Auctions with an asking price

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Abstract

This paper studies a sales mechanism, prevalent in housing markets, where the seller does not reveal or commit to a reserve price but instead publicly announces an asking price. We show that the seller sets an asking price such that, in equilibrium, buyers of certain types would accept it with positive probability. We also show that this sales mechanism, with an optimally chosen asking price set above the seller's reservation value, does better than any standard auction with a reserve price equal to the seller's reservation value. We then extend the analysis to the case where the asking price reveals information about the seller's reservation value. We show that in this case there is a separating equilibrium with fully-revealing asking prices, which is revenue-equivalent to a standard auction with a reserve price set at the seller's reservation value.

Keywords: asking price; auctions.

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1 Introduction

A standard result in auction theory is that the optimal auction, which maximises the seller's expected revenue, can be implemented using any standard format (such as a first or second-price sealed-bid auction) with a suitably chosen reserve price[1].

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[1] Myerson (1981), Riley and Samuelson (1981).

This optimally chosen reserve price is above the seller's valuation and requires a commitment not to sell the object below the reserve[2].

However, in practice, there are many markets where sellers seem unable to commit to a public reserve or prefer not to do so. Indeed, it is common to see the same items being listed for sale again with lower reserves or items being listed without the announcement of a reserve price[3]. Of course, this lack of commitment has an impact on the seller's expected revenue. Menezes and Ryan (2005), for example, show that when the seller negotiates with the highest bidder, if the reserve is not met, a feature of some house auctions, then the impact of a lack of commitment depends on the bargaining power of the different parties. If the buyer has all the bargaining power, then bargaining undermines the reserve completely: the effective reserve becomes the option value of re-auctioning. In contrast, if the seller has all the bargaining power, then bargaining strength substitutes perfectly for the reserve price commitment. In general, however, the implications of a lack of commitment to a reserve are not well understood especially for secret reserves in which the seller has no ability or incentive to commit.

The contribution of this paper is to study a particular sales mechanism, which is prevalent in the US housing market (see, for example, Albrecht *et al.* (2014)). Under this mechanism, the seller does not reveal or commit to a reserve price but instead publicly announces an asking price. The asking price is a partial commitment device. The seller commits to sell her house at the asking price if there is only one buyer who is willing to pay this price and the asking price is above the seller's reservation value. If two or more buyers offer the asking price, then the seller runs an auction amongst those buyers and sells the house to the highest bidder as long as her bid is higher than the seller's reservation value.

We first consider the case where the seller sets an asking price prior to learning her reservation value. This is the case, for example, where the asking price is proposed by the real estate agent. Once the property is advertised, then the seller will come to a view about her reservation value perhaps once she observes how many buyers have inspected the property or shown some interest in purchasing it.

In this case, the asking price reveals no information about the seller's reservation value, and we show that, in equilibrium, buyers of certain types would accept it with positive probability. Then we show that this sales mechanism, with an optimally chosen asking price set above the seller's reservation value, does better than any standard auction with a reserve price equal to the seller's reservation value.

[2]See Menezes and Monteiro (2005) chapter 3, pages 22-25.
[3]See Ashenfelter (1989).

The reason is that in such mechanism, the only time in equilibrium that a sale occurs at the asking price is when there is only one bidder with value greater than the maximum of the asking price and the seller's reservation value. In this case, the seller's revenue is equal to the asking price. Otherwise, the seller's revenue is equal to the expected value of the bidder with the second highest valuation among those with values greater than the maximum of the asking price and the seller's reservation value. As in Menezes and Monteiro (2003), bidders bid more aggressively under this mechanism than in a standard auction with a reserve lower than the asking price. In addition, in the case when all buyers have values lower than the asking price, buyers make a counteroffer and the winner is the buyer with the highest counteroffer if it is also higher than the seller's reservation value. We show these two facts combined mean that, in expectation, the sales mechanism we examine yields higher expected revenue than a standard auction with a reserve set at the seller's reservation value.

A second contribution is to provide a rationale for why transaction prices can sometimes be higher than asking prices in housing markets.[4] This is in contrast with a theoretical literature that assumes that the asking price is a ceiling for transaction prices[5].

A third and final contribution relates to the analysis of equilibrium behaviour when the seller announces her asking price after learning her reservation value. In this instance, we show that there is a fully-revealing unique asking price equilibrium. Moreover, this equilibrium yields the same expected revenue of a standard auction with a reserve price set at the seller's reservation value.

Albrecht *et al.* (2014) study a similar selling mechanism. However, there are major differences in the approach we take. Albrecht *et al.* (2014) consider a directed search environment where buyers search for a seller, a match is made, and then a transaction is undertaken. They show that when sellers are homogeneous, that is, they all have the same known reservation value, there is a continuum of equilibrium asking prices higher than the seller's reservation value. When sellers are heterogeneous, the types become private information and there are only two possible types of sellers. They show that there is a unique separating equilibrium in which sellers signal their types via asking prices. In contrast, we take an auction theoretical approach. We consider the seller as a monopolist and focus on the pricing mechanism. We are interested in understanding the capacity of this pricing mechanism to generate revenue for the seller. The seller's type is a continuous random variable in our model. Both

[4]See Case and Shiller (2003) for US data, de Wita and van der Klaauw (2013) for Europe and Khezr (2015) for Australia. The data shows that asking prices can be higher or lower than transaction prices.

[5]See Chen and Rosenthal (1996b), Chen and Rosenthal (1996a) and Carrillo (2012).

approaches, directed search and auction theory, may help understand the prevalence of such sales methods in housing markets.

We also add to a literature that focuses on existing auction formats and seek to understand why they have survived the test of time and remain popular as selling mechanisms. For example, Goeree and Offerman (2004) study Amsterdam real estate auctions. These are a popular auction format consisting of two stages. In the first stage, the auction price rises until only two bidders remain. Then in the second stage, the seller uses the highest standing bid from the previous stage as the reserve price and both bidders submit sealed bids. The highest bidder wins but both bidders receive a monetary payment proportional to the difference between the lowest bid and the reserve price (Goeree and Offerman (2004)). These authors suggest that this format helps low valuation bidders and increases the competition between the two highest bidders. In a series of experiments, Goeree and Offerman (2004) show that under some conditions second-price Amsterdam auctions generate almost as much revenue as the optimal auction.

Mathews and Katzman (2006) also investigate another type of real world auction where sellers advertise a buyout price. In this type of auction, if a buyer accepts the buyout price, she wins the object immediately. Mathews and Katzman (2006) show that by varying the seller's risk attitude, this auction can generate higher expected revenue to the seller than the optimal auction[6].

In a similar vein, by showing that, under certain conditions, an auction with an asking price can be better for the seller than a standard auction, we provide a rationale for the prevalence of asking price mechanisms.

2 Model

A seller of an object faces $n \geq 2$ potential buyers. Each buyer i's privately known value for the object is denoted by v_i, which is drawn independently from a known continuous distribution function F with support $[0, \bar{v}]$, twice differentiable, and with density $f < \infty$. The seller's privately known reservation value for the object is equal to s, which is drawn from a known, continuous and twice differentiable distribution function G with the same support and density $g < \infty$.

The game is described as follows. First, the seller announces an asking price p_A. Then buyers, after observing the asking price, can either accept it or make a counteroffer (a bid) less than p_A. We assume these decisions are made simultaneously.

[6]See Reynolds and Wooders (2009) for another examples of auctions with a buyout option.

This assumption is consistent with practice, where the seller may set a deadline for offers to be received but buyers will not be aware of their competitors' decisions.

Furthermore, in what follows we assume that the seller chooses p_A before learning her reservation value s. Then once buyers make their decisions, the seller learns s, and then makes her decision as described below. Note that in Section 6, we relax this assumption and consider the case where the seller learns her value prior to setting p_A. Both approaches in our view provide possible descriptions of real world markets. Real estate agents often play a major role in setting p_A and it is reasonable to think of sellers learning their reservation value once they observe the number of people who attended the open houses or the number of offers that they have received. Similarly, it seems plausible that, alternatively, sellers may invest some effort into learning their reservation value prior to setting p_A. In this paper we take an agonistic view and study both cases.

To continue the description of the game, we note that at the end of the first stage – where the seller announces the asking price and buyers either accept it or make a counteroffer – there are three possible outcomes:

1. No buyer accepts the asking price and some (or all) buyers make counteroffers at a value lower than the asking price. The seller accepts the highest offer if it is higher than her reservation value, otherwise the auction ends and the seller retains the object;

2. If only one buyer accepts the asking price, then the seller sells the object to this buyer at the asking price if $p_A \geq s$, otherwise she keeps the object at its value; and

3. If more than one buyer accept the asking price in the first stage, then the game goes to a second stage where the seller runs an auction with the maximum of the asking price and her reservation value as the reserve price.

Finally, we assume that the second stage takes the format of an English auction, implemented as a button auction as in Milgrom and Weber (1982). However, the number of participants in this auction is only known at the end of stage 1.

3 The Second Stage

To solve for the equilibrium, we start at the second stage. Suppose $2 \leq n' \leq n$ bidders offer the asking price in the first stage. This results in a second stage English auction with n' bidders and reserve price equal to $\max\{s, p_A\}$. Second-stage participants, by construction, have values greater than p_A. Bidding starts at $\max\{s, p_A\}$ and stops when only one bidder remains. The last bidder wins the auction and pays the highest standing bid. The winner's payoff is equal to:

$$u_i = v_i - \beta,$$

where β is the highest bid and v_i the winner's value.

It is well known that, in this setting, a weakly dominant strategy for bidders is to continue to bid until the auction price reaches their valuation. To simplify matters, from now on we use the strategic equivalence between the English auction and the second-price sealed-bid auction[7], and consider that bidders bid their true values in the second stage.

4 The First Stage

To determine the equilibrium behaviour in the first stage, we posit, and later establish, the existence (and uniqueness) of a value v^* which is the minimum value for a bidder to offer the asking price in the first stage. Bidders with values lower than v^* would not offer the asking price and instead would make a counteroffer. Bidders with values higher than v^* offer the asking price.

Now we consider the expected payoff, for Bidder i, who has a value v_i, and offers the asking price. If she is the only buyer to offer the asking price, her expected payoff is equal to:

$$\pi_a(v_i, v^*) = F(v^*)^{n-1}(v_i - p_A)G(p_A), \tag{1}$$

where $G(p_A)$ is the probability that the asking price exceeds the seller's reservation value, and $F(v^*)^{n-1}$ is the probability of being the only buyer who offers the asking price.

If, instead, there are m buyers, $2 \leq m \leq n$, who offer the asking price, her expected payoff from the second stage is equal to:

$$\pi_b(v_i, v^*) = \rho_{(m)} \int\limits_{\max\{v^*, s\}}^{v_i} \Big(v_i - x\Big)(m-1)F(x)^{m-2}f(x)dx, \tag{2}$$

[7]See Krishna (2002) Ch. 1, page 4-5.

where $\rho_{(m)}$ is the probability that m buyers offer the asking price, and as established in the previous section, bidders who continue to the second stage bid their values.

If we know the threshold value v^*, then it will be possible to use the distribution of the m^{th} order statistics of F to calculate the probability that there are exactly m bidders with values greater than or equal to v^* :

$$F_{(m)}(v^*) = \binom{n}{m} F(v^*)^{n-m}(1 - F(v^*))^m$$

We can now calculate the expected profit of bidder i who offers the asking price in the first stage as the sum of the expected payoffs from being the only bidder who offers the asking price and from participating in the second-stage auction with an uncertain number of bidders m, $2 \leq m \leq n$, when at least one additional bidder offers the asking price:

$$\pi^2(v_i, v^*) = F(v^*)^{n-1}(v_i - p_A)G(p_A) + \sum_{m=2}^{n} \rho_{(m)} \int\limits_{\max\{v^*, s\}}^{v_i} \big(v_i - t\big)(m-1)F(t)^{m-2}f(t)dt. \tag{3}$$

To continue the analysis for the first stage, we need to examine bidders' behaviour for those with a value lower than v^*. Suppose bidders $j \neq i$ follow the symmetric bidding strategy $b(v)$. Then Bidder i's expected payoff from bidding $b(z)$ is equal to:

$$\pi^1(v_i, z) = (v_i - b(z))F(z)^{n-1}G(b(z)), \tag{4}$$

where $G(b(z))$, is the probability of the bid being higher than the seller's reservation value. The next proposition establishes bidding behaviour for those bidders with values lower than v^*.

Proposition 1. *The following bidding function characterises the symmetric bidding strategy for bidders who make a counteroffer less than the asking price with the boundary condition $b(0) = 0$.*

$$b(v) = v - \frac{\int_0^v F(x)^{n-1}G(b(x))dx}{F(v)^{n-1}G(b(v))} \tag{5}$$

Proof. The First order condition from the maximisation of (4) at $z = v$ yields:

$$
\begin{aligned}
&-\frac{db(v)}{dv}\Big(F(v)^{n-1}G(b(v))\Big)+ \\
&(v-b(v))\Big((n-1)F(v)^{n-2}f(v)G(b(v))+F(v)^{n-1}g(b(v))\frac{db(v)}{dv}\Big)=0.
\end{aligned} \tag{6}
$$

This can be rewritten as:

$$
\begin{aligned}
&\frac{db(v)}{dv}\Big(F(v)G(b(v))\Big)= \\
&(v-b(v))\Big((n-1)f(v)G(b(v))+F(v)g(b(v))\frac{db(v)}{dv}\Big)
\end{aligned} \tag{7}
$$

The above differential equation can be solved with the boundary condition $b(0) = 0$. To do so, we first multiply both sides of (7) by $F^{n-2}(v)$:

$$
\frac{d}{dv}\Big(b(v)F(v)^{n-1}G(b(v))\Big)=v\frac{d}{dv}\Big(F(v)^{n-1}G(b(v))\Big). \tag{8}
$$

We then use integration by parts of the right hand side to yield equation (5). □

Equation (5) defines the equilibrium bidding function implicitly as there is no closed form solution.[8]. The next proposition establishes that the implied bidding strategy is unique. First, however, we need to establish a sufficient condition for $b(v)$ to be an increasing function:

Lemma 4.1. *The equilibrium bidding function for the first-stage, $b(v)$ is increasing in v if the following condition is satisfied.*

$$
\frac{G(b(v))}{g(b(v))}>\frac{\int_0^v F(x)^{n-1}G(b(x))dx}{F(v)^{n-1}G(b(v))}
$$

Proof: From (7), we have:

$$
\begin{aligned}
&\frac{db(v)}{dv}\Big(F(v)G(b(v))\Big)+(b(v)-v))F(v)g(b(v))\frac{db(v)}{dv}= \\
&(v-b(v))\Big((n-1)f(v)G(b(v))\Big).
\end{aligned} \tag{9}
$$

[8]While there is an important literature on the existence of equilibrium in asymmetric auctions (see for example, Maskin and Riley (2000), Lebrun (1996) and Plum (1992)), it is not directly applicable to our setting.

Then, it follows that:

$$\frac{db(v)}{dv} = \frac{(v - b(v))\Big((n-1)f(v)G(b(v))\Big)}{F(v)G(b(v)) + (b(v) - v))F(v)g(b(v))}. \tag{10}$$

The numerator is clearly positive. If the denominator is also positive then $b(v)$ is increasing. That is, we need that:

$$F(v)G(b(v)) + (b(v) - v))F(v)g(b(v)) > 0,$$

or

$$\frac{G(b(v))}{g(b(v))} > v - b(v).$$

Another way to represent the above condition is:

$$\frac{G(b(v))}{g(b(v))} > \frac{\int_0^v F(x)^{n-1}G(b(x))dx}{F(v)^{n-1}G(b(v))}.$$

It is easy to check that this condition is satisfied if both distributions are uniform.[9] □

Proposition 2. *The solution defined implicitly by (5) is the unique increasing bidding strategy.*

Proof: We need to show that the following differential equation has a unique solution:

$$\frac{db(v)}{dv} = \frac{(v - b(v))\Big((n-1)f(v)G(b(v))\Big)}{F(v)G(b(v)) + (b(v) - v))F(v)g(b(v))}. \tag{11}$$

Consider an initial point $\underline{\underline{v}}$ such that, $b(\underline{\underline{v}}) = s_0$.[10] Let us rewrite the above equation as follows:

$$b'(v) = \frac{(n-1)(v - b(v))\frac{f(v)}{F(v)}}{1 - (v - b(v))\frac{g(b(v))}{G(b(v))}} \tag{12}$$

[9] See section 5.1.

[10] Since $b(0) = 0$, and $\underline{\underline{v}} > 0$, we have $F(s_0) > 0$.

Continuity of $b'(v)$ requires:

$$\frac{G(b(v))}{g(b(v))} \neq v - b(v),$$

or

$$\frac{G(b(v))}{g(b(v))} \neq \frac{\int_0^v F(x)^{n-1} G(b(x))dx}{F(v)^{n-1} G(b(v))}.$$

At the initial point $\underline{\underline{v}} = s_0$ we have:

$$b'(\underline{\underline{v}}) = \frac{(n-1)(\underline{\underline{v}} - s_0)\frac{f(\underline{\underline{v}})}{F(\underline{\underline{v}})}}{1 - (\underline{\underline{v}} - s_0)\frac{g(s_0)}{G(s_0)}} > 0 \tag{13}$$

Therefore, the Lipschitz condition is satisfied, that is, if $b'(v) = H(v, b(v))$, there exist a k such that:

$$|\frac{dH}{db}| < k.$$

We can then apply Picard's theorem, which ensures that there is a unique solution around $\underline{\underline{v}}$. To obtain a unique solution for our target interval, we define an initial point $\underline{\underline{v}}$ such that $\underline{\underline{v}} = \frac{s+v^*}{2}$. It follows that the continuity condition is automatically satisfied as $b(v)$ is increasing in v. Beyond v^*, $b'(v)$ becomes zero so the Lipschitz condition is no longer satisfied and we might have multiple solutions. However, these solutions are not equilibria as they are not in the defined interval.□

As expounded above, our approach entails characterising a symmetric equilibrium that includes a cut off value such that buyers with values below the cut off bid according to (5), while buyers with value higher than the cut off offer the asking price. If such an equilibrium exists, then a bidder with value equal to the cut off must be indifferent between offering the asking price or making a counter offer in the first stage according to (5).

We will use this indifference to characterise the optimal asking price as a function of v^*. Define $p_A(v^*)$ as the asking price such that a buyer with value v^* is indifferent between offering it or making a counter offer according to (5). It follows that such an asking price must solve the following equation:

$$\int_0^{v^*} F(x)^{n-1}G(b(x))dx$$

$$= F(v^*)^{n-1}(v^* - p_A(v^*))G(p_A(v^*)) + \sum_{m=2}^{n} \rho_{(m)} \int\limits_{\max\{v^*,s\}}^{v^*} \big(v^* - t\big)(m-1)F(t)^{m-2}f(t)dt.$$

The left hand side is the expected payoff of making a counter offer, which is the expression in (4) with the equilibrium bidding function substituted from (5). And the right hand side is the expected payoff of accepting the asking price for type v^*. Solving for $p_A(v^*)$ yields:

$$p_A(v^*) = v^* - \frac{\int_0^{v^*} F(x)^{n-1}G(b(x))dx}{F(v^*)^{n-1}G(p_A(v^*))}. \tag{14}$$

There are two important corollaries of the above equation. First, we can conclude that the asking price is equal to the counter offer of the buyer with value v^*. And second, as long as $b(v)$ is increasing in v, $p_A(v^*)$ will also be increasing in v^*.

The next proposition summarises the equilibrium behaviour of buyers.

Proposition 3. *For every v^* in the interval, there is a unique $p_A(v^*)$ given by (14), which characterises a unique symmetric equilibrium where buyers with values lower than v^* bid according to (5), and buyers with value higher than v^* offer $p_A(v^*)$ and bid their values in the subsequent auction if two or more buyers offer $p_A(v^*)$.*

Proof: We need to show that both π^1 and π^2 are increasing in v and

$$\frac{\partial \pi^2}{\partial v} > \frac{\partial \pi^1}{\partial v},$$

for every v greater than v^*. But this is straightforward as:

$$\frac{\partial \pi^1}{\partial v}|_{v=v^*} = F(v^*)^{n-1}G(b(v^*)) \tag{15}$$

and

$$\frac{\partial \pi^2}{\partial v}|_{v=v^*} = F(v^*)^{n-1}G(p_A(v^*)). \tag{16}$$

The second equation is the differentiation of the term in (3) with respect to v_i at the point v^*. Since $\pi^1 = \pi^2$ at v^*, and π^1 is flat and π^2 is increasing in v for values higher than v^*, then the uniqueness follows. $\square$

5 The Seller's Behaviour

In our setting, the seller faces different decisions. First, the seller chooses an asking price prior to learning her reservation value. Second, after learning her reservation value, she decides whether to accept or reject the highest counteroffer in the event none of the buyers offer the asking price. The second decision is simple. If no one offers the asking price in the first stage, then the optimal decision for the seller is to accept the highest counteroffer, if it is higher than her reservation value. Otherwise, she rejects the counteroffer and retains the object. This follows from the assumption that the seller cannot commit to a reserve price.

We now turn to the first decision and characterise the seller's *ex ante* optimal choice of the asking price, $p_A^*(v^*)$. First, we analyse the seller's interim expected payoff for the case where her realised signal is lower than the asking price. In this case, the seller's expected payoff covers four possible events: (i) where no buyer accepts the asking price and the highest counteroffer is less than s; (ii) where no buyer accepts the asking price but the highest counteroffer is higher than s; (iii) where only one buyer accepts the asking price, wins the object and pays the asking price; and (iv) where there are two or more buyers who accept the asking price. The seller's interim expected profits in this case are given by the following expression:

$$\begin{aligned}\Pi_1(v^*, s) = n\int_0^{\underline{v}} sF(x)^{n-1}f(x)dx + n\int_{\underline{v}}^{v^*} b(x)F(x)^{n-1}f(x)dx \\ + nF(v^*)^{n-1}(1-F(v^*))(p_A) + \int_{v^*}^{\bar{v}} n(n-1)v(1-F(v))F(v)^{n-2}f(v)dv,\end{aligned} \tag{17}$$

where $\underline{v} = b^{-1}(s)$. The last integral captures the expectation of the second highest value among buyers in the event that at least two buyers have values higher than v^*.

We now consider the complementary case where the seller's realised signal is higher than the asking price. If only one buyer accepts the asking price, the seller rejects the offer and receives s. When more than one buyer accept the asking price, buyers will bid in an auction with s as the reserve price[11]. The following expresses the seller's interim expected payoff when her realised signal is higher than the asking price:

[11] Of course, in this scenario no counter offer would be acceptable to the seller.

$$\Pi_2(v^*,s) = nF(v^*)^{n-1}(1-F(v^*))s + \int_{\max[v^*,s]}^{\bar{v}} n(n-1)v(1-F(v))F(v)^{n-2}f(v)dv. \quad (18)$$

Thus, the total *ex ante* expected payoff for the seller can be expressed as follows;

$$\Pi(v^*, p_A(v^*)) = \int_0^{p_A(v^*)} \Pi_1(v^*,s)dG(s) + \int_{p_A(v^*)}^{\bar{v}} \Pi_2(v^*,s)dG(s). \quad (19)$$

The seller's problem is to find v^* and $p_A(v^*)$ that maximises her total expected payoff.

Differentiating (19) with respect to v^* yields:

$$\begin{aligned}\frac{\partial \Pi(v^*, p_A(v^*))}{\partial v^*} &= p'_A(v^*)\Pi_1(v^*, p_A(v^*))g(p_A(v^*)) + \int_0^{p_A(v^*)} \frac{\partial \Pi_1(v^*,s)}{\partial v^*}dG(s) \\ &\quad - p'_A(v^*)\Pi_2(v^*, p_A(v^*))g(p_A(v^*)) + \int_{p_A(v^*)}^{\bar{v}} \frac{\partial \Pi_2(v^*,s)}{\partial v^*}dG(s)\end{aligned} \quad (20)$$

Theorem 1. The seller's *ex ante* expected payoff from an optimally chosen asking price, set prior to the realisation of her reservation value, is higher than the expected payoff from an auction with s as the reserve price set after the seller learns her reservation value.

Proof: When the seller posts an asking price equal to zero and sets $v^* = s$, the mechanism becomes equivalent to an auction with a reserve price equal to s. It suffices to show the *ex ante* expected payoff at this point is increasing in v^*. The differentiation of (20) at this point is:

$$\begin{aligned}\frac{\partial \Pi(v^*, p_A(v^*))}{\partial v^*}|_{v^*=s} &= p'_A(v^*)\Pi_1(s,0)g(0) + \int_0^0 \frac{\partial \Pi_1(v^*,s)}{\partial v^*}|_{v^*=s}dG(s) \\ &\quad - p'_A(v^*)\Pi_2(s,0)g(0) + \int_0^{\bar{v}} \frac{\partial \Pi_2(v^*,s)}{\partial v^*}|_{v^*=s}dG(s)\end{aligned} \quad (21)$$

It is possible to check $\Pi_1(s,0) = \Pi_2(s,0)$. Thus, we have,

$$\frac{\partial \Pi(v^*, p_A(v^*))}{\partial v^*}|_{v^*=s} = \int_0^{\bar{v}} \frac{\partial \Pi_2(v^*, s)}{\partial v^*}|_{v^*=s} dG(s) \tag{22}$$

$\frac{\partial \Pi_2(v^*,s)}{\partial v^*}|_{v^*=s}$ is the differentiation of the expected payoff of an auction with respect to its reserve price. From Riley and Samuelson (1981) we know the optimal reserve price that maximises the expected payoff is $v_* = s + \frac{1-F(v_*)}{f(v_*)}$ which is greater than s. Therefore, at $v^* = s$ the expected payoff is increasing to its maximum. □

Theorem 1 shows that the seller is better off in expectation by setting an optimal asking price, even before knowing her true valuation, rather than waiting until she realises her value and running an auction with this value as the reserve price. The next theorem establishes an upper bound for the optimal asking price.

Theorem 2. Given the buyers' equilibrium behaviour, the seller maximises her *ex ante* expected payoff by setting an asking price $p_A^*(v^*) < p_A(\bar{v})$.

Proof: First, it is easy to check that at $p_A(\bar{v})$, the asking price mechanism is equivalent in an ex-ante sense (i.e., prior to the realisation of the seller's reservation value) to a first-price auction with a secret reserve price equal to the seller's value. This is because no buyer would accept this asking price and all buyers would make a counter offer. According to Elyakime *et al.* (1994), proposition 1, a first-price auction with a secret reserve price is revenue equivalent to a first-price (or a second-price) auction with a public reserve price of $b^{-1}(s)$. That is, the seller does not gain, from an ex-ante viewpoint, from keeping s secret. Finally, Theorem 1 shows that an optimal asking price mechanism does better, in expected revenue terms, than an auction with a reserve price s. It follows that the optimal asking price, $p_A^*(v^*)$, cannot be larger than or equal to $p_A(\bar{v})$. □

Theorem 2 provides a rationale for observing transactions at prices higher than the asking price. In equilibrium, there is always a positive probability that there will be more than two bidders with values greater than $p_A^*(v^*)$ and, therefore, an auction will ensue. We note that this extends the existing literature that suggests that the transaction price should always be lower than or equal to the asking price and is consistent with empirical evidence.[12]

More broadly, however, we do not know how the expected revenue generated by the optimal asking price mechanisms compares with that generated by the optimal auction, which is no longer the optimal mechanism in this environment. The next example shows that the former can be greater than the latter.

5.1 Example

Suppose the values for both the seller and the $n = 2$ buyers are distributed uniformly on $[0, 1]$. It follows from (5) that the first-stage bidding strategy can be determined by solving:

$$b(v) = v - \frac{\int_0^v xb(x)dx}{vb(v)}. \tag{23}$$

Solving (23) for $b(v)$ results in the following bidding function:

$$b(v) = \frac{2}{3}v \tag{24}$$

We can use the above bidding function and the expression in (14) to rewrite the optimal asking price as follows.

$$p_A(v^*) = v^*\left(1 - \frac{2}{9}\frac{v^*}{p_A}\right), \tag{25}$$

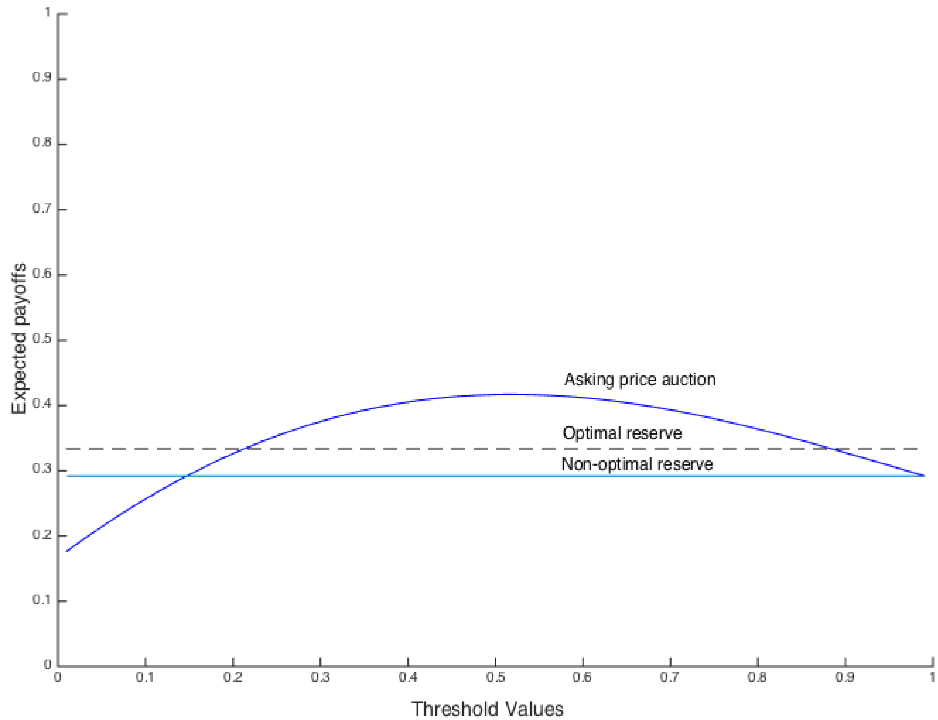
We can also use the counteroffer bidding function to write $p_A(v^*) = \frac{2}{3}v^*$. Using the expression of the total *ex ante* expected payoff, we can solve numerically for v^* and p_A^* yielding:

$$v^* = \frac{1}{2} \ , p_A^* = \frac{1}{3}.$$

Figure 1 shows the seller's *ex ante* expected revenue from different values for the asking price, compared to a standard second-price auction with a reserve price equal to the seller's reservation value and an auction with an optimal reserve price (dashed line). In this example, the optimal asking price generates more expected revenue

[12]For example, Han and Strange (2014) shows the number of properties sold in the US in which the transaction price exceeded the asking price tripled over the period from 1995 to 2005, and reached 30% of the total in some local markets

Figure 1: Asking price vs a regular auction, *ex ante* comparison.



than the optimal auction. This contrasts to the results in Albrecht *et al.* (2014) where an asking price mechanism always yields (weakly) less expected revenue than an auction with an optimally chosen reserve. The difference stems from the distinct informational assumptions. In particular, in our setting when the seller does not learn her reservation value until later, committing to an auction with a reserve entails the risk that the object will be sold below the seller's reservation value. Such risk is eliminated in the asking price mechanism. The next section considers the alternative informational setting where the seller learns her reservation value prior to setting the asking price.

6 Separating with the Asking price

This section considers the case when the asking price is set subsequently to the realisation of s. In this instance, the choice of asking price may convey some information about s. In particular, suppose buyers update their information regarding the seller's value upon observation of the asking price and $\hat{s}$ is their posterior beliefs regarding the seller's reservation value.

It follows that the expected payoffs to a buyers with value v_i who bids in the first stage is equal to:

$$\pi^1(v_i, z) = \begin{cases} (v_i - b(z))F(z)^{n-1} & \text{if} \quad b(z) \geq \hat{s} \\ 0 & \text{otherwise} \end{cases}. \tag{26}$$

We can now characterise the equilibrium bidding strategy for the bidders who make a counteroffer. Define $\underline{\underline{v}}$ such that, $b(\underline{\underline{v}}) = \hat{s}$.

Lemma 6.1. *The symmetric bidding strategy for bidders who make a counteroffer less than the asking price is given by:*

$$b(v) = v - \frac{\int_{\underline{\underline{v}}}^{v} F(x)^{n-1} dx}{F(v)^{n-1}}, \tag{27}$$

if their valuation is higher than $\underline{\underline{v}}$. Otherwise they do not bid.

Proof. This follows the same steps as deriving the bidding function for a first-price auction with a reserve price equal to $\hat{s}$. For instance, see Menezes and Monteiro (2005), Chapter 3, page 23. $\square$

The buyers' optimal strategy for the second stage remains unchanged as they still have a weakly dominant strategy to bid their valuation at a second-price auction with a reserve price equal to the asking price, which is now greater than or equal to the realised value of s. It is possible to check that the seller would not post an asking price lower than s. Suppose a seller posts $p_A < s$. Then by the construction of the game we know that she is effectively runs an auction with a reserve price s. This is because no counter offer would be accepted. But we know this is weakly dominated by a $p_A = s$.

Following the same argument as in section 4, we can characterise the symmetric equilibrium by a cut off value $v^\dagger$ such that the buyer with this value is indifferent between making a counteroffer and accepting the asking price. Thus, we have:

$$\int_{\underline{\underline{v}}}^{v^\dagger} F(x)^{n-1}dx = F(v^\dagger)^{n-1}(v^\dagger - p_A(v^\dagger)) + \sum_{m=2}^{n} \rho_{(m)} \int_{v^\dagger}^{v^\dagger} \big(v^\dagger - t\big)(m-1)F(t)^{m-2}f(t)dt.$$

This allows us to compute the optimal asking price as follows:

$$p_A(v^\dagger, \hat{s}) = v^\dagger - \frac{\int_{\underline{\underline{v}}}^{v^\dagger} F(x)^{n-1}dx}{F(v^\dagger)^{n-1}}. \tag{28}$$

Equation (28) represents the counter offer of a buyer with value $v^\dagger$. The following lemma is useful for the characterisation of the equilibrium.

Lemma 6.2. *The asking price (28) is increasing in $\hat{s}$ for every $v > \underline{\underline{v}}$.*

Proof: First we show $b(v)$ is increasing in v.

$$\begin{aligned} b'(v) &= 1 - \frac{F(v)^{n-1}F(v)^{n-1} - (n-1)f(v)F(v)^{n-2}\int_{\underline{\underline{v}}}^{v} F(x)^{n-1}dx}{F(v)^{2n-2}} \\ &= \frac{(n-1)f(v)\int_{\underline{\underline{v}}}^{v} F(x)^{n-1}dx}{F(v)^{n}} > 0 \end{aligned} \tag{29}$$

Therefore, b^{-1} is also increasing and we have,

$$\frac{\partial p_A(v^\dagger, \hat{s})}{\partial \hat{s}} = \frac{(b^{-1})'(\hat{s})F(\underline{\underline{v}})^{n-1}}{F(v^\dagger)^{n-1}} > 0.$$

□

The seller's expected payoff has the following expression.

$$\begin{aligned} \Pi(v^\dagger, s, \hat{s}) = {} & \underbrace{n\int_{0}^{\underline{\underline{v}}} sF(x)^{n-1}f(x)dx}_{A} + \underbrace{n\int_{\underline{\underline{v}}}^{v^\dagger} b(x)F(x)^{n-1}f(x)dx}_{B} \\ & + \underbrace{nF(v^\dagger)^{n-1}(1-F(v^\dagger))(p_A(v^\dagger, \hat{s}))}_{C} + \underbrace{\int_{v^\dagger}^{\bar{v}} n(n-1)v(1-F(v))F(v)^{n-2}f(v)dv}_{D}. \end{aligned} \tag{30}$$

Differentiating this expression with respect to $v^\dagger$ and $\hat{s}$ gives us,

$$\begin{aligned}
\frac{\partial \Pi(v^\dagger, s, \hat{s})}{\partial v^\dagger} &= nb(v^\dagger)F(v^\dagger)^{n-1}f(v^\dagger) + n(n-1)F(v^\dagger)^{n-2}f(v^\dagger)(1-F(v^\dagger))(p_A(v^\dagger,\hat{s})) \\
&- nF(v^\dagger)^{n-1}f(v^\dagger)p_A(v^\dagger,\hat{s}) + nF(v^\dagger)^{n-1}(1-F(v^\dagger))\frac{\partial p_A(v^\dagger,\hat{s})}{\partial v^\dagger} \\
&- n(n-1)v^\dagger(1-F(v^\dagger))F(v^\dagger)^{n-2}f(v^\dagger) \\
&= -n(n-1)(1-F(v^\dagger))\frac{f(v^\dagger)}{F(v^\dagger)}\int_{\underline{v}}^{v^\dagger} F(x)^{n-1}dx + nF(v^\dagger)^{n-1}(1-F(v^\dagger))
\end{aligned} \tag{31}$$

$$\begin{aligned}
\frac{\partial \Pi(v^\dagger, s, \hat{s})}{\partial \hat{s}} &= n(b^{-1})'(\hat{s})sF(\underline{v})^{n-1}f(\underline{v}) - n(b^{-1})'(\hat{s})b(\underline{v})F(\underline{v})^{n-1}f(\underline{v}) \\
&+ nF(v^\dagger)^{n-1}(1-F(v^\dagger))\frac{(b^{-1})'(\hat{s})F(\underline{v})^{n-1}}{F(v^\dagger)^{n-1}} \\
&= n(s-\hat{s})\big(b^{-1})'(\hat{s})F(\underline{v})^{n-1}f(\underline{v}\big) + n(1-F(v^\dagger))(b^{-1})'(\hat{s})F(\underline{v})^{n-1}.
\end{aligned} \tag{32}$$

The first part in the last line of (32) follows as $b(\underline{v}) = \hat{s}$.

To check the single crossing condition, we check how $v^\dagger$ and $\hat{s}$ change when s changes. From (31) we know $\frac{\partial \Pi(v^\dagger,s,\hat{s})}{\partial v^\dagger}$ does not depend on s and (32) implies that $\frac{\partial \Pi(v^\dagger,s,\hat{s})}{\partial \hat{s}}$ is increasing in s. Therefore, fixing Π at $\bar{\Pi}$, we have,

$$\frac{d}{ds}\left(\frac{\frac{\partial \Pi}{\partial v^\dagger}}{\frac{\partial \Pi}{\partial \hat{s}}}\right) < 0 \tag{33}$$

Now consider the seller with the lowest type $s = 0$. If there was full information, then we would have:

$$\begin{aligned}
\Pi(v^0, s, s) &= n\int_0^{v^0} b(x)F(x)^{n-1}f(x)dx + nF(v^0)^{n-1}(1-F(v^0))(p_A(v^0,0)) \\
&+ \int_{v^0}^{\bar{v}} n(n-1)v(1-F(v))F(v)^{n-2}f(v)dv.
\end{aligned} \tag{34}$$

Where $p_A(v^0, 0) = v^0 - \frac{\int_0^{v^0} F(x)^{n-1}dx}{F(v^0)^{n-1}}$, and v^0 satisfies the first-order condition of (34) as well as the second-order condition for a maximum point.

Proposition 4. *There is a unique separating equilibrium at the lowest type full information asking price, $p_A(v^0, 0)$, where each seller posts an asking price, such that buyers infer the true valuation of the seller, that is, $\hat{s} = s$.*

Proof: To characterise the separating equilibrium, consider a $\hat{s}$ such that,

$$\Pi(v^\dagger, s, s) = \max_{\hat{s}} \Pi(v^\dagger(\hat{s}), s, \hat{s})$$

Then we have,

$$\frac{\partial \Pi(v^\dagger, s, \hat{s})}{\partial v^\dagger} + \frac{\partial \Pi(v^\dagger, s, \hat{s})}{\partial \hat{s}}|_{\hat{s}=s} = 0$$

According to (31) and (32), $v^\dagger = \bar{v}$ is a solution. For every s, this gives an asking price equal to $p_A(\bar{v}, s)$, which is increasing in s. We further check whether any type wants to deviate from this equilibrium.

First, assume that a seller with value s_2 wants to deviate and announces a lower type, that is, she posts an asking price $p_A(v_1, s_1)$ with $s_1 < s_2$. In this case, $\hat{s} = s_1$, and parts A and B of expression (30) correspond to the expected payoff of an auction with a reserve price equal to $b^{-1}(s_1)$. We also know that $b^{-1}(s_1) < b^{-1}(s_2)$. Therefore, buyers with values between $b^{-1}(s_1) < v < b^{-1}(s_2)$ submit bids that are less than the seller's reservation value but can win the object with positive probability. But if the seller announces her true type via the asking price, that is, increases the asking price to $p_A(v_1, s_2)$, then $\hat{s}$ becomes equal to s_2 and the sum of A and B will increase. Since both parts C and D are zero, the overall expected payoff increases by announcing the true type and no type benefits from announcing a lower asking price.

Second, consider the possibility that a lower type s_1 deviates by setting an asking price of a higher type $p_A(v_2, s_2)$, with $s_1 < s_2$. If $v^\dagger = \bar{v}$, from (32) we know that no $\hat{s}$ higher than the seller's actual value is profitable as the differentiation of the expected payoff for $\hat{s} > s$ is negative at $v^\dagger = \bar{v}$. Thus, no type of seller would benefit by announcing a higher asking price. $\square$

The above result suggests that in the separating equilibrium every seller type would set the asking price high enough so that in equilibrium no buyers would accept it with positive probability, except the buyer with value $\bar{v}$ who is indifferent between accepting or rejecting. The next result characterises the expected revenue in the separating equilibrium. [13]

[13] In this environment, there are other pooling and partial pooling equilibria. However, there

Theorem 3. The separating equilibrium of the asking price auction is revenue equivalent to an auction with the seller's value as the reserve price.

Proof. Set $v^{\dagger} = \bar{v}$ in equation (30) makes both C and D equal to zero. Since the seller signals her exact value via the asking price, that is, $s = \hat{s}$, then part A and B are equivalent to the expected payoff of a first price auction with the seller's value as the reserve price. $\square$

Theorem 3 suggest that the asking price mechanism in the signalling environment is revenue equivalent to an auction with a reserve price equal to s. However, in the current set up it is not straightforward to check whether there exists a reserve price $r > s$ which is revenue superior for all seller types.

7 Conclusion

This paper shows that an asking price mechanism, which reveals no information about the seller's reservation value, can generate more expected revenue for the seller than a standard auction with a reserve. While this sale method may not strictly maximise the seller's revenue, there are many practical reasons why the optimal auction – a standard auction with an optimally chosen reserve – is not observed in practice. These reasons include resale costs, which makes committing not to sell below the reserve difficult, and a lack of the information needed to set an optimal reserve.

A second contribution of this paper is to show that it is possible for the asking price to be below the transaction price as observed in practice. This contrasts with a theoretical literature that assumes that the asking price is a ceiling for transaction prices.

We also provided an example where the asking price mechanism yields higher expected revenue than an optimal auction when the reserve price is set prior to the seller learning her reservation value. This provides another reason for the prevalence of the asking price mechanisms when sellers may be unsure about the value of the asking price or reserve price they should set.

A third and final contribution relates to the analysis of equilibrium behaviour when the seller announces her asking price after learning her reservation value. In this instance, we show that there is a fully-revealing asking price equilibrium. Moreover,

are methods to eliminate them from the equilibrium set. For instance, we can use Banks and Sobel (1987), D1 *criterion* with the same argument as in Albrecht *et al.* (2014) to eliminate those equilibria from the set.

this equilibrium yields the same expected revenue as a standard auction with a reserve price set at the seller's reservation value.

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