Optimists, Pessimists, and the Stock Market: The Role of Preferences, Dynamics, and Market (In)Completeness

Nicole Branger* Patrick Konermann‡ Christian Schlag§

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Abstract

We show that a sizable equity premium is compatible with risk sharing between an optimistic and a pessimistic Epstein-Zin investor in a model featuring jumps in expected consumption growth. Our model generates a positive correlation between return volatility and trading volume as in the data. It reproduces the stylized facts of a positive link between disagreement and expected returns, volatilities, and trading volume. We analyze the impact of preferences, fundamental dynamics, and market incompleteness in detail and highlight their respective importance for our results.

Keywords: Epstein-Zin utility, long-run risk, jumps in expected consumption growth, heterogeneous beliefs, disagreement, market incompleteness

JEL: D52, D53, G12

*Finance Center Muenster, University of Muenster, Universitaetsstr. 14-16, 48143 Muenster, Germany. E-mail: nicole.branger@wiwi.uni-muenster.de.

‡Department of Finance, BI Norwegian Business School, 0442 Oslo, Norway. E-mail: patrick.konermann@bi.no.

§Goethe University and Research Center SAFE, Theodor-W.-Adorno-Platz 3, 60323 Frankfurt am Main, Germany. E-mail: schlag@finance.uni-frankfurt.de.

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I. Introduction

One prominent idea to explain the equity premium puzzle, originally put forward by Rietz (1988) and later taken up by Barro (2006, 2009), is that rare, but large negative jumps in the consumption process ('consumption disasters') make the stock market, i.e., the claim on levered aggregate consumption, very risky, so that a high premium is required to make the representative agent hold this asset in equilibrium.

However, as for example shown by Julliard and Ghosh (2012), the ability of these models to match the equity premium together with other stylized facts is limited. Moreover, in the long time series of US consumption growth analyzed by Beeler and Campbell (2012), the largest negative rate of consumption growth is -7.7% during the Great Depression. This makes the jump sizes usually assumed in the disaster literature look rather extreme, and consequently these parameter choices raise doubts expressed, e.g., by Constantinides (2008).

An alternative approach are long-run risk models with downward jumps in the expected growth rate of aggregate consumption instead of consumption disasters. For example, Benzoni et al. (2011) show that such a model is able to match the equity premium.[1]

An obvious but open question in this context is whether these explanations for the equity premium are robust to the introduction of heterogeneous investors who can trade with each other to share risks. A natural source of heterogeneity are the investors' beliefs about the probability of a consumption disaster or a jump in the expected growth rate, because they are rare by definition so that their probability is hard to estimate.

In a long-run risk model with jumps in expected consumption growth and recursive preferences we show that a substantial equity premium is indeed compatible with pronounced risk sharing between an optimist (with a low subjective disaster probability) and a pessimist (with a high subjective disaster probability).

This is not a trivial result. In models with consumption disasters and constant relative risk aversion (CRRA) preferences Dieckmann (2011), assuming log utility, finds that variations in the potential degree of risk sharing have little impact on the equity premium, while Chen et al. (2012) show that for the general CRRA case the equity premium more or less vanishes once moderate risk sharing is possible. Figure 1 presents these results graphically, it shows the equity premium as a function of the pessimistic investor's consumption share

[1]To understand why these models work, consider a simple Gordon growth model with an interest rate of 5%. Here a *permanent* change in the dividend growth rate from, e.g., 3% down to 0% would make price-dividend ratio decrease from 50 to 20, so the effect would be even more drastic than those of a -40% shock to dividends.

for the models suggested by Chen et al. (2012), Dieckmann (2011), and our model.[2]

Since risk sharing occurs through trading, it is natural to ask how the investors' ability to trade certain risks with each other impacts equilibrium. Technically speaking, this is the issue of market completeness or incompleteness. It is often argued that the availability of options puts investors into a position where they can comfortably insure against disastrous risks. Bates (2008) though shows that the market for insurance against 'large' risks suffers from substantial under-capitalization, i.e., there are not enough insurers willing to offer the necessary contracts (like deep-out-of-the-money put options) to actually make the market complete. We therefore do not restrict ourselves to the complete market, but explicitly take market incompleteness into account.

In our analysis we consider the equity premium as just one target moment for equilibrium models. We therefore look at a number of additional stylized facts, which can serve in a sense as 'over-identifying restrictions', and show that the incomplete markets version of our model is able to reproduce these as well. As documented empirically in a recent paper by Carlin et al. (2014), there is a positive relation between the amount of disagreement in the economy and expected returns, return volatility, and trading volume in the data. Furthermore, Karpoff (1987) documents a positive association between return volatility and trading volume in the data. While this positive correlation between volatility and trading volume has been rationalized in an equilibrium model by, e.g., Li (2007), we match this stylized fact *and* the empirical results in Carlin et al. (2014). A key result of our analysis is that market incompleteness is crucial when it comes to generating a positive link between disagreement and expected returns. Table 1 provides an overview of different models and their ability to match the properties of the data.

All of the central features (preferences, dynamics, incompleteness) of our model are important for our key results. With respect to the properties of the equity risk premium it is mostly the presence of jumps in expected consumption growth combined with recursive utility which makes a high equity premium compatible with substantial risk sharing between investors. In a model with CRRA preferences and consumption disasters the main reason for the tension between the equity premium and risk sharing is the fact that jump risk represents a large share of total equity market risk and can be easily traded by investors.

With regard to the positive link between return volatility and trading volume, recursive preferences are the main feature responsible for the ability of the model to match the data. The key reason is that with disagreement about jump intensities and recursive preferences

[2]Risk sharing is maximal when both investors are of roughly the same size ($w \approx 0.5$) and obviously zero in a one-investor economy ($w = 0$ or $w = 1$).

the return volatility is highest when the optimist and the pessimist are of roughly the same size, while for CRRA the opposite is true. Trading volume behaves almost the same in the two models.

When we vary the amount of disagreement between the two equally large investors, market incompleteness is necessary to match the reaction of expected return, return volatility, and trading volume. On the incomplete market the decrease in the expected excess return for increasing disagreement is overcompensated by the increase in the risk-free rate, leading to an expected return, which is increasing in disagreement.

To get a feeling for the impact of market incompleteness, we look at the equity premium and the risk-free rate as the two key asset pricing quantities in four models from the literature. Our general finding with respect to the risk-free rate is that there is hardly a difference between the complete and the incomplete market case. With respect to the equity premium, however, we observe that the risk structure of the economy (in the sense of the relative importance of jump and diffusive risks) is the key determinant for potential differences between complete and incomplete markets. More precisely, when there is a dominant source of risk (like consumption jump risk in the model by Chen et al. (2012)) then incompleteness hardly matters as long as this risk source remains tradable via the consumption claim which loads heavily on jump risk. On the other hand, when the different types of risk are of roughly equal importance, as it is the case in our model with jumps in the expected growth rate of consumption, incompleteness matters more.

Finally, in a model with heterogeneous investors, long-run investor survival is an issue.[3] With CRRA preferences, the simultaneous survival of both investors is a knife-edge case, and, with identical preferences, it is always the investor with the less biased beliefs who survives in the long run. For the case of EZ preferences Borovicka (2015) shows that this is not necessarily true. Instead, there can be many parameter combinations for which both investor types or even the investor with the worse bias survives.

A Monte Carlo simulation shows that for our parametrization, it is the pessimistic investor with (by assumption) correct beliefs, who vanishes in the long run. The pessimist is right, but she is right only concerning very rare events, occurring on average once every 50 years. In the other periods, the pessimist has to pay the insurance premium to the optimist. Furthermore, anticipating relatively more disasters than the optimist in the future, she consumes more and saves less than the optimist. So being right does not pay off for her over the long term.

Whether the market is complete or incomplete does not change the overall results on

[3]See, e.g., Dumas et al. (2009), Yan (2008), and Kogan et al. (2006, 2009).

survival, but has a significant impact on the speed of extinction. Starting from a share of 50%, the pessimist's consumption decreases to about 35% of total output after 50 years on a complete and to 25% on an incomplete market. This is because the pessimist benefits less (in terms of her wealth) from jumps and suffers more from diffusive shocks than on the complete market.

We would like to mention that the solution method we use in this paper is innovative in a number of ways. First, we do not apply a social planner approach, but rather solve the individual investors' optimization problems, which are linked through market clearing conditions. Technically, this amounts to solving a system of coupled partial differential equations for the individual investors' wealth-consumption ratios. We then extend this approach to the incomplete market case, where the model solution has to satisfy additional partial differential equations and constraints related to the investors' individual exposures to the risk factors in the economy as well as their subjective price-consumption ratios. Compared to, e.g., the technique suggested by Collin-Dufresne et al. (2013, 2016), our approach is more flexible, since it can be applied to problems featuring incomplete market problems. Furthermore an advantage relative to the solution method developed by Dumas and Lyasoff (2012) is that our method is applicable also in the infinite horizon case and does not require the discretization of continuous-time stochastic processes.

II. Model Setup

We consider two investors with identical EZ preferences.[4] Investor i's ($i = 1, 2$) individual value function at time t is given as

$$J_{i,t} = E_{i,t}\left[\int_t^\infty f_i(C_{i,s}, J_{i,s})\, ds\right], \tag{1}$$

where $f_i(C_i, J_i)$ is her normalized aggregator function with

$$f_i\left(C_{i,t}, J_{i,t}\right) = \frac{\beta\, C_{i,t}^{1-\frac{1}{\psi}}}{\left(1-\frac{1}{\psi}\right)\,\left[\left(1-\gamma\right)\, J_{i,t}\right]^{\frac{1}{\theta}-1}} - \beta\,\theta\, J_{i,t}. \tag{2}$$

β is the subjective time preference rate, γ is the coefficient of relative risk aversion, ψ denotes the intertemporal elasticity of substitution (IES), and $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. The well-known advantage

[4]See e.g. Epstein and Zin (1989) for the discrete-time setup and Duffie and Epstein (1992) for the extension to continuous time.

of recursive utility over CRRA is that it allows to disentangle the relative risk aversion and the IES, which in the CRRA case would be linked via $\gamma \equiv \psi^{-1}$, implying $\theta = 1$. In the following, we assume $\gamma > 1$ and $\psi > 1$, which implies $\gamma > \frac{1}{\psi}$ (and thus $\theta < 0$), so that both investors exhibit a preference for early resolution of uncertainty.

Under the true probability measure $\mathbb{P}$ aggregate consumption C and the stochastic component of its expected growth rate X follow the system of stochastic differential equations

$$\begin{aligned} \frac{dC_t}{C_t} &= (\bar{\mu}_C + X_t)\, dt + \sigma_C' \, dW_t \\ dX_t &= -\kappa_X \, X_t \, dt + \sigma_X' \, dW_t + L_X \, dN_t (\lambda) \, , \end{aligned}$$

where $W = (W^C, W^X)'$ is a two-dimensional standard Brownian motion, and N represents a non-compensated Poisson process with constant intensity λ. The jump size $L_X < 0$ is also constant. With the exception of the jump component this is the classical long-run risk setup from Bansal and Yaron (2004) written in continuous time. The volatility vectors are specified as $\sigma_C' = (\sigma_c, 0)$ and $\sigma_X' = (0, \sigma_x)$, so that consumption and the long-run growth rate are locally uncorrelated. The key feature of this model is that there are jumps representing disasters, which, however, do not occur in the consumption process itself, but in the state variable X.

The investors agree on all parameters of the model except the intensity of the Poisson process, i.e., roughly speaking they disagree about the likelihood of a disaster in the growth process over the next time interval. This implies that under investor i's subjective probability measure $\mathbb{P}^i$ $(i = 1, 2)$ the stochastic growth rate evolves as

$$dX_t = -\kappa_X \, X_t \, dt + \sigma_X' \, dW_t + L_X \, dN_t (\lambda_i) \, .$$

Hence, the investors also disagree on the expectation $\frac{1}{dt}\mathbb{E}_t^i \left[dX_t\right] = -\kappa_X \, X + L_X \, \lambda_i$. Since the investors in our model do not learn about the unobservable intensity, they 'agree to disagree', i.e., they observe the same information flow, but interpret it differently.[5]

Note that the expected growth rate X is assumed to be observable, while the jump intensity λ is not. The assumption of observable X is standard in the long-run risk literature (see, e.g., Bansal and Yaron (2004) and Benzoni et al. (2011)). Nevertheless, even when X

[5]The 'agree to disagree' assumption is justified theoretically, e.g., in Acemoglu et al. (2007), who show that when investors are simultaneously uncertain about a random variable and the informativeness of an associated signal, even an infinite sequence of signals does not lead investors' heterogeneous prior beliefs about the random variable to converge. The reason is that investors have to update beliefs about two sources of uncertainty (namely the latent random variable and the informativeness of the signal regarding this variable) using one sequence of signals.

itself is observable, it seems reasonable to assume that the investors do not have perfect knowledge about λ. In fact, it would be very difficult or basically impossible for the agents to infer the unknown intensity exactly from observations on X. Especially for low values of the true intensity, as they are commonly used in the literature, the uncertainty around estimates even from long samples would still be rather large.[6]

Since it is central to our analysis whether the market is complete or not, we have to fix the set of traded assets. When the market is complete, the investors can trade the claim on aggregate consumption, the money market account, and two 'insurance products' linked to the Brownian motion and the jump component in X, respectively.[7] The consumption claim is in unit net supply, while the other three assets are all in zero net supply. When we consider the incomplete market case, the insurance products will not be available.[8]

III. Equilibrium

All the equilibrium quantities in our model will be functions of the investors' relative share in aggregate consumption as the endogenous and of the expected growth rate X as the exogenous state variable. Investors differ with respect to their assessment of jump risk in X. Let investor 1 be the pessimistic investor, and let w denote her share of aggregate consumption, i.e., $w = \frac{C_1}{C}$.[9] Its dynamics can be written as a jump-diffusion process

$$dw = \mu_w(w, X)\, dt + \sigma_w(w, X)'\, dW + L_w(w, X)\, dN(\lambda_1), \tag{3}$$

where the coefficient functions $\mu_w(w, X)$, $\sigma_w(w, X)$, and $L_w(w, X)$ will be determined in equilibrium. The dynamics of investor 1's and investor 2's level of consumption then follow

[6] In a situation with an unobservable parameter one could of course consider adding learning to the model, but due to the fact that the investor's current estimates of the jump intensity would then become two additional state variables, this would make the analysis extremely complicated. Therefore we consider this an extension, which is significantly beyond the scope of this paper.

[7] The two insurance products are characterized by their cash flows. We assume that the first insurance claim has some (given) cash flow exposure to diffusion risk in X and no exposure to jumps, while it is the other way around for the second. For details, see Appendix A.

[8] One could of course also analyze the case of intermediate incompleteness, where only one of the insurance products is available to the investors. The results we achieve in this setup typically lie between the two special cases we analyze in Section IV.

[9] In what follows we suppress the time index to simplify notation.

from Ito's lemma:

$$\begin{aligned}
\frac{dC_1}{C_1} &= \left\{\bar{\mu}_C + X + \frac{1}{w}\,\mu_w + \frac{1}{w}\,\sigma_w'\,\sigma_C\right\}\,dt \\
&\quad + \left\{\sigma_C + \frac{1}{w}\,\sigma_w\right\}'\,dW + \left\{\frac{1}{w}\,L_w\right\}\,dN\left(\lambda_1\right) \\
&\equiv \mu_{C_1}\,dt + \sigma_{C_1}'\,dW + L_{C_1}\,dN\left(\lambda_1\right) \qquad (4) \\
\frac{dC_2}{C_2} &= \left\{\bar{\mu}_C + X - \frac{1}{1-w}\,\mu_w - \frac{1}{1-w}\,\sigma_w'\,\sigma_C\right\}\,dt \\
&\quad + \left\{\sigma_C - \frac{1}{1-w}\,\sigma_w\right\}'\,dW + \left\{-\frac{1}{1-w}\,L_w\right\}\,dN\left(\lambda_2\right) \\
&\equiv \mu_{C_2}\,dt + \sigma_{C_2}'\,dW + L_{C_2}\,dN\left(\lambda_2\right). \qquad (5)
\end{aligned}$$

A key element of the solution will be the investors' individual log wealth-consumption ratios $v_i \equiv v_i(w, X)$. From Equation (1) we get

$$\mathbb{E}_{i,t}\left[dJ_i + f_i\left(C_i, J_i\right)\,dt\right] = 0. \qquad (6)$$

Motivated by Campbell et al. (2004) and Benzoni et al. (2011), we employ the following guess for the individual value function J_i:

$$J_i = \frac{C_i^{1-\gamma}}{1-\gamma}\,\beta^{\theta}\,e^{\theta\,v_i} \qquad (7)$$

where, as shown in these papers, v_i is indeed investor i's log wealth-consumption ratio. An application of Ito's lemma to $v_i \equiv v_i\left(w, X\right)$ $(i = 1, 2)$ yields

$$\begin{aligned}
dv_i &= \left\{\frac{\partial v_i}{\partial w}\,\mu_w + \frac{1}{2}\,\frac{\partial^2 v_i}{\partial w^2}\,\sigma_w'\,\sigma_w - \frac{\partial v_i}{\partial X}\,\kappa_X\,X + \frac{1}{2}\,\frac{\partial^2 v_i}{\partial X^2}\,\sigma_X'\,\sigma_X + \frac{\partial^2 v_i}{\partial w\,\partial X}\,\sigma_w'\,\sigma_X\right\}\,dt \\
&\quad + \left\{\frac{\partial v_i}{\partial w}\,\sigma_w + \frac{\partial v_i}{\partial X}\,\sigma_X\right\}'\,dW + \left\{v_i\left(w + L_w, X + L_X\right) - v_i\left(w, X\right)\right\}\,dN\left(\lambda_i\right) \\
&\equiv \mu_{v_i}\,dt + \sigma_{v_i}'\,dW + L_{v_i}\,dN\left(\lambda_i\right). \qquad (8)
\end{aligned}$$

Plugging the guess in (7) into (6) results in the following partial differential equation (PDE) for v_i:

$$\begin{aligned}
0 &= e^{-v_i} - \beta + \left(1 - \frac{1}{\psi}\right)\,\left[\mu_{C_i} - \frac{1}{2}\,\gamma\,\sigma_{C_i}'\,\sigma_{C_i}\right] + \mu_{v_i} + \frac{1}{2}\,\theta\,\sigma_{v_i}'\,\sigma_{v_i} \\
&\quad + \left(1-\gamma\right)\,\sigma_{C_i}'\,\sigma_{v_i} + \frac{1}{\theta}\,\left[\left(1 + L_{C_i}\right)^{1-\gamma}\,e^{\theta\,L_{v_i}} - 1\right]\,\lambda_i. \qquad (9)
\end{aligned}$$

Following Duffie and Skiadas (1994), the pricing kernel ξ_i of investor i at time t is given as

$$\xi_i = e^{-\beta\,\theta\,t-(1-\theta)\,\int_0^t e^{-v_{i,s}}\,ds}\,e^{(\theta-1)\,v_i}\,C_i^{-\gamma}\,\beta^\theta$$

with dynamics

$$\begin{aligned}\frac{d\xi_i}{\xi_i} \;=\; & -\left\{\beta+\frac{1}{\psi}\,\mu_{C_i}-\frac{1}{2}\left(1+\frac{1}{\psi}\right)\,\gamma\,\sigma_{C_i}'\,\sigma_{C_i}-\frac{1}{2}\,(1-\theta)\,\sigma_{v_i}'\,\sigma_{v_i}\right.\\ & \left.-\,(1-\theta)\,\sigma_{C_i}'\,\sigma_{v_i}+\left(1-\frac{1}{\theta}\right)\,\left[(1+L_{C_i})^{1-\gamma}\,e^{\theta\,L_{v_i}}-1\right]\,\lambda_i\right\}\,dt\\ & -\left\{\gamma\,\sigma_{C_i}+(1-\theta)\,\sigma_{v_i}\right\}'\,dW+\left\{(1+L_{C_i})^{-\gamma}\,e^{(\theta-1)\,L_{v_i}}-1\right\}\,dN\,(\lambda_i)\,. \qquad (10)\end{aligned}$$

From this we obtain the investor-specific market prices of risk η_i as the exposures of the pricing kernel to the different risk factors. For the diffusion risks this yields

$$\eta_i^W \;=\; \gamma\,\sigma_{C_i}+(1-\theta)\,\sigma_{v_i}. \qquad (11)$$

The first term is the standard market price of (individual) consumption risk, which would also result in a CRRA economy, while the second term gives the extra market prices of risk for the diffusive volatility of the (log) wealth-consumption ratio and thus for the state variables w and X. Analogously, the individual market prices of jump risk η_i^N are given by

$$\eta_i^N \;=\; (1+L_{C_i})^{-\gamma}\,e^{(\theta-1)\,L_{v_i}}-1, \qquad (12)$$

where the first term on the right-hand side is the product of the market price of consumption jump risk with CRRA utility and an adjustment for jump risk in the individual wealth-consumption ratios.

Finally, the subjective risk-free rate r_i^f equals the expected value of the pricing kernel, i.e.,

$$\begin{aligned}r_i^f \;=\; & \beta+\frac{1}{\psi}\,\mu_{C_i}-\frac{1}{2}\left(1+\frac{1}{\psi}\right)\,\gamma\,\sigma_{C_i}'\,\sigma_{C_i}-\frac{1}{2}\,(1-\theta)\,\sigma_{v_i}'\,\sigma_{v_i}-(1-\theta)\,\sigma_{C_i}'\,\sigma_{v_i}\\ & -\left[\eta_i^N-\left(1-\frac{1}{\theta}\right)\,\left[(1+L_{C_i})^{1-\gamma}\,e^{\theta\,L_{v_i}}-1\right]\right]\,\lambda_i, \qquad (13)\end{aligned}$$

where the terms have the usual interpretation as reflecting the impact of impatience, the individual consumption growth rate, and precautionary savings.

The equilibrium is characterized by the fact that markets for the traded assets have to clear and that the investors agree on their prices. The exact procedures to compute the equilibria for the complete and the incomplete market economy are described in Appendix A.

IV. Quantitative Analysis of the Model

A. Parameters

The parameters used in the quantitative analysis of the model are given in Table 2. They mostly represent standard values from the long-run risk literature.[10] When a disaster occurs, expected consumption growth drops by $L_X = -0.03$ which is about the same size as in Benzoni et al. (2011).[11] With $\kappa_X = 0.1$, shocks have a half-life of about 6.9 years.[12] The key new element in the model is given by the agents' heterogeneous beliefs with respect to the intensity of jumps in X. The pessimistic investor assumes an intensity of $\lambda_1 = 0.020$, i.e., on average one X-jump every 50 years, while the optimist thinks there will be on average one jump only every 1,000 years, i.e., $\lambda_2 = 0.001$. So the probabilities for rare events are about the same size as in Chen et al. (2012). In what follows we will assume that the pessimist's belief represents the true model.

Equity is considered a claim on levered aggregate consumption with a leverage factor of $\phi = 1.3$. When the market is complete the investor also has access to two insurance products, which make the diffusive and the jump risk in X tradable.[13] All the model results are shown for the stochastic part of the expected growth rate of consumption at its long run mean of -0.006.[14]

We present the quantitative analysis of our model in each of the following subsections in two parts. In the first part we present the main findings with respect to the certain aspect of the model, i.e., the equity premium, the link between trading volume and return volatility, the implications of varying degrees of disagreement between investors, etc., while

[10] See, among others, Benzoni et al. (2011) or Bansal et al. (2012).

[11] When a jump occurs these parameter values lead to respective wealth losses of up to 37% and 7% for the optimist and the pessimist on the complete market. Although aggregate consumption itself remains unchanged, the value of levered aggregate consumption falls by up to 9%.

[12] We have chosen this rather small value for κ_X mainly to match the equity premium. However, the results presented in the following sections remain qualitatively unchanged when we follow Benzoni et al. (2011) and set $\kappa_X = 0.2785$.

[13] The cash flows of these insurance products are given exogenously, but their price-to-cash flow ratios are of course determined endogenously in equilibrium. See Appendix A.1 for details. Note that the cash flow exposures of these assets to the risk factors in the model merely represent scaling factors, which do not have an impact on the equilibrium. However, they do have an impact on the positions and the trading volume in the insurance assets.

[14] The long-run mean is computed as the value of X, where the expected change in X is equal to zero. This value is given by $\frac{\lambda L_X}{\kappa_X}$, which is equal to -0.006 for our choice of parameters.

in the second we take a more detailed and somewhat more technical look at the mechanism within the model, which actually generates the respective result.[15]

B. Equity Risk Premium

B.1. Results

Figure 2 presents the equity risk premium and the expected excess returns on the investors' respective individual wealth as well as their components for a complete and an incomplete market. From left to right the graphs show the parts of the equity premium due to diffusive consumption risk, diffusive growth rate risk, and growth rate jump risk, and finally the total equity risk premium. All these quantities are presented as observed by an econometrician, i.e., under the true measure.[16] The equity risk premium ERP is determined as the wealth-weighted average of individual expected excess returns on wealth EER_i $(i = 1, 2)$:

$$ERP \quad = \quad \frac{w\, e^{v_1}}{e^{v}}\, EER_1 + \frac{(1-w)\ e^{v_2}}{e^{v}}\, EER_2 \tag{14}$$

with

$$EER_i \quad = \quad \phi \left(\frac{1}{dt}\, \mathbb{E}\left[\frac{dV_i}{V_i} + e^{-v_i}\, dt \right] - r_i^f \right) \quad = \quad \phi\, \sigma_{V_i}'\, \eta_i^W + \phi\, L_{V_i}\, \left[\lambda - \lambda_i^{\mathbb{Q}} \right]. \tag{15}$$

where λ and $\lambda_i^{\mathbb{Q}} = \lambda_i\, \left(1 + \eta_i^N\right)$ are the jump intensities under the true and the risk-neutral measure, respectively. The two summands on the right-hand side of (15) represent the usual compensation for diffusive and jump risk.

On the complete market (upper row of graphs) the part of the premium due to consumption risk is the same as in a CRRA economy, and it is furthermore equal for the two investors. The investors also agree on the market price of risk for diffusive shocks to the stochastic growth rate X, but the part of the equity premium due to this factor is nevertheless not exactly constant across w. The reason for the small variation is that the exposure of the return on individual wealth to W^X is higher for the pessimist.[17] Overall, with a value

[15] Here we also consider a new model, our so-called 'extended' Chen et al. (2012) model with recursive preferences. Appendix F describes this model and explains how to solve for the equilibrium in this case.

[16] Appendix E contains the results for all the basic equilibrium quantities, such as the wealth-consumption ratio, the dynamics of consumption shares, the risk-free rate, the market prices of risk, the exposures of individual and aggregate wealth to the risk factors in the model, and the expected excess return on individual wealth. These results are not discussed here in detail, but delegated to Appendix E, since they only represent preliminary steps for the analysis of our model in this section.

[17] See Appendix E.4 and the third graph from the left in the lower row of Figure 17.

of around 3% the part of the equity premium due to the long-run risk factor X is sizable.

Since the two diffusive premia are basically independent of the investors' consumption shares, the variation in the equity premium as a function of w is almost exclusively caused by the jump part. ERP increases in the pessimist's consumption share and ranges from 3.8% to 5%. As Figure 1 shows our model generates a rather flat and close to linear relation between the equity premium and the share of optimists. This shows that a sizable equity premium can occur in equilibrium even when investors substantially share risk, i.e., when neither of them is much larger than the other.

The incomplete market case is shown in the lower row of graphs in Figure 2. All the curves for ERP are pretty similar to the complete market case. Of course, the two boundary values for $w = 0$ and $w = 1$ coincide with the complete market case, so that also the range of the premium remains the same. Like on the complete market it is also monotonically increasing in w. The impact of market incompleteness on the premia on individual wealth is much more pronounced.

B.2. Mechanism

To identify the main drivers for our results presented in Figure 1, we will now discuss the impact of the key elements of our model (preferences, consumption dynamics, and market incompleteness) on the equity premium.[18]

First, we look at the role of the preference specification in models with consumption jumps and complete markets. In these models the dominant component of the equity premium is the compensation for jump risk, given by the second term of the sum in Equation (15). In the Dieckmann (2011) model with log utility the investors' individual wealth-consumption ratios do not vary with w, i.e., they are constant (and equal to $\frac{1}{\beta}$). Consequently, the jump size in aggregate wealth is constant and therefore the linear increase of the equity premium in w is exclusively driven by the risk-neutral jump intensity $\lambda_i^{\mathbb{Q}} = \lambda_i \left(1 + \eta_i^N\right)$ (with $\gamma = 1$ in the expression for η_i^N in Equation (12)). In a general CRRA model like Chen et al. (2012) with $\gamma > 1$ the wealth-consumption ratios are no longer constant, but the pessimist saves less when her consumption share is small than when it is large. As a result, the risk-neutral jump intensity increases in w in a non-linear fashion due to $\gamma > 1$ in Equation (12), and so does the equity premium. This creates the tension described above between the size of the equity premium and the degree of risk sharing between investors. To see whether the restrictions of CRRA utility ($\gamma = 4, \psi = \frac{1}{4}$) are the key driver behind

[18]In terms of the market structure (complete vs. incomplete) this section focuses on our model. A general analysis of the impact of incompleteness across a variety of models is provided in Section IV.E.

this result we solve our "extended version" of the Chen et al. (2012) model with recursive preferences ($\gamma = 4, \psi = 1.5$). This model generates the proper investor behavior (i.e., the pessimist saves more when her consumption share is small), but it still does not solve the bigger problem of a pronounced conflict between equity premium and risk sharing.

When it comes to the role of the dynamics of fundamentals, i.e., whether there are jumps in consumption itself or in its expected growth rate, we compare the 'EZ version' of Chen et al. (2012) ($\gamma = 4, \psi = 1.5$) with our model ($\gamma = 10, \psi = 1.5$) on a complete market. In both cases, the risk-neutral jump intensities are rather similar. With consumption jumps, however, the jump size in aggregate wealth is mainly caused by the jump in aggregate consumption, and this large jump exposure, together with the non-linear shape of the jump intensity, leads to a highly non-linear equity premium, which is almost exclusively due to consumption jump risk.

With jumps in expected consumption growth, the picture is markedly different, since there is an additional diffusive source of risk driving the expected growth rate of aggregate consumption. Now the jump in aggregate wealth is caused by the jump of the wealth-consumption ratio only. Thus the part of the equity premium for X-jump risk has a maximum of around 1.3% for an all-pessimist economy, while the premia for diffusive growth rate risk varies between 2.9% and 3.1% for $w \approx 1$ and $w \approx 0$, respectively. So having jumps in X instead of C is the key point needed to have a substantial equity premium and a pronounced degree of risk sharing at the same time.

Finally, when the market is incomplete and investors can only trade the consumption claim in our model, the pessimist will try to get rid of the associated risk, and individual and aggregate exposures to all risk factors are almost linear in w. Furthermore, the non-linearity in individual jump intensities is much less pronounced than on the complete market. This ultimately results in a higher equity premium which also depends on w in a more linear fashion.

So with respect to the relation between equity premium and the investors' consumption shares the decisive element of the model is whether there are jumps in consumption or in its expected growth rate combined with recursive preferences.

C. Trading Volume and Return Volatility

C.1. Results

Trading volume is generated by changes in the investor's asset holdings. So as a first step we take a look at the investors' portfolio compositions. The fractions of individual wealth invested in the different assets on a complete market are shown in the upper row of Figure 3. Both agents invest 100% of their respective wealth into the consumption claim. To implement their desired exposures to the different risk sources they use the other three assets. The investors disagree on the amount of jump risk, and sharing this risk is their primary trading motive. The pessimist buys the jump insurance product I from the optimist and thereby reduces her exposure to jump risk. Furthermore, she offsets most of the exposure to diffusive X-risk by selling the diffusive insurance product Z, and finally she takes a long position in the money market account.

To analyze trading volume in our model we follow Longstaff and Wang (2012) and Xiong and Yan (2010), who measure trading volume by the absolute volatility of the number of shares of an asset held by an investor. We generalize their measure to take into account the fact that there are multiple sources of risk in our model. More precisely, let $n_1^j = \frac{\pi_{1,j} V_1}{P^j}$ denote the number of shares of asset j held by investor 1 with dynamics

$$dn_1^j = \mu_{n_1^j}\, dt + \sigma'_{n_1^j}\, dW + L_{n_1^j}\, dN\left(\lambda\right),$$

where the drift, volatility, and jump size are given in Appendix C. Then trading volume in asset j is given as

$$TV_j = \sqrt{\sigma'_{n_1^j}\, \sigma_{n_1^j} + \lambda \left(L_{n_1^j}\right)^2}, \tag{16}$$

where λ is the jump intensity under the true measure.

Looking at the complete market case first, the trading volume for each of the two insurance products shown in Figure 4 is inversely U-shaped in the pessimist's consumption share. This is also true for the consumption claim as shown in the middle graph in the upper row of Figure 5. This may seem surprising, given that, as discussed above, both investors hold 100% of their respective wealth in the consumption claim. However, when the investors' wealth *levels* change, this implies that also the number of shares of the consumption claim in their respective portfolios changes.

The volatility of the return on levered aggregate consumption claim, RV, is obtained as

$$RV = \phi \sqrt{(\sigma_C + \sigma_v)' \ (\sigma_C + \sigma_v) + \lambda \ (e^{L_v} - 1)^2} \tag{17}$$

where $v \equiv \log\left(w\, e^{v_1} + (1-w)\ e^{v_2}\right)$ denotes the aggregate (log) wealth-consumption ratio. As can be seen from the upper left graph, RV is around 5.2% and thus greater than consumption volatility of around 2.8%, so there is excess volatility.[19] Like trading volume, it is higher in an heterogeneous investor economy than with only optimists or only pessimists.

When the insurance products are not available the pessimist reduces her jump exposure by selling the consumption claim to the optimist and invests the proceeds in the money market account (see the lower row in Figure 3). Consequently the trading volume in the consumption claim is positive and inversely U-shaped in w, as shown in the middle graph in the lower row of Figure 5.[20] The return volatility of the consumption claim is determined as before and looks similar to the one on a complete market.

Comparing our results for the complete and the incomplete market case, we find that the return volatility on an incomplete market is higher than on a complete market. The worse possibilities for risk sharing thus increase the overall risk of the only traded risky asset. This is in contrast to the findings presented by Kübler and Schmedders (2012) for an overlapping generations framework featuring two log utility investors with heterogeneous beliefs on the probability of exogenous i.i.d. shocks.

In the next step, we look at the relation between the trading volume in the consumption claim and its return volatility. As Karpoff (1987) points out, the positive correlation between these two quantities is one of the most robust patterns related to trading activity in equity markets. It turns out that our model can match this pattern for both market structures. The properties of trading volume and return volatility as functions of w imply a positive relation between the two quantities as can be seen in the right graphs in the upper and lower rows of Figure 5.

C.2. Mechanism

Like in the case of the equity premium we want to highlight the role of preferences (in models with consumption jumps), dynamics (on a complete market with EZ preferences),

[19] Note that there would be excess volatility even without the leverage factor, since stock return volatility is greater than 1.3 times consumption volatility.

[20] The fact that the trading volume on a complete market is larger than on an incomplete market is due to the effect of jumps. The key thing to note is that the jump size L_w of the consumption share is much larger on the complete than on the incomplete market.

and market structure (in a model with X jumps and EZ preferences) for our results.

In all the models we discuss below trading volume is inversely u-shaped in w. To explain the correlation between trading volume and return volatility we can focus on the properties of return volatility which is mainly driven by the jump size (L_v) of the aggregate wealth-consumption ratio since investors only share jump risk.

The log utility specification in Dieckmann (2011) implies that the individual wealth-consumption ratios do not vary with w, so the the jump size L_v and consequently also the volatility of equity returns are constant, automatically implying a zero correlation with trading volume. With general CRRA as in Chen et al. (2012) ($\gamma > 1$) the negative jump size in the price of the consumption claim is larger in absolute value in an economy with only optimists or pessimists than when both types of investors are present. The jump size in the price is driven by the jump size in aggregate consumption and the jump size of the aggregate wealth-consumption ratio.[21] Since the former is constant, the effect has to be driven by the latter, which in turn implies that L_v has to be negative. Consequently, return volatility is u-shaped in w, which generates a negative instead of a positive relation with trading volume in the Chen et al. (2012) model as shown in Figure 6.

Generalizing the Chen et al. (2012) model from CRRA to EZ preferences as above still does not generate diffusive risk sharing, but now the jump size in the price of the consumption claim is "more negative" in the middle of the range for w than at the boundaries. This implies $L_v > 0$, so that return volatility is inversely u-shaped in w. Since trading volume exhibits the same functional relationship with w, we find a positive relation for these two quantities in Figure 6. Thus it is the more general EZ preferences, which are needed to reconcile this correlation in the model with the one observed in the data.

The specification of consumption dynamics and the market structure (complete versus incomplete) on the other hand are not crucial in getting the sign of the correlation between volatility and volume right. For both consumption and expected growth rate jumps trading volume and return volatility are inversely u-shaped in w, and so the two quantities are positively related. Compared to the complete market investors share more diffusive, but less jump risk on the incomplete market, but again return volatility and volume are both inversely u-shaped in w, implying a positive correlation.

[21] See Equation (F.2) in Appendix F.

D. Varying the Degree of Disagreement

D.1. Results

Disagreement increases in our model when the difference between the subjective jump intensities becomes larger. The difference increases when the beliefs of one or both investors become more extreme. In the following we will interpret an increase in disagreement as a mean preserving spread. In doing so we follow Carlin et al. (2014) who investigate the link between disagreement and asset prices empirically in a model-free fashion and thus provide very robust results.[22] Their key findings are that higher disagreement leads to higher expected returns, higher return volatility, and higher trading volume.

To analyze the effect of varying disagreement we consider variations of the model, in all of which the average of the subjective intensities remains constant at $\bar{\lambda} = 0.02$, the true parameter value in our model. We start with the case in which the investors agree on the jump intensity with $\lambda_1 = \lambda_2 = 0.02$ and increase the disagreement step-by-step up to a maximum of $\lambda_1 = 0.038$ and $\lambda_2 = 0.002$. The other parameters of the model remain unchanged.

Figure 7 presents the results for the complete and the incomplete market. On the complete market higher disagreement leads to a lower expected return (again under the true measure), a higher trading volume in the consumption claim, and a higher return volatility. The lower expected return is mainly caused by the smaller compensation for jump risk, which in turn is caused mainly by the increased risk sharing and thus the lower impact of the pessimist on the jump risk premium. When the degree of disagreement increases, the amount of diffusive growth rate and jump risk shared between the investors increases, leading to a higher return volatility. So, in summary, on a complete market two of the three variables are reacting to higher disagreement in the direction suggested by the empirical findings in Carlin et al. (2014).

On the incomplete market higher disagreement leads to a higher expected return (under the true measure), a higher return volatility, and a higher trading volume. To get why the expected return increases in disagreement on the incomplete, but decreases on the complete market, consider its two components, the expected excess return and the risk-free rate. With increasing disagreement (and thus a more intensive risk sharing between investors), the risk-free rate increases (due to less precautionary saving) on both markets. Note that there is hardly a difference between the complete and incomplete market case for the risk-free rate. Consequently, the expected excess return is the main driver behind the different

[22] Their disagreement index is based on the *standard deviation* of a normalized change in prepayment forecasts across dealers in the mortgage backed security market.

results for the expected return. The expected excess return decreases, but as we had already seen in Section IV.B, the equity premium is higher on the incomplete than on the complete market. Therefore, the expected excess return on the incomplete market also decreases more slowly than on the complete market. In particular, this decrease is now slower than the accompanying increase in the risk-free rate. When disagreement increases, the amount of shared diffusion and jump risk increases and this leads to a higher return volatility as well as to a higher trading volume. Overall, we find that our model with market incompleteness matches all the stylized facts presented in Carlin et al. (2014) very well.

From their empirical findings Carlin et al. (2014) draw the conclusion that there is a positive premium for disagreement. They do so, however, without explicitly showing that not only expected returns but also expected *excess* returns exhibit the relevant characteristics, i.e., they implicitly assume that the impact on the interest rate does not over-compensate the effects on expected returns. In our general equilibrium model disagreement necessarily also has an effect on the risk-free rate, and, as can be seen from Figure 7, higher disagreement indeed leads to a higher risk-free rate on both complete and incomplete markets. On the incomplete market this increase is actually larger than the increase in the expected return, thus leading to a lower overall equity risk premium. Without putting too much emphasis on this result, it may nevertheless be interpreted as an indication that the equilibrium effects on all relevant quantities, including interest rates, should be taken into account.

It may be of interest to see whether the result of the expected return increasing with disagreement holds across different values for the investors' risk aversion and their elasticity of intertemporal substitution.[23] We consider two variations of our basecase parametrization, one with a lower risk aversion ($\gamma = 4, \psi = 1.5$) and one with a lower EIS ($\gamma = 10, \psi = 1.1$). Note that in both cases the EIS is greater than one, which is the common choice in the finance literature to make sure that asset prices react in an intuitive way to changes in state variables. Figure 8 shows the results for the first case, and we see that the results remain qualitatively unchanged relative to our standard parametrization. When the EIS is lowered relative to the basecase, we can see from Figure 9 that the expected return now increases in disagreement on the complete market, since the decrease in the expected excess return decreases is so slow that it is more than compensated by the increase in the risk-free rate. However, in this scenario the equity premium generated by the model does not match the data anymore. The results on the incomplete market remain qualitatively unchanged compared to the basecase.[24]

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[23]We thank an anonymous referee for suggesting this exercise.

[24]In the Online Appendix we provide plots for a wide range of the pessimist's subjective intensity and the full range of the state variable X (see Figures A.1 to A.3).

D.2. Mechanism

Like in the previous sections we want to identify those elements of our model, i.e., preferences, consumption dynamics, and market incompleteness, which drive the results concerning the effect of varying degrees of disagreement between investors.

In other papers like Chen et al. (2012) and Dieckmann (2011) varying degrees of disagreement are represented via more extreme beliefs on the part of one investor while the other investor's beliefs do not change. For the purpose of confronting these models with the data from Carlin et al. (2014) in the same way as our own model we vary disagreement by introducing a mean-preserving spread.

That higher disagreement leads to higher trading volume is true already in the Dieckmann (2011) model with log utility, since more pronounced disagreement generates more pronounced risk sharing even in this simple setting with two myopic investors. Based on the mechanism explained above for the link between return volatility and trading volume our extended Chen et al. (2012) model with recursive utility will generate higher return volatility with higher disagreement, as can be seen in Figure 10. Thus neither jumps in X nor incomplete markets are needed to generate this effect.

To obtain the result that higher disagreement leads to a higher expected return one needs all the features of our most general model with recursive utility, X-jumps, and incomplete markets. With more pronounced disagreement the risk-free rate will go up, so to obtain a higher expected return this increase must be larger than the decrease in the expected *excess* return on equity. The main element needed for this is recursive utility, so that the desired result is already obtained in our extended Chen et al. (2012) model, but quantitatively it is close to negligible, as can be seen in Figure 10. The setup that finally gets it done is our long-run risk model with equity represented by a levered consumption claim and incomplete markets. According to Equation (14), the equity risk premium is the wealth-weighted average of individual expected excess returns on wealth. Via more intensive risk sharing and the fact that investors can only trade the consumption claim higher disagreement leads to an increase in the two diffusive premia in the optimist's total risk premium, while the opposite happens in the pessimist's case. Overall, the diffusive premia in the aggregate equity premium increases, since the optimist's wealth share increases, and it increases in a much more pronounced fashion than on the complete market. So higher disagreement still decreases the equity premium on the incomplete market due to the smaller jump risk premium, but to a lesser degree than on the complete market. The decrease in the equity premium is small enough to be more than compensated by the increase in the risk-free rate, which finally leads to a higher expected return on equity.

E. Market Incompleteness across Models

In the following we provide a detailed analysis of the impact of market incompleteness across four different models.[25] We regard the equity premium and the risk-free rate as the two most important asset pricing moments, so that we focus our discussion on them. The models we consider are those suggested by Dieckmann (2011) and Chen et al. (2012) as well as our version of the Chen et al. (2012) model with recursive preferences and our own. The results are presented in Figures 11 to 14.

Concerning the risk-free rate there is no unique sign for the difference between complete and incomplete market across models. In two of the four models the rate is higher on the incomplete market (Dieckmann (2011) and Chen et al. (2012) model with recursive preferences), in the other two we observe the opposite. Overall, however, the differences are rather small, so that in terms of the risk-free rate incompleteness does not seem to matter so much.

This changes when we look at the equity risk premium. Our novel result is that the risk structure of the economy in the sense of the relative importance of jump and diffusive risks is highly relevant for the impact of incompleteness, while preferences only play a minor role. As one can see from the figures, the difference between the equity premia on the incomplete and the complete market is largest for our model, followed by Dieckmann (2011), while for the two versions of the Chen et al. (2012) model with CRRA and recursive preferences it does not matter so much whether the market is complete or not.

At first sight, the latter result may seem surprising. To get the intuition behind this result, take the Chen et al. (2012) model (with either CRRA or EZ utility) as an example. Its parametrization with a consumption jump size of about -40% and on average one jump every 58 years makes jump risk vastly more important than diffusive consumption risk with an annualized volatility of 2%. Even if jump risk cannot be traded separately via some sort of insurance contract, as long as the consumption claim with its pronounced loading on jump risk is available to investors they can still share risks almost perfectly. This can be seen from a comparison of the left and the right graphs in the lower row of Figures 12 and 13. The fact that risk sharing between investors is hardly affected by the market structure then also implies very similar equity risk premia.

In contrast to this the importance of jump and diffusive risk in the expected growth is much more similar in our model (diffusive consumption risk is less relevant), and consequently it affects investor risk sharing much more when one or multiple sources of risk cannot be

[25] We thank an anonymous referee for motivating us to analyze alternative model specifications.

traded anymore (see the lower row of graphs in Figure 14). The direct implication of this decreasing quality of risk sharing is a higher equity premium on the incomplete market.

As indicated above, the analysis points towards the risk structure of the economy being the most important determinant of the impact of market incompleteness, at least much more important than investor preferences.

F. Investor Survival

F.1. Results

Our economy is populated by investors with different and agnostic beliefs, i.e., they do not update their subjective estimate of the jump intensity given the realizations of consumption growth. This might cause a divergence from a heterogeneous to a homogeneous (one-)investor economy in the long run in the sense that only one of the two investors will have a non-negligible consumption share.

There is a rich literature dealing with natural selection in financial markets,[26] and it can be shown analytically in a model with CRRA investors and i.i.d. consumption growth, that the investor with the 'worse' model will lose all her consumption in the long run and disappear from the economy. 'Worse' here means that when otherwise identical investors disagree about one parameter in the model, the one whose assumed value is further away from the true model will vanish in the long run. This is not necessarily true in models with EZ investors. As shown by Borovicka (2015), two investors with identical EZ preferences, who differ with respect to the expected growth rate of consumption, can both have non-zero expected consumption shares in the long run despite the fact that the beliefs of only one of them represent the true model. Furthermore, it may even happen that only the investor with the worse model survives in the long run.

Under the given parametrization our model represents an example for this last case. This becomes clear from Table 3, which shows the results of a Monte Carlo simulation of the pessimist's consumption share over a period of 50, 100, 200, 500, and 1,000 years. Over time the pessimist's expected consumption share becomes smaller and smaller. This survival of the optimist in our model is thus another example for the special features of models with recursive utility.

Figure 15 shows the density of the pessimist's consumption share on the complete and incomplete market. The speed of extinction is larger in the incomplete than in the complete

[26] Among others, see Dumas et al. (2009), Yan (2008), and Kogan et al. (2006, 2009).

market. On the incomplete market, the pessimist earns the lower expected excess return on wealth (see Figure 2) and consumes more out of her wealth (see Figure 16). This also holds true when she is close to extinction, and by earning a lower risk premium and consuming more, she cannot escape extinction. On the complete market, she still consumes more when she is small, but now earns a higher expected return on her wealth. However, the higher risk premium is not large enough to compensate the larger propensity to consume, and again, it is the pessimist who vanishes from the market in the long run. The pessimist is indeed right, but she is right only concerning very rare events, occurring on average once every 50 years. So being right does not pay off for her over the long term.

F.2. Mechanism

It is obvious that with respect to survival the preference specification is the most important element of the model. As can be seen from Table 4, in the model proposed by Chen et al. (2012) the pessimist's expected consumption share increases over time and the pessimist survives, while in our extension of their model with recursive preferences it is the optimist who takes over the economy in the long run.

With both consumption and expected growth rate jumps the optimist's wealth decreases after a jump, while the pessimist's goes up. This effect is much stronger for jumps in C than for jumps in X, so that the pessimist benefits much more from a jump in consumption than from a jump in the expected growth rate, so that her chances for survival are much better under the former specification.

Investors can share risk only to a limited degree on the incomplete market, namely by trading the consumption claim. Given that she wants to get rid of the bad jump risk the pessimist will sell this asset to the optimist, thereby automatically reducing her exposure to the diffusive sources of risk as well. In total the pessimist benefits less in terms of her wealth from jumps and suffers more from diffusive shocks, so that she will lose consumption share faster on the incomplete than on the complete market.

V. Conclusion

Long run risk models with downward jumps in the expected growth rate of aggregate consumption, as proposed by, e.g., Benzoni et al. (2011), are one way to explain the equity premium puzzle. An obvious and yet still open question is whether this explanation is robust to the introduction of heterogeneous investors who can trade with each other to share risks.

In this paper we propose and analyze a model featuring an optimistic and a pessimistic investor with recursive preferences, who differ in their beliefs about the intensity of jumps in expected consumption growth. Furthermore, we explicitly take market incompleteness into account when solving the model. To the best of our knowledge we are the first to combine all these features in one model.

We find a rather flat and almost linear relation between the equity risk premium and the investors' consumption shares. So even when the investors are about equally large and can share risk to a substantial degree, we see a sizable equity risk premium.

We take the analysis one step further by considering not only moments of returns like the equity premium and return volatilities, but also trading volumes and the relation between all these quantities and the amount of disagreement in the economy. The empirical findings concerning these quantities provided by Karpoff (1987) and Carlin et al. (2014) represent important over-identifying restrictions for equilibrium asset pricing models. The incomplete markets version of our model successfully reproduces the stylized facts described in these papers, i.e., trading volume and return volatility are positively correlated, and higher disagreement leads to higher expected returns, higher return volatility, and higher trading volume.

We analyze the relevance of market incompleteness in great detail by comparing the equity risk premium and the risk-free rate in four different models, namely the log utility specification with consumption disasters in Dieckmann (2011), the original Chen et al. (2012) model, our recursive utility version of their approach, and our own model. While the impact of the market structure on the risk-free rate is basically negligible in all four models, the key determinant for the impact of market incompleteness on the equity risk premium is the risk structure of the economy (in the sense of the relative importance of jump and diffusive risk), but not investor preferences. When there is a dominant source of risk (like jumps in the models with consumption disasters and large jump sizes), incompleteness does not have a major impact as long as this source of risk remains tradable, which is the case when the consumption claim loading heavily on jump risk is available to the investors. In models where the overall risk is more evenly distributed across diffusions and jumps, the equity premium can exhibit very different behaviors on the complete and the incomplete market.

Finally, in a model with two otherwise identical investors, who differ in their beliefs about a key parameter, survival becomes an important issue. In the usual diffusion-driven models under CRRA preferences it will always be the investor whose beliefs are closer to the true model (or even coincide with it) who survives. From the analysis in Borovicka (2015) we know that under EZ preferences both investors or even the one with the worse beliefs can survive in such a case. In our setup, the pessimist has beliefs which coincide with the true model, but nevertheless she is the one who loses all her consumption share in the long run. The speed of extinction is faster on an incomplete than on a complete market.

The results of our analysis are also relevant for the interpretation of the results found in other papers, like Backus et al. (2011) and Julliard and Ghosh (2012), where the authors determine implied disaster probabilities assuming CRRA preferences. Backus et al. (2011) point out explicitly that changes in investor preferences or the introduction of heterogeneity could potentially also change their conclusions, and our results give a strong indication in that direction. A generalization of their analysis with respect to recursive preferences and heterogeneous beliefs is left for future research.

Appendix

A. Solving for the Equilibrium

1. Complete Market

When the market is complete, the investors will in equilibrium agree on the risk-free rate, the market prices of diffusion risk, and the risk-neutral jump intensity $\lambda_i^{\mathbb{Q}} = \lambda_i \left(1 + \eta_i^N\right)$. We will use these restrictions to solve for the coefficients μ_w, σ_w, and L_w of the consumption share process (3).

First, σ_w is obtained by equating the investors' market prices of diffusion risk η_i^W, yielding

$$\sigma_w = \frac{w\,(1-w)\,(1-\theta)\,\left[\frac{\partial v_2}{\partial X} - \frac{\partial v_1}{\partial X}\right]}{\gamma + w\,(1-w)\,(1-\theta)\,\left[\frac{\partial v_1}{\partial w} - \frac{\partial v_2}{\partial w}\right]}\,\sigma_X. \tag{A.1}$$

The drift μ_w follows from the condition that the investors must agree on the risk-free rate, so that

$$\begin{aligned}
\mu_w \;=\; & -\sigma_w'\,\sigma_C + \psi\,w\,(1-w) \\
& \times\Bigg\{\frac{1}{2}\left(1+\frac{1}{\psi}\right)\gamma\,\left[\sigma_{C_1}'\,\sigma_{C_1} - \sigma_{C_2}'\,\sigma_{C_2}\right] + \frac{1}{2}\,(1-\theta)\,\left[\sigma_{v_1}'\,\sigma_{v_1} - \sigma_{v_2}'\,\sigma_{v_2}\right] \\
& \quad + (1-\theta)\,\left[\sigma_{C_1}'\,\sigma_{v_1} - \sigma_{C_2}'\,\sigma_{v_2}\right] \\
& \quad + \left[\eta_1^N - \left(1-\frac{1}{\theta}\right)\,\left[(1+L_{C_1})^{1-\gamma}\,e^{\theta\,L_{v_1}} - 1\right]\right]\,\lambda_1 \\
& \quad - \left[\eta_2^N - \left(1-\frac{1}{\theta}\right)\,\left[(1+L_{C_2})^{1-\gamma}\,e^{\theta\,L_{v_2}} - 1\right]\right]\,\lambda_2\Bigg\}.
\end{aligned} \tag{A.2}$$

Finally, the jump size L_w is found by using the condition that the investor-specific risk-neutral jump intensities $\lambda_i^{\mathbb{Q}}$ must be equal, implying

$$L_w = \frac{e^{\frac{1}{\gamma}\left[(\theta-1)\left(L_{v_1}-L_{v_2}\right)+\ln\frac{\lambda_1}{\lambda_2}\right]} - 1}{\frac{1}{w} + \frac{1}{1-w}\,e^{\frac{1}{\gamma}\left[(\theta-1)\left(L_{v_1}-L_{v_2}\right)+\ln\frac{\lambda_1}{\lambda_2}\right]}}. \tag{A.3}$$

The equilibrium solution is then found by simultaneously solving the two PDEs in (9) for v_1 and v_2 using the above equations for μ_w, σ_w, and L_w.

Given these coefficients as well as the individual wealth-consumption ratios we can then compute other equilibrium quantities. For example, the total return (including consumption) on investor i's wealth follows the process

$$\begin{aligned}\frac{dV_i}{V_i} + e^{-v_i}\,dt &= \left\{\left[\mu_{C_i} + \mu_{v_i} + \frac{1}{2}\,\sigma'_{v_i}\,\sigma_{v_i} + \sigma'_{C_i}\,\sigma_{v_i}\right] + e^{-v_i}\right\} dt + (\sigma_{C_i} + \sigma_{v_i})'\,dW \\ &\quad + \left[(1 + L_{C_i})\; e^{L_{v_i}} - 1\right]\, dN\left(\lambda_i\right) \\ &\equiv \left(\mu_{V_i} + e^{-v_i}\right) dt + \sigma'_{V_i}\,dW + L_{V_i}\,dN(\lambda_i), \end{aligned} \tag{A.4}$$

where $V_i = C_i e^{v_i}$.

To find the agents' portfolio weights one has to set the dynamics of individual wealth component by component equal to the dynamics of a portfolio containing the set of tradable assets, i.e., wealth changes and changes in the value of the portfolio have to have identical exposures to each risk factor. In the complete market case the tradable assets are the claim on aggregate consumption, the money market account, and the insurance products linked to jump and diffusion risk in X, respectively.

We now specify the insurance products in more detail. To trade the diffusion risk in X the agents can use a claim labeled Z with cash flow dynamics

$$\frac{dZ}{Z} = \mu_Z\,dt + \sigma'_Z\,dW,$$

where the drift μ_Z and the volatility $\sigma'_Z = (0, \sigma_z)$ are exogenous constants. Let ζ_i denote the log price-to-cash-flow ratio ζ_i of this asset from investor i's perspective. We write the dynamics of ζ_i as

$$d\zeta_i = \mu_{\zeta_i}\,dt + \sigma'_{\zeta_i}\,dW + L_{\zeta_i}\,dN\left(\lambda_i\right).$$

The coefficients as well as the PDE satisfied by ζ_i are presented in Appendix B. Since the investors agree on the price of the instrument, $\zeta_1 = \zeta_2 \equiv \zeta$.

In an analogous fashion, the payoff from the jump-linked instrument is denoted by I and evolves as

$$\frac{dI}{I} = \mu_I\,dt + L_I\,dN(\lambda_i),$$

where the coefficients μ_I and L_I are again given exogenously. The log price-to-cash-flow ratio ϖ_i follows the process

$$d\varpi_i = \mu_{\varpi_i}\,dt + \sigma'_{\varpi_i}\,dW + L_{\varpi_i}\,dN(\lambda_i).$$

The coefficients and the PDE satisfied by ϖ_i are again shown in Appendix B. As for Z, the investors must agree on the price of I, i.e., $\varpi_1 = \varpi_2 \equiv \varpi$.

Finally, the aggregate log wealth-consumption ratio $v = \log\left(w\, e^{v_1} + (1-w)\; e^{v_2}\right)$ has dynamics

$$\begin{aligned} dv &= \left\{ \frac{\partial v}{\partial w}\,\mu_w + \frac{1}{2}\,\frac{\partial^2 v}{\partial w^2}\,\sigma_w'\,\sigma_w - \frac{\partial v}{\partial X}\,\kappa_X\,X + \frac{1}{2}\,\frac{\partial^2 v}{\partial X^2}\,\sigma_X'\,\sigma_X + \frac{\partial^2 v}{\partial w\,\partial X}\,\sigma_w'\,\sigma_X \right\} dt \\ &\quad + \left\{ \frac{\partial v}{\partial w}\,\sigma_w + \frac{\partial v}{\partial X}\,\sigma_X \right\}' dW + \left\{ v\,(w + L_w, X + L_X) - v\,(w, X) \right\}\, dN\,(\lambda_i) \\ &\equiv \mu_v\,dt + \sigma_v'\,dW + L_v\,dN(\lambda_i). \end{aligned} \tag{A.5}$$

Investor i's total wealth V_i is equal to the value of her holdings (in units) $Q_{i,C}$, $Q_{i,M}$, $Q_{i,Z}$, and $Q_{i,I}$ in the consumption claim, the money market account, and the two insurance products with prices P^C, P^M, P^Z, and P^I, respectively. Let Π_i denote the value of this portfolio. With $\pi_{i,C}$, $\pi_{i,M}$ $\pi_{i,Z}$ and $\pi_{i,I}$ denoting the relative share of investor i's wealth invested in the four assets, the total return dR_i^Π on her portfolio can be represented as

$$\begin{aligned} dR_i^\Pi &= \pi_{i,C}\left(\frac{dP^C}{P^C} + e^{-v}dt\right) + \pi_{i,M}\,r dt + \pi_{i,Z}\left(\frac{dP^Z}{P^Z} + e^{-\zeta}dt\right) + \pi_{i,I}\left(\frac{dP^I}{P^I} + e^{-\varpi}dt\right) \\ &= \Bigg\{ \pi_{i,C}\left(\bar{\mu}_C + X_t + \mu_v + \frac{1}{2}\,\sigma_v'\,\sigma_v + \sigma_C'\,\sigma_v + e^{-v}\right) + \pi_{i,M}\,r \\ &\qquad + \pi_{i,Z}\left(\mu_Z + \mu_\zeta + \frac{1}{2}\,\sigma_\zeta'\,\sigma_\zeta + \sigma_Z'\,\sigma_\zeta + e^{-\zeta}\right) \\ &\qquad + \pi_{i,I}\left(\mu_I + \mu_\varpi + \frac{1}{2}\,\sigma_\varpi'\,\sigma_\varpi + e^{-\varpi}\right)\Bigg\}\, dt \\ &\quad + \left\{\pi_{i,C}\;(\sigma_C + \sigma_v) + \pi_{i,Z}\;(\sigma_Z + \sigma_\zeta) + \pi_{i,I}\,\sigma_\varpi\right\}'\, dW \\ &\quad + \left\{\pi_{i,C}\;\left(e^{L_v} - 1\right) + \pi_{i,Z}\;\left(e^{L_\zeta} - 1\right) + \pi_{i,I}\;\left[(1 + L_I)\;e^{L_\varpi} - 1\right]\right\}\, dN(\lambda_i). \end{aligned}$$

The portfolio shares are determined by the condition that investor i's wealth and her financing portfolio have to react in the same way to the shocks in the model. With respect to the diffusions the condition is thus

$$\sigma_{V_i} \equiv \sigma_{C_i} + \sigma_{v_i} \;\stackrel{!}{=}\; \pi_{i,C}\;(\sigma_C + \sigma_v) + \pi_{i,Z}\;(\sigma_Z + \sigma_\zeta) + \pi_{i,I}\,\sigma_\varpi, \tag{A.6}$$

where the diffusion coefficients for investor i's wealth were derived in (A.4).

Look at $\pi_{i,C}$ first. From Equations (4), (5), and (A.1) one can see that the first component of the vector σ_{C_i} is equal to σ_c, since the first component of σ_w (a multiple of σ_X) is equal to zero. Equation (8) furthermore shows that σ_{v_i} is a multiple of σ_X, so that its first component is also equal to zero. Overall, the first component of the vector $\sigma_{C_i} + \sigma_{v_i}$ is thus equal to σ_c. The same is true for the volatility vectors of investor i's portfolio, as can be seen from the definitions of σ_C and σ_Z and from Equations (B.1), (B.2), and (A.5). Taken together this implies $\pi_{i,C} = 1$ $(i = 1, 2)$. So both agents invest 100% of their respective wealth into the claim on aggregate consumption, implying that the positions in the other three assets add up to zero in value *for each agent individually.*

$\pi_{i,Z}$ and $\pi_{i,I}$ follow from equating the reactions of wealth and the portfolio to W_X-shocks and jumps. This gives two conditions, where the first one refers to the second components of the vectors $\sigma_{C_i} + \sigma_{v_i}$ and $(\sigma_C + \sigma_v) + \pi_{i,Z}\,(\sigma_Z + \sigma_\zeta) + \pi_{i,I}\,\sigma_\varpi$, respectively. The second one is obtained by matching the terms in front of dN in the total return on wealth and on the financing portfolio, using $\pi_{i,C} = 1$. This implies

$$(1 + L_{C_i})\, e^{L_{v_i}} - 1 \;\; \equiv \;\; \left[e^{L_v} - 1\right] + \pi_{i,Z}\,\left[e^{L_\zeta} - 1\right] + \pi_{i,I}\,\left[(1 + L_I)\, e^{L_\varpi} - 1\right]. \qquad \text{(A.7)}$$

The resulting two equations can then be solved numerically for $\pi_{1,Z}$ and $\pi_{1,I}$. The portfolio weights for investor 2 are found via the aggregate supply condition for the insurance products, which says that their total value in the economy has to equal zero, i.e., $\pi_{1,Z}\, V_1 + \pi_{2,Z}\, V_2 \equiv 0$ and $\pi_{1,I}\, V_1 + \pi_{2,I}\, V_2 \equiv 0$. Finally, investor i's position in the money market account is given as

$$\pi_{i,M} \;\; = \;\; -(\pi_{i,Z} + \pi_{i,I}). \qquad \text{(A.8)}$$

2. Incomplete Market

On the incomplete market the insurance products are no longer available to the investors, but they still have to agree on the prices of the claim on aggregate consumption and the money market account. Let ν_i denote investor i's subjective log price-dividend ratio of the claim on aggregate consumption. Its dynamics are given as follows:

$$\begin{aligned} d\nu_i \;\; &= \;\; \left\{ \frac{\partial \nu_i}{\partial w}\, \mu_w + \frac{1}{2}\, \frac{\partial^2 \nu_i}{\partial w^2}\, \sigma_w'\, \sigma_w - \frac{\partial \nu_i}{\partial X}\, \kappa_X\, X + \frac{1}{2}\, \frac{\partial^2 \nu_i}{\partial X^2}\, \sigma_X'\, \sigma_X + \frac{\partial^2 \nu_i}{\partial w\, \partial X}\, \sigma_w'\, \sigma_X \right\} dt \\ &\quad + \left\{ \frac{\partial \nu_i}{\partial w}\, \sigma_w + \frac{\partial \nu_i}{\partial X}\, \sigma_X \right\}' dW + \left\{ \nu_i\,(w + L_w, X + L_X) - \nu_i\,(w, X) \right\}\, dN\,(\lambda_i)\,. \end{aligned}$$

Furthermore ν_i solves the following PDE

$$\begin{aligned} 0 \;\; &= \;\; e^{-\nu_i} + \mu_{\xi_i} + \mu_C + \mu_{\nu_i} + \frac{1}{2}\, \sigma_{\nu_i}'\, \sigma_{\nu_i} + \sigma_{\xi_i}'\, \sigma_C + \sigma_{\xi_i}'\, \sigma_{\nu_i} + \sigma_C'\, \sigma_{\nu_i} \\ &\quad + \left[(1 + L_{C_i})^{-\gamma}\, e^{(\theta - 1)\, L_{\nu_i}}\, e^{L_{\nu_i}} - 1\right]\, \lambda_i, \end{aligned} \qquad \text{(A.9)}$$

which is obtained by first computing the differential $\frac{d(\xi_i\, C\, e^{\nu_i})}{\xi_i\, C\, e^{\nu_i}}$ and then using the fact that the sum of expected price change and cash flow must be equal to zero, i.e., $E^{\mathbb{P}^i}\left[\frac{d(\xi_i\, C\, e^{\nu_i})}{\xi_i\, C\, e^{\nu_i}}\right] + e^{-\nu_i} = 0$. Since the investors agree on the price of the dividend claim, $\nu_i = \nu_2 \equiv \nu$.

Like before on the complete market the investor constructs her financing portfolio so that its return equals the return on her individual wealth. The return on wealth is the same as on a complete market, while the return on the financing portfolio is now given as

$$
\begin{aligned}
dR_i^{\Pi} &= \pi_{i,C}\left(\frac{dP_i^C}{P_i^C} + e^{-\nu_i}\, dt\right) + \pi_{i,M}\, r\, dt \\
&= \left\{\pi_{i,C}\left[\bar{\mu}_C + X_t + \mu_{\nu_i} + \frac{1}{2}\,\sigma'_{\nu_i}\,\sigma_{\nu_i} + \sigma'_C\,\sigma_{\nu_i} + e^{-\nu_i}\right] + \pi_{i,M}\, r\right\} dt \\
&\quad + \left\{\pi_{i,C}\,\left(\sigma_C + \sigma_{\nu_i}\right)\right\}'\, dW + \left\{\pi_{i,C}\,\left(e^{L_{\nu_i}} - 1\right)\right\}\, dN(\lambda_i).
\end{aligned}
$$

Since the investors' individual wealth and their financing portfolios have to have the same exposure to the two diffusions and the jump component, the following conditions have to hold for each investor:

$$
\begin{aligned}
\pi_{i,C}\,\left(\sigma_c + \sigma_{\nu_i,C}\right) &= \sigma_{C_i,C} + \sigma_{v_i,C} && \text{(A.10)} \\
\pi_{i,C}\,\sigma_{\nu_i,X} &= \sigma_{C_i,X} + \sigma_{v_i,X} && \text{(A.11)} \\
\pi_{i,C}\,\left(e^{L_{\nu_i}} - 1\right) &= \left(1 + L_{C_i}\right)\, e^{L_{v_i}} - 1, && \text{(A.12)}
\end{aligned}
$$

where $\sigma_{\cdot,C}$ and $\sigma_{\cdot,X}$ refer to the first and second component of the respective volatility vector.

We want to solve for the following eight variables of interest: the two individual log wealth-consumption ratios v_1 and v_2, the log price-dividend ratio of the traded consumption claim ν, the drift μ_w, the two elements of the volatility vector σ_w, and the jump size L_w of the consumption share process, and the portfolio weight for the claim on aggregate consumption $\pi_{1,C}$. The portfolio weight for investor 2 is determined via the market clearing condition $\pi_{1,C}\, C_1\, e^{v_1} + \pi_{2,C}\, C_2\, e^{v_2} = C\, e^{v}$, and the weight of the money market account is given by $\pi_{i,M} \equiv 1 - \pi_{i,C}$.

There are eight equations we can use to find these quantities: the two PDEs for the individual log wealth-consumption ratios represented by Equation (9) for $i = 1, 2$, the two PDEs for the individual log price-dividend ratios of the claim on aggregate consumption given in (A.9) for $i = 1, 2$, the equation obtained through the restriction that the individual risk-free rates given in (13) have to be equal, and the three equations for the portfolio weights (A.10) – (A.12).

B. Pricing the Insurance Assets

Analogously to Equations (8) and (9) the dynamics of the log price-to-cash-flow ratio ζ_i of the insurance asset Z are given by

$$\begin{aligned} d\zeta_i &= \left\{ \frac{\partial \zeta_i}{\partial w} \mu_w + \frac{1}{2} \frac{\partial^2 \zeta_i}{\partial w^2} \sigma_w' \sigma_w - \frac{\partial \zeta_i}{\partial X} \kappa_X X + \frac{1}{2} \frac{\partial^2 \zeta_i}{\partial X^2} \sigma_X' \sigma_X + \frac{\partial^2 \zeta_i}{\partial w \partial X} \sigma_w' \sigma_X \right\} dt \\ &\quad + \left\{ \frac{\partial \zeta_i}{\partial w} \sigma_w + \frac{\partial \zeta_i}{\partial X} \sigma_X \right\}' dW + \left\{ \zeta_i (w + L_w, X + L_X) - \zeta_i (w, X) \right\} dN(\lambda_i) \\ &\equiv \mu_{\zeta_i} dt + \sigma_{\zeta_i}' dW + L_{\zeta_i} dN(\lambda_i), \end{aligned} \tag{B.1}$$

and ζ_i solves the PDE

$$\begin{aligned} 0 &= e^{-\zeta_i} + \mu_{\xi_i} + \mu_Z + \mu_{\zeta_i} + \frac{1}{2} \sigma_{\zeta_i}' \sigma_{\zeta_i} + \sigma_{\xi_i}' \sigma_Z + \sigma_{\xi_i}' \sigma_{\zeta_i} + \sigma_Z' \sigma_{\zeta_i} \\ &\quad + \left[(1 + L_{C_i})^{-\gamma} e^{(\theta - 1) L_{v_i}} e^{L_{\zeta_i}} - 1 \right] \lambda_i. \end{aligned}$$

The insurance product I has a price-to-cash flow ratio denoted by ϖ_i with dynamics

$$\begin{aligned} d\varpi_i &= \left\{ \frac{\partial \varpi_i}{\partial w} \mu_w + \frac{1}{2} \frac{\partial^2 \varpi_i}{\partial w^2} \sigma_w' \sigma_w - \frac{\partial \varpi_i}{\partial X} \kappa_X X + \frac{1}{2} \frac{\partial^2 \varpi_i}{\partial X^2} \sigma_X' \sigma_X + \frac{\partial^2 \varpi_i}{\partial w \partial X} \sigma_w' \sigma_X \right\} dt \\ &\quad + \left\{ \frac{\partial \varpi_i}{\partial w} \sigma_w + \frac{\partial \varpi_i}{\partial X} \sigma_X \right\}' dW + \left\{ \varpi_i (w + L_w, X + L_X) - \varpi_i (w, X) \right\} dN(\lambda_i) \\ &\equiv \mu_{\varpi_i} dt + \sigma_{\varpi_i}' dW + L_{\varpi_i} dN(\lambda_i). \end{aligned} \tag{B.2}$$

ϖ_i solves the PDE

$$\begin{aligned} 0 &= e^{-\varpi_i} + \mu_{\xi_i} + \mu_I + \mu_{\varpi_i} + \frac{1}{2} \sigma_{\varpi_i}' \sigma_{\varpi_i} + \sigma_{\xi_i}' \sigma_{\varpi_i} \\ &\quad + \left[(1 + L_I) (1 + L_{C_i})^{-\gamma} e^{(\theta - 1) L_{v_i}} e^{L_{\varpi_i}} - 1 \right] \lambda_i. \end{aligned}$$

C. Trading Volume

The number of shares of asset j held by the pessimistic investor 1 is given by $n_1^j = \frac{\pi_{1,j} V_1}{P^j}$. Its dynamics are

$$\begin{aligned} dn_1^j &= \left\{ \frac{\partial n_1^j}{\partial w} \mu_w + \frac{1}{2} \frac{\partial^2 n_1^j}{\partial w^2} \sigma_w' \sigma_w - \frac{\partial n_1^j}{\partial X} \kappa_X X + \frac{1}{2} \frac{\partial^2 n_1^j}{\partial X^2} \sigma_X' \sigma_X + \frac{\partial^2 n_1^j}{\partial w \partial X} \sigma_w' \sigma_X \right\} dt \\ &\quad + \left\{ \frac{\partial n_1^j}{\partial w} \sigma_w + \frac{\partial n_1^j}{\partial X} \sigma_X \right\}' dW + \left\{ n_1^j (w + L_w, X + L_X) - n_1^j (w, X) \right\} dN(\lambda) \\ &= \mu_{n_1^j} dt + \sigma_{n_1^j}' dW + L_{n_1^j} dN(\lambda), \end{aligned}$$

where $\mu_{n_1^j}$, $\sigma_{n_1^j}$, and $L_{n_1^j}$ denote the drift, the volatility, and jump size of n_1^j.

D. Numerical Implementation

We now briefly describe how we implemented the model in MATLAB, using the corresponding toolbox provided by the Numerical Algorithms Group (NAG).

We solve the model numerically on a two-dimensional grid for the pessimist's consumption share w and the long-run growth rate X. For w we use 41 points over the interval $(0, 1)$, while there are 39 points over the interval $[-0.1560, 0.1440]$ for X.[27] The vectors of grid points are given as follows:

$$\begin{aligned} [w_1, \dots, w_{41}] &= \left[\cos \left(\frac{(2\,s - 1)\,\pi}{2 \cdot 41} \right) + 1 \right] \frac{1}{2} \\ [X_1, \dots, X_{39}] &= \left[\cos \left(\frac{(2\,t - 1)\,\pi}{2 \cdot 39} \right) + 1 \right] \frac{0.1440 + 0.1560}{2} - 0.1560 \end{aligned}$$

with $s = [41, 40, 39, \dots, 2, 1]$ and $t = [39, 38, 37, \dots, 2, 1]$.

1. Complete market

To obtain boundary conditions for the PDE in (9) we study the limiting cases $w \to 0$ and $w \to 1$. In either case we have one very large and one very small investor. This is based

[27] Table B.1 in the Online Appendix shows that the interval we use for our numerical implementation is actually larger than an interval ranging from the 0.0001% to the 99.9999% quantile of the long-run distribution of X. Widening this interval has no impact on the results.

on the assumption that the large investor sets the prices and risk premia, while the small one takes these quantities as exogenous.

When w is very close to zero, we are basically in a one investor economy, so that $\frac{1}{1-w}\,\mu_w$, $\frac{1}{1-w}\,\sigma_w$ and L_w are zero as well. In this case the PDE (9) for the log wealth-consumption ratio of the large investor 2, using Equations (4), (5), and (8), simplifies to

$$\begin{aligned} 0 &= e^{-v_2} - \beta + \left(1 - \frac{1}{\psi}\right) \left[\bar{\mu}_C + X_t - \frac{1}{2}\,\gamma\,\sigma_C'\,\sigma_C\right] - \frac{\partial v_2}{\partial X}\,\kappa_X\,X + \frac{1}{2}\,\frac{\partial^2 v_2}{\partial X^2}\,\sigma_X'\,\sigma_X \\ &\quad + \frac{1}{2}\,\theta\,\left(\frac{\partial v_2}{\partial X}\right)^2\,\sigma_X'\,\sigma_X + (1-\gamma)\,\frac{\partial v_2}{\partial X}\,\sigma_C'\,\sigma_X + \frac{1}{\theta}\,\left[e^{\theta\,[v_2(w,X+L_w)-v_2(w,X_t)]} - 1\right]\,\lambda_2. \end{aligned}$$

We use $v_2 = -\log\left[\beta - \left(1-\frac{1}{\psi}\right)\,\left(\bar{\mu}_C - \frac{1}{2}\,\gamma\,\sigma_C'\,\sigma_C\right)\right]$, the solution for the wealth-consumption ratio in a one-investor economy without a state variable, as starting value for our numerical optimization.

For the small investor the fact that $1-w$ is very close to one implies (based on Equations (A.1), (A.2), and (A.3)) in the limit as w tends to 0 that

$$\begin{aligned} \frac{1}{w}\,\mu_w &= \psi\,\Bigg\{\frac{1}{2}\,\left(1+\frac{1}{\psi}\right)\,\gamma\,\left[\sigma_{C_1}'\,\sigma_{C_1} - \sigma_{C_2}'\,\sigma_{C_2}\right] + \frac{1}{2}\,(1-\theta)\,\left[\sigma_{v_1}'\,\sigma_{v_1} - \sigma_{v_2}'\,\sigma_{v_2}\right] \\ &\quad + (1-\theta)\,\left[\sigma_{C_1}'\,\sigma_{v_1} - \sigma_{C_2}\,\sigma_{v_2}\right] \\ &\quad + \left[e^{(\theta-1)\,[v_1(w,X+L_X)-v_1(w,X)]} - 1 - \left(1-\frac{1}{\theta}\right)\,\left[e^{\theta\,[v_1(w,X+L_X)-v_1(w,X)]} - 1\right]\right]\,\lambda_1 \\ &\quad - \left[e^{(\theta-1)\,[v_2(w,X+L_X)-v_2(w,X)]} - 1 - \left(1-\frac{1}{\theta}\right)\,\left[e^{\theta\,[v_2(w,X+L_X)-v_2(w,X)]} - 1\right]\right]\,\lambda_2\Bigg\} \\ \frac{1}{w}\,\sigma_w &= \frac{1}{\gamma}\,(1-\theta)\,\left(\frac{\partial v_2}{\partial X} - \frac{\partial v_1}{\partial X}\right)\,\sigma_X \end{aligned}$$

and that $v_1\,(w, X+L_X) - v_1\,(w,X) = v_2\,(w,X+L_X) - v_2\,(w,X) - \frac{1}{\theta-1}\,\ln\left(\frac{\lambda_1}{\lambda_2}\right)$. Thus the PDE is given by

$$\begin{aligned} 0 &= e^{-v_1} - \beta + \left(1-\frac{1}{\psi}\right)\,\left[\bar{\mu}_C + X_t + \frac{1}{w}\,\mu_w - \frac{1}{2}\,\gamma\,\left(\sigma_C'\,\sigma_C - \frac{1}{w^2}\,\sigma_w'\,\sigma_w\right)\right] - \frac{\partial v_1}{\partial X}\,\kappa_X\,X \\ &\quad + \frac{1}{2}\,\frac{\partial^2 v_1}{\partial X^2}\,\sigma_X'\,\sigma_X + \frac{1}{2}\,\theta\,\left(\frac{\partial v_1}{\partial X}\right)^2\,\sigma_X'\,\sigma_X + (1-\gamma)\,\frac{\partial v_2}{\partial X}\,\left(\sigma_C + \frac{1}{w}\,\sigma_w\right)'\,\sigma_X \\ &\quad + \frac{1}{\theta}\,\left[e^{\theta\left[v_2(w,X+L_w)-v_2(w,X_t)-\frac{1}{\theta-1}\,\ln\left(\frac{\lambda_1}{\lambda_2}\right)\right]} - 1\right]\,\lambda_2. \end{aligned}$$

Again we rely on $v_1 = -\log\left[\beta - \left(1-\frac{1}{\psi}\right)\,\left[\bar{\mu}_C - \frac{1}{2}\,\gamma\,\sigma_C'\,\sigma_C\right]\right]$ as starting value for our numerical optimization. The maximum errors in the solutions of the investor-specific partial differential equations (9) are always less that 10^{-6}.

2. Incomplete market

On an incomplete market we also need starting values for the optimization problem described in Section A.2. In addition to the complete market solution we use $\nu = \frac{1}{2}\,\upsilon_1 + \frac{1}{2}\,\upsilon_2$ and $\pi_1^C = 1$ (for $w < 0.5$) resp. $\pi_2^C = 1$ (for $w \leq 0.5$). The maximum errors in the solutions for the investor-specific partial differential equations (9) and (A.9), the equilibrium conditions (13) and the conditions for the portfolio weights in Equations (A.10) to (A.12) are always smaller than 10^{-13}.

E. Auxiliary Quantitative Results

1. Wealth-consumption ratios

Due to the recursive utility specification wealth-consumption ratios are key ingredients to asset pricing. The aggregate and individual wealth-consumption ratios on a complete market are shown in the upper row of Figure 16. Looking at the dependence on the pessimist's consumption share w first we see that when the optimist becomes small (i.e., when w tends to 1), she consumes less and saves more. The analogous logic (now for w going to 0) applies to the pessimist which is represented by the dotted line, although the pessimist reacts in a much less extreme fashion than the optimist. The aggregate wealth-consumption ratio is downward sloping in w, since the optimist would save more in the respective single investor economy (left boundary) than the pessimist (right boundary). The right graph in the upper row confirms the intuition that a higher long-run growth rate implies more attractive investment opportunities which lead to less consumption and higher savings. The slope of all three curves in this graph is about the same, so the optimist and the pessimist react in pretty much the same relative fashion to changes in X, although the level is higher for the optimist.

In terms of the dependence on X the results for the incomplete market are very similar (right graph in the lower row). Concerning the dependence on w, however, incompleteness matters, at least for the pessimist. Her individual wealth-consumption ratio is now increasing in w. Since the insurance products are not present anymore, saving becomes so unattractive for the pessimist that, even in the face of extinction, she still prefers to consume more than when she is large. Also the optimist is affected, but to a much lesser degree than the pessimist, and also for the market as a whole the results are quite similar to the case of completeness.

2. Consumption share dynamics

The upper row in Figure 17 shows (from left to right) the coefficients μ_w, $\sigma_{w,C}$, $\sigma_{w,X}$, and L_w of the consumption share dynamics from Equation (3) on a complete market. The curves for the optimist, the pessimist, and the aggregate market are all identical, so that there is only one line in the graphs. Note that the boundary values for $w = 0$ and $w = 1$ are equal to zero.

The graph for μ_w shows that in times without jumps the pessimist's consumption share decreases on average due to the compensation for risk sharing. Since jumps increase the pessimist's consumption share due to the payoffs from the associated insurance contract, the average compensation in times without jumps has to be negative.

As we can see from the graph for $\sigma_{w,C}$, consumption risk is not shared, since the investors have identical beliefs with respect to this source of risk. So the investors' consumption shares remain unchanged following a consumption shock. Also the reaction of w to a diffusive shock in X is not very pronounced, as we can see from the graph for $\sigma_{w,X}$. The small non-zero values for larger w are due to the fact that the optimist reacts stronger to an increase in the long-run growth rate than the pessimist as can be seen from the upper right graph in Figure 16.

The picture is quite different for jumps in X. For $w \approx 0.5$ the reaction to jumps is in the order of 10%. When a disaster strikes the long-run growth rate in the economy drops and due to the less attractive investment opportunities, both investors save less and consume more. With the pessimist's reaction being much stronger than the optimist's, the term $L_{v_1} - L_{v_2}$ in Equation (A.3) is negative, leading to the shape of the curve shown in the graph. So on a complete market investors almost exclusively share jump risk.

The upper row of Figure 18 shows the results for the incomplete markets case. Compared to the complete market the situation changes significantly. The reactions to the two types of diffusion risk become much more pronounced, whereas the reaction to jumps becomes much smaller. The reason is that on an incomplete market the only risky asset which the pessimist can use to reduce her jump exposure is the consumption claim. Reducing this exposure by reducing the amount of wealth invested in the consumption claim *automatically* implies a reduction in the diffusive exposure as well, so that in the end the investors mainly share diffusive risk. Of course, as indicated by the first graph in this row, the pessimist still accepts a decrease in her consumption share on average.

3. *Risk-free rate and market prices of risk*

On a complete market the investors have to agree on the risk-free rate, on the market prices for the diffusion risks W^C and W^X, and on the risk-neutral jump intensity, which are shown from left to right in the upper row of Figure 19.

The graph for the risk-free rate from Equation (13) shows that precautionary savings due to jump risk overcompensate the impact of the individual consumption growth rate for the optimist (and vice versa for the pessimist). Overall the risk-free rate decreases slightly in w and varies between 0.6% and 1.1%.

Next, the market price of risk for W^C is constant in w. Since X does not load on W^C and the investors do not share consumption risk, we end up with the usual CRRA result that the market price of risk is equal to $\gamma\,\sigma_C$ for both investors.

Also the market price of risk for W^X is basically a constant. It decreases only very slowly in w, and this is due to the fact that in the respective one-investor economies for $w = 0$ and $w = 1$ there is also only a small difference between the respective market prices of risk, since investors only disagree on the jump intensity.

While disagreement about the jump intensity has a negligible impact on the market prices of diffusion risk, it has a dramatic effect on the risk-neutral jump intensity $\lambda^{\mathbb{Q}}$, which ranges from below 1% in an all-optimist economy to almost 16% for $w = 1$. The investors' subjective market prices for *positive* jump exposure can be determined by comparing the risk-neutral with the subjective jump intensities, and here we see significant differences between the optimist and the pessimist. While the optimist has a negative market price of risk throughout, the sign switches for the pessimist, once she has reached a certain size in the economy.

The lower row of graphs in Figure 19 presents the result for the incomplete markets case. First, the risk-free rate on the incomplete market is basically indistinguishable from the one on the complete market. Next, the market price of consumption risk, represented by the first component of the vector shown in Equation (11), is mainly driven by the 'market price of risk for relative size', i.e., the influence of σ_w on individual consumption and on the log wealth-consumption ratio, which is negative for the pessimist and positive for the optimist. These terms are increasing in w, and so is the market price of consumption risk. Note that now, on an incomplete market, the individual market prices of risk no longer coincide. The story is basically the same for the market price of diffusive risk in X, only the numbers are different.

Finally, the pronounced differences between the optimist's and the pessimist's risk-neutral jump intensities are mostly determined by the different physical jump intensities assumed by the investors. In contrast to the complete markets case the market price of jump risk is now negative for both investors across the full range of w (and more negative for the pessimist). Both investors' risk-neutral jump intensities are much closer to linear in w than on the complete market and are increasing much less in w.

4. Wealth exposures

To analyze the properties of the return on individual and aggregate wealth we go back to Figure 17 and look at the lower row of graphs, which show the drift and sensitivity of these returns with respect to diffusive consumption risk, diffusive growth rate risk, and jump risk (see Equation (A.4)).

The exposure of all the returns to consumption risk is constant and equal to σ_c, again since X does not load on W^C and the investors do not share consumption risk. They do, however, share the risk of diffusive shocks in the long-run growth rate, and this is why we see in the third graph that the exposure of the optimist's wealth is decreasing in w, and the opposite is true for the pessimist. In the aggregate the exposure to diffusive growth rate risk decreases slightly.

In terms of the jump exposure of individual and aggregate wealth we see that the sensitivity of the optimist's wealth to jumps in X is always negative, whereas the pessimist's is mostly positive, but also becomes negative when she is sufficiently large. Both exposures decrease in w, whereas in the aggregate the jump sensitivity is more or less constant.

The graph at the very left of the lower panel of Figure 17 illustrates the average return on wealth in times without jumps. We have already seen that on a complete market, investors mainly share jump risk, so that the drift is mainly a compensation for diffusive growth rate risk and jump risk.

The return on individual wealth and aggregate wealth on an incomplete market are presented in the lower row of Figure 18. The second and third graph show the exposures to consumption risk and to diffusive shocks in the long-run growth rate. The curves look very similar due to the similar way in which the investors share these two sources of risk by shifting exposures from the pessimist to the optimist. This is also true for jump risk. Concerning the drift of the wealth process the pessimist has to compensate the optimist for sharing mainly diffusion risks and a small amount of jump risk. Therefore, the optimist's wealth increases in times without jumps, whereas it decreases for the pessimist.

Whether the market is complete or not obviously only has a very small effect on the results for aggregate wealth. It does, however, matter for the properties of the investors' individual wealth processes. On a complete market the investors mainly share mainly jump risk, which shows up in the wide range of the jump sizes of individual wealth as a function of w.

On an incomplete market the investors cannot adjust the exposures to the different sources of risk separately because they only have access to the money market account and the claim on aggregate consumption. So if the pessimist reduces her jump exposure this means both diffusion exposures will decrease automatically. Since jump risk is only a small fraction of the risk embedded in the aggregate consumption claim while it is mainly influenced by diffusion risk, we see large differences in the diffusion coefficients of the return on individual wealth.

5. *Expected Excess Return on Individual Wealth*

Besides the equity risk premium, Figure 2 also shows the expected excess returns on individual wealth both on a complete (upper row) and an incomplete market (lower row). From left to right, the graphs give the risk premia due to diffusive consumption risk, diffusive growth rate risk, and growth rate jump risk, and finally the total expected excess return.

On the complete market the premium on diffusive consumption risk coincides with the usual premium we would also obtain in a CRRA economy.

Since both investors agree on the market price of risk for diffusive shocks to X (which is hardly varying with w), the small differences between the two premia are caused by the corresponding small differences in exposures due to risk sharing. The overall size of this premium is around 3.5% for both investors and thus accounts for a large part of the overall expected excess return.

The premia for jump risk differ significantly between the two investors. The premium earned by the optimist (under the true measure) is the larger the smaller her consumption share, i.e., the more valuable the insurance against jump risk which she offers to the pessimist. This premium is negative for small w, which may seem surprising. The reason is that the premium is shown here under the true measure (i.e., the pessimist's beliefs), while under the optimist's own beliefs it would of course be positive.

The premium for the pessimist is more involved. When she becomes larger, her exposure to jump risk switches sign from positive to negative (for $w \approx 0.25$), and the market price of jump risk also changes sign from positive to negative (for $w \approx 0.8$). This results in a

U-shaped premium for jump risk, which is positive for rather small and large consumption shares of the pessimist and negative in between.

On the incomplete market the premia on consumption risk and diffusive shocks in X reflect the exposures to consumption risk and X-risk from Figure 18. They are increasing in w and larger for the optimist (who provides insurance to the pessimist).

The premia on jump risk are shown in the third graph. The positive premium earned by the pessimist is the smaller the smaller her consumption share, reflecting the fact that she can reduce her exposure to jump risk best when her consumption share is small. As in the complete market case, the premium on jump risk is negative for the optimist under the true measure (but still positive under her subjective measure), since her subjective risk-neutral jump intensity is less than the true jump intensity, but still greater than her subjective physical jump intensity.

The total expected excess return (fourth graph) inherits the linear shape from its components. The optimist's excess return is always higher than the pessimist's since investors mainly share diffusion risk, on which the optimist earns the larger premium.

The impact of incompleteness on expected excess returns is small for the claim to aggregate consumption, but can be rather large for individual wealth, in particular for when the pessimist is small. When she has access to insurance products she is able to offer some risk sharing to the optimist when the optimist is large. Since this is not possible on an incomplete market the pessimist's excess return is up to 40% smaller here than when the market is complete.

F. The 'extended' Chen et al. (2012) model with recursive preferences

As in Section II we consider two investors with identical EZ preferences. Her individual value function is given in Equation (1) and her normalized aggregator function in Equation (2).

Under the true probability measure $\mathbb{P}$ aggregate consumption follows

$$\frac{dC_t}{C_t} = \mu_C \, dt + \sigma_C \, dW_t + L_C \, dN_t\,(\lambda)$$

where W is a one-dimensional standard Brownian motion and N represents a Poisson process with constant intensity λ and constant jump size L_C. Both investors agree on all parameters of the model except the intensity of the Poisson process. This implies that under investor i's subjective probability measure $\mathbb{P}^i$ aggregate consumption evolves as

$$\frac{dC_t}{C_t} = \mu_C\, dt + \sigma_C\, dW_t + L_C\, dN_t\left(\lambda_i\right).$$

The investors can trade the claim on aggregate consumption, the money market account, and an 'insurance product' linked to the jump component. The consumption claim is in unit net supply, while the other two assets are in zero net supply.

To solve for the equilibrium, we proceed in a similar way as in Section III. The dynamics of the pessimistic investor's consumption share are given by

$$dw = \mu_w\left(w\right)\, dt + \sigma_w\left(w\right)\, dW + L_w\left(w\right) dN\left(\lambda_1\right), \tag{F.1}$$

where all coefficients will be determined in equilibrium. The dynamics of investor 1's and investor 2's level of consumption then follow from Ito's lemma:

$$\begin{aligned}
\frac{dC_1}{C_1} &= \left\{\mu_C + \frac{1}{w}\,\mu_w + \frac{1}{w}\,\sigma_w\,\sigma_C\right\} dt + \left\{\sigma_C + \frac{1}{w}\,\sigma_w\right\} dW + \left\{\frac{L_w}{w} + L_C\left(1 + \frac{L_w}{w}\right)\right\} dN_t\left(\lambda_1\right) \\
&\equiv \mu_{C_1}\, dt + \sigma_{C_1}\, dW + L_{C_1}\, dN\left(\lambda_1\right) \\
\frac{dC_2}{C_2} &= \left\{\mu_C - \frac{1}{1-w}\,\mu_w - \frac{1}{1-w}\,\sigma_w\,\sigma_C\right\} dt + \left\{\sigma_C - \frac{1}{1-w}\,\sigma_w\right\} dW \\
&\quad + \left\{-\frac{L_w}{1-w} + L_C\left(1 - \frac{L_w}{1-w}\right)\right\} dN\left(\lambda_2\right) \\
&\equiv \mu_{C_2}\, dt + \sigma_{C_2}\, dW + L_{C_2}\, dN\left(\lambda_2\right).
\end{aligned}$$

The dynamics of investor i's log wealth-consumption ratio $v_i = v_i\left(w\right)$ are given as

$$\begin{aligned}
dv_i &= \left\{\frac{\partial v_i}{\partial w}\,\mu_w + \frac{1}{2}\,\frac{\partial^2 v_i}{\partial w^2}\,\sigma_w\,\sigma_w\right\} dt + \left\{\frac{\partial v_i}{\partial w}\,\sigma_w\right\} dW + \left\{v_i\left(w + L_w\right) - v_i\left(w\right)\right\} dN_t\left(\lambda_i\right) \\
&\equiv \mu_{v_i}\, dt + \sigma_{v_i}\, dW + L_{v_i}\, dN\left(\lambda_i\right).
\end{aligned}$$

Following the other steps explained in Section III, leads to the same PDE for v_i as in Equation (9). The dynamics pricing kernel ξ_i of investor i at time t coincide with those in Equation (10), so that the investor-specific market prices of diffusion risk η_i, the market prices of jump risk η_i^N, and the subjective risk-free rate r_i^f given in Equations (11)-(13) coincide, too.

Both investors' market prices of diffusion risk coincide if $\sigma_w = 0$, i.e., investors share no diffusive risks.[28] μ_w follows from equating both investors' risk-free rates and has the same form as in Equation (A.2). L_w is determined by equating the risk-neutral jump intensity $\lambda_i^{\mathbb{Q}} = \lambda_i \left(1 + \eta_i^N\right)$ for both investors and coincides with Equation (A.3). The equilibrium solution is then found by simultaneously solving the two PDEs in (9) for v_1 and v_2 using the equations for μ_w, σ_w, and L_w.

Given these coefficients as well as the individual wealth-consumption ratios, we can then compute other equilibrium quantities. As for example, the total return (including consumption) on investor i's wealth $V_i = C_i\, e^{v_i}$ follows the process shown in Equation (A.4).

While the formulas for the equity premium, the individual expected excess returns on wealth, and the trading volume given in Equations (14) to (16) remain unchanged, the number of shares of asset j held by investor 1 now follows

$$\begin{aligned} dn_1^j &= \left\{ \frac{\partial n_1^j}{\partial w}\, \mu_w + \frac{1}{2}\, \frac{\partial^2 n_1^j}{\partial w^2}\, \sigma_w^2 \right\} dt + \left\{ \frac{\partial n_1^j}{\partial w}\, \sigma_w \right\}' dW + \left\{ n_1^j \left(w + L_w\right) - n_1^j \left(w\right) \right\} dN\left(\lambda\right) \\ &= \mu_{n_1^j}\, dt + \sigma_{n_1^j}\, dW + L_{n_1^j}\, dN\left(\lambda\right). \end{aligned}$$

The volatility of the return on the aggregate consumption, RV, also changes slightly to

$$RV = \sqrt{\left(\sigma_C + \sigma_v\right)^2 + \lambda\, \left[\left(1 + L_C\right)\, e^{L_v} - 1\right]^2}. \tag{F.2}$$

[28] An alternative condition which may lead to coinciding market prices of risk is $0 = \frac{\gamma}{w\,(1-w)} + (1-\theta)\left[\frac{\partial v_1}{\partial w} - \frac{\partial v_2}{\partial w}\right]$. Our numerical results show, however, that this does not hold for the parameterizations we look at.

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	Dieckmann (2011)		Chen et al. (2012)	Our paper	
Preferences	log utility		CRRA utility	recursive utility	
Disaster in	aggregate consumption		aggregate consumption	long-run growth rate	
Market	complete	incomplete	complete	complete	incomplete
Positive correlation between return volatility and trading volume (Karpoff (1987))					
	✗	✗	✗	✓	✓
Higher disagreement leads to higher expected returns, higher return volatility, and higher trading volume (Carlin et al. (2014))					
Expected Return	✗	✗	✗	✗	✓
Return Volatility	✗	✗	✗	✓	✓
Trading Volume	✓	✓	✓	✓	✓

Table 1. Overview of Results

The table provides an overview of the results generated by our model and the related approaches suggested by Dieckmann (2011) and Chen et al. (2012) with respect to the link between disagreement, return volatility, trading volume and expected returns. The relevant empirical findings are presented in the papers by Karpoff (1987) and Carlin et al. (2014). In the table '✓' ('✗') indicates that the respective model produces (cannot procuce) a pattern like the one observed empirically.

Investor preferences		
Relative risk aversion	γ	10
Elasticity intertemporal of substitution	ψ	1.5
Subjective discount rate	β	0.02
Aggregate consumption		
Expected growth rate of aggregate consumption	$\bar{\mu}_C$	0.02
Volatility of aggregate consumption	σ_C	0.0252
Stochastic growth rate		
Mean reversion speed	κ_X	0.1
Volatility	σ_x	0.0114
Jump size	L_X	-0.03
Jump intensity of the pessimistic investor 1	λ_1	0.020
Jump intensity of the optimistic investor 2	λ_2	0.001
Further parameters		
Leverage factor for dividends	ϕ	1.3
Drift of insurance product Z	μ_Z	-0.1
Volatility of insurance product Z	σ_Z	0.001
Drift of insurance product I	μ_I	-0.1
Jump size of insurance product I	L_I	0.01

Table 2. Parameters

The table reports the basecase parametrization of our economy. These parameters are held constant throughout the paper, except γ and ψ for which we also look at different values in Section IV.D. We assume that the pessimist's belief represents the true model.

T (years)	Complete market	Incomplete market
50	0.349	0.250
100	0.260	0.111
200	0.161	0.019
500	0.056	0.000
1,000	0.013	0.000

Table 3. Investor survival

The table shows the pessimist's expected consumption share $E[w_T]$ for T years into the future under the true measure. The expectation is computed via a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (3) with a starting value of $w_0 = 0.5$. The coefficients μ_w, σ_w, and L_w are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters for our model are given in Table 2.

	Chen et al. (2012)	
T (years)	CRRA	EZ
50	0.588	0.3866
100	0.661	0.3287
200	0.777	0.2736
500	0.934	0.2248
1,000	0.992	0.2162

Table 4. Investor survival in Chen et al. (2012) and our recursive utility version of their model

The table shows the pessimist's expected consumption share $E[w_T]$ for T years into the future under the true measure. The second (third) column gives the results in the model proposed by Chen et al. (2012) and our 'extended' version of their model in which CRRA utility ($\gamma = 4, \psi = \frac{1}{4}$) has been replaced by recursive preferences ($\gamma = 4, \psi = 1.5$). The expectation is computed via a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (F.1) with a starting value of $w_0 = 0.5$. The coefficients μ_w, σ_w, and L_w are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution.

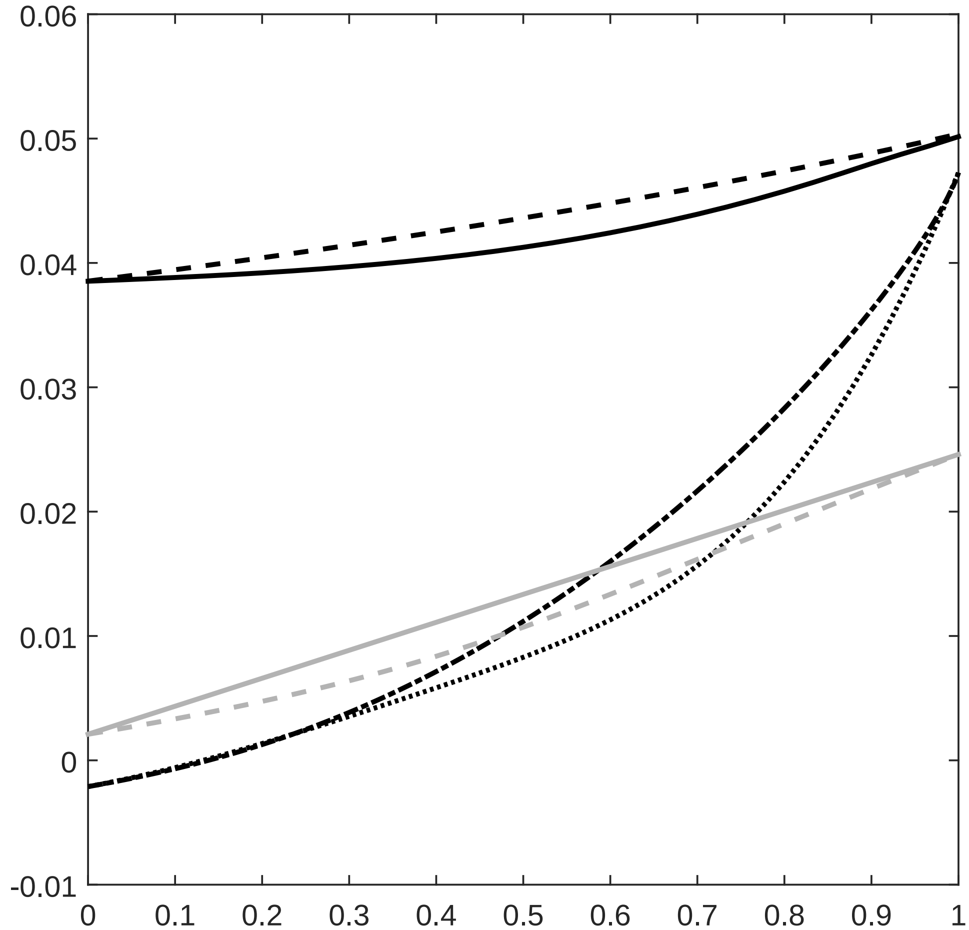

Figure 1. Equity premium in different models

The figure shows the equity premium under the respective true probability measure (y-axis) for different heterogeneous investor models as a function of the pessimist's consumption share (x-axis). The different lines in the picture represent the following models:

- original Chen et al. (2012) model with jumps in aggregate consumption and CRRA utility ($\gamma = 4, \psi = \frac{1}{4}$) — black dotted line
- 'extended' Chen et al. (2012) model with jumps in aggregate consumption and recursive preferences ($\gamma = 4, \psi = 1.5$) — black dashed-dotted line
- Dieckmann (2011) model with jumps in aggregate consumption and log utility on complete (incomplete) market — gray solid (dashed) line
- our model with jumps in expected growth rate of aggregate consumption and recursive preferences ($\gamma = 10, \psi = 1.5$) on complete (incomplete) market — black solid (dashed) line

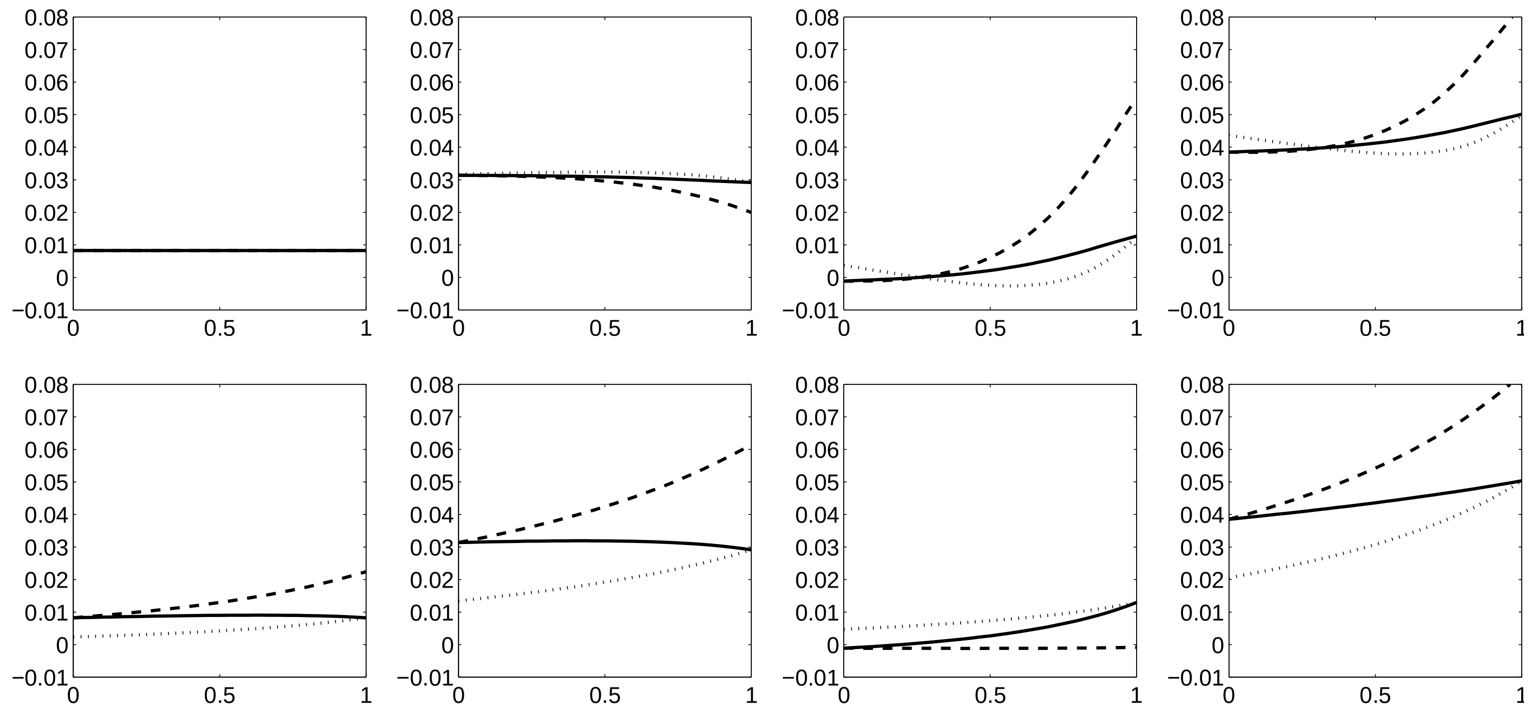

Figure 2. Equity premium and its components

The figure shows (from left to right) the equity premium due to diffusive consumption risk, diffusive growth rate risk, and jump risk, as well as the total equity premium given by the sum of the three components (see Equation (15)). The first and the second component of $\phi\,\sigma_{V_i}'\,\eta_i^W$ give the equity premium due to diffusive consumption risk and diffusive growth rate risk. The part due to jump risk is determined via $\phi\,L_{V_i}\,(\lambda_i - \lambda_i^{\mathbb{Q}})$. The dotted (dashed) line depicts the expected excess return on the individual wealth of the pessimist (optimist) and the solid line the excess return on aggregate wealth as defined in Equation (14). The upper (lower) row of graphs shows the results for the complete (incomplete) market. All quantities are determined under the true measure and shown as functions of the pessimist's consumption share with the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$. The parameters are given in Table 2.

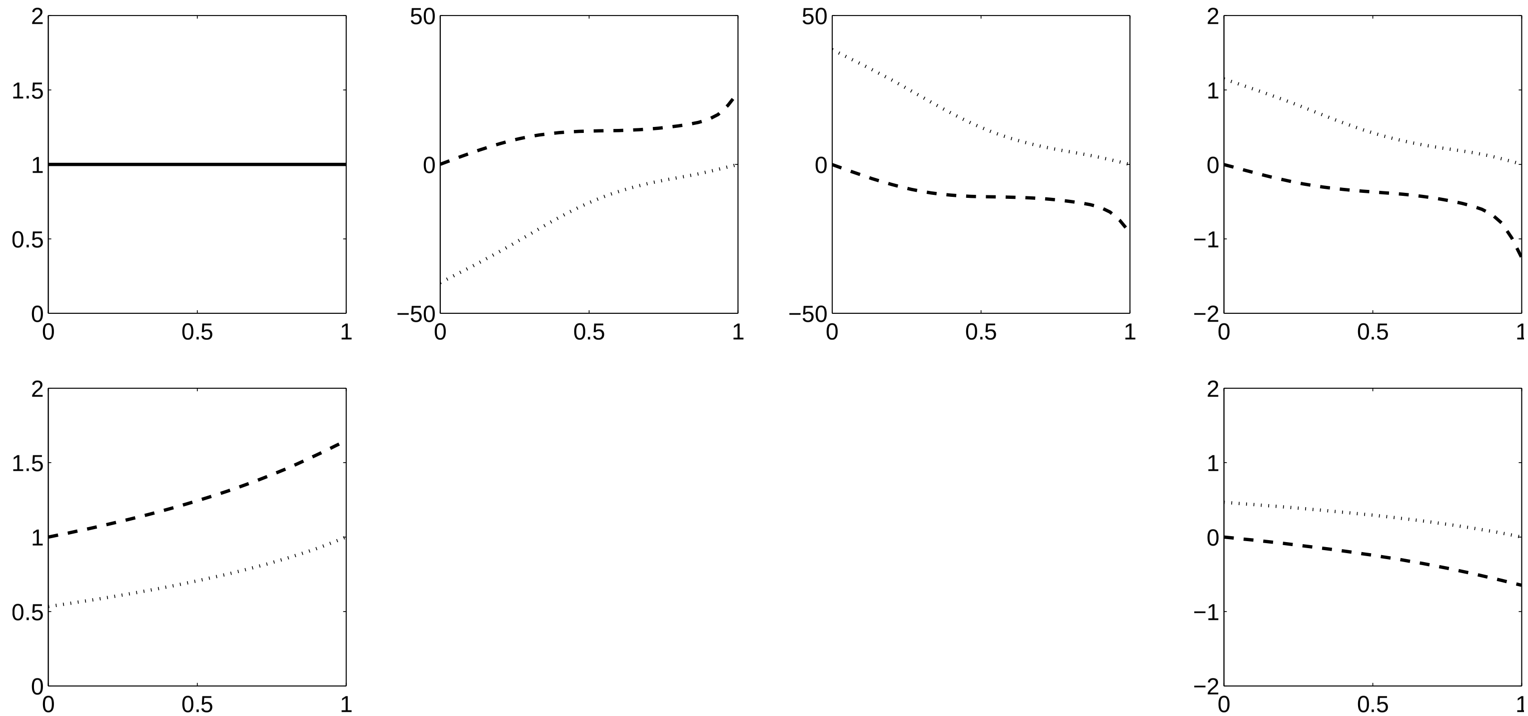

Figure 3. Asset holdings

The figure shows the investors' asset holdings on the complete (upper row) and the incomplete market (lower row), respectively. In the upper row the graphs show from left to right show the fraction of wealth invested in the consumption claim, $\pi_{i,C}$, the diffusion insurance product Z, $\pi_{i,Z}$, the jump insurance product I, $\pi_{i,I}$, and the money market account, $\pi_{i,M}$, which can be determined using Equations (A.6)-(A.8). In the lower row the left (right) graph is for the consumption claim, $\pi_{i,C}$ (money market account, $\pi_{i,M}$). Both quantities follow from Equations (A.10)-(A.12). The pessimist's (optimist's) portfolio holdings are indicated by the dotted (dashed) line. All quantities are determined under the true measure and shown as functions of the pessimist's consumption share with the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$. The parameters are given in Table 2.

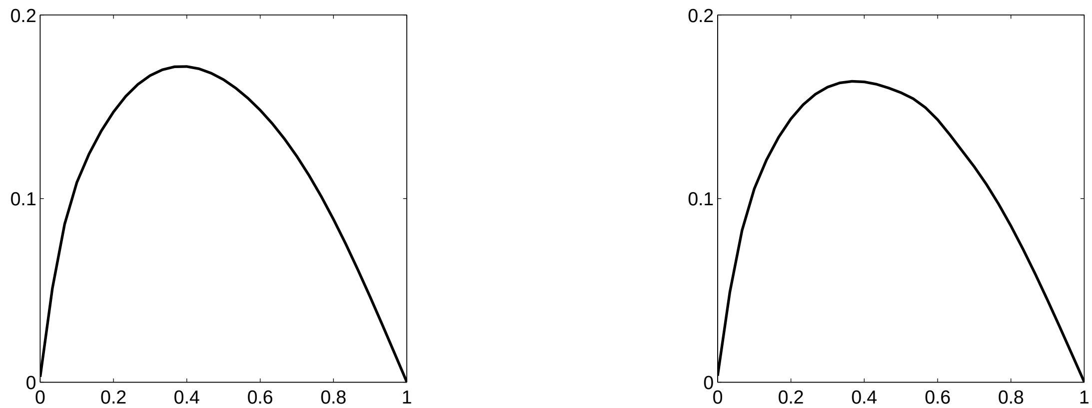

Figure 4. Trading volume in the insurance assets

The figure shows the trading volume in the insurance products Z (on the left) and I (on the right) as defined in Equation (16). All quantities are determined under the true measure and shown as functions of the pessimist's consumption share with the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$. The parameters are given in Table 2.

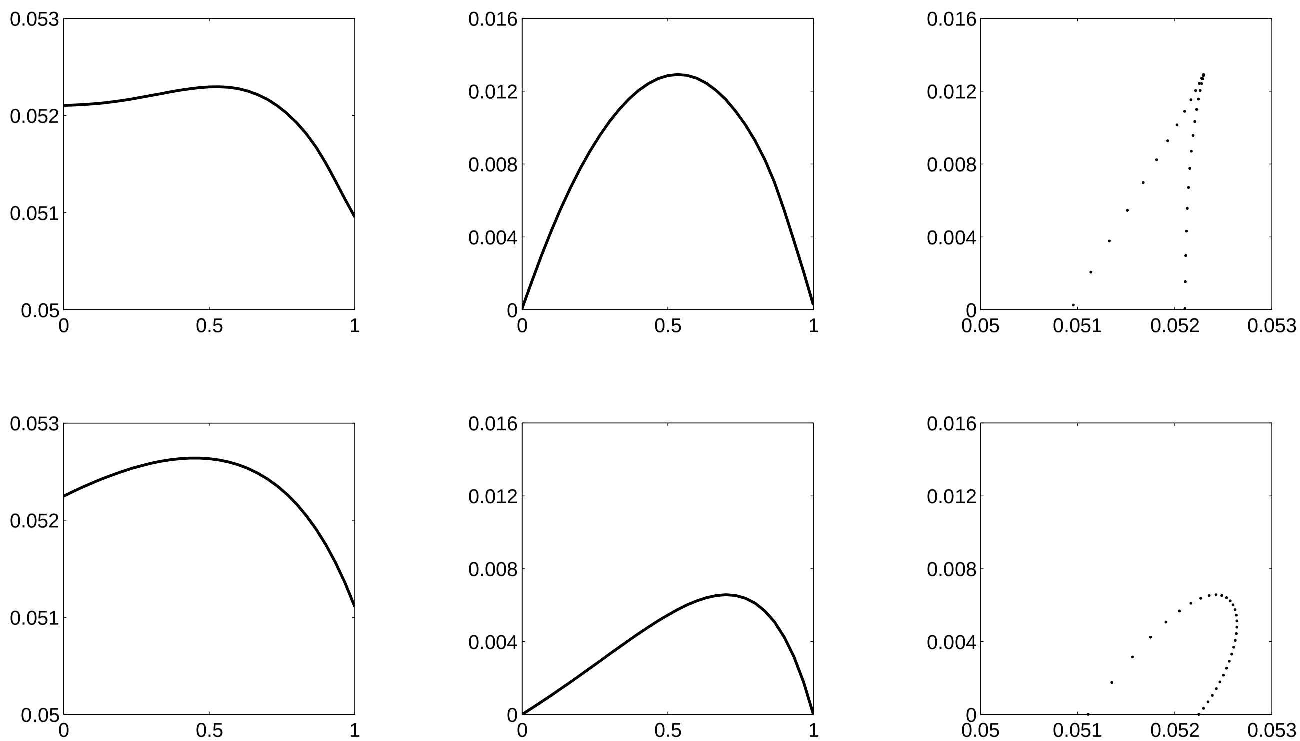

Figure 5. Return volatility and trading volume in our model

The figure depicts (from left to right) the return volatility of the consumption claim, as defined in Equation (16), the trading volume in the consumption claim, as defined in Equation (17), and a scatter plot showing the relation between these two quantities. The upper (lower) row of graphs gives our results for the complete (incomplete) market. The graphs in the first and second column are depicted as functions of the pessimist's consumption share, whereas those in the third column are shown as functions of the return volatility of the consumption claim. All quantities are determined under the true measure and for the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$. The parameters are given in Table 2.

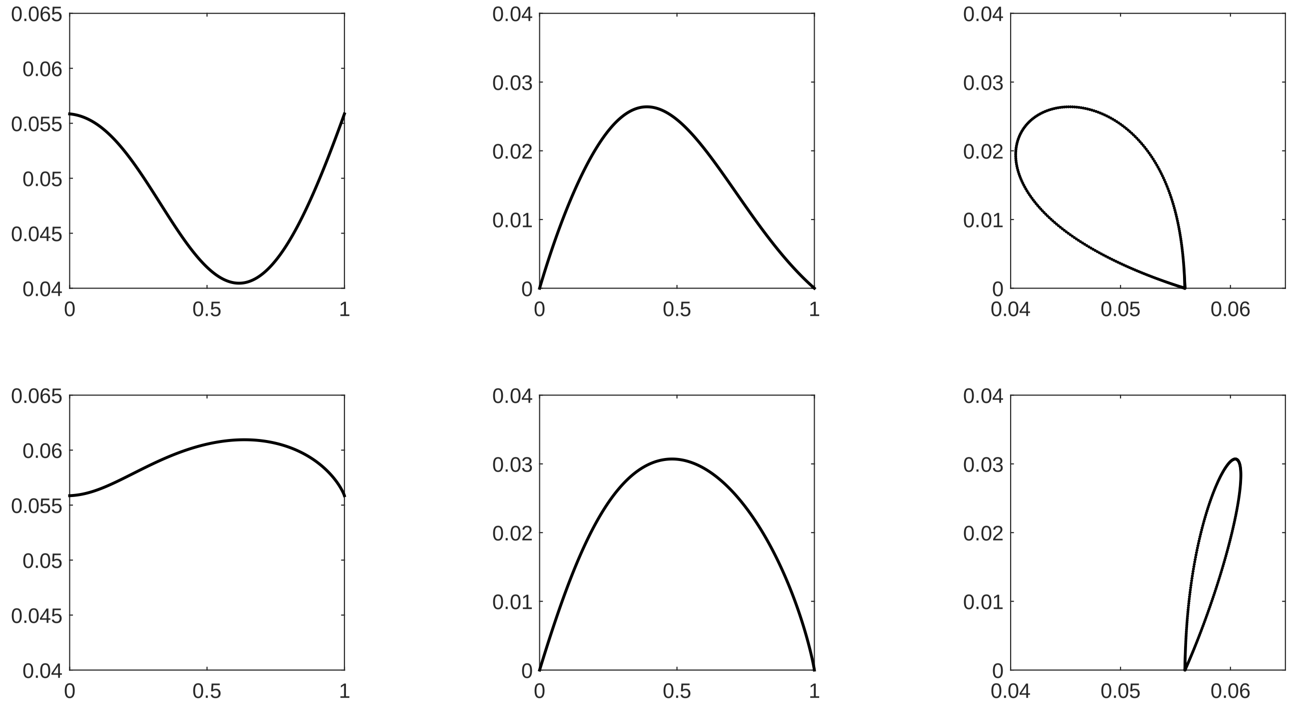

Figure 6. Return volatility and trading volume in Chen et al. (2012) and our recursive utility version of their model

The figure depicts (from left to right) the return volatility of the consumption claim, as defined in Equation (F.2), the trading volume in the consumption claim, as defined in Equation (16), and a scatter plot showing the relation between these two quantities. The upper (lower) row of graphs gives the results in the model proposed by Chen et al. (2012) and our 'extended' version of their model in which CRRA utility ($\gamma = 4, \psi = \frac{1}{4}$) has been replaced by recursive preferences ($\gamma = 4, \psi = 1.5$). The graphs in the first and second column are depicted as functions of the pessimist's consumption share, whereas those in the third column are shown as functions of the return volatility of the consumption claim. All quantities are determined under the true measure.

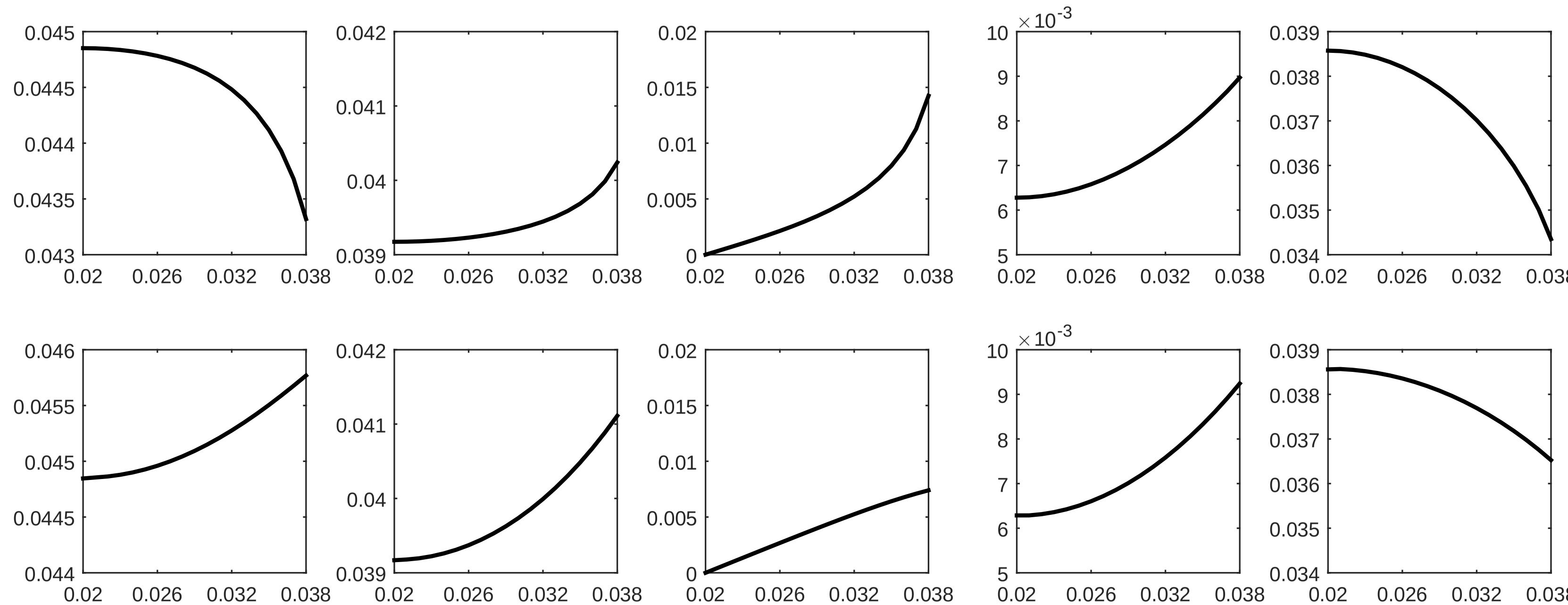

Figure 7. Varying the degree of disagreement in our model

The figure depicts (from left to right) the expected return, the return volatility of the consumption claim, the trading volume in the consumption claim, the risk-free rate, and the equity risk premium. The upper (lower) row of graphs gives our results on the complete (incomplete) market. The return volatility of the consumption claim is defined in Equation (17), the trading volume in the consumption claim in Equation (16), the risk-free rate in Equation (13), and the equity risk premium in Equation (14). The expected return is calculated as the sum of the risk-free rate and the equity risk premium. All quantities are determined under the true measure and shown as functions of the pessimist's belief λ_1. The optimist's belief is calculated such that the average belief $\bar{\lambda} = 0.0200$, which represents the true measure, remains unchanged. The model is solved on a grid for the consumption share w and the long-run growth rare X with $w \in (0, 1)$ and $X \in [-0.1560, 0.1440]$, where the range of X is determined via a Monte Carlo simulation. The quantiles of X are shown in Table B.1 in the online appendix. The figures show the quantities from the solution of the model evaluated at the grid point $w = 0.5$ and $X = -0.0060$. The parameters are given in Table 2.

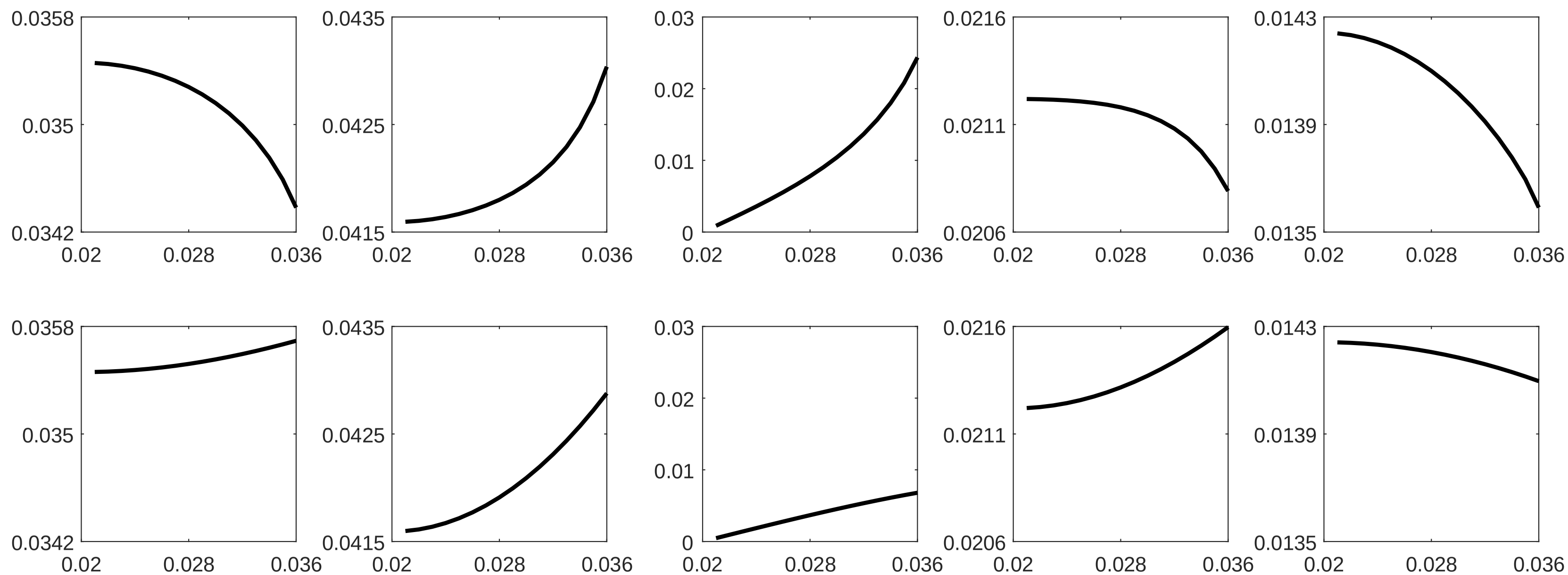

Figure 8. Varying the degree of disagreement in our model for $\gamma = 4$ and $\psi = 1.5$

The figure depicts (from left to right) the expected return, the return volatility of the consumption claim, the trading volume in the consumption claim, the risk-free rate, and the equity risk premium for $\gamma = 4$ and $\psi = 1.5$. The upper (lower) row of graphs gives our results on the complete (incomplete) market. The return volatility of the consumption claim is defined in Equation (17), the trading volume in the consumption claim in Equation (16), the risk-free rate in Equation (13), and the equity risk premium in Equation (14). The expected return is calculated as the sum of the risk-free rate and the equity risk premium. All quantities are determined under the true measure and shown as functions of the pessimist's belief λ_1. The optimist's belief is calculated such that the average belief $\bar{\lambda} = 0.0200$, which represents the true measure, remains unchanged. The model is solved on a grid for the consumption share w and the long-run growth rare X with $w \in (0, 1)$ and $X \in [-0.1560, 0.1440]$, where the range of X is determined via a Monte Carlo simulation. The quantiles of X are shown in Table B.1 in the online appendix. The figures show the quantities from the solution of the model evaluated at the grid point $w = 0.5$ and $X = -0.0060$. The remaining parameters are given in Table 2.

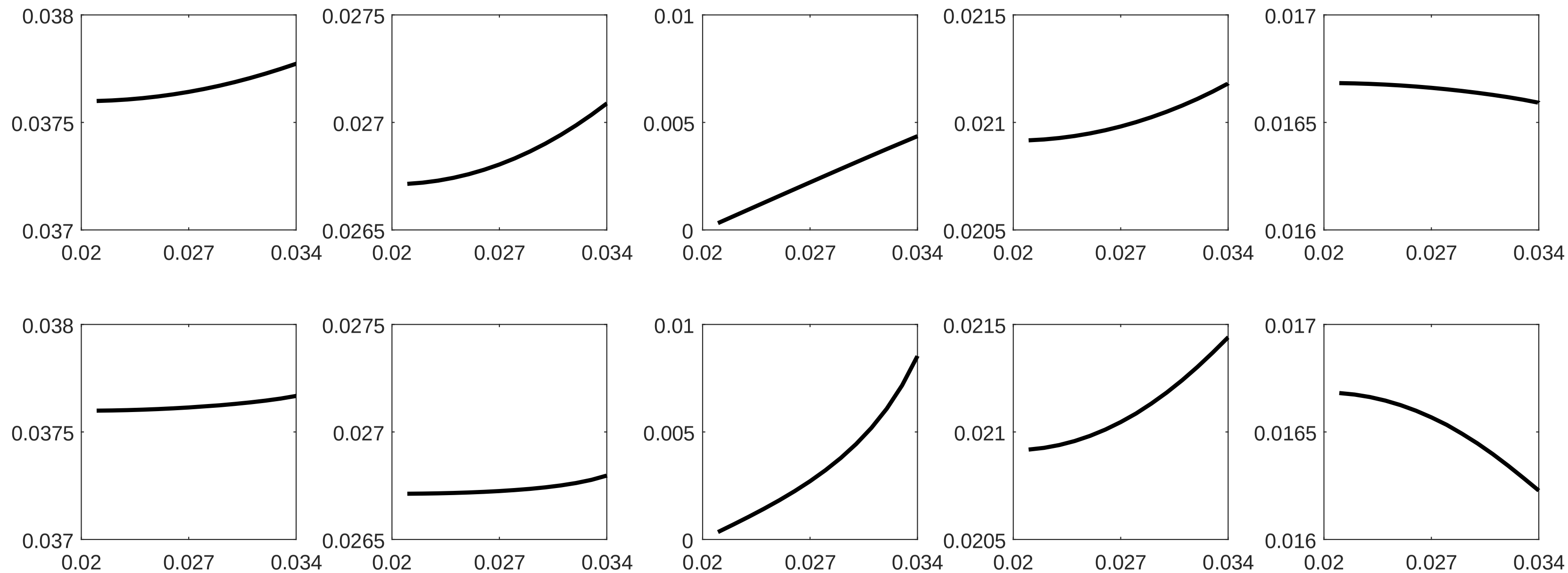

Figure 9. Varying the degree of disagreement in our model for $\gamma = 10$ and $\psi = 1.1$

The figure depicts (from left to right) the expected return, the return volatility of the consumption claim, the trading volume in the consumption claim, the risk-free rate, and the equity risk premium for $\gamma = 10$ and $\psi = 1.1$. The upper (lower) row of graphs gives our results on the complete (incomplete) market. The return volatility of the consumption claim is defined in Equation (17), the trading volume in the consumption claim in Equation (16), the risk-free rate in Equation (13), and the equity risk premium in Equation (14). The expected return is calculated as the sum of the risk-free rate and the equity risk premium. All quantities are determined under the true measure and shown as functions of the pessimist's belief λ_1. The optimist's belief is calculated such that the average belief $\bar{\lambda} = 0.0200$, which represents the true measure, remains unchanged. The model is solved on a grid for the consumption share w and the long-run growth rare X with $w \in (0, 1)$ and $X \in [-0.1560, 0.1440]$, where the range of X is determined via a Monte Carlo simulation. The quantiles of X are shown in Table B.1 in the online appendix. The figures show the quantities from the solution of the model evaluated at the grid point $w = 0.5$ and $X = -0.0060$. The remaining parameters are given in Table 2.

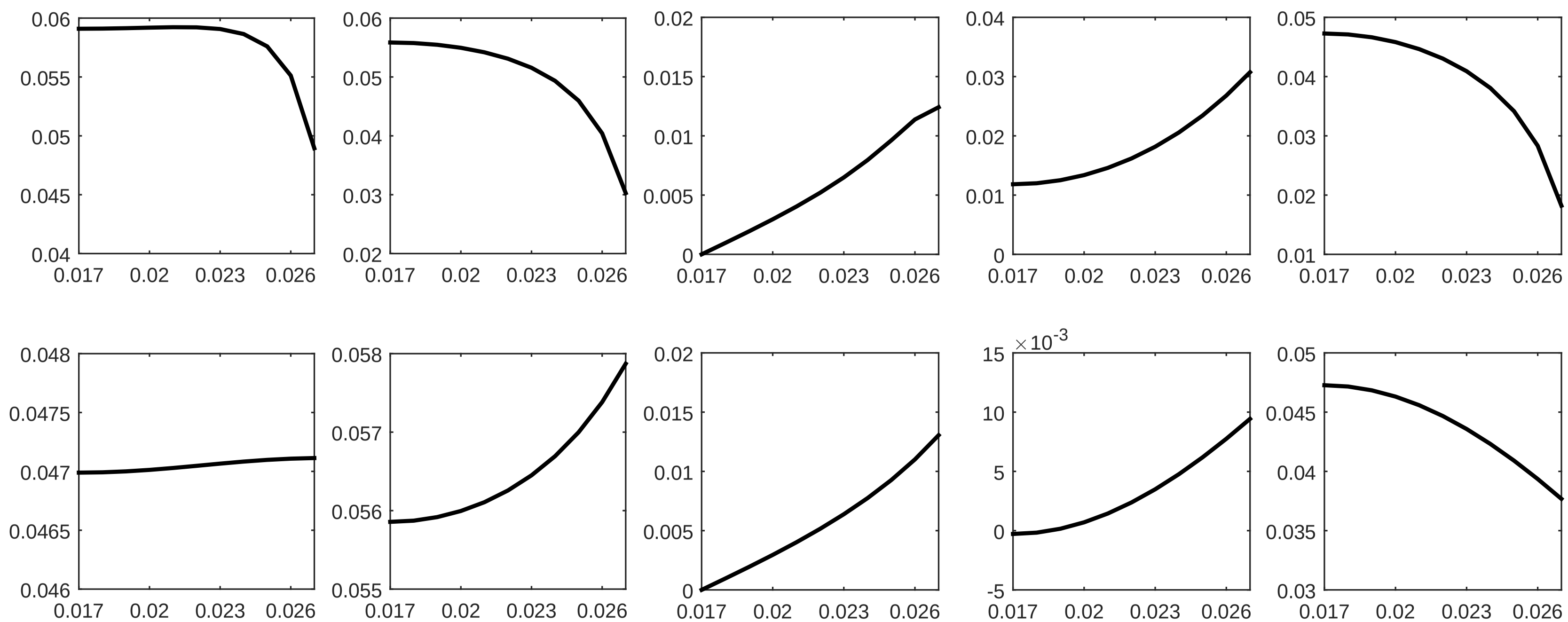

Figure 10. Varying the degree of disagreement in Chen et al. (2012) and the recursive utility version of their model

The figure depicts (from left to right) the expected return, the return volatility of the consumption claim, the trading volume in the consumption claim, the risk-free rate, and the equity risk premium. The upper (lower) row of graphs gives the results in the model proposed by Chen et al. (2012) and an 'extended' version of their model in which CRRA utility ($\gamma = 4, \psi = \frac{1}{4}$) has been replaced by recursive preferences ($\gamma = 4, \psi = 1.5$). The return volatility of the consumption claim is defined in Equation (F.2), the trading volume in the consumption claim in Equation (16), the risk-free rate in Equation (13), and the equity risk premium in Equation (14). All quantities are determined under the true measure and shown as functions of the pessimist's belief λ_1. The optimist's belief is calculated such that the average belief $\bar{\lambda} = 0.0170$, which represents the true measure, remains unchanged. The pessimist's consumption share is fixed at $w = 0.5$.

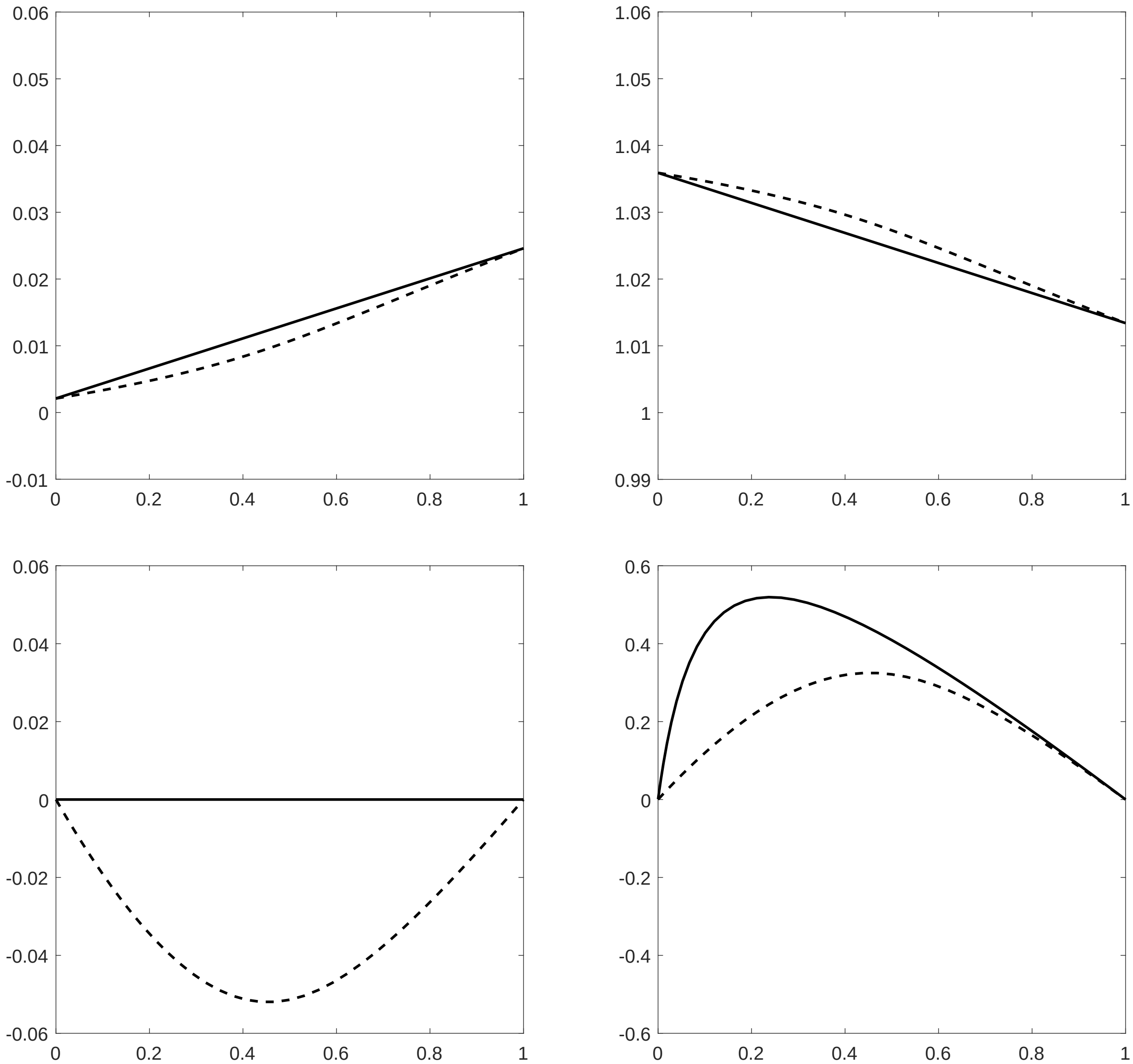

Figure 11. Expected excess return, risk-free rate, and risk sharing in Dieckmann (2011)

The figure depicts in the upper row the expected excess return on aggregate wealth under the true probability measure (on the left) and the risk-free rate (on the right) in the Dieckmann (2011) model with jumps in aggregate consumption and log utility. The lower row shows the coefficients in the dynamics of the pessimist's consumption share, i.e., the coefficients for diffusive consumption shocks (on the left) and jumps in aggregate consumption (on the right). The solid line represents the results on a complete and the dashed line those on an incomplete market. All quantities are shown as functions of the pessimist's consumption share.

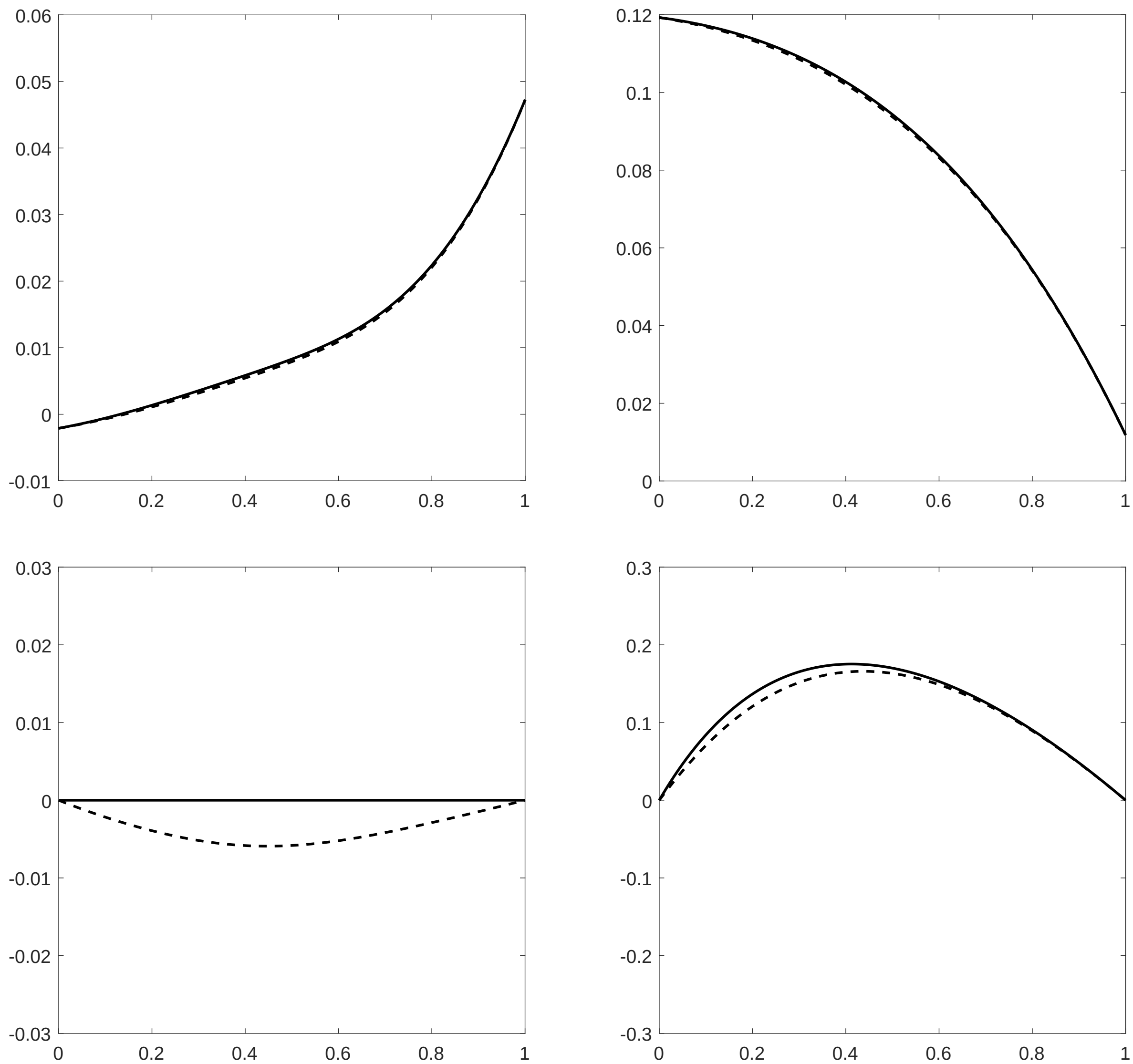

Figure 12. Expected excess return, risk-free rate, and risk sharing in Chen et al. (2012)

The figure depicts in the upper row the expected excess return on aggregate wealth under the true probability measure (on the left) and the risk-free rate (on the right) in the original Chen et al. (2012) model with jumps in aggregate consumption and CRRA utility ($\gamma = 4$, $\psi = \frac{1}{4}$). The lower row shows the coefficients in the dynamics of the pessimist's consumption share, i.e., the coefficients for diffusive consumption shocks (on the left) and jumps in aggregate consumption (on the right). The solid line represents the results on a complete and the dashed line those on an incomplete market. All quantities are shown as functions of the pessimist's consumption share.

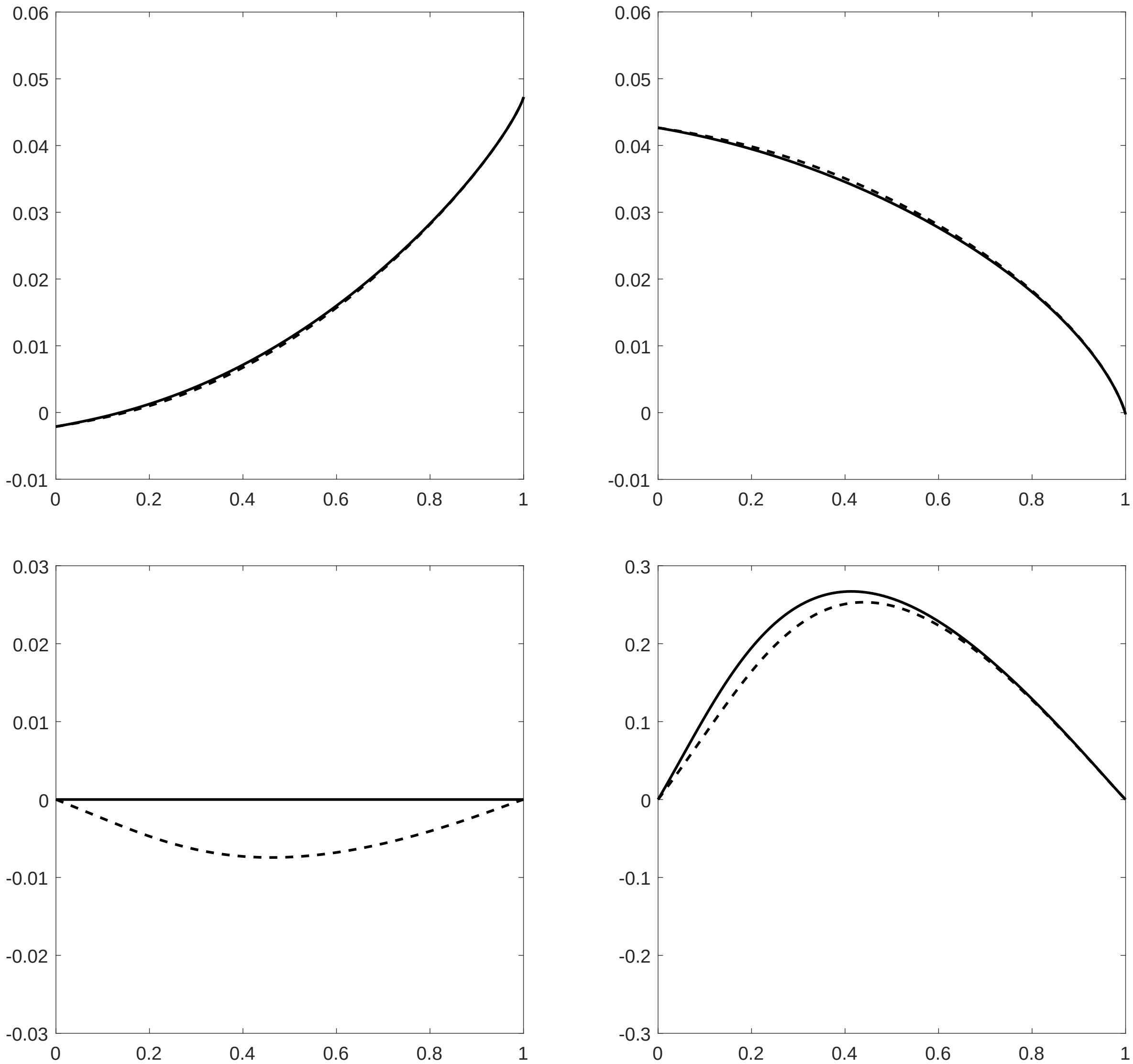

Figure 13. Expected excess return, risk-free rate, and risk sharing in the recursive utility version of Chen et al. (2012)

The figure depicts in the upper row the expected excess return on aggregate wealth under the true probability measure (on the left) and the risk-free rate (on the right) in the 'extended' Chen et al. (2012) model with jumps in aggregate consumption and recursive preferences ($\gamma = 4$, $\psi = 1.5$). The lower row shows the coefficients in the dynamics of the pessimist's consumption share, i.e., the coefficients for diffusive consumption shocks (on the left) and jumps in aggregate consumption (on the right). The solid line represents the results on a complete and the dashed line those on an incomplete market. All quantities are shown as functions of the pessimist's consumption share.

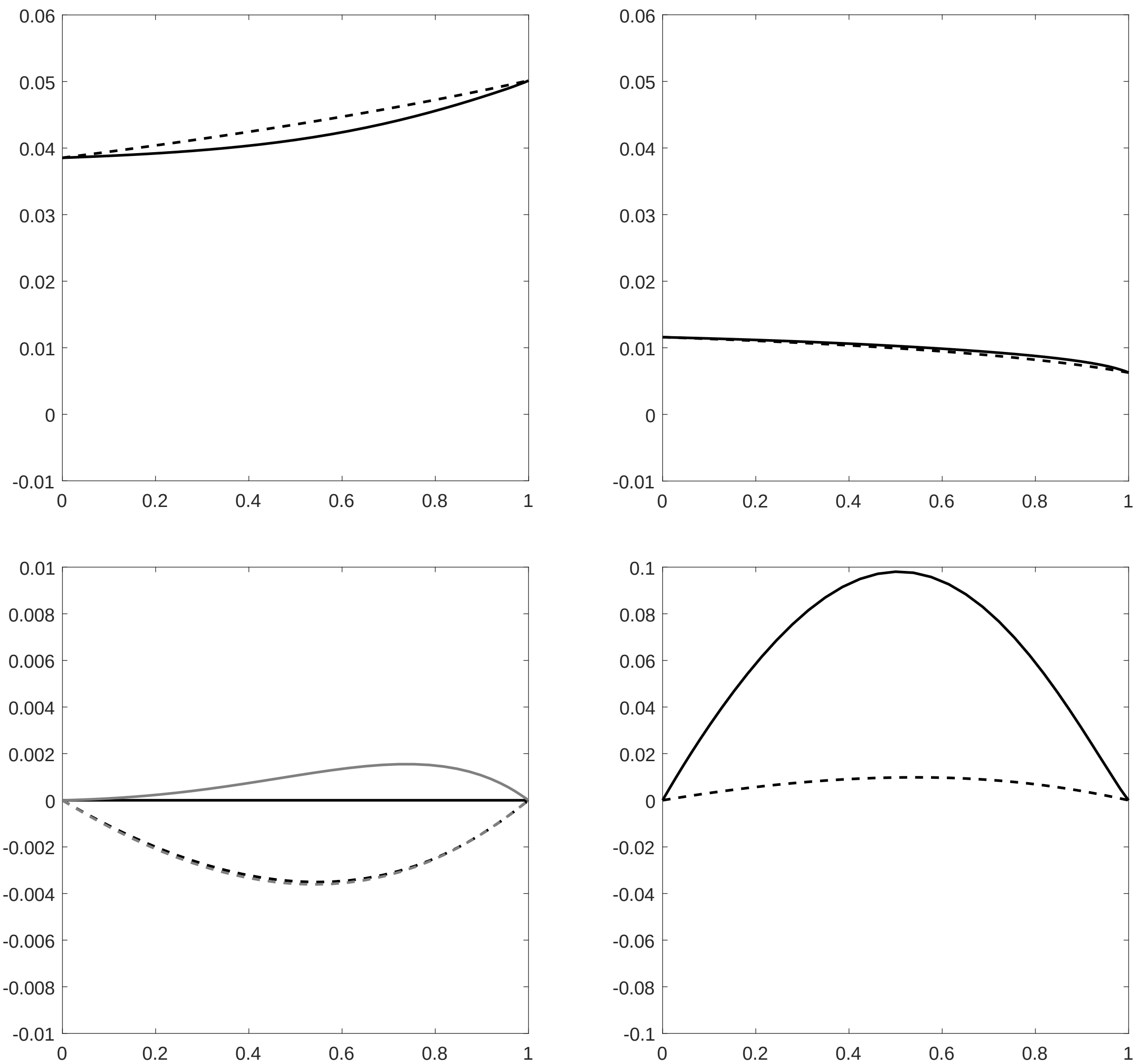

Figure 14. Expected excess return, risk-free rate, and risk sharing in our model

The figure depicts in the upper row the expected excess return on aggregate wealth under the true probability measure (on the left) and the risk-free rate (on the right) in our model with jumps in the expected growth rate of aggregate consumption and recursive preferences ($\gamma = 10$, $\psi = 1.5$). The lower row shows the coefficients in the dynamics of the pessimist's consumption share, i.e., the coefficients for diffusive consumption shocks (black lines in left plot), diffusive expected growth rate shocks (gray lines in left plot), and jumps in expected growth rate (on the right). The solid line represents the results on a complete and the dashed line those on an incomplete market. All quantities are shown as functions of the pessimist's consumption share.

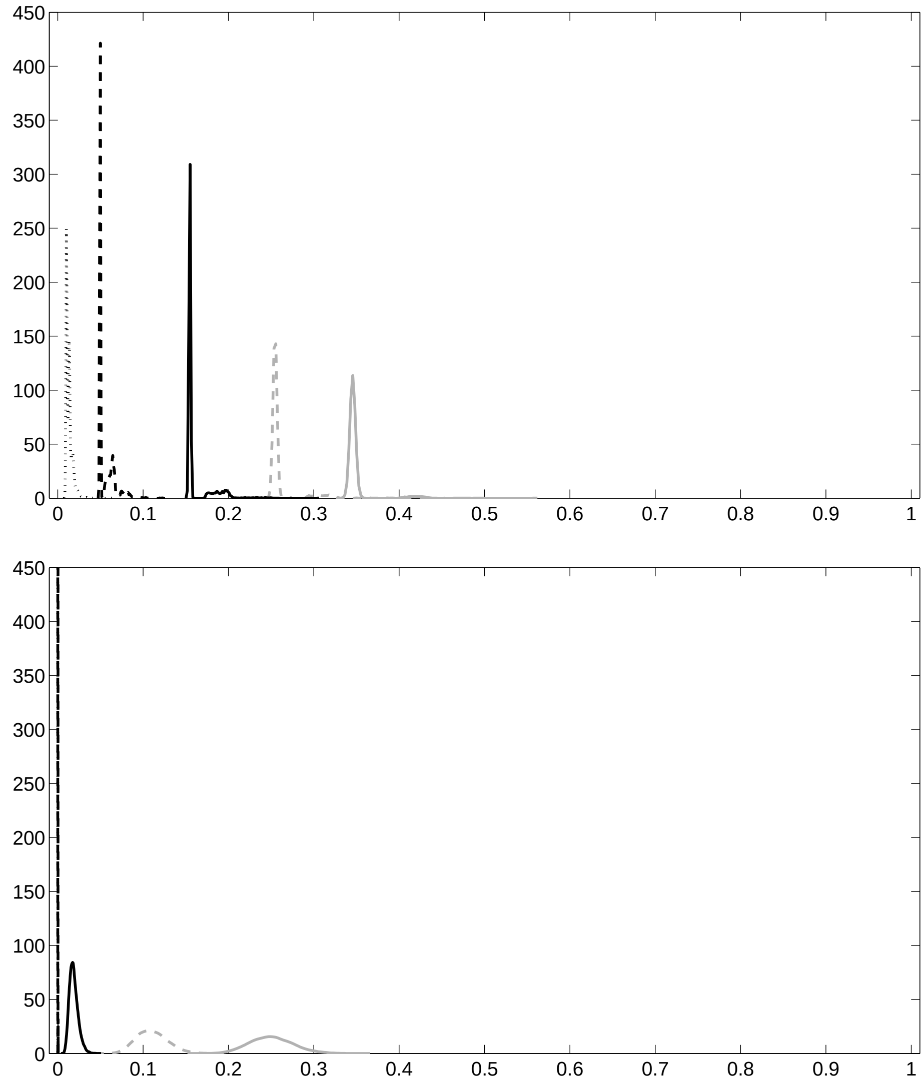

Figure 15. Survival

The figure shows the kernel density estimates for the pessimist's consumption share w_T for T years into the future under the respective true measure after 50 years (gray solid line), 100 years (gray dashed line), 200 years (black solid line), 500 years (black dashed line) and 1,000 years (black dotted line). The upper graph shows the results on a complete market, the lower one those on an incomplete market. All quantities are determined by a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (3) over 10,000 paths with a starting value of $w_0 = 0.5$. The coefficients μ_w, σ_w, and L_w are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters are given in Table 2.

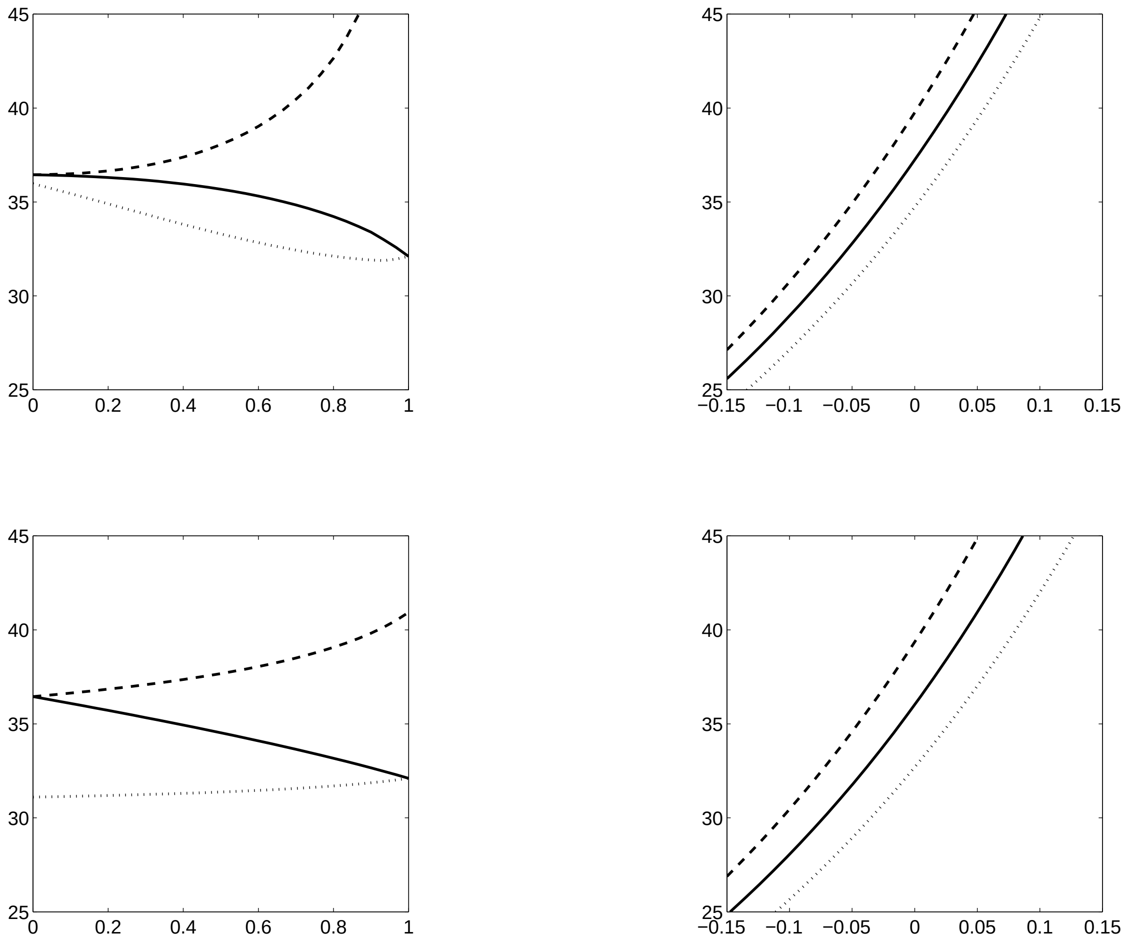

Figure 16. Wealth-consumption ratios

The figure shows the aggregate and individual wealth-consumption ratios, i.e., $v = \log\left(w\,e^{v_1} + (1-w)\;e^{v_2}\right)$ and v_i. The solid line represents the aggregate, the dotted (dashed) line shows the pessimist's (optimist's) individual wealth-consumption ratio. The graphs in the upper (lower) row show the results for the complete (incomplete) market. The individual wealth-consumption ratios are determined by solving for the equilibrium as explained in Appendix A. All quantities are shown in the left column as functions of the pessimist's consumption share with the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$ and in the right column as function of the long-run growth rate for $w = 0.5$. The parameters are given in Table 2.

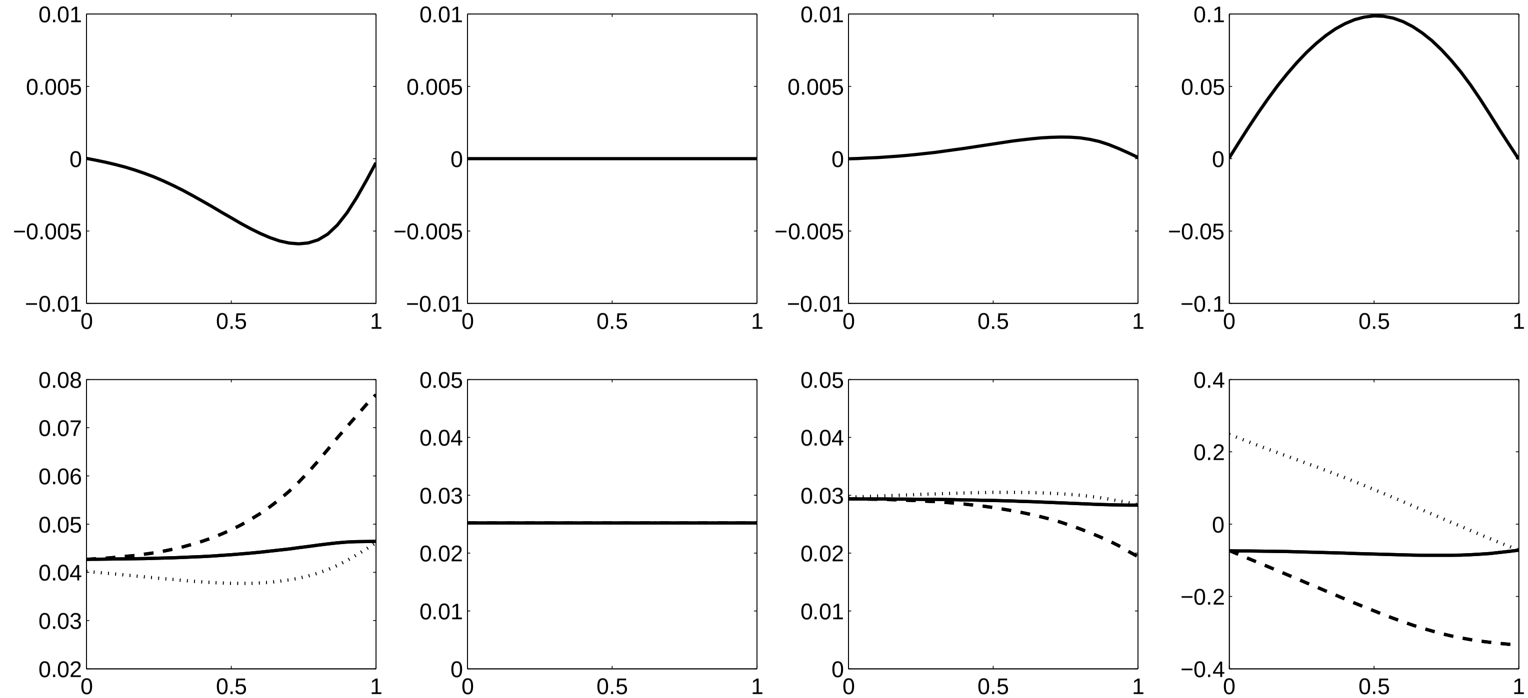

Figure 17. Consumption share and wealth dynamics on a complete market

The figure depicts in the upper row the coefficients in the dynamics of the pessimist's consumption share, as defined in Equation (3), and in the lower row the coefficients in the dynamics of the return on aggregate and individual wealth in the economy, as defined in Equation (A.4), for the case of a complete market. In the lower row of graphs the solid line represents aggregate wealth, and the dotted (dashed) line shows the results for the pessimist (optimist). From left to right the graphs show the drift, and the coefficients for diffusive consumption shocks, diffusive expected growth rate shocks, and jumps in the expected growth rate, respectively. All quantities are determined under the true measure and shown as functions of the pessimist's consumption share with the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$. The parameters are given in Table 2.

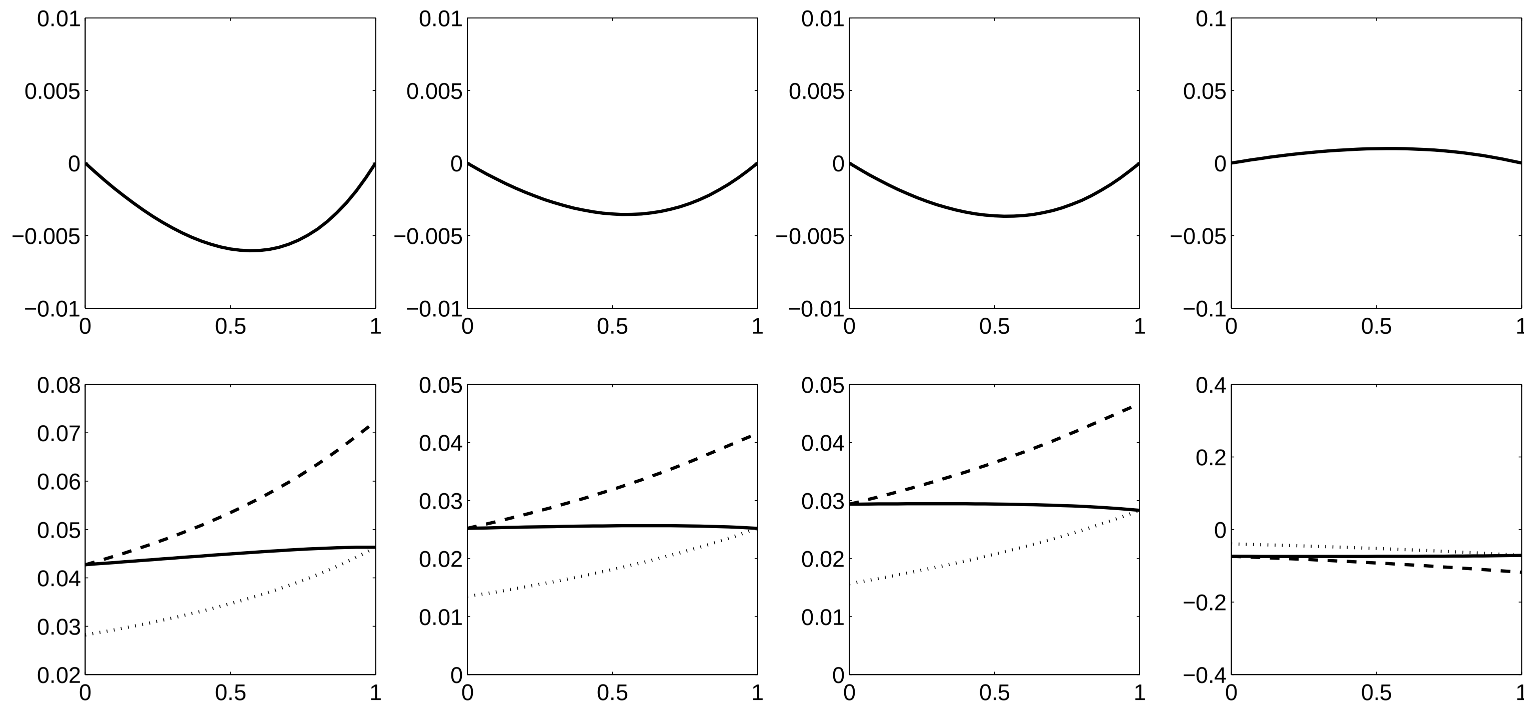

Figure 18. Consumption share and wealth dynamics on an incomplete market

The figure depicts in the upper row the coefficients in the dynamics of the pessimist's consumption share, as defined in Equation (3), and in the lower row the coefficients in the dynamics of the return on aggregate and individual wealth in the economy for the case of an incomplete market. In the lower row of graphs the solid line represents aggregate wealth, and the dotted (dashed) line shows the results for the pessimist (optimist). In the lower row of graphs the solid line represents aggregate wealth, and the dotted (dashed) line shows the results for the pessimist (optimist). From left to right the graphs show the drift, and the coefficients for diffusive consumption shocks, diffusive expected growth rate shocks, and jumps in the expected growth rate, respectively. All quantities are determined under the true measure and shown as functions of the pessimist's consumption share with the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$. The parameters are given in Table 2.

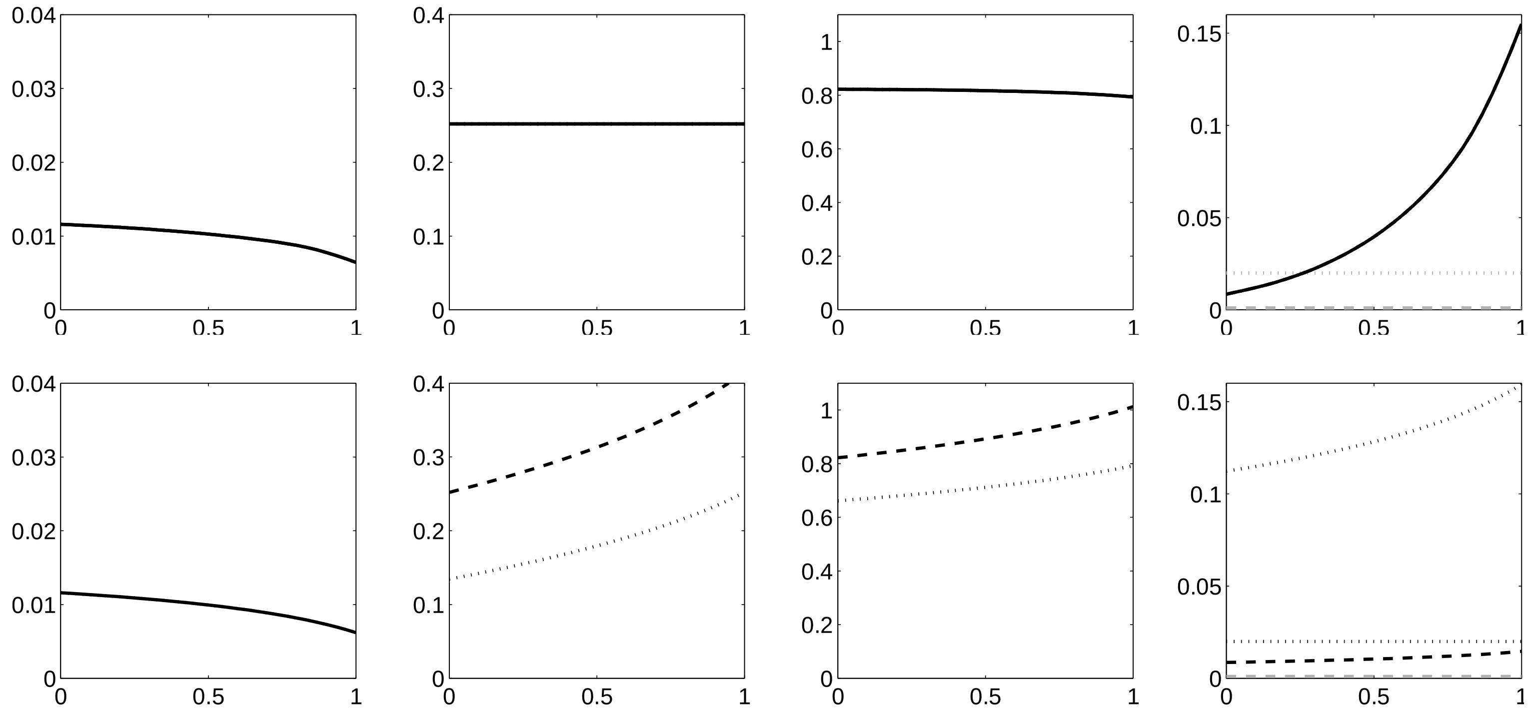

Figure 19. Risk-free rate, market prices of risk, and risk-neutral jump intensities

The figure shows (from left to right) the risk-free rate, the market prices of risk for diffusive consumption and diffusive growth rate risk, and risk-neutral jump intensities on a complete (upper row) and an incomplete market (lower row). A solid black line indicates that both investors agree on the respective quantity. Otherwise, the dotted (dashed) black line represents the pessimist's (optimist's) view. The gray dotted (dashed) line shows the pessimist's (optimist's) subjective jump intensity. Investor i's individual risk-free rate r_i^f is defined in Equation (13), her market prices for diffusive risks η_i^W in (11), and her risk-neutral jump intensity can be determined by plugging η_i^N from (12) into $\lambda_i^{\mathbb{Q}} \equiv \lambda_i \left(1 + \eta_i^N\right)$. All quantities are shown as functions of the pessimist's consumption share with the stochastic part of the expected growth rate of consumption fixed at $X = -0.0060$. The parameters are given in Table 2.

ONLINE APPENDIX

Optimists, Pessimists, and the Stock Market: The Role of Preferences, Dynamics, and Market (In)Completeness

Nicole Branger* Patrick Konermann‡ Christian Schlag§

This version: October 31, 2016

Abstract

This Internet Appendix serves as a companion to our paper "Optimists, Pessimists, and the Stock Market: The Role of Preferences, Dynamics, and Market (In)Completeness". It provides additional results not reported in the main text due to space constraints.

A. Varying Investor Preferences

This section provides additional results with respect to the analysis of disagreement in Section IV.D of the main text. There we present the solution of our model for the different disagreement scenarios for the point $X = -0.060$ (the long.run mean of X) and $w = 0.5$. Figures A.1 to A.3 present plots of the results for different preference specifications and for the full range of the pessimist's consumption share.

*Finance Center Muenster, University of Muenster, Universitaetsstr. 14-16, 48143 Muenster, Germany. E-mail: nicole.branger@wiwi.uni-muenster.de.

‡Department of Finance, BI Norwegian Business School, 0442 Oslo, Norway. E-mail: patrick.konermann@bi.no.

§House of Finance, Goethe-University, Theodor-W.-Adorno-Platz 3, 60323 Frankfurt am Main, Germany. E-mail: schlag@finance.uni-frankfurt.de.

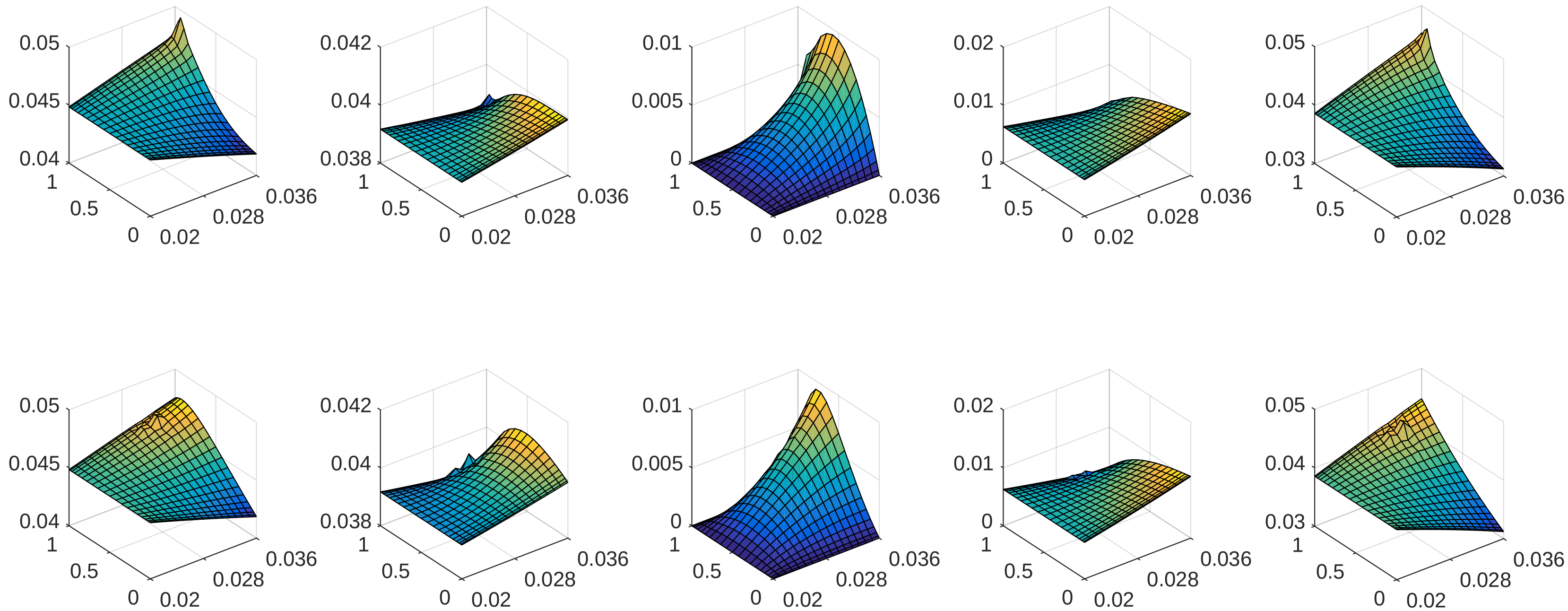

Figure A.1. Varying the degree of disagreement in our model, $\gamma = 10$, $\psi = 1.5$

The figure depicts (from left to right) the expected return, the return volatility of the consumption claim, the trading volume in the consumption claim, the risk-free rate, and the equity premium. The upper (lower) row of graphs gives our results on the complete (incomplete) market. The return volatility of the consumption claim is defined in Equation (17), the trading volume in the consumption claim in Equation (16), the risk-free rate in Equation (13), and the equity risk premium in Equation (14). The expected return is calculated as the sum of the risk-free rate and the equity risk premium. All quantities are determined under the true measure and shown as functions of the pessimist's consumption share w and the pessimist's belief λ_1. The optimist's belief is calculated such that the average belief $\bar{\lambda} = 0.0200$, which represents the true measure, remains unchanged. The model is solved on a grid for the consumption share w and the long-run growth rare X with $w \in (0, 1)$ and $X \in [-0.1560, 0.1440]$, where the range of X is determined via a Monte Carlo simulation. The quantiles of X are shown in Table B.1 in the online appendix. The figures show the quantities from the solution of the model evaluated at the grid points for $X = -0.0060$. The parameters are given in Table 2.

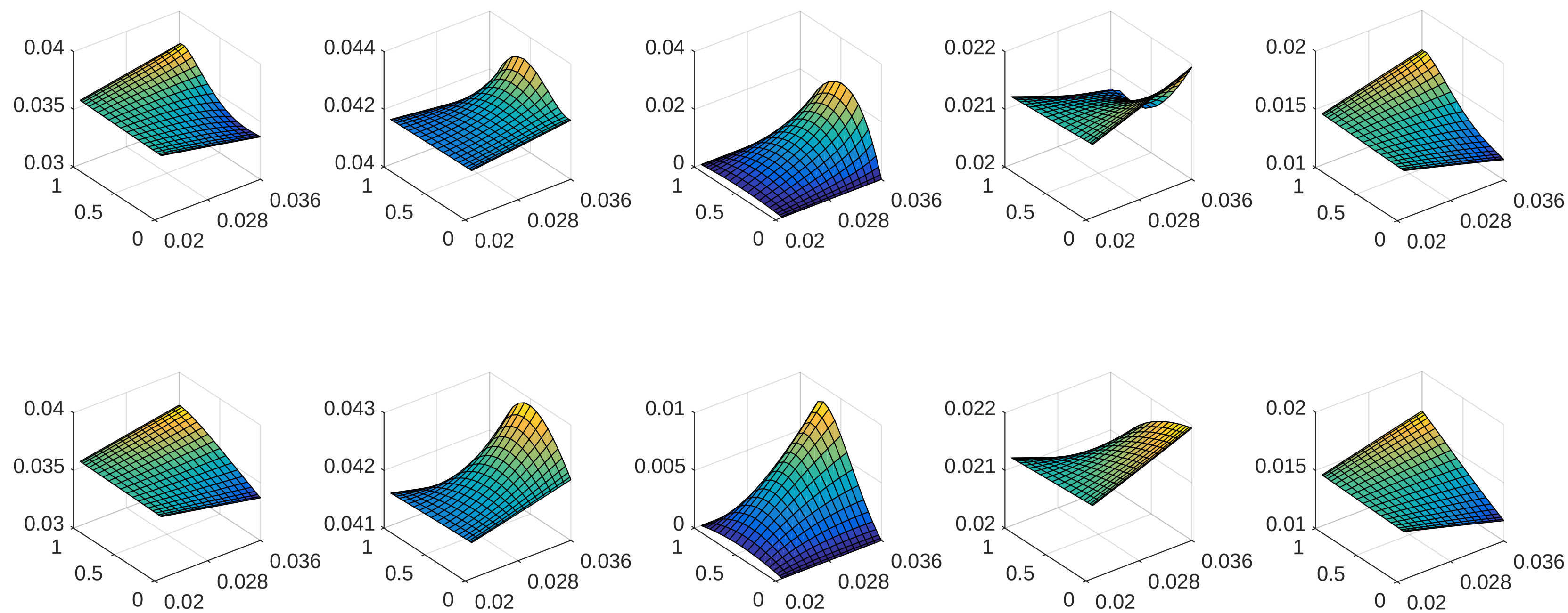

Figure A.2. Varying the degree of disagreement in our model, $\gamma = 4$**,** $\psi = 1.5$

The figure depicts (from left to right) the expected return, the return volatility of the consumption claim, the trading volume in the consumption claim, the risk-free rate, and the equity premium for $\gamma = 4$ and $\psi = 1.5$. The upper (lower) row of graphs gives our results on the complete (incomplete) market. The return volatility of the consumption claim is defined in Equation (17), the trading volume in the consumption claim in Equation (16), the risk-free rate in Equation (13), and the equity risk premium in Equation (14). The expected return is calculated as the sum of the risk-free rate and the equity risk premium. All quantities are determined under the true measure and shown as functions of the pessimist's consumption share w and the pessimist's belief λ_1. The optimist's belief is calculated such that the average belief $\bar{\lambda} = 0.0200$, which represents the true measure, remains unchanged. The model is solved on a grid for the consumption share w and the long-run growth rare X with $w \in (0, 1)$ and $X \in [-0.1560, 0.1440]$, where the range of X is determined via a Monte Carlo simulation. The quantiles of X are shown in Table B.1 in the online appendix. The figures show the quantities from the solution of the model evaluated at the grid points for $X = -0.0060$. The remaining parameters are given in Table 2.

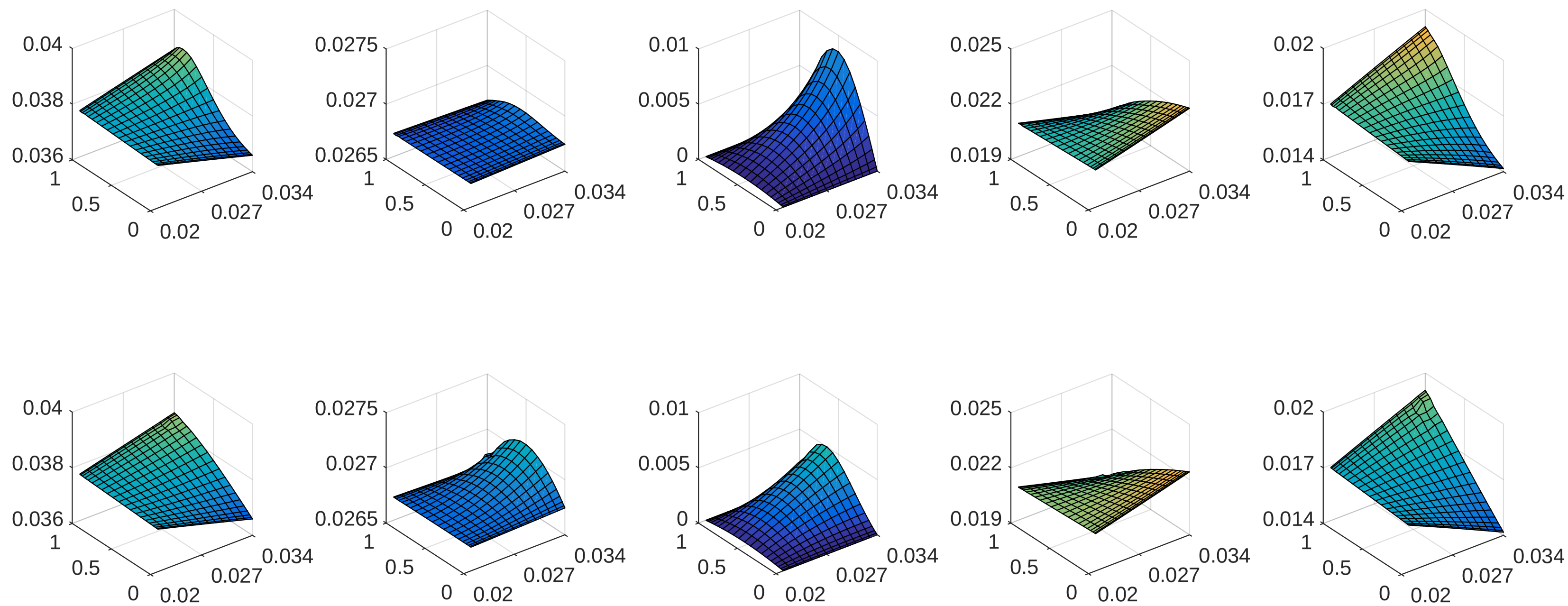

Figure A.3. Varying the degree of disagreement in our model, $\gamma = 10$, $\psi = 1.1$

The figure depicts (from left to right) the expected return, the return volatility of the consumption claim, the trading volume in the consumption claim, the risk-free rate, and the equity premium for $\gamma = 10$ and $\psi = 1.1$. The upper (lower) row of graphs gives our results on the complete (incomplete) market. The return volatility of the consumption claim is defined in Equation (17), the trading volume in the consumption claim in Equation (16), the risk-free rate in Equation (13), and the equity risk premium in Equation (14). The expected return is calculated as the sum of the risk-free rate and the equity risk premium. All quantities are determined under the true measure and shown as functions of the pessimist's consumption share w and the pessimist's belief λ_1. The optimist's belief is calculated such that the average belief $\bar{\lambda} = 0.0200$, which represents the true measure, remains unchanged. The model is solved on a grid for the consumption share w and the long-run growth rare X with $w \in (0, 1)$ and $X \in [-0.1560, 0.1440]$, where the range of X is determined via a Monte Carlo simulation. The quantiles of X are shown in Table B.1 in the online appendix. The figures show the quantities from the solution of the model evaluated at the grid points for $X = -0.0060$. The remaining parameters are given in Table 2.

B. Range for the State Variable X

Quantiles	X
0.0001%	-0.1526
0.0010%	-0.1342
0.0100%	-0.1151
0.1000%	-0.0948
1.0000%	-0.0714
10.0000%	-0.0409
50.0000%	-0.0056
90.0000%	0.0285
99.0000%	0.0559
99.9000%	0.0760
99.9900%	0.0922
99.9990%	0.1061
99.9999%	0.1185

Table B.1. Quantiles

The table shows the quantiles of the long-run growth rate determined by a Monte Carlo simulation. We simulate the X process daily and take the end of the month values over 10,000 paths of length 1,000 years each under the true measure. The parameters for our model are given in Table 2.