Endogenous Regime Shifts in a New Keynesian Model with a Time-varying Natural Rate of Interest*

(Preliminary)

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Abstract

This paper develops a New Keynesian model with a time-varying natural rate of interest (r-star), i.e., the real interest rate that is consistent with full utilization of economic resources and steady inflation at the central bank's target rate. The time series process for r-star is calibrated to approximate the path of the U.S. natural rate series estimated by Laubach and Williams (2015). The zero lower bound (ZLB) on the nominal interest rate gives rise to two long-run endpoints, as in Benhabib, Schmitt-Grohé and Uribe (2001a,b). The representative agent in the model employs forecast rules that are constructed as a weighted-average of the forecast rules associated with each of two local rational expectations equilibria, labeled the "targeted" and the "deflation" solutions, respectively. The time-varying forecast rule weights are determined by recent performance, as measured by the root mean squared forecast errors for inflation, the output gap, and the desired nominal interest rate. Sustained periods when the exogenous real interest rate remains below the central bank's estimate of r-star can induce the agent to place a substantially higher probability on the deflation equilibrium, causing it to occasionally become self-fulfilling. These rare episodes are accompanied by highly negative output gaps and a binding ZLB, reminiscent of the U.S. Great Recession. But even outside of recessions and when the ZLB is not binding, the agent may continue to assign a nontrivial probability to the deflation equilibrium, causing the central bank to consistently undershoot its inflation target, similar to the U.S. economy since mid-2012. I show that raising the central bank's inflation target to 4% from 2% can mostly eliminate switches to the deflation equilibrium.

Keywords: *Natural rate of interest, New Keynesian, Liquidity trap, Zero lower bound, Taylor rule, Deflation.*

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*The views in this paper are my own and not necessarily those of the Federal Reserve Bank of San Francisco or of the Federal Reserve System.

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1 Introduction

The period from 1988 onwards is generally viewed as an example of consistent U.S. monetary. The nature of this policy is typically described in standard New Keynesian macroeconomic models by a Taylor (1993, 1999) type rule in which movements in the federal funds rate are driven by fluctuations in inflation and some real activity variable. Amazingly, the U.S. federal funds rate has been pinned close to zero for about one-fourth of the elapsed time since 1988. Moreover, the U.S. economy is not alone in experiencing an extended period of zero (or slightly negative) nominal interest rates in recent decades.

Figure 1 plots three-month nominal Treasury bill yields in four countries, namely, the United States, Japan, Switzerland, and the United Kingdom. Nominal interest rates in the United States encountered the zero lower bound during the 1930s and from 2008.Q4 though 2015.Q4. Since 1998.Q3, nominal interest rates in Japan have remained near zero, except for the period from 2006.Q4 to 2008.Q3. Nominal interest rates in Switzerland have been zero or slightly negative since 2008.Q4. Nominal interest rates in the United Kingdom have been approximately zero since 2009.Q1. Notice however, that outside of these episodes, all four countries exhibit a strong correlation between nominal interest rates and inflation, consistent with the Fisher relationship.

Benhabib, Schmitt-Grohé and Uribe (2001a,b) show that imposing a zero lower bound (ZLB) on the nominal interest rate in a standard New Keynesian model gives rise to two long-run endpoints (steady states).[1] The basic idea is illustrated in Figure 2, which is adapted from Bullard (2010). The two intersections of the ZLB-augmented monetary policy rule (solid red line) with the Fisher relationship (dashed black line) define two long-run endpoints. I refer to these as the "targeted equilibrium" and "deflation equilibrium," respectively. Data since 2008.Q4 lie closer to the deflation equilibrium.

This paper develops a New Keynesian model with a time-varying natural rate of interest (r-star), i.e., the real interest rate that is consistent with full utilization of economic resources and steady inflation at the central bank's target rate. The times series process for r-star is calibrated to closely approximate the path of the U.S. natural rate series estimated by Laubach and Williams (2015). The representative agent in the model employs forecast rules that are constructed as a weighted-average of the forecast rules associated with each of the two local

[1] I use the terminology "long-run endpoints" rather than "steady states" because the model developed here allows for permanent shifts in the natural rate of interest which, in turn, can shift the long-run values of some macroeconomic variables.

equilibria. The time-varying forecast rule weights are determined by recent performance, as measured by the root mean squared forecast errors for inflation, the output gap, and the desired nominal interest rate.

The forecast rules associated with the deflation equilibrium induce more volatility in inflation and the real output gap in response to real interest rate shocks. Model variables in the deflation equilibrium have distributions with lower means and higher variances than those in the targeted equilibrium. But the significant overlap in the various distributions creates a dilemma for an agent who seeks to determine the likelihood that a string of recent quarterly observations comes from one equilibrium or the other. Sustained periods when the exogenous real interest rate remains below the central bank's estimate of r-star can induce the agent to place a substantially higher probability on the deflation equilibrium, causing it to occasionally become self-fulfilling. These rare episodes are accompanied by highly negative output gaps and a binding ZLB, reminiscent of the U.S. Great Recession. But even outside of recessions and when the ZLB is not binding, the agent may continue to assign a nontrivial probability to the deflation equilibrium, causing the central bank to consistently undershoot its inflation target, similar to the U.S. economy since mid-2012. I show that raising the central bank's inflation target to 4% from 2% can mostly eliminate switches to the deflation equilibrium.

The setup considered here is similar to that of Aruoba, Cuba-Borda, and Schorfheide (2014). These authors construct a stochastic two-regime equilibrium in which the economy may alternate between a targeted-inflation regime and a deflation regime, depending on the realization of a sunspot variable. The probability of transitioning from one regime to the other is independent of the realization of fundamental shocks or the level of macroeconomic variables. In contrast, the transition between regimes here is not driven by a sunspot, but rather by the recent performance of forecast rules that employ recent data on macroeconomic variables. Hence, the transition probabilities that govern the switching between regimes are endogenous, depending on the realization of fundamental shocks.

Another closely related paper is one by Dordal-i-Carrera et al. (2016). These authors develop a New Keynesian model with volatile and persistent "risk shocks" (i.e., shocks that drive a wedge between the nominal policy rate and the short-term bond rate) to account for infrequent but long-lived ZLB episodes. A risk shock in their model is isomorphic to a real interest rate shock. Large adverse risk shocks are themselves infrequent but long-lived. As the binding ZLB becomes more frequent or more long-lived, the optimal inflation target increases. Unlike here, their analysis does not consider model solutions near the deflation equilibrium, but rather focuses only on the targeted equilibrium.[2] In contrast, the model developed here accounts for infrequent but long-lived ZLB episodes via endogenous switching between two local equilibria, i.e., the shock process itself is not the source of the infrequent, long-lived ZLB episodes.[3]

1.1 Related Literature

A number of papers introduce backward-looking learning type mechanisms to examine the dynamics of convergence to either the targeted or the deflation equilibrium. Examples include Evans and Honkapohja (2005), Eusepi (2007), Evans, Guse, and Honkapohja (2008), and Benhabib, Evans and Honkapohja (2014). Unlike here, these frameworks do not entertain the possibility of switching between equilibria.

Armenter (2014) considers an extension of Benhabib, Schmitt-Grohé and Uribe (2001b) in which monetary policy is governed not by a Taylor-type rule, but rather by the optimal time-consistent rule that minimizes the central bank's loss function. He shows that it may not be possible to achieve the targeted equilibrium if agents' initial inflation expectations are below the central bank's inflation target.

Alstadheim and Henderson (2006) and Sugo and Ueda (2008) describe interest rate rules that can preclude the deflation equilibrium.

Numerous papers consider optimal monetary policy in response to a time-varying natural rate of interest. The models typically impose the ZLB (or effective lower bound), but the deflation equilibrium is ignored, i.e., the analysis is local to the targeted equilibrium. Examples include Eggertsson and Woodford (2003), Adam and Billi (2007), Nakov (2008), Nakata (2013), Gust, Johannsen, López-Salido (2015), Hamilton, et al. (2016), Basu & Bundick (2015), and Evans, et al. (2015). One finding of this literature is that more uncertainty about the future natural rate implies looser monetary policy today or more policy inertia.

Finally, the model developed here shares some similarities with the work of Sargent (1999) in which the model economy can endogenously switch between regimes of high versus low inflation, depending on monetary policymakers' perceptions about the slope of the long-run Phillips curve in light of recent data. Here, the endogenous regime switching depends on agents' perceptions about whether recent data are more likely to have been generated by the

[2] This is also the methodology pursued by Reifschneider and Williams (2000), Schmitt-Grohé and Uribe (2010), Chung et al. (2012) and Coibion, Gorodnichenko, and Wieland (2012).

[3] In a New Keynesian model with physical capital, Dennis (2016) shows that the introduction of capital adjustment costs can help to generate infrequent, long-lived ZLB episodes.

targeted versus the deflation equilibrium.

2 Model

The framework for the analysis is a standard New Keynesian model, augmented by a zero lower bound (ZLB) on the short-term nominal interest rate. The log-linearized version of the New Keynesian model is taken to represent a set of global equilibrium conditions, with the only nonlinearity coming from the ZLB.[4] Private-sector behavior is governed by the following equilibrium conditions:

$$y_t = E_t\, y_{t+1} - \alpha[i_t - E_t\, \pi_{t+1} - r_t] + v_t, \qquad v_t \sim N\left(0,\, \sigma_v^2\right), \tag{1}$$

$$\pi_t = \beta E_t\, \pi_{t+1} + \kappa y_t + u_t, \qquad u_t \sim N\left(0,\, \sigma_u^2\right), \tag{2}$$

where y_t is the output gap (the log deviation of real output from potential output), π_t is the inflation rate, i_t is the short-term nominal interest rate, r_t is the exogenous real interest rate, and E_t is the rational expectations operator. Fluctuations in r_t can be interpreted as arising from changes in the representative agent's rate of time preference or changes in the expected growth rate of potential output.[5] The terms v_t and u_t represent an aggregate demand shock and a cost-push shock, respectively.

The time series process for the real rate of interest is given by

$$r_t = \rho_r\, r_{t-1} + (1 - \rho_r)\, r_t^* + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0,\, \sigma_\varepsilon^2\right), \tag{3}$$

$$r_t^* = r_{t-1}^* + \eta_t, \qquad \eta_t \sim N\left(0,\, \sigma_\eta^2\right), \tag{4}$$

where r_t^* is the unobservable "natural rate of interest," i.e., the real interest rate that is consistent with full utilization of economic resources and steady inflation at the central bank's target rate. Equations (3) and (4) summarize a "shifting endpoint" time series process since the long-run endpoint r_t^* can vary over time due to the permanent shock η_t. In any given period, r_t can deviate from r_t^* due to the temporary shock ε_t. The persistence of the "real interest rate gap" $r_t - r_t^*$ is governed by the parameter ρ_r, where $|\rho_r| < 1$. Kozicki and Tinsely (2012) employ this type of time series process to describe U.S. inflation. When $\rho_r = 1$, we

[4] Armenter (2016) adopts a similar approach in computing the optimal monetary policy in the presence of two steady states. Eggertsson and Sing (2016) show that the log-linearized New Keynesian model behaves very similar to the true nonlinear model in the vicinity of the targeted equilibrium.

[5] For the derivation, see Hamilton, et al. (2016) or Gust, Johannsen, and Lopez-Salido (2015).

recover the random walk plus noise specification employed by Stock and Watson (2007) to describe U.S. inflation.[6]

The real interest rate gap captures a concept that has been emphasized by Fed policymakers in recent speeches, namely, a distinction between estimates of the "short-term natural of interest" and its long-term counterpart (Yellen 2015, Dudley 2015, and Fischer 2016). In the model, the agent's rational forecast for the real interest rate gap at any horizon $k \geq 1$ is given by

$$E_t\left(r_{t+k} - r^*_{t+k}\right) \;=\; \left(\rho_r\right)^k \left(r_t - E_t r^*_t\right), \tag{5}$$

where $E_t r^*_t$ represents the agent's current estimate of the natural rate computed using the Kalman filter so as to minimize the mean squared forecast error. When $|\rho_r| < 1$, the real rate gap is expected to shrink to zero as the forecast horizon increases. In Appendix A, I show that the Kalman filter expression for $E_t r^*_t$ is

$$E_t r^*_t \;=\; \lambda \left[\frac{r_t - \rho_r\, r_{t-1}}{1-\rho_r}\right] \;+\; (1-\lambda)\; E_{t-1} r^*_{t-1} \tag{6}$$

$$\lambda \;=\; \frac{-\left(1-\rho_r\right)^2 \phi + \left(1-\rho_r\right)\sqrt{\left(1-\rho_r\right)^2 \phi^2 + 4\phi}}{2}, \tag{7}$$

where λ is the Kalman gain parameter and $\phi \equiv \sigma^2_\eta / \sigma^2_\varepsilon$ is the signal-to-noise ratio. For the quantitative analysis, the values of ρ_r, σ^2_η, and σ^2_ε are chosen so that the time path of $E_t r^*_t$ from equation (6) approximates the path of the U.S. natural rate series estimated by Laubach and Williams (2015) for the sample period 1988.Q1 to 2015.Q4. Their estimation strategy assumes that the natural rate of interest exhibits a unit root, consistent with equation (4). Hamilton, et al. (2016) present evidence that the U.S. ex-ante real rate of interest $i_t - E_t \pi_{t+1}$ is nonstationary, but they find that the gap between the ex-ante real rate and their estimate of the world long-run real rate appears to be stationary. This evidence is also consistent with equations (3) and (4) which imply that real interest rate gap $r_t - r^*_t$ is stationary.

The central bank's monetary policy rule is given by

$$i^*_t \;=\; \rho i^*_{t-1} \;+\; (1-\rho)\left[E_t\, r^*_t + \pi^* + g_\pi \left(\overline{\pi}_t - \pi^*\right) + g_y \left(y_t - y^*\right)\right], \tag{8}$$

$$\overline{\pi}_t \;=\; \omega\, \pi_t \;+ (1-\omega)\; \overline{\pi}_{t-1}, \tag{9}$$

$$i_t \;=\; \max\left\{0,\; i^*_t\right\}, \tag{10}$$

[6]But unlike here, Stock and Watson (2007) allow for stochastic volatility in the permanent and temporary shocks.

where i_t^* is the desired nominal interest rate that responds to deviations of recent inflation $\overline{\pi}_t$ from the central bank's target rate π^* and to deviations of the output gap from its targeted long-run endpoint y^*. Recent inflation $\overline{\pi}_t$ is a moving average of past inflation rates so as to approximate the average inflation rate over the past 4 quarters—a typical central bank target variable.[7] The quantity $E_t\, r_t^* + \pi^*$ represents the long-run endpoint of i_t^* which depends on the central bank's current estimate of the unobservable natural rate of interest. The parameter ρ governs the degree of interest rate smoothing as i_t^* adjusts partially each period toward the value implied by the terms in square brackets. Equation (10) is the ZLB that constrains the actual nominal interest rate i_t to be non-negative.

2.1 Long-run endpoints

The Fisher relationship is embedded in the non-stochastic version of equation (1). Consequently, when $g_\pi > 1$, the model has two long-run endpoints (steady states) as shown originally by Benhabib, Schmitt-Grohé, and Uribe (2001a,b). The novelty here is that the long-run endpoints can shift due to shifts in r_t^*. Straightforward computations using the model equations yields the following two sets of long-run endpoints that characterize the "targeted equilibrium" and the "deflation equilibrium," respectively.

Table 1. Long-run Endpoints

Targeted equilibrium	Deflation equilibrium
$\pi_t = \pi^*$	$\pi_t = -r_t^*$
$y_t = y^* \equiv \pi^* (1-\beta)/\kappa$	$y_t = -r_t^* (1-\beta)/\kappa$
$i_t^* = r_t^* + \pi^*$	$i_t^* = (r_t^* + \pi^*)[1 - g_\pi - g_y (1-\beta)/\kappa]$
$i_t = r_t^* + \pi^*$	$i_t = 0$

In the targeted equilibrium, long-run inflation is at the central bank's target rate π^* and the long-run output gap y^* is slightly positive for typical calibrations with $0.99 < \beta < 1$. The long-run desired nominal interest rate i_t^* conforms to the Fisher relationship.[8] The ZLB is not binding such that $i_t = i_t^* > 0$, provided that $r_t^* > -\pi^*$. In the model simulations, I impose bounds on fluctuations in r_t^* that ensure $r_t^* \geq 0$, consistent with the natural rate estimates of Laubach and Williams (2015) for the sample period 1988.Q1 to 2015.Q4. In the deflation equilibrium, the long-run inflation rate, the long-run output gap, and the long-run desired nominal interest rate are all negative when $r_t^* > 0$.[9]

[7] Specifically, the value of ω is set to achieve $\overline{\pi}_t \simeq 0.25\,(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})$.

[8] Cochrane (2015) shows that Fisherian effects appear to dominate Phillips curve effects for determining the comovement between the nominal interest rate and inflation in the standard New Keynesian model.

[9] Evans Honkopoja and Mitra (2016) develop a New Keynesian models that imposes a lower bound on the inflation rate that is more negative than $-r^*$ (which is assumed to be constant in their model). They show that

2.2 Local forecast rules

Given the linearity of the model aside from the ZLB, it is straightforward to derive the agent's rational decision rules for π_t, y_t, and i_t^* in the vicinity of the long-run endpoints associated with each of the two equilibria. For the targeted equilibrium, the local decision rules are unique linear functions of the four state variables $r_t - E_t r_t^*$, $\overline{\pi}_{t-1} - \pi^*$, v_t, and u_t. For the deflation equilibrium, I solve for the minimum state variable (MSV) solution in terms of fundamental state variables only, i.e., abstracting from extraneous sunspot variables.[10] Given the decision rules, we can construct the agent's conditional forecast rules for $E_t \pi_{t+1}$, $E_t y_{t+1}$, and $E_t i_{t+1}^*$ for each of the two equilibria. Details are contained in Appendices B and C.

The decision rule coefficients applied to the state variable $r_t - E_t r_t^*$ are much larger in magnitude in the deflation equilibrium than in the targeted equilibrium. Consequently, the deflation equilibrium exhibits more volatility and undergoes a more severe recession in response to an adverse shock sequence that causes $r_t - E_t r_t^*$ to become negative. This is due to the binding ZLB in the deflation equilibrium which prevents the central bank from taking action to mitigate the consequences of the adverse shock sequence.

2.3 Endogenous regime switching

The linear forecast rules for the targeted equilibrium are derived under the assumption that $i_t^* > 0$ and hence do not take into account the possibility that a shock could be large enough to cause the ZLB to become binding in the future. The error induced by this assumption will depend on the frequency and duration of ZLB episodes in the targeted equilibrium. Based on model simulations, the targeted equilibrium experiences a binding ZLB in only 2.6% of the periods, with an average duration of only 2.2 quarters. Consequently, the agent's use of forecast rules that assume $i_t^* > 0$ in the targeted equilibrium seems reasonable.[11] Similarly, the linear forecast rules for the deflation equilibrium are derived under the assumption that $i_t^* \leq 0$ and hence do not take into account the possibility that a shock could be large enough to cause the ZLB to become slack in the future. Based on model simulations, the targeted equilibrium experiences a binding ZLB in 63% of the periods, with an average duration of 7.6 quarters. The higher volatility of the deflation equilibrium causes the assumption of $i_t^* \leq 0$

this additional constraint gives rise to a third steady state in which the ZLB binds but the Fisher relationship does not hold.

[10] For background on MSV solutions, see McCallum (1999).

[11] Richter and Throckmorton (2016) compare linear model solutions for the targeted equilibrium in which agents ignore the possibility of future ZLB episodes to nonlinear model solutions that account for this possibility.

to be violated in 37% of the periods. Hence, the error induced by the agent's use of linear forecast rules would appear to be more significant in the deflation equilibrium.[12] Nevertheless, as shown in the quantitative analysis of Section 4, the agent's forecast errors in the deflation equilibrium are close to white noise.

Now consider a more sophisticated agent who contemplates the possibility of switching between equilibria, implying that one set of linear forecast rules might perform better than the other. Along the lines of Brock and Hommes (1997, 1998), I postulate that the agent employs forecast rules that are constructed as a weighted-average of the forecast rules associated with different future scenarios. Here, the scenarios pertain to the two local rational expectations equilibria. The time-varying forecast rule weights are determined by recent performance, as measured by the root mean squared forecast errors for inflation, the output gap, and the desired nominal interest rate.

$$\widehat{E}_t\, y_{t+1} \;=\; \mu_t E_t^{\mathrm{targ}}\, y_{t+1} \;+\; (1-\mu_t)\, E_t^{\mathrm{defl}}\, y_{t+1}, \tag{11}$$

$$\widehat{E}_t\, \pi_{t+1} \;=\; \mu_t E_t^{\mathrm{targ}}\, \pi_{t+1} \;+\; (1-\mu_t)\, E_t^{\mathrm{defl}}\, \pi_{t+1}, \tag{12}$$

$$\widehat{E}_t\, i^*_{t+1} \;=\; \mu_t E_t^{\mathrm{targ}}\, i^*_{t+1} \;+\; (1-\mu_t)\, E_t^{\mathrm{defl}}\, i^*_{t+1}, \tag{13}$$

$$\mu_t = \frac{\exp\left[\psi\left(RMSFE_{t-1}^{\mathrm{defl}} - RMSFE_{t-1}^{\mathrm{targ}}\right)\right]}{1 \;+\; \exp\left[\psi\left(RMSFE_{t-1}^{\mathrm{defl}} - RMSFE_{t-1}^{\mathrm{targ}}\right)\right]}, \tag{14}$$

where $RMSFE$ is the root mean squared forecast error computed over a moving window of recent data and ψ is the "intensity of choice" parameter. As ψ becomes larger, the resulting sequence for μ_t takes on values approaching either 1 or 0, with intermediate values less likely. In the simulations, forecast performance is computed as follows:

$$RMSFE_{t-1}^{\mathrm{i}} \;=\; \frac{1}{T_w}\sum_{i=1}^{T_w} \left[\left(y_{t-j} - E_{t-j-1}^{\mathrm{i}}\, y_{t-j}\right)^2 + \left(\pi_{t-j} - E_{t-j-1}^{\mathrm{i}}\, \pi_{t-j}\right)^2 + \left(i^*_{t-j} + E_{t-j-1}^{\mathrm{i}}\, i^*_{t-j}\right)^2\right], \tag{15}$$

where the superscript "i" indicates either the targeted or the deflation equilibrium.

Given the current forecasts from equations (11) through (13), the realizations of the macro-

[12] Aruoba Cuba-Borda, Schorfheide (2014) solve for piece-wise linear decision rules in both the targeted equilibrium and the deflation equilibrium to account for the occasionally binding nature of the ZLB constraint.

economic variables are determined by the following global equilibrium conditions:

$$\begin{aligned} i_t^* &= \frac{1}{\rho}\left\{\widehat{E}_t i_{t+1}^* - (1-\rho)\left[E_t r_t^* + \pi^* + g_\pi\,\omega\left(\widehat{E}_t \pi_{t+1} - \pi^*\right)\right.\right. \\ &\qquad \left.\left. + (1-\omega)\, g_\pi\left(\overline{\pi}_t - \pi^*\right) + g_y\left(\widehat{E}_t y_{t+1} - y^*\right)\right]\right\}, \qquad (16) \\ i_t &= \max\left\{0,\ i_t^*\right\}, \qquad (17) \\ y_t &= \widehat{E}_t y_{t+1} \,-\, \alpha\left[i_t - \widehat{E}_t \pi_{t+1} - r_t\right] \,+\, v_t, \qquad (18) \\ \pi_t &= \beta \widehat{E}_t \pi_{t+1} \,+\, \kappa y_t \,+\, u_t, \qquad (19) \end{aligned}$$

where $\overline{\pi}_t = \omega\,\pi_t + (1-\omega)\;\overline{\pi}_{t-1}$. Equation (16) is obtained by iterating equation (8) ahead one period, taking expectations of both sides, and then solving for i_t^*.

As a check, I also compute the time-varying weight μ_t using the standard classification formula for the conditional probability that a given observation comes from one of two populations with known densities.[13] In this model, the standard formula takes the form

$$\mu_t \;=\; \frac{\mu_{t-1}\, pdf^{\,\mathrm{targ}}\left(\pi_{t-1}\right)}{\mu_{t-1}\, pdf^{\,\mathrm{targ}}\left(\pi_{t-1}\right) \;+\; \left(1-\mu_{t-1}\right)\, pdf^{\,\mathrm{defl}}\left(\pi_{t-1}\right)}, \qquad (20)$$

where $pdf^{\,\mathrm{targ}}$ and $pdf^{\,\mathrm{defl}}$ are the probability density functions for the inflation distributions under the targeted equilibrium and the deflation equilibrium, respectively, which are assumed known to the agent. For the quantitative analysis, I run a pre-simulation to compute the moments of the inflation distributions in each of the two equilibria.

3 Parameter Values

Table 2 shows the parameter values used in the model simulations.

[13] See Anderson (1958), Chapter 6.

Table 2. Parameter Values

Parameter	Value	Description/Target
α	0.2	Interest rate coefficient in Euler equation.
β	0.995	Discount factor in Phillips curve.
κ	0.025	Output gap coefficient in Phillips curve.
π^*	0.02	Central bank inflation target.
ω	0.684	$\overline{\pi}_t \simeq$ 4-quarter inflation rate.
g_π	1.5	Policy rule response to inflation.
g_y	0.5	Policy rule response to output gap.
ρ	0.80	Interest rate smoothing parameter.
ρ_r	0.857	Persistence parameter for natural rate.
σ_ε	0.0099	Std. dev. of temporary shock to natural rate.
σ_η	0.0016	Std. dev. of permanent shock to natural rate.
λ	0.0226	Optimal Kalman gain for $E_t r_t^*$.
σ_v	0.008	Std. dev. of aggregate demand shock.
σ_u	0.016	Std. dev. of cost push shock.
T_w	8	Window length in qtrs. for forecast evaluation.
ψ	75	Intensity of choice parameter for forecast evaluation.

The low value of α implies a very small sensitivity of consumption to changes in the interest rate, consistent with the empirical findings of Campbell and Mankiw (1989). Evans et al. (2015) employ $\beta = 0.995$ and $\kappa = 0.025$.

Table 3 compares the properties of the U.S. real interest rate to those implied by the model. The U.S. real interest rate is defined as the nominal federal funds rate minus expected inflation computed from a rolling 40-quarter vector autoregression that includes four lags each of the annualized funds rate, annualized PCE inflation, and the output gap computed using real GDP and potential output from the Congressional Budget Office (CBO).

Table 3. Properties of Real Interest Rate: Data versus Model

Statistic	U.S. Data 1988.Q1 to 2015.Q4	Model
Std Dev (Δr_t)	0.0103	0.0103
Std Dev $\left(\Delta^2 r_t\right)$	0.0151	0.0178
Corr $(\Delta r_t \, \Delta r_{t-1})$	-0.088	-0.070
Corr $(\Delta r_t \, \Delta r_{t-2})$	-0.194	-0.060

Notes: The real interest rate in U.S. data is defined as the nominal federal funds rate minus expected inflation from a vector autoregression. Model statistics are computed analytically from the laws of motion (3) and (4).

Figure 3 plots the two-sided estimate of the U.S. natural rate series (dashed red line) from Laubach and Williams (2015, updated). The series shows a downward-sloping trend. This pattern is consistent with the declines in global real interest rates observed over the same period (International Monetary Fund 2014, Rachel and Smith 2015). The time series process for the natural rate in the model (dotted green line) is calibrated so that $E_t r_t^*$ from equation (6) approximates the Laubach-Williams series for the data sample 1988.Q1 to 2015.Q4, with r_t given by the U.S. real interest rate (blue line).[14] For the simulations, I impose the bounds $0.002 \leq r_t^* \leq 0.0298$, but $E_t r_t^*$ can exceed these bounds.

4 Quantitative analysis

Figure 4 shows that the U.S. real interest rate has remained below the Laubach-Williams estimate of the natural rate since early 2009. The nominal federal funds rate was pinned at zero from 2008.Q4 through 2015.Q4. A Taylor-type rule using the parameter values shown in Table 2 and the Laubach-Williams estimate of the natural rate of interest implies that the desired nominal funds rate was negative during this time. PCE inflation was briefly negative in 2009 and has remained below the Fed's 2% inflation target since 2012.Q2. The Great Recession was very severe, pushing the CBO output gap down to -6.5% at the business cycle trough in 2009.Q2. The output gap remains negative at -1.7% in 2015.Q4, more than six years after the Great Recession ended. The endpoints plotted in the figure are computed using the expressions in Table 1, with r_t^* given by the Laubach-Williams estimate.

Figure 5 shows that when the exogenous real interest gap $r_t - E_t r_t^*$ is negative for a sustained period, the resulting downward pressure on π_t, y_t , and i_t^* serves to reduce the recent $RMSFE$ of the deflation forecast rules and increase the recent $RMSFE$ of the targeted forecast rules. The shift in relative forecast performance can induce the agent to place a substantially higher weight on the deflation forecast rules, causing the deflation equilibrium to occasionally become self-fulfilling. Qualitatively similar results are obtained if the agent employs Bayes law (20) to compute the likelihood that a string of recent quarterly inflation observations comes from one equilibrium or the other.

Figure 6 shows that model variables in the deflation equilibrium have distributions with lower means but higher variances than those in the targeted equilibrium. But the significant overlap in the various distributions creates a dilemma for an agent who seeks to determine the likelihood that a string of recent quarterly observations comes from one equilibrium or the other. Variables in the switching model have means that are somewhat lower and variances

[14] Similar results are obtained if the model is calibrated to approximate the natural rate series estimated by Lubik and Matthes (2015). For a comparison of Lubik-Mathhes series to the Laubach-Williams series, see Lansing (2016).

that are somewhat higher than those in the targeted equilibrium. Consequently, the central bank in the switching model undershoots its inflation target and the economy underperforms, as measured by the average value of the output gap.

Hills, Nakata, and Schmidt (2016) show that the risk of encountering the zero lower bound in the future can shift agents' expectations such that the central bank undershoots its inflation target in the present. Something similar is at work here; when the agent increases the subjective weight on the deflation forecast rules, this can cause realized inflation to undershoot the central bank's target for a sustained period even when the ZLB is not binding.

Table 4 provides a quantitative comparison between the U.S. data and the results of model simulations. Figure 7 plots the distribution of ZLB durations in each model version. Unlike the targeted equilibrium, the switching model can produce infrequent, but long-lived ZLB episodes in response to normally distributed shocks. To account for infrequent but long-lived ZLB episodes, the targeted equilibrium would require large shocks that are themselves infrequent but long-lived, as in Dordal-i-Carreras, et al. (2016).

Table 4. Unconditional Moments: Data versus Model

Statistic	U.S. Data 1988.Q1-2015.Q4	Model Simulations Targeted	Deflation	Switching
Mean $\sum_{i=0}^{3} \pi_{t-i}$	2.20%	1.99%	−1.60%	1.21%
Std. Dev.	1.09%	0.81%	1.27%	1.08%
Corr. Lag 1	0.89	0.75	0.90	0.86
Mean y_t	−1.51%	0.40%	−0.32%	0.24%
Std. Dev.	2.02%	0.97%	2.83%	1.34%
Corr. Lag 1	0.96	0.27	0.78	0.55
Mean i_t^*	3.45%	3.59%	−2.15%	2.42%
Std. Dev.	2.84%	1.84%	6.35%	3.46%
Corr. Lag 1	0.99	0.88	0.85	0.89
% periods $i_t = 0$	25.9%	2.59%	63.3%	17.5%
Mean ZLB duration	29 qtrs.	2.2 qtrs.	7.6 qtrs.	4.0 qtrs.
Max. ZLB duration	29 qtrs.	20 qtrs.	96 qtrs.	67 qtrs.

Notes: The ZLB episode in U.S. data is from 2008.Q4 through 2015.Q4. Model results are computed from a 300,000 period simulation.

Figure 8 plots simulations from each of the three model versions: targeted, deflation, and switching. All versions employ the same sequence of stochastic shocks. Around period 1455 in the switching model, the weight on the targeted forecast rules approaches zero, causing the deflation equilibrium to become temporarily self-fulfilling. The episode results in a negative desired nominal interest rate, brief deflation followed by below-target inflation, and a highly negative output gap, reminiscent of the U.S. Great Recession and its aftermath.

The severity of the recession is due to the higher response coefficient on the real interest rate gap in the deflation equilibrium decisions rules. These decision rules receive more weight as $\mu_t \to 0$, causing the effects of an adverse real interest rate shock to be transmitted much more forcefully to macro variables in the deflation equilibrium. Evans, Honkapohja, and Mitra (2015) argue that the deflation equilibrium is not a convincing explanation of the U.S. Great Recession since the steady state level of real activity in the deflation equilibrium is not much below the steady state level of real activity in the targeted equilibrium. However, their analysis fails to consider the significant difference in the dynamic responses to an adverse real interest rate shock that is implied by the two sets of local decision rules.

Table 5 summarizes the properties of the agent's forecast errors in each of the three model versions. Equations (16) through (19) show that there are three endogenous variables that the agent must forecast: $\pi_{t+1},\ y_{t+1}$, and i^*_{t+1}. For variable $x_{t+1} \in \left\{\pi_{t+1},\ y_{t+1},\ i^*_{t+1}\right\}$, the forecast error is given by $err^x_{t+1} = x_{t+1} - F_t\, x_{t+1}$, where $F_t\, x_{t+1}$ is the value predicted by the linear forecast rule. As noted earlier in Section 2.3, the agent's use of linear forecast rules in a nonlinear environment subject to an occasionally binding ZLB would be expected to introduce errors, particularly in the deflation equilibrium, which is more volatile. Nevertheless, Table 5 shows that the agent's forecast errors in all three model versions are close to white noise, giving no clear indication to the agent that the linear forecast rules are misspecified.

Table 5. Comparison of Forecast Errors

	Model Simulations		
	Targeted	Deflation	Switching
$Corr(err^{\pi}_{t+1},\ err^{\pi}_t)$	0.0004	0.0005	0.0175
$Corr(err^{y}_{t+1},\ err^{y}_t)$	−0.0002	−0.0029	0.0134
$Corr(err^{i^*}_{t+1},\ err^{i^*}_t)$	−0.0032	−0.0031	0.0572

Notes: Model results are computed from a 300,000 period simulation.

4.1 Effect of Raising the Inflation Target

Table 6 shows that raising the central bank's inflation target to 4% from 2% can mostly eliminate switches to the deflation equilibrium.

Table 6. Effect of Raising the Inflation Target

Statistic	Switching Model $\pi^* = 0.02$	$\pi^* = 0.03$	$\pi^* = 0.04$	$\pi^* = 0.05$
Std. Dev. $\frac{1}{4}\sum_{i=0}^{3}\pi_{t-i}$	1.08%	1.04%	0.91%	0.83%
Std. Dev. y_t	1.34%	1.12%	1.01%	0.98%
Std. Dev. i_t^*	3.46%	2.72%	2.14%	1.92%
% periods $i_t = 0$	17.5%	5.72%	0.99%	0.11%
Mean ZLB duration	4.0 qtrs.	3.3 qtrs.	2.9 qtrs.	3.1 qtrs
Max. ZLB duration	67 qtrs.	55 qtrs.	38 qtrs.	32 qtrs

Note: Model results computed from a 300,000 period simulation.

Numerous papers make the case for a higher inflation target using frameworks that ignore the deflation equilibrium.[15] This methodology likely understates the benefits of a higher inflation target because the analysis does not take into account the important model feature that a higher target can prevent switching to the volatile deflation equilibrium where recessions are more severe.

Aruoba and Schorfheide (2015) consider the welfare implications of a 4% inflation target in a framework that considers the possibility of switching to the deflation equilibrium via a sunspot shock. They conclude (p. 40) that "the overall benefits of this policy are far from clear."

5 Conclusion

This paper develops a New Keynesian model with a shifting natural rate of interest and an occasionally binding ZLB. It is well known that this class of models exhibits two long-run endpoints associated with two local rational expectations equilibria. I examine a version of this setup with endogenous forecast rule switching based on past performance. The model can produce severe recessions when the real interest rate gap is negative, causing the agent to place a significant weight on the forecast rules associated with the deflation equilibrium. Escape from the deflation equilibrium occurs endogenously when the real interest rate gap eventually starts rising. In normal times, a non-trivial weight on the deflation forecast rules may cause central bank to undershoot its inflation target. But with an inflation target of 4%, the probability of ZLB episode is very small $\simeq 1\%$ and the average duration of a ZLB episode is only 2.9 quarters.

[15] See, for example, Blanchard, Dell'Ariccia, and Mauro (2010), Ball and Mazumder (2011), and Ball (2013).

A Appendix: Kalman filter estimate of r-star

To be added.

B Appendix: Targeted equilibrium forecast rules

The targeted equilibrium assumes $i_t^* = i_t > 0$. Iterating equation (8) ahead one period, taking expectations of both sides, and then solving for i_t^* yields:

$$\begin{aligned} i_t^* \;=\; & \frac{1}{\rho}\Big\{E_t i_{t+1}^* - (1-\rho)\Big[E_t r_{t+1}^* + \pi^* + g_\pi\,\omega\,(E_t\pi_{t+1} - \pi^*) + g_\pi\,\omega\,(1-\omega)\,(\pi_t - \pi^*) \\ & + g_\pi\,(1-\omega)^2\,(\overline{\pi}_{t-1} - \pi^*) + g_y\,(E_t y_{t+1} - y^*)\Big]\Big\}, \end{aligned} \tag{A.1}$$

where I have used equation (9) to eliminate $\overline{\pi}_{t+1}$ and $\overline{\pi}_t$. The law of motion (4) implies $E_t r_{t+1}^* = E_t r_t^*$, where $E_t r_t^*$ is the Kalman filter estimate from equation (6).

Equation (A.1) together with the Euler equation (1) and the Phillips curve equation (2) form a linear system of three equations in the three unknown decision rules for π_t, y_t, and i_t^*.The four state variables are $r_t - E_t r_t^*$, $\overline{\pi}_{t-1} - \pi^*$, v_t, and u_t. Standard techniques yield a set of linear decision rules of the form

$$\begin{bmatrix} \pi_t - \pi^* \\ y_t - y^* \\ i_t^* - E_t r_t^* - \pi^* \end{bmatrix} = \mathbf{A} \begin{bmatrix} r_t - E_t r_t^* \\ \overline{\pi}_{t-1} - \pi^* \\ u_t \\ v_t \end{bmatrix}, \tag{A.2}$$

where $y^* \equiv \pi^*(1-\beta)/\kappa$ and $\mathbf{A}$ is a 3×4 matrix of decision rule coefficients. For the parameter values shown in Table 2, the matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} 0.0582 & 0.0007 & 1.0015 & 0.0250 \\ 0.3421 & 0.0186 & 0.0403 & 1.0010 \\ 0.8086 & -0.0635 & -0.1373 & -0.0034 \end{bmatrix}. \tag{A.3}$$

Iterating the decision rules in (A.2) ahead one period and then taking the conditional expectation of both sides yields the following set of forecast rules for the targeted equilibrium:

$$E_t\pi_{t+1} = \pi^* + \mathbf{A}_{11}\rho_r\,(r_t - E_t r_t^*) + \mathbf{A}_{12}\,(\overline{\pi}_t - \pi^*), \tag{A.4}$$

$$E_t y_{t+1} = y^* + \mathbf{A}_{21}\rho_r\,(r_t - E_t r_t^*) + \mathbf{A}_{22}\,(\overline{\pi}_t - \pi^*), \tag{A.5}$$

$$E_t i_{t+1}^* = E_t r_t^* + \pi^* + \mathbf{A}_{31}\rho_r\,(r_t - E_t r_t^*) + \mathbf{A}_{32}\,(\overline{\pi}_t - \pi^*), \tag{A.6}$$

where $\mathbf{A}_{ij}$ represents the corresponding element of the matrix $\mathbf{A}$ and I have substituted in $E_t\left(r_{t+1} - E_{t+1} r_{t+1}^*\right) = \rho_r\,(r_t - E_t r_t^*)$.

C Appendix: Deflation equilibrium forecast rules

The deflation equilibrium assumes $i_t^* \leq 0$ such that $i_t = 0$, i.e., the ZLB is always binding. Equation (A.1) applies unchanged to the deflation equilibrium, as does the Phillips curve equation (2). However, due to the binding ZLB, the Euler equation (1) now becomes

$$y_t \;=\; E_t\, y_{t+1} \,+\, \alpha[E_t\, \pi_{t+1} - r_t] \,+\, v_t. \tag{B.1}$$

Equation (B.1) together with equations (A.1) and (2) form a linear system of three equations in the three unknown decision rules for π_t, y_t, and i_t^*.The four state variables are $r_t - E_t r_t^*$, $\overline{\pi}_{t-1} - \pi^*$, v_t, and u_t. The minimum state variable (MSV) solution yields a set of linear decision rules of the form

$$\begin{bmatrix} \pi_t - (-E_t\, r_t^*) \\ y_t - (-E_t\, r_t^*)\,(1-\beta)\,/\kappa \\ i_t^* - (E_t\, r_t^* + \pi^*)\left[1 - g_\pi - \frac{g_y(1-\beta)}{\kappa}\right] \end{bmatrix} = \mathbf{B} \begin{bmatrix} r_t - E_t r_t^* \\ \overline{\pi}_{t-1} - (-E_t\, r_t^*) \\ u_t \\ v_t \end{bmatrix}, \tag{B.2}$$

where $\mathbf{B}$ is a 3×4 matrix of constant coefficients. The MSV solution implies $\mathbf{B}_{12} = \mathbf{B}_{22} = 0$. For the parameter values shown in Table 2, the matrix $\mathbf{B}$ is

$$\mathbf{B} = \begin{bmatrix} 0.2969 & 0 & 1 & 0.0250 \\ 1.7516 & 0 & 0 & 1 \\ 4.1230 & -0.0620 & -0.1341 & -0.0034 \end{bmatrix}. \tag{B.3}$$

Iterating the decision rules in (B.2) ahead one period and then taking the conditional expectation of both sides yields the following set of forecast rules for the deflation equilibrium:

$$E_t \pi_{t+1} \;=\; -E_t\, r_t^* + \mathbf{B}_{11} \rho_r\,(r_t - E_t r_t^*)\,, \tag{B.4}$$

$$E_t y_{t+1} \;=\; -E_t\, r_t^*\,(1-\beta)\,/\kappa + \mathbf{B}_{21} \rho_r\,(r_t - E_t r_t^*)\,, \tag{B.5}$$

$$E_t i_{t+1}^* \;=\; (E_t\, r_t^* + \pi^*)\left[1 - g_\pi - \frac{g_y(1-\beta)}{\kappa}\right] + \mathbf{B}_{31} \rho_r\,(r_t - E_t r_t^*) + \mathbf{B}_{32}\,[\overline{\pi}_t - (-E_t\, r_t^*)]\,, \tag{B.6}$$

where I have imposed the MSV restriction $\mathbf{B}_{12} = \mathbf{B}_{22} = 0$ and substituted in $E_t\left(r_{t+1} - E_{t+1} r_{t+1}^*\right) = \rho_r\,(r_t - E_t r_t^*)$.

For the special case when $\beta, \omega \to 1$ and $g_y \to 0$, it is straightforward to derive the following analytical relationship between the decision rule coefficients for the two local equilibria:

$$\frac{\mathbf{B}_{11}}{\mathbf{A}_{11}} = \frac{\mathbf{B}_{21}}{\mathbf{A}_{21}} = \frac{\mathbf{B}_{31}}{\mathbf{A}_{31}} \;=\; 1 \;+\; \frac{(1-\rho)\, g_\pi}{(\rho_r - \rho)} \frac{\alpha\kappa\rho_r}{\left[(1-\rho_r)^2 - \alpha\kappa\rho_r\right]}. \tag{B.7}$$

For the the baseline calibration, the right side of equation (B.5) is approximately equal to 5, which means that a shock to $r_t - E_t r_t^*$ will be transmitted much more forcefully to macro variables in the deflation equilibrium.

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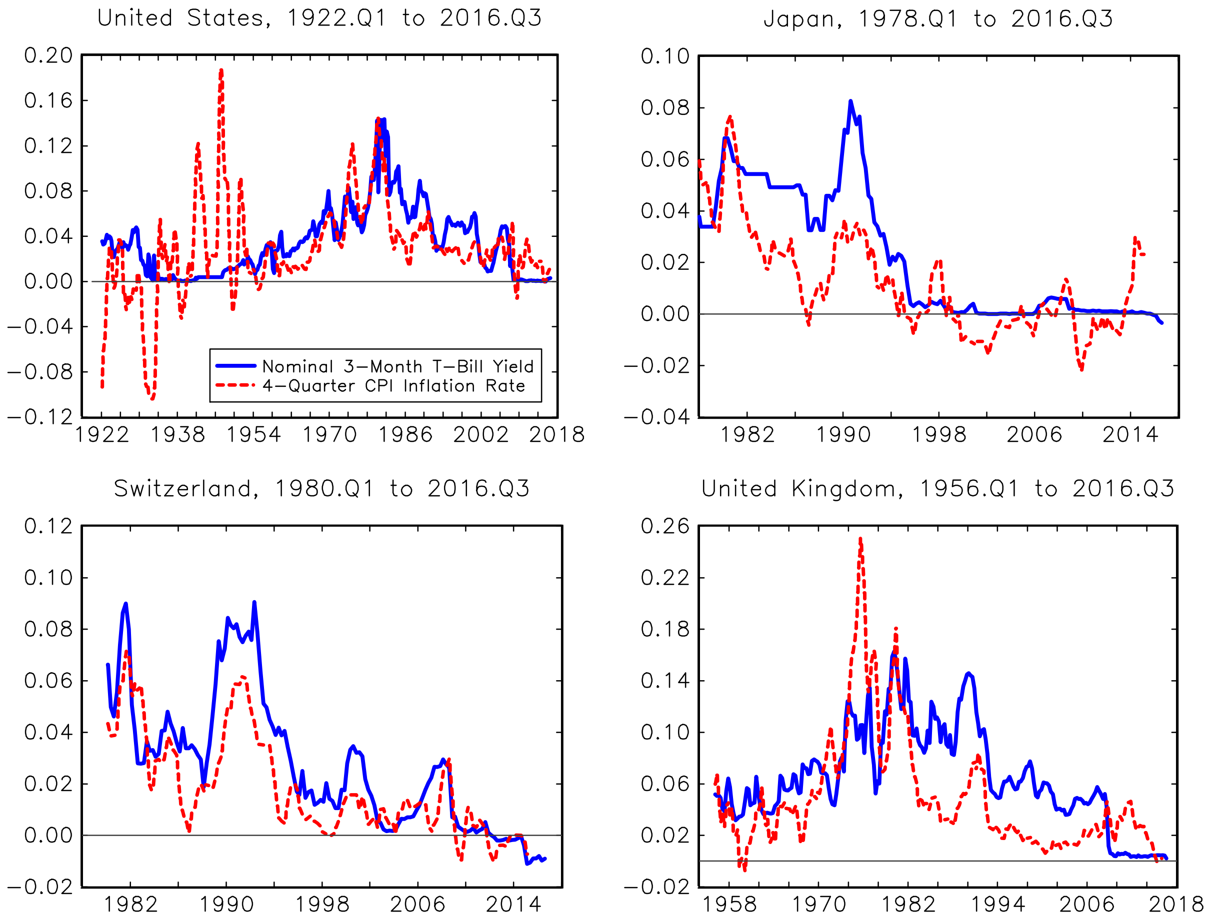
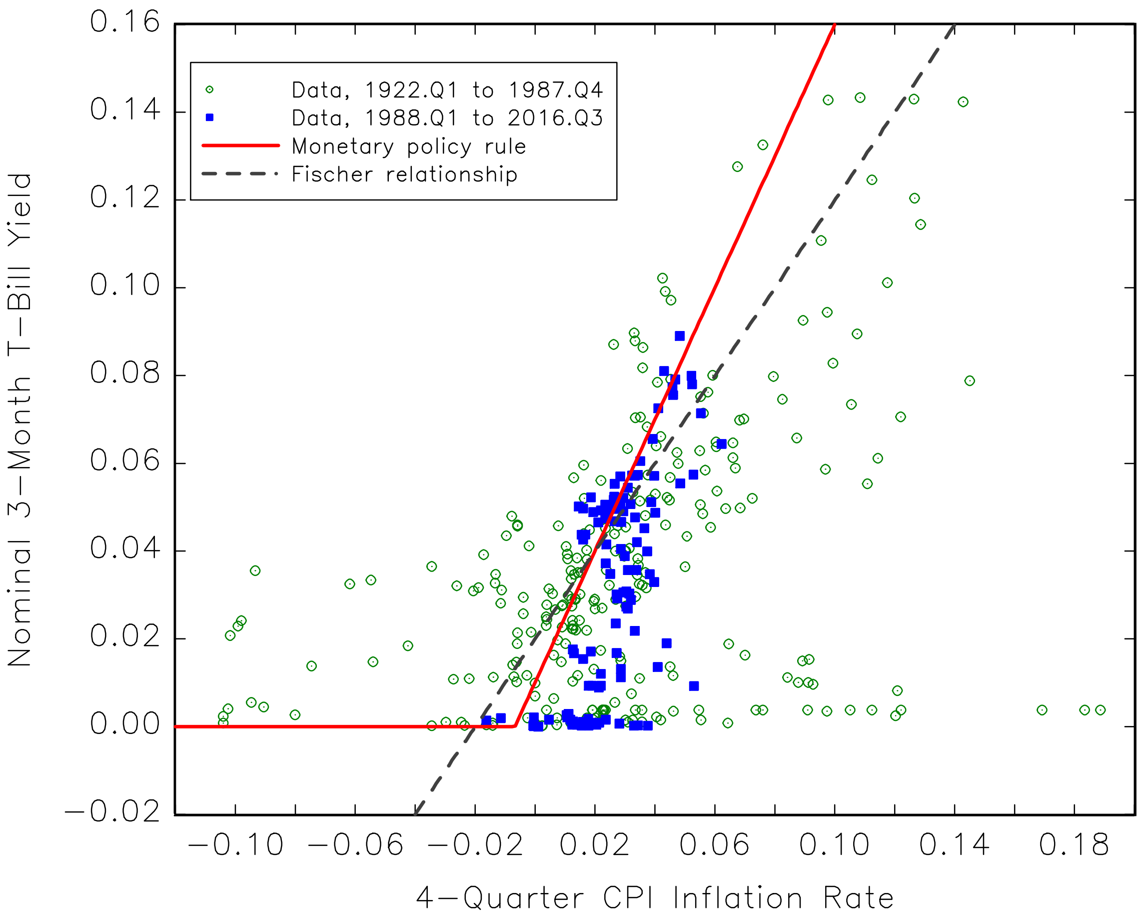
Figure 1: *Nominal Interest Rates and Inflation in Four Countries*



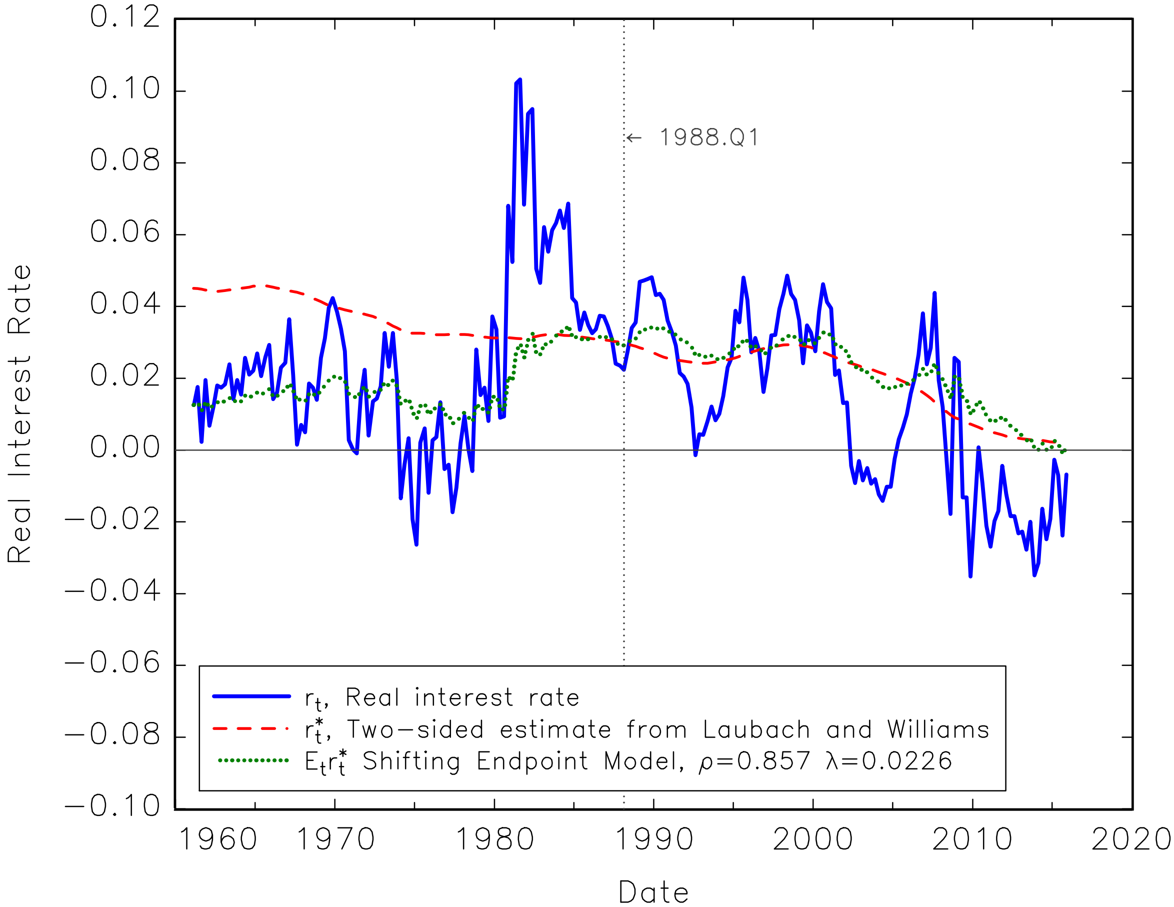
Nominal interest rates in the United States encountered the zero lower bound during the 1930s and from 2008.Q4 though 2015.Q4. Since 1998.Q3, nominal interest rates in Japan have remained near zero, except for the period from 2006.Q4 to 2008.Q3. Nominal interest rates in Switzerland have been zero or slightly negative since 2008.Q4. Nominal interest rates in the United Kingdom have been approximately zero since 2009.Q1. Outside of these episodes, all four countries exhibit a strong correlation between nominal interest rates and inflation, consistent with the Fisher relationship.

Figure 2: *U.S. Nominal Interest Rates and Inflation*



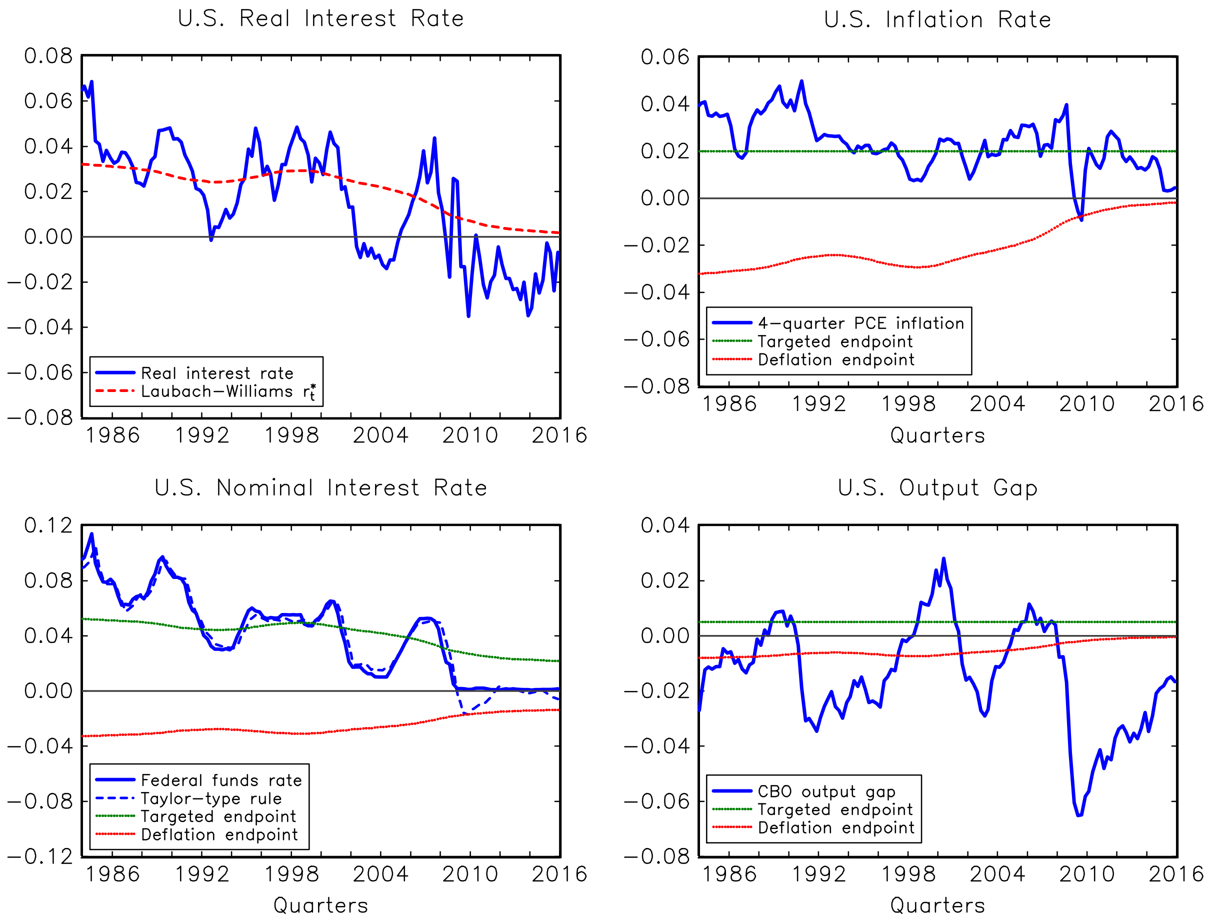
The two intersections of the ZLB-augmented monetary policy rule (solid red line) with the Fisher relationship (dashed black line) define two long-run endpoints, labeled the "targeted equilibrium" and "deflation equilibrium," respectively. The monetary policy rule is $i_t = r^* + \pi^* + g_\pi (\pi_t - \pi^*)$ with $r^* = \pi^* = 0.02$ and $g_\pi = 1.5$. The Fisher relationship is $i_t = r^* + \pi_t$. Data since 2008.Q4 lie closer to the deflation equilibrium.

Figure 3: *U.S. Real Interest Rates*



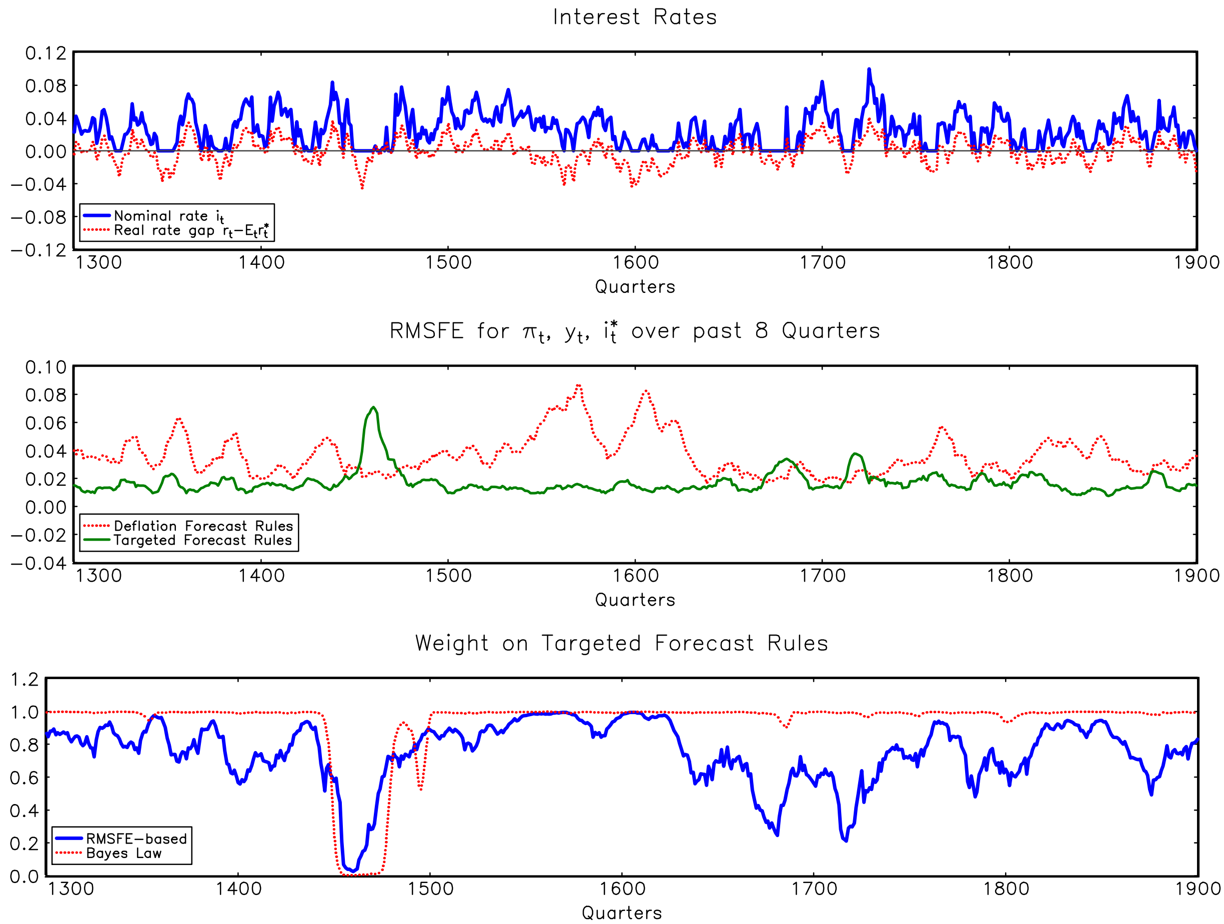
The real interest rate (blue line) is defined as the nominal federal funds rate minus expected quarterly inflation computed from a rolling 40-quarter vector autoregression that includes the funds rate, PCE inflation, and the CBO output gap. The time series process for the natural rate of interest in the model (dotted green line) is calibrated to approximate the two-sided estimate of the U.S. natural rate series (dashed red line) from Laubach and Williams (2015, updated) for the data sample 1988.Q1 to 2015.Q4—a period of consistent monetary policy.

Figure 4: *U.S. Data*



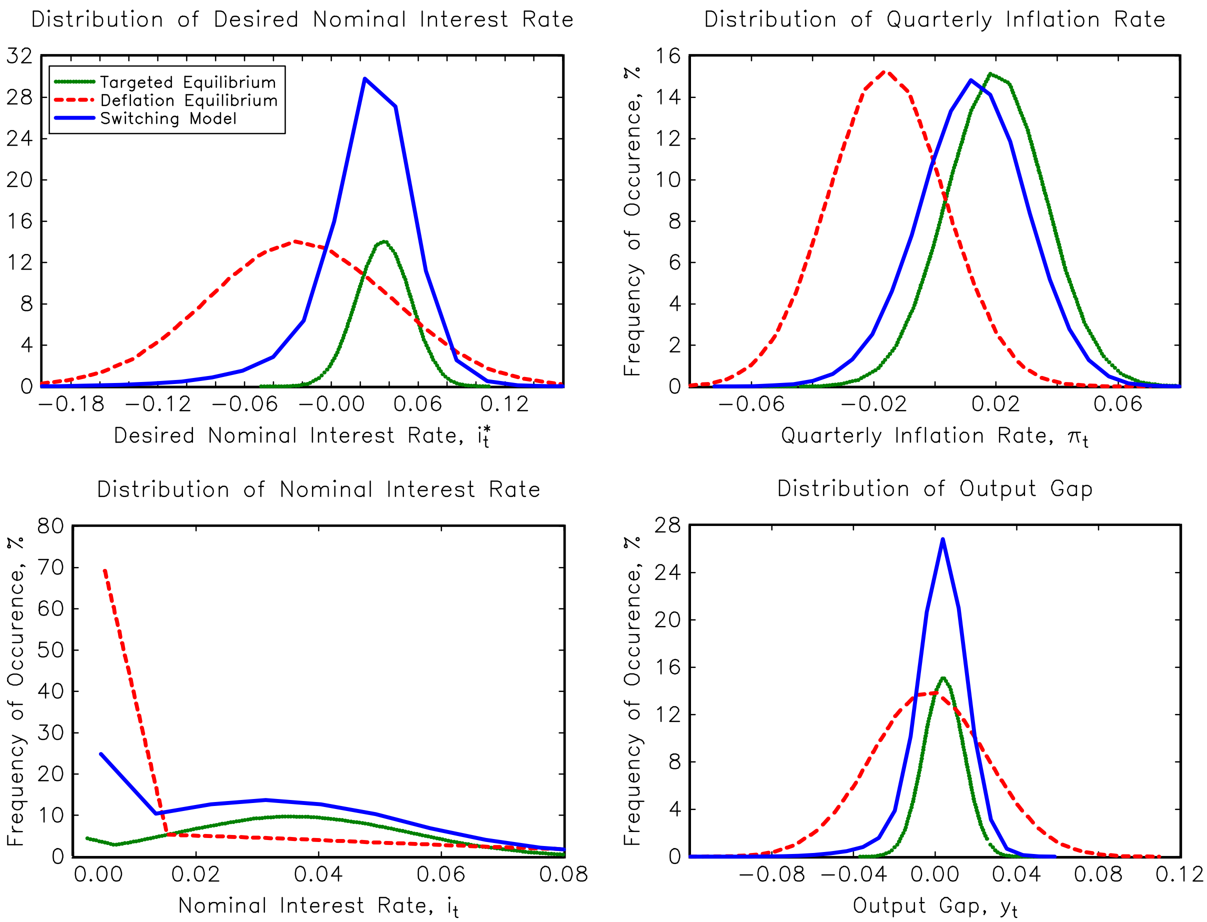
The U.S. real interest rate has remained below the Laubach-Williams estimate of r-star since early 2009. The nominal federal funds rate was pinned at zero from 2008.Q4 through 2015.Q4. A Taylor-type rule using the parameter values shown in Table 2 and the Laubach-Williams estimate of the natural rate of interest implies that the desired nominal funds rate was negative during this time. PCE inflation was briefly negative in 2009 and has remained below the Fed's 2% inflation target since 2012.Q2. The Great Recession was very severe, pushing the CBO output gap down to −6.5% at the business cycle trough in 2009.Q2. The output gap remains negative at −1.7% in 2015.Q4, more than six years after the Great Recession ended. The endpoints plotted in the figure are computed using the expressions in Table 1, with r_t^* given by the Laubach-Williams estimate.

Figure 5: *Model Simulation: Endogenous Regime Switching*



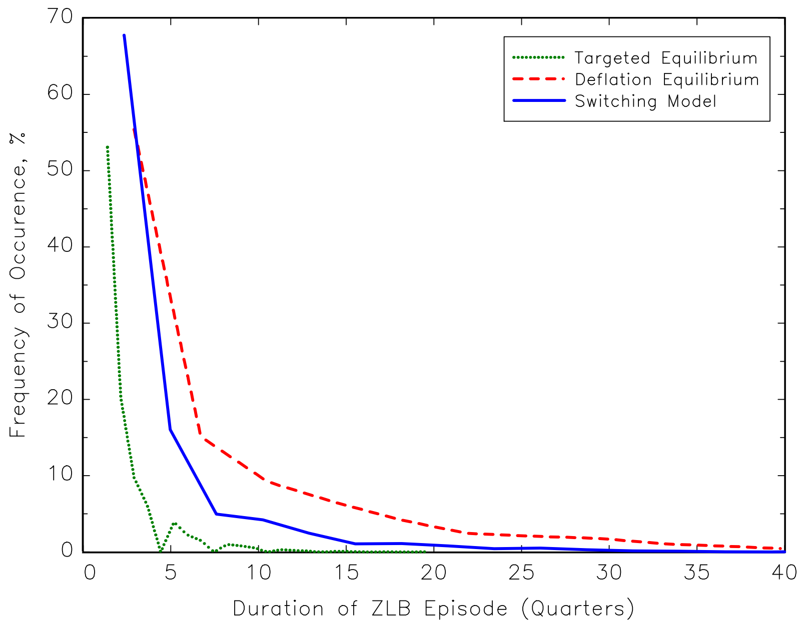
When the exogenous real interest gap $r_t - E_t r_t^*$ is negative for a sustained period, the resulting downward pressure on π_t, y_t , and i_t^* serves to reduce the recent $RMSFE$ of the deflation forecast rules and increase the recent $RMSFE$ of the targeted forecast rules. The shift in relative forecast performance can induce the agent to place a substantially higher weight on the deflation forecast rules, causing the deflation equilibrium to occasionally become self-fulfilling. Qualitatively similar results are obtained if the agent employs Bayes law (20) to compute the likelihood that a string of recent quarterly inflation observations comes from one equilibrium or the other.

Figure 6: *Model Simulations: Distributions of Endogenous Variables*



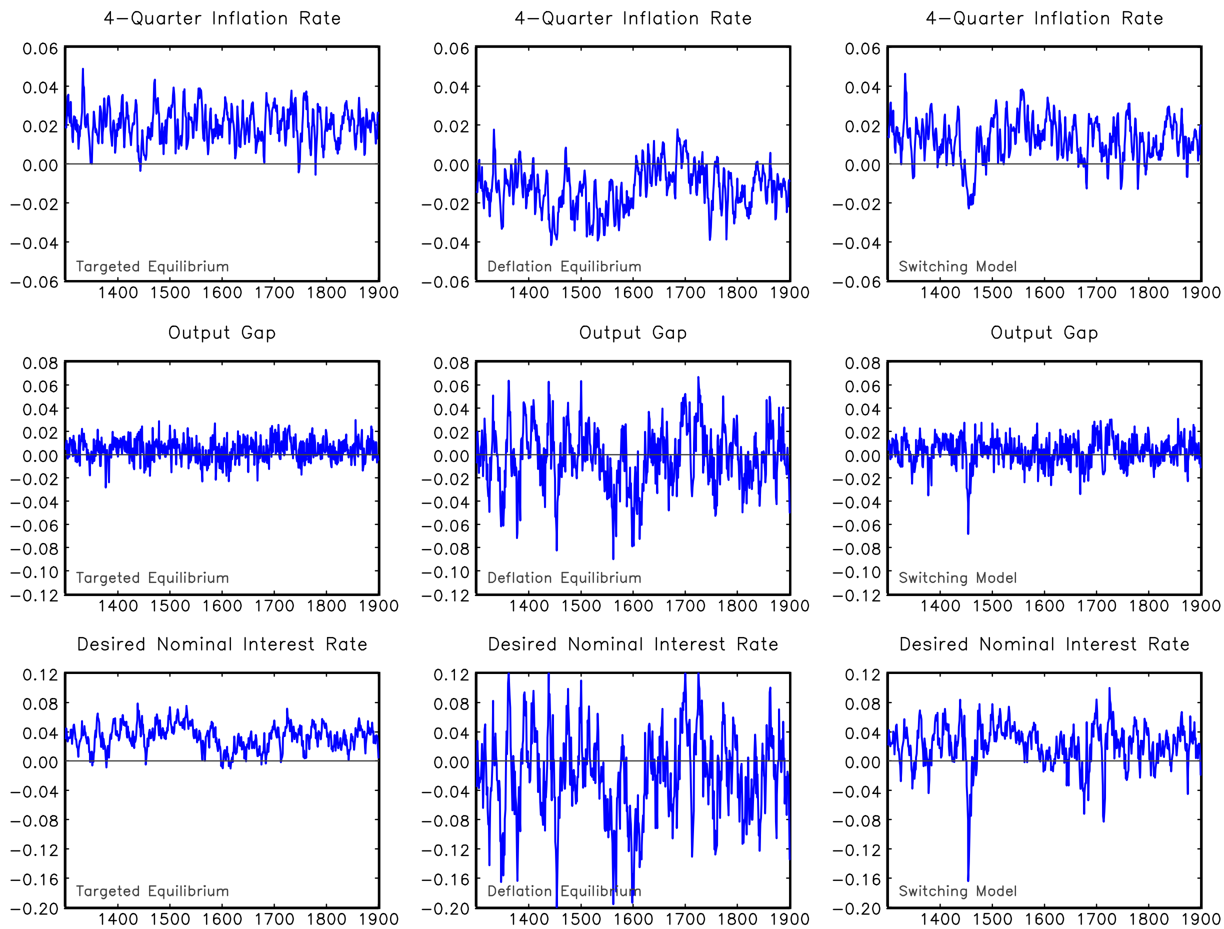
Model variables in the deflation equilibrium have distributions with lower means but higher variances than those in the targeted equilibrium. But the significant overlap in the various distributions creates a dilemma for an agent who seeks to determine the likelihood that a string of recent quarterly observations comes from one equilibrium or the other. Variables in the switching model have means that are somewhat lower and variances that are somewhat higher than those in the targeted equilibrium. Consequently, the central bank in the switching model undershoots its inflation target and the economy underperforms, as measured by the average value of the output gap.

Figure 7: *Model Simulations: Distribution of ZLB Durations*



Unlike the targeted equilibrium, the switching model can produce infrequent, but long-lived ZLB episodes in response to normally distributed shocks. To account for infrequent but long-lived ZLB episodes, the targeted equilibrium would require large shocks that are themselves infrequent but long-lived, as in Dordal-i-Carreras, et al. (2016).

Figure 8: *Model Simulations: Comparing Three Model Versions*



All three model versions employ the same sequence of stochastic shocks. Around period 1455 in the switching model, the weight on the targeted forecast rules approaches zero, causing the deflation equilibrium to become temporarily self-fulfilling. The episode results in a negative desired nominal interest rate, brief deflation followed by below-target inflation, and a highly negative output gap, reminiscent of the U.S. Great Recession and its aftermath.