On the joint evolution of culture and institutions*
(incomplete version)

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Abstract

What accounts for economic growth and prosperity? What stands at their origin? Recent literature typically searches for single univariate causal explanations: institutions, culture, human capital, geography. In this paper we provide instead a first theoretical modeling of the interaction between different possible explanations for growth and prosperity (in particular, between culture and institutions) and their effects on economic activity. Depending on the economic environment, culture and institutions might complement each other, giving rise to a *multiplier* effect, or on the contrary they can act as substitutes, contrasting each other and limiting their combined ability to spur economic activity. By means of examples we show how the dynamics may display non-ergodic behavior, hysteresis, oscillating behaviors and interesting comparatve dynamics .

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1 Introduction

> "Era questo un ordine buono, quando i cittadini erano buoni [...] ma diventati i cittadini cattivi, divento' tale ordine pessimo."[1]; Niccolo' Machiavelli, *Discorsi,* I. 16, 1531

> [...] "among a people generally corrupt, liberty cannot long exist." Edmund Burke, Letter to the Sheriffs of Bristol (1777-04-03).

> "If there be no virtue among us, no form of government can render us secure. To suppose that any form of government will secure liberty or happiness without any virtue in the people is an illusion." James Madison, 20 June 1788, Papers 11:163

The distribution of income across countries in the world is very unequal: according to World Bank data 2015, U.S. GDP per capita in international dollars is 71 times that of the Democratic Republic of Congo, 58 times that of Niger, 9 times that of India and 3 times that of Brazil, for instance. But what makes a poor country poor and a rich country rich? What accounts for economic growth and prosperity? What stands at its *origin*?

The question of *origin* is typically translated, in the economic literature, into one of causation in the language statistics and econometrics. Furthermore, often a single univariate cause is searched for and different possible causes are run one against each other. Acemoglu and Robinson (2012), for instance, argue explicitly against each one of several potential alternative causes (geography, culture, ignorance; Ch. 2, *Theories that don't work*) before laying their argument in favor of institutions in the rest of the volume.

Establishing institutions as the main cause of economic growth, even if in different specific contexts, is not always devoid of problems.[2] It essentially requires historical natural experiments where institutions are varied in geographical units with common geographical characteristics, culture, and other possible socio-economic determinants of future prosperity.[3] However, in many

[1] This was a good institutional order when citizens were good [...] but when citizen became bad, it turned into an horrible order; our translation

[2] Besides the arguments following, there are also methodological reasons to be skeptical about the concept of causation when facing slow-moving non-stationary processes (as, arguably is long-run history). For instance, the origin of the Mafia in Sicily has been reduced with good arguments to a price shock on sulfur an lemon in the 1850's (Buonanno, Durante, Prarolo, and Vanin, 2012); to the lack of city states in the XIV'th century - in turn a consequence of Norman domination (Guiso, Sapienza, and Zingales, 2007); to the Paleolithic split into nomadic pastoralism in 7th millenium B.C. (Alinei, 2007).

We might even suggest ironically that a single origin of economic growth and prosperity is a myth, like the one about the birth of all languages after the Christian God's destruction of the Babel's Tower, an "event" which was indeed "accurately" dated, allegedly on May 5th, 1491 B.C. by James Ussher, in 1650.

[3] Successful examples include: the institutional design of colonial empires, the more extractive the higher settlers' mortality rates (Acemoglu and Robinson, 2001); the spanish colonial policy regarding the forced mining labor system in Peru' (Dell, 2010); the U.S.-Mexico border separating the city of Nogales (Acemoglu and Robinson, 2010); the border separating the island of Hispaniola into two distinct political and institutional systems, Haiti and the Dominican Republic (Diamond, 2010).

of these examples, the assumption that the distinct institutions originated in the natural experiment arise in otherwise common cultural, geographical, environments is disputable. For instance, settlers' mortality rates could be correlated with natives mortality rates and hence pre-colonial development (see e.g., Alsan, 2012, on the habitat for the Tse-Tse fly in Africa). Furthermore, even the identification of the historical natural experiment as a change in institution is often debatable, as institutions generally reflect the cultural attitudes of the institution builders. Fischer (1989), for instance, studies institution formation during the early immigration waves in North America, showing how the cultural origins of the different groups of migrants (Puritans, Cavaliers, Quakers, Scots-Irish) affected the institutions they set in place; see also the well-know analysis by Greif (1994) of the institutional set-up of the Genoese and Maghrebi traders and Ben-Ner and Putterman (1998).[4] Finally, similar arguments have been produced for culture as the cause of prosperity, historically identifying instances of cultural variation in environments with a common institutional set-up.[5]

Even when not problematic, these causal analyses disregard the interactions between various determinant of economic growth: for instance, the same institutional change may have differential effects according to different cultural environments. Instances where this is the case have indeed been extensively documented.[6] The main reference in this respect of course is the work of Putnam on social capital, following the differential effects in the North and in the South of Italy of the institutional decentralization of the 60's and 70's (Putnam, 1993).[7] More generally, instances where institutions and cultural traits have manifestly jointly contributed to the development or the disruption of economic activity are common. This is the case for instance of Italian independent city states in the Renaissance (Guiso, Sapienza, and Zingales, 2007, 2008), industrialization and social capital in Indonesia (Miguel, 2003), the technology of plough, patriarchal institutions and gender attitudes (Alesina and Giuliano, 2011), the authoritarian culture of the sugar plantation regions of Cuba operated with slave labor as opposed to the with liberal culture of the

[4]Even the institutional changes in Medieval England from the Magna Charta onwards, which arguably stand at the origin of British prosperity and the Industrial Revolution (Acemoglu and Robinson, 2010) could be attributed to a general bourgeois culture as forcefully argued by McCloskey (2006, 2010). The same can be said for the formation of Italian independent city states in the Renaissance (Guiso, Sapienza, and Zingales, 2007 and 2008).

[5]While perhaps the first example of such kind of analysis is Weber's protestant ethic arguments (Weber, 1930), recent examples include the effects of the slave trade on trust within african tribes differently exposed to it but with similar institutional set-up (Nunn and Wantchekon, 2009); individual values about the scope of application of norms of good conduct in Europe (Tabellini, 2008a).

[6]See Alesina and Giuliano (2015) for a related argument, accompanied by a comprehensive survey of this literature; along similar lines, see also Nunn (2012).

[7]More recently, see e.g., Durante, Labartino, and Perotti, 2011, on university reform in Italy; Nannicini, Stella, Tabellini, and Troiano, 2010 on voting reform again in Italy; Mauro and Pigliaru (2012) on how culture has different effects when political institutions are centralized or decentralized; Grosjean (2011) on the traditonal (Scottish-Irish) pastoral society honor code in the U.S.; Minasyan (2014) on the effects of development aid institutions depending on donor-recipient cultural differences.

tobacco farms (Ortiz, 1963).[8] Even the presumption that culture is fundamentally immutable in the relevant time-frame, that is, changing at a much more slower pace than institutions, seems unfounded. Attitudes towards redistribution after the institution of welfare states in Europe, for instance, (Alesina and Angeletos, 2005; Alesina and Giuliano, 2010) and in East Germany after unification (Alesina and Fuchs Schuendeln, 2005) also changed very rapidly. So did in various instances the applications of the honor code studied by Appiah (2010). This is also arguably the case for social/civic/human capital after colonization (Glaeser, La Porta, Lopez-de-Silanez, Shleifer, 2004; Easterly and Levine, 2012; and Bisin and Kulkarni, 2012) and for various social preferences after the creation of the Kuba Kingdom in the early 17th century Africa (Lowes et al., 2015).[9]

Motivated by (this reading of) this literature, therefore, we study socio-economic environments in which culture and institutions jointly evolve and interact. While the existing related theoretical literature is very thin, one such environment has been studied by Greif and Tabellini (2010, 2011), where norms of coorperation (local vs. global) interact with institutional set-ups (informal vs. formal, clan vs. cities) to determine distinct paths of economic activity (China vs. Europe). Our objective is to develop an abstract model of culture, institutions, and their joint dynamics. While we aim at an abstract model, we are not after full generality. Rather we aim at a simple model which could help identify conditions under which the interaction of culture and institutions produces specific outcomes of interest. In these environments the *origin*, and hence the causation, question loses most of its interest: culture and institutions are jointly and endogenously determined and they jointly affect economic growth and prosperity, indeed all sorts of economic activity.[10] The focus is moved from the cause (both culture and/or institution can have causal effects) to the process as determined by the interaction.

By means of specific examples, we then characterize conditions under which cultural and institutional dynamics reinforce a specific (e.g., desirable) socio-economic equilibrium pattern, and economies in which on the contrary the interaction between culture and institutions ends-up weakening this equilibrium outcome. In this context, we can define the *cultural multiplier*, as the ratio of the total effect of institutional change divided by the direct effect, that is, the

[8]Relatedly, there is also evidence on the complementarity between culture and organization of firms: see e.g., La Porta, Lopez de Silanes, Shleifer, and Vishny (1997) on the effect of trust on firm size; Aghion, Algan, Cajuc, and Shleifer (2010) on the complementarity between distrust and regulation in a model with multiple equilibria; Bloom, Sadun, and Van Reenen (2012) on the organization of firms across countries, and in particular its relationship to culture.

[9]An example of a rapid joint change of institutions and culture induce by pro-active policies is the case of the fight against corruption in Hong Kong in the last decades which was driven by institutional change but engendered a deep modification of norms and attitudes towards corruption in the population in just a few years (Clark, 1987 and 1989; see also Hauk and Saez-Marti', 2002).

[10]This view is already clear in N. Machiavelli, as well as in E. Burke and in J. Madison, as the quotes at the outset demonstrates.

counterfactual effect which would have occurred had the distribution of cultural traits in the population remained constant after the instritutional change. These examples also display other interesting properties of the dynamics of culture and institutions, e.g., non-ergodic behavior, in which initial conditions determine important qualitative properties of their evolution, as well as of the stationary state the process converges to. Finally, we indicate how oscillations, cycles and other interesting complex behaviors can emerge from the interaction of culture and institutions.

We proceed, in turn, with an abstract model of the dynamics of of institutions (Section 2) and then with an abstract model of cultural evolution (Section 3). We then study the interaction of the two (Section 4). Finally, three examples aim at illustrating the analyis and the different forms of interactions (Section 5).

2 A simple model of the dynamics of institutions

We conceptualize institutions as mechanisms through which social choices are delineated and implemented. This is in line with the recent work effort by Acemoglu, Johnson, and Robinson in various pathbreaking contributions (surveyed in Acemoglu, Johnson, and Robinson, 2006) on economic and political institutions; see also Acemoglu, Egorov, and Sonin (2014). In their view, political institutions are mechanisms for the distribution of political power across different socio-economic groups. It is in turn political power which determines economic institutions which govern (incentivize and constrain) economic activity. In most of their analysis, political institutions represent the mechanism through which the conflict between *de jure* and *de facto* political power is resolved into a social choice problem whose outcome are specific economic institutions. More specifically, in Acemoglu (2003) e.g., institutions are represented by an indicator of which political pressure group has the power to control social choice. Institutional change is then the result of voluntary concessions by the controlling group typically under threats of social conflict.[11] This is also the approach taken by Acemoglu and Robinson (2006) to study more specifically the shift between dictatorship and democracy and viceversa. More generally, institutional change can represent an effective commitment mechanism on the part of one political group to extract resources from the others; this is the case, for instance in Besley and Persson (2009a,b, 2010), who study a society with pressure groups alternating in the power to control economic institutions regarding taxation and contractual enforcement.[12]

While we share with this literature the view of institutional change as a commitment mechanism, we depart from its notion of political power and control as embedded in one single group. Specifically, we model institutions as Pareto weights associated to the different groups in the so-

[11] Levine and Modica (2012) and Belloc and Bowles (2012) take a different, explicitly evolutionary, approach to the dynamics of institutions.

[12] Along these lines, Angelucci and Meraglia (2013) study charters to city states in the early Renaissance in Europe as concessions from the king to citizens to check and control the extractive power of fiscal bureaucracies.

cial choice problem.[13] This allows us to view institutional change as more incremental (formally, a continuous rather than a discrete change in political control) than just revolutions and regime changes[14]. It also allows us to eschew relying necessarily on social conflict as an explanation of institutional change: institutional change can much more generally occur as a mechanism to imperfectly and indirectly internalize the lack of commitment and the externalities which plague social choice problems; social conflict being only one of them and not necessarily the most prevalent in history.[15]

Consider a society with a continuum of agents separated into distinct groups defined in terms of relevant characteristics, i.e., political power and cultural traits.[16] We shall assume first that political and cultural groups are aligned and indexed by $i \in I$. In Section 5 and in several of the examples in Section 6 we allow for distinct groups (with no substantial effects on the general analysis). In this paper we also restrict for simplicity to dychotomous groups, that is $I = \{1, 2\}$.[17]

Let a^i denote the action of agents of group i and $\mathbf{a} = \{a^1, a^2\}$ the vector profile of actions, which we assume lies in some compact set. Let p denote economic policy in society, also in some compact set.[18] Let q^i denote the fraction of agents of group i in the population, with $\sum_{i \in I} q^i = 1$. We adopt the shorthand $q^1 = q,\ q^2 = 1 - q$.

The preferences of the fraction of agents belonging to group (i) are represented by an indirect utility function:

$$u^i\left(a^i, p; \mathbf{a}, q\right). \tag{1}$$

[13] See also Guimaraes and Sheedy (2010) who ground the study of institutions in the theory of coalition formation; and Lagunoff (2008) who provides a general study of the theoretical properties of political economy equilibria with dynamic endogenous institutions.

[14] This approach is consistent with the view expanded by Mahoney and Thelen (2010) whereby institutional change occurs through gradual and piecemeal changes that only 'show up' or 'register' as change if a somewhat long time frame is considered. Mahoney and Thelen (2010) distinguish between four modal types of such institutional change: displacement, layering, drift, and conversion.

[15] E.g., Lizzeri and Persico (2004) challenge Acemoglu and Robinson (2000, 2001, 2003) and Conley and Temimi (2001)'s rationalization of the extension of the franchise in early nineteenth century England, as an effect of threats to the stablished order. They argue instead that such institutional change had been motivated by the necessary evolution of public spending which required a commitment to limit particularistic politics in favor of public programs.

[16] Groups can of course be defined also in terms of resources, technologies, and so on. But we shall abstract from these characteristics for simplicity in the paper.

[17] With more than two groups the issue of coalition formation in institutional set-up and change becomes central. We leave this for a subsequent paper. The dynamics of $n \geq 2$ cultural traits has been studied by Bisin, Topa and Verdier (2009) and Montgomery (2009).

[18] Of course policies might be multi-dimensional, an extension we avoid for simplicity. Also, without loss of generality we could add a parametrization of the component of economic institutions which acts directly on the economic environment. We avoid clogging the notation when not necessary.

The dependence of u^i on a captures indirectly any externality in the economy. The dependence of u^i on q captures instead indirectly the dependence of technologies and resources on the distribution of the population by groups. A natural example would have the externality being represented by the mean action in the population: $A = qa^1 + (1-q)a^2$.

In this society, we identify political institutions with the weights of the groups $i \in I = \{1, 2\}$ in the social choice problem which determines economic policies. Let $\beta^i \geq 0$, denote the weight associated to group i, with $\sum_{i \in I} \beta^i = 1$. Again, we adopt the shorthand $\beta^1 = \beta$, $\beta^2 = 1 - \beta$.

2.1 Societal optimum and equilibria (given institutions and cultural distribution)

> "[...] gli assai uomini non si accordano mai ad una legge nuova che riguardi uno nuovo ordine nella citta' se non e' mostro loro da una necessita' che bisogni farlo; e non potendo venire questa necessita' sanza pericolo, e' facil cosa che quella republica rovini, avanti che la si sia condotta a una perfezione d'ordine."[19]; Niccolo' Machiavelli, *Discorsi,* I. 2, 1531.

The *societal optimum given institutions β and cultural distribution q* is a tuple $\{\mathbf{a}^{eff}, p^{eff}\}$ such that:

$$\{\mathbf{a}^{eff}, p^{eff}\} \in \arg\max \ \beta \, u^1\left(a^1, p; \mathbf{a}, q\right) + (1-\beta) \, u^2\left(a^2, p; \mathbf{a}, q\right). \tag{2}$$

The social optimum will be generally unattainable in our economy. We introduce instead two distinct equilibrium concept which will play a fundamental role in our analysis. The *societal equilibrium given institutions β and cultural distribution q* is a tuple $\{\mathbf{a}, p\}$ such that:

$$\begin{array}{l} p \in \arg\max_p \ \beta \, u^1\left(a^1, p; \mathbf{a}, q\right) + (1-\beta) \, u^2\left(a^2, p; \mathbf{a}, q\right) \\ a^i \in \arg\max_a \ u^i\left(a, p; \mathbf{a}, q\right) \quad i \in I = \{1, 2\} \end{array}. \tag{3}$$

That is, the *societal equilibrium* is a Nash equilibrium of the societal game between agents of the two groups and the policy maker operating in an institutional set-up characterized by weights β and cultural distribution q. Note that this simple formulation of the *societal equilibrium* in fact captures lack of commitment on the part of the policy maker, who is not allowed to pick the policy p in advance of the choices of the economic agents.[20]

To model a policy maker with commitment, we define instead the *societal commitment equilibrium given institutions β and cultural distribution q* as the Stackelberg Nash equilibrium of the

[19] [...] the majority of people will never agree to a new institutional order for the city unless necessary; and since necessity cannot come without danger, it is easily the case that institutions get into ruins before being perfected in a new order; our translation.

[20] No issues other than notational ones are involved in modeling a policy maker choosing after the economic agents, thereby strengthening its lack of commitment.

same game, where the policy maker is assumed to be the leader; that is, as a tuple $\{\mathbf{a}^{com}, p^{com}\}$ such that:

$$\begin{array}{ll} \{\mathbf{a}^{com}, p^{com}\} & \in \arg\max \ \beta\, u^1\left(a^1, p; \mathbf{a}, q\right) + (1-\beta)\, u^2\left(a^2, p; \mathbf{a}, q\right) \\ & s.t. \quad a^i \in \arg\max_a \ u^i\left(a, p; \mathbf{a}, q\right), \quad i \in I = \{1, 2\} \end{array} . \tag{4}$$

Under general conditions the *societal optimum*,[21] the *societal equilibrium,* and the *societal commitment equilibrium* are distinct. More precisely,

Proposition 1 *Given any institutions β and cultural distribution q, the* societal equilibrium *and the* societal commitment equilibrium *are both weakly inefficient, that is, they are weakly dominated by the* societal optimum. *On the other hand, the* societal commitment equilibrium *weakly dominates the* societal equilibrium.

Proof. The statement is a straightforward consequence of the fact that, for any (β, q): i) problem (4), which defines a *societal commitment equilibrium* , is a constrained version of problem (2), which in turn defines a *societal optimum*; ii) any *societal equilibrium* satisfying (3) is always contained in the constrained feasible set of problem (4), which defines a *societal commitment equilibrium.*[22] ■

2.2 Institutional design (given cultural distribution)

Future political and economic institutions are designed each generation by the present institutional set-up. We assume that institutional design is myopic, that is, institutions are designed for the future as if they would never be designed anew in the forward future.[23]

Making the dependence on (β, q) explicit, the *societal equilibrium*, the *societal commitment equilibrium*, and the *societal optimum* can be denoted, respectively:

$$[\mathbf{a}(\beta, q), p(\beta, q)]\,; \ \ [\mathbf{a}^{com}(\beta, q), p^{com}(\beta, q)]\,; \ \ \left[\mathbf{a}^{eff}(\beta, q), p^{eff}(\beta, q)\right].$$

A simple formulation of the design and hence of the dynamics of institutions can be obtained under the following regularity assumptions.[24]

[21]In the interest of lightness, we drop, from now on the qualifier "given institutions β and cultural distribution q" when referring to the equilibrium concepts in the paper.

[22]Of course, under robust conditions - in particular in all examples we study - domination holds strictly.

[23]It would be natural to consider a greater degree of institutional forward lookingness whereby current institutional changes would take into account the possibility for institutions to eventually continue to evolve in the future. The assumption of one step-forward institutional myopia allows our analysis of the coevolution between institutions and culture to remain tractable. For an analysis of the institutional evolution in a completely rational model but without cultural evolution see Lagunoff (2008), and Acemoglu, Egorov and Sonin (2015).

[24]Assumptions 1-2 require obvious but stringent monotone comparative statics requirements for *societal equilibria*. In the Appendix we spell out regularity conditions on utility functions that ensure the desired comparative statics properties.

Assumption 1 *Utility functions are sufficiently regular so that*

$$\mathbf{a}(\beta, q),\ p(\beta, q),\ \mathbf{a}^{com}(\beta, q),\ p^{com}(\beta, q)\ \textit{are continuous functions.}$$

Assumption 2 *Utility functions are sufficiently regular so that $p(\beta, q)$ is monotonic in β.*

Institutions then evolve as a solution to the following design problem:

$$\max_{\beta'}\ \beta\ u^1\left(a^1(\beta', q'), p(\beta', q'); a(\beta', q')\right) + (1-\beta)\ u^2\left(a^2(\beta', q'), p(\beta', q'); a(\beta', q')\right). \tag{5}$$

Adding an index t to denote time,

Proposition 2 *Under Assumptions 1-2, and given (q_t, q_{t+1}), the dynamics of institutions β_t is governed by the following implicit difference equation:*

$$\beta_{t+1} = \begin{cases} \beta \ \textit{such that}\ p^{com}(\beta_t, q_{t+1}) = p(\beta, q_{t+1}) & \textit{if it exists} \\ \left[\begin{array}{ll} 1 & \textit{if}\ p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}),\ \forall 0 \leq \beta \leq 1 \\ 0 & \textit{if}\ p^{com}(\beta_t, q_{t+1}) < p(\beta, q_{t+1}),\ \forall 0 \leq \beta \leq 1 \end{array}\right. & \textit{else} \end{cases}. \tag{6}$$

These dynamics can be intuitively interpreted as follows. At any time t, current institutions β_t induce the choice $p(\beta_t, q_t)$ at equilibrium. But they would rather prefer the choice $p^{com}(\beta_t, q_t)$. Therefore, when designing institutions for time $t+1$, current institutions design ("delegate to") institutions guaranteeing $p^{com}(\beta_t, q_{t+1})$ whenever possible at equilibrium; that is, they design ("delegate to") institutions β_{t+1} such that $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_{t+1})$. Whenever this is not possible, under our assumptions, they will design ("delegate to") institutions guaranteeing at equilibrium a policy choice p as close as possible to $p^{com}(\beta_t, q_{t+1})$.

To characterize the stationary states of the dynamics of institutions and their stability properties, it is convenient to define $P(\beta, q) := p^{com}(\beta, q) - p(\beta, q)$.[25]For a given society characterized by institutions β and cultural population q, $P(\beta, q)$ is an indicator of the extent of the policy commitment problem faced by such society, and how institutional change may resolve such problem. Intuitively, the absolute value of $P(\beta, q)$ indicates the intensity of the commitment problem as it reflects the distance between what can best be achieved under commitment and what is actually achieved in the policy game. The sign of $P(\beta, q)$ on the other hand indicates the direction of institutional change in β that needs to be implemented to resolve the commitment problem.

[25]We collect here the properties of (6) which are most relevant in our subsequent analysis. A more complete global stability analysis is not particularly complex but is tedious. We relegate it to the Appendix.

Proposition 3 *Under Assumption 1-2, for any given q, the dynamics of institutions governed by (6) have at least one stationary state. An interior stationary state β^* obtains as a solution to $P(\beta, q) = 0$. The boundary stationary state $\beta = 1$ obtains when $P(\beta, q) \mid_{\beta=1} > 0$; while the boundary stationary state $\beta = 0$ obtains when $P(\beta, q) \mid_{\beta=0} < 0$.*[26] *In the continuous time limit, the dynamics governed by (6) satisfies the following properties:*

- *if $P(\beta, q) > 0$ for any $\beta \in [0, 1]$, then $\beta = 1$ is a globally stable stationary state;*
- *if $P(\beta, q) < 0$ for any $\beta \in [0, 1]$, then $\beta = 0$ is a globally stable stationary state;*
- *any boundary stationary state is always locally stable;*
- *if an interior stationary state β^* exists, it is locally stable if $\frac{\partial P(\beta^*, q)}{\partial \beta} < 0$.*

Remark 1 *Assumption 2 implies that the extremal stationary states can only correspond to the corners of the dynamics; that is, $\beta^i = 0, 1$.*[27] *More importantly, when Assumption 2 is not satisfied the dynamics of institution might generally be undetermined, as $P(\beta, q) = 0$ might not have a unique solution in β. Furthermore, in this case, the dynamics of institutions can easily give rise to limit cycles. Consider for instance the example in Figure 1, with initial condition β_0, where the path $\beta_1 -> \beta_2 -> \beta_1$ constitutes such a limit cycle for a particular selection of the solutions to $P(\beta, q) = 0$.*

[Figure 1 about here]

2.3 Inefficient institutions

It is not generally the case in our set-up that institutions are efficient in a stationary state. By combining the results of Proposition 1 and Proposition 3 we obtain that a stationary *societal equilibrium* at best constitutes a *societal commitment equilibrium* for some institutions. In particular, the *societal commitment equilibrium* will not represent a *societal optimum* when the government policy p does not span the whole set of possible values of the vector profile $\mathbf{a}$. In other words, the institutional dynamics provides a tendency towards efficiency but i) generally not all the way towards a *societal optimal,* and ii) for a specific institutional set-up, that is, not necessarily towards a Pareto improvement. Several of the examples we study clearly demonstrate these points.

[26] Note that we arbitrarily define $\beta = 1$ (resp. $\beta = 0$) as an *interior stationary state* if $P(\beta, q^i) \mid_{\beta=1} = 0$ (resp. $P(\beta, q^i) \mid_{\beta=0} = 0$).

[27] See the Appendix A for a formal generalization of equation (6).

3 A simple model of the dynamics of cultural traits

We conceptualize culture as preference traits, norms, and attitudes which can be transmitted across generations by means of various socialization practices or can be acquired through socio-economic interactions between peers. Models of the population dynamics of cultural traits along these lines have been extensively studied in the social sciences and in biology.[28]

Cultural transmission is modeled as the result of *direct vertical* (parental) socialization and *horizontal/oblique socialization* in society at large:

i) direct vertical socialization to the parent's trait $i \in I = \{1, 2\}$ occurs with probability d^i;

ii) if a child from a family with trait i is not directly socialized, which occurs with probability $1 - d^i$, he/she is horizontally/obliquely socialized by picking the trait of a role model chosen randomly in the population inside the political group (i.e., he/she picks trait i with probability q^i and trait $i' \neq i$ with probability $q^{i'}$.

If we let $P^{ii'}$ denote the probability that a child, in (a family in) group $i \in I$ is socialized to trait i', we obtain:

$$P^{ii'} = d^i + (1 - d^i)q^{i'} \tag{7}$$

Let $V^{ii'}(\beta, q)$ denote the utility to a cultural trait i parent of a type i' child. It depends on the institutional set-up and the cultural distribution the child will face when he/she will make his/her economic decision $a^{i'}$:

$$V^{ii'}(\beta, q) = u^i \left(a^{i'}(\beta, q), p(\beta, q); a(\beta, q), q \right) \tag{8}$$

Let $C(d^i)$ denote socialization costs. Direct socialization, for any $i \in I = \{1, 2\}$, is then the solution to the following parental socialization problem:

$$\max_{d^i \in [0,1]} -C(d^i) + \sum_{i' \in I} P^{ii'} V^{ii'}(\beta, q), \text{ s. t. } (7).$$

As usual in this literature, define $\Delta V^i(\beta, q) = V^{ii}(\beta, q) - V^{ii'}(\beta, q)$ as the *cultural intolerance* of trait i. It follows that the direct socialization, with some notational abuse, has the form:

$$d^i = d^i(q, \Delta V^i(\beta, q)) = d^i(\beta, q), \; i \in I = \{1, 2\}. \tag{9}$$

Let $D(\beta, q) = d^1(\beta, q) - d^2(\beta, q)$.

[28]For an economic approach see a series of papers by Bisin and Verdier (1998, 2000a, 2001a) which build on the work of Cavalli-Sforza and Feldman (1973, 1981) in evolutionary biology and of Boyd and Richerson (1985) in anthropology; see Bisin and Verdier (2010) for a recent survey. We briefly introduce them here again for completeness and we refer the reader to the survey for the many details and extensions omitted here.

Assumption 3 *Utility and socialization cost functions are sufficiently regular so that* $d^i = d^i(\beta, q)$ *is continuous in* (β, q), *for any* $i \in I = \{1, 2\}$.

Adding an index t, the dynamics of the distribution of the population by cultural trait q_t is straightforwardly determined by:

Proposition 4 *Under Assumption 3, and given* β_{t+1}, *the dynamics of culture* q_t *is governed by the following difference equation:*

$$q_{t+1} - q_t = q_t(1 - q_t)D(\beta_{t+1}, q_{t+1}) \tag{10}$$

- The dynamics of culture governed by (10) have at least the two boundary stationary states, $q = 0$ *and* $q = 1$. *An* interior stationary states $0 < q^* < 1$ *obtains as a solution to* $D^i(\beta, q) = 0$.*In the continuous time limit, the dynamics governed by (10) satisfies the following properties:*

- *if* $D(\beta, q) > 0$ *for any* $q \in [0, 1]$, *then* q_t *converges to* $q = 1$ *from any initial condition* $q_0 > 0$;
- *if* $D(\beta, q) < 0$ *for any* $q \in [0, 1]$, *then* q_t *converges to* $q = 0$ *from any initial condition* $q_0 < 1$;
- *if* $D(\beta, 1) > 0$, *then* $q = 1$ *is locally stable ;*
- *if* $D(\beta, 0) < 0$, *then* $q = 0$ *is locally stable;*
- *if an interior stationary state* q^* *exists, and* $\frac{\partial D(\beta, q^*)}{\partial q} < 0$, *it is locally stable.*

It is often convenient to impose the following assumption (we do so in the examples as it simplifies the study of of the dynamics of culture essentially without loss of generality).

Assumption 4 *Socialization costs are quadratic:*

$$C(d^i) = \frac{1}{2} \left(d^i\right)^2.$$

The following corollary characterizes the resulting simplification:

Corollary 1 *Under Assumption 4,*

$$D(\beta, q) = \Delta V^1(\beta, q)q - \Delta V^2(\beta, q)(1 - q),$$

and hence interior steady states are characterized by solutions to:

$$\frac{\Delta V^1(\beta, q)}{\Delta V^2(\beta, q)} = \frac{q}{1 - q} \tag{11}$$

4 Joint evolution of culture and institutions

Under Assumptions 1-3, the joint dynamics of institutions and culture is governed by the system (6,10), which we report here for convenience:

$$\begin{aligned}
\beta_{t+1} &= \begin{cases} \beta \text{ such that } p^{com}(\beta_t, q_{t+1}) = p(\beta, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{l} 1 \quad \text{if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}),\ \forall 0 \leq \beta \leq 1 \\ 0 \quad \text{if } p^{com}(\beta_t, q_{t+1}) < p(\beta, q_{t+1}),\ \forall 0 \leq \beta \leq 1 \end{array} \right. & \text{else} \end{cases} \\
q_{t+1} - q_t &= q(1 - q_t) D(\beta_{t+1}, q_{t+1}).
\end{aligned}$$

Very little can be proved in general about the non-linear dynamical system (6,10); as a consequence we will turn to phase-diagrams in specific examples. We can nonetheless show the following:[29]

Proposition 5 *Under Assumptions 1-3 the dynamical system (6,10) has at least one stationary state.*

Furthermore, any *interior* stationary state (β^*, q^*) solves the following system of equations:

$$\begin{aligned}
P(\beta, q) &= p^{com}(\beta, q) - p^{com}(\beta, q) = 0 \\
\frac{\Delta V^1(p)}{\Delta V^2(p)} &= \frac{q}{1-q} \text{ and } p = p(\beta, q)
\end{aligned} \quad (12)$$

Let $\beta = \beta(q)$ be the steady state manifold associated with the first equation in (56) and $q(\beta) = \widehat{q}(p)$ with $p = p(\beta, q)$, with some notational abuse, be the steady state interior cultural manifold associated with the second equation. Note that $\widehat{q}(p)$ is actually a well defined function taking values in $[0, 1]$.[30]

A more detailed analysis of the stability properties of the non-linear dynamical system (6,10), is possible under more stringent assumptions. First to simplify the analysis, we may consider the continuous time limit of the system, where the change in institutional set-up and cultural composition between time t and $t + dt$ are λdt and μdt, for $dt \to 0$.[31](see the appendix). Second, we also impose the following preference separability condition:

[29] The proof is detailed in Appendix B.

[30] $\widehat{q}(p) \in [0, 1]$ is the unique solution of the following equation

$$\frac{\Delta V^1(p)}{\Delta V^2(p)} = \frac{q}{1-q}$$

[31] As is well known, discrete time dynamics may generate complex dynamic behaviors that are difficult to characterize and go beyond the points we want to emphasize about the co-evolution between culture and institutions.

Assumption 5 *Agents' preferences satisfy*

$$u^i\left(a^i, p; a, q^i\right) = v^i\left(a^i, p\right) + H^i(p; a, q^i)\,, \qquad SP$$

Assumption 5 implies that the policy instrument p affects the optimal private actions, a^i, independently of the economy-level aggregates a and q^i. This in turn implies that the socialization incentives ΔV^i depend only on the equilibrium policy level p.

We say that institutional and cultural dynamics are complementary at (β^*, q^*) when the steady state manifolds $\beta(q)$ and $q(\beta)$ have slopes of the same sign.

$$\frac{d\beta(q)}{dq} \text{ and } \frac{dq(\beta)}{d\beta} \text{ have the same sign.}$$

Conversely we say that institutional and cultural dynamics are substitutes at (β^*, q^*) when the slopes have opposite signs.

The non linear nature of the interaction between culture and institutions suggests that even in the context of a local stable steady state, the joint of evolution of cultural and institutional change may not be monotonic over time; that is the system may exhibit oscillations and cycles. Sufficient conditions to rule out oscillatory dynamics are obtained in the following:

Proposition 6 *Assume that (β^*, q^*) is a locally stable interior steady state of the (continuous-time approximation to the) non-linear dynamical system (6, 10). Then the local dynamics of culture and institutions show no converging cycles (dampening oscillations) if institutional and cultural dynamics are complementary.*

From this result, it follows that a necessary condition for the existence of dampening oscillations in institutional and cultural changes is that the steady state manifolds $q(\beta)$ and $\beta\,(q)$ are of opposite signs, namely that institutions and culture are dynamic substitutes

One may actually derive a specific condition to ensure the existence of a stable spiral steady state (see appendix). Such condition essentially states that when institutional and cultural dynamics are substitutes, there is an intermediate range of relative rates of change μ/λ between culture and institutions such that the system displays non monotonic dynamics in institutional and cultural change close to the stable steady state (β^*, q^*).

An interesting question is of course whether the dynamical system (6,10) (25) can generate limit cycles (periodic orbits) in institutional and cultural dynamics. In this respect, a simple application of the Bendixon Negative Criterion provides also have the following negative result:

Proposition 7 *Assume that the steady state institution β^* is locally stable when culture remains constant at $q = q^*$ along the institutional dynamics, and conversely that the cultural steady state $q = q^*$ is locally stable when the institutional context remains constant at $\beta = \beta^*$along the* *cultural dynamics. Then there cannot be periodic orbits and limit cycles in institutional and cultural evolution in the neighborhood of* (β^*, q^*).[32]

4.1 The cultural multiplier

In this section we study the comparative dynamics on institutions and culture. More specifically, we study conditions under which the effects of a shock are reinforced by the interaction of culture and institutions. To this end, we introduce and study the concept of *cultural* (resp. *institutional*) *multiplier*, the ratio of the long run change in institutions (resp. culture) relatively to the counterfactual long run change that would have happened had the cultural composition (resp. institutional set-up) of society remained fixed. In fact, motivated by the literature discussed in the Introduction, which stresses the economic effects of institutions for given cultural composition, we shall concentrate on the *cultural multiplier*, under the understanding that symmetric arguments and conditions hold for the institutional multiplier.

Consider the effects of a change in a parameter γ at a *stable interior* stationary state of the dynamics, $(\beta^*, q^*) \in (0,1)^2$. Adding explicit reference to γ in the notation, we restrict the arbitrary components of the environment as follows. First of all, we define the parameter γ so that, locally at the steady state, it increases both the policy p as well as the extent of the commitment problem:

$$\frac{dp^{com}(\beta^*, q^*; \gamma)}{d\gamma} > \frac{dp(\beta^*, q^*; \gamma)}{d\gamma} > 0,$$

Furthermore, we define the relative characteristics of the groups, so that the members of group 1 (with institutional power β) aim at a relatively larger policy level, p:

$$\frac{\partial p(\beta^*, q^*, \gamma)}{\partial \beta} > 0.$$

As a consequence, a positive change in γ induces a process of convergence to a new steady state with a larger β to promote a larger value of the *societal equilibrium* policy p; that is, in the absence of cultural change β would increase: $\left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} > 0$.

We can now define the *Cultural multiplier* on institutional change m, at (β^*, q^*) as

$$m = \left(\frac{d\beta^*}{d\gamma}\right) / \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} - 1 \tag{13}$$

The following proposition characterizes then conditions under which the *cultural multiplier* is positive:

[32] By the same token, when the institutional dynamics admit a global stable steady state $\beta(q)$ for any value $q \in [0,1]$ and the cultural dynamics admit a global steady state $q(\beta)$ for any value $\beta \in [0,1]$, one can also show the global result that in the full domain $(\beta, q) \in [0,1]^2$, there is no periodic orbits and limit cycles in institutional and cultural evolution.

Proposition 8 *The cultural multiplier m is positive if and only if the institutional and cultural dynamics are complementary (resp. substitute).*

The concept of *cultural multiplier* is related to whether institutional change and cultural change are dynamic complements or substitutes. When the slopes of $\beta(q)$ and $q(\beta)$ have the same sign, institutional and cultural evolutions are dynamic complements and the cultural multiplier is positive. Suppose that culture and institutions are complements in the sense that $\frac{d\beta(q)}{dq}$ and $\frac{dq(\beta)}{d\beta} > 0$. Then in our environment an increase in γ is set to induce an increase in β. Because of complementarity, this in turn promotes an increase in q which feedbacks positively on the institutional weight β. Any exogenous change in institutions is amplified by the associated cultural dynamics that co-evolve with institutions. Conversely, an institutional change would be mitigated by cultural changes (i.e., the *cultural multiplier* is negative) when culture and institution are substitutes, that is, when the slopes of $\beta(q)$ and $q(p)$ have opposite signs.

It is worth to develop more of an intuition for the mechanisms driving complementarity and hence the cultural multiplier. The complementarity condition on the slopes of $\beta(q)$ and $q(\beta)$ at an interior locally stable steady state (β^*, q^*), under assumption 5, can be shown to require that[33]

$$\frac{\partial P(\beta^*, q^*)}{\partial q}, \quad \frac{d\,\frac{\Delta V^1(p)}{\Delta V^2(p)}}{dp} \text{ have the same sign.}$$

The term $\frac{\partial P(\beta^*,q^*)}{\partial q}$ reflects how the institutional problem of commitment in policymaking is affected by a change of the size of the cultural groups. Institutional change represents a mechanism to solve the policy commitment problem by inducing an increase in the societal equilibrium policy p. This is obtained by giving more institutional weight to the group supporting relatively more the policy p, that is, by increasing β. Conversely, the second term $\frac{d\,\frac{\Delta V^1(p)}{\Delta V^2(p)}}{dp}$ reflects how a change in the equilibrium policy p affects the process of cultural evolution in the population. A higher level of the policy p is associated with a larger steady state frequency of the trait that is relatively more in favor of that policy, that is promoting the cultural diffusion of the trait that supports more intensively that policy, an increase in q.

Consider now an economic variable of interest, e.g., per capita income, public good provision, or any other measure of economic activity of interest in the model which depends on the joint dynamics of institutions and culture. In the context of the model, by the same logic it is straightforward to decompose the effects of culture from those of institutions and vice-versa. For this, formally define an aggregate variable $A(p, q, a^1(p), a^2(p))$. The *cultural multiplier on A* can then be defined as

$$m_A = \frac{dA}{d\gamma} / \left(\frac{dA}{d\gamma}\right)_{q=q^*} - 1.$$

[33] See the Appendix for details.

Typically, when one assumes no cultural evolution (i.e., q remaining fixed at its steady state level q^*), a change in γ is going to affect A through the way γ impacts institutions (ie . $\left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*}$) which in turn have effects on the *equilibrium societal policy* $p(\beta, q^*)$ and socio-economic behaviors $a^1(p), a^2(p)$. Compared to such situation, the expression of m_A highlights three additional effects of γ on A (see appendix). First, the total change in institutions $\left(\frac{d\beta^*}{d\gamma}\right)$ is affected by the cultural multiplier m as $\left(\frac{d\beta^*}{d\gamma}\right) = (1+m)\left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*}$. Second, the change in institutions triggers a policy change $\frac{\partial p}{\partial \beta}$ which in turn triggers cultural evolution and leads therefore to a change in the cultural composition of the population. Finally, cultural change also feedbacks on the *equilibrium societal policy* p, which in turn has again an additional impact on aggregate behavior A.

5 Extension: Distinct political and cultural types

In this section we extend our analysis to consider a society in which political and cultural groups are distinct. Let $i \in I$ index the political groups and $j \in J$ the cultural groups. Let a^{ij} denote the action of agents of subgroup (i, j) and $a = \{a^{ij}\}_{i,j}$ the vector profile of actions. Let q^{ij} denote the distribution of the population by cultural group and by $q = \{q^{ij}\}_{i,j}$ the vector profile satisfying satisfies $\sum_{j \in J} q^{ij} = 1$, for $i \in I$. Let λ^i denote the fraction of agents in political group i. Utility functions are then written $u^{ij}\left(a^{ij}, p; a, q\right)$.

In this society, we continue to identify political institutions with the weights of the groups $i \in I$ in the social choice problem which determines economic policy, $\beta = \{\beta^i\}_i$ satisfying $\sum_{i \in I} \beta^i = 1$.

As for cultural transmission, we assume for simplicity that political groups are perfectly segregated, so that the reference population for an agent in subgroup (i, j) is the subgroup itself. Fixing a political group $i \in I$, direct vertical socialization to the parent's trait, say $j \in J$, occurs with probability d^{ij}; $P_t^{i,jj}$ (resp. $P_t^{i,jj'}$) denote the probability that a child, in (a family in) political group $i \in I$ with trait j is socialized to trait j (resp. j') at t; $V^{i,jj}(\beta_{t+1}, q_{t+1})$ (resp. $V^{i,jj'}(\beta_{t+1}, q_{t+1})$) denotes the utility to a cultural trait j parent in political group i of a type j (resp. j') child.

It is then straightforward to extend the analysis of the previous sections to this society, with distinct political and cultural groups, to obtain the following system for the joint dynamics of culture and institutions:

$$
\begin{aligned}
\beta^i_{t+1} &= \begin{cases} \beta \text{ such that } p^{com}(\beta^i_t, q_{t+1}) - p(\beta, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{ll} 1 & \text{if } p^{com}(\beta^i_t, q_{t+1}) > p(\beta, q_{t+1}),\ \forall 0 \leq \beta \leq 1 \\ 0 & \text{if } p^{com}(\beta^i_{t,}, q_{t+1}) < p(\beta, q_{t+1}),\ \forall 0 \leq \beta \leq 1 \end{array}\right. & \text{else} \end{cases} \\
q^{ij}_{t+1} - q^{ij}_t &= q^{ij}(1 - q^{ij}_t)\left(d^{ij} - d^{ij'}\right), \text{ with } d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1})).
\end{aligned}
$$

6 Examples

In this section we work out several main examples, rich enough to display some interesting cultural and institutional dynamics.[34]

6.1 Elites, workers, and extractive institutions

Consider a society populated by two groups, workers and members of the elite, with distinct cultural traits and technologies. In particular, the preferences of the members of the elite are shaped by cultural norms which let them value leisure greatly, more than workers. Furthermore, while both the elite and workers are endowed with the same technology which transforms labor into private consumption goods, only the elite is endowed with initial resources. As a consequence, in equilibrium workers will work to survive, while the elite will generally eschew labor and constitute a leisure class. Finally, (fiscal) institutions collect taxes on income to finance public good consumption, which is valued by both groups. In this society, institutions are extractive inasmuch as members of the elite extract resources from society, and workers in particular, by means of fiscal policies.

Institutions, however, lack commitment; that is, fiscal authorities choose the tax rate ex-post, after workers have exterted their labor effort. This gives institutions generally an incentive to tax labor excessively. In this society, therefore, the elite might have in turn an interest in establishing less-extractive institutions, by devolving part of the fiscal authority to workers. This would indirectly commit institutions to a lower tax rate, in turn inducing workers to exert an higher labor effort and hence to contribute more to the public good. Indeed, we will show that, in this society, culture and institutions are complements, reinforcing each other: institutional change devolving fiscal authorities to the workers weakens any cultural predominance of the leisure class; while a smaller leisure class in society augments the incentives of the elite to devolve fiscal authorities to workers.

Formally, let workers be group $i = 1$ and the elite be $i = 2$. Both groups can transform labor ono-for-one into private consumption goods. Let a^i denote labor exerted by any member of group i. Let s denote the initial resources each elite member is endowed with. Let p denote the (income) tax rate and G the public good provided by fiscal institutions. Preferences for group i are represented by the following utility function:

$$u^i\left(a^i, G, p\right) = u\left(a^i(1-p) + s^i\right) + \theta^i v(1-a^i) + \Omega \cdot G.$$

Our characterization of the distinction between workers and the elite in terms of cultural values and technologies requires that:

[34] In all the examples we impose and exploit various regularity conditions without explicit mentioning them. We discuss however all the details in Appendix B.

i) the parameter θ^i representing the preference for leisure satisfies $\theta^1 < \theta^2$

ii) Initial resources s^i satisfy : $s^1 = 0$, $s^2 = s$ii) Initial resources s^i satisfy: $s^1 = 0, s^2 = s$.

To better illustrate the dynamics of culture and institutions in this society we consider extreme preferences for leisure of the elite, $\theta^2 > \frac{u'(s)}{v'(1)} > 1 = \theta^1$. In this case, members of the elite never work, $a^2 = 0$, and consume their resources, s. Workers instead exert some effort level $a^1 > 0$ and consume in fact a^1 units of the private consumption good. Both groups consume the public good G, in an amount equal the tax burden, to balance the budget of the fiscal institutions: $G = p\left[a^1 q + a^2(1-q)\right]$ where q is again the fraction of workers type $i = 1$.

The *societal equilibrium* and the *societal commitment equilibrium* are then easily characterized, for any institutional set-up, β, and distribution of the society by cultural traits, q. Equilibrium policies, that is, tax rate p, are as in Figure 2. Consider first the *societal equilibrium.* Typically, for β small enough, $\beta \leq \underline{\beta}(q)$, all policies $p\,(\beta, q)$ inducing no labor effort, that is, p larger than a threshold $\geq p_0$, are a *societal equilibrium* policy. In this case, workers have so little power that the natural ex post incentive is to tax them to the extent that they do not provide any labor supply. On the contrary, for $\beta \geq \overline{\beta}(q)$, workers have effectively control of the fiscal authority. In this case, labor income is either not taxed for all $\beta > \overline{\beta}(q)$; or else taxed only inasmuch as it is necessary to finance the amount of public good preferred by workers themselves, p^* (in this last case $\overline{\beta} = 1$). For intermediate values of $\beta \in \left(\underline{\beta}(q), \overline{\beta}(q)\right)$, the *societal equilibrium policy* $p(\beta, q)$ takes interior values and is a decreasing function of β. Indeed the ex-post incentives to finance the public good through labor income taxes are lower when the workers' interest are better represented.

The *societal commitment equilibrium,* $p^{com}(\beta, q)$, is also a decreasing function of β, always smaller than the tax rate $p^{\max}$ which maximizes tax revenue. Furthermore, $p^{com}(\beta, q) = 0$ when β is larger than the threshold $\overline{\beta}(q)$ (or $p^{com}(\beta, q) = p^*$ if $\overline{\beta}(q) = 1$). Most importantly,

$$p^{com}(\beta, q) < p(\beta, q),\ \forall \beta < \overline{\beta}(q),\ 0 < q < 1.$$

In the *societal equilibrium*, the policy maker does not internalize the negative distortion of taxation on the tax base. Hence taxes at a *societal equilibrium* are systematically (and inefficiently) higher than at the *societal commitment equilibrium* where such effect is internalized.

[Figure 2 about here]

Institutional dynamics. The institutional dynamics tend to internalize the inefficiency of the *societal equilibrium* which is due to lack of commitment, that is, to decrease $p(\beta, q) - p^{com}(\beta, q)$ for any given cultural distribution in the population q. In this society, therefore, fixing $0 < q < 1$

and $\forall \beta < \overline{\beta}(q)$, the institutional dynamics tend towards increasing the fiscal authority of workers, that is towards increasing β. This leads to reducing the excessive (and inefficient) tax rate p until it is optimal for the workers to do so. At the stationary equlibrium, therefore, all fiscal authority ends up effectively in the hands of workers.[35] Importantly, the public good consumption G^*, at the stationary equilibrium, is efficient under the stationary institutional set-up β^* (which effectively does not account for the preferences of the elite), but not generally under the initial institutional set-up.

Cultural dynamics. For every value of β the cultural dynamics tend to an interior stationary state $q(\beta)$, whereby q increases when $q < q(\beta)$ and decreases instead when $q > q(\beta)$. In other words, cultural substitution (see Bisin and Verdier, 2001) is the main driver of cultural dynamics in this society: given the institutional set-up, both group tend to engage in more intense cultural transmission when their trait is relatively minoritarian in society. Furthermore, the relative incentives to socialization $\Delta V^1(p)/\Delta V^2(p)$ are decreasing in p. Indeed, as taxation leads to increased rent extraction on labor, leisure class norms are more likely to be transmitted than those of the workers: the larger the rents of the elites, the larger their socialization advantage.

Joint evolution of culture and institutions. The joint cultural and institutional dynamics of this society are concisely represented in the phase diagram in Figure 3. The curve along which β is constant, $\overline{\beta}(q)$, is weakly increasing in q: to a larger fraction of the workers' trait in the distribution,(ie. a larger q), is associated a (weakly) larger β in the long run dynamics of institutions. Indeed, the larger is q, the larger are the incentives of the elite to extract resources from workers by taxing labor income. Consequently a larger fiscal authority to workers is necessary to reach their most preferred tax rate. For large enough values of q, this is only possible when $\beta = 1$, that is, all fiscal authority is devolved to the workers, $p = 0$, and no public good is consumed in the society, $G = 0$.

[Figure 3 about here]

The curve along which q is constant, $q(\beta)$, is also weakly increasing in β: more fiscal authority to workers leads to a lower equilibrium tax on labor p and therefore to an increase in the prevalence of the workers' cultural trait in the population in the long run dynamics of culture. Indeed, the lower is the tax rate, the higher are the relative gains of workers in the socialization process.

Indeed since both $\overline{\beta}(q)$ and $q(\beta)$ are increasing, culture and institutions are cultural complements in this society, reinforcing each other. The cultural multiplier is positive (Proposition 11). Furthermore, the joint evolution of culture and institutions displays a unique ergodic stationary state. The parameter configuration of the society determines whether any of the public good is

[35] Even though $\beta < 1$, an higher β would have no effect on equilibrium policies.

provided in the long run, that is whether the society is in case a) or b) in Figures 2 and 3. In either case, however, *extractive* institutions are undermined by their own inefficiency (due to the lack of commitment of the policy maker). The transition away from *Extractive* institutions is inevitable, from any initial condition.

Interestingly, this transition is triggered independently of any technology on the part of the workers to threaten, e.g., by means of a revolution, the power of the aristocrats. In this sense, the mechanism driving the evolution of institutions is distinct from the one stressed by Acemoglu (2003), Acemoglu and Robinson (2006, 2010), and Acemoglu, Johnson, and Robinson (2006); it is instead closer to the mechanism proposed for the extention of the suffrage in Britain by Lizzeri and Persico (2004). Furthermore, in this society, extractive institutions are not stable independently of the population distribution between workers and elites. In particular, this is the case even if the relative power of workers is unaffected by their relative size (or even relative income) in society.

6.1.1 The transition away from extractive institutions.

In the society we have just studied in the previous section, the elites' lack of commitment leads to very inefficient equilibrium outcomes when the elites themselves control the institutional set-up. As a consequence the joint dynamics of culture and institutions necessarily drive the society towards less extractive institutions where fiscal authority is devolved mostly (or even completely, in some parameters' configurations) to workers. In this section we study a simple extension of this society with the objective of providing a more articulate and interesting representation of the transition away from extractive institutions, one which delves deeper into the cultural preferences and the incentives of the elites.

Consider a society alike to the one studied in the previous section except in that i) members of the elite might hold a cultural trait which specifies work-ethic norms akin to those of the workers, rather than those of the leisure class;[36] ii) workers face a survival constraint, a minimum level of consumption necessary for survival. Furthermore, in the society we study in this section, taxes are not raised to finance the public good but are instead purely extractive, being redistributed pro-capita to the members of the elite.[37]

As in the previous section, the elites, as a political group, have the power of taxing workers, but cannot commit ex-ante on the tax rate. In this society, however, their incentives and preferences are heterogeneous: the members of the elite who share work-ethic norms (the *bourgeois*) are more aligned with workers' interests than those who do not (the *aristocrats*). Depending on their

[36] Note that in this society, therefore, political groups (workers and elites) are not aligned with cultural groups (bourgeois and aristocrats, inside the elite). The example is then a special case of the class of societies introduced in Section 5 and therefore follows the notational stucture laid out there.

[37] This is not substantial to the analysis. It is just for the sake of variation.

distribution by cultural trait and the political control they exert on the fiscal authority in society, the elites might impose a tax rate such that workers are constrained to subsistence (an *extractive regime*) (the survival constraint can be binding only for workers, as we maintain the assumption that members of the elites are endowed with initial resources which we postulate are enough for survival). The institutional dynamics of this economy will in general be *non-ergodic*, depending crucially on initial conditions. When the initial institutional set-up guarantees enough control on fiscal authority on the part of the workers, the institutional dynamics will tend to transition away from the *extractive regime*. This transition will generally induce the formation of a sizeable bourgeoisie. Interestingly, it is also the case that a larger bourgeoisie at the initial conditions favors the transition away from the *extractive* regime.

The detailed analysis of this society follows. Workers, group $i = 1$, are in proportion $1 - \lambda$ and members of the elites, group $i = 2$, in proportion λ. Members of the elite carry one of two possible cultural traits, $j = a, b$: i) the *bourgeois*, in proportion $q^{2b} = q$ of the total elite size λ, have the same preferences as workers; ii) the *aristocrats* are instead in proportion $q^{2a} = 1 - q$ of the elite and have preferences with extreme disutility for work.

All agents have preferences over a consumption good c^{ij} and labor effort a^{ij}, where $i = 1, 2$ indexes the group and j the cultural trait.[38] The production technology converts effort one-to-one in the consumption good. Let $\beta^1 = \beta$ denote the institutional weight of the workers, and p, the policy choice, represent the tax rate on workers' output, a^1. Let T denote the lump sum fiscal transfer received by each member of the elite, by budget balance. Let finally $\bar{c}$ denote the subsistence level required for survival. .

Preferences are represented by the following utility functions, respectively for workers and elites:

$$u^1(a^1, T^1, p) = u\left(a^1(1-p) + s^1 + T^1\right) + \theta^1 v(1 - a^1)$$

$$u^{2j}(a^{2j}, T^2, p) = u\left(a^{2j}(1-p) + s^2 + T^2\right) + \theta^{2j} v(1 - a^{2j})$$

Our characterization of the distinction between the political groups (workers and elites) and the cultural groups (bourgeois and aristocrats) in terms of cultural values and technologies requires that:

i) the parameter θ^{2j} representing the preference for leisure of the elites satisfy $\theta^{2a} > \theta^{2b} = \theta^1$
ii) Initial resources s^i satisfy : $s^1 = 0$, $s^2 = s > \bar{c}$
iii) Tax Transfers T^i satisfy $T^1 = 0$, $T^2 = T$

Again we assume that the aristocrats have extreme preferences for leisure $\theta^{2a} > \frac{u'(s)}{v'(1)} > 1 = \theta^1$ so that again they never work, $a^{2a} = 0$.

[38] Abusing notation the apex j is omitted for workers, $i = 1$, since they are culturally homegeneous.

In this society, the labor effort exerted by workets, $a^1(p)$, is non-monotonic in the tax rate p, depending on whether the survival constraint is binding, as shown in Figure 4. When the survival constraint is not binding, $a^1(p)$ is decreasing in p, because of the disincentive effects of the tax rate on effort. When instead the survival constraint is binding (in the *extractive* regime),[39]

[Figure 4 about here]

$a^1(p) = \frac{\overline{c}}{1-p}$ for $p \in [\widehat{p}, 1-\overline{c}]$; that is, $a^1(p)$ is increasing in p. As for the labor effort choice of the bourgeoisie, $a^{2b}(T) \geq 0$ is decreasing in the transfer level T. Aristocrats continue not exerting any effort (for any value of $T \geq 0$). The pro-capite fiscal transfer to the members of the elite is set to balance the budget of the fiscal institutions: $T =: \frac{1-\lambda}{\lambda} pa^1$.

The *societal equilibrium policy* $p(\beta, q)$, and the *societal commitment policy* $p^{com}(\beta, q)$ are illustrated in Figure 5. When the institutional weight of the workers is low enough, below a threshold $\underline{\beta}(q)$, at the *societal equilibrium*, the fiscal authorities tax the workers to a level that forces them to an extractive regime where the survival constraint $\overline{c}$ is binding.[40] Indeed, when workers are at the survival constraint, more extractive institutions will not necessarily reduce their labor effort, as workers will always have to exert enough effort to satisfy the survival constraint. On the other hand, when this is not the case, the elites might have an incentive to establish less-extractive institutions, to indirectly commit on a lower tax rate, in turn inducing workers to extend an higher production effort, as in the society studied in the previous section. If workers are sufficiently powerful, therefore, their behavior is the same as in the previous section: $p = 0$ for $\beta \geq \overline{\beta}(q)$; while $p(\beta, q) > 0$ and declining in β for $\beta \in \left(\underline{\beta}(q), \overline{\beta}(q)\right)$.

[Figure 5 about here]

As the *societal equilibrium policy*, the optimal policy at the *societal equilibrium with commitment*, is decreasing in the fiscal authority of workers β and is $= 0$ when $\beta \geq \overline{\beta}(q)$. Furthermore, when β is small enough, $p^{com}(\beta, q)$ is also high enough so that workers are kept at subsistence and the societal equilibrium with commitment is in the *extractive* regime. The transition away from the *extractive* regime occurs at $\beta = \hat{\beta}(q)$ at the *societal equilibrium with commitment*, a lower β than at the *societal equilibrium*, as in the first case the distortionary effects of taxation are internalized.

Not surprisingly, when the optimal policy at the *societal equilibrium with commitment* induces a non-extractive regime, $\beta > \hat{\beta}(q)$, it is the case that $p^{com}(\beta, q) < p(\beta, q)$. Indeed, without

[39] The policy space is assumed bounded in such a way as to always have make survival of the workers feasible.

[40] The maximal feasible tax rate is $p = 1 - \overline{c}$. At this rate workers have to supply their full time endowment $a^1 = 1$ to maintain their consumption level at the survival limit.

commitment the fiscal authorities do not internalize the effects of taxation on the labor effort of workers and therefore induce an equilibrium tax that is inefficiently high. This is not the case, however in the *extractive* regime. In this regime, in fact the effect of taxation on the effort of workers is positive. This tends to make the *societal equilibrium policy* $p(\beta, q)$ too low compared to the *societal equilibrium policy with commitment* $p^{com}(\beta, q)$. On the other hand, in this regime it is also the case that the fiscal authorities do not internalize the distortionary effects of taxes on the effort choice of the members of the bourgeoisie, which tends to make $p(\beta, q)$ too high compared to $p^{com}(\beta, q)$. When β is very low, the distortionary effect on the bourgeoisie dominates: indeed the effect of taxation on the workers' labor supply is minimal, in this case, as a^1 is already very close to its maximal value. As β increases, however the distortion on workers' labor effort turns to be larger, so that $p^{com}(\beta, q) > p(\beta, q)$. By continuity of the equilibrium policy functions $p(\beta, q)$ and $p^{com}(\beta, q)$ in the range $\left(0, \hat{\beta}(q)\right)$, there is a point $\beta = \beta^e(q)$ where the two curves cross, as depicted in Figure 5. But as β increases even more, to the point where the fiscal authorities turn to taxes p which do not constrain workers at survival, the distortionary effects of taxation turns to have a negative effect on both the workers and on the bourgeoisie. As we noted, in fact, in this case $p^{com}(\beta, q) < p(\beta, q)$ (it turns out that this happens discountinuously, as in the figure).

Institutional dynamics. From the previous discussion, the non-ergodic behavior of the institutional dynamics is apparent. Fixing a cultural distribution $0 < q < 1$, for all initial value $\beta_0 \in [0, \hat{\beta}(q))$, the institutional dynamics converge to a unique steady state $\beta = \beta^e(q)$ and the society ends up in an *extractive* regime with low political representation of the workers who are maintained at their survival constraint by extractive taxation on the part of the elites.[41] Conversely for initial values $\beta_0 \in (\hat{\beta}(q), \overline{\beta}(q)]$, the institutional dynamics are very different. The weight of the workers on the institutional setting converge to the unique steady state $\beta = \overline{\beta}(q)$, characterized by no taxation, in a non-extractive regime.[42]

Cultural dynamics. The dynamics of cultural evolution within the elite are driven by the relative incentives to socialization $\Delta V^b(p)/\Delta V^a(p)$, which is generally,[43] decreasing in p. Indeed,

[41] Interestingly in the *extractive* regime, higher taxation may actually increase the efficiency of the rent extraction process as the survival constraint prevents the traditional disincentives on labor supply to kick in. This local effect is arguably instrumental in maintaining such an extractive regime for workers. This is reminiscent of an argument in Clark (2009), suggesting that policies that would otherwise appear as having inefficiency costs in a non extractive world, on the contrary may find some efficiency rationale under extractive conditions.

[42] The dynamics from $\beta_0 = \hat{\beta}(q)$ are indetermined. Also, for initial values $\beta_0 > \overline{\beta}(q)$, the institutional weight of the workers is already large enough to induce no taxation and therefore no distortions. Institutions do not change and stay at their initial value $\beta_t = \beta_0$ for all $t > 0$.

[43] More precisely, when the tax rate $\widehat{p}$ at which an *extractive* regime is triggered is below the tax rate that maximizes total tax revenues.

aristocratic norms are more likely to be transmitted than those of the bourgeoisie the larger the rents of the elites. Since equilibrium taxation is a decreasing function of the institutional weight of workers, β, the more fiscal authority the workers possess in society, the larger the diffusion of (the norms of) the bourgeoisie inside the elite, and hence in society.

Joint evolution of culture and institutions. The joint cultural and institutional dynamics of this society are concisely represented in the phase diagram in Figure 6. The institutional stationary curve, $\overline{\beta}(q)$ is decreasing in q: in this society, in fact, the workers are at least in part supported by the bourgeoisie in their preferences over fical policy and as a consequence, when the fraction of the elite with bourgeois values is larger, the institutions support a no-tax policy even with less power to the workers. The cultural stationary state curve, $q(\beta) \in (0,1)$, is instead as in the society studied in the previous section: an upward sloping curve in the region $\beta \in \left[0, \overline{\beta}(q)\right]$ and a vertical line $q = q^*$ in the region $\beta \geq \overline{\beta}(q)$ for which there is no redistribution, $T = p = 0$. This is essentially for the same reasons: an higher β leads to a lower p and hence to greater socialization gains to the bourgeoisie and greater q in the long run.

[Figure 6 about here]

Differently from the case of the society studied in the previous section, therefore, culture and institutions are not cultural complements in this society. Furthermore, the joint evolution of culture and institutions does not display a unique stationary state, but rather two: an *extractive* state, (β^e, q^e), and a stationary state with no-tax, characterized by $\beta \geq \beta^*$ and $q = q^*$.

The dynamics of culture and institutions in this society will in general be *non-ergodic*: which stationary state they will converge to in the long-run depends on initial conditions. A transition away from *extractive* institutions is not inevitable in this society, as higher taxes do not decrease the fiscal rents of the elites when workers are at or around the survival constraint. *Extractive* institutions are therefore not any more undermined by their own inefficiency and could be supported in the long-run. Whether *extractive* institutions are supported in the long-run or whether the dynamics transition away depends on the political control the elites exert on the fiscal authority in society but also, crucially and interestingly, on their distribution by cultural trait, that is, on the relative size of the bourgeoisie, which is partly aligned with workers' interests. When the initial institutional set-up guarantees enough control on fiscal authority on the part of the workers, the dynamics will tend to transition away from the *extractive regime*. But a larger bourgeoisie at the initial conditions also favors the transition away from the *extractive* regime. Formally, the basin of attraction of the $(\beta \geq \beta^*, q^*)$ stationary state comprises all (β, q) strictly above the line $\hat{(\beta)}(q)$ and those on the line with $q < q^*)$. It is larger in β for higher q and it is also larger in q for higher β.

Importantly, even though culture and institutions are not cultural complements in this society, along a transition away from *extractive* institutions, with a powerful elites and small bourgeoisie (that is, in the region above $\hat{\beta}(q)$ and on the left of $q(\beta)$ in Figure 6), the dynamics display the devolution of power to workers jointly with the formation of a sizeable bourgeoisie (β and q both increase along the path).

6.2 Civic Culture and institutions

In this section, we discuss the interaction between the evolution of civic culture and institutional dynamics. We consider a society in which some workers may be endowed with intrinsic motivations ("civic culture") inducing them to take actions that may have beneficial implications for other worker members of society. Specifically, workers may exert civic participation efforts that are complements to the provision of some public goods and generate positive externalities on the rest of society. As well, individuals may also react negatively to inefficient public policies enacted by the government in favor of the elite. As these types of actions are costly at the individual level and produce some group level effect, they typically induce a common pool problem. In equilibrium only workers characterized by civic-minded attitudes will exert participation and civic control efforts.

In this society, the size of the public budget (the public policy decided by the government) has both positive and negative consequences on the economy, with opposing implications for the role of policy commitments, and therefore institutional dynamics.

On the one hand, larger public expenditures are associated to more provision of public goods in the society. This in turn stimulates civic participation by *civic minded* workers and consequently positive externalities on other agents. Given this, a commitment to increase the size of public expenditures helps internalize these positive externalities and therefore motivates some institutional change in that direction.

On the other hand, a large public sector also creates opportunities for administrative rents and corruption transfers that benefit the elite. These in turn provoke monitoring reactions by civic-minded individuals, and as a consequence diverted resources and increased transaction costs to reduce these transfers. For these reasons, a policy commitment to shrink the size of the public sector helps internalize the resource waste associated to the increased transaction costs of corruption and public leakages.

As a result of the tradeoff between these two opposite motives for policy commitment, the institutional dynamics lead to a long run balanced allocation of power between workers and elite over decision rights on public policy.

The most interesting aspect of this society consists however in the way institutions and culture interact. Typically, under some configuration of the shape of the fundamentals, culture and institutions may act as dynamic substitutes, providing therefore an illustration of a case where some exogenous variations on institutions can be mitigated by the cultural dynamics induced by these changes.

More in detail, the society is populated by the two political groups in the previous examples: workers, type $i = 1$, and the elite, $i = 2$, in fractions $\lambda^1 = 1 - \lambda^2 = \lambda$. All individuals (workers and elite) are endowed with a fixed amount of resources, ω that can be taxed lump-sum to finance public expenditures g necessary to produce a public good. We assume that in the process, a fraction μ of these public expenditures leaks into corruption generating diverted rents $T = \mu g \geq 0$, to the benefits of the members of the elite[44] while the residual share $(1 - \mu) g$ generates some effective amount of public good $G = (1 - \mu) g$. The rents T are subject to transaction costs, e.g., inefficient administrative procedures, secret kick-backs, corruption schemes, hidden accounts, and other "creative" fiscal accounting.

Workers can exert two types of efforts. First they may exert some sort of civic control, to monitor the government. Monitoring by the civil society tends to increase transparency, so that the transaction costs required to transfer resources to the elite on the part of the government are an increasing function of the civic society monitoring. More precisely, $\theta(A)$ denotes the transaction costs: a transfer T produces $\theta(A) T < T$ consumption units available for the elite; where $\theta(A)$ is decreasing in A, the total amount of civic monitoring effort exerted by the workers. For simplicity we take a linear specification: $\theta(A) = \varphi \cdot A$ with $\varphi > 0$.[45]

Second, workers may also exert civic participation efforts (contributing privately to public goods, creating social associations, volunteering in social activities) that generate positive allocative externalities over the whole society. More precisely, a total amount of civic participation effort E produces a society wide externality augmenting each individual's endowment by $\kappa \cdot E$ with $\kappa > 0$.[46]

We assume that the elite has standard preferences over consumption and the public good, $U^2(c^2, G) = c^2 + v(G)$ The workers belong to one of two cultural groups.The first, $j = c$ in proportion q, is composed of *civic-minded* individuals with both an intrinsic motivation for exerting civic control effort, a^{1c} and civic participation effort e^{1c}. More precisely, their preferences are given by:

$$\begin{aligned} U^{1c}\left(c^{1c}, G, a^{1c}, e^{1c}, T\right) &= c^{1c} + v(G) - (\alpha \cdot T)\left(1 - a^{1c}\right) - C(a^{1c}) \\ &\quad + G \cdot e^{1c} - \Phi(e^{1c}) \end{aligned}$$

where $c^{1c} + v(G)$ is the direct utility of private consumption and the public good. $-(\alpha \cdot T)(1 - a^{1c})$

[44] Alternatively one may think of members of roup $i = 2$ as public sector employees or bureaucrats, and μ as some degree of bureaucratic slack or inefficiency generating rents μG to these agents.

[45] Restrictions can be imposed on the preferences and the cost structures such that $\varphi \cdot A < 1$ for the relevant feasible range of aggregate monitoring A in society.

[46] One possible interpretation is that E is some sort of trust building mechanism that facilitates transactions and contracts and therefore increases productivity in the economy.

is the intrinsic motivation for civic control, $C(a^{1c})$ is the utility cost of undertaking civic control, $G \cdot e^{1c}$ is the intrinsic motivation to contribute e^{1c} to civic participation, while $\Phi(e^{1c})$ is the disutility cost of civic participation.[47] A level of rents T generates a direct intrinsic utility loss $-\alpha \cdot T$ to a *civic-minded* individual that is assumed to be linearly increasing in T. Undertaking a civic control effort a^{1c} reduces this utility loss to $-(\alpha \cdot T)(1 - a^{1c})$. Similarly, the intrinsic motivation $G.e^{1c}$ of a *civic-minded* individual to contribute e^{1c} to the society is increasing in the effective level of public good G supplied by the government[48].

This formulation therefore captures in a simple way the fact that a *civic-minded* worker is sensitive both to the positive part of public sector activity (the public good component G) as well as the negative part of it (the rent component T). He is therefore ready to react to both dimensions by providing some costly efforts $e^{1c}(G)$ and $a^{1c}(T)$ respectively increasing in G and T. Given that the size of the public sector g is affecting positively both dimensions G and T, an increase in g therefore stimulates both types of civic efforts e^{1c} and a^{1c}.

The second type of worker $j = p$, in proportion $1 - q$, do not have intrinsic motivations for civic participation and civic monitoring. We call them *passive* members and their preferences are given by

$$U^{1p}\left(c^{1p}, G, a^{1p}, e^{1p}, T\right) = c^{1p} + v(G) - C(a^{1p}) - \Phi\left(e^{1p}\right)$$

The policy choice $p = g$ (the budget size) depends on the workers' efforts a^{1c}, a^{1p}, e^{1c}, e^{1p} and only through the total amount of civic monitoring $A = \lambda\left[q \cdot a^{1c} + (1 - q) \cdot a^{1p}\right]$ and civic participation $E = \lambda\left[q \cdot e^{1c} + (1 - q) \cdot e^{1p}\right]$ they exert through respectively the transaction costs $\theta(A)$ and the positive externality $\kappa \cdot E$. As a consequence, the total amount of civic actions is a public good, the contribution of each worker effort is negligible, and hence passive workers always choose not to exert any effort, $a^{1p} = e^{1p} = 0$, and civic-minded workers contribute according to their intrinsic motivations.

Given any institutional weight $\beta \geq 0$, we may characterize the *societal equilibrium* $p(\beta, q)$and *societal commitment* $p^{com}(\beta, q)$ policies. In the appendix, we show that under some reasonable regularity conditions the shape of the policy functions $p(\beta, q)$ and $p^{com}(\beta, q)$ are as in Figure 7. The two curves are downward sloping in the weight β of workers. Intuitively, all individuals in society enjoy some direct utility of the public good as well as the positive externalities associated

[47]The utility of public good $\nu(G)$ is strictly increasing concave in G, the component $\alpha \cdot T$ of intrinsic motivation for civic control is linearly increasing in T, the utility costs of undertaking civic control and civic participation , $C(a^{1c})$ and $\Phi(e^{1c})$ are increasing convex functions satisfying $C(0) = \Phi(0) = 0$ and inada conditions $C'(0) = \Phi'(0) = 0$ and sufficiently convex to ensure our maximization policy problems to be enough regular.

To simplify our explicit computations, in the appendix we parametrize the utility cost functions to take the following form $C(a^{1c}) = \phi_A \frac{(a^{1c})^{1+\epsilon_A}}{1+\epsilon_A}$ and $\Phi(e^{1c}) = \phi_E \frac{(e^{1c})^{1+\epsilon_E}}{1+\epsilon_E}$ with ϕ_A, $\phi_E > 0$ and $\epsilon_A, \epsilon_E > 0$.

[48]Again we assume for analytical convenience that the intrinsic motivation for civic participation is linearly increasing in G. The important aspect is the fact that G enters as a complement to civic participation e^{1c} in the intrinsic motivation of *civic-minded* workers.

to the civic participation of *civic-minded* workers. The elite however additionally values a larger public sector because of the diverted rents it can obtain from it. For the workers, two other features come into play when they are *civic-minded.* First, these workers get some additional value from the public good because this enhances their intrinsic motivation to supply some civic participation action e^{1c}. At the same time though, a larger public sector also generates additional public leakages. This produces a higher utility cost $(\alpha \cdot T)(1 - a^{1c})$ connected to the intrinsic motivation to exert civic control a^{1c}. When civic participation is not too large[49], and/or workers are quite sensitive to public corruption[50], the positive contribution of the public good on civic participation is overwhelmed by the negative utility impact associated to public sector leakages. In such a situation, civic-oriented workers are less in favor of a large public sector than the elite members. As a consequence, an increase in the weight β of the workers' group tends to reduce the size of the public sector that the policymaker wants to implement at equilibrium. For the same reason, at a given value of β, an increase in the fraction of *civic-oriented* workers also leads to a decrease of the *societal equilibrium* $p(\beta, q)$ and the *societal commitment* $p^{com}(\beta, q)$.

Most importantly, it is apparent that the *societal equilibrium* curve $p(\beta, q)$ crosses the *societal commitment* curve $p^{com}(\beta, q)$ from above at some interior point $\widehat{\beta}(q)$. To understand this, notice that the discrepancy between $p(\beta, q)$ and $p^{com}(\beta, q)$ comes from the fact that the commitment issue that the policymaker has to deal with involves two externalities associated to the size of the public sector. The first externality is positive and economy-wide, and associated to the aggregate civic participation $E(p) = \lambda q e^{1c}(p)$ of *civic-oriented* workers. At the margin, the internalization of this positive externality leads the *societal commitment* policy $p^{com}(\beta, q)$ to be larger than the *societal equilibrium* curve $p(\beta, q)$. The second externality is negative and relates to the transaction costs $\theta(A)$ associated to the civic control effort $A = \lambda q a^{1c}(p)$ undertaken by *civic-oriented* workers to mitigate public leakages. At the margin, this negative externality leads the *societal commitment* policy $p^{com}(\beta, q)$ to be smaller than the *societal equilibrium* curve $p(\beta, q)$.

The negative externality is born out only by elite members while the positive externality is enjoyed by the whole society. As a consequence, when the weight of the Elite is large (ie. β small) in the policymaker objective function, the need to internalize the negative externality outweights the need to internalize the positive one. Therefore $p^{com}(\beta, q) < p(\beta, q)$. Conversely, when the weight of the Elite is small (ie. β large), the need to internalize the positive externality dominates the need to do so for the negative one, and consequently $p^{com}(\beta, q) > p(\beta, q)$. By continuity, there is therefore a threshold weight $\widehat{\beta}(q)$ such that $p(\beta, q)$ crosses $p^{com}(\beta, q)$ from above, as indicated in figure 7. Importantly at this threshold the internalization of the two externalities balance out at the margin.

[49]This occurs when the civic participation cost function $\Phi(.)$ is sufficiently convex.

[50]This occurs when the function $\alpha(T)$ is sufficiently convex in the corruption transfers T.

Institutional dynamics. As can be seen in figure 7, for all initial value β_0 the institutional dynamics converge to a unique steady state $\beta = \widehat{\beta}(q)$.

[Figure 7 about here]

In the steady state institutions, to balance out the two opposing externalities associated to the size of the public sector, there is power sharing between the workers and the elite.

How does the steady state weight $\widehat{\beta}(q)$ depend on the fraction q of *civic-minded* workers? A priori this depends on details of the preferences and technology fundamentals. Interestingly, when the level of civic participation e^{1c} is not much sensitive to the provision of public goods G, and conversely the level of civic control a^{1c} is sufficiently sensitive to public sector leakage T, then formal political power of workers in the steady state can be shown to be negatively related to the extent of informal monitoring provided by *civic-minded* individuals (ie. $\widehat{\beta}(q)$ is decreasing in q). [51] In such a case, from an institutional perspective, an active civic society acts as a substitute to formal political power.

The intuition for such a situation is the following. At the institutional steady state situation $\widehat{\beta}(q)$, one has $p^{com}\left(\widehat{\beta}, q\right) = p\left(\widehat{\beta}, q\right)$ and the positive and negative externalities associated to public sector size balance out at the margin. An increase Δq of the fraction of *civic-oriented* workers leads to a reduced equilibrium size of the public sector and less public leakages, as these workers are more concerned than the rest of society by corruption. A reduced level of the public sector then leads to smaller equilibrium levels of civic participation e^{1c} and civic monitoring a^{1c}. This in turn reduces both the marginal positive externality associated to civic participation and the marginal transaction cost externality associated to civic monitoring. Now, when civic participation e^{1c} (resp. civic monitoring a^{1c}) is not much sensitive to public good provision (resp. quite sensitive to public leakages), the marginal externality associated to civic participation is less impacted than the marginal externality associated to civic monitoring. As a consequence, $p^{com}(\beta, q)$ becomes larger than $p(\beta, q)$. Institutional dynamics move then in the direction of a larger public sector size and less power delegation to the workers: the new equilibrium steady state $\widehat{\beta}(q + \Delta q)$ is smaller than $\widehat{\beta}(q)$.

Cultural dynamics. The elite is culturally homogenous and hence displays no cultural dynamics, members of the elite remain such. The cultural dynamics within workers are determined by the the relative incentives to socialization $\Delta V^{1c}(p)/\Delta V^{1p}(p)$ as they depend on the equilibrium policy instrument p. When civic participation e^{1c} is less sensitive to public good provision than civic

[51] When such conditions are not satisfied, one may clearly have $\widehat{\beta}(q)$ to be positively related to q, at least in some range. In such a case, an increase in the extent of civic culture in society (ie. a larger value of q) translates into more formal delegation power given to workers.

monitoring a^{1c} is to public leakage, then $\Delta V^{1c}(p)/\Delta V^{1p}(p)$ is decreasing in p. As the societal equilibrium $p(\beta, q)$ is itself a decreasing function of β and q, the relative advantage of cultural transmission of civic-mindedness is then positively affected both by the formal political power of workers β and the fraction of civic-minded workers q in society.

Given this, for a fixed institutional weight β, the cultural steady states are characterized by a manifold $q(\beta) \in [0, 1]$. Whenever this manifold is characterizes by a well defined function, $q(\beta)$ is upward sloping in β. That is formal delegation of power to the workers tends to trigger a larger diffusion of civic culture into the workers' group. This is presented in Figure (8).[52].

[Figure 8 about here]

Joint evolution of culture and institutions. From our previous discussion, the joint evolution of culture and institutions is illustrated in Figure 8. The locus of institutional steady states is represented by the downward sloping curve $\widehat{\beta}(q)$ indicating that civic culture acts as a substitute to formal political power inside institutions. The locus of the stable cultural steady states $q(\beta)$ is upward sloping and reflects the fact that formal political power to workers promotes the diffusion of civic culture inside that group. The intersection point A of these two curves characterizes the steady state (β^*, q^*) associated to the joint dynamics of culture and institutions. As the two manifolds at that steady state have slopes of opposite signs, this is a case of dynamic substituability between culture and institutions.

Because of this, the effect of an exogenous shock on one of the two variables is mitigated by the dynamics induced on the other variable. To see that in figure 9, suppose that the society settled at the steady state point A. Consider then for instance an increase in the coefficient κ of the positive externality associated to civic participation E. The institutional steady state manifold $\widehat{\beta}(q)$ is shifted down. Indeed, as the value of civic participation is increased, this stimulates a policy commitment to a larger size of the public sector, inducing more civic participation by *civic-minded* workers. Institutionally, this is achieved by a reduced weight β of the group of workers. Given that the cultural manifold $q(\beta)$ is not affected by this parameter shock, the new steady state obtains at point B with a lower steady state value of β and also a smaller steady state fraction q of *civic minded* individuals.

[Figure 9 about here]

[52] For a given value of β, the cultural dynamics may exhibit multiple steady states. The branches of the cultural manifold $q(\beta)$ associated to stable steady states are then also positively sloped in β, while those associated to unstable steady states are negatively sloped in β.

Importantly, the final change in β at point B is less than the downward shift of the steady state manifold $\widehat{\beta}(q)$ visualized at point A'. Because culture and institutions are dynamic substitutes, the cultural dynamics mitigate the impact of the exogenous shock on institutions and the cultural multiplier is less than one. The intuition for this result is the following. A change in κ triggers some institutional dynamics biased against the workers' group. This institutional change in turn reduces the relative incentives to transmit the *civic-minded* cultural trait inside the population and leads to a reduction of q. As civic culture is reduced, there is less monitoring effort against public sector leakages. This feature calls then for some institutional change giving back some degree of formal power to workers, mitigating therefore the initial impact effect of the shock on institutions.

Interestingly, depending on the relative speeds of the two dynamics, the change in institutions may not necessarily evolve in a monotonic way after the shock. To see that in the clearest way, assume for instance that institutions adjust much faster than culture, meaning in the phase diagram of figure 10 that the dynamic system has to remain permanently on the institutional manifold $\widehat{\beta}(q)$. As can be seen, after the shock on κ, civic culture remains at its initial pre-shock value q_A^* while institutions jump downward to $\beta_{A'} < \beta_A$. Afterwards, there is cultural evolution: the fraction of civic-minded individuals decreases progressively from q_A^* to $q_B^* < q_A^*$ along the institutional manifold $\widehat{\beta}(q)$. Correspondingly, the institutional weight β of workers moves back up to the steady state value β_B illustrating the fact that along this trajectory, institutions evolved in a non monotonic way.

6.3 Property rights and conflict

The society studied in this section provides yet another interesting example of non-ergodic behavior, in which the steady state of the joint dynamics of culture and institutions depends on initial conditions. The society is characterized by socio-economic interactions consisting in agents contesting each other's resource endowment under imcomplete protection of property rights. Political and cultural groups coincide as in the previous society and agents are only differentiated by their propensity to act into conflict. Along the lines of the specific groups described by Nisbett (1993), Cohen and Nisbett (1994), or more recently Grosjean (2014), as displaying a *culture of honor,* one of the cultural groups in society is more prone to violence than the other individuals, after rituals and practices that individuals partake into in order to be culturally legitimized.

More in detail, in this society people are matched randomly in a contest. Each agent's endowment prior to the contest is $\omega > 0$. Property right protection is the main policy variable, represented by the fraction $p \in [0, 1]$ of each agent's endowment which is protected in the contest. After two agents match, their relative effort determines the probability that each of them succeeds in the contest, hence winning the fraction of the endowment of the opponent which is not protected by property rights. More specifically, let a^{hk} denote the effort exerted by an agent h when matching with an agent k. The probability of agent h winning the contest is $\frac{a^{hk}}{a^{hk}+a^{kh}}$.[53] The winner of the contest appropriates of the fraction of the total endowment not protected by property rights, $2(1-p)\omega$.

We assume that there are two political groups $i \in \{1,2\}$ which are fully identified to cultural groups $j \in \{1,2\}$ so that $i = j \in \{1,2\}$. The size of group 1 in society is therefore $\lambda^1 = 1 - \lambda^2 = q^1 = 1 - q^2 = q$.[54]. The two groups are culturally differentiated by their propensity to act into conflict. Specifically group 1 reflects individuals with a *culture of honor*, more prone to violence. Formally, these individuals are endowed with culture in which individuals have to pay a resource cost $F > 0$ allowing them to enjoy afterwards a higher propensity for violent action (ie. a low marginal cost of effort, c^1 in our contest setting). One could see the resource cost F as the typical cost associated to "beating" and "fighting" training sessions, rituals and practices that individuals with such culture go through in order to be legitimized. These violent sessions give them afterwards a "taste" or an ability to engage more easily into violent actions in potential contests with others. On the opposite, group 2 is composed of *conflict-averse,* individuals who do not have such a culture of a ritualized fighting capacity and consequently these individuals are less effective in violent contests. In terms of the model, they do not pay the resource cost F of violence ritualization but have a higher marginal cost of contest effort $c^2 > c^1$ when fighting in contests.

Denote for convenience $\alpha = \left(c^2 - c^1\right)/c^1$. With these notations, q is the fraction of "conflict-prone" individuals in society.

Agents observe the opponent type before choosing their effort.[55] and the Nash equilibrium effort of an agent of type i in his contest with an agent of type j can be solved for straightforwardly, for given property rights p. Denoting by slight abuse of notation such effort as a^{ij}, this is given by:

$$a^{ij} = 2(1-p)\omega \frac{c^j}{\left(c^i + c^j\right)^2},$$

Matching is random, so that an agent in group i will match another agent in the same group with probability q^i and an agent in the other group with probability $1 - q^i$. Let the ex-ante expected payoff for agents of each of the groups at equilibrium be denoted $\Omega_i(p, q)$. It is decreasing in the fraction q of "conflict-prone" indviduals as a larger fraction of "conflict-prone" agents hurts both groups ex-ante. It induces a larger rent dissipation for the conflict-prone agents and a larger probability of extortion (loss of endowment) for the "conflict averse" individuals.

[53] Formally, this is the case if $a^{hk}, a^{kh} > 0$; while the probability of winning is 1/2 if $a^{hk} = a^{kh} = 0$.

[54] As compared to the general setting in section (?), in this example political groups have some endogenous size. The dynamics however still remain tractable, as poltical groups are fully alligned with cultural groups..

[55] That is, the contest is a complete information game. The expected payoffs of an agent of cultural group j matching with an agent of group l is $W^j(a^{jl}, a^{lj}) = pV + 2(1-p)V\frac{a^{jl}}{a^{ijl}+a^{lj}} - c^j a^{jl}$. This example also represents an extension of the general analysis in Section 2 in that a^j is a multi-dimensional vector; again the same methods apply however.

On the other hand, while $\Omega_2(p,q)$ is always increasing in p, $\Omega_1(p,q)$ is increasing in p only for a large enough fraction q of *conflict-prone* agents. Indeed "conflict-averse" individuals always benefit from property right protection, "conflict-prone" agents favor better property rights protection only when their fraction in the population is large enough.

We assume that implementing a level p of property rights protection requires a resource cost $C(p)$ satisfying standard convexity properties (see the Appendix for details).

Denote by $\beta^1 = 1 - \beta^2 = \beta$, the institutional weight of the "conflict prone" group. At the *societal equilibrium* property right protection p is chosen, taking as given the effort choices in contests for agents of the two groups. It can be easily shown that[56] when $\beta \geq q$, *the societal equilibrium* involves no property right protection and $p(\beta,q) = 0$; while for $\beta < q$, a positive level protection of property right is implemented with $p(\beta,q) > 0$. Moreover in such a case $p(\beta,q)$ is a decreasing function of β and is increasing in q. The larger the weight of the "conflict prone" group, the smaller the level of property right protection, as such group benefits less from this protection. On the other hand, the larger the fraction of the conflict prone individuals in society, the larger the social need for a reduction of conflict efforts dissipated into resource contests and therefore some enhanced degree of protection of property rights.

Similarly one can characterize the *societal equilibrium with commitment,* which now internalizes the impact of property right protection on the effort choices of the two groups in their contests. Specifically, one can show that at a *societal equilibrium with commitment*, there exist a threshold $\widetilde{q}(\alpha) \in]0,1[$ and an increasing function $\beta = \widetilde{\beta}(q)$ with $\widetilde{\beta}(0) < 1$ such that $p^{com}(\beta) = 0$ and there is no protection of property rights if and only if $(\beta,q) \in [0,1]^2$ are such that $q < \widetilde{q}(\alpha)$ and $\beta \geq \widetilde{\beta}(q)$. When conversely β, q do not satisfy such relations, *the societal equilibrium with commitment* involves positive protection of property rights and $p^{com}(\beta,q) > 0$. In such a case, $p^{com}(\beta,q)$ is again decreasing in β and increasing in q. Furthermore, one can show that the boundary $\widetilde{\beta}(q)$ is increasing and equal to 1 for $q < 1$ large enough.

In other words and not surprisingly, the *societal equilibrium with commitment* involves no property right protection when the institutional set-up is favoring the *conflict-prone* group, that is, when β is large enough [57]. More interestingly, however, when the fraction of conflict-prone agents is too high, q is large enough, then such group is always in favor of instituting property rights as a form of self-protection and $p^{com}(\beta,q) > 0$ at any level of β. Finally $p(\beta,q) \leq p^{com}(\beta,q)$, simply reflecting the fact an institutional commitment at property right protection prevent costly efforts to be undertaken into the rentseeking contests.

The two policy schedules $p(\beta,q)$ and $p^{com}(\beta,q)$ are represented in Figure 10, out of which it is then straightforward to study the dynamics of institutions. More specifically, one can immediately

[56] As usual see Appendix C for details.

[57] More precisely, a *societal commitment equilibrium* with no property rights also requires that $\frac{c_2}{c_1}$ be large enough; see Appendix C for details.

see that for any given q, if $\beta_0 > \widetilde{\beta}(q)$, then $\beta_{t+1} = \beta_t = \beta_0$. If instead $\beta_0 < \widetilde{\beta}(q)$, then β_t converges towards $\beta = 0$.

[Figure 10 about here]

We turn now to the dynamics of culture and hence to the socialization incentives of the two cultural traits ΔV^1 and ΔV^2. The "conflict-prone" agents have positive incentives $\Delta V^1(\beta, q)$ to transmit their trait but such incentives decrease with the fraction q of "conflict-prone" individuals in society. Similarly the incentives for the "conflict-averse" agents $\Delta V^2(\beta, q)$ are also positive. But, interestingly, their incentives are increasing with the fraction q of "conflict prone" agents in the population. A larger value of q reduces the expected payoff of "conflict-averse" agents matched with "conflict-prone" agents, thereby reducing the incentives to transmit their own trait; at the same time however, a larger q also increases the cost of effort for "conflict-averse" agents whose children turn out to be "conflict-prone" and undertake the high effort a^{11} when facing other "conflict-prone" agents in a contest. This effect tends to increase the value $\Delta V^2(\beta, q)$. It turns out that this second positive effect actually dominates the first negative one and therefore the incentives for "conflict-averse" agents to transmit their trait are positively associated to the fraction of "conflict-prone" agents in the population.

With regards to socialization incentives, property rights protection affects negatively the socialization incentives of the conflict-prone and promotes on the opposite the socialization incentives of conflict-averse agents.[58]

As a consequence, the cultural dynamics has a unique interior stationary state $q(\beta)$ which is increasing in the weight of the "violence prone" group β. Indeed a larger weight of that group implies less protection of property rights and therefore a larger diffusion of a culture of violence in society. Furthermore, $q(\beta) < 1/2$.

With respect to the joint dynamics of culture and institutions, we distinguish two cases (that we represent in Figures 11a) and 11b)).

The first case corresponds to $\alpha = \frac{c_2}{c_1} - 1$ large enough (the culture of violence gives a significant advantage in conflicts) and is represented in Figure (11a). Suppose first the initial conditions (β_0, q_0) are in the stripped region $[1, \beta_A, A, B]$[59]; that is, the "conflict-prone" are well represented in the initial institutions but not as a fraction of the population. The dynamics of

[58]This feature is consistent with the observation by Cohen and Nisbett (1994) and Grosjean (2013) that a "culture of honor" and violence in the US South has persisted and be transmitted because of weak institutions of property rights protection and a need to enforce individually by contest and violence such property rights. It is also consistent with various anthropological observations that suggest that cultures of violence are more likely to develop in pastoralist and herders' societies where property rights protection on cattle is more difficult to enforce than property rights on land in agrarian societies (see Campbell (1965), Edgerton 1971, Peristiany1965).

[59]where $\beta^*(\alpha)$ is defined as: $\beta^*(\alpha) = \widetilde{\beta}(q^*(\alpha))$.

culture is not undermining then the institutional set-up serving the interests of the “conflict-prone” group and the system remains with the initial institutional set-up, $\beta_{t+1} = \beta_t = \beta_0$, with no property rights protection. The dynamics of culture converge towards the fraction $\widehat{q}(\alpha)$ of ”conflict-prone” individuals.

[Figures 11a) and 11b) about here]

On the other hand when the initial conditions are outside of the stripped region in the figure, ”conflict prone” agents are not well represented in the initial institutional set-up and/or they are too numerous. Then the institutional dynamics evolves towards an increased representation of the ”conflict-averse” agents, increased property rights protection, and a long run fraction of ”conflict-prone” individuals is given by $q(0) < \widehat{q}(\alpha)$. It should be noted that $p(\beta, q) > 0$ along the equilibrium path, the interaction of the dynamics of institutions and culture leads progressively towards a reduction of a culture of violence and also less resources spent in these conficts.

Interestingly when $\beta_0 \leq \beta_A$, even a smaller fraction of conflict-prone individuals is ultimately self-defeating in terms of institutional dynamics. While for some time the system does not exhibit any institutional change and $\beta_{t+1} = \beta_t = \beta_0$, the underlying cultural dynamics tend to favor the socialization of the conflict-prone agents towards $\widehat{q}(\alpha)$. As soon as q_t passes the threshold of $\widetilde{\beta}^{-1}(\beta_0)$, endogenous institutional dynamics are triggered inducing the implementation of more extensive property rights and institutions biased towards the conflict-averse group. As a consequence of this, the transmission of a culture of violence also regress towards to long run steady state $q(0)$. This example shows therefore cleary the importance of initial institutional and cultural conditions for the long run of society and the non ergodicity properties of this system. Importantly, a temporary exogenous institutional shock that gives more formal power to the conflict-averse group may trigger a very different long run trajectory of the institutional and cultural dynamics. Indeed suppose that the society has settled to a point like point A in Figure (11a) with no property rights and a culture of violence at $\widehat{q}(\alpha)$. Then a reduction of β below β_A, leads an endogenous institutional response towards further power to the ”conflict averse” individuals. This in turn triggers reinforcing cultural dynamics towards that ”conflict averse” group. After a while an inverse institutional shock of similar amplitude will not however bring back the system towards to region without property rights. Indeed even when the conflict prone group regains back some formal power for some exogenous reasons, the cultural dynamics have irreversibly driven the system in a region where property rights are protected and there are less individual conflicts for the contest of resources. This suggests that external interventions (colonization, foreign aid, invasions) that changes the balance of power domestically between groups may have long term effects in terms of institutional and cultural evolution.

The second case corresponds to $\alpha = \frac{c_2}{c_1} - 1$ not too large so that $\widehat{q}(\alpha) > \widetilde{q}(\alpha)$. In this case, represented in Figure (11b), the marginal effort costs c_i are similar across groups and hence "conflict-averse" agents are not much different than "conflict prone" agents. The dynamics of culture and institutions are such that $p(\beta_0, q_0) > 0$, that is property rights are protected for any initial conditions $(\beta_0, q_0) \in [0,1]^2$. The joint dynamics of culture and institutions converge to the stationary state $(0, q(0))$, characterized by institution giving all power to the "conflict-averse" agents and hence a maximal protection of property rights and a small fraction of "conflict-prone" agents in the population.

7 Conclusions

Motivated by the recent literature on the root causes of long term development, this paper proposed a simple theoretical perspective to analyze how culture and institutions evolve, interact and jointly determine socio-economic outcomes. Our framework highlights two major components On the one hand, institutional change obtains as a coordinated process through which existing social and political power structures may strategically increase their policy commitment capacity in order to resolve fundamental socio-economic externalities. On the other hand, cultural dynamics emanate from decentralized population level cultural evolutionary processes due to voluntary and involuntary activities between (and within) generations of individuals.

Our approach allows a simple and easily applicable description of the joint interactions between culture and institutions. Particularly, we provide conditions under which cultural and institutional dynamics tend to strengthen each other in a complementary way, or on the contrary, tend to mitigate each other in terms of their effects on socio-economic aggregate variables. Exogenous historical accidents propagate over the joint dynamics induced by institutions and culture, and may therefore have magnified or mitigated effects on long run socioeconomic outcomes. Importantly, our discussion indicates the extent of the comparative dynamic biais that can be generated by neglecting one of the two dynamics, when the other one is affected by an exogenous shock (the so-called *cultural and institutional multipliers*) .

Conceptually, our framework also suggests that in general the joint evolution of culture and institutions is highly non-linear. This feature has a number of implications such as: the non ergodic character of the underlying dynamic processes between culture and institutions, the sensitivity of equilibrium trajectories to initial conditions, the existence of irreversibility and thresholds effects and the non-monotonicity of cultural and institutional changes over transition paths. From an empirical point of view, these phenomena are consistent with the observation of (and the difficulty of explaining) the great diversity of development experiences encountered across the world. As well, they suggest that linear regression methods may be at a disadvantage over more structural analyses of the data.

Overall, our analysis underlines the fact that the search for a deep and unique origin for the long-term development can be quite an arduous and probably sterile undertaking. Focusing more systematically on the positive or negative interactions between culture and institutions along the development process may appear to be more fruitful in terms of historical understanding, and as well as in terms of policy implications.

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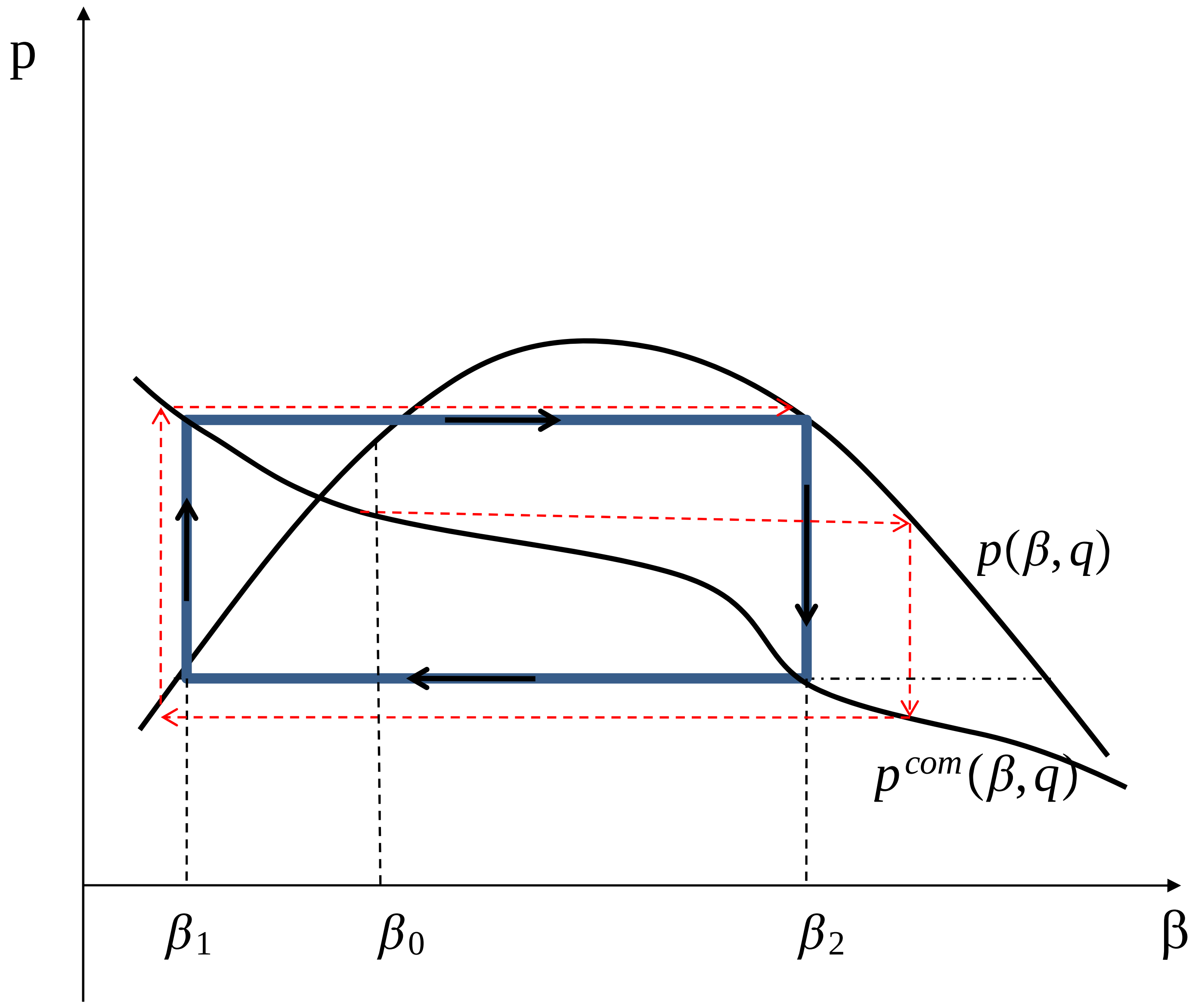


Figure 1: Limit cycle (β_1 , β_2) for $p(\beta,q)$ non-monotonic

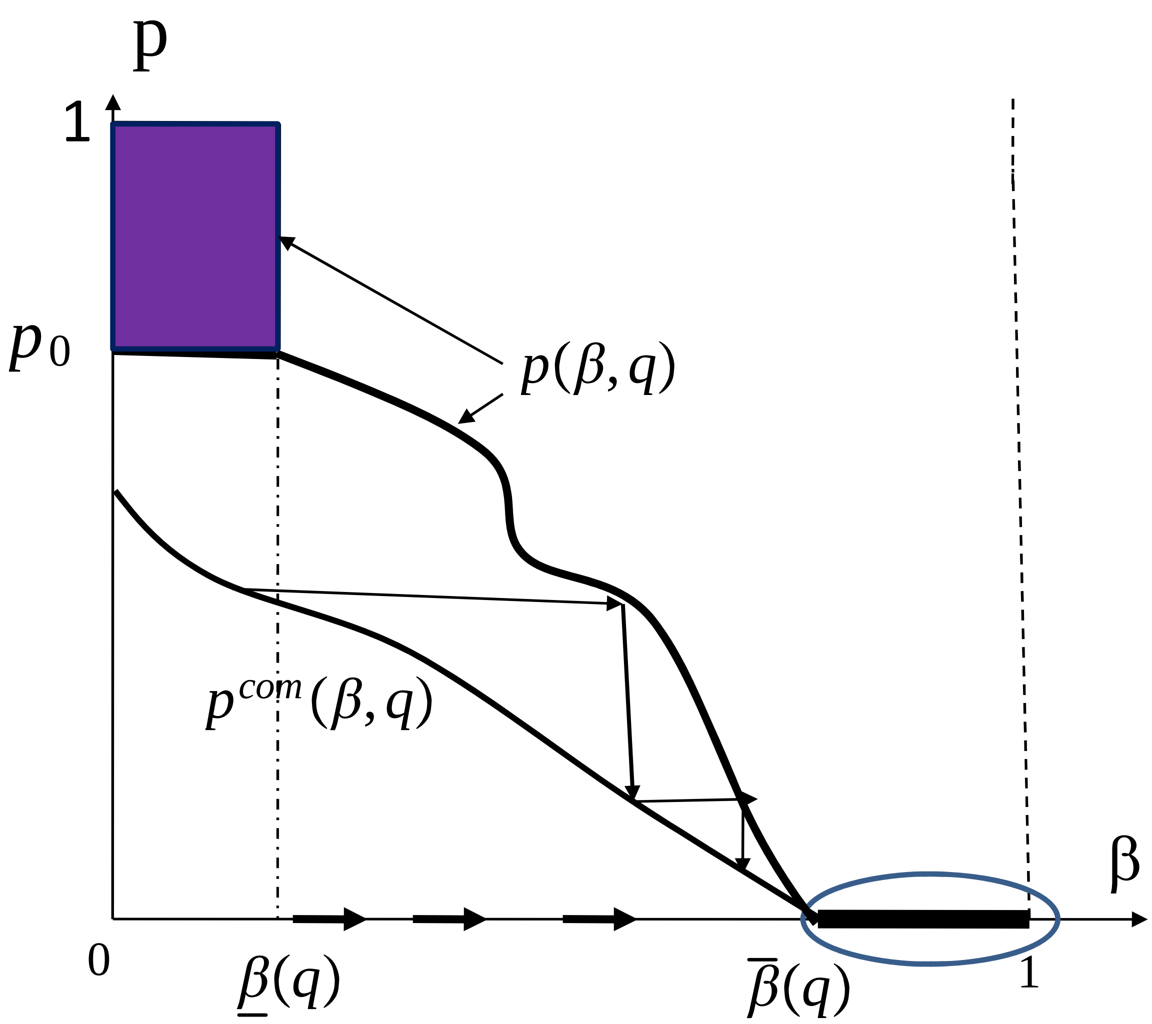


Figure 2a) : Elites, Workers, and Extractive Institutions
No Public Good Provision at Stationary State

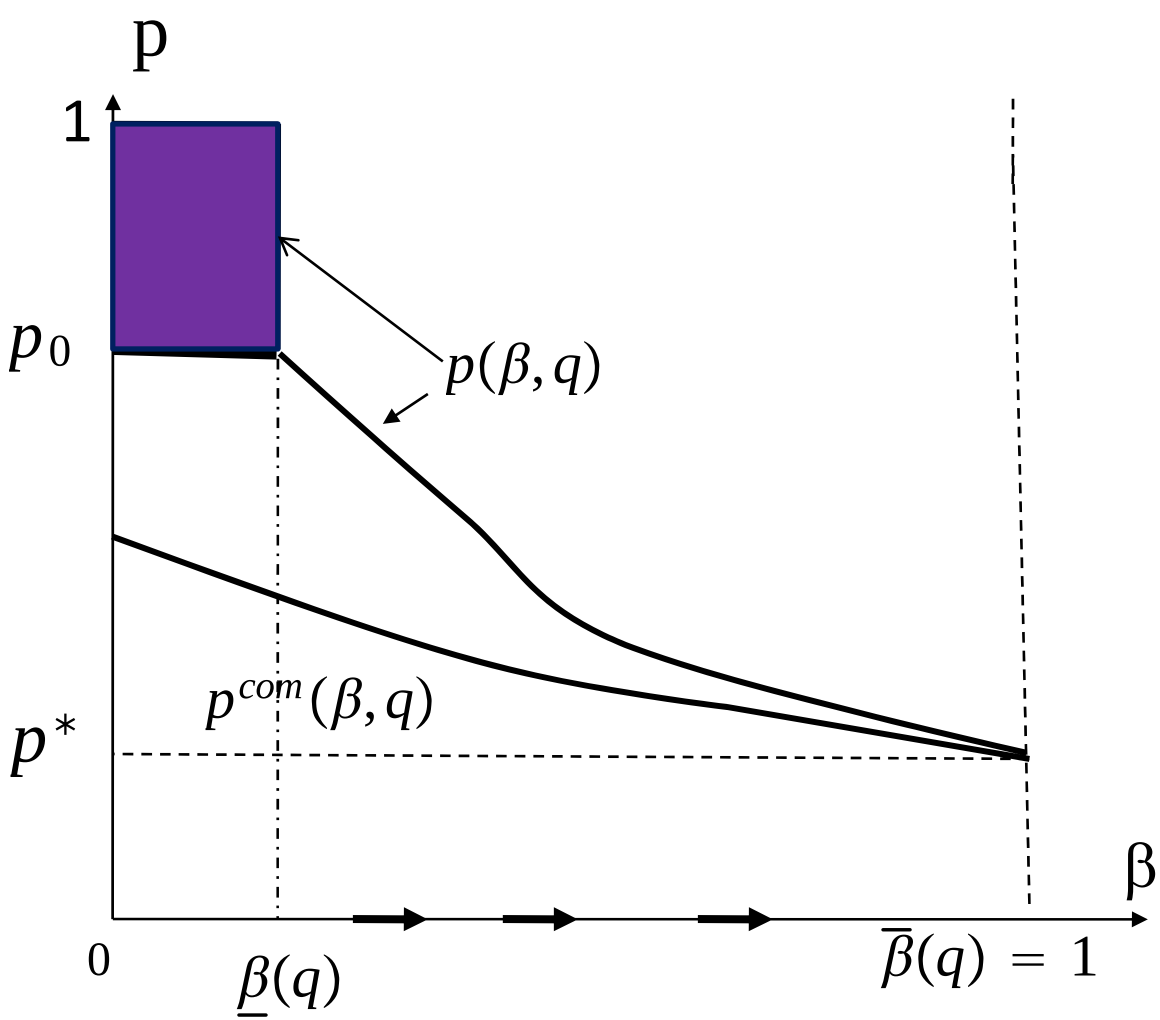


Figure 2b) : Elites, Workers, and Extractive Institutions
Public Good Provision at Stationary State

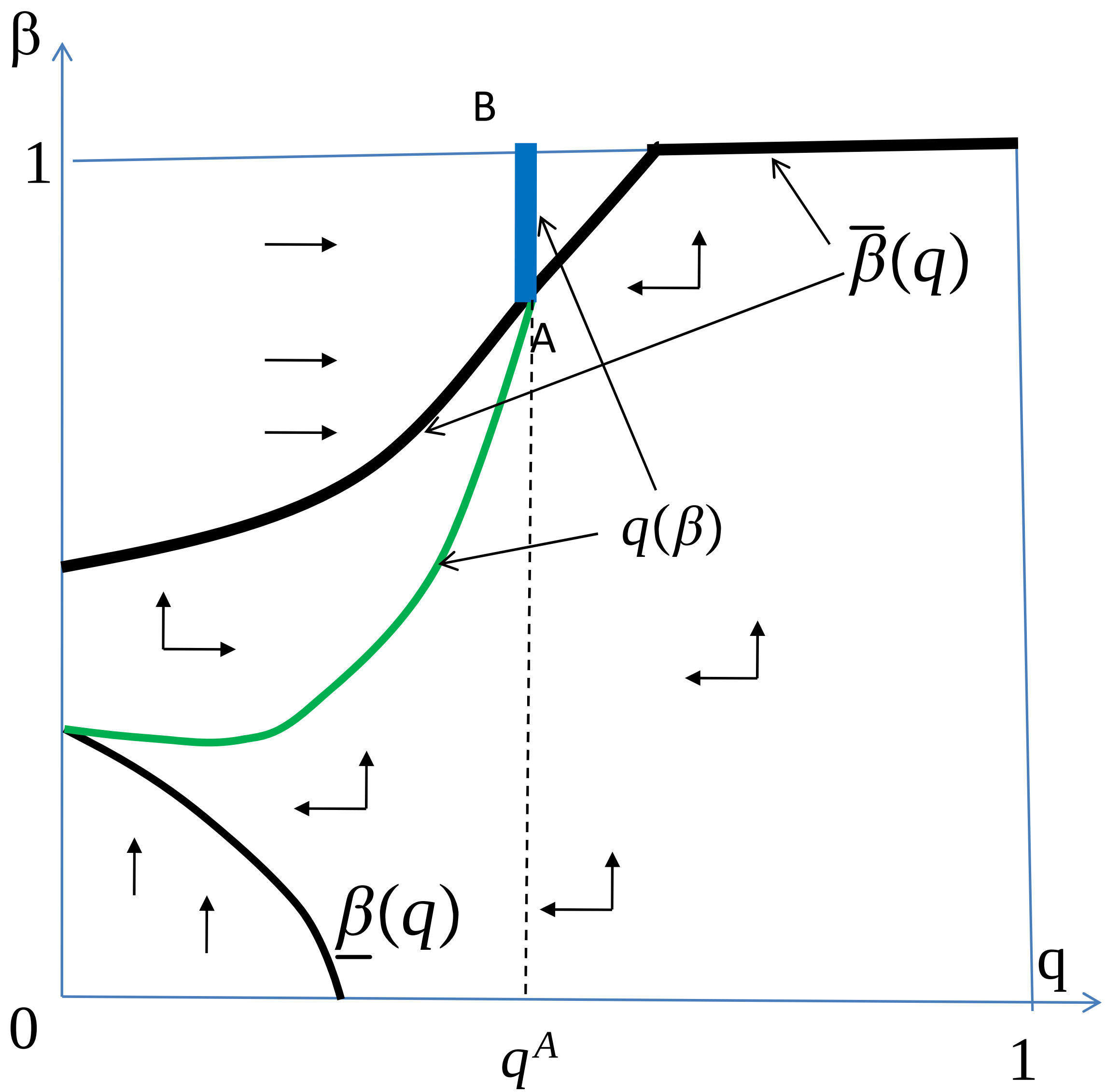


Figure 3a: Elites, workers, and Extractive Institutions
No Provision of Public Good at Stationary State

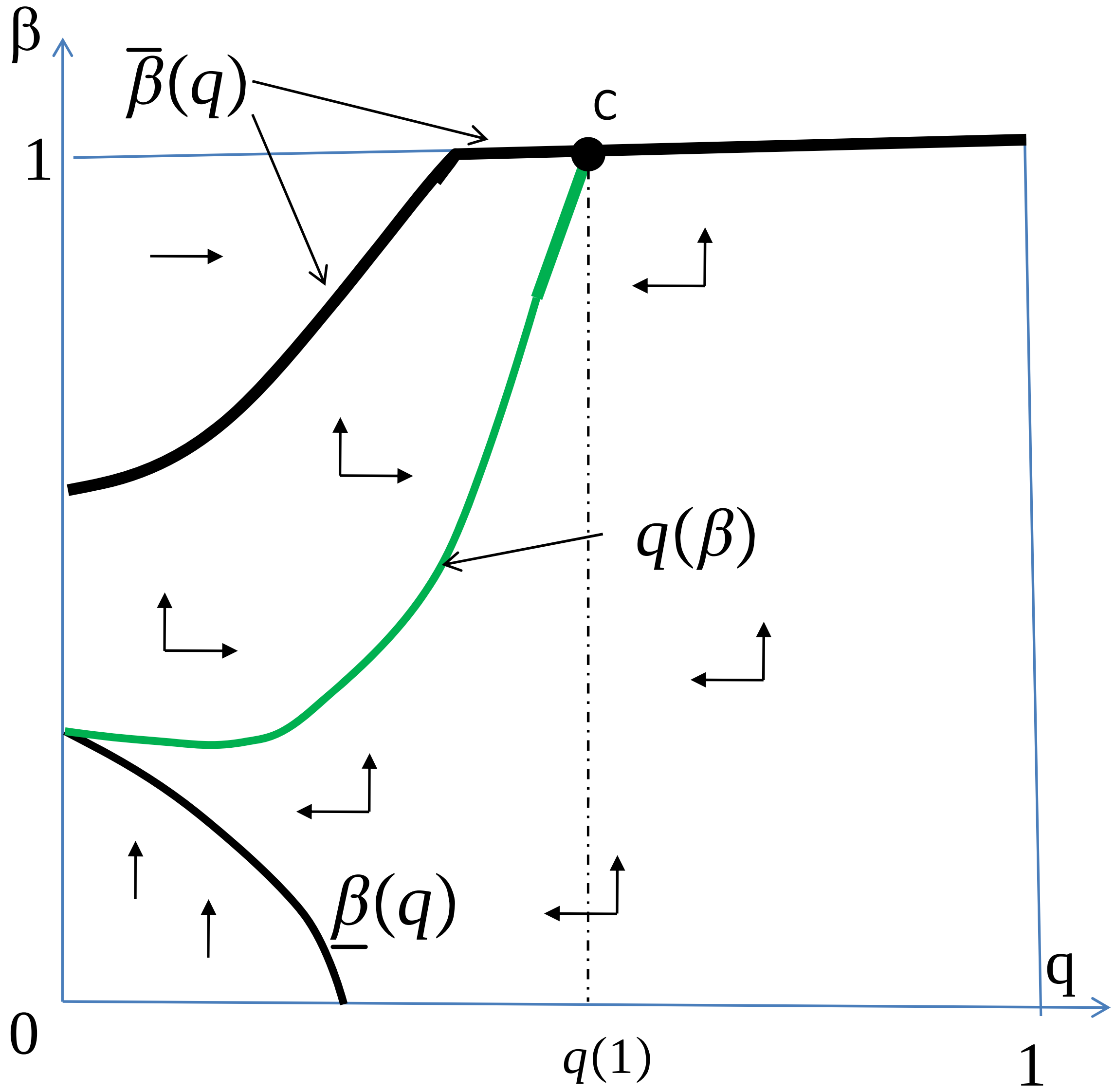


Figure 3b: Elites, Workers, and Extractive Institutions
Public Good Provision at Stationary State

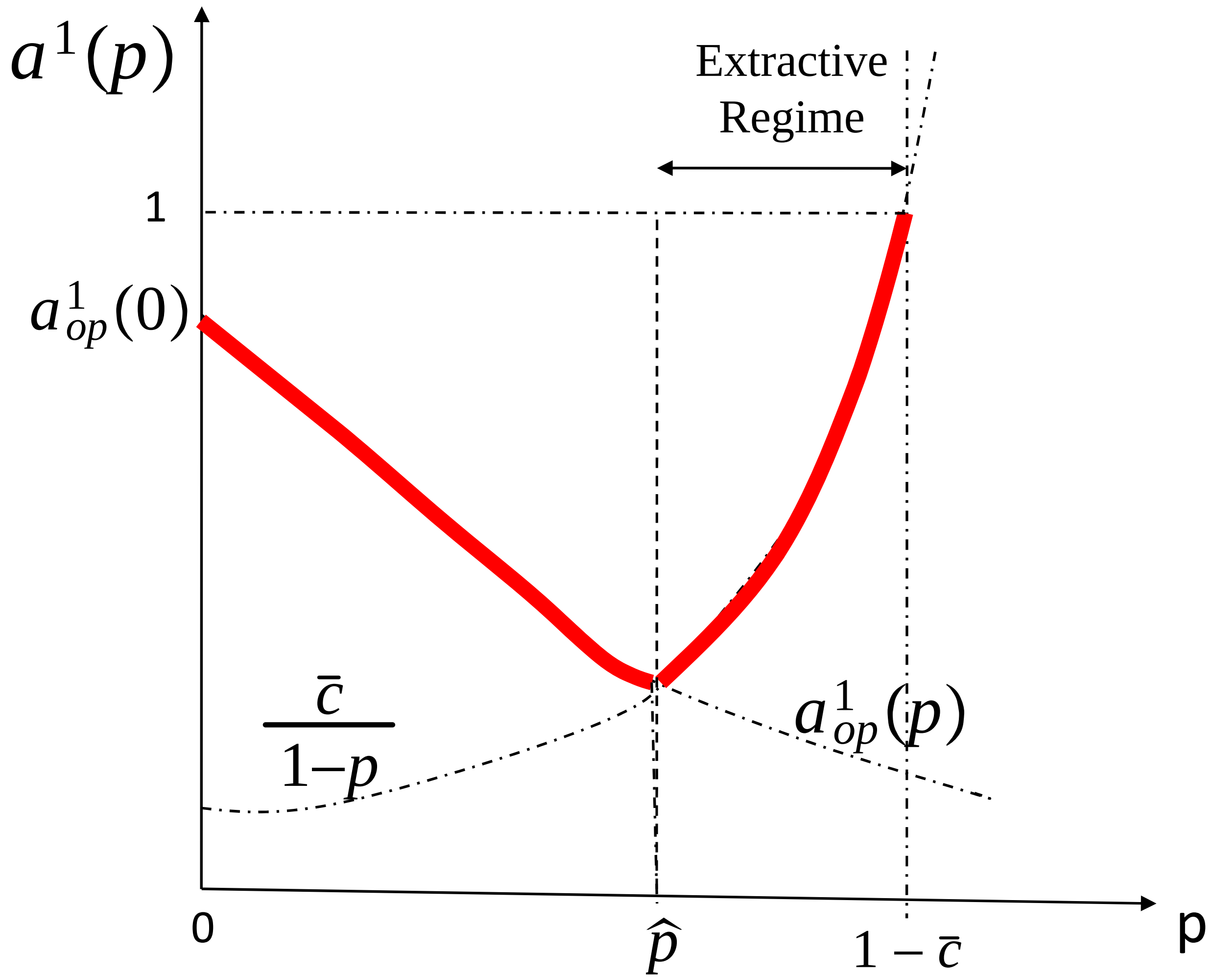


Figure 4: Elite, Workers and Extractive institutions
Optimal Effort of the Mass workers

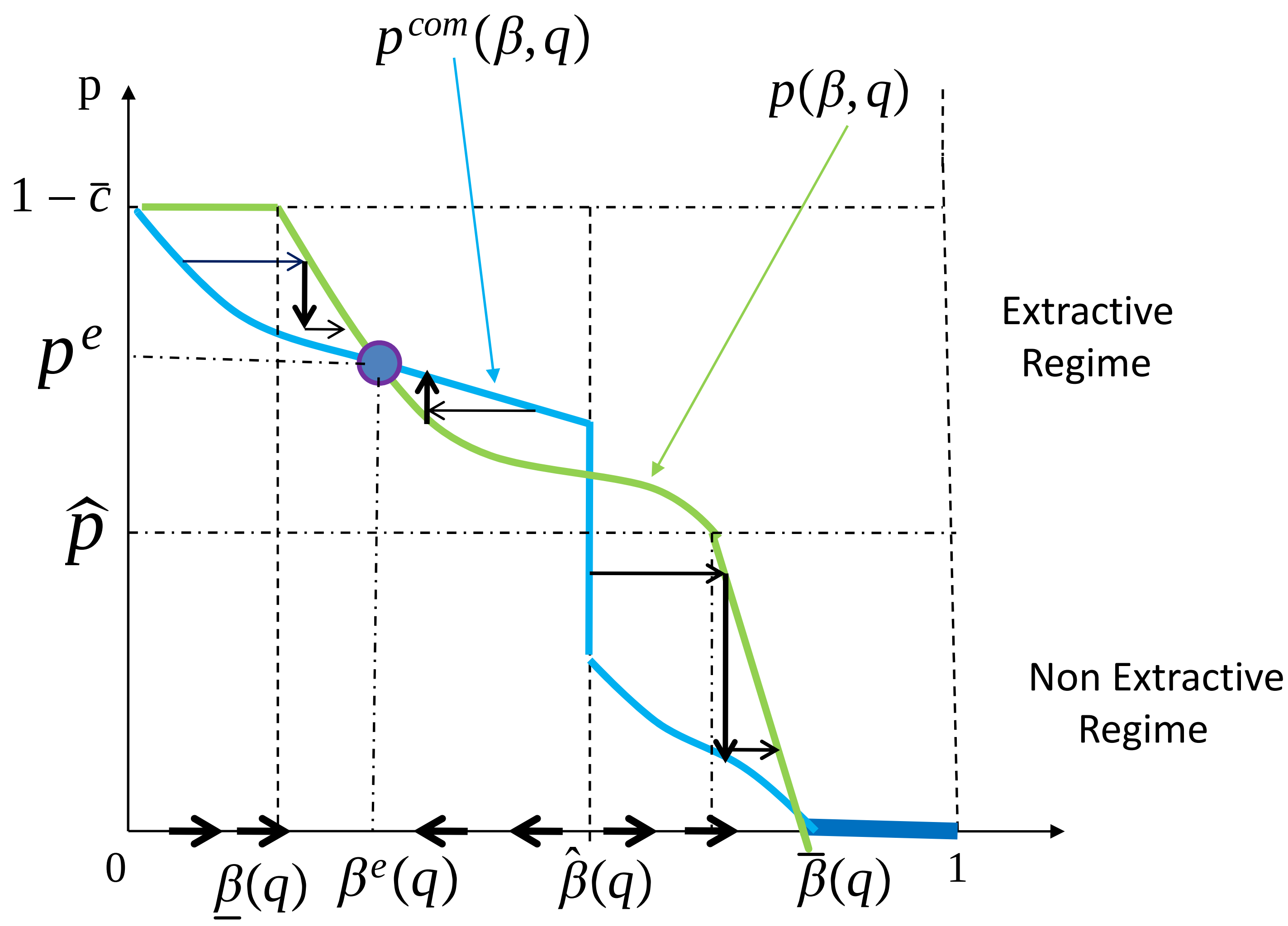


Figure 5 : Elite, Workers and Extractive institutions with Survival Constraint
Equilibrium Policies and Institutional Dynamics

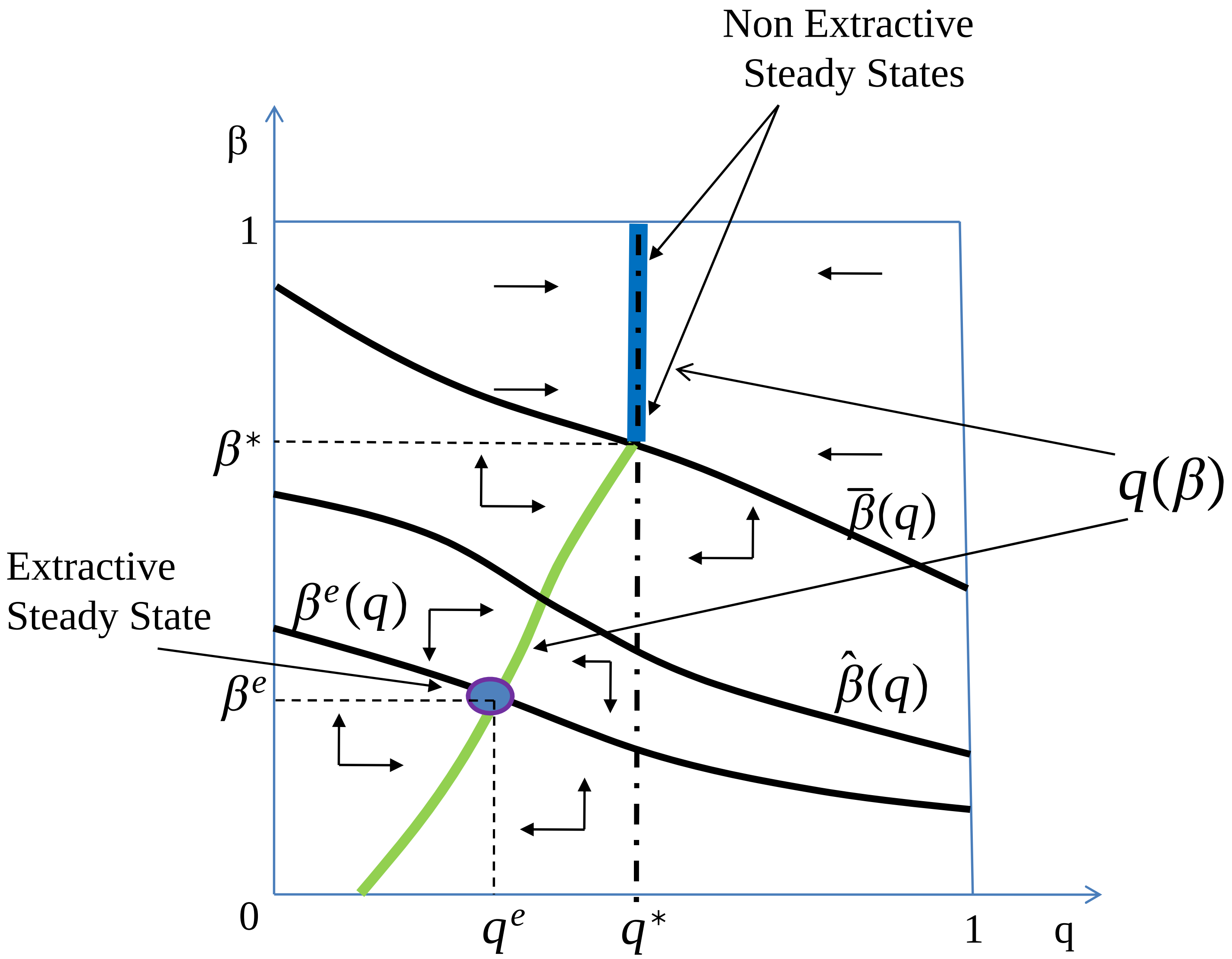


Figure 6: Elite, Workers and Extractive institutions with Survival Constraint
Phase Diagram Coevolution institutions-culture

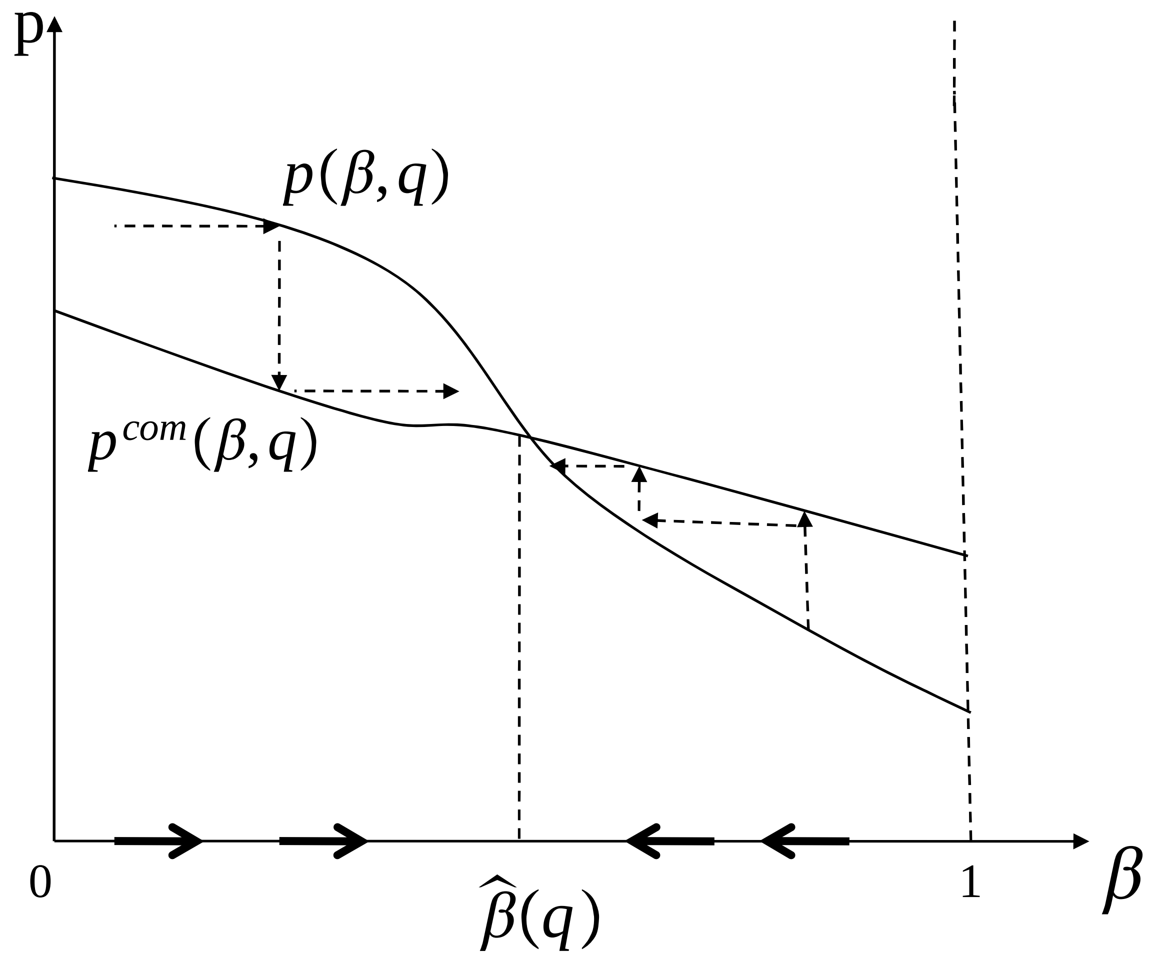
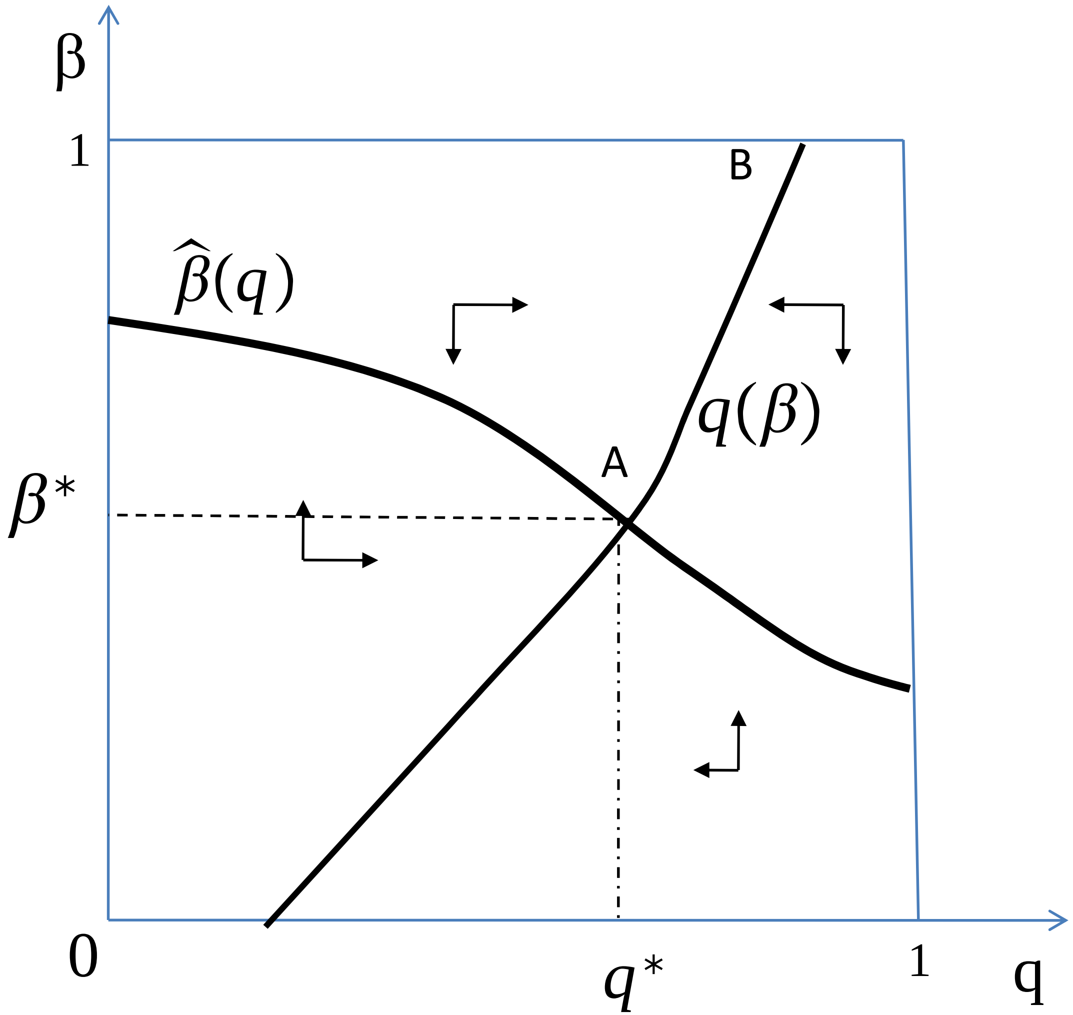
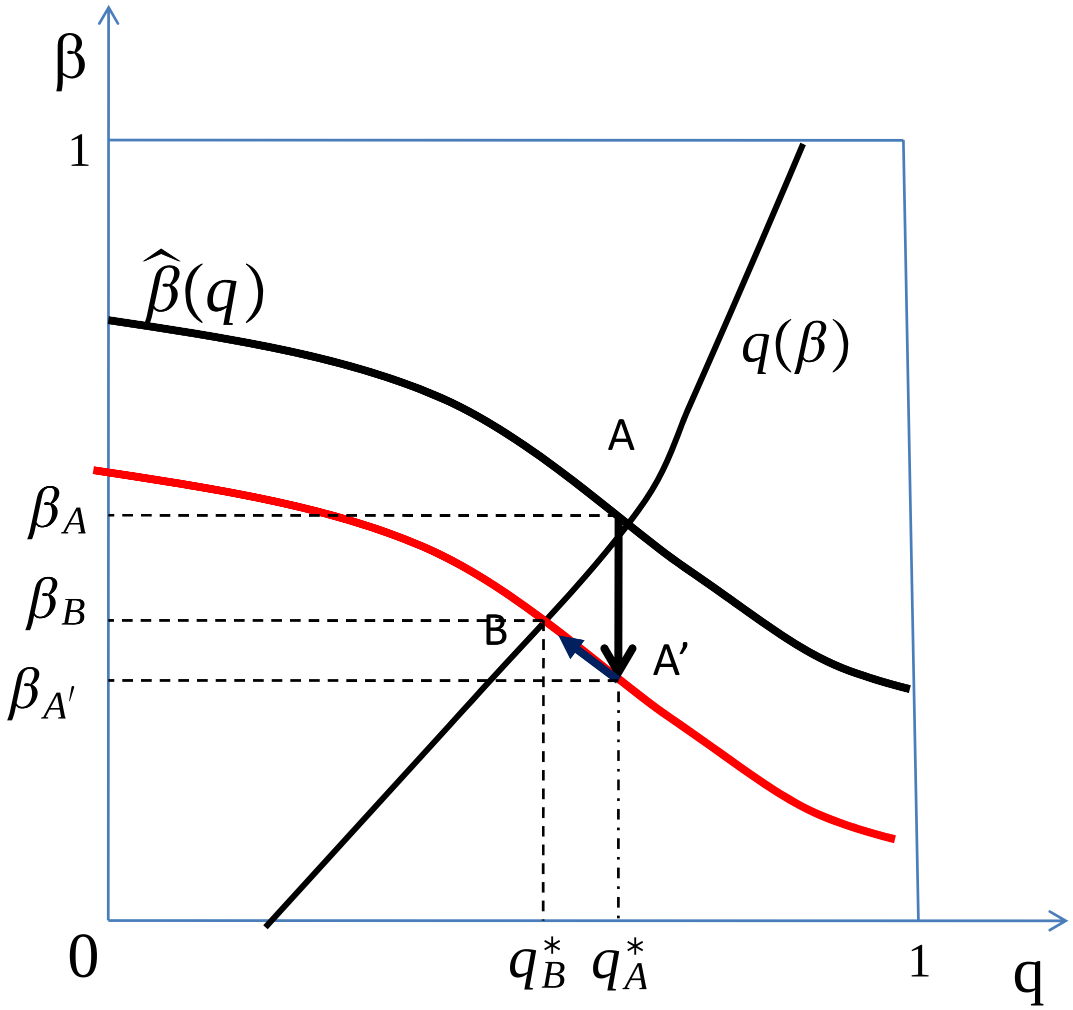


Figure 7 : Civic Culture and Externalities
Institutional Dynamics



Figures 8 : Civic Culture and Externalities
Institutions and Culture are Dynamic Substitutes



Figures 9: Cultural Evolution of Civic Culture
Dynamic substitutes and the Cultural Multiplier

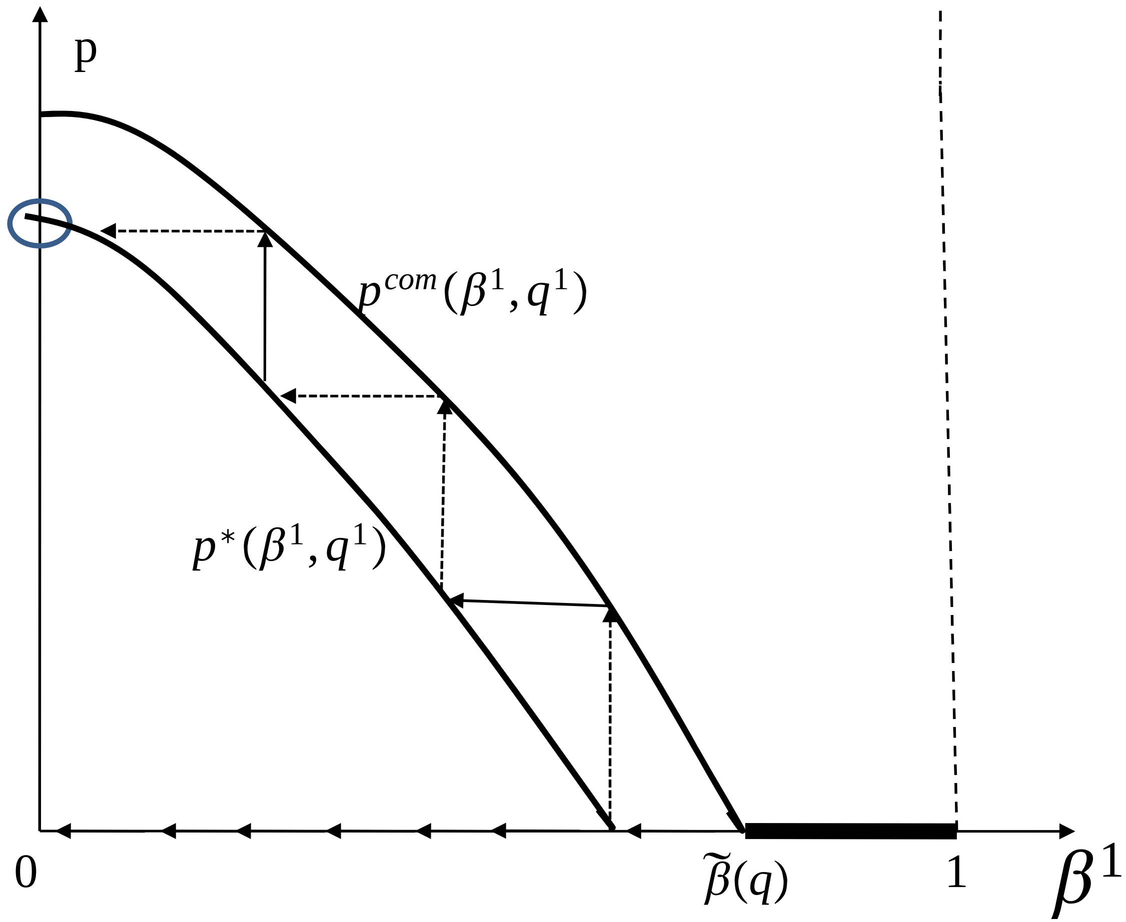
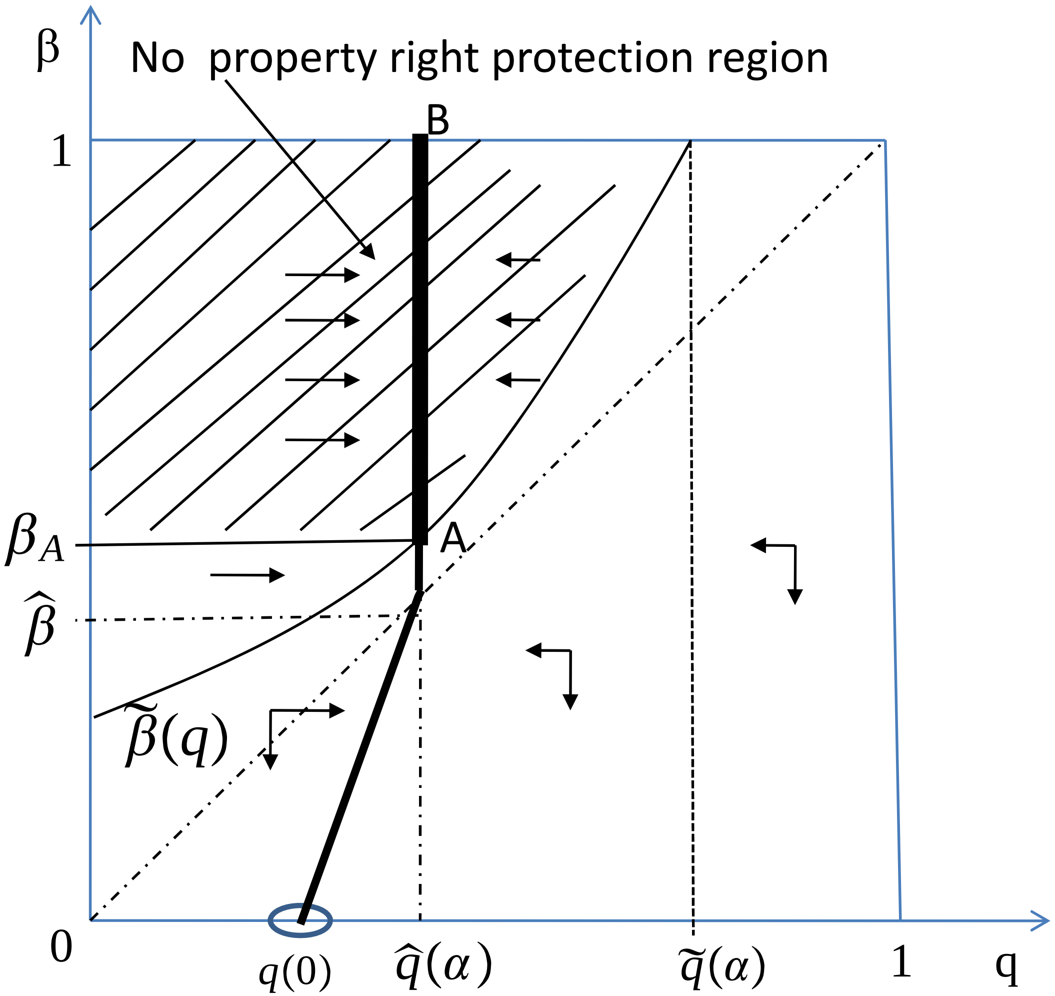
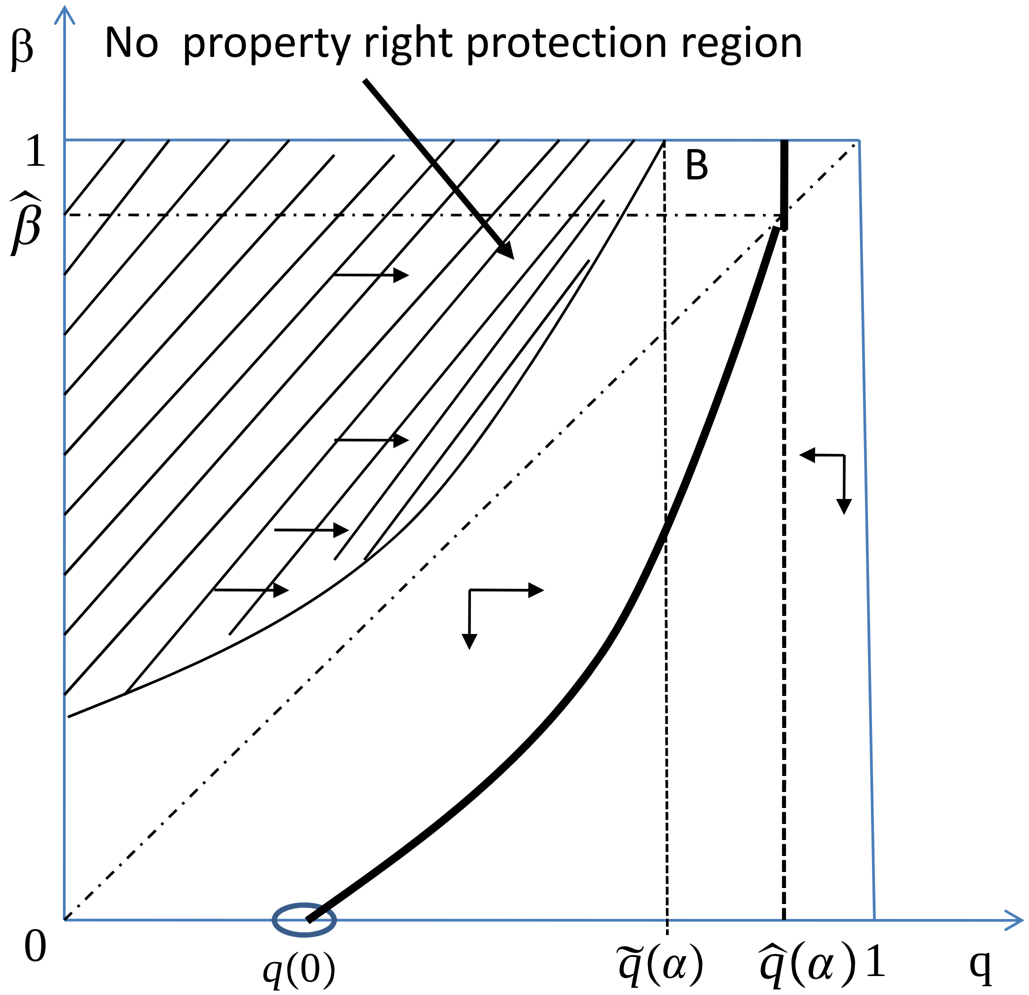


Figure 10: Property Rights and Conflicts
Equilibrium Policies and Institutional Dynamics



Figures 11a): Property Rights and Conflicts
Phase Diagram (α large enough)



Figures 12b: Property rights and conflict
Phase Diagram (α close to α_{min})

Appendix A: Extensions

Non-monotonic $p(\beta, q_{t+1})$**.** Consider the case in which Assumption 7 is not imposed and hence $p(\beta, q_{t+1})$ can be non-monotonic. Then the dynamical system for β^i is characterized by the following implicit difference equation:

$$\beta^i_{t+1} = \begin{cases} \beta \text{ such that } p^{com}(\beta^i_t, q_{t+1}) = p(\beta, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{ll} \arg\max p(\beta, q_{t+1}) & \text{if } p^{com}(\beta^I_t, q_{t+1}) > p(\beta, q_{t+1}),\ \forall 0 \le \beta \le 1 \\ \arg\min p(\beta, q_{t+1}) & \text{if } p^{com}(\beta^I_t, , q_{t+1}) < p(\beta, q_{t+1}),\ \forall 0 \le \beta \le 1 \end{array} \right. & \text{else} \end{cases} \tag{14}$$

In this case it is straightforward to show that the institutional dynamics might be underdetermined, that is, equation 14 might define an implicit map $(\beta^i_t, q_t) \to \beta^i_{t+1}$ which is multi-valued in an open set of the domain. In this case it is easy to construct dynamics of β^i_t which converge to cycles, as it is illustrated in figure 1.

Distinct political and cultural groups. The *societal equilibrium given institutions* β *and cultural distribution* q is a tuple $\{a, p\}$ such that:

$$\begin{array}{l} p \in \arg\max_p \sum_i \beta^i \sum_j q^{ij} u^{ij}\left(a^{ij}, p; a, q\right) \\ a^{ij} \in \arg\max\ u^{ij}\left(a^{ij}, p; a, q\right) \quad i \in I,\ j \in J. \end{array} \tag{15}$$

The *societal commitment equilibrium given institutions* β *and cultural distribution* q is a tuple $\{a^{com}, p^{com}\}$ such that:

$$\begin{array}{rl} \{a^{com}, p^{com}\} & \in \arg\max \sum_i \beta^i \sum_j q^{ij} u^{ij}\left(a^{ij}, p; a, q\right) \\ & s.t. \quad a^{ij} \in \arg\max\ u^{ij}\left(a^{ij}, p; a, q\right), \quad i \in I,\ j \in J \end{array} \tag{16}$$

Restricting to dychotomous groups, that is $I = \{1, 2\}$ and $J = \{a, b\}$ the *societal equilibrium*, the *societal commitment equilibrium*, and the *societal optimum* can be denoted, respectively:

$$[a(\beta, q), p(\beta, q)]\,; \quad [a^{com}(\beta, q), p^{com}(\beta, q)]\,; \quad \left[a^{eff}(\beta, q), p^{eff}(\beta, q)\right]$$

Assumption 6 *Utility functions are sufficiently regular so that*
$a(\beta, q),\ p(\beta, q),\ a^{com}(\beta, q),\ p^{com}(\beta, q)$ *are continuous functions.*

Assumption 7 *Utility functions are sufficiently regular so that* $p(\beta, q)$ *is monotonic in* β*.*

Adding an index t to denote time, institutions evolve as a solution to the following design problem:

$$\max_{\beta_{t+1}} \sum_{i \in I} \beta^i_t \sum_{j \in J} q^{ij}_{t+1} u^{ij}\left(a^{ij}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1})\right) \tag{17}$$

Proposition 9 *Under Assumption 6-7, and given* (q_t, q_{t+1}), *the dynamics of institutions* β_t^i, $i \in I$, *is governed by the following implicit difference equation:*

$$\beta_{t+1}^i = \begin{cases} \beta^i \text{ such that } p^{com}(\beta, q_{t+1}) = p(\beta_t, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{ll} 1 & \text{if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}),\ \forall 0 \leq \beta^i \leq 1 \\ 0 & \text{if } p^{com}(\beta_t, q_{t+1}) < p(\beta, q_{t+1}),\ \forall 0 \leq \beta^i \leq 1 \end{array} \right. & \text{else} \end{cases} \tag{18}$$

It is convenient to define $P(\beta, q) := p^{com}(\beta, q) - p(\beta, q)$.

Proposition 10 *Under Assumption 6-7, for any given* q, *the dynamics of institutions governed by (18) have at least one stationary state. An* interior stationary states β^* *obtains as a solution to* $P(\beta, q) = 0$. *The* boundary stationary state $\beta^i = 1$ *obtains when* $P(\beta, q) \mid_{\beta^i=1} > 0$; *while the* boundary stationary state $\beta^i = 0$ *obtains when* $P(\beta, q) \mid_{\beta^i=0} < 0$.

Proposition 11 *Under Assumption 6-7, for any given* q, *in the continuous time limit, the dynamics governed by (18) satisfies the following properties:*

if $P(\beta, q) > 0$ *for any* $\beta^i \in [0, 1]$, *then* $\beta^i = 1$ *is a globally stable stationary state.*

if $P(\beta, q) < 0$ *for any* $\beta^i \in [0, 1]$, *then* $\beta^i = 0$ *is a globally stable stationary state;*

any boundary stationary state is always locally stable;

if an interior stationary state β^* *exists, it is locally stable if* $\frac{\partial P(\beta^*, q)}{\partial \beta^i} < 0$.

Cultural transmission implies:

$$\begin{aligned} P_t^{i,jj} &= d^{ij} + (1 - d^{ij}) q_t^{ij} \\ P^{i,jj'} &= (1 - d^{ij})(1 - q_t^{ij}) \end{aligned}$$

$$V^{i,jj}(\beta_{t+1}, q_{t+1}) = u^{ij}\left(a^{ij}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1}), q_{t+1}\right) \tag{19}$$

$$V^{i,j\neq j}(\beta_{t+1}, q_{t+1}) = u^{ij}\left(a^{ij'}(\beta_{t+1}, q_{t+1}), p(\beta_{t+1}, q_{t+1}); a(\beta_{t+1}, q_{t+1}), q_{t+1}\right) \tag{20}$$

Let $C(d^{ij})$ denote socialization costs. Direct socialization is then the solution to the following parental socialization problem:

$$\max_{d^{ij} \in [0,1]} -C(d^{ij}) + P_t^{i,jj} V^{i,jj}(\beta_{t+1}, q_{t+1}) + P_t^{i,jj'} V^{i,jj'}(\beta_{t+1}, q_{t+1}), \text{ s. t. } 1) .$$

Calling $\Delta V^{ij}(\beta_{t+1}, q_{t+1}) = V^{i,jj}(\beta_{t+1}, q_{t+1}) - V^{i,j\neq j}(\beta_{t+1}, q_{t+1})$, the *cultural intolerance* of trait j in political group i, it follows that the direct socialization, with some notational abuse, has the form:

$$d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1})) = d^{ij}(\beta, q),\ i \in I,\ j \in J \tag{21}$$

Assumption 8 *Utility and socialization cost functions are sufficiently regular so that* $d^{ij} = d^{ij}(\beta, q)$ *is continuous.*

Proposition 12 *Under Assumption 8, and given* β_{t+1}*, the dynamics of culture* q_t^{ij} *is governed by the following difference equation:*

$$q_{t+1}^{ij} - q_t^{ij} = q_t^{ij}(1 - q_t^{ij}) \left(d^{ij} - d^{ij'} \right). \tag{22}$$

evaluated at $d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1}))$ *satisfying (21).*

It is convenient to define $D^{ij}(\beta, q) := d^{ij}(\beta, q) - d^{ij'}(\beta, q)$.

Proposition 13 *Under Assumption 8, for any given* β*, the dynamics of institutions governed by (22) have at least the two boundary stationary states,* $q^{ij} = 0$ *and* $q^{ij} = 1$*. An* interior stationary states $0 < q^{ij*} < 1$ *obtains as a solution to* $D(\beta, q) = 0$.

Proposition 14 *Under Assumption 8, for any given* β*, in the continuous time limit, the dynamics governed by (22) satisfies the following properties:*

if $D^{ij}(\beta, q) > 0$ *for any* $q^{ij} \in [0, 1]$*, then* q_t^{ij} *converges to* $q^{ij} = 1$ *from any initial condition* $q_0^{ij} > 0$*;*

if $D^{ij}(\beta, q) < 0$ *for any* $q^{ij} \in [0, 1]$*, then* q_t^{ij} *converges to* $q^{ij} = 0$ *from any initial condition* $q_0^{ij} < 1$*;*

if $D^{ij}(\beta, 1) > 0$*, then* $q^{ij} = 1$ *is locally stable ;*

if $D(\beta, 0) < 0$*, then* $q^{ij} = 0$ *is locally stable;*

if an interior stationary state q^{ij*} *exists, and* $\frac{\partial D^{ij}(\beta, q^{ij*})}{\partial q^{ij}} < 0$*, it is locally stable.*

If we impose

Assumption 9 *Socialization costs are quadratic:*

$$C(d^{ij}) = \frac{1}{2} \left(d^{ij} \right)^2.$$

we obtain

Corollary 2 *Under Assumption 9,*

$$D^{ij}(\beta, q) = \Delta V^{ij}(\beta, q) q^{ij'} - \Delta V^{ij'}(\beta, q) q^{ij},$$

and hence interior steady states are characterized by solutions to:

$$\frac{\Delta V^{ij}(\beta, q)}{\Delta V^{ij'}(\beta, 1-q)} = \frac{q^{ij}}{q^{ij'}} \tag{23}$$

Under Assumptions 6-8, the joint dynamics of institutions and culture is governed by the system (18,22), which we report here for convenience:

$$\begin{aligned}
\beta^i_{t+1} &= \begin{cases} \beta^i \text{ such that } p^{com}(\beta, q_{t+1}) - p(\beta_t, q_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{ll} 1 & \text{if } p^{com}(\beta_t, q_{t+1}) > p(\beta, q_{t+1}), \ \forall 0 \leq \beta^i \leq 1 \\ 0 & \text{if } p^{com}(\beta_t, , q_{t+1}) < p(\beta, q_{t+1}), \ \forall 0 \leq \beta^i \leq 1 \end{array} \right. & \text{else} \end{cases} \\
q^{ij}_{t+1} - q^{ij}_t &= q^{ij}(1 - q^{ij}_t) \left(d^{ij} - d^{ij'} \right), \text{ with } d^{ij} = d^{ij}(q_t, \Delta V^{ij}(\beta_{t+1}, q_{t+1})).
\end{aligned}$$

Proposition 15 *Under Assumptions 6-8 the dynamical system (18,22) has at least one stationary state. Furthermore, if both the institutional and the cultural dynamics display an interior stationary state, respectively, for all $0 \leq q^{ij} \leq$ and all $0 \leq \beta \leq 1$, then the dynamical system (18,22) has at least one interior stationary state.*

Appendix B: Results on the Dynamical system

[changed the definition in the text - now $P(.) = p^{com}(.) - p(.)$ - all signs in this proof must be reversed!!!!!]

In this Appendix we study in some detail the dynamics of our economy. We shall restrict to the case in which $j = i$, as introduced in footnote **??**. Furthermore, we keep adopting the convention that, when an apex i is omitted, it refers to $i = 1$.

We study then the dynamics of $(\beta_t, q_t) \in [0, 1]^2$. We shall study the dynamical system in continuous time, but it is simpler to describe it in discrete time, as we do in the text. The fundamental dynamics equation, as reported in the text as equations (18,22), are the following:

$$\beta^i_{t+1} = \begin{cases} \beta \text{ such that } p^{com}(\beta^i_T, q^i_{t+1}) = p(\beta, q^i_{t+1}) & \text{if it exists,} \\ \left[\begin{array}{ll} 1 & \text{if } p^{com}(\beta^i_t, q^i_{t+1}) > p(\beta, q^i_{t+1}), \ \forall 0 \leq \beta \leq 1 \\ 0 & \text{if } p^{com}(\beta^i_t, , q^i_{t+1}) < p(\beta, , q^i_{t+1}), \ \forall 0 \leq \beta \leq 1 \end{array} \right. & \text{else} \end{cases} \tag{β}$$

$$q^i_{t+1} - q^i_t = q^i(1 - q^i_t) \left(d^i - d^j \right), \text{ with } d^i = d(q^i_t, \Delta V^i(\beta^i_{t+1}, q^i_{t+1})). \tag{q}$$

We impose Assumptions 6-8 and we further assume for regularity that all maps, $p(\beta, q)$, $p^{com}(\beta, q)$, $\Delta V^i(\beta, q)$, $\Delta V^j(\beta, q)$ are smooth.[60]

[60] As in the text, we continue to drop the index i when convenient, under the convention that $(\beta, q) = (\beta^1, q^1)$.

The dynamics of β_t given q. Let $f : [0,1]^2 \to [0,1]$ denote the map which governs the dynamics of β_{t+1}; that is, which satisfies equation (β) and, in the continuous time limit:

$$\dot{\beta}_t = f(\beta_t, q_t).$$

Lemma A. 1 *Under our assumptions, $f : [0,1]^2 \to [0,1]$ is a continuous function in $(\beta_t, q_t) \in [0,1]^2$.*

Proof. Consider equation (β). First of all note that when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is not satisfied for any β_{t+1}, for some (q_t, q_{t+1}), the assumption that $p(\beta, q)$ is monotonic implies that β_{t+1} is $= 0$ or $= 1$, depending on the sign of $p^{com}(\beta_{t+1}, q_{t+1}) - p(\beta_t, q_t)$. In the continuous time limit $q_{t+1} = q_t$ and hence, in this case, trivially, f maps continuously $(\beta_t, q_t) \in [0,1]^2$ into $\{0\}$.

Consider equation (β), again. We show that β_{t+1} is a continuous function of β_t, q_t, q_{t+1} when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is satisfied. To this end note that the assumed monotonicity in β of $p(\beta, q)$ implies that, when $p(\beta_{t+1}, q_{t+1}) = p^{com}(\beta_t, q_t)$ is satisfied, we can write $\beta_{t+1} = p^{-1}(p, q_t, q_{t+1})$ and hence $\beta_{t+1} = p^{-1}(p^{com}(\beta_t, q_t), q_t, q_{t+1})$, a continuous function. Again, in the continuous time limit $q_{t+1} = q_t$ and hence we can construct a continuous function $f : [0,1]^2 \to \mathbb{R}$ such that $\dot{\beta}_t = f(\beta_t, q_t)$.

Finally, it is straightforward to see that as $p^{com}(\beta_{t+1}, q_{t+1}) - p(\beta_t, q_t)$ crosses 0 $\beta_{t+1} = p^{-1}(p^{com}(\beta_t, q_t), q_t, q_{t+1})$ converges continuously to 0 or 1 depending on the direction of the crossing so as to preserve continuity. ■

Let the $\beta : [0,1] \to [0,1]$ map $q \in [0,1]$ into the stationary states of f; that is, $\beta : [0,1] \to [0,1]$ satisfies

$$0 = f(\beta, q), \text{ for any } \beta \in \beta(q)$$

Lemma A. 2 *Under our assumptions, the map $\beta : [0,1] \to [0,1]$ is an non empty and compact valued upper-hemi-continuous correspondence with connected components.*

Proof. The proof is a direct consequence of the continuity of f proved in the Lemma A.1. ■

Let $P(\beta, q) := p^{com}(\beta, q) - p(\beta, q)$. We consider only the regular case in which $P(\beta, q) \neq 0$ at the vertices of $[0,1]^2$, leaving the simple but tedious analysis of the singular cases to the reader. Also, we say that q is a regular point of $\beta \in \beta(q)$ if any stationary stationary state $\beta \in \beta(q)$ satisfies that property that $\frac{\partial P(\beta,q)}{\partial \beta} \neq 0$; that is if $p(\beta, q)$ and $p^{com}(\beta, q)$ intersect transversally. The characterization of $\beta : [0,1] \to [0,1]$ depends crucially on the topological properties of the zeros of $P(\beta, q)$. Let $\pi : [0,1] \to [0,1]$ map q into the stationary states β such that $P(\beta, q) = 0$; that is, the map π satisfies $P(\pi(q), q) = 0$.

Proposition A. 1 *The dynamics of β_t as a function of $q \in [0,1]$ has the following properties,*

1. *$P(0,q) > 0$, $P(1,q) < 0$, for any $q \in [0,1]$, and $p(\beta,q)$ is increasing; or $P(0,q) < 0$, $P(1,q) > 0$, for any $q \in [0,1]$, and $p(\beta,q)$ is decreasing. For any given regular $q \in [0,1]$ there exist an odd number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0,1$ are also stationary states for given $q \in [0,1]$. The stability properties of the regular stationary states alternate with the smallest and the larger being always locally stable; the boundaries $\beta = 0,1$ are locally unstable for all $q \in [0,1]$.*

2. *$P(0,q) < 0$, $P(1,q) > 0$, for any $q \in [0,1]$, and $p(\beta,q)$ is increasing; or $P(0,q) > 0$, $P(1,q) < 0$, for any $q \in [0,1]$, and $p(\beta,q)$ is decreasing. For any given $q \in [0,1]$ there exist an odd number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0,1$ are also stationary states for given $q \in [0,1]$. The stability properties of the regular stationary states alternate with the smallest and the larger being always locally unstable; the boundaries $\beta = 0,1$ are locally stable.*

3. *$P(0,q) < 0$, $P(1,q) < 0$, for any $q \in [0,1]$. For any given $q \in [0,1]$ there exist either none or an even number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 0$ is also a stationary state for given $q \in [0,1]$. The stability properties of the regular stationary states alternate with the smallest always locally unstable; the boundary $\beta = 0$ is locally stable.*

4. *$P(0,q) > 0$, $P(1,q) > 0$, for any $q \in [0,1]$. For any given $q \in [0,1]$ there exist either none or an even number of regular stationary states $\beta \in \pi(q)$; furthermore $\beta = 1$ is also a stationary state for given $q \in [0,1]$. The stability properties of the regular stationary states alternate with the smallest always locally stable; the boundary $\beta = 1$ is locally stable.*

5. *$P(0,q)$ and/or $P(1,q)$ change sign with $q \in [0,1]$. The characterization obtained above then can be repeated for each sub-interval of $[0,1]$ in which the Brouwer degree of the manifold $\pi(q)$ is invariant (see the proof). We leave the tedious cathegorization of all possible cases to the reader.*

- **Continuous time joint dynamics between institutions and culture.**

Let us define first the following function $\widetilde{\beta}(\beta, q)$:

$$\widetilde{\beta}(\beta,q) = \begin{cases} \widetilde{\beta} \text{ such that } p^{com}(\beta,q) - p(\widetilde{\beta},q) = 0 & \text{if it exists,} \\ \left[\begin{array}{ll} 1 & \text{if } p^{com}(\beta,q) > p(\beta',q),\ \forall 0 \leq \beta' \leq 1 \\ 0 & \text{if } p^{com}(\beta,q) < p(\beta',q),\ \forall 0 \leq \beta' \leq 1 \end{array} \right. & \text{else} \end{cases} \tag{24}$$

and consider the continuous time limit of the system, where the change in institutional set-up and cultural composition between time t and $t + dt$ are λdt and μdt, for $dt \to 0$.[61]

$$\begin{aligned} \dot{\beta}_t &= \lambda \left[\widetilde{\beta}(\beta_t, q_t) - \beta_t \right] \quad \text{with } \widetilde{\beta}(\beta_t, q_t) \text{ defined in (24)} \\ \dot{q}_t &= \mu q_t (1 - q_t) \Theta(\beta_t, q_t) \quad \text{with } \Theta(\beta_t, q_t) = d^1(q_t, \Delta V^1(\beta_t, q_t)) - d^2(q_t, \Delta V^2(\beta_t, q_t)) \end{aligned} \tag{25}$$

given the initial conditions (β_0, q_0).

As usual, we denote the partial derivative of a variable x on another variable y as $\partial x/\partial y = x_y$.

The linearized local dynamics around the interior steady state (β^*, q^*) can then easily be obtained by

$$\begin{pmatrix} \dot{\beta} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} \lambda \left[\frac{p^{com}_{\beta} - p_{\beta}}{p_{\beta}} \right]_{(\beta^*, q^*)} & \lambda \left[\frac{p^{com}_{q} - p_{q}}{p_{\beta}} \right]_{(\beta^*, q^*)} \\ -\mu G q^*(1 - q^*) \cdot \widehat{q}_p \cdot p_{\beta} & \mu G q^*(1 - q^*) \left[1 - \widehat{q}_p \cdot p_q \right] \end{pmatrix} \begin{pmatrix} \beta \\ q \end{pmatrix} \tag{26}$$

where $G = -(\Delta V^1(p(\beta^*, q^*)) + \Delta V^2(p(\beta^*, q^*)) < 0$ and where

The local stability of the interior steady state (β^*, q^*) of (56) is obtained under the standard Hessian conditions:

$$\begin{aligned} \left[\frac{p_{\beta} - p^{com}_{\beta}}{p_{\beta}} \right]_{(\beta^*, q^*)} &> 0 \\ 1 - \left[p_q \cdot \widehat{q}_p \right]_{(\beta^*, q^*)} &> 0 \\ \left[(1 - p_q \cdot \widehat{q}_p) \cdot \left[\frac{p_{\beta} - p^{com}_{\beta}}{p_{\beta}} \right] + \widehat{q}_p \cdot \left(p_q - p^{com}_q \right) \right]_{(\beta^*, q^*)} &> 0 \end{aligned} \tag{27}$$

- **Dynamic Complementarity and Substitution between institutions and culture**

Let us show the following lemma:

[61] As is well known, discrete time dynamics may generate complex dynamic behaviors that are difficult to characterize and go beyond the points we want to emphasize about the co-evolution between culture and institutions.

Lemma A. 3 *Under Assumption 5, institutional and cultural* dynamics are complementary *at a locally stable interior steady state* (β^*, q^*) *if*

$$\frac{dP(\beta^*, q^*)}{dq} \text{ has the same sign as } \left[\frac{d\left(\frac{\Delta V^1(p)}{\Delta V^2(p)}\right)}{dp}\right]_{p(\beta^*, q^*)}; \tag{28}$$

they are instead substitute *if the signs are opposite.*

Proof: As said in the main text, we say that institutional and cultural dynamics are complementary at (β^*, q^*) when

$$\frac{d\beta(q)}{dq} \text{ and } \frac{dq(\beta)}{d\beta} \text{ have the same sign.} \tag{29}$$

Simple differentiation provides that

$$\begin{aligned} \frac{d\beta(q)}{dq} &= -\frac{\left(p_q - p_q^{com}\right)}{p_\beta - p_\beta^{com}} \\ \frac{dq(\beta)}{d\beta} &= \frac{\widehat{q}_p p_\beta}{1 - p_q \cdot \widehat{q}_p} \end{aligned}$$

thus condition (29) is equivalent to

$$\frac{d\beta(q)}{dq} \cdot \frac{dq(\beta)}{d\beta} \geq 0$$

or

$$-\frac{\left(p_q - p_q^{com}\right)}{p_\beta - p_\beta^{com}} \cdot \frac{\widehat{q}_p p_\beta}{1 - p_q \cdot \widehat{q}_p} \geq 0$$

Given the Hessian conditions (27) for local stability, at an interior locally stable steady state (β^*, q^*), this condition is equivalent to

$$\left[\left(p_q^{com} - p_q\right) \cdot \widehat{q}_p\right]_{(\beta^*, q^*)} \geq 0$$

Recalling that the cultural manifold $q(\beta)$ is obtained from

$$\frac{\Delta V^1(p)}{\Delta V^2(p)} = \frac{q}{1-q} \text{ and } p = p(\beta, q)$$

and that $P(\beta, q) := p^{com}(\beta, q) - p(\beta, q)$, differentiation provides immediately

$$\left[\left(p_q^{com} - p_q\right) \cdot \widehat{q}_p\right]_{(\beta^*, q^*)} = \left[P_q(\beta, q) \cdot \left[\frac{d\left(\frac{\Delta V^1(p)}{\Delta V^2(p)}\right)}{dp}\right]_{p(\beta, q)} (1-q)^2\right]_{(\beta^*, q^*)}$$

and thus institutional and cultural *dynamics are complementary* at a locally stable interior steady state (β^*, q^*) when P_q and $\frac{d\left(\frac{\Delta V^1(p)}{\Delta V^2(p)}\right)}{dp}$ have the same sign at (β^*, q^*). Obviously they are dynamic substitute otherwise. **QED**

- **Oscillations and Cycles**

- Proof of proposition 9: Consider first the situation of an interior steady state (β^*, q^*) of (25) that is locally stable.

As we know the local stability conditions ensure that the trace $T < 0$ and that the determinant $\Delta > 0$. Standard considerations provide moreover that one has dampened oscillations (a stable spiral steady state equilibrium) in cultural and institutional change when $T^2 < 4\Delta$. This last condition writes as:

$$\left[\lambda \frac{p_\beta^{com} - p_\beta}{p_\beta} + \mu G^* q^*(1-q^*)\left[1 - \widehat{q}_p \cdot p_q\right]\right]^2 < 4\lambda\mu G^* q^*(1-q^*) \left[\begin{array}{c} \frac{p_\beta^{com} - p_\beta}{p_\beta}\left[1 - \widehat{q}_p \cdot p_q\right] \\ + \widehat{q}_p \cdot (p_q^{com} - p_q) \end{array}\right]$$

or after manipulations

$$\left[\lambda \frac{p_\beta^{com} - p_\beta}{p_\beta} - \mu G^* q^*(1-q^*)\left[1 - \widehat{q}_p \cdot p_q\right]\right]^2 < 4\lambda\mu G^* q^*(1-q^*)\widehat{q}_p \cdot (p_q^{com} - p_q)$$

with $G^* = -(\Delta V^1(p^*) + \Delta V^2(p^*)) < 0$. Using (56) and substituting provides that $G^* q^*(1 - q^*) = -\frac{\Delta V^1(p^*)\Delta V^2(p^*)}{\Delta V^1(p^*) + \Delta V^2(p^*)}$

The condition for dampened oscillations then rewrites as:

$$\left[\lambda \frac{p_\beta^{com} - p_\beta}{p_\beta} + \mu \frac{\Delta V^{1*}\Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}}\left[1 - \widehat{q}_p \cdot p_q\right]\right]^2 < -4\lambda\mu \frac{\Delta V^{1*}\Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}} \cdot \widehat{q}_p \cdot (p_q^{com} - p_q) \tag{30}$$

Given that the left hand side of this inequality is positive, it follows that there are no dampening oscillations in cultural and institutional change when institutions and culture are dynamic complements (ie. $\left[\widehat{q}_p \cdot \left(p_q^{com} - p_q\right)\right]_{(\beta^*, q^*)} > 0$) at the interior locally stable steady state (β^*, q^*).

- Condition for dampening oscillations: Conversely we have dampening oscillations when (30) is satisfied. In the case of dynamic substituability $\left|\widehat{q}_p \cdot (p_q^{com} - p_q)\right| = -\widehat{q}_p \cdot (p_q^{com} - p_q)$ and thus there are non monotonic dynamics in culture and institutions when

$$\left|\widehat{q}_p \cdot (p_q^{com} - p_q)\right| > \frac{\Delta V^{1*} + \Delta V^{2*}}{\Delta V^{1*}\Delta V^{2*}} \frac{1}{4\lambda\mu} \left[\lambda \frac{p_\beta^{com} - p_\beta}{p_\beta} + \mu \frac{\Delta V^{1*}\Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}}\left[1 - \widehat{q}_p \cdot p_q\right]\right]^2$$

or

$$\frac{\lambda}{\mu}\left[\frac{p_\beta^{com} - p_\beta}{p_\beta} + \frac{\mu}{\lambda}\frac{\Delta V^{1*}\Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}}\left[1 - \widehat{q}_p \cdot p_q\right]\right]^2 < 4\frac{\Delta V^{1*}\Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}}\left|\widehat{q}_p \cdot (p_q^{com} - p_q)\right| \tag{31}$$

Using the local stability conditions for the Hessien at (β^*, q^*), we can pose

$$\begin{aligned} \frac{p_\beta^{com} - p_\beta}{p_\beta} &= -a < 0 \\ \frac{\Delta V^{1*} \Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}} \left[1 - \widehat{q}_p \cdot p_q\right] &= b > 0 \\ 4 \frac{\Delta V^{1*} \Delta V^{2*}}{\Delta V^{1*} + \Delta V^{2*}} \left|\widehat{q}_p \cdot (p_q^{com} - p_q)\right| &= M > 0 \end{aligned}$$

and denoting $x = \mu/\lambda$ the relative rate of change between culture and institutions, condition (31) rewrites as

$$(-a + bx)^2 < Mx \tag{32}$$

Simple examination of this condition reveals that (32) is satisfied when $x \in (x_- \,; x_+)$ with

$$x_\pm = \frac{(2ab + M) \pm \sqrt{(2ab + M)^2 - 4(ab)^2}}{2b^2} > 0$$

Hence we get non monotonic dynamics of institutions and culture around the locally stable steady state (β^*, q^*) when institutions and culture are dynamic substitutes and the relative rate of change between culture and institutions is neither too low, neither too high. **QED**

Proof of proposition 10: Assume the "partial" stability of the steady state, in the sense that the steady state institution β^* is locally stable when culture remains constant at $q = q^*$ along the institutional dynamics, and conversely the cultural steady state $q = q^*$ is locally stable when the institutional context remains constant along the cultural dynamics. Formally this implies that:

$$\begin{aligned} \left[\frac{p_\beta^{com} - p_\beta}{p_\beta}\right]_{(\beta^*, q^*)} &< 0 \\ 1 - \left[p_q \cdot \widehat{q}_p\right]_{(\beta^*, q^*)} &> 0 \end{aligned} \tag{33}$$

Suppose that conditions (33) are satisfied at an interior steady state (β^*, q^*). Then with enough regularity of the policy functions p^{com} and p, there is a connected neighborhood of (β^*, q^*) such that the trace $T = \lambda \left[\frac{p_\beta^{com} - p_\beta}{p_\beta}\right] - \mu G q^*(1 - q^*) \left[1 - \widehat{q}_p \cdot p_q\right]$does not change sign on that domain. The Bendixson Negative Criterion precludes then the existence of local periodic orbits or limit cycles around (β^*, q^*) in that domain.

Note that when (33) are globally satisfied for all $(\beta, q) \in [0,1] \times [0,1]$,.it is not possible to get globally periodic orbits and limit cycles in institutional and cultural evolution. Indeed given that in the simple connected domain $D = [0,1] \times [0,1]$.the sign of the trace $T = \lambda \left[\frac{p_\beta^{com} - p_\beta}{p_\beta}\right] + \mu G q^*(1 - q^*) \left[1 - \widehat{q}_p \cdot p_q\right]$ is always strictly negative, the Bendixson Negative Criterion again precludes the existence of periodic orbits of (25) in this domain. **QED**

- **Cultural Multiplier**

- Proof of proposition 11:

We assume condition (**??**) holds which can be restated compactly as:

$$p_\beta = \frac{\partial p(\beta^*, q^*, \gamma)}{\partial \beta} > 0,\ p_\gamma^{com} - p_\gamma = \frac{\partial P(\beta^*, q^*, \gamma)}{\partial \gamma} > 0 \tag{34}$$

The comparative statics on (β^*, q^*) on the parameter are then easily obtained by differentiation of (56). Tedious computations provide

$$\begin{aligned}
\frac{d\beta^*}{d\gamma} &= \frac{\left(p_\gamma^{com} - p_\gamma\right) + \left(p_q^{com} - p_q\right)\widehat{q}_p \frac{p_\gamma}{1-p_q\widehat{q}_p}}{\left(p_\beta - p_\beta^{com}\right) - \left(p_q^{com} - p_q\right)\widehat{q}_p \frac{p_\beta}{1-p_q\widehat{q}_p}} \\
\frac{dq^*}{d\gamma} &= \frac{\widehat{q}_p p_\beta}{1 - p_q\widehat{q}_p} \cdot \frac{\left(p_\gamma^{com} - p_\gamma\right) + \left(p_q^{com} - p_q\right)\widehat{q}_p \frac{p_\gamma}{1-p_q\widehat{q}_p}}{\left(p_\beta - p_\beta^{com}\right) - \left(p_q^{com} - p_q\right)\widehat{q}_p \frac{p_\beta}{1-p_q\widehat{q}_p}}
\end{aligned}$$

Consider now the impact of a change in γ on institutional change, if there were no cultural co-evolution. (ie fixing q to its pre-shock value). Differentiation of the first equation of (56). provides immediately

$$\left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} = \frac{\left(p_\gamma^{com} - p_\gamma\right)}{\left(p_\beta - p_\beta^{com}\right)} > 0$$

The last inequality comes from our stability condition (27) $\left(p_\beta - p_\beta^{com}\right)/p_\beta > 0$ coupled with condition (34. This tells us that a change in γ leads to a positive change in the institutional weight β, taking q as constant. Using this, one may obtain the full impact of γ on institutional change (ie. taking into account the joint evolution with culture) as the full change of institutions

$$\frac{d\beta^*}{d\gamma} = \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} + \frac{\left(p_q^{com} - p_q\right)}{\left(p_\beta - p_\beta^{com}\right)} \frac{\widehat{q}_p}{1 - p_q\widehat{q}_p} \frac{\frac{\left(p_\gamma^{com} - p_\gamma\right)}{\left(p_\beta - p_\beta^{com}\right)} p_\beta + p_\gamma}{1 - \frac{\left(p_q^{com} - p_q\right)}{\left(p_\beta - p_\beta^{com}\right)}\widehat{q}_p \frac{p_\beta}{1-p_q\widehat{q}_p}}$$

Hence the *Cultural multiplier* on institutional change μ, at (β^*, q^*)

$$m = \left(\frac{d\beta^*}{d\gamma}\right) / \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} - 1$$

is positive if and only if

$$\frac{\left(p_q^{com} - p_q\right)}{\left(p_\beta - p_\beta^{com}\right)} \frac{\widehat{q}_p}{1 - p_q\widehat{q}_p} \frac{\frac{\left(p_\gamma^{com} - p_\gamma\right)}{\left(p_\beta - p_\beta^{com}\right)} p_\beta + p_\gamma}{1 - \frac{\left(p_q^{com} - p_q\right)}{\left(p_\beta - p_\beta^{com}\right)}\widehat{q}_p \frac{p_\beta}{1-p_q\widehat{q}_p}} > 0$$

The stability condition coupled with (34) imply that $\left(p_\beta - p_\beta^{com}\right) > 0$, $1 - p_q\widehat{q}_p > 0$, $1 - \frac{\left(p_q^{com} - p_q\right)}{\left(p_\beta - p_\beta^{com}\right)}\widehat{q}_p \frac{p_\beta}{1-p_q\widehat{q}_p} > 0$, and $\frac{\left(p_\gamma^{com} - p_\gamma\right)}{\left(p_\beta - p_\beta^{com}\right)} p_\beta > 0$. When $p_\gamma > 0$, it follows that $\frac{\left(p_\gamma^{com} - p_\gamma\right)}{\left(p_\beta - p_\beta^{com}\right)} p_\beta + p_\gamma > 0$.

Therefore $m > 0$ if and only if $\left(p_q^{com} - p_q\right) \widehat{q}_p > 0$, which defines exactly the condition for institutions and culture to be dynamic complements. **QED**

- Cultural Multiplier on an aggregate variable $A(p, q, a^1(p), a^2(p))$:

We may decompose the effects of a shock as follows:

$$\frac{dA}{d\gamma} = \left\{ \underbrace{\left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_\beta}_{\text{direct effect}} + \underbrace{\left[A_q + \left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_q\right] \frac{\widehat{q}_p p_\beta}{1 - p_q \widehat{q}_p}}_{\text{indirect effect}} \right\} \frac{d\beta^*}{d\gamma}$$

The effect of γ on institutions will come from a direct effect as well as an indirect one. The direct effect in turn will be composed of two terms: a direct effect of the policy change induced by an institutional change p_β on the aggregate variable A (i.e., the term A_p), and the impact of changes in private actions $a^1(p)$ and $a^2(p)$ as induced also by the policy change p_β), the term $\left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right) p_\beta$). The indirect effect of cultural evolution will come from the compositional effect of changing the cultural group sizes (A_q), plus again the change in policy and private actions $\left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_q$ which such a cultural compositional change induces.

Furthermore,

$$\left(\frac{dA}{d\gamma}\right)_{q=q^*} = \left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_\beta \cdot \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*}$$

recalling the cultural multiplier on institutions as $m = \left[\left(\frac{d\beta^*}{d\gamma}\right) / \left(\frac{d\beta^*}{d\gamma}\right)_{q=q^*} - 1\right]$, one obtains easily the cultural multiplier m_A on the aggregate variable A as

$$m_A = \frac{dA}{d\gamma} / \left(\frac{dA}{d\gamma}\right)_{q=q^*} - 1.$$

It rewrites as :

$$m_A = \frac{A_q + \left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_q}{\left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right] p_\beta} \frac{\widehat{q}_p p_\beta}{1 - p_q \widehat{q}_p} \cdot (1 + m) \tag{35}$$

or finally the following expression:

$$m_A = \left[\frac{A_q}{\left[A_p + \left(A_{a^1} a_p^1 + A_{a^2} a_p^2\right)\right]} + p_q\right] \cdot \frac{\widehat{q}_p}{1 - p_q \widehat{q}_p} (1 + m) \tag{36}$$

Online Appendix:

- **Sufficient conditions for the existence and monotonicity of the societal equilibrium** $p(\beta, q)$

We consider the case with 2 cultural groups $i \in (1,2)$ with respective population fractions $q^1 = 1 - q^2 = q \in [0,1]$. The policy p is unidimensional in a closed interval domain. Without loss of generality we note $p \in [0,1]$. The indirect utility function write as

$$u^i(a^i, p; A, q)$$

where the individual private action $a^i \in [0,1]$, A is an aggregate population level index $A = A(a^1, a^2, p, q)$. depending on individual actions a^1, a^2, the policy variable p and the population structure q.

We assume that $u^i(.)$ is twice differentiable in $(a^i, p; A, q)$ and strictly concave in a^i (ie. $u^i_{11} < 0$). The aggregator function $A(.)$ is differentiable in (a^1, a^2, p, q) and such that the image of $[0,1]^4$ by $A(.)$ is an interval $[A_{\min}; A_{\max}]$ .We assume the following boundary conditions

$$u^i_1(0, p; A, q) \geq 0, \; u^i_1(1, p; A, q) \leq 0 \text{ for all } (p, A, q) \in [0,1] \times [A_{\min}; A_{\max}] \times [0,1]$$

These conditions and the fact that $u^i(.)$ is a strictly concave function in a^i ensure that the optimal individual behavior for a given value of p and A is characterized by a continuous function $a^i(p, A, q) \in [0,1]$ obtained from the First Order Condition:

$$u^i_1(a^i, p; A, q) = 0.$$

For given values of $p \in P$ and $q \in [0,1]$, a Nash equilibrium in private actions a^{1N}, a^{2N} and aggregate index $A^N(p,q)$ is characterized by the solution of the following system :

$$a^{iN} = a^i\left(p, A^N, q\right) \text{ for } i \in (1,2) \text{ and } A^N = A(a^{1N}, a^{2N}, p, q)$$

which translates into the following condition for A^N:

$$A^N = A(a^1\left(p, A^N, q\right), \; a^2\left(p, A^N, q\right), p, q) \tag{37}$$

The following sufficient conditions ensure the existenc of a unique Nash equilibrium in private actions $a^{1N}(p,q)$, $a^{2N}(p,q)$, $A^N(p,q)$:

$$\begin{aligned} 1 - \sum_{i=1,2} A'_i \frac{u^i_{13}}{-u^i_{11}} &> 0 \text{ for all } (a^1, a^{2,}, A, p, q) \\ A(a^1(p, A_{\min}, q), a^2(p, A_{\min}, q), q) &> A_{\min} \text{ for all } (p,q) \in [0,1]^2 \\ A(a^1(p, A_{\max}, q), a^2(p, A_{\max}, q), q) &< A_{\max} \text{ for all } (p,q) \in [0,1]^2 \end{aligned}$$

The first condition ensures that the function $\Gamma(x,p,q) = x - A(a^1(p,x,q), \ a^2(p,x,q), p, q)$ is increasing for all $(p,q) \in [0,1]^2$. The second and the third conditions ensure that $\Gamma(A_{\min},p,q) < 0 < \Gamma(A_{\max},p,q)$; Together these conditions ensure the existence of a unique value $A^N(p,q)$ satisfying (37) and thus correspondingly a unique Nash equilibrium profile $a^{1N}(p,q), \ a^{2N}(p,q)$.

Moreover simple differentiation provides

$$\begin{aligned}
\frac{dA^N}{dp} &= \frac{A_p + \sum_{j=1,2} A'_j \frac{u^j_{12}}{-u^j_{11}}}{\left[1 - \sum_{j=1,2} A'_j \frac{u^j_{13}}{-u^j_{11}}\right]} \\
\frac{da^{iN}}{dp} &= \frac{u^i_{12}}{-u^i_{11}} + \frac{u^i_{13}}{-u^i_{11}} \frac{A_p + \sum_{j=1,2} A'_j \frac{u^j_{12}}{-u^j_{11}}}{\left[1 - \sum_{j=1,2} A'_j \frac{u^j_{13}}{-u^j_{11}}\right]}
\end{aligned}$$

The condition for an interior *societal equilibrium* $p(\beta,q)$ is obtained from the FOC of the policymaker

$$\beta \, u^1_2(a^1,p,A,q) + (1-\beta) \, u^2_2\left(a^2,p,A,q\right) = 0$$

and after substitution of the Nash equibrium private actions $a^{1N}(p,q), \ a^{2N}(p,q), \ A^N(p,q)$ rewrites as :

$$\Psi(p,q,\beta) = 0 \tag{38}$$

with

$$\Psi(p,q,\beta) = \beta \, u^1_2(a^{1N}(p,q),p,A^N(p,q),q) + (1-\beta) \, u^2_2\left(a^{2N}(p,q),p,A^N(p,q),q\right)$$

Moreover a corner societal equilibrium $p(\beta,q) = 0$ (resp. $p(\beta,q) = 1$) obtains when $\Psi(0,q,\beta) \leq 0$ (resp. $\Psi(1,q,\beta) \geq 1$).

A sufficient condition for the existence of a unique *societal equilibrium* $p(\beta,q)$ is the fact that the function $\Psi(p,q,\beta)$ is decreasing in p for all $q \in [0,1]$. Given the smoothness assumptions on the functions $u^i(.)$ and $A(.)$ this is satisfied when the following condition holds

$$u^i_{12}\frac{da^i}{dp} + u^i_{22} + u^i_{23}\frac{dA}{dp} < 0 \quad \text{for all } i \in (1,2)$$

This rewrites in terms of the fundamentals as

$$\frac{u^i_{12}}{-u^i_{22}}\left[\frac{u^i_{12}}{-u^i_{11}} + \frac{u^i_{13}}{-u^i_{11}} \frac{A_p + \sum_{j=1,2} A'_j \frac{u^j_{12}}{-u^j_{11}}}{\left[1 - \sum_{j=1,2} A'_j \frac{u^j_{13}}{-u^j_{11}}\right]}\right] + \frac{u^i_{23}}{-u^i_{22}} \frac{A_p + \sum_{j=1,2} A'_j \frac{u^j_{12}}{-u^j_{11}}}{\left[1 - \sum_{j=1,2} A'_j \frac{u^j_{13}}{-u^j_{11}}\right]} < 1$$

or

$$\frac{\left(u_{12}^i\right)^2}{u_{22}^i u_{11}^i} + \left(\frac{u_{12}^i}{u_{22}^i}\frac{u_{13}^i}{u_{11}^i} + \frac{u_{23}^i}{(-u_{22}^i)}\right) \frac{A_p + \sum_{j=1,2} A'_j \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A'_j \frac{u_{13}^j}{-u_{11}^j}\right]} < 1 \text{ for } i \in (1,2)$$

with $A_p = \partial A/\partial p$, and $A_j = \partial A/\partial a^j$,. This condition is more likely to be satisfied when $\left|u_{11}^i\right|$ and $\left|u_{22}^i\right|$ are large enough.

To obtain a condition ensuring the monotonicity in β of the *societal equilibrium* $p(\beta, q)$, we may differentiate (**??**). We get

$$\frac{\partial p}{\partial \beta} = \frac{u_2^1 - u_2^2}{-\Psi_p}$$

Thus $p(\beta, q)$ is monotonic in β when $u_2^1 - u_2^2$ has a constant sign.

- These conditions simplify for the case of preferences structure with characterized by some degree of separability:

$$\begin{aligned} u^i(a, p; A, q) &= u(a, p; A, q, \theta_i) \\ &= v(a, p, \theta_i) + H(p, A) \end{aligned}$$

and $\theta_1 > \theta_2$. Such preferences lead to

$$\begin{aligned} a^{1N} &= a(p, \theta_1) \\ a^{2N} &= a(p, \theta_2) \\ A^N &= A(a(p, \theta_1), a(p, \theta_2), p, q) \end{aligned}$$

A sufficient condition for the existence of a unique *societal equilibrium* write as (given that $u_{13}^j = 0$)

$$\frac{\left(v_{12}^i\right)^2}{\left[v_{22}^i + H_{pp}\right] v_{11}^i} + \left(\frac{H_{pA}}{-(v_{22}^i + H_{pp})}\right) \left[A_p + \sum_{j=1,2} A'_j \frac{v_{12}^j}{-v_{11}^j}\right] < 1 \text{ for } i \in (1,2)$$

where $v_{kl}^i = v"_{kl}(a, p, \theta_i)$. Now

$$\begin{aligned} u_2^1 - u_2^2 &= u_2(a^{1N}, p, A^N, q, \theta_1) - u_2\left(a^{2N}, p, A^N, q, \theta_2\right) \\ &= v_2(a(p, \theta_1), p, \theta_1) - v_2(a(p, \theta_2), p, \theta_2) \end{aligned}$$

Thus a sufficient conditions for the monotonicity of the *societal equilibrium* $p(\beta, q)$ is that $v_p(a(p, \theta), p, \theta)$ is monotonic in θ, or after manipulations that $v_{ap}\frac{v_{a\theta}}{(-v_{aa})} + v_{p\theta}$ has a constant sign.

Consider as an example the preference structure

$$u(a,p,A,q,\theta) = (1-p)a + \theta W(1-a) + H(p,A)$$

with $W(.)$ a strictly increasing and concave function, $A = qa^1 + (1-q)a^2$, $H(p,A)$ concave in p. We have

$$\begin{aligned} v_a &= (1-p) - \theta W'(1-a),\ v_p = -a \\ v_{ap} &= -1, v_{a\theta} = -W'(1-a) \\ -v_{aa} &= -\theta W"(1-a) \\ v_{p\theta} &= 0,\ v_{pp} = 0 \end{aligned}$$

Then the sufficient condition for a well defined *societal equilibrium* $p(\beta,q)$ writes as:

$$\frac{1}{H_{pp}\theta_i W"} + \left(\frac{H_{pA}}{-(H_{pp})}\right)\left[\sum_{j=1,2} q_j \frac{1}{\theta_j W"}\right] < 1 \text{ for } i = 1,2$$

When $H_{pA} > 0$, given that $-(H_{pp}) > 0$ and $\left[\sum_{j=1,2} q_j \frac{1}{\theta_j W"}\right] < 0$, this condition is satisfied when $\frac{1}{H_{pp}\theta_i W"} < 1$, which in turn holds when $1 < H_{pp}W"\theta_2$. This is satisfied when $H_{pp}W"$ is sufficiently large (enough concavity of W and H respectively in a and p).

When $H_{pA} < 0$,this sufficient condition can be rewritten as

$$\frac{1}{\theta_i} - H_{pA}\left[\sum_{j=1,2} q_j \frac{1}{\theta_j}\right] < H_{pp}W"$$

which again will be satisfied when H_{pA} is bounded from below on the relevant domain $[0,1] \times [A_{\min}, A_{\max}]$ (ie. $H_{pA} > -K$, with $K > 0$) and $H_{pp}W" > (1+K)/\theta_2$. This will be also satisfied when there is enough concavity of W and H respectively in a and p.

Finally the monotonicity of the *societal equilibrium* function $p(\beta,q)$ holds when $v_{ap}\frac{v_{a\theta}}{(-v_{aa})} + v_{p\theta}$ has a constant sign. here

$$v_{ap}\frac{v_{a\theta}}{(-v_{aa})} + v_{p\theta} = \frac{W'}{-\theta W"} > 0$$

Thus the *societal equilibrium* function is monotonically increasing in β.

- **Sufficient conditions for the existence of the societal commitment equilibrium** $p^{com}(\beta,q)$

The *societal commitment equilibrium given institutions β and cultural distribution q* is obtained from the following programme

$$\begin{array}{l} \max\ \beta\, u^1\left(a^{1N}, p; A^N, q\right) + (1-\beta)\, u^2\left(a^{2^N}, p; A^N, q\right) \\ s.t.\ \ a^{iN} = a^{iN}(p,q) \ \text{ for } \ i \in (1,2) \ \text{ and } A^N = A^N(p,q) \end{array} . \tag{39}$$

Denote then the function

$$\begin{aligned}\Omega(p,\beta,q) &= \beta\, u^1\left(a^{1N}(p,q), p; A^N(p,q), q\right)\\ &\quad +(1-\beta)\, u^2\left(a^{1N}(p,q), p; A^N(p,q), q\right)\end{aligned}$$

The first order condition for an interior *societal commitment equilibrium* $p^{com}(\beta,q)$ writes as

$$\begin{aligned}\Omega_p(p,\beta,q) &= \beta\, u_2^1(a^{1N}, p, A^N, q) + (1-\beta)\, u_2^2\left(a^{2N}, p, A^N, q\right)\\ &\quad + \left(\beta\, u_3^1(a^{1N}, p, A^N, q) + (1-\beta)\, u_3^2\left(a^{2N}, p, A^N, q\right)\right)\frac{dA^N}{dp}\\ &= 0\end{aligned}$$

To ensure an optimum in $\Omega(p,\beta,q)$, a sufficient (strong) condition is to have that $\Omega_{pp}(p,\beta,q) < 0$ for all $p \in [0,1]$. Differentiation provides (with the notation $\beta^1 = 1-\beta^2 = \beta$)

$$\begin{aligned}\Omega_{pp}(p,\beta,q) &= \sum_{i=1,2}\beta^i\left(u_{12}^i\frac{da^{iN}}{dp} + u_{22}^i + u_{23}^i\frac{dA^N}{dp}\right)\\ &\quad + \sum_{i=1,2}\beta^i\left[u_{13}^i\frac{da^{iN}}{dp} + u_{23}^i + u_{33}^i\right]\frac{dA^N}{dp}\\ &\quad + \sum_{i=1,2}\beta^i u_3^i\frac{d^2A^N}{dp^2}\end{aligned}$$

Thus a sufficient condition for $\Omega_{pp}(p,\beta,q) < 0$ is that for $i = 1,2$ one has

$$u_{12}^i\frac{da^{iN}}{dp} + u_{22}^i + u_{23}^i\frac{dA^N}{dp} + \left[u_{13}^i\frac{da^{iN}}{dp} + u_{23}^i + u_{33}^i\right]\frac{dA^N}{dp} + u_3^i\frac{d^2A^N}{dp^2} < 0$$

recalling that

$$\frac{da^{iN}}{dp} = \frac{u_{12}^i}{-u_{11}^i} + \frac{u_{13}^i}{-u_{11}^i}\frac{A_p + \sum_{j=1,2}A_j'\frac{u_{12}^j}{-u_{11}^j}}{\left[1-\sum_{j=1,2}A_j'\frac{u_{13}^j}{-u_{11}^j}\right]}$$

and

$$\frac{dA^N}{dp} = \frac{A_p + \sum_{j=1,2}A_j'\frac{u_{12}^j}{-u_{11}^j}}{\left[1-\sum_{j=1,2}A_j'\frac{u_{13}^j}{-u_{11}^j}\right]}$$

Tedious manipulations provide then that a sufficient condition for $\Omega_{pp}(p,\beta,q) < 0$ is that the following expression

$$D^i = \frac{\left(u_{12}^i\right)^2}{-u_{11}^i} + u_{22}^i + \left(2(\frac{u_{13}^i u_{12}^i}{-u_{11}^i} + u_{23}^i) + u_{33}^i\right) \frac{A_p + \sum_{j=1,2} A_j' \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A_j' \frac{u_{13}^j}{-u_{11}^j}\right]}$$

$$+\frac{\left(u_{13}^i\right)^2}{-u_{11}^i} \left[\frac{A_p + \sum_{j=1,2} A_j' \frac{u_{12}^j}{-u_{11}^j}}{\left[1 - \sum_{j=1,2} A_j' \frac{u_{13}^j}{-u_{11}^j}\right]}\right]^2 + u_3^i \frac{d^2 A^N}{dp^2}$$

is negative for $i = 1, 2$. Because of the term in $d^2 A^N / dp^2$, this involves complicated conditions on the third derivatives of the indirect preference functions. When preferences are separable of the form

$$\begin{aligned} u^i(a, p; A, q) &= u(a, p; A, q, \theta_i) \\ &= v(a, p, \theta_i) + H(p, A) \end{aligned}$$

the expression D^i simplifies somewhat to

$$\begin{aligned} D^i &= \frac{\left(v_{ap}^i\right)^2}{-v_{aa}^i} + v_{pp}^i + (2H_{pA} + H_{AA})\,(A_p + \sum_{j=1,2} A_j' \frac{v_{ap}^j}{-v_{pp}^j}) \\ &\quad + H_A \frac{d^2 A^N}{dp^2} \end{aligned}$$

and $\Omega(p, \beta, q)$ is strictly concave in p when $v(a, p, \theta_i)$ will be sufficiently concave in (a, p).

Appendix for Examples

1) Elites, workers, and extractive institutions

Let workers be group $i = 1$ and the elite be $i = 2$. Both groups can transform labor ono-for-one into private consumption goods. Let a^i denote labor exerted by any member of group i. Let s denote the initial resources each elite member is endowed with. Let p denote the (income) tax rate and G the public good provided by fiscal institutions. Preferences for group i are represented by the following utility function:

$$u^i\left(a^i, G, p\right) = u\left(a^i(1-p) + s^i\right) + \theta^i v(1-a^i) + \Omega \cdot G.$$

Our characterization of the distinction between workers and the elite in terms of cultural values and technologies requires that:

i) the parameter θ^i representing the preference for leisure satisfies $\theta^1 < \theta^2$
ii) Initial resources s^i satisfy : $s^1 = 0$, $s^2 = s$ii) Initial resources s^i satisfy: $s^1 = 0, s^2 = s$.

Furthermore we assume extreme preferences for leisure of the elite, $\theta^2 > \frac{v'(0)}{u'(s)} > 1 = \theta^1$. In this case, members of the elite never work, $a^2 = 0$, and consume their resources, s.

The optimal behavior of the mass workers is determined by:

$$a^1 = a(p) = \arg\max_a u\left((1-p)\,a\right) + v(1-a)$$

and thus

$$a(p)\begin{cases} \in (0,1) \text{ and determined by } (1-p)u'\left((1-p)\,a\right) = v'(1-a) \text{ when } p \leq p_0 \\ = 0 \quad \text{when } p \geq p_0 \end{cases}$$

with p_0 the tax rate over which the optimal effort is equal to 0 :

$$p_0 = 1 - \frac{v'(1)}{u'(0)}$$

- **The societal equilibrium policy:**

The *societal equilibrium policy* $p(\beta, q)$ is characterized by the following conditions

$$\begin{aligned} p &\in \arg\max_{p\in[0,1]} W(p, a^1, a^2, G) = \beta u^1\left(a^1, G, p\right) + (1-\beta)u^2\left(a^2; G\right) \quad (40) \\ s.c.G &= p\left[qa^1 + (1-q)a^2\right], \text{ for given } a^1, a^2. \end{aligned}$$

$$\begin{aligned} a^1 &= a(p) = \arg\max_a u\left((1-p)\,a\right) + v(1-a) \\ a^2 &= 0 = \arg\max_a u(s + a(1-p)) + \theta^2 v(1-a) \end{aligned} \tag{41}$$

We then get the following characterization:

Characterization of societal equilibrium policy:

Assume $u'(s + a(0)) < \Omega < u'(s)$, there exists two thresholds $\underline{\beta}(q)$ and $\overline{\beta}(q)$ such that $\underline{\beta}(q) < \overline{\beta}(q)$, for all $q \in [0,1]$ and such that

i) *For* $0 < \beta < \underline{\beta}(q)$, $p(\beta, q) \in [p_0, 1]$, $a(p) = 0$ *and* $G = 0$.

ii) *For* $\beta \in \left(\underline{\beta}(q), \overline{\beta}(q)\right)$ then $p(\beta, q) \in (0, p_0)$ and $a(p) > 0$ and $G > 0$. $p(\beta, q)$ is decreasing in β and decreasing in q.

iii) *For* $\beta \geq \overline{\beta}(q)$, then $p(\beta, q) = 0$, $a = a(0) > 0$ and $G = 0$.

Proof. i) Substituting (41) into the first order condition of problem (40) provides the following conditions characterizing a stable interior *societal equilibrium policy:*

$$\Xi(p, q, \beta) = -\beta \cdot u'\left((1-p)a(p)\right) \cdot a(p) + \Omega q \cdot a(p) = 0$$

Thus when $a(p) \neq 0$ (that is for $p \leq p_0$), this is equivalent to

$$\Theta(p, q, \beta) = -\beta \cdot u'\left((1-p)a(p)\right) + \Omega q = 0$$

Now the function $\Theta(p, q, \beta)$ is decreasing in p as long as $a(p)$ is decreasing in p (something that is ensured when the utility function $u(.)$ satisfies $\frac{-u''(x)x}{u'(x)} < 1$). As well we have :

$$\Theta(0, q, \beta) = -\beta \cdot u'\left(a(0)\right) + \Omega q < 0$$

if and only if

$$\beta > \overline{\beta}(q) = \frac{\Omega q}{u'\left(a(0)\right)}$$

Note that $\overline{\beta}(q)$ is increasing in q. It follows immediately that for $\beta \geq \overline{\beta}(q)$, $\Theta(p, q, \beta) < 0$ for all $p \in [0,1]$ and the only possible societal equilibrium is the corner solution $p(\beta, q) = 0$.

ii) Similarly note that

$$\Theta(p_0, q, \beta) = -\beta \cdot u'(0) + \Omega q < 0$$

when $\beta > \underline{\beta}(q) = \frac{\Omega q}{u'(0)}$. It is easy to see that $\underline{\beta}(q) < \overline{\beta}(q)$. Hence *for* $\beta \in \left(\underline{\beta}(q), \overline{\beta}(q)\right)$, we then get that $\Theta(0, q, \beta) > 0$, $\Theta(p_0, q, \beta) < 0$ and $\Theta(p, q, \beta)$ decreasing in p. Therefore there exists a unique $p(\beta, q) \in (0, p_0)$ such that $\Theta(p, q, \beta) = 0$. At this point we obviously have $a(p) > 0$ and $G = q \cdot a(p) > 0$. Moreover, one can immediately see that $\Theta_\beta(p, q, \beta) < 0$ and that $\Theta_q(p(\beta, q), q, \beta) = \Omega > 0$ and thus $p(\beta, q)$ is decreasing in β and increasing in q.

iii) Finally note that when $\beta < \underline{\beta}(q)$, $\Theta(p_0, q, \beta) > 0$ and $\Xi(p, q, \beta) > 0$ for all $p < p_0$. From this it follows the best response of the policy maker to any effort of workers $a(p) > 0$ is to choose a policy $p \geq p_0$. Given that with $a(p) = 0$, the policy maker is indifferent with any policy $p \geq p_0$. Moreover for all policies $p \geq p_0$, one has $a(p) = 0$. It follows therefore that for $\beta < \underline{\beta}(q)$ $p(\beta, q)$ can be any policy in $[p_0, 1]$ associated to $a = 0$, no production and no provision of the public good (ie. $G = 0$). A natural selection of this equilibrium correspondence is $p(\beta, q) = p_0$. **QED.** ■

- **Societal commitment equilibrium policy:**

Denote now

$$\begin{aligned} \widetilde{W}(p, \beta, q) &= \beta u^1 \left(a^1(p), G, p\right) + (1-\beta) u^2 \left(a^2(p), G, p\right) \\ &= \beta \cdot \left[u\left(a(p)(1-p)\right) + v(1-a(p)\right] + (1-\beta) \cdot \left[u(s) + \theta^2 v(1)\right] \\ &\quad + \Omega q \left[p \cdot a(p)\right] \end{aligned}$$

with $G = q\left[p \cdot a(p)\right]$. Then the *societal commitment equilibrium policy* $p^{com}(\beta, q)$ for any value of $(\beta, q) \in [0, 1]^2$ is the solution of the following program:

$$p \in \arg\max_{p} \widetilde{W}(p, \beta, q).$$

We get the first order condition characterizing an interior solution as

$$\begin{aligned} \widetilde{W}_p(p, \beta, q) &= -\beta \cdot u'\left((1-p)a(p)\right) \cdot a(p) \\ &\quad + \Omega q \cdot \left[a(p) + p a_p(p)\right] \end{aligned}$$

We assume that $\widetilde{W}(p, \beta, q)$ is strictly concave in p. This holds when $|v''|$ is large enough and $p \cdot a(p)$ is concave in p[62]. Then we have the folllowing characterization:

[62] Indeed

$$\begin{aligned} \widetilde{W}_{pp} &= -\beta \left[u''\left(-a + (1-p)a_p\right)a + u' a_p\right] \\ &\quad + \Omega\left[\lambda + (1-\lambda)q\right] \cdot \left[pa(p)\right]'' \\ &= -\beta \left[-u''\left(a\right)^2 + \left[u''(1-p)a + u'\right] a_p\right] \\ &\quad + \Omega\left[\lambda + (1-\lambda)q\right] \cdot \left[pa(p)\right]'' \end{aligned}$$

differentiation of the workers' first order condition provides:

$$a_p^1 = \frac{da^1}{dp} = \frac{u' + (1-p)au''}{u''(1-p)^2 + v''}$$

now after substitution, one gets

$$-u''\left(a\right)^2 + \left[u''(1-p)a + u'\right] a_p = \frac{\left(u'\right)^2 + 2(1-p)u'u''a - v''u''(a)^2}{u''(1-p)^2 + v''} < 0$$

when $|v''|$ large enough. Hence $\widetilde{W}_{pp} < 0$ when $|v''|$ is large enough and $pa(p)$ is concave in p.

Characterization of societal commitment equilibrium policy:

i) *For* $\beta < \overline{\beta}(q)$, $p^{com}(\beta, q) \in (0, p^{\max}]$ *and is characterized by the following equation:*

$$-\beta \cdot u'\left((1-p)a(p)\right) \cdot a(p) + \Omega q \cdot [a(p) + pa_p(p)] = 0$$

and $p^{\max} = \arg\max_p pa(p)$.

ii) *For* $\beta \geq \overline{\beta}(q)$, $p^{com}(\beta, q) = 0$.

iii) $p^{com}(\beta, q)$ *is decreasing in* β *and* increasing in q.

iv) $p^{com}(\beta, q) < p(\beta, q)$, $\forall \beta < \overline{\beta}(q)$, *and* $q \in (0, 1]$.

Proof. i) It is easy to see that for $\beta < \overline{\beta}(q)$,

$$\begin{aligned} \widetilde{W}_p(0, \beta, q) &= -\beta \cdot u'\left(a(0)\right) \cdot a(0) \\ &\quad + \Omega q \cdot [a(0)] > 0 \end{aligned}$$

and

$$\widetilde{W}_p(p^{\max}, \beta, q) = -\beta \cdot u'\left((1 - p^{\max})a(p^{\max})\right) \cdot a(p^{\max}) \leq 0$$

Thus $p^{com}(\beta, q) \in (0, p^{\max}]$ is obtained and is characterized by $\widetilde{W}_p(p, \beta, q) = 0$.

ii) Conversely for $\beta \geq \overline{\beta}(q)$, $\widetilde{W}_p(0, \beta, q) \leq 0$ and $p^{com}(\beta, q) = 0$. Finally given that $\widetilde{W}_{p\beta}(p, \beta, q) < 0$ one has $p^{com}(\beta, q)$ is decreasing in β. Similarly $\widetilde{W}_{pq}(p^{com}, \beta, q) = \Omega \cdot [a(p) + pa_p(p)] > 0$ for $p < p^{\max}$. Hence $p^{com}(\beta, q)$ is increasing in q.

iii) Finally at $p = p(\beta, q) \in (0, 1)$**,** it is easy to see that $\widetilde{W}_p(p(\beta, q), \beta, q) = \Omega q \cdot [pa_p(p)] < 0$ for $\beta < \overline{\beta}(q)$) and $q \in (0, 1]$. Given that $\widetilde{W}(p, \beta, q)$ is strictly concave, this implies that $p(\beta, q) > p^{com}(\beta, q)$ for that range of parameters β and q. **QED.** ■

- **Cultural Dynamics :**

\- For $\beta \in \left(\underline{\beta}(q), \overline{\beta}(q)\right)$, the parameters of cultural intolerance $\Delta V^1(p)$ and $\Delta V^2(p)$ are given by

$$\begin{aligned} \Delta V^1(p) &= u\left((1-p)\, a(p)\right) + v(1 - a(p)) \\ &\quad - \left[u\left(0\right) + v(1)\right] \end{aligned}$$

$$\begin{aligned} \Delta V^2(p) &= u\left(s\right) + \theta^2 v(1) \\ &\quad - \left[u\left(s + (1-p)\, a(p)\right) + \theta^2 v(1 - a(p))\right] \end{aligned}$$

$\Delta V^1(p)$ is obviously decreasing in p. For $\Delta V^2(p)$ we get

$$\begin{aligned}\frac{d\Delta V^2}{dp} &= -\left[-u'a + \left(u'(1-p) - \theta^2 v'\right) a_p\right] \\ &= u'a + (\theta^2 - 1)v'a_p \\ &= u'a + (\theta^2 - 1)(1-p) \cdot u' \cdot \frac{u' + (1-p)au''}{u''(1-p)^2 + v''} > 0\end{aligned}$$

when $|v''|$ large enough .

We conclude that $\Delta V^1(p)/\Delta V^2(p)$ is decreasing in p when $v(.)$ is concave enough. In that region as $p = p(\beta, q)$ is in fact a decreasing function of β and increasing function of q, it follows that $\Delta V^1/\Delta V^2$ is an increasing function of β and an increasing function of q.

- For $\beta \geq \overline{\beta}(q)$, $\Delta V^1(p) = \Delta V^1(0)$ and $\Delta V^2(p) = \Delta V^2(0)$ are constant.

- For $\beta < \underline{\beta}(q)$, It is easy to see that $\Delta V^1(p_0) = \Delta V^2(p_0) = 0$. and there is no cultural dynamics in that region.

Cultural steady states are determined by:

$$\begin{aligned}\frac{\Delta V^1(p(\beta,q))}{\Delta V^2(p(\beta,q))} &= \frac{q}{1-q} \text{ for } \beta \in \left(\underline{\beta}(q), \overline{\beta}(q)\right) \\ \frac{\Delta V^1(0)}{\Delta V^2(0)} &= \frac{q}{1-q} \text{ for } \beta \geq \overline{\beta}(q) \\ q &= q_0 \text{ for } \beta < \underline{\beta}(q)\end{aligned}$$

In the region $\beta \in \left(\underline{\beta}(q), \overline{\beta}(q)\right)$ it is easy to see that when $|v''|$ large enough, this determines an upward sloping curve $q(\beta)$ in β. In the region $\beta > \overline{\beta}(q)$, whenever it exists, this determines a vertical sloping curve $q = q_0$.[63]

Finally note that Note that as $\beta \to \underline{\beta}(q)_+$

$$\begin{aligned}\Delta V^1(p) &= \Delta V^1(p_0) + (p - p_0)\left(\frac{d\Delta V^1}{dp}\right)_{p_0} + \frac{(p-p_0)^2}{2}\left(\frac{d^2\Delta V^1}{dp^2}\right)_{p_0} \\ &= -\frac{(p-p_0)^2}{2}\left[u'(0)\right] a'_p(p_0)\end{aligned}$$

and

$$\Delta V^2(p) = \Delta V^2(p_0) + (p - p_0)\left(\frac{d\Delta V^2}{dp}\right)_{p_0} + \frac{(p-p_0)^2}{2}\left(\frac{d^2\Delta V^2}{dp^2}\right)_{p_0}$$

$$\Delta V^2(p) = (p - p_0)\left(\theta^2 - 1\right) a'_p(p_0) + \frac{(p-p_0)^2}{2}\left(\frac{d^2\Delta V^2}{dp^2}\right)_{p_0}$$

[63]The condition for such vertical part to exist is simply

$$q_0 = \frac{\Delta V^1(0)}{\Delta V^1(0) + \Delta V^2(0)} < \frac{u'(a^1(0))}{\Omega}$$

Thus

$$\lim_{\beta \to \underline{\beta}(q)+} \frac{\Delta V^1(p\,(\beta, q))}{\Delta V^2(p(\beta; q))} \simeq \lim_{\beta \to \underline{\beta}(q)+} \frac{-\frac{(p-p_0)}{2}\,[u'(0)]\,a'_p(p_0)}{\left(\theta^2 - 1\right) a'_p(p_0) + \frac{(p-p_0)}{2}\left(\frac{d^2 \Delta V^2}{dp^2}\right)_{p_0}} = 0$$

and thus therefore the manifold $q\,(\beta)$ charactering the cultural steady states for $\beta \in \left(\underline{\beta}(q), \overline{\beta}(q)\right)$ touches the region $\beta \leq \underline{\beta}(q)$ at the point $q = 0$ and $\beta = \underline{\beta}(0)$.

The description on the two configurations of steady states (case a) and (case b) depends on whether the cultural manifold $q(\beta)$ intersect the curve $\overline{\beta}(q)$ before the point $\widetilde{q}$ at which $\overline{\beta}(\widetilde{q}) = 1$. Notice that this point $\widetilde{q}$ is given by

$$\widetilde{q} = \frac{u'(a(0))}{\Omega}$$

Therefore we are in case a) when

$$\frac{\Delta V^1(0)}{\Delta V^2(0)} < \frac{\widetilde{q}}{1 - \widetilde{q}} \tag{42}$$

given that

$$\begin{aligned} \Delta V^1(0) &= u\,(a(0)) + v(1 - a(0)) \\ &\quad - [u\,(0) + v(1)] \end{aligned}$$

$$\begin{aligned} \Delta V^2(0) &= u\,(s) + \theta^2 v(1) \\ &\quad - \left[u\,(s + a(0)) + \theta^2 v(1 - a(0))\right] \end{aligned}$$

and substituting the value of $\widetilde{q}$ and $\Delta V^1(0)$ and $\Delta V^2(0)$ into (42) and rearranging provides the following condition for case a): $\Omega < \widetilde{\Omega}$ with

$$\widetilde{\Omega} = u'(a(0))\left[1 + \frac{\Delta V^2(0)}{\Delta V^1(0)}\right]$$

QED.

The transition away from extractive institutions.

Preferences are represented by the following utility functions, respectively for workers and elites:

$$u^1(a^1, T^1, p) = u\left(a^1(1-p) + s^1 + T^1\right) + \theta^1 v(1-a^1)$$

$$u^{2j}(a^{2j}, T^2, p) = u\left(a^{2j}(1-p) + s^2 + T^2\right) + \theta^{2j} v(1-a^{2j})$$

Our characterization of the distinction between the political groups (workers and elites) and the cultural groups (bourgeois and aristocrats) in terms of cultural values and technologies requires that:

i) the parameter θ^{2j} representing the preference for leisure of the elites satisfy $\theta^{2a} > \theta^{2b} = \theta^1$
ii) Initial resources s^i satisfy : $s^1 = 0$, $s^2 = s > \bar{c}$
iii) Tax Transfers T^i satisfy $T^1 = 0$, $T^2 = T$

We make the following regularity conditions: $u(.)$ and $v(.)$ are smooth increasing concave functions with the inada conditions $u'(0) = v'(0) = +\infty$, $u'(\infty) = 0$.

Moreover we assume that*:*

$$\text{Assumption (P1):} \quad \theta^{2a} > \frac{u'(s)}{v'(1)} > 1 = \theta^1$$

This ensures that with no transfer from the workers to the elite, "bourgeois" elite members do work while "aristocrat" elite members do not.

- **Optimal Behaviors of workers and elite members:**

1) Given a linear tax rate p, the optimal behavior $a^1(p)$ of the workers depends on whether the survival constraint $c^1 \geq \bar{c}$ is binding or not.

1i) Consider first the "non extractive" regime NE where the survival constraint is not binding. The optimal behavior $a^1 = a^1_{op}(p)$ of the workers is obtained by the following condition for $p < 1$:

$$u'\left((1-p)\,a^1\right)(1-p) = v'(1-a^1)$$

and $a^1_{op}(1) = 0$. Simple differentiation provides that $a^1_{op}(p)$ is decreasing in p when the utility function $u(.)$ satisfies the following property:

$$\text{Assumption (P2):} \quad \frac{-xu''(x)}{u'(x)} < 1 \text{ for all } \; x \geq 0$$

Indeed

$$\frac{da^1_{op}}{dp} = \frac{u''_1(1-p)a^1_{op} + u'_1}{u''_1(1-p)^2 + v''_1} < 0 \tag{43}$$

with the convenient notations $u_1' = u'\left((1-p)\,a_{op}^1\right)$, $u_1'' = u''\left((1-p)\,a_{op}^1\right)$, $v_1'' = v''(1-a_{op}^1)$.

1ii) Consider now the "extractive regime" where the survival constraint is binding. In that case we the workers' consumption is given by $c^1 = (1-p)\,a^1 = \overline{c}$ and the optimal behavior $a^1(p)$ of the workers is given by the relationship:

$$a^1 = \widetilde{a}^1(p) = \frac{\overline{c}}{1-p} \text{ for } p \leq 1-\overline{c}$$

This regime prevails when the tax rate p is larger than a threshold $\widehat{p}$ given by the condition $a_{op}^1(\widehat{p}) = \frac{\overline{c}}{1-\widehat{p}}$ and characterized by the following condition $u'(\overline{c})(1-\widehat{p}) = v'(1-\frac{\overline{c}}{1-\widehat{p}}))$

1iii) The full characterization of the optimal behavior of the workers is then obtained as follows : assume that $u'(\overline{c}) > v'(1-\overline{c})$ (which ensures that $a_{op}^1(0) > \overline{c}$) and assumption (P2) holds, then the optimal effort of a worker writes as follows :

$$\begin{aligned} \text{non extractive regime} \quad &: \quad a^1(p) = a_{op}^1(p) \quad \text{for } p \in [0,\widehat{p}] \\ \text{extractive regime:} \quad & \quad a^1(p) = \widetilde{a}^1(p) = \frac{\overline{c}}{1-p} \quad \text{for } p \in [\widehat{p}, 1-\overline{c}] \end{aligned}$$

Notice this effort $a^1(p)$ is decreasing in p in the Non Malthusian regime and is increasing in p in the Malthusian regime.

2) Given a lump sum transfer T, the optimal behavior $a^{2b}(T)$ of a "bourgeois" elite member (for an internal solution) is obtained by the following condition:

$$u'\left(a^{2b} + T + s\right) = v'(1-a^{2b})$$

Assumption (P1) ensures that $a^{2b}(0) > 0$ while there exists a transfer level $\overline{T} = u'^{-1}(v'(1)) - s > 0$ such that $a^{2b}(T) = 0$ for all $T \geq \overline{T}$. Moreover it is immediate to see that

$$a_T^{2b} = \frac{da^{2b}}{dT} = -\frac{u_{2b}''}{u_{2b}'' + v_{2b}''} < 0 \quad \text{and } 1 + a_T^{2b} = \frac{v_{2b}''}{u_{2b}'' + v_{2b}''} > 0 \tag{44}$$

with the usual notations $u_{2b}'' = u''\left(a^{2b} + T + s\right)$, $v_{2b}'' = v''(1-a^{2b})$

3) For an "aristocrat" elite member, given that $\theta^2 v'(1) > u'(s)$, we immediately get $a^{2a}(T) = 0$ for all $T \geq 0$.

- **Societal equilibrium policy:**

- Define the policy maker objective function

$$\begin{aligned} W(p, a^1, a^{2b}, T,) \quad &= \quad \beta\left[u((1-p)a^1) + v(1-a^1)\right] \\ & \quad +(1-\beta)\cdot\left[\begin{array}{c} q\cdot\left(u(T + a^{2b} + s) + v(1-a^{2b})\right) \\ +(1-q)\cdot(u(T+s) + \theta v(1)) \end{array}\right] \end{aligned}$$

Then the *societal equilibrium policy* $p(\beta, q)$ is characterized by the following conditions:

$$p \in \arg\max_{p\in[0,1-\overline{c}]} W(p, a^1, a^{2b}, pa^1\frac{1-\lambda}{\lambda})) \quad (45)$$
$$\text{for given } a^1, a^{2b}$$

$$a^1 = a^1(p); \quad a^{2b} = a^2(T), \ a^{2a} = 0 \text{ and } T = pa^1\frac{1-\lambda}{\lambda}$$

Note that under our regularity conditions, for given a^1, a^{2b}, $W(p, a^1, a^{2b}, pa^1\frac{1-\lambda}{\lambda}))$ is a well defined, smooth and concave function of p. Hence there is a unique well defined value $p \in [0, 1-\overline{c}]$ that solves problem (45).

To characterize a *societal equilibrium policy* $p(\beta, q)$, we can distinguish between the two possible regimes (extractive and non extractive).

1) Consider first a "non extractive" *societal equilibrium* . Such a "non extractive" *societal equilibrium policy* $p(\beta, q) \leq \widehat{p}$ should then satisfy the following conditions:

$$\begin{array}{rcll} p & = & 0 & \text{when } W_p'(0, a^1(0), a^{2b}(0), 0) \leq 0 \\ p & \in & (0, \widehat{p}) & \text{when } \begin{array}{c} W_p'(p, a^1(p), a^{2b}(T(p)), T(p)) = 0 \\ \text{with } T(p) = pa^1(p)\frac{1-\lambda}{\lambda} \end{array} \end{array}$$

To analyze the structure of a "non extractive" *societal equilibria,* observe first that

$$W_p'(p, a^1(p), a^{2b}(T(p)), T(p)) = a^1(p) \cdot \Psi(p, \beta, q)$$

with the "auxiliary" function $\Psi(p, \beta, q)$:

$$\begin{aligned} \Psi(p, \beta, q) &= -\beta u'\left((1-p)\, a^1(p)\right) \\ &\quad +(1-\beta)\frac{(1-\lambda)}{\lambda}\left[\begin{array}{c} qu'(T(p) + a^{2b}(T(p)) + s) \\ +(1-q)u'(T(p) + s) \end{array}\right] \end{aligned}$$

and $T(p) = pa^1(p)\frac{1-\lambda}{\lambda}$. We are now in a position to provide sufficient regularity conditions ensuring the existence and uniqueness of a "Non extractive" societal equilibrium in this economy.

Characterization of a "non extractive" societal equilibrium policy: *Assume that the function* $\Psi(p, \beta, q)$ *is convex in* p. *Then for all* $q \in [0, 1]$, *there exists* $\beta^m(q) \in (0, 1)$ and $\overline{\beta}(q) \in (0, 1)$ with $\left(\beta^m(q) < \overline{\beta}(q)\right)$ such that there exists *a unique "non extractive" societal equilibrium* $p(\beta, q) \in [0, \widehat{p}]$. *It is characterized in the following way:*

- *For* $\beta \in \left(\beta^m(q), \overline{\beta}(q)\right)$. $p(\beta, q) \in (0, \widehat{p})$ and is a *decreasing function in* β *and* q.

- *For* $\beta \geq \overline{\beta}(q)$, $p(\beta, q) = 0$.

Proof. Note first that $p = 0$ is a societal equilibrium when $\Psi(0, \beta, q) \leq 0$. This condition writes as:

$$\Psi(0, \beta, q) = -\beta u'\left(a^1(0)\right) + (1-\beta)\frac{(1-\lambda)}{\lambda}\left[qu'(a^{2b}(0) + s) + (1-q)u'(s)\right] \leq 0$$

which is satisfied if and only if $\beta \geq \overline{\beta}(q)$ where

$$\overline{\beta}(q) = \frac{(1-\lambda)\left[qu'(a^{2b}(0) + s) + (1-q)u'(s)\right]}{(1-\lambda)\left[qu'(a^{2b}(0) + s) + (1-q)u'(s)\right] + \lambda u'\left(a^1(0)\right)}$$

which is clearly a decreasing function of q.

- Now consider $\beta < \overline{\beta}(q)$. We have $\Psi(0, \beta, q) > 0$ and $\Psi(\widehat{p}, \beta, q) \leq 0$ with

$$\begin{aligned}\Psi(\widehat{p}, \beta, q) &= -\beta u'\left(\overline{c}\right) \\ &\quad +(1-\beta)\frac{(1-\lambda)}{\lambda}\left[\begin{array}{c} qu'(\frac{\widehat{p}}{1-\widehat{p}}\overline{c}\frac{1-\lambda}{\lambda} + a^{2b}(\frac{\widehat{p}}{1-\widehat{p}}\overline{c}\frac{1-\lambda}{\lambda}) + s) \\ +(1-q)u'(\frac{\widehat{p}}{1-\widehat{p}}\overline{c}\frac{1-\lambda}{\lambda} + s) \end{array}\right] < 0\end{aligned}$$

when

$$\frac{(1-\lambda)}{\lambda}\left[\begin{array}{c} qu'(T(\widehat{p}) + a^{2b}(T(\widehat{p})) + s) \\ +(1-q)u'(T(\widehat{p}) + s) \end{array}\right] \leq \beta\left[u'\left(\overline{c}\right) + \frac{(1-\lambda)}{\lambda}\left[\begin{array}{c} qu'(T(\widehat{p}) + a^{2b}(T(\widehat{p})) + s) \\ +(1-q)u'(T(\widehat{p}) + s) \end{array}\right]\right]$$

or

$$\beta \geq \beta^m(q) = \frac{\frac{(1-\lambda)}{\lambda}\left[qu'(T(\widehat{p}) + a^{2b}(T(\widehat{p})) + s) + (1-q)u'(T(\widehat{p}) + s)\right]}{u'\left(\overline{c}\right) + \frac{(1-\lambda)}{\lambda}\left[qu'(T(\widehat{p}) + a^{2b}(T(\widehat{p})) + s) + (1-q)u'(T(\widehat{p}) + s)\right]}$$

$$\begin{aligned}\frac{\partial \beta^m(q)}{\partial q} &\propto \frac{(1-\lambda)}{\lambda}\left[u'(T(\widehat{p}) + a^{2b}(T(\widehat{p}) + s) - u'(T(\widehat{p}) + s)\right]\left[u'\left(\overline{c}\right) + \frac{(1-\lambda)}{\lambda}u'(T(\widehat{p}) + s)\right] \\ &\quad -\frac{(1-\lambda)}{\lambda}\left[u'(T(\widehat{p}) + a^{2b}(T(\widehat{p}) + s) - u'(T(\widehat{p}) + s)\right]\left[\frac{(1-\lambda)}{\lambda}u'(T(\widehat{p}) + s)\right] \\ &\propto \frac{(1-\lambda)}{\lambda}\left[u'(T(\widehat{p}) + a^{2b}(T(\widehat{p}) + s) - u'(T(\widehat{p}) + s)\right]u'\left(\overline{c}\right) < 0\end{aligned}$$

and $\beta^m(q)$ is a decreasing function of q.

As the function $\Psi(p, \beta, q)$ is continuously differentiable on $p \in [0, \widehat{p}]$, there exists an interior value $p \in (0, \widehat{p})$ such that $\Psi(p, \beta, q) = 0$ and at which $\Psi_p'(p, \beta, q) < 0$. Unicity of such a point is ensured when the function $\Psi(p, \beta, q)$ is convex in p. Indeed simple differentiation provides that

$$\begin{aligned}\Psi_p &= -\beta u''\left((1-p)\, a^1(p)\right)\left[(1-p)a_p^1 - a^1\right] \\ &\quad +(1-\beta)\frac{(1-\lambda)}{\lambda}\left[\begin{array}{c} q^b u''(T + a^{2b}(T) + s)\left(1 + a_T^{2b}\right) \\ +(1-q^b)u''(T + s) \end{array}\right]\frac{dT}{dp}\end{aligned}$$

and $dT/dp = \left[a^1(p) + pa_p^1(p)\right]\frac{1-\lambda}{\lambda}$. Then using (43) and (44), it is immediate to see that $\Psi_p(0, \beta, q) < 0$ and

$$\Psi_p(\widehat{p}, \beta, q) = -\beta u''(\overline{c})\left[(1-\widehat{p})a_p^1 - \frac{\overline{c}}{1-\widehat{p}}\right] + (1-\beta)\frac{(1-\lambda)}{\lambda}\left[\begin{array}{c} q^b u''(\widehat{T} + a^{2b}(\widehat{T}) + s)\left(1 + a_T^{2b}\right) \\ +(1-q^b)u''(\widehat{T} + s) \end{array}\right]\frac{dT}{dp} > 0$$

when $\widehat{p} > p^m = \arg\max_p T(p)$

- When $\Psi(p, \beta, q)$ is convex in p, Ψ_p is an increasing function of p and given that $\Psi_p(0, \beta, q) < 0$ and $\Psi_p(\widehat{p}, \beta, q) > 0$, there exists a unique $p_0 \in (0, \widehat{p})$ such that $\Psi_p(p_0, \beta, q) = 0$ with consequently $\Psi(.)$ is decreasing for $p \leq p_0$ and increasing for $p \geq p_0$, reaching the minimum at $\Psi(p_0, \beta, q)$. From this it follows that necessarily $\Psi(p_0, \beta, q) < 0$ (as we know that there exists an interior value $p \in (0, \widehat{p})$ such that $\Psi(p, \beta, q) = 0$). Consequently for all $p \in [p_0, \widehat{p}]$, $\Psi(p, \beta, q)$ takes negative value. As for $\beta \in \left(\underline{\beta}^n(q), \overline{\beta}(q)\right)$,we know that $\Psi(0, \beta, q) > 0$. Hence there exists a unique $p \in (0, p_0)$ such that $\Psi(p, \beta, q) = 0$. Also it is clear that at such point $p < p_0$, one has $\Psi_p'(p, \beta, q) < 0$.

- Finally regularity conditions for the function $\Psi(p, \beta, q)$ to be convex in p are $u''' \geq 0$, $v''' \geq 0$ and the tax revenue $T(p)$ sufficiently concave in the tax rate p.[64]

- The fact that for $\beta \in \left(\beta^m(q), \overline{\beta}(q)\right)$, $p(\beta, q)$ is a decreasing function of β and q comes immediately from the fact that $\Psi(p, \beta, q)$ is a decreasing function of β and q. **QED** ■

2) Consider now the "extractive" societal equilibrium policy. It is characterized by:

$$p \in \arg\max_{p\in[0,1-\overline{c}]} W(p, a^1, a^{2b}, pa^1\frac{1-\lambda}{\lambda}))$$

for given a^1, a^{2b}

$$a^1 = \frac{\overline{c}}{1-p};\quad a^{2b} = a^2(T),\ a^{2a} = 0 \text{ and } T = T(p) = \frac{p\overline{c}}{1-p}\frac{1-\lambda}{\lambda}$$

observe now that for $p \geq \widehat{p}$

$$W_p'(p, a^1(p), a^{2b}(T(p)), T(p)) = \frac{\overline{c}}{1-p} \cdot \widehat{\Psi}(p, \beta, q)$$

[64]Indeed differentiation provides:

$$\begin{aligned}\Psi_{pp} &= -\beta u_1''' \left[(1-p)a_p^1 - a^1\right]^2 - \beta u_1'' \left[-2a_p^1 + (1-p)a_{pp}^1\right] \\ &+(1-\beta)\frac{(1-\lambda)}{\lambda}\left[q^b u_{2b}'''\left(1+a_T^{2b}\right)^2 + q^b u_{2b}'' a_{TT}^{2b} + (1-q^b)u_{2a}'''\right]\left(\frac{dT}{dp}\right)^2 \\ &+(1-\beta)\frac{(1-\lambda)}{\lambda}\left[q^b u_{2b}''\left(1+a_T^{2b}\right) + (1-q^b)u_{2a}''\right]\frac{d^2T}{dp^2}\end{aligned}$$

Now it can be seen that when $u'' < 0$ and $u''' = 0$ and $0 < v'''$ and $2v'' + (1-p)v''' < 0$, one has $-2a_p^1 + (1-p)a_{pp}^1 > 0$ and $a_{TT}^{2b} > 0$, implying the convexity of Ψ.

with the "auxiliary" function $\widehat{\Psi}(p,\beta,q)$:

$$\begin{aligned}\widehat{\Psi}(p,\beta,q) &= -\beta u'(\overline{c}) \\ &\quad +(1-\beta)\frac{(1-\lambda)}{\lambda}\begin{bmatrix} qu'(T(p)+a^{2b}(T(p))+s) \\ +(1-q)u'(T(p)+s)\end{bmatrix}\end{aligned}$$

note that $\Psi(\widehat{p},\beta,q)=\widehat{\Psi}(\widehat{p},\beta,q)>0$ for all $\beta<\underline{\beta}(q)$. Moreover

$$\widehat{\Psi}_p(p,\beta,q)=(1-\beta)\frac{(1-\lambda)}{\lambda}\left[qu"(T(p)+a^{2b}(T(p))+s)(1+a_T^{2b})+(1-q)u"(T(p)+s)\right]\frac{dT}{dp}<0 \text{ for } p>\widehat{p}$$

Moreover

$$\widehat{\Psi}(1-\overline{c},\beta,q)=-\beta u'(\overline{c})+(1-\beta)\frac{(1-\lambda)}{\lambda}\begin{bmatrix} qu'((1-\overline{c})\frac{1-\lambda}{\lambda}+a^{2b}((1-\overline{c})\frac{1-\lambda}{\lambda})+s) \\ +(1-q)u'((1-\overline{c})\frac{1-\lambda}{\lambda}+s)\end{bmatrix}$$

which is positive if $(1-\beta)\frac{(1-\lambda)}{\lambda}\begin{bmatrix} qu'((1-\overline{c})\frac{1-\lambda}{\lambda}+a^{2b}((1-\overline{c})\frac{1-\lambda}{\lambda})+s) \\ +(1-q)u'((1-\overline{c})\frac{1-\lambda}{\lambda}+s)\end{bmatrix}>\beta u'(\overline{c})$ or

$$\beta\leq\underline{\beta}(q)=\frac{\frac{(1-\lambda)}{\lambda}\left[qu'((1-\overline{c})\frac{1-\lambda}{\lambda}+a^{2b}((1-\overline{c})\frac{1-\lambda}{\lambda})+s)+(1-q)u'((1-\overline{c})\frac{1-\lambda}{\lambda}+s)\right]}{u'(\overline{c})+\frac{(1-\lambda)}{\lambda}\left[qu'((1-\overline{c})\frac{1-\lambda}{\lambda}+a^{2b}((1-\overline{c})\frac{1-\lambda}{\lambda})+s)+(1-q)u'((1-\overline{c})\frac{1-\lambda}{\lambda}+s)\right]}$$

which is also decreasing in q and $\underline{\beta}(q)<\beta^m(q)$. Thus

when $\beta\leq\underline{\beta}(q)$, $\widehat{\Psi}(p,\beta,q)>0$ for all $p\in(\widehat{p},1-\overline{c})$ and the societal equilibrium policy is $p(\beta,q)=1-\overline{c}$.

For $\beta\in\left(\underline{\beta}(q),\beta^m(q)\right)$, there exists a unique $p(\beta,q)\in(\widehat{p},1-\overline{c})$ such that $\widehat{\Psi}(p(\beta,q),\beta,q)=0$. $p(\beta,q)$ is the extractive societal equilibrium policy.

Characterization of "Extractive" societal equilibrium policy: *For all* $q\in[0,1]$, *there exists* $\underline{\beta}(q)\in(0,1)$ with $\underline{\beta}(q)<\beta^m(q)$ such for $\beta<\beta^m(q)$ that there exists *a unique "Extractive" societal equilibrium* $p(\beta,q)\in[\widehat{p},1-\overline{c}]$. *It is characterized in the following way:*

- *For* $\beta\in\left(\underline{\beta}(q),\beta^m(q)\right)$ $p(\beta,q)\in(\widehat{p},1-\overline{c})$ and is a *decreasing function in* β *and* q. Workers are at their survival consumption constraint and provide an equilibrium effort $\frac{\overline{c}}{1-p(\beta,q)}<1$

- *For* $\beta\leq\underline{\beta}(q)$

, $p(\beta,q)=1-\overline{c}$. Workers are at their survival consumption constraint and provide an equilibrium effort equal to 1.

- **Societal commitment equilibrium policy:**

Denote in the same way :

$$\begin{aligned}\widetilde{W}(p,\beta,q) &= \beta\left[u((1-p)a^1(p)) + v(1-a^1(p))\right] \\ &+(1-\beta)\left[\begin{array}{c} q\cdot\left(u(T(p)+a^{2b}(T(p)+s)+v(1-a^{2b}(T(p)))\right) \\ +(1-q)\cdot(u(T(p)+s)+\theta v(1)) \end{array}\right]\end{aligned}$$

with

$$T(p) = pa^1(p)\frac{1-\lambda}{\lambda}$$

The *societal commitment equilibrium* $p^{com}(\beta,q)$ for any value of $\beta\in[0,1]$ is the solution of the following program:

$$p \in \arg\max_p \widetilde{W}(p,\beta,q)$$

Note that the function $\widetilde{W}(p,\beta,q)$ takes different shapes depending on whether we are in a "non extractive" or an "extractive" regime. More precisely, we have:

- for $p<\widehat{p}$, the "non extractive" objective funciton $\widetilde{W}(p,\beta,q)=\widetilde{W}^{nm}(p,\beta,q)$ given by

$$\begin{aligned}\widetilde{W}^{nm}(p,\beta,q) &= \beta\left[u((1-p)a^1_{op}(p)) + v(1-a^1_{op}(p))\right] \\ &+(1-\beta)\left[\begin{array}{c} q\cdot\left(u(T(p)+a^{2b}(T(p)+s)+v(1-a^{2b}(T(p)))\right) \\ +(1-q)\cdot(u(T(p)+s)+\theta v(1)) \end{array}\right]\end{aligned}$$

and $T(p)=pa^1_{op}(p)\frac{1-\lambda}{\lambda}$.

- for $p>\widehat{p}$, we have the "extractive" objective function $\widetilde{W}(p,\beta,q)=\widetilde{W}^{m}(p,\beta,q)$ given by

$$\begin{aligned}\widetilde{W}^{m}(p,\beta,q) &= \beta\left[u(\overline{c}) + v(1-\frac{\overline{c}}{1-p})\right] \\ &+(1-\beta)\left[\begin{array}{c} q\cdot\left(u(T^m(p)+a^{2b}(T(p)+s)+v(1-a^{2b}(T(p)))\right) \\ +(1-q)\cdot(u(T(p)+s)+\theta v(1)) \end{array}\right]\end{aligned}$$

with now $T(p)=\frac{p\overline{c}}{1-p}\frac{1-\lambda}{\lambda}$. We then get the following characterization of the *societal commitment equilibrium policy*:

Characterization of societal commitment equilibrium policy:

When $T(p)=a^1_{op}(p)p$ is concave and reaching its maximum at some value $p^{\max}>\widehat{p}$. Then $p=p^{com}(\beta,q)$ is a well defined function characterized in the following way. For all $q\in[0,1]$, there *exists a threshold* $\widehat{\beta}(q)$ with $\widehat{\beta}(q)<\overline{\beta}(q)$ such that

- *For $\beta\in\left[0,\widehat{\beta}(q)\right)$ $p^{com}(\beta,q)=p_E(\beta,q)\in[\widehat{p},1-\overline{c}]$ and is a decreasing function of β* and q. We are in the extractive regime

- For $\beta\in(\widehat{\beta}(q),\overline{\beta}(q))$, $p^{com}(\beta,q)=p_{NE}(\beta,q)\in(0,\widehat{p})$ *and is a decreasing function of β* and q. We are in the "non extractive" regime

- For $\beta \geq \overline{\beta}(q)$, $p^{com}(\beta, q) = 0$
- *For* $\beta = \widehat{\beta}(q)$, $p^{com}(\beta, q)$ can be any randomization between $p_E(\beta, q)$ and $p_{NE}(\beta, q)$.
- The thresholds $\widehat{\beta}(q)$ is decreasing in q.

Proof:
i) Consider first the "non extractive" objective function $\widetilde{W}^{NE}(p, \beta, q)$. It is given by

$$\begin{aligned}\widetilde{W}^{NE}(p,\beta,q) &= \beta\left[u((1-p)a^1_{op}(p)) + v(1-a^1_{op}(p))\right] \\ &\quad +(1-\beta)\left[\begin{array}{c} q\cdot\left(u(T(p)+a^{2b}(T(p)+s)+v(1-a^{2b}(T(p))\right) \\ +(1-q)\cdot(u(T(p)+s)+\theta v(1)) \end{array}\right]\end{aligned}$$

with $T(p) = pa^1_{op}(p)\frac{1-\lambda}{\lambda}$. Note that

$$\begin{aligned}\widetilde{W}^{NE}_p &= -\beta u'\left((1-p)\,a^1_{op}(p)\right)\cdot a^1_{op}(p) \\ &\quad +(1-\beta)\frac{(1-\lambda)}{\lambda}\left[qu'(T(p)+a^{2b}(T(p))+s)+(1-q)u'(T(p)+s)\right]\frac{d\left[pa^1_{op}(p)\right]}{dp}\end{aligned}$$

and when $T(p)$ is concave, it is straightforward to see that $\widetilde{W}^{NE}(p, \beta, q)$ is a strictly concave function of p.

ii) It is also a simple matter to observe that $\widetilde{W}^{NE}_p(0, \beta, q) \leq 0$ if and only if $\beta \geq \overline{\beta}(q)$. When $\beta \geq \overline{\beta}(q)$ as $\widetilde{W}^{NE}(p, \beta, q)$ is a concave function of p, we have that $\widetilde{W}^{NE}_p(p, \beta, q) < \widetilde{W}^{NE}_p(0, \beta, q) \leq 0$,and therefore $\widetilde{W}^{NE}(p, \beta, q)$ reaches its maximum at $p_{NE}(\beta, q) = 0$. For $\beta < \overline{\beta}(q)$, one has $\widetilde{W}^{NE}_p(0, \beta, q) > 0$. Moreover $\widetilde{W}^{NE}_p(p^{\max}, \beta, q) < 0$. Hence there is a unique value $p_{NE}(\beta, q) \in (0, p^{\max})$ at which $\widetilde{W}^{NE}(p, \beta, q)$ reaches a maximum and it is determined by the first order condition $\widetilde{W}^{NE}_p(p, \beta, q) = 0$. Obviously as $\widetilde{W}^{NE}_\beta(p, \beta, q) < 0$ and $\widetilde{W}^{NE}_q(p, \beta, q) < 0$, it follows immediately that $p_{NE}(\beta, q)$ is decreasing both in β and q.

iii) Consider next the "Extractive" objective function $\widetilde{W}(p, \beta, q) = \widetilde{W}^E(p, \beta, q)$ given by

$$\begin{aligned}\widetilde{W}^{E}(p,\beta,q) &= \beta\left[u(\overline{c}) + v(1-\frac{\overline{c}}{1-p})\right] \\ &\quad +(1-\beta)\left[\begin{array}{c} q\cdot\left(u(T^m(p)+a^{2b}(T(p)+s)+v(1-a^{2b}(T(p))\right) \\ +(1-q)\cdot(u(T(p)+s)+\theta v(1)) \end{array}\right]\end{aligned}$$

with now $T(p) = \frac{p\overline{c}}{1-p}\frac{1-\lambda}{\lambda}$. Differentiation of $\widetilde{W}^E(p, \beta, q)$ gives as well :

$$\widetilde{W}^E_p = \Omega\left(p,\beta,q)\right)\cdot\frac{\overline{c}}{(1-p)^2}$$

with

$$\begin{aligned}\Lambda(p,\beta,q)) &= -\beta v'(1-\frac{\overline{c}}{1-p}) \\ &+(1-\beta)\frac{1-\lambda}{\lambda}\left[q\cdot\left(u'(T(p)+a^{2b}(T(p)+s)\right)+(1-q)\cdot\left(u'(T(p)+s)\right)\right]\end{aligned}$$

and $\widetilde{W}_p^m \gtrless 0$ if and only if $\Lambda(p,\beta,q)) \gtrless 0$.

It is easy to see that $\Lambda(p,\beta,q))$ is decreasing in p. Moreover for $\beta > \overline{\beta}_{\max}(q)$ such that

$$\beta > \overline{\beta}_{\max}(q) = \frac{\frac{1-\lambda}{\lambda}\left[q\cdot\left(u'(a^{2b}(0)+s)\right)+(1-q)\cdot(u'(s))\right]}{v'(1-\overline{c})+\frac{1-\lambda}{\lambda}\left[q\cdot(u'(a^{2b}(0)+s))+(1-q)\cdot(u'(s))\right]}$$

we have $\Lambda(0,\beta,q)) < 0$ and therefore $\widetilde{W}_p^E < 0$ for all $p\in[0,1-\overline{c}]$. Note as well that $\lim_{p\to 1-\overline{c}}\Lambda(p,\beta,q)) = -\infty$ for $\beta > 0$. Hence when $\beta \leq \overline{\beta}_{\max}(q)$, $\Lambda(0,\beta,q)) > 0$ and there is a unique $p = p_E(\beta,q) < 1-\overline{c}$ such that $\Lambda(p_E,\beta,q)) = 0$ and the "extractive" objective function is maximized at this point $p_E(\beta,q)$. Moreover $p_E(\beta,q)$ is decreasing in β and q. From this it follows that for all β, q such that $p_E(\beta,q) \leq \widehat{p}$, necessarily $\widetilde{W}_p^E \leq 0$ in the extractive region $p > \widehat{p}$. This last condition can be stated as $\beta \geq \overline{\beta}_M(q) \in [0,1]$ with necessarily $\overline{\beta}_M(q) < \overline{\beta}_{\max}(q)$. Conversely for $\beta \leq \overline{\beta}_M(q) < \overline{\beta}_{\max}(q)$, the extractive objective function $\widetilde{W}_p^E$ is maximized at some $p_E(\beta,q)\in[\widehat{p},1-\overline{c}]$ in the extractive policy region. Note as well that $\lim_{\beta\to 0}p_E(\beta,q) = 1-\overline{c}$.

iv) One may compute also the right side and left side derivative of the social objective function at the borderline of the "extractive" and the "non extractive" regimes $p=\widehat{p}$. For $p=\widehat{p}^-$, we have:

$$\begin{aligned}\widetilde{W}_p^- &= \widetilde{W}_p^{NE}(\widehat{p}) = -\beta u'(\overline{c})\frac{\overline{c}}{(1-\widehat{p})} \\ &+(1-\beta)\frac{(1-\lambda)}{\lambda}\left[qu'(T(\widehat{p})+a^{2b}(T(\widehat{p}))+s)+(1-q)u'(T(\widehat{p})+s)\right]\left[\frac{\overline{c}}{1-\widehat{p}}+\widehat{p}\frac{da^1_{op}(\widehat{p})}{dp}\right]\end{aligned}$$

while for $p=\widehat{p}^+$,

$$\begin{aligned}\widetilde{W}_p^+ &= \widetilde{W}_p^E(\widehat{p}) = \beta\left[-v'(1-\frac{\overline{c}}{1-\widehat{p}})\frac{\overline{c}}{(1-\widehat{p})^2}\right] \\ &+(1-\beta)\frac{1-\lambda}{\lambda}\left[q\cdot\left(u'(T(\widehat{p})+a^{2b}(T(\widehat{p})+s)\right)+(1-q)\cdot\left(u'(T(\widehat{p})+s)\right)\right]\frac{\overline{c}}{(1-\widehat{p})^2}\end{aligned}$$

Using the fact that $u'(\overline{c})(1-\widehat{p}) = v'(1-\frac{\overline{c}}{1-\widehat{p}})$, it follows that

$$\begin{aligned}\widetilde{W}_p^- &= \widetilde{W}_p^+ \\ &+(1-\beta)\frac{(1-\lambda)}{\lambda}\left[qu'(T(\widehat{p})+a^{2b}(T(\widehat{p}))+s)+(1-q)u'(T(\widehat{p})+s)\right]\underbrace{(-\frac{\widehat{p}\overline{c}}{1-\widehat{p}}+\widehat{p}\frac{da^1(\widehat{p})}{dp})}_{-} \\ &< \widetilde{W}_p^+\end{aligned}$$

Note also that $\widetilde{W}_p^- > 0$ when

$$\beta < \overline{\beta}_c(q) = \frac{\frac{(1-\lambda)}{\lambda}\left[qu'(T(\widehat{p}) + a^{2b}(T(\widehat{p})) + s) + (1-q)u'(T(\widehat{p}) + s)\right]\left[\frac{\overline{c}}{1-\widehat{p}} + \widehat{p}\frac{da^1(\widehat{p})}{dp}\right]}{u'(\overline{c})\frac{\overline{c}}{(1-\widehat{p})} + \frac{(1-\lambda)}{\lambda}\left[qu'(T(\widehat{p}) + a^{2b}(T(\widehat{p})) + s) + (1-q)u'(T(\widehat{p}) + s)\right]\left[\frac{\overline{c}}{1-\widehat{p}} + \widehat{p}\frac{da^1(\widehat{p})}{dp}\right]}$$

with $\overline{\beta}_c(q) < \overline{\beta}_M(q)$. This discussion tells us that the social policy objective function $\widetilde{W}(p, \beta, q)$ is a continuously differentiable function on the interval $p \in [0, \widehat{p})$ and $(\widehat{p}, 1-\overline{c}]$ but not at $\widehat{p}$. Hence it is not necessarily a concave function of p. More precisely we have the following characterization for the global optimum:

v) Collecting the previous information, we have the following:

- For $\beta \leq \overline{\beta}_c(q)$, given that $\widetilde{W}_p^- = \widetilde{W}_p^{NE}(\widehat{p}, \beta, q) > 0$ and that the function $\widetilde{W}_p^{NE}(p, \beta, q)$ is a concave function of p in the interval $[0, \widehat{p}]$, it follows that for all $p \in [0, \widehat{p}]$ $\widetilde{W}_p^{NE}(p, \beta, q) > 0$ and therefore the global optimum can only be in the "extractive" region $[\widehat{p}, 1-\overline{c}]$. Therefore $p^{com}(\beta, q) = p_m(\beta, q) \in [\widehat{p}, 1-\overline{c}]$.

- Similarly for $\beta \geq \overline{\beta}_M(q)$, the optimum policy $p_E(\beta, q)$ of the "extractive" objective function $\widetilde{W}_p^E(p, \beta, q)$ is smaller than $\widehat{p}$, Hence in the "extractive" region $p \in [\widehat{p}, 1-\overline{c}]$, necessarily $\widetilde{W}_p^E \leq 0$, therefore the global optimum can only be in the "non extractive region $[0, \widehat{p})$. As a consequence, $p^{com}(\beta, q) = p_{NE}(\beta, q) \in [0, \widehat{p})$ with $p_{NE}(\beta, q) = \arg\max_{[0,\widehat{p}]} \widetilde{W}_p^{NE}(p, \beta, q)$.

- Finally consider the last case where $\beta \in \left(\overline{\beta}_c(q), \overline{\beta}_M(q)\right)$. In such a situation, we need to compare $\max_{p \leq \widehat{p}} \widetilde{W}^{NE}(p, \beta, q)$ to $\max_{\widehat{p} \leq p \leq 1-\overline{c}} \widetilde{W}^E(p, \beta, q)$ to characterize the global optimum. For this consider the function

$$\Delta(\beta, q) = \widetilde{W}^{NE}(p_{NE}(\beta, q), \beta, q) - \widetilde{W}^E(p_E(\beta, q), \beta, q)$$

in the interval $\beta \in \left[\overline{\beta}_c(q), \overline{\beta}_M(q)\right]$. We see that $\Delta\left(\overline{\beta}_c(q), q\right) < 0$ while $\Delta\left(\overline{\beta}_M(q), q\right) > 0$. Moreover differentiation of $\Delta(\beta, d)$ gives:

$$\begin{aligned}
\Delta_\beta(\beta, q) &= \widetilde{W}_\beta^{NE}(p_{NE}(\beta, q), \beta, q) - \widetilde{W}_\beta^E(p_E(\beta, q), \beta, q) \qquad (46)\\
&= u((1-p_{NE})a_{op}^1(p_{NE})) + v(1 - a_{op}^1(p_{NE}))\\
&\quad - \left[\begin{array}{c} q \cdot \left(u(T_{op}(p_{NE}) + a^{2b}(T_{op}(p_{NE}) + s) + v(1 - a^{2b}(T_{op}(p_{NE})))\right) \\ +(1-q) \cdot (u(T_{op}(p_{NE}) + s) + \theta v(1)) \end{array}\right]\\
&\quad - \left[u(\overline{c}) + v(1 - \frac{\overline{c}}{1-p_E})\right]\\
&\quad + \left[\begin{array}{c} q \cdot \left(u(T_E(p_E) + a^{2b}(T_E(p_E) + s) + v(1 - a^{2b}(T_E(p_E)))\right) \\ +(1-q) \cdot (u(T_E(p_E) + s) + \theta v(1)) \end{array}\right]
\end{aligned}$$

with $T_{op}(p) = \frac{1-\lambda}{\lambda} p a_{op}^1(p)$, $T_E(p) = \frac{1-\lambda}{\lambda}\frac{\overline{c}p}{1-p}$. Note that $p_{NE}(\beta, q) < \widehat{p} < p_E(\beta, q)$ and as well that $\widehat{p} < p^{\max}$ where $p^{\max} = \arg\max_p p a_{op}^1(p)$. Also we have $u((1-p_{NE})a_{op}^1(p_{NE})) + v(1-$

$a^1_{op}(p_{NE})) > u(\overline{c}) + v(1 - \frac{\overline{c}}{1-p_E})$ and that

$$\begin{aligned} T_{op}(p_{NE}) &= p_{NE} a^1_{op}(p_{NE}) \frac{1-\lambda}{\lambda} \\ &< \widehat{p} a^1_{op}(\widehat{p}) \frac{1-\lambda}{\lambda} \\ &= \frac{\widehat{p}\widehat{c}}{1-\widehat{p}} \frac{1-\lambda}{\lambda} \\ &< \frac{p_E \overline{c}}{1-p_E} \frac{1-\lambda}{\lambda} \\ &= T_E(p_E) \end{aligned}$$

From this it follows that in (46) the term

$$q \cdot \left(u(T_{op}(p_{NE}) + a^{2b}(T_{op}(p_{NE}) + s) + v(1 - a^{2b}(T_{op}(p_{NE}))\right) \\ +(1-q) \cdot (u(T_{op}(p_{NE}) + s) + \theta v(1))$$

is smaller than the term

$$q \cdot \left(u(T_E(p_E) + a^{2b}(T_E(p_E) + s) + v(1 - a^{2b}(T_E(p_E))\right) \\ +(1-q) \cdot (u(T_E(p_E) + s) + \theta v(1))$$

Therefore $\Delta_\beta(\beta, q) > 0$ and there exists a unique threshold $\widehat{\beta}(q) \in \left(\overline{\beta}_c(q), \overline{\beta}_M(q)\right)$ such that $\Delta\left(\widehat{\beta}(q), q\right) = 0$ and $\Delta_\beta\left(\widehat{\beta}(q), q\right) > 0$. Moreover differentiation by q provides

$$\begin{aligned} \Delta_q(\beta, q) &> 0 = \widetilde{W}^{NE}_q(p_{NE}(\beta, q), \beta, q) - \widetilde{W}^{E}_q(p_E(\beta, q), \beta, q) \\ &= (1-\beta) \left[\begin{array}{c} \left[\left(\begin{array}{c} u(T_{op}(p_{NE}) + a^{2b}(T_{op}(p_{NE}) + s) + v(1 - a^{2b}(T_{op}(p_{NE})) \\ -(u(T_E(p_E) + a^{2b}(T_E(p_E) + s) + v(1 - a^{2b}(T_E(p_E)) \end{array} \right)\right] \\ -\left[(u(T_{op}(p_{NE}) + s) + \theta v(1)) - (u(T_E(p_E) + s) + \theta v(1))\right] \end{array} \right] \end{aligned}$$

Denote

$$\Sigma(T) = u(T+s) - \left[u(T + a^{2b}(T) + s) + v(1 - a^{2b}(T))\right]$$

Then it follows that $\Sigma'(T) = u'(T+s) - u'(T + a^{2b}(T) + s) > 0$. As $\Delta_q(\beta, q) = (1-\beta)\left[\Sigma(T_E(p_E)) - \Sigma(T_{op}(p_{NE})\right.$ and that $T_{op}(p_{NE}) < T_E(p_E)$, it follows that $\Delta_q(\beta, q) > 0$ and therefore $\widehat{\beta}(q)$ is decreasing in q.

The previous discussion implies that for $\beta \in \left(\overline{\beta}_c(q), \widehat{\beta}(q)\right)$ the global optimum is in the "extractive" region $[\widehat{p}, 1-\overline{c}]$. with $p^{com}(\beta, q) = p_E(\beta, q) \in (\widehat{p}, 1-\overline{c}]$, while for $\beta \in \left(\widehat{\beta}(q), \overline{\beta}_M(q)\right)$ the global optimum is in the "non extractive" region $[0, \widehat{p}]$. with $p^{com}(\beta, q) = p_{NE}(\beta, q) \in (0, \widehat{p})$

Finally at $\beta = \widehat{\beta}(q)$, the objective function is maximized both at $p_E(\widehat{\beta}(q), q) > \widehat{p}$ and at $p_{NE}(\widehat{\beta}(q), q) < \widehat{p}$. The optimal policy can be any randomization between these two policies.

Collecting all the results in **i)**, **ii)** and **v)** gives the characterization of the *societal commitment equilibrium policy.* **QED.**

- **Comparaison between $p(\beta,q)$ and $p^{com}(\beta,q)$:**

At $\beta \geq \widehat{\beta}\,(q)$ we have the *societal equilibrium policy* $p(\beta,q) < \widehat{p}$ and at such point $\widetilde{W}_p^{NE}(p(\beta,q,)\beta,q)$ is equal to

$$(1-\beta)\cdot\left[qu'(T(p)+a^{2b}(T(p)+s)+(1-q)u'(T(p)+s)\right]p\cdot a_p^1 < 0$$

Hence $p(\beta,q) > p^{com}(\beta,q)$.

On the other hand for $\beta \in \left(0,\widehat{\beta}\,(q)\right)$ the *societal equilibrium policy* $p(\beta,q) > \widehat{p}$ and determined by the equation

$$\beta u'\left(\overline{c}\right) = (1-\beta)\frac{(1-\lambda)}{\lambda}\left[\begin{array}{c} qu'(T(p)+a^{2b}(T(p))+s) \\ +(1-q)u'(T(p)+s) \end{array}\right]$$

at such point $\widetilde{W}_p^E$ is equal to

$$\widetilde{W}_p^E = \Omega\left(p(\beta,q),\beta,q)\right)\frac{\overline{c}}{\left(1-p(\beta,q)\right)^2}$$

with $\Lambda\left(p,\beta,q)\right)$ given by

$$\begin{aligned}\Lambda\left(p,\beta,q)\right) &= -\beta v'(1-\frac{\overline{c}}{1-p}) \\ &+(1-\beta)\frac{1-\lambda}{\lambda}\left[q\cdot\left(u'(T(p)+a^{2b}(T(p)+s)\right)+(1-q)\cdot\left(u'(T(p)+s)\right)\right]\end{aligned}$$

Thus at such point $p(\beta,q)$ one has:

$$\Lambda\left(p(\beta,q),\beta,q)\right) = \beta\left[u'\left(\overline{c}\right)-v'(1-\frac{\overline{c}}{1-p})\right]$$

Denote p^e such that $u'\left(\overline{c}\right) = v'(1-\frac{\overline{c}}{1-p^e})$.Such point exists as the RHS of the equation is increasing in p taking value $v'(1-\overline{c}) < u'(\overline{c})$ at $p=0$ and value $v'(0)=+\infty$ at $p=1-\overline{c}$. Moreover

$$v'(1-\frac{\overline{c}}{1-p^e}) = u'\left(\overline{c}\right) > u'\left(\overline{c}\right)(1-\widehat{p}) = v'(1-\frac{\overline{c}}{1-\widehat{p}})$$

and thus $p^e > \widehat{p}$. Note that there exists a unique value $\beta^e\,(q) \in \left(\underline{\beta}(q),\min\left\{\beta^m(q),\widehat{\beta}(q)\right\}\right)$ such that $p(\beta^e,q) = p^e = p_E(\beta^e,q) = p^{com}(\beta^e,q)$. Note as well that the two curves $p(\beta,q)$ and $p^{com}(\beta,q)$ can only cross at that value p^e and therefore at this point β^e in the interval $\left(0,\min\left\{\beta^m(q),\widehat{\beta}(q)\right\}\right)$. Then it follows that for ϵ small enough $\beta=\beta^e+\epsilon$, one has $p(\beta^e+\epsilon,q) < p^e$. Therefore

$$\Lambda\left(p(\beta^e+\epsilon,q),\beta^e+\epsilon,q)\right) > 0 = \Lambda\left(p_E(\beta^e+\epsilon,q),\beta^e+\epsilon,q)\right)$$

and $p(\beta^e+\epsilon,q) < p_E(\beta^e+\epsilon,q)$ because $\Lambda(p,\beta,q)$ is a decreasing function of p. From this at the value point $\beta=\beta^e+\epsilon < \min\left\{\beta^m(q),\widehat{\beta}(q)\right\}$ such that $p(\beta^e+\epsilon,q) < p_m(\beta^e+\epsilon,q) = p^{com}(\beta^e+\epsilon,q)$.

By continuity of the functions $p(\beta,q)$ and $p^{com}(\beta,q)$ on the interval $(0,\beta^*(q))$, and given that there is at most one crossing point in the interval $\left(0,\min\left\{\beta^m(q),\widehat{\beta}(q)\right\}\right)$ the curve $p(\beta,q)$ has to be below the curve $p^{com}(\beta,q)$ for all points $\beta\in(\beta^e,\ \widehat{\beta}(q))$.

Similarly for the set of points $\beta\in\left[\underline{\beta}(q),\beta^e\right]$, the curve $p(\beta,q)$ has to be above the curve $p^{com}(\beta,q)$. Moreover for the set of points $\beta\in\left(0,\underline{\beta}(q)\right)$, $p(\beta,q)=1-\overline{c}>p_E(\beta,q)=p^{com}(\beta,q)$. From this it follows the comparison between $p(\beta,q)$ and $p^{com}(\beta,q)$:

- For $\beta\in(0,\beta^e(q))$ one has $p(\beta,q)>p^{com}(\beta,q)$
- For $\beta\in(\beta^e(q),\widehat{\beta}(q))$ one has $p(\beta,q)<p^{com}(\beta,q)$
- For $\beta\in(\widehat{\beta}(q),1)$ one has $p(\beta,q)>p^{com}(\beta,q)$

Finally note that $\beta^e(q)$ is given by

$$p^e=1-\frac{\overline{c}}{1-v'^{-1}(u'(\overline{c}))}=p(\beta^e(q),q)$$

and thus $\beta^e(q)$ is decreasing in q.**QED**

- **Cultural Dynamics :**

The parameters of cultural intolerance of the two types of elite members (bourgeois and aristocrats) can be written simply as:

$$\begin{aligned}\Delta V^b(p) &= u\left(T(p)+a^{2b}(T(p))+s\right)+v(1-a^{2b}(T(p)))\\ &\quad -\left[u\left(T(p)+s\right)+v(1)\right]\end{aligned}$$

$$\begin{aligned}\Delta V^a(p) &= u\left(T(p)+s\right)+\theta v(1)\\ &\quad -\left[u\left(T(p)+a^{2b}(T(p))+s\right)+\theta v(1-a^{2b}(T(p))\right]\end{aligned}$$

We then have

$$\Delta V^{b\prime}(p)=\left[u'\left(T(p)+a^{2b}(T(p))+s\right)-u'\left(T(p)+s\right)\right]\frac{dT}{dp}$$

The bracket term is negative and therefore $\Delta V^{b\prime}(p)$ has the sign opposite to dT/dp.

Similarly

$$\Delta V^{a\prime}(p)=\left[\begin{array}{c}\left[u'\left(T(p)+s\right)-u'\left(T(p)+a^{2b}(T(p))+s\right)\right]\\ +(\theta-1)v'(1-a^{2b}(T(p)))a_T^{2b}\end{array}\right]\cdot\frac{dT}{dp}$$

The first term inside the bracket is positive while the second term is negative as $a_T^{2b} < 0$ However the larger the degree of concavity of $v(.)$, the smaller is $\left\|a_T^{2b}\right\|$. It follows that when $|v''|$ large enough, the bracket term has also positive sign and therefore $\Delta V^{a\prime}(p)$ has the sign of dT/dp

From this discussion, it follows that when $|v''|$ is large enough, $\Delta V^b/\Delta V^a$ has the same sign of variation as $-dT/dp$. As for both in the "non extractive" region (ie. $p \leq \widehat{p}$) and in the "extractive" region (ie. $p \geq \widehat{p}$) we have $dT/dp > 0$, then $\Delta V^b(p)/\Delta V^a(p)$ is decreasing in p. Now given that $p(\beta, q)$ is a decreasing function of β and q, that $p(0, q) = 1 - \overline{c}$, that $p(\overline{\beta}(q), q) = 0$, then for $\beta \in \left[0, \overline{\beta}(q)\right]$, we get that $\Delta V^b/\Delta V^a$ is increasing in β and in q, Moreover for $\beta \geq \overline{\beta}(q)$, $p(\beta, q) = 0$ and $\Delta V^b(p)/\Delta V^a(p)$ is a constant given by

$$\frac{\Delta V^b(0)}{\Delta V^a(0)} = \frac{u\left(a^{2b}(0) + s\right) - u\left(s\right) - \left[v(1) - v(1 - a^{2b}(0))\right]}{\theta\left[v(1) - v(1 - a^{2b}(0))\right] - \left[u\left(a^{2b}(0) + s\right) - u\left(s\right)\right]}$$

Cultural steady states are determined by:

$$\frac{\Delta V^b(p(\beta, q))}{\Delta V^a(p(\beta, q))} = \frac{q}{1 - q} \tag{47}$$

In the region $\beta > \overline{\beta}(q)$, there is a unique solution q^* of equation (47) independent from β (as $\Delta V^b/\Delta V^a$ is just a constant in that region). In the region $\beta \in \left[0, \overline{\beta}_1(q)\right]$, given that $p(\beta, q)$ is decreasing in q, there could exist more than one value of q satisfying equation (47) in the relevant range of parameter β. Note however that the LHS of (47) is always a strictly positive and bounded continuous function of $q \in [0, 1]$ while the RHS is a continous function vanishing to 0 at $q = 0$ and unbounded at $q = 1$. Hence by continuity, there is always one value $q(\beta)$ that satisfies (47) and is necessarily increasing in β. [65]. Finally the point q^* such that the curve $q\left(\beta\right)$ crosses $\overline{\beta}\left(q\right)$, by continuity is determined by

$$\frac{\Delta V^b(0)}{\Delta V^a(0)} = \frac{q^*}{1 - q^*}$$

QED.

[65] If there are more than one solution, there exists an odd number $2k + 1$ of solutions $\left(q_{2j+1}(\beta)\right)_{j \in [0,k]}$ with $q_{2j-1}(\beta)$ increasing in β and $q_{2j}(\beta)$ decreasing in β for $j \in [0, k]$.

Civic culture and Institutions

Recall the preference profiles of the different agents write as :

$$\text{civic minded workers:} \quad U^{1c}\left(e^{1c}, a^{1c}, p, E\right) = \omega - p + v(p(1-\mu)) + \kappa E + \left[p(1-\mu)\right] e^{1c} - \phi_E \frac{\left(e^{1c}\right)^{1+\epsilon_E}}{1+\epsilon_E} - \alpha\left(\mu p\right)\left(1 - a^{1c}\right) - \phi_A \frac{\left(a^{1c}\right)^{1+\epsilon_A}}{1+\epsilon_A}$$

$$\text{passive worker:} \quad U^{1p}\left(e^{1p}, a^{1p}, E\right) = \omega - p + v(p(1-\mu)) + \kappa E - \phi_E \frac{\left(e^{1p}\right)^{1+\epsilon_E}}{1+\epsilon_E} - \phi_A \frac{\left(a^{1p}\right)^{1+\epsilon_A}}{1+\epsilon_A}$$

$$\text{Elite member :} \quad U^2(p, E, A) = \omega - p + \mu p(1 - \theta \cdot \left(\lambda q a^{1c}\right)) + v(p(1-\mu)) + \kappa E$$

Therefore the optimal action of a "civic minded" worker is obtained from

$$\max_{e^{1c}, a^{1c}} U^{1c}\left(e^{1c}, a^{1c}, p, E\right)$$

which provides the optimal behaviors

$$e^{1c}(p) = \left[p \frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}} \quad \text{and} \quad a^{1c}(p) = \left[\frac{\alpha \cdot (\mu p)}{\phi_A}\right]^{\frac{1}{\epsilon_A}} \tag{48}$$

- **Characterization of the societal equilibrium** $p\left(\beta, q\right)$:

Denote next for given institutions β, the policy maker objective function as:

$$\begin{aligned} W(p, A, E, e^{1c}, e^{1p}, a^{1c}, a^{1p}) &= \beta\left\{qU^{1c} + (1-q)U^{1p}\right\} + (1-\beta)U^2 \\ \text{with } A &= \lambda \cdot \left[q \cdot a^{1c} + (1-q) \cdot a^{1p}\right] \\ E &= \lambda \cdot \left[q \cdot e^{1c} + (1-q) \cdot e^{1p}\right] \end{aligned}$$

The first order condition of the policymaker for an interior solution writes as

$$W_p(p, A, E, e^{1c}, e^{1p}, a^{1c}, a^{1p}) = 0$$

or

$$-1 + (1-\mu)v'(p(1-\mu)) + \beta q\left[(1-\mu)e^{1c} - \mu\alpha(1 - a^{1c})\right] + (1-\beta)\mu\left[1 - \theta \cdot \lambda q a^{1c}\right] = 0$$

Substitution of the optimal individual behaviors of the private agents provides the condition for an interior *societal equilibrium policy* $p\left(\beta, q\right)$ $\Psi(p, \beta, q) = 0$ with

$$\begin{aligned}\Psi(p,\beta,q) &= -1+(1-\mu)v'(p(1-\mu))+\beta q\left[(1-\mu)e^{1c}(p)-\mu\alpha(1-a^{1c}(p))\right] \\ &\quad +(1-\beta)\mu\left[1-\theta\cdot\lambda q a^{1c}(p)\right]\end{aligned}$$

$\Psi(0,\beta,q)=\infty$ as $\nu'(0)=\infty$ and $\Psi(\omega,\beta,q)<0$ for large enough ω when ϕ_E large enough that the following condition is satisfied . $v'(\omega(1-\mu))<\left[1-\left[\omega\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}}\right]\cdot(1-\mu)$.

As well substitution of (48) gives immediately that

$$\begin{aligned}\Psi_p(p,\beta,q) &= (1-\mu)\left(\beta q\frac{1}{\epsilon_E}\left[\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}}p^{\frac{1}{\epsilon_E}-1}+(1-\mu)v''(p(1-\mu))\right) \\ &\quad +\beta\mu q\alpha\frac{1}{\epsilon_A}\left[\frac{\mu\alpha}{\phi_A}\right]^{\frac{1}{\epsilon_A}}p^{\frac{1}{\epsilon_A}-1}-(1-\beta)\theta\cdot\lambda q\mu\frac{1}{\epsilon_A}\left[\frac{\mu\alpha}{\phi_A}\right]^{\frac{1}{\epsilon_A}}p^{\frac{1}{\epsilon_A}-1}\end{aligned}$$

Hence Ψ_p is negative when $\nu(.)$ concave enough and ϕ_E and ϕ_A are large enough. From this it follows that there exists a unique *societal equilibrium policy* $p(\beta,q)\in(0,\omega)$ when $\nu(.)$ is concave enough and ω, ϕ_E and ϕ_A are large enough.

Differentiation provides then immediately that

$$\begin{aligned}\Psi_\beta &= q\left[(1-\mu)e^{1c}(p)-\mu\alpha(1-a^{1c}(p))\right]-\mu\left[1-\theta\cdot\lambda q a^{1c}(p)\right] \\ \Psi_q &= \beta\left[(1-\mu)e^{1c}(p)-\mu\alpha(1-a^{1c}(p)\right]-(1-\beta)\theta\cdot\lambda\mu a^{1c}(p)\end{aligned} \tag{49}$$

Now it can be seen that

$$(1-\mu)e^{1c}(p)-\mu\alpha\cdot(1-a^{1c}(p))=(1-\mu)\left[p\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}}-\mu\alpha\cdot(1-\left[\frac{\alpha\cdot(\mu p)}{\phi_A}\right]^{\frac{1}{\epsilon_A}})$$

consequently $(1-\mu)e^{1c}(p)-\mu\alpha\cdot(1-a^{1c}(p))<0$ when ϕ_E and ϕ_A are large enough that the following condition $(1-\mu)\left[\omega\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}}+\mu\alpha\left[\frac{\alpha\cdot\mu\omega}{\phi_A}\right]^{\frac{1}{\epsilon_A}}<\mu\alpha$ is satisfied.

From this and (49) we obtain that Ψ_β and Ψ_q are negative when ϕ_E and ϕ_A are large enough. Differentiation of the *societal equilibrium policy* condition $\Psi(p,\beta,q)=0$ provides then immediately that:

$$\frac{\partial p}{\partial\beta}=\frac{\Psi_\beta}{-\Psi_p}<0 \text{ and } \frac{\partial p}{\partial q}=\frac{\Psi_q}{-\Psi_p}<0$$

Hence there exists a unique *societal equilibrium policy* $p(\beta,q)\in(0,\omega)$ and it is decreasing in β and in q when $\nu(.)$ is concave enough, and ω, ϕ_E and ϕ_A are large enough.

- **Characterization of the commitment societal equilibrium** $p^{com}(\beta,q)$:

Consider the policy objective function for given institutions β as:

$$\begin{aligned}
\widetilde{W}(p,\beta,q) &= \beta\left\{qU^{1c}+(1-q)U^{1p}\right\}+(1-\beta)U^{2}(p,A) \\
\text{with } A &= \lambda\cdot q\cdot a^{1c}(p) \\
E &= \lambda\cdot q\cdot e^{1c}(p) \\
e^{1c}(p) &= \left[p\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}} \text{ and } a^{1c}(p)=\left[\frac{\alpha\cdot(\mu p)}{\phi_C}\right]^{\frac{1}{\epsilon_C}}
\end{aligned}$$

one obtains the *societal equilibrium with commitment* $p^{com}(\beta,q)$ for any value of $(\beta,q)\in[0,1]^2$ as the solution of the following program:

$$p\in\arg\max_{p\in[0,\omega]}\widetilde{W}(p,\beta,q)$$

The first order condition characterizing an interior solution for $p^{com}(\beta,q)$ is given by:

$$\begin{aligned}
\widetilde{W}_p &= \kappa\lambda q e^{1c}_p+\beta q(1-\mu)e^{1c}-1+(1-\mu)v'(p(1-\mu)) \\
&\quad -\beta q\mu\alpha(1-a^{1c})+(1-\beta)\mu\left[1-\theta\lambda q a^{1c}-p\lambda q\theta a^{1c}_p\right]
\end{aligned}$$

Differentiating another time provides

$$\begin{aligned}
\widetilde{W}_{pp} &= \kappa\lambda q e^{1c}_{pp}+\beta q(1-\mu)e^{1c}_p+(1-\mu)^2v''(p(1-\mu)) \\
&\quad +\beta q\mu\alpha a^{1c}_p-(1-\beta)\mu\lambda q\theta\left[2a^{1c}_p+pa^{1c}_{pp}\right]
\end{aligned}$$

or after substitution

$$\begin{aligned}
\widetilde{W}_{pp} &= \kappa\lambda q\frac{1-\epsilon_E}{(\epsilon_E)^2}\left[\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}}p^{\frac{1}{\epsilon_E}-2}+\beta q(1-\mu)\frac{1}{\epsilon_E}\left[\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}}p^{\frac{1}{\epsilon_E}-1} \\
&\quad +(1-\mu)^2v''(p(1-\mu))+\beta q\mu\alpha\frac{1}{\epsilon_A}\left[\frac{\mu\alpha}{\phi_A}\right]^{\frac{1}{\epsilon_A}}p^{\frac{1}{\epsilon_A}-1}-(1-\beta)\ \frac{\mu\lambda q\theta\left(1+\epsilon_A\right)}{(\epsilon_A)^2}\left[\frac{\mu\alpha}{\phi_A}\right]^{\frac{1}{\epsilon_A}}p^{\frac{1}{\epsilon_A}-1}
\end{aligned}$$

Again when $\nu(.)$ is concave enough and ϕ_E and ϕ_A are large enough, we obtain that $\widetilde{W}_{pp}<0$ and the function $\widetilde{W}(p,\beta,q)$ is a strictly concave in p, ensuring the existence of the *societal equilibrium with commitment* $p^{com}(\beta,q)$. Moreover the cross derivative $\widetilde{W}_{p\beta}$ writes as

$$\widetilde{W}_{p\beta}=q\left[(1-\mu)e^{1c}-\mu\alpha(1-a^{1c})\right]-\mu\theta\lambda q\left[a^{1c}+pa^{1c}_p\right]<0$$

Hence $\widetilde{W}_{p\beta}<0$ when $(1-\mu)e^{1c}-\mu\alpha(1-a^{1c})<0$, something that is ensured when ϕ_E and ϕ_A are large enough. Consequently $p^{com}(\beta,q)$ is decreasing in β.

- **Comparison between $p^{com}(\beta,q)$ and $p(\beta,q)$**

Note that at the point $p(\beta,q))$,

$$\begin{aligned}\widetilde{W}_p(p(\beta,q)) &= \kappa\lambda q\cdot e_p^{1c}-(1-\beta)\lambda q\theta\cdot pa_p^{1c}\\ &= \lambda q\left[\kappa\cdot\frac{1}{\epsilon_E}\left[\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}}p^{\frac{1}{\epsilon_E}-1}-(1-\beta)\theta\cdot\frac{1}{\epsilon_A}\left[\frac{\mu\alpha}{\phi_A}\right]^{\frac{1}{\epsilon_A}}p^{\frac{1}{\epsilon_A}}\right]\\ &= \lambda q p^{\frac{1}{\epsilon_P}-1}\left[\kappa\cdot\frac{1}{\epsilon_E}\left[\frac{(1-\mu)}{\phi_E}\right]^{\frac{1}{\epsilon_E}}-(1-\beta)\theta\cdot\frac{1}{\epsilon_A}\left[\frac{\mu\alpha}{\phi_A}\right]^{\frac{1}{\epsilon_A}}p^{\left(\frac{1}{\epsilon_A}-\frac{1}{\epsilon_E}\right)+1}\right]\end{aligned}$$

and assume $\frac{1}{\epsilon_A}-\frac{1}{\epsilon_E}>0$ or $\epsilon_E>\epsilon_A$ (ie. civic monitoring is more sensitive to public leakages than civic participation is sensitive to public good provision). Then the function

$$\Theta(\beta,q)=(1-\beta)\theta\cdot\frac{1}{\epsilon_A}\left[\frac{\mu\alpha}{\phi_A}\right]^{\frac{1}{\epsilon_A}}[p(\beta,q)]^{\left(\frac{1}{\epsilon_A}-\frac{1}{\epsilon_E}\right)+1}$$

is decreasing in β and q. Thus the value $\widehat{\beta}(q)$ such that

$$\Theta(\beta,q)=\sigma=\kappa\cdot\frac{1}{\epsilon_P}\left[\frac{(1-\mu)}{\phi_P}\right]^{\frac{1}{\epsilon_P}}$$

is decreasing in q. Moreover $\widetilde{W}_p(p(\beta,q),\beta,q)\gtrless 0$ if and only if $\Theta(\beta,q)\lessgtr\sigma$ or $\beta\gtrless\widehat{\beta}(q)$. Thus as $\widetilde{W}(p,\beta,q)$ is a concave function and reaches a maximum at $p^{com}(\beta,q)$, one finally obtains $p^{com}(\beta,q)\gtrless p(\beta,q)$ if and only if $\beta\gtrless\widehat{\beta}(q)$.

Collection of the previous discussion on the properties of $p^{com}(\beta,q)$ and $p(\beta,q)$ provides figure 8 in the main text.

- **Characterization of the cultural manifold $q(\beta)$**

$$\begin{aligned}\Delta V^{1c} &= [p(1-\mu)]\,e^{1c}-\Phi_E\left(e^{1c}\right)+\alpha(\mu p)\,a^{1c}-\Phi_A(a^{1c})\\ \Delta V^{1p} &= \Phi_E\left(e^{1c}\right)+\Phi_A(a^{1c})\\ \frac{\Delta V^{1c}}{\Delta V^{1p}} &= \frac{[p(1-\mu)]\,e^{1c}-\Phi_E\left(e^{1c}\right)+\alpha(\mu p)\,a^{1c}-\Phi_A(a^{1c})}{\Phi_E\left(e^{1c}\right)+\Phi_A(a^{1c})}\end{aligned}$$

or

$$\frac{\Delta V^{1c}}{\Delta V^{1p}} = \frac{\frac{\Phi_E'\left(e^{1c}\right)e^{1c}}{\Phi_E(e^{1c})} - 1}{1 + \frac{\Phi_A(a^{1c})}{\Phi_E(e^{1c})}} + \frac{\frac{\Phi_A'\left(a^{1c}\right)a^{1c}}{\Phi_A(a^{1c})} - 1}{\frac{\Phi_E(e^{1c})}{\Phi_A(a^{1c})} + 1}$$

$$= \frac{\frac{\Phi_E(e^{1c})}{\Phi_A(a^{1c})}\left[\frac{\Phi_E'\left(e^{1c}\right)e^{1c}}{\Phi_E(e^{1c})} - 1\right] + \frac{\Phi_A'\left(a^{1c}\right)a^{1c}}{\Phi_A(a^{1c})} - 1}{\frac{\Phi_E(e^{1c})}{\Phi_A(a^{1c})} + 1}$$

$$\frac{\Delta V^{1c}}{\Delta V^{1p}} = \frac{\frac{\Phi_E(e^{1c})}{\Phi_A(a^{1c})}\epsilon_E + \epsilon_A}{\frac{\Phi_E(e^{1c})}{\Phi_A(a^{1c})} + 1}$$

which is an increasing function of $\frac{\Phi_E(e^{1c})}{\Phi_A(a^{1c})}$ as $\epsilon_E > \epsilon_A$. Now

$$\frac{\Phi_E(e^{1c})}{\Phi_A\left(a^{1c}\right)} = \frac{\phi_E}{\phi_A}\frac{1+\epsilon_A}{1+\epsilon_P}\frac{\left(e^{1c}\right)^{1+\epsilon_E}}{\left(a^{1c}\right)^{1+\epsilon_A}}$$

$$= \frac{\phi_A^{\frac{1}{\epsilon_A}}}{\phi_E^{\frac{1}{\epsilon_E}}}\frac{1+\epsilon_A}{1+\epsilon_P}\frac{(1-\mu)^{\frac{1+\epsilon_E}{\epsilon_E}}}{(\alpha\mu)^{\frac{1+\epsilon_A}{\epsilon_A}}}p^{\left(\frac{1}{\epsilon_E}-\frac{1}{\epsilon_A}\right)}$$

whic is a decreasing function of p. Thus $\frac{\Delta V^{1c}}{\Delta V^{1p}} = \Gamma\left(p\right)$ is a decreasing function of p.

Thus the cultural manifold $q\left(\beta\right)$ is characterized by the relationship:

$$\Gamma\left(p\left(\beta, q\right)\right) = \frac{q}{1-q} \tag{50}$$

There always exists at least solution $q = q\left(\beta\right)$ to this equation as $\Pi(q) = \Gamma\left(p\left(\beta, q\right)\right) - \frac{q}{1-q}$ is a continuous function of q such that $\Pi(0) > 0$ and $\Pi\left(1\right) < 1$. When the solution $q = q\left(\beta\right)$ is unique,one necessarily has $\Pi_q(q = q\left(\beta\right)) = \Gamma' p_q - \frac{1}{(1-q)^2} < 0$ and consequently the cultural manifold is depicted by a well defined function $q = q\left(\beta\right)$ such that

$$\frac{dq}{d\beta} = \frac{\Gamma' p_\beta}{\frac{1}{(1-q)^2} - \Gamma' p_q} > 0$$

as $\Gamma' < 0$ and $p_\beta < 0$.

The solution to (50) may have more than one solution. In that case the sign of Π_q at these solutions alternate between negative and positive, starting with a negative sign for the smallest solution. The branches (in even number) of the cultural manifold associated to solutions with $\Pi_q < 0$ are upward sloping in β while those (in uneven number) for which $\Pi_q > 0$ are downward sloping in β.

Property Rights and Conflict

Given there is random matching before each contest game, one may compute the expected payoff of "conflict-prone" and "conflict-averse" individuals as

$$\begin{aligned}\Omega_1(p,q) &= p\omega + 2(1-p)\omega\left[\frac{q}{4} + (1-q)\left(\frac{c^2}{c^1+c^2}\right)^2\right] - F \\ \Omega_2(p,q) &= p\omega + 2(1-p)\omega\left[q\left(\frac{c^1}{c^1+c^2}\right)^2 + \frac{(1-q)}{4}\right]\end{aligned} \tag{51}$$

Writing $c^1 = c$ and $c^2 = c(1+\alpha)$ with $a > 0$. Then (51) rewrites as

$$\begin{aligned}\Omega_1(p,q) &= p\omega + 2(1-p)\omega\left[\frac{q}{4} + (1-q)\left(\frac{1+\alpha}{2+\alpha}\right)^2\right] - F \\ \Omega_2(p,q) &= p\omega + 2(1-p)\omega\left[q\left(\frac{1}{2+\alpha}\right)^2 + \frac{(1-q)}{4}\right]\end{aligned} \tag{52}$$

It is immediate to see that $\Omega_1(p,q)$ is decreasing in q and that

$$\frac{\partial\Omega_1(p,q)}{\partial p} \geq 0 \text{ iff } q \geq \widetilde{q}(\alpha) = \frac{\left(\frac{1+\alpha}{2+\alpha}\right)^2 - \frac{1}{2}}{\left(\frac{1+\alpha}{2+\alpha}\right)^2 - \frac{1}{4}}$$

and $\Omega_2(p,q)$ is decreasing in q and that

$$\frac{\partial\Omega_2(p,q)}{\partial p} \geq 0$$

Property rights costs $C(p)$ satisfy: $C(0) = 0, C(p)$ is increasing convex (ie. $C'(p) \geq 0$, $C"(p) > 0$ and $C'(0) = 0$, $C'(1) = +\infty$).

- **The societal equilibrium**

To compute the *societal equilibrium* outcome, note first that the expected payoff of an agent of "conflict-prone" type in a *societal equilibrium* is given by:

$$G_1(p,q,a^{11},a^{12},a^{21}) = p\omega + q\left(2(1-p)\omega\frac{a^{11}}{a^{11}+a^{11}} - c^1a^{11}\right) + (1-q)\left(2(1-p)\omega\frac{a^{12}}{a^{12}+a^{21}} - c^1a^{12}\right) - F$$

and that of an agent of type "conflict averse",

$$G_2(p,q,a^{21},a^{12},a^{22}) = p\omega + q\left(2(1-p)\omega\frac{a^{21}}{a^{21}+a^{12}} - c^2a^{21}\right) + (1-q)\left(2(1-p)\omega\frac{a^{22}}{a^{22}+a^{22}} - c^2a^{22}\right)$$

where a^{11}, a^{12}, a^{21}, a^{22} are respectively the Nash contest efforts of a "conflict-prone" type 1 agent playing against another type 1 agent, a type-1 agent playing against a "conflict-averse" type-2 agent, a type-2 agent playing against a type-1 agent, and a type-2 agent playing against another type 2 agent.

The social planner in the policy game will then choose p to solve the following problem:

$$\max_{\gamma} \beta G_1(p,q,a^{11},a^{12},a^{21}) + (1-\beta)G_2(p,q,a^{21},a^{12},a^{22}) - C(p)$$

taking as given the values of a^{11}, a^{12}, a^{21}, a^{22}. One gets the following FOC:

$$\begin{aligned} & \beta\omega\left[1-2\left(q\frac{a^{11}}{a^{11}+a^{11}}+(1-q)\frac{a^{12}}{a^{12}+a^{21}}\right)\right] \\ & +(1-\beta)\omega\left[1-2\left(q\frac{a^{21}}{a^{21}+a^{12}}+(1-q)\frac{a^{22}}{a^{22}+a^{22}}\right)\right] \\ = \; & C'(p) \end{aligned}$$

with the Nash equilibrium levels of contest efforts obtained from:

$$a^{11}=\frac{2(1-p)\omega}{4c},\; a^{22}=\frac{2(1-p)\omega}{4c(1+\alpha)},\; a^{12}=2(1-p)\omega\frac{1+\alpha}{c\,(2+\alpha)^2},\; a^{21}=2(1-p)\omega\frac{1}{c\,(2+\alpha)^2}$$

Hence the societal equilibrium level of property right protection is characterized by the following condition:

$$\beta\omega\left[1-2\left(\frac{q}{2}+(1-q)\frac{1+\alpha}{2+\alpha}\right)\right]+(1-\beta)\omega\left[1-2\left(q\frac{1}{2+\alpha}+\frac{(1-q)}{2}\right)\right]=C'(p) \tag{53}$$

It is easy to see that:

Characterization of societal equilibrium policy: *When $\beta < q$, the societal equilibrium policy outcome involves strictly positive protection of property right with $p(\beta, q) > 0$. Moreover $p(\beta, q)$ is decreasing in β and increasing in q. When $\beta \geq q$, there is no property right protection in the societal equilibrium (ie. $p = 0$)*

- **The societal commitment equiibrium:**

The *societal commitment equilibrium* with property rights protection p satisfies the following program:

$$\max_p \beta\Omega_1(p,q) + (1-\beta)\Omega_2(p,q)$$

the FOC of this problem writes as:

$$\beta\omega\left[1-2\left(\frac{q}{4}+(1-q)\left(\frac{1+\alpha}{2+\alpha}\right)^2\right)\right]+(1-\beta)\omega\left[1-2\left(q\left(\frac{1}{2+\alpha}\right)^2+\frac{(1-q)}{4}\right)\right]=C'(p^{com}) \tag{54}$$

One easily gets

Characterization of societal commitment equilibrium policy: *Denote* $\phi(\alpha)=\frac{1+\alpha}{2+\alpha}$ and assume that $1/\sqrt{2}<\phi(\alpha)$, *i) then there exist a threshold* $\widetilde{q}(\alpha)\in\,]0,1[$ *and an increasing function* $\beta=\widetilde{\beta}(q)$ *with* $\widetilde{\beta}(0)<1$ *such that the societal commitment equilibrium involves "no-property rights" (ie.* $p^{com}=0$*) if and only if* $(\beta,q)\in[0,1]^2$ are such that $q<\widetilde{q}(\alpha)$ and $\beta\geq\widetilde{\beta}(q)$.

ii) When the societal commitment equilibrium policy $p^{com}(\beta,q)>0$, *then* $p^{com}(\beta,q)$ *is decreasing in* β *and increasing in* q.

iii) One has $p(\beta,q)\leq p^{com}(\beta,q)$.

Proof. i) Inspection of the FOC reveals that

$$p^{com}=0 \quad \text{when} \quad \beta\geq\widetilde{\beta}(q)=\frac{\frac{1}{4}+q\left[\frac{1}{4}-(1-\phi(\alpha))^2\right]}{q\left[\frac{1}{4}-(1-\phi(\alpha))^2\right]+(1-q)\left[\phi(\alpha)^2-\frac{1}{4}\right]}$$

with $\widetilde{\beta}(q)>q$ for all $q\in[0,1]$

$$\phi(\alpha)=\frac{1+\alpha}{2+\alpha}$$

is an increasing function of α. Notice as well that for all $\alpha>0$, one has

$$\frac{1}{4}-(1-\phi(\alpha))^2>0 \text{ and } \phi(\alpha)^2+(1-\phi(\alpha))^2>\frac{1}{2}$$

Moreover $\widetilde{\beta}(q)=1$ at a value $\widetilde{q}(\alpha)\in(0,1)$ given by

$$\widetilde{q}(\alpha)=\frac{\phi(\alpha)^2-\frac{1}{2}}{\phi(\alpha)^2-\frac{1}{4}}$$

Hence it follows that a *societal equilibrium with commitment* region of "no-property rights" exists (ie. $p^{com}=0$) if and only if

$$\frac{1}{\sqrt{2}}<\phi(\alpha) \text{ and } q<\widetilde{q}(\alpha)$$

It is also immediate to see that $\widetilde{\beta}(q)$ is increasing in q with

$$\widetilde{\beta}(0)=\frac{\frac{1}{4}}{\left[\phi(\alpha)^2-\frac{1}{4}\right]} \quad \text{and} \quad \widetilde{\beta}(1)>1$$

ii) Differentiation immediately provides that for an interior solution of property rights $p^{com}(\beta, q)$ one has:

$$\frac{\partial p^{com}}{\partial \beta} < 0 \quad \text{and} \quad \frac{\partial p^{com}}{\partial q} > 0$$

iii) To show that $p(\beta, q) \leq p^{com}(\beta, q)$, consider the difference of the LHS of the two equations (54) and(53 :

$$\beta\omega\left[1-2\left(\frac{q}{4}+(1-q)\left(\frac{1+\alpha}{2+\alpha}\right)^2\right)\right]+(1-\beta)\omega\left[1-2\left(q\left(\frac{1}{2+\alpha}\right)^2+\frac{(1-q)}{4}\right)\right]$$
$$-\beta\omega\left[1-2\left(\frac{q}{2}+(1-q)\frac{1+\alpha}{2+\alpha}\right)\right]-(1-\beta)\omega\left[1-2\left(q\frac{1}{2+\alpha}+\frac{(1-q)}{2}\right)\right]$$

which gives:

$$2\beta\omega\left[\frac{q}{2}+(1-q)\frac{1+\alpha}{2+\alpha}-\left(\frac{q}{4}+(1-q)\left(\frac{1+\alpha}{2+\alpha}\right)^2\right)\right]$$
$$+2(1-\beta)\omega\left[\left(q\frac{1}{2+\alpha}+\frac{(1-q)}{2}\right)-\left(q\left(\frac{1}{2+\alpha}\right)^2+\frac{(1-q)}{4}\right)\right]$$

or finally

$$2\beta\omega\left[\frac{q}{4}+(1-q)\frac{1+\alpha}{(2+\alpha)^2}\right]+2(1-\beta)\omega\left[q\frac{1+\alpha}{(2+\alpha)^2}+\frac{(1-q)}{4}\right]>0$$

Hence $C'(p^{com}) > C'(p)$ and the result $p(\beta, q) < p^{com}(\beta, q)$ for the case of interior solutions. Obviously for $\beta > \widetilde{\beta}(q)$ one has $p(\beta, q) = p^{com}(\beta, q) = 0$.**QED.** ■

Given that "conflict-prone" individuals are not always in favor of property right protection, an increase of their weight in the social welfare function implies a lower *societal equilibrium with commitment* value of p. Also the larger the fraction of "conflict-prone" individuals, the larger the occurence of rent-dissipation for the "conflict prone" agents and the larger the risk of rent expropriation for the " conflict-averse" individuals. Hence the more efficient it is to commit to protection of property rights.The previous discussion suggests that there may actually be a region in which the *societal equilibrium with commitment* is characterized by no property right protection.

- **Cultural dynamics:**

Turning to cultural dynamics, we have the socialization incentives $\Delta V^1(p, q)$ of "violence-prone" individuals as:

$$
\begin{aligned}
\Delta V^1(p,q) &= q\left(2(1-p)\frac{\omega}{2} - c^1 a^{11}\right) + (1-q)\left(2(1-p)\omega\frac{1+\alpha}{2+\alpha} - c^1 a^{12}\right) - F \\
&\quad - \left[q\left(2(1-p)\omega\frac{1}{2+\alpha} - c^1 a^{21}\right) + (1-q)\left(2(1-p)\frac{\omega}{2} - c^1 a^{22}\right)\right]
\end{aligned}
$$

which after substitution of $a^{11} = \frac{2(1-p)\omega}{4c}$, $a^{22} = \frac{2(1-p)\omega}{4c(1+\alpha)}$, $a^{12} = 2(1-p)\omega\frac{1+\alpha}{c(2+\alpha)^2}$, $a^{21} = 2(1-p)\omega\frac{1}{c(2+\alpha)^2}$ provides

$$
\begin{aligned}
\Delta V^1 &= 2(1-p)\omega\left[q\left(\frac{1}{4}\right) + (1-q)\left(\frac{1+\alpha}{2+\alpha}\right)^2\right] - F \\
&\quad -2(1-p)\omega\left[q\frac{1+\alpha}{(2+\alpha)^2} + (1-q)\frac{1+2\alpha}{4(1+\alpha)}\right] \\
&= 2(1-p)\omega\left[q\left(\frac{1}{4} - \frac{1+\alpha}{(2+\alpha)^2}\right) + (1-q)\left(\left(\frac{1+\alpha}{2+\alpha}\right)^2 - \frac{1+2\alpha}{4(1+\alpha)}\right)\right] - F \\
&= \frac{2(1-p)\omega\alpha^2}{4(1+\alpha)(2+\alpha)^2}\left[(3+\alpha) - q(2+\alpha)\right] - F
\end{aligned}
$$

Now for the "conflict-averse" individual, one similarly has :

$$
\begin{aligned}
\Delta V^2 &= q\left(2(1-p)\omega\frac{1}{2+\alpha} - c^2 a^{21}\right) + (1-q)\left(2(1-p)\omega\frac{1}{2} - c^2 a^{22}\right) \\
&\quad - \left[q\left(2(1-p)\omega\frac{1}{2} - c^2 a^{11}\right) + (1-q)\left(2(1-p)\omega\frac{1+\alpha}{2+\alpha} - c^2 a^{12}\right) - F\right]
\end{aligned}
$$

or

$$
\begin{aligned}
\Delta V^2 &= 2(1-p)\omega\left[q\frac{1}{(2+\alpha)^2} + (1-q)\frac{1}{4}\right] \\
&\quad -2(1-p)\omega\left[q\left(\frac{1}{2} - (1+\alpha)\frac{1}{4}\right) + (1-q)\frac{1+\alpha}{(2+\alpha)^2}\right] + F \\
&= 2(1-p)\omega\left[q\frac{\alpha^2}{4(2+\alpha)} + \frac{\alpha^2}{4(2+\alpha)^2}\right] + F > 0
\end{aligned}
$$

Finally for the locus $\dot{q}_t = 0$, one can compute:

$$
\begin{aligned}
\frac{\Delta V^1}{\Delta V^2} &= \frac{\frac{\alpha^2}{4(1+\alpha)(2+\alpha)^2}\left[(3+\alpha) - q(2+\alpha)\right] - \frac{F}{2(1-p)\omega}}{q\frac{\alpha^2}{4(2+\alpha)} + \frac{\alpha^2}{4(2+\alpha)^2} + \frac{F}{2(1-p)\omega}} \\
&= \frac{\frac{\alpha^2}{4(1+\alpha)(2+\alpha)^2}\left[(3+\alpha) - q(2+\alpha)\right] - \frac{F}{2(1-p)\omega}}{q\frac{\alpha^2}{4(2+\alpha)} + \frac{\alpha^2}{4(2+\alpha)^2} + \frac{F}{2(1-p)\omega}}
\end{aligned}
$$

or finally

$$\frac{\Delta V^1}{\Delta V^2} = \frac{\frac{1}{(1+\alpha)}\left[(2+\alpha)(1-q)+1\right] - \frac{2F(2+\alpha)^2}{(1-p)\omega\alpha^2}}{q(2+\alpha)+1+\frac{2F(2+\alpha)^2}{(1-p)\omega\alpha^2}} = \Phi(q,p,\alpha)$$

It is a simple matter to see that $\Phi(q,p,\alpha)$ is a decreasing function of p and q. We assume that F/ω is small enough

$$\frac{F}{\omega} < \frac{(1-p(0,0))}{2(1+\alpha)}\left(\frac{\alpha}{2+\alpha}\right)^2 [3+\alpha] \tag{55}$$

to ensure that for all $\beta, q) \in [0,1]^2$, one has $\Delta V^1/\Delta V^2$ to be strictly positive[66]. The characterization of the interior cultural steady state is obtained from :

$$\Phi(q, p(\beta,q), \alpha) = \frac{q}{1-q} \tag{56}$$

It is immediate to see that the LHS of equation (56) is a decreasing function of q (as $p(\beta,q)$ is increasing in q and $\Phi(q,p,\alpha)$ is decreasing in p). The RHS of (56) is increasing in q and goes from 0 to ∞ when q goes from 0 to 1. Hence given by (55) that $\Phi(0,p(\beta,0),\alpha) > 0$, then it is immediate to see that equation (56) has a unique solution $q(\beta)$ and that $q(\beta) < 1/2$. Moreover $q(\beta)$ is an increasing function of β. Moreover there is a unique value $\widehat{\beta}(\alpha)$ such that $q(\beta) = \beta$. Indeed such β is determined by

$$\Phi(\beta, p(\beta,\beta), \alpha) = \Phi(\beta, 0, \alpha) = \frac{\beta}{1-\beta}$$

or after substitution

$$\frac{1-\beta}{(1+\alpha)}\left[(2+\alpha)(1-\beta)+1\right] - \beta(\beta(2+\alpha)+1) = \frac{2F(2+\alpha)^2}{\omega\alpha^2} \tag{57}$$

Denote the LHS of (57) as a function $\Sigma(\beta,\alpha)$. Simple differentiation shows that $\Sigma(\beta,\alpha)$ is decreasing in β and takes value $\Sigma(0,\alpha) = \frac{4+\alpha}{1+\alpha} > 0$ and $\Sigma(1,\alpha) = -((2+\alpha)+1) < 0$. Now (55) implies

$$\frac{2F(2+\alpha)^2}{\omega\alpha^2} < \frac{(1-p(0,0))}{(1+\alpha)}[3+\alpha] < \frac{4+\alpha}{1+\alpha} = \Sigma(0)$$

Ttherefore there exists a unique value $\widehat{\beta}(\alpha) \in (0,1)$ satisfying (57). To such value $\widehat{\beta}(\alpha)$, there exists a corresponding unique value $\widehat{q}(\alpha) = q(\widehat{\beta}(\alpha))$. **QED.**

- **Institutional and cultural co-evolution**

We have the following results that correspond to the phase diagrams 11a) and 11b):

[66] Otherwise we could get the possibility of a cultural steady state without "conflict-prone" individuals.

Result : *i) The dynamics of case 1 as shown in the phase diagram (11a) (ie.* $\widehat{q}(\alpha) < \widetilde{q}(\alpha)$*) holds for* $\alpha = \frac{c^2}{c^1} - 1$ *large enough.*

ii) The dynamics of case 2 as shown in the phase diagram (11b) (ie. $\widehat{q}(\alpha) > \widetilde{q}(\alpha)$*) holds for* $\alpha = \frac{c^2}{c^1} - 1$ *close enough to* $\alpha_{\min}$*.*

Proof. Consider

$$\Sigma\left(\widetilde{q}(\alpha), \alpha\right) = \frac{1 - \widetilde{q}(\alpha)}{(1+\alpha)}\left[(2+\alpha)\left(1 - \widetilde{q}(\alpha)\right) + 1\right] - \widetilde{q}(\alpha)\left(\widetilde{q}(\alpha)\left(2+\alpha\right) + 1\right)$$

recalling that $\widetilde{q}(\alpha_{\min}) = 0$ and $\lim_{\alpha\to\infty} \widetilde{q}(\alpha) = \frac{2}{3}$ with

$$\alpha_{\min} = \frac{2 - \sqrt{2}}{(\sqrt{2} - 1)}$$

then it follows that

$$\Sigma\left(\widetilde{q}(\alpha_{\min}, \alpha_{\min})\right) = \frac{(3 + \alpha_{\min})}{(1 + \alpha_{\min})} > \frac{(1 - p\,(0,0))}{(1 + \alpha_{\min})}\left[3 + \alpha_{\min}\right] > \frac{2F(2 + \alpha_{\min})^2}{\omega\alpha_{\min}^2}$$

and therefore that $\widetilde{q}(\alpha_{\min}) < \widehat{\beta}\,(\alpha_{\min}) = \widehat{q}(\alpha_{\min})$ and this holds as well for α close enough to $\alpha_{\min}$ by continuity.

Similarly ■

$$\lim_{\alpha\to\infty}\left[\Sigma\left(\widetilde{q}(\alpha), \alpha\right) - \frac{2F(2+\alpha)^2}{\omega\alpha^2}\right] = \lim_{\alpha\to\infty} \Sigma\left(\widetilde{q}(\alpha), \alpha\right) - \frac{2F}{\omega} = -\infty$$

Thus for α large enough, one has $\Sigma\left(\widetilde{q}(\alpha), \alpha\right) < 0 < \frac{2F(2+\alpha)^2}{\omega\alpha^2} = \Sigma\left(\widehat{q}(\alpha), \alpha\right)$ and therefore $\widehat{q}(\alpha) < \widetilde{q}(\alpha)$. **QED.**