The Welfare Impact of High Frequency Trading*

Preliminary draft

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Abstract

We show in a dynamic trading model that market fragmentation, induced by an informational friction resulting from high frequency trading, may generate market instability (flash crashes) and deleterious welfare consequences from increased trading platform competition. In this context an increase in the mass of dealers with continuous presence in the market can decrease liquidity and welfare. However, with transparent markets, the market is stable, and maximal market participation induces the highest levels of liquidity and welfare.

Keywords: Market fragmentation, high frequency trading, flash crash, welfare, asymmetric information, endogenous market structure.

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1 Introduction

Technology and deregulation have paved the way for financial market fragmentation, in two different dimensions. Along a "cross-sectional" dimension, competition among trading platforms has led to the fragmentation in the supply of trading services.[1] Along a "time-series" dimension, the increased automation of the trading process has induced the fragmentation of liquidity supply, in that some liquidity providers' market participation is limited (Duffie (2010) and SEC (2010)), endogenous (Anand and Venkataraman (2015)), or impaired by the existence of limits to the access of reliable and timely market information (Ding, Hanna, and Hendershott (2014)).[2] Fragmentation and recurrent "flash crash" episodes have increased regulatory concerns about the potential for market fragility.[3]

Is fragmentation detrimental for market quality? How does it impact the welfare of market participants? Under what conditions can it induce "flash crash" episodes? Are these episodes an upshot of excessive competition among exchanges, or are they due to "excessive" liquidity provision fragmentation?

We analyze the equilibrium and welfare implications of fragmentation. In our baseline model two classes of risk-averse dealers provide liquidity to two cohorts of risk-averse, short-term traders who receive an endowment shock, in a two-period market. In the first round of trade both dealers' types absorb the (market) orders of the first traders' cohort. In the second trading round, only one class of dealers, named 'full,' is able to participate. Full dealers, like stylized High Frequency Traders (HFT), are continuously in the market and can therefore accommodate the reverting orders of the first traders' cohort, as well as those of the incoming second cohort who observe an imperfect signal about the first period order imbalance. We then embed the baseline model in a simple platform competition setup in which exchanges compete in the supply of trading services (co-location capacity). In this framework we endogenize the decision of a dealer to acquire the technology to be continuously in the market, and the

[1]This, in turn, has spurred the fragmentation of trading volume across different venues. In the US, equity trading occurs in 11 exchanges and more than 50 ATS (Alternative Trading Systems, which include crossing-networks and dark pools). As a consequence, traditional markets, such as the NYSE, have lost market share to new entrants. For example, the fraction of NYSE-listed stocks actually traded at the NYSE went from about 80% in 2004 to something close to 20% in 2009 (see O'Hara (2015), Pagnotta and Philippon (2015), and SEC (2013)). Fragmentation affects equity trading across the world. In Europe, the fraction of total turnover on the Stoxx Europe 600 Index stocks traded in Regulated Markets (the incumbents) went from about 64% in 2008 to 45% in February 2011. The lost market share was absorbed by the Multilateral Trading Facilities (MTFs, the entrants introduced by the MiFID in 2007, equivalent to ECNs in the US), whose market share went from close to 0 in 2008 to about 18% in 2011 (see Fioravanti and Gentile (2011)). According to FIDESSA, incumbents' market share in EU countries ranges from 51% to 77%, while the total market share of then 2nd and 3rd closest competitors (most of the time both MTFs) ranges from 17% to 38%.

[2]Ding, Hanna, and Hendershott (2014) argue that in the U.S. "...not all market participants have equal access to trade and quote information. Both physical proximity to the exchange and the technology of the trading system contribute to the latency."

[3]The list of events where markets suddenly crash and recover is by now quite extensive. Starting with the May 6, 2010 U.S. "flash-crash" where U.S. equity indices dropped by 5-6% and recovered within half an hour; moving to the October 15, 2014 Treasury Bond crash, where the yield on the benchmark 10-year U.S. government bond, dipped 33 basis points to 1.86% and reversed to 2.13% by the end of the trading day; to end with the August 25, 2015 ETF market freeze, during which more than a fifth of all U.S.-listed exchange traded funds and products were forced to stop trading. More evidence of flash events is provided by NANEX.

number of exchanges supplying trading services. Thus, our setup captures the two features of fragmentation discussed above and enables us to pin down the structure of two industries: HFT and trading services.

A central finding of our analysis is that dealers' limited market participation favors the propagation of endowment shocks across time. This is because when first period traders load their positions, part of their orders are absorbed by standard dealers. These agents, however, are not in the market in the second period, when first period traders unwind. As a consequence, an order imbalance (induced by first period traders' unwinding orders and) affecting the second period price, arises. As standard dealers are unable to rebalance in the second period, they require a larger price concession to absorb traders' orders. This implies that as liquidity dries up, standard dealers absorb more of the imbalance, magnifying the propagation effect.

We first study a benchmark market in which second period traders have access to a perfect signal on the first period imbalance. This situation is likely to arise at low trading frequencies (e.g., intradaily), or in a transparent setup where all market participants have access to the same type of feed, even at high frequencies. In this case we show that in the unique equilibrium of the market, maximizing the mass of full dealers always has a beneficial effect on market liquidity and total welfare. Furthermore, an increase in the number of competing exchanges implements this outcome. When exchanges bear a fixed set up cost, however, our preliminary results show that, with transparent markets, provision of co-location services is insufficient despite the fact that a regime of platform free entry improves over a monopoly exchange solution and yields excessive entry when platforms compete à la Cournot in co-location capacity. If the regulator can control the platform fee, then entry is allowed basically as in the market; if it can control entry, then the latter is restricted. In both cases the provision of co-location services is increased.

In contrast, when the market is opaque—for example because access to imbalance information is impaired—a self-sustaining loop leading to multiple equilibria can arise. To see this, note that due to propagation, second period traders speculate against the imbalance generated by their first period peers the more, the stronger is such propagation. Suppose now that liquidity evaporates in the first period market. As a consequence, standard dealers intermediate more of the outstanding imbalance, magnifying the propagation of the first period endowment shock, and leading second period traders to trade more aggressively against it. However, as information on the first period imbalance is noisy, these trades increase the first period uncertainty about the second period price. This can lead first period traders to consume more liquidity (as holding exposure to the asset becomes riskier), and liquidity suppliers to charge more to absorb the order imbalance (as their inventory of the risky asset increases), eventually reinforcing the initial shock to market liquidity.

Multiplicity induces three levels of liquidity, and tends to occur when second period traders trade aggressively against the propagated imbalance, with a signal of intermediate precision. This is because in this case such trades have a powerful uncertainty creation effect on the second period price, which magnifies the reaction of first period traders.

The presence of multiple equilibria highlights that market liquidity can be fragile in our setup. We show this using two numerical examples. In the first one we study the consequence of a shock that disconnects a small mass of full dealers from the market (a technological 'glitch'). In our second example, we analyze the effect of a positive shock to the volatility of first period traders' demand, which captures an increase in the likelihood that a large, liquidity-consuming order hits the market. Both our examples show that the effects of these shocks can move the market from the high to the low liquidity equilibrium, generating large liquidity withdrawals.

We then focus on the case in which the market is strongly opaque, in that second period traders' imbalance information is so noisy to become useless. In this case, second period traders refrain from speculating on the propagated imbalance, and equilibrium uniqueness is reestablished. This equilibrium has a number of interesting properties. First, it features a higher liquidity level than the one that obtains with transparency. Next, along this equilibrium, liquidity can decrease in the mass of full dealers. Finally, total welfare may not necessarily be maximized when only full dealers' are in the market.

To understand the first two results, note that in our setup liquidity measures the risk compensation that dealers demand to hold the asset inventory. In turn, such inventory depends on the interaction between full dealers 'speculative' trades and the hedging needs of first period traders. Indeed, as the first period endowment shock has a predictable impact on the second period price, full dealers devote part of their activity to speculate on it. This partially offsets traders' orders, lowering dealers' inventory, and improving liquidity. However, first period trading decisions depend on the anticipated volatility of the second period price, which is also affected by second period trades.

When the market is transparent, second period traders face little price uncertainty, and hedge more aggressively their endowment shock.[4] This increases the second period price volatility, inducing first period traders to hedge more, and full dealers to speculate less, ultimately having a negative impact on market liquidity. In contrast, when the market is opaque, second period traders face higher price uncertainty, which leads them to scale down their hedging, and having a beneficial effect on price volatility, which improves the liquidity of the market.

Consider now the effect of an increase in the mass of full dealers in an opaque market. In such a market, as argued above, first period traders face little price uncertainty, and owing to the propagation effect can predict the second period price. This implies that they find it profitable to hold a larger part of their endowment, to benefit from the potential capital gain. As the mass of full dealers increases, however, less of the first period endowment shock propagates to the second period, which impairs first period traders forecast. As a consequence, these traders can start holding a smaller fraction of their endowment, consuming more liquidity.

Finally, increased full dealer participation can have a negative effect on first period traders' welfare. Indeed, as argued above, because of the propagation effect these traders enjoy a capital gain that has a positive impact on their utility. When the mass of full dealers increases, less of the first period shock propagates to the second period, which lowers the capital gains component of these traders' utility. When the market is strongly opaque, we find that this effect can be so

[4]As their signal is perfect, they can exactly anticipate the price at which their order is executed.

strong to make aggregate welfare higher with a limited presence of full dealers. In such cases, increasing full dealers' market participation may not only impair liquidity, but also reduce total welfare.

The rest of the paper is organized as follows. In the next section we review the literature related to the paper. We introduce the model, and show that with limited market participation, endowment shocks propagate across trading dates. Next, we analyze the benchmark with a transparent market, which yields the result that an increase in trading platform competition is both liquidity and welfare improving. We then illustrate how the presence of an informational friction can generate a loop responsible for equilibrium multiplicity and for liquidity fragility. Finally, we turn the attention to the strongly opaque market case, and show that most of the conclusions obtained in the transparent market benchmark may not necessarily hold.

2 Related literature

This paper is related to four strands of the literature. First, equilibrium multiplicity, liquidity complementarities, and liquidity fragilities are known to obtain in economies where asset prices are driven by fundamentals information and noise trading (see, e.g., Cespa and Foucault (2014), Cespa and Vives (2015), Goldstein, Li, and Yang (2014), and Goldstein and Yang (2014)). In this setup, in contrast, asset prices are exclusively driven by endowment shocks. However, the demand of all the traders is responsive to the volatility of the price at which these agents unwind their positions. In turn, such volatility depends on traders' demand. As we argued above, this two-sided loop—which in a noise traders' economy cannot possibly arise—is partly responsible for the multiplicity result. Other authors obtain multiple equilibria in setups where order flows are driven by only one type of information (see, e.g., Spiegel (1998)). However, multiplicity there arises from the bootstrap nature of expectations in the steady-state equilibrium of an overlapping generations (OLG) model in which investors live for two periods. Our setup, in contrast, considers an economy with a finite number of trading rounds.

Second, this paper is also related to the literature that assesses the impact of high frequency trading on market performance. The HFT literature has concentrated on modeling risk neutral agents (e.g., Hoffmann (2014), and Du and Zhu (2014)), and typically does not explicitly address welfare (see, e.g. Foucault, Hombert, and Rosu (2015), Baruch and Glosten (2013), Menkveld and Zoican (2015), Bongaerts and Van Achter (2015), and Aït-Sahalia and Saglam (2013); see O'Hara (2015) for a survey). In a calibrated model, Jovanovic and Menkveld (2015) find that HFT improves liquidity provision and even welfare. Budish, Cramton, and Shim (2014) in contrast, argue that HFT thrives in the continuous limit order book, which is however a flawed market structure, that generates a socially wasteful arms' race to respond fuller to (symmetrically observed) public signals. The authors advocate a switch to frequent batch auctions instead of a continuous market. Biais, Foucault, and Moinas (2015) study the welfare implications of investment in the acquisition of HFT technology. In their model HFTs have a superior ability to match orders, and possess superior information compared to human (slow) traders. They find excessive incentives to invest in HFT technology, which, in view of the negative externality generated by HFT, can be welfare reducing. Pagnotta and Philippon (2015) find that competition among exchanges increases investor participation but may lead to excessive fragmentation and entry in trading venues.

Third, the paper relates to the literature that measures the economic impact of limited market participation. Heston, Korajczyk, and Sadka (2010) and Bogousslavsky (2014) find that some liquidity providers' limited market participation can have implications for return predictability. Chien, Cole, and Lustig (2012) focus instead on the time-series properties of risk premium volatility. Finally, Hendershott, Li, Menkveld, and Seasholes (2014) concentrate on the effect of limited market participation for price departures from semi-strong efficiency.

Fourth, by highlighting the first order asset pricing impact of uninformed traders' imbalance predictability, this paper shares features of our previous work (Cespa and Vives (2012)). In that setup, however, predictability obtained because of the assumed statistical properties of noise traders' demands, whereas in this paper it arises endogenously, because of a participation friction. A growing literature investigates the asset pricing implications of noise trading predictability. Collin-Dufresne and Vos (2015) argue that informed traders time their entry to the presence of noise traders in the market. This, in turn, implies that standard measures of liquidity (e.g., Kyle's lambda), may fail to pick up the presence of such traders. Peress and Schmidt (2015) estimate the statistical properties of a noise trading process, finding support for the presence of serial correlation in demand shocks.

3 The model

A single risky asset with liquidation value $v \sim N(0, \tau_v^{-1})$, and a risk-less asset with unit return are exchanged in a market during two trading rounds. Three classes of traders are in the market. First, a continuum of competitive, risk-averse, High Frequency Traders (which we refer to as "Full Dealers" and denote by FD) in the interval $(0, \mu)$, are active at both dates. Second, competitive, risk-averse dealers (D) in the interval $[\mu, 1]$, are active only in the first period. Finally, a unit mass of short-term traders enters the market at date 1. At date 2, these traders unwind their position, and are replaced by a new cohort of short-term traders (of unit mass). The asset is liquidated at date 3.

We now illustrate the preferences and orders of the different players.

3.1 Liquidity providers

A FD has CARA preferences (we denote by γ his risk-tolerance coefficient) and submits price-contingent orders x_t^{FD}, $t = 1, 2$, to maximize the expected utility of his final wealth: $W^{FD} = (v - p_2)x_2^{FD} + (p_2 - p_1)x_1^{FD}$.[5] A Dealer also has CARA preferences with risk-tolerance γ, but is in the market only in the first period. He thus submits a price-contingent order x_1^D to maximize

[5]We assume, without loss of generality with CARA preferences, that the non-random endowment of FDs and dealers is zero. Also, as equilibrium strategies will be symmetric, we drop the subindex i.

the expected utility of his wealth $W^D = (v - p_1)x_1^D$. The inability of D to trade in the second period is a way to capture limited market participation in our model. This friction could be due to technological reasons (as, e.g. in the case of standard dealers with impaired access to a technology that allows trading at high frequencies).

3.2 Short-term traders

In the first period a unit mass of short-term traders is in the market. A short-term trader receives a random endowment of the risky asset u_1 and posts a market order x_1^L anticipating that it will unwind its holdings in the following period, and leave the market. We assume $u_1 \sim N(0, \tau_{u_1}^{-1})$, and $\mathrm{Cov}[u_1, v] = 0$. First period traders have identical CARA preferences (we denote by γ_1^L the common risk-tolerance coefficient). Formally, a trader maximizes the expected utility of his "short-term" profit $\pi_1^L = u_1 p_2 + (p_2 - p_1)x_1^L$:

$$E\left[-\exp\{-\pi_1^L/\gamma_1^L\}|\Omega_1^L\right],$$

where Ω_1^L denotes his information set. In period 2, first period traders are replaced by a new (unit) mass of traders receiving a random endowment of the risky asset $u_2 \sim N(0, \tau_{u_2}^{-1})$, where $\mathrm{Cov}[u_2, v] = \mathrm{Cov}[u_2, u_1] = 0$. A second period trader has CARA utility function with risk-tolerance γ_2^L, and submits a market order to maximize the expected utility of his profit $\pi_2^L = u_2 v + (v - p_2)x_2^L$:

$$E\left[-\exp\{-\pi_2^L/\gamma_2^L\}|\Omega_2^L\right],$$

where Ω_2^L denotes his information set.[6]

3.3 Information sets

We now describe the information sets of the different market participants. At equilibrium, we conjecture that a period 1 trader submits an order $x_1^L = b_1^L u_1$, where b_1^L denotes the first period "hedging" aggressiveness, to be determined in equilibrium, while a FD and a dealer respectively post a limit order $x_1^{FD} = \varphi_1^{FD}(p_1)$, $x_1^D = \varphi_1^D(p_1)$ where $\varphi_1^{FD}(\cdot)$, $\varphi_1^D(\cdot)$ are linear functions of p_1. In the second period, we assume that a FD submits a limit order $x_2^{FD} = \varphi_2(p_1, p_2)$, where $\varphi_2(\cdot)$ is a linear function of prices. A second period trader observes a signal of the first period endowment shock $s_{u_1} = u_1 + \eta$, with $\eta \sim N(0, \tau_\eta^{-1})$, and independent from all the other random variables in the model, and submits a market order $x_2^L = b_{21}^L u_2 + b_{22}^L s_{u_1}$, where b_{21}^L and b_{22}^L denote respectively the second period hedging and speculative aggressiveness. With these assumptions, we obtain

Lemma 1. *At equilibrium, p_1 is observationally equivalent to u_1, and the sequence $\{p_1, p_2\}$ is observationally equivalent to $\{u_1, x_2^L\}$.*

[6] Our results are robust to the case in which the first period market is populated by a mass β of short-term traders, that unwind at date 2, and a mass $(1 - \beta)$ of long-term ones that hold their position until liquidation.

A first period trader observes the endowment shock u_1. Therefore, his information set coincides with the one of Ds and FDs: $\Omega_1^L = \Omega_1^{FD} = \Omega_1^D = \{u_1\}$. A second period trader receives an endowment shock u_2, and can observe a signal s_{u_1}. Thus, his information set is $\Omega_2^L = \{u_2, s_{u_1}\}$. Finally, a FD in period 2 observes the sequence of prices: $\Omega_2^{FD} = \{p_1, p_2\}$ from which he retrieves $\{u_1, x_2^L\}$.

This model captures the time dimension of fragmentation we discussed in the introduction. Indeed, liquidity provision is fragmented because (i) only one class of dealers is able to participate in the second period and (ii) some traders (the second cohort of short-term traders) have access to opaque information on the first period price. This assumption is consistent with the evidence that exchanges sell fuller access to their matching engine, as well as direct feeds of their market information at a premium (see, e.g., O'Hara (2015)).[7] To account for the time-series dimension of fragmentation, we assume that before the first trading round (date 0), N exchanges compete in the supply of co-location services to FDs. At the same date, dealers decide whether to acquire the technology to be continuously in the market (we defer the details of this part of the model to Section 4.1). Figure 1 displays the timeline of the model.

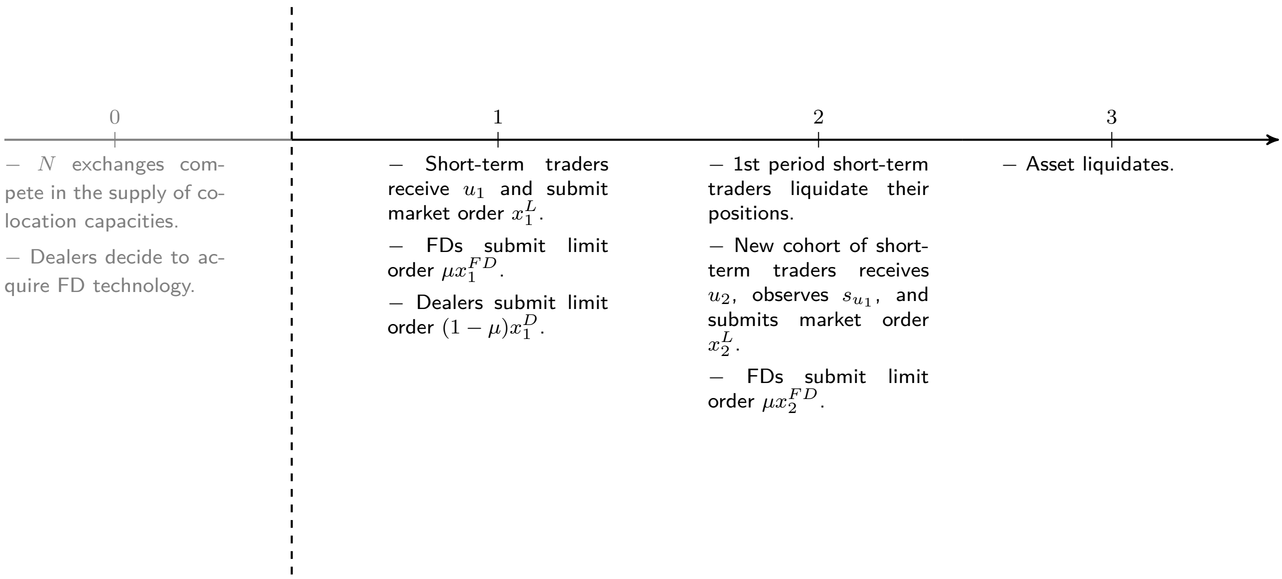


Figure 1: The timeline.

[7]This assumption is also similar to Foucault, Hombert, and Rosu (2015) who posit that HFTs receive market information slightly ahead of the rest of the market Ding, Hanna, and Hendershott (2014) compare the NBBO (National Best Bid and Offer, which is the price feed computed by the Security Industry Processors in the US) to the fuller feeds market participants obtain via a direct access to different trading platforms. Their findings point to sizeable price differences that can yield substantial profits to HFTs. Latency in the reporting of market data can also be profitably exploited for securities with centralized trading, see "High-speed traders exploit loophole," *Wall Street Journal*, May 1, 2013.

3.4 Limited market participation and the propagation of endowment shocks

Due to limited market participation, the first period endowment shock propagates to the second trading round, thereby affecting p_2. To see this, consider the first period market clearing equation

$$\mu x_1^{FD} + (1-\mu)x_1^D + x_1^L = 0. \tag{1}$$

At equilibrium the orders of first period traders are absorbed by both FDs and Ds. Thus, when $\mu < 1$, FDs' aggregate position falls short of x_1^L: $\mu x_1^{FD} + x_1^L \neq 0$. As a consequence, the inventory FDs carry over from the first period is insufficient to absorb the reverting orders that first period traders post in period 2. This creates an order imbalance driven by the first period endowment shock u_1 that adds to the one originating from second period trades, and affects the second period price. Formally, from the second period market clearing equation we have

$$\mu(x_2^{FD} - x_1^{FD}) + (x_2^L - x_1^L) = 0.$$

Substituting (1) in the latter and rearranging yields:

$$\mu x_2^{FD} + x_2^L + (1-\mu)x_1^D = 0. \tag{2}$$

According to Lemma 1, at equilibrium x_1^D depends on u_1. Thus, when $\mu < 1$, p_2 also reflects the first period endowment shock.

3.5 Strategies

We now discuss the strategies of the different market participants. In the second period, FDs act like in a static market:

$$X_2^{FD}(p_1, p_2) = -\gamma\tau_v p_2.$$

Therefore, they speculate on the asset payoff (recall that $E[v] = 0$), and supply liquidity, demanding a compensation that is inversely related to the risk they bear. In the first period, as we show in the appendix, we have

$$X_1^{FD}(p_1) = \underbrace{\gamma\frac{E[p_2 - p_1|u_1]}{\text{Var}[p_2|u_1]}}_{\text{Speculation}} - \underbrace{\frac{\gamma}{\text{Var}[v]}p_1}_{\text{Market making}}. \tag{3}$$

The above expression implies that FDs speculate on short term returns, and accommodate the residual order imbalance, demanding a compensation that is inversely related to the overall risk they bear. A traditional dealer in the first period trades according to $X_1^D(p_1) = -\gamma\tau_v p_1$.

Importantly, the slope of FDs demand function is smaller than the one of traditional dealers:

$$\frac{1}{\gamma}\left(\frac{1}{\mathrm{Var}[p_2|u_1]}+\frac{1}{\mathrm{Var}[v]}\right)^{-1}<\frac{\mathrm{Var}[v]}{\gamma}. \tag{4}$$

This is because FDs can rebalance their position at interim, and thus manage more efficiently their asset inventory.[8]

Consider now short-term traders. In the appendix we show that a second period trader trades according to

$$\begin{aligned} X_2^L(u_2, s_{u_1}) &= \underbrace{\gamma_2^L\frac{E[v-p_2|\Omega_2^L]}{\mathrm{Var}[v-p_2|\Omega_2^L]}}_{\text{Speculation}}-\underbrace{\frac{\mathrm{Cov}[v-p_2,v|\Omega_2^L]}{\mathrm{Var}[v-p_2|\Omega_2^L]}u_2}_{\text{Hedging}} \\ &= \underbrace{\frac{\gamma_2^L\mathrm{Cov}[v-p_2,u_2]}{\mathrm{Var}[v-p_2|\Omega_2^L]\mathrm{Var}[u_2]}u_2}_{\text{Speculation on } u_2}+\underbrace{\frac{\gamma_2^L\mathrm{Cov}[v-p_2,s_{u_1}|u_2]}{\mathrm{Var}[v-p_2|\Omega_2^L]\mathrm{Var}[s_{u_1}|u_2]}s_{u_1}}_{\text{Speculation on } u_1}-\underbrace{\frac{\mathrm{Cov}[v-p_2,v|\Omega_2^L]}{\mathrm{Var}[v-p_2|\Omega_2^L]}u_2}_{\text{Hedging}}. \end{aligned} \tag{5}$$

Thus, a trader's strategy has a speculative and a hedging component. According to the first line in (5), a trader speculates on value change the more, the less liquid is the market (see the first term on the r.h.s. in (5)), while lowering his exposure to the asset risk the more, the higher is the covariance between the return on his position (i.e. $v-p_2$) and the final liquidation value (v), given his information. In this way he reduces the risk that his speculative strategy goes sour precisely when the value of his endowment collapses. Expanding the expectation operator at the numerator of (5) shows that there are two sources of speculation. Other things equal, given u_2 a trader retains part of his asset exposure to the extent that this is positively correlated with the capital gain $v-p_2$, to profit from the latter. Additionally, he uses his information on u_1 to speculate on the reverting orders of first period traders. First period traders' strategies are similar to (5):

$$X_1^L(u_1)=\gamma_1^L\frac{E[p_2-p_1|u_1]}{\mathrm{Var}[p_2|u_1]}-\frac{\mathrm{Cov}[p_2-p_1,p_2|u_1]}{\mathrm{Var}[p_2|u_1]}u_1. \tag{6}$$

First period traders can partially anticipate the second period price, and thus speculate on it, e.g. by holding part of their endowment when $u_1>0$ (see the numerator of the first term on the right hand side of (6)). At the same time, due to the impact of second period traders' demand on p_2, first period traders face uncertainty on the liquidation price, which is reflected in the conditional variance at the denominator of their strategies (6).

[8]Thus, the price change needed by FDs to accommodate an increase in the aggregate demand for the asset is smaller than the one demanded by traditional dealers.

4 A benchmark with a transparent market

We start our analysis of the equilibrium, by assuming that second period traders have a perfect signal on the first period endowment shock: $\tau_\eta \to \infty$. This captures a scenario in which information on the first period imbalance is public, as is the case in a low frequency trade environment (e.g., intradaily). Alternatively, it represents an ideal setup in which second period traders have access to the same information as FDs.

In this case, we obtain the following result:

Proposition 1. *When the market is transparent there exists a unique equilibrium in linear strategies, where*

$$p_2 = \lambda_2(b_{21}^L u_2 + b_{22}^L s_{u_1}) + \lambda_2(1-\mu)\gamma\tau_v\Lambda_1^* u_1 \tag{7a}$$

$$p_1 = -\Lambda_1^* u_1, \tag{7b}$$

$\lambda_2 = 1/(\mu\gamma\tau_v) > 0$, *and*

$$\Lambda_1^* = \frac{1}{\gamma\tau_v}\left(1 - \frac{(\mu\gamma+\gamma_1^L)(1+b_1^L)}{\gamma_1^L}\right) \tag{8a}$$

$$b_1^L = \gamma_1^L(\gamma_2^L+\gamma)(\gamma_2^L+\mu\gamma)\Lambda_1^*\tau_{u_2}\tau_v^2 - 1 \in \left(-1, -\frac{\mu\gamma}{\gamma_1^L+\mu\gamma}\right) \tag{8b}$$

$$b_{21}^L = -\frac{\mu\gamma}{\mu\gamma+\gamma_2^L} \tag{8c}$$

$$b_{22}^L = \frac{\gamma_2^L b_{21}^L(1-\mu)\Lambda_1^*\tau_v}{\mu}. \tag{8d}$$

The coefficient Λ_1^*, i.e. the negative of the price impact of the first period endowment shock, is our measure of liquidity:

$$\Lambda_1^* = -\frac{\partial p_1}{\partial u_1}. \tag{9}$$

According to (8b) and (8c), first and second period traders only hedge a fraction of their endowment, thus keeping exposure to benefit from the potential capital gains. According to (8d) second period traders also speculate on the propagated order imbalance by putting a negative weight on their signal ($b_{22}^L < 0$), which is increasing in Λ_1^*. Indeed, if $s_{u_1} > 0$ the first period endowment is likely to be positive ($u_1 > 0$), which leads first period traders to shed part of it. Due to reversion, this creates a positive imbalance at date 2, which prompts second period traders to short the asset. A less liquid first period market makes it more profitable for Ds to absorb u_1, which strengthens the linear dependence between p_2, and u_1. Indeed, using (7a) we have

$$\mathrm{Cov}[p_2, u_1] = \frac{(1-\mu)\lambda_2\tau_v\Lambda_1^*}{\tau_{u_1}}\left(\frac{\gamma_2^L b_{21}^L}{\mu} + \gamma\right). \tag{10}$$

Thus, as Λ_1^* increases, second period traders have more speculative opportunities and step up b_{22}^L.

Importantly, and differently from a noise traders' setup, dealers' inventory and market liquidity depend on the trading decisions of FDs and first period traders. To see this, consider (8a). In view of (6) and (8b), at equilibrium first period traders hold a fraction

$$1 + b_1^L = \gamma_1^L \frac{\text{Cov}[p_2, u_1]\tau_{u_1} + \Lambda_1^*}{\text{Var}[p_2|u_1]} = \gamma_1^L(\gamma_2^L + \gamma)(\gamma_2^L + \mu\gamma)\Lambda_1^*\tau_{u_2}\tau_v^2, \tag{11}$$

of their endowment shock. At the same time, using (7a), one can verify that FDs aggregate speculative position per unit of endowment shock is given by

$$\mu\gamma \frac{E[p_2 - p_1|u_1]}{\text{Var}[p_2|p_1]u_1} = \mu\gamma \frac{1 + b_1^L}{\gamma_1^L}. \tag{12}$$

Thus, the sum of (11) and (12):

$$1 + b_1^L + \mu\gamma \frac{1 + b_1^L}{\gamma_1^L} = \frac{(\mu\gamma + \gamma_1^L)(1 + b_1^L)}{\gamma_1^L},$$

represents the fraction of the endowment shock that is not absorbed by liquidity suppliers, while its complement to one is dealers' inventory (per unit of endowment shock). Therefore, liquidity in this setup measures the compensation (per unit of endowment shock) that dealers demand to hold an inventory

$$1 - \frac{(\mu\gamma + \gamma_1^L)(1 + b_1^L)}{\gamma_1^L},$$

of the asset, and bear the payoff risk to which they are exposed.[9]

Using (8a) we can analyze the effect of an increase in the mass of full dealers on Λ_1^*. For given b_1^L, this has a positive effect on liquidity, since, according to (12), the aggregate speculative position of FDs increases, lowering dealers' inventory. However, from (11), a larger μ has two contrasting effects on b_1^L: on the one hand, as one can compute using (7a) and (7b),

$$\text{Var}[p_2|p_1] = \frac{(\lambda_2 b_{21}^L)^2}{\tau_{u_2}} = \frac{1}{(\mu\gamma + \gamma_2^L)^2\tau_v^2\tau_{u_2}}, \tag{13}$$

which is decreasing in μ. Therefore, a larger μ lowers first period traders' uncertainty about p_2, and makes them hold a larger portion of u_1, lowering dealers' inventory, and consuming less liquidity. However, according to (10),

$$\frac{\partial \text{Cov}[p_2, u_1]}{\partial \mu} < 0 \tag{14}$$

and a higher μ lowers the predictability of the second period price, which in turn pushes first period traders to shed a larger fraction of their endowment, increasing dealers' inventory, and

[9] Why is payoff risk relevant? At date 1 FDs can perfectly anticipate the reverting demand of first period traders they face in the following period. Thus, they know that they are able to unwind their inventory, and should absorb x_1^L at no cost (the risk-free rate is null in the model). However, in the second period a new generation of traders enters the market. These traders hedge an endowment shock and speculate on the u_1-related price pressure. This exposes FDs to the risk of holding their initial inventory until the liquidation date, and to additional price volatility, making the first period market liquidity finite.

consuming more liquidity. When the market is transparent, this latter effect is never strong enough to offset the former two and differentiating (8a) we obtain:

Corollary 2. *In a transparent market* $\partial \Lambda_1^*/\partial \mu < 0$.

4.1 Welfare and entry

Liquidity is often taken as a proxy for market welfare, and empirical analysis has in many occasions highlighted the positive effect of HFT on liquidity, informing a benign policy view of HFT. However, a proper welfare analysis has to account for traders' utilities, which we compute in the next result:

Corollary 3. *In a transparent market the expected utilities of FDs and Ds are given by*

$$EU^{FD} = -\left(1 + \frac{1}{(\gamma\mu + \gamma_2^L)^2 \tau_{u_2} \tau_v}\right)^{-1/2} \left(1 + \frac{\Lambda_1^2 \tau_v}{\tau_{u_1}} \left(1 + (\gamma + \gamma_2^L)^2 \tau_{u_2} \tau_v\right)\right)^{-1/2} \tag{15a}$$

$$EU^{D} = -\left(1 + \frac{\Lambda_1^2 \tau_v}{\tau_{u_1}}\right)^{-1/2}, \tag{15b}$$

where $EU^{FD} > EU^D$. *The expected utilities of first and second period traders are given by*

$$EU_1^L = -\left(1 + \frac{\mathrm{Var}[p_2|u_1]}{(\gamma_1^L)^2 \tau_{u_1}} \left((b_1^L)^2 - 1\right) + \frac{2\gamma(1-\mu)\Lambda_1}{\gamma_1^L \tau_{u_1}(\mu\gamma + \gamma_2^L)}\right)^{-1/2} \tag{16a}$$

$$EU_2^L = -\Bigg(\left((\gamma_2^L)^2 \tau_{u_2} \tau_v - 1\right) \times \left(\frac{1}{(\gamma_2^L)^2 \tau_{u_2} \tau_v} + \frac{\gamma^2 (1-\mu)^2 \Lambda_1^2}{(\gamma_2^L)^2 (\mu\gamma + \gamma_2^L)^2 \tau_{u_1} \tau_{u_2}}\right) + \frac{\mu^2 \gamma^2}{(\gamma_2^L)^2 (\mu\gamma + \gamma_2^L)^2 \tau_{u_2} \tau_v}\Bigg)^{-1/2}. \tag{16b}$$

A sufficient condition for (16a) and (16b) to be well defined is

$$(\gamma_2^L)^2 > \max\left\{\frac{1}{(\gamma_1^L)^2 \tau_v^2 \tau_{u_1} \tau_{u_2}}, \frac{1}{\tau_v \tau_{u_2}}\right\}. \tag{17}$$

According to (15a) and (15b), a larger mass of FDs lowers the utility of liquidity suppliers. This is because as μ increases, liquidity improves and a lower fraction of the endowment shock propagates to the second period, eroding FDs' profits from short-term speculation. Consider now first period traders. According to (16a), an increase in μ has two contrasting effects on EU_1^L. On the one hand as it lowers $\mathrm{Var}[p_2|p_1]$ (see (13)), first period traders face lower uncertainty on p_2, which works to make them better off (recall that $(b_1^L)^2 < 1$). On the other hand, as a larger μ reduces the predictability of p_2, it diminishes the capital gain these traders make on the fraction of u_1 they hold, lowering their utility. Finally, consider the utility of second period traders. When (17) holds, an increase in the mass of FDs has a direct negative effect on EU_2^L (see the first term in the parenthesis under the square root in (16b)). Indeed, second period traders have perfect information on u_1 and can speculate on the propagated order imbalance. As μ increases, propagation wanes, which lowers traders' speculative profits. However, a larger μ improves risk-sharing, boosting the fraction of u_2 that second period traders hedge (see (8c)). This, in turn, has a positive effect on EU_2^L (see the second term in the parenthesis under the square root in (16b)).

Overall, an increase in the mass of FDs worsens dealers' welfare while having a mixed effect on the welfare of first and second period traders. To compute the aggregate welfare effect, we express expected utilities in terms of certainty equivalents (a monotone transformation of expected utilities):

$$CE^{FD} \equiv -\gamma \ln(-EU^{FD}),\ CE^{D} \equiv -\gamma \ln(-EU^{D}),\ CE_t^L \equiv -\gamma_t^L \ln(-EU_t^L),\ t \in \{1, 2\},$$

and define the following total welfare function:

$$TW(\mu) \equiv \mu CE^{FD} + (1-\mu)CE^{D} + CE_1^L + CE_2^L. \tag{18}$$

A formal analysis of the impact of μ on $TW(\mu)$ is complicated by the number of different effects this has on market participants' welfare. Thus, we resort to numerical simulations, obtaining the following result:[10]

Numerical Result 1. *In a transparent market, $\mu = 1$ maximizes $TW(\mu)$.*

Thus, when the market is only populated by FDs, a policy of maximizing liquidity is also welfare maximizing.

One way to implement an equilibrium with maximal liquidity is to augment the supply of trading services, facilitating the entry of different exchange platforms. This is the approach adopted both in the US and EU, where spurred by regulatory changes the number of trading venues has dramatically increased in the past fifteen years (O'Hara and Ye (2011)). For given supply of trading services, however, the supply of liquidity depends on the ability of market participants to absorb demand shocks, which in our setup is related to the decision to acquire the needed technology to provide liquidity at higher trading frequency. In turn, such decision depends on the FDs technology's comparative advantage. We now proceed to endogenize a dealer's decision to become a FD by purchasing co-location services, assuming that there is a set of competing exchanges offering such services.

We start from the demand for FD technology. According to Corollary 3, $CE^{FD} > CE^{D}$.

[10] Numerical simulations were run with the following set of parameters: $\gamma, \gamma_1^L \in \{0.1, 0.5, 0.9\}$, $\tau_{u_1}, \tau_{u_2}, \tau_v \in \{1, 5, 9\}$, $c \in \{0.01, 0.05\}$, $\gamma_2^L \in \{0.21, 0.51, 0.81\}$.

Thus, we can define the value of becoming a FD as follows:

$$\begin{aligned}\phi(\mu) &\equiv CE^{FD} - CE^{D} \qquad (19)\\ &= \frac{\gamma}{2}\ln\frac{\left(1+\dfrac{1}{(\gamma\mu+\gamma_2^L)^2\tau_{u_2}\tau_v}\right)\left(1+\dfrac{(\Lambda_1^*)^2\tau_v}{\tau_{u_1}}\left(1+(\gamma+\gamma_2^L)^2\tau_{u_2}\tau_v\right)\right)}{\left(1+\dfrac{(\Lambda_1^*)^2\tau_v}{\tau_{u_1}}\right)}.\end{aligned}$$

The function $\phi(\mu)$ can be interpreted as the inverse demand function of a dealer for FD technology. It can be verified that in a transparent market, the inverse demand for co-location services is decreasing in μ:

Corollary 4. *In a transparent market, $\phi'(\mu) < 0$.*

On the supply side, suppose that there are $i = 1, 2, \ldots, N$ exchanges competing to offer co-location services in the second period market, and assume that each exchange i bears a marginal cost $c > 0$ to produce co-location capacity μ_i and a fixed set up cost $f > 0$. Finally, suppose that there is a best price rule ensuring that the second period price is identical across all the competing trading platforms. In this setup, the total co-location capacity is given by

$$\sum_{i=1}^{N} \mu_i = \mu, \qquad (20)$$

and corresponds to the mass of FDs in the market. An exchange i's profit is then given by

$$\pi(\mu_i;\mu_j) = (\phi(\mu) - c)\mu_i - f, \; j \neq i. \qquad (21)$$

We assume that exchanges compete à la Cournot to offer co-location capacities, and study the symmetric equilibrium of this game.

Define a symmetric equilibrium in co-location capacities as the set of μ_i^C, $i = 1, 2, \ldots, N$, such that (i) $\mu_1^C = \mu_2^C = \cdots = \mu_N^C$ and (ii) each μ_i^C maximizes (21), for given capacity choice of other exchanges μ_j^C, $j \neq i$:

$$\mu_i^C \in \arg\max_{\mu_i} \pi(\mu_i;\mu_j). \qquad (22)$$

Due to symmetry, $\mu_i^C = \mu^C/N$, where μ^C satisfies the first order condition

$$\frac{\partial\phi(\mu^C)}{\partial\mu_i}\frac{\mu^C}{N} + \phi(\mu^C) - c = 0, \qquad (23)$$

and the second order condition $\phi''(\mu^C) \leq 0$. Equation (23) implicitly defines the supply curve for co-location services. To analyze entry in this setup, we once again resort to numerical simulations and obtain the following result:[11]

Numerical Result 2. *In a transparent market, when exchanges compete in co-location capacities, a symmetric Cournot equilibrium $\mu^C(c)$:*

[11]We extend our numerical simulations, letting $N = 1, 2, \ldots, 100$.

1. *Generically exists.*

2. *$\mu^C(c)$ is decreasing in c.*

3. *$\mu^C(c)$ tends to 1 for c sufficiently small and a sufficiently large and finite N.*

Consider now the equilibrium that arises in the Cournot market with free entry. At equilibrium, the available co-location capacity μ^C must satisfy the following condition:

$$(\phi(\mu^C) - c)\frac{\mu^C}{N} = f. \tag{24}$$

We now compare the Cournot solution in (24) with the solution of a social Planner that internalizes the effects of FDs participation, incurring the co-location cost $c\mu$ and the total fixed cost fN. Define the planner objective as follows:

$$\mathcal{P}(\mu, N) \equiv TW(\mu) - c\mu - fN. \tag{25}$$

The first best solution to the planner's problem obtains when the planner is able to choose the number of competing exchanges N as well as the total co-location capacity μ (since $\phi'(\mu) < 0$, this corresponds to the planner setting the co-location fee):

$$\max_{\mu,N} \mathcal{P}(\mu, N). \tag{26}$$

Suppose instead that the planner can only regulate the co-location fee. In this case, the planner sets a fee that is high enough to make exchanges break even, and chooses the co-location capacity μ that maximizes (25) which is compatible with such constraint:

$$\max_{0\leq\mu\leq1} \mathcal{P}(\mu, N) \text{ s.t. } \pi_i(\mu_i; \mu_j) = f. \tag{27}$$

Finally, suppose that the planner is unable to affect the way in which exchanges compete, but can set the number of exchanges entering the intermediation industry. In this case the planner's problem becomes

$$\max_{N} \mathcal{P}(\mu, N) \text{ s.t. } \mu = \mu^C, \tag{28}$$

where μ^C is the co-location capacity obtained in (24).

Our preliminary simulations indicate that

Numerical Result 3. *In a transparent market*

1. *The co-location capacity that obtains in the Cournot equilibrium with free entry (see (24)) is larger compared to the monopolistic exchange solution.*

2. *The number of exhanges that solves (28) is never larger than the number of exchanges that obtains in the Cournot equilibrium with free entry (see (24)).*

3. *The co-location capacity that solves (27) is typically larger than the co-location capacity that obtains in the Cournot equilibrium with free entry (see (24)), except when c is very small (in which case, the two coincide).*

4. *For c sufficiently low the first best solution is to set $\mu = N = 1$. When c increases, the planner sets $\mu < 1$ and $N = 1$.*

Co-location services in our setup are akin to a homogeneous good. Thus, Cournot competing exchanges will always produce at least as much as a monopolist. The second result is reminiscent of Mankiw and Whinston (1986) who show that in homogeneous product markets, when an entrant reduces the output produced by incumbent firms, there is a natural tendency to have "excessive" entry.[12] This is because the entrant, differently from a social planner maximizing total surplus, does not internalize the negative effect of its decision on its peers. In our setup a similar effect is at work. Indeed, although the social welfare function also encompasses the utilities of short term traders and standard dealers, these traders' welfare only depends on the total capacity μ (and is thus independent of N), and turns out to be maximal when $\mu = 1$. Thus, a planner increases liquidity supply, minimizing the negative impact of excessive exchange entry. The same intuition explains why a planner that can choose both N and μ decides to impose fee regulation on a monopolistic exchange, as implied by the last result.

Thus, we can summarize our findings so far by saying that in the transparent market benchmark, a unique equilibrium exists. In this equilibrium (i) liquidity increases in the mass of FDs, (ii) total welfare is maximized when liquidity is maximal, (iii) the provision of co-location services is insufficient despite the fact that free entry of platforms is excessive (iv) if the regulator can control the platform fee, then entry is allowed basically as in the market while if it can control entry, then the latter is restricted. In both cases the provision of co-location services is increased.

5 The effect of informational frictions

Suppose now that second period traders' signal on u_1 has a bounded precision ($\tau_\eta < \infty$), and restrict attention to the case $N = 1$. This setup characterizes a scenario where some traders (FDs, in our setup) have access to better market information compared to others (the second cohort of traders), and given our previous discussion, is likely to hold at a high trading frequency.

In this case, we obtain the following result:

Proposition 5. *When $\tau_\eta < \infty$, there exists an equilibrium in linear strategies where*

$$p_2 = \lambda_2(b_{21}^L u_2 + b_{22}^L s_{u_1}) + \lambda_2(1-\mu)\gamma\tau_v\Lambda_1^* u_1 \tag{29a}$$

$$p_1 = -\Lambda_1^* u_1, \tag{29b}$$

[12] In our setup, at a symmetric equilibrium a larger N implies a smaller co-location capacity for each exchange, since $\mu_i = \mu^C/N$.

$\lambda_2 = 1/(\mu\gamma\tau_v) > 0$,

$$\Lambda_1^* = \frac{1}{\gamma\tau_v}\left(1 - \frac{(\mu\gamma + \gamma_1^L)(1+b_1^L)}{\gamma_1^L}\right) \in \left(0, \frac{1}{\gamma\tau_v}\right) \tag{30a}$$

$$b_1^L = \gamma_1^L \frac{\text{Cov}[p_2, u_1]\tau_{u_1} + \Lambda_1^*}{\text{Var}[p_2|u_1]} - 1 \in \left(-1, -\frac{\mu\gamma}{\gamma_1^L + \mu\gamma}\right), \tag{30b}$$

$$b_{21}^L = -\frac{\mu\gamma}{\gamma_2^L + \mu\gamma\kappa} \tag{30c}$$

$$b_{22}^L = \gamma_2^L b_{21}^L \tau_v \tau_\eta \text{Cov}[p_2, u_1|\Omega_2^L], \tag{30d}$$

$\kappa \equiv \tau_v \text{Var}[v - p_2|\Omega_2^L]$, $\text{Var}[p_2|u_1] = \lambda_2^2((b_{21}^L)^2/\tau_{u_2} + (b_{22}^L)^2/\tau_\eta)$, $\text{Var}[v - p_2|\Omega_2^L] = \text{Var}[v] + (\lambda_2(1-\mu)\gamma\tau_v\Lambda_1^*)^2\text{Var}[u_1|s_{u_1}]$, *and* Λ_1^* *obtains as a fixed point of the following mapping:*

$$\psi(\Lambda_1^*) = -\frac{\mu\gamma\text{Cov}[p_2, u_1]\tau_{u_1} + b_1^L\text{Var}[p_2|u_1]}{\gamma(\mu + \tau_v\text{Var}[p_2|u_1])}. \tag{31}$$

Define the asset supply in period 1 as $z_1 \equiv x_1^L$. In the second period, given our discussion in Section 3.4, the asset supply is given by $z_2 \equiv x_2^L + (1-\mu)x_1^D$. The next result characterizes some important properties of the equilibrium prices, asset supply, and equilibrium coefficients:

Corollary 6. *At equilibrium,* $\text{Cov}[p_2 - p_1, p_1] < 0$*, and* $\text{Cov}[p_2, u_1] \geq 0$, $\text{Cov}[p_2, u_1|\Omega_2^L] \geq 0$, $\text{Cov}[z_1, z_2] \leq 0$, *for* $\mu \leq 1$*. If* $\mu = 1$, $\text{Var}[v - p_2|\Omega_2^L] = \text{Var}[v]$*, and* $\text{Cov}[p_2, u_1] = 0$.

To interpret the sign of $\text{Cov}[z_1, z_2]$ and $\text{Cov}[p_2, u_1]$, suppose $u_1 > 0$. Then, due to (30b), first period traders short the asset, creating a negative imbalance that is absorbed by Ds and FDs. In period 2, first period traders unwind their position (buying back the asset) and the inventory held by FDs falls short of their demand (as $0 < \mu \leq 1$). This induces a positive imbalance that is responsible for the negative (positive) correlation between asset supplies (p_2 and u_1).

Most of the intuitions for the strategies' coefficients of the transparent market benchmark extend to the present setup. In particular, first and second period traders hedge only a fraction of their endowment (see (30b) and (30c)). Differently from the transparent market benchmark, second period traders now face uncertainty on the price at which their order is executed–besides that on the liquidation value. This additional source of uncertainty is captured by the coefficient κ:

$$\kappa \equiv \tau_v \text{Var}[v - p_2|\Omega_2^L] > 1.$$

For $\tau_\eta < \infty$, second period traders cannot perfectly anticipate p_2, and hedge a lower fraction of their endowment shock (see (30c)). Other things equal, as μ increases, κ tends to 1, as u_1 propagates less to period 2, and second period traders hedge more of their endowment shock. Finally, because of (30d) a consequence of Corollary 6 is that at equilibrium $b_{22}^L < 0$, like in Proposition 1. When $\tau_\eta \to 0$, second period traders are uninformed about u_1 and

$$\lim_{\tau_\eta \to 0} b_{22}^L = 0. \tag{32}$$

We are now ready to analyze the effect of a shock to first period liquidity on the equilibrium coefficients:

Corollary 7. *At equilibrium,*

$$\frac{\partial \mathrm{Cov}[p_2, u_1]}{\partial \Lambda_1^*} > 0, \; \frac{\partial \mathrm{Var}[v - p_2 | \Omega_2^L]}{\partial \Lambda_1^*} > 0, \; \frac{\partial b_{21}^L}{\partial \Lambda_1^*} > 0. \tag{33}$$

An increase in Λ_1^* has an ambiguous effect on b_{22}^L, $\mathrm{Var}[p_2|u_1]$, and b_1^L.

According to (33) as in the transparent market case, a less liquid first period market increases the linear dependence between p_2 and u_1. Furthermore, as second period traders do not perfectly observe u_1, this also augments these traders' uncertainty and, according to (33), lowers their hedging responsiveness (recall that $b_{21}^L < 0$).

Importantly, and differently from the transparent market case, an increase in Λ_1^* has two contrasting effects on b_{22}^L. Indeed, direct computation yields: $\mathrm{Cov}[p_2, u_1|\Omega_2^L] = \lambda_2(1-\mu)\gamma\tau_v\Lambda_1^*\mathrm{Var}[u_1|s_{u_1}]$. Thus, differentiating b_{22}^L we obtain:

$$\frac{\partial b_{22}^L}{\partial \Lambda_1^*} = \gamma_2^L \tau_v \tau_\eta \left(\underbrace{\mathrm{Cov}[p_2, u_1|\Omega_2^L] \frac{\partial b_{21}^L}{\partial \Lambda_1^*}}_{\text{Uncertainty effect } (+)} + \underbrace{b_{21}^L \frac{\partial \mathrm{Cov}[p_2, u_1|\Omega_2^L]}{\partial \Lambda_1^*}}_{\text{Speculation effect } (-)} \right). \tag{34}$$

On the one hand, like in the transparent market benchmark, an increase in Λ_1^* augments second period traders' speculative opportunities, and drives them to trade more against the u_1-led imbalance (the second term in the parenthesis in (34)). On the other hand, a higher Λ_1^* augments second period traders return uncertainty, and makes them speculate less (the first term in the parenthesis). Consider now the effect of an increase in Λ_1^* on $\mathrm{Var}[p_2|u_1]$:

$$\frac{\partial \mathrm{Var}[p_2|u_1]}{\partial \Lambda_1^*} = 2\lambda_2^2 \left(\underbrace{\frac{b_{21}^L}{\tau_{u_2}} \frac{\partial b_{21}^L}{\partial \Lambda_1^*}}_{(-)} + \underbrace{\frac{b_{22}^L}{\tau_\eta} \frac{\partial b_{22}^L}{\partial \Lambda_1^*}}_{(\pm)} \right). \tag{35}$$

In the transparent market benchmark, an increase in Λ_1^* has no impact on first period traders' uncertainty over p_2 (see (13)). In contrast, according to (33), due to the informational friction, when Λ_1^* increases, second period traders scale down their hedging activity, which reduces one source of price impact, and lowers $\mathrm{Var}[p_2|u_1]$. However, as we argued above, a less liquid first period market can spur more speculation by second period traders. Thus, according to (35), the ultimate impact of a shock to Λ_1^* on first period traders' uncertainty depends on the strength of the speculation effect. Finally, an increase in Λ_1^* can also have two contrasting effects on b_1^L:

$$\frac{\partial b_1^L}{\partial \Lambda_1^*} = \frac{\gamma_1^L}{\mathrm{Var}[p_2|u_1]^2} \times \tag{36}$$
$$\left(\underbrace{\left(\frac{\partial \mathrm{Cov}[p_2, u_1]}{\partial \Lambda_1^*} \tau_{u_1} + 1 \right) \mathrm{Var}[p_2|u_1]}_{(+)} - \underbrace{\frac{\partial \mathrm{Var}[p_2|u_1]}{\partial \Lambda_1^*} (\mathrm{Cov}[p_2, u_1]\tau_{u_1} + \Lambda_1)}_{(\pm)} \right).$$

For given $\mathrm{Var}[p_2|u_1]$, as first period traders can better predict p_2, a larger Λ_1^* leads them to speculate more, and hedge less. However, when $\mathrm{Var}[p_2|u_1]$ increases in Λ_1^*, as this increases the risk to which first period traders are exposed, a less liquid market can lead them to hedge more, and speculate less.

As a result of the above effects, strategic complementarities in liquidity supply can arise and yield multiple equilibria. The intuition is as follows. According to Corollary 7, a less liquid first period market heightens the time-propagation of the first period shock. This, in turn, can lead second period traders to speculate more aggressively on the u_1-led imbalance (see (34)), which can increase the uncertainty faced by first period traders on p_2 (see (35)). As a consequence, first period traders can decide to hedge more, and FDs to speculate less (see (36)).[13] This magnifies liquidity suppliers' inventory, reinforcing the initial liquidity shock. Indeed, we obtain the following result:

Corollary 8. *When $0 < \tau_\eta < \infty$, there can be up to three equilibria which can be ranked in terms of first period liquidity.*

In Figure 2 we plot the function $\psi(\Lambda_1)$ for a set of parameter values that yields multiple equilibria. We will refer to the equilibrium where Λ_1^* is low (resp., intermediate, and high) as the high, (resp., intermediate, and low) liquidity equilibrium.

In Figures 3 and 4, we illustrate the effect of different parameters' changes on the equilibrium set.

1. An increase in μ increases the risk bearing capacity of the market, and lowers the second period imbalance due to u_1. This weakens the strength of the loop. A similar effect obtains when γ or τ_v increase (see Figure 3, panel (a), (b), and (c)).

2. An increase in γ_1^L or in τ_{u_1} works instead to lower the supply shock that Ds and FDs absorb in the first period, and thus the second period imbalance due to u_1, again weakening the loop (see Figure 3, panel (d) and (e)).

3. An increase in τ_η has two contrasting effects on the strength of the loop. For τ_η small, a more precise signal on u_1 boosts second period traders' speculation on the u_1-induced

[13] Because of (12), whenever first period traders consume more liquidity, FDs speculate less, increasing the inventory held by liquidity suppliers.

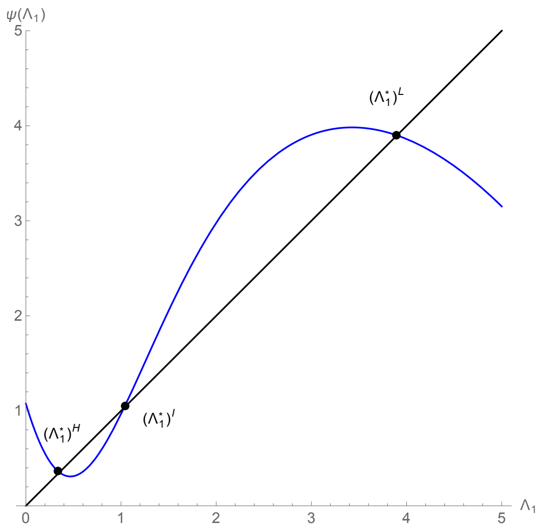


Figure 2: Equilibrium multiplicity. The figure displays the function $\psi(\Lambda_1)$ against the 45-degree line for $\tau_v = 0.1$, $\tau_{u_1} = 2$, $\tau_{u_2} = 400$, $\tau_\eta = 10$, $\gamma = 0.9$, $\gamma_1^L = 0.2$, $\gamma_2^L = 0.9$, and $\mu = 0.2$. At equilibrium $\Lambda_1^* \in \{0.31, 0.99, 6.67\}$.

imbalance, heightening first period traders' uncertainty on p_2, increasing Λ_1^*, and strengthening the loop. As τ_η increases further, $\text{Var}[p_2|u_1]$ starts decreasing (see the expression in Proposition 5), leading first period traders to hold more of their endowment shock, increasing the liquidity of the first period market, reducing the size of the u_1-led imbalance, and weakening the loop. When $\tau_\eta \to \infty$, (i) the impact of second period traders' speculation on $\text{Var}[p_2|u_1]$ disappears, and (ii) second period traders' uncertainty no longer depends on Λ_1^* (see the expression for $\text{Var}[v - p_2|\Omega_2^L]$ in Proposition 5). This severs the link between trading decisions at the two dates, yielding a unique equilibrium (as we know from the analysis of Section 4). Figure 4, Panel (a), illustrates this effect. Note that as τ_η increases, the intermediate and low liquidity equilibria eventually disappear, but liquidity at the high liquidity equilibrium diminishes (compared with the case with low signal precision).

To understand the effect of a change in τ_{u_2}, consider first the extreme case in which the second period endowment shock is null (almost surely):

Corollary 9. *If $\tau_{u_2} \to \infty$, there always exists an equilibrium with $\Lambda_1^* = 0$. When $\tau_\eta \to \infty$, $\Lambda_1^* = 0$ is the unique equilibrium.*

When $\tau_{u_2} \to \infty$, second period traders have no endowment to hedge, and only trade to speculate on the u_1-induced imbalance. In the equilibrium where $\Lambda_1^* = 0$, $x_1^D = 0$, so that first period traders' orders are absorbed by FDs' speculative trades, which implies that no imbalance arises in the second period, and $b_{22}^L = 0$ (see (30d)).[14] When second period traders' signal on u_1 is fully revealing, this equilibrium is unique. For τ_η finite, however, first period traders cannot

[14] A full analytical characterization of this equilibrium is complex. Numerically, it can be seen that first period liquidity traders hedge the smallest possible fraction of their endowment shock: $b_1^L \to -\mu\gamma/(\mu\gamma + \gamma_1^L)$.

rule out the possibility that second period traders trade on a certain realization of s_{u_1} that gives an incorrect signal about u_1 (e.g., $s_{u_1} > 0$, while $u_1 < 0$). This increases the price risk faced by FDs, increasing their cost of liquidity provision, possibly leading first period traders to hedge more, and allowing for multiple equilibria. For $\tau_{u_2} < \infty$, a lower τ_{u_2} magnifies the risk to which first period traders are exposed, and works to weaken the loop, as first period traders tend to behave as noise traders. The smaller τ_{u_2} becomes, the harder it is to sustain the equilibrium with high liquidity. When τ_{u_2} becomes very small, our simulations show that only the low liquidity equilibrium survives. Figure 4, Panel (b) illustrates this effect.

Finally, an increase in γ_2^L has two contrasting effects on the strength of the loop. Other things equal, according to (30c) and (30d), when second period traders become more risk tolerant they hedge less of their endowment shock, and speculate more aggressively against the u_1-led imbalance. The first of the above effects works to weaken the loop, because it reduces the impact of the endowment shock on first period traders' uncertainty. Conversely, the second effect strengthens the loop. For γ_2^L small (large), the first (second) effect prevails, as illustrated in Figure 4, Panel (c).

5.1 Fragility

Liquidity complementarities imply that liquidity can be "fragile," in the sense that a small shock to one of the model's parameters can lead to a disproportionately large change in liquidity. We show this with two examples, in which we shock the mass of FDs, and the volatility of first period traders' demand. The first example is meant to capture the effect of a technological 'glitch' that disconnects a fraction full dealers, replacing them with standard dealers. The second example captures instead the effect of an increase in the likelihood that a large, liquidity-consuming order hits the first period market.

Consider again Panel (a) in Figure 3, and suppose that initially the market is at the high liquidity equilibrium, where $\Lambda_1^* = 0.35$, as shown in Figure 5, Panel (a). Suppose now that the proportion of FDs is shocked, and that it moves to $\mu = 0.18$ (a 10% decrease). In this new situation, a unique equilibrium arises, with $\Lambda_1^* = 4.35$, which corresponds to a 12-fold liquidity decrease.

Consider now Panel (e) in Figure 3, and again assume that the market is at the high liquidity equilibrium (as shown if Figure 5, Panel (b)). Suppose now that we introduce a 30% shock to the volatility of first period traders' endowment (bringing τ_{u_1} to 1.4). In this new situation there is a unique equilibrium, where $\Lambda_1^* = 4.27$ which corresponds to a liquidity dry-up comparable to the one of the previous example.

Summarizing: this section highlights the role of informational frictions in generating strategic complementarities in liquidity supply. Second period traders, endowed with a noisy signal on the first period endowment shock, speculate against the propagated order imbalance, generating additional volatility. This can feed back on first period traders' strategies, inducing short-term traders to consume more liquidity and FDs to retreat from speculation, thereby magnifying the inventory held by liquidity suppliers, and further lowering market liquidity. This can trigger a self-sustaining loop, inducing multiple equilibria and liquidity fragility.

6 A strongly opaque market

In this section we consider the case in which the market is "strongly" opaque in that second period traders' signal is uninformative: $\tau_\eta \to 0$. In this case, second period traders do not speculate on the propagated imbalance (see (32)), and uniqueness is restored:

Corollary 10. *When the market is strongly opaque, there exists a unique equilibrium in linear strategies.*

Importantly, a strongly opaque market is more liquid than its transparent counterpart:

Corollary 11. *The strongly opaque market is more liquid than the fully transparent one:*

$$\Lambda_1^*|_{\tau_\eta \to 0} < \Lambda_1^*|_{\tau_\eta \to \infty}. \tag{37}$$

According to (30c), due to execution risk second period traders hedge less of their shock when their signal is uninformative (in which case $\kappa > 1$) than when it is perfect (in which case, instead, $\kappa = 1$). At the same time, when $\tau_\eta \to 0$, $b_{22}^L \to 0$, implying that first period traders' uncertainty only depends on second period hedging aggressiveness. But the same is true also when $\tau_\eta \to \infty$, since in this case the impact of second period speculation can be fully anticipated in the first period. As a consequence, when $\tau_\eta \to 0$ first period traders face lower uncertainty than when $\tau_\eta \to \infty$. Thus, in the latter case first period traders hedge more, and FDs speculate less, lowering market liquidity. Hence, market transparency, alone, won't necessarily enhance market liquidity.

Consider now the effect of an increase in the mass of FDs on liquidity. As we argued in Section 4, an increase in μ triggers two potentially contrasting effects:

$$\frac{\partial \Lambda_1^*}{\partial \mu} = -\frac{1}{\gamma \tau_v}\left(\underbrace{\frac{\gamma(1+b_1^L)}{\gamma_1^L}}_{(+)} + \frac{\mu\gamma}{\gamma_1^L}\underbrace{\frac{\partial b_1^L}{\partial \mu}}_{(\pm)} + \underbrace{\frac{\partial b_1^L}{\partial \mu}}_{(\pm)}\right).$$

First, for given b_1^L, FDs speculate more aggressively against the orders of first period short-term traders, which works to make the market more liquid; second, an increase in the mass of FDs can lower the impact of second period traders' orders, thereby lowering $\mathrm{Var}[p_2|p_1]$ and leading first period traders to hold more of their endowment;[15] at the same time, however, it can lower the propagation of u_1 to the second period, lowering the predictability of p_2, and inducing traders to shed more of their endowment. In a transparent market, we know that this

[15]The volatility reduction can happen for two different reasons. As μ increases, (i) less of the first period endowment shock propagates to the second period, and (ii) more FDs absorb second period liquidity traders' orders, enhancing risk sharing.

latter effect is never strong enough to overcome the previous two, and liquidity increases in the mass of FDs (see Corollary 2). In contrast, when the market is strongly opaque, we obtain the following result:

Corollary 12. *When $\tau_\eta \to 0$, $\Lambda_1^*(\mu)$ can be increasing in μ.*

When the signal on u_1 is uninformative, second period traders face high execution risk, which tames their hedging aggressiveness (see (30c)), and lowers first period traders' uncertainty. In this situation, the uncertainty reduction effect of μ on b_1^L can be dwarfed by the one due to reduced predictability. As a consequence, when μ increases, first period traders can start to hedge more, swamping the increase in FDs' speculative trades, and for low values of μ, impairing liquidity. When the distribution of the asset payoff has little dispersion, as μ grows large, the effect of FDs speculative trades tends to prevail, implying that Λ_1^* can be hump-shaped. However, when τ_v is sufficiently small, the effect of execution risk on second period liquidity traders' hedging aggressiveness is stronger, and Λ_1^* can be increasing in μ, for all $\mu \in (0, 1)$. This implies that when information on prices and/or order imbalances is opaque, an increase in the mass of HFTs (promoting full participation), can lower market liquidity. This finding is consistent with Boehmer, Fong, and Wu (2015) who provide international evidence on the impact of an increase in algorithmic trading on market quality. An important findings is that while on average greater algorithmic trading intensity is associated with more liquidity, the same is not true for small market cap firms. For these firms, when algorithmic trading increases, liquidity declines.[16] Figure 6 provides a graphical illustration of Corollary 12.

Finally, the following result provides sufficient conditions for first or second period traders to behave like noise traders:

Corollary 13. *When*

1. *$\tau_\eta \to \infty$, and $\gamma_2^L \to 0$, $b_{21}^L \to -1$, $b_{22}^L = 0$, and*

$$\Lambda_1^* = \frac{1}{\gamma\tau_v(1 + (\gamma_1^L + \gamma\mu)\gamma\mu\tau_{u_2}\tau_v)}. \tag{38}$$

2. *$\tau_{u_2} \to 0$, $\Lambda_1^* = 1/\gamma\tau_v$, and $b_1^L = -1$.*

When markets are informationally integrated, second period traders face no execution risk on their orders ($\kappa = 1$). Hence, when they become infinitely risk averse they rather hedge all of their endowment, behaving like noise traders. When $\tau_{u_2} \to 0$, second period traders' hedging activity is so aggressive that first period traders' uncertainty diverges ($\lim_{\tau_{u_2}\to 0} \mathrm{Var}[p_2|u_1] = \infty$). As a result, the risk of speculating on short term returns is so large that both FDs and first period traders refrain from it. Hence, $b_1^L = -1$, and equilibrium liquidity reaches its minimum.

Importantly,

$$\Lambda_1^*\big|_{\tau_{u_2}\to 0} > \Lambda_1^*\big|_{\substack{\tau_\eta\to\infty \\ \gamma_2^L\to 0}}.$$

[16]See also Breckenfelder (2014) for other evidence on the negative impact of an increase in HFT competition on market liquidity for a sample of stocks traded on the Stockholm Stock Exchange.

This is because when the second period endowment shock explodes, as argued above, the risk of first period speculation is so large that all first period trades are absorbed by liquidity providers. Conversely, when the conditions leading second period traders to behave like noise traders are in place, $\mathrm{Var}[p_2|u_1] < \infty$, which makes speculation possible in the first period.

6.1 Welfare

We now turn to the welfare analysis, providing general expressions for the expected utilities of market participants:

Proposition 14. *At a linear equilibrium the unconditional expected utilities of market participants are given by*

$$EU^D = -\left(1 + \frac{\mathrm{Var}[p_1]}{\mathrm{Var}[v]}\right)^{-1/2} \tag{39a}$$

$$EU^{FD} = -\left(1 + \frac{\mathrm{Var}[p_2|p_1]}{\mathrm{Var}[v]}\right)^{-1/2}\left(1 + \frac{\mathrm{Var}[p_1]}{\mathrm{Var}[v]} + \frac{\mathrm{Var}[E[p_2|p_1] - p_1]}{\mathrm{Var}[p_2|p_1]}\right)^{-1/2}, \tag{39b}$$

respectively for Ds and FDs, where $EU^{FD} > EU^D$*, and*

$$EU_1^L = -\left(1 + \frac{\mathrm{Var}[x_1^L]\mathrm{Var}[p_2|u_1]}{(\gamma_1^L)^2} + \frac{2\mathrm{Cov}[p_2, u_1]}{\gamma_1^L} - \frac{\mathrm{Var}[u_1]\mathrm{Var}[p_2|u_1]}{(\gamma_1^L)^2}\right)^{-1/2}, \tag{40a}$$

for first period traders,

$$EU_2^L = -\Bigg(1 + \frac{\mathrm{Var}[x_2^L]\mathrm{Var}[v - p_2|\Omega_2^L]}{(\gamma_2^L)^2} - \frac{\mathrm{Var}[u_2]\mathrm{Var}[v]}{(\gamma_2^L)^2} + \left(\rho^2_{x_2^L,u_2} - 1\right)\frac{\mathrm{Var}[v]\mathrm{Var}[u_2]\mathrm{Var}[x_2^L]\mathrm{Var}[v - p_2|\Omega_2^L]}{(\gamma_2^L)^4}\Bigg)^{-1/2}, \tag{40b}$$

where $\rho^2_{x_2^L,u_2} \equiv (b_{21}^L)^2/\mathrm{Var}[x_2^L]\tau_{u_2}$*, for second period traders.*

Expression (39a) shows that dealers derive utility from liquidity provision to first period traders: the less liquid is the first period market, the higher is their welfare.[17]

From (39b) we can instead see that FDs derive utility from three trading activities: (i) liquidity supply to first traders like traditional dealers, (ii) speculation on short term returns, and (iii) liquidity supply to second period traders. A change in the conditional volatility of returns has two contrasting effects on FDs' utility: the higher is $\mathrm{Var}[p_2|p_1]$, the larger are the potential returns from second period liquidity provision (as this implies a larger liquidity demand in the future), but also the higher is the risk of speculating on short-term price changes (the short term return variance unexplained by p_1).[18]

[17] More in detail, $\mathrm{Var}[p_1]$, and $\mathrm{Var}[v]$ are respectively the variance of the profit from liquidity provision that is explained and unexplained by p_1 (that is $\mathrm{Var}[E[v - p_1|p_1]] = \mathrm{Var}[p_1]$, and $\mathrm{Var}[v] = \mathrm{Var}[v - p_1|p_1]$). Therefore, the higher is $\mathrm{Var}[p_1]$ ($\mathrm{Var}[v]$) the more (less) accurately dealers can anticipate their profit — namely $(v - p_1)$ — from the knowledge of p_1.

[18] At date 1, the short term profit of a FD is $(p_2 - p_1)$. The variance of this profit that is explained (unexplained)

Expression (40a) shows that first period traders are negatively affected by the uncertainty over the value of their endowment (due to the effect of $\text{Var}[u_1]\text{Var}[p_2|u_1]$) and benefit from trading (due to the effect of $\text{Var}[x_1^L]\text{Var}[p_2|u_1]$). As $-1 < b_1^L < 0$, $(\text{Var}[x_1^L] - \text{Var}[u_1])\text{Var}[p_2|u_1] < 0$, and for given b_1^L an increase in return uncertainty lowers first period traders' welfare. Furthermore, (40a) shows that these traders also benefit from keeping part of their exposure to the payoff risk, as p_2 is positively correlated with their endowment. To see why, suppose that $u_1 > 0$. Then, traders reduce their exposure to the asset (as $b_1^L < 0$), selling part of their endowment in the first period, and keeping $(1 + b_1^L)$ of it in their portfolio. In this case, the higher is p_2, the more they earn out of such position. Thus, as in the transparent market case, traders' utility contains a *capital gains* component.

According to (40b) second period traders also benefit from trading and suffer from the uncertainty of their endowment value. As argued in Section 3.5, these traders hedge u_2, and speculate on the u_1-led imbalance. Therefore, endowment shock propagation has a positive 'speculative' effect on EU_2^L (through $\text{Var}[x_2^L]$). Additionally, it also has a negative 'risk-exposure' effect. To see this, note that the lower is the correlation between these traders' endowment shock and their strategy ($\rho^2_{x_2^L,u_2}$), the lower is their utility. This is because speculating on the u_1-led imbalance, increases these traders' exposure to payoff-risk, lowering their ex-ante utility. Accordingly, this effect disappears if $\tau_\eta \to 0$, in which case $\rho^2_{x_2^L,u_2} \to 1$ (their strategy only loads on u_2), and they only hedge endowment risk.

Overall, we can say that (i) traders benefit when they can trade more and suffer from the uncertainty of their endowment value, (ii) first period traders are adversely affected by short term price reversion, and (iii) endowment shock propagation has two contrasting effects on second period traders' welfare.

To investigate the welfare effect of an increase in μ with a strongly opaque market we use numerical simulations. Starting from Ds. We know from Corollary 12 that when $\tau_\eta \to 0$, Λ_1^* can be hump-shaped in μ. As EU_1^D is a monotone transformation of Λ_1^* (see (39a)), the same applies to Ds' welfare. For FDs, our simulations show that their welfare is decreasing in μ. Figure 8 (panels (a) and (b)) provides a graphical illustration.

Consider short-term traders. When $\tau_\eta \to 0$, second period traders do not trade on the u_1-led imbalance ($b_{22}^L \to 0$). This implies that they do not bear any of the welfare effects that are related to it (e.g., $\rho^2_{x_2^L,u_2} \to 1$), and benefit from an increase in risk sharing opportunities. As a consequence, in our simulations, their welfare increases in μ. Lack of information on u_1 also exposes second period traders to execution risk ($\kappa > 1$), leading them to hedge less (compared to the case of transparent markets). This, in turn, implies that first period traders face low uncertainty on the liquidation price, and the risk of their asset exposure weighs less in their welfare function, compared to the capital gains component. This explains why an increase in μ, that lowers the propagation of u_1 to p_2, has a negative impact on their welfare.

by p_1 is $\text{Var}[E[p_2|p_1] - p_1]$ $(\text{Var}[p_2 - p_1|p_1] = \text{Var}[p_2|p_1])$. Thus, the higher is $\text{Var}[E[p_2|p_1] - p_1]$ $(\text{Var}[p_2|p_1])$, the more (less) accurately FDs can anticipate their short term profit based on p_1.

As a consequence, in our simulations, CE_1^L is always decreasing in μ (see panels (c) and (d) in Figure 8).

Aggregating across market participants' welfare, highlights two important welfare implications of opaqueness. First, a more liquid market is not necessarily welfare improving. To see this, consider Panels (e) and (f) in Figure 8. The parameters values of this figure coincide with the ones used in Figure 6, which shows that liquidity is maximal with strong limits to market participation (μ close to zero).[19] This yields the following conclusion:

Numerical Result 4. *With opaque markets, a maximally liquid market can be welfare decreasing.*

Second, for a highly dispersed endowment shock to first period traders, the negative effect of a larger μ on the capital gains component can be so large, that strong limits to market participation (μ close to–but higher than–zero) become dominant compared to full participation ($\mu = 1$). This is in stark contrast with the transparent markets case. Figure 9 displays an example of this situation. Therefore,

Numerical Result 5. *With opaque markets, limited market participation can be welfare improving.*

	Transparent ($\tau_\eta \to \infty$)	Opaque ($0 < \tau_\eta < \infty$)	Strongly opaque ($\tau_\eta \to 0$)
Equilibrium	Unique	Possible ME*	Unique
Liquidity	Increasing in μ	Can be 'fragile'*	Hump-shaped/decreasing in μ*
Welfare	Increasing in μ*		Increasing/decreasing in μ*
Entry	N large/c small implement welfare optimum		

*Numerical result.

Table 1: Summary of results.

7 Concluding remarks

In this paper we study a 2-period model in which two classes of dealers—full and standard—intermediate short-term traders' orders, in a context where markets are fragmented due to an informational and a participation friction. We show that full dealers' limited market participation favors the propagation of first period traders' endowment shock across time. This, in turn, leads second period traders to speculate against the order imbalance due to their first period

[19]With strong opaqueness, second period traders do not speculate on the u_1-led imbalance and, absent FDs, no one absorbs the reverting orders of first period traders. As a consequence, if $\mu \to 0$, there is no equilibrium. Analytically, this can be seen from the fact that $\lim_{\mu\to 0}(\lim_{\tau_\eta \to 0} \psi(\Lambda_1)) = \infty$.

peers, affecting the latter uncertainty, and feeding back on these traders' liquidity consumption decision. As a result, a self-fulfilling loop can arise in which multiple levels of liquidity can be sustained in equilibrium.

Our main findings can be summarized as follows (see Table 1):

- When markets are either strongly opaque or fully transparent there is a unique equilibrium. Liquidity is increasing in market participation when markets are transparent, but can be hump-shaped when they are opaque (often with a maximum at the level in which there is as little market participation as possible). This implies that if markets are opaque, favoring FDs' entry doesn't necessarily enhance liquidity.

- When markets are opaque, multiple equilibria can arise. These equilibria can be ranked in terms of market liquidity, and we can show that a shock to market participation and/or to the volatility of the market participants' endowment shocks can move the market from the high to the low liquidity equilibrium. Hence, a second implication of our work is that market fragmentation can make liquidity fragile.

- When markets are transparent, total welfare is maximized when only full dealers are in the market. Together with the corresponding result for liquidity, this implies that a policy of improving liquidity when markets are transparent achieves a welfare optimum. In particular, as in this case increasing platform competition spurs full dealers' entry, an increase in the number of trading venues is both a liquidity and a welfare maximizing policy.

- With strong opaqueness, welfare can be increasing or decreasing in market participation. Coupling this with the corresponding liquidity results, this offers two further implications: on the one hand, we can find examples in which maximal liquidity and welfare are negatively correlated. On the other hand, there are situations in which strong limits to market participation can be welfare optimal with opaqueness, in contrast with the result that obtains with transparency.

A number of important issues are left for future work. In particular, understanding the impact of a change in opaqueness for a given level of market participation, would allow to give a better grounding to policies aimed at improving the prompt dissemination of market information (see, e.g., SEC (2010)). Also, assessing the welfare impact of abrupt changes in liquidity, can shed light on the consequences of market instability episodes for traders' participation.

References

Aït-Sahalia, Y. and M. Saglam (2013). High frequency traders: Taking advantage of speed. Working Paper 19531, National Bureau of Economic Research.

Anand, A. and K. Venkataraman (2015). Market conditions, fragility and the economics of market making. *Journal of Financial Economics (Forthcoming).*

Baruch, S. and L. Glosten (2013). Fleeting orders. *Working paper*.

Biais, B., T. Foucault, and S. Moinas (2015). Equilibrium fast trading. *Journal of Financial Economics (Forthcoming).*

Boehmer, E., K. Fong, and J. Wu (2015). International evidence on algorithmic trading. *Working paper*.

Bogousslavsky, V. (2014). Infrequent rebalancing and short-term return predictability. *Working Paper*.

Bongaerts, D. and M. Van Achter (2015). High frequency trading and market stability. *Working Paper, Erasmus University*.

Breckenfelder, H. J. (2014). Competition between high-frequency traders, and market quality. *Working Paper*.

Budish, E., P. Cramton, and J. Shim (2014). The high-frequency trading arms race: Frequent batch auctions as a market design response. *Working Paper*.

Cespa, G. and T. Foucault (2014). Illiquidity contagion and liquidity crashes. *Review of Financial Studies 27*(6), 1615–1660.

Cespa, G. and X. Vives (2012). Dynamic trading and asset prices: Keynes vs. Hayek. *Review of Economic Studies 79*, 539–580.

Cespa, G. and X. Vives (2015). The Beauty Contest and short-term trading. *Journal of Finance 70*(5), 2099–2154.

Chien, Y., H. Cole, and H. Lustig (2012). Is the volatility of the market price of risk due to intermittent portfolio rebalancing? *American Economic Review 102*(6), 2859–96.

Collin-Dufresne, P. and V. Vos (2015). Do prices reveal the presence of informed trading? *Journal of Finance (Forthcoming).*

Ding, S., J. Hanna, and T. Hendershott (2014). How slow is the nbbo? a comparison with direct exchange feeds. *Financial Review* (49), 313–332.

Du, S. and H. Zhu (2014). Welfare and optimal trading frequency in dynamic double auctions. *Working paper*.

Duffie, D. (2010). Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance 65*(4), 1237–1267.

Fioravanti, S. and M. Gentile (2011). The impact of market fragmentation on European stock exchanges. Technical report, CONSOB.

Foucault, T., J. Hombert, and I. Rosu (2015). News trading and speed. *Journal of Finance (Forthcoming)*.

Goldstein, I., Y. Li, and L. Yang (2014). Speculation and hedging in segmented markets. *Review of Financial Studies 27*(3), 881–922.

Goldstein, I. and L. Yang (2014). Information diversity and complementarities in trading and information acquisition. *Journal of Finance, Forthcoming*.

Hendershott, T., S. Li, A. Menkveld, and M. Seasholes (2014). Asset price dynamics with limited attention. *Working Paper*.

Heston, S. L., R. A. Korajczyk, and R. Sadka (2010). Intraday patterns in the cross-section of stock returns. *The Journal of Finance 65*(4), 1369–1407.

Hoffmann, P. (2014). A dynamic limit order market with fast and slow traders. *Journal of Financial Economics 113*(1), 156–169.

Jovanovic, B. and A. Menkveld (2015). Middlemen in limit order markets. *Working Paper*.

Mankiw, N. G. and M. D. Whinston (1986). Free entry and social inefficiency. *RAND Journal of Economics 17*(1), 48–58.

Menkveld, A. J. and M. A. Zoican (2015). Need for speed? exchange latency and liquidity. *Working Paper, Timbergen Institute*.

O'Hara, M. (2015). High frequency market microstructure. *Journal of Financial Economics 116*, 257–270.

O'Hara, M. and M. Ye (2011). Is market fragmentation harming market quality? *Journal of Financial Economics 100*(3), 459–474.

Pagnotta, E. and T. Philippon (2015). Competing on speed. *Working Paper*.

Peress, J. and D. Schmidt (2015). Noise traders incarnate: Describing a realistic noise trading process. *Working paper*.

SEC (2010). Concept release on equity market structure. *Federal Register*.

SEC (2013). Equity market structure literature review. part i: Market fragmentation. Technical report, Staff of the Division of Trading and Markets.

Spiegel, M. (1998). Stock price volatility in a multiple security overlapping generations model. *Review of Financial Studies 11*(2), 419–447.

Vives, X. (2008). *Information and Learning in Markets: The Impact of Market Microstructure*. Princeton University Press.

A Appendix

The following is a standard results (see, e.g. Vives (2008), Technical Appendix, pp. 382–383) that allows us to compute the unconditional expected utility of market participants.

Lemma 2. *Let the n-dimensional random vector $z \sim N(0, \Sigma)$, and $w = c + b'z + z'Az$, where $c \in \mathbb{R}$, $b \in \mathbb{R}^n$, and A is a $n \times n$ matrix. If the matrix $\Sigma^{-1} + 2\rho A$ is positive definite, and $\rho > 0$, then*

$$E[-\exp\{-\rho w\}] = -|I + 2\rho\Sigma A|^{-1/2} \exp\{-\rho(c - \rho b'(\Sigma + 2\rho A)^{-1} b)\}.$$

Proof of Lemma 1

Denote by $\mu x_1^{FD} = \int_0^\mu x_1^{FD} di$, $(1-\mu)x_1^D = \int_\mu^1 x_1^D di$, and by x_1^L respectively the aggregate position of FDs, dealers and liquidity traders in the first period. Imposing market clearing yields:

$$\mu x_1^{FD} + (1-\mu)x_1^D + x_1^L = 0 \iff \mu\varphi_1^{FD}(p_1) + (1-\mu)\varphi^D(p_1) + b_1^L u_1 = 0. \tag{A.1}$$

At equilibrium the coefficients of traders' strategies are known, which implies that p_1 is observationally equivalent to u_1 and that both FDs and dealers can retrieve u_1 from the price. Therefore, the information set of a FD and a dealer in the first period coincide and are given by $\Omega_1^{FD} = \Omega_1^D = \{u_1\}$. In the second period, denote by $\mu x_2^{FD} = \int_0^\mu x_2^{FD} di$ and by x_2^L, respectively the aggregate position of FDs and second period liquidity traders. Impose market clearing:

$$\mu(x_2^{FD} - x_1^{FD}) + (x_2^L - x_1^L) = 0,$$

and rearrange the first period market clearing condition as follows

$$(1-\mu)x_1^D = -\left(\mu x_1^{FD} + x_1^L\right).$$

Substitute the latter in the second period clearing equation to obtain

$$\mu x_2^{FD} + x_2^L + (1-\mu)x_1^D = 0. \tag{A.2}$$

Once again, at a linear equilibrium the coefficient of traders' strategies are known, which implies that the price sequence $\{p_1, p_2\}$ is observationally equivalent to $\{u_1, x_2^L\}$. Thus, the second period information set of a FD is given by $\Omega_2^{FD} = \{s_v, u_1, u_2\}$. $\square$

Proof of Proposition 1

The result comes as a special case of Proposition 5, when $\tau_\eta \to \infty$.

$\square$

Proof of Corollary 2

Using (8a) and (8b), the following explicit expression can be obtained for Λ_1^*:

$$\Lambda_1^* = \frac{1}{\tau_v(\gamma + (\mu\gamma + \gamma_1^L)(\mu\gamma + \gamma_2^L)(\gamma_2^L + \gamma)\tau_{u_2}\tau_v)}. \tag{A.3}$$

As one can verify, the above expression is decreasing in μ.

□

Proof of Corollary 3

The result comes as a special case of Proposition 14, when $\tau_\eta \to \infty$. More in detail, when $\tau_\eta \to \infty$ at equilibrium we have:

$$b_1^L = \gamma_1^L(\gamma_2^L + \gamma)(\gamma_2^L + \mu\gamma)\Lambda_1^*\tau_{u_2}\tau_v^2 - 1 \tag{A.4a}$$

$$b_{21}^L = -\frac{\mu\gamma}{\mu\gamma + \gamma_2^L} \tag{A.4b}$$

$$b_{22}^L = \frac{\gamma_2^L b_{21}^L(1-\mu)\Lambda_1^*\tau_v}{\mu} \tag{A.4c}$$

$$\mathrm{Var}[p_2|u_1] = \frac{1}{(\mu\gamma + \gamma_2^L)^2\tau_v^2\tau_{u_2}}, \tag{A.4d}$$

where Λ_1^* is given by (A.3). Furthermore,

$$E[p_2 - p_1|p_1] = \frac{(\gamma + \gamma_2^L)\Lambda_1^*}{\mu\gamma + \gamma_2^L}u_1,$$

so that

$$\mathrm{Var}\left[E[p_2 - p_1|p_1]\right] = \left(\frac{(\gamma + \gamma_2^L)\Lambda_1^*}{\mu\gamma + \gamma_2^L}\right)^2 \tau_{u_1}^{-1}. \tag{A.5}$$

The first two parts of the corollary come from substituting the expressions in (A.4a)–(A.4d) and (A.5) in (A.40) and (A.42). Note that since $\Lambda_1'(\mu) < 0$, both EU^D and EU^{FD} are decreasing in μ.

□

Proof of Corollary 4

The result comes as a special case of Proposition 14, when $\tau_\eta \to \infty$. In detail, differentiating the argument of the logarithm in (19) yields

$$-\frac{2}{(\gamma_2^L + \gamma\mu)^3\tau_v(\tau_{u_1} + \tau_v\Lambda_1^2(\mu))^2} \times \Bigg(\frac{\gamma(\tau_{u_1} + \tau_v\Lambda_1(\mu)^2)(\tau_{u_1} + \tau_v(1 + (\gamma + \gamma_2^L)^2\tau_{u_2}\tau_v)\Lambda_1(\mu)^2)}{\tau_{u_2}} \\ - (\gamma + \gamma_2^L)^2(\gamma_2^L + \gamma\mu)\tau_{u_1}\tau_v^2(1 + (\gamma_2^L + \gamma\mu)^2\tau_{u_2}\tau_v)\Lambda_1(\mu)\Lambda_1'(\mu)\Bigg),$$

which, given that Λ_1^* is decreasing in μ, is negative.

□

Proof of Proposition 5

In the second period a new mass of liquidity traders endowed with risk-tolerance coefficient $\gamma_2^L > 0$ enter the market. A date-2 liquidity trader submits a market order

$$X_2^L(u_2, s_{u_1}) = b_{21}^L u_2 + b_{22}^L s_{u_1}, \tag{A.6}$$

with $u_2 \sim N(0, \tau_{u_2}^{-1})$, and $s_{u_1} = u_1 + \eta$, with $\eta \sim N(0, \tau_\eta^{-1})$ and u_2, η independent of all the other random variables in the model. Consider the sequence of market clearing equations

$$\mu x_1^{FD} + (1-\mu)x_1^D + x_1^L = 0 \tag{A.7a}$$
$$\mu(x_2^{FD} - x_1^{FD}) + (b_{21}^L u_2 + b_{22}^L s_{u_1} - x_1^L) = 0. \tag{A.7b}$$

Rearrange (A.7a) as follows:

$$(1-\mu)x_1^D = -\left(\mu x_1^{FD} + x_1^L\right).$$

Substitute the latter in (A.7b):

$$\mu x_2^{FD} + b_{21}^L u_2 + b_{22}^L s_{u_1} + (1-\mu)x_1^D = 0. \tag{A.8}$$

A FD maximizes the expected utility of his second period wealth:

$$\begin{aligned}
&E\Big[-\exp\Big\{-\frac{1}{\gamma}\Big((p_2-p_1)x_1^{FD} + (v-p_2)x_2^{FD}\Big)\Big\}|p_1,p_2\Big] = \\
&= E\left[\exp\Big\{-\frac{1}{\gamma}(p_2-p_1)x_1^{FD}\Big\}\Big(-\exp\Big\{-\frac{1}{\gamma}(v-p_2)x_2^{FD}\Big\}\Big)|p_1,p_2\right] \\
&= \exp\Big\{-\frac{1}{\gamma}(p_2-p_1)x_1^{FD}\Big\}E\left[-\exp\Big\{-\frac{1}{\gamma}(v-p_2)x_2^{FD}\Big\}|p_1,p_2\right] \qquad \text{(A.9)}\\
&= \exp\Big\{-\frac{1}{\gamma}(p_2-p_1)x_1^{FD}\Big\}\Big(-\exp\Big\{-\frac{1}{\gamma}\Big(E[v-p_2|p_1,p_2]x_2^{FD} - \frac{(x_2^{FD})^2}{2\gamma}\mathrm{Var}[v-p_2|p_1,p_2]\Big)\Big\}\Big),
\end{aligned}$$

where the last expression in (A.9) is due to CARA and normality. For given x_1^{FD} the above is a concave function of the second period strategy x_2^{FD}. Solving the FOC, yields that in the second period a FD's limit order is given by $X_2^{FD}(p_1,p_2) = -\gamma\tau_v p_2$. Similarly, due to CARA and normality, in the first period a traditional market maker maximizes

$$E\left[-\exp\Big\{-\frac{1}{\gamma}(v-p_1)x_1^D\Big\}|p_1\right] = -\exp\Big\{-\frac{1}{\gamma}\Big(E[v-p_1|p_1]x_1^D - \frac{(x_1^D)^2}{2\gamma}\mathrm{Var}[v-p_1|p_1]\Big)\Big\}. \tag{A.10}$$

Hence, his strategy is given by $X_1^D(p_1) = -\gamma\tau_v p_1$. Substituting these strategies in (A.8) and solving for p_2 yields

$$p_2 = \lambda_2 \left(b_{21}^L u_2 + b_{22}^L s_{u_1} \right) - \frac{1-\mu}{\mu} p_1, \tag{A.11}$$

where $\lambda_2 = 1/\mu\gamma\tau_v$. The assumption that first period liquidity traders' strategies are linear implies that $p_1 = -\Lambda_1 u_1$ (see below). As a consequence we can rewrite (A.11) as follows:

$$p_2 = \lambda_2 (b_{21}^L u_2 + b_{22}^L s_{u_1}) + \lambda_2 (1-\mu)\gamma\tau_v \Lambda_1 u_1. \tag{A.12}$$

CARA and normality assumptions imply that the objective function of a second period liquidity trader is given by

$$E[-\exp\{-\pi_2^L/\gamma_2^L\}|\Omega_2^L] = -\exp\left\{ -\frac{1}{\gamma}\left(E[\pi_2^L|\Omega_2^L] - \frac{1}{2\gamma}\text{Var}[\pi_2^L|\Omega_2^L] \right) \right\}, \tag{A.13}$$

where $\Omega_2^L = \{u_2, s_{u_1}\}$, and $\pi_2^L \equiv (v - p_2)x_2^L + u_2 v$. Maximizing (A.13) with respect to x_2^L, the strategy of a second period liquidity trader is given by

$$X_2^L(u_2, s_{u_1}) = \gamma_2^L \frac{E[v - p_2|\Omega_2^L]}{\text{Var}[v - p_2|\Omega_2^L]} - \frac{\text{Cov}[v - p_2, v|\Omega_2^L]}{\text{Var}[v - p_2|\Omega_2^L]} u_2. \tag{A.14}$$

Computing

$$E[v - p_2|\Omega_2^L] = -\left(\lambda_2 (b_{21}^L u_2 + b_{22}^L s_{u_1}) + \frac{1-\mu}{\mu}\Lambda_1 \frac{\tau_\eta}{\tau_\eta + \tau_{u_1}} s_{u_1} \right) \tag{A.15a}$$

$$\text{Var}[v - p_2|\Omega_2^L] = \frac{\mu^2(\tau_{u_1} + \tau_\eta) + ((1-\mu)\Lambda_1)^2 \tau_v}{\mu^2(\tau_{u_1} + \tau_\eta)\tau_v} \tag{A.15b}$$

$$\text{Cov}[v - p_2, v|\Omega_2^L] = \frac{1}{\tau_v}. \tag{A.15c}$$

Substituting (A.15a), (A.15b), and (A.15c) in (A.14) and identifying coefficients yields

$$X_2^L(u_2, s_{u_1}) = b_{21}^L u_2 + b_{22}^L s_{u_1},$$

where

$$b_{21}^L = -\frac{1}{\tau_v(\gamma_2^L \lambda_2 + \text{Var}[v - p_2|\Omega_2^L])} \tag{A.16a}$$

$$b_{22}^L = -\frac{\gamma_2^L \tau_\eta \lambda_2 (1-\mu)\gamma\tau_v \Lambda_1}{(\tau_\eta + \tau_{u_1})(\gamma_2^L \lambda_2 + \text{Var}[v - p_2|\Omega_2^L])}. \tag{A.16b}$$

According to (A.14) second period liquidity traders' strategies react both to endowment and informational shocks. Thus, there are *two* measures of the price impact of trades in the second period (see (A.12)):

$$\lambda_{21} \equiv \frac{\partial p_2}{\partial u_2} = \lambda_2 b_{21}^L \tag{A.17a}$$

$$\lambda_{22} \equiv \frac{\partial p_2}{\partial s_{u_1}} = \lambda_2 b_{22}^L. \tag{A.17b}$$

Expressions (A.17a) and (A.17b) respectively correspond to the price impact of a marginal increase in the endowment shock and in the realization of the signal about u_1 observed by second period liquidity traders.

Consider now the first period. We start by characterizing the strategy of a FD. Substituting the optimal strategy in (A.9), rearranging and applying Lemma 2 yields the following expression for the first period objective function of a FD:

$$\begin{aligned} E[U((p_2-p_1)x_1^{FD}+(v-p_2)x_2^{FD})|u_1] = -\left(1+\frac{\text{Var}[p_2|u_1]}{\text{Var}[v]}\right)^{-1/2} \times \\ \exp\left\{-\frac{1}{\gamma}\left(\frac{\gamma\tau_v}{2}\nu^2+(\nu-p_1)x_1^{FD}-\frac{(x_1^{FD}+\gamma\tau_v\nu)^2}{2\gamma}\left(\frac{1}{\text{Var}[p_2|u_1]}+\frac{1}{\text{Var}[v]}\right)^{-1}\right)\right\}, \end{aligned} \tag{A.18}$$

where

$$\nu \equiv E[p_2|u_1] = \left(\lambda_2 b_{22}^L + \frac{1-\mu}{\mu}\Lambda_1\right)u_1 \tag{A.19a}$$

$$\text{Var}[p_2|u_1] = \lambda_2^2\left(\frac{(b_{21}^L)^2}{\tau_{u_2}}+\frac{(b_{22}^L)^2}{\tau_\eta}\right). \tag{A.19b}$$

Maximizing (A.18) with respect to x_1^{FD} and solving for the first period strategy yields

$$X_1^{FD}(p_1) = \frac{\gamma}{\text{Var}[p_2|u_1]}\nu - \gamma\left(\frac{1}{\text{Var}[p_2|u_1]}+\frac{1}{\text{Var}[v]}\right)p_1. \tag{A.20}$$

As we argued above, due to CARA and normality, for traditional market makers at date 1 we have $X_1^D(p_1) = -\gamma\tau_v p_1$. At equilibrium we then have

$$\mu\left(\frac{\gamma}{\text{Var}[p_2|u_1]}\nu - \gamma\left(\frac{1}{\text{Var}[p_2|u_1]}+\frac{1}{\text{Var}[v]}\right)p_1\right) + (-(1-\mu)\gamma\tau_v p_1) + b_1^L u_1 = 0,$$

implying that p_1 is linear in u_1: $p_1 = -\Lambda_1 u_1$, with Λ_1 to be determined.

We now turn to the characterization of first period liquidity traders' strategies. CARA and normality imply

$$E[-\exp\{-\pi_1^L/\gamma_1^L\}] = -\exp\left\{-\frac{1}{\gamma}\left(E[\pi_1^L|u_1]-\frac{1}{2\gamma}\text{Var}[\pi_1^L|u_1]\right)\right\}, \tag{A.21}$$

where $\pi_1^L \equiv (p_2-p_1)x_1^L + u_1 p_2$. Maximizing (A.21) with respect to x_1^L, and solving for the optimal strategy, yields

$$X_1^L(u_1) = \gamma_1^L \frac{E[p_2 - p_1|u_1]}{\mathrm{Var}[p_2 - p_1|u_1]} - \frac{\mathrm{Cov}[p_2 - p_1, p_2|u_1]}{\mathrm{Var}[p_2 - p_1|u_1]} u_1. \tag{A.22}$$

Computing

$$p_2 - p_1 = \left(\lambda_2 b_{22}^L + \frac{\Lambda_1}{\mu}\right) u_1 + \lambda_2 \left(b_{21}^L u_2 + b_{22}^L \eta\right),$$

and

$$E[p_2 - p_1|u_1] = \left(\lambda_2 b_{22}^L + \frac{\Lambda_1}{\mu}\right) u_1 \tag{A.23a}$$

$$\mathrm{Cov}[p_2 - p_1, p_2|u_1] = \mathrm{Var}[p_2|u_1]. \tag{A.23b}$$

Substituting the above in the strategy of a first period liquidity trader and identifying yields

$$X_1^L(u_1) = b_1^L u_1, \tag{A.24}$$

where

$$b_1^L = \gamma_1^L \frac{\mu\lambda_2 b_{22}^L + \Lambda_1}{\mu \mathrm{Var}[p_2|u_1]} - 1. \tag{A.25}$$

Substituting (A.20), x_1^D, and (A.24) in the first period market clearing condition and solving for the price yields $p_1 = -\Lambda_1 u_1$, where

$$\Lambda_1 = \psi(\Lambda_1) \equiv -\left(\mu\gamma\left(\frac{1}{\mathrm{Var}[p_2|u_1]} + \frac{1}{\mathrm{Var}[v]}\right) + (1-\mu)\gamma\frac{1}{\mathrm{Var}[v]}\right)^{-1} \times \left(\mu\frac{\gamma\mathrm{Cov}[p_2, u_1]}{\mathrm{Var}[p_2|u_1]\mathrm{Var}[u_1]} + b_1^L\right). \tag{A.26}$$

According to (A.25), the equilibrium coefficient of a first period liquidity trader depends on b_{21}^L, and b_{22}^L. Therefore, recursive substitution of the equilibrium strategies' coefficients in (A.26) shows that Λ_1 is pinned down by the solution of the following equation in Λ_1:

$$\psi(\Lambda_1) - \Lambda_1 = \frac{(\mu\gamma + \gamma_1^L)(\mathrm{Cov}[p_2, u_1]\tau_{u_1} + \Lambda_1) + \mathrm{Var}[p_2|u_1](\gamma\tau_v\Lambda_1 - 1)}{\gamma(\mu + \tau_v \mathrm{Var}[p_2|u_1])} = 0. \tag{A.27}$$

For $\mu \in (0, 1]$ the denominator in the above expression is positive, which implies that equilibria are pinned down by solutions to the quintic at the numerator of (A.27):

$$f(\Lambda_1) \equiv g_1(\Lambda_1) + g_2(\Lambda_1) + g_3(\Lambda_1) = 0, \tag{A.28}$$

where

$$g_1(\Lambda_1) \equiv -\mu^3\tau_{u_1}^2(1 - \gamma\tau_v\Lambda_1) + \Lambda_1\tau_{u_2}\tau_v^2(\gamma_1^L + \mu\gamma)(\gamma\tau_v\Lambda_1^2(1-\mu)^2 + \mu\tau_{u_1}(\gamma_2^L + \mu\gamma))^2 \tag{A.29a}$$

$$g_2(\Lambda_1) \equiv \mu^3\tau_\eta^2(\Lambda_1\tau_v(\gamma + (\gamma_1^L + \gamma)(\gamma_1^L + \mu\gamma)(\gamma_2^L + \mu\gamma)\tau_{u_2}\tau_v) - 1), \tag{A.29b}$$

and

$$g_3(\Lambda_1) \equiv \tau_\eta\big(- 2\mu^3\tau_{u_1} - (\gamma_2^L)^2\Lambda_1^2(1-\mu)^2\mu\tau_{u_2}\tau_v^2 + \gamma\Lambda_1^3(1-\mu)^2\mu\tau_{u_2}\tau_v^3 \times \\ (\gamma_2^L(\gamma_1^L+\gamma_2^L) + (2\gamma\gamma_1^L + (\gamma+\gamma_1^L)\gamma_2^L)\mu + \mu^2\gamma(\gamma_2^L+2\gamma)) + \\ + \mu^2\Lambda_1\tau_{u_1}\tau_v(2\mu\gamma + (\gamma_1^L+\mu\gamma)(\gamma_2^L+\mu\gamma)(\gamma_2^L+2\mu\gamma+\mu\gamma_2^L)\tau_{u_2}\tau_v)\big). \tag{A.29c}$$

Using (29a) we can compute $\text{Cov}[p_2, u_1]$:

$$\text{Cov}[p_2, u_1] = \frac{(1-\mu)\Lambda_1(\tau_{u_1}\gamma_2^L\lambda_2 + (\tau_{u_1}+\tau_\eta)\text{Var}[v-p_2|\Omega_2^L])}{\mu\tau_{u_1}(\tau_{u_1}+\tau_\eta)(\gamma_2^L\lambda_2 + \text{Var}[v-p_2|\Omega_2^L])}, \tag{A.30}$$

which is positive if and only if $\Lambda_1 > 0$. Consider (A.27) and suppose that at equilibrium $\Lambda_1^* < 0$. From (A.30), $\text{Cov}[p_2, u_1] < 0$. Due to (A.27) this implies $f(\Lambda_1^*) < 0$, which is impossible. Thus, at equilibrium, $\Lambda_1^* > 0$, and $\text{Cov}[p_2, u_1] \geq 0$. Similarly,

$$\text{Cov}[p_2, u_1|\Omega_2^L] = \lambda_2(1-\mu)\gamma\tau_v\Lambda_1\text{Var}[u_1|s_{u_1}] \geq 0.$$

To sign the strategy coefficient of a first period liquidity trader, we use (A.25):

$$b_1^L = \gamma_1^L\frac{\text{Cov}[p_2, u_1]\tau_{u_1} + \Lambda_1}{\text{Var}[p_2|u_1]} - 1. \tag{A.31}$$

From (A.31) we obtain

$$\frac{\text{Var}[p_2|u_1]}{\gamma_1^L}(1+b_1^L) = \text{Cov}[p_2, u_1]\tau_{u_1} + \Lambda_1,$$

which substituted in (A.27) yields

$$f(\Lambda_1) = \frac{\text{Var}[p_2|u_1]}{\gamma_1^L}\big((\mu\gamma+\gamma_1^L)(1+b_1^L) + \gamma_1^L(\gamma\tau_v\Lambda_1 - 1)\big) = 0. \tag{A.32}$$

Solving the above for Λ_1 yields:

$$\Lambda_1^* = \frac{1}{\gamma\tau_v}\left(1 - \frac{(1+b_1^L)(\mu\gamma+\gamma_1^L)}{\gamma_1^L}\right) \tag{A.33a}$$

$$= \frac{1}{\gamma_1^L\gamma\tau_v}(-\mu\gamma - b_1^L(\mu\gamma+\gamma_1^L)). \tag{A.33b}$$

Since $\Lambda_1^* > 0$, the last expression in (A.33a) implies that at equilibrium $b_1^L < 0$. Furthermore, using (A.31), $1 + b_1^L > 0$, which proves our result.

If we let $\tau_\eta \to 0$, (A.28) reads as follows:

$$f(\Lambda_1) = -\mu^3\tau_{u_1}^2(1-\gamma\tau_v\Lambda_1) + \\ \Lambda_1(\gamma_1^L+\mu\gamma)\tau_{u_2}\tau_v^2(\gamma\tau_v\Lambda_1^2(1-\mu)^2 + \mu\tau_{u_1}(\gamma_2^L+\mu\gamma))^2 = 0. \tag{A.34}$$

By inspection, $f(0) < 0$, $f(1/\gamma\tau_v) > 0$, and $f'(\Lambda_1) > 0$, which implies that in this case there exists a unique positive root.

Taking the limit for $\tau_\eta \to \infty$ in $\psi(\Lambda_1)$ yields:

$$\lim_{\tau_\eta\to\infty} \psi(\Lambda_1) = \frac{1 - \Lambda_1(\gamma_2^L + \mu\gamma)(\gamma_1^L(\gamma + \gamma_2^L) + \mu\gamma^2(1-\mu))\tau_{u_2}\tau_v^2}{\gamma\tau_v(1 + \mu(\gamma_2^L + \mu\gamma)^2\tau_{u_2}\tau_v)}. \tag{A.35}$$

Identifying Λ_1:

$$f(\Lambda_1) = \Lambda_1(\tau_v(\gamma + (\gamma + \gamma_2^L)(\gamma_1^L + \mu\gamma)(\gamma_2^L + \gamma\mu)\tau_{u_2}\tau_v) - 1 = 0,$$

and a unique solution with

$$\Lambda_1^*|_{\tau_\eta\to\infty} \equiv \frac{1}{\tau_v(\gamma + (\mu\gamma + \gamma_1^L)(\mu\gamma + \gamma_2^L)(\gamma_2^L + \gamma)\tau_{u_2}\tau_v)}, \tag{A.36}$$

obtains. Note that liquidity is in this case increasing in μ. Also, according to (A.35) we have $\psi'(\Lambda_1^*) < 0$.

□

Proof of Corollary 6

The proof of Proposition 5 has already established that at equilibrium $\mathrm{Cov}[p_2, u_1] \geq 0$, and $\mathrm{Cov}[p_2, u_1|\Omega_2^L] \geq 0$, for $\mu \leq 1$. Using the definition of z_1 and z_2 and computing the covariance yields:

$$\mathrm{Cov}[z_1, z_2] = \frac{b_1^L(b_{22}^L + (1-\mu)\gamma\tau_v\Lambda_1)}{\tau_{u_1}},$$

and the sign of the above expression depends on the sign of the second factor at its numerator. From the expression for b_{22}^L we have

$$b_{22}^L + (1-\mu)\gamma\tau_v\Lambda_1 = (1-\mu)\gamma\tau_v\Lambda_1\left(1 - \frac{\gamma_2^L\lambda_2\tau_\eta}{(\tau_\eta + \tau_{u_1})(\gamma_2^L\lambda_2 + \mathrm{Var}[v - p_2|\Omega_2^L])}\right) > 0,$$

which implies that $\mathrm{Cov}[z_1, z_2] \leq 0$, for $\mu \leq 1$. Finally, the expression for $\mathrm{Var}[v - p_2|\Omega_2^L]$ in Proposition 5 implies that for $\mu = 1$, $\mathrm{Var}[v - p_2|\Omega_2^L] = 1/\tau_v$.

□

Proof of Corollary 7

From Proposition 5 it is immediate that $\mathrm{Var}[v - p_2|\Omega_2^L]$ is increasing in Λ_1. Differentiating $\mathrm{Cov}[p_2, u_1]$ yields

$$\frac{\partial \mathrm{Cov}[p_2, u_1]}{\partial \Lambda_1} = \frac{1-\mu}{\mu(\tau_{u_1} + \tau_\eta)} + \frac{(1-\mu)\tau_\eta(\mathrm{Var}[v - p_2|\Omega_2^L](\gamma_2^L\lambda_2 + \mathrm{Var}[v - p_2|\Omega_2^L]) + \gamma_2^L\lambda_2\Lambda_1\mathrm{Var}[v - p_2|\Omega_2^L])}{\mu\tau_{u_1}(\tau_{u_1} + \tau_\eta)(\gamma_2^L\lambda_2 + \mathrm{Var}[v - p_2|\Omega_2^L])^2} \geq 0,$$

for $\mu \leq 1$. Finally,

$$\text{Cov}[p_2 - p_1, p_1] = -\frac{\Lambda_1^2}{\mu\tau_{u_1}}\left(\frac{\gamma_2^L\lambda_2(\mu\tau_\eta + \tau_{u_1}) + \text{Var}[v - p_2|\Omega_2^L](\tau_\eta + \tau_{u_1})}{(\tau_\eta + \tau_{u_1})(\gamma_2^L\lambda_2 + \text{Var}[v - p_2|\Omega_2^L])}\right) < 0.$$

□

Proof of Corollary 8

Collecting the terms in Λ_1 in the quintic equation (A.28) yields

$$\begin{aligned}
f(\Lambda_1) = {} & \gamma^2(1-\mu)^4\Lambda_1^5(\gamma_1^L + \mu\gamma)\tau_{u_2}\tau_v^4 \\
& + \mu\gamma(1-\mu)^2\tau_{u_2}\tau_v^3\Lambda_1^3\big(((\mu\gamma + \gamma_1^L)(2\mu\gamma + \gamma_2^L(1+\mu)) + (\gamma_2^L)^2)\tau_\eta + 2\tau_{u_1}(\mu\gamma + \gamma_1^L)(\mu\gamma + \gamma_2^L)\big) \\
& - (\gamma_2^L)^2(1-\mu)^2\mu\tau_\eta\tau_{u_2}\tau_v^2\Lambda_1^2 \qquad\qquad (\text{A.37}) \\
& + \mu^2(\tau_\eta + \tau_{u_1})\tau_v\Lambda_1\big(\mu\gamma(\tau_\eta + \tau_{u_1}) + (\mu\gamma + \gamma_1^L)(\mu\gamma + \gamma_2^L)(\mu\gamma(\tau_\eta + \tau_{u_1}) + \gamma_2^L(\mu\tau_\eta + \tau_{u_1}))\tau_{u_2}\tau_v\big) \\
& - \mu^3(\tau_\eta + \tau_{u_1})^2 = 0.
\end{aligned}$$

The above expression shows that there are three sign changes in the sequence formed by the quintic coefficients. Therefore, by Descartes' rule of sign, there are up to three positive roots of the equation $f(\Lambda_1) = 0$.

□

Proof of Corollary 9

If we let $\tau_{u_2} \to \infty$, the quintic that solves the equilibrium becomes

$$\begin{aligned}
f(\Lambda_1) = {} & \Lambda_1\big(\gamma^2\tau_v^2\Lambda_1^4(1-\mu)^4(\gamma_1^L + \mu\gamma) - (\gamma_2^L)^2(1-\mu)^2\mu\tau_\eta\Lambda_1 + \\
& \mu^2(\gamma_1^L + \mu\gamma)(\gamma_2^L + \mu\gamma)(\tau_{u_1} + \tau_\eta)(\mu\gamma(\tau_{u_1} + \tau_\eta) + \gamma_2^L(\tau_{u_1} + \mu\tau_\eta)) + \\
& \mu\gamma\tau_v(1-\mu)^2\Lambda_1^2((\gamma_2^L)^2\tau_\eta + 2\mu\gamma(\gamma_1^L + \mu\gamma)(\tau_{u_1} + \tau_\eta) + \gamma_2^L(\gamma_1^L + \mu\gamma)(\tau_\eta(1+\mu) + 2\tau_{u_1}))\big) = 0.
\end{aligned}$$

The above equation always has one null root. Inspection of (A.36) shows that in this case, when $\tau_\eta \to \infty$, $\Lambda_1^* = 0$ is the unique equilibrium.

Proof of Corollary 10

Follows from the proof of Proposition 5 in the case $\tau_\eta \to 0$.

□

Proof of Corollary 11

Evaluating (A.34) at $\Lambda_1^*|_{\tau_\eta\to\infty}$ yields

$$f(\Lambda_1^*|_{\tau_\eta\to\infty}) > 0,$$

which implies that $\Lambda_1^*|_{\tau_\eta\to 0} < \Lambda_1^*|_{\tau_\eta\to\infty}$.

□

Proof of Corollary 12

Differentiating (31) yields

$$\begin{aligned}\psi'(\Lambda_1) = &-\frac{1}{\gamma(\mu+\tau_v \mathrm{Var}[p_2|\theta_1])^2} \times \Bigg(\mu\gamma\tau_u(\mu+\tau_v\mathrm{Var}[p_2|\theta_1])\frac{\partial \mathrm{Cov}[p_2,\theta_1]}{\partial\Lambda_1} \\ &+\frac{\partial b_1^L}{\partial\Lambda_1}\mathrm{Var}[p_2|\theta_1](\mu+\tau_v\mathrm{Var}[p_2|\theta_1]) + \mu\frac{\partial\mathrm{Var}[p_2|\theta_1]}{\partial\Lambda_1}(b_1^L-\gamma\tau_u\tau_v\mathrm{Cov}[p_2,\theta_1])\Bigg).\end{aligned} \tag{A.38}$$

The first term inside the parenthesis in the above expression is positive as $\mathrm{Cov}[p_2,\theta_1]$ increases in Λ_1. Consider the remaining two terms:

$$\begin{aligned}\frac{\partial b_1^L}{\partial\Lambda_1} &= \frac{\gamma_1^L}{(\mathrm{Var}[p_2|\theta_1])^2}\left(\left(\frac{\partial\mathrm{Cov}[p_2,\theta_1]}{\partial\Lambda_1}\tau_u+1\right)\mathrm{Var}[p_2|\theta_1]-(\mathrm{Cov}[p_2,\theta_1]\tau_u+\Lambda_1)\frac{\partial\mathrm{Var}[p_2|\theta_1]}{\partial\Lambda_1}\right) \\ \frac{\partial\mathrm{Var}[p_2|\theta_1]}{\partial\Lambda_1} &= 2\lambda_2\left(\frac{b_{21}^L}{\tau_u}\frac{\partial b_{21}^L}{\partial\Lambda_1}+\frac{b_{22}^L}{\tau_\eta}\frac{\partial b_{22}^L}{\partial\Lambda_1}\right).\end{aligned}$$

At equilibrium $b_{21}^L < 0$ and, according to (30c) an increase in Λ_1 leads second period traders to hedge less their endowment shock. Also, when $\tau_\eta = 0$, according to (30d), $b_{21}^L = 0$. Therefore, in this case an increase in Λ_1 lowers $\mathrm{Var}[p_2|\theta_1]$. But then, this implies that b_1^L increases in Λ_1, which in turn implies that the remaining two terms in the parenthesis of (A.38) are positive too. Thus, $\psi'(\Lambda_1) < 0$.

□

Proof of Proposition 14

We start by obtaining a formula for the unconditional expected utility of Ds and FDs. Because of CARA and normality, a dealer's conditional expected utility evaluated at the optimal strategy is given by

$$\begin{aligned}E[U((v-p_1)x_1^D)|p_1] &= -\exp\left\{-\frac{(E[v|p_1]-p_1)^2}{2\mathrm{Var}[v]}\right\} \\ &= -\exp\left\{-\frac{\tau_v\Lambda_1^2}{2}u_1^2\right\}.\end{aligned} \tag{A.39}$$

Thus, traditional dealers derive utility from the expected, long term capital gain obtained supplying liquidity to first period hedgers.

$$
\begin{aligned}
EU^D \equiv E\left[U\left((v-p_1)x_1^D\right)\right] &= -\left(1+\frac{\text{Var}[p_1]}{\text{Var}[v]}\right)^{-1/2} \\
&= -\left(\frac{\tau_{u_1}}{\tau_{u_1}+\tau_v\Lambda_1^2}\right)^{1/2}.
\end{aligned}
\tag{A.40}
$$

Turning to FDs. Replacing (A.20) in (A.18) and rearranging yields

$$
E[U((p_2-p_1)x_1^{HF}+(v-p_2)x_2^{HF})|\theta_1] = -\left(1+\frac{\text{Var}[p_2|u_1]}{\text{Var}[v]}\right)^{-1/2} \times \exp\left\{-\frac{g(u_1)}{\gamma}\right\}, \tag{A.41}
$$

where

$$
g(u_1) = \frac{\gamma}{2}\left(\frac{(E[p_2|p_1]-p_1)^2}{\text{Var}[p_2|p_1]} + \frac{(E[v|p_1]-p_1)^2}{\text{Var}[v]}\right).
$$

Comparing the latter with (A.39) shows that given their second period utility, at date 1 FDs derive utility from two sources: the “long term” capital gain due to liquidity supply to first period hedgers *and* the “short term” capital gain due to the anticipation of p_2. Based on the expression for $g(u_1)$ we can see that the argument at the exponential of (A.41) is a quadratic form of the first period endowment shock. We can therefore apply Lemma 2 and obtain

$$
\begin{aligned}
EU^{FD} &\equiv E[U((p_2-p_1)x_1^{FD}+(v-p_2)x_2^{FD})] = \\
&= -\left(1+\frac{\text{Var}[p_2|p_1]}{\text{Var}[v]}\right)^{-1/2}\left(1+\frac{\text{Var}[p_1]}{\text{Var}[v]}+\frac{\text{Var}[E[p_2|p_1]-p_1]}{\text{Var}[p_2|p_1]}\right)^{-1/2},
\end{aligned}
\tag{A.42}
$$

where

$$
\text{Var}[E[p_2|p_1]-p_1] = \left(\lambda_2 b_{22}^L + \frac{\Lambda_1}{\mu}\right)^2 \frac{1}{\tau_{u_1}}.
$$

Consider now first period liquidity traders. Evaluating the objective function at optimum and rearranging yields

$$
-\exp\left\{-\frac{1}{\gamma_1^L}\left(E[\pi_1^L|u_1]-\frac{1}{2\gamma_1^L}\text{Var}[\pi_1^{HL}|u_1]\right)\right\} = -\exp\left\{-\frac{u_1^2}{\gamma_1^L}\left(\frac{(b_1^L)^2-1}{2\gamma_1^L\tau_v}\right)\right\},
$$

where $u_1 \sim N(0,\tau_{u_1}^{-1})$. The argument at the exponential is a quadratic form of a normal random variable. Therefore, applying again Lemma 2 yields

$$
E[-\exp\{\pi_1^L/\gamma_1^L\}] = -\left(\frac{\gamma_1^L\tau_{u_1}}{\gamma_1^L\tau_{u_1}+2C^L}\right)^{1/2}, \tag{A.43}
$$

where

$$
C^L = \frac{((b_1^L)^2-1)\text{Var}[p_2|u_1]}{2\gamma_1^L} + \frac{\text{Cov}[p_2,u_1]}{\text{Var}[u_1]}. \tag{A.44}
$$

Finally, for second period liquidity traders, substituting the optimal strategy (A.14) in the objective function (A.13) yields

$$E\big[-\exp\{-\pi_2^L/\gamma_2^L\}|\Omega_2^L\big] = -\exp\left\{-\frac{1}{\gamma_2^L}\left(\frac{\mathrm{Var}[v-p_2|\Omega_2^L](x_2^L)^2-\mathrm{Var}[v]u_2^2}{2\gamma_2^L}\right)\right\} \tag{A.45}$$

$$= -\exp\left\{-\frac{1}{\gamma_2^L}\begin{pmatrix} x_2^L & u_2 \end{pmatrix}\left(\frac{1}{2\gamma_2^L}\begin{pmatrix} \mathrm{Var}[v-p_2|\Omega_2^L] & 0 \\ 0 & -\mathrm{Var}[v] \end{pmatrix}\right)\begin{pmatrix} x_2^L \\ u_2 \end{pmatrix}\right\}.$$

The argument of the exponential is a quadratic form of the normally distributed random vector

$$\begin{pmatrix} x_2^L & u_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 & 0 \end{pmatrix}, \Sigma\right),$$

where

$$\Sigma \equiv \begin{pmatrix} \mathrm{Var}[x_2^L] & b_{21}^L\mathrm{Var}[u_2] \\ b_{21}^L\mathrm{Var}[u_2] & \mathrm{Var}[u_2] \end{pmatrix}.$$

Therefore, we can again apply Lemma 2 to (A.45), obtaining

$$E\left[E\left[-\exp\left\{-\pi_2^L/\gamma_2^L\right\}|\Omega_2^L\right]\right] = -|I+(2/\gamma_2^L)\Sigma A|^{-1/2}, \tag{A.46}$$

where

$$A \equiv \frac{1}{2\gamma_2^L}\begin{pmatrix} \mathrm{Var}[v-p_2|\Omega_2^L] & 0 \\ 0 & -\mathrm{Var}[v] \end{pmatrix}.$$

Rearranging (A.46) yields (40b), where we use

$$\rho^2_{x_2^L,u_2} = \frac{(b_{21}^L)^2}{\mathrm{Var}[x_2^L]\tau_{u_2}}.$$

□

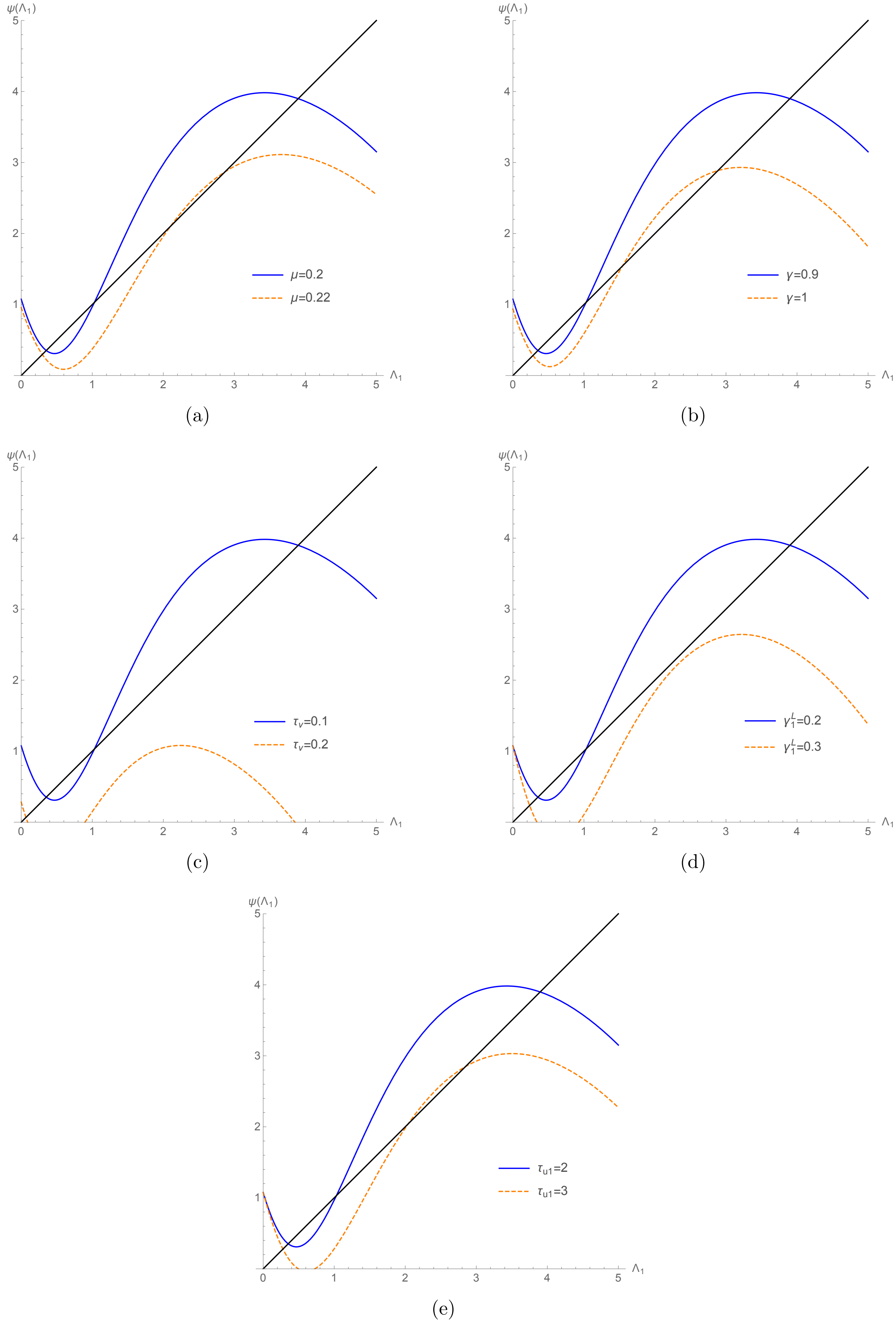
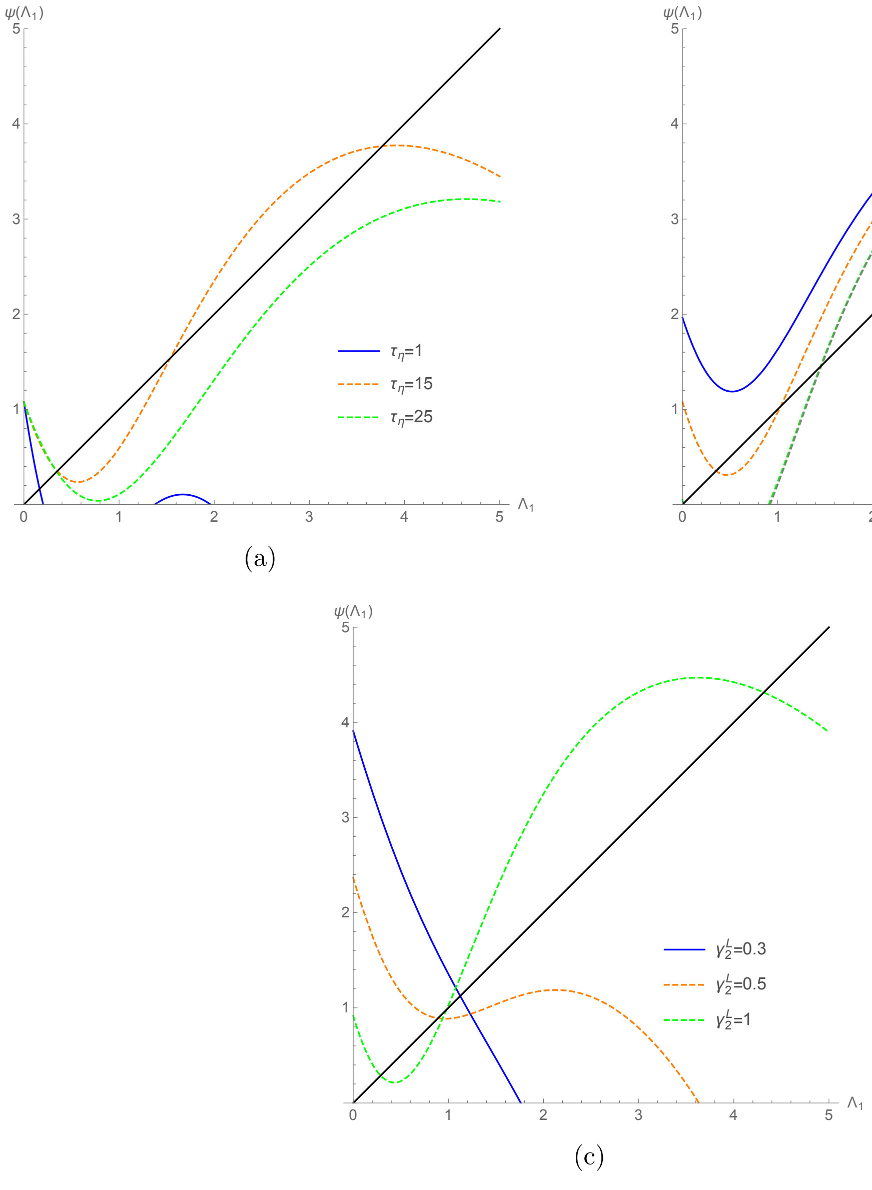
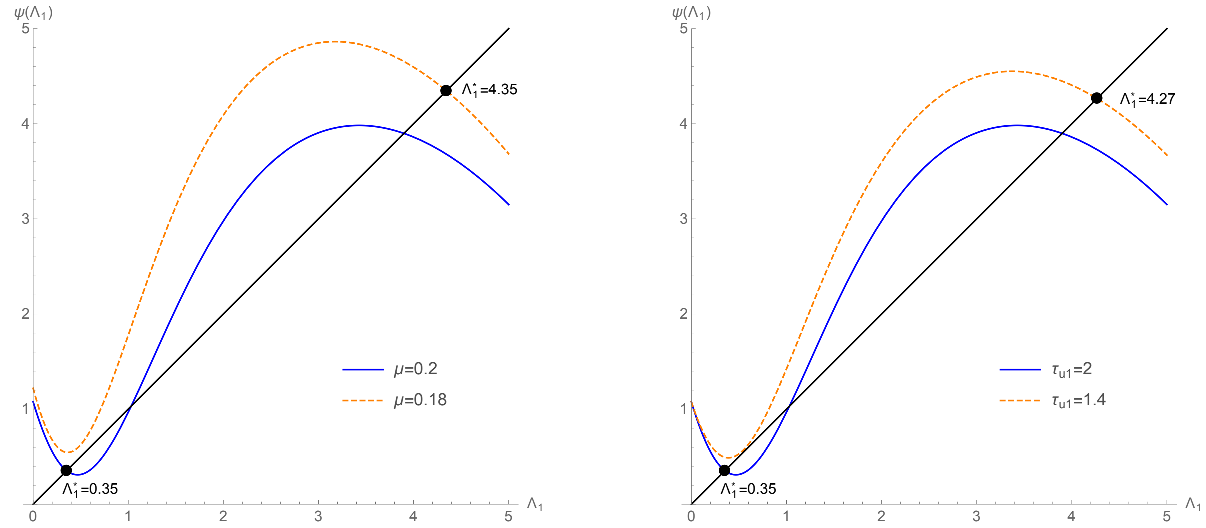


Figure 3: The effect of a change in parameters' values on the equilibrium set. The continuous blue curve represents the function $\psi(\Lambda_1)$ for $\tau_v = 0.1$, $\tau_{u_1} = 2$, $\tau_{u_2} = 400$, $\tau_\eta = 10$, $\gamma = 0.9$, $\gamma_1^L = 0.2$, $\gamma_2^L = 0.9$, and $\mu = 0.2$.



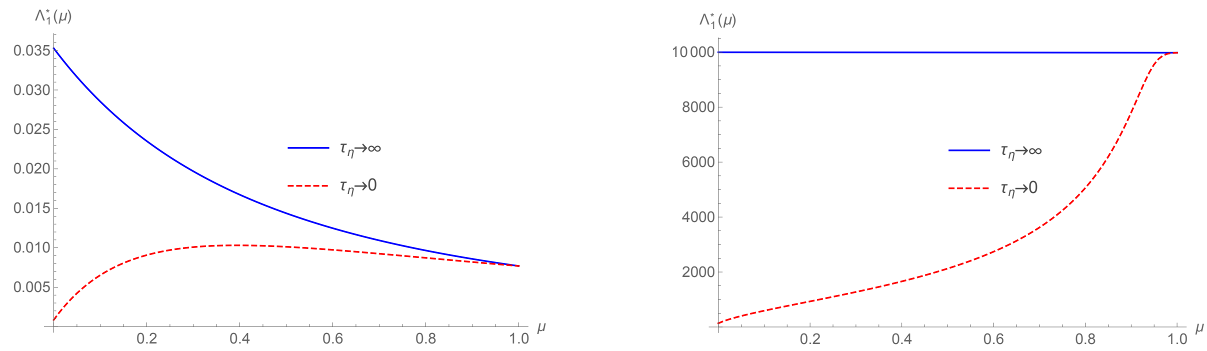
(a) (b) (c)

Figure 4: Comparative statics. The figure displays the effect of a change in the precision of second period liquidity traders' signal (Panel (a)), the dispersion of second period liquidity traders' endowment shock (Panel (b)), and second period liquidity traders' risk tolerance (Panel (c)). The other parameters' values are as in Figure 2.



(a) (b)

Figure 5: Liquidity fragility. In panel (a) we illustrate the effect of a decrease in the mass of FDs, and in panel (b) that of an increase in the volatility of first period liquidity traders' demand. Other parameters' values are as in Figure 3.



(a) (b)

Figure 6: The impact of an increase in the mass of FDs on market liquidity. When $\tau_\eta \to 0$, Λ_1^* can be hump-shaped, or always increasing in μ, depending on the dispersion of the payoff distribution (respectively, Panel (a) and (b)). Parameters' values: $\tau_{u_1} = \tau_{u_2} = 3.5$, $\gamma = 1$, $\gamma_1^L = \gamma_2^L = .8$, and $\tau_v = 2.5$ (Panel (a)), $\tau_v = 0.0001$ (Panel (b)).

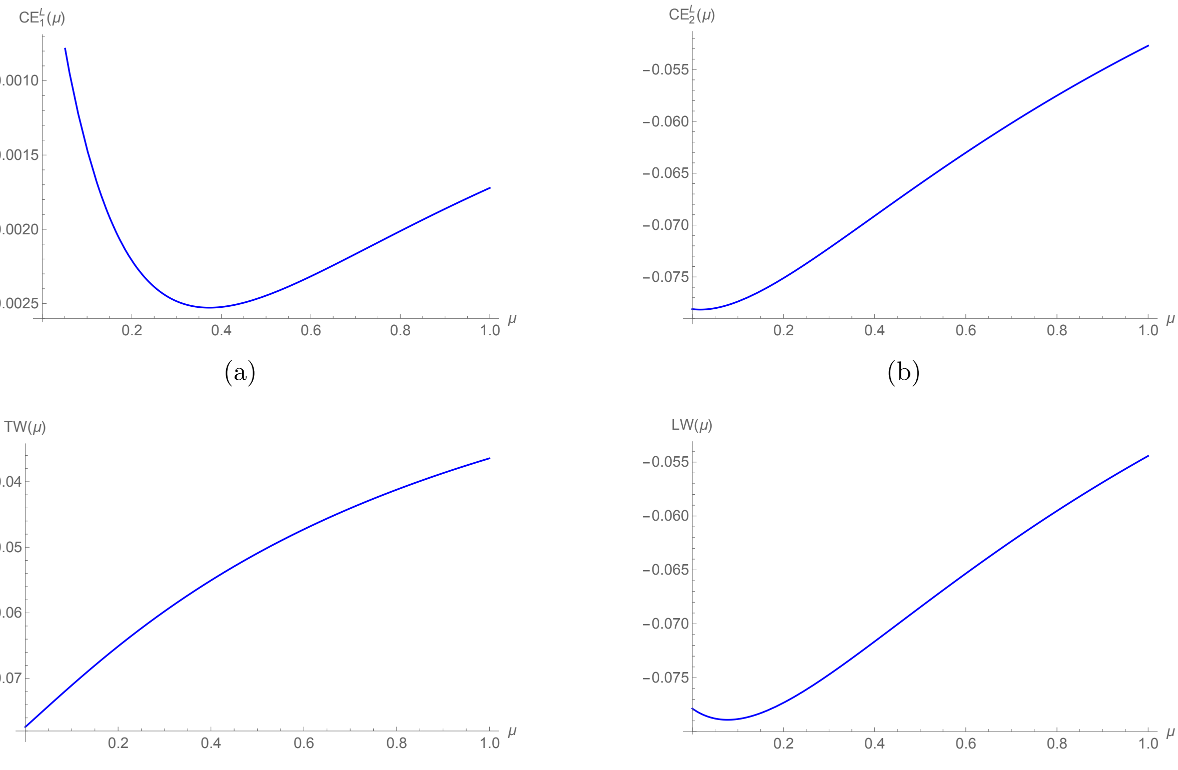


Figure 7: The impact of an increase in the mass of FDs on traders' welfare, when $\tau_\eta \to \infty$. Parameters' values: $\tau_{u_1} = \tau_{u_2} = 3.5$, $\gamma = 1$, $\gamma_1^L = \gamma_2^L = .8$, and $\tau_v = 2.5$.

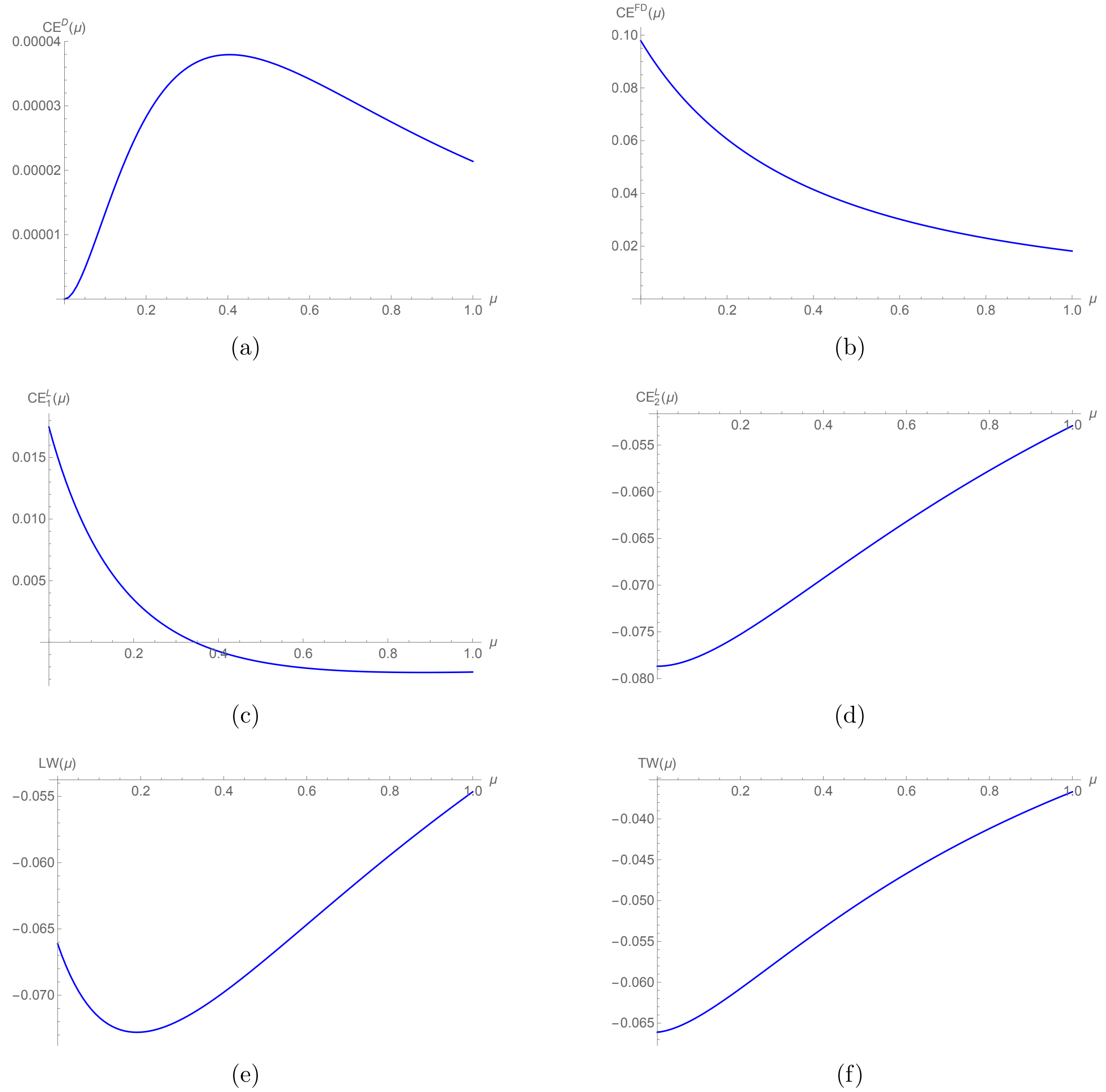


Figure 8: The impact of an increase in the mass of FDs on traders' welfare, with strongly opaque markets. Parameters' values: $\tau_{u_1} = \tau_{u_2} = 3.5$, $\gamma = 1$, $\gamma_1^L = \gamma_2^L = .8$, and $\tau_v = 2.5$.

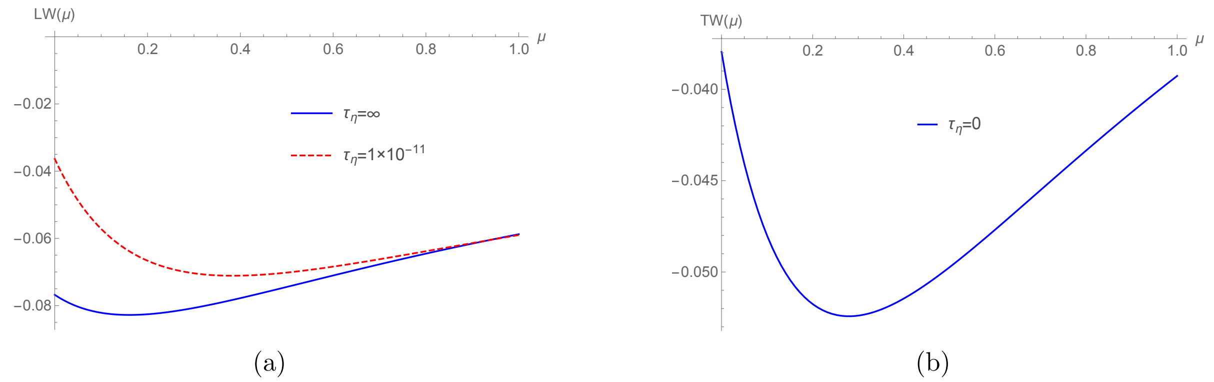


Figure 9: The impact of an increase in the mass of FDs on liquidity traders' and total welfare, when $\tau_\eta \to 0$. Parameters' values: $\tau_{u_1} = 1$, $\tau_{u_2} = 3.5$, $\gamma = 1$, $\gamma_1^L = \gamma_2^L = .8$, and $\tau_v = 2.5$.