Does Easing Monetary Policy Increase Financial Instability?☆

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Abstract

This paper develops a model featuring both a macroeconomic and a financial friction that speaks to the interaction between monetary and macro-prudential policy and to the role of U.S. monetary and regulatory policy in the run up to the Great Recession. There are two main results. First, real interest rate rigidities in a monopolistic banking system increase the probability of a financial crisis (relative to the case of flexible interest rate) in response to contractionary shocks to the economy, while they act as automatic macro-prudential stabilizers in response to expansionary shocks. Second, when the interest rate is the only available policy instrument, a monetary authority subject to the same constraints as private agents cannot always achieve a (constrained) efficient allocation and faces a trade-off between macroeconomic and financial stability in response to contractionary shocks. An implication of our analysis is that the weak link in the U.S. policy framework in the run up to the Global Recession was not excessively lax monetary policy after 2002, but rather the absence of an effective second policy instrument aimed at preserving financial stability.

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JEL codes: E44, E52, E61.

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1 Introduction

The global financial crisis and ensuing Great Recession of 2007-09 have ignited a debate on the role of policies for the stability of the financial system or the economy as a whole (i.e., so called macro-prudential policies). In advanced economies, this debate is revolving around the role of monetary and regulatory policies in causing the global crisis and how the conduct of monetary policy and the supervision of financial intermediaries should be altered in the future to avoid the recurrence of such a catastrophic event.

In this paper we develop a simple model featuring both a macroeconomic and a financial friction —i.e., an interest rate rigidity that give rise to a traditional macroeconomic stabilization objective and a pecuniary externality that give rise to a more novel financial stability objective— which speaks to the necessity to complement monetary policy with macro-prudential policies in response to contractionary shocks.

Some observers have assigned to monetary policy a key role in exacerbating the severity of the global financial crisis of 2007-09. Taylor (2007), in particular, noticed that during the period from 2002 to 2006 the US federal funds rate was well below what a good rule of thumb for US monetary policy would have predicted. Figure 1 displays the actual federal funds rate (solid line) and the counterfactual policy rate that would have prevailed if monetary policy had followed a standard Taylor rule (dashed line). Indeed, the interest rate implied by the Taylor rule is well above the actual federal funds rate, starting from the second quarter of 2002. Taylor (2007) argues that such a counterfactual policy rate would have contained the housing market bubble. Taylor also supports the idea that deviating from this rule-based monetary policy framework has been a major factor in determining the likelihood and the severity of the 2007-09 crisis (Taylor, 2010).

Despite a somewhat widely shared common sentiment that the Federal Reserve is partly to blame for the housing bubble, the issue is highly controversial in academia and the policy community. Besides Taylor (2007, 2010), Borio and White (2003), Gordon (2005), and Borio (2006) support the idea that monetary policy contributed significantly to the boom that preceded the global financial crisis. In contrast, Posen (2009), Bean (2010), and Svensson (2010) argue against this thesis.[1]

To address this issue, we develop a simple model of consumption-based asset pricing with collateralized borrowing and pecuniary externalities, monopolistic banking and real interest rates rigidities. The presence of real and financial frictions give rise to both a traditional macroeconomic stabilization role for government policy and a more novel financial stability objective.

[1]Bernanke (2010) recently said that "the best response to the housing bubble would have been regulatory, rather than monetary".

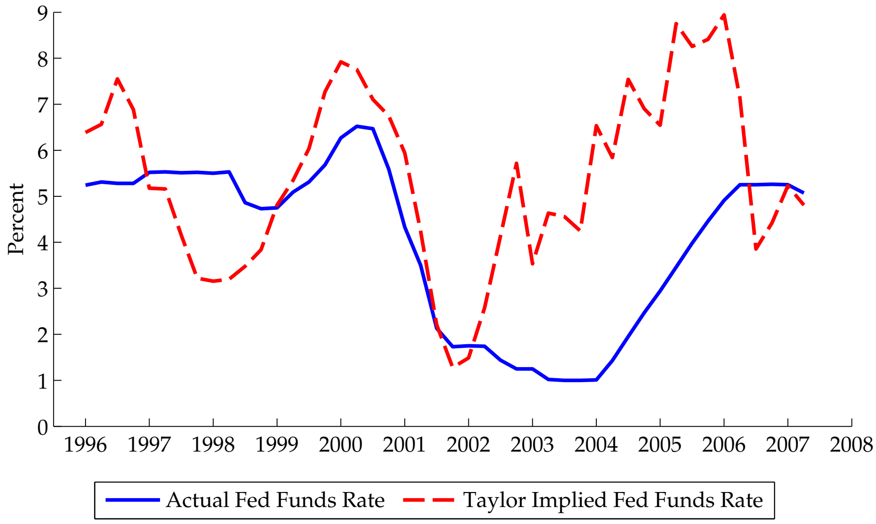

Figure 1 A COUNTERFACTUAL PATH FOR THE U.S. POLICY RATE. This chart replicates the counterfactual federal funds rate reported by Taylor (2007). The counterfactual path for the policy rate from 1996 to 2007 is obtained with a Taylor rule of the type: $i_t = r_t + \pi_t + 1.5(\pi_t - \pi) + 0.5(y_t - y_t^*)$, where r_t (the long-run, real value of the federal funds rate) is set to 2 percent, π_t is CPI inflation, π is target inflation (assumed at 2 percent), y_t is real GDP growth, and y_t^* is real potential GDP growth.

The macroeconomic stabilization objective arises from the presence of monopolistic competition and (real) interest rates rigidities in the banking sector. Due to monopolistic power, banks apply a markup on their funding cost. When they cannot fully adjust lending rates in response to macroeconomic shocks, the economy displays distortions typical of models with staggered price setting, generating an equilibrium that is not efficient (Hannan and Berger, 1991, Neumark and Sharpe, 1992, Kwapil and Scharler, 2010, Gerali, Neri, Sessa, and Signoretti, 2010, Espinosa-Vega and Rebucci, 2004).

The financial stability objective stems from the fact that the model endogenously generates financial crises when a collateral or leverage constraint occasionally binds. When access to bank credit is subject to an occasionally binding collateral constraint, a pecuniary externality arises. Individual borrowers do not internalize the effect of their decisions on the market price of collateral, and hence borrow and consume more than socially efficient, thereby increasing the frequency and the severity of financial crises (see, between others, Korinek, 2010, Bianchi, 2011, Jeanne and Korinek, 2010a,b, Benigno, Chen, Otrok, Rebucci, and Young, 2013).

There are two main results of the analysis. First, the analysis of our model economy shows that the interest rate rigidity has a different impact on financial stability (measured by the probability of a financial crisis) depending on the sign of the shock hitting the economy. In response to expansionary shocks that increase the funding cost of banks (i.e., a positive shock) aggregate bank lending rates rise too. However, because of interest rate stickiness, they increase less than in a flexible interest rate equilibrium. This affects next period net worth through two channels. On the one hand, relatively lower lending rates prompt consumers to borrow more than in the flexible-rate case, and thus lowering next period net worth; on the other hand, interest rate repayments are lower relative to the flexible case, thus increasing next period net worth. As the second effect dominates the first one in our model for a wide range of parameter values, the probability of a crisis is lower with interest rate stickiness. Thus, interest rate rigidity acts as an automatic macro-prudential stabilizer in response to shocks that require bank funding costs to increase.

In contrast, in the presence of a contractionary shock that lowers the funding cost of banks (i.e., a negative shock), aggregate lending interest rates fall relatively less than in the flexible-rate equilibrium. Because of the same mechanisms working in reverse, interest rate rigidity leads to a higher probability of financial crisis than in the flexible interest rate equilibrium in response to negative shocks. Thus, in our model, interest rate rigidity magnifies financial stability concerns (relative to the flexible-rate case) in response to shocks that push down bank funding costs. Note here that, while the effects of a positive or a negative shock on the equilibrium allocations of the model are perfectly symmetric, the implications of interest rate rigidity for financial stability depend on the sign of the shock and, in this sense, are 'asymmetric' in the model.

Second, our analysis shows that, if the government has only one policy instrument (say the monetary policy interest rate) and faces both the financial and the macroeconomic friction, efficiency cannot be achieved when a negative shocks hits the economy. However, when the government has two instruments (such as for instance a tax on debt and the monetary policy interest rate), efficiency can be achieved in response to both positive and negative shocks. Specifically, when both set of frictions are at work, achieving efficiency requires interventions of opposite direction on the same policy tool at the government's disposal in response to shocks that lower the funding cost of banks (in our case, the monetary policy interest rate). The model therefore entails a stark trade off between macroeconomic and financial stability in response to negative shocks that can be resolved only with a second policy instrument.[2]

Our analysis has an important implication regarding the role of US monetary policy for the stability of the financial system in the run-up to the Great Recession. In particular, in the last section of the paper, we use our model to assess Taylor's argument that higher interest rates would have reduced both the likelihood and the severity of the Great Recession in the US. In our model, Taylor's argument holds only with the auxiliary (but counterfactual) assumption that the US policy authority had only one instrument (namely the policy interest rate) in the 2002-2006 period to address both a macroeconomic and

[2] As we shall see, the second, macro-prudential policy instrument needs not necessarily be a tax on debt. Other tools can achieve the same results.

financial stability objectives. In this case our model calls for a higher monetary policy rate than what would be needed to address only macroeconomic stability. However, Taylor's argument does not hold if we assume, more realistically, that the US policy authority had two different instruments at its disposal, the policy rate and other regulatory tools. With two instruments, in our model, the monetary policy rate is freed to pursue macroeconomic stability, while the second tool can address financial stability.

As suggested by Bernanke (2010) and Blanchard, Dell'Ariccia, and Mauro (2010), this suggests that the same monetary policy stance as the one adopted by the Fed during the 2002-06 period, accompanied by stronger regulation and supervision of the financial system than we observed, might have been more effective in reducing the likelihood and the severity of the crisis, relative to a tighter monetary policy stance with the same financial supervision and regulation observed during the 2002-06 period that Taylor (2007, 2010) advocated.

This paper is related to several strands of literature. The first is the branch of the New Keynesian literature that considers financial frictions and Taylor-type interest rate rules (see Angelini, Neri, and Panetta, 2011, Beau, Clerc, and Mojon, 2012, Kannan, Rabanal, and Scott, 2012, for example). These papers consider either interest rate rules augmented with macro-prudential arguments —such as credit growth, asset prices, loan-to-value limits— or a combination of interest and macro-prudential rules in order to allow monetary policy to "lean against financial winds". However, in this class of models, macro-prudential regulation is taken for granted, in the sense that it does not target a clearly identified market failure giving rise to a well defined financial stability objective. In our model, there is a well defined pecuniary externality that justify government intervention for financial stability purposes.

The second is a growing literature on pecuniary externalities that interprets financial crises as episodes of financial amplification in environments where credit constraints are only occasionally binding (see, between others, Korinek, 2010, Bianchi, 2011, Jeanne and Korinek, 2010a,b, Benigno, Chen, Otrok, Rebucci, and Young, 2013). In this class of models the need for macro-prudential policies stems from a well-defined market failure: a pecuniary externality originating from the presence of the price of collateral in the aggregate borrowing constraint faced by private agents. However, in all these models, the financial friction is the only distortion in the economy. The question of how the pursuit of financial stability may affect and interact with the macroeconomic stability objective is therefore novel relative to this literature.

The third and final strand is a small, but growing literature that considers both macroeconomic and financial frictions at the same time. Kashyap and Stein (2012) use a modified version of the pecuniary externality framework of Stein (2012) where the central bank has both a price stability and a financial stability objective. Similar to our findings, a trade-off emerges between the two objectives when the policy interest rate is the only instrument and it disappears when there is a second tool (reserve requirements on bank deposits, which is equivalent to taxing debt in our framework as we shall see). However, they do not model the price stability objective explicitly. Woodford (2012), in contrast, sets up a New Keynesian model with credit frictions in which the probability of a financial crisis is endogenous (i.e., it is a regime-switching process that depends on the model variables). He then characterizes optimal policy in this environment, showing that —under certain circumstances— the central bank may face a trade-off between macroeconomic and financial stability. However, he does not explicitly model financial stability.

In contrast, in our paper, both the macroeconomic and the financial stability objective are well defined and each originates from a friction that we model explicitly. The interaction between the macroeconomic and the financial friction delivers a stark trade-off between objectives that helps rationalize the role of monetary policy and macro-prudential policy (or the lack of thereof) in the run-up to the Great Recession in the United States.

The rest of the paper is organized as follows. Section 2 describes the model economy. Sections 3 and 4 characterize the decentralized and the socially planned equilibrium of the economy, respectively. In Section 5 we discuss the implications of our model for the debate on the role of US monetary policy for the stability of the financial system in the run-up to the Great Recession. Section 6 concludes. An Appendix reports additional results discussed in the text.

2 The Model

We include monopolistic banking and interest rate rigidities in the pecuniary externality framework of Jeanne and Korinek (2010a). In their set-up, consumers borrow directly from international capital markets (or foreign banks) at the gross world interest rate R. In our model, consumers must borrow from a stylized monopolistic banking sector that intermediates foreign saving.[3]

The financial friction is given by the presence of *collateralized borrowing*. The real friction, strictly speaking, has two parts: the first is the presence of *market power* in the loan market exercised by monopolistically competitive banks; the second is *infrequent adjustment of real lending rates* by banks.[4]

[3]While this is a stark assumption, it is a simple way to introduce heterogeneity in a model which features another important source of complexity such as an occasionally binding collateral constraint. Alternatively, borrowers could be interpreted as entrepreneurs/households in a closed economy enjoying a comparative advantage in owning the asset. This implies that the lending rate faced by borrowers is affected by both domestic and external factors in the model, i.e., the behaviour of banks and government policy as well as the supply of foreign saving.

[4]Note that real and nominal interest rates coincide when expected inflation is zero. So here we are implicitly assuming that good prices are completely fixed in the short run.

The economy is populated by two sets of agents: a continuum of monopolistically competitive banks and a continuum of identical and atomistic consumers. Each set of agents has a unit mass. There are only three periods, denoted $t = 0, 1, 2$, representing the short, medium and long term respectively.

At the beginning of period 0 consumers own an asset in fixed, unit supply. In order to consume, they must either sell a fraction $(1 - \theta_{i,1})$ of the asset or borrow from banks $(b_{i,1})$. They have a well-defined loan demand that is decreasing in the lending interest rate (R_{L1}). Monopolistic banks collect deposits from foreign savers at the interest rate $(R_t = R)$ and —given loans demand— optimally set their lending rates. We assume that, when the cost of funds (R) changes because of shocks or policy interventions, only a fraction of banks (μ) can reset their lending rates, while the remaining banks $(1 - \mu)$ need to keep their lending rates fixed. The purpose of this key assumption is to introduce macroeconomic stabilization considerations in a relatively simple manner, but it can be justified by both theoretical and empirical grounds.[5]

At the beginning of period 0, the funding cost of banks is hit by a temporary shock $(R \pm \upsilon)$. While this is a reduced form shock in our model, a drop (increase) in the funding cost of banks can be triggered by a negative (positive) aggregate demand shock in a standard New-Keynesian framework. In our model, the source of such shock must be external, but in a more general (heterogeneous agents) set-up it could be domestic or external. In terms of a standard, Taylor interest rate rule such as the one plotted in Figure 1, this shock can be interpreted as a change in the real, long-run value of the policy rate (the so-called natural interest rate). In the rest of the paper we label an increase (decrease) in R a positive (negative) shock and will assume that it is driven by an increase (decrease) in aggregate demand. The loan market clears after the realization of this shock, which all agents observe. Then households consume $(c_{i,0})$ at the end of period 0.

In period 1, consumers have a stochastic endowment (e), they repay their debt $(b_{i,1}R_{L1})$, borrow an additional amount from banks $(b_{i,2})$, realize bank profits $(\pi_{i,1})$, and consume $(c_{i,1})$. Borrowing in period 1 is subject to a collateral constraint, with $(b_{i,2})$ limited to a fraction of the market value of the consumers' assets. This assumption is realistic and its purpose is to introduce explicit financial stability considerations in the model. If hit by a shock in period 0, the bank funding cost returns to long term value value (R) in period 1.

In period 2, consumers receive a deterministic return on the asset that they own (y), repay their debt $(b_{i,2}R_{L2})$, realize banks profits $(\pi_{i,2})$, and consume $(c_{i,2})$.

[5]Note here that nominal interest rate rigidity translates into real rate rigidity to an extent that depends on the degree of good price rigidity. As we noted, we implicitly assume that good prices are completely fixed in the short term. For examples of nominal interest rate rigidities see, between others, Hannan and Berger (1991), Neumark and Sharpe (1992), Kwapil and Scharler (2010), Gerali, Neri, Sessa, and Signoretti (2010), Espinosa-Vega and Rebucci (2004).

We now discuss the consumers' and banks' problems in turn.

2.1 Consumers and loan demand

The utility of each consumer, indexed by $i \in [0,1]$, is given by:

$$u(c_{i,0}) + u(c_{i,1}) + c_{i,2}, \tag{1}$$

where, for simplicity, we assume a unitary discount factor. The period utility function, $u(\cdot)$, is a standard CES function:

$$u(c) = \frac{c^{1-\varrho}}{1-\varrho}. \tag{2}$$

The budget constraint can be written as:

$$\begin{cases} c_{i,0} = b_{i,1} + (1-\theta_{i,1})p_0, \\ c_{i,1} + b_{i,1}R_{L1} = e + b_{i,2} + (\theta_{i,1} - \theta_{i,2})p_1 + \pi_{i,1}, \\ c_{i,2} + b_{i,2}R_{L2} = \theta_{i,2}y + \pi_{i,2}. \end{cases} \tag{3}$$

Consumer enter period 0 endowed with a unit of the asset $\theta_{i,0} = 1$ with price p_0. In order to consume in period 0, they need to either sell a fraction of their assets $(1 - \theta_{i,1})$ or borrow from banks $(b_{i,1})$. And as they are identical, in a symmetric equilibrium, we will have $\theta_{i,0} = \theta_{i,1} = \theta_{i,2} = 1$.[6]

In period 1, consumers faces a collateral constraint of the form:

$$b_{i,2} \leq \theta_{i,1}p_1, \tag{4}$$

where $\theta_{i,1}$ is the share of the asset held at the beginning of period 1.

This specification of the collateral constraint follows Mendoza (2010), Jeanne and Korinek (2010a), and Jeanne and Korinek (2010b). Equation (4) can also be interpreted as a Loan-to-Value constraint (LTV) of a margin loan or a cash-out refinancing. The LTV constraint implies that households can borrow up to a fraction (Θ) of the value of the collateral. Following Jeanne and Korinek (2010a), in our model Θ is set to 1 for simplicity. Note that the fraction Θ determines households' maximum leverage:

$$L^{\max} = \frac{1}{1-\Theta}. \tag{5}$$

where $L = L^{\max}$ when the LTV constraint is binding and $L < L^{\max}$ otherwise (see Appendix A). According to equation 5, this implies an unbounded maximum leverage ratio. In equilibrium, however, leverage is pinned down by preferences, interest rates, the

[6] As in Jeanne and Korinek (2010b), we assume that consumers derive some benefits from owning the asset. For instance, this asset can be interpreted as house.

deterministic return on the asset and the shadow price of the constraint. As we discuss below (and show in Appendix A), therefore, in our model equilibrium leverage is well defined and bounded.[7]

As Mendoza (2010) discusses and illustrates at length, when the collateral or leverage constraint occasionally binds, it triggers a process of asset price deflation and deleveraging in which small business cycle shocks are amplified, like in a financial crisis. In this sense, collateralized borrowing brings into the analysis a stylized set of financial stability considerations.

Consumers maximize (1) subject to the budget constraint (3) and the collateral constraint (4). Dropping the subscript i, the utility maximization problem of the *representative consumer* can be written as:

$$\max_{b_1,b_2,\theta_2} \left\{ \begin{array}{c} u\Big(b_1 + (1-\theta_1)p_0\Big) + \mathbb{E}_0\Big[u\Big(e + b_2 + (\theta_1 - \theta_2)p_1 + \pi_1 - \\ -b_1 R_{L1}\Big) + \theta_2 y + \pi_2 - b_2 R_{L2} - \lambda(b_2 - \theta_1 p_1)\Big] \end{array} \right\}. \tag{6}$$

Because of the occasionally binding constraint, the solution of this problem is non trivial. Solving the maximization problem backward, the first order conditions are:

$$\begin{array}{l} p_1 = \frac{y}{u'(c_1)}, \\ u'(c_1) = R_{L2} + \lambda, \\ u'(c_0) = R_{L1}\mathbb{E}_0\big[u'(c_1)\big]. \end{array} \tag{7}$$

The first equation represents the asset pricing condition for the economy. The second and third equations are the Euler equation for consumption in period 1 and 0, respectively. The global numerical solution of this problem is derived in the Appendix, where we also show that the problem has a well behaved closed-form solution when the constraint is not binding.

Consumers' demand of loans

In order to allow for market power in the banking sector, we model the loan market a-la Dixit and Stiglitz (1977) following Gerali, Neri, Sessa, and Signoretti (2010). Thus, we assume that loan contracts bought by consumers are a constant elasticity of substitution composite of differentiated financial products, each supplied by a bank j with an elasticity of substitution ζ. In particular, in order to obtain a loan of size $b_{i,t}$, the consumer i needs

[7] Alternatively, we can specify a Debt-To-Income (DTI) constraint, where total expected repayment (interest + principal) next period cannot be larger than a fraction of income. As we show in the Appendix B, the qualitative properties of our model are the same in the LTV economy and the DTI economy. The only difference is a quantitative one, stemming from the fact that the shadow price of the constraint will always be higher in the DTI economy than in the LTV economy.

to take out a continuum of loans $b_{ij,t}$ from all existing banks j, such that:

$$b_{i,t} \leq \left(\int_0^1 b_{ij,t}^{\frac{\zeta-1}{\zeta}} \, dj \right)^{\frac{\zeta}{\zeta-1}} \tag{8}$$

Demand by consumer i seeking a loan of size $b_{i,t}$ can be derived by minimizing the total repayment due to the continuum of banks j over $b_{ij,t}$. Aggregating over symmetric consumers, yields the following downward-sloping loan demand curve:

$$b_{j,t} = \left(\frac{R_{Lj,t}}{R_{Lt}} \right)^{-\zeta} b_t. \tag{9}$$

with the aggregate lending interest rate given by:

$$R_{Lt} = \left(\int_0^1 R_{Lj,t}^{1-\zeta} dj \right)^{\frac{1}{1-\zeta}}. \tag{10}$$

2.2 Banks and loan supply

There is a continuum of monopolistically competitive domestic banks indexed by $j \in [0,1]$ and owned by households.[8] In particular, we assume that each bank j supplies a differentiated financial product, and no other bank produces the same variety. However, banks competes with each other since costumers perceives each variety as an imperfect substitute. Because of market power banks can set the lending rate to maximize profits, taking into account the elasticity of demand for their variety.

Each bank j collects deposits $d_{j,t}$ from foreign savers at the funding cost $R_t = R$, where R is exogenous. We further assume that foreign savers can supply an infinite amount of deposits, so that banks can satisfy any demand for loans. Finally, banks use deposits to supply loans to consumers with the following constant return to scale production function:[9]

$$b_{j,t} = d_{j,t}. \tag{11}$$

In each period, bank j maximizes its profits by choosing price and quantity for given funding cost:

$$\max_{R_{Lj,t}, b_{j,t}} b_{j,t} R_{Lj,t} - d_{j,t} R_t,$$

[8] Market power is a standard assumption in banking. It can be justified by the presence of switching costs that lead to long-term relationships between banks and their costumers (Diamond (1984) for example). Empirically, the presence of market power in banking and its determinants over the business cycle are well documented. See, for example, Berger, Demirguc-Kunt, Levine, and Haubrich (2004) and Degryse and Ongena (2008).

[9] More realistic balance sheet assumptions with bank reserves and bank capital are discussed below when we specify government policy.

subject to the demand schedule in (9) and to the production function in (11). The first order condition for this problem implies that the optimal lending rate is a constant markup ($\mathcal{M}$) over the marginal cost of funds:

$$R_{Lt}(j) = \frac{\zeta}{\zeta - 1} R_t = \mathcal{M} R_t. \tag{12}$$

Together with consumers' optimality conditions, equation (12) determines the equilibrium of the economy: once the lending rate is set, households choose consumption (and hence borrowing), and loan market clearing closes the model.

As we noted earlier, we also assume interest rate rigidity: banks cannot always adjust lending rates in response to changes in their funding costs, which in turn can be affected by various macroeconomic shocks. The presence of interest rate stickiness in the banking sector can be justified by the presence of adjustment costs and monopolistic power. For example, Hannan and Berger (1991) show that, in the presence of fixed adjustment costs, banks reset their lending rates only if the costs of changing the interest rate are lower than the costs of maintaining a non-equilibrium rate (see also Neumark and Sharpe, 1992). Empirically, it is a well documented that the adjustment of lending rates to changes in the funding cost of banks is only partial and heterogeneous, especially in the short run. For example, Kwapil and Scharler (2010) find that the pass-through of changes in the policy rate to consumer loan rates in the US can be as low as 0.3, implying that banks smooth lending rates significantly. Espinosa-Vega and Rebucci (2004) find similar evidence for small open economies.

In particular, when the funding cost changes because of the shock (υ) in period 0, we assume that only a fraction μ of the banks can reset the lending rate, whereas the remaining $1 - \mu$ banks cannot. This entails that, following a shock to the deposit rate, the *aggregate* lending rate will be different from the one desired by banks. Given that consumers are price takers and that their loan demands depend on the *average* interest rate in the economy, this friction leads to a distortion in the competitive equilibrium of the economy that creates scope for policy intervention to restore efficiency. However, lending rates are again fully flexible in the long-run (i.e. in period 2).

Note here that all agents (banks, consumers, and the government) observe the shock to the deposit rate in period 0 before making their decisions. We consider three states: no shock ($\upsilon = 0$), a temporary increase in funding costs ($\upsilon > 0$), and a temporary reduction in funding costs ($\upsilon < 0$). As we noted earlier, these states can be interpreted as the result of temporary aggregate demand shocks, driven by changes in preferences or fiscal policy in a closed economy or as a foreign demand shock in a small open economy.

2.3 Government

The government is an agent that has two instruments, a macroprudential tool and a monetary policy tool. It can use either of them or both of them at the same time. Revenues (or financing) associated with the use of these tools are always rebated in lump-sum manner to households.[10]

Prudential policy is conducted with a Pigouvian tax on credit (bank loans) in our analysis.[11] Prudential policy could also be implemented with other instruments such as for instance reserve or capital requirements (see Bianchi (2011) and Stein (2012) and Appendix D for a more detailed discussion).

Following Kashyap and Stein (2012), monetary policy is conducted with an additive factor (ψ) on the bank funding cost R. While this is a stylized and reduced form representation of monetary policy, given that R is exogenous and the supply of foreign saving is perfectly elastic in the model, it is simple and intuitive. Indeed, when a central bank changes its interbank interest rate target, it is raising the banks' cost of meeting their daily liquidity needs. In the terminology of Stein (2012), this increases the scarcity value of bank reserves.

To see this, Kashyap and Stein (2012) show that the interbank rate, or bank funding cost (R) in our model, can always be decomposed in the sum of the interest rate on reserves (IOR, if any is paid) and a scarcity value term (SV), where R is exogenous but nominal in their model, and interpreted as following a standard Taylor rule like in Figure 1. If the IOR is zero (because there are no reserve requirements or they are not remunerated) then $R = SV$. So our additive factor (ψ) can be interpreted as monetary policy intervention affecting SV under the assumption that prices are perfectly fixed.[12]

Note finally that, following the New-Keynesian tradition, the government also has the possibility to use a separate subsidy (η) to remove the distortion due to monopolistic competition. As we shall see, the main results of the analysis are not affected by this policy action.

2.4 Shocks and Parameter Values

Table 1 summarizes the assumptions we make on the structural parameters and the stochastic processes of the shocks in the model. We choose a parametrization to study

[10] As we discuss below, the use of the two instruments give rise to no coordination problem.

[11] Alternatively, prudential policy could be conducted with a tax on foreign debt, i.e. with capital controls. This policy would be needed if the exchange rate were fixed so that monetary policy could not be conducted independently. But we do not model the exchange rate explicitly.

[12] In Appendix C we show how, in our model with a system of remunerated required reserves, the effects of changing the additive factor (ψ) are the same as increasing the coefficient of reserve requirements or lowering the rate of remuneration of those reserves.

the solution of the model in the case where there are financial stability considerations at play because of the pecuniary externality. This happens when the borrowing constraint does not bind today, but can bind tomorrow with positive probability. Given the stylized nature of the model, we do not use it for quantitative purposes. In the last section of the paper, however, we use its *qualitative* predictions to interpret the US experience in the run-up to the Great Recession.

Table 1 Calibration of Model's Parameter and Shocks

Variable	*Symbol*	*Value*	*Source/Target*
Average Endowment	$\bar{e}$	1.3	Jeanne and Korinek (2010a)
Asset return	y	0.8	Jeanne and Korinek (2010a)
Risk free rate	R^*	1.015	Average 3M US T-Bill
Elasticity of Subst. (Loans)	ζ	33.3	250 b.p. spread of R_L on R^*
Risk Aversion Coefficient	ϱ	2	Standard value
Interest rate stickiness	μ	0.5	Borio and Fritz (1995)
Shocks			
Shock to the endowment	$\tilde{\epsilon}$	$[-\epsilon, +\epsilon]$	
Shock to the interest rate	υ	$[-0.02, +0.02]$	St. Deviation 3M US T-Bill

Note. 3M US T-Bill is the the average 3-Month Treasury Bill deflated with US CPI; R_L is the 15-Year mortgage fixed rate deflated with US CPI. U.S. monthly data from 1985:M1 to 2007:M3.

We assume that endowment e is uniformly distributed over the $[\bar{e} - \varepsilon, \bar{e} + \varepsilon]$ interval. We then analyze the model's properties for different values of the maximum size of the endowment shock (ε), which controls the variance of the shock. We consider values for ε and the expected value $\bar{e}$ such that the economy is not constrained in the absence of disturbances, but it may be constrained for a sufficiently large negative realization of the shock.

Following Jeanne and Korinek (2010a), we set the return of the asset (y) and the expected value of the endowment to $\bar{e} = 1.3$ and $y = 0.8$. Jeanne and Korinek (2010a) choose these two parameters jointly with the maximum size of the endowment shock (ε) to control when the borrowing constraint binds. We take these two parameters as given, and set the maximum value of the endowment shock following the same strategy.

We calibrate the remaining parameters using US data from 1985 to 2007, i.e., from the beginning of the Great Moderation to the beginning of the Great Recession. The gross rate on deposit, i.e., the cost of bank funds, is set to $R = 1.015$, matching the average real yield of the 3-Month Treasury Bill over the period 1985-2007 (deflated nominal yields with US CPI) . The elasticity of substitution between financial products is set to $\zeta = 33.3$, implying a gross markup of $\mathcal{M} \simeq 1.03$. This markup generates a spread between deposit and lending rates of about 250 basis points, which matches the average spread in the data between the 15-year mortgage fixed rate and the the 3-Month Treasury Bill rate. The period utility is CES with relative risk aversion coefficient $\varrho = 2$, which is a conventional value.[13]

As we show in the Appendix, under these assumptions, the model economy is never constrained when $\varepsilon \leq \varepsilon^b = 0.095$, and the probability of observing a crisis in period 1 is zero. In this case, the model has a closed-form solution given by optimality conditions (7) together with $\lambda = 0$. In contrast, when $\varepsilon > 0.095$ the probability that the constraint will bind in period 1 positive. In this second case, the model must be solved numerically as shown in the Appendix).

The calibration of the degree of interest rate stickiness (μ) is more difficult. Although there is compelling evidence on the imperfect adjustment of retail interest rates rate to movements in policy rates, the precise degree of such rigidity varies across studies. For the US, Kwapil and Scharler (2010) estimate a short-run pass-through of changes in the policy rate to consumer loans of 0.3.[14] Based on this evidence we assume that only 50 percent of the banks can adjust their lending rates in response to a change in their funding costs. Note also that, as long as there is some interest rate stickiness (i.e., $\mu > 0$), the calibration of this parameter does not affect the qualitative behaviour of our model. In the long-run, in contrast, pass-through is assumed to be complete.

Finally, we assume that the deposit rate shock υ in period 0,

$$R_1 = R + \upsilon, \tag{13}$$

can take three values, namely $\upsilon = \{0, +0.02, -0.02\}$. The size of the shock matches the standard deviation of the yield on the US 3-Month Treasury Bill over the 1985-2007 period.

3 Decentralized Equilibrium

We can now analyze the decentralized equilibrium of the economy without any government intervention. In order to build intuition, we consider first the effects of the financial friction (which manifests itself conditional on the endowment shock) by comparing the allocation in our model economy with an economy in which the collateral constraint is never binding. Second, we will analyze the effect of the macroeconomic friction (which manifests itself conditional on the bank funding cost shock) by comparing the allocation in our model

[13] As discussed in more detail below, a coefficient of relative risk aversion (ϱ) larger than 1 is crucial for some of our results.

[14] These estimates are in line with older studies on interest rate pass-through in the U.S.. For example, Cottarelli and Kourelis (1994) estimate a short run pass through of 0.32 and a long run pass through of 1; Moazzami (1999) and Borio and Fritz (1995) report a short run coefficient of 0.4 and 0.34, respectively.

economy with an economy with fully flexible interest rates. Third, and finally, we will analyze the full model, when both frictions are at work simultaneously.

3.1 Financial Friction

The financial friction affects the economy only when the collateral constraint is not binding today but can bind tomorrow with a positive probability. We label states in which the collateral constraint is binding as "crisis states" and interpret the probability that the constraint will bind in period 1 (i.e., the crisis probability) as our measure of financial stability.[15]

Figure 2 displays the behaviour of key endogenous variables for different values of the maximum size of the shock (ε, displayed on the horizontal axis), which parametrizes the volatility of the shock. As we discussed earlier, for given parameter values and hence level of consumption and borrowing in period zero, the threshold for ε that makes the collateral constraint bind in period one with positive probability is $\varepsilon^b \simeq 0.095$.

The upper-left panel of Figure 2 plots the equilibrium level of borrowing in period 0 (b_1). Conditional on b_1, it is possible to compute net worth ($e - b_1 R_{L1}$), consumption (c_1), and the probability of observing a crisis (π) in period 1, which are plotted in the other three panels.

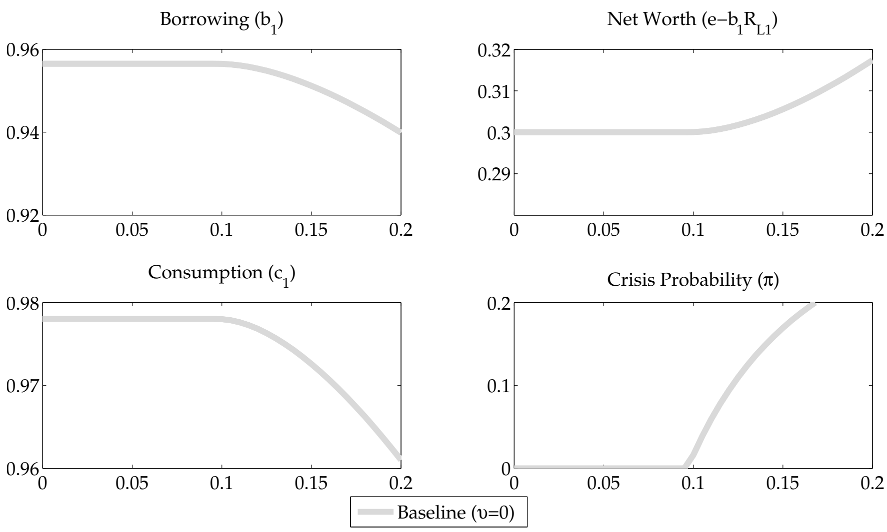

Figure 2 Model Equilibrium with Financial Friction. On the horizontal axis is the maximum size of the endowment shock (ϵ), which parametrizes its volatility.

[15] Woodford (2012), Stein (2012), and Benigno, Chen, Otrok, Rebucci, and Young (2013) define financial stability in the same way.

When $\varepsilon \leq \varepsilon^b$ the economy is never constrained, and households' decisions are not affected by the endowment volatility in period one ε: if faced with a larger than expected shock in period one, households can borrow from banks to smooth consumption. In contrast, if there is high enough volatility that the constraint can bind in period one with positive probability, consumers insure against this possibility with precautionary saving. Thus, they reduce borrowing and consumption in period 0 as well as in period 1 itself. The probability of a crisis (π) is positive and increasing in the shock variance, but less than one-for-one as lower consumption and borrowing at time zero, all else equal, make the collateral constraint less likely to bind in period one.

The mechanics of the comparative statics in Figure 2 is the following. The Lagrangian multiplier (λ) in the Euler equation (7) represents the shadow value of the collateral constraint. When λ is positive, the marginal utility of consumption in period one is higher and hence consumption will be lower than the case in which (λ) is zero. Higher marginal utility of period 1 consumption will also drive up marginal utility of consumption in period zero and will lower the asset price.

3.2 Macroeconomic Friction

Let us now analyze how the macroeconomic friction affects our model economy. We know from the New Keynesian literature that there are two sources of distortion in models with monopolistic competition and staggered pricing. As we shall see below, our model displays a similar behaviour.

First, monopolistic power forces average output below the socially optimal level. In our model, monopolistic competition in the banking sector implies an inefficiently low level of consumption, because lending interest rates are, on average, higher than under perfect competition.

Second, staggered pricing implies that both the economy's average markup and the relative price of different goods will vary over time *in response to shocks*, violating efficiency conditions.[16] To see how this distortion works in our model, assume for the moment that interest rates can freely adjust and that lending rates at the beginning of period 0 are at the desired level, set as a markup over the marginal cost ($R_{L1} = \mathcal{M}R$). If a positive shock $\upsilon > 0$ hits the economy, banks face a new, higher marginal cost and update their lending interest rates such that $R_{L1} = \mathcal{M}(R + \upsilon)$. Households update their loans demand accordingly, and the loans market clears at a higher lending rate. In response to the higher interest rate, consumption and borrowing in period 0 will be lower relative to the case in which $\upsilon = 0$.

[16] Note here that, if no shock pushes the economy away from its steady state equilibrium, the average markup would be equal to the constant frictionless level and the price of all goods in the economy would be the same, implying that no efficiency condition would be violated.

In a sticky-rate environment, not all banks can reset their lending rates so as to be consistent with the new marginal cost. The fraction μ of banks that can reset lending rates will set:

$$R_{L1}^{\mu} = \mathcal{M}(R + \upsilon).$$

The remaining $1 - \mu$ banks will not be allowed to reset their lending rates, implying that:

$$R_{L1}^{1-\mu} = \mathcal{M}R < R_{L1}^{\mu}.$$

As a consequence, the aggregate lending rate in the economy would differ from its flexible-rate counterpart. In fact, according to equation 10, the aggregate lending rate in the sticky-rate economy becomes:

$$R_{L1} = \mathcal{M}(R + \mu\upsilon),$$

which is lower than the lending rate prevailing under flexible rates in the case of a positive bunk funding shock.

A similar gap of opposite sign emerges when the υ shock is negative. In general, interest rates stickiness results in an average interest rate, R_{L1}, which differs from the one required to obtain the same allocation obtained under flexible interest rates (henceforth "flex-rates" allocation), thus also affecting the aggregate level of borrowing and consumption. With a positive shock, debt and consumption are higher than in the flex-rates economy, because interest rates increase by less than they would in a fully flexible world. But, with a negative shock, debt and consumption are lower than in the flex-rates economy, because interest rates decrease by less than they would in a flexible world. As we shall see, this property has crucial implications for the results of our analysis when the macroeconomic frictions interact with the financial friction.

3.3 The Interaction between the Financial Friction and the Macroeconomic Friction

In this section we show that the impact of staggered interest rates setting on the crisis probability depends on the sign of the shock hitting the economy. In response to a positive shock to the deposit rate, for instance as a consequence of an aggregate demand expansion, the probability of a crisis in the sticky-rate economy is lower (increases less) than in the in the flex-rate economy. In contrast, in response to a negative shock to the deposit rate, for instance as a consequence of an aggregate demand contraction, the crisis probability is higher (it falls less) than in the flex-rate economy.

We first analyze the effect of a positive shock to the risk-free interest rate (Figure 3). The benchmark is the economy with both frictions and no shock (solid line, i.e., the same allocation as in Figure 2). The thin line with asterisk markers and the thin line with circle markers display the equilibrium after the shock has hit, under flexible and sticky interest rates respectively.

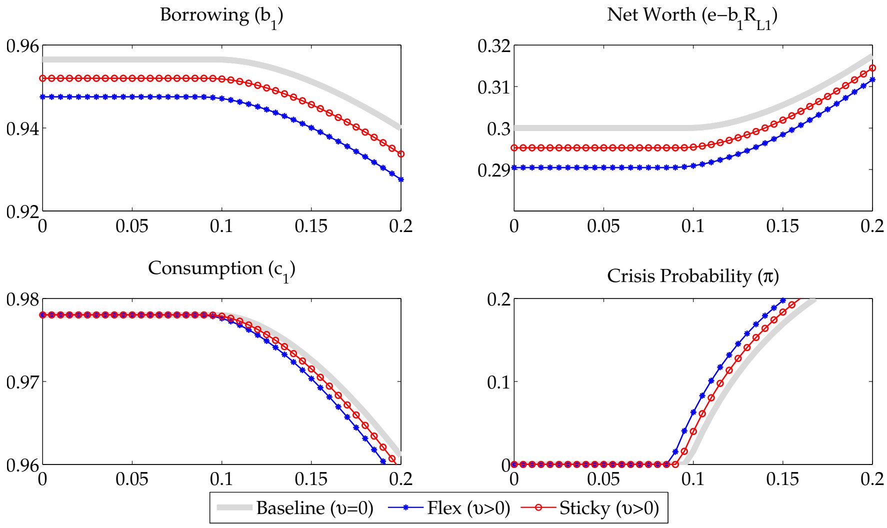

Figure 3 MODEL EQUILIBRIUM WITH BOTH FRICTIONS: POSITIVE SHOCK TO THE DEPOSIT RATE. On the horizontal axis is the maximum size of the endowment shock (ϵ). The thick solid line displays the equilibriums in the absence of shocks; the thin line with asterisk markers and the thin line with circle markers display the equilibrium after a positive shock under flex-rates and sticky-rates, respectively.

As we showed before, under the assumption of sticky rates, the aggregate lending rate in the economy does not increase as much as the bank funding cost following a positive shock. On the one hand, lower lending rates —relative to the flex-rates case— prompt consumers to borrow more ($\mathbf{b_1}$) in period 0 and to consume more (c_1) in period 1, as shown by the difference between the circles line and the asterisks line. All else equal, this implies higher expected next-period refinancing needs (b_2) and, therefore, a higher expected probability that the constraint will be binding in period 1. On the other hand, and despite the higher level of borrowing in period 0, expected net worth ($e - b_1 R_{L1}$) in period 1 is larger under sticky rates than under flex rates, because of lower interest rate repayments. All else equal, this implies a relaxation of the borrowing constraint in period 1. As the effect of the lower interest rates on net worth dominates the effect on borrowing and consumption, the probability that the constraint will bind in period 1 increases by less than in the flex-rates case in equilibrium —as we can see from the bottom right-hand panel of Figure 3.[17]

Consider now a negative shock. As we can see from Figure 4, in the case of a negative shock, sticky interest rates exacerbate the effects of the financial friction rather than

[17]The effect of the lower interest rates on net worth dominates the effect on borrowing and consumption as long as the coefficient of relative risk aversion (ϱ) is larger than 1. With log-utility the two effects cancel out.

dampening it. Under interest rate stickiness (circles line), the average lending rate now falls by less than the risk-free interest rate. In the sticky rate economy, consumption and borrowing are lower (or increase less) than in the flex-rate economy (asterisks line) in response to the shock, but next period interest payments are higher. As a result next-period net worth in the sticky rates economy is lower than in the flex-rate economy, and the crisis probability is higher (or it falls less).

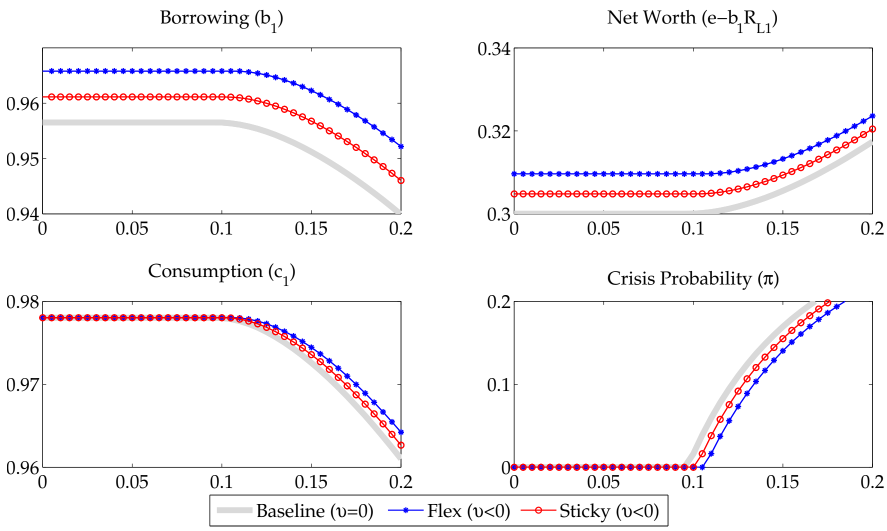

Figure 4 Model Equilibrium with Both Frictions: Negative Shock to the Deposit Rate. On the horizontal axis is the maximum size of the endowment shock (ϵ). The thick solid line displays the equilibriums in the absence of shocks; the thin line with asterisk markers and the thin line with circle markers display the equilibrium after a negative shock under flex-rates and sticky-rates, respectively.

In conclusion, when both the macroeconomic and the financial friction are present in the model, interest rate stickiness reduces the crisis probability relative to the flex-rate equilibrium in response to an increase in bank funding costs (such as when aggregate demand is expanding). However, it increases the probability of a crisis relative to the flex-rate equilibrium in response to a fall of bank funding costs (such as when aggregate demand is contracting).

Thus, in our model, sticky rates act as automatic macro-prudential stabilizer in response to positive shocks, while they exacerbate financial stability concerns in response to a negative shock. In this sense, the interaction of the two frictions has an asymmetric impact on financial stability in the model, although the allocation is perfectly symmetric in response to positive and negative shocks.

These results are robust to assuming different values for all other parameters of the model, including the size of the shock to the interest rate (υ) and the degree of interest rate stickiness (μ). Changing these parameters does not affect the mechanisms driving the result, but only the magnitude of the effects. In other words, for every possible value of υ and μ the allocation under sticky-rates (circles line) is bounded between the allocation under flex-rates (asterisks line) and the allocation where no shock hits the economy (solid line).

4 Restoring Efficiency

In this section we discuss how government intervention, and in particular monetary and macro-prudential policies, can address the market failures of our model economy.

To build understanding and intuition for the main results, we first analyze the case in which there is only the financial friction or the macroeconomic friction. We then consider the case in which the policy authority faces both frictions with either one or two policy instruments.

A key result of this section is that a policy-maker with only a monetary policy instrument (say the interest rate) faces a trade-off between macroeconomic and financial stability when the economy is hit by negative shock to bank funding costs and cannot achieve efficiency. In contrast, when the policy-maker has two instruments (e.g., a macro-prudential instrument, such as a tax on bank credit; and a monetary policy instrument, like for instance the policy interest rate), she/he can address both distortions and achieve efficiency.

4.1 Addressing the Pecuniary Externality

As it is well known, the occasionally binding constraint that is in our model generates a pecuniary externality. This pecuniary externality drives a wedge between private and socially optimal allocations because private agents do not internalize the effect of their decisions on the asset price that enters the specification of the borrowing constraint. Unlike private agents in a decentralized economy, a social planner internalizes that consumption decisions affect the asset price —as shown by the asset price equation in (7)— which, in turn, affects the aggregate collateral constraint in (4).[18]

To see this, following Jeanne and Korinek (2010a), we write the planner problem for this economy as follows:

$$\max_{b_1,b_2} \left\{ \begin{array}{c} u(b_1) + \mathbb{E}_0 \Big[u\big(e + b_2 + \pi_1 - b_1 R_{L1}\big) + y - b_2 R_{L2} \\ -\lambda^{sp} \left(b_2 - p_1(e + b_2 - b_1 R_{L1})\right) \Big] \end{array} \right\},$$

[18] See Bianchi (2011), Jeanne and Korinek (2010a,b), Benigno, Chen, Otrok, Rebucci, and Young (2013) for a more detailed discussion.

where the maximization is subject to the budget constraint (3), the aggregate borrowing constraint (4), and the pricing rule of the competitive equilibrium allocation:

$$p_1(c_1) = \frac{y}{u'(c_1)}.$$

The the asset price, $p_1(c_1)$, therefore, depends on aggregate consumption in the planner problem.

The corresponding first order conditions are:

$$\begin{aligned} u'(c_0) &= R_{L1}\mathbb{E}_0\big[u'(c_1) + \lambda^{sp} p'(c_1)\big], \\ u'(c_1) &= R_{L2} + \lambda^{sp}(1 - p'(c_1)). \end{aligned} \tag{14}$$

By comparing (7) and (14) and noting that $p'(c_1) > 0$, it is clear that there is a wedge between the decentralized and the planned allocation: the social planner saves more than private agents in period zero whenever the borrowing constraint is expected to bind in period 1 with positive probability. She/he internalizes the endogeneity of next period's asset price to this period's aggregate saving. Consumption and borrowing in period zero are also excessive relative to the planned allocation and the crisis probability will be higher. So the decentralized equilibrium is (constrained) inefficient relative to the social planner one. Instead, when the constraint is not expected to bind, the two allocations coincide (if we ignore the other friction in the model).

Alternative macro-prudential tools

In this set-up, Jeanne and Korinek (2010a) show that efficiency in the economy can be restored by imposing a state-contingent Pigouvian tax on borrowing in period 0 (namely $b_1(1 - \tau)$), rebating the proceeds with lump-sum transfers. The optimal tax is given by:

$$\tau = \mathbb{E}_0\left[\frac{\lambda^{sp} p'(c_1)}{u'(c_1)}\right], \tag{15}$$

This equation states that whenever the borrowing constraint is expected to bind in period 1 with positive probability, the policy-maker imposes a tax on borrowing in period 0. The tax induces private agents to consume and borrow less in period 0, relative to the equilibrium without government intervention.

A Pigouvian tax on debt may be difficult to implement. This tax, however, is not the only policy instrument that can decentralize the social planner allocation above. For instance, the same allocation could be implemented by imposing reserve or capital requirements on banks (Bianchi, 2011, Stein, 2012). An alternative way to decentralize this equilibrium is by increasing directly the banks' costs of funds, i.e. by using monetary policy. For instance, the policy-maker (e.g., a central bank in this specific case) can increase the bank funding cost by an additive factor ψ at the beginning of period 0, affecting banks' marginal costs and, therefore, consumers' borrowing and consumption decisions.

In Appendix D, we show that, if rebated in a lump-sum manner, this policy action has the same effect of the Pigouvian tax above. Specifically, we prove that the value of ψ that equates the two margins is given by:

$$\psi = \mathbb{E}_0 \left[\frac{\lambda^{sp} p'(c_1)}{u'(c_1)} \right] R. \tag{16}$$

This equation says that, as long as the shadow value of the collateral constraint (λ^{sp}) is different from zero, ψ is positive and can be interpreted as a prudential component to interest rate policy. This, in turn, implies that whenever the constraint is binding with positive probability, the central bank would raise interest rates so that households consume less and issue less debt in period 0, reducing the probability of hitting the constraint in case of an adverse shock in period 1.

So, when the borrowing constraint is the only friction in the economy, a social planner can achieve constrained efficiency by using a menu of policy instruments: by increasing interest rates in period 0 (i.e., by introducing reserve requirements, but also by decreasing the interest rate on remunerated reserves).[19] These instruments are substitutes for the Pigouvian tax on debt.

4.2 Addressing Monopolistic Competition and Interest Rate Stickiness

Our model embeds two macroeconomic distortions. The first is the presence of market power in loan markets. The second is staggered adjustment of lending rates. Staggered interest rate setting implies an inefficient level of borrowing and consumption because the economy's aggregate lending rate generally differs from the one prevailing under flexible rates.

In this section we want to study policies that address interest rate staggering in isolation from the distortions induced by market power. Therefore, following the New-Keynesian tradition, we remove the effects of market power introducing a subsidy (η) to interest rate repayments such that $\mathcal{M}(1 - \eta_t) = 1$. Note however that removing the subsidy does not affect our results.

A simple way to address the consequences of interest rate stickiness is *via* direct intervention on bank funding costs. Assume that the central bank can intervene directly on R by an additive factor ψ. Thus, the marginal cost of funds for banks —conditional on a shock to the risk free interest rate— would be given by $R + v + \psi$. Then, the central

[19] We show the equivalence of these instruments in Appendix D.

bank could set:

$$\psi : R_{L1} = \mathcal{M}(R + \upsilon),$$

which is the flexible level of the lending interest rate. Solving this equality yields:

$$\psi = \frac{1-\mu}{\mu}\upsilon. \tag{17}$$

Hence, in response to a positive shock ($\upsilon > 0$), the central bank would raise interest rates above the competitive equilibrium level by the factor $\psi > 0$; while in response to a negative shock the central bank would lower interest rates below the competitive equilibrium level by the factor $\psi < 0$.

Like in the case of the pecuniary externality, this allocation could be also achieved with other instruments. For instance, we could achieve it with reserve requirements or taxes on borrowing. In the rest of the analysis we focus on monetary policy (and, hence, on the direct intervention on the bank funding cost) as this is more consistent with how macroeconomic distortions are targeted in practice by policy-makers.

4.3 Addressing Both Frictions with Two Instruments

We now analyze how to achieve efficiency when both frictions are present. We consider a policy-maker who maximizes the expected utility of consumers (6), subject to their budget constraints (3), the borrowing constraint (4), and interest rate staggering.[20]

The two instruments that address the two distortions in the economy are the interest rate wedge (ψ) and the prudential tax on debt (τ). The prudential tax targets the pecuniary externality. The interest rate wedge targets interest rate staggering.

Consider first a positive bank funding cost shock. Figure 5 summarizes the results. The dashed line displays the efficient equilibrium. The line with triangles plots the competitive equilibrium allocation in which interest rates are flexible. The line with squares plots the competitive equilibrium in which interest rates are sticky. In both allocations there is the pecuniary externality at work.[21]

With two instruments, the policy-maker can address the macroeconomic and financial friction separately.[22]

[20] Again, remember that we remove the effects of the market power distortion introducing a subsidy (η) to interest rate repayments such that $\mathcal{M}(1 - \eta_t) = 1$. Our results in Figure 5 would be the same if we were not to do this.

[21] As we are removing the effects of the markup with the subsidy (η), borrowing under flexible interest rates in Figure 5 (triangles line) is now slightly larger than borrowing in Figure 3 (asterisks line).

[22] This is regardless of whether a single policy authority is in charge of both monetary and financial-stability policy (e.g., a central bank) or whether one authority is in charge of monetary policy and the other is in charge of macroprudential policy. In other words, in our set-up, there are no incentives for a central bank and a financial stability authority to deviate from a coordinated equilibrium.

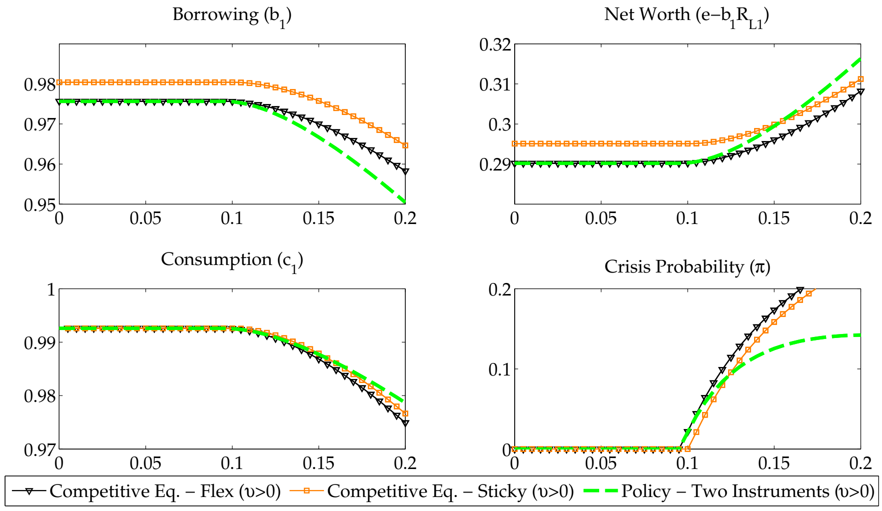

Figure 5 Efficient Allocation with Both Frictions: Positive Shock. On the horizontal axis is the maximum size of the endowment shock (ϵ). The thin lines with triangle and square markers display the equilibrium after a positive bank funding cost shock under flexible and sticky interest rates, respectively. In both of these two allocations, the pecuniary externality is at work. The dashed line displays the efficient allocation. The subsidy η is in place to remove the effects of market power.

The policy-maker can first increase the deposit rate by a factor $\psi > 0$ restoring the aggregate lending rate that would prevail under flex rates.[23] This moves the economy from the sticky-rates equilibrium (squares line) to the flex-rates competitive equilibrium (triangles line) with pecuniary externality. The policy-maker can then impose a distortionary tax on borrowing (τ) to address the pecuniary externality, moving the economy to the constrained efficient equilibrium (dashed line).

Note here that when $\varepsilon \leq \varepsilon^b$ in the flex-rate economy there is no inefficiency due to the externality (i.e., the line with triangles and the dashed line coincide). This is because there is no externality when the constraint never binds. However, when $\varepsilon > \varepsilon^b$, the efficient level of borrowing in period 0 is lower than in the flexible rates equilibrium (upper-left panel of Figure 5), while consumption in period 1 is larger (lower-left panel of Figure 5). This is because whenever the collateral constraint is expected to bind with a positive probability, the tax on credit forces private agents to borrow less in period 0. This increases their net worth and consumption in period 1, thereby also reducing the probability of a financial crisis (upper and lower-right panels of Figure 5, respectively).

As we saw before, in the sticky-rate equilibrium the net worth (and the crisis probability) is higher (lower) than in the flex-rate one because of the higher debt repayment in period 1. With flexible interest rates, borrowing is lower, but it is more costly to service, so net worth in period 1 is lower, and the probability of a crisis is higher. The tax on

[23]Note that changing the order of the policy actions, or inverting the assignment of the instruments would not alter the results.

credit curtails borrowing without increasing debt service costs. As a result, in the efficient allocation, the crisis probability is always lower than the flex-rate equilibrium, and can fall below the sticky-rate when the endowment volatility is high enough.

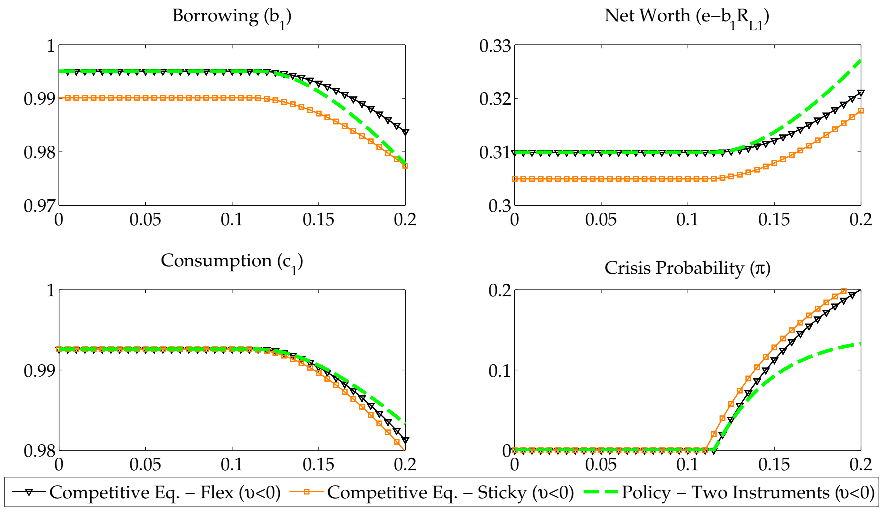

Figure 6 Efficient Allocation with Both Frictions: Positive Shock. On the horizontal axis is the maximum size of the endowment shock (ϵ). The thin lines with triangle and square markers display the equilibrium after a positive bank funding cost shock under flexible and sticky interest rates, respectively. In both of these two allocations, the pecuniary externality is at work. The dashed line displays the efficient allocation. The subsidy η is in place to remove the effects of market power.

Consider now a negative bank funding cost (Figure 6). To address the pecuniary externality, the policy-maker can impose the tax on debt whenever there is a positive probability that the constraint will bind in period 1, regardless of the sign of the funding cost shock. To address the interest rate rigidity, when a negative shock hits the economy, the policy-maker can lower interest rates by a factor ψ. Unlike the case of a positive shock, however, achieving the flex-rate equilibrium already reduces the probability of a crisis, as the higher borrowing than in the sticky-rate case is more than compensated by the lower interest payment. So, when the policy-maker also uses the tax on debt, the probability of a crisis is even lower than in the sticky-rate case, and it is always below it regardless of the level of endowment volatility.

In summary, with two instruments such as a tax on borrowing and the monetary policy interest rate, a policy-maker can address both the financial and the macroeconomic friction, thereby achieving constrained efficiency, independently of the sign of the shock hitting the economy.

4.4 The Trade-off: Addressing Both Frictions with One Instrument

Let us now consider the case in which both frictions are present in the model but the policy-maker has only one instrument, namely the interest rate. When both macroeconomic and financial frictions are present, if the policy interest rate is the only available instrument, a policy-maker that aims to achieve both macroeconomic and financial stability faces a policy trade-off. The trade-off emerges when the economy is hit by negative shock, because addressing both frictions requires interventions of opposite sign on the same policy instrument.

Consider a positive bank funding cost shock. As we showed earlier, both the macroeconomic and the financial friction result in higher borrowing in period 0 relative to the socially efficient allocation. To address the macroeconomic friction, the policy-maker can raise interest rates by the factor $\psi = (1-\mu)\,\upsilon/\mu > 0$, as implied by equation (17). To address the financial friction, she/he can further raise interest rates by the factor $\psi = \mathbb{E}_0\left[R(\lambda^{sp} p'(c_1))/u'(c_1)\right] > 0$, as implied by equation (16). Therefore, when a positive shock hits the economy, a single instrument can restore efficiency.[24]

When a negative shock hits the economy, however, the macroeconomic friction and the financial friction require opposite actions on the interest rate. The macroeconomic friction requires a decrease in interest rates: given that interest rates fall by less than in the flexible rate case, the social planner intervenes to lower interest rates by the factor $\psi = -(1-\mu)\,\upsilon/\mu < 0$. In contrast, the financial friction still requires an increase in interest rates independently of the sign of the shock. Hence, if the interest rate is the only instrument, the social planner would try to lower interest rates to address the macroeconomic friction and, at the same time, to raise the interest rate to address the financial friction. As a result the optimal level of the policy interest rate, and hence of the aggregate lending rate prevailing in the economy, would be higher than in the case in which there are two instruments.

In our model, the financial friction results in more borrowing than socially desirable in period 0 when the collateral constraint has a positive probability to bind in period 1, regardless of the sign of the shock. In contrast, the macroeconomic friction generates either more or less borrowing than socially desirable depending on whether the economy is hit by a positive or a negative shock. It is thus evident that, if the policy-maker has only one instrument, she/he may face a trade off in the face of negative shocks when the economy requires interventions in opposite direction.[25]

[24] As Kashyap and Stein (2012) note, a second instrument might be needed if the level of the interest needed to address the pecuniary externality is lower than the one needed to address the interest rate stickiness. In general, however, the two frictions require movements of the interest rate in the same direction.

[25] This case is different than the examples considered by Kashyap and Stein (2012) as it is a general results that hold for all values of the interest rate needed to restore macroeconomic efficiency.

5 US Monetary Policy and The Great Recession

Under former Chairman Alan Greenspan, the Federal Reserve lowered its benchmark rate from 6.5 percent to about 2 percent in 2000-01 as a response to the bursting of the dot-com bubble. It further lowered interest rates to 1 percent in 2002-03 in response to a deflationary scare, and finally started a sequence of tightening actions in June 2004, bringing the Fed funds rate back to 5 percent by 2006 (see Figure 1).

Against this background, Taylor (2007) put forth the idea that the Federal Reserve helped inflate US housing prices in the mid-2000s by keeping rates too low for too long after 2002. His main argument was based on the observation that the policy rate was well below what implied by a standard Taylor rule, a good approximation to the successful conduct of monetary policy in the previous several years (Figure 1). As a consequence, "*those low interest rates were not only unusually low but they logically were a factor in the housing boom and therefore ultimately the bust.*"[26] Therefore, according to this view, higher interest rates would have reduced both the probability and the severity of the bust that led to the Great Recession.

In this section, we evaluate this claim against the *qualitative* predictions of our model. In particular, we will show that Taylor's argument can be rationalized within the logic of our model only if we make the auxiliary assumption that the Fed had only one policy instrument at its disposal to pursue macroeconomic and financial stability. However, Taylor's argument is no longer valid within the logic of our model if we assume that the Fed had two instruments to address the macroeconomic and the financial friction in the model or, alternatively, that other government agencies were primarily responsible for financial stability. In the latter case, which is the institutional set-up prevailing in the United States, in response to a negative aggregate demand shock, the "optimal" best response of the central bank is to lower interest rates without concern for financial stability, which should be addressed by the second instrument (or by the another policy authority with a different instrument).

In this section, we also briefly review some evidence on what US financial regulation did during the same period as a way to see whether or not a second policy instrument was at work alongside the monetary policy interest rate. Albeit only descriptive and circumstantial, this evidence suggests that the US regulators were at best ineffective in curbing the continued expansion in subprime mortgage lending, well past the point at which prime lending had started to respond to the ongoing monetary tightening. We conclude from this analysis that the claim that US monetary policy is to blame for the US Great Recession is not justified within the logic of our model, given the regulatory regime prevailing in the United States and the evidence we report.

[26] John Taylor, interviewed by Bloomberg at the American Economic Association's annual meeting, Atlanta, January 5, 2010, available at:
http://www.bloomberg.com/apps/news?pid=newsarchive&sid=a44P5KTDjWWY

5.1 Model predictions

To assess Taylor's contention through the lens of our model, consider a negative aggregate demand shock hitting our model economy, such as the one that occurred in March 2000 when the dot-com bubble burst in the United States. Our simple model of financial intermediation, with banks that fund loans with foreign deposits, represents this shock in reduced form like a negative bank funding costs change, or a decline of the deposit interest rate.[27]

Then set the beginning of period 0 in the model as the year 2000, and assume that the economy comes back to its pre-shock level of economic activity after four years, namely at the beginning of 2004 (the beginning of period 1 in the model), consistent with the fact that the policy interest rate was raised for the first time in June 2004.

Figure 7 summarizes the qualitative behaviour of the lending interest rate as implied by our model when a negative bank funding cost hits the economy, and both frictions are at work. We consider two policy regimes. In the first one, the policy-maker has just one instrument at disposal, namely the monetary policy interest rate. In the second one, the policy-maker has two instruments to address the macroeconomic and the financial friction separately, namely a macro-prudential tool and the monetary policy interest rate.

As we saw in the previous section, the policy-maker can achieve the allocation in which there is no interest rate stickiness and no pecuniary externality in the economy when she/he uses two instruments, regardless of the sign of the shock. However, if the interest rate is the only policy instrument, there is a trade-off between macroeconomic and financial stability in response to negative shocks.

When there is only one instrument, the two frictions require interventions in opposite directions on the policy interest rate (not reported) in response to a negative shock. On the one hand, she/he would have to lower the policy interest rate to restore the aggregate lending rate that would prevail in the absence of interest rate stickiness. On the other hand, he/she would have to raise it to contain the excess borrowing generated by the pecuniary externality. As a result, the lending interest rate in this environment would be higher than the level prevailing when a macro-prudential tool can address the pecuniary externality separately.

[27] While the bursting of the dot.com bubble was a domestic shock, as we discussed earlier, bank funding costs and deposit rates would be lowered also by en external demand shock. The key difference is that a negative domestic aggregate demand shock works mostly via consumption and investment. An external demand shock works via exports. In both cases, we need to assume that good prices are rigid for a change in nominal interest rates to translate into a real interest rate change. As we noted earlier, our model assumes that good prices are completely fixed in the short term. For simplicity, the model also abstracts completely from the aggregate demand channels through which contractionary and expansionary shocks are propagated to the economy or from exchange rate movements.

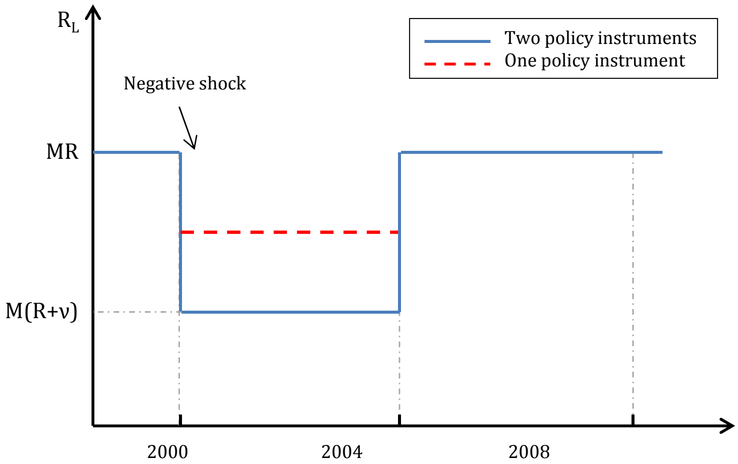

Figure 7 THE LENDING INTEREST RATE WITH $\nu < 0$ AND DIFFERENT ASSUMPTIONS ON THE NUMBER OF POLICY INSTRUMENTS. *Two policy instruments* and *One Policy Instrument* plot the lending interest rate that would prevail when the policy maker addresses both the macroeconomic and the financial friction with two or one instruments, respectively.

If we assume that the policy-maker has only the interest rate as a policy instrument, our model supports Taylor's argument. In fact, it suggests to keep interest rates higher than the case in which a separate instrument targets financial stability in period 0. This is to avoid excessive borrowing, an asset price increase, and thereby reducing the probability of a financial crisis if the economy were to be hit by another negative shock in period 1.

The implications for the Taylor's argument are different if we assume that the policy-maker has two separate policy instruments to address financial and macroeconomic stability. In this case, the policy-maker can achieve efficiency regardless of the sign of the shock. If the excess borrowing generated by the pecuniary externality in response to a negative shock is addressed by the macro-prudential tool, then it is optimal for the central bank to lower interest rates in period 0 as much as needed to address interest rate stickiness. In this second case, therefore, lowering the Fed funds rate as much as needed to respond to the bursting of the dot.com bubble in 2001 and the deflationary scare in 2002-03 can be rationalized as an optimal policy response within the logic of the model, while the Taylor's argument cannot.

5.2 Some Evidence

Did the Fed have one or two policy instruments to respond to the busting of the dot.com bubble and the subsequent deflationary scare? In the US, prior to the crisis, institutional responsibility for financial stability was shared among a multiplicity of agencies, including but not limited to the Fed. For instance, since the Glass-Steagall Act of 1932, US depository institutions (e.g., banks, thrifts, credit unions, savings and loans, etc.) have been regulated by different federal agencies: the Office of the Comptroller of the Currency being in charge of nationally chartered banks and their subsidiaries; the Federal Reserve covering affiliates of nationally chartered banks; the Office of Thrift Supervision overseeing savings institutions; the Federal Deposit Insurance Corporation insuring deposits of both state-chartered and nationally chartered banks. The Security and Exchange Commission (SEC) being responsible for capital markets regulation, other agencies cover other intermediaries and markets. So we can safely assume that a "second" policy instrument was available to the US policy-maker to address financial stability.

The question then becomes whether or not regulatory policy was used to address financial stability while monetary policy was focusing on macroeconomic stability. In particular, Taylor's contention could be justified within the logic of our model, even assuming that there are two policy instruments, if we were to observe an ineffective regulatory clampdown on mortgage lending during the period in which monetary policy was unusually lax by the standard of the Taylor rule. Indeed, as we shall see below, the regulatory effort to contain mortgage lending during the period 2003-06 was at best ineffective, if not absent altogether.

Figure 8 provides a picture of the evolution of the US mortgage market and the Fed funds rate over the 2000-07 period. The picture also reports the last important deregulation measure and the first tightening regulatory measures we can identify clearly (vertical bars). Broadly speaking, the picture shows that as the Federal Reserve started to tighten its monetary policy stance, the prime segment of the mortgage market turned around, as one would expect. In contrast, the subprime segment of the market continued to boom, with increased perceived risk of loans portfolios and declining lending standards. Despite this, the first restrictive regulatory action we can identify was undertaken only in late 2006, after almost two years of steady increase in the federal funds rate.

The upper-left panel of Figure 8 (Panel a) reports the evolution of the federal funds rate (annual average) together with mortgage originations by category over the period 2001-2007. While prime mortgage originations started to fall in 2003, non-prime mortgage originations continued to increase in 2004 and 2005.[28] The share of non-prime mortgage over total mortgage originations went from about 20 percent in 2001 to more than 50 percent in 2006, experiencing the largest increase in 2004, while the Federal Reserve was already tightening its monetary policy stance. A similar pattern emerges by looking at the issuance of mortgage backed securities (MBS).[29] The upper-right panel of Figure 8

[28]By prime mortgage we refer to loans that conform to Government Sponsored Enterprises (GSE) guidelines; by non-prime mortgage we refer to Alt-A, Home Equity, FHA/VA, and subprime mortgages.

[29]MBS which can be issued or guaranteed by a government sponsored enterprise (GSE) such as Fannie Mae or Freddie Mac and are usually referred to as "'agency MBS'." Or they can be issued by private institutions, such as subsidiaries of investment banks, commercial banks, financial institutions, non-bank mortgage lenders and home builders usually called "private label" MBS.

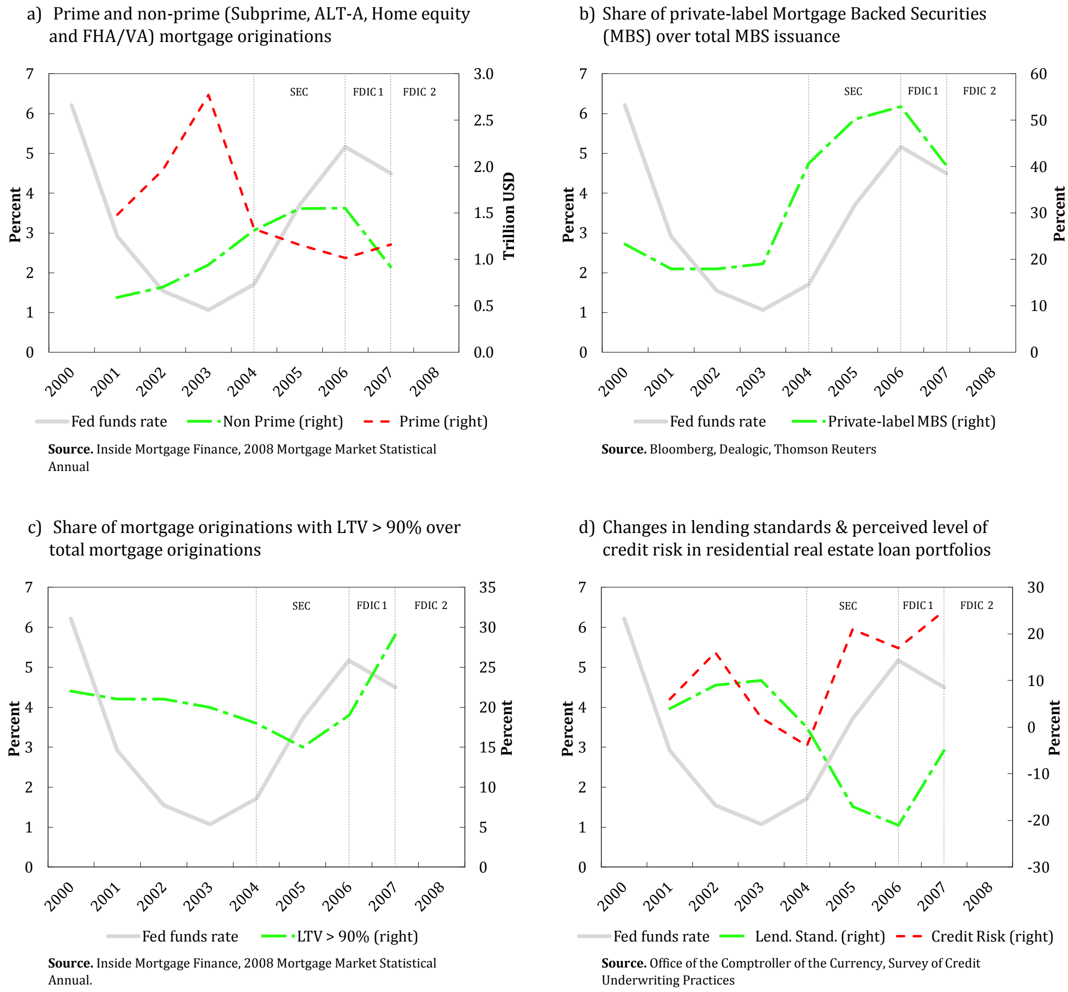

Figure 8 U.S. Mortgage Markets, the Federal Funds Rate, and Selected Regulatory Measures. The figure reports the Fed funds rate together with prime and sub-prime mortgage marekt indicators from 2000 to 2008. The figures also reports (vertical lines) the last important deregulatory measure and the first measures that regulatory measures that we can clearly identify (see text for more details).

(Panel b) also shows how the share of private label MBS increased sharply in the 2003-06 period.

The lower-left panel of Figure 8 (Panel c) reports the federal funds rate together with the share of mortgage originations with a Loan-to-Value (LTV) ratio greater than 90 percent. While in some countries a countercyclical maximum LTV limit is used as a macro-prudential policy instrument, here we see that in the United States the share of high-LTV mortgages spiked in 2005, two years after the beginning of the monetary policy tightening.

Finally, the lower-right panel of Figure 8 (Panel d) reports additional evidence on the fact that, while loan quality was relatively stable or improving from 2000 to 2003, it deteriorated sharply from 2004 to 2007. The Office of the Comptroller of the Currency publishes an annual underwriting survey to identify trends in lending standards and credit risk for the most common types of commercial and retail credit offered by national banks. Using data from the 2009 survey, which covered 52 banks engaged in residential real estate lending, Panel (d) reports the evolution of changes in underwriting standards (dash-dotted line) and the perceived level of credit risk (dashed line) in residential real estate loan portfolios.[30] The figure shows that, while the level of perceived risk was sharply increasing starting from 2004, banks started easing their lending standards from 2003 and did even more so in the 2004-05 period.

Despite this evidence, US regulatory agencies, including the regulatory arm of the Fed, did not take action while monetary policy was being tightened. On the contrary, some agencies provided additional deregulatory momentum while monetary policy was being tightened. For instance, the SEC proposed a system of voluntary regulation under the Consolidated Supervised Entities Program in 2004 that allowed investment banks to hold less capital (vertical line in our charts labelled SEC).

When regulators finally decided to act, it was too late. It was not until September 2006 that they agreed on new guidelines aimed at tightening "non-traditional" mortgage lending practices (vertical line labelled FDIC 1). However, these new underwriting criteria did not apply to subprime loans, whose standards were modified in a subsequent regulatory action that was introduced in June 2007 (vertical line under label FDIC 2). By that time, more than 30 subprime lenders had gone bankrupt and many more followed suit.

This evidence, albeit circumstantial, suggests that regulatory function of the US financial system was at best ineffective if not completely absent in addressing the financial imbalances that continued to grow in the subprime mortgage market while monetary policy was tightened in 2004-05. Even though the variables plotted are equilibrium outcomes, Figure 8 shows that policy measures aimed at tightening the subprime sector of the US mortgage market kicked in much later than the tightening of monetary policy enacted by the Federal Reserve.

This evidence therefore suggests that, after 2004, US monetary policy was indeed fighting a battle against an overheated housing and mortgage market with a single instrument, the policy interest rate. With no aid from the regulatory arm of the Fed itself or other agencies, consistent with our model's qualitative predictions when there is only one policy instrument, one could argue that the Fed should have raised interest rate more than it actually did. In this sense, Taylor's contention that excessively lax monetary policy contributed to the likelihood and the severity of the great recession can be justified within the logic of our model. Notice, however, that —given the US institutional context

[30]Net percentage calculated by subtracting the percent of banks tightening from the percent of banks easing. Negative values, therefore, indicate easing.

and the evidence we report— it is regulation (or the lack thereof) rather than monetary policy *per se* that should be blamed for the subprime mortgage crisis.

6 Conclusions

In this paper, we develop a model with both a macroeconomic and a financial friction (i.e., a bank lending interest rate rigidity that gives rise to a macroeconomic stabilization objective and an occasionally and endogenously binding collateral constraint that gives rise to a pecuniary externality) that speaks to the interaction between macroeconomic and financial stability, and to the role of US monetary policy and regulatory policy in the run up to the Great Recession.

There are two main results. First, we find that real interest rate rigidity has a different impact on financial stability (defined as the probability that the collateral constraint binds) depending on the sign of the shock hitting the economy. In response to expansionary shocks that raise the funding cost of banks (in short, a positive shock), interest rate rigidity acts as an automatic macro-prudential stabilizer. This is because higher debt today, induced by lower interest rates (relative to the flexible interest rate equilibrium), is offset by lower interest repayments tomorrow, resulting in higher net worth and lower probability of a crisis in the future. In contrast, when the economy is contracting and bank funding costs decline (in short, in response to a negative shock), real interest rate rigidity leads to a relatively higher crisis probability through the same mechanisms working in reverse: borrowing and consumption are relatively lower today, but they are offset by relatively higher debt service tomorrow, resulting in lower future net-worth and higher crisis probability. While the allocations in response to positive and negative shocks are fully symmetric, the implications for financial stability are asymmetric.

Second, we find that, when the interest rate is the only policy instrument to address both the macroeconomic and the financial friction, and a negative shock hits the economy, a policy trade-off emerges. This is because the two frictions require interventions of opposite direction on the same instrument. Other instruments, however, may be at the policy-maker's disposal to pursue financial stability. Our model shows that when two instruments are available this trade-off disappears and efficiency can be restored.

Our analysis has implications regarding the role of US monetary policy in the run-up to the Great Recession. In a series of recent papers Taylor (2007, 2010) suggested that higher interest rates in the 2002-2006 period would have reduced the likelihood and the severity of the Great Recession. Our theoretical findings support this argument only if we make the auxiliary assumption that the US policy authority was seeking to address both the macroeconomic and the financial distortion in the model with a single instrument, namely the policy interest rate. In contrast, when the policy authority is endowed with two different instruments, interest rates can be lowered as much as needed in response to a contractionary shock without concerns for financial stability. This is consistent with the view of Bernanke (2010) that additional policy tools were needed to prevent the global financial crisis from happening.

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A Appendix. Leverage

Consider a loan-to-value (LTV) constraint where one can borrow up to a fraction (Θ) of the value of the collateral, say a house:

$$b \leq \Theta p. \tag{A.1}$$

When the LTV constraint is binding with equality, the balance sheet is:

Assets	**Liabilities**
House price, p	Mortgage, $b = \Theta p$
	Downpayment, $(1 - \Theta)p$

The LTV constraint implies the following leverage ratio:

$$L^{\max} = \frac{p}{p - \Theta p} = \frac{1}{1 - \Theta}. \tag{A.2}$$

Here, leverage L is well defined only if $\Theta < 1$ (i.e., for positive levels of "equity"), and $L = L^{\max}$ when the LTV constraint is binding and $L < L^{\max}$ otherwise.

How does leverage behave in our simple model? In our model $\Theta = 1$. So equation (A.2) implies a unbounded maximum leverage ratio. In equilibrium, however, leverage is pinned down by preferences, interest rates, the deterministic return on the asset and the shadow price of the constraint and is lower than ∞.

Figure A.1 reports the equilibrium leverage in our baseline model. Consider leverage in period 1 (the period in which there is the constraint) before the shock to the endowment realizes. Consistent with the above definition and the notation in the text, we have:

$$L_1 = \frac{p_1}{p_1 - b_2}. \tag{A.3}$$

When the collateral constraint is never binding, equilibrium leverage in our baseline model is equal to 8, even if for simplicity we set $\Theta = 1$. When the collateral constraint is binding with positive probability, leverage is lower and decreases with the maximum value of the shock.

Why does leverage (L_1) is lower with more volatile endowment shocks? When the collateral constraint is binding with positive probability (i.e., when $\pi > 0$) the Lagrange multiplier (λ) becomes positive. As a result, households optimally reduce their desired consumption and borrowing in both periods (b_1 and b_2). Lower borrowing in period 1 (b_2) implies lower leverage. Lower consumption implies a lower asset price (p_1), and a lower asset price (p_1) implies higher leverage as we can see from the partial derivative of L_1 with respect to p_1. The former effect dominates in our model.

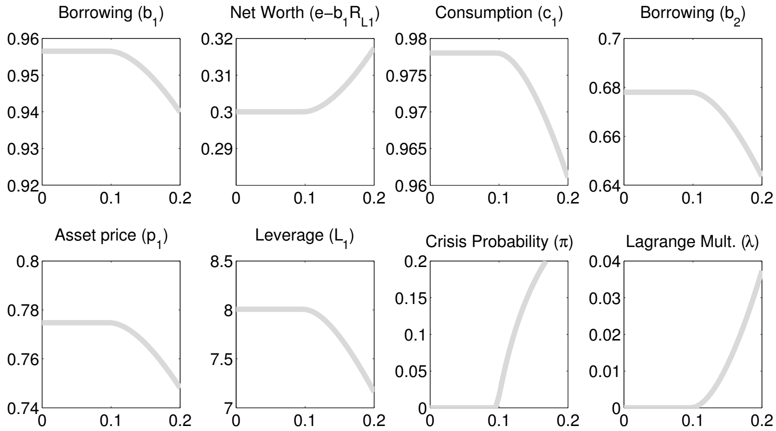

Figure A.1 MODEL EQUILIBRIUM WITH FINANCIAL FRICTION. On the horizontal axis is the maximum size of the endowment shock (ϵ).

B Appendix. Debt-To-Income constraint

Consider a debt-to-income constraint rather than a LTV constraint:

$$R_{L2}b_2 \leq \chi e,$$

where total expected repayment period 2—interest plus principal, $R_{L2}b_2$—cannot be larger than a fraction (χ) of expected income.

The problem for the representative household therefore is:

$$\mathcal{V}_1 = \max_{b_2,\theta_2} \left\{ u\Big(e + b_2 + (\theta_1 - \theta_2)p_1 + \pi_1 - b_1 R_{L1} \Big) + \theta_2 y + \pi_2 - b_2 R_{L2} - \lambda(R_{L2}b_2 - \chi e) \right\},$$

The first order conditions for the competitive equilibrium (CE) therefore are:

$$\begin{cases} FOC(b_1): & u'(c_0) = R_{L1}\mathbb{E}_0\big[u'(c_1)\big], \\ FOC(b_2): & u'(c_1) = R_{L2} + \lambda R_{L2}, \\ FOC(\theta_2): & p_1 = y/\mathbb{E}_0\left[u'(c_1)\right]. \end{cases}$$

The only difference relative to the LTV economy is that R_{L2} now multiplies the Lagrange multiplier in $FOC(b_2)$. The shadow price of the constraint will always be higher in the DTI economy than in the LTV economy.

This implies that the qualitative properties of the model are the same as in the LTV economy. The allocations in the competitive equilibrium will be different, as well as the maximum value of the endowment for which the constraint binds with positive probability, but the behaviour of the model is unchanged.

C Appendix. Characterizing Monetary Policy

For simplicity, in the model, we represent monetary policy by assuming that the central bank can affect bank funding costs with an additive factor ψ, so that the marginal cost of funds for banks becomes $R + \psi$. In this appendix, we show that the effects of adding (subtracting) ψ are the same as those of increasing (decreasing) the coefficient of reserve requirements or lowering (increasing) the rate of remuneration of those reserves in a system of remunerated required reserves.

Suppose that banks must hold a fraction ϕ of their deposits in the form of *unremunerated* reserves:

$$f(j) \geq \phi d(j). \tag{C.1}$$

Bank j's balance sheet will be:

Assets	**Liabilities**
Loans $b(j)$	Deposits $d(j)$
Reserves $f(j)$	

In each period, bank j maximizes its profits:

$$\max_{R_{Lt}(j), b_t(j), f_t(j), d_t(j)} b_t(j)R_{Lt}(j) + f_t(j) - d_t(j)R_t,$$

subject to the demand schedule in (9), the balance sheet constraint $b_t(j) + f_t(j) = d_t(j)$, and the regulatory constraint $f_t(j) \geq \phi d_t(j)$. Solving banks' maximization problems yields the following optimal level of the lending rate:

$$R_{Lt}(j) = \frac{\zeta}{\zeta - 1} \frac{R_t - \phi}{1 - \phi}.$$

This shows that the lending rate charged by banks is increasing in the coefficient of reserve requirement (i.e., the reserve requirement is a tax on banks). Increasing (decreasing) reserve requirements, therefore, has the same effect as adding (subtracting) ψ to R.

Suppose now that banks are required to hold a fraction ϕ of deposits in the form of *remunerated* reserves:

$$f(j) \geq \phi d(j), \tag{C.2}$$

where R^f is the interest rate at which required reserves are remunerated (i.e., r_{IOR} in the notation of Kashyap and Stein (2012)).

Bank j's balance sheet continues is unchanged. But bank $j's$ maximization problem becomes:

$$\max_{R_{Lt}(j), b_t(j), f_t(j), d_t(j)} b_t(j)R_{Lt}(j) + f_t(j)R^f - d_t(j)R_t,$$

subject to the demand schedule in (9), the balance sheet constraint $b_t(j) + f_t(j) = d_t(j)$, and the regulatory constraint $f_t(j) \geq \phi d_t(j)$.

Solving banks' maximization problems yields the following optimal level of the lending rate:

$$R_{Lt}(j) = \frac{\zeta}{\zeta - 1} \frac{R_t - \phi R^f}{1 - \phi}.$$

We can now see from this expression that the lending rate, for given coefficient of reserve requirement ϕ, is decreasing in the rate of remuneration of these reserves, R^f. So remunerating these reserves can partially or completely offset the implicit "reserve requirement tax". Indeed, in the limiting case in which $R^f = R_t$, the lending rate coincides to the case in which there are no reserve requirements.

D Appendix. Alternative Policy Instruments

If there is only one friction in the economy, either the pecuniary externality or interest rate rigidity, all instruments discussed in the paper can be used to address them individually. And hence they are substitute in the sense that we can use one or the other to achieve the same allocation.

For instance, we could use monetary policy represented by the additive factor ψ to address the pecuniary externality. To see this, note that the consumers' maximization problem becomes:

$$\max_{b_1,b_2}\left\{\begin{array}{c} u(b_1)+\mathbb{E}_0\Big[u\big(e+b_2+\pi_1-b_1\mathcal{M}(R+\psi)+TR\big)+ \\ +y-b_2R_{L2}-\lambda(b_2-p_1)\Big] \end{array}\right\}.$$

By equalizing the first order condition with respect to b_1 of the decentralized equilibrium and the social planner equilibrium, we can derive the level of ψ which closes the wedge:

$$\begin{cases} u'(c_0)=R_{L1}\mathbb{E}_0\big[u'(c_1)+\lambda^{sp}p'(c_1)\big], \\ u'(c_0)=\mathcal{M}(R+\psi)\mathbb{E}_0\left[u'(c_1)\right], \end{cases}$$

Solving for ψ yields:

$$\psi=\mathbb{E}_0\left[\frac{\lambda^{sp}p'(c_1)}{u'(c_1)}\right]R. \tag{D.1}$$

Consider now a system of unremunerated reserve requirements. Reserve requirements can be used to address the pecuniary externality or interest rate rigidity. With reserve requirements, the consumers' maximization problem becomes:

$$\max_{b_1,b_2}\left\{\begin{array}{c} u(b_1)+\mathbb{E}_0\Big[u\big(e+b_2+\pi_1-b_1\mathcal{M}\left(\frac{R_t-\phi}{1-\phi}\right)+TR\big)+ \\ +y-b_2R_{L2}-\lambda(b_2-p_1)\Big] \end{array}\right\}.$$

By equalizing the first order condition with respect to b_1 of the decentralized equilibrium and the social planner equilibrium, we can derive the level of ϕ which closes the wedge:

$$\begin{cases} u'(c_0)=R_{L1}\mathbb{E}_0\big[u'(c_1)+\lambda^{sp}p'(c_1)\big], \\ u'(c_0)=\mathcal{M}\left(\frac{R_t-\phi}{1-\phi}\right)\mathbb{E}_0\left[u'(c_1)\right], \end{cases}$$

Solving for ϕ yields:

$$\phi=\frac{\mathbb{E}_0\big[\lambda^{sp}p'(c_1)\big]}{\mathbb{E}_0\big[u'(c_1)+\lambda^{sp}p'(c_1)\big]-\frac{\mathbb{E}_0[u'(c_1)]}{R_t}}. \tag{D.2}$$

Similarly, in a system of *remunerated* reserve requirements, the consumers' maximization problem becomes:

$$\max_{b_1,b_2}\left\{\begin{array}{c} u(b_1)+\mathbb{E}_0\Big[u\big(e+b_2+\pi_1-b_1\mathcal{M}\left(\frac{R_t-\phi}{1-\phi}\right)+TR\big)+ \\ +y-b_2R_{L2}-\lambda(b_2-p_1)\Big] \end{array}\right\}.$$

By equalizing the first order condition with respect to b_1 in the decentralized equilibrium and the social planner equilibrium, we can derive the level of R^f which closes the wedge:

$$\begin{cases} u'(c_0) = R_{L1}\mathbb{E}_0\big[u'(c_1) + \lambda^{sp}p'(c_1)\big], \\ u'(c_0) = \mathcal{M}\left(\frac{R_t - \phi R^f}{1-\phi}\right)\mathbb{E}_0\left[u'(c_1)\right], \end{cases}$$

Solving for R^f yields:

$$R^f = R_t - (1-\phi)R_t\mathbb{E}_0\left[\lambda^{sp}p'(c_1)\right].$$

In contrast, if there are both frictions in the economy, the pecuniary externality *and* interest rate rigidity, not all pairs of instruments can be used to restore efficiency. In particular, in our model, only the tax on credit can complement the other instruments in restoring efficiency, while ϕ and R^f cannot do so. This is because both these two instruments act on the same wedge, i.e. banks' marginal cost of funds, and hence R_{Lt}. As a result, they cannot be used independently of each other. To restore efficiency, we need an instrument that affects the supply of credit (like all the monetary tools we considered) and one that affects the demand, like the tax on borrowing.

In this respect, our results differ from those of Kashyap and Stein (2012). This is because, in the case studied by Kashyap and Stein (2012) $R = IOR + SV$, where R is assumed to follow a posited Taylor rule. The macroeconomic environment is not fully specified. So for a given level of R and SV derived from the Taylor rule, IOR can always be adjusted to match the required level of the policy rate. In our model, the macroeconomic friction, albeit simply, the macroeconomic friction is modeled explicitly. And as we saw, when both frictions are at work, they require movements in opposite directions in response to negative shocks. As a result, introducing remunerated reserves is not sufficient to resolve this trade off.

E Appendix. Numerical solution

This appendix describes the solution of the model and the solution method for the case when the collateral constraint is occasionally binding with a positive probability. We closely follow the approach proposed by Jeanne and Korinek (2010a). First, describe the first order conditions of the model. Second, we derive the combination of parameter values that make the collateral constraint binding with a positive probability. Finally, we show how to solve for the optimal allocations of the decentralized equilibrium and the social planner equilibrium.

First order conditions. We solve for the equilibrium backward, as in Jeanne and Korinek (2010a). As stated in the main text, in period 1 consumers maximize their utility:

$$u(c_{i,0}) + u(c_{i,1}) + c_{i,2},$$

where, for simplicity, we assume a unitary discount factor. The period utility function, $u(\cdot)$, is a standard CES function:

$$u(c) = \frac{c^{1-\varrho}}{1-\varrho}.$$

Consumers are subject to the following budget constraint:

$$\begin{cases} c_{i,0} = b_{i,1} + (1-\theta_{i,1})p_0, \\ c_{i,1} + b_{i,1}R_{L1} = e + b_{i,2} + (\theta_{i,1} - \theta_{i,2})p_1 + \pi_{i,1}, \\ c_{i,2} + b_{i,2}R_{L2} = \theta_{i,2}y + \pi_{i,2}, \end{cases}$$

and the following collateral constraint:

$$b_{i,2} \leq \theta_{i,1}p_1.$$

The problem for the representative consumer therefore is:

$$\mathcal{V}_1 = \max_{b_2,\theta_2} \left\{ u\Big(e + b_2 + (\theta_1 - \theta_2)p_1 + \pi_1 - b_1 R_{L1} \Big) + \theta_2 y + \pi_2 - b_2 R_{L2} - \lambda(b_2 - \theta_1 p_1) \right\},$$

where net worth $(e - b_1 R_{L1})$ is taken as given. The first order conditions are:

$$\begin{cases} FOC(b_2): & u'(c_1) = R_{L2} + \lambda, \\ FOC(\theta_2): & p_1 = y/u'(c_1). \end{cases}$$

In period 0, consumers solve the following problem:

$$\max_{b_1} \left\{ u(b_1) + \mathbb{E}_0 \left[\mathcal{V}_1 \right] \right\},$$

where we make use of the fact that, in equilibrium, $\theta_t = 1$. The maximization yields:

$$u'(c_0) = R_{L1}\mathbb{E}_0\big[u'(c_1)\big].$$

The first order conditions of the competitive equilibrium (CE) therefore are:

$$\begin{cases} FOC(b_1): & u'(c_0) = R_{L1}\mathbb{E}_0\big[u'(c_1)\big], \\ FOC(b_2): & u'(c_1) = R_{L2} + \lambda, \\ FOC(\theta_2): & p_1 = y/u'(c_1). \end{cases}$$

When the economy is not constrained ($\lambda = 0$) the model has the following close form solution:

$$\begin{cases} u'(c_1) = R_{L2} \\ u'(c_0) = \mathbb{E}_0\left[R_{L2}R_{L1}\right], \\ p_1 = \frac{y}{R_{L2}} \end{cases} \implies \begin{cases} c_1^* = (R_{L2})^{-\frac{1}{\varrho}} \\ c_0^* = b_1^* = (R_{L2}R_{L1})^{-\frac{1}{\varrho}}, \\ p_1^* = \frac{y}{R_{L2}}. \end{cases}$$

Moreover, by definition, the collateral constraint must hold when the economy is not constrained:[31]

$$\underbrace{b_2^*}_{c_1^* + b_1^* R_{L1} - e} \leq \underbrace{p_1^*}_{\frac{y}{R_{L2}}},$$

which we can rewrite as:

$$e \geq e^b \quad = c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}}.$$

That is, whenever the endowment is above a certain threshold ($e \geq e^b$) the economy is

[31]Note here that we are assuming that profits are realized at the end of the period so that they have no effect on the borrowing constraint.

never constrained. When the economy is constrained ($e < e^b$) consumers borrow up to the limit and maximize consumption in period 1. In this case, $b_2 = p_1$, so that:

$$c_1 + b_1 R_{L1} - e = \frac{y}{u'(c_1)}.$$

And using the fact that the utility function is in CES we have:

$$c_1 + b_1 R_{L1} - e = y c_1^{\varrho}. \tag{E.1}$$

Therefore, depending whether the constraint is binding or not, we can express borrowing in period 0 as:

$$b_1 = \begin{cases} (R_{L2}R_{L1})^{-\frac{1}{\varrho}} & e \geq e^b \\ \frac{y c_1^{\varrho} - c_1 + e}{R_{L1}} & e < e^b \end{cases} \tag{E.2}$$

We finally assume that the endowment is stochastic and follows a uniform distribution $e \sim U(\bar{e} - \varepsilon, \bar{e} + \varepsilon)$.

Assumption on parameter values. As we discussed in the text, to be able to solve the model we need to make assumptions on the value of two parameters: y and $\bar{e}$. In particular, we will consider values such that the economy may be constrained for sufficiently large negative shocks, but is not constrained in the absence of uncertainty.

First, we find a condition that is necessary and sufficient for the economy to be constrained with positive probability, conditional on $e \sim U(\bar{e} - \varepsilon, \bar{e} + \varepsilon)$. We know that the economy is always unconstrained in period 1 if and only if:

$$e \geq e^b = c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}}.$$

When e is stochastic, the economy is unconstrained if and only if the above inequality holds for all possible realizations of e. So it must be the case that:

$$\begin{aligned} e - \varepsilon &\geq c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}}, \\ \bar{e} &\geq c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}} + \varepsilon. \end{aligned}$$

Therefore, when $\bar{e} < c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}} + \varepsilon$ there is positive probability that the constraint binds.

Second, we need a condition that is necessary and sufficient for the economy to be unconstrained when there is no uncertainty (i.e., $\varepsilon = 0$ and $\bar{e} = e$). When $\varepsilon = 0$, the constraint is not binding in period 1 if and only if $e = \bar{e} \geq e^b$; that is:

$$\bar{e} \geq c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}}.$$

Therefore, with no uncertainty, when $\bar{e} \geq c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}}$ the constraint never binds.

Summarizing, we choose an $\bar{e}$ such that the economy will not be constrained in the absence of uncertainty, but it may be constrained for sufficiently large negative shocks:

$$(R_{L2})^{-\frac{1}{\varrho}} + (R_{L2}R_{L1})^{-\frac{1}{\varrho}} R_{L1} - \frac{y}{R_{L2}} \leq \bar{e} < (R_{L2})^{-\frac{1}{\varrho}} + (R_{L2}R_{L1})^{-\frac{1}{\varrho}} R_{L1} - \frac{y}{R_{L2}} + \varepsilon.$$

This implies that there is a threshold for the size of the shock (ε^b) above which the collateral constraint will start to be bind with positive probability. Specifically, the collateral constraint can bind with positive probability for realizations of e in the interval $[\bar{e}-\varepsilon, \bar{e}-\varepsilon^b]$. The level of ε^b can be easily computed as:

$$\varepsilon^b = \bar{e} - e^b = \bar{e} - c_1^* - b_1^* R_{L1} + \frac{y}{R_{L2}}.$$

Competitive equilibrium. We find numerical values for consumption at time 1 (c_1) from the Euler equation $FOC(b_1)$.[32] In order to be able to solve this equation we need to find an expression for borrowing as a function of consumption in both constrained and unconstrained states, as we already did in equation (E.2), and then to weight those states with their probability.

Combining $FOC(b_1)$, the budget constraint, and the expression for b_1 derived earlier in equation (E.2) we get the following system of equations:

$$\begin{cases} b_1^{-\varrho} = R_{L1}\mathbb{E}_0\left[c_1^{-\varrho}\right], \\ b_1 = \begin{cases} (R_{L2}R_{L1})^{-\frac{1}{\varrho}} & e \geq e^b, \\ \frac{yc_1^{\varrho} - c_1 + e}{R_{L1}} & e < e^b. \end{cases} \end{cases}$$

By plugging the second equation in the first one we can write:

$$\Pr(e < e^b) \cdot \left[b_1^{-\varrho}\right]^{\text{binding}} + \Pr(e \geq e^b) \cdot \left[b_1^{-\varrho}\right]^{\text{non-binding}} = R_{L1}\mathbb{E}_0\left[c_1^{-\varrho}\right].$$

Now, by substituting for b_1, the left hand side (LHS) of this equation can be expressed as follows:[33]

$$\begin{aligned} b_1^{-\varrho} &= \frac{1}{2\varepsilon}\int_{\bar{e}-\varepsilon}^{\bar{e}-\varepsilon^b}\left(\frac{yc_1^{\varrho} - c_1 + e}{R_{L1}}\right)^{-\varrho} de + \frac{1}{2\varepsilon}\int_{\bar{e}-\varepsilon^b}^{\bar{e}+\varepsilon} R_{L2}R_{L1} de = \\ &= \frac{1}{2\varepsilon}\int_{\bar{e}-\varepsilon}^{\bar{e}-\varepsilon^b}\left(\frac{yc_1^{\varrho} - c_1}{R_{L1}} + \frac{e}{R_{L1}}\right)^{-\varrho} de + \frac{R_{L2}R_{L1}}{2\varepsilon}\Big[e\Big]_{\bar{e}-\varepsilon^b}^{\bar{e}+\varepsilon} = \\ &= \frac{1}{2\varepsilon}\left[R_{L1}\frac{\left(\frac{yc_1^{\varrho}-c_1}{R_{L1}} + \frac{e}{R_{L1}}\right)^{-\varrho+1}}{-\varrho+1}\right]_{\bar{e}-\varepsilon}^{\bar{e}-\varepsilon^b} + \frac{R_{L2}R_{L1}}{2\varepsilon}\left[\varepsilon + \varepsilon^b\right] \\ &= \frac{R_{L1}^{\varrho}}{2\varepsilon(1-\varrho)}\left[\left(yc_1^{\varrho} - c_1 + e\right)^{-\varrho+1}\right]_{\bar{e}-\varepsilon}^{\bar{e}-\varepsilon^b} + \frac{R_{L2}R_{L1}}{2\varepsilon}\left[\varepsilon + \varepsilon^b\right]. \end{aligned}$$

By equating LHS and RHS numerically, we obtain the competitive equilibrium level of consumption at time 1, where:

$$\begin{aligned} \text{LHS} &= \frac{R_{L1}^{\varrho}}{2\varepsilon(1-\varrho)}\left[\left(yc_1^{\varrho} - c_1 + \bar{e} - \varepsilon^b\right)^{-\varrho+1} - \left(yc_1^{\varrho} - c_1 + \bar{e} - \varepsilon\right)^{-\varrho+1}\right] + \frac{R_{L2}R_{L1}}{2\varepsilon}\left[\varepsilon + \varepsilon^b\right] \\ \text{RHS} &= R_{L1}\mathbb{E}_0\left[c_1^{-\varrho}\right]. \end{aligned}$$

[32] Rember that $c_0 = b_1$ from the budget constraint.

[33] If X is uniformely distributed with $U(a,b)$, then the nth moment of X is given by $\mathbb{E}\left[X^n\right] = \frac{1}{b-a}\int_a^b x^n dx$.

Finally, one can also derive the level of debt at time 0, by using again $FOC(b_1)$:

$$b_1 = \mathbb{E}_0 \left[\left(R_{L1} c_1^{-\varrho} \right)^{-\frac{1}{\varrho}} \right].$$

Social planner. The social planner problem is solved with the same strategy. The first order conditions are:

$$\begin{cases} FOC(b_1): & u'(c_0) = R_{L1}\mathbb{E}_0[u'(c_1) + \lambda p'(c_1)], \\ FOC(b_2): & u'(c_1) = R_{L2} + \lambda(1 - p'(c_1)), \\ FOC(\theta_2): & p_1 = \frac{y}{u'(c_1)}. \end{cases}$$

First we find an expression for $p'(c_1)$. From $FOC(\theta_2)$ we get:

$$p(c_1) = \frac{y}{u'(c_1)} = y c_1^{\varrho},$$

and computing the derivative:

$$p'(c_1) = \frac{\partial \left(y c_1 \right)}{\partial c_1} = \varrho y c_1^{\varrho - 1}.$$

Notice here that the $p'(c_1)$ is positive and decreasing. By looking at $FOC(b_1)$ for the social planner problem, we can see that the she/he borrows less than in the competitive equilibrium. In fact, given that λ is positive only when the constraint binds, $u'(c_1)^{SP} > u'(c_1)^{CE}$ implying that consumption and, therefore, borrowing at time 1 are lower relative to the competitive equilibrium. On the other hand, the planner increases consumption in period 1: given that $p'(c_1) > 0$, from $FOC(b_2)$ we see that $u'(c_1)^{SP} < u'(c_1)^{CE}$.

We also need a value of λ. Notice that the Lagrange multiplier of the social planner is numerically different from the one of the competitive equilibrium problem. In fact, from $FOC(b_2)$ we get

$$\lambda = \frac{c_1^{-\varrho} - R_{L2}}{1 + y}.$$

Combining these two results we can compute:

$$\lambda p'(c_1) = \begin{cases} 0 & e \geq e^b, \\ \frac{\varrho y}{1+y} \left(c_1^{-1} - R_{L2} c_1^{\varrho - 1} \right) & e < e^b. \end{cases}$$

We can now solve for the level of c_1. The $FOC(b_1)$ can be written:

$$b_1^{-\varrho} = R_{L1}\mathbb{E}_0 \left[c_1^{-\varrho} + \lambda p'(c_1) \right].$$

The LHS is the same as before. The RHS now is:

$$
\begin{aligned}
&\frac{R_{L1}}{2\varepsilon}\int\limits_{\bar{e}-\varepsilon}^{\bar{e}-\varepsilon^b}\left(c_1^{-\varrho}+\frac{\varrho y}{1+y}\left(c_1^{-1}-R_{L2}c_1^{\varrho-1}\right)\right)de+\frac{R_{L1}}{2\varepsilon}\int\limits_{\bar{e}-\varepsilon^b}^{\bar{e}+\varepsilon}c_1^{-\varrho}de,\\
&\frac{R_{L1}}{2\varepsilon}\left[\left(c_1^{-\varrho}+\frac{\varrho y}{1+y}\left(c_1^{-1}-R_{L2}c_1^{\varrho-1}\right)\right)\left(\varepsilon-\varepsilon^b\right)+c_1^{-\varrho}\left(\varepsilon+\varepsilon^b\right)\right],\\
&\frac{R_{L1}}{2\varepsilon}\left[\left(\frac{\varrho y}{1+y}\left(c_1^{-1}-R_{L2}c_1^{\varrho-1}\right)\right)\left(\varepsilon-\varepsilon^b\right)+2c_1^{-\varrho}\varepsilon\right]
\end{aligned}
$$

And by equalizing LHS to RHS numerically, we obtain consumption at time 1, where:

$$
\begin{aligned}
\text{LHS} &= \frac{R_{L1}^{\varrho}}{2\varepsilon(1-\varrho)}\left[\left(yc_1^{\varrho}-c_1+\bar{e}-\varepsilon^b\right)^{-\varrho+1}-\left(yc_1^{\varrho}-c_1+\bar{e}-\varepsilon\right)^{-\varrho+1}\right]+\frac{R_{L2}R_{L1}}{2\varepsilon}\left[\varepsilon+\varepsilon^b\right]\\
\text{RHS} &= \frac{R_{L1}}{2\varepsilon}\left[\left(\frac{\varrho y}{1+y}\left(c_1^{-1}-R_{L2}c_1^{\varrho-1}\right)\right)\left(\varepsilon-\varepsilon^b\right)+2c_1^{-\varrho}\varepsilon\right].
\end{aligned}
$$

Finally, we can derive the optimal expression for borrowing at time 1 from the social planner $FOC(b_1)$:

$$
b_1=\left(R_{L1}\mathbb{E}_0\left[c_1^{-\varrho}+\lambda p'(c_1)\right]\right)^{-\frac{1}{\varrho}}.
$$

Crisis Probability. The crisis probability is defined as the probability that the constraint binds. Therefore:

$$
\begin{aligned}
\Pr\left[b_2\right. &> \left. p_1\right]\\
&= \frac{1}{2\varepsilon}\int\limits_{\bar{e}-\varepsilon}^{\bar{e}-\varepsilon^b}de=\frac{1}{2\varepsilon}\left(\varepsilon-\varepsilon^b\right).
\end{aligned}
$$

By using the optimality conditions and the budget constraint, this expression can be written as

$$
\Pr\left[(c_1-(e-b_1R_{L1})>\frac{y}{u'(c_1)}\right].
$$

Now, knowing that $e=\bar{e}+\tilde{\varepsilon}$ and that $\tilde{\varepsilon}\sim\mathcal{U}(-\varepsilon,\varepsilon)$, we can write

$$
\Pr\left[\tilde{\varepsilon}<\underbrace{c_1-\bar{e}+b_1R_{L1}-\frac{y}{u'(c_1)}}_{x}\right].
$$

In particular, the probability that the constraint binds is given by:

$$
\Pr\left[-\varepsilon\le\tilde{\varepsilon}<x\right]=\frac{x-(-\varepsilon)}{2\varepsilon}=\frac{c_1-\bar{e}+b_1R_{L1}-y/u'(c_1)+\varepsilon}{2\varepsilon}.
$$