

# Misallocation Cycles\*

Cedric Ehouarne<sup>†</sup>    Lars-Alexander Kuehn<sup>‡</sup>    David Schreindorfer<sup>§</sup>

September 13, 2016

## Abstract

The goal of this paper is to quantify the cyclical variation in firm-specific risk and study its aggregate consequences via the allocative efficiency of capital resources across firms. To this end, we estimate a general equilibrium model with firm heterogeneity and a representative household with Epstein-Zin preferences. Firms face investment frictions and permanent shocks, which feature time-variation in common idiosyncratic skewness. Quantitatively, the model replicates well the cyclical dynamics of the cross-sectional output growth and investment rate distributions. Economically, the model generates business cycles through inefficiencies in the allocation of capital across firms, which amounts to an average output gap of 4.5% relative to a frictionless model. These cycles arise because *(i)* permanent Gaussian shocks give rise to a power law distribution in firm size and *(ii)* rare negative Poisson shocks cause time-variation in common idiosyncratic skewness. Despite the absence of firm-level granularity, a power law in the firm size distribution implies that large inefficient firms dominate the economy, which hinders the household's ability to smooth consumption.

---

\*We thank Rüdiger Bachmann, Yasser Boualam, Scott Cederburg, Andrea Eisfeldt, Lukas Schmid, seminar participants at Arizona State University (Economics), Carnegie Mellon University, and the University of Southern California, and conference participants at the 6th Advances in Macro-Finance Tepper-LAEF Conference, the 1st Arizona Junior Finance Conference, the 2016 Duke-UNC Asset Pricing Conference, the 2016 Society for Economic Dynamics Meeting, the 2016 European Finance Association Meeting, and the BYU Red Rock Finance Conference 2016 for helpful comments and suggestions. The views expressed in this paper are the authors' and do not necessarily reflect the views of Bank of America.

<sup>†</sup>Bank of America, Model Risk Management, 1133 Av. of the Americas, 39th floor, New York, NY 10036, ehouarne@gmail.com

<sup>‡</sup>Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA15213, kuehn@cmu.edu

<sup>§</sup>W.P. Carey School of Business, Arizona State University, PO Box 873906, Tempe, AZ 85287, david.schreindorfer@asu.edu

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Model</b>	<b>5</b>
2.1	Production . . . . .	5
2.2	Firms . . . . .	8
2.3	Household . . . . .	9
2.4	Equilibrium . . . . .	10
<b>3</b>	<b>Analysis</b>	<b>10</b>
3.1	Household Optimization . . . . .	10
3.2	Firm Optimization . . . . .	11
3.3	Aggregation . . . . .	13
3.4	Capital Misallocation . . . . .	15
3.5	Numerical Method . . . . .	18
<b>4</b>	<b>Estimation</b>	<b>21</b>
4.1	Data . . . . .	21
4.2	Cyclical Properties of the Cross-Section of Firms . . . . .	22
4.3	Simulated Method of Moments . . . . .	23
4.4	Moment Selection and Parameter Identification . . . . .	25
4.5	Baseline Estimates . . . . .	26
4.6	Alternative Specifications . . . . .	27
<b>5</b>	<b>Model Implications</b>	<b>28</b>
5.1	Firms' Life Cycle . . . . .	29
5.2	Power Law and Consumption Dynamics . . . . .	29
5.3	Measuring Misallocation Empirically . . . . .	31
5.4	Shock persistence in the data . . . . .	32
<b>6</b>	<b>Conclusion</b>	<b>33</b>

# 1 Introduction

A large body of research has shown that the cross-section of firms is characterized by a substantial degree of productivity and capital heterogeneity (e.g., [Eisfeldt and Rampini \(2006\)](#)). While the empirical facts about firm heterogeneity are well known, the aggregate consequences are not well understood. In this paper, we develop and estimate a simple general equilibrium model to illustrate how the dynamics of the cross-section of firms impact aggregate fluctuations and risk premia via the misallocation of capital resources. The key implication of our general equilibrium model is that idiosyncratic shocks do not integrate out at the aggregate level but instead generate cyclical movements in the higher moments of consumption growth and risk premia.

Our model is driven by a cross-section of heterogeneous firms, which face irreversible investment decisions, exit, and permanent idiosyncratic and aggregate productivity shocks. The representative household has recursive preferences and consumes aggregate dividends. To generate aggregate consequences from a continuum of idiosyncratic shocks via capital misallocation, our model mechanism requires both a *power law distribution* as well as *common idiosyncratic skewness* in productivity.

While most of the literature assumes a log-normal idiosyncratic productivity distribution arising from mean-reverting Gaussian shocks, idiosyncratic shocks are permanent and follow random walks in our model. With firm exit, distributions of random walks generate power laws as emphasized by [Gabaix \(1999\)](#) and [Luttmer \(2007\)](#). Quantitatively, the endogenous power law for firm size is consistent with the data, as reported in [Axtell \(2001\)](#), such that the largest 5% of firms generate more than 30% of consumption and output in our model.

In addition to the random walk assumption, we model innovations to idiosyncratic productivity not only with Gaussian but also with negative Poisson shocks, which induce common idiosyncratic skewness. These negative Poisson shocks do not capture rare aggregate disaster, as in [Gourio \(2012\)](#), because they wash out at the aggregate level in a frictionless model.<sup>1</sup> Instead, time variation in the size of common idiosyncratic skewness allows us to capture

---

<sup>1</sup>For disaster risk in consumption see [Barro \(2006\)](#), [Gabaix \(2012\)](#), and [Wachter \(2013\)](#).

the cyclicity in the skewness of cross-sectional sales growth, consistent with the evidence in [Salgado et al. \(2015\)](#).

In the model, these features lead to large occasional inefficiencies in the allocation of capital across firms and it hinders the representative agent's ability to smooth consumption. Intuitively, in recessions aggregate productivity falls and the distribution of output growth becomes negatively skewed. Firms with negative idiosyncratic productivity draws find it difficult to disinvest unproductive capital to raise dividends. At the same time, the representative household would like to reallocate capital to smooth consumption.

Because of the power law distribution in firm size, the share of output coming from large firms contributes disproportionately to aggregate consumption, so that negatively skewed shocks to their productivity are particularly painful. Consequently, the drop in dividends from the mass of constrained firms is large, given that they are large in size. While unconstrained firms increase dividends by reducing investment, they are smaller so that they are not able to offset the impact of large constrained firms on aggregate consumption. This effect implies that in recessions aggregate consumption falls by more than aggregate productivity, causing negative skewness and kurtosis, and it arises purely from the cross-sectional misallocation. In contrast, in models with log-normal productivity distributions, the size difference between constrained and unconstrained firms is small so that the groups offset each other.

While the impact of capital misallocation on output and consumption are short lived under temporary mean-reverting shocks, permanent Poisson shocks render misallocation distortions long lasting. Quantitatively, output and consumption growth become more volatile and persistent, even though the model is only driven by i.i.d. innovations. Importantly, consumption growth is left skewed and leptokurtic, as in the data. Because the household cares about long lasting consumption distortions due to Epstein-Zin preferences, the welfare costs of capital misallocation and aggregate risk premia are large.

Our mechanism to generate aggregate fluctuations from idiosyncratic shocks obeying a power law is distinct from the granular hypothesis of [Gabaix \(2011\)](#). While Gabaix also argues that the dynamics of large firms matters for business cycles, he relies on the fact that

the number of firms is finite in an economy so that a few very large firms dominate aggregate output. The impact of these very large firms does not wash at the aggregate level when firm size follows a power law. In contrast, we model a continuum of firms such that each individual firm has zero mass. In our model, the power law in firm size generates aggregate fluctuations based on capital misallocation, arising from the investment friction, and not because the economy is populated by a finite number of firms. In reality, both effects are at work to shape business cycles.<sup>2</sup>

Methodologically, we build on [Veracierto \(2002\)](#) and [Khan and Thomas \(2008\)](#), who find that microeconomic investment frictions are inconsequential for aggregate fluctuations in models with mean-reverting idiosyncratic productivity.<sup>3</sup> We show that a model with permanent shocks and a more realistic firm size distribution not only breaks this irrelevance result, but also produces risk premia that are closer to the data. We are not the first to model permanent idiosyncratic shocks, e.g., [Caballero and Engel \(1999\)](#) and [Bloom \(2009\)](#) do so, but these papers study investment dynamics in partial equilibrium frameworks.

There is substantial empirical evidence that the riskiness of the economy is countercyclical, both at the aggregate and at the firm level. Starting with [Bloom \(2009\)](#), a large literature has used this observation to argue that shocks to uncertainty generate business cycles via wait-and-see effects in firms' investment and hiring policies. Our work differs from this prior literature in two important ways.

First, our theory does not feature wait-and-see effects because we deliberately model shocks to idiosyncratic risk as i.i.d. and unobservable ex ante. While recessions in our model are characterized by higher micro and macro volatility, risk shocks do not *cause* recessions. Rather, they lead to an amplification and propagation of downturns via their persistent effect on capital misallocation. Additionally, while aggregate shocks in our model are symmetrically distributed, aggregate output and consumption growth rates are unconditionally left-skewed because measured aggregate productivity falls more than true productivity during recessions. Our work thus provides an endogenous mechanism for the channel in [Berger et al. \(2016\)](#), who

---

<sup>2</sup>Related to the granular notion, [Kelly et al. \(2013\)](#) derive firm volatility in sparse networks.

<sup>3</sup>[Bachmann and Bayer \(2014\)](#) show that the same irrelevance result holds with idiosyncratic volatility shocks.

argue that the increased volatility observed during recessions is a consequence of negatively-skewed aggregate productivity technology shocks as opposed to a causal driver of recessions.

Second, while [Bloom \(2009\)](#) models risk shocks as a symmetric increase in the volatility of idiosyncratic risk, we assume that they operate through its left-skewness. In particular, we model idiosyncratic shocks as a combination of homoscedastic Gaussian innovations and negative Poisson jumps with time-varying size. As in Bloom’s model, the dispersion of idiosyncratic shocks therefore increases during recessions in our model, but the effect is driven by a widening of the left tail of the shock distribution only. This makes our model consistent with the empirically observed distribution of firms’ sales growth, which becomes strongly negatively skewed during recessions.

This empirical fact is also reminiscent of [Guvenen et al. \(2014\)](#), who document that households’ income shocks feature procyclical skewness. [Constantinides and Ghosh \(2015\)](#) and [Schmidt \(2015\)](#) show that procyclical skewness is quantitatively important for aggregate asset prices in incomplete market economies. Different from these papers, our paper focuses on heterogeneity on the productive side of the economy and analyzes the effect of skewed shocks on capital misallocation.

The first study to quantify capital misallocation is [Olley and Pakes \(1996\)](#). More recent contributions include [Hsieh and Klenow \(2009\)](#) and [Bartelsman et al. \(2013\)](#). We extend their measure of capital misallocation and derive a frictionless benchmark in a general equilibrium framework. The importance of capital misallocation for business cycles is illustrated by [Eisfeldt and Rampini \(2006\)](#).

Our study also relates to the literature on production-based asset pricing, including [Jermann \(1998\)](#), [Boldrin et al. \(2001\)](#), and [Kaltenbrunner and Lochstoer \(2010\)](#), which aims to make the real business cycle model consistent with properties of aggregate asset prices. While these models feature a representative firm, we incorporate a continuum of firms. This allows us to pay close attention to cross-sectional aspects of the data, thereby providing a more realistic micro foundation for the sources of aggregate risk premia. While [Kogan \(2001\)](#) and [Gomes et al. \(2003\)](#) also model firm heterogeneity, our model provides a tighter link to

firm fundamentals such that we estimate model parameters.

Our model mechanism is also related to the works of [Gabaix \(1999\)](#) and [Luttmer \(2007\)](#). [Gabaix \(1999\)](#) explains the power law of city sizes with random walks reflected at a lower bound. Using a similar mechanism, [Luttmer \(2007\)](#) generates a power law in firm size in a steady-state model. We extend this literature by studying the impact of a power law in firm size in a business cycle model with common idiosyncratic skewness shocks.

Starting with the influential paper by [Berk et al. \(1999\)](#), there exists a large literature, which studies the cross-section of returns in the neoclassical investment framework, e.g., [Carlson et al. \(2004\)](#), [Zhang \(2005\)](#), [Cooper \(2006\)](#), and [Gomes and Schmid \(2010\)](#). For tractability, these papers assume an exogenous pricing kernel and link firm cash flows and the pricing kernel directly via aggregate shocks. In contrast, we provide a micro foundation for the link between investment frictions and aggregate consumption.

## 2 Model

Time is discrete and infinite. The economy is populated by a unit mass of firms. Firms own capital, produce output with a neoclassical technology subject to investment being partially irreversible, and face permanent idiosyncratic and aggregate shocks. The representative household has recursive preferences and consumes aggregate dividends. This section elaborates on these model elements and defines the recursive competitive equilibrium of the economy.

### 2.1 Production

Firms produce output  $Y$  with the neoclassical technology

$$Y = (X\mathcal{E})^{1-\alpha}K^\alpha, \tag{1}$$

where  $X$  is aggregate productivity,  $\mathcal{E}$  is idiosyncratic productivity,  $K$  is the firm's capital stock and  $\alpha < 1$  is a parameter that reflects diminishing returns to scale. Aggregate productivity  $X$  follows a geometric random walk

$$X' = \exp \{ g_x - \sigma_x^2/2 + \sigma_x \eta'_x \} X, \tag{2}$$

where  $g_x$  denotes the average growth rate of the economy,  $\sigma_x$  the volatility of log aggregate productivity growth, and  $\eta_x$  an i.i.d. standard normal innovation.

Idiosyncratic productivity growth is a mixture of a normal and a Poisson distribution, allowing for rare but large negative productivity draws. These negative jumps capture, for instance, sudden drops in demand, increases in competition, the exit of key human capital, or changes in regulation. As we will see, they are also essential for allowing the model to replicate the cross-sectional distribution of firms' sales growth. Specifically, idiosyncratic productivity  $\mathcal{E}$  follows a geometric random walk modulated with idiosyncratic jumps

$$\mathcal{E}' = \exp \left\{ g_\varepsilon - \sigma_\varepsilon^2/2 + \sigma_\varepsilon \eta'_\varepsilon + \chi' J' - \lambda (e^{\chi'} - 1) \right\} \mathcal{E}, \quad (3)$$

where  $g_\varepsilon$  denotes the average firm-specific growth rate,  $\sigma_\varepsilon$  the volatility of the normal innovations in firm-specific productivity,  $\eta$  an i.i.d. idiosyncratic standard normal shock, and  $J$  an i.i.d. idiosyncratic Poisson shock with constant intensity  $\lambda$ . The jump size  $\chi$  varies with aggregate conditions  $\eta_x$ , which we capture with the exponential function

$$\chi(\eta_x) = -\chi_0 e^{-\chi_1 \eta_x} \quad (4)$$

with strictly positive coefficients  $\chi_0$  and  $\chi_1$ . This specification implies that jumps are negative and larger in worse aggregate times, i.e., for low values of  $\eta_x$ .

Our specification for idiosyncratic productivity warrants a few comments. First, [Bloom \(2009\)](#) structurally estimates the cyclicalities in the dispersion of idiosyncratic productivity, which is a symmetric measure of uncertainty. Our specification also leads to time variation in the higher moments of idiosyncratic productivity growth. In particular, equation (4) implies that firm-specific productivity shocks become more left skewed in recessions. Second, different from the uncertainty shocks in [Bloom \(2009\)](#) and [Bloom et al. \(2014\)](#), our assumptions imply that changes in idiosyncratic jump risk are neither known to firms ex ante nor persistent, and therefore do not cause wait-and-see effects. As we will show, however, they induce large changes in measured aggregate productivity via their effect on the efficiency of the cross-sectional capital distribution. Third, in contrast to the consumption-based asset pricing literature with disaster risk in consumption, for instance [Barro \(2006\)](#), [Gabaix \(2012\)](#), and

Wachter (2013), we do not model time variation in the jump probability  $\lambda$ . If the jump probability were increasing in recessions, it would induce rising skewness in productivity and sales growth, while in the data it is falling.<sup>4</sup> Fourth, the idiosyncratic jump risk term  $\chi J$  is compensated by its mean  $\lambda(e^\chi - 1)$ , so that the cross-sectional mean of idiosyncratic productivity is constant (see equation (5) below). This normalization implies that aggregate productivity is determined solely by  $\eta_x$ -shocks, so that our model does not generate aggregate jumps in productivity as emphasized by, e.g., Gourio (2012). Because the size of the jump risk is common across firms, we refer to it as *common idiosyncratic skewness* in productivity.

Given the geometric growth in idiosyncratic productivity, the cross-sectional mean of idiosyncratic productivity is unbounded unless firms exit. We therefore assume that at the beginning of a period – before production takes place and investment decisions are made – each firm exits the economy with probability  $\pi \in (0, 1)$ . Exiting firms are replaced by an identical mass of entrants who draws their initial productivity level from a log-normal distribution with location parameter  $g_0 - \sigma_0^2/2$  and scale parameter  $\sigma_0$ . Whenever firms exit, their capital stock scrapped and entrants start with zero initial capital.

Since the idiosyncratic productivity distribution is a mixture of Gaussian and Poisson innovations, it cannot be characterized by a known distribution.<sup>5</sup> But two features are noteworthy. First, due to random growth and exit, the idiosyncratic productivity distribution and thus firm size features a power law, as shown by Gabaix (2009). A power law holds when the upper tail of the firm size distribution obeys a Pareto distribution such that the probability of size  $S$  greater than  $x$  is proportional to  $1/x^\zeta$  with tail (power law) coefficient  $\zeta$ .<sup>6</sup>

Second, even though the distribution is unknown, we can compute its higher moments. Let  $\mathbb{M}_n$  denote the  $n$ -th cross-sectional raw moment of the idiosyncratic productivity distribution

---

<sup>4</sup>Note that skewness of Poisson jumps  $J$  equals  $\lambda^{-1/2}$ .

<sup>5</sup>Dixit and Pindyck (1994) assume a similar process without Poisson jumps in continuous time and solve for the shape of the cross-sectional density numerically; see their chapter 8.4.

<sup>6</sup>In our model, the tail coefficient solves the nonlinear equation  $1 = (1 - \pi)Z(\zeta)$ , where  $Z(\zeta) = \exp\{\zeta g_\varepsilon - \zeta \sigma_\varepsilon^2/2 + \zeta^2 \sigma_\varepsilon^2/2 + \lambda(e^{\zeta \chi} - 1) - \zeta \lambda(e^\chi - 1)\}$ .

$\mathcal{E}$ . It has the following recursive structure

$$\begin{aligned} \mathbb{M}'_n &= (1 - \pi) \exp\{ng_\varepsilon - n\sigma_\varepsilon^2/2 + n^2\sigma_\varepsilon^2/2 + \lambda(e^{n\chi'} - 1) - n\lambda(e^{\chi'} - 1)\}\mathbb{M}_n \\ &\quad + \pi \exp\{ng_0 - n\sigma_0^2/2 + n^2\sigma_0^2/2\}. \end{aligned} \quad (5)$$

The integral over idiosyncratic productivity and capital determines aggregate output. To ensure that aggregate output is finite, we require that the productivity distribution has a finite mean.<sup>7</sup> Equation (5) states that the mean evolves according to  $\mathbb{M}'_1 = (1 - \pi)e^{g_\varepsilon}\mathbb{M}_1 + \pi e^{g_0}$ , which is finite if

$$g_\varepsilon < -\ln(1 - \pi) \approx \pi. \quad (6)$$

In words, the firm-specific productivity growth rate has to be smaller than the exit rate. In this case, the first moment is constant and, for convenience, we normalize it to one by setting

$$g_0 = \ln(1 - e^{g_\varepsilon}(1 - \pi)) - \ln(\pi). \quad (7)$$

## 2.2 Firms

To take advantage of higher productivity, firms make optimal investment decisions. Capital evolves according to

$$K' = (1 - \delta)K + I, \quad (8)$$

where  $\delta$  is the depreciation rate and  $I$  is investment. As in [Khan and Thomas \(2013\)](#) and [Bloom et al. \(2014\)](#), we assume investment is partially irreversible, which generates spikes and positive autocorrelation in investment rates as observed in firm level data. Quadratic adjustment costs can achieve the latter only at the expense of the former, since they imply an increasing marginal cost of adjustment. Partial irreversibility means that firms recover only a fraction  $\xi$  of the book value of capital when they choose to disinvest. These costs arise from resale losses due to transactions costs, asset specificity, and the physical costs of resale.

We show in [Section 3](#) that partial irreversibility yields an  $(S, s)$  investment policy such that firms have nonzero investment only when their capital falls outside an  $(S, s)$  inactivity

---

<sup>7</sup>[Luttmer \(2007\)](#) makes a related assumption (Assumption 4), which states that “a firm is not expected to grow faster than the population growth rate” to ensure that the firm size distribution has finite mean.

band.<sup>8</sup> A firm with an unacceptably high capital stock relative to its current productivity will reduce its stock only to the upper bound of its inactivity range. Similarly, a firm with too little capital invests only to the lower bound of its inactivity range to reduce the linear penalty it will incur if it later chooses to shed capital. Thus, partial irreversibility can deliver persistence in firms investment rates by encouraging repeated small investments at the edges of inactivity bands.

We summarize the distribution of firms over the idiosyncratic states  $(K, \mathcal{E})$  using the probability measure  $\mu$  and note that the aggregate state of the economy is given by  $(X, \mu)$ . The distribution of firms evolves according to a mapping  $\Gamma$ , which we derive in Section 3. Intuitively, the dynamics of  $\mu$  are shaped by the exogenous dynamics of  $\mathcal{E}$  and  $X$ , the endogenous dynamics of  $K$  resulting from firms' investment decisions, and firm entry and exit.

Firms maximize the present value of their dividend payments to shareholders by solving

$$V(K, \mathcal{E}, X, \mu) = \max_I \{D + (1 - \pi)\mathbb{E}[M'V(K', \mathcal{E}', X', \mu')]\}, \quad (9)$$

where

$$D = Y - I 1_{\{I \geq 0\}} - \xi I 1_{\{I < 0\}} \quad (10)$$

denotes the firm's dividends and  $M$  is the equilibrium pricing kernel based on aggregate consumption and the household's preferences, which we derive in Section 3.1.

### 2.3 Household

The representative household of the economy maximizes recursive utility  $U$  over consumption  $C$  as in [Epstein and Zin \(1989\)](#):

$$U(X, \mu) = \max_C \left\{ (1 - \beta)C^{1 - \frac{1}{\psi}} + \beta (\mathbb{E}[U(X', \mu')^{1 - \gamma}])^{(1 - \frac{1}{\psi})/(1 - \gamma)} \right\}^{1/(1 - \frac{1}{\psi})} \quad (11)$$

where  $\psi > 0$  denotes the elasticity of intertemporal substitution (EIS),  $\beta \in (0, 1)$  the subjective discount factor, and  $\gamma > 0$  the coefficient of relative risk aversion. In the special case when risk aversion equals the inverse of EIS, the preferences reduce to the common power

---

<sup>8</sup>See [Andrew B. Abel \(1996\)](#) for a continuous time model of partial irreversibility.

utility specification. The household's resource constraint is

$$C = \int D \, d\mu. \tag{12}$$

## 2.4 Equilibrium

A recursive competitive equilibrium for this economy is a set of functions  $(C, U, V, K, \Gamma)$  such that:

- (i) Firm optimality: Taking  $M$  and  $\Gamma$  as given, firms maximize firm value (9) with policy function  $K$  subject to (8) and (10).
- (ii) Household optimality: Taking  $V$  as given, household maximize utility (11) subject to (12) with policy function  $C$ .
- (iii) The good market clears according to (12).
- (iv) Model consistency: The transition function  $\Gamma$  is induced by  $K$ , aggregate productivity  $X$ , equation (2), idiosyncratic productivity  $\mathcal{E}$ , equation (3), and entry and exit.

## 3 Analysis

In this section, we characterize firms' optimal investment policy and the transition dynamics of the cross-sectional distribution of firms. We also derive closed-form solutions for a frictionless version of the model, which serves as a benchmark for quantifying the degree of capital misallocation and the wedge between actual and measured aggregate productivity. Because aggregate productivity contains a unit root, we solve the model in detrended units, such that detrended consumption  $c$  and wealth  $w$  are given by

$$c = C/X \quad w = W/X.$$

### 3.1 Household Optimization

The household's first order condition with respect to the optimal asset allocation implies the usual Euler equation

$$\mathbb{E}[M'R'] = 1 \tag{13}$$

where  $M'$  is the pricing kernel and  $R'$  is the return on equity, defined by  $V'/(V - D)$ . The pricing kernel is given by

$$M' = \beta^\theta (x')^{-\gamma} \left(\frac{c'}{c}\right)^{-\theta/\psi} \left(\frac{w'}{w - c}\right)^{\theta-1}, \quad (14)$$

where  $\theta = \frac{1-\gamma}{1-1/\psi}$  is a preference parameter and  $x' = X'/X$  is i.i.d. log-normal distributed. In the case of power utility,  $\theta$  equals one and wealth drops out of the pricing kernel. With Epstein-Zin preferences, the dynamics of both consumption and wealth evolve endogenously and are part of the equilibrium solution.

Consistent with the Euler equation (13), wealth is defined recursively as the present value of future aggregate consumption:

$$w = c + \beta \mathbb{E} \left[ (x')^{1-\gamma} (w')^\theta \left(\frac{c'}{c}\right)^{-\theta/\psi} \right]^{1/\theta}. \quad (15)$$

Firm exit introduces a wedge between wealth and the aggregate market value of firms. This stems from the fact that wealth captures the present value of both incumbents and entrants, whereas aggregate firm value relates to the present value of dividends of incumbent firms only.

### 3.2 Firm Optimization

Having solved for the functional form of the pricing kernel, we can characterize firms' optimal investment policy. The homogeneity of the value function and the linearity of the constraints imply that we can detrend the firm problem by the product of both permanent shocks  $X\mathcal{E}$ , as for instance in Bloom (2009). We define the firm-specific capital to productivity ratio  $\kappa = K/(X\mathcal{E})$ , the capital target to productivity ratio  $\tau = K'/(X\mathcal{E})$ , and the firm value to productivity ratio  $v = V/(X\mathcal{E})$ .

Given the linear cost structure, one can divide the value function into three regions. In the investing region ( $(1 - \delta)\kappa \leq \tau$ ), firms increase their capital to productivity ratio and the optimal firm value solves  $v_u$ ; in the disinvesting region ( $\tau \leq (1 - \delta)\kappa$ ), firms decrease their capital to productivity ratio and the optimal firm value solves  $v_d$ ; otherwise, firms are inactive.

Firm value  $v$  is thus the maximum of the value of investing  $v_u$ , disinvesting  $v_d$ , or inactivity:

$$v_u(\kappa, \mu) = \max_{(1-\delta)\kappa \leq \tau} \left\{ \kappa^\alpha - (\tau - (1-\delta)\kappa) + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v(\kappa', \mu')\right] \right\}, \quad (16)$$

$$v_d(\kappa, \mu) = \max_{\tau \leq (1-\delta)\kappa} \left\{ \kappa^\alpha - \xi(\tau - (1-\delta)\kappa) + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v(\kappa', \mu')\right] \right\}, \quad (17)$$

$$v(\kappa, \mu) = \max \left\{ v_u(\kappa, \mu), v_d(\kappa, \mu), \kappa^\alpha + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v((1-\delta)\kappa/(x'\varepsilon'), \mu')\right] \right\}, \quad (18)$$

where  $\varepsilon' = \mathcal{E}'/\mathcal{E}$ . Because both growth rates  $\varepsilon'$  and  $x'$  are i.i.d., the state space of the detrended firm problem reduces to  $(\kappa, \mu)$ . Importantly, for adjusting firms next period's capital to productivity ratio  $\kappa' = \tau/(x'\varepsilon')$  is independent of the current capital to productivity ratio. This fact implies that firms share a common time-varying capital target  $\tau$ , which is independent of their own characteristic  $\kappa$ . The optimal capital targets for the investing and disinvesting regions is given by  $\mathcal{T}_u(\mu)$  and  $\mathcal{T}_d(\mu)$ , respectively, and solves

$$\begin{aligned} \mathcal{T}_u(\mu) &= \arg \max_{\tau} \left\{ -\tau + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v(\tau/(x'\varepsilon'), \mu')\right] \right\}, \\ \mathcal{T}_d(\mu) &= \arg \max_{\tau} \left\{ -\xi\tau + (1-\pi)\mathbb{E}\left[M'x'\varepsilon'v(\tau/(x'\varepsilon'), \mu')\right] \right\}. \end{aligned}$$

Given these capital targets, the optimal policy of the firm-specific capital to productivity ratio can be characterized by an  $(S, s)$  policy and is given by

$$\kappa' = \max\{\mathcal{T}_u(\mu), \min\{\mathcal{T}_d(\mu), (1-\delta)\kappa\}\}/(x'\varepsilon') \quad (19)$$

where the max operator characterizes the investing region and the min operator the disinvesting one. Conditional on adjusting, the capital to productivity ratio of every firm is either  $\mathcal{T}_u$  or  $\mathcal{T}_d$ , independent of their own characteristic  $\kappa$  but dependent on the aggregate firm distribution  $\mu$ .

The optimal investment rate policy, implied by (19), can be summarized by the same three regions of investment, inactivity, and disinvestment:

$$\frac{I}{K} = \begin{cases} \frac{\mathcal{T}_u(\mu) - \kappa}{\kappa} + \delta & (1-\delta)\kappa < \mathcal{T}_u & \text{investing,} \\ 0 & \mathcal{T}_u \leq (1-\delta)\kappa \leq \mathcal{T}_d & \text{inactive,} \\ \frac{\mathcal{T}_d(\mu) - \kappa}{\kappa} + \delta & \mathcal{T}_d < (1-\delta)\kappa & \text{disinvesting.} \end{cases}$$

In Figure 1, we plot both the optimal capital to productivity and investment rate policies for two arbitrary capital targets. Intuitively, when a firm receives a positive idiosyncratic

productivity draw, its capital to productivity ratio  $\kappa$  falls. If the shock is large enough and depreciated  $\kappa$  is less than  $\mathcal{T}_u$ , it will choose a positive investment rate, which reflects the relative difference between target and current capital to productivity ratio as well as the depreciation rate. As a result, next period's capital to productivity ratio will reach  $\mathcal{T}_u$  in the investment region.

When a firm experiences an adverse idiosyncratic productivity draw, its capital to productivity ratio  $\kappa$  increases and it owns excess capital. If the shock is severe enough and depreciated  $\kappa$  is greater than  $\mathcal{T}_d$ , it will choose a negative investment rate, which reflects the relative difference between target and current capital to productivity ratio as well as the depreciation rate. As a result, next period's capital to productivity ratio will fall to  $\mathcal{T}_d$  in the disinvestment region. For small enough innovations, the depreciated capital to productivity ratio remains within  $\mathcal{T}_u$  and  $\mathcal{T}_d$ . In this region, firms are inactive and have a zero investment rate.

An important features of our model is that there is heterogeneity in the duration of disinvestment constraintness. This feature arises because adverse idiosyncratic productivity shocks can arise either from a normal distribution or from a Poisson distribution. While adverse normal distributed shocks are short lasting, Poisson shocks are rare and large and therefore long lasting. As a result of Poisson shocks, the capital to productivity ratio rises dramatically, indicating a long duration of disinvestment constraintness.

### 3.3 Aggregation

In the previous section, we have shown that the firm-specific state space of the firm's problem reduces to the univariate capital to productivity ratio  $\kappa$ . One might therefore conjecture that the household only cares about the distribution of the capital to productivity ratio across firms. Yet the univariate distribution of the capital to productivity ratio is not sufficient to solve for equilibrium consumption.

Aggregate output is the integral over the product of capital and idiosyncratic productivity and thus the correlation between capital and idiosyncratic productivity matters. While capital and idiosyncratic productivity are perfectly correlated in the frictionless economy, the

investment friction renders capital and idiosyncratic productivity imperfectly correlated. As a result, the joint distribution of capital and idiosyncratic productivity matters for aggregate consumption because it captures the degree of capital misallocation across firms, whereas in the frictionless model aggregate capital is a sufficient variable for the firm-level distribution.

Instead of normalizing capital by both permanent shocks  $X\mathcal{E}$ , we define detrended capital  $k$  by  $k = K/X$ . We can then summarize the distribution of firms over the idiosyncratic states by  $(k, \mathcal{E})$  using the probability measure  $\mu$ , which is defined on the Borel algebra  $\mathcal{S}$  for the product space  $\mathbf{S} = \mathbb{R}_0^+ \times \mathbb{R}^+$ .<sup>9</sup> The distribution of firms evolves according to a mapping  $\Gamma$ , which is derived from the dynamics of idiosyncratic productivity  $\mathcal{E}$  in equation (3) and capital. The law of motion for detrended capital can be obtained by multiplying firms' optimal policies (19) with idiosyncratic productivity  $\mathcal{E}$  and is given by

$$k' = \max\{\mathcal{E}\mathcal{T}_u(\mu), \min\{\mathcal{E}\mathcal{T}_d(\mu), (1 - \delta)k\}\}/x'. \quad (20)$$

In Figure 2, we illustrate the three regions of  $\mu$  implied by the optimal capital policy (20). For the majority of firms, capital and idiosyncratic productivity are closely aligned such that these firms are optimally inactive. When idiosyncratic productivity exceeds capital, firms optimally invest such that next period's capital lies on the (blue) boundary to inactivity. Similarly, when capital exceeds idiosyncratic productivity, firms optimally disinvest such that next period's capital lies on the (red) boundary to inactivity.

Given the capital policy (20), the aggregate resource constraint detrended by  $X$  yields detrended consumption

$$c = \int \mathcal{E}^{1-\alpha} k^\alpha d\mu + (1+\xi)(1-\delta)\bar{k} - \int \max\{\mathcal{E}\mathcal{T}_u, (1-\delta)k\} d\mu - \xi \int \min\{\mathcal{E}\mathcal{T}_d, (1-\delta)k\} d\mu, \quad (21)$$

where  $\bar{k} = \int k d\mu$  denotes the aggregate capital stock. The first term is aggregate output, the second one is the book value of depreciated capital, the third one captures aggregate investment, and the fourth one aggregate disinvestment.

The distribution  $\mu$  evolves over time according to the mapping  $\Gamma : (\mu, \eta'_x) \mapsto \mu'$ . To derive this mapping, note that the capital policy  $k'$  in equation (20) is predetermined with

---

<sup>9</sup>Note that, with a slight abuse of notation but for better readability, we continue to use the symbols  $\mu$  and  $\Gamma$  to denote the distribution of firms and its transition in the detrended economy.

respect to the firm-level productivity shocks  $(\eta', J')$ . This implies that, conditional on current information and next period's aggregate shock  $\eta'_x$ , next period's characteristics  $(k', \mathcal{E}')$  are cross-sectionally independent of one another. Therefore, for any  $(\mathbf{K}, \mathbf{E}) \in \mathbf{S}$ ,

$$\mu'(\mathbf{K}, \mathbf{E}|\eta'_x) = \mu'_k(\mathbf{K}|\eta'_x) \times \mu'_\mathcal{E}(\mathbf{E}|\eta'_x), \quad (22)$$

where  $\mu_k$  and  $\mu_\mathcal{E}$  are the marginal distributions of capital and productivity, respectively.

The measure of firms with a capital stock of  $k' \in \mathbf{K}$  next period is simply the integral over the measure of firms who choose  $k'$  as their optimal policy this period and survive, plus the mass of entrants in the case  $0 \in \mathbf{K}$ .

$$\mu'_k(\mathbf{K}|\eta'_x) = (1 - \pi) \int 1_{\{k' \in \mathbf{K}\}} d\mu + \pi 1_{\{0 \in \mathbf{K}\}} \quad (23)$$

The measure of firms with an idiosyncratic productivity of  $\mathcal{E}' \in \mathbf{E}$  next period follows from the fact that, conditional on  $(\mathcal{E}, J', \eta'_x)$ ,  $\mathcal{E}'$  is log-normally distributed for continuing firms. For entrants, idiosyncratic productivity is log-normally distributed as well. The distribution of  $\mathcal{E}'$  conditional on  $\eta'_x$  can therefore be computed as follows

$$\begin{aligned} \mu'_\mathcal{E}(\mathbf{E}|\eta'_x) &= \int_{\mathcal{E}' \in \mathbf{E}} \left\{ (1 - \pi) \int \sum_{j=0}^{\infty} p_j \phi \left( \frac{\ln(\mathcal{E}') - \left( \ln(\mathcal{E}) + g_\varepsilon - \frac{\sigma_\varepsilon^2}{2} + \chi' j - \lambda(e^{\mathcal{X}'} - 1) \right)}{\sigma_\varepsilon} \right) \right. \\ &\quad \left. + \pi \phi \left( \frac{\ln(\mathcal{E}') - (g_0 - \sigma_\varepsilon^2/2)}{\sigma_\varepsilon} \right) \right\} d\mu_\mathcal{E} \end{aligned} \quad (24)$$

where  $p_j = \lambda^j e^{-\lambda} / j!$  is the Poisson probability of receiving  $j$  jumps and  $\phi$  the standard normal density. Equations (22)–(24) define the transition function  $\Gamma$ .

### 3.4 Capital Misallocation

We are interested in the extend to which aggregate output and consumption dynamics are affected by time-variation in the efficiency of the capital allocation across firms. A natural benchmark is therefore an allocation that maximizes aggregate output by equating the marginal products of capital across firms. Deviations from this benchmark arise in our model from interplay of idiosyncratic risk and investment frictions.

There are two channels. First, the assumption that capital stocks are predetermined implies that there is always a contemporaneous mismatch between firms' productivity levels and

the capital stocks that would equate their marginal products. The severity of this mismatch increases in the dispersion and skewness of idiosyncratic shocks. Similarly, time-variation in these higher moments induces time-variation in misallocation by altering the fraction of firms that become inactive in a given period.

Second, existing mismatches between capital and productivity carry over to future periods for firms that do not adjust their capital stocks. Because partial investment irreversibility creates an inactivity region in firms' investment policies, it amplifies the degree of misallocation. It also induces persistence by making the extensive margin a function of the recent history of shocks to the higher moments of idiosyncratic risk.

We show below that a model with no investment friction other than predetermined capital stocks is isomorphic to a representative firm model. The part of misallocation that arises from predetermined capital stocks in our heterogeneous firm economy is therefore equivalent to the ex-post sub-optimality of the aggregate capital stock in a representative firm model with predetermined capital. Because our interest lies in the part of misallocation that originates uniquely from cross-sectional features of the model, we measure misallocation relative to a benchmark economy without proportional costs but *with* predetermined capital stocks. In particular, we define the output gap as the percentage difference between output in the frictionless benchmark model and output in the full model.<sup>10</sup>

The firm's optimal investment target in the frictionless benchmark economy can be solved for analytically and is given by

$$\mathcal{T}(\mu) = \left( \frac{(1 - \pi)\alpha\mathbb{E}[M'(x'\varepsilon')^{1-\alpha}]}{1 - (1 - \pi)(1 - \delta)\mathbb{E}[M']} \right)^{1/(1-\alpha)}.$$

The optimal capital policy (20) simplifies to

$$k' = \mathcal{E}\mathcal{T}(\mu)/x'. \quad (25)$$

Intuitively, without the irreversibility constraint, firms are at their optimal capital target in every period.

---

<sup>10</sup>A second consideration that supports our choice of a benchmark is the fact that misallocation arising from predetermined capital stocks is a feature of nature that cannot be altered by policies, whereas misallocation arising from proportional costs can be attenuated by policies that increase the efficiency of the market for used capital.

	<b>Case I</b>		<b>Case II</b>		
$k_{high}$	0	0.5	$k_{high}$	0.5	0
$k_{low}$	0.5	0	$k_{low}$	0	0.5
	$\varepsilon_{low}$	$\varepsilon_{high}$		$\varepsilon_{low}$	$\varepsilon_{high}$

**Table 1: Two stylized firm-level distributions**

In the frictionless case, it is feasible to derive a closed-form expression for the law of motion of aggregate capital. Specifically, by aggregating the optimal capital policy (25) across firms, it follows that

$$\bar{k}' = (1 - \pi)\mathcal{T}/x'.$$

Intuitively, aggregate capital is a weighted average of the investment target of incumbents and average capital of entrants. This aggregation result fails in the full model because the optimal capital policy under partial irreversible investment (20) implies that future capital is a function of past shocks, rendering capital and idiosyncratic shocks correlated.

Similarly, the detrended aggregate resource constraint (21) in the frictionless economy simplifies to  $c = y + (1 - \delta)\bar{k} - \mathcal{T}$ . In contrast to the full model, aggregate output in the frictionless economy collapses to a function of aggregate capital  $\bar{k}$  and is given by

$$y = A(1 - \pi)^{1-\alpha}\bar{k}^\alpha, \tag{26}$$

where  $A$  is a Jensen term coming from the curvature in the production function.

In the full model, capital is misallocated across firms because unproductive firms find it costly to disinvest. Following the literature, we use two measures of resource misallocation: a capital misallocation measure  $\mathcal{M}$  and an output distortion measure  $\mathcal{D}$ .

The first misallocation measure is the correlation between log capital and log productivity

$$\mathcal{M} = 1 - \text{Corr}(\ln K', \ln \mathcal{E}). \tag{27}$$

In the frictionless case, capital is never misallocated and  $\mathcal{M} = 0$  in each period. In the full model, the more capital is misallocated across firms the larger is  $\mathcal{M}$ . Importantly, the correlation measure takes into account that capital is predetermined. This measure is motivated by [Olley and Pakes \(1996\)](#), who studied the covariance between productivity and output share.

To illustrate capital misallocation in the context of our model, we present two stylized firm-level distributions in Table 1, where both idiosyncratic productivity and capital can only take on two values. The table entries are the probability mass for each point in the support of  $\mu$ , in line with the intuition of Figure 2. In Case I, productive firms hold a high capital stock, while unproductive firms hold a low capital stock. Consequently, there is no capital misallocation and  $\mathcal{M} = 0$ . In Case II, the scenario is reversed and capital completely misallocated with  $\mathcal{M} = 2$ .

The second measure is the output gap due to capital misallocation, as in [Hsieh and Klenow \(2009\)](#). The output gap  $\mathcal{D}$  is defined as the one minus ratio of output in the full model relative to output on the frictionless economy and is given by

$$\mathcal{D} = 1 - \frac{\int \mathcal{E}^{1-\alpha} k^\alpha d\mu}{A(1-\pi)^{1-\alpha} \bar{k}^\alpha}. \quad (28)$$

While [Hsieh and Klenow \(2009\)](#) are agnostic about the source of distortions that drive a wedge between the marginal products of capital across firms, we assume proportional adjustment costs.

### 3.5 Numerical Method

As in [Krusell and Smith \(1998\)](#), we approximate the firm-level distribution  $\mu$  with a finite-dimensional aggregate state variable to make the model solution computable. Krusell and Smith and most of the subsequent literature have relied on moments of capital to summarize the policy-relevant information in  $\mu$ . Instead, we use detrended aggregate consumption  $c$ . For two reasons, this approach is better suited for models with significant time-variation in the higher moments of the cross-sectional distribution. First, consumption captures the joint distribution of capital and productivity, whereas aggregate capital (and higher moments of capital) only capture the marginal distribution of capital. Second, using consumption as a state variable eliminates the need for a second approximation rule that maps capital into marginal utilities. Below, we discuss each of these points in more detail and summarize our numerical approach.

To illustrate the importance of capturing both dimensions of  $\mu$ , consider again the stylized

example in Table 1. In both cases, the marginal distribution of capital is identical and aggregate capital stock equals  $(k_{low} + k_{high})/2$ . Aggregate output, however, is higher in Case I, where capital is not misallocated and productive firms hold a high capital stock. For many preference specifications, a higher output level also implies higher consumption and investment levels, and therefore a higher future capital stock. Regardless of preferences, consumption and investment will generally differ across the two cases. Because the Krusell and Smith algorithm predicts next period’s capital stock solely based on today’s capital stock, it incorrectly predicts the same value in both cases. In contrast, consumption reflects firms’ investment policies today and is therefore better suited for predicting tomorrow’s aggregate state.

The second issue related to the Krusell and Smith algorithm arises only when it is applied to models with firm heterogeneity, as in Khan and Thomas (2008, 2013) and Bloom et al. (2014). Because the decentralized firm problem involves the pricing kernel, it is necessary to compute the representative agent’s marginal utility. When  $\mu$  is approximated with  $\bar{k}$ , one has to introduce a second, contemporaneous approximation that maps  $\bar{k}$  into marginal utility. For example, Khan and Thomas (2008) specify  $u'(c)$  as a log-linear function of  $\bar{k}$ . When misallocation becomes quantitatively important, this approximation is poor because  $c$  is in general a function of both dimensions of  $\mu$ , whereas  $\bar{k}$  only reflects one marginal distribution.<sup>11</sup> In contrast, specifying  $c$  as an aggregate state variable implies that no additional approximation is required.

Methodologically, the main difference between aggregate capital compared to consumption as state variable arises when specifying their law of motions. Tomorrow’s capital stock for each firm is contained in the current information set, which implies that tomorrow’s aggregate capital stock is contained in the current information set as well. Consequently, it is possible to approximate the law of motion for aggregate capital with a deterministic function. On the contrary, tomorrow’s consumption is not known today but depends on tomorrow’s realization of the aggregate shock  $\eta'_x$ . We approximate the law of motion for consumption with an affine

---

<sup>11</sup>For example, in Bloom et al. (2014), the mapping to marginal utility results in  $R^2$ ’s as low as 88% for some states – see their Table B1.

function in log consumption

$$\ln(c') = \varphi_0(\eta'_x) + \varphi_1(\eta'_x) \ln(c). \quad (29)$$

These forecasting functions imply intercepts and slope coefficients that depend on the future shock to aggregate productivity, i.e., they yield forecasts conditional on  $\eta'_x$ . As we illustrate quantitatively in Section 5, this functional form for aggregate consumption is not very restrictive as it allows for time variation in conditional moments of consumption growth.

In a model based on a representative household with power utility, the consumption rule (29) is sufficient to close the model. Because we model a representative household with recursive utility, we also have to solve for the wealth dynamics to be able to compute the pricing kernel (14). In the absence of arbitrage opportunities, the Euler equation for the return on wealth (13) implies a strict consistency requirement between the dynamics of consumption and wealth. In particular, wealth has to equal the present value of future consumption. To impose this requirement, we define wealth as a nonparametric function of current consumption,  $w(c)$ , that we determine by iterating on the Euler equation (15). To do so, we specify a fine grid for current consumption, impose the dynamics specified in (29), and use cubic splines to evaluate  $w(c)$  on off grid values. In contrast to the algorithm used by Khan and Thomas (2008) and many subsequent papers, our model therefore does not allow for dynamic inconsistencies in the form of arbitrage opportunities.

To summarize, our algorithm works as follows. Starting with a guess for the coefficients of the equilibrium consumption rule (29), we first solve for the wealth rule and then the firm's problem (16)–(18) by value function iteration. To update the coefficients in the equilibrium rule (29), we simulate a continuum of firms. Following Khan and Thomas (2008), we impose market clearing in the simulation, meaning that firm policies have to satisfy the aggregate resource constraint (21). The simulation allows us to update the consumption dynamics and we iterate on the procedure until the consumption dynamics have converged.

## 4 Estimation

The main goal of our paper is to relate aggregate fluctuations and risk premia to time variation in the efficiency of factor allocations at the firm level. Because such variation results from the interplay of idiosyncratic risk and frictions, it is crucial for our model to capture the cyclical-ity in the shocks that individual firms face. We therefore estimate productivity parameters based on a set of moments that reflects both the shape and cyclical-ity of the cross-sectional distribution. In particular, our simulated method of moments (SMM) estimation targets the cross-sectional distribution of firms' sales growth and investment rates, along with a set of aggregate quantity moments. Our paper is the first to estimate a general equilibrium model with substantial heterogeneity based on such a set of endogenous moments. This estimation is made feasible largely due to modeling shocks as permanent, which allows us to reduce the dimensionality of the state space relative to earlier studies such as [Khan and Thomas \(2008\)](#), [Bachmann and Bayer \(2014\)](#), or [Bloom et al. \(2014\)](#).

### 4.1 Data

Our estimation relies on both aggregate and firm-level data over the period from 1976 to 2014. We use quarterly data but the moments are annual (based on four quarters). This allows us to make use of the higher information content of quarterly relative to annual data, while avoiding the seasonal variation of quarterly moments.

We define aggregate output as gross value added of the non-financial corporate sector, aggregate investment as private nonresidential fixed investment, and aggregate consumption as the difference between output and investment. All series are per capita and deflated with their respective price indices. Aggregate moments are based on four quarter log growth rates.

Firm-level data is taken from the merged CRSP-Compustat database. We eliminate financial firms (SIC codes 6000-6999) and utilities (SIC codes 4900-4999), because our model is inappropriate for these firms. Additionally, we only consider firms with at least 10 years of data. While this filter induces a sample selection bias, it also ensures that the time-variation in cross-sectional statistics is mostly driven by shocks to existing firms as opposed to changes

in the composition of firms. In reality, such changes are driven by firms’ endogenous entry and exit decisions, but this channel is outside of our model.

We estimate the model based on cross-sectional moments of sales growth and investment rates. Sales growth is defined as the four quarter change in log SALEQ, deflated by the implicit price deflator for GDP. The investment rate is defined as the sum of four quarterly investment observations divided by the beginning capital stock. We compute quarterly investment as the difference in net property, plant and equipment (PPENTQ), deflated by the implicit price deflator for private nonresidential fixed investment.<sup>12</sup> Capital is computed using a perpetual inventory method.<sup>13</sup> The cross-sectional dimension of our final sample grows from 915 firms in the first quarter of 1977 to 1501 firms in the last quarter of 2014.

## 4.2 Cyclical Properties of the Cross-Section of Firms

In this section, we document how the cross-section of firms moves over the business cycle, and we discuss implication of the associated empirical facts. Figure 3 shows the evolution of the cross-sectional distributions of firms’ sales growth (left column) and investment rates (right column) over time. We summarize both distributions with robust versions of their first three moments, i.e., we measure centrality with the median, dispersion with the inter quartile range (IQR), and asymmetry with Kelly skewness.<sup>14</sup> The two top panels of the figure show that recessions are characterized by sizable declines in sales growth and investment rates for the median firm. This observation is unsurprising. However, recessions are further characterized by pronounced changes in the *shape* of the cross-sectional distributions.

Sales growth becomes more disperse during recessions and its skewness switches sign from positive to negative. This evidence suggests that recessions coincide with an increase in idiosyncratic risk. Bloom (2009) and Bloom et al. (2014) provide ample additional evidence

<sup>12</sup>This assumes that economic depreciation is equal to accounting depreciation. A preferable approach would be to define investment as the difference in gross PPE and subtract economic depreciation. Yet individual firms’ economic depreciation is not observable.

<sup>13</sup>The perpetual inventory method assumes that  $K_{i,t} = (1 - \delta)K_{i,t-1} + I_{i,t}$ , initialized using PPENTQ deflated by the implicit price deflator for private nonresidential fixed investment. As in our calibration, we assume a quarterly depreciation rate of  $\delta = 2.5\%$ .

<sup>14</sup>Kelly skewness is defined as  $KSK = \frac{(p_{90} - p_{50}) - (p_{50} - p_{10})}{p_{90} - p_{10}}$ , where  $p_x$  denotes the  $x$ -th percentile of the distribution. It measures asymmetry in the center of the distribution as opposed to skewness that can result from tail observations. Similar to the median and IQR, Kelly skewness is thus robust to outliers.

for the increase in dispersion and model it as an increase in the volatility of firms' Gaussian productivity shocks. However, the pronounced change in the skewness of sales growth shows that the countercyclicality of idiosyncratic risk is better described as resulting from an expansion of the left tail of the shock distribution as opposed to a symmetric widening of the whole distribution. Intuitively, recessions are times where a subset of firms receives very negative shocks, but it is not the case that an equal proportion of firms receives very positive shocks.

Another characteristic of recessions is the fact – first documented by [Bachmann and Bayer \(2014\)](#) – that the dispersion in firms' investment rates *declines*. This procyclicality is suggestive of nonconvexities in firms' capital adjustment cost because in the absence of such frictions, the increase in the dispersion of firms' productivity would lead to a *larger* dispersion in investment rates.

[Bachmann and Bayer \(2014\)](#) argue that this fact is also informative about the cyclicality of idiosyncratic risk. Intuitively, when Gaussian volatility increases during recessions, the subset of firms receiving large positive shocks will undertake large positive investments, which leads to an increase in the cross-sectional dispersion of investment rates. When changes in uncertainty are large enough, this effect dominates the real options effect that causes firms to delay their investments in the face of increased uncertainty, which all else equal reduces the dispersion in investment rates. On the other hand, when uncertainty shocks are more moderately sized, the model becomes consistent with the procyclical investment rate dispersion, but uncertainty shocks no longer induce business cycles. Note, however, that this conclusion relies on a model with (a) real option effects and (2) time-variation in idiosyncratic risk that results from a *symmetric* change in the dispersion of shocks. Neither of these features are present in our model.

### 4.3 Simulated Method of Moments

This section explains how we estimate the model parameters. The full set of model parameters includes preference  $(\beta, \gamma, \psi)$ , technology  $(\delta, \alpha)$ , entry and exit  $(\pi, \sigma_0)$ , and productivity  $\theta = (\chi_0, \chi_1, \lambda, g_\varepsilon, \sigma_\varepsilon, g_x, \sigma_x)$  parameters. Since it is not feasible computationally to estimate the full set of parameters, we focus on estimating the vector of productivity parameters  $\theta$ .

Values for the remaining parameters are taken from previous literature and are summarized in Panel A of Table 2. Following [Bansal and Yaron \(2004\)](#), we assume that the representative agent is fairly risk averse,  $\gamma = 10$ , and has a large EIS,  $\psi = 2$ . The time discount rate of  $\beta = 0.995$  is chosen to achieve a low average risk-free rate. Capital depreciates at a rate of 2.5% and the curvature of the production function equals 0.65, similar to [Cooper and Haltiwanger \(2006\)](#). Firms exit the economy with a rate of 2%, similar to the value reported in [Dunne et al. \(1988\)](#). The productivity draws of entrants has a mean pinned down by condition (7) and a volatility of 10%. As estimated by [Bloom \(2009\)](#), we assume partial irreversibility costs of  $\xi = 0.7$ .

Productivity parameters are estimated with SMM, which minimizes a distance metric between key moments from actual data,  $\Psi^D$ , and moments from simulated model data,  $\Psi^M(\theta)$ . Given an arbitrary parameter vector  $\theta$ , the model is solved numerically as described in Section 3.5. In solving the model, we use an equilibrium simulation of length 3040 + 600 quarters, which equals twenty times the time dimension of the actual data plus an initial 600 periods that we discard to eliminate dependence on initial conditions. We then fix the equilibrium path of consumption that results from the equilibrium simulation and simulate a finite panel of firms for the same path of the economy.<sup>15</sup> Based on the simulated data panel, we calculate the model moments  $\Psi^M(\theta)$  as well as the objective function  $[\Psi^D - \Psi^M(\theta)]'W[\Psi^D - \Psi^M(\theta)]$ . The parameter estimate  $\hat{\theta}$  is found by searching globally over the parameter space. We use an identity weighting matrix and implement the global minimization via a genetic algorithm. Computing standard errors for the parameter estimate requires the Jacobian of the moment vector, which we find numerically via a finite difference method.

---

<sup>15</sup>While the simulation step of the solution algorithm is based on a continuum of firms that are tracked on a bivariate histogram, this approach is not feasible for determining cross-sectional moments that span multiple quarters. The reason is that multi-period transition functions become too high dimensional to be computationally manageable. However, the Monte Carlo sample can be interpreted as a subsample of the continuum of firms because it is based on the same path for aggregate shocks and aggregate consumption as the model solution. We choose the number of simulated firms high enough to ensure that the simulated cross-sectional moments are not significantly affected by Monte Carlo noise.

#### 4.4 Moment Selection and Parameter Identification

To identify the parameter vector  $\theta$ , we rely on a combination of aggregate and cross-sectional moments. First, we include the time series means of the six cross-sectional moments depicted in Figure 3: median, IQR, and Kelly skewness of sales growth and investment rates. Doing so ensures that we capture the average shape of the conditional distributions of sales growth and investment rates, and therefore also the shape of their long run distributions. Second, we include the time series standard deviations of the same six cross-sectional moments to capture the amount of time-variation in the conditional cross-sectional distributions. Third, we rely on time series correlations between three cross-sectional moments and aggregate output growth to capture the cyclicity of the cross-section. In particular, we include the cyclicity of the dispersion in sales growth documented by Bloom (2009), the cyclicity of the skewness in sales growth documented by Salgado et al. (2015), and the cyclicity in the dispersion of investment rates documented by Bachmann and Bayer (2014). Relying on three measures of cyclicity jointly ensures that we capture various aspects of how the cross section co-moves with the cycle.

At the aggregate level, we include the mean growth rate of aggregate output, and the standard deviations of aggregate output, consumption, and investment to ensure that productivity parameters reflect not only the cross-section but also remain consistent with macro aggregates. In total, we estimate 7 productivity parameters based on the 19 moments shown in the data column of Table 3. In what follows, we discuss the main sources of identification for each estimated parameter.

The drift and volatility parameters of aggregate productivity,  $g_x$  and  $\sigma_x$ , are pinned down by the mean and volatility of aggregate output growth. While the drift and volatility parameters of idiosyncratic productivity,  $g_\varepsilon$  and  $\sigma_\varepsilon$ , are identified by the average median, IQR, and Kelly skewness of sales growth and investment rates, the jump process parameters,  $\chi_0$ ,  $\chi_1$  and  $\lambda$ , affect the standard deviation of IQR and Kelly skewness of sales growth and investment rates as well as their cyclicity.

## 4.5 Baseline Estimates

SMM parameter estimates are shown in Table 2, whereas data and model moments are shown in Table 3. Our benchmark specification is shown in the columns labeled Spec-1 (we will return to the alternative specifications below). As Table 2 shows, all estimated parameters are well-identified as indicated by very small standard errors. The estimated jump intensity of  $\lambda = 0.0941$  implies that firms receive negative jumps in productivity about once every 11 quarters, whereas the estimated parameters of the jump size function ( $\chi_0 = 0.2384$  and  $\chi_1 = 0.7027$ ) imply that the average jump size is about  $-31\%$ . The log growth rate of idiosyncratic productivity has a Gaussian volatility of  $\sigma_\varepsilon = 5.39\%$  and drift parameter of  $g_\varepsilon = 1.46\%$  per quarter, well below the threshold of  $\pi = 2\%$  that is required to ensure finiteness of the cross-sectional mean of productivity – see Equation 6. Lastly, the log growth rate of aggregate productivity has a drift parameter of  $g_x = 0.30\%$  and a volatility of  $\sigma_x = 3.56\%$  per quarter.

To gain a better understanding of how the model matches the sales growth and investment rate distributions, we plot the idiosyncratic productivity, sales growth, and investment rate distributions averaged across expansions (black line) and recessions (red line) in Figure 4.<sup>16</sup> Intuitively, sales growth is a weighted average of idiosyncratic productivity shocks and changes in capital, i.e., investment rates. Due to costly reversibility, a large fraction of firms is inactive with zero investment rates. Firms with large positive idiosyncratic productivity draws have positive investment rates, causing a positively skewed investment rate distribution. At the same time, investment rates are lower on average in recessions but more dispersed in expansions.

The positive skewness in investment rates also makes sales growth positively skewed in expansions. When the economy switches into a recession due to a sequence of adverse aggregate productivity draw, the jump size  $\chi$  increases and the productivity distribution becomes strongly left skewed. Importantly, the mass of firms receiving a negative Poisson draw does not increase. This negative skewness in productivity dominates the positive skewness in invest-

---

<sup>16</sup>We define an expansion as a period where four quarter aggregate output growth falls below its' unconditional 25-th percentile. Correspondingly, expansions are defined as output being above its' 75-th percentile.

ment rates and thus sales growth becomes left skewed as in the data. The negative skewness in productivity also raises the dispersion in productivity and sales growth.

This mechanism explains why the model cannot fit all target moments in Table 3 perfectly. While the average median sales growth and investment rate are too small relative to the data, increasing the drift of idiosyncratic productivity would yield a better fit. But the drift also has a positive impact on the average IQR of sales growth and investment rates and especially the latter is already too large. Increasing idiosyncratic volatility would yield a better fit for the average IQR of sales growth and investment rates but, as explained above, it also raises the average skewness of sales growth and investment rates, which are closely matched.

The parameters that govern Poisson jumps in idiosyncratic productivity have a strong effect on the volatility of cross-sectional moments and their cyclicalities. While making Poisson jumps more likely increase the volatility of IQR and Kelly skewness of sales growth and investment rates, this effect also leads to counterfactual large aggregate consumption volatility because the capital misallocation hinders consumption smoothing – see our discussion in Section 5.2. The cyclicalities and size of Poisson jumps also have a strong positive impact on the volatility of IQR and Kelly skewness of sales growth and investment rates, these parameters also affect the cyclicalities of IQR and Kelly skewness of sales growth. Increasing the jump size parameters would add more time variation in terms of IQR of sales growth and investment rates at the cost of too much cyclicalities.

## 4.6 Alternative Specifications

In this section, we illustrate estimation results for a number of alternative model specifications in order to highlight the role of pre-specified parameters. The results of these experiments are contained in the additional columns of Tables 2 and 4.

Columns labeled Spec-2 show results for a preference parameter calibration that implies time-separable utility. In particular, we calibrate the EIS to a low value of  $\psi = 0.5$  and the relative risk aversion coefficient to 2, similar to the values typically assumed in the macroeconomics literature. Table 2 shows that the estimated productivity parameters are very similar to those in our benchmark specification, with the exception of  $\chi_1$  and  $g_x$ . The estimated pa-

parameter values imply that the size of productivity jumps is less cyclical than in the benchmark, whereas aggregate quantities grow at a lower rate. Table 3 shows that, while cross-sectional moments are matched similarly well as in the benchmark specification, the volatilities of aggregate output, consumption, and investment are very different from their data counterparts. Therefore, the low EIS leads to a tension between matching cross-sectional and aggregate facts. While one could certainly match quantity volatilities by assigning a larger weight to these moments in the criterion function, this would come at the expense of no longer matching cross-sectional facts.

The next alternative specification labeled Spec-3 changes the benchmark specification by assuming a higher exit rate of  $\pi = 3\%$  as opposed to 2% in the benchmark. While this only leads to small differences in estimated productivity parameters and fit of the targeted moments, the higher exit rate implies a lower power law coefficient, i.e., a lower concentration of large firms. As shown in Table 4 and Figure 8 (both of which we discuss below), this results in an improved ability to smooth consumption, lower risk premia, and less misallocation relative to the benchmark.

## 5 Model Implications

In this section, we study the consumption dynamics and asset pricing implications, arising from capital misallocation. Quantitatively, the amount of capital misallocation in the model depends on the firms' life cycle effect combined with power law in firms size. Intuitively, the aggregate consequences of capital misallocation are worse when large old firms dominate the economy because older firms tend to be less efficient than young firms.

Finally, we can also study the empirical misallocation through the lens of over model. To this end, we fit the output gap to cross-sectional moments of the sales growth and investment rate densities in the model. Given this fit, we use the empirical counterparts to construct an empirical output gap measure, as in [Eisfeldt and Muir \(2016\)](#).

## 5.1 Firms' Life Cycle

To understand the nature of aggregate fluctuations in our model, it is useful to first characterize the behavior of a typical firm over its life cycle. Figure 6 shows how various firm characteristics change as firms age. Firms enter the economy with zero capital and positive productivity, so that young firms tend to have high investment (top-left) and sales growth rates (top-right), and low payout (bottom-left) and capital-to-productivity ratios (bottom-right). Due to the geometric growth in idiosyncratic productivity, firms' investment rates continue to exceed their depreciation rates as they age, and older firms are larger on average. At the same time, older firms are more likely to have experienced a negative Poisson shock to productivity during their lifetime. After experiencing such a shock, firms' capital-to-productivity ratio increases and, because disinvestment is costly, firms tend to become inactive and let their capital stock depreciate over the coming periods. A large reason why older firms have lower investment rates is therefore that they are more likely to be constrained, i.e. the extensive margin is lower among older firms. To summarize, the typical old firm is larger and more constrained relative to the typical young firm.

## 5.2 Power Law and Consumption Dynamics

The combination of a unit root in idiosyncratic productivity and random exit in our model results in a firm size distribution, whose right tail exhibits a power law. In particular, the log right tail probabilities (above the 50th percentile), with firm size measured by log capital, lie on a straight line with a slope of  $-1.24$  for our benchmark estimation results. This means that the firm size distribution is well approximated in its right tail by a Pareto distribution with right tail probabilities of the form  $1/x^\zeta$ , with a tail index  $\zeta$  of 1.24. To illustrate the economic effect of the power law, Figure 7 shows the degree of output concentration implied by our benchmark parameter estimates. Specifically, it shows the fraction of aggregate output produced by firms in various percentiles of the capital distribution.<sup>17</sup> On average, the largest 5% of firms (in terms of capital) produce almost 50% of aggregate output.

---

<sup>17</sup>To produce the figure, we simulate a continuum of firms and record the fraction of output produced by the 5% smallest firms (in terms of capital), firms between the 5th and 10th size percentiles, etc. in each period. We then average these fractions across all periods in the simulation.

Computing the data analogue of this statistic requires a count of the number of firms in the U.S. economy. The 2013 U.S. census reported a total of 5.8 million firms. The majority of firms has between 0 and 4 employees. Given a minimum of 100 employees, the US economy consisted of 103,900 firms; for a minimum of 500 employees, it had 18,636 firms. In the same year, CRSP-Compustat universe included about 2,500 firms (excluding financial firms and utilities), whose sales accounted for 55% of U.S. GDP. Depending on the definition of a firm in the data, the output concentration in the data is even larger than in our model.

Due to the importance of large firms for output and consumption in the model, permanent negative shocks to their productivity are particularly painful. The discussion in Section 5.1 highlighted that large firms tend to be more constrained on average. Thus, constrained firms are responsible for a large fraction of the drop in consumption during recessions. While unconstrained firms increase dividends by reducing investment, they are smaller on average so that they are not able to offset the impact of large constrained firms on aggregate consumption. In other words, the firm size distribution in combination with negative jumps in productivity implies that it is difficult for the representative household to smooth consumption during recessions. In contrast, in models with log-normal productivity distributions the size difference between constrained and unconstrained firms is small so that the groups offset each other.

Figure 8 illustrates this channel quantitatively via a comparative statics exercise, that varies the exit probability  $\pi$ . A larger exit probability implies that firms survive shorter on average, which reduces the mass of large firms. The top-left panel shows that the power law coefficient ( $-\zeta$ ) decreases as  $\pi$  increases, meaning that the right tail of the firm size distribution becomes thinner. This implies that it becomes easier to smooth consumption by offsetting the losses in consumption from large firms during recessions with the dividend payments of unconstrained (and relatively smaller) firms. As a consequence, the unconditional left skewness of consumption growth is reduced (top-right panel), the loss in output relative to the frictionless benchmark decreases (bottom-right panel), and risk premia decline (bottom-left panel).

In Table 4, we summarize consumption growth moments, risk premia, and misallocation

measures across the three specifications of Tables 2 and 3. In the benchmark specification 3, consumption growth is positively autocorrelated, left skewed, and exhibits excess kurtosis. In the frictionless economy, consumption growth is normal distributed. Relative to the power utility case (specification 2), this increased tail risk has a significant effect on the expected return on wealth, which increases to 1.8%, on the risk-free rate, which decreases to 1.5%, and on the Sharpe ratio, which increases to 0.57.

We next explore the effect of misallocation in the model. As discussed in Section 3.4, the frictionless model does not feature any misallocation, so that  $\mathcal{M}$  and  $\mathcal{D}$  are both constant and equal to zero. Given our parameter estimates for productivity, we find that capital misallocation is 4.6% on average. It is also very volatile, persistent, and cyclical. Capital misallocation also leads to a significant output gap of the full model relative to the frictionless economy of 4.5%. By eliminating capital frictions, the economy would exhibit a large one time rise in wealth.

### 5.3 Measuring Misallocation Empirically

Our model economy allows us to quantify misallocation in the data by providing a link between the output gap and cross-sectional densities. In particular, the output gap in the model is well-described by a function of cross-sectional moments. We can therefore estimate this relationship in the model and then apply it to the empirically observed moments to back out the output gap in the data. Based on model-generated data, we estimate a linear projection of the output gap on its first lag and the six moments that summarize the cross-sectional sales growth and investment rate densities.<sup>18</sup> We then compute a fitted value based on the estimated coefficients and the cross-sectional moments in the data. The initial value of the lagged output gap is set to the model-implied unconditional average.

The resulting time series for the empirical output gap in Figure 5 shows that misallocation spiked up during all recessions in our sample period. The recessions of 2001 and 2008 were associated with particularly high levels of misallocation, suggesting that cross-sectional inefficiencies were an important driver of these downturns. This view is further supported by the

---

<sup>18</sup>The population  $R^2$  in the model is approximately 82%.

fact that misallocation remained elevated for an extended period in both episodes, thereby contributing to the observed slow recoveries.

#### 5.4 Shock persistence in the data

In our model, firms that are hit by large negative jump shocks either sell capital or become inactive for an extended period, thereby contributing to an increase in misallocation cross-sectionally. Because this effect results from the assumption that shocks are permanent, a potential concern is that the model overstates the true effect that these shocks have on firms' investment. If extreme shocks to sales were completely transitory for example, one would expect to see little to no effect in investment rates.

To investigate this issue in the data, we analyze the investment behavior of firms around severe shocks to their sales. Both in the model and the data, the most extreme such shocks occur during periods in which the cross-sectional skewness of sales growth becomes very negative. We thus create a pooled sample of all firm-quarter observations for which (1) the cross-sectional skewness falls below -0.1 and (2) the sales growth rate of the individual firm is at the 10-th percentile of the cross-sectional distribution – the cutoff that defines the left tail in Kelly's skewness.<sup>19</sup> To isolate the effect of firm-specific shocks from aggregate conditions, we further demean investment rates with the period-specific cross-sectional average.

The top panel of Figure 9 shows the average demeaned investment rate of our pooled sample, along with a 95% confidence region around the cross sectional average (which is normalized to zero). The gray-shaded region defines the event window and extends from -3 to +3 because investment rates are based on four quarters of data. The figure shows that the average investment rate of affected firms differs insignificantly from the cross-sectional average pre event, drops to about 10% below average on impact of the sales shock, and then remains significantly below average for the following 20 quarters. This evidence supports the assumption that extreme shocks to sales are well-described as being very persistent.

---

<sup>19</sup>There were four episodes during which Kelly's skewness fell below -0.1 in our sample (see figure 3), during some of which it remained below the threshold for a few quarters. Because we are interested in the response to the initial shock, we only include the first quarter during each such sequence. Additionally, we include all firms between the 5-th and 15-th percentile to capture the behavior of firms around the 10-th percentile in order to reduce noise.

To examine whether the effect is quantitatively consistent with the model, we repeat the same experiment based on simulated model data, using both a large cross-section of firms and a long time series. The bottom panel of Figure 9 shows that the dynamic response of investment rates in the model, while slightly more pronounced than in the data, is very comparable to that in the data. We interpret this evidence as very supportive of the main mechanism for generating misallocation in our model.

## 6 Conclusion

[to be added]

## References

- ANDREW B. ABEL, J. C. E. (1996): “Optimal Investment with Costly Reversibility,” *Review of Economic Studies*, 63, 581–593.
- AXTELL, R. L. (2001): “Zipf Distribution of U.S. Firm Sizes,” *Science*, 293, 1818–1820.
- BACHMANN, R. AND C. BAYER (2014): “Investment Dispersion and the Business Cycle,” *American Economic Review*, 104, 1392–1416.
- BANSAL, R. AND A. YARON (2004): “Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles,” *Journal of Finance*, 59, 1481–1509.
- BARRO, R. J. (2006): “Rare Disasters and Asset Markets in the Twentieth Century,” *Quarterly Journal of Economics*, 121, 823–866.
- BARTELSMAN, E., J. HALTIWANGER, AND S. SCARPETTA (2013): “Cross-Country Differences in Productivity: The Role of Allocation and Selection,” *American Economic Review*, 103, 305–334.
- BERGER, D., I. DEW-BECKER, AND S. GIGLIO (2016): “Contractionary volatility or volatile contractions?” Working paper.
- BERK, J., R. C. GREEN, AND V. NAIK (1999): “Optimal Investment, Growth Options, and Security Returns,” *Journal of Finance*, 54, 1553–1607.
- BLOOM, N. (2009): “The Impact of Uncertainty Shocks,” *Econometrica*, 77, 623–685.
- BLOOM, N., M. FLOETOTTO, N. JAIMOVICH, I. SAPORTA-EKSTEN, AND S. J. TERRY (2014): “Really Uncertain Business Cycles,” Working paper.
- BOLDRIN, M., L. J. CHRISTIANO, AND J. D. M. FISHER (2001): “Habit Persistence, Asset Returns and the Business Cycle,” *American Economic Review*, 91, 149–166.
- CABALLERO, R. J. AND E. M. ENGEL (1999): “Explaining investment dynamics in US manufacturing: A generalized (S, s) approach,” *Econometrica*, 67, 783–826.
- CARLSON, M., A. FISHER, AND R. GIAMMARINO (2004): “Corporate Investment and Asset Price Dynamics: Implications for the Cross-section of Returns,” *Journal of Finance*, 59, 2577–2603.
- CONSTANTINIDES, G. M. AND A. GHOSH (2015): “Asset Pricing with Countercyclical Household Consumption Risk,” Working Paper.
- COOPER, I. (2006): “Asset Pricing Implications of Non-Convex Adjustment Costs of Investment,” *Journal of Finance*, 61, 139–170.

- COOPER, R. W. AND J. C. HALTIWANGER (2006): “On the Nature of Capital Adjustment Costs,” *Review of Economic Studies*, 73, 611–633.
- DIXIT, A. K. AND R. S. PINDYCK (1994): *Investment under Uncertainty*, Princeton, New Jersey: Princeton University Press.
- DUNNE, T., M. J. ROBERTS, AND L. SAMUELSON (1988): “Patterns of Firm Entry and Exit in U.S. Manufacturing Industries,” *RAND Journal of Economics*, 19, 495–515.
- EISFELDT, A. AND T. MUIR (2016): “Aggregate External Financing and Savings Waves,” *Journal of Monetary Economics*.
- EISFELDT, A. L. AND A. RAMPINI (2006): “Capital Reallocation and Liquidity,” *Journal of Monetary Economics*, 53, 269–399.
- EPSTEIN, L. AND S. ZIN (1989): “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” *Econometrica*, 57, 937–969.
- GABAIX, X. (1999): “Zipf’s Law for Cities: An Explanation,” *Quarterly Journal of Economics*, 114, 739–767.
- (2009): “Power Laws in Economics and Finance,” *Annual Review of Economics*, 1, 255–293.
- (2011): “The Granular Origins of Aggregate Fluctuations,” *Econometrica*, 79, 733–772.
- (2012): “Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance.” *Quarterly Journal of Economics*, 127.
- GOMES, J. F., L. KOGAN, AND L. ZHANG (2003): “Equilibrium Cross Section of Returns,” *Journal of Political Economy*, 111, 693–732.
- GOMES, J. F. AND L. SCHMID (2010): “Levered Returns,” *Journal of Finance*, 65, 467–494.
- GOURIO, F. (2012): “Disaster Risk and Business Cycles,” *American Economic Review*, 102, 2734–2766.
- GUVENEN, F., S. OZKAN, AND J. SONG (2014): “The Nature of Countercyclical Income Risk,” *Journal of Political Economy*, 122, 621–660.
- HSIEH, C.-T. AND P. J. KLENOW (2009): “Misallocation and Manufacturing TFP in China and India,” *Quarterly Journal of Economics*, 124, 1403–1448.
- JERMANN, U. J. (1998): “Asset Pricing in Production Economies,” *Journal of Monetary Economics*, 41, 257–275.

- KALTENBRUNNER, G. AND L. LOCHSTOER (2010): “Long-Run Risk through Consumption Smoothing,” *Review of Financial Studies*, 23, 3141–3189.
- KELLY, B., H. LUSTIG, AND S. V. NIEUWERBURGH (2013): “Firm Volatility in Granular Networks,” Working Paper.
- KHAN, A. AND J. K. THOMAS (2008): “Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics,” *Econometrica*, 76, 395–436.
- (2013): “Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity,” *Journal of Political Economy*, 121, 1055–1107.
- KOGAN, L. (2001): “An Equilibrium Model of Irreversible Investment,” *Journal of Financial Economics*, 62, 201–245.
- KRUSELL, P. AND J. SMITH, ANTHONY A. (1998): “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy*, 106, 867–96.
- LUTTMER, E. G. (2007): “Selection, Growth, and the Size Distribution of Firms,” *Quarterly Journal of Economics*, 1103–1144.
- OLLEY, G. S. AND A. PAKES (1996): “The Dynamics of Productivity in the Telecommunications Equipment Industry,” *Econometrica*, 64, 1263–1297.
- SALGADO, S., F. GUVENEN, AND N. BLOOM (2015): “Skewed Business Cycles,” Working Paper.
- SCHMIDT, L. (2015): “Climbing and Falling Off the Ladder: Asset Pricing Implications of Labor Market Event Risk,” Working Paper.
- VERACIERTO, M. L. (2002): “Plant-Level Irreversible Investment and Equilibrium Business Cycles,” *American Economic Review*, 92, 181–97.
- WACHTER, J. A. (2013): “Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?” *Journal of Finance*, 68, 987–1035.
- ZHANG, L. (2005): “The Value Premium,” *Journal of Finance*, 60, 67–103.

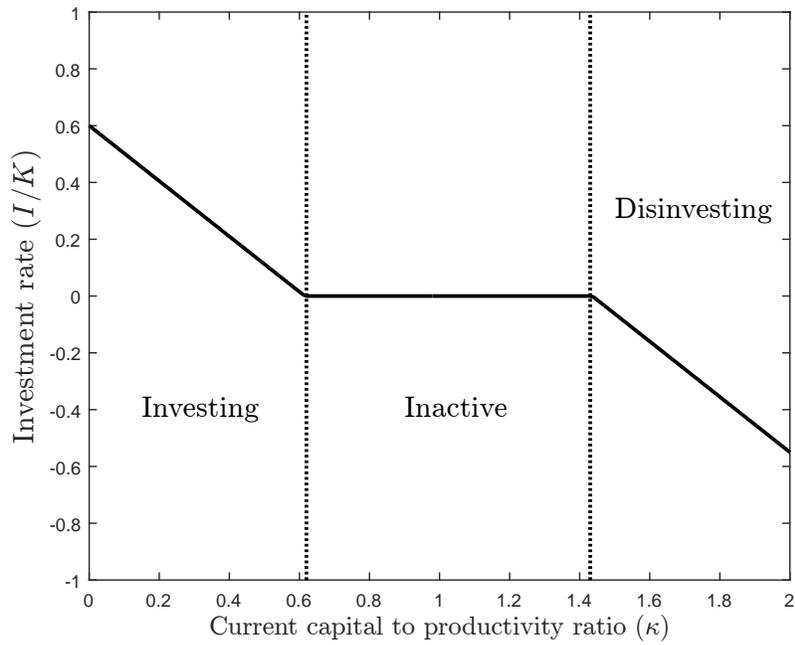
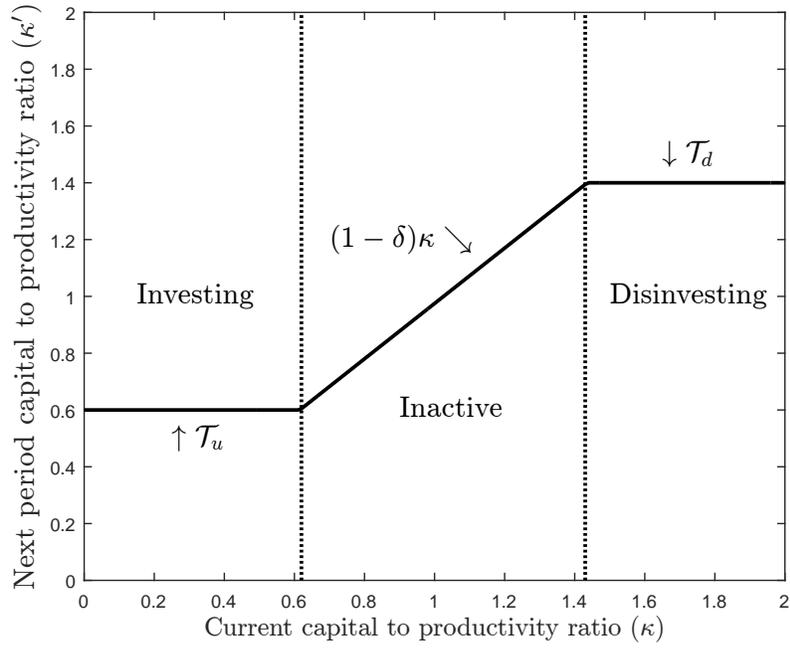
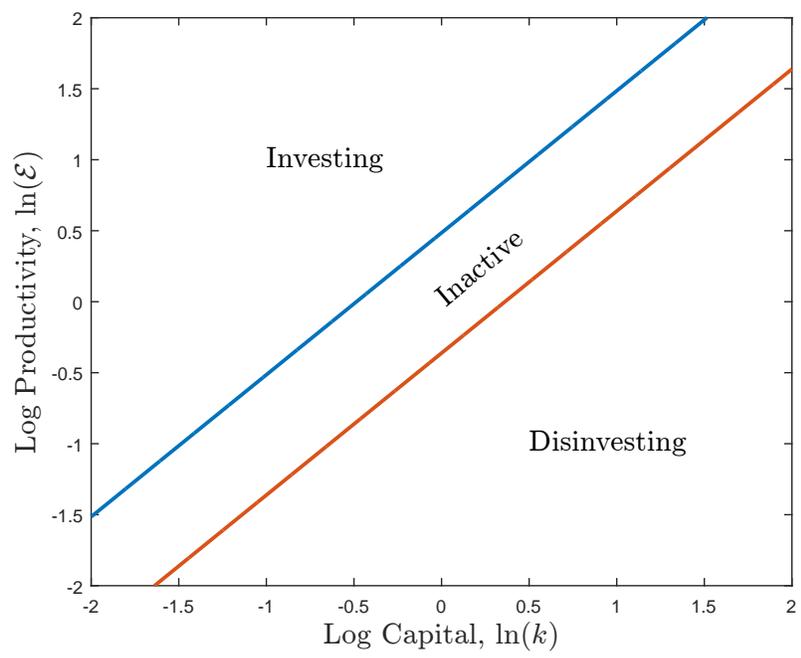
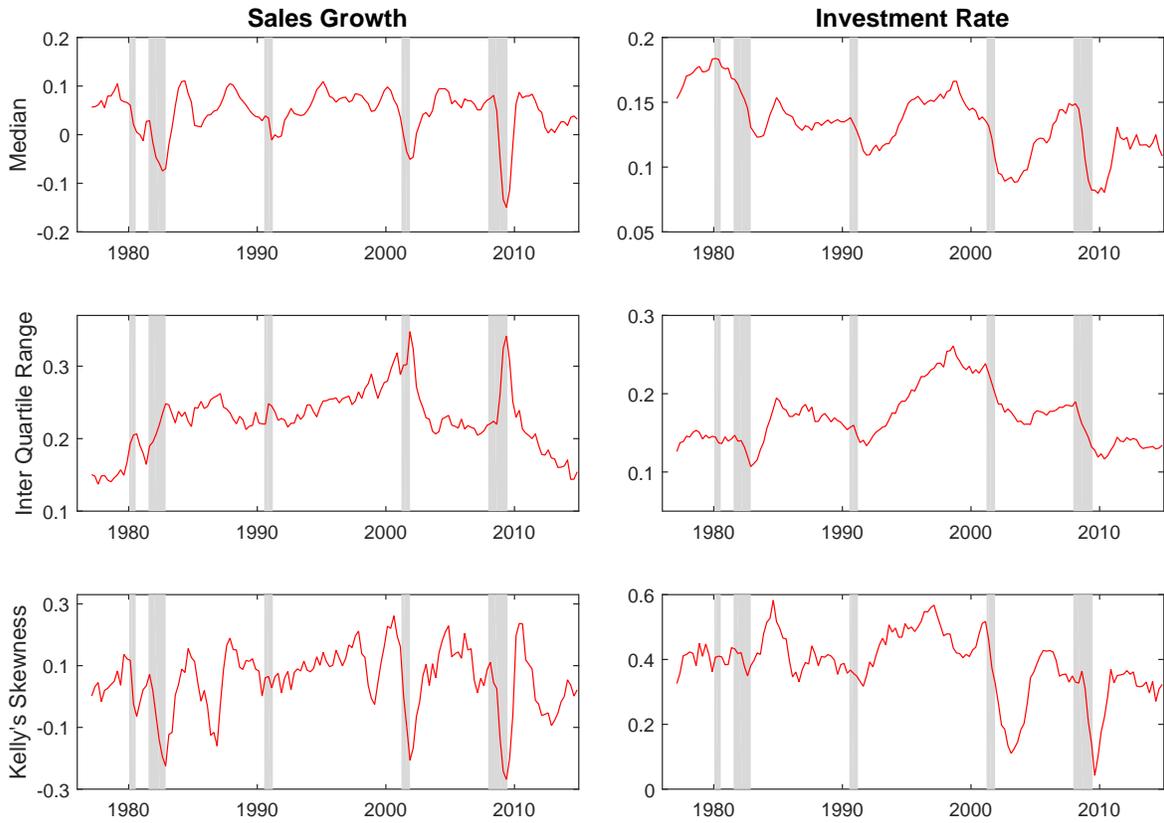


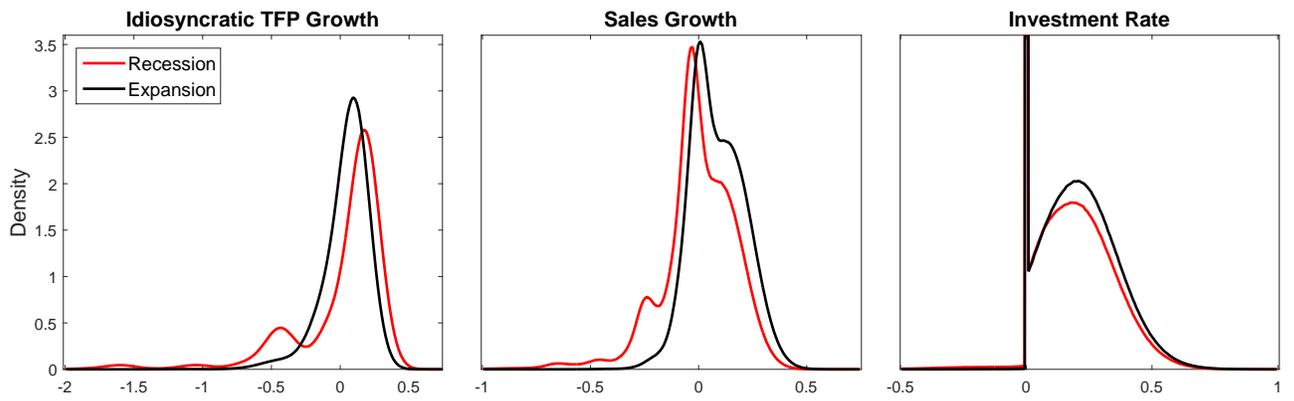
Figure 1: Optimal capital and investment rate policies



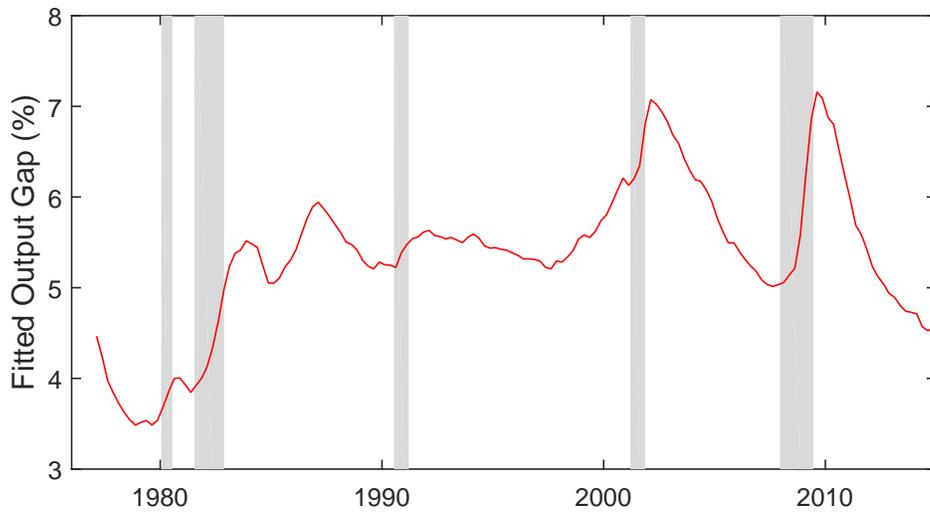
**Figure 2:** Optimal capital policies in the  $\mu$ -distribution



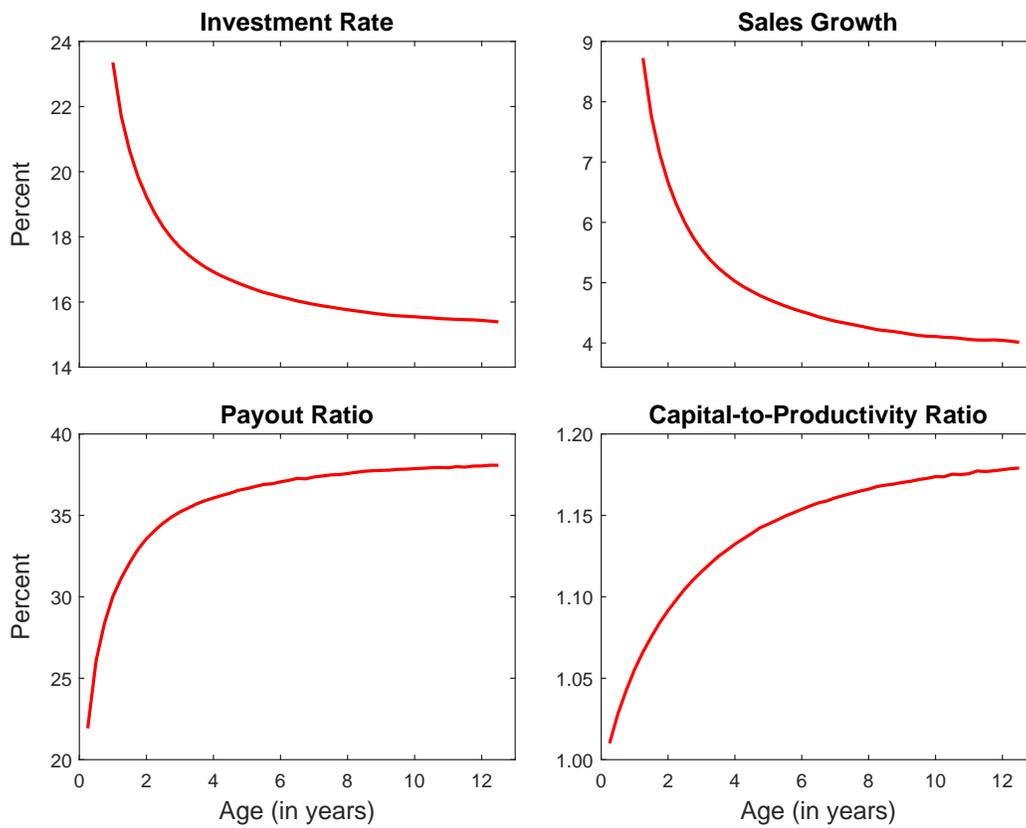
**Figure 3: Robust cross-sectional moments of annual sales growth and investment rates**



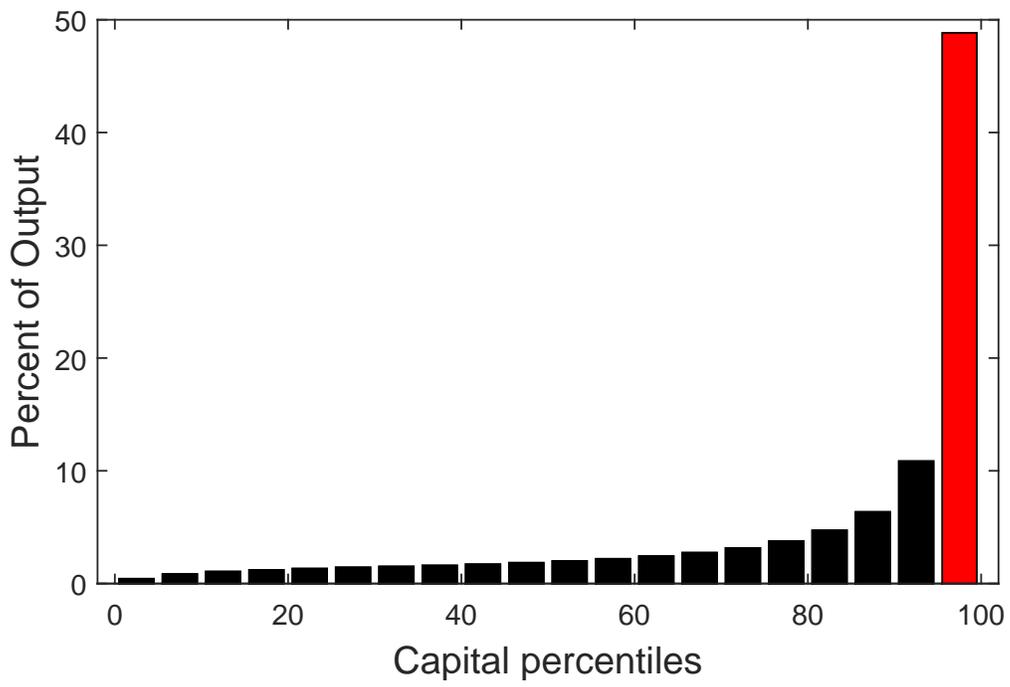
**Figure 4: Average cross-sectional idiosyncratic productivity, sales growth, and investment rate distributions in expansions and recessions**



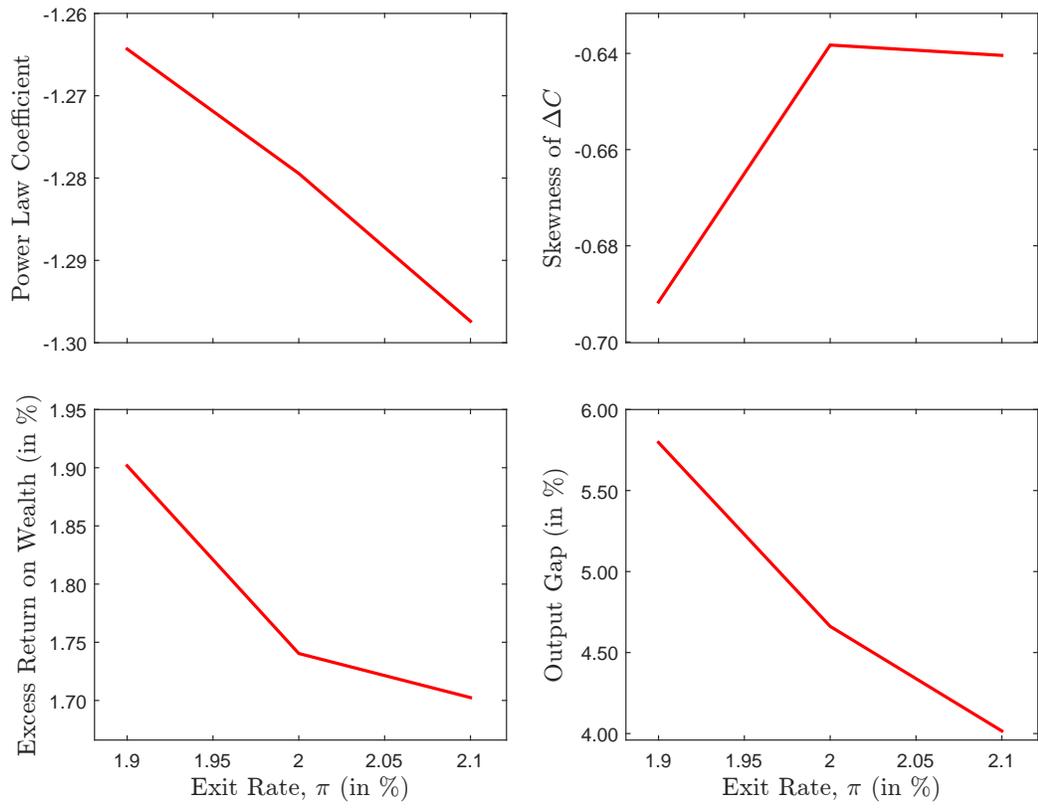
**Figure 5: Estimated output gap in the data**



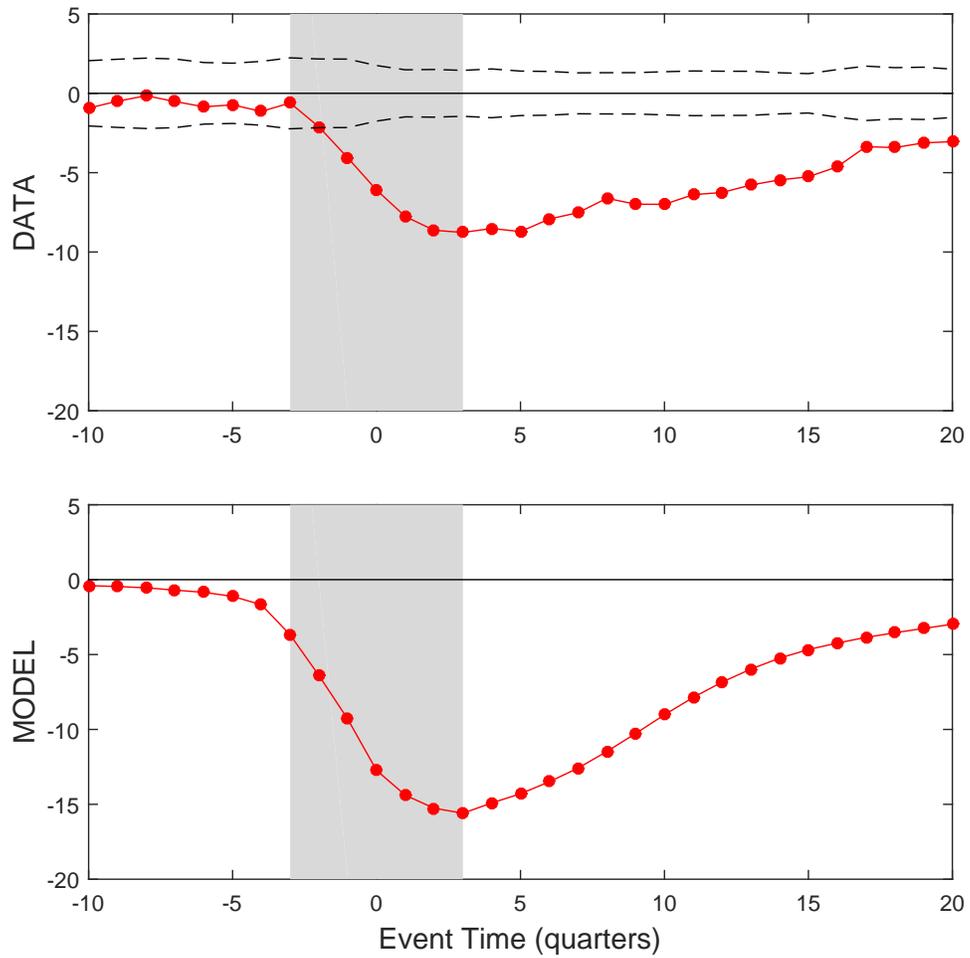
**Figure 6: Life cycle of the average firm**



**Figure 7: Output concentration**



**Figure 8: Comparative statics for the exit rate**



**Figure 9: Investment rates for firms with extreme shocks to sales growth**

**Table 2: Predefined and Estimated Parameter Values**

Parameter	Spec-1	Spec-2	Spec-3	Description
<b>A: Predefined Parameters</b>				
$\delta$	0.025	0.025	0.025	Depreciation rate
$\alpha$	0.65	0.65	0.65	Curvature in production function
$\sigma_0$	0.1	0.1	0.1	Volatility of productivity of new entrants
$\pi$	0.02	0.02	0.03	Exit probability
$\beta$	0.995	0.995	0.995	Time discount factor
$\xi$	0.7	0.7	0.7	Proportional resale value of capital
$\gamma$	10	2	10	Risk aversion
$\psi$	2	0.5	2	Elasticity of intertemporal substitution
<b>B: Estimated Parameters</b>				
$\chi_0$	0.2384 [0.0000]	0.2915	0.2421	Parameter of jump size function
$\chi_1$	0.7027 [0.0000]	0.4189	0.7300	Parameter of jump size function
$\lambda$	0.0941 [0.0000]	0.0896	0.0900	Parameter of jump intensity function
$g_\varepsilon$	0.0146 [0.0003]	0.0149	0.0139	Mean idio. productivity growth rate
$\sigma_\varepsilon$	0.0496 [0.0001]	0.0416	0.0541	Parameter of idiosyncratic volatility function
$g_x$	0.0030 [0.0012]	0.0011	0.0029	Mean aggregate productivity growth rate
$\sigma_x$	0.0356 [0.0001]	0.0369	0.0334	Volatility of aggregate productivity growth rate

Notes: Panel A shows calibrated parameters and Panel B shows parameters estimated via SMM with standard errors in brackets. The model is solved at a quarterly frequency. Spec-1 equals the benchmark specification. Spec-2 replaces the recursive utility function with a time-separable power utility function with a low value for the relative risk aversion parameter. Spec-3 allows for time-variation not only in the jump size, but also in the jump intensity and the volatility of Gaussian idiosyncratic shocks.

**Table 3: Moments Targeted in SMM Estimation**

		Data	Spec-1	Spec-2	Spec-3
<b>A: Cross-Sectional Sales Growth Moments</b>					
Median	<i>mean</i>	0.045	0.035	0.030	0.034
	<i>std</i>	0.046	0.031	0.030	0.029
IQR	<i>mean</i>	0.225	0.186	0.183	0.184
	<i>std</i>	0.042	0.016	0.011	0.016
	<i>corr</i>	-0.332	-0.332	-0.351	-0.331
Kelly	<i>mean</i>	0.046	0.075	0.077	0.070
	<i>std</i>	0.104	0.128	0.082	0.133
	<i>corr</i>	0.586	0.588	0.597	0.594
<b>B: Cross-Sectional Investment Rate Moments</b>					
Median	<i>mean</i>	0.142	0.126	0.125	0.125
	<i>std</i>	0.032	0.043	0.029	0.041
IQR	<i>mean</i>	0.207	0.256	0.255	0.253
	<i>std</i>	0.043	0.024	0.019	0.023
	<i>corr</i>	0.244	0.249	0.266	0.244
Kelly	<i>mean</i>	0.352	0.337	0.310	0.335
	<i>std</i>	0.104	0.200	0.135	0.191
<b>C: Aggregate Quantity Moments</b>					
Output Growth	<i>mean</i>	0.015	0.009	0.002	0.009
	<i>std</i>	0.030	0.033	0.031	0.032
Consumption Growth	<i>std</i>	0.026	0.023	0.039	0.023
Investment Growth	<i>std</i>	0.066	0.046	0.027	0.047

Notes: The table summarizes the moments used in the SMM estimation. Panels A and B contain time series statistics of cross-sectional moments. For example, the row for IQR *mean* in Panel A contains the time series mean of the cross-sectional sales growth IQR. Panel C contains time series moments of aggregate quantity growth rates. All statistics refer to annual moments, i.e. annual sales growth rates, annual investment rates, as well as annual aggregate quantity growth rates. The model parameters related to each specification are shown in Table 2.

Table 4: Consumption Growth and Asset Prices

	Spec-1	Spec-2	Spec-3
<b>Panel A: Consumption Growth</b>			
Autocorrelation	0.330	0.058	0.462
Skewness	-0.600	-0.440	-0.395
Kurtosis	3.541	3.323	3.411
<b>Panel B: Returns</b>			
Excess return on wealth	1.79%	0.07%	1.60%
Risk-free Rate	1.48%	2.06%	1.62%
Sharpe Ratio	0.573	0.062	0.543
<b>Panel C: Misallocation</b>			
Mean of power law coef. $\zeta$	-1.241	-1.273	-1.614
Mean of misallocation $\mathcal{M}$	0.046	0.037	0.063
Std of misallocation $\mathcal{M}$	0.024	0.014	0.030
AC4 of misallocation $\mathcal{M}$	0.701	0.750	0.635
corr[g $\mathcal{M}$ ,gY]	-0.589	-0.704	-0.559
Mean of output gap $\mathcal{D}$	0.045	0.064	0.036
Std of output gap $\mathcal{D}$	0.004	0.002	0.003
AC4 of output gap $\mathcal{D}$	0.744	0.749	0.707
corr[g $\mathcal{D}$ ,gY]	-0.511	-0.561	-0.504

Notes: The table summarizes moments related to consumption risks and risk premia. These moments were not targeted in the SMM estimation. The model parameters related to each specification are shown in Table 2.  $g\mathcal{M}$ ,  $g\mathcal{D}$ , and  $gY$  denote the four quarter log changes in the misallocation measure, the output gap, and aggregate output.