Short-term Debt and Systemic Risk*

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Abstract

We study the liability structure decisions(short-term debt choices) of banks in a general equilibrium setting with multiple banks, depositors and external investors. The externalities appear through secondary market prices (positive) and overall systemic risk (negative). The private optimal choices are positively (negatively) correlated when the sign of the aggregate externality is positive (negative). The private optimum is inefficient relative to the first best because banks do not internalize the probability of other bank's failure (or the deadweight costs associated with bankruptcies) when it increases its own debt. Ex-post interventions tend to be suboptimal and time-inconsistent, and in fact can accentuate collective moral hazard and systemic risk.

JEL Classification:
Keywords: Systemic Risk, Short-term Debt, Pecuniary Externalities

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"It is difficult to establish any archetype for failure. Banks with high capital ratios imploded while those with lower ratios survived. Plain vanilla retail bank blew up while some black-box trading shops prospered. Both small and big firms collapsed. Yet there was a common ingredient in most failures: an over reliance on (short-term) wholesale funding." The Economist, September 5, 2009

1 Introduction

In the wake of the recent crisis, the over-reliance of financial institutions on short-term debt and its debilitating effects on the financial stability has become an important topic of debate both in academia and policy alike. On the one hand, short-term debt plays an important monitoring role in curbing excessive risk-shifting by managers (e.g., Calomiris and Kahn (1991) and Diamond and Rajan (2001)) and mitigating ex-ante debt-overhang. It also acts as a provider of liquidity needs for the economy (see for example, Stein (2012), Dang et al. (2015), Hanson et al. (2015), Krishnamurthy and Vissing-Jorgensen (2013)). On the other, excessive short-term debt exposes banks to unnecessary rollover risks and panic runs and can be detrimental to the overall systemic stability(see Acharya and Viswanathan (2011), Acharya et al. (2011)). There is a growing body of literature which argues that very high short-term leverage induced banks to engage in excessive risk-taking and ultimately led to their failure (see Acharya et al. (2013), Adrian and Shin (2010), Goel et al. (2014) among others).

To understand the reasons behind the prevalence of short-term debt in the system, several papers have looked at a single bank setting. For example, Brunnermeier and Oehmke (2013) show that there can be a *maturity rat race* arising among various depositors of a bank. This leads to a downward spiral in the average debt maturity. However, the literature is mostly silent on the externalities imposed by maturity structure of one institution on that of the other. Can the high prevalence of short-term debt in the economy be a result of correlated debt choices of institutions? More importantly, can it lead to an increase in systemic risk? This paper seeks to fill this important gap.

We consider a two period model with two banks, a continuum of risk-neutral depositors and an external buyer. The bank deposits can take the form of a short-term or long-term debt contract. At date 0, the banks choose the fraction of short-term debt and invests the borrowed amount in safe or risky assets[1]. This risky asset generates a random cash-flow at the interim date and a fixed liquidating divided at date 2 conditional on success. There is also aggregate uncertainty as the overall state of the economy can either be good (in which case the project success is a certainty) or bad (the project succeeds with a small probability). The short-term creditors of each bank observe the interim realizations and decide whether to reinvest in the bank or withdraw. We show that this rollover/withdraw decision follows a switching strategy which is positively correlated with the fraction of short-term debt of the bank. If there is a

[1]For tractability issues, we have focused on the case where investment takes solely in risky assets. Generalizing this remains an important future objective of this paper.

partial run, it can still survive till the final date by selling some of the assets to the external investor.

The multi-bank setting allows us to capture the externality imposed by actions of one bank on the other. When a bank makes a certain liability structure choice, there can be two conflicting effects on the other bank. First, there is an increase in the possibility that both banks may need to sell assets in the market. This state is referred to as the *systemic risk* state where only one bank can successfully survive till date 2. *Contagion* arises in this state, not because of information spillovers or interbank connections, but through fire sale externalities and secondary market restrictions. Second, high short-term debt of one bank has a positive impact on the prices obtained in the secondary market. This allows the other bank to increase it's own short-term debt as well.

If the positive externality exceeds the negative, the resulting equilibrium has very high short-term debts. The bank owners capture all the upside gain, being protected from downside risks due to limited liability. This high level of debt is inefficient compared to the first best where the central bank or the regulator maximizes the utility of the overall banking system net of distress costs. On the other hand, if the negative externality and the systemic risk is more severe, banks make prudent debt-choices and an asymmetric equilibrium with low aggregate short-term debt results. This characterization is the first important contribution of this paper.

Our second contribution is to show that any ex-post forbearance mechanism may fail to correct this externality. We design the forbearance as a bail-out policy where the central bank provides the necessary liquidity to the distressed banks in the systemic risk state and takes up an equity position in the banks. If the bailout is more lenient compared to the market, moral hazard in part of banks creates an adverse feedback loop and increases the likelihood of the systemic state instead.

1.1 Related Literature

This paper shares the general theme adopted in two recent papers which highlight the inefficiencies related to excessive short-term debt. The first is Perotti and Suarez (2011). They consider a representative bank model with only short-term debt but include a negative spillover factor to capture the associated refinancing risk. Our model, in contrast, explicitly models the interactions among multiple banks and discusses the origin and the extent of systemic risk. The negative spillovers arise naturally in our model as banks fail to satisfy their liquidity demand at the intermediate date. The second, Stein (2012), includes the bank's choice between different debt maturities to show that private equilibrium may create *too much money* relative to the social optimum. Though our model borrows some features from this paper, there are crucial differences between his analysis and ours. First, Stein considers a representative bank model, precluding any discussion of spillovers - an idea central to ours. Second, our model has a more realistic incomplete information structure whereas his setting reflects common knowledge of states. Finally, the wedge between the social and private optimum arises only when the yield curve is sufficiently positive sloping. In our model, the social optimum deviates from private because banks does not internalize the social costs imposed by bank failures.

Our paper is also related to Acharya and Thakor (2015). They consider a similar setting in which excessive leverage increases systemic risk through information contagion. However, there are two important differences between their model and ours. First, their leverage is essentially short-term where the investor decides whether to continue/liquidate at the interim date. Excess leverage in their model refers to the total face value of debt. In contrast, we are more concerned with the choice between the debts of differing maturities. Second, the asset liquidation value in their model is exogenous while it is endogenous here and presents a very important source of externality.

We also contribute to the literature on systemic risk (see De Bandt and Hartmann (2000) for an excellent survey). Acharya (2009) considers asset side correlations as a generator of systemic risk in a multi-bank setting. He shows that when ex-post charter values are high, banks may ex-ante choose high asset-side correlations. This propensity to correlate increases with the positive externalities from the failure of competing banks (arising from decreasing competition and migration of depositors to the surviving banks). In our model, the positive externality appears through secondary market prices and increases with the ex-ante expectation of good state at the intermediate date. Banks' debt-choice become positively correlated if this positive externality is high. Allen and Gale (2000) shows that financial fragility may increase in an incomplete network with increasing linkages (though interbank debt holdings, also see Dasgupta (2004)). In contrast, our paper shows that interbank connections are not necessary for systemic risk via contagion to arise. This holds more generally, as long as the secondary market is credit constrained or is not willing to extend credit. Allen et al. (2012) model the origin of systemic risk in a financial network when banks finance themselves through short-term debt. However, in their model, systemic risk appears through cross-holdings of assets and not through debt per se.

Finally our paper touches upon the literature which suggests that too much competition may induce excessive risk taking as it reduces margins(see Hellmann et al. (2000), Allen and Gale (2004) as leading early contributions to this view and Carletti (2008) for a comprehensive review). Proponents of this view suggests that increased competition may lead banks to *gamble* on risky assets, thereby accentuating the fragility of the financial system. Also, lower rents in a more competitive economy may reduce incentives for monitoring (Boot and Thakor (1993)). This paper is different is two important aspects. First, the cited papers consider a single bank model which eliminates the possibility to study the spillover effects on other banks. Second, these papers are more concerned with asset side impact of fluctuations whereas, the present paper focuses on liability side correlations arising in a more competitive environment.

The remainder of this paper proceeds as follows. Section 2 discusses the model setup. Section 3 characterizes the equilibrium and contains our main results on the externality and systemic risk. Section 4 shows how the private choices may be inefficient compared to the social optimum. Section 5 considers the effects of possible regulatory interventions. Finally, section 6 concludes and discusses the future extensions for this project.

2 The Model

2.1 Environment

The economy consists of two periods ($t = 0, 1, 2$) with three types of agents - a finite set of $N \geq 2$ banks, a continuum of households with unit measure and uninformed investors. We assume that all the agents are risk neutral and normalize the risk-free interest rate to be zero. Risk-neutrality is chosen for convenience and enables us to focus on liquidity driven security choices in the absence of risk-sharing incentives.

The household's utility is linear and time separable in consumption and is given by

$$U(c_0, c_1, c_2) = c_0 + \mathbb{E}(c_1) + \mathbb{E}(c_2)$$

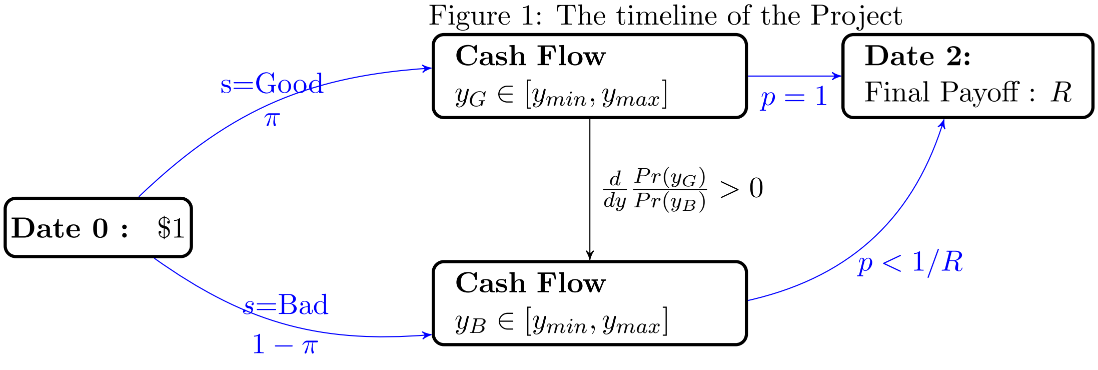
where the consumption at each date must be non-negative $c_t \geq 0$.

At time $t = 0$, the banks have access to two investment technologies, each requiring \$1 of initial investment. The first is a risk-less storage which yields \$1 at date $(t+1)$ per dollar invested at date t. The second is a risky technology and produces two cash-flow streams $\{y_{1s}, y_{2s}\}$ at dates $t = 1$ and $t = 2$ respectively. Here, $s \in \{G, B\}$ represents the underlying state and can be good with an exogenous probability π or bad with probability $(1 - \pi)$. The final cash flow y_{2s} is random and has a two point distribution $\{R, 0\}$ with the associated probabilities

$$\mathbb{P}\left[y_{2G} = R\right] = 1, \quad \mathbb{P}\left[y_{2B} = R\right] = p \in (0, 1/R)$$

The intermediate cash-flow y_{1s}, on the other hand, is assumed to be a continuous, i.i.d random variable with cdf F_s and pdf f_s, conditional on the underlying state. $f_s \forall s \in \{G, B\}$ is continuously differentiable and has the support $[\mathrm{y_{min}}, \mathrm{y_{max}}]$ with $0 \leq \mathrm{y_{min}} < \mathrm{y_{max}} \leq 1$. The following figure 1 shows the sequence of events.

Figure 1: The timeline of the Project



We assume that the higher cash-flows are more likely in the good state than in the bad in the sense of Monotone Likelihood Ratio Property (MLRP) : given any pair of cash-flows $(\mathbf{y}, \mathbf{y}') \in [0,1]$ with $\mathbf{y}' > \mathbf{y}$, the probability that the underlying state is good is higher when the observed cash-flow is $\mathbf{y}'$ than when it is $\mathbf{y}$. In other words,

Assumption 1. $\dfrac{\mathbb{P}(G|\mathbf{y}')}{\mathbb{P}(B|\mathbf{y}')} > \dfrac{\mathbb{P}(G|\mathbf{y})}{\mathbb{P}(B|\mathbf{y})} \ \forall \ \mathbf{y}' = (y_1', y_2') > \mathbf{y} = (y_1, y_2)$.

Accordingly, higher intermediate cash flows are good news for all the parties involved as it puts a higher probability on the underlying state being good and consequently on the probability of success of the project.

We add a friction to this environment by considering the possibility that the manager may choose to shift to a riskier asset at date $t = 1$. It provides a private benefit of B and reduces the probability of success by Δ irrespective of the underlying state. Thus an asset substitution at the intermediate date yields an observable cash flow R with probability $1-\Delta$ when the underlying state is good (respectively $p-\Delta$ in the bad state) and a private payoff B per unit of initial investment. Moreover, only the good technology is socially efficient. This moral hazard friction reduces the pledgeability of the final cash-flows as sufficient rents should be left to the manager to preserve incentives. We model this situation in a reduced form by representing this rent as a fraction ψ of the final cash-flows.

Assumption 2. *The maximum pledgeable income is only a fraction* $(1-\psi)R, \quad \psi \in (0,1)$.

Finally, we assume that the risky project is efficient ex-ante. However, if the intermediate state turns out to be bad, then the expected cash-flow is not enough to cover the costs.

2.2 Contract Choices

In order to invest in the project, the penniless bank manager needs to raise the requisite funds from the competitive capital markets populated by a continuum of households. We will restrict attention to simple contracts comprising of short-term debt, long-term zero coupon debt and equity. Although we will not solve the optimal contracting problem, we do present some justification for the choices of contracts. First, let us consider long-term contracts where all the payoffs are at date $t = 2$. Essentially, this entails a comparison between equity and long-term zero coupon debt. The following lemma proves that the optimal security which satisfies the asset-substitution moral hazard described earlier and bank's incentives to hide cash-flows in a costly state verification (CSV) regime is a simple debt contract (SDC) whose face value binds the investor's participation constraint.

Lemma 1. *The optimal long term contract is a SDC with face value* r_D.

The proof of the above lemma along with the requisite expression for r_D is provided in Appendix A.2.. The basic intuition for the result is simple. In the CSV regime, when the project is funded with equity, verification happens over the entire interval of the combined cash-flows $y_1 + y_2$, while with debt, the verification is restricted to the bankruptcy region when the reported cash-flow falls short of the face value of debt. Since all the agents are risk-neutral in our model, long-term debt emerges as the optimal security which minimizes the expected verification cost. Debt trivially arises as the favored security since agency costs of debt are not incorporated.

In general, the optimal dynamic contract in this setting will involve transfers $(f_1(y_1), f_2(y_1, y_2))$ at dates $t = 1$ and $t = 2$ respectively. A similar problem has been tackled in Chang (1990). Without the threat of intermediate liquidation, the optimal contract has the following structure[2]:

$$\begin{aligned} f_1(y_1) + f_2(y_1, y_2) &= M \text{ if no verification takes place at date 1} \\ f_1'(y_1) + f_2'(y_1, y_2) &= y_1 + k \text{ if audit occurs at date } t = 1 \\ f_1'(y_1) &= y_1 \quad y_1 \le m \in [\mathrm{y}_{\min}, \mathrm{y}_{\max}] \text{ if verfication happens at date } t = 1 \end{aligned}$$

This optimal security (or in this case a menu of securities unique up to a constant) can be interpreted as a coupon bearing debt or a combination of strips yielding payoffs at times $t = 1$ and $t = 2$. The possibility of interim liquidations, however, renders the problem more complicated. Using a three period model with possible liquidations, Repullo and Suarez (1998) show that the optimal security is a mixture of an information-sensitive component (interpreted as short-term debt with rollover in our context) and an information- insensitive component(two period zero coupon debt).

With these motivations at hand, we choose the preferred contract as a mixture of short-term and long-term debt. Let $(\phi, 1-\phi)$ denote the chosen liability structure of the bank, where ϕ represents the fraction of one-period debt with face value r_M and the remaining fraction is a two period debt with face value r_D. Stein (2012) argues that banks have an inherent preference for short-term debt because of the underlying *moneyness*. It implies that this debt is implicitly backed by the project and can be sold at the secondary market to repay the creditors, if rollover does not happen at the intermediate date. Thus, the short-term debt can be made largely risk-less implying

$$r_D \gg r_M \sim 1.$$

However, with short-term debt, the manager runs the risk of failure to roll-over. As we show in the next section, such failure can happen when the short-term creditors get adverse news affecting the final payoff of the project. Thus, the optimal mix ϕ^* depends upon the trade off between the liquidation risk and the carry (the spread between the two maturities) and will be the focus of our analysis in a general equilibrium setting. We start by addressing the decision problem of the short-term creditors of a single bank at date $t = 1$.

[2]See Propositions 4, 5 and 6 in Chang (1990) for the proofs of the following result.

2.3 Creditors' Decision Problem

At date 1, the short-term creditors observe the cash-flow realization and decide to roll-over or withdraw their debt. We make a simplifying assumption that the short-term creditors of bank i sees the realization of the debtor bank perfectly but not for the other bank.

Assumption 3. *Creditors observe the interim cash-flow of their own bank perfectly. The realization of the other bank remains completely opaque.*

This assumption can be easily relaxed to the case where the creditors observe a noisy signal (s_i, s_j) of the joint realization at the interim date. Standard global-games approach will then lead to a switching strategy of the following type

$$\begin{cases} \text{Roll-over} & \text{if } (s_i, s_j) \geq (s_i^*, s_j^*) \\ \text{Withdraw} & \text{otherwise} \end{cases}$$

with (s_i^*, s_j^*) representing the threshold level of the joint signal. We obtain a similar action profile for the short-term creditors in the present model. However, in this case, the threshold action depends only on the conditional cash-flows of the debtor bank and not on the joint realization.
At date $t = 1$, the short-term investors must decide whether to rollover the debt for another period (and the required interest rate) or withdraw and demand payment. We assume that the cash-flow of the bank is observed perfectly by the creditors (this avoids the coordination problem among investors). However, even with perfect observability of the interim cash-flow, there is incomplete information as the underlying state of nature remains unobserved.

Given that the creditors know about the distributions and their dominance property, they can conjecture about the underlying state from the interim realization y. Let $\alpha(y)$ denote the posterior that the state is good, that is

$$\alpha(y) = \mathbb{P}(s = G | \tilde{y} \in (y, y + dy))$$

Since higher cash-flows are more likely in the good state, the creditors are more willing to rollover when the interim cash flows are high than when they are low. Thus, we expect $\alpha(y)$ to be increasing in y. The following lemma verifies this.

Lemma 2. *The posterior $\alpha(y)$ is monotonically increasing in y.*

If the creditor decides to rollover the debt, let the second period interest rate demanded be $r_2(y)$. For a given cash-flow y, the posterior that good state has occurred is given by $\alpha(y)$. Thus the participation condition of the short- term creditor must satisfy

$$\alpha(y) r_2(y) + (1 - \alpha(y)) p r_2(y) + (1 - \alpha(y))(1 - p)\beta y \geq 1$$

The first two terms reflects the returns to the creditor conditional on success. The final term is the product of the probability of failure $(1 - \alpha(y))(1 - p)$ and the residual cash accruing to the creditors in a bankruptcy event βy where a fraction $(1-\beta)$ is lost in bankruptcy proceedings. To simplify the expressions, take $\beta = 0$. This implies that the creditors demand an interest rate

$$r_2(y) = \frac{1}{\alpha(y) + (1-\alpha(y))p} \tag{1}$$

However, given r_D to be delivered to the long-term investors, second period rate must be such that the bank survives at date 2, implying the restriction

$$\phi r_2(y) + (1-\phi) r_D \leq (1-\psi) R$$

Thus there exists a cutoff y_c such that the creditor does not rollover below this cutoff. The following proposition summarizes this analysis.

Proposition 1. *(i) There exists a lower-threshold value $\underline{\phi}$ given by*

$$\underline{\phi} = \frac{(1-\psi)R - r_D}{\frac{1}{p} - r_D}$$

such that short-term creditors always roll-over when $\phi < \underline{\phi}$.
(ii) For all $\phi \geq \underline{\phi}$, there exists a critical value y_c such that the short-term creditors withdraw if and only if $y < y_c$ and roll-over for all $y \geq y_c$. It is given by solution of the following equation

$$y_c = \alpha^{-1}\left(\frac{1}{1-p}\left[\frac{\phi}{R-(1-\phi)r_D} - p\right]\right). \tag{2}$$

Thus, information asymmetry leads to ex-post rationing in the credit markets as banks a higher interest rate in good state as compared to the full-information benchmark (risk-less refinancing). How does short-term debt affect the threshold value? To see this we differentiate (2) with respect to ϕ.

$$\alpha'(y_c)\frac{dy_c}{d\phi} = \frac{1}{1-p}\frac{R - r_D}{(R-(1-\phi)r_D)^2}$$

Since $\alpha'(\cdot) \geq 0$ and the RHS is also positive, $\frac{dy_c}{d\phi}$ must be non-negative.

Corollary 1. *The threshold value y_c increases with the fraction of short-term debt ϕ.*

The following figure 2 provides an example of the results derived in this section. Given the interim cash-flow distributions in sub-figure 1 (the solid red line represents the cash-flows in the good state and follows $\mathcal{N}(0.6, 0.2^2)$ while the dashed black line represents the cash-flows in the bad state and follows $\mathcal{N}(0.2, 0.2^2)$), the figure shows the posteriors of the short-term creditors, the second period interest rate charged $r_2(y_i)$ and finally, the switching threshold level.

In this section, we focused on the rollover decisions of the creditors of a single bank. In the next section we move to the general equilibrium setting with multiple banks. Given the MLRP assumption

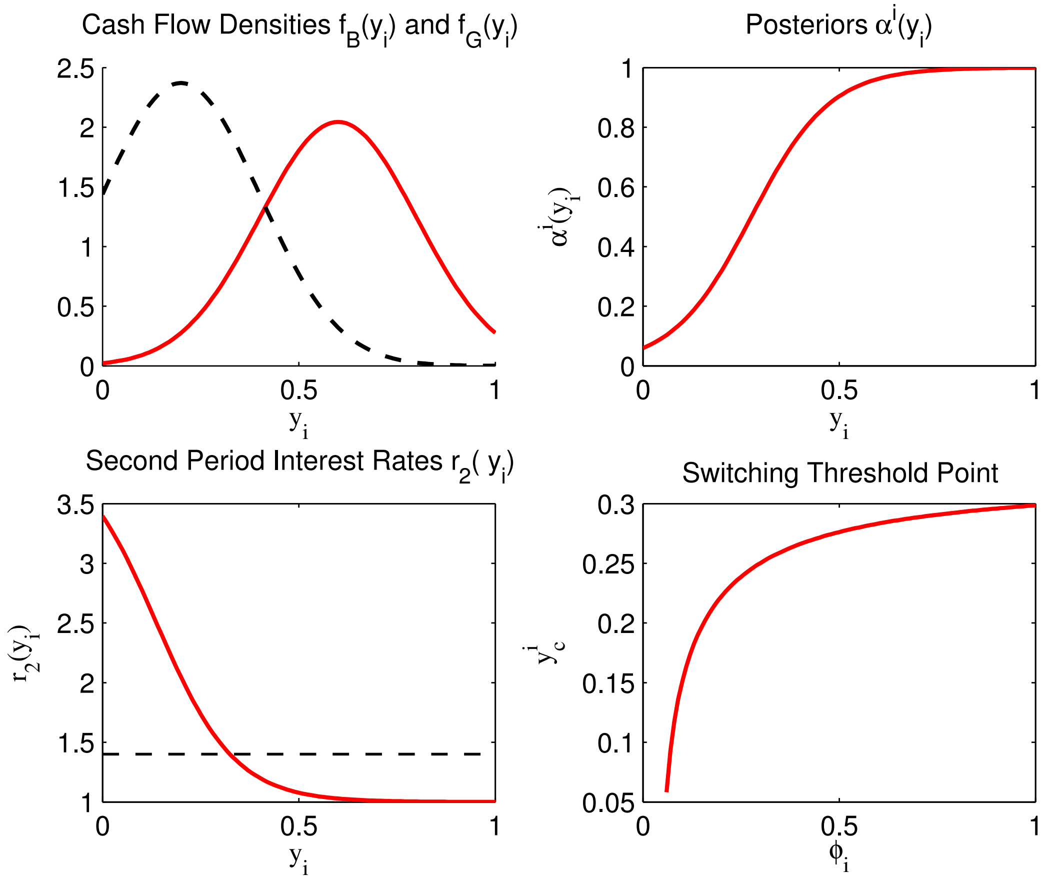


Figure 2: Let $\mathrm{y_{min}} = 0, \mathrm{y_{max}} = 1$. The conditional distributions follow $\mathcal{N}(\mu_s, 0.2)$ where $\mu_B = 0.2, \mu_G = 0.6$. The second figure plots the posteriors $\alpha^i(y_i)$ conditional on the cash flow y_i and prior $\pi = 0.8$. Also let $R = 2, \psi = 0.2, r_D = 1.4, p = 0.25$. The third figure plots the second term interest rates demanded by the short term creditors. The final figure shows the cutoff interest rates as a function of the short-term debt.

of the intermediate cash-flows, the threshold criterion holds for each individual bank. For simplicity of exposition, we will focus on the case where there are two banks in the system $N = 2$. The same conclusions are derived in the general case as well.

3 The Intermediary Equilibrium

Let the vector $\mathbf{y_c} = \{y_c^1, y_c^2, ..., y_c^N\}$ denote the threshold level of the intermediate cash-flows such that the short-term creditors of bank i withdraw funding at date $t = 1$ if and only if $y_i < y_c^i$. As indicated above, we restrict attention to the case when $N = 2$. Given two banks in the system, there are four possible states at date 1 depending on the interim realizations. They are depicted in the table 1 below.

We refer to states $HL(LH)$ as the *idiosyncratic* risk states and state LL as the *systemic* risk state.

State	Description	Probability
HH	Both banks refinanced	$p_1 = \pi \int_{\mathrm{y_c^1}}^{y_{max}} \int_{\mathrm{y_c^2}}^{y_{max}} dF_G + (1-\pi) \int_{\mathrm{y_c^1}}^{y_{max}} \int_{\mathrm{y_c^2}}^{y_{max}} dF_B$
HL	Bank 1 is refinanced, Bank 2 not	$p_2 = \pi \int_{\mathrm{y_c^1}}^{y_{max}} \int_{y_{min}}^{\mathrm{y_c^2}} dF_G + (1-\pi) \int_{\mathrm{y_c^1}}^{y_{max}} \int_{y_{min}}^{\mathrm{y_c^2}} dF_B$
LH	Bank 2 is refinanced, Bank 1 not	$p_3 = \pi \int_{y_{min}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{y_{max}} dF_G + (1-\pi) \int_{y_{min}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{y_{max}} dF_B$
LL	Both Banks not refinanced	$p_4 = \pi \int_{y_{min}}^{\mathrm{y_c^1}} \int_{y_{min}}^{\mathrm{y_c^2}} dF_G + (1-\pi) \int_{y_{min}}^{\mathrm{y_c^1}} \int_{y_{min}}^{\mathrm{y_c^2}} dF_B$

Table 1: Four Possible States with Probabilities

3.1 Secondary Market at Date 1

At date $t = 1$, a secondary market for the assets exists where the banks can sell their projects for liquidity. For simplicity, we will assume that there is a representative external investor endowed with one unit of money. He can be thought of as an external banker who takes over the project and continues to operate till maturity. Since the original managers are better informed and better equipped to operate the project, it suffers a loss of revenue under the second-best operator.

Assumption 4. *The external investor has wealth $W = 1$ at date 1 with which he can buy assets in the secondary market. She is a second best user of the project and can only generate a cash-flow $\gamma R < R$ if the project succeeds.*

Let us focus on a particular bank, say 1 to fix ideas. With probability $p_3 + p_4$, the short-term creditors don't refinance at date 1 and it has to access the secondary market to obtain the liquidity requirement $(\phi_1 - y_1)$. In state LH, the total demand is $\phi_1 - y_1 < 1$, while in state LL, it is $\sum_{i\in\{1,2\}}(\phi_i - y_i)$. The total supply of liquidity is 1 as assumed before. Thus, in the *idiosyncratic* risk state, the banks can get enough liquidity to survive till date 2, when it might become insolvent. On the other hand, in the *systemic* risk state, there can be a liquidity crisis if the total demand exceeds 1 and solvent banks may be liquidated. This captures the key externality in this setup. Here, both idiosyncratic and the systemic risks increase with the short-term debts of the banks. In the *idiosyncratic* or *systemic* risk-states, the banks need to sell assets to an uninformed investor who does not observe the cash-flows[3].
But he can conjecture the threshold levels $(\mathrm{y_c^1}, \mathrm{y_c^2})$ which turns out to be correct in equilibrium. Thus, if bank 1 posts a liquidity demand of $\phi_1 - y_1$, the investor forms expectations based on $y_1 < \mathrm{y_c^1}, y_2 \geq \mathrm{y_c^2}$. The following lemma shows that the posterior of a good state is increasing in the short-term debt of either banks.

Lemma 3. *The uninformed investor's posterior of good state occurring increases in the fraction of short-term debt of either banks.*

Call the uninformed investor's posterior of the underlying good state by

$$\begin{cases} \alpha_{id}(\phi_1, \phi_2) & \text{if the observed state is LH or HL} \\ \alpha_{sys}(\phi_1, \phi_2) & \text{if the observed state is LL.} \end{cases}$$

[3] The only restriction required for this model to work is that the external investors cannot exactly predict the cash-flow when pricing assets. This assumption is not unreasonable and becomes more realistic in the general version when the total liquidity requirement is given by $\phi - yX - (1 - X)$ for the general asset structure $(X, 1 - X)$ of the bank.

Thus the fundamental price paid by the investor for the assets in states $LH(HL)$ and LL are Q_{id} and Q_{sys} respectively, where

$$\begin{aligned} Q_{id} &= \left[\alpha_{id} + (1-\alpha_{id})p\right]\gamma R \\ Q_{sys} &= \left[\alpha_{sys} + (1-\alpha_{sys})p\right]\gamma R \end{aligned} \tag{3}$$

Given lemma (3), the price paid by the investor increases with ϕ. This explains banks' propensity to take excessive short-term debt. To get the intuition for the result, consider the information of the external investors. Since they are the least informed, they update their beliefs conditional on the actions of the short-term creditors. The higher this fraction, the more informative the actions become[4].

It is incumbent upon us to discuss at this point, the two opposing forces which may restrict the bank's short-debt choice in equilibrium. Higher short-term debt directly increases the probability of a bank run and simultaneously, the bank's liquidity demand. More importantly, it increases the probability of systemic failure, where investors run on both banks. This represents the key negative externality in the model. On the other hand, larger fraction of short-term debt implies higher prices in equilibrium and less asset sale. The second factor represents the positive externality in our model. The trade-off between the two forces is the pivotal factor driving the equilibrium allocations.

3.2 Bank's Problem at Date 1

In order to understand the bank's problem at date 1, we need to consider three cases separately.

Case 1 : Short-term debt rolled over: The first state refers to the one where the short-term debt is rolled over for one more period and the bank does not face any immediate liquidity need. In this case, the bank i has extra cash $y_i \geq y_c^i$ and can use it to retire some fraction of short-term debt. Let f_s^i denote the marginal distribution of the cash-flow for bank $i \in \{1,2\}$ in state $s \in \{G, B\}$. Then the date 1 value of the bank in this case can be written as

$$v_{is}^1(\phi_i) = \pi_s \left[\int_{\phi_i}^{\mathrm{y_{max}}} [R + u - \phi_i - (1-\phi_i)r_D] f_s^i(u)du + \int_{y_c^i}^{\phi_i} [R - (\phi_i - u)r_2(u) - (1-\phi_i)r_D] f_s^i(u)du \right] \tag{4}$$

Here, π_s denotes the probability of success conditional on the underlying state s and takes values $\pi_G = 1$ and $\pi_B = p$ respectively. The first integral captures the state when the bank is able to retire the entire short term debt while the second term denotes the situation where it can only retire a fraction $\phi_i - u$ and pays an interest rate of $r_2(u)$ on the remaining.

Will banks go to the market with above threshold cash-flows? Suppose that they do. To fix ideas, assume that bank i, with an intermediate cash-flow $y_i > y_c^i$, decides the retire the fraction $(\phi_i - y_i)$

[4]For example, if ϕ is very small, $r_2(y)$ can be made very large and still the bank will be able to pay this high debt with the profit $(1-\phi)(R - r_D)$ from the long-term component. Thus for low enough ϕ, the running threshold is very low. This means that the investors form very low posteriors in this case and the bid price for the assets become very low.

of short-term debt instead of refinancing. The external investor does not know about the deviation and will assume $y_i < y_c^i$. It is easy to see that this move does not work in the bank's favor as the investor's posteriors will be categorically worse than that of the short-term creditor of the bank. The contradiction follows easily from this point.

Case 2 : Partial Run in *Idiosyncratic Risk State*: In this case, the bank needs to sell some assets to raise the required liquidity $(\phi_i - y_i)$. Given the price Q_{id} for the assets in the secondary market, the fraction the bank needs to sell is

$$\Delta_i(\phi_i, y_i) = \frac{\phi_i - y_i}{Q_{id}}$$

The date-2 payoff to the bank if the project succeeds is

$$(1 - \Delta_i)R - (1 - \phi_i)r_D.$$

Let $\underline{y}_2(\phi_i)$ denote the lowest value of the date-1 realization such that the bank survives in date 2, that is

$$(1 - \frac{\phi_i - \underline{y}_2(\phi_i)}{Q_{id}})R = (1 - \phi_i)r_D \iff \underline{y}_2(\phi_i) = \max(0, \phi_i - Q_{id}\frac{R - (1 - \phi_i)r_D}{R}).$$

The date-1 value of the bank i in state s is given by

$$v_{is}^2(\phi_i) = \pi_s \left[\int_{\underline{y}_2(\phi_i)}^{y_c^i} [R(1 - \Delta_i(\phi_i, u)) - (1 - \phi_i)r_D] \int_{y_c^j}^{\mathrm{y}_{\max}} f_s(u, v) dv du \right] \tag{5}$$

Case 3 : Partial Run in *Systemic Risk State*: In this case, both banks face a liquidity shortage and must sell assets to repay the short-term creditors. Since the total supply of liquidity is capped at 1, two separate cases may arise.

In the first situation, the debt choices of the banks are *correlated* in the sense that

$$\phi_1 + \phi_2 \leq 1$$

In this case, there is enough liquidity to cover the shortfall even for the realization $(\mathrm{y}_{\min}, \mathrm{y}_{\min})$ and the banks never fail at date 1. Bank i raises the required amount by selling a fraction Δ_i where

$$\Delta_i(\phi_i, y_i) = \frac{\phi_i - y_i}{Q_{sys}}$$

As in the previous case, the bank survives at date 2 if and only if

$$\Delta_i(\phi_i, y_i) \leq \frac{R - (1 - \phi_i)r_D}{R}.$$

This yields a lower cutoff value of the cash-flow $\underline{y}_3(\phi_i)$

$$\underline{y}_3(\phi_i) = \max(0, \phi_i - Q_{sys}\frac{R - (1-\phi_i)r_D}{R}).$$

The date-1 value of the bank i in state s is

$$v_{is}^3(\phi_i) = \pi_j\Big[\int_{\underline{y}_3(\phi_i)}^{y_c^i}[R(1-\Delta_i(\phi_i,u)) - (1-\phi_i)r_D]\int_{\mathrm{y_{min}}}^{y_c^j} f_s(u,v)dvdu\Big] \tag{6}$$

In the second situation, the debt choices are such that the following restriction holds:

$$2 \geq \phi_1 + \phi_2 > 1 + 2\,\mathrm{y_{min}}$$

Here, for some realizations of the interim cash-flow (for example, $(y_1, y_2) : y_1 + y_2 < \phi_1 + \phi_2 - 1$) the total liquidity demand exceeds the supply and both banks cannot be saved together. In such a situation, assume that one of the banks is selected at random with probability $\frac{1}{2}$ and is given the priority. The other bank does not get the required liquidity and declares bankruptcy in which case the shareholders get nothing.

If the joint realization is high enough, both banks can acquire the required liquidity at date 1 and operate till date 2. On the other hand, for low enough cash-flows, a single bank survives with probability $\frac{1}{2}$. Thus, the date-1 value of the bank can be written as

$$\begin{aligned} v_{is}^3(\phi_i) = \pi_j\Big[&\iint_{u+v\geq\phi_1+\phi_2-1}[R(1-\Delta_i) - (1-\phi_i)r_D]f_s(u,v)dvdu \\ &+\frac{1}{2}\iint_{u+v<\phi_1+\phi_2-1}[R(1-\Delta_i) - (1-\phi_i)r_D]f_s(u,v)dvdu\Big] \end{aligned} \tag{7}$$

3.3 Bank's Leverage Choices at Date 0

The time-0 optimization problem of the banks can be written by collecting the individual terms obtained in equations (4), (5), and (6) (or (7)). Thus at date 0, the bank i's best response function $\hat{\phi}_i(\phi_j)$ can be written as the solution to the following optimization problem :

$$V_i(\phi_i, \phi_j) = \max_{\phi_i}\Big[\pi(v_{iG}^1 + v_{iG}^2 + v_{iG}^3) + (1-\pi)p(v_{iB}^1 + v_{iB}^2 + v_{iB}^3)\Big] \tag{8}$$

The value function of the bank i for a generic state s is given as follows

$$\begin{aligned}
V_i(\phi_i,\phi_j) = &\int_{\phi_i}^{\mathrm{y_{max}}}\int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}}[R+u-\phi_i-r_D(1-\phi_i)]f_s(u,v)dvdu+ \\
&+\int_{y_c^i}^{\phi_i}\int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}}[R-(\phi_i-u)r_2(u)-r_D(1-\phi_i)]f_s(u,v)dvdu \\
&+\int_{\underline{\mathrm{y}}_2}^{y_c^i}\int_{y_c^j}^{\mathrm{y_{max}}}[R(1-\frac{\phi_i-u}{Q_{id}})-r_D(1-\phi_i)]f_s(u,v)dvdu \\
&+\int_{\underline{\mathrm{y}}_3}^{y_c^i}\int_{\phi_i+\phi_j-1-u}^{y_c^j}[R(1-\frac{\phi_i-u}{Q_{sys}})-r_D(1-\phi_i)]f_s(u,v)dvdu \\
&+\frac{1}{2}\int_{\underline{\mathrm{y}}_3}^{y_c^i}\int_{\underline{\mathrm{y}}_3}^{\phi_i+\phi_j-1-u}[R(1-\frac{\phi_i-u}{Q_{sys}})-r_D(1-\phi_i)]f_s(u,v)dvdu
\end{aligned} \tag{9}$$

The first order condition can be written as [5]:

$$\begin{aligned}
0 = &(r_D-1)(1-F_s^i(\phi_i))+\int_{y_c^i}^{\phi_i}(r_D-r_2(u))f_s^i(u)du \\
&+\int_{\underline{\mathrm{y}}_2}^{y_c^i}[(r_D-\frac{R}{Q_{id}})+\frac{R(\phi_i-u)}{Q_{id}^2}\frac{\partial Q_{id}}{\partial y_c^i}\frac{dy_c^i}{d\phi_i}]\int_{y_c^j}^{\mathrm{y_{max}}}f_s(u,v)dvdu \\
&+\int_{\underline{\mathrm{y}}_3(\phi_i)}^{y_c^i}\left((r_D-\frac{R}{Q_{sys}}+\frac{R(\phi_i-u)}{Q_{sys}^2}\frac{\partial Q_{sys}}{\partial y_c^i}\frac{dy_c^i}{d\phi_i})g(\cdot)+(R(1-\Delta_i)-r_D(1-\phi_i))\frac{\partial g}{\partial \phi_i}\right)f_s^i(u)du \\
&+\frac{1}{2}\int_{\underline{\mathrm{y}}_3(\phi_i)}^{y_c^i}\left((r_D-\frac{R}{Q_{sys}}+\frac{R(\phi_i-u)}{Q_{sys}^2}\frac{\partial Q_{sys}}{\partial y_c^i}\frac{dy_c^i}{d\phi_i})h(\cdot)+(R(1-\Delta_i)-r_D(1-\phi_i))\frac{\partial h}{\partial \phi_i}\right)f_s^i(u)du
\end{aligned} \tag{10}$$

where $g(\phi_i,\phi_j,u)=\int_{\phi_i+\phi_j-1-u}^{y_c^j}f_s(u,v)dv$ and $h(\phi_i,\phi_j,u)=\int_{\underline{\mathrm{y}}_3(\phi_j)}^{\phi_i+\phi_j-1-u}f_s(u,v)dv$. The point-mass terms due to Leibniz rule cancel out and have been ignored. The effect of the short-term debt ϕ_j on the debt-choice $\hat{\phi}_i$ of bank i depends on the last three terms of the first-order condition. The third term corresponds to the *idiosyncratic risk* state where only bank j is refinanced while bank i is not. The fourth term corresponds to the *systemic risk* state in correlated choices where neither banks are refinanced but there is sufficient liquidity in the market to cover the joint needs of the banks. The fifth term refers to the *contagion* case.

The equilibrium at $t=0$ depends critically on the externality due to bank j's debt-choice ϕ_j on bank i. In the next subsection, we talk about the sources and the extent of this externality.

[5] To make the equation less cumbersome, we deliberately remove the explicit dependence on the underlying state and the corresponding probabilities, with the understanding that the expectation operator is present.

3.3.1 The Externality of Bank's Debt Choices

The externalities imposed on bank i by the liability structure of bank j stem from two sources. The first relates to the probability of the systemic risk state occurring, which increases with the short-term debt of bank j. To see this directly, we differentiate the probability p_4 with respect to ϕ_j.

$$\frac{\partial p_4}{\partial \phi_j} = \left[\pi \int_{\underline{y}}^{y_c^i} f_G(u, y_c^j)du + (1-\pi) \int_{\underline{y}}^{y_c^i} f_B(u, y_c^j)du\right] \frac{dy_c^j}{d\phi_j} > 0$$

The second externality comes from the intermediate liquidation (inefficient) of bank i when a higher ϕ_j leads the inequality $\phi_i + \phi_j \leq 1$ to break down. This is the classic *contagion* effect which increases the probability of bank i's failure at date 1. These effects are captured in terms (3) - (5) of (10).

On the other hand, lemma 3 shows that the investor's posteriors and hence the equilibrium price increases with ϕ_j. Should bank i need to liquidate assets at the intermediate date, it can do so at a higher price. This creates incentives for bank i to increase its own debt. However, as ϕ_i increases, the probability of systemic state also goes up simultaneously. The dominant effect depends on both ϕ_i and ϕ_j. To capture the externality, fix ϕ_i for the time being. As ϕ_j increases, the price effect dominates as long as the systemic risk is not too high. After a certain threshold level, the sign of the externality reverses. The following proposition formalizes the argument laid above:

Proposition 2. *For every π, $\exists \phi_c^j(\phi_i, \pi) \in (0,1)$ such that*

$$\frac{\partial V_i(\phi_i, \phi_j)}{\partial \phi_j} = \begin{cases} \geq 0 & \text{if } \phi_j \leq \phi_c^j \\ < 0 & \text{if } \phi_j > \phi_c^j \end{cases}$$

3.3.2 Equilibrium at $t = 0$

The short-term debt choices of the banks at time $t = 0$ anticipate the possible states at time $t = 1$ and the continuation values in those states. There are four possible states at date $t = 1$, labeled HH, HL, LH, and LL respectively. The short-term debt choices of the banks together with the exogenous parameters of the model (π) jointly determine the likelihood of the states. For example, when π is high, monotone likelihood property predicts that the probability of low cash-flows (and hence the probability of the systemic state) is small, the banks can take on more debt.

The following key result proves the existence of a Nash Equilibrium in pure strategies and characterizes its nature.

Theorem 1. *The equilibrium at $t = 0$, characterized by $({\phi_1}^*, {\phi_2}^*)$ exists and is characterized by the fixed* *point of the equations*

$$\phi_1^* = \min(\phi_c(\phi_2^*, \pi), \bar{\phi}(\phi_2^*, \pi))$$
$$\phi_2^* = \min(\phi_c(\phi_1^*, \pi), \bar{\phi}(\phi_1^*, \pi))$$

where $\bar{\phi}$ represents the upper threshold of each bank i such that

$$\frac{\partial V_i}{\partial \phi_i} \begin{cases} \geq 0 & \text{if } \phi_i \leq \bar{\phi}_i \\ < 0 & \text{otherwise} \end{cases}$$

In the competitive equilibrium, banks take on too much short-term debt because they don't internalize the contagion costs borne by other banks in the systemic states of nature and the dead-weight costs to the society in case of a premature liquidation. If the good state is more likely, the Nash Equilibrium debt choices of both banks tend to be high. This however also increases the likelihood of both banks not being refinanced by the creditors. Further, with both banks taking large short-term positions, the probability of one of the banks failing because of pure liquidity shock also increases. This is the key source of inefficiency in the decentralized equilibrium. In the next section, we study the central planner's problem and show that the first-best equilibrium corresponds to a lower level of short-term debt.

4 The First Best

The central planner's problem is to maximize the overall utility of the bank-owners and depositors net of distress costs. Thus, unlike individual banks, the planner cares about the dead-weight costs associated with intermediate bank failures. To introduce the notion of this loss meaningfully and in a tractable fashion, we assume an increasing and convex cost function $c(\cdot)$ which depends on the debt-choices of individual banks[6]. The probability of such financial distress occurring is given by

$$\eta(\phi_1, \phi_2) = \pi \iint_{u+v \leq \phi_1 + \phi_2 - 1} f_G(u,v) dv du + (1-\pi) \iint_{u+v \leq \phi_1 + \phi_2 - 1} f_B(u,v) dv du$$

The planner chooses the vector (ϕ_1, ϕ_2) to maximize the joint value of the banks. Thus the time-0 optimization problem can be written as follows:

$$\max_{\phi_1, \phi_2} \sum_{i \in \{1,2\}} V_i - c(\phi_1, \phi_2) \eta(\phi_1, \phi_2) \tag{11}$$

with the associated FOC for all $j \in \{1, 2\}$:

[6] The results would not change if we replace the cost function by the project value R. The key externality is captured by the distress probability η.

$$0 = \sum_i \frac{\partial V_i}{\partial \phi_j} - \eta \frac{\partial c}{\partial \phi_j} - c \frac{\partial \eta}{\partial \phi_j} \tag{12}$$

The externality of high short-term debt enters into this equation explicitly through two channels - higher probability of bank failure and larger dead-weight losses. The first effect can be seen from the fact

$$\frac{\partial \eta}{\partial \phi_i} > 0; \quad \frac{\partial^2 \eta}{\partial \phi_i \partial \phi_j} > 0$$

and the second is also evident from the increasing property of the losses :

$$\frac{\partial c}{\partial \phi_i} > 0.$$

An interior solution to the above optimization problem is guaranteed if cost function has finite derivatives with respect to the short-term debt $\phi_i \quad \forall i \in \{1, 2\}$. It is trivial to see that the first derivatives with respect to η are finite. Also observe that $c(\phi_1, \phi_2) = 0$ if $\phi_1 + \phi_2 \leq 1$. These restrictions coupled with the proof outlined in proposition **??** guarantees the existence of socially optimal allocations.

The presence of the deadweight cost term in equation (11) implies that the privately optimal solution $({\phi_1}^*, {\phi_2}^*)$ will not coincide with the socially optimal allocation $({\phi_1}^o, {\phi_2}^o)$ over any positive measure set of (ϕ_1, ϕ_2). To see this directly, contrast the social first order condition with the individual $\frac{\partial V_i}{\partial \phi_i} = 0$. Given the nature of the private equilibrium as in proposition **??**, we have

$$\sum_i \frac{\partial V_i}{\partial \phi_j} = 0$$

However, the expected dead-weight cost is strictly positive since $\eta c'(\cdot) + c\eta'(\cdot) > 0$ as explained earlier.

The presence of the negative externality through the bank's contribution to the aggregate systemic risk, captured by $\eta(\cdot)$, and the associated bankruptcy costs, modeled with $c(\cdot)$, suggests that the social optimum will entail a lower level of short-term debt. We can consider a social switching function $\phi^j_{soc}(\phi_i; \pi, c)$ as a counterpart to the one given in proposition 2. Since the negative externality is much larger in this case, it is clear that the following relationship holds true:

$$\phi^j_{soc}(\phi_i; \pi, c) < \phi^j_c(\phi_i; \pi)$$

The rest follows in an analogous fashion, giving the social equilibrium as the fixed point

$$\phi^o_1 = \phi_{soc}(\phi^o_2; \pi, c)$$
$$\phi^o_2 = \phi_{soc}(\phi^o_1; \pi, c)$$

This analysis is summarized in the following proposition which highlights the inefficiency of the private optimum.

Proposition 3. *The socially optimal equilibrium characterized by* (ϕ_1^o, ϕ_2^o) *exists and is characterized by the fixed points of the following system:*

$$\phi_1^o = \phi_{soc}(\phi_2^o; \pi, c)$$
$$\phi_2^o = \phi_{soc}(\phi_1^o; \pi, c)$$

Moreover, the private optimum is socially inefficient with higher systemic risk.

Proposition 3 discusses the origin of the wedge between the private and the social optimum. To understand this intuitively, note that when solving for the privately optimal liability structure, bank i does not take into account the negative externality it imposes on the other banks. In the systemic risk state, with probability $\frac{1}{2}$, bank j may fail because of very high debt choices for bank i - a factor which does not enter its own optimization program.

4.1 Effect of Competition

How does competition in the banking economy affect banks' choices? We will not attempt to review the extant literature here, though some leading papers are cited in the introduction. Hellmann et al. (2000) show that increasing competition lowers bank's profits, inducing excessive risk taking. Since we have assumed a single risky asset and that banks solely invest in them, competition in our model affects banks' profits through the costs involved. Thus increased competition in the banking industry increases the long-term interest rate r_D offered by the banks to the depositors.

To see the effect on bank's profits, differentiate the value function $V_i(\phi_i^*, \phi_j^*; r_D)$ with respect to r_D. The use of envelope theorem removes the indirect terms involving $\frac{\partial V_i}{\partial \phi_i}\frac{d\phi_i}{dr_D}$ leaving only the direct effect

$$\frac{dV_i}{dr_D} = \frac{\partial V_i}{\partial r_D} = -(1 - \phi_i).$$

It implies that in a sufficiently competitive economy, banks prefer to take higher fraction of short-term debt and the wedge between the socially efficient and privately efficient choices increases.

5 Regulatory Interventions

5.1 Sub-optimality of Ex-post Bank Bailouts

A central bank (or regulator, used interchangeably) forbearance mechanism takes the form of a bailout at the intermediate date where the regulator covers the liquidity short-fall faced by the banks and allows the banks to continue. If the systemic costs of bank failures (loss of charter value, likelihood of contagion impairing the financial system in general) is greater than the opportunity cost, it is always ex-post optimal to save the banks at the intermediate date. We are more interested in analyzing the effects of this bailout guarantee on the ex-ante debt choices of the banks. We assume that the central bank bails out the troubled bank with probability 1 but takes an equity state of $(1-\tau)$ in the bank by diluting other shareholders who now retain τ[7].

Further, to simplify exposition, we assume that when the market liquidity is insufficient to meet the total demand

$$(\phi_1 - y_1) + (\phi_2 - y_2) > 1$$

the central bank takes over and provides the liquidity instead. This enables us to ignore the issues related to the bank's preference for one form of liquidity over the other. Also, we assume that $\tau'(\phi_i) < 0$. This simply states that the central bank's acquired equity increases with the amount of liquidity assistance provided $(\phi_i - y_i)$.

We rewrite the maximization problem of the bank as follows:

$$\begin{aligned}
V_i(\phi_i, \phi_j) = \max_{\phi_i} & \int_{\phi_i}^{\mathrm{y_{max}}} \int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}} [R + u - \phi_i - r_D(1-\phi_i)] f_s(u,v)\,dvdu + \\
& + \int_{y_c^i}^{\phi_i} \int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}} [R - (\phi_i - u) r_2(u) - r_D(1-\phi_i)] f_s(u,v)\,dvdu \\
& + \int_{\underline{\mathrm{y}}_2}^{y_c^i} \int_{y_c^j}^{\mathrm{y_{max}}} [R(1 - \frac{\phi_i - u}{Q_{id}}) - r_D(1-\phi_i)] f_s(u,v)\,dvdu \\
& + \int_{\underline{\mathrm{y}}_3}^{y_c^i} \int_{\phi_i+\phi_j-1-u}^{y_c^j} [R(1 - \frac{\phi_i - u}{Q_{sys}}) - r_D(1-\phi_i)] f_s(u,v)\,dvdu \\
& + \int_{\mathrm{y_{min}}}^{y_c^i} \int_{\mathrm{y_{min}}}^{\phi_i+\phi_j-1-u} \tau(\phi_i)[R - r_D(1-\phi_i)] f_s(u,v)\,dvdu
\end{aligned} \tag{13}$$

The exact nature of the solution depends on the expected terms and conditions of the bailout program, but at the very least it can be assumed to be better than the market rate offered. Recent anecdotal evidence (TARP, GM bailout institutionalized by the US government) supports this premise. A bailout policy is said to be more forbearing if the fraction of assets surrendered to the central bank for a specific liquidity need is smaller compared to that sold to the external investor. The following proposition underscores the sub-optimality of such actions

Proposition 4. *A bailout policy which shows greater forbearance compared to the market induces more correlated and higher short-term debt compared to the private optimum, i.e.* $\phi_i(\tau) \geq \phi_i^*$.

[7] We could also have considered mixed strategy games where the central bank employs a *constructive ambiguity* policy. Here a troubled bank is bailed out with probability $\tau \in (0,1)$. But the underlying time-inconsistency of such policies is well understood in the literature(see Acharya and Yorulmazer (2007) and Chari and Kehoe (2013)).

Intuitively, if the terms offered by the regulator are better than the uninformed investor, the banks would prefer to correlate on debt choices so that the probability of getting *nicer deals* increases. Importantly, this proposition highlights the adverse feedback effects of such government guarantees on systemic risk. This time inconsistency of implicit or explicit government guarantees reverberates in the following testimony given by Richmond Fed President Jeffrey Lacker at the House Judiciary Committee's hearing [8] :

[Second], policymakers may well worry that if a large financial firm with a high reliance on short-term funding were to file for bankruptcy under the U.S. bankruptcy code, it would result in undesirable effects on counterparties, financial markets, and economic activity. This expectation induces policymakers to intervene in ways that allow short-term creditors to escape losses, such as through central bank lending or public sector capital injections. This reinforces creditors' expectations of support and firms' incentives to grow large and rely on short-term funding, resulting in more financial fragility and more rescues.

6 Conclusion

This paper develops a theory of systemic risk with multiple banks, depositors and an external investor where systemic risk arises from banks making correlated short-term debt choices. When the expectation of future good state is high, banks tend to increase the asset-liability mismatch to profit from the *moneyness* of the short-term liquid contracts. This in turn increases the likelihood of joint failures and systemic risk ensues. Our model highlights the perils of symmetric choices by banks where a negative shock can lead to the collapse of the entire banking system.

The second important contribution of our model is to highlight the time-inconsistency nature of ex-post regulatory policies like bailouts or LOLR facilities. When the banks anticipate future forbearance, systemic moral hazard incentivizes them to take on more debt ex-ante. We point out that an optimal regulation schedule should aim to break the symmetric choices by banks as risk-sharing is minimal under such circumstances. Designing of such regulatory policies remain an important extension of this paper.

[8] http://judiciary.house.gov/_files/hearings/113th/12032013_2/Lacker%20Testimony.pdf

A Appendix

A.1. Omitted Proofs

Proof of Lemma 2

Proof. Consider the general case where the cash-flows have the same support $[y_{min}, y_{max}]$ and are generated by the distributions F_G and F_B respectively with the subscripts denoting the underlying state. Denote by $\alpha(y)$, the posterior that good state prevails given the observed interim cash flow $c \in (y, y+dy)$. Thus,

$$\begin{aligned}\alpha(y) = \mathbb{P}(s = G | c \in (y, y + dy)) &= \frac{\pi[F_G(y + dy) - F_G(y)]}{\pi[F_G(y + dy) - F_G(y)] + (1 - \pi)[F_B(y + dy) - F_B(y)]} \\ &= \frac{\pi f_G(y)}{\pi f_G(y) + (1 - \pi) f_B(y)}.\end{aligned} \tag{14}$$

where π denotes the prior of a good state occurring.

Differentiating with respect to y,

$$\alpha'(y) = \pi(1 - \pi)\frac{f'_G(y)f_B(y) - f_G(y)f'_B(y)}{[\pi f_G(y) + (1 - \pi)f_B(y)]^2}$$

From monotone likelihood property of assumption (1), we have for all y in the support,

$$\frac{f_G(y + dy)}{f_B(y + dy)} \geq \frac{f_G(y)}{f_B(y)}$$

Using linear Taylor expansions, we obtain

$$f'_G(y)f_B(y) - f'_B(y)f_G(y) \geq 0$$

implying $\alpha'(y) \geq 0$ for all y. □

Proposition 1

Proof. Consider the decision problem of the short-term creditors who observe an interim cash-flow y. Here for the simplicity of exposition, assume that the cash-flow is perfectly observed by the creditors[9] of the bank, but not by the outsiders. They form posteriors $\alpha(y)$ according to lemma (**??**) and require a second period interest rate of $r_2(y)$ in order to roll over. The individual rationality constraint can be written as

[9] The case when the signal is noisy can be tackled in a global-games setup. Given the Laplacian belief formation by the creditors, the general case also yields a switching threshold (albeit a different one) at which the creditors are indifferent between running and rolling over their debt.

$$\alpha(y)r_2(y) + (1-\alpha(y))pr_2(y) + (1-\alpha(y))(1-p)\beta y \geq 1 \tag{15}$$

Thus,

$$r_2(y) = \frac{1-(1-\alpha(y))(1-p)\beta y}{\alpha(y)+(1-\alpha(y))p}$$

subject to the restriction $r_2(y) \leq \frac{(1-\psi)R-(1-\phi)r_D}{\phi}$.

Under very general conditions, the following assumption holds, given the monotone property of $\alpha(y)$

$$\lim_{y \uparrow y_{max}} \alpha(y) = 1-\epsilon; \qquad \lim_{y \downarrow y_{min}} \alpha(y) = \epsilon \qquad \text{where } \epsilon \text{ small}$$

Thus,

$$\lim_{y \to y_{max}} r_2(y) = \frac{1-\epsilon(1-p)\,\mathrm{y_{max}}}{1-\epsilon+p\epsilon} \to 1 < \frac{(1-\psi)R-(1-\phi)r_D}{\phi}$$

On the other hand,

$$\lim_{y \to y_{min}} r_2(y) = \frac{1-(1-\epsilon)(1-p)\,\mathrm{y_{min}}}{\epsilon+(1-\epsilon)p} \to \frac{1-(1-p)\,\mathrm{y_{min}}}{p} \sim \frac{1}{p}$$

Clearly, $\frac{1}{p}$ is the maximum period 2 interest rate that the short-term creditors can demand from the bank. Contrast this with the maximum period-2 interest rate that the bank is willing to pay; i.e., $\frac{(1-\psi)R-(1-\phi)r_D}{\phi}$. When ϕ is very small, the short-term creditors have no incentive to run on the bank irrespective of the actual realization. To see why, note that the bank's profit from the long term creditors (conditional on project success) is $((1-\psi)R-r_D)(1-\phi)$. If ϕ is small, this profit can be used to pay a potentially large interest rate to the creditors.

It implies that there exists a lower bound $\underline{\phi}$ such that there is no run for a lower fraction of short-term debt. This cutoff value meets the condition

$$(1-\psi)R = (1-\underline{\phi})r_D + \underline{\phi}\frac{1}{p}$$

For all $\phi > \underline{\phi}$, there exists a critical threshold cash-flow[10] below which the creditors run on the bank. It is given by y_c where

$$\frac{1}{\alpha(y_c)+(1-\alpha(y_c))p} = r_2(y_c) = \frac{(1-\psi)R-(1-\phi)r_D}{\phi}$$

□

[10] The monotonicity condition can be seen by differentiating r_2 with respect to y and using the fact $\alpha'(y) \geq 0$.

$$\frac{dr_2(y)}{dy} = -\frac{(1-p)\alpha'(y)}{(\alpha(y)+(1-\alpha(y))p)^2} \leq 0$$

Lemma 3

Proof. Let $\alpha_{id}(\phi_1, \phi_2) = \mathbb{P}(s = G|\text{One bank needs liquidity})$ denote the posterior of the uninformed investors given that only one bank needs to raise liquidity from the market. To fix ideas, let it be the state LH (by symmetry, the case HL will be identical). It can be written as

$$\begin{aligned}\alpha_{id}(\phi_1, \phi_2) &= \mathbb{P}(s = G|LH) \\ &= \frac{\pi \int_{y_{min}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{y_{max}} dF_G}{\pi \int_{y_{min}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{y_{max}} dF_G + (1-\pi) \int_{y_{min}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{y_{max}} dF_B}\end{aligned}$$

Differentiating with respect to ϕ_1,

$$\begin{aligned}\frac{\partial \alpha_{id}}{\partial \phi_1} = \frac{\pi(1-\pi)}{p_3^2} \Bigg[&\int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_G(\mathrm{y_c^1}, v) dv \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_B(u, v) dv du \\ &- \int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_B(\mathrm{y_c^1}, v) dv \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_G(u, v) dv du \Bigg] \frac{\partial \mathrm{y_c^1}}{\partial \phi_1}\end{aligned}$$

Thus $\dfrac{\partial \alpha_{id}}{\partial \phi_1} \geq 0$ if and only if

$$\frac{\int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_G(\mathrm{y_c^1}, v) dv}{\int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_B(\mathrm{y_c^1}, v) dv} \geq \frac{\int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_G(u, v) dv du}{\int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_B(u, v) dv du}$$

By the monotone likelihood property of the joint distributions, this is always true.

Differentiating with respect to ϕ_2 yields,

$$\begin{aligned}\frac{\partial \alpha_{id}}{\partial \phi_2} = -\frac{\pi(1-\pi)}{p_3^2} \Bigg[&\int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} f_G(u, \mathrm{y_c^2}) du \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_B(u, v) dv du \\ &- \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} f_B(u, \mathrm{y_c^2}) du \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_c^2}}^{\mathrm{y_{max}}} f_G(u, v) dv du \Bigg] \frac{\partial \mathrm{y_c^2}}{\partial \phi_2}\end{aligned}$$

Since the term inside the brackets is ≤ 0,it implies $\dfrac{\partial \alpha_{id}}{\partial \phi_2} \geq 0$.

Now consider the *systemic* state LL. Let $\alpha_{sys}(\phi_1, \phi_2)$ denote the probability that the state is good when both the banks are not refinanced. It is given by

$$\alpha_{sys}(\phi_1, \phi_2) = \frac{\pi \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_{min}}}^{\mathrm{y_c^2}} f_G(u, v) dv du}{\pi \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_{min}}}^{\mathrm{y_c^2}} f_G(u, v) dv du + (1-\pi) \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} \int_{\mathrm{y_{min}}}^{\mathrm{y_c^2}} f_B(u, v) dv du}$$

Differentiating with respect to ϕ_1, we obtain

$$
\begin{aligned}
\frac{\partial \alpha_{sys}}{\partial \phi_1} = \frac{\pi(1-\pi)}{p_4^2}\Big[&\int_{\mathrm{y_{min}}}^{\mathrm{y_c^2}} f_G(\mathrm{y_c^1}, v)dv \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}}\int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} f_B(u,v)dvdu - \\
&\int_{\mathrm{y_{min}}}^{\mathrm{y_c^2}} f_B(\mathrm{y_c^1}, v)dv \int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}}\int_{\mathrm{y_{min}}}^{\mathrm{y_c^1}} f_G(u,v)dvdu\Big]\frac{\partial\, \mathrm{y_c^1}}{\partial\, \phi_1} \geq 0
\end{aligned}
$$

Similarly the same logic also implies $\frac{\partial \alpha_{sys}}{\partial\, \phi_2} \geq 0$ □

Proposition 2

Proof. Consider the effect of the competitor bank's debt choice ϕ_j on the value function of the bank i. It is clear that ϕ_j does not affect the high state continuation value v_{is}^1. Next consider the value v_{is}^2. To simplify the exposition, we rewrite it as follows (using Fubini's Theorem to change the order of integration if necessary):

$$
\begin{aligned}
v_{is}^2 &= \int_{y_c^j}^{\mathrm{y_{max}}}\int_{\underline{\mathrm{y}}_2}^{y_c^i} [R(1 - \frac{\phi_i - u}{Q_{id}(y_c^i, y_c^j)}) - (1-\phi_i)r_D] f_s(u,v)dudv \\
&= \int_{y_c^j}^{\mathrm{y_{max}}} \Gamma(Q_{id}, \phi_i) f_s^j(v)dv
\end{aligned}
$$

Differentiating with respect to ϕ_j, we obtain

$$
\frac{\partial v_{is}^2}{\partial \phi_j} = \int_{y_c^j}^{\mathrm{y_{max}}} \frac{\partial \Gamma}{\partial Q_{id}} \frac{\partial Q_{id}}{\partial y_c^j}\frac{dy_c^j}{d\phi_j} f_s^j(v)dv - \Gamma(Q_{id}, \phi_i)\mid_{y_c^j} \frac{dy_c^j}{d\phi_j}
$$

The first term is positive and captures the *positive externality* through higher prices. On the other hand, the second term is captures the lower probability of the idiosyncratic risk state when ϕ_j increases. When the first term dominates, we have an overall positive externality appearing through higher liquidation prices.

Similarly consider the value function v_{is}^3 in the systemic risk state. We can rewrite it as follows:

$$
\begin{aligned}
v_{is}^3 &= \int_{\underline{\mathrm{y}}_3}^{y_c^j}\int_{\phi_i+\phi_j-1-v}^{y_c^i} [R(1 - \frac{\phi_i - u}{Q_{sys}(y_c^i, y_c^j)}) - (1-\phi_i)r_D] f_s(u,v)dudv \\
&+ \frac{1}{2}\int_{\underline{\mathrm{y}}_3}^{y_c^j}\int_{\underline{\mathrm{y}}_3}^{\phi_i+\phi_j-1-v} [R(1 - \frac{\phi_i - u}{Q_{sys}(y_c^i, y_c^j)}) - (1-\phi_i)r_D] f_s(u,v)dudv \\
&= \int_{\underline{\mathrm{y}}_3}^{y_c^j} \Lambda_1(\phi_i, \phi_j, v) f_s^j(v)dv + \int_{\underline{\mathrm{y}}_3}^{y_c^j} \Lambda_2(\phi_i, \phi_j, v) f_s^j(v)dv
\end{aligned}
$$

where

$$\Lambda_1 = \int_{\phi_i+\phi_j-1-v}^{y_c^i} [R(1 - \frac{\phi_i - u}{Q_{sys}(y_c^i, y_c^j)}) - (1-\phi_i) r_D] f_s(u|v) du$$
$$\Lambda_2 = \frac{1}{2}\int_{\underline{y}_3}^{\phi_i+\phi_j-1-v} [R(1 - \frac{\phi_i - u}{Q_{sys}(y_c^i, y_c^j)}) - (1-\phi_i) r_D] f_s(u|v) du$$

Differentiating with respect to ϕ_j,

$$\frac{\partial v_{is}^3}{\partial \phi_j} = \int_{\underline{y}_3}^{y_c^j} \frac{\partial \Lambda_1}{\partial Q_{sys}} \frac{\partial Q_{sys}}{\partial y_c^j} \frac{dy_c^j}{d\phi_j} f_s^j(v) dv + \Lambda_1(y_c^i, y_c^j) \frac{dy_c^j}{d\phi_j} - \Lambda_1(\underline{y}_3, y_c^j) \frac{d\,\underline{y}_3}{d\phi_j}$$
$$+ \int_{\underline{y}_3}^{y_c^j} \frac{\partial \Lambda_2}{\partial Q_{sys}} \frac{\partial Q_{sys}}{\partial y_c^j} \frac{dy_c^j}{d\phi_j} f_s^j(v) dv + \Lambda_2(y_c^i, y_c^j) \frac{dy_c^j}{d\phi_j} - \Lambda_2(\underline{y}_3, y_c^j) \frac{d\,\underline{y}_3}{d\phi_j}$$

Two important observations are in order. First, the similar positive and negative externalities are also apparent in this case. However, the effects are much starker in the systemic risk state. For example, as ϕ_j increases, the measure of the set $[\phi_i + \phi_j - 1 - v, y_c^i]$ goes down. Also, the total probability that bank i fails at the interim date increases with ϕ_j in the systemic risk state. These two effects together make the negative impact more substantial in this case as compared to the idiosyncratic state.

Thus the total effect of ϕ_j on the value-function $V_i(\phi_i, \phi_j)$ is

$$\frac{\partial V_i}{\partial \phi_j} = \pi \left[\frac{\partial v_{iG}^2}{\partial \phi_j} + \frac{\partial v_{iG}^3}{\partial \phi_j}\right] + (1-\pi) p \left[\frac{\partial v_{iB}^2}{\partial \phi_j} + \frac{\partial v_{iB}^3}{\partial \phi_j}\right]$$

The dominant effect depends on (ϕ_i, ϕ_j) for every prior π. Thus there exists a threshold point $\phi_c^j(\phi_i, \pi)$ such that

$$\frac{\partial V_i(\phi_i, \phi_j)}{\partial \phi_j} = \begin{cases} \geq 0 & \text{if } \phi_j \leq \phi_c^j(\phi_i, \pi) \\ < 0 & \text{if } \phi_j > \phi_c^j(\phi_i, \pi) \end{cases}$$

□

Proof of Theorem 1

Proof. The generic state-s version of the first order condition is given in (10). The dependence of the best-response of bank i on the short-term debt choice of bank j is captured indirectly through the prices and the threshold values in terms 3 through 5.

We provide an outline of the proof of existence:

Step 1: There is no equilibrium with $\phi_i^* \in (0, \underline{\phi})$. If any such equilibrium exists, it is similar in nature to that with only long-term debt (as creditors always refinance the bank at date 1). However, the interest rate $r_2(y)$ is large compared to r_D. Thus, the bank always profits by reducing the fraction of short-term debt by ϵ with the corresponding increase in value being $(r_2 - r_D)\epsilon > 0$. Thus in equilibrium, banks with always prefer $\phi = 0$ over any $\phi \in (0, \underline{\phi})$.

Step 2: Consider the auxiliary function

$$W_i(\phi_i) = V_i(\phi_i, \phi_j; \pi)$$

where we fix the decision variable ϕ_j of bank j. Start with a low value of ϕ_i such that the systemic risk given (ϕ_i, ϕ_j) is small. Using the logic outlined in proposition 2, we can define a critical threshold $\bar{\phi}_i$ such that

$$\frac{dW_i}{d\phi_i} \begin{cases} \geq 0 & \text{if } \phi_i \leq \bar{\phi}_i \\ < 0 & \text{otherwise} \end{cases}$$

This step is important in two aspects. First, it proves the quasi-concavity of the value function - an essential requirement for the equilibrium to exist. Second, it implies that $\bar{\phi}_i$ acts as the upper bound of the best response function $\hat{\phi}_i(\phi_j)$.

Step 3: The strategy space for bank i is given by $\Sigma_i = [\underline{\phi}, \bar{\phi}_i]$. The strategy space for the game is simply $\Sigma = \prod_{i \in \{1,2\}} \Sigma_i$. Since Σ_i is closed and bounded, it is compact. The product Σ is also compact.

The best response correspondence is given by

$$\hat{\phi}_i(\phi_j) = \arg\max_{\phi_i \in \Sigma_i} V_i(\phi_i, \phi_j)$$

where Σ_i is compact and convex and V_i is continuous in ϕ_i. Thus by Weierstrass' theorem, $\hat{\phi}_i(\phi_j)$ is non-empty. Finally, it is easy to see that V_i is continuous in ϕ_j.

Step 4: We show that V_i is quasi-concave in ϕ_i. The first derivative of the value function is given by equation 10. Differentiating again with respect to ϕ_i we obtain

$$\begin{aligned}
\frac{\partial^2 V_i}{\partial \phi_i^2} &= -[r_D - r_2(y_c^i)] f_s^i(y_c^i) \frac{dy_c^i}{d\phi_i} \\
&\quad - \int_{\underline{\mathrm{y}}_2}^{y_c^i} \left[\frac{2R}{Q_{id}^2} \frac{\partial Q_{id}}{\partial \phi_i} \left(\frac{\phi_i - u}{Q_{id}} \frac{\partial Q_{id}}{\partial \phi_i} - 1 \right) \right] \int_{y_c^j}^{\mathrm{y}_{\max}} f_s(u,v) dv du \\
&\quad - \int_{\underline{\mathrm{y}}_3}^{y_c^i} \left[\frac{2R}{Q_{sys}^2} \frac{\partial Q_{sys}}{\partial \phi_i} \left(\frac{\phi_i - u}{Q_{sys}} \frac{\partial Q_{sys}}{\partial \phi_i} - 1 \right) \right] g(\cdot) f_s^i(u) du \\
&\quad - \frac{1}{2} \int_{\underline{\mathrm{y}}_3}^{y_c^i} \left[\frac{2R}{Q_{sys}^2} \frac{\partial Q_{sys}}{\partial \phi_i} \left(\frac{\phi_i - u}{Q_{sys}} \frac{\partial Q_{sys}}{\partial \phi_i} - 1 \right) \right] h(\cdot) f_s^i(u) du \\
&\quad + \int_{\underline{\mathrm{y}}_3}^{y_c^i} [r_D - \frac{R}{Q_{sys}} + \frac{R(\phi_i - u)}{Q_{sys}^2} \frac{\partial Q_{sys}}{\partial y_c^i} \frac{dy_c^i}{d\phi_i}] g'(\phi_i) f_s^i(u) du \\
&\quad + \frac{1}{2} \int_{\underline{\mathrm{y}}_3}^{y_c^i} [r_D - \frac{R}{Q_{sys}} + \frac{R(\phi_i - u)}{Q_{sys}^2} \frac{\partial Q_{sys}}{\partial y_c^i} \frac{dy_c^i}{d\phi_i}] h'(\phi_i) f_s^i(u) du \quad \leq 0
\end{aligned}$$

where we use $g_{\phi_i} < 0, h_{\phi_i} > 0$.

Another easier and direct approach is shown in step 2. This analysis supplants the earlier step by proving a stronger concavity result.

Step 5: All the previous steps show that the requirements for the Debreu-Glicksberg-Fan theorem hold true in our case. Thus by the application of the theorem, an equilibrium exists in pure strategies.

To find the equilibrium, we use proposition 2. Let the debt choice of bank j be ϕ_j. The externality on bank i is positive if and only if

$$\phi_j \leq \phi_c^j(\phi_i, \pi).$$

When this condition is true, bank i is incentivized to increase ϕ_i. This iteration stops when the sign of the externality reverses. The reversal happens when either of the two following bounds are hit. The first is the upper threshold $\bar{\phi}_i$ and the second is the critical threshold ϕ_c.

The same logic applies for the other bank as well. Thus, the Nash equilibrium in the two bank case has the following form

$$\phi_1^* = \min(\phi_c(\phi_2^*, \pi), \bar{\phi}(\phi_2^*, \pi))$$
$$\phi_2^* = \min(\phi_c(\phi_1^*, \pi), \bar{\phi}(\phi_1^*, \pi))$$

The symmetric equilibrium, if it exists, is given by the solution of the following fixed point problem

$$\phi^* = \min(\phi_c(\phi^*; \pi), \bar{\phi}(\phi^*; \pi))$$

The other equilibria will be asymmetric in nature. □

Proposition 4

Proof. Since the first four terms remain unchanged (refer Equation 9), we focus on the final term where the central bank plays a role. Differentiating with respect to ϕ_i, we obtain

$$\frac{\partial T_5}{\partial \phi_i} = \int_{\mathrm{y_{min}}}^{y_c^i} [\tau(\phi_i - u)r_D + \tau'(\phi_i - u)(R - (1-\phi_i)r_D)]h_2(\cdot) + \tau(\phi_i)[R - r_D(1-\phi_i)]\frac{\partial h_2}{\partial \phi_i}$$

where

$$h_2(\phi_i, \phi_j, u) = \int_{\mathrm{y_{min}}}^{\phi_i + \phi_j - 1 - u} f_s(u, v)dv \implies \frac{\partial h_2}{\partial \phi_i} > 0; \quad \frac{\partial h_2}{\partial \phi_j} > 0$$

Contrast this with the last term of the first order condition (10) reproduced here for convenience.

$$\frac{1}{2}\int_{\underline{\mathrm{y}}_3(\phi_i)}^{y_c^i} \left((r_D - \frac{R}{Q_{sys}} + \frac{R(\phi_i - u)}{Q_{sys}^2}\frac{\partial Q_{sys}}{\partial y_c^i}\frac{dy_c^i}{d\phi_i})h(\cdot) + (R(1-\Delta_i) - r_D(1-\phi_i))\frac{\partial h}{\partial \phi_i} \right) f_s^i(u)du$$

If the first term in T_5 multiplying $h_2(\cdot)$ is positive[11] then it is easy to see that $\frac{\partial T_5}{\partial \phi_i} > 0$ and it is increasing in ϕ_j. It follows from the arguments laid out in propositions 2 and **??**, that the debt choices of the banks will become more correlated in this case.

This result holds more generally. A bailout term is considered lenient compared to the market if the fraction of assets surrendered to the regulator is less compared to the fraction of assets sold to the uninformed investor for the same amount of liquidity. It follows from the comparison of terms T_5 in equation 13 with the corresponding term in 9 that banks prefer higher short-term debt with bailout. □

A.2. Optimal Financial Contracting

Although we take the debt contracts as primitives in the model, this section provides some motivation for considering such in the context of optimal contract choices at the initial date. As specified in section 2 of the main text, the bank needs to raise the initial investment cost from the households at date 0 in order to fund the risky technology. The project yields random cash-flows at dates 1 and 2.

In the essence of costly state verification (CSV) models, we will assume that the contract cannot be made contingent upon the underlying cash-flow unless it is verifiable. Verification for outsiders is costly and costs $c_t(y_t)$ where $c_t(\cdot)$ is a smooth and non-decreasing function.

[11] The sufficient condition to ensure this is

$$\frac{\tau'}{\tau} \geq -\frac{r_D}{R}.$$

At date $t = 0$, the bank manager designs a contract to maximize his expected profit subject to his incentive compatibility constraint, the individual rationality or the participation constraint of the investor and the limited liability constraints which imply that the contract transfers cannot exceed the total available cash in any state. We continue to assume that the capital market is competitive and the household earns the competitive rate (breaks even in our case, as the risk-free rate is assumed to be 0).

Before proceeding to derive the optimal contracts, we address the asset substitution moral hazard problem. As outlined in the main text, the bank manager has the opportunity to switch to a personal benefit project at date $t = 1$. This issue is most pertinent when we are dealing with long-term contracts and restricts the maximum pledgeable income to the investors. Denote by date-2 payout to the external creditors by F.

The incentive compatibility constraint is

$$\big[\pi + (1-\pi)p\big](R - F) \geq \big[\pi(1-\Delta) + (1-\pi)(p-\Delta)\big](R - F) + B$$

This implies

$$F \leq R - \frac{B}{\Delta} = (1-\psi)R, \text{ where } \psi = \frac{B}{\Delta R}$$

providing an expression for ψ as in assumption 2. We will only focus on the pledgeable part of the second period income while deriving the features of the optimal contract. We will also make the following assumptions

Assumption 5. *The bank manager cannot pay himself a dividend until the contractual obligations are met and will not be able to borrow again from the market.*

Optimal Long Term Contracts: Here we focus on optimal long term contracts which entail a single payout to the external agent at date $t = 2$. What we have in mind is essentially a comparison between equity and two period zero coupon debt. Although the contracts cannot be contingent on the underlying cash-flows y_t unless the investor verifies, they can depend on the manager's report $\tilde{y}_t$. Let $\nu_t(\tilde{y}_t)$ be the outside investor's verification schedule at date t depending on the level of the reported cash-flows. ν_t is the probability that verification takes place at date t. Here, we will consider simple verification schedules whence ν_t will assume the values 1 or 0. Let $f_t(\tilde{y}_t)$ denote the transfer to the investor when there is no verification and $f'_t(y_t, \tilde{y}_t)$ be the corresponding transfer when the investor audits the bank. With long-term contracts, the problem reduces to that of a single period optimal contract design studied in Gale and Hellwig (1985) and Townsend (1979).

With long term contracts, the verification (if any) happens only at date $t = 2$. Thus we can drop the time subscripts from the contractual terms with the understanding that they apply only at the time of maturity. The date 2 cash-flow can be easily observed from the outcome of the project (success yields R, while there is no payoff if the project fails). However, the intermediate state-dependent stream y_1 and can be observed with precision if and only if an audit happens.

Partition the combined cash-flow interval $\mathcal{I} \equiv [\mathrm{y_{min}}, \mathrm{y_{max}} + (1-\psi)R]$ into disjoint intervals $\mathcal{I}_0$ and $\mathcal{I}_1$ such that

$$\nu_2(y_1 + y_2 \in \mathcal{I}_0) = 0$$
$$\nu_2(y_1 + y_2 \in \mathcal{I}_1) = 1$$

Thus, audit happens if and only if the total cash-flow from the project lies in the interval $\mathcal{I}_1$. We will look for incentive compatibility conditions which induce truth telling in equilibrium enabling us to focus on direct truthful contracts (using Revelation Principle). First, limited liability restricts the second period transfer to

$$f_2(y) \leq y; \quad f_2'(y, \tilde{y}) \leq y. \tag{LL}$$

Second, the transfer in the non-verification region cannot be contingent on the cash-level. Call this fixed level $\bar{F}$. Third, the investor's participation constraint must be satisfied for the project to be undertaken

$$\begin{aligned} &\pi \int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}} \Big\{ \nu_2(y)[f'(y,y) - c_2(y)] + (1 - \nu_2(y))f(y) \Big\} f_G(y)dy \\ &\quad + (1-\pi) \int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}} \Big\{ \nu_2(y)[f'(y,y) - c_2(y)] + (1 - \nu_2(y))f(y) \Big\} f_B(y)dy \geq 1 \end{aligned} \tag{PC}$$

Finally, given true cash-flow $y \in \mathcal{I}_1$, the bank manager will not lie and report $\tilde{y} \in \mathcal{I}_0$ if and only if

$$f_2'(y, y) \leq f_2(\tilde{y}) \tag{IC}$$

Since the manager has no incentives to lie if it gets audited, these three are the only constraints we need to solve for the optimal contract. Since all the parties are risk-neutral, the optimal contract which maximizes the manager's profit is also the one which minimizes the expected verification cost. Thus the optimal contracting problem is:

$$\min_{f_2(\cdot), f_2'(\cdot,\cdot)} \pi \int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}} \nu_2(y)c_2(y)f_G(y)dy + (1-\pi) \int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}} \nu_2(y)c_2(y)f_B(y)dy \tag{16}$$

subject to (LL), (PC) and (IC). Following Gale and Hellwig (1985) and Townsend (1979), the optimal contract will have the following features :

$$f_2'(y, y) = y \quad \forall y \in \mathcal{I}_1$$
$$f_2(y) = r_D \quad \forall y \in \mathcal{I}_0$$

where r_D is the minimum value of $\bar{R}$ which solves (PC) with equality

$$\pi \int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}} \left\{ \nu_2(y)[y - c_2(y)] + (1 - \nu_2(y))\bar{R} \right\} f_G(y)dy$$
$$+ (1-\pi) \int_{\mathrm{y_{min}}}^{\mathrm{y_{max}}} \left\{ \nu_2(y)[y - c_2(y)] + (1 - \nu_2(y))\bar{R} \right\} f_B(y)dy = 1$$

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