Sustainable Housing Policy

Preliminary and Incomplete

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December 16, 2016

Abstract

Investment in housing is often thought of as an important tool for household wealth accumulation and for stimulating economic activity through its effect on household equity. As such, many policies that subsidize investment in housing also care about the resulting level of housing prices besides just the quantity of new investment in housing. Empirical research on the effect of such policies on wealth accumulation and investment in the economy is mixed. In this paper, we develop a comprehensive framework for studying the effect of housing market policies on household wealth accumulation and the form that such policies should take. We find that when households are financially constrained, the price of housing generates externalities on investment in the economy and subsequently on household wealth accumulation. At times, it can be optimal to decrease the price of housing rather than to support high housing prices. Contrary to standard economic theory, we find that subsidizing the demand-side of the housing market and the supply-side of the market have different effects on welfare. When the return from new real estate investment is high, a combination of subsidies for construction companies and taxes on purchases of houses can be optimal.

1 Introduction

Many policies, such as the mortgage interest rate tax deduction and credits for first time home-buyers, aim to subsidize investment in housing. A unique feature of subsidization schemes in the housing market, as opposed to other markets, is that policy makers often care not just about the amount of investment in housing but also about the impact of policy on the level of house prices. In fact, following the collapse of the housing market in 2008, many government interventions such as the large scale repurchase of mortgage-backed securities were made with the explicit goal of supporting housing prices. The level of housing prices matters since home-equity is considered to be an important tool for wealth accumulation for existing home-owners. The rationale behind this argument is that households obtain a durable asset when they buy a house and they can use this asset to obtain funds to invest and build up their wealth. In a 2013 policy brief, the White House explained the importance of these policy measures in building household wealth and as a result stimulating the economy, ”Housing wealth is growing again, with owners equity up $2.8 trillion since hitting a low at the beginning of 2009. This in turn has contributed to increased economic activity through consumer spending, small business investment, and more.”

The typical channel through which investment in real estate leads to wealth accumulation for a household is the collateral channel. Households are able to borrow against the value of housing stock that they own and spend or invest the proceeds. This leads to policy goals by the government taking the form of supporting real estate investment and keeping house prices high in an attempt to keep the value of household equity high. During a press conference on September, 2012, announcing the FOMC action Ben Bernanke mentioned: ”To the extent that home prices begin to rise, consumers will feel wealthier, they'll feel more disposed to spend...if people feel that their financial situation is better...or for whatever reason, their house is worth more, they are more willing to go out and spend, and that's going to provide the demand that firms need in order to be willing to hire and to invest.”

Empirical research on the effect of the housing collateral channel on household wealth accumulation and economic activity has found mixed results. Chaney, Sraer, and Thesmar (2012) and Gan (2007) find a positive effect of increases in collateral value on investment. On the other hand, recent work by Chakraborty, Goldstein, and MacKinlay (2014) and Jorda, Schularick, and Taylor (2014) find that banks in areas that experienced a significant real estate price boom, increased mortgage lending while simultaneously decreasing commercial lending. The effect of an increase in collateral values on total investment in the economy may therefore be uncertain. Furthermore, standard economic theory on subsidization policies typically cares about the final quantity of the good in the market in question and not the level of prices. Since in housing markets the level of of prices is often a policy goal in itself, this approach to evaluating subsidization schemes may not be appropriate. These issues call for a comprehensive framework for studying the effect of housing investment on household wealth accumulation, the value of policies subsidizing investment in housing and the form that such policies should take.

In this paper, we analyze the optimal design of housing subsidization programs when governments wish to build household wealth and stimulate investment in the economy. Subsidization programs are often targeted at increasing the *demand* for mortgages by either incentivizing banks to lend more to real estate loans or incentivizing people to buy housing. Examples include the mortgage interest tax deduction, tax credits to first-time home buyers, low insurance payments to qualify for government guarantees on mortgages in the United States and right-to-buy and help-to-buy schemes in the United Kingdom. Contrary to this common practice, our analysis demonstrates that in many cases, it can be preferable to actually tax investment in real estate and subsidize the *supply* of housing instead. Such subsidies include but are not limited to subsidizing construction companies directly, a tax-credit to the sellers of houses and decreasing red-tape around land rights.

We develop a model in which a representative household borrows from banks to invest in the real estate and commercial sectors. The household is financially constrained because of an inability to commit to repaying loans using future income and therefore may not be able to invest in all positive investment opportunities. The household can use housing as collateral to help it obtain loans from a bank and can relax its financial constraint. Housing thus serves a dual purpose as an asset in the model - giving the household returns on investing and also relaxing the household's borrowing constraint. A government wishing to help the household build more wealth can implement demand side subsidies by increasing the return from investing in housing and supply side subsidies by subsidizing the production of houses.

The market price of housing in the model generates externalities on investment and subsequently household wealth accumulation when the household is financially constrained and cannot invest in all productive NPV opportunities. An increase in housing prices allows households to borrow more against home-equity thus generating a positive externality on the funds available for investment. However, an increase in housing prices also increases the cost of investment in real estate, taking away funds that could be used to invest in the commercial sector. This generates a negative externality on investment. When the household is a price taker and is financially constrained, these externalities prevent the first-best level of investment from being achieved.

The key novel insight of the model is that an increase in real estate prices does not necessarily increase household wealth even when the household actively uses it's house as an asset to borrow against and invests the proceeds in positive NPV opportunities. In our model the typical narrative of rising real estate prices helping to relax borrowing constraints by increasing the value of collateral and thus encouraging greater lending and investment is indeed true. However in addition to this, we also show that the effect of an increase in real estate prices on household wealth accumulation and aggregate investment can be negative due to a possible *crowding-out* of commercial investment as households are incentivized to invest more in mortgages. Although in our framework, an increase in real estate prices relaxes borrowing constraints, it also increases the amount of money that has to be spent on new housing purchases. This second effect takes resources away from commercial investment in the presence of financial constraints and if big enough can decrease the aggregate level of investment in the economy, subsequently reducing household wealth. The government may want to either subsidize or tax the investment in real estate depending on which effect dominates.

This negative investment effect of a boom in asset prices is in opposition to the collateral effect that is usually discussed in the literature. We additionally study the difference in subsidizing the supply-side of the real estate market rather than the demand-side and find that the two are not equal in our model due to their opposite effect on housing prices. A key insight of the model is that demand- and supply-side mortgage subsidization policies affect real estate and commercial investment in distinct ways. Interestingly this result goes against the traditional economic insight that it does not matter what side of the market is taxed or subsidized - the real effects of such interventions are the same. The difference between the two interventions arises in the model because of their opposite effects on the price of housing. While demand-side subsidies increase the price of housing by shifting the demand curve out, supply-side subsidies decrease the price of housing by shifting the housing supply curve out. In the presence of financial constraints and price externalities from housing, the real effects in the economy are not price-insensitive.

Supply- and demand- side subsidies are not equivalent in the model because they affect the agents' budget constraints differently when agents are financially constrained through their effect on housing prices. A change in price affects the agents' budget constraint in two ways. The first effect is a *collateral* effect coming through the borrowing constraint. The representative agent can use real estate assets as collateral allowing loans to be made against future earnings. An increase in real estate prices helps encourage investment by relaxing borrowing constraints since the value of the agents' collateral increases. This makes demand-side subsidies preferable. The second effect is a *crowding-out* effect. A decrease in prices makes housing cheaper and less resources need to be spent on the same quantity of housing investment. This frees up funds to increase investment in other productive opportunities.

This channel makes supply-side subsidies preferable. Interestingly, we find that in our setting, a social planner will want to combine expansionary supply-side policies like subsidies to construction companies with contractionary demand-side policies such as a tax on household real-estate investment and vice-versa. In particular, supply-side subsidies are optimally combined with demand-side taxes when the return from real-estate investment is high or alternatively when the government wishes to increase purchases of new houses over increasing existing home-owner equity. This leads to a very interesting and counter-intuitive implication of our framework - when the government believes that there are high returns from investment in housing, it is optimal to tax investment in housing and subsidize the production of housing due to the externalities generated by housing prices. On the other hand, when existing levels of home-ownership are high and the government wishes to increase household wealth through higher home equity, it is optimal to tax the supply-side of the housing market and subsidize the demand.

In the model, all government taxation and subsidies are paid out in the future. The demand and supply of housing respond to these future taxes and subsidies and subsequently affect the equilibrium price of housing. The benefits and costs of real estate price movements thus accrue earlier than the benefits and costs of government collections and payouts. When there are inter-temporal benefits from investment, these price movements affect welfare. Supply subsidies can be preferable when the net benefit from a decrease in the price of real estate is higher than that from an increase in price even if the future cost of such a subsidy is higher than that of a demand subsidy.

We also find that if the government can additionally control when taxes and subsidies are implemented, policies that front-load their benefits are generally preferable. This feature can help us rank different demand- or supply-based interventions. For example, our analysis suggests that policies that directly subsidize the down-payment of houses such as UK's help-to-buy scheme are preferable to policies such as the mortgage interest rate tax deduction whose benefits accrue over time. These policy recommendations are in line with those recently proposed by Mian and Sufi (2015). However, while they propose this based on the inherent risk associated with taking on debt we abstract from risk entirely and demonstrate another channel that would favour this policy. Interestingly, even without any risk, our model demonstrates that investing in housing is not always an efficient way to build wealth.

We propose a policy that can achieve the first best level of welfare and investment. This requires intervening in both sides of the market. Two-sided intervention is required to achieve the first best, because the social planner cares not only about the resulting level of housing investment, but also the resulting price of housing. Intervening in only the demand (supply) side of the market, shifts the demand (supply) curve for housing. Such a policy is restricted to a housing quantity/price combination that lie on the housing supply (demand) curve. An additional tax or subsidy on the supply (demand) side of the housing market, allows for additional shifts of the housing supply (demand) curve as well as the demand (supply) curve. This increases the combination of price/quantity combinations that are permissible in equilibrium allowing the social planner to target the optimal housing quantity as well as an optimal housing price. Targeting a housing price can help achieve the optimal level of commercial investment since housing prices affect the household budget constraint.

The optimal policy in our model depends on the optimal levels of investment in the commercial and real-estate sectors which cannot be observed and are hard to measure due to their dependency on functional form assumptions. We can, however, calculate a sufficient statistic that depends on three easily observable parameters - the existing stock of real estate that is owned by households, new housing purchases and the amount households can borrow against a dollar of home-equity - which tells us whether the collateral effect or the crowding-out effect is larger. The relative magnitude of these two effects can then tell us which side of the market should be taxed and which side should be subsidized in the optimal policy. Thus we follow Chetty (2009) and derive formulas so that the optimal policy is a function of high-level observables without the need to measure deep primitives by using a sufficient statistic approach.

Our model is able to reconcile the differing empirical findings of the collateral effect on investment since the effect of high housing prices on investment can be positive or negative depending on the three statistics we identify. Our model is also able to theoretically support the existence of a crowding-out channel as documented by Chakraborty et al. (2014). Finally, we also contribute to the way different policies affect household debt. Policies that decrease the price of housing lead to the household having a smaller debt burden than policies that increase the price of housing. This is because the crowding-out effect channel encourages investment by effectively making the household richer while the collateral effect channel increases investment by allowing the household to borrow more. In our framework, there are no negative consequences of taking on debt. However, many recent papers have highlighted the role high household debt plays in generating fragility and deeper recessions (Mian and Sufi (2011), Mian, Sufi, and Verner (2015), Shularick and Taylor (2012)). Our paper can contribute to this strand of literature by highlighting the effect different housing interventions have on household debt.

Literature: Our paper contributes to the literature on the real effects of real estate price fluctuations. In particular, the collateral effect whereby an increase in real estate prices helps alleviate borrowing constraints in the economy has been shown theoretically by Holmstrom and Tirole (1997). Owners of real estate will be able to post more collateral and raise more capital for investment when prices of real estate are high. Chaney et al. (2012) and Gan (2007) provide some empirical support for this. While the collateral effect does operate in our model, we also find an investment crowd-out can occur in the commercial sector as the price of real estate increases due to resources being diverted away from firms. Chakraborty et al. (2014) and Jorda et al (2014) find empirical support for this by documenting negative real effects on commercial investment of housing price appreciation. Theoretically, our model highlights a substitution effect amongst investments in a world with financial constraints which resembles the papers on internal capital markets by Stein (1997) and Scharfstein and Stein (2000). In these papers, a financially constrained headquarters makes an investment decision on how to allocate resources across divisions. A positive shock to the investment opportunities in one division diverts resources away from other divisions. In our model, agents face similar constrains and allocate resources to mortgages at the expense of commercial investment following price appreciations.

This negative real crowding-out effect of price booms we find in our paper also relates to Tirole (1985) in which bubbles in asset prices crowd out productive real investment by raising interest rates and reducing firm incentives to invest. In a similar vein, in a recent paper Farhi and Tirole (2012) find that the rise in interest rates might further restrict credit availability for financially constrained firms. In our paper, we do not require a bubble to produce the negative real effects accompanying a price-boom. We simply require that the household is financially constrained.

Our paper also contributes to the macro literature which tries to understand the role of asset prices for the real economy and how price changes amplify shocks to investment. Seminal papers in this field such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) discuss the amplification of choice via the effect of price on collateral and the ability of firms to borrow. Our proposed mechanism suggests that asset price booms may also serve to cause negative shocks to investment.

Finally our paper contributes to a large and body of literature in Economics on optimal subsidies and taxation policies. To the best of our knowledge, there are not many papers that consider the differences in subsidizing the supply versus the demand side of markets. The literature considering the supply side of the housing market when considering housing policy is quite sparse. Glaeser, Gyourko, and Saiz (2008) argue that we must consider the supply side of the housing market to understand fluctuations in housing cycles. They present a model in which areas with more elastic housing supply have fewer and shorter housing bubbles. They also do an empirical analysis in which they find that areas with a more inelastic housing supply were the ones that experiences a large run-up in housing prices in the 1980s. Our model predictions are in line with theirs, also predicting that housing prices in areas with a more elastic supply will be less affected by shifts in housing demand. However, we focus on the optimal housing policy given different elasticities of the supply curve and actually consider supply-side policies which may not cause a price boom at all in order to increase homeownership. Another paper worth mentioning is by Romer (2000), in which he argues that government policy programs aimed at increasing innovation should focus on subsidizing the supply of scientists and engineers rather than the demand for them. Romer makes the simple observation that if the supply of scientists and engineers is inelastic, subsidizing their labor demand may simply push wages up without increasing the equilibrium amount of innovation by much. He therefore recommends policies that would make the supply of such labour more elastic. While this is obviously true in our model, we find a difference in demand- and supply- side subsidies even if we keep the elasticity of the supply curve constant. This is due to the fact that financial constraints respond differently to increases versus decreases in price.

The rest of the paper is arranged as follows. Section 2 outlines the main model. Section 3 discusses the features of the model equilibrium. Section 4 discusses optimal housing policy and provides an implementable mechanism that can achieve the first-best level of investment and welfare. Section 5 compares demand and supply based policy interventions. The last section concludes. All proofs are in the Appendix.

2 The Model

There are two time periods $(1, 2)$, a representative risk-neutral household, a representative bank, and two investment goods in the economy. The household consumes at $t = 2$ and tries to maximize the terminal value of its wealth. At $t = 1$, the household receives an endowment ω which it can invest in two sectors - housing and commercial firms. The representative household is also born with an existing housing stock, B, and can use this stock along with any new housing purchases, x_m, as collateral to borrow an amount l from banks. The household additionally has access to a storage technology that has a return of 1.

At $t = 1$, a representative bank can make a loan, l, to the household to invest in housing and commercial firms. Loans need to be collateralized because of moral hazard in the repayment of loans. The household can use its housing stock as collateral and can commit to repaying a portion of the value of this stock $\phi(B + x_m)P$ at $t = 2$, where P represents the price of housing in the economy and $0 < \phi < 1$. ϕ represents the degree of pledgability of collateral. This formulation of the collateral constraint is similar to that in Gertler and

Karadi (2011) and in Gertler and Kiyotaki (2015).[1]

Investing in firms gives a gross return of $r_f(x_f)$ at $t = 2$ for every x_f units invested at $t = 1$. The function r_f has the following standard properties, $r_f^{'}(x_f) > 0$ and $r_f^{''}(x_f) < 0$ for all x_f, $r_f^{'}(0) = \infty$ and $r_f^{'}(\infty) = 0$. Investing in housing gives a return $r_m(x_m)$ at $t = 2$ for every x_m units invested at $t = 1$. The function r_m has the following standard properties, $r_m^{'}(x_m) > 0$ and $r_m^{''}(x_m) < 0$ for all x_m, $r_m^{'}(0) = \infty$ and $r_m^{'}(\infty) = 0$. The price per-unit of housing, P, is determined by demand and supply in the housing market. The representative construction firm takes the price of housing as given and has a strictly increasing and convex cost of housing production given by $K(x_m)$. All firm profits, π, are rebated back to the household at $t = 2$.

The return from housing can be interpreted as a convenience yield derived from the benefits of owning rather than renting a house such as the ability to customize one's residence, entering into long-term service contracts and the option of reselling in the future amongst others. As an investment good, the return on housing can also reflect beneficial tax treatment of owning property. Alternatively, the return from housing can be viewed as innovations in rental income which take the form of savings for the household if it lives in the house.

The government may subsidize or tax the real-estate sector to affect the amount of investment in the economy. This can be done by a demand side-subsidy (tax) which increases (decreases) the $t = 2$ per-unit return on housing by an amount r_g. In practice most government interventions to increase real-estate investment tend to fall into demand-side interventions. We also consider supply-side subsidies (taxes) in the form of a per-unit tax-rebate (cost), b, that the government can give to real estate firms as an alternative intervention. The government must have a balanced budget and households are taxed an amount τ at $t = 2$ to cover the cost of the subsidy. All subsidy/taxation payments are made at $t = 2$. We assume that firms can operate at a loss between period 1 and 2 without any additional costs.

To close the model, we finally make two more assumptions. First, households, firms and banks are price-takers in the economy. They therefore do not internalize their effect on P and τ when making decisions. Second, households have limited funds. We define x_m^* and x_f^* as the first best levels of investment in the economy. These are the quantities of x_m and x_f when the marginal return from investing is 1, i.e. $\frac{r_m^{'}(x_m^*)}{K^{'}(x_m^*)} = 1$ and $r_f^{'}(x_f^*) = 1$. Then we assume that,

[1]In particular, each period t, the household can abscond immediately with the funds it borrows. In this case the bank is able to recover a fraction ϕ of the household's housing stock. For the bank to lend to the household therefore, $l_t \leq \phi(B + x_m)P$.

$$K'(x_m^*)x_m^* + x_f^* > \phi(B + x_m^*)K'(x_m^*) + \omega \qquad \text{(Limited Funds)}$$

The above equation implies that the household is financially constrained and cannot invest in all positive investment opportunities even if it borrows to its full capacity. [2] This is a key assumption that drives the main results of the model. We will discuss its importance throughout the paper.

2.1 The Household's Problem

The representative household borrows an amount l from banks and maximizes the terminal value of its wealth. It therefore solves the following portfolio allocation problem, where x_m, x_f and x_s are the units of housing, commercial investment and storage purchased,

$$\begin{aligned} max_{(x_f,x_m,x_s,l)\geq 0} \quad & r_f(x_f) + r_m(x_m) + r_g x_m + x_s - \tau - l(1 + r_l) + \pi \\ s.t \quad & x_f + Px_m + x_s \leq \omega + l \\ s.t \quad & l \leq \phi\,(B + x_m)\,P \end{aligned}$$

The first four terms in the household's $t = 2$ wealth represent the value of the household's investment portfolio and the last two terms represent the payments that the household must make at $t = 2$ to the government and the bank. The first constraint is the household's budget constraint while the second constraint is its borrowing constraint.

Since there is no uncertainty in the economy, the household never defaults in the model. At $t = 2$, a household's funds are deterministic conditional on its chosen investment portfolio at $t = 1$. In equilibrium the bank will never lend more than the household's ability to repay and therefore default will never occur. Competition between banks therefore drives the equilibrium rate of interest on loans, r_l, to 0.

Additionally, if the Limited Funds assumption is satisfied, the household's borrowing constraint binds in equilibrium and $l = \phi(B + x_m)P$. In this case, the household will never invest in the storage technology since it has unexploited NPV projects at $t = 1$ that yield a return strictly greater than 1. Since this is the pertinent case for government intervention, we focus on this going forward and assume the Limited Funds assumption always holds. This allows us to simplify the portfolio allocation problem to the following,

[2] Note that if the equilibrium quantity of housing demanded by the household is x_m^*, then in equilibrium (unless a construction firm is subsidized), $K'(x_m^*) = P^*$, where P^* is the equilibrium price of housing. Therefore in an equilibrium in which the equilibrium quantity of housing demanded by the household is x_m^*, the above assumption can also be written as $P^*x_m^* + x_f^* > \phi(B + x_m^*)P^* + \omega$.

$$max_{(x_f,x_m)\geq 0} \qquad r_f(x_f) + r_m(x_m) + r_g x_m - \tau - \phi(B + x_m)P + \pi$$

$$s.t \qquad x_f + (1-\phi)Px_m \leq \omega + \phi BP$$

This yields the following first order conditions,

$$r_m^{'}(x_m) + r_g = \lambda P(1-\phi) + \phi P$$

$$r_f^{'}(x_f) = \lambda$$

λ is the lagrange multiplier on the budget constraint. We can combine the two FOCs to obtain the following equation that determines the amount of investment given the price of housing,

$$r_m^{'}(x_m) + r_g = (r_f^{'}(x_f)(1-\phi) + \phi)P \tag{1}$$

2.2 The Firm's Problem

The representative construction firm solves the following maximization problem,

$$max_{(x_m)\geq 0} \qquad Px_m - K(x_m) + bx_m$$

The first order condition for the firm gives us the equilibrium quantity of housing supplied,

$$K^{'}(x_m^s) = P + b$$

2.3 Equilibrium

An equilibrium of this economy is given by, (i) The household's portfolio allocation x_m^d, x_f^e and x_s^e given the price of housing P^e, (ii) The firm's choice of housing production x_m^s given price P^e, (iii) Price P^e such that the housing market clears, $x_m^s = x_m^d = x_m^e$.

3 Equilibrium Analysis

We begin the analysis of the equilibrium in this model by discussing a benchmark case in which the household does not face any financial constraints. In this case, the household is able to achieve the first best level of investment, x_m^* and x_f^*, and consumption and there is no need for government intervention. We then discuss the equilibrium when the household is financially constrained and show how price externalities in the housing market generate inefficiencies in investment and consumption.

3.1 Benchmark Equilibrium without Financial Constraints

We start by outlining a benchmark case in which the household is not financially constrained. In the model, this is equivalent to the household being able to borrow from the bank without the need for posting collateral as long as it has sufficient funds at $t = 2$ to cover its $t = 1$ loan. Therefore conditional on the expected return from an investment being above 1, the household can borrow the funds available to invest in it. The presence of the storage technology ensures the household never invests with an expected return of below 1 and therefore the household is always able to repay its loan. In the benchmark case, the household solves the following problem,

$$max_{(x_f, x_m, x_s, l) \geq 0} \qquad r_f(x_f) + r_m(x_m) + x_s - l + \pi$$

$$s.t \qquad x_f + Px_m + x_s \leq \omega + l$$

The first order conditions yield the following equilibrium quantities of housing and commercial investment in the economy where the superscript b refers to the benchmark case,

$$r_f^{'}(x_f^b) = 1$$

$$\frac{r_m^{'}(x_m^b)}{P^b} = 1$$

Assuming households do not borrow simply to store (i.e. they have a weak preference for not taking a loan), storage is used in equilibrium if $\omega \geq P^b x_m^b + x_f^b$. If $\omega < P^b x_m^b + x_f^b$, then the household will borrow an amount $l = P^b x_m^b + x_f^b - \omega$ from banks to fund its additional investment. The limited funds assumption corresponds to this second case with the household borrowing in equilibrium. Since $P^b = K^{'}(x_m^b)$, the decentralized equilibrium allocation corresponds to the first-best levels of investment. The usual welfare theorems apply in this benchmark economy.

Note that, the decentralized equilibrium result only differs from the benchmark case when the household is financially constrained and has limited funds that are not enough to fund all productive investment. The fact that the household has more positive NPV investment opportunities than the funds necessary to invest in these opportunities is thus a crucial assumption for the results.

Now that we understand the benchmark equilibrium when the household is not financially constrained, we can see how in the presence of financial constraints the household not internalizing its effects on the price of housing causes inefficiencies in equilibrium.

3.2 Equilibrium with Financial Constraints

We now move to the equilibrium analysis of the main model in which the household is financially constrained. The decentralized equilibrium allocation here simply corresponds to setting $r_g = 0$ and $b = 0$ in (1). To establish the inefficiencies in the decentralized equilibrium, we first solve the constrained social planner's problem in this economy.

The social planner in this economy takes into account how housing prices affect the financial constraint of the household. The social planner therefore solves,

$$\begin{aligned} &max_{(x_f,x_m,x_s)\geq 0} && r_f(x_f) + r_m(x_m) + x_s - l(1+r_l) + \pi(x_m) \\ &s.t && x_f + P(x_m)x_m + x_s \leq \omega + l \\ &s.t && l \leq \phi\,(B + x_m)\,P(x_m) \end{aligned}$$

When the Limited Funds assumption is satisfied, the household is financially constrained in equilibrium, it's borrowing constraint binds and $x_s = 0$. This problem simplifies to,

$$\begin{aligned} &max_{(x_f,x_m)\geq 0} && r_f(x_f) + r_m(x_m) - \phi(B + x_m)P(x_m) + P(x_m)x_m - K(x_m) \\ &s.t && x_f + P(x_m)(1-\phi)x_m \leq \omega + \phi BP(x_m) \end{aligned}$$

The FOCs for the constrained social planner are,

$$r_f^{'}(x_f) = \lambda$$

$$r_m^{'}(x_m) + P + P^{'}x_m - K^{'}(x_m) = \lambda(P(1-\phi) + P^{'}(1-\phi)x_m - P^{'}\phi B) + \phi P + \phi P^{'}(B + x_m)$$

To compare the constrained social planner's allocation to that of the household's in the decentralized equilibrium, we impose the firm's equilibrium condition i.e. $K^{'}(x_m) = P$. Then, the social planner's optimal allocation is given by,

$$r'_m(x_m) = P(r'_f(x_f)(1-\phi)+\phi) + P'x_m(r'_f(x_f)-1) - P'\phi(B+x_m)(r'_f(x_f)-1) \quad (2)$$

This differs from the allocation chosen by the household when $r'_f(x_f) > 1$. This is the case when the Limited Funds assumption holds. Comparing the social planner's allocation in (2) with the household's in (1), there are two additional terms on the RHS. The term $P'x_m(r'_f(x_f)-1)$ captures the *crowding-out effect*. As the representative household demands more housing, the price rises and it has to pay a greater amount for all units of housing, leaving it with less funds to invest into the commercial sector. This decreases the optimal x_m chosen by the social planner. The term $P'\phi(B+x_m)(r'_f(x_f)-1)$ captures the *collateral effect*. As the representative household demands more housing, the price rises and loosens the household's borrowing constraint, giving it more funds to invest. This increases the optimal x_m. Internalizing the price effects of housing, will makes the social planner's optimum differ from that of the household's when the household is financially constrained. Therefore, the first welfare theorem does not hold in this case. [3]

Let x_m^{dc} be the decentralized equilibrium demand for housing and x_m^{sp} be the optimal amount of housing in the constrained social planner equilibrium. Based on the above analysis, we can establish the following proposition,

Proposition 1 *If $x_m^{sp} > \phi(B + x_m^{sp})$, the crowding-out effect is larger than the collateral effect. In this case, the decentralized equilibrium features inefficiently high investment in housing and $x_m^{sp} < x_m^{dc}$.*

Conversely, if $x_m^{sp} < \phi(B + x_m^{sp})$, the crowding-out effect is smaller than the collateral effect. In this case, the decentralized equilibrium features inefficiently low investment in housing and $x_m^{sp} > x_m^{dc}$.

The inefficiency in the decentralized equilibrium in this model arises because the representative household is acting like a price-taker and does not internalize the effect his housing demand has on prices. The price of housing affects both the value of the household's collateral and the amount of funds the household has to fund investment opportunities. When the existing amount of housing is high or investment in housing relaxes borrowing constraints for households, an increase in the price of housing affects the household's ability to borrow.

[3] The above analysis accounts for how a social planner would change the portfolio allocation picked by the household when the firm acts competitively and picks $K'(x_m) = P$. A similar analysis can be done if the social planner was choosing how much the firm produces with the household acting competitively and picking $r'(x_m) = P(r'_f(x_f)(1-\phi)+\phi)$. In this case, the constrained optimum can be restored with a supply-based subsidization or taxation scheme.

The social planner internalizes this effect and therefore wants to increase the demand for real-estate investment to push up the aggregate price of housing. When the existing amount of housing in the economy is low or the household is not able to use its housing as collateral efficiently, an increase in the amount of housing causes resources to be diverted away from investment in the commercial sector when the household is financially constrained. The social planner internalizes this effect and therefore wants to decrease the demand for real-estate investment to decrease the aggregate price of housing.

An increase in the price of housing affects commercial investment in two ways as discussed above- the *collateral effect* and the *crowding-out effect*. The collateral effect captures the fact that an increase in the price of housing loosens the households budget constraint as the existing stock of homeownership is now worth more. The household can therefore borrow more against their future income and invest more in commercial firms at $t = 1$. At the same time, the boom in the price of houses causes the household to spend relatively more on housing and therefore they must compensate by reducing the amount spent on firm investment. This is the crowding-out effect. Which effect dominates depends on relative increase in the value of collateral versus the relative increase in the amount that the household needs to pay for the extra investment in housing. If the existing housing in an economy is high (a high B) and it is easy to borrow (ϕ), an increase in housing prices can relax borrowing constraints enough to lead to a crowding-in of commercial investment.

We can see that when $\phi = 0$ and the household cannot borrow at all against it's housing stock, the collateral effect dissapears and only the crowding-out effect remains. Increasing investment in housing therefore always leads to a substitution away from commercial investment. When $\phi = 1$, the crowding-out effect is zero and only the collateral effect remains. In this case, increasing investment in housing always allows the household to increase commercial investment. In Section 5, we will look into these special cases more and discuss what they imply for optimal policy.

Based on the above analysis, we can find a demand-based subsidization scheme that can restore the constrained social planner optimum. The following proposition states this,

Proposition 2 *A demand-based subsidization scheme in which* $r_g^* = K''(x_m^{sp})(\phi B - (1 - \phi)x_m^{sp})(r'_f(x_f^{sp}) - 1)$ *and* $b^* = 0$ *restores the socially optimum level of housing and commercial investment chosen by the constrained social planner.*

A particularly interesting aspect of this proposition is that if this investment-driven collateral effect is not large enough in the economy (a low B or ϕ), the optimum policy for the government would be to have a negative tax on real estate investment. The size of the government subsidy or tax is larger when quality of investment opportunities in the commercial investment sector $(r_f^{'})$ is higher. This is because better commercial investment opportunities increase the social cost of the externality on investment coming from housing prices. The size of the government subsidy or tax is also larger in magnitude when the supply curve is more inelastic $(K^{''}(x_m^{sp}))$. This arises because of the non-neutrality of prices in the model. The price effects of housing drive the investment externalities and a more inelastic supply curve leads to greater price movements as we change the amount of real estate investment.

One of the key takeaways of our model so far is that when the household is financially constrained, price effects are not welfare-neutral. Therefore when we consider government interventions, how they affect the price of housing is critical. In the next section, we show that we can restore the first-best level of welfare corresponding to the unconstrained social planner's outcome with a mix of demand-side and supply-side interventions.

4 Optimal Housing Policy

In our setting housing price movements can create externalities on commercial investment. Therefore when picking policy that taxes or subsidizes investment in housing, we must not only consider the resulting level of housing investment, but also the resulting price of housing as this affects commercial investment. In the previous section, we show that a tax or subsidy on the demand for housing can restore a constrained social planner's optimal allocation by making the household internalize the price externalities generated from housing investment. Such a scheme shifts the household demand for housing out by a subsidy when the collateral effect dominates, and shifts household demand in via a tax when the crowding-out effect dominates. Such a policy is restricted to a housing quantity/price combination that is permitted by the housing supply curve, i.e. $K^{'}(x_m) = P$. A tax or subsidy on the supply side of the housing market, allows for additional shifts of the housing supply curve as well as the demand curve. This increases the combination of price/quantity combinations that are permissible in equilibrium.

We can show that a combination of demand- and supply-side interventions can help restore the first-best level of investment and consumption. Formally, we can establish the key proposition of the paper,

Proposition 3 *The first-best level of welfare can be achieved by a subsidy pair*

$$\{r_g, b\} = \left\{ \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} - r_m^{'}(x_m^*), r^{'}(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} \right\}$$

This proposition is the key result of our paper and yields a very surprising result. In the presence of price externalities from housing, the optimal way to build wealth in the economy is to combine expansionary housing supply subsidies with contractionary housing demand taxes and vice-versa. From the proposition we can see that in the optimal subsidy scheme $r_g = -b$. This is simply due to the linearity of the demand and supply subsidies. In equilibrium it has to be the case that $K'(x_m^*) - b = P = r'_m(x_m^*) + r_g$. Since at the optimal level of housing investment $K'(x_m^*) = r'_m(x_m^*)$, this implies that $-b = r_g$.

Since price externalities are the key source of inefficiency in the model, the optimal policy needs to target price movements which require opposite subsidies to demand and supply. The government's problem can thus be thought of as wanting to achieve a certain price level at the optimal level of housing investment. Whenever the collateral effect is stronger than the crowding-out effect at x_m^*, the government taxes supply and subsidizes demand, both of which put upward pressure on the price of housing. The price is increased until the collateral value of housing is high enough such that the household can borrow enough to fund the optimal level of commercial investment. Conversely, when the crowding-out effect is stronger than the collateral effect at x_m^* the government needs to put downward pressure on the price of housing to relax the household's budget constraint and provide it with enough funds to undertake the optimal level of commercial investment. In this case, the government taxes demand and subsidizes supply, both of which put downward pressure on the price of housing.

When the crowding-out effect is stronger at x_m^*, i.e. when $(1-\phi)x_m^* > \phi B$, the government wants to push house prices down which can be achieved through supply-side subsidies and demand-side taxes. Thus, when x_m^* is quite high relative to B and the government wants to increase new housing purchases, we should be pushing housing prices down rather than trying to support housing prices. This involves measures such as a tax on taking out a mortgage and rebates to construction companies.

This result highlights that when the return to new housing investment is high (high x_m^*), it is preferable to try and reduce the price of housing, rather than to make borrowing for households easier by allowing them to take on more leverage. Other arguments in the literature also support household's taking on less leverage. We get the same in our model but through a completely different channel. We abstract away from the risk created by taking on more leverage and show that even from a wealth accumulation perspective taking on leverage may not be optimal for households. This is driven by the fact that if additional leverage is generated due to an increase in prices, that rise in prices also reduces the funds available for a household to invest in other productive opportunities. Reducing the costs of downpayments of houses can instead free up funds and increase a household's ability to invest. Demand subsidies in the model induce taking on leverage while supply subsidies cause a reduction in the leverage required. The crowding-out effect works as a counter to taking on more leverage.

Implementation: The optimal policy in our model requires knowledge of x_f^* and x_m^* which depend upon the exact functional forms r_m and r_f. These quantities are hard to determine without a full structural model. However we show that when the crowding-out effect is stronger than the collateral effect, housing prices should optimally be decreased. The relative magnitude of the crowding-out effect and the collateral effect depend on three parameters of the model, B, ϕ and x_m, where x_m is the observed level of new investment in housing. These three parameters are sufficient to determine which effect is relatively larger, we simply have to compare $(1-\phi)x_m$ to ϕB.[4] This determines whether we should subsidize the supply of housing and tax the demand or vice-versa. This approach is limited in that it cannot tell us the exact magnitude of the subsidies and taxes. However, these three statistics are sufficient to guide us in the direction of the intervention and have the benefit of being easily observable. This approach follows Chetty (2009) and derives formulas based on sufficient statistics to guide policy without the need to estimate deeper parameters of the model.

In the next section, we discuss supply- and demand-side interventions in more detail and explain why the two are not equivalent in our framework. We also provide some special cases which help understand the key forces in our model and how they drive different policy interventions.

5 Comparing Demand and Supply Subsidies

Traditional economic theory has long established that under general conditions, it does not matter whether we subsidize (tax) supply or demand from a welfare perspective. The gains to consumers and suppliers are the same and depend only on the relative elasticities of the supply and demand curves. However in our model, we find that taxing the supply and demand sides of the market can be quite different due to the effects that are caused by the increase or decrease in the price of housing when households are financially constrained and when there is an inter-temporal opportunity cost of capital. In our model, this cost is the presence of a second sector in the economy, commercial investment, since this seems to be an

[4]This follows from Proposition 1. When $(1-\phi)x_m^{sp} < \phi B$, the proposition states that $x_m^{dc} < x_m^{sp}$ and it is optimal to decrease the price of housing. It follows that when $(1-\phi)x_m^{sp} < \phi B$, $(1-\phi)x_m^{dc} < \phi B$ and it is also optimal to decrease the price of housing. We can therefore compare level of equilibrium investment in housing rather than the planner's optimal.

empirically relevant case. However, this cost can be much more general, and could include labor income costs, costs associated with the moral hazard of lending such as monitoring costs, etc.

In this economy, the introduction of subsidies ($r_g > 0$) that increase the demand for housing always lead to an appreciation in the price of housing. The price for housing increases because each additional home is more expensive to produce giving rise to an upward sloping supply curve in the housing market. For the housing market to clear and respond to the increase in demand, the price of housing must consequently appreciate. Conversely, the introduction of subsidies ($b > 0$) that increase the supply for housing have an opposite effect on price to that of demand subsidies. They lead to a decrease in the price of housing. Supply side interventions hence do not cause the boom in housing prices that demand subsidies do.

In most literature on externalities, the socially optimum level of the good in question is affected by the existence of externalities. This level is typically independent of price movements. However, in our paper the externalities themselves are generated due to prices and therefore this causes demand- and supply- subsidies to have different welfare implications. This will be discussed more and formalized in the rest of this section. Since typically externalities lead the social planner to choose a particular optimal level of a good, in this case housing, we will compare supply and demand subsidies holding fixed the level of housing investment in the economy. This will help clarify the main forces in the model that are driving the difference between these two policy interventions.

5.1 Demand and Supply Equivalence

As discussed earlier, in classic economic theory the welfare implications of taxing or subsidizing the supply and demand side of the market are the same. We will therefore start this section with a discussion of when subsidizing the supply and the demand side of the market in our model are welfare equivalent. We will then discuss which frictions cause the welfare implications of these two policies to differ.

If the household is not financially constrained, then demand and supply subsidization are equivalent policies. For any level of housing that can be achieved in the economy, subsidizing either the supply or demand generate the same utility for the representative household. This is because the household will simply invest until all the productive investment opportunities in the economy are realized. We can then establish the following proposition,

Proposition 4 (Demand and Supply Equivalence) *Suppose the household is unconstrained. Then for any r_g' generates x_m', x_f' and U', there exists a b' that also generates x_m', x_f' and U'. The converse is also true.*

Proposition 4 states the conditions under which supply and demand subsidies are welfare equivalent. The household in this model is financially constrained which prevents all productive investment opportunities from being realized in equilibrium. Once this constraint is taken away, the costs of being financially constrained i.e. the investment externalities that investing more in housing causes on the commercial sector, dissapear as well. The welfare gains from supply and demand subsidies therefore come from how they each affect the household's ability to borrow and invest.

From Proposition 4 , we see that under no financial constraints, subsidizing the demand- and supply-side are welfare equivalent. The household's constraints prevent it from investing in all productive investment opportunities. To understand the difference in the two policy interventions, we therefore need to look at their effect on financial constraints and subsequent investment in the economy. Having established when demand and supply subsidies produce the same effects, we now establish two more propositions that explain why they differ in our model.

In the model, supply and demand respond to future government subsidies at $t = 2$. Demand subsidies increase the demand for housing resulting in an increase in the price of housing while supply subsidies increase the supply of housing resulting in a decrease in the price of housing. Prices therefore respond to future subsidies. However, a key part of the benefits and costs of subsidies are provided to households and firms through price movements. Supply subsidies may allow the household to invest more by lowering the price of housing at $t = 1$. This effectively allows the household to "borrow" against some of its future income since taxes to pay for the subsidies are paid in the future. Alternatively demand side subsidies directly increase the value of collateral that households have by increasing the price of housing thus helping them to borrow more.

In the absence of a collateral effect in the model demand subsidies will thus lose their advantage over supply-side subsidies. The following proposition formalizes this result,

Proposition 5 *If $\phi = 0$, subsidizing (taxing) the supply (demand) side of the housing market pareto dominates subsidizing (taxing) the demand (supply) side of the housing market. That is, for any r_g^+ that is associated with utility U, there exists a supply-subsidy b^+ that generates higher $U' > U$ and for any b^- that is associated with utility U, there exists a demand-tax r_g^- that generates higher $U' > U$.*

This proposition states that without a collateral effect, supply subsidies are always preferable to demand subsidies. The key assumptions driving this result are financial constraints and an inter-temporal advantage of having more capital early. In the case of our model, the advantage is being able to invest in positive NPV projects of commercial firms. Consider the case when $\phi = 0$. In this case the borrowing capacity of the household does not change with the price of housing. Therefore the introduction of demand subsidies will push up price and always cause a reallocation of household investment in favor of mortgages and away from firm investment due to a negative crowding-out effect. Supply subsidies, on other hand, will free up household funds to invest in more projects by pushing the cost of housing down and have a positive crowding-out effect.

We can establish an analogous proposition for demand-based subsidization schemes. Namely,

Proposition 6 *If $\phi = 1$, subsidizing (taxing) the demand (supply) side of the housing market pareto dominates subsidizing (taxing) the supply (demand) side of the housing market. That is, for any b^+ that is associated with utility U, there exists a demand-subsidy r_g^+ that generates higher $U' > U$ and for any r_g^- that is associated with utility U, there exists a supply-tax b^- that generates higher $U' > U$.*

When $\phi = 1$, the household is able to borrow against the full value of its housing stock. Therefore investment in housing does not require the household to substitute away from funds that it would otherwise use for commercial investment. When $\phi = 1$, the household can effectively borrow to fund all its new housing purchases. This neutralizes the crowding-out effect of price movements while keeping the stronger collateral effect that demand subsidies have since they push up the price of housing.

It is worth noting that when $\phi = 1$, and the household can borrow up to the full value of its housing stock from the bank and therefore the household will always pick $x_m^s = x_m^d = x_m^*$ and the household picks the optimal level of housing. However, if the limited funds assumption continues to hold, then the household will still not be able to pick the optimal level of commercial investment still creating room for government intervention.

The above propositions highlight the usefulness of front-loading benefits and back-loading costs of policies when there are intertemporal opportunity costs of capital. They can also be used to evaluate the effectiveness of different interventions which affect the same side of the market. Within demand-based subsidization schemes, an intervention in which the government provides aid to households to help in the downpayment of their mortgage helps counter-balance the negative crowding-out effect created by subsidies that back-load benefits such as the mortgage interest tax deduction.

6 Concluding Remarks

In this paper we develop a comprehensive framework for studying the effect of housing policy on household wealth accumulation. We find that an increase in household home equity does not necessarily lead to efficient wealth accumulation, housing prices can generate externalities on investment, and supply and demand subsidies are not equivalent in the presence of price externalities. When the return to investing in real estate in the economy is high, the optimal policy involves an expansion of supply subsidies and a tax on housing demand. We summarize below some of the key insights of the paper.

(i) **Investment:** In the model understanding how supply and demand subsidies affect investment through their effect on price is the key to understanding the differences between these policy interventions. This may provide a theoretical basis for the empirical results found in Chakraborty et al. (2014). We find that a housing price increase is only good for investment if the existing stock of home-ownership is large. In this case, households can use housing as collateral effectively and increasing house prices provide them the ability to increase invest in investment profitably. When the existing stock of housing is low, increasing the price of housing can lead to negative externalities on investment and crowd-out investment from the commercial sector *even when the household actively uses its house as collateral to fund investment*. In such a case, policies aimed at reducing the price of housing are preferable for investment.

(ii) **Price Externalities:** A novel feature of our model is looking at price externalities in a general equilibrium framework. When externalities exist because of prices the standard result of the irrelevance of using supply or demand subsidies no longer applies when subsidies and taxes are paid out and collected in the future. This is because supply and demand curve movements caused by move prices in the opposite direction.

(iii) **Inter-temporal Effects:** A key mechanism at play in our model is that the costs and benefits of supply and demand subsidies are distributed differently across time. Supply subsidies have some of their benefits delivered today in terms of lower prices while demand subsidies have some of their benefits accrue in the future in terms of a higher return on housing. Without financial constraints, the different inter-temporal properties of the two schemes do not matter. However, in the presence of financial constraints, the inter-temporal properties matter because there is a positive effect of front-loading benefits of subsidies since the household can use the extra funds to invest.

(iv) **Debt:** Different policies in our model have different implications for the level of household debt. A positive *crowding-out effect* effectively gives the household more funds at $t = 1$, leading it to achieve higher levels of investment without taking on additional debt. In fact, due to downward price movements associated with a positive crowding-out effect, household debt actually decreases. On the other hand, a positive *collateral effect* allows the household to achieve higher levels of investment by increasing its debt capacity. In our framework, since there is no uncertainty or default, there are no negative consequences to taking on more debt. However, many recent papers have found that a buildup in household debt can cause increased economic fragility. Our analysis contributes to this literature by explaining how different housing policies can have different effects on household debt. In particular, if the government wishes to expand investment in real estate (a high x_m^*) which is a common policy objective for many governments, focusing on supply subsidies that have a positive crowding-out effect and negative collateral effect, may be a more sustainable intervention.

In this paper, we abstract away from the risk of investing in real estate and focus on a different channel highlighting the effect of housing subsidies on the financial constraints of a household. Our results support policies aimed at reducing the housing debt taken on by households and encourage policies that front-load their benefits even in the absence of risk. In practice, it appears that many households used household equity to fund consumption rather than investment. Our framework can be extended to include a measure of impatient households who would rather borrow against their home to fund immediate consumption over investment and may be an interesting area for future research.

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7 Appendix

Proof of Proposition 1.

To begin the proof first note that, $P^{'}(x_m) = K^{''}(x_m) > 0$ in both the decentralized and the SP equilibrium. The equilibrium condition for the SP equilibrium requires,

$$r_m^{'}(x_m^{sp}) = K^{'}(x_m^{sp})(r_f^{'}(x_f^{sp})(1-\phi)+\phi) + K^{''}(x_m^{sp})x_m^{sp}(r_f^{'}(x_f^{sp})-1) - K^{''}(x_m^{sp})\phi(B+x_m^{sp})(r_f^{'}(x_f^{sp})-1) \tag{3}$$

The equilibrium condition for the decentralized equilibrium requires,

$$r_m^{'}(x_m^{dc}) = K^{'}(x_m^{dc})(r_f^{'}(x_f^{dc})(1-\phi)+\phi) \tag{4}$$

When $x_m^{sp} > \phi(B+x_m^{sp})$, then the last two terms together in (3) are positive and therefore,

$$r_m^{'}(x_m^{sp}) > K^{'}(x_m^{sp})(r_f^{'}(x_f^{sp})(1-\phi)+\phi) \tag{5}$$

To prove the first part of the proposition, we want to show that when $x_m^{sp} > \phi(B+x_m^{sp})$, $x_m^{sp} < x_m^{dc}$.

We prove this by contradiction by showing that (4) and (5) cannot simultaneously hold if $x_m^{sp} > x_m^{dc}$. Note that we can rule out the case in which $x_m^{sp} = x_m^{dc}$ because (4) and (5) cannot simultaneously hold in this case since the LHS and the RHS of both equations would be identical.

The proof is as follows. Suppose $x_m^{sp} > x_m^{dc}$. Then since $r_m^{''} < 0$, $r_m^{'}(x_m^{sp}) < r_m^{'}(x_m^{dc})$. Therefore using (4) and (5),

$$K^{'}(x_m^{dc})(r_f^{'}(x_f^{dc})(1-\phi)+\phi) = r_m^{'}(x_m^{dc}) > r_m^{'}(x_m^{sp}) > K^{'}(x_m^{sp})(r_f^{'}(x_f^{sp})(1-\phi)+\phi) \tag{6}$$

Since $K^{''} > 0$, $K^{'}(x_m^{sp}) > K^{'}(x_m^{dc})$. Therefore for (11) to hold,

$$\begin{aligned} r_f^{'}(x_f^{dc}) &> r_f^{'}(x_f^{sp}) \\ \Rightarrow x_f^{dc} &< x_f^{sp} \end{aligned} \tag{7}$$

where the last line follows from the concavity of r_f. Using the budget constraint to substitute in for x_f^{dc} and x_f^{sp},

$$\omega + K^{'}(x_m^{dc})(\phi B - (1-\phi)x_m^{dc}) < \omega + K^{'}(x_m^{sp})(\phi B - (1-\phi)x_m^{sp}) \tag{8}$$

Since $(1-\phi)x_m^{sp} > \phi B$ and $K^{'}(x_m^{sp}) > K^{'}(x_m^{dc})$, for (8) to hold we require that,

$$\begin{aligned} \phi B - (1-\phi)x_m^{dc} &< \phi B - (1-\phi)x_m^{sp} \\ \Rightarrow x_m^{dc} &> x_m^{sp} \end{aligned} \tag{9}$$

This is a contradiction. Therefore it must be the case that $x_m^{sp} < x_m^{dc}$ when $x_m^{sp} > \phi(B + x_m^{sp})$. This proves the first part of the proposition.

to prove the second part of the proposition, note that when $x_m^{sp} < \phi(B + x_m^{sp})$, then the last two terms together in (3) are negative and therefore,

$$r_m^{'}(x_m^{sp}) < K^{'}(x_m^{sp})(r_f^{'}(x_f^{sp})(1-\phi) + \phi) \tag{10}$$

To prove the second part of the proposition, we want to show that when $x_m^{sp} < \phi(B+x_m^{sp})$, $x_m^{sp} > x_m^{dc}$.

We prove this by contradiction by showing that if $x_m^{sp} < x_m^{dc}$, (4) and (10) imply that household welfare is lower under the social planner's allocation than the decentralized equilibrium which is a contradiction. We can one again ignore the case of $x_m^{sp} = x_m^{dc}$. The proof is as follows. Suppose $x_m^{sp} < x_m^{dc}$. Then since $r_m^{''} < 0$, $r_m^{'}(x_m^{sp}) > r_m^{'}(x_m^{dc})$. Therefore using (4) and (5),

$$K^{'}(x_m^{dc})(r_f^{'}(x_f^{dc})(1-\phi) + \phi) = r_m^{'}(x_m^{dc}) < r_m^{'}(x_m^{sp}) < K^{'}(x_m^{sp})(r_f^{'}(x_f^{sp})(1-\phi) + \phi) \tag{11}$$

Since $K^{''} > 0$, $K^{'}(x_m^{sp}) < K^{'}(x_m^{dc})$. Therefore for (11) to hold,

$$\begin{aligned} r_f^{'}(x_f^{dc}) &< r_f^{'}(x_f^{sp}) \\ \Rightarrow x_f^{dc} &> x_f^{sp} \end{aligned} \tag{12}$$

However, if $x_m^{dc} > x_m^{sp}$ and $x_f^{dc} > x_f^{sp}$, then household welfare is higher under decentralized equilibrium allocation than under the social planner allocation (this is always when the Limited Funds assumption holds). This is a contradiction and therefore when $x_m^{sp} < \phi(B + x_m^{sp})$, $x_m^{sp} > x_m^{dc}$.

■

Proof of Proposition 2. When $r_g^* = K^{''}(x_m^{sp})(\phi B - (1-\phi)x_m^{sp})(r_f^{'}(x_f^{sp}) - 1)$ and $b^* = 0$, the household's first order condition is,

$$r'_m(x_m) + K''(x_m^{sp})(\phi B - (1-\phi)x_m^{sp})(r'_f(x_f^{sp}) - 1) = (r'_f(x_f)(1-\phi) + \phi)P \tag{13}$$

When $b = 0$, $P = K'(x_m)$ in equilibrium. Substituting that into (13),

$$r'_m(x_m) = K'(x_m)(r'_f(x_f)(1-\phi) + \phi) + K''(x_m^{sp})x_m^{sp}(r'_f(x_f^{sp}) - 1) - K''(x_m^{sp})\phi(B + x_m^{sp})(r'_f(x_f^{sp}) - 1)$$

For this equation to hold, $x_m = x_m^{sp}$ and $x_f = x_f^{sp}$ since it is identical to (2) since the household's problem when the household is a price-taker has a unique solution. For the government to have a balanced budget, $\tau = r_g x_m^{sp}$. Substituting this into household utility, we see that the utility is the same as that of the constrained social planner. ■

Proof of Proposition 3. At optimal from the household's budget constraint,

$$x_f^* = \omega + \phi BP^* - P^*(1-\phi)x_m^*$$

This gives us

$$P^* = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*}$$

At that level household doesn't invest anymore in x_f since $r'(x_f^*) = 1$. Looking at the household's FOC, we also require that,

$$\frac{r'_m(x_m^*) + r_g^*}{P^*} = 1$$

This gives us an r_g^* of

$$r_g^* = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} - r'_m(x_m^*)$$

and it gives us a b^* of,

$$K'(x_m^*) = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} + b^*$$

therefore b^* is,

$$b^* = K'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*}$$

We know that at x_m^*, $K'(x_m^*) = r'_m(x_m^*)$ and therefore we can rewrite b^* as,

$$b^* = r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} = -r_g^*$$

Therefore, expansionary supply-side policy (positive b) have to be accompanied by contractionary demand-side intervention (negative r_g) to achieve the optimum. b^* is positive when,

$$r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} > 0$$

Rewriting,

$$r'(x_m^*) > \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*}$$

Under this subsidy scheme the household's utility is given by,

$$U = r_m(x_m^*) + r_f(x_f^*) + r_g^* x_m^* - \tau - l + (P^{b^*} + b^*)x_m^* - K(x_m^*)$$

where $l = \phi(B + x_m^*)P^{b^*} = -P^{b^*}x_m^* + x_f^* - \omega$. For the government to have a balanced budget, $\tau = (b^* + r_g^*)x_m^*$. Substituting this into household utility and using the fact that $P^{b^*} = K'(x_m) - b^*$, the above simplifies to,

$$U = r_m(x_m^*) + r_f(x_f^*) - K(x_m^*) - x_f^* + \omega$$

We see that the utility is the same as that of the unconstrained social planner.

■

Proof of Proposition 4. When the household can invest as it likes and is not financially constrained, the household chooses x_f^* s.t. $r'(x_f^*) = 1 \; \forall \; b, r_g$. This can be proven by contradiction. Suppose the household chooses an $x_f < x_f^*$. Then taking a loan of $x_f - x_f^*$ and deviating to x_f^* will provide higher terminal wealth and therefore an $x_f < x_f^*$ cannot be optimal. Suppose the household chooses an $x_f > x_f^*$. Then reducing its loan by $x_f^* - x_f$ and deviating to x_f^* will provide higher terminal wealth and therefore an $x_f > x_f^*$ cannot be optimal for the household. In the analysis that follows, superscript d refers to a demand-side quantities, while superscript s refers to supply-side quantities.

A demand side intervention r_g that generates x_m' will be associated with household utility U^d given by,

$$U^d = r_m(x_m') + r_f(x_f^*) + r_g x_m' - \tau^d - l^d + P^d x_m' - K(x_m')$$

Substituting in for τ^d and $l^d = P^d x_m' + x_f^* - \omega$, this simplifies to

$$U^d = r_m(x_m') + r_f(x_f^*) - x_f^* + \omega - K(x_m')$$

Consider the supply side subsidy b' given by,

$$r_g(x'_m) = (K'(x'_m) - b')(r'_f(x^*_f)(1-\phi) + \phi)$$

$$b' = K'(x'_m) - r_g(x'_m)$$

This supply side subsidy b that generates housing demand of x'_m. It is associated with household utility U^s that is given by,

$$U^s = r_m(x'_m) + r_f(x^*_f) - \tau^s - l^s + (P^s + b)x'_m - K(x'_m)$$

Substituting in for τ^s and $l^s = P^s x'_m + x^*_f - \omega$, this simplifies to

$$U^s = r_m(x'_m) + r_f(x^*_f) - x^*_f + \omega - K(x'_m)$$

Therefore, for any r'_g generates x'_m, x'_f and U', there exists a b' that also generates x'_m, x'_f and U'. The converse can be proven similarly. ■

Proof of Proposition 5. In the analysis that follows, superscript d refers to a demand-side quantities, while superscript s refers to supply-side quantities. Say we want to achieve a level of x'_m. Then a demand side intervention will require r_g such that,

$$r'_m(x'_m) + r_g = K'(x'_m)r'_f(r^d_f)$$

A supply side intervention will require b such that,

$$r'_m(x'_m) = \Big(K'(x'_m) - b)\Big)\, r'_f(r^s_f)$$

Using the fact that $l = 0$ and that $\tau = r_g x'_m$, the utility of the household under demand-side intervention is then given by,

$$U^d = r_m(x'_m) + r_f(x^d_f) - K(x'_m) + \omega - x^d_f$$

Similarly, the utility of the household under supply-side intervention is given by,

$$U^s = r_m(x'_m) + r_f(x^s_f) - K(x'_m) + \omega - x^s_f \tag{14}$$

Using the budget constraint and that fact that in equilibrium $P^d = K^{'}(x^{'}_m)$ and $P^s = K^{'}(x^{'}_m) - b$, we see that,

$$x^d_f = \omega - K^{'}(x^{'}_m)x^{'}_m$$

$$x^s_f = \omega - (K^{'}(x^{'}_m) - b)x^{'}_m = x^d_f + bx^{'}_m$$

We can rewrite (14) as,

$$U^s = r_m(x^{'}_m) + r_f(x^d_f + bx^{'}_m) - x^d_f - bx^{'}_m - K(x^{'}_m) + \omega$$

Since $r^{'}_f(x^d_f + bx^{'}_m) \geq 1$ (since the household will prefer to invest in storage when $r^{'}_f < 1$) and $r^{''}_f < 0$, when $b > 0$, $r_f(x^d_f + bx^{'}_m) - x^d_f - bx^{'}_m > r_f(x^d_f) - x^d_f$. Therefore $U^s > U^d$. Conversely, when $b < 0$, $r_f(x^d_f + bx^{'}_m) - x^d_f - bx^{'}_m < r_f(x^d_f) - x^d_f$. Therefore $U^d > U^s$ and demand taxes pareto dominate supply taxes. ■

Proof of Proposition 6. In the analysis that follows, superscript d refers to a demand-side quantities, while superscript s refers to supply-side quantities. Say we want to achieve a level of $x^{'}_m$. Then a demand side intervention will require r_g such that,

$$r^{'}_m(x^{'}_m) + r_g = K^{'}(x^{'}_m)r^{'}_f(r^d_f)$$

A supply side intervention will require b such that,

$$r^{'}_m(x^{'}_m) = \Big(K^{'}(x^{'}_m) - b)\Big) r^{'}_f(r^s_f)$$

Using the fact that $\tau = r_g x^{'}_m$, the utility of the household under demand-side intervention is then given by,

$$U^d = r_m(x^{'}_m) + r_f(x^d_f) - K(x^{'}_m) + \omega - x^d_f$$

Similarly, the utility of the household under supply-side intervention is given by,

$$U^s = r_m(x^{'}_m) + r_f(x^s_f) - K(x^{'}_m) + \omega - x^s_f$$

Using the budget constraint and that fact that in equilibrium $P^d = K^{'}(x^{'}_m)$ and $P^s =$

$K^{'}(x^{'}_{m}) - b$, we see that,

$$x_f^d = \omega - K^{'}(x^{'}_{m})x^{'}_{m} + K^{'}(x^{'}_{m})(x^{'}_{m} + B) = \omega + K^{'}(x^{'}_{m})B$$

$$x_f^s = \omega - (K^{'}(x^{'}_{m}) - b)x^{'}_{m} + (K^{'}(x^{'}_{m}) - b)(x^{'}_{m} + B) = \omega + (K^{'}(x^{'}_{m}) - b)B$$

Following the same steps as in the proof of Proposition 5, we can show that in this case demand subsidies pareto dominate supply-side subsides and supply taxes pareto dominate demand taxes. ■