\left\{-\alpha\frac{\delta^2}{2}\tau - \alpha\delta^2\left(\frac{1}{v_t} - \frac{1}{v_T}\right)\tilde{\rho}_t + \frac{\alpha^2\delta^4}{2v_T^2}(v_t - v_T)\right\}, \end{aligned} \tag{44}$$

which results in (39) after some algebraic manipulations. □

Lemma A2. *Let $z \sim \mathbf{N}(0, \sigma_z^2)$, and let $\check{\rho}, c, \bar{z} \in \mathbb{R}$. We have:*

$$E\left[e^{-\check{\rho}(z-c)^2}\mathbb{1}_{\{z\le\bar{z}\}}\right] = \frac{e^{-\frac{\check{\rho}c^2}{1+2\check{\rho}\sigma_z^2}}}{\sqrt{1+2\check{\rho}\sigma_z^2}}\mathcal{N}\left(\frac{\bar{z}-\frac{2\check{\rho}\sigma_z^2}{1+2\check{\rho}\sigma_z^2}c}{\sigma_z/\sqrt{1+2\check{\rho}\sigma_z^2}}\right), \tag{45}$$

where $\mathcal{N}(.)$ is the standard normal cumulative distribution function.

Proof. Follows from direct integration against the normal density, using the change of variables $\check{z} = \frac{z-\frac{2\check{\rho}\sigma_z^2}{1+2\check{\rho}\sigma_z^2}}{\sigma_z/\sqrt{1+2\check{\rho}\sigma_z^2}}$, $\check{z}_l = \frac{\bar{z}-\frac{2\check{\rho}\sigma_z^2}{1+2\check{\rho}\sigma_z^2}}{\sigma_z/\sqrt{1+2\check{\rho}\sigma_z^2}}$. □

Proof of Proposition 1. Problem (13) can be solved as in an equivalent full-information framework. In particular, the dynamic budget constraint (14) can be restated (see e.g. Karatzas and Shreve (1998)) as:

$$\tilde{E}_0\left[\tilde{\pi}_T W_T^U\right] = w_0. \tag{46}$$

Using the martingale/duality approach of Cox and Huang (1989) and Karatzas, Lehoczky, and Shreve (1987), the dynamic optimization problem (13) can be solved as a static problem over final payoffs W_T^U. The standard solution to uninformed investors' optimization problem is then:

$$\hat{W}_T^U = (\lambda\tilde{\pi}_T)^{-\frac{1}{\gamma}} \Rightarrow \tilde{\pi}_T = \frac{1}{\lambda}(\hat{W}_T^U)^{-\gamma}. \tag{47}$$

By market clearing condition (15):

$$\hat{W}_T^U = D_T \Rightarrow \tilde{\pi}_T = \frac{1}{\lambda}D_T^{-\gamma}. \tag{48}$$

Uninformed investors' equilibrium SPD is:

$$\tilde{\pi}_t = e^{r(T-t)}\tilde{E}_t[\tilde{\pi}_T] = e^{r(T-t)}\tilde{E}_t\left[D_T^{-\gamma}\right]. \tag{49}$$

Applying Lemma A1 for $\alpha = -\gamma$, uninformed investors' equilibrium SPD is then:

$$\tilde{\pi}_t = \lambda^{-1}D_t^{-\gamma}\exp\left\{\left(r - \gamma\tilde{\rho}_t + \frac{1+\gamma}{2}\gamma\delta^2 + \frac{\gamma^2}{2}v_t\tau\right)\tau\right\}. \tag{50}$$

Using (50) to solve for λ in the equation $\tilde{\pi}_0 = 1$:

$$\lambda = D_0^{-\gamma}\exp\left\{\left(r - \gamma\tilde{\rho}_0 + \frac{1+\gamma}{2}\gamma\delta^2 + \frac{\gamma^2}{2}v_0 T\right)T\right\}. \tag{51}$$

By no-arbitrage, equilibrium stock prices are:

$$S_t = \tilde{\pi}_t^{-1}\tilde{E}_t[\tilde{\pi}_T D_T] = (\lambda\tilde{\pi}_t)^{-1}\tilde{E}_t\left[D_T^{1-\gamma}\right] \tag{52}$$

Using Lemma A1 for $\alpha = 1-\gamma$ and equation (50):

$$S_t = D_t\exp\left\{\left(\tilde{\rho}_t - r - \gamma\delta^2 - \left(\gamma - \frac{1}{2}\right)v_t\tau\right)\right\}. \tag{53}$$

Applying Itô's Lemma to (53)

$$dS_t = \left(r + \gamma \left(\delta + \frac{v_t}{\delta} \tau \right)^2 \right) S_t dt + \left(\delta + \frac{v_t}{\delta} \tau \right) S_t d\tilde{B}_t. \tag{54}$$

Under the $\tilde{P}$-probability, the stock dynamics (2) can be rewritten as:

$$dS_t = \tilde{\mu}_t S_t dt + \sigma_t S_t d\tilde{B}_t. \tag{55}$$

Comparing the drift and diffusion terms of (54) and (55) we get equations (18). □

Proof of Corollary 1. Equations (19)-(20) follow by letting $\rho_0 \to \rho$ and $v_0 \to 0$ in Proposition 1. To obtain (21), we divide (16) by (19) to get:

$$\frac{S_t}{S_t^{CI}} = \exp \left\{ \left(\tilde{\rho}_t - \rho - \left(\gamma - \frac{1}{2} \right) v_t \tau \right) \tau \right\}. \tag{56}$$

The result then follows from the definition of OV_t. □

Proof of Proposition 2. For an informed direct investor with RRA coefficient $\tilde{\gamma}$, the normal optimization problem is:

$$\max_{W_{\tilde{\gamma},T}} E_0 \left[\frac{(W_{\tilde{\gamma},T})^{1-\tilde{\gamma}}}{1-\tilde{\gamma}} \right], \tag{57}$$

subject to:

$$E_0 \left[\pi_T W_{\tilde{\gamma},T} \right] = w_0. \tag{58}$$

Attaching Lagrange multiplier λ_N to the budget constraint (58),the normal time-T optimal wealth profile is given by the first order condition:

$$W^N_{\tilde{\gamma},T} = (\lambda_N \pi_T)^{-\frac{1}{\tilde{\gamma}}}, \tag{59}$$

where the Lagrange multiplier λ_N is given by:

$$\lambda_N = w_0^{-\tilde{\gamma}} \left(E_0 \left[(\pi_T)^{1-\frac{1}{\tilde{\gamma}}} \right] \right)^{\tilde{\gamma}} = \left(\frac{Z_{1-\frac{1}{\tilde{\gamma}},0,T}}{w_0} \right)^{\tilde{\gamma}}. \tag{60}$$

The normal time-t ($0 \le t \le T$) portfolio value $W^N_{\tilde{\gamma},t}$ is given by the no-arbitrage condition:

$$\begin{aligned} \pi_t W^N_{\tilde{\gamma},t} &= E_t \left[\pi_T W^N_{\tilde{\gamma},T} \right] \\ \Rightarrow W^N_{\tilde{\gamma},t} &= (\lambda_N \pi_t)^{-\frac{1}{\tilde{\gamma}}} E_t \left[\left(\frac{\pi_T}{\pi_t} \right)^{1-\frac{1}{\tilde{\gamma}}} \right] = (\lambda_N \pi_t)^{-\frac{1}{\tilde{\gamma}}} Z_{1-\frac{1}{\tilde{\gamma}},t,T}, \end{aligned} \tag{61}$$

with $Z_{1-\frac{1}{\tilde{\gamma}},t,T} = E_t \left[\left(\frac{\pi_T}{\pi_t} \right)^{1-\frac{1}{\tilde{\gamma}}} \right]$. The following lemma provides a closed-form expression for $Z_{1-\frac{1}{\tilde{\gamma}},t,T}$:

Lemma A3. *Let $\psi \in \mathbb{R}$. For $0 \le t \le t' \le T$:*

$$\begin{aligned} Z_{\psi,t,t'} \equiv & E_t \left[\left(\frac{\pi_{t'}}{\pi_t} \right)^{\psi} \right] \\ = & \delta^{\psi} \sqrt{\frac{(\delta^2 + v_t(t'-t))^{1-\psi}}{\delta^2 + (1-\psi) v_t (t'-t)}} \exp \left\{ -\psi r (t'-t) - \frac{\psi(1-\psi)\delta^2(t'-t)}{\delta^2 + (1-\psi) v_t (t'-t)} \frac{\eta_t^2}{2} \right\} \end{aligned} \tag{62}$$

Proof. We first simplify the expression of the likelihood process ξ_t. Defining $\kappa_t \equiv \frac{\rho-\tilde{\rho}_t}{\delta}$, the likelihood process can be rewritten as:

$$\xi_t = e^{-\frac{1}{2}\int_0^t \kappa_s^2 ds - \int_0^t \kappa_s dB_s}. \tag{63}$$

The manager sees the dynamics of $\tilde{\rho}$ in (7) as:

$$d\tilde{\rho}_t = \frac{v_t}{\delta}\left(\frac{\rho-\tilde{\rho}_t}{\delta} + dB_t\right) = \frac{v_t}{\delta^2}(\rho-\tilde{\rho}_t)dt + \frac{v_t}{\delta}dB_t. \tag{64}$$

An application of Itô's Lemma gives the dynamics of $\frac{\tilde{\rho}_t}{v_t}$ as:

$$d\left(\frac{\tilde{\rho}_t}{v_t}\right) = \frac{\rho}{\delta^2}dt + \frac{1}{\delta}dB_t \tag{65}$$

A further application of Itô's Lemma to the product $\kappa_t\frac{\tilde{\rho}_t}{v_t}$ leads to:

$$\frac{\kappa^2}{2}dt + \kappa_t dB_t = \frac{\delta}{2}d\left(\kappa_t\frac{\tilde{\rho}_t}{v_t}\right) + \frac{1}{2}\left(\frac{v_t}{\delta^2}dt + \frac{\rho}{\delta}dB_t\right). \tag{66}$$

Integrating both side from 0 to t allows us to re-express the likelihood process as:

$$\xi_t = e^{-\frac{1}{2}\int_0^t \kappa_s^2 ds - \int_0^t \kappa_s dB_s} = \sqrt{\frac{v_t}{v_0}}e^{-\frac{\delta}{2}\left(\frac{\rho-\tilde{\rho}_t}{\delta}\frac{\tilde{\rho}_t}{v_t} - \frac{\rho-\tilde{\rho}_0}{\delta}\frac{\tilde{\rho}_0}{v_0}\right) - \frac{\rho}{2\delta}B_t}, \tag{67}$$

or, for $t' \geq t$:

$$\frac{\xi_{t'}}{\xi_t} = \frac{\delta}{\sqrt{\delta^2 + v_t(t'-t)}}e^{-\frac{1}{2}\left((\rho-\tilde{\rho}_{t'})\frac{\tilde{\rho}_{t'}}{v_{t'}} - (\rho-\tilde{\rho}_t)\frac{\tilde{\rho}_t}{v_t} - \frac{\rho}{\delta}(B_{t'}-B_t)\right)}. \tag{68}$$

Integrating both sides of (65) from t to t' and replacing back in (68) we get:

$$\frac{\xi_{t'}}{\xi_t} = \frac{\delta}{\sqrt{\delta^2 + v_t(t'-t)}}e^{\frac{1}{2v_{t'}}\left(\tilde{\rho}_{t'}^2 - 2\rho\tilde{\rho}_{t'}\right) - \frac{1}{2v_t}\left(\tilde{\rho}_t^2 - 2\rho\tilde{\rho}_t\right) + \frac{\rho^2}{2\delta^2}(t'-t)}. \tag{69}$$

Given expressions (10) and (17) for the manager's and uninformed investors' state-price deflators, and equation (69) for the likelihood process, we can write:

$$\begin{aligned} E_t\left[\left(\frac{\pi_{t'}}{\pi_t}\right)^{\psi}\right] =& \delta^{\psi}\sqrt{\delta^2 + (1-\psi)v_t(t'-t)}\exp\left\{-\psi\left(r + \gamma(\rho+\gamma\delta^2) + \frac{\gamma^2}{2}v_t(t'-t) - \gamma\tilde{\rho}_t\right)\right. \\ & \left.\times (t'-t) - \frac{\psi}{2v_t}\left(\tilde{\rho}_t - (\rho+\gamma\delta^2)\right)^2\right\} \\ & \times E_t\left[\exp\left\{\frac{\psi}{2v_{t'}}\left(\tilde{\rho}_{t'} - (\rho+\gamma\delta^2)\right)^2\right\}\right]. \end{aligned} \tag{70}$$

We can work (65) to show that:

$$\tilde{\rho}_{t'} = \tilde{\rho}_t + (\rho-\tilde{\rho}_t)\frac{v_{t'}}{\delta^2}(t'-t) + \frac{v_{t'}}{\delta}(B_{t'}-B_t), \tag{71}$$

so the under P $\tilde{\rho}_{t'}$ is normally distributed with conditional mean and variance:

$$\begin{cases} E_t[\tilde{\rho}_{t'}] = \frac{\delta^2}{\delta^2+v_t(t'-t)}\tilde{\rho}_t + \frac{v_t(t'-t)}{\delta^2+v_t(t'-t)}\rho, \\ Var_t[\tilde{\rho}_{t'}] = \frac{\delta^2 v_t^2}{(\delta^2+v_t(t'-t))^2}(t'-t). \end{cases} \tag{72}$$

We can then rewrite the expectation on the RHS of (70) as:

$$E_t\left[\exp\left\{\frac{\psi}{2v_{t'}}\left(\tilde{\rho}_{t'}-(\rho+\gamma\delta^2)\right)^2\right\}\right]=E_t\left[\exp\left\{\frac{\psi}{2v_{t'}}\left(\tilde{\rho}_{t'}-E_t[\tilde{\rho}_{t'}]\right.\right.\right.$$
$$\left.\left.\left.-\frac{\delta^2}{\delta^2+v_t(t'-t)}(\rho-\tilde{\rho}_t)-\gamma\delta^2\right)^2\right\}\right]. \tag{73}$$

Using Lemma A2 for $z=\tilde{\rho}_{t'}-E_t[\tilde{\rho}_{t'}]$, $\sigma_z^2=Var_t[\tilde{\rho}_{t'}]$, $\check{\rho}=\frac{\psi}{2v_{t'}}$, $c=\frac{\delta^2}{\delta^2+v_t(t'-t)}(\rho-\tilde{\rho}_t)+\gamma\delta^2$,$\bar{z}=+\infty$, we can compute this expectation as:

$$E_t\left[\exp\left\{\frac{\psi}{2v_{t'}}\left(\tilde{\rho}_{t'}-(\rho+\gamma\delta^2)\right)^2\right\}\right]=\sqrt{\frac{\delta^2+v_t(t'-t)}{\delta^2+(1-\psi)v_t(t'-t)}}$$
$$\times\exp\left\{\frac{\psi\frac{\delta^2+v_t(t'-t)}{2v_{t'}}\left(\frac{\delta^2}{\delta^2+v_t(t'-t)}(\rho-\tilde{\rho}_t)+\gamma\delta^2\right)^2}{\delta^2+(1-\psi)v_t(t'-t)}\right\}. \tag{74}$$

Plugging (74) in (70) we get, after some algebraic manipulation, equation (62). □

In order to derive the investment policy (22) replicating the optimal portfolio value (23), note that this can be rewritten as $W^N_{\tilde{\gamma},t}=f(t,\eta_t)$, where the diffusion term σ_η of η can be computed as $\sigma_\eta=-v_t/\delta^2$ and $f\in C^{1,2}$. Applying Itô's Lemma the diffusion term of $dW^N_{\tilde{\gamma},t}$ is:

$$-\frac{v_t}{\delta^2}\frac{\partial W^N_{\tilde{\gamma},t}}{\partial\eta_t}=W^N_{\tilde{\gamma},t}\frac{\delta^2+v_t\tau}{\delta^2+\frac{v_t}{\tilde{\gamma}}\tau}\frac{\eta_t}{\tilde{\gamma}} \tag{75}$$

Equating (75) to the diffusion term of W_t in (3) gives the optimal portfolio (22).

□

Proof of Corollary 2. Equation (25) follows from plugging in the equilibrium values $\eta_t=\tilde{\eta}_t+\frac{\rho-\tilde{\rho}_t}{\delta}$ and $\tilde{\eta}_t=\gamma\sigma_t$ in equation (22), letting $\tilde{\gamma}=\gamma$, subtracting 1 from $\phi^N_{\gamma,t}$ and rearranging. Since $\delta^2+v_t/\gamma\tau$ and γ are positive, the LHS of (25) is negative iff the numerator on the RHS is negative, i.e.:

$$\tilde{\rho}_t>\rho+(\gamma-1)v_t\tau. \tag{76}$$

To obtain condition (26), we apply the natural logarithm on both sides of equation (56) to get:

$$\frac{1}{\tau}\ln\left(\frac{S_t}{S_t^{CI}}\right)-\frac{1}{2}v_t\tau=(\rho-\tilde{\rho}_t)+(\gamma-1)\,v_t\tau. \tag{77}$$

Therefore, condition (76) holds iff condition (26) holds. □

Proof of Proposition 3. Let $\zeta\equiv\bar{\zeta}W_0/Y_0$ be the normalized performance fee threshold, and let

$$U_T(W_T)\equiv\frac{(f_TW_T)^{1-\gamma}}{1-\gamma}. \tag{78}$$

At $t=0$, the problem of the informed money manager is then:

$$\max_{W_T}E_0\left[U_T(W_T)\right]\qquad\text{s.t. }E_0[\pi_TW_T]=w_0. \tag{79}$$

The objective function (78) in the manager's problem (79) is locally non-concave in a neighborhood of $\delta W_T = \zeta Y_T$ Standard optimization techniques cannot be applied directly to this problem. Following Basak and Makarov (2014), the first step consists in constructing the concavification $\tilde{U}_T(.)$ of the manager's utility function $U_T(.)$ (i.e. the smallest concave function $\tilde{U}_T(w)$ satisfying $\tilde{U}_T(w) \geq U_T(w)$ for all $w \geq 0$), restate and solve the original problem (79) in terms of $\tilde{U}_T(.)$.

In order to construct the concavified function, we look for functions $\underline{W}(\zeta Y_T)$, $\overline{W}(\zeta Y_T)$, $a(\zeta Y_T)$ and $b(\zeta Y_T)$ so that (omitting the arguments for notational simplicity):

$$\tilde{U}_T(W_T) = \begin{cases} U_T(W_T), & \text{if } W_T < \underline{W} \leq \zeta Y_T, \\ a + bW_T, & \text{if } \underline{W} \leq W_T < \overline{W}, \\ U_T(W_T), & \text{if } \zeta Y_T \leq \overline{W} \leq W_T, \end{cases} \tag{80}$$

and

$$\tilde{U}_T^{'}(W_T) = \begin{cases} U_T^{'}(W_T), & \text{if } W_T < \underline{W} \leq \zeta Y_T, \\ b, & \text{if } \underline{W} \leq W_T < \overline{W}, \\ U_T^{'}(W_T), & \text{if } \zeta Y_T \leq \overline{W} \leq W_T. \end{cases} \tag{81}$$

where:

$$U_T^{'}(W_T) = \begin{cases} (1+\alpha_1) W_T^{-\gamma_1} (\zeta Y_T)^{\gamma_1 - \gamma}, & \text{if } W_T < \zeta Y_T, \\ (1+\alpha_2) W_T^{-\gamma_2} (\zeta Y_T)^{\gamma_2 - \gamma}, & \text{if } W_T > \zeta Y_T. \end{cases} \tag{82}$$

Eqs. (82) and eqrefeq:MIU give us a system of 4 equations in our 4 unknowns $\underline{W}$, $\overline{W}$, a and b:

$$\begin{cases} a + b\underline{W} = \frac{1}{1-\gamma} \underline{W}^{1-\gamma_1} (\zeta Y_T)^{\gamma_1 - \gamma} \\ a + b\overline{W} = \frac{1}{1-\gamma} \overline{W}^{1-\gamma_2} (\zeta Y_T)^{\gamma_2 - \gamma} \\ b = (1+\alpha_1) \underline{W}^{-\gamma_1} (\zeta Y_T)^{\gamma_1 - \gamma} \\ b = (1+\alpha_2) \overline{W}^{-\gamma_2} (\zeta Y_T)^{\gamma_2 - \gamma}. \end{cases} \tag{83}$$

The solution to this system of equation yields

$$b(\zeta Y_T) = \left[\frac{(1+\alpha_2)^{\gamma_1(\gamma_2 - 1)}}{(1+\alpha_1)^{\gamma_2(\gamma_1 - 1)}} \left(\frac{\gamma_1}{\gamma_2} \right)^{\gamma_1 \gamma_2} \right]^{\frac{1}{\gamma_2 - \gamma_1}} (\zeta Y_T)^{-\gamma}, \tag{84}$$

$$\underline{W}(\zeta Y_T) = \left[\left(\frac{1+\alpha_2}{1+\alpha_1} \right)^{\gamma_2 - 1} \left(\frac{\gamma_2}{\gamma_1} \right)^{\gamma_2} \right]^{\frac{1}{\gamma_2 - \gamma_1}} \zeta Y_T, \tag{85}$$

$$\overline{W}(\zeta Y_T) = \left[\left(\frac{1+\alpha_1}{1+\alpha_2} \right)^{\gamma_1 - 1} \left(\frac{\gamma_2}{\gamma_1} \right)^{\gamma_1} \right]^{\frac{1}{\gamma_2 - \gamma_1}} \zeta Y_T. \tag{86}$$

In order to verify that (84) to (86) are indeed the solutions we are after, it remains to verify that $\underline{W}$ and $\overline{W}$ satisfy the condition:

$$\underline{W} \leq \zeta Y_T \leq \overline{W}, \tag{87}$$

which holds iff:

$$\left[\left(\frac{(1+\alpha_2)}{(1+\alpha_1)} \right)^{\gamma_2 - 1} \left(\frac{\gamma_2}{\gamma_1} \right)^{\gamma_2} \right]^{\frac{1}{\gamma_2 - \gamma_1}} < 1, \tag{88}$$

and

$$\left[\left(\frac{(1+\alpha_1)}{(1+\alpha_2)} \right)^{\gamma_1 - 1} \left(\frac{\gamma_2}{\gamma_1} \right)^{\gamma_1} \right]^{\frac{1}{\gamma_2 - \gamma_1}} > 1. \tag{89}$$

Since

$$\frac{1+\alpha_2}{1+\alpha_1} < \left(\frac{1+\alpha_2}{1+\alpha_1}\frac{\gamma_1}{\gamma_2}\right)^{\gamma_2}, \tag{90}$$

and

$$\frac{1+\alpha_1}{1+\alpha_2} < \left(\frac{1+\alpha_1}{1+\alpha_2}\frac{\gamma_2}{\gamma_1}\right)^{\gamma_1}, \tag{91}$$

both conditions indeed verify.

We can now restate the manager's optimization problem (79) at $t = 0$ as:

$$\max_{W_T} E_0\left[\tilde{U}_T(W_T)\right] \qquad \text{s.t. } E_0[\pi_T W_T] = w_0. \tag{92}$$

Attaching Lagrange multiplier λ_M to the budget constraint, the solution to the concavified problem (92) is given by the standard (state-by-state) first order condition:

$$\tilde{U}_T'(W_T) = \lambda_M \pi_T. \tag{93}$$

Using (82):

$$\lambda_M \pi_T = \begin{cases} (1+\alpha_1) W_T^{-\gamma_1} (\zeta Y_T)^{\gamma_1-\gamma}, & \text{if } W_T < \zeta Y_T, \\ b, & \text{if } \underline{W} \le W_T < \overline{W}, \\ (1+\alpha_2) W_T^{-\gamma_2} (\zeta Y_T)^{\gamma_2-\gamma}, & \text{if } W_T > \zeta Y_T, \end{cases} \tag{94}$$

which gives the manager's optimal time-T AUM as:

$$\hat{W}_T = \begin{cases} (1+\alpha_1)^{\frac{1}{\gamma_1}} (\zeta Y_T)^{\frac{\gamma_1-\gamma}{\gamma_1}} (\lambda_M \pi_T)^{-\frac{1}{\gamma_1}}, & \text{if } W_T < \underline{W}, \\ W \in \left[\underline{W}, \overline{W}\right], & \text{if } \underline{W} \le W_T < \overline{W}, \\ (1+\alpha_2)^{\frac{1}{\gamma_2}} (\zeta Y_T)^{\frac{\gamma_2-\gamma}{\gamma_2}} (\lambda_M \pi_T)^{-\frac{1}{\gamma_2}}, & \text{if } \overline{W} \le W_T. \end{cases} \tag{95}$$

Using Eqs. (84) through (86), we note that:

$$\hat{W}_T < \underline{W} \Leftrightarrow \lambda_M \pi_T > b, \tag{96}$$

and

$$\hat{W}_T \ge \overline{W} \Leftrightarrow \lambda_M \pi_T \le b, \tag{97}$$

which allows us to re-express (95) as:

$$\hat{W}_T = \begin{cases} (1+\alpha_1)^{\frac{1}{\gamma_1}} (\zeta Y_T)^{\frac{\gamma_1-\gamma}{\gamma_1}} (\lambda_M \pi_T)^{-\frac{1}{\gamma_1}}, & \text{if } \lambda_M \pi_T > b \quad (\mathcal{R}_1), \\ (1+\alpha_2)^{\frac{1}{\gamma_2}} (\zeta Y_T)^{\frac{\gamma_2-\gamma}{\gamma_2}} (\lambda_M \pi_T)^{-\frac{1}{\gamma_2}}, & \text{if } \lambda_M \pi_T \le b \quad (\mathcal{R}_2). \end{cases} \tag{98}$$

In order to define regions $\mathcal{R}_1$ and $\mathcal{R}_2$, we need to obtain an explicit expression for the benchmark Y_T. This can be done more easily by first writing the dynamics of Y_t under the uninformed investors' probability $\tilde{P}$:

$$dY_t = Y_t \left(r + \phi^Y \sigma_t \tilde{\eta}_t\right) dt + Y_t \phi^Y \sigma_t d\tilde{B}_t, \tag{99}$$

which implies:

$$
\begin{aligned}
Y_T &= Y_t \exp\left\{ r\tau + \phi^Y\left(\gamma - \frac{\phi^Y}{2}\right)\int_t^T \sigma_s^2 ds + \phi^Y \int_t^T \sigma_s d\tilde{B}_s \right\} \\
&= Y_t \exp\left\{ \left(r + \phi^Y\left(\gamma - \frac{\phi^Y}{2}\right)(\delta^2 + v_t\tau)\phi^Y \right)\tau + \frac{\delta^2\phi^Y}{v_T}(\tilde{\rho}_T - \tilde{\rho}_t) \right\}, \qquad (100)
\end{aligned}
$$

where we used expressions (7) and (18) to solve for the integrals in (100). Defining:

$$
\zeta_0 \equiv \left[\frac{(1+\alpha_2)^{\gamma_1(\gamma_2-1)}}{(1+\alpha_1)^{\gamma_2(\gamma_1-1)}} \left(\frac{\gamma_1}{\gamma_2}\right)^{\gamma_1\gamma_2} \right]^{\frac{1}{\gamma_2-\gamma_1}}, \qquad (101)
$$

we can express $b(\zeta Y_T) = \zeta_0 (\zeta Y_T)^{-\gamma}$. Region $\mathcal{R}_1$ is then given by:

$$
\lambda_M \pi_T > b(\zeta Y_T) \Leftrightarrow \lambda_M \pi_T > \zeta_0 (\zeta Y_T)^{-\gamma}. \qquad (102)
$$

Using the above closed-form expressions for π_T and Y_T we can express region $\mathcal{R}_1$ as:

$$
\left\{\tilde{\rho}_T < \rho + (1-\phi^Y)\gamma\delta^2 + \Gamma\right\} \cup \left\{\tilde{\rho}_T > \rho + (1-\phi^Y)\gamma\delta^2 + \Gamma\right\}, \qquad (103)
$$

where:

$$
\Gamma \equiv \sqrt{v_T \Delta(\tilde{\rho}_0, v_0)}, \qquad (104)
$$

and

$$
\begin{aligned}
\Delta(\tilde{\rho}_0, v_0) \equiv& \frac{1}{v_0}\left(\tilde{\rho}_0 - \rho - (1-\phi^Y)\gamma\delta^2\right)^2 \\
&+ 2(1-\phi^Y)\gamma\left\{ \ln\left(\frac{D_0}{\beta_0}\right) - \left[r + \left(1 - (1-\phi^Y)\gamma\right)\frac{\delta^2}{2} - \rho \right] T \right\} \\
&+ 2\ln\left(\frac{\lambda\zeta_0}{\lambda_M \delta \eta^\gamma} \sqrt{\delta^2 + v_0 T} \right). \qquad (105)
\end{aligned}
$$

The existence of a solution to the manager's problem (79) requires $\Delta(\tilde{\rho}_0, v_0) > 0$, implying $\Gamma \geq 0$. Region $\mathcal{R}_2$ is just the relative complement in $\mathbb{R}$ of $\mathcal{R}_1$. We can now derive the interim AUM (106). By no-arbitrage, the deflated wealth process $\pi_t \hat{W}_t$ is a martingale, so using (98) the optimal wealth $\hat{W}_t$ for all $t \in [0, T]$ is given by:

$$
\begin{aligned}
\pi_t \hat{W}_t &= E_t\left[\pi_T \hat{W}_T\right] \\
\Rightarrow \hat{W}_t &= f_{1,t} + f_{2,t}, \qquad (106)
\end{aligned}
$$

where :

$$
f_{i,t} = \pi_t E_t\left[(1+\alpha_i)^{\frac{1}{\gamma_i}} (\zeta Y_T)^{\frac{\gamma_i - \gamma}{\gamma_i}} (\lambda_M \pi_T)^{-\frac{1}{\gamma_i}} \mathbb{1}_{\mathcal{R}_i} \right]. \qquad (107)
$$

Using the closed-form expressions above for π_T and Y_T, $\mathcal{R}_1$ and $\mathcal{R}_2$, and applying Lemma A2 to compute the expectation in (107) we get, for $i = 1, 2$:

$$f_{i,t} \equiv \left(\frac{1+\alpha_i}{\lambda_M \xi_t}\right)^{\frac{1}{\gamma_i}} \zeta^{1-\frac{\gamma}{\gamma_i}} \left(\frac{\delta}{\lambda}\right)^{1-\frac{1}{\gamma_i}} \sqrt{\frac{(\delta^2 + v_t\tau)^{\frac{1}{\gamma_i}}}{\delta^2 + \frac{v_t}{\gamma_i}\tau}} \beta_t^{\left(1-\frac{\gamma}{\gamma_i}\right)(1-\phi^Y)} D_t^{\left(1-\frac{\gamma}{\gamma_i}\right)\phi^Y - \left(1-\frac{1}{\gamma_i}\right)\gamma} \tilde{\pi}_t^{-1}$$
$$\times \exp\left\{\left[\left(1-\frac{\gamma}{\gamma_i}\right)\left(1-\phi^Y\right) r - \left(1-\frac{1}{\gamma_i}\right)\left(k_i\left(\rho - ((\gamma_i - 1)k_i + 1)\frac{\delta^2}{2}\right) + \frac{1}{2\gamma_i}\frac{\left(\rho - \tilde{\rho}_t - (\gamma_i - 1)k_i\delta^2\right)^2}{\delta^2 + \frac{v_t}{\gamma_i}\tau}\right)\right]\tau\right\} \Pi_{i,t}, \tag{108}$$

The Lagrange multiplier λ_M is the solution to the equation $\hat{W}_0 = w_0$.

In order to derive the investment policy (28) replicating the optimal portfolio value (106), note that this can be rewritten as $\hat{W}_t = h(t, D_t, \tilde{\pi}_t, \xi_t, X_{1,t}, X_{2,t}, \underline{d}_{1,t}, \underline{d}_{2,t}, \overline{d}_{1,t}, \overline{d}_{2,t})$, where for $i = 1, 2$:

$$X_{i,t} \equiv \exp\left\{\left[\left(1-\frac{\gamma}{\gamma_i}\right)\left(1-\phi^Y\right) r - \left(1-\frac{1}{\gamma_i}\right)\left(k_i\left(\rho - ((\gamma_i - 1)k_i + 1)\frac{\delta^2}{2}\right) + \frac{1}{2\gamma_i}\frac{\left(\rho - \tilde{\rho}_t - (\gamma_i - 1)k_i\delta^2\right)^2}{\delta^2 + \frac{v_t}{\gamma_i}\tau}\right)\right]\tau\right\}, \tag{109}$$

for some function $h \in C^{1,2}$. Applying Itô's Lemma the diffusion term $\sigma_{\hat{W}}$ of $d\hat{W}_t$ is:

$$\sigma_{\hat{W}} = \sigma_D h_D + \sigma_{\tilde{\pi}} h_{\tilde{\pi}} + \sigma_\xi h_\xi + \sigma_{X_1} h_{X_1} + \sigma_{X_2} h_{X_2} + \sigma_{\underline{d}_1} h_{\underline{d}_1} + \sigma_{\underline{d}_2} h_{\underline{d}_2} + \sigma_{\overline{d}_1} h_{\overline{d}_1} + \sigma_{\overline{d}_2} h_{\overline{d}_2}, \tag{110}$$

where h_x denotes the partial derivative of h w.r.t. x and σ_X is the diffusion term in the SDE characterizing the dynamics of the process X. Computing the diffusion terms in (110) explicitly and equating the result to the diffusion term of W_t in (3) gives the optimal portfolio (28).

□

Proof of Corollary 3. From equation (29), for $i = 1, 2$ the sign of each risk-shifting component $\Phi_{i,t}$ equals the sign of $\mathcal{N}'(\underline{d}_{i,t}) - \mathcal{N}'(\overline{d}_{i,t})$. By the symmetry of the standard normal density, $\mathcal{N}'(\underline{d}_{i,t}) \geq \mathcal{N}'(\overline{d}_{i,t})$ if and only if $|\underline{d}_{i,t}| \leq |\overline{d}_{i,t}|$. Since $\underline{d}_{i,t} < \overline{d}_{i,t}$,

$$\begin{aligned} |\underline{d}_{i,t}| \leq |\overline{d}_{i,t}| &\Leftrightarrow \underline{d}_{i,t} + \overline{d}_{i,t} \geq 0 \\ &\Leftrightarrow 2\frac{\gamma\delta^2 \underline{\sigma}_{\gamma,t}}{v_t\sqrt{\tau}} \sqrt{\frac{\delta^2 + v_t\tau}{\delta^2 + \frac{v_t}{\gamma_i}\tau}} \left(\phi^N_{\gamma,t} - \phi^Y\right) \geq 0. \end{aligned} \tag{111}$$

Since the factor multiplying the difference $(\phi^N_{\gamma,t} - \phi^Y)$ above is positive, we conclude that $\mathcal{N}'(\underline{d}_{i,t}) \geq \mathcal{N}'(\overline{d}_{i,t})$ if and only if $\phi^N_{\gamma,t} \geq \phi^Y$. Thus, for $i = 1, 2$, $\operatorname{sgn}(\Phi_{i,t}) = \operatorname{sgn}(\phi^N_{\gamma,t} - \phi^Y)$, which leads to equation (31).

□

Proof of Proposition 4. Following Cuoco and Kaniel (2011), the time-t optimal wealth of a manager perceiving the compensation (33) is:

$$\hat{W}_T = \begin{cases} (kT)^{\frac{1}{\gamma}-1}(\lambda_M \pi_T)^{-\frac{1}{\gamma}}, & \text{if } D_T < \underline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right), \\ P_T \underline{W} + (1 - P_T)\overline{W}, & \text{if } \underline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right) \leq D_T < \overline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right), \\ (kT(1+\alpha_2))^{\frac{1}{\gamma}-1}(\lambda_M \pi_T)^{-\frac{1}{\gamma}} + \frac{\alpha_2}{1+\alpha_2}\zeta\beta_T, & \text{if } \overline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right) \leq D_T, \end{cases} \tag{112}$$

where λ_M is the Lagrange multiplier attached to the manager's budget constraint in (79),

$$\underline{D}(\xi) = \left((kT)^{\frac{1}{\gamma}-1} + \xi^{\frac{1}{\gamma}}\right)(kT(1+\alpha_2))^{1-\frac{1}{\gamma}} \frac{\gamma-1}{\gamma} \frac{1}{\kappa-1} \frac{\alpha_2}{1+\alpha_2} \zeta \beta_T, \tag{113}$$

$$\overline{D}(\xi) = \left((kT(1+\alpha_2))^{\frac{1}{\gamma}-1} + \xi^{\frac{1}{\gamma}}\right)(kT(1+\alpha_2))^{1-\frac{1}{\gamma}} \left(\frac{\gamma-1}{\gamma} \frac{1}{\kappa-1} + 1\right) \frac{\alpha_2}{1+\alpha_2} \zeta \beta_T, \tag{114}$$

$$\underline{W} = \frac{\gamma-1}{\gamma} \frac{\kappa}{\kappa-1} \frac{\alpha_2}{1+\alpha_2} \zeta \beta_T, \tag{115}$$

$$\overline{W} = \left(\frac{\gamma-1}{\gamma} \frac{1}{\kappa-1} + 1\right) \frac{\alpha_2}{1+\alpha_2} \zeta \beta_T, \tag{116}$$

and the equilibrium randomizing probability P_T is:

$$P_T = \frac{\overline{W} + \left(\frac{\lambda_M}{\lambda_U}\xi_T\right)^{\frac{1}{\gamma}} (kT(1+\alpha_2))^{1-\frac{1}{\gamma}} \frac{\gamma-1}{\gamma} \frac{1}{\kappa-1} \frac{\alpha_2}{1+\alpha_2} \zeta \beta_T - D_T}{\overline{W} - \underline{W}} \times \mathbb{1}_{\left\{\underline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right) \le D_T < \overline{D}\left(\frac{\lambda_M}{\lambda_U}\xi_T\right)\right\}} \tag{117}$$

Attaching a Lagrange multiplier λ_U to the U-investor's budget constraint, the optimal time-t wealth of a U-investor is:

$$\hat{W}_T^U = (\lambda_U \tilde{\pi}_T)^{-\frac{1}{\gamma}} = (\xi_T^{-1} \lambda_U \tilde{\pi}_T)^{-\frac{1}{\gamma}}, \tag{118}$$

Where we use the Abstract Bayes Theorem for the likelihood process ξ_t for the second equality in (118). Using the market clearing condition (34) and rearranging, we obtain the equilibrium SPD (37). $\square$

B Parameterization of Money Managers' Incentives

B.1 (Piece-wise) Linearization of Hedge Fund Managers' Incentives

Given the continuously-compounded rates r_T, r_T^Y and the threshold $\bar{\zeta} = e^{hT}$ as defined in Section 3.2.3, for any $\alpha > 0$ we can write:

$$k\left(\frac{R_T}{\bar{\zeta}R_T^Y}\right)^{\alpha} = ke^{\alpha(r_T-(r+h))T}. \tag{119}$$

A first-order approximation of the RHS of (119) around $r_T = r + h$ gives:

$$k\left(\frac{R_T}{\bar{\zeta}R_T^Y}\right)^{\alpha} \approx kT + k\alpha(r_T - (r+h))T, \tag{120}$$

Applying (120) to the two terms in the RHS of (5) and setting $\alpha_1 = 0$ implies a fee rate:

$$f_T \approx kT + kT\alpha_2(r_T - (r+h))^+, \tag{121}$$

where $x^+ \equiv \max(0, x)$.

B.2 Flow-Performance Relationship of Mutual Funds

Most mutual funds in the U.S. charge a management fee proportional to AUM but no performance fees. Let m be this base fee. We assume that At $t = T$, but at no other $0 \le t < T$, mutual fund investors purchase or redeem additional fund shares depending on the manager's performance during $[0, T]$ relative to the benchmark Y according to an exogenously given flow-to-relative performance relationship (F-PR) q_T:

$$q_T = \underline{q}\left(\frac{R_T}{\bar{\zeta}R_T^Y}\right)^{\alpha_1} \mathbb{1}_{\{R_T < \bar{\zeta}R_T^Y\}} + \underline{q}\left(\frac{R_T}{\bar{\zeta}R_T^Y}\right)^{\alpha_2} \mathbb{1}_{\{R_T \ge \bar{\zeta}R_T^Y\}}, \tag{122}$$

with $\underline{q} > 0$. Defining $k \equiv m\underline{q}$, the mutual fund manager's fee rate $f_T = mq_T$ follows the specification (5). This functional form allows flows to be sensitive (and potentially locally concave, if $\alpha_1 < 1$) to medium and low relative performance. At the same time, f_T reflects the well-documented convexity in the sensitivity of flows to performance (see, e.g., Chevalier and Ellison (1997), and Sirri and Tufano (1998)) for $\alpha_1 < \alpha_2$, according to which outperforming funds receive a disproportionally high amount of inflows.[28] The F-PR (5) can also capture linear relationships ($\alpha_1 = \alpha_2 = 1$), log-linear relationship ($\alpha_1 = \alpha_2 \ne 1$), as well as no relationship ($\alpha_1 = \alpha_2 = 0$). Although our theoretical results in Section 3.2 apply to general benchmarks, we specialize our analysis to the case of an all-equity mutual fund for which $\phi^Y = 1$.

We choose our parameterization of the fee rate f_T to reflect the typical flow-performance relationships in the mutual fund industry.[29] Specifically, we assume a moderately high flow sensitivity to top performance, no sensitivity of flows to bottom performance, and high sensitivity to medium performance: $\alpha_1 = 0, \alpha_2 = 1.5, \bar{\zeta} = .94$.

---

[28] While many empirical studies document no sensitivity of flows to poor relative past performance (e.g. Sirri and Tufano (1998)), many others (e.g. Huang, Wei, and Yan (2007)) find it is positive, although lower than the sensitivity to medium or high relative returns.

[29] See, e.g., Berk and Green (2004) and Huang, Wei, and Yan (2007) for models that identify mutual fund characteristics associated with differences in the flow relationship. The latter authors provide empirical support for their predictions.