On Enhanced Alternatives for the Hodrick-Prescott Filter

Marlon Fritz* Thomas Gries Yuanhua Feng

Abstract

The trend estimation for macroeconomic time series is crucial due to many different model specifications. One of the standard methods for detrending macroeconomic time series in the business cycle literature is the Hodrick-Prescott (HP) filter. In order to address difficulties of the HP filter, especially the choice of the smoothing parameter λ and the boundary problem, we argue that a data-driven local linear trend estimation could be a preferred method. Unlike the HP filter, using exogenously chosen and frequency-dependent smoothing parameters, we suggest a local linear trend estimation which determines the bandwidth endogenously, corrects automatically at boundary points and allows for short-range dependence. For this endogenous bandwidth selection, a data-driven iterative plug-in (IPI) algorithm is used. A comparison between the HP filter and the local linear trend estimation demonstrates similar trend results if λ for the HP filter is chosen appropriately. However, no exogenous choice is needed within the data-driven local polynomial estimation approach where the optimal bandwidth is estimated by minimizing the asymptotic mean integrated squared error. Further, with the endogenously determined bandwidth we can determine the length of the local linear trend, which we regard as continuously Moving Trend (MT), lasting 16 years for the quarterly US GDP data from 1947.1 to 2016.1. Moreover, a linear moving trend estimation method perfectly corresponds to log-linear growth processes often considered in growth theories.

JEL Classification: C14, C22, E32

Keywords: Nonparametric Methods; Time Series, Hodrick-Prescott Filter; Business Cycles

* Corresponding author: Marlon Fritz, Department of Economics, Paderborn University, Germany, e-mail: marlon.fritz@uni-paderborn.de

Co-authors: Thomas Gries, Department of Economics, Paderborn University, Germany

Yuanhua Feng, Department of Economics, Paderborn University, Germany

1. Introduction

For business cycle analysis, it is common sense to decompose the time series into trend and cyclical movements. Morley and Piger (2012) state that the estimated business cycle depends on model specification as different methods lead to different trend and cycle decompositions. The reason is that there is no generalized method of separating both components. In addition, trend and business cycles interact. In practice, several decomposition methods gained attention over the last decades. Thus, the Beveridge-Nelson decomposition (Beveridge and Nelson, 1981), Morley and Piger's (2008) regime-dependent steady-state approach, linear trends, the Hodrick-Prescott filter[1] (Hodrick and Prescott, 1997) and more recently spline smoothing are popular methods. Besides the simple linear trend, the HP filter, which is in accordance with Paige and Trindade (2010) a type of penalized spline smoothing, becomes the standard method for estimating the trend and the cyclical component. Ravn and Uhlig (2002) argue that "it is likely that the HP filter will remain one of the standard methods for detrending" (Ravn and Uhlig, 2002, p. 371).[2] However, among others, Harvey and Jaeger (1993) as well as King and Rebelo (1993) show that some problems arise with the application of the HP filter. These main problems contain the poor behavior at an unknown amount of boundary points and the arbitrary selection of the smoothing parameter λ, which leads to different trend and cycle estimations. In order to improve the properties of the HP filter, we propose a general trend and cycle decomposition method, which is local linear. This *data-driven local linear trend* estimation can lead to a similar trend compared to the HP trend, if λ for the HP filter is chosen appropriately. Moreover, our method is endogenous and improves the behavior/sensitivity at boundary points. The bandwidth selection, which is based on the data-driven IPI algorithm of Feng and Gries (2016), determines

[1] Here after HP filter.

[2] For a comparison of different filters the reader is referred to Baxter and King (1999), Alexandrov et al. (2012).

a trend period which identifies the current trend. This trend period may lead to the interpretation of a stable time range supporting the momentary growth trend. Hence, it may identify a kind of stable growth periods in accordance with the underlying observations. Those periods last around 16 years for postwar quarterly US GDP data and the estimated trend will be called continuously *Moving Trend (MT)* in the further analysis. Thus, the proposed trend identification has an economically meaningful interpretation and is consistent with log-linear growth theories. The remainder of this paper is structured as follows. Section 2 presents the HP filter and the related literature, including the criticism. Section 3 shortly demonstrates the data-driven local linear trend estimation. Section 4 provides results from a short simulation study. In section 5 our local linear trend is compared to the HP trend using US GDP data at quarterly and annual frequencies and section 6 concludes.

2. The HP filter

A good trend "should be influenced by the cyclical movements in the data but it should also be smooth" (Zarnowitz and Ozyildirim, 2006, p. 1732). In 1981, Hodrick and Prescott introduce a method for the decomposition of time series into a trend and a cyclical component. Over the years, the HP filter becomes a standard tool for extracting trend and cycle in macroeconomic time series. For example, Kydland and Prescott (1990), Backus and Kehoe (1992) and many economic institutions (IMF, OECD (Giorno et al., 1995) and ECB) use or have used the HP filter for detrending actual data. In accordance with Hodrick and Prescott (1997), the HP filter considers that the series can be separated into a nonlinear growth or trend component g_t and a cyclical component c_t. Let y_t be a time series, which is defined as

$$y_t = g_t + c_t, \tag{1}$$

where $t = 1, \dots, T$ denotes the time. By solving the following optimization problem, a smooth trend is estimated and removed from the data:

$$\min_{g_t} \left\{ \sum_{t=1}^{T} (y_t - g_t)^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\}. \tag{2}$$

The remaining stationary residual series $c_t = y_t - g_t$ is known as the cyclical component or business cycle. A crucial point is the prior, exogenous selection of the positive smoothing parameter λ, which enters equation (2) as a penalization of the growth component variability. Furthermore, Kaiser and Maravall (1999) as well as Maravall and del Río (2007) demonstrate that λ, which has no intuitive economic interpretation (Wynne and Koo, 2000), determines the main period of the cycle that will be produced by the HP filter. That is, the larger the value of λ, the smoother becomes the HP trend. Hodrick and Prescott (1997) propose to use $\lambda = 1600$ for quarterly data, which follows from their business cycle definition and which needs to be adjusted in accordance with the frequency of the underlying observations. However, no general agreement is reached on which value of λ may be used for annual data. Thus, it is chosen arbitrarily, leading to values of the smoothing parameter of $\lambda \in [6.25; 1600]$. Baxter and King (1999) use a value around 10, whereas Backus and Kehoe (1992) show that 100 works well for their purpose. In contrast, Correia et al. (1992) argue for a higher value of $\lambda = 400$ for data on an annual frequency. Ravn and Uhlig (2002) have recently proposed to use $\lambda = 6.25$, which is based on the idea that the filter representation for quarterly data have to be equal to the filter representation of an alternative frequency. That is, the smoothing parameter is adjusted in accordance to the fourth power of the frequency change, but following Kauermann et al. (2011) it does not use information available from the data set. Furthermore, the adjustment of Ravn and Uhlig (2002) is based on the initial cycle definition of Hodrick and Prescott (1997). Thus, Hamilton (2016) argues that the adjustment is only correct, if it would be accurate to use $\lambda =$

1600 for quarterly data. Besides Ravn and Uhlig (2002), many authors propose adjustments for the smoothing parameter in the HP filter. For example, Pedersen (2001) determines λ data-driven by minimizing the mean squared error (MSE).

In addition, Mise et al. (2005) criticize the HP filter for its suboptimality at an unknown amount of boundary points. This behavior results from the fact that the HP filter is non-causal, meaning that the value at one point in time depends on future values of the underlying time series. Reeves et al. (2000) state that "no apparent economic reason why the cyclical residual should be proportional to the fourth difference of the trend" exists (Reeves et al., 2000, p. 3). An additional point concerns the dependence structure of the remaining stationary part, which is not considered at all. Kauermann et al. (2011) demonstrate that the performance of the HP filter may be poor, if the errors are dependent. Moreover, the HP filter is only optimal in the mean squared error-sense in particular circumstances (Ravn and Uhlig, 2002). Thus, the HP filter is just an ad-hoc approach, practicable if the time series is an I(2) process. In accordance with Kauermann et al. (2011), even the asymptotic properties of this filter under independent errors are not yet studied.

3. Data-driven local polynomial trend estimation

These properties of the well-known HP filter cause doubt for the application to real data in practice. In order to overcome the difficulties of the HP filter, a *local polynomial trend* estimation is introduced as an alternative. Let Y_t be a sequence of macroeconomic time series with time $t = 1, \ldots, T$. An additive component model, in accordance with Beran and Feng (2002), is given by slightly adjusting equation (1):

$$Y_t = m(x_t) + \xi_t, \tag{3}$$

where $x_t = \frac{t}{T}$ denotes the rescaled time, $m(x)$ is some smooth function and ξ_t denotes a stationary time series. Beran and Feng (2002) propose a data-driven local polynomial estimator for the trend function in time series with short- or long-range dependence using a data-driven *IPI algorithm* for selecting the optimal bandwidth endogenously. This approach with short-range dependence is extended by Feng and Gries (2016) for the application to macroeconomic time series. In accordance with their idea, the v-th derivative of $m(x)$, defined as $m^{(v)}(x)$ $(v \leq p)$, can be estimated by minimizing the locally weighted least squares

$$Q = \sum_{t=1}^{T} \left\{ y_t - \sum_{j=0}^{p} \beta_j (x_t - x)^j \right\}^2 W\left(\frac{x_t - x}{h}\right), \tag{4}$$

where $W(u) = C_{\mu}(1-u^2)^{\mu} 1 I_{[-1,1]}(u), \mu = 0, 1, \ldots$ is the weight function and h is the bandwidth, respectively. W and h need to be selected in accordance with the data (Feng et al., 2015). For the application, we use a second order kernel as the weight function $W(u)$, more precisely an Epanechnikov (optimal) kernel. The obtained trend estimates are $\hat{m}^{(v)}(x) = v!\, \hat{\beta}_v$, where $v = 0, 1, \ldots, p$. For our purpose, the local linear estimator with $p = 1$ will be used in accordance with log-linear growth theory. A crucial problem is to choose the optimal bandwidth h. In this paper h will be selected by minimizing the asymptotic mean integrated squared error (AMISE) given by:

$$AMISE(h) = h^{2(k-v)} \frac{I\left[m^{(k)}\right]\beta^2}{[k!]^2} + \frac{2\pi c_f (d_b - c_b) R(K)}{T h^{2v+1}}, \tag{5}$$

where $I[m^k] = \int_{c_b}^{d_b} \left[m^{(k)}(x)\right]^2 dx\,, \beta_{(v,k)} = \int_{-1}^{1} u^k K(u) du$, $R(K) = \int K^2(u) du$ and $c_f = f(0)$ denotes the spectral density of ξ_t with $f(\lambda) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} \gamma_{\xi}(l) e^{-il\lambda}, -\pi \leq \lambda \leq \pi$. For boundary correction c_b and d_b are introduced in order to use only 90% of the observations for bandwidth selection. Moreover, an asymmetric boundary kernel is used for weighting the boundary points, and the bandwidth at the boundary is kept constant such that the asymptotic properties at the boundary are the same as in the interior (Feng et al., 2015). The asymptotical global optimal bandwidth h_A for estimating $m(x)$ is the one that minimizes the AMISE. As required by Morley and Piger (2012), our estimates of trend and cycle are optimal in the minimum mean squared error-sense. For estimating $m(x)$ on $[0, 1]$, the asymptotical global optimal bandwidth is given by:

$$h_A = \left(\frac{2v+1}{2(k-v)} \frac{2\pi c_f [k!]^2 (d_b - c_b) R(K)}{I[m^{(k)}] \beta^2{}_{(v,k)}} \right)^{1/(2k+1)} T^{-1/(2k+1)}, \tag{6}$$

provided that $I\left[m^{(k)}\right] > 0$. Details on the IPI algorithms for estimating h and c_f are presented in Feng and Gries (2016). Furthermore, h_A is adjusted in accordance with the sample size and its selection under dependent errors is well studied, for example in Francisco-Fernández and Vilar-Fernández (2005), whereas λ is fixed for any T in the HP filter. Moreover, the dependence structure is reflected in the bandwidth.

4. Simulation study

In order to demonstrate the appropriateness of the local linear trend and to compare it to the HP trend a simulation study is carried out. Therefore, four regression functions for the true data generating processes are considered including a linear trend and a random walk process. For the error dependence structure a simple AR(2) model is used. The simulation is carried out using four different sample sizes, $n_1 = 50, n_2 = 100, n_3 = 200, n_4 = 400$, with 1000 replications in each case. For estimating the trend function, the Epanechnikov (optimal) kernel is used. For a direct comparison, the average squared errors (ASE) are calculated for our proposed alternative and for the HP trend. In most cases the ASE is smaller using the local linear trend estimation method.[3] Moreover, the simulation study demonstrates that the IPI algorithm works well, even for small sample sizes.

5. Comparison of the HP trend and the local linear trend

In order to demonstrate the advantages of the local linear trend estimation in practice, both trends are compared. Therefore, US GDP data at a quarterly frequency covering a period from 1947.1-2016.1 and at an annual frequency covering a period from 1790 to 2015 are used. The quarterly data are extracted from the US Bureau of Economic Analysis (2016) and the annual data are from Johnston and Williamson (2016) “What Was the U.S. GDP Then?”, Measuring Worth 2016. Figure 1 shows the quarterly data (labeled in red) together with the linear trend (blue), the HP trend for $\lambda = 1600$ (green) and the local linear trend (black) for the endogenously determined bandwidth of 16 years. Figure 2 displays the same results zooming into the period from 1990.1 to 2016.1. As can be seen from Figure 2, some major differences for the trends at the boundary points exist in the sense that our local linear trend is more stable and not as sensitive as the HP trend.

[3] To save space detailed results are not shown in this paper, but are available from the authors up on request.

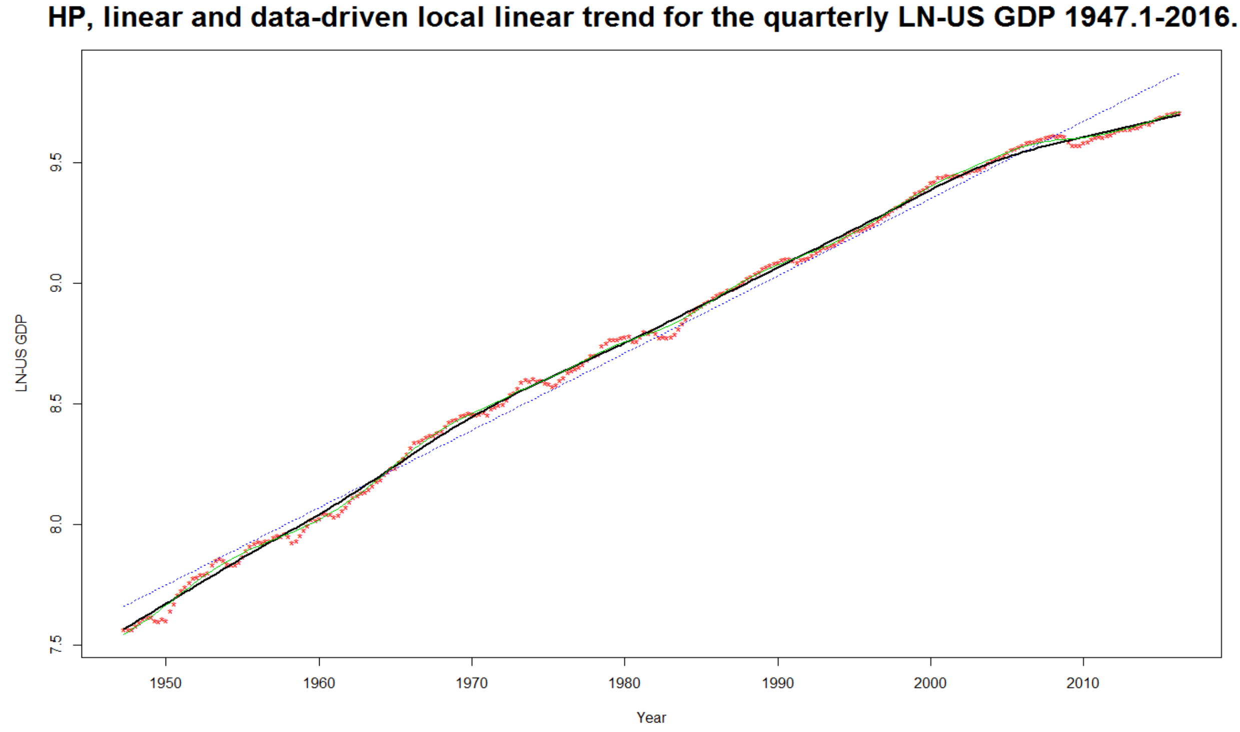

Figure 1: Local linear (black), linear (blue) compared to HP λ=1600 (green) trend for quarterly LN-US GDP 1947.1-2016.1 data (red).

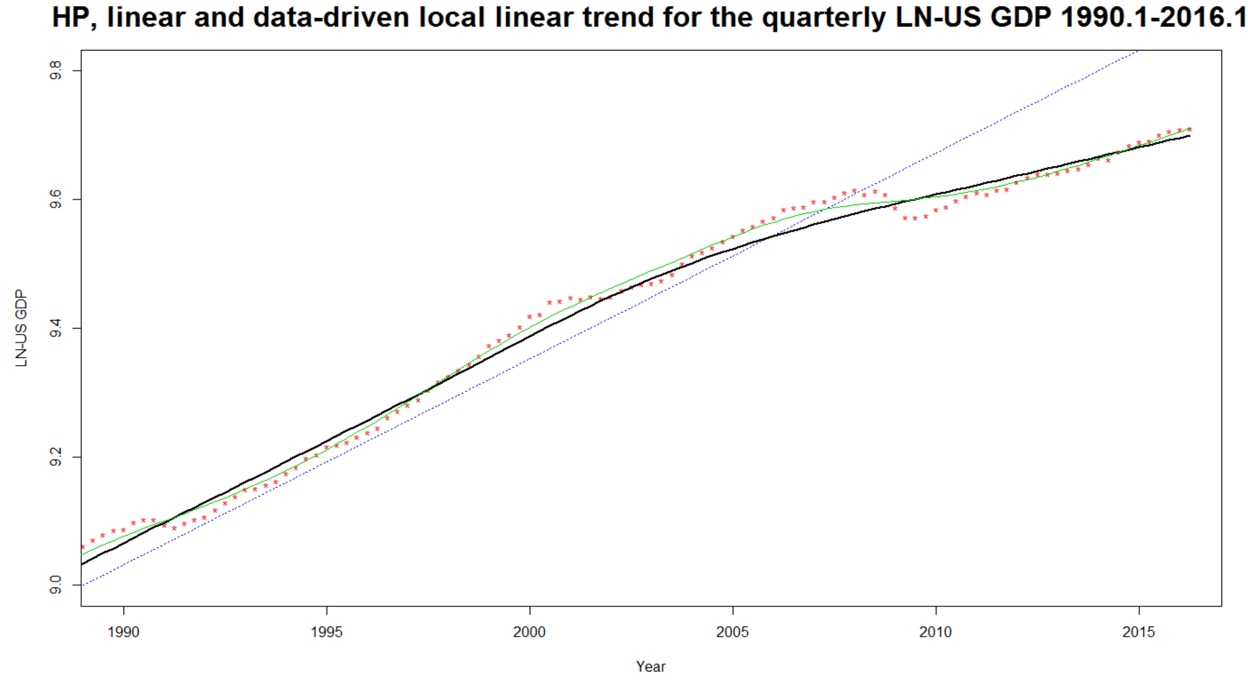

Figure 2: Zoom in local linear (black), linear (blue) compared to HP λ=1600 (green) trend for quarterly LN-US GDP 1990.1-2016.1 data (red).

Figure 3 shows the annual data (labeled in red) together with the linear trend (blue), the HP trend for $\lambda = 6.25$ (turquoise), $\lambda = 100$ (yellow), $\lambda = 400$ (pink), and the local linear trend (black). Again, Figure 4 demonstrates the same results zooming into the period from 1920 to 1970.

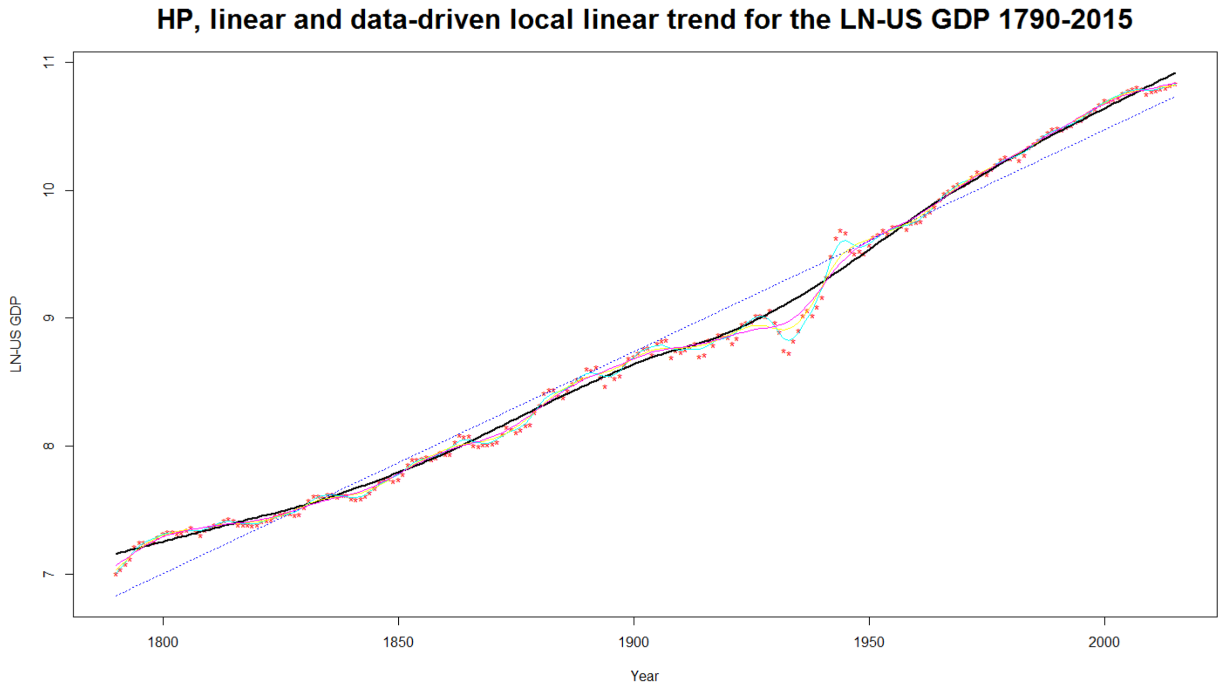

Figure 3: Local linear (black), linear (blue) compared to HP λ=6.25 (turquoise), λ=100 (yellow), λ=400 (pink) trend for annual LN-US GDP 1790-2015 data (red).

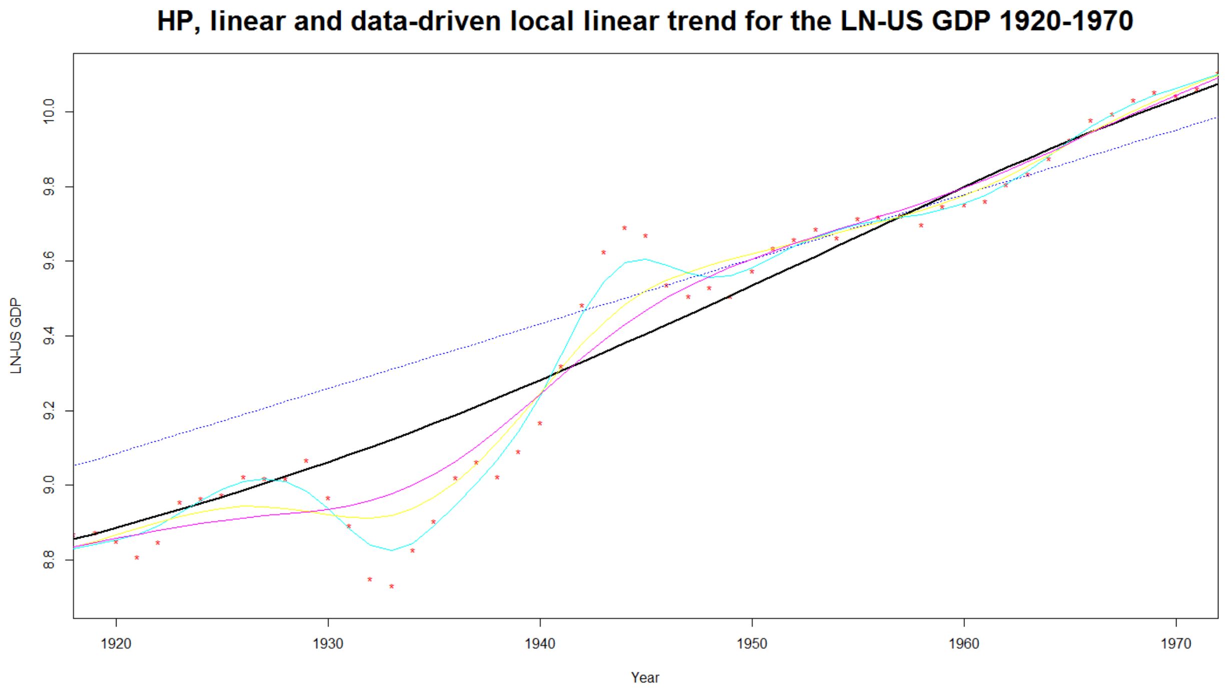

Figure 4: Zoom in Local linear (black), linear (blue) compared to HP λ=6.25 (turquoise), λ=100 (yellow), λ=400 (pink) trend for annual LN-US GDP 1920-1970 data (red).

Obviously, the HP trend highly depends on the value of the smoothing parameter, which is chosen arbitrarily. As a result, the extracted cycle highly depends on λ and is therefore extracted somehow arbitrarily as well. From Figure 3 it is obvious that the HP filter with $\lambda = 400$ and the local linear method yield quite similar trends, in the sense that our trend estimation results are in line with the most frequently used method of Hodrick and Prescott (1997), if λ for the HP filter is chosen appropriately. Consequently, the extracted cyclical components are quite similar for this specific smoothing parameter. However, as can be seen from Figure 4, some major differences for the trends during 1925 to 1950 exist, where the local linear trend is smoother and more robust against outliers. Thus, compared to our MT λ needs to be larger than most suggestions in the literature. Moreover, the main advantage of the local linear trend is the economic interpretation of the trend with the endogenously selected bandwidth, as *MT*. Thus, there are periods of continuous growth trend segments arguing in favor of moving steady states in contrast to the one overall steady state growth process. Consequently, economic conditions changes smoothly over the observation period. This directly corresponds to log-linear growth models often discussed in growth theory. Hence, this linear moving trend (MT) allows to directly link up to theoretical interpretations.

6. Conclusion

This paper demonstrates an alternative method for trend and cycle decomposition. Therefore, the HP trend and its drawbacks are presented. Afterwards, the data-driven local linear estimation approach is explained with an extended data-driven IPI algorithm for selecting endogenously the optimal bandwidth. Besides a short simulation study, real US GDP data are used to demonstrate the usefulness of the local linear trend and to compare it with different HP trends. The comparison shows that the trend curves become similar for proper choices of λ. Thus, the local linear trend *rationalizes* the HP trend by improving some of its disadvantages as the arbitrary selection of λ and the poor behavior at boundary points. Instead of the independent errors assumption, the local linear estimation allows for dependent errors, where the dependence structure is reflected in the bandwidth. Moreover, the endogenously determined bandwidth, specified for an optimal trend and cycle decomposition, allows for the determination of continuously moving trend (MT) processes, which last around 16 years for quarterly data. Those periods represent stable macroeconomic decision-making segments. Hence, the proposed endogenous local linear trend estimation could be a preferred method as it refines the HP trend and directly relates to (log) linear growth theories.

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