

on the interaction of growth, trade and international macroeconomics*

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Abstract

Standard economic theories have severe difficulties in simultaneously explaining a number of key aggregate empirical facts: i) there are substantial differences in capital-labor ratios across space and time ii) both production factors earn non-negligible shares in income iii) labor hours per capita are rather stable iv) price levels are higher in more developed countries v) there are no large gains from factor-proportions trade vi) the world trade-to-output ratio increases over time. I present an augmented standard theory that incorporates improvements in goods quality and new products in a straightforward manner and can jointly rationalize these six empirical facts.

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“*We’re richer than we realize.*” — Martin Feldstein

1 Introduction

Differences in economic development across space and time are ubiquitous and intimately connected to variations in capital-labor ratios,¹ see e.g. Panel (i) of Figure 1. Importantly, they raise a number of puzzling theoretical questions. (ii) Why do capital and labor both still earn non-negligible shares in income despite the vast increases in capital-labor ratios? (iii) Why do people still work amid the considerable increases in consumption possibilities? (iv) Why are price levels higher in more developed countries? (v) Why do the data show no clear specialization patterns between countries with high and countries with low capital-labor ratios? (vi) Why does the world trade-to-GDP ratio increase over time?

Standard economic theories provide a set of mutually exclusive and otherwise rather unsatisfying answers to these questions. In this paper, I argue that the key pitfall of standard theories is to ignore the vast improvements in goods quality and new products. I provide an augmented standard two-country model that incorporates these features in a straightforward manner and can jointly rationalize these six empirical facts.

This augmented model starts from the empirically plausible assumption that both the productivity of capital and labor improve over time. It thus contrasts with the neoclassical growth theory which, in the light of facts (ii) and (i), implies that only the productivity of labor but not the productivity of capital improves in development.² This implication is rather difficult to reconcile with the substantial increases in capital productivity that have arguably appeared over time.

By contrast, in the augmented model, more developed countries produce and consume more advanced goods. Producing these goods requires increasingly sophisticated capital. Increasingly sophisticated capital remains as scarce as labor along the growth path. That is why both labor and capital maintain a substantial share in overall income (ii). The real (quality-adjusted) capital-labor ratio, however, increases in development (i).³

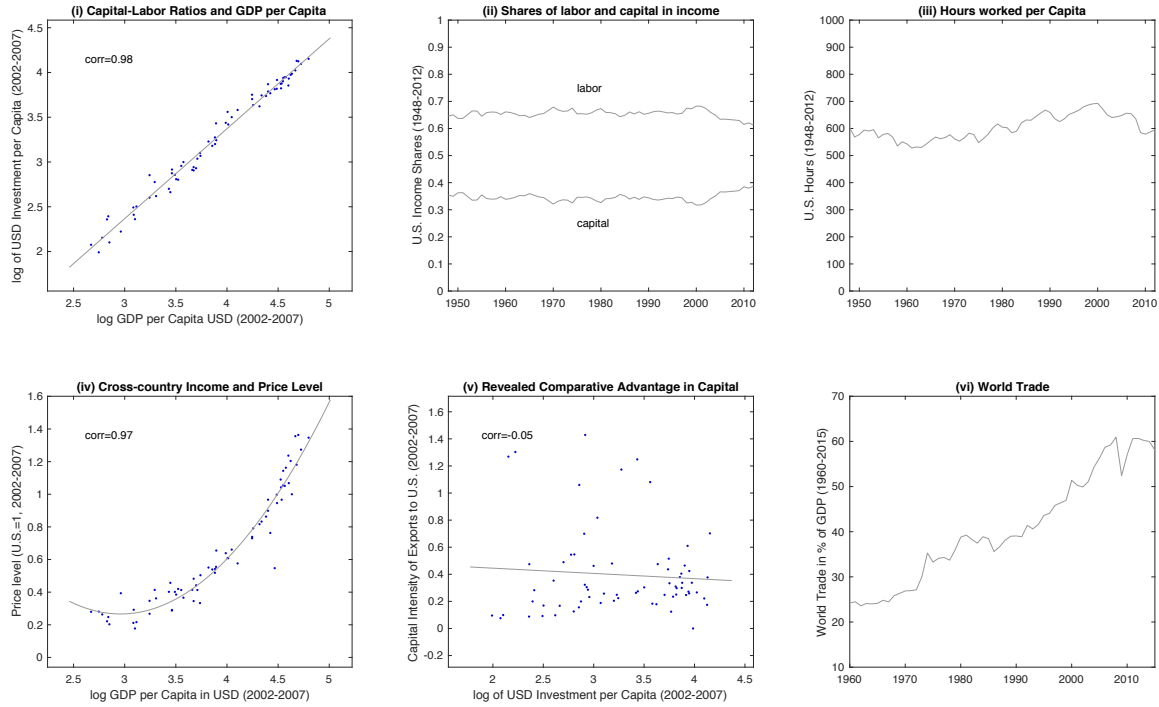
In this model, consumers gain utility from the improvements in product quality and not the level of consumption *per se*. Continuously improving product quality keeps their appetite for consumption high and motivates them to work (iii). In contrast, standard neoclassical theories imply that the expansion in production capacities (i) increases consumption possibilities and thereby induces

¹A more thorough description of the evidence is provided by [Hall & Jones \(1999\)](#), [Caselli \(2005\)](#) and [Jones \(2016\)](#).

²See e.g. [Uzawa \(1961\)](#), [Acemoglu \(2002\)](#) and [Jones \(2005\)](#). The only other alternative explanation within the neoclassical growth framework - that the capital labor elasticity equals unity - has virtually no empirical support. Estimates of this elasticity are difficult to obtain as pointed out by [Diamond *et al.* \(1978\)](#) and [León-Ledesma *et al.* \(2010\)](#), usually significantly different from unity, and rather complements than substitutes, see e.g. [Klump *et al.* \(2007\)](#) and [Oberfeld & Raval \(2014\)](#).

³As emphasized in [Struck & Velic \(2017b\)](#), neoclassical models typically employ real USD capital (as the capital input) and the number of hours or workers (as the labor input). Since the quality of work increases over time but the number of hours worked remains rather stable, using the number of hours as the labor input means that the labor input is quality-*un*adjusted. By contrast, using real capital (deflated capital) as the capital input means that the capital input is quality-adjusted. This is because quality improvements are reflected in inflation.

Figure 1: Six aggregate stylized facts in economics.



Notes: The figure shows the author's calculations. Panel (i) shows the 2002-2007 average relation between USD capital per capita and USD GDP per capita across 70 countries. The data are drawn from the World Bank Development Indicators. Panel (ii) shows the labor and capital shares in the U.S. Business Sector between 1948-2012. Panel (iii) shows the number of hours worked in the U.S. between 1948-2012. The data for Panels (ii) and (iii) are drawn from the U.S. Bureau of Labor Statistics. Panel (iv) shows the 2002-2007 average relation between the price level and the USD GDP per capita across 70 countries. The data are drawn from the Worldbank Development Indicators. Panel (v) shows the 2002-2007 avg. relation between a measure of revealed comparative advantage and a proxy for USD capital per capita across 70 countries. The data are drawn from [Schott \(2008\)](#), [Bartelsman & Gray \(1996\)](#) and the World Bank Development Indicators. Panel (vi) shows the 1960-2015 world trade to GDP ratio in percent. The data are drawn from the Worldbank Development Indicators. More details on the data sources and the data construction can be found in [Appendix A](#).

a decline in the labor supply. This decline results from falling marginal utility of consumption of higher numbers of the same varieties.⁴

In the augmented model, more sophisticated goods require more advanced resources in production. Therefore, their relative prices are high. As a result, more developed countries, which tend to use more sophisticated goods also have higher price levels.⁵ By contrast, international neoclassical

⁴[Keynes \(1930\)](#) famously predicted that people will work only 3 hours per day in the generation of his grandchildren. Yet, this prediction which is an ingredient of many of today's macroeconomic models has not materialized. On the puzzle, see e.g. [Prescott \(2004\)](#), [Prescott & Wallenius \(2012\)](#), [Epstein & Kimball \(2014\)](#), [Struck \(2014\)](#) and [Bick et al. \(2016\)](#).

⁵I implicitly assume that the official price statistics used in Panel (iv) do not appropriately account for quality and variety differences across various stages of development. In particular I assume that they underestimate the quality and variety improvements over time. There is good reason to believe that this is the case, see e.g. [Bils & Klenow \(2001\)](#), [Schott \(2004\)](#), [Erickson & Pakes \(2011\)](#), [Syverson \(2017\)](#), [Byrne et al. \(2016\)](#), [Struck & Velic \(2017a\)](#), [Feldstein \(2017\)](#) and [Aghion et al. \(2017\)](#).

models that account for fact (iv) assume that productivity growth is not particularly high in labor intensive tasks such as the non-tradable service sector. This assumption is difficult to reconcile with neoclassical growth models that account for fact (ii) and an increasing world-trade-to-GDP ratio (vi).⁶

Similarly, because of differences in assumptions about the allocation of productivity within the economy, the neoclassical growth and trade theories collide. According to the latter theory, cross-country differences in capital-labor ratios (i) lead to pronounced specialization patterns in capital and labor intensive goods across countries. This implication is difficult to reconcile with the lack of trade specialization patterns shown in Panel (v).⁷ It is further difficult to reconcile with the prediction of neoclassical growth models that capital and labor earn rather stable shares in income (ii).

The augmented model also puts heterogeneity in goods quality and varieties at the center state of fact (v). Developed countries have more capital than less developed ones in quality-adjusted terms (in real terms). Quality capital and quality labor however, remain equally scarce along the growth path. Therefore, relative output prices equalize across countries even in autarky such that there are no gains from factor-proportions trade. That is why pronounced specialization patterns remain unobserved.

Yet, the world trade to GDP ratio rises in the augmented model (vi) as convergence between developed and developing countries raises the preferences for varieties trade of non-homothetic consumers. By contrast, standard models in trade and international economics imply that the world-trade-to-GDP ratio declines over time. In neoclassical trade models, convergence between countries makes countries more similar. Thus, it eradicates the gains from (neoclassical) trade by reducing specialization opportunities across countries.

This paper contributes to three main literatures. First, it is a contribution to the literature that attempts to give a proper place to quality and variety improvements in standard aggregate economic analysis. In line with the arguments of [Houthakker \(1957\)](#) and [Linder \(1961\)](#), a growing

⁶With relatively modest productivity growth in the non-tradable sector, the price level of a country increases in its level of development, see e.g. [Balassa \(1964\)](#), [Samuelson \(1964\)](#), [Baumol \(1967\)](#) and [Ghironi & Melitz \(2005\)](#). This explanation of price level differences, however, is problematic itself. First, it implies that the labor share in income converges to unity as labor and capital intensive goods are rather complements than substitutes. This implication conflicts with recent evidence that the labor share has rather declined over time, see e.g. [Karabarbounis & Neiman \(2014\)](#) and [Piketty & Zucman \(2014\)](#), or has remained approximately stable as fact (ii) suggests, see e.g. [Rognlie \(2015\)](#) and [Auerbach & Hassett \(2015\)](#). Second, the data from which the productivity estimates are drawn are not reliable because they are plagued with inflation measurement issues as [Erickson & Pakes \(2011\)](#), [Syverson \(2017\)](#), [Byrne et al. \(2016\)](#), [Struck & Velic \(2017a\)](#), [Aghion et al. \(2017\)](#) point out. Third, it is difficult to square with the neoclassical as well as the endogenous growth theory. After a period of relative price growth in the non-tradable sector, incentives for innovation become so large in this sector that productivity growth must increase again, see e.g. [Uzawa \(1961\)](#) or [Acemoglu \(2002\)](#). Fourth, it is also difficult to reconcile with the fact that the world trade to output ratio grows over time (vi). In a standard international neoclassical model, higher productivity growth in the tradable sector means that the world trade to GDP ratio converges to zero over time as tradable and non-tradable goods are complements, see e.g. the closed-economy model of [Acemoglu & Guerrieri \(2008\)](#).

⁷Initially observed by [Leontief \(1953\)](#), it is now well recognized that the evidence in support of the factor-proportions trade theory is rather weak, see e.g. [Trefler & Zhu \(2010\)](#), [Caron et al. \(2014\)](#), [Struck & Velic \(2016\)](#) and [Sorg-Langhans et al. \(2017\)](#). However, the reasons behind this outcome are not well understood.

literature recognizes the importance of heterogeneity in goods quality across different income levels, see e.g. [Bils & Klenow \(2001\)](#), [Schott \(2004\)](#), [Hallak & Schott \(2011\)](#), [Fajgelbaum *et al.* \(2011\)](#), [Caron *et al.* \(2014\)](#), [Aguiar & Bils \(2015\)](#) and [Aghion *et al.* \(2017\)](#). I view this literature as key to resolving many deeper issues in macroeconomics. I consider the augmented model as an exercise that demonstrates that quality and variety improvements can play a crucial role in jointly explaining several empirical puzzles across different fields.

Second, it is a contribution to the literature that attempts to bring growth, trade and international macroeconomics closer together. [Redding & Weinstein \(2017\)](#) provide a thorough discussion of the deep inconsistencies between these fields. In particular, I contribute to this literature by providing a model that offers a consistent explanation of some of the core empirical regularities across these three fields. Previous explanations are usually only consistent with one dimension of the data at the expense of explaining other dimensions. [Ventura \(1997\)](#), [Antràs & Caballero \(2009\)](#), [Jin \(2012\)](#) and [Zymek \(2015\)](#), among others, use neoclassical growth models with capital-labor heterogeneity across countries and industries to analyze various empirical phenomena at the intersection of the three fields. Their models imply strong specialization patterns and thus naturally struggle in accounting for facts (v) and (ii). I view my model as an attempt to help resolve these inconsistencies.

Third, it is a contribution to the literature that is concerned with the effects of structural change on aggregate economic outcomes. This literature proposes both supply and demand side mechanisms of structural change. On the former mechanism, see e.g. [Ngai & Pissarides \(2007\)](#) and [Acemoglu & Guerrieri \(2008\)](#). On the latter mechanism, see e.g. [Kongsamut *et al.* \(2001\)](#), [Matsuyama \(2002\)](#) and [Foellmi & Zweimüller \(2008\)](#). I follow more recent contributions that tend to jointly study both mechanisms, see e.g. [Buera & Kaboski \(2012\)](#), [Herrendorf *et al.* \(2013\)](#), [Boppart \(2014\)](#) and [Comin *et al.* \(2015\)](#). Relative to these closed economy papers, I follow [Matsuyama \(2017\)](#) in that I focus on the open economy and attempt to jointly explain several stylized facts from both the closed and open economy literatures. In line with [Grossman *et al.* \(2017\)](#), my approach builds on a microfoundation of the aggregate production function and manages to explain fact (ii) without relying on the inconvenient assumption that only the productivity of labor improves over time.

2 The model

In the model, there are two countries, Home and Foreign, denoted by subscripts $i, j \in \{H, F\}$, $i \neq j$. The countries differ in terms of their level of development. In each country, there are two production factors, capital and labor, denoted by subscript $n \in \{K, L\}$. The goods produced in each country differ in their sophistication. The upper-case variables $Y_i, Y_i^i, Y_i^j, Q_i, Q_{n,i}, P_i, P_{Q,i}, P_{Q,n,i}, R_i, K_i, C_i, I_i$ are quality-adjusted. The lower-case variables $q_i, q_{n,i}, p_i, p_{Q,i}, p_{n,i}, w_i, r_i, k_i, l_i$ are quality-*un*adjusted.

I split the theoretical analysis into two parts. First, I analyze, in autarky, two countries with different development levels. It helps to think about this setup either as a single country at different stages of development across two points in time or as two countries at different stages of development at a single point in time. In autarky, I analytically show that this model can rationalize all facts except (vi). Second, I extend the model to allow for trade between the two countries. All

propositions from the autarky analysis still hold, yet this extension also accounts for fact (vi).

For simplicity, I use the term “quality” as synonymous with “quality and variety” in this section.

2.1 A basic analysis without trade

One unit of a good with relatively high sophistication provides more units in real terms (quality-adjusted terms) than one unit of a good with relatively low sophistication. The relation between units of quality-*un*adjusted output, $q_{n,i,t}$ and units of quality-adjusted (real) output, $Q_{n,i,t}$, is given by

$$Q_{n,i,t} = \Phi_{i,t} q_{n,i,t}, \quad (1)$$

where $\Phi_{i,t}$ denotes a catchall variable that reflects the level of sophistication (the quality and the varieties) of output in country i at time t . Meanwhile, I define the relation between the corresponding price indexes as

$$P_{n,i,t} \Phi_{i,t} = p_{n,i,t}. \quad (2)$$

The relation between the level of sophistication of the goods produced in a country, $\Phi_{i,t}$ and the country’s level of development, $A_{i,t}$, is given by

$$\Phi_{i,t} = \gamma A_{i,t}, \quad (3)$$

where $\gamma > 0$ is a constant.

Assumption 1. *The quality of goods produced in country i at time t , $\Phi_{i,t}$, linearly depends on i ’s level of development, $A_{i,t}$.*

I rather realistically assume that the productivity of both capital and labor commensurately improves over time. In contrast to neoclassical models, I use quality-*un*adjusted inputs for both production factors.³ Quality-*un*adjusted capital and labor intensive output is given by

$$q_{n,i,t} = \begin{cases} k_{i,t}, & \text{if } n=K \\ l_{i,t}, & \text{if } n=L \end{cases} \quad (4)$$

where $k_{i,t}$ and $l_{i,t}$ denote the quality-*un*adjusted capital and labor inputs, respectively.⁸ Capital and labor intensive inputs are substitutable and can be combined to form quality-adjusted aggregate output,

$$Q_{i,t} = \left[Q_{K,i,t}^{\frac{\theta_{KL}-1}{\theta_{KL}}} + Q_{L,i,t}^{\frac{\theta_{KL}-1}{\theta_{KL}}} \right]^{\frac{\theta_{KL}}{\theta_{KL}-1}}, \quad (5)$$

⁸In the production setup, Eqs. (1), (3) and (4), there is an implicit assumption of non-homothetic preferences. At a higher levels of development an economy produces higher quality output. One unit of $q_{n,i,t}$ therefore yields greater real output $Q_{n,i,t}$. However, producing one unit of higher quality output, $q_{n,i,t}$, also requires more real resources as Eq. (9) further below shows. A more explicit micro-foundation of the production side of this model is provided in [Struck & Velic \(2017b\)](#).

where θ_{KL} denotes the elasticity of substitution between capital and labor inputs. Quality-*un*adjusted aggregate output is given by

$$q_{i,t} = \left[q_{K,i,t}^{\frac{\theta_{KL}-1}{\theta_{KL}}} + q_{L,i,t}^{\frac{\theta_{KL}-1}{\theta_{KL}}} \right]^{\frac{\theta_{KL}}{\theta_{KL}-1}}, \quad (6)$$

where $Q_{i,t} = \Phi_{i,t} q_{i,t}$. Demand for capital and labor input is given by

$$Q_{K,i,t} = \left(\frac{P_{K,i,t}}{P_{Q,i,t}} \right)^{-\theta_{KL}} Q_{i,t}, \quad \text{and} \quad Q_{L,i,t} = \left(\frac{P_{L,i,t}}{P_{Q,i,t}} \right)^{-\theta_{KL}} Q_{i,t}, \quad (7)$$

where $P_{Q,i,t}$ is the price of the aggregate good produced in country i at time t . The aggregate good can be used for both consumption and investment,

$$P_{Q,i,t} Q_{i,t} = P_{i,t} I_{i,t} + P_{i,t} C_{i,t}, \quad (8)$$

where $I_{i,t}$ and $C_{i,t}$ denote quality-adjusted consumption and investment; $P_{i,t}$ is the quality-adjusted price of the aggregate final good and is equal to $P_{Q,i,t}$, the quality-adjusted price of aggregate output. Quality capital accumulation is given by

$$k_{i,t+1} = (1 - \delta) k_{i,t} + \frac{1}{\Phi_{i,t}} I_{i,t}, \quad (9)$$

where δ denotes the depreciation rate. Intuitively, the last equation implies that more units of real output are required to accumulate one unit of new capital in a more advanced economy. The representative consumer's present discounted value of lifetime utility is given by

$$U_{i,t} = \sum_{s=0}^{\infty} \beta^{t+s} \frac{(C_{i,t+s}/aC_{i,t+s-1})^{1-\phi}}{1-\phi} - \frac{l_{i,t}^{1+\phi_L}}{1+\phi_L}, \quad (10)$$

where β is the discount factor; ϕ and ϕ_L are parameters governing the intertemporal elasticity of substitution and the labor elasticity, respectively; a is a parameter that determines the sensitivity of habit persistence.

To illustrate the main properties of this model analytically, I begin the analysis assuming a zero growth steady state (and autarky).⁹ In particular, I assume that the productivity of Home is greater than the productivity of Foreign, i.e. $A_{F,t-1} = A_{F,t} = A_{F,t+1} < A_{H,t-1} = A_{H,t} = A_{H,t+1}$. It thus might help to think about Home as a group of developed countries and Foreign as a group of developing countries, or as Foreign as Home at an earlier stage of development. The representative firm in the perfectly competitive sector n maximizes profits, $\pi_{n,i,t}$, given by

$$\pi_{n,i,t} = \begin{cases} p_{K,i,t} q_{K,i,t} - r_{i,t} k_{i,t}, & \text{if } n=K \\ p_{L,i,t} q_{L,i,t} - w_{i,t} l_{i,t}, & \text{if } n=L \end{cases} \quad (11)$$

⁹Adding a common trend growth rate to the model unnecessarily complicates the analysis without adding any additional analytical insight.

where $p_{n,i,t}$ is the quality-*un*adjusted price of output n ; $r_{i,t}$ and $w_{i,t}$ denote the returns to capital and labor, respectively. Firm optimization yields,

$$r_{i,t} = p_{K,i,t}, \quad (12)$$

$$w_{i,t} = p_{L,i,t}. \quad (13)$$

In period t , the representative consumer in country i maximizes the Lagrangian function

$$\begin{aligned} \mathcal{L}_{i,t} = & \sum_{s=0}^{\infty} \beta^{t+s} \left[\frac{\left(\frac{C_{i,t+s}}{aC_{i,t+s-1}} \right)^{1-\phi}}{1-\phi} - \frac{l_{i,t+s}^{1+\phi_L}}{1+\phi_L} \right. \\ & + \lambda_{i,t+s} (w_{i,t+s} l_{i,t+s} + r_{i,t+s} k_{i,t+s} - P_{i,t+s} I_{i,t+s} - P_{i,t+s} C_{i,t+s}) \\ & \left. + \chi_{i,t+s} \left((1-\delta) k_{i,t+s} + \frac{1}{\Phi_{i,t}} I_{i,t+s} - k_{i,t+s+1} \right) \right], \end{aligned}$$

where $\chi_{i,t+s}$, and $\lambda_{i,t+s}$ denote Lagrange multipliers. Optimization with respect to $k_{i,t+s+1}$, $I_{i,t+s}$, $C_{i,t+s}$ and $l_{i,t+s}$ yields the first-order conditions

$$\mathcal{L}_{k_{i,t+s+1}} = (1-\delta)\beta\chi_{i,t+s+1} + \beta\lambda_{i,t+s+1}r_{i,t+s+1} - \chi_{i,t+s} = 0, \quad (14)$$

$$\mathcal{L}_{I_{i,t+s}} = -\lambda_{i,t+s}P_{i,t+s} + \frac{1}{\Phi_{i,t}}\chi_{i,t+s} = 0, \quad (15)$$

$$\mathcal{L}_{C_{i,t+s}} = \frac{C_{i,t+s}^{-\phi}}{a^{1-\phi}C_{i,t+s-1}^{1-\phi}} - \beta\frac{C_{i,t+s+1}^{1-\phi}}{a^{1-\phi}C_{i,t+s}^{2-\phi}} - P_{i,t+s}\lambda_{i,t+s} = 0, \quad (16)$$

$$\mathcal{L}_{l_{i,t+s}} = \lambda_{i,t+s}w_{i,t+s} - l_{i,t+s}^{\phi_L} = 0. \quad (17)$$

Let me introduce two quality-adjusted variables. The quality-adjusted capital stock is given by $K_{i,t} = \Phi_{i,t}k_{i,t}$. The quality-adjusted return to capital is given by $R_{i,t} = (1/\Phi_{i,t})r_{i,t}$. Then, combining equations (14) and (15) yields

$$R_{i,t} = r_{i,t} \frac{1}{\Phi_{i,t}} = P_{i,t} \left(\frac{1}{\beta} - 1 + \delta \right) \Leftrightarrow \frac{R_{i,t}}{P_{i,t}} = \left(\frac{1}{\beta} - 1 + \delta \right). \quad (18)$$

Capital accumulation, Eq. (9), simplifies to

$$K_{i,t} = \Phi_{i,t}k_{i,t} = \frac{I_{i,t}}{\delta}. \quad (19)$$

Combining Eqs. (2), (12) with the previously defined quality-adjusted variable, $R_{i,t}$, yields a simple expression for the quality-adjusted capital price:

$$R_{i,t} = P_{K,i,t}. \quad (20)$$

Using Eqs. (1), (3), (4), (5) and (19), I can express aggregate production as the neoclassical production function with labor-augmenting productivity,

$$Q_{i,t} = \left[(K_{i,t})^{\frac{\theta_{KL}-1}{\theta_{KL}}} + (\gamma A_{i,t} l_{i,t})^{\frac{\theta_{KL}-1}{\theta_{KL}}} \right]^{\frac{\theta_{KL}}{\theta_{KL}-1}}, \quad (21)$$

What is the intuition behind this surprising result? As alluded to earlier,³ in the neoclassical model, capital (real USD capital) is capital adjusted for quality improvements. By contrast, real labor (the number of hours or workers) is labor *un*adjusted for quality improvements. In the augmented model, both inputs are quality-*un*adjusted. I assume that the productivity of both of these inputs commensurately improves. Therefore, if I represent aggregate production asymmetrically with one factor being quality-adjusted and one factor being quality-*un*adjusted, as in Eq. (21), I get an asymmetry in the bias of technical change.

Proposition 1. *Despite factor-neutral productivity at the firm level, aggregate production takes the form of a neoclassical production function with labor-augmenting productivity. ■*

Using $P_{Q,i,t} = P_{i,t}$ and substituting Eqs. (1), (4), (5), (18) and (20) into Eq. (7), yields the quality-*un*adjusted capital-labor ratio:

$$\frac{k_{i,t}}{l_{i,t}} = \left[\left(\frac{1}{\beta} - 1 + \delta \right)^{\theta_{KL}-1} - 1 \right]^{-\frac{\theta_{KL}}{\theta_{KL}-1}}. \quad (22)$$

Thus, the ratio of quality-adjusted capital to quality-*un*adjusted labor is given by

$$\frac{K_{i,t}}{l_{i,t}} = \gamma A_{i,t} \left[\left(\frac{1}{\beta} - 1 + \delta \right)^{\theta_{KL}-1} - 1 \right]^{-\frac{\theta_{KL}}{\theta_{KL}-1}}. \quad (23)$$

This latter equation shows that the capital per capita ratio positively depends on the level of development. Hence, $K_{F,t}/l_{F,t} < K_{H,t}/l_{H,t}$.

Proposition 2 (Fact i). *The capital-labor ratio increases in the level of development, $A_{i,t}$. A more developed country therefore has a higher capital-labor ratio. ■*

It is now straightforward to show that both production factors maintain substantial shares in income despite factor-neutral productivity growth. The intuition is that both factors remain scarce along the growth path. Plugging Eq. (18) into (7) using (20) yields

$$\frac{P_{K,i,t} Q_{K,i,t}}{P_{Q,i,t} Q_{i,t}} = \left(\frac{P_{K,i,t}}{P_{Q,i,t}} \right)^{1-\theta_{KL}} = \left(\frac{1}{\beta} - 1 + \delta \right)^{1-\theta_{KL}}. \quad (24)$$

Since the share of labor is given by $(P_{L,i,t} Q_{L,i,t})/(P_{Q,i,t} Q_{i,t}) = 1 - (P_{K,i,t} Q_{K,i,t})/(P_{Q,i,t} Q_{i,t})$, both shares are constant as the RHS of Eq. (24) only depends on given parameters.

Proposition 3 (Fact ii). *Both labor and capital maintain a substantial share in income independent of the level of development, $A_{i,t}$. ■*

To show that the labor supply is constant, I first need to derive four intermediate equations. Combining Eqs. (17) and (16) yields

$$C_{i,t} = \frac{1}{P_{i,t}} \frac{w_{i,t}}{l_{i,t}^{\phi_L}} \frac{1-\beta}{a^{1-\phi}}. \quad (25)$$

Rearranging Eq. (23) and using Eq. (19) yields

$$I_{i,t} = \gamma A_{i,t} \delta \left[\left(\frac{1}{\beta} - 1 + \delta \right)^{\theta-1} - 1 \right]^{-\frac{\theta}{\theta-1}} l_{i,t}. \quad (26)$$

Plugging Eq. (22) into (5) using Eqs. (1) and (4) yields

$$Q_{i,t} = \gamma A_{i,t} \left[\left[\left(\frac{1}{\beta} - 1 + \delta \right)^{\theta-1} - 1 \right]^{-1} + 1 \right]^{\frac{\theta}{\theta-1}} l_{i,t}. \quad (27)$$

Substituting Eqs. (1), (2), (3), (4), (5), (13) and (22) into Eq. (7) yields

$$w_{i,t} = \gamma A_{i,t} P_{Q,i,t} \left[\left[\left(\frac{1}{\beta} - 1 + \delta \right)^{\theta-1} - 1 \right]^{-1} + 1 \right]^{\frac{1}{\theta-1}}. \quad (28)$$

Substituting the previous four equations into the resource constraint (8) and solving for $l_{i,t}$ yields

$$l_{i,t} = \frac{\left[\frac{1-\beta}{a^{1-\phi}} \left[\left(\frac{k_{i,t}}{l_{i,t}} \right)^{\frac{\theta-1}{\theta}} + 1 \right]^{\frac{1}{\theta-1}} \right]^{\frac{1}{1+\phi_L}}}{\left[\left(\frac{k_{i,t}}{l_{i,t}} \right)^{\frac{\theta-1}{\theta}} + 1 \right]^{\frac{\theta}{\theta-1}} - \delta \frac{k_{i,t}}{l_{i,t}}}. \quad (29)$$

Having previously established that $k_{i,t}/l_{i,t}$ is a constant, the RHS of this last equation must also be a constant. Therefore, $l_{i,t}$ must be constant.

Proposition 4 (Fact iii). *The labor supply maintains its level independent of the level of development, $A_{i,t}$. ■*

Let me define $p_{i,t}$ as the quality-*unadjusted* aggregate price level in country i . The aggregate quality-adjusted price of output is given by $P_{Q,i,t} = [P_{K,i,t}^{1-\theta_{KL}} + P_{L,i,t}^{1-\theta_{KL}}]^{1/(1-\theta_{KL})}$. Following Eq. (2), the quality-*unadjusted* price of output is therefore given by $p_{Q,i,t} = [p_{K,i,t}^{1-\theta_{KL}} + p_{L,i,t}^{1-\theta_{KL}}]^{1/(1-\theta_{KL})}$. Normalizing the quality-adjusted output price to unity, i.e. setting $P_{Q,i,t} = 1$, from Eqs. (2) and (3), it follows that

$$p_{Q,i,t} = \gamma A_{i,t} \Rightarrow p_{F,t} < p_{H,t}. \quad (30)$$

The price level of Home (the developed country group) is higher than the price level of Foreign (the developing country group). The relatively higher price level is the result of the relatively higher goods quality that is used in Home. As higher quality goods are more difficult to produce, they

require more resources and are therefore more costly.

Proposition 5 (Fact iv). *The quality-unadjusted price level increases in the level of development, $A_{i,t}$. ■*

Combining Eqs. (1), (4), (7) and (22) yields an expression for relative prices for the output of the two tasks which is

$$\frac{P_{K,i,t}}{P_{L,i,t}} = \left[\left(\frac{1}{\beta} - 1 + \delta \right)^{\theta_{KL}-1} - 1 \right]^{\frac{1}{\theta_{KL}-1}}. \quad (31)$$

As I previously illustrated in Eq. (22), quality-unadjusted capital and labor are equally scarce across different levels of development. Therefore, relative prices for capital and labor intensive output do not vary in the level of development.¹⁰ Since there are no differences in relative prices across countries, there are no incentives for trade specialization in capital and labor intensive goods.

Proposition 6 (Fact vi). *The relative price of capital and labor intensive output does not depend on the level of development, $A_{i,t}$. ■*

2.2 An extended analysis with trade

To initiate trade between the two countries, I follow [Armington \(1969\)](#) in assuming that goods produced in different countries are imperfect substitutes. This assumption makes sense in the light of Assumption 1 given that countries at different stages of development produce goods that differ vastly in terms of their quality, see e.g. [Schott \(2004\)](#) and [Caron et al. \(2014\)](#). More specifically, I assume that aggregate quality-adjusted demand, $Y_{i,t}$, is given by

$$Y_{i,t} = \left[(1 - \omega_{i,t})^{\frac{1}{\theta}} [Y_{i,t}^i]^{\frac{\theta-1}{\theta}} + \omega_{i,t}^{\frac{1}{\theta}} [Y_{i,t}^j]^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (32)$$

where $Y_{i,t}^i$ denotes the amount of the quality-adjusted good that is produced and used (for consumption or investment) in country i ; $Y_{i,t}^j$ denotes the amount of the quality-adjusted good that is used in country i but produced in country j ; θ denotes the elasticity of substitution between Home and Foreign goods; $\omega_{i,t}$ denotes the share of the imported good in total demand.¹¹

Assumption 2. *Home and Foreign varieties are imperfect substitutes with $\theta \ll \infty$.*

¹⁰I could also form intermediate goods using various combinations of the capital and labor output. The relative price of such goods also does not vary in the level of development since the relative price of raw capital and labor output is constant.

¹¹Technically, to account for fact (v), I should allow for trade in intermediate goods. For simplicity, I only allow for trade in final goods however. In [Struck & Velic \(2016\)](#) and [Sorg-Langhans et al. \(2017\)](#), I show in versions of the model studied here that the mechanism works even when I allow for trade in intermediate goods. However, in these two papers I cannot obtain an analytical solution and therefore have to rely on numerical exercises.

Demand for domestic and imported goods are given by

$$Y_{i,t}^i = (1 - \omega_{i,t}) \left(\frac{P_{Q,i,t}}{P_{i,t}} \right)^{-\theta} Y_{i,t} \quad \text{and} \quad Y_{i,t}^j = \omega_{i,t} \left(\frac{\xi_{i,t} P_{Q,j,t}}{P_{i,t}} \right)^{-\theta} Y_{i,t} \quad (33)$$

where $\xi_{i,t} = 1/\xi_{j,t}$ denotes the nominal exchange rate of country i . I now make an additional assumption that complements the assumptions that I previously made. In line with [Caron *et al.* \(2014\)](#) and [Aguiar & Bilal \(2015\)](#), among others, I assume that there are non-homothetic preferences. That is, the further the countries are apart in terms of their level of development, the lower their import demand for goods from each other. To capture this intuition, I take a simple approach and make $\omega_{i,t}$ directly depend on the cross-country difference of the level of development,

$$\omega_{i,t} = \frac{e^{-|\ln(A_{j,t}) - \ln(A_{i,t})|}}{z} \frac{Q_{j,t}}{Q_{i,t} + Q_{j,t}}. \quad (34)$$

where $z \geq 1$ is an indicator of home bias.¹² $e^{-|\ln(A_{j,t}) - \ln(A_{i,t})|}$ is an indicator of the development distance between the two countries.¹³ It is important to understand that the second term on the RHS is just a country size adjustment that has no particular economic interpretation. It ensures that both, the small and the large country, export and import the same amount in real terms.¹⁴

Assumption 3. *Preferences are non-homothetic.*

Market clearing is given by

$$Q_{i,t} = Y_{i,t}^i + Y_{j,t}^i. \quad (35)$$

I assume that trade is balanced, i.e. that

$$P_{Q,i,t} Q_{i,t} = P_{i,t} Y_{i,t}. \quad (36)$$

As I show in [Appendix B](#), in equilibrium, $P_{Q,i,t} = P_{i,t}$ and $\xi_{i,t} P_{Q,j,t} = P_{i,t}$. Thus, [Eq. \(8\)](#) and all previous propositions hold in this extension. Using [Eq. \(36\)](#), the demand for domestic and imported varieties, [Eq. \(33\)](#), then simplifies to

$$Y_{i,t}^i = (1 - \omega_{i,t}) Q_{i,t} \quad \text{and} \quad Y_{i,t}^j = \omega_{i,t} Q_{i,t}. \quad (37)$$

The world trade-to-GDP ratio is given by

¹²If $z = 1$ and both countries have an equal level of development, then $\omega_{i,t} = 1/2$. If $z = 2$ and both countries have an equal level of development, then $\omega_{i,t} = 0.25$. Thus, the greater z the greater the home bias.

¹³If countries are equally developed, i.e. $A_{j,t} = A_{i,t}$, the term equals unity. If, for example, i is only half as developed as j , i.e. $A_{j,t} = 2A_{i,t}$, the term is $1/2$. If i is only one third as developed as j , i.e. $A_{j,t} = 3A_{i,t}$, the term is $1/3$.

¹⁴For example, suppose Home is twice as large as Foreign and Foreign imports 5% of its GDP from Home. Then Home will import only 2.5% of its GDP from Foreign.

$$\tau_t \equiv \frac{\xi_{j,t} P_{Q,i,t} Y_{j,t}^i + P_{Q,j,t} Y_{i,t}^j}{\xi_{j,t} P_{Q,i,t} Q_{i,t} + P_{Q,j,t} Q_{j,t}} = \frac{\omega_{i,t} Q_{i,t} + \omega_{j,t} Q_{j,t}}{Q_{i,t} + Q_{j,t}} = \frac{2 \times A_{i,t}^2}{[A_{i,t} + A_{j,t}]^2} = \frac{2}{\left[1 + \frac{1}{A_{i,t}/A_{j,t}}\right]^2}, \quad (38)$$

where i is assumed to be the less developed country, i.e. $A^i < A^j$. This ratio is increasing when country i grows relative to country j as the derivative shows:

$$\frac{\partial \tau_t}{\partial (A_{i,t}/A_{j,t})} = \frac{4}{\left[1 + \frac{1}{A_{i,t}/A_{j,t}}\right]^3} \frac{1}{(A_{i,t}/A_{j,t})^2} > 0 \quad \forall A_{i,t}, A_{j,t} > 0 \wedge A_{i,t} < A_{j,t}. \quad (39)$$

Proposition 7 (Fact vi). *The world trade-to-GDP ratio increases when the level of development of the less developed country, $A_{i,t}$, converges to the level of development of the more developed country, $A_{j,t}$. ■*

What is the intuition behind this result? According to the neoclassical trade theory, the world-trade-to-GDP ratio falls when countries converge. Why? When countries converge they become similar. Greater differences across countries however, allow for greater specialization opportunities. Thus, gains from trade are high when countries have rather different levels of development. By contrast, in the model presented in this paper, convergence raises world trade. When countries are dissimilar, consumers in each country have rather low demand for the goods from one another. This is because of non-homothetic preferences. People in more developed countries prefer to consume expensive varieties while people in less developed countries prefer to consume inexpensive varieties. When countries converge they become more similar. As a consequence, demand for imported varieties from abroad rises.

3 Conclusion

Standard economic theories have severe difficulties in jointly accounting for a number of key aggregate stylized facts. I present an augmented standard two-country model that can simultaneously generate i) substantial differences in capital-labor ratios across countries ii) non-negligible factor shares in income despite continuously increasing capital-labor ratios iii) labor hours per capita that are stable amid expanding consumption possibilities iv) price levels that are higher in more developed countries v) no large gains from factor-proportions trade vi) an increasing world trade-to-GDP ratio.

Key to the empirical success of this model is to recognize that standard models do not explicitly account for quality and variety differences across various income levels. It incorporates these features in a straightforward manner and sheds new light on these six well-known facts. For example, it implies that, when official price statistics underestimate quality and variety improvements, more developed countries also have higher price levels (iv).

I consider this model as an exercise that underlines the vital role that quality and variety improvements play in understanding various empirical phenomena. It calls for future research to give a more appropriate place to these forces in the analysis.

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A Data Construction

Table 1 provides a summary of the data underlying Figure 1. The sample for the cross section is determined following the procedure of [Lane & Milesi-Ferretti \(2012\)](#) which omits small and oil-dominated countries. In total the sample incorporates 70 developing and developed economies.

Fact (i). The data are logs of current USD GDP and investment per capita. Since investment is (in theory) proportional to the capital stock, it serves as an approximation for the capital-labor ratio. The latter variable is much more difficult to measure. All data are taken from the World Bank development indicators. The period, 2002-2007, is chosen to reflect a single point in time. A 5-year average is taken to avoid any inference from country-specific business cycles.

Fact (ii). The data reflect the shares of labor and capital income as a fraction of the U.S. Private Business Sector (excluding government enterprises) total income. The data are taken from the U.S. Bureau of Labor Statistics. The period 1948-2012 is naturally governed by data limitations.

Fact (iii). The data reflect the number of hours worked in the U.S. Non-farm Business Sector divided by the U.S. population. The data for the hours worked are drawn from the U.S. Bureau of Labor Statistics. The data for the U.S. population are taken from the World Bank Development Indicators. The period 1947-2016 is naturally governed by data limitations.

Fact (iv). The data are the log of current USD GDP per capita and an approximation of purchasing power parity. Purchasing power parity (PPP) is the number of units of a country’s currency required to buy the same amount of goods and services in the domestic market as a U.S. dollar would buy in the United States. It tells how many dollars are needed to buy a dollar’s worth of goods in the country as compared to the United States. All data are taken from the World Bank development indicators. The period, 2002-2007, is chosen to reflect a single point in time. A 5-year average is taken to avoid any inference from country-specific business cycles.

Fact (v). The data for the log of current USD investment per capita are taken from the World Bank Development Indicators. Since investment is (in theory) proportional to the capital stock, it serves as an approximation for the capital-labor ratio. The period 2002-2007 is chosen to reflect a point in time. A 5-year average is taken to avoid any inference from country specific business cycles. To construct a measure of revealed comparative advantage in capital intensive goods (RCA), I adopt the non-parametric methodology of [Sorg-Langhans et al. \(2017\)](#). I obtain a measure of inter-industry trade using bilateral U.S. trade data.

The U.S. trade dataset is constructed as follows. First, I combine the U.S. 6-digit North American Industry Classification System (NAICS) trade data of Schott (2008) with Census trade data in order to produce the extended sample period 2002-2007. The raw dataset is then rectangularized by treating any missing values as zero import or export flows. Subsequently, I match this dataset up with the NBER-CES Manufacturing Industry data (Bartelsman & Gray (1996)) which comprises subsectoral information on variables such as employment, payroll, investment, capital stock and value added.

The degree of trade specialization in capital- and labor-intensive manufacturing industries across countries is captured by a trade-weighted average measure. More precisely, for country i at time t , I define revealed comparative advantage in capital-intensive goods as the trade-weighted capital intensity of exports

$$RCA_{i,t} = \sum_{z \in Z} \frac{x_{i,z,t}}{X_{i,t}} k_{z,t}$$

where $x_{i,z,t}$ denotes the exports of country i in industry $z \in Z$ to the U.S. in period t , $X_{i,t}$ represents the total exports of country i to the U.S. in period t and $k_{z,t}$ is the capital intensity of industry z in period t . The trade-weighted nature of the measure implies that the index is insensitive to the digit level of the trade data. In addition, this measure of trade specialization can be derived directly from theory (see e.g. Struck & Velic (2016)). This definition makes the standard assumption that industry factor intensities are the same across countries. The implication is that factor intensity can be consistently ranked using factor share data for just one country, namely the U.S.. U.S. capital intensity data is used due to its availability and attractiveness, given the size and diversity of the industrial economy.

I put forward a simple measure of the capital intensity: the logarithm of the capital to labor expenditure ratio. Thus, the RCA variable is given by

$$RCA_{i,t} = \sum_{z \in Z} \frac{x_{i,z,t}}{X_{i,t}} \ln \left(\frac{\text{invest}}{\text{pay}} \right)_{z,t} \tag{40a}$$

where “pay” is total payroll and “invest” is total capital expenditure.

Fact (vi). The data are the share of trade (exports+imports) as a fraction of World GDP. The data are taken from the World Bank development indicators. The period 1960-2015 is naturally governed by data limitations.

Table 1: Summary Statistics: Cross-Section and Time-Series

Cross-Country					Time-Series				
country	2002-2007				year	1947-2016			
	log(Y)	log(I)	PPP	RCA		l	rk/PY	wl/PY	τ
ARG	3.67	2.91	0.35	0.70	1947	602.68	-	-	-
AUS	4.49	3.92	0.95	0.25	1948	601.83	0.35	0.65	-
AUT	4.57	3.95	1.05	0.26	1949	568.65	0.35	0.65	-
BEL	4.55	3.90	1.05	0.46	1950	577.24	0.36	0.64	-
BGD	2.67	2.08	0.28	0.08	1951	594.22	0.36	0.64	-
BGR	3.57	3.00	0.37	0.46	1952	590.56	0.35	0.65	-
BLR	3.46	2.92	0.29	1.43	1953	595.59	0.34	0.66	-
BRA	3.66	2.91	0.41	0.32	1954	565.34	0.34	0.66	-
CAN	4.53	3.88	0.97	0.34	1955	577.82	0.36	0.64	-

CHE	4.72	4.10	1.27	0.22	1956	581.73	0.34	0.66	-
CHL	3.86	3.18	0.54	0.48	1957	568.16	0.34	0.66	-
CHN	3.24	2.85	0.35	0.16	1958	535.40	0.34	0.66	-
CMR	2.96	2.22	0.39	1.30	1959	550.78	0.35	0.65	-
COL	3.50	2.81	0.39	0.55	1960	542.80	0.34	0.66	24.2
CYP	4.38	3.74	0.86	0.44	1961	528.31	0.34	0.66	24.5
CZE	4.10	3.58	0.58	0.18	1962	531.67	0.35	0.65	23.6
DEU	4.53	3.82	1.04	0.30	1963	529.96	0.35	0.65	24.1
DNK	4.67	4.02	1.36	0.27	1964	541.87	0.35	0.65	24.0
DOM	3.52	2.80	0.42	0.13	1965	555.83	0.36	0.64	24.1
EGY	3.10	2.36	0.18	0.47	1966	568.48	0.35	0.65	24.8
ESP	4.40	3.87	0.90	0.40	1967	562.20	0.35	0.65	24.5
EST	4.01	3.56	0.61	1.08	1968	566.44	0.34	0.66	25.8
FIN	4.58	3.95	1.16	0.42	1969	577.00	0.33	0.67	26.4
FRA	4.52	3.87	1.09	0.30	1970	561.37	0.32	0.68	26.9
GBR	4.60	3.85	1.24	0.38	1971	553.30	0.33	0.67	26.9
GHA	2.78	2.15	0.26	1.27	1972	565.05	0.34	0.66	27.1
GRC	4.34	3.74	0.83	0.52	1973	582.39	0.34	0.66	30.0
GTM	3.31	2.62	0.36	0.10	1974	577.90	0.32	0.68	35.3
HKG	4.42	3.77	0.76	0.12	1975	547.91	0.35	0.65	33.3
HRV	3.99	3.44	0.64	0.27	1976	561.93	0.35	0.65	34.1
HUN	4.01	3.42	0.61	0.26	1977	578.26	0.35	0.65	34.3
IDN	3.11	2.50	0.22	0.17	1978	602.01	0.34	0.66	33.7
IND	2.84	2.39	0.25	0.20	1979	616.64	0.33	0.67	36.0
IRL	4.68	4.13	1.18	0.38	1980	604.61	0.33	0.67	38.8
ISL	4.70	4.12	1.36	0.17	1981	602.88	0.34	0.66	39.2
ISR	4.32	3.62	0.82	0.18	1982	584.09	0.34	0.66	38.3
ITA	4.49	3.82	1.00	0.25	1983	590.11	0.35	0.65	37.5
JPN	4.55	3.94	1.14	0.27	1984	620.59	0.36	0.64	38.9
KEN	2.75	1.99	0.28	0.10	1985	631.31	0.35	0.65	38.5
KOR	4.25	3.75	0.73	0.33	1986	630.46	0.34	0.66	35.6
LBN	3.74	3.07	0.50	0.19	1987	643.79	0.35	0.65	36.7
LKA	3.08	2.49	0.21	0.09	1988	656.26	0.34	0.66	38.1
LTU	3.88	3.27	0.52	1.17	1989	667.95	0.35	0.65	39.0
LVA	3.89	3.43	0.55	1.25	1990	659.31	0.34	0.66	39.1
MAR	3.29	2.78	0.41	0.54	1991	636.07	0.34	0.66	38.9
MEX	3.90	3.24	0.65	0.22	1992	625.25	0.34	0.66	41.4
MYS	3.74	3.10	0.33	0.26	1993	635.48	0.34	0.66	40.6
NLD	4.60	3.93	1.07	0.61	1994	651.84	0.34	0.66	41.6
NOR	4.80	4.15	1.35	0.70	1995	659.08	0.34	0.66	43.6
NZL	4.40	3.79	0.97	0.23	1996	666.66	0.35	0.65	44.1
PAK	2.85	2.10	0.20	0.10	1997	679.57	0.34	0.66	45.8
PER	3.43	2.70	0.40	0.49	1998	686.57	0.33	0.67	46.4
PHL	3.09	2.41	0.29	0.28	1999	691.01	0.33	0.67	47.0
POL	3.88	3.20	0.54	0.20	2000	692.77	0.32	0.68	51.4
PRT	4.26	3.64	0.79	0.48	2001	672.92	0.32	0.68	50.3
RUS	3.71	3.04	0.41	0.82	2002	650.09	0.33	0.67	49.9
SGP	4.47	3.81	0.55	0.31	2003	640.54	0.34	0.66	51.1
SLV	3.45	2.66	0.46	0.17	2004	643.21	0.35	0.65	54.3
SRB	3.55	2.96	0.41	0.23	2005	647.84	0.37	0.63	56.3
SVK	4.05	3.50	0.66	0.30	2006	656.44	0.37	0.63	58.7
SVN	4.25	3.70	0.74	0.25	2007	654.77	0.37	0.63	59.2
SWE	4.62	3.97	1.20	0.34	2008	635.18	0.37	0.63	60.9

THA	3.46	2.87	0.29	0.20	2009	584.43	0.37	0.63	52.4
TUN	3.49	2.86	0.40	1.06	2010	579.12	0.38	0.62	57.0
TUR	3.82	3.23	0.55	0.25	2011	586.67	0.38	0.62	60.6
UKR	3.24	2.60	0.27	0.35	2012	595.33	0.39	0.61	60.7
URY	3.70	2.93	0.44	0.31	2013	601.15	-	-	60.2
USA	4.63	3.99	1.00	-	2014	610.11	-	-	60.0
VNM	2.83	2.36	0.22	0.09	2015	618.69	-	-	58.0
ZAF	3.67	2.94	0.48	0.29	2016	623.99	-	-	-

Notes: $\log(Y)$ denotes the log of current USD GDP per capita. $\log(I)$ denotes the log of current USD investment per capita. PPP denotes the measure of the price level. RCA denotes the measure of revealed comparative advantage in capital intensive goods. l denotes the number of hours worked per year per person. rk/PY denotes the share of capital in income. wl/PY denotes the share of labor in income. τ denotes the ratio of trade to world GDP in %.

B Derivation Details of the Analysis with Trade

In this section I omit the time subscript as the system is in a zero-growth steady state. To show that $P_{Q,i} = P_i$ and $\xi_i P_{Q,j} = P_i$, I solve the following system of equations that describes the trade structure in the augmented model:

$$Y_H = \left[(1 - \omega_H)^{\frac{1}{\theta}} [Y_H^H]^{\frac{\theta-1}{\theta}} + \omega_H^{\frac{1}{\theta}} [Y_H^F]^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad Y_F = \left[(1 - \omega_F)^{\frac{1}{\theta}} [Y_F^F]^{\frac{\theta-1}{\theta}} + \omega_F^{\frac{1}{\theta}} [Y_F^H]^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (41)$$

$$Q_H = Y_H^H + Y_F^H \quad \text{and} \quad Q_F = Y_F^F + Y_H^F \quad (42)$$

$$\frac{P_{Q,H}}{P_H} Q_H = Y_H \quad \text{and} \quad \frac{P_{Q,F}}{P_F} Q_F = Y_F \quad (43)$$

$$Y_H^H = (1 - \omega_H) \left(\frac{P_{Q,H}}{P_H} \right)^{-\theta} Y_H \quad \text{and} \quad Y_F^F = (1 - \omega_F) \left(\frac{P_{Q,F}}{P_F} \right)^{-\theta} Y_F \quad (44)$$

$$Y_H^F = \omega_H \left(\frac{\xi_H P_{Q,F}}{P_H} \right)^{-\theta} Y_H \quad \text{and} \quad Y_F^H = \omega_F \left(\frac{\xi_F P_{Q,H}}{P_F} \right)^{-\theta} Y_F \quad (45)$$

There are ten variables 1) Y_H , 2) Y_F , 3) Y_H^H , 4) Y_F^F , 5) Y_H^F , 6) Y_F^H , 7) $P_{Q,F}/P_F$, 8) $P_{Q,H}/P_H$, 9) $\xi_H P_{Q,F}/P_H$, 10) $\xi_F P_{Q,H}/P_F$, that depend on Q_F , Q_H , θ , ω_F and ω_H . As a solution strategy, I substitute in a guess solution,

$$\frac{P_{Q,F}}{P_F} = 1 \quad \text{and} \quad \frac{P_{Q,H}}{P_H} = 1, \quad (46)$$

and see if this guess solution yields any contradictions. More specifically, I substitute Eq. (46) into (43) to obtain

$$Q_H = Y_H \quad \text{and} \quad Q_F = Y_F. \quad (47)$$

I then substitute Eq. (46) and (47) into (44) which yields

$$Y_H^H = (1 - \omega_H)Q_H \quad \text{and} \quad Y_F^F = (1 - \omega_F)Q_F. \quad (48)$$

Substituting Eq. (48) into Eq. (42) yields

$$Y_F^H = \omega_H Q_H \quad \text{and} \quad Y_H^F = \omega_F Q_F. \quad (49)$$

Substituting Eqs. (49), (47) and (34) into Eq. (45) yields

$$\frac{\xi_H P_{Q,F}}{P_H} = 1 \quad \text{and} \quad \frac{\xi_F P_{Q,H}}{P_F} = 1. \quad (50)$$

Finally, I substitute Eqs. (47), (48) and (49) into Eq. (41) yields

$$1 = 1 \quad \text{and} \quad 1 = 1. \quad (51)$$

Hence, the system solves as there is no contradiction. Finally, combining Eqs. (50) and (46) yields

$$\xi_F P_H = P_F \quad \text{and} \quad \xi_H P_F = P_H. \quad (52)$$