# Benefits from non-competing persuaders*

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**Abstract**

This paper shows that biased persuaders can provide better information to a decision maker due to cooperative, and not competitive, motives. I study Bayesian persuasion games with persuaders who all want a decision maker to take the same action unconditionally. While the optimal information policy from a unique persuader never benefits the decision maker, I show that this is not the case when there are multiple identical persuaders. Despite the fact that all persuaders share a common goal, there always exist strict equilibria in which they endogenously design highly informative policies that benefit the decision maker. The benefit of an additional non-competing persuader is as high as the value difference between full information and no information. The persuaders' motivation to provide extra information is cooperative; the extra information helps offset their colleagues' potential negative news. Consequently, a highly informative equilibrium not only benefits the decision maker, but also can result in a high payoff for the persuaders. In particular, when the persuaders' information is intrinsically imperfect, their payoff can be maximized in an equilibrium with highly revealing information policies.

Keywords: Bayesian persuasion, multiple identical persuaders, endogenous information design, imperfect information

JEL classifications: C72, D83

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# 1 Introduction

When persuaders want a decision maker to unconditionally take a particular action, they can bring this about effectively by designing a biased information-collection process. For example, a recruiter can influence his boss to hire a friend by giving the friend easy interview questions; a doctor can promote surgery over conservative treatment by prescribing medical tests that are likely to give bad results. In these cases, persuaders' information changes the decision maker's choice without necessarily increasing her utility. Indeed, when there is only one persuader, his optimal information design keeps all value of the information completely away from the decision maker (Kamenica and Gentzkow, 2011). How can the decision maker do better? Traditional wisdom suggests that the decision maker should seek information from multiple persuaders who have different preferences (think of opposing sides in court). It seems plausible that, only when information providers have different objectives, they are incentivized to collect extra information in order to induce their own preferred outcomes. As a result of this competition, the decision maker benefits from the the extra information. (Gentzkow and Kamenica, 2017a, 2017b)

However, is competition among persuaders really necessary to induce better outcomes for the decision maker? This paper shows that the answer is "no." Non-competing, identically biased persuaders can endogenously collect extra information that benefits the decision maker, and it is often in their best interest to do so. To make the point clear, this paper focuses on a scenario in which multiple persuaders collect information independently and share the identical state-independent preference: they all want the decision maker to take the same action unconditionally.

This paper makes two contributions. The first is to show that, with as few as two non-competing persuaders, beneficial equilibria with outcomes arbitrarily close to full information revelation always exist. Moreover, these equilibria are robust in the sense that they are strict perfect Bayesian equilibria that do not reply on specific tie-breaking rules. While the literature (Gentzkow and Kamenica 2017a, Gentzkow and Kamenica 2017b, Li and Norman 2017a) has acknowledged the existence of a non-strict fully revealing equilibrium that is induced entirely by a truth-telling tie-breaking rule,[1] this paper is the first to characterize the set of strict beneficial equilibria with non-competing persuaders. The paper's second contribution is to show that, when persuaders' information is intrinsically imperfect, the persuaders' utility can be maximized in an informative equilibrium that benefits the decision maker. This second contribution speaks to realistic situations in which the persuaders can only indirectly learn the true state. For example, recruiters can

[1]When there are multiple persuaders, regardless of their preferences, there always exists a fully revealing equilibrium in which all persuaders choose to reveal the true state. A persuader is indifferent towards all information policies when all the other persuaders are revealing the true state. Full revelation is an equilibrium if the persuader also chooses to reveal the true state when indifferent. However, this equilibrium is an unlikely prediction because all persuaders strictly prefer a less informative outcome.

only indirectly learn a job candidate's ability by measuring his task performance; doctors can only indirectly learn a patient's condition by testing blood and tissue samples. In these cases, I show that the best equilibrium for the biased persuaders can be one in which they endogenously collect high-quality information to benefit the decision maker.

The following example illustrates the intuition behind these two contributions.

## Example

Consider a patient who is choosing between surgery and a more conservative treatment for her condition. For personal reasons, she prefers the surgery if and only if her current condition is already severe. Her utility function, described by the table below, implies that she will choose surgery if and only if the probability that her current condition is severe is at least 0.8. The unconditional probability of the current severity is $\Pr(\textit{severe}) = 0.5$. Without additional information, the patient's default choice is the conservative treatment.

| | | current condition | |
|---|---|---|---|
| | | severe | not severe |
| treatment | surgery | 1 | 0.2 |
| | conservative | 0.8 | 1 |

The patient seeks advice from doctors and communicates her preference honestly with all of them. The doctors want the patient to take the most effective treatment independent of whom she gets the treatment from. Knowing the nature of her condition, the patient understands that all of the doctors prefer that she chooses the surgery, regardless of the current severity.

To give credible advice, doctors ask the patient to take some medical tests that will yield either *positive* or *negative* results. The strategy of each doctor is to design an independent medical test with endogenous type I and type II errors. I separately analyze a case in which the patient's true state can be tested directly and a case in which it can be tested only indirectly.

In the first case, when doctors can directly test the true state, the medical tests are arbitrarily informative. A doctor's strategy is to design the conditional probabilities of binary test outcomes given the true state of the patient - i.e., each doctor chooses $\Pr(\textit{positive}|\textit{severe})$ and $\Pr(\textit{positive}|\textit{not severe})$.

In the second and more realistic case, doctors do not have the technology to directly examine the true severity of the patient's condition. Instead, each doctor takes a blood sample, which is imperfectly correlated with the true state. Assume that each blood sample is i.i.d. with

$$\Pr(\textit{normal blood}|\textit{not severe}) = \Pr(\textit{abnormal blood}|\textit{severe}) = 0.85.$$

In this case, a doctor's strategy is to design a test on the blood sample by choosing condition probabilities $\Pr(\textit{positive}|\textit{abnormal blood})$ and $\Pr(\textit{positive}|\textit{normal blood})$. (For example, the doctors can endogenously define the threshold for "an alarming number" of white blood cells in the sample.)

In both cases, the patient sees the design of the medical tests, as well as the test results. She updates her belief based on this information and makes a decision. All doctors receive payoff 1 if she chooses the surgery and 0 otherwise.

If the patient seeks advice from only one doctor, then, in either case, the doctor will design a test such that $\Pr(\textit{severe}|\textit{positive}) = 0.8$ to maximize the probability of a surgery. The patient is indifferent between the two treatments when the test result is positive. She chooses the surgery to break the tie, but her expected payoff is the same as that if she ignores the doctor's information and always chooses the default conservative treatment. In other words, the unique doctor's information does not increase her expected payoff. This case establishes the lower bound on the patient's expected payoff, which is equal to 0.9 in this example. Next, I will introduce equilibria in which the patient's payoff is higher than 0.9 when she visits more than one doctor. I refer to these equilibria as "*beneficial*" equilibria.

**Observation 1: Strict beneficial equilibria exist with direct tests**

Suppose that the patient visits two doctors, and the doctors can directly test the true state. Each doctor designs an independent test by choosing $(\Pr(\textit{positive}|\textit{not severe}), \Pr(\textit{positive}|\textit{severe}))$ as a best response to the design of the other doctor's test. The doctors understand that if they design tests with sufficiently low false-positive rates, they need only one positive result to make the patient choose surgery. Thus, there are infinitely many strict perfect Bayesian equilibria in which tests are relatively informative with low false positive-rates. Here are two examples.

> *Beneficial equilibrium 1:* Both doctors choose $\Pr(\textit{positive}|\textit{not severe}) \approx 0.001$ and $\Pr(\textit{positive}|\textit{severe}) \approx 0.996$. The patient chooses the surgery if at least one test result is positive and her expected utility is 0.999. Each doctor's expected utility is 0.501.
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> *Beneficial equilibrium 2:* Both doctors choose $\Pr(\textit{positive}|\textit{not severe}) \approx 0.067$ and $\Pr(\textit{positive}|\textit{severe}) \approx 0.5$. The patient chooses the surgery if at least one test result is positive and her expected utility is 0.923. Each doctor's expected utility is 0.440.

The conditional probabilities in both equilibria are chosen such that the patient is indifferent when exactly one test result is positive. When both results are positive, the patient strictly prefers the surgery. Therefore, she strictly benefits from the doctors' information. Indeed, her expected utility in both equilibria is strictly higher than 0.9.

In both equilibria, each doctor's design is the unique best response to the other doctor's (symmetric) design. Therefore, these are strict perfect Bayesian equilibria. A deviation to some higher $\Pr(positive|not\ severe)$ decreases $\Pr(severe|positive)$, and an increase in $\Pr(positive|severe)$ leads to a decrease in $\Pr(severe|negative)$. If either doctor deviates to a higher $\Pr(positive|not\ severe)$ or $\Pr(positive|severe)$, the patient must see two positive results to choose surgery, which decreases her overall probability of choosing surgery.

An interesting observation is that both the patient and the doctors favor equilibrium 1 over equilibrium 2. The tests in equilibrium 1 are almost fully revealing, so it is not surprising that the patient prefers it. The less trivial finding is that the persuaders prefer this almost fully revealing equilibrium, too. They like it because the chance of getting at least one positive result is higher in this equilibrium, due to a significantly higher $\Pr(positive|severe)$. In general, cases in which the decision maker and the persuaders have aligned ranking over equilibria are common. In an arbitrary game with two persuaders, among all symmetric equilibria such that one positive result is sufficient to induce the persuaders' preferred action, the persuaders' most preferred equilibrium never minimizes the decision maker's utility. Moreover, as the decision maker's threshold of doubt increases, the persuaders' most preferred equilibrium converges to the fully-revealing equilibrium.

While beneficial equilibria exist, it is true that a non-beneficial equilibrium also exists. In this example, there is a symmetric equilibrium in which both doctors choose $\Pr(positive|not\ severe) = 0.5$ and $\Pr(positive|severe) = 1$, and the patient chooses the surgery only when both test results are positive. In this equilibrium, the patient's expected utility (= 0.9) is the lowest, but the doctors' expected utility (= 0.625) is higher than those in the beneficial equilibria.

Given this observation, the paper imposes a further question: among all symmetric equilibria, can the doctors' payoff be maximized in a beneficial equilibrium? The answer is "yes" when medical tests are indirect measures of the true condition. Below is an example.

**Observation 2: Beneficial equilibrium is Pareto dominant with indirect tests**

Suppose now that the patient visits three doctors, and the doctors must indirectly test the true state. Each doctor takes an independent blood sample from the patient and then performs an endogenous test of the sample. Recall that this means that each doctor chooses $\Pr(positive|normal\ blood)$ and $\Pr(positive|abnormal\ blood)$ as best responses to the choices of the other two doctors, knowing that $\Pr(normal\ blood|not\ severe)$ = $\Pr(abnormal\ blood|severe)$ = 0.85.

To show that the best symmetric equilibrium for the doctors is a beneficial one, it suffices to show that this is true for the subset of symmetric equilibria such that $\Pr(positive|abnormal\ blood) = 1$ for all doctors. This is because any equilibrium with $\Pr(positive|abnormal\ blood) < 1$ must be beneficial to the patient.[2]

[2] In a perfect Bayesian equilibrium, if some doctor chooses $\Pr(positive|abnormal\ blood) < 1$, then the patient

With three doctors, there are two symmetric equilibria with $\Pr(\textit{positive}|\textit{abnormal blood}) = 1$. One is beneficial to the patient; the other is not.

> *Beneficial equilibrium:* all three doctors choose $\Pr(\textit{positive}|\textit{normal blood}) \approx 0.03$ and $\Pr(\textit{positive}|\textit{abnormal blood}) = 1$. The patient chooses surgery if at least two test results are positive. The patient's expected utility is 0.96. Each doctor's expected utility is 0.514.
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> *Non-beneficial equilibrium:* all three doctors choose $\Pr(\textit{positive}|\textit{normal blood}) \approx 0.51$ and $\Pr(\textit{positive}|\textit{abnormal blood}) = 1$. The patient chooses surgery only when all test results are positive. The patient's expected utility is 0.9. Each doctor's expected utility is 0.497.

The tests in the beneficial equilibrium are more informative than those in the non-beneficial equilibrium. As a result, the patient's expected utility in the beneficial equilibrium (0.96) exceeds the lower bound (0.9), and she is willing to choose surgery even if one of the test results turns out to be negative. The most important observation from this example is that the doctors strictly prefer the beneficial equilibrium, too. In other words, the beneficial equilibrium Pareto dominates the non-beneficial one. Since the latter is the only non-beneficial symmetric equilibrium in this game, this implies that the doctors' favorite symmetric equilibrium must be beneficial to the patient.

I show that this result is generally true when the samples are sufficiently noisy (e.g., when the correlation between the blood sample and the true state is relatively low) or when the decision maker's threshold of doubt is sufficiently high (e.g., the patient chooses the surgery only when $\Pr(\textit{severe})$ is above a relatively high threshold). In the former case, negative test results are common because of the exogenous noise; in the latter case, negative test results are common because persuaders endogenously choose tests with low false-positive rates. In either case, persuaders expect to see negative test results often, so they highly value the fact that, in beneficial equilibria, the decision maker is willing to forgo a few negative results. This is why the persuaders' payoff achieves its maximum in a beneficial equilibrium.

Overall, this paper generalizes results from this motivating example to games with multiple persuaders who have the identical state-independent bias. There always exists a set of strict equilibria that benefit the decision maker. Elements of this set can be arbitrarily close to the fully revealing equilibrium if the biased persuaders directly test the true state. These beneficial equilibria exist

must be wiling to choose surgery even when some test results are negative. If the patient chooses surgery only when all results are positive, the doctor will be better off deviating to $\Pr(\textit{positive}|\textit{abnormal blood}) = 1$. Hence, the patient is at least indifferent when there is only one negative result. Then, in the possible case of no negative result, the patient must strictly prefer to switch from the default conservative treatment to the surgery. This implies that, in expectation, the doctors' information must strictly benefit the patient.

because when persuaders cannot guarantee perfect synchronization of their test results, they want to be able to persuade the decision maker despite a few negative test results. This is achievable only if positive test results are sufficiently informative. Therefore, each persuader lowers the false-positive rate of his test in the presence of other persuaders. When persuaders must indirectly learn the true state by testing noisy samples, unsynchronized test results are more common, and persuaders' incentives to lower the false positive-rates are even stronger. As a result, a strict beneficial equilibrium not only exists but it can also be the most efficient equilibrium for the persuaders.

The novelty of this paper is its emphasis on the fact that these beneficial equilibria do not at all result from any inter-persuader misalignment or competition. Persuaders in this paper share a common interest, but they still choose to collect extra information. Therefore, this paper sheds light on a new avenue that explains why having more than one information provider is beneficial to decision-making. Multiple persuaders provide extra information not because they want to compete with their colleagues, but because they want to help offset their colleagues' negative results.

The remainder of the paper is organized as follows. Section 2 summarizes related papers. Section 3 characterizes the set of strict beneficial equilibria when persuaders directly test the true state. Section 4 shows that, when persuaders learn about the true state through indirect tests, beneficial equilibria not only exist but can also be persuader-optimal. Section 5 discusses alternative modeling choices.

# 2 Related papers

This paper is closely related to three papers on Bayesian persuasion games: Kamenica and Gentzkow (2011), Gentzkow and Kamenica (2017a), and Gentzkow and Kamenica (2017b). I extend Kamenica and Gentzkow (2011) by introducing multiple independent persuaders with identical state-independent preferences. This extension gives rise to a wide range of equilibrium outcomes. In particular, if there is only one persuader who wants the decision maker to switch from a default action to a new action, then the decision maker never benefits from the collected information (Kamenica and Gentzkow, 2011). This is the case because the unique persuader can and, indeed, will design his test in such a way that a positive result leaves the decision maker precisely indifferent. Since the decision maker never strictly prefers to switch her action, the test never strictly increases her expected utility. In contrast, in games with multiple identical persuaders, there always exist strict equilibria in which the decision maker strictly prefers to switch her action. Moreover, in the single-persuader equilibria studied in Kamenica and Gentzkow (2011), if the decision maker chooses the default action, she is always certain of her choice. This is no longer the case in equilibria with multiple identical persuaders.

Gentzkow and Kamenica (2017b) study multiple persuaders with arbitrarily different preferences. They show that, regardless of persuaders' preferences, the competition among persuaders is guaranteed to benefit the decision maker if and only if the information environment is Blackwell-connected. That is, each persuader can unilaterally deviate to induce any feasible distribution of belief that is more informative. In contrast, the current paper assumes a fixed set of preferences (all persuaders unconditionally prefer a particular action), and its information environment is not Blackwell-connected because the persuaders are independent[3]. Gentzkow and Kamenica's result does not apply in this setup. For comparison, an environment is Blackwell-connected if the tests chosen by the persuaders are arbitrarily correlated. Gentzkow and Kamenica (2017a) particularly focus on this situation. In this case, misaligned incentives among persuaders are necessary for persuaders to reveal sufficient information that benefits the decision maker. If persuaders share identical preferences, then, regardless of the total number of persuaders, all strict equilibria of the game are outcome-equivalent to the single-persuader game, and the decision maker never gains. This is why heterogeneous preferences among persuaders are necessary to induce more information revelation in Gentzkow and Kamenica (2017a): a persuader will reveal extra information as a means to induce his preferred action only when persuaders differ in their preferences. In contrast, when persuaders are independent, I show that even if persuaders have identical preferences, there always exist strict equilibria in which the persuaders endogenously design informative tests that strictly benefit the decision maker. The persuaders act in this way not because they are competing under different motives, but because when they design those informative tests, the Bayesian decision maker switches her action upon seeing relatively few counts of positive results. Since the persuaders cannot guarantee that their test results are always synchronized, they find it desirable to lower the decision maker's ex post switching standard at a cost of fewer false positive results. Section 5.B shows that the same result holds when persuaders are partially correlated.

Two papers by Li and Norman extend Gentzkow and Kamenica (2017a) by studying misaligned persuaders in slightly different settings. Li and Norman (2017a) look at independent persuaders with different preferences who choose tests simultaneously. The paper provides an example in which two persuaders release less information than one. Li and Norman (2017b) look at arbitrarily

[3]For example, let the true state be $H$ or $L$ with equal probabilities. Suppose that the first persuader chooses a test with $\Pr(positive|H) = 0.8$ and $\Pr(positive|L) = 0.2$; the second persuader chooses an uninformative test with $\Pr(positive|H) = \Pr(positive|L) = 1$. The induced posterior belief for the state $H$ is 0.8 with probability 0.5 and 0.2 with probability 0.5. Blackwell-connectedness requires that, given the strategy of the first persuader, the second persuader can unilaterally deviate to a different test, so that the two tests induce a posterior belief of 0.9 with probability 0.5 and 0.1 with probability 0.5. However, since the two persuaders choose tests independently, such a belief distribution is unattainable by a unilateral deviation. If the second persuader deviates to a more informative test so that he can sometimes induce a posterior belief of 0.9 or 0.1, then it is always possible that his test result is negative when the test result from the first persuader is positive, or vice versa. Either case induces a posterior belief between 0.8 and 0.2 with positive probability.

correlated persuaders with different preferences who choose their tests in a sequence. It shows that while adding an extra persuader at the end or in the middle of the sequence can result in an information loss, adding a persuader at the beginning of the sequence will not. These papers imply that, when persuaders have different preferences, it can be worse for the decision maker to add a persuader. In my paper, adding a second same-minded persuader never hurts the decision maker.

Other papers that discuss competitive persuasion include Board and Lu (2017) and Au and Kawai (2017). Board and Lu (2017) study competing sellers of the same product who try to attract searching buyers by disclosing information about the product. They show that the effect of competition on information disclosure depends on whether the buyers' beliefs are private. Au and Kawai (2017) study the competition between sellers of different products who try to attract a single buyer by disclosing information about their own products. The effect of competition is ambiguous in general.

On the topic of noisy test results, a paper by Rick (2013) studies persuasion games with exogenous noise. He studies a one-persuader game and shows that if the persuader cannot choose the test design but can repeat the test an arbitrary number of times and report only the final result, then the decision maker can be better off if, with some probability, she falsely receives a positive result, even if the persuader did not send one. This is because the error induces a posterior belief that the persuader favors even when he has not exerted effort to harvest false-positive evidence. This gives the persuader an incentive to reduce false-positive reports and deliver more-truthful information. In this paper, because the persuaders are able to choose the test design, the intuition from Rick (2013) does not apply. When a persuader must learn the true state through indirectly testing a noisy sample, the noise in the sample does not make him release more information when he is the only persuader. Beneficial equilibria exist only when there exist other persuaders, too. Persuaders are incentivized to design informative tests because they want to make their own positive results powerful enough to outweigh others' negative results. This incentive disappears if a persuader is alone.

There are papers on cheap talk persuasion games with multiple persuaders (e.g., Battaglini, 2002; Ambrus and Takahashi, 2008; Ambrus and Lu, 2014). However, note that for any game in which the persuaders' preferences are state-independent, if the decision maker observes only the results of the tests and not the design, the only equilibrium is a trivial one in which the persuaders always conduct completely uninformative tests with uniformly positive results. The decision maker is never persuaded (Sobel, 2013). Therefore, in this paper, it is crucial that the decision maker observes both the design and the outcome of the test.

Other papers (e.g., Bhattacharya and Mukherjee, 2013; Felgenhauer and Schulte, 2014; Hart, Kremer, and Perry, 2016) study persuasion games in which state-independent persuaders cannot choose the test design but can hide unfavorable test results. A key distinction is that the persuaders in those papers decide whether to report a result only after they see the results, whereas the persuaders in this paper unconditionally commit to report all results. Therefore, the persuaders in their settings report only good evidence, and they dislike tests that can yield negative results in a good state because that means fewer good reports. In contrast, all beneficial equilibria in this paper feature persuaders who optimally choose tests that can yield negative results in a good state. They are incentivized to do so because it increases the decision maker's posterior belief when she sees negative results, which makes it possible to persuade her even when some tests results are negative.

Many assumptions in this paper are similar to those in the standard voting literature, such as Feddersen and Pesendorfer (1998). But there is one crucial difference that leads to very different results. The decision maker in this paper does not commit to any decision rule that is based only on test results. In the voting literature, the decision maker takes a certain action if the number of votes passes an exogenous threshold, regardless of the voting strategy (e.g., the unanimity rule or the majority rule). In contrast, the decision maker in this paper chooses the action that best responds to both the test results and the test design. In particular, if the decision maker were to commit to a fixed result-based standard (e.g., two positive results out of three tests), the persuaders would simply choose uninformative tests that always yield positive results. If that were the case, the decision maker would rather ignore the persuaders and always choose the default action. This outcome is undesirable for both the decision maker and the persuaders.

# 3 Direct tests

In this section, I study a general case with $n$ identical persuaders who endogenously design direct tests on the true state. Strict beneficial equilibria with high payoffs for the decision maker always exist.

## 3.1 Setup

There are two states of the world: $\omega \in \{L, H\}$. [4] There are $n$ persuaders and a decision maker. The decision maker can choose one of two actions, $a_L$ or $a_H$. (Think of $a_L$ as "conservative treatment" and $a_H$ as "surgery" in the motivating example.) Her preference is described by a utility function $u$ that depends on her action and the true state: $u(a_L, L) = u(a_H, H) = 1$, $u(a_H, L) = 1 - p_d$, and $u(a_L, H) = p_d$, for some $p_d \in \left(\frac{1}{2}, 1\right)$. With these preferences, the decision maker prefers $a_H$ iff. the posterior probability for state $H$ is above $p_d$. Thus, $p_d$ can be viewed as the decision maker's "threshold of doubt." I assume here that the decision maker chooses $a_H$ when she is indifferent.

[4] The main result of the paper is robust when the state space is a continuum; see discussion in Section 5.

The persuaders, on the other hand, all prefer that the decision maker chooses $a_H$, regardless of the true state. Their preference can be represented by a common utility function $v$ with $v(a_H) = 1$, and $v(a_L) = 0$.

The persuaders and the decision maker share a common prior: $\Pr(H) = \Pr(L) = \frac{1}{2}$. Each persuader $i$ can design an endogenous test on the true state. A test is a garbling of the true state that generates a message $m_i \in \{positive, \ negative\}$ with probabilities conditional on $\omega$. Results of this paper do not hinge on the assumption of a binary message space. [5] The strategy of persuader $i$ is to choose the conditional probabilities $(x_i, y_i)$, where $x_i \equiv \Pr(positive|L)$ and $y_i \equiv \Pr(positive|H)$. Assume that $x_i \leq y_i$ for all $i$ so that "positive" is positively associated with state $H$.

All persuaders choose their tests simultaneously. The decision maker observes both the tests $((x_1, y_1), ..., (x_n, y_n))$ and their results $(m_1, ..., m_n)$.

The timeline of the game is summarized below.

1. $N$ persuaders simultaneously design tests $(x_1, y_1), ..., (x_n, y_n)$.

2. Nature chooses the state of the world.

3. Each test generates a result $m_i$.

4. After observing the test designs and the test results, the decision maker Bayesian updates her belief about the true state and chooses an action $a$.

Let $U$ denote the expected utility of the decision maker and let $V$ denote the expected utility of each persuader before test outcomes are revealed.

Let $\underline{U} \equiv \frac{1}{2}(1 + p_d)$ be *the decision maker's expected utility when she receives no information from any persuader*. In this case, she always chooses $a_L$. Then, $U \geq \underline{U}$ in any equilibrium because the decision maker can always ignore the persuaders' information to guarantee $\underline{U}$. Let $\overline{U} \equiv 1$ be *the decision maker's expected utility when she learns the true state*. $U \leq \overline{U}$ always.

The *solution concept* used in this paper is *strict* perfect Bayesian equilibrium. The requirement of strictness eliminates "nuisance" equilibria such as the one in which every persuader chooses the fully-revealing test. This fully-revealing equilibrium relies on a strong tie-breaking assumption that each persuader perfectly reveals the true state whenever indifferent; but this is an unlikely prediction because all persuaders strictly prefer a less informative outcome. Therefore, by restricting attention to perfect Bayesian equilibria that are strict,[6] this paper emphasizes that any beneficial equilibrium studied below is robust.

[5] Results of this paper hold when the message space is larger. See Section 5.E for details.

[6] It is not sufficient to eliminate the fully-revealing equilibrium by focusing only on admissible equilibria because full revelation is not weakly dominated by any other strategy. For example, suppose that there are only two persuaders. Let $(x_1, y_1) \neq (0, 1)$ be any arbitrary strategy from persuader 1 that is not fully revealing. Then, there exists some strategy $x_2 = 0$, $y_2 < 1$ from persuader 2 such that 1) persuader 2 always reports "negative" state L, and 2) persuader 2

## 3.2 Equilibrium

In this section, I start by identifying the necessary and sufficient condition for the equilibria that benefit the decision maker. Then, I analyze the welfare implications of all symmetric equilibria that satisfy these conditions when $n = 2$. The decision maker's payoff can be arbitrarily close to $\overline{U}$ in these equilibria; the persuaders and the decision maker often share the same ranking over equilibria. At the end of this section, I prove the existence of beneficial equilibria for any $n > 2$.

To make equilibrium analysis easier, I first introduce a notation that represents the decision maker's decision rule after she sees the test results.

Note that if a persuader chooses a test whose result is positive with the same probability in either state, then his test result is simply white noise. Call him *an uninformative persuader*. Since persuaders are independent, adding or deleting uninformative persuaders has no impact on other players' equilibrium strategies. Therefore, the decision maker's action in equilibrium depends only on the strategy of informative persuaders.

**Definition 1.** A persuader $i$ is *informative* if and only if $x_i < y_i$. In an equilibrium, let $N_I$ denote the set of all informative persuaders.

In general, persuaders' test designs can be asymmetric. The decision maker's decision rule is characterized by a set; she chooses action $a_H$ if and only if the positive results come from persuaders belonging to this set.

**Definition 2.** In an equilibrium, let $a \subseteq N_I$ denote the set of informative persuaders whose test results are positive. Then, $A \in \mathscr{P}(N_I)$ is called the *acceptance set* for this equilibrium when the decision maker chooses $a_H$ if and only if $a \in A$.

When persuaders' test designs are symmetric, the decision maker's decision rule can also be characterized by a single number.

**Definition 3.** In a symmetric equilibrium, define the *acceptance fraction* $\alpha \in [0, 1]$ such that the decision maker chooses $a_H$ if and only if the fraction of positive results from informative persuaders' tests is at least $\alpha$ in this equilibrium.

*Remark* 1. Since the decision maker is Bayesian, an acceptance set must satisfy this: if $a_1 \in A$ and $a_1 \subset a_2$, then $a_2 \in A$. That is, more positive results cannot be less persuasive. The analogy of a higher acceptance fraction in symmetric equilibria is a smaller acceptance set in asymmetric equilibria.

sometimes reports "negative" in state H with probability $y_2$. $y_2$ is a function of $(x_1, y_1)$ and is chosen to be sufficiently low so that the decision maker chooses $a_L$ when the result is "positive" from persuader 1 and "negative" from persuader 2. Given persuader 2's strategy, persuader 1 is strictly better off with the fully-revealing strategy (0, 1) than with $(x_1, y_1)$. Therefore, the fully-revealing strategy is not weakly dominated.

Next, Proposition 1 provides two ways to identify whether the decision maker strictly benefits from persuaders' tests in an equilibrium. A beneficial equilibrium is identified by a minimum acceptance set and the existence of informative persuaders whose tests may yield negative results in state $H$. The key intuition behind the proof is simple yet important - the decision maker strictly benefits from a set of tests if and only if some test outcomes make her strictly prefer action $a_H$. Proposition 1 is the backbone of all the other results in the paper.

**Proposition 1.** *In any equilibrium with $p_d \in \left(\frac{1}{2}, 1\right)$ and $n \geq 1$, the following statements are equivalent:*

*(1) $U = \underline{U}$.*

*(2) $A = N_I$.*

*(3) $y_i = 1$ for all $i \in N_I$.*

*Proof.* (1) implies (2): Prove by contraposition. Suppose that $A \subsetneq N_I$. This implies that there exists an informative persuader $i$ such that the decision maker chooses $a_H$ when $i$'s test result is negative, but every other persuader's test result is positive. When only $i$'s test result is negative, let $\mu_1$ denote the decision maker's posterior belief and $u_1$ denote her expected utility. When everyone's test result is positive, let $\mu_2$ denote the decision maker's posterior belief and $u_2$ denote her expected utility. Then, $\mu_2 > \mu_1 \geq p_d$ and $u_2 > u_1 \geq \underline{U}$. Since persuaders are informative, the event that every test result is positive occurs with positive probability. This implies that the decision maker obtains $u_2 > \underline{U}$ with positive probability. Therefore, the decision maker's ex ante expected utility $U$ must be strictly higher than $\underline{U}$, which proves that (1) cannot hold when (2) fails.

(2) implies (3): Prove by contradiction. Suppose that $y_i < 1$ for some $i$. Then, persuader $i$ must be strictly better off when he increases $y_i$ because, all else equal, it increases both the probability of a positive result from his test and the posterior belief when his test result is, indeed, positive. The acceptance set does not shrink after the deviation since $A = N_I$ is already the smallest acceptance set. Therefore, such a deviation strictly increases the probability of $a_H$, and persuader $i$ strictly prefers to deviate to $y_i = 1$.

(3) implies (1) and (2): If informative tests never yield negative results in state $H$, a negative result from a single informative test perfectly reveals state $L$. Therefore, the decision maker never chooses $a_H$ when seeing an informative negative result - i.e., $A = N_I$.

When $A = N_I$ and $y_i = 1$ for all $i \in N_I$, the persuaders must choose tests such that the decision maker is indifferent when all results are positive. That is, tests $(x_i, y_i)$ satisfy

$$\prod_{i \in N_I} \frac{y_i}{x_i} = \frac{p_d}{1 - p_d}.$$

The decision maker never strictly prefers to choose $a_H$. Her ex ante expected utility is equivalent to the amount when she chooses $a_L$ unconditionally - that is, $\underline{U}$. Therefore, $U = \underline{U}$. $\square$

Proposition 1 immediately implies that when $n = 1$, the decision maker never benefits from the persuader's optimal test.

**Corollary 1.** *When $n = 1$, $U = \underline{U}$ for all $p_d \in \left(\frac{1}{2}, 1\right)$.*

*Proof.* When there is a single persuader with a state-independent utility function, the optimal test design assigns $x = \frac{1-p_d}{p_d}$ and $y = 1$. By Proposition 1, this implies that $U = \underline{U}$. $\square$

If one persuader is useless, how about two? Theorem 1 confirms that when there are two persuaders, despite their identical state-independent preference, there always exist infinitely many strict equilibria that benefit the decision maker. In these equilibria, both persuaders choose to design tests that are more revealing than the single-persuader test, and, consequently, the decision maker lowers her acceptance fraction.

**Theorem 1.** *Let $n = 2$. For all $p_d \in \left(\frac{1}{2}, 1\right)$, any pair of $(x, y)$ such that*

$$0 < x \leq \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}} \text{ and } y = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{p_d}{1 - p_d}(x - x^2)} < 1 \tag{1}$$

*constitutes a strict symmetric equilibrium in which both persuaders choose the same test $(x, y)$; the decision maker's acceptance fraction is $\frac{1}{2}$ and $U > \underline{U}$.*

The condition $y = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{p_d}{1-p_d}(x - x^2)}$ comes from the requirement that the decision maker is indifferent when exactly one test result is positive - i.e.,

$$\frac{y}{x} \cdot \frac{1-y}{1-x} = \frac{p_d}{1-p_d}. \tag{2}$$

For $y$ to be well-defined, $x$ must be smaller than $\frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}$. The proof in the Appendix verifies that, when the first persuader chooses some $(x, y)$ that satisfies (1), the second persuader's unique best response is always to choose the same $(x, y)$. If the second persuader deviates to a higher false-positive rate ($x$), then a positive result from his test is no longer informative enough to outweigh a negative result from his colleague. If the second persuader deviates to a higher true-positive rate ($y$), then a negative result from his test is too revealing to be outweighed by a positive result from his colleague. Both types of deviation will make the decision maker raise her acceptance fraction from $\frac{1}{2}$ to 1, which always decreases the expected probability of $a_H$. In other words, a persuader optimally makes his positive test result informative enough to offset bad news from his colleague, and he optimally makes his negative result uninformative enough so that it can be offset by good news from the colleague. This is why these informative but noisy test designs in Theorem 1 constitute strict equilibria.

As Theorem 1 points out, there are infinitely many equilibria with test designs that benefit the decision maker. The next Proposition ranks these equilibria by their corresponding utility outcomes. The result shows that the decision maker is better off in an equilibrium associated with a lower false-positive rate ($x$). Her expected utility converges to the value associated with full revelation $\left(\overline{U}\right)$ as $x$ converges to zero. In other words, with only two persuaders, the strict equilibrium outcome is arbitrarily close to full revelation. Interestingly, the persuaders' ranking over these equilibria is partially aligned with the decision maker's. They never prefer the least informative equilibrium; in fact, their favorite equilibrium converges to the most revealing one as $p_d$ converges to 1.

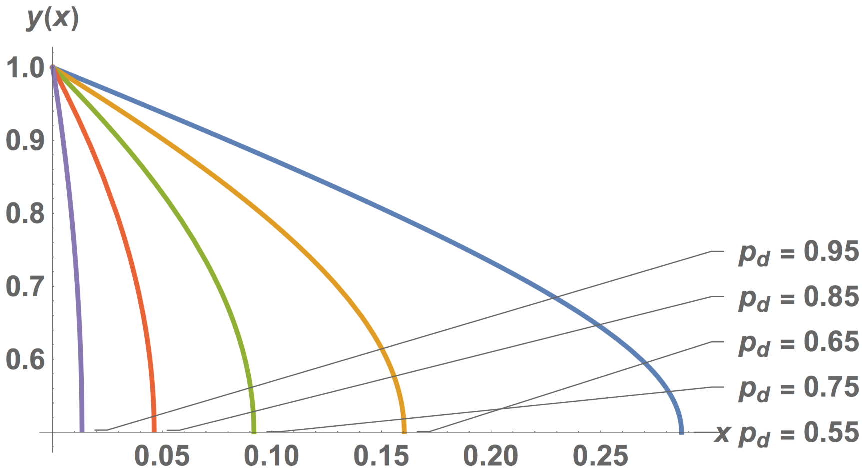
**Proposition 2.** *Let $n = 2$. Suppose that $x$ and $y$ satisfy condition (1) in Theorem 1. Let $U$ and $V$ denote the expected utility of the decision maker and the persuaders in the symmetric equilibrium associated with $x$ and $y$. Then, for all $p_d \in \left(\frac{1}{2}, 1\right)$,*

1. *$y$ strictly decreases in $x$ and $\lim_{x \to 0} y = 1$.*
2. *$U$ strictly decreases in $x$ and $\lim_{x \to 0} U(x) = \overline{U}$.*
3. *$V$ is strictly concave in $x$. Let $x^*(p_d)$ be the maximizer of $V$ and let $\overline{x}(p_d) = \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}$ denote the upper bound of $x$. Then,*
   (a) *$U(x^*(p_d)) > \underline{U}$ for all $p_d$;*
   (b) *$\frac{x^*(p_d)}{\overline{x}(p_d)}$ strictly decreases in $p_d$ and $\lim_{p_d \to 1} \frac{x^*(p_d)}{\overline{x}(p_d)} = 0$. Among all equilibria characterized in Theorem 1, the persuaders' most preferred equilibrium converges to the most revealing one as $p_d \to 1$.*

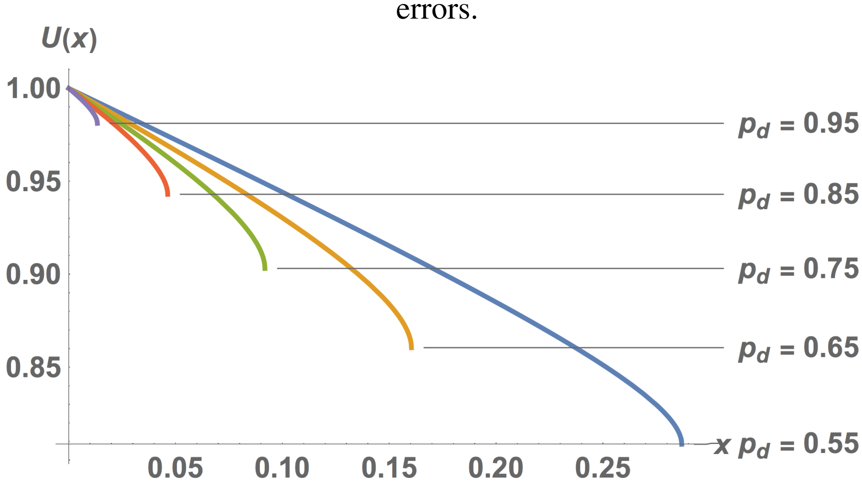
Figures 1 and 2 illustrate the comparative statics in Proposition 2. The formal proof can be found in the Appendix. Here, I describe the intuition behind these results.

In equilibrium, a high false-positive rate $x$ weakens the persuasiveness of a positive result, so in order for the decision maker to be indifferent after seeing one positive result and one negative result, the persuaders must weaken the informativeness of a negative result by increasing $\Pr(negative|H)$. As a result, (1) shows that $y = 1 - \Pr(negative|H)$ decreases with $x$.

An equilibrium test design associated with a low $x$ (and, consequently, a high $y$) is relatively informative because of the high correlation between the test result and the true state. This directly leads to result (2): the decision maker is better off in an equilibrium with a lower $x$. In particular, as $x$ converges to 0, the equilibrium test almost fully reveals the true state, and, consequently, the




1. $y$ strictly decreases in $x$. An equilibrium associated with a higher $x$ has higher type 1 *and* type 2 errors.

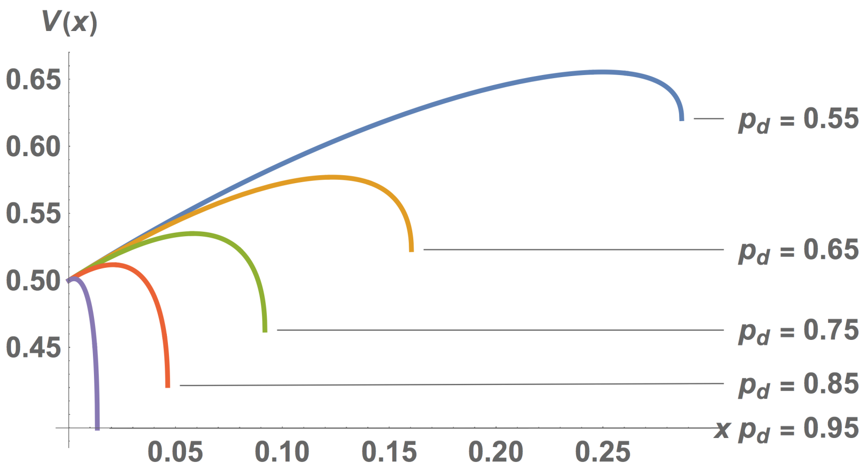



2. $U$ strictly decreases in $x$. The decision maker prefers an equilibrium associated with a lower $x$.

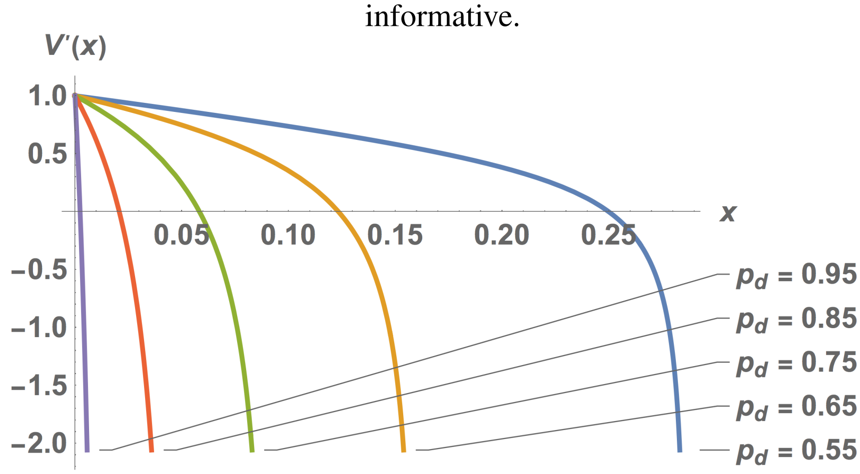
Figure 1: Comparative statics of symmetric equilibria with two persuaders and acceptance fraction $\frac{1}{2}$.

decision maker's utility is infinitesimally close to $\overline{U}$ (i.e., her utility when she learns the true state). Recall that the decision maker's utility is always $\underline{U}$ when there is only one persuader. This result in Proposition 2 emphasizes that the benefit of just one additional identical persuader is as large as $\overline{U} - \underline{U}$.

From the perspective of the persuaders, their ranking of the equilibria is partially aligned with the decision maker's. Part 3 shows that their favorite equilibrium is never the least informative one. Since $y$ must decrease with $x$ in equilibrium, the least informative equilibrium is associated with the lowest rate of positive results in state $H$ and, therefore, does not induce the highest unconditional probability of $a_H$. When the decision maker's threshold of doubt $p_d$ is high, as the false-positive rate in state $L$ increases, the corresponding true positive rate in state $H$ must drop significantly in




3(a). The maximizer of $V$ does not minimize $U$. The persuaders' favorite equilibrium is relatively informative.




3(b). The maxizer of $V$ (the $x$-intercept) decreases faster than $\bar{x}$. As $p_d$ converges to 1, the persuaders' most preferred equilibrium converges to the most revealing one.

Figure 2: (Continue) Comparative statics of symmetric equilibria with two persuaders and acceptance fraction $\frac{1}{2}$.

order to maintain the same acceptance fraction. This significant drop in $y$ results in a low expected utility for the persuaders in equilibria with high $x$. This is why they are better off in a more informative equilibrium.

Finally, Theorem 1 can be generalized to cases with more than two persuaders.

**Proposition 3.** *Let $n > 2$. For all $p_d \in \left(\frac{1}{2}, 1\right)$, there exists a strict equilibrium in which the decision maker's expected utility is strictly higher than $\underline{U}$.*

The proof of Proposition 3 takes the set of two-persuader equilibria as a starting point. It shows that a non-empty subset of these equilibria can be extended into an $n$-persuader equilibrium in which the first two persuaders play the same strategy as in the two-persuader equilibrium, while the other persuaders choose uninformative tests. Whether symmetric equilibria with high $U$ exist for arbitrary $n$ and $p_d$ remains an open question; the exponentially increasing number of potential deviations makes proving this statement a challenge. However, when $n$ is small, examples of symmetric equilibria with high $U$ are easily found. Here is an example with $n = 3$.

**Example 1.** Let $n = 3$ and $p_d = \frac{81}{113}$. There exists an equilibrium in which all persuaders choose the test $(0.2, 0.9)$, and the decision maker's acceptance fraction is $\frac{2}{3}$. $U = 0.96 > \underline{U} = 0.86$.

I verify that this is a strict equilibrium in the Appendix.

The results in this section show that robust equilibria with high payoffs for the decision maker always exist as long as $n > 1$. The information revealed in these equilibria is arbitrarily close to full revelation and, in cases in which the decision maker's threshold of doubt is high, the persuaders rank the revealing equilibria highly, too.

However, while equilibria with $U > \underline{U}$ always exist for $n \geq 2$, non-beneficial equilibria that induce $U = \underline{U}$ also exist. For example, in the symmetric non-beneficial equilibrium, each of the $n$ persuaders chooses the test with $x = \left(\frac{1-p_d}{p_d}\right)^{\frac{1}{n}}$ and $y = 1$. In this equilibrium, either the decision maker is indifferent or she learns that the state is $L$. Results from Kamenica and Gentzkow (2011) imply that the persuaders' expected utility is maximized in this equilibrium. In other words, the persuaders prefer this non-beneficial equilibrium over the ones with $U > \underline{U}$.

Is there a scenario in which both the decision maker and the persuaders prefer an equilibrium with $U > \underline{U}$ over one with $U = \underline{U}$? The next section shows that the answer is "yes" when persuaders indirectly learn the true state through endogenously testing imperfect samples.

# 4 Indirect tests

In this section, I study multiple identical persuaders who learn the true state through endogenous indirect tests. Strict beneficial equilibria with high payoffs for the decision maker always exist, and, moreover, the persuaders' payoffs can be maximized in a beneficial equilibrium, too.

## 4.1 Setup with indirect tests

Suppose that persuaders cannot directly test the true state. Instead, they must indirectly learn the true state by designing tests on some samples that are imperfectly correlated with the true state. (In the motivating example, doctors design medical tests on the patient's blood samples, which are imperfectly correlated with the patient's true condition.)

Formally, assume that persuader $i$ can perform an endogenous test on an i.i.d. *sample* $s_i \in \{s_H, s_L\}$ that is correlated with the true state with $\Pr(s_H|H) = \Pr(s_L|L) = p$.[7] Assume that $p_d \leq p < 1$; the decision maker prefers $a_H$ if she directly observes $s_H$. The strategy of persuader $i$ is to choose conditional probabilities $\tilde{x}_i \equiv \Pr(positive|s_L)$ and $\tilde{y}_i \equiv \Pr(positive|s_H)$. (Recall that in the last section, persuader $i$ chose $x_i = \Pr(positive|L)$ and $y_i = \Pr(positive|H)$. )

Everything else is identical to the setup in the last section with direct tests. With the new assumption, a test result can be negative in state $H$ even if the persuader chooses $\tilde{y} = 1$.

## 4.2 Equilibrium with indirect tests

In this section, I first identify the necessary and sufficient condition for equilibria that benefit the decision maker. Then, I focus on the set of symmetric equilibria for an arbitrary number of persuaders. I characterize elements of this set and show that the payoff-maximizing symmetric equilibrium for the persuaders can be one that benefits the decision maker.

The first proposition is an analogy of Proposition 1. It states that, even when tests are indirect, non-beneficial equilibria with $U = \underline{U}$ can be identified by the smallest acceptance set. There is one difference between Proposition 4 and Proposition 1: $\tilde{y}_i = 1$ for all $i$ no longer implies that the equilibrium is not beneficial to the decision maker. This is because a negative result from a test with $\tilde{y} = 1$ no longer implies that the state is $L$ for certain, and the decision maker may still choose $a_H$ if there are enough positive results from the other tests. This gives rise to equilibria in which all persuaders choose $\tilde{y} = 1$, but the decision maker's acceptance set is large.

**Proposition 4.** *In any equilibrium with $p_d \in \left(\frac{1}{2}, 1\right)$, $p_d \leq p < 1$, and $n \geq 1$,*

*(1) $U = \underline{U}$ if and only if $A = N_I$.*

*(2) $\tilde{y}_i = 1$ for all $i$ is a necessary but not sufficient condition for $U = \underline{U}$.*

*Proof.* (1) The proof for the "only if" part is identical to the proof in Proposition 1. For the "if" part, note that $A = N_I$ means that the decision maker chooses $a_H$ only when all tests results are positive. Thus, either she chooses $a_L$ (the default action under her prior belief), or she is indifferent between $a_H$ and $a_L$. As a result, the decision maker's ex ante expected utility is equal to her utility when she chooses $a_L$ unconditionally - i.e., $U = \underline{U}$.

(2) The proof to show that $U = \underline{U} \Rightarrow \tilde{y}_i = 1$ for all $i$ is identical to the proof in Proposition 1. The motivating example in the Introduction and in Theorem 2 in this section show that there generally exist equilibria in which $\tilde{y}_i = 1$ for all $i$, but $U > \underline{U}$. □

[7] Although exogenous noise is modeled to be symmetric (i.e., $\Pr(s_H|H) = \Pr(s_L|L) = p$) in this section, the symmetry is not necessary for the results. To see why, note that the symmetric constraint puts an upper bound on both posterior probabilities $\Pr(H|positive)$ and $\Pr(L|negative)$. In equilibrium, only the latter constraint on $\Pr(L|negative)$ binds since the persuaders endogenously choose tests with relatively low $\Pr(H|positive)$ anyway. Therefore, relaxing the constraint on $\Pr(s_L|L)$ does not change the equilibrium outcome.

The next corollary is an analogy of Corollary 1. The optimal test of a unique persuader is still useless for the decision maker in the case with indirect tests.

**Corollary 2.** *When $n = 1$, $U = \underline{U}$ for all $p_d \in \left(\frac{1}{2}, 1\right)$ and $p_d \le p < 1$.*

*Proof.* When there is a single persuader with a state-independent utility function, the optimal test design assigns $\tilde{x} = \frac{1-p_d}{p_a}$ and $\tilde{y} = 1$. The decision maker chooses $a_H$ only when the test result is positive. By Proposition 4, this implies that $U = \underline{U}$. $\square$

In the remainder of this section, I focus on the set of strict symmetric equilibria.[8] In particular, I show that the persuaders' favorite strict symmetric equilibrium can be one with $U > \underline{U}$. By Proposition 4, it is sufficient to prove this statement if the persuaders' favorite equilibrium among all *strict symmetric equilibria s.t.* $\tilde{y}_i = 1$ *for all* $i$ is one with $U > \underline{U}$. This is because any equilibrium with $y_i < 1$ for some $i$ must imply that $U > \underline{U}$ (part 2 of Proposition 4).

The results below generalize the observations from the motivating example in the Introduction. Theorem 2 characterizes all strict symmetric equilibria with $\tilde{y}_i = 1$ for all $i$. Proposition 5 ranks these equilibria by the decision maker's payoff. An equilibrium is more beneficial to the decision maker if it is associated with a lower acceptance fraction. Proposition 6 ranks these equilibria by the persuaders' payoff. It shows that when the accuracy of the samples ($p$) is sufficiently close to the decision maker's threshold of doubt ($p_d$), the persuaders prefer an equilibrium with $U > \underline{U}$. Finally, Theorem 3 concludes that the best symmetric equilibrium for the persuaders is one that strictly benefits the decision maker when $p$ is close to $p_d$.

First note that when there are $n \ge 3$ persuaders, there are $\frac{n+1}{2}$ possible acceptance fractions if $n$ is odd, and $\frac{n}{2}$ possible acceptance fractions if $n$ is even. Each acceptance fraction corresponds to a unique symmetric equilibrium with test designs $(\tilde{x}, 1)$ where $\tilde{x}$ is a function of the acceptance fraction.

**Theorem 2.** *Let $p < 1$. Given any $n > 2$ and any integer $k$ such that $\frac{n}{2} < k \le n$, there exists a strict symmetric equilibrium in which $(\tilde{x}_i, \tilde{y}_i) = (\tilde{x}, 1)$ for all $i$, where*

$$\tilde{x} = \frac{p - (1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{k}} \left(\frac{p}{1-p}\right)^{\frac{n-k}{k}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{k}} \left(\frac{p}{1-p}\right)^{\frac{n-k}{k}} - (1-p)}, \tag{3}$$

*and the decision-maker's acceptance fraction is $\frac{k}{n}$.*

[8] While I restrict the remaining discussion to symmetric equilibria, it is worth mentioning that asymmetric equilibria with $U > \underline{U}$ generally exist as well. For example, when $n = 3$, $p = 0.8$, and $p_d = \frac{2}{3}$, there is a strict equilibrium in which the first two persuaders choose the most revealing test $(0, 1)$, and the last persuader chooses a less revealing test $\left(\frac{2}{7}, 1\right)$. The decision maker's acceptance set is $\{\{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$, and her ex ante expected utility is strictly above $\underline{U}$.

Condition (3) is derived from the fact that the decision maker is indifferent when exactly $k$ out of $n$ tests results are positive. The proof in the Appendix verifies that no profitable deviation exists for any persuader. In particular, a deviation to some higher $\tilde{x}$ raises the acceptance fraction instantly, and this always lowers the persuader's utility.

To better understand what the equilibria in Theorem 2 are like, Proposition 5 ranks them by the informativeness of their corresponding test designs, as well as by the decision maker's expected utility. In equilibrium, a higher acceptance fraction is associated with less-informative tests. Therefore, the decision maker's expected utility is higher in an equilibrium with a lower acceptance fraction, and she strictly benefits in all equilibria except for the one with the highest acceptance fraction $(\alpha = 1)$. As the number of persuaders goes to infinity, the decision maker almost always learns the true state and gets the highest expected utility $\overline{U}$. The only exception is the case in which her acceptance fraction is one.

**Proposition 5.** *Let $p_d \in \left(\frac{1}{2}, 1\right)$ and $p_d \le p < 1$. Given $n > 2$ and some integer $k$ such that $\frac{n}{2} < k \le n$, let $\tilde{x}$ satisfy equation (3) in Theorem 2. Then:*

*1. Fixing $n$, $\tilde{x}$ strictly increases in $k$.*

*2. Let $U$ denote the decision maker's expected utility in equilibrium. Fixing $n$, $U$ strictly decreases in $k$. When $k = n$, $U = \underline{U}$.*

*Let $\alpha$ be the acceptance fraction in some equilibrium with $N < \infty$ persuaders. For each $\alpha$, define sequences $\{\tilde{x}_n^\alpha\}_{n=N}^\infty$ and $\{U_n^\alpha\}_{n=N}^\infty$ such that $(\tilde{x}_n^\alpha, 1)$ is the test in the n-persuader equilibrium associated with the same acceptance fraction $\alpha$, and $U_n^\alpha$ is the corresponding expected utility of the decision maker. Then:*

*3. If $\alpha < 1$, then $\lim\limits_{n\to\infty} \tilde{x}_n^\alpha < 1$ ; if $\alpha = 1$, then $\lim\limits_{n\to\infty} \tilde{x}_n^\alpha = 1$ . Asymptotically, the state is revealed if and only if $\alpha < 1$.*

*4. If $\alpha < 1$, then $\lim\limits_{n\to\infty} U_n^\alpha = \overline{U}$ ; if $\alpha = 1$, then $U_n^\alpha = \underline{U}$ for all $n \ge N$ .*

To see why (1) and (2) are true, note that a high acceptance fraction (high $\frac{k}{n}$) implies that each positive result is relatively weak due to frequent false positives (high $\tilde{x}$). This explains why $\tilde{x}$ and $k$ always move in the same direction. The fact that $\tilde{x}$ increases in $k$ leads directly to the results in (2). Since a higher equilibrium acceptance fraction is associated with less-informative tests, the decision maker's expected utility is lower in an equilibrium with a higher acceptance fraction.

To see why (3) and (4) are true, note that, in equilibria with the highest acceptance fraction $(\alpha = 1)$, tests are chosen so that the decision maker is merely indifferent when all test results are positive. By the law of large numbers, when the number of persuaders is infinite, if there is any informativeness in each persuader's test, the accumulated information will reveal the true state to the decision maker. Therefore, the decision maker remains indifferent in the limit only if each persuader's test converges to the uninformative one - i.e., $\lim\limits_{n\to\infty} \tilde{x}_n^\alpha = 1$. In equilibria with lower acceptance fractions ($\alpha < 1$), tests remain informative even when the number of persuaders is large. As the number of persuaders grows, the decision maker eventually learns the true state by comparing the different fractions of positive and negative results.

Proposition 5 establishes that the decision maker's expected utility monotonically decreases with the equilibrium acceptance fraction. She strictly benefits from an equilibrium if and only if the acceptance fraction $\alpha \neq 1$. What about the persuaders? To prove that persuaders can prefer an equilibrium that strictly benefits the decision maker is equivalent to showing that persuaders' expected utility is not maximized in the equilibrium with acceptance fraction $\alpha = 1$. Proposition 6 and Remark 2 below confirm this.

**Proposition 6.** *Focus on equilibria characterized in Theorem 2.*

*Let $\alpha$ be the acceptance fraction in some equilibrium with $N < \infty$ persuaders. For each $\alpha$, define sequence $\{V_n^\alpha\}_{n=N}^\infty$ such that $V_n^\alpha$ is the persuaders' expected utility in the n-persuader equilibrium associated with the same acceptance fraction $\alpha$. Then:*

*If $\alpha < 1$, then $\lim_{n\to\infty} V_n^\alpha = \frac{1}{2}$ ; if $\alpha = 1$, then $\lim_{n\to\infty} V_n^\alpha$ is equal to*

$$V_\infty^{\alpha=1} \equiv \frac{1}{2}\left[\left(\frac{p_d}{1-p_d}\right)^{\frac{p-1}{2p-1}} + \left(\frac{p_d}{1-p_d}\right)^{\frac{-p}{2p-1}}\right].$$

*Moreover, there exist $B \in [0,1]^2$ s.t. when $(p_d, p) \in B$, $V_\infty^{\alpha=1} < \frac{1}{2}$ - i.e., the persuaders are strictly better off in the state-revealing symmetric equilibria with $\alpha < 1$.*

The set B in the last statement is visualized in Figure 3.

Proposition 6 shows that asymptotically, when $p_d$ and $p$ are sufficiently close, all players rank state-revealing symmetric equilibria ($\alpha < 1$ and $U = \overline{U}$) over the non-revealing equilibrium ($\alpha = 1$ and $U = \underline{U}$). To see why persuaders can prefer the state-revealing equilibria, note that the key trade-off for the persuaders is benefits from a high frequency of positive results (high $\tilde{x}$) versus benefits from a low acceptance fraction. The existence of set $B$ shows that the the latter can dominate the former. Specifically, when $p$ is not significantly higher than $p_d$, the decision maker is hard to persuade and the test samples are noisy. For positive test results to be sufficiently persuasive, $\tilde{x}$ is generally kept low across all acceptance fractions. This makes negative results common. Therefore, the persuaders benefit more from the lower acceptance fraction in state-revealing equilibria.

The same result from Proposition 6 extends to finite $n$, too. When $n$ is finite, the persuaders' expected utility $V$ is different across all acceptance fractions. Making a statement about the full ranking of $V$ across all acceptance fractions may be challenging. Nevertheless, to prove that persuaders can prefer a beneficial equilibrium, it is sufficient to compare the two equilibria with $\alpha = 1$

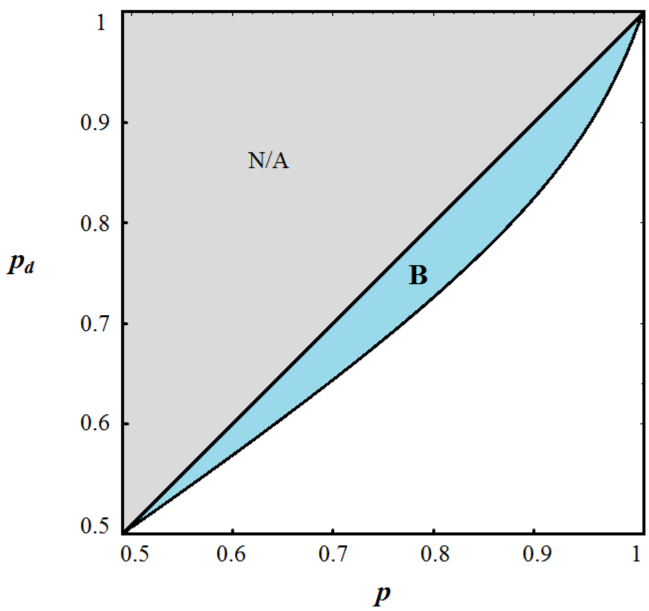



Figure 3: Persuaders prefer the state-revealing symmetric equilibria over the non-revealing symmetric equilibrium asymptotically iff. $(p_d, p) \in B$.

and $\alpha = \frac{n-1}{n}$. The persuaders' expected utilities in these two equilibria, $V^{\alpha=1}$ and $V^{\alpha=\frac{n-1}{n}}$, can be expressed analytically as functions of $n$, $p$, and $p_d$. If $V^{\alpha=1} < V^{\alpha=\frac{n-1}{n}}$, then the persuaders' favorite equilibrium must be one with $\alpha < 1$ and $U > \underline{U}$. Remark 2 verifies that for small $n$, this is, indeed, true when $p_d$ and $p$ are sufficiently close.

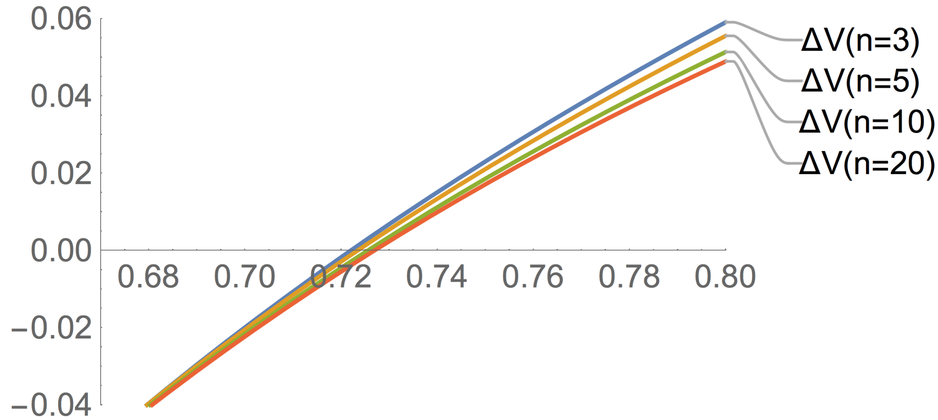
*Remark* 2. Let $V^{\alpha=1}$ and $V^{\alpha=\frac{n-1}{n}}$ denote the persuaders' expected utilities in the symmetric equilibria with $\tilde{y} = 1$ and acceptance fractions $\alpha = 1$, $\alpha = \frac{n-1}{n}$, respectively.

For $n = 3, 5, 10, 20$ and $p = 0.8, 0.99$, $V^{\alpha=1} < V^{\alpha=\frac{n-1}{n}}$ when $p_d$ is sufficiently close to $p$.

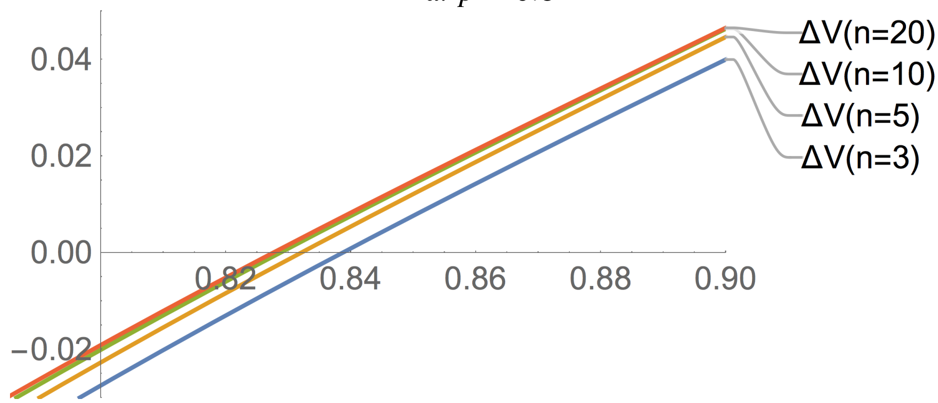
Let $\Delta V \equiv V^{\alpha=\frac{n-1}{n}} - V^{\alpha=1}$. Figure 4 plots $\Delta V$ against $p_d$ for the above values of $p$ and $n$. In every case, the plots show that $\Delta V > 0$ when $p_d$ is sufficiently close to its upper bound $p$.

The same result holds if the values of $n$ and $p$ are different those from above. I omit the (infinitely many) other combinations simply due to space limitations.

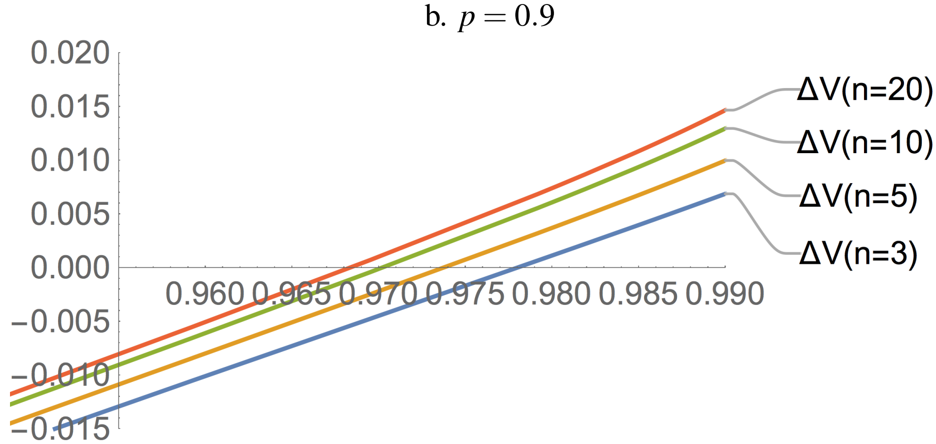
Finally, Theorem 3 incorporates Theorem 2, Propositions 4, 5, 6 and Remark 2 to show that the best symmetric equilibrium for persuaders can be one that strictly benefits the decision maker.




a. $p = 0.8$




b. $p = 0.9$




c. $p = 0.99$

Figure 4: When $p_d$ is sufficiently close to $p$, $\Delta V = V^{\alpha=\frac{n-1}{n}} - V^{\alpha=1} > 0$ - i.e., the persuaders' expected utility in the non-beneficial equilibrium with $\alpha = 1$ is less than that in the beneficial equilibrium with $\alpha = \frac{n-1}{n}$ .

**Theorem 3.** *For $n > 2$, the best symmetric equilibrium for persuaders is one with $U > \underline{U}$ when $p_d$ and $p$ are sufficiently close.*

*Proof.* When $n > 2$, there is a unique symmetric equilibrium with $U = \underline{U}$. It is the one with $\tilde{y} = 1$

and $\tilde{x} = \frac{p-(1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}-(1-p)}$ (Proposition 4 and 5 and Theorem 2). There also exist other symmetric equilibria with $U > \underline{U}$ (Theorem 2 and Proposition 5). Proposition 6 and Remark 2 show that the unique equilibrium with $U = \underline{U}$ does not maximize the persuaders' expected utility when $p_d$ and $p$ are sufficiently close. Therefore, in these cases, the best symmetric equilibrium for the persuaders must be one with $U > \underline{U}$. $\square$

# 5 Discussion

***A. Single persuader with multiple tests***

This paper features games with multiple identical persuaders, each of whom independently conducts one test. One might ask what happens if there is only one persuader in charge of multiple independent tests. In the case of one persuader, the equilibrium is unique: the persuader chooses tests that maximize $V$. If this persuader directly tests the true state, the result from Kamenica and Gentzkow (2011) implies that the decision maker never benefits because the optimal tests that maximize $V$ induce a two-point posterior belief distribution; either the decision maker learns that the state is $L$ or she is indifferent. However, if the persuader tests the true state only indirectly and the tests must be identical, Theorem 3 implies that the persuader can optimally design fairly informative tests that benefit the decision maker.

Overall, when there is a single persuader conducting multiple tests (or, alternatively, when all persuaders collaborate), outcomes with a high payoff for the decision maker are more rare. This comparison emphasizes that it is the strategic interaction between independent persuaders that facilitates the beneficial outcomes for the decision maker.

***B. Correlated persuaders***

In this paper, I assume that the persuaders are independent. Section 2 mentioned that Gentzkow and Kamenica's studies (2017a, 2017b) applied to the opposite case, in which persuaders are arbitrarily correlated. When identical persuaders are independent, there are many strict equilibria that benefit the decision maker; when these persuaders are arbitrarily correlated, there are none. A natural question is to ask what happens when persuaders are partially correlated. Here, I argue that equilibrium outcomes with partial correlation are more likely to resemble the outcomes in this paper.

Borrowing the modeling language from Li and Norman (2017a), say that the result of a test depends on the true state and the realization of a sunspot variable independent of the state (e.g., a random variable uniformly distributed on $[0,1]$). The sunspot variable is responsible for the randomness of the test result. Then, $n$ persuaders are *arbitrarily correlated* if there is a public sunspot variable and all $n$ test results are conditioned on it; persuaders are *independent* if there are $n$ i.i.d. private sunspot variables, and each persuaders' test result is conditioned on his private sunspot. One way to model partial correlation is to say that $n$ persuaders are *partially correlated* if their test results are conditioned on the public sunspot with probability $z$ and the private sunspots with probability $1-z$ for some $z \in (0,1)$, assuming that all players, including the decision maker, know which sunspot(s) is(/are) actually used. It does not matter if the persuaders are restricted to designing tests independent of the sunspot selection or if their test designs can be conditional on the sunspot selection. In either case, it is with positive probability that persuader $i$'s test result will be different from $j$'s. In particular, the number of positive results in state $L$ will be random, and this randomness incentivizes the persuaders to lower the decision maker's acceptance fraction by designing more-revealing tests. This gives rise to equilibria that benefit the decision maker.

***C. Sequential persuaders***

The persuaders in this paper choose tests designs simultaneously. Suppose, instead, that they choose the test designs sequentially. Since all persuaders share a common interest, the outcome of this scenario resembles that of a one-persuader game. Results from Kamenica and Gentzkow (2011) predict that the first persuader optimally chooses a test design as if he were the only persuader, and all later persuaders choose uninformative tests. This unique subgame perfect equilibrium maximizes the persuaders' expected utility. The decision maker does not benefit from persuaders' information.

***D. Continuous state space***

This paper does not hinge on the assumption of the binary state space. Suppose that the true state is a continuous variable $z \in \mathbb{R}$. If the action space of the decision maker is still $\{a_H, a_L\}$, and the persuaders still strictly prefer $a_H$ regardless of the true state, then the persuaders adopt a threshold strategy (Kolotilin, 2015) and the main results of the paper still apply.

For example, when there are two persuaders, there exists a symmetric equilibrium in which both persuaders choose tests that yield positive results when $z \geq \bar{z}$ and negative results when $z < \bar{z}$, where $\bar{z}$ is a relatively high[9] threshold chosen in such a way that the decision maker is indifferent when exactly one test result is positive. Since the acceptance fraction is less than one, the decision maker strictly benefits from the tests.

[9] compared to the threshold in the game with only one persuader.

### *E. Non-binary test results*

Recall that a test generates a message $m \in M$. In this paper, $M = \{positive,\ negative\}$. In general, this binary assumption is not without loss of generality when there are multiple persuaders, but it does not affect this paper's results.

For example, suppose that $M_1 = \{positive,\ negative\}$ for persuader 1, and he chooses $\Pr(positive\,|\,H) \approx 0.067$ and $\Pr(positive\,|\,L) \approx 0.5$ (this is the same test as that of "Beneficial equilibrium 2" in the Introduction). Assume that $M_2 = \{A, B, C\}$ for persuader 2. Consider the following strategy: $\Pr(A|H) = \frac{3}{5}$, $\Pr(A|L) \approx 0.08$, $\Pr(B|H) = \frac{2}{5}$, $\Pr(B|L) \approx 0.75$, $\Pr(C|H) = 0$, $\Pr(C|L) \approx 0.17$. These numbers are chosen so that the decision maker is just indifferent when she sees $(negative,\ A)$ or $(positive,\ B)$. Hence, the persuader's payoff is the unconditional probability $\Pr(A) + \Pr(positive,\ B) \approx 0.465$. This value is higher than the maximum payoff that a persuader can get when $M_2 = \{positive,\ negative\}$ (see "Beneficial equilibrium 2" in Introduction). Therefore, there does not exist any feasible test under $M_2 = \{positive,\ negative\}$ that is outcome-equivalent to the proposed test with $M_2 = \{A, B, C\}$.

However, any equilibrium with $M \subseteq \mathbb{R}$ and $U = \underline{U}$ is outcome-equivalent to some equilibrium with $M = \{positive,\ negative\}$. To see why, note that, in equilibria with $M \subseteq \mathbb{R}$ and $U = \underline{U}$, the decision maker chooses $a_H$ with positive probability and is always indifferent when choosing $a_H$. This implies that each persuader $i$'s test has a most positive message $m_i^*$ with the highest $\frac{\Pr\left(m_i^*|H\right)}{\Pr\left(m_i^*|L\right)}$ among all possible messages, and the decision maker chooses $a_H$ if and only if the realized messages are $\{m_1^*, m_2^*, ..., m_n^*\}$. In this case, it is possible to construct an outcome-equivalent test with $M' = \{positive, negative\}$ for each persuader $i$. Let $\Pr\left(positive|\omega\right) = \Pr\left(m_i^*|\omega\right)$ and $\Pr\left(negative|\omega\right) = \sum_{m \neq m_i^*} \Pr\left(m|\omega\right)$. The distribution of the decision maker's actions conditional on the state under the new tests is the same as under the original tests.

Therefore, relaxing the binary restriction on $M$ does not change the set of equilibria outcomes with $U = \underline{U}$; it only increases the number of equilibrium outcomes with $U > \underline{U}$. Since equilibria with $U > \underline{U}$ already exist when $M$ is binary, they continue to exist when $M$ is larger; if persuaders prefer some equilibrium with $U > \underline{U}$ over those with $U = \underline{U}$ when $M$ is binary, they continue to exhibit this preference when $M$ is larger. Hence, the results of this paper are robust when the binary restriction of $M$ is relaxed.

# 6 Appendix

## 6.1 Proof of Theorem 1

Let $x$ and $y$ satisfy

$$0 < x \leq \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}} \text{ and } y = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{p_d}{1-p_d}\left(x - x^2\right)} < 1.$$

Given that the decision maker's acceptance set is $A = \{\{1\},\{2\},\{1,2\}\}$ (i.e., chooses $a_H$ upon seeing at least one positive result), each persuader's expected utility is

$$\begin{aligned} V^* &= \frac{1}{2}\left[2y(1-y) + y^2 + 2x(1-x) + x^2\right] \\ &= \frac{1}{2}\left(2y - y^2 + 2x - x^2\right). \end{aligned}$$

I now go through each case of unilateral deviation to identify the conditions for $x$ and $y$ such that the proposed strategies, indeed, form a strict equilibrium.

a. Suppose that persuader 1 deviates to some $(x_a, y_a)$ s.t. the decision maker's acceptance set becomes $A_a = \{\{1\},\{2\},\{1,2\}\}$. $A_a$ implies that $x_a$ and $y_a$ must satisfy

$$\begin{aligned} \frac{y_a}{x_a} \cdot \frac{1-y}{1-x} &\geq \frac{y}{x} \cdot \frac{1-y}{1-x}, \\ \frac{y}{x} \cdot \frac{1-y_a}{1-x_a} &\geq \frac{y}{x} \cdot \frac{1-y}{1-x}. \end{aligned}$$

The inequalities imply that

$$\frac{y}{x} \cdot x_a \leq y_a \leq \frac{y - x + (1-y)x_a}{1-x_a},$$

which, in turn, implies that

$$x_a \leq x \text{ and } y_a \leq y.$$

Following this deviation, persuader 1's expected utility is

$$\begin{aligned} V_a &= \frac{1}{2}\left[y_a(1-y) + x_a(1-x) + (1-y_a)y + (1-x_a)x + y_a y + x_a x\right] \\ &= \frac{1}{2}\left[(1-y)y_a + (1-x)x_a + x + y\right]. \end{aligned}$$

Since $V_a$ is increasing in both $x_a$ and $y_a$, $V_a$ is maximized when $x_a = x$ and $y_a = y$, i.e. there is no profitable deviation to some $(x_a, y_a) \neq (x, y)$ s.t. $A_a = \{\{1\}, \{2\}, \{1,2\}\}$.

b. Suppose that persuader 1 deviates to some $(x_b, y_b)$ s.t. $A_b = \{\{2\}, \{1,2\}\}$. $A_b$ implies that $x_b$ and $y_b$ must satisfy

$$\frac{y}{x} \cdot \frac{1-y_b}{1-x_b} \geq \frac{y}{x} \cdot \frac{1-y}{1-x},$$

and persuader 1's expected utility is

$$\begin{aligned} V_b &= \frac{1}{2}(y+x) \\ &< \frac{1}{2}\left[y+x+\left(y-y^2\right)+\left(x-x^2\right)\right] = V^*. \end{aligned}$$

Therefore, there is no profitable deviation to some $(x_b, y_b)$ s.t. $A_b = \{\{2\}, \{1,2\}\}$.

c. Suppose that persuader 1 deviates to some $(x_c, y_c)$ s.t. $A_c = \{\{1,2\}\}$. Then, persuader 1's expected utility following this deviation is

$$\begin{aligned} V_c &= \frac{1}{2}(y \cdot y_c + x \cdot x_c) \\ &\leq \frac{1}{2}(y+x) \\ &\leq V_b \\ &< V^*. \end{aligned}$$

Therefore, there is no profitable deviation to some $(x_c, y_c)$ s.t. $A_c = \{\{1,2\}\}$.

d. Finally, suppose that persuader 1 deviates to some $(x_d, y_d)$ s.t. $A_d = \{\{1\}, \{1,2\}\}$. $A_d$ implies that $x_d$ and $y_d$ must satisfy

$$\begin{aligned} &\frac{y_d}{x_d} \cdot \frac{1-y}{1-x} \geq \frac{y}{x} \cdot \frac{1-y}{1-x} \\ \Rightarrow & x_d \leq \frac{x}{y} \text{ and } y \leq 1. \end{aligned}$$

Following this deviation, persuader 1's expected utility is

$$\begin{aligned} V_d &= \frac{1}{2}(y_d + x_d) \\ &\leq \frac{1}{2}\left(1+\frac{x}{y}\right). \end{aligned}$$

A sufficient condition for $V_d \leq V^*$ is $\frac{1}{2}\left(1+\frac{x}{y}\right) \leq V^*$.

Summarizing the four cases of possible deviation, it suffices to prove Theorem 1 if $\frac{1}{2}\left(1+\frac{x}{y}\right) \leq$

$V^*$. This inequality can be written as

$$\left(2y - y^2 + 2x - x^2\right) - \left(1 + \frac{x}{y}\right) \geq 0. \tag{4}$$

Replacing $y$ with $\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{p_d}{1-p_d}\left(x - x^2\right)}$, the left-hand side of (4) can be rewritten as

$$g(x) \equiv \left(\frac{p_d}{1-p_d} + 1\right)\left(x - x^2\right) + x + \frac{1}{2} + \left[\frac{1}{4} - \frac{p_d}{1-p_d}\left(x - x^2\right)\right]^{\frac{1}{2}},$$

which is always non-negative whenever $0 < x \leq \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}$.

This completes the proof.

## 6.2 Proof of Proposition 2

Let $n = 2$ and $0 < x \leq \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}$. Let $y(x) = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{p_d}{1-p_d}\left(x - x^2\right)}$.

1.

$$y^{'}(x) = -\frac{p_d\,(1-2x)}{2\,(1-p_d)\sqrt{\frac{1}{4} - \frac{p_d\left(x-x^2\right)}{1-p_d}}} < 0$$

whenever $0 < x \leq \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}$ .

2. The decision maker's ex ante expected utility is

$$U(x) = \frac{1}{2}\left\{\left[2y(x)\,(1-y(x)) + y(x)^2\right] + (1-p_d)\left[2x(1-x) + x^2\right] + p_d\,[1-y(x)]^2 + (1-x)^2\right\}.$$

Take the derivative of $U$ with respect to $x$ to get

$$U^{'}(x) = \frac{p_d\left[x - \frac{1}{2} - \frac{1}{2}\sqrt{\frac{p_d(2x-1)^2+1-2p_d}{1-p_d}}\right]}{\sqrt{\frac{p_d(2x-1)^2+1-2p_d}{1-p_d}}} < 0$$

whenever $0 < x \leq \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}$ .

3. The persuaders' ex ante expected utility is

$$V(x) = \frac{1}{2}\left\{2y(x)\,[1-y(x)] + y(x)^2 + 2x(1-x) + x^2\right\}.$$

The derivative of $V$ with respect to $x$ is

$$V^{'}(x) = 1 - x + \frac{p_d (2x-1) \left[ \sqrt{\frac{p_d (2x-1)^2 + 1 - 2p_d}{1-p_d}} - 1 \right]}{2(1-p_d) \sqrt{\frac{p_d (2x-1)^2 + 1 - 2p_d}{1-p_d}}},$$

which is positive for small values of $x$ and negative for large values of $x$.

The second derivative of $V$ with respect to $x$ is

$$V^{''}(x) = \frac{\left[p_d \left(1 - 4x^2 + 4x\right) - 1\right] \sqrt{\frac{p_d[1-4x(x-1)]-1}{p_{d-1}}} - p_d (2p_d - 1)}{\sqrt{(1-p_d)\left[p_d \left(4x^2 - 4x - 1\right) + 1\right]} \cdot \left[p_d \left(4x^2 - 4x - 1\right) + 1\right]} < 0$$

whenever $0 < x \leq \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}$.

Since $V$ is strictly concave, the maximizer of $V$ can be found by solving

$$V^{'}(x) = 0,$$

which yields

$$x^*(p_d) \equiv argmaxV$$

$$= \frac{8\left(p_d^2 + p_d - 3\right)(2p_d - 1)^{\frac{2}{3}}}{48 \cdot \sqrt[3]{p_d^2 \{[(14 - 5p_d)p_d - 17]p_d + 9\} - 3\sqrt{3}\sqrt{(p_d - 1)^3 p_d^3 \{(p_d - 1)p_d[(p_d - 1)p_d + 3] - 1\}}}} - \frac{p_d - 2}{3}$$

$$- \frac{1}{6p_d} \sqrt[3]{(2p_d - 1)\left\{p_d^2\left[\left(14p_d - 5p_d^2 - 17\right)p_d + 9\right] - 3\sqrt{3}\sqrt{(p_d - 1)^3 p_d^3 \left[(p_d - 1)p_d\left(p_d^2 - p_d + 3\right) - 1\right]}\right\}}.$$

Note that since $\bar{x}(p_d) = \frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}$ converges to 0 as $p_d \to 1$, $x^*(p_d)$ converges to 0 trivially. Therefore, to examine which equilibrium the persuaders prefer as $p_d \to 1$, I need to look at the ratio between $x^*$ and $\bar{x}$. A small ratio implies that the persuaders prefer a relatively informative equilibrium. Formally, define $x_r \equiv \frac{x^*(p_d)}{\frac{1}{2} - \frac{1}{2}\sqrt{2 - \frac{1}{p_d}}}$. $x_r$ strictly decreases in $p_d$ and $\lim_{p_d \to 1} x_r = 0$. (See Figure 5 for the graphical illustration.) Therefore, among all equilibria characterized in Theorem 1, the persuaders prefer the most informative one when $p_d \to 1$.

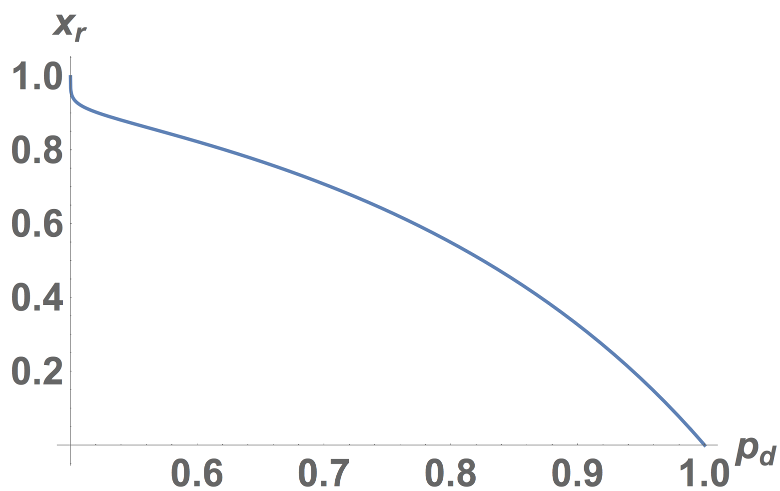



Figure 5: $x_r = \frac{x^*}{\bar{x}}$ strictly decreases in $p_d$ and converges to 0 as $p_d \to 1$. Smaller $x_r$ implies that the persuaders prefer a more informative equilibrium.

## 6.3 Proof of Proposition 3

Let $n > 2$. Let $0 < x \leq \sqrt{\frac{2p_d - 1}{4p_d}}$ and $y = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{p_d}{1-p_d}(x - x^2)} < 1$. It suffices to show that there exists an equilibrium in which $x_1 = x_2 = x$, $y_1 = y_2 = y$, $x_i = y_i = 1$ for all $i > 2$, and the acceptance set is $A = \{\{1\}, \{2\}, \{1,2\}\}$. Let

$$V^* = \frac{1}{2}\left(2y - y^2 + 2x - x^2\right)$$

denote the persuaders' expected utility induced by this strategy profile.

Theorem 1 implies that there is no profitable deviation for the first or the second persuader. Here, I prove that without loss of generality, among all pairs of $(x, y)$ that satisfy the condition in Theorem 1, there always exists some $(x, y)$ such that no profitable deviation exists for the third persuader.

First, note that a unilateral deviation by the third persuader may be profitable only if a positive result from his test dominates two negative results from both the first and the second persuaders - i.e.,

$$\frac{(1-y)^2}{(1-x)^2} \cdot \frac{y_3}{x_3} \geq \frac{p_d}{1-p_d}.$$

Since $\frac{p_d}{1-p_d} = \frac{y}{x} \cdot \frac{1-y}{1-x}$, the above condition is equivalent to

$$\frac{y_3}{x_3} \geq \frac{y}{x} \cdot \frac{1-x}{1-y}. \tag{5}$$

Since the decision maker is indifferent after one positive result and one negative result from the first two persuaders, it is impossible for the decision maker to choose $a_H$ when, in addition to one positive and one negative, the result from the third persuader is negative as well. Therefore, it is sufficient to check only the following two cases of deviation:

a. The third persuader deviates to some $(x_a, y_a)$ such that $A_a = \{\{3\}, \{3,1\}, \{3,2\}, \{3,1,2\}\}$. In this case, it is most profitable for the third persuader to choose $x_a \in (0,1)$ and $y_a = 1$, which yields an expected utility of $V_a = \frac{1}{2}(1+x_a) = \frac{1}{2}\left(1+\frac{x}{y}\cdot\frac{1-y}{1-x}\right)$.

The proof of Theorem 1 shows that $2y - y^2 + 2x - x^2 \geq 1 + \frac{x}{y}$. Since $x < y$, $\frac{1-y}{1-x} \in (0,1)$. Hence, it must be the case that $V^* = \frac{1}{2}\left(2y - y^2 + 2x - x^2\right) > \frac{1}{2}\left(1+\frac{x}{y}\cdot\frac{1-y}{1-x}\right) = V_a$. Therefore, it is not profitable for the third persuader to deviate in this way.

b. Persuader 3 deviates to some $x_b \in (0,1)$ and $y_b \in (0,1)$ s.t.

$A_b = \{\{1,2\}, \{3\}, \{3,1\}, \{3,2\}, \{3,1,2\}\}$. $A_b$ implies that, in addition to (5), $x_b$ and $y_b$ must also satisfy

$$\frac{1-y_b}{1-x_b} \geq \frac{x}{y}\cdot\frac{1-y}{1-x}. \tag{6}$$

Since $V_b = \frac{1}{2}\left[y_b + x_b + (1-y_b)y^2 + (1-x_b)x^2\right]$ increases in $x_b$ and $y_b$, $V_b$ is maximized when both (5) and (6) hold with equality - i.e.,

$$\frac{y_b}{x_b} = \frac{y}{x}\cdot\frac{1-x}{1-y} \text{ and } \frac{1-y_b}{1-x_b} = \frac{x}{y}\cdot\frac{1-y}{1-x},$$

which yields

$$x_b = \frac{x - xy}{x + y - 2xy} \text{ and } y_b = \frac{y - xy}{x + y - 2xy}.$$

$$V_b(x,y) = \frac{y + x^2 y - x^3 y + x\left(1 - 2y + y^2 - y^3\right)}{2(x + y - 2xy)}$$

$$V^* - V_b = \frac{x^3(1-3y) + (1-y)^2 y + x^2(6y-2) + x\left(1 - 6y + 6y^2 - 3y^3\right)}{-2y + x(4y-2)}$$

Replace $y$ with $y(x) = \frac{1}{2} + \left[\frac{1}{4} - \frac{p_d}{1-p_d}\left(x - x^2\right)\right]^{\frac{1}{2}}$ and express $V^* - V_b$ as a single function of $x$. Define $h(x) \equiv V^* - V_b$ then,

$$h(x) = -\frac{(x-1)x\left\{\left[\frac{p_d}{1-p_d}(3x-1) + 3(x-1)\right]\sqrt{1 + 4\frac{p_d}{1-p_d}(-1+x)x} - \frac{p_d}{1-p_d}(3x-1) + (x-1)\right\}}{(4x-2)\sqrt{1 + 4\frac{p_d}{1-p_d}(x-1)x} - 2}.$$

Note that $h(0) = 0$ and $h'(0) = 1$. In other words, there always exists some $x$ in the positive neighborhood of 0 s.t. $V^* - V_b > 0$, and no profitable deviation exists for the third persuader. This completes the proof.

## 6.4 Verifying Example 1

Let $n = 3$, $p_d = \frac{81}{113}$, and $p = 1$. I verify that $\mathbf{t} = ((0.2, 0.9), (0.2, 0.9), (0.2, 0.9))$ and $A(\mathbf{t}) = \{\{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$ is an equilibrium.

When players choose the above strategies, $V(\mathbf{t}) = 0.538$. Given $(x_2, y_2) = (x_3, y_3) = (0.2, 0.9)$, I show that persuader 1 is strictly worse off if he deviates to some $\left(x_1^{'}, y_1^{'}\right) \neq (0.2, 0.9)$. I use $\mathbf{t}'$ to denote the tests after persuader 1's deviation.

Among all deviations that induce $A(\mathbf{t}') = \{\{1,2,3\}\}$, the most profitable is $\left(x_1^{'}, y_1^{'}\right) = (1,1)$, which yields $V(\mathbf{t}') = 0.425 < V(\mathbf{t})$.

If a deviation by persuader 1 induces $A(\mathbf{t}') = \{\{1,2,3\}, \{2,3\}\}$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{0.9^2}{0.2^2} \cdot \frac{1 - y_1^{'}}{1 - x_1^{'}} \geq \frac{p_d}{1 - p_d}.$$

Among all deviations that satisfy the above inequality, persuaders' payoff is maximized when $\left(x_1^{'}, y_1^{'}\right) \to (1,1)$, and $V(\mathbf{t}') \to 0.425 < V(\mathbf{t})$.

If a deviation by persuader 1 induces $A(\mathbf{t}') = \{\{1,2,3\}, \{1,2\}, \{1,3\}\}$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{0.9}{0.2} \cdot \frac{0.1}{0.8} \cdot \frac{y_1^{'}}{x_1^{'}} \geq \frac{p_d}{1 - p_d}.$$

Among all deviations that satisfy the above inequality, persuaders' payoff is maximized when $\left(x_1^{'}, y_1^{'}\right) = (\frac{2}{9}, 1)$, and $V(\mathbf{t}') = 0.535 < V(\mathbf{t})$.

If a deviation by persuader 1 induces $A(\mathbf{t}') = \{\{1,2,3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{y_1^{'}}{x_1^{'}} \geq \frac{0.9}{0.2}, \; \frac{1 - y_1^{'}}{1 - x_1^{'}} \geq \frac{0.1}{0.8}.$$

Among all deviations that satisfy the above inequalities, persuaders' payoff is uniquely maximized when $\left(x_1^{'}, y_1^{'}\right) = (x_1, y_1) = (0.2, 0.9)$.

If $\{1\} \in A(\mathbf{t}')$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{y_1^{'}}{x_1^{'}} \cdot \frac{0.1^2}{0.8^2} \geq \frac{p_d}{1 - p_d} \Rightarrow \frac{y_1^{'}}{x_1^{'}} \geq 162.$$

Among all deviations that induce $A(\mathbf{t}') = \{\{1\},\{1,2\},\{1,3\},\{1,2,3\}\}$, the most profitable is $\left(x_1^{'}, y_1^{'}\right) = (\frac{1}{162}, 1)$, which yields $V(\mathbf{t}') = 0.503 < V(\mathbf{t})$.

Among all deviations that induce $A(\mathbf{t}') = \{\{1\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$, the most profitable is $\left(x_1^{'}, y_1^{'}\right) = (\frac{7}{1295}, \frac{162}{185})$, which yields $V(\mathbf{t}') = 0.511 < V(\mathbf{t})$.

$\{2\} \notin A(\mathbf{t}')$ and $\{3\} \notin A(\mathbf{t}')$ for all $\mathbf{t}'$. Therefore, persuader 1 is strictly worse off after a deviation to some $\left(x_1^{'}, y_1^{'}\right) \neq (0.2, 0.9)$.

Finally, $\underline{U} = \frac{1}{2}(1 + p_d) \approx 0.86$, and the expected utility of the decision maker is

$$U = \frac{1}{2}\left[\Pr(a_L|L) + \Pr(a_H|H) + (1 - p_d)\Pr(a_H|L) + p_d \Pr(a_L|H)\right],$$

where $\Pr(a_L|L) = (1-x)^3 + 3x(1-x)^2$, $\Pr(a_H|H) = 3y^2(1-y) + y^3$,
$\Pr(a_H|L) = 3x^2(1-x) + x^3$, and $\Pr(a_L|H) = (1-y)^3 + 3y(1-y)$. Plug in values of $p_d$, $x$, and $y$ to get $U \approx 0.96$. Hence, $U > \underline{U}$.

## 6.5 Proof of Theorem 2

Given $n$, $k$, suppose that all persuaders choose $\tilde{x}$ that satisfies condition (3) in Theorem 2. Then,

$$\frac{\Pr(H|\text{exactly } k \text{ positive results})}{\Pr(L|\text{exactly } k \text{ positive results})} = \left[\frac{p + (1-p)\tilde{x}}{(1-p) + p\tilde{x}}\right]^k \left(\frac{1-p}{p}\right)^{n-k} = \frac{p_d}{1-p_d}. \tag{7}$$

The decision maker is indifferent (and, hence, chooses $a_H$) when exactly $k$ out of $n$ test results are positive. Hence, her acceptance fraction is $\frac{k}{n}$.

In this equilibrium, each persuader $i$ strictly prefers to choose $\tilde{y} = 1$. To see why, note that a deviation to some $\left(x^{'}, y^{'}\right)$ s.t. $y^{'} < 1$ is profitable only when it induces the decision maker to choose $a_H$, even if $n - k + 1$ test results, including the one from persuader $i$, are negative. However, this is never the case for any $x' \leq y' < 1$. Therefore, a downward deviation in $y^{'}$ always strictly decreases the persuader's expected utility because it lowers the rate of positive results and makes each positive result weaker.

Next, I show that a persuader strictly prefers to choose $\tilde{x}$ when everyone else chooses $\tilde{x}$.

Suppose that persuader $i$ deviates to a test with $x' > \tilde{x}$. The equilibrium outcome is affected only when the test result of $i$ is positive. Given the higher $x^{'}$, a positive result from $i$'s test is less informative. This implies that the decision maker now strictly prefers $a_L$ when exactly $k$ test results (including $i$'s) are positive. In other words, when $i$ reports a positive result, the decision maker needs to see at least $k$ more positive results from the other persuaders in order to choose $a_H$. To find out the decision maker's exact response to the deviation, consider the extreme case in which $x' = 1$. That is, persuader $i$ deviates to the least informative test, which always has a positive result. Under this extreme case, persuader $i$ is completely uninformative, and the decision maker chooses an action based only on information delivered by the other persuaders. Moreover, among the rest of the $n-1$ persuaders, $k$ positive results and $n-k-1$ negative results are sufficient to induce action $a_H$. Therefore, when $i$ reports a positive result, the decision maker requires exactly $k$ more positive results from the other persuaders to choose $a_H$, and this is true for all $x' \in (\tilde{x}, 1]$. Knowing how the decision maker responds to an upward deviation in $\tilde{x}$, it is most profitable for the persuader to choose $x' = 1$. Then, the new expected utility for the persuaders is equal to the probability of having at least $k$ positive results among the rest of the $n-1$ persuaders. Since those $n-1$ persuaders are still choosing the test $\tilde{x}$, $\Pr(\text{at least } k \text{ positive results} | n-1 \text{ tests with } \tilde{x}) < \Pr(\text{at least } k \text{ positive results} | n \text{ tests with } \tilde{x})$. Therefore, any deviation to some $x' > \tilde{x}$ strictly decreases $i$'s expected utility.

Suppose that persuader $i$ deviates to a test with $x' < \tilde{x}$. Again, the equilibrium outcome is affected only when the test result of $i$ is positive. A deviation to a more informative test with lower $\Pr(positive|s_L)$ can be profitable only if it induces the decision maker to choose $a_H$ upon seeing fewer positive results - i.e., she chooses $a_H$ when there are only $k-1$ positive results. This never happens. Suppose that $i$ deviates to the most informative test, $x' = 0$, and his test result is positive. Suppose, further, that among the rest of the $n-1$ persuaders, $k-2$ report positive results, and $n-k+1$ report negative results. Then, the posterior likelihood in this case is

$$
\begin{aligned}
\frac{\Pr(H)}{\Pr(L)} &= \left(\frac{p}{p-1}\right)\left[\frac{p+(1-p)\tilde{x}}{(1-p)+p\tilde{x}}\right]^{k-2}\left(\frac{1-p}{p}\right)^{n-k+1} \\
&= \left[\frac{p+(1-p)\tilde{x}}{(1-p)+p\tilde{x}}\right]^{k-2}\left(\frac{1-p}{p}\right)^{n-k} \\
&< \frac{p_d}{1-p_d}.
\end{aligned}
$$

Even if $i$ deviates to the most informative test, the decision maker's acceptance fraction is still $\frac{k}{n}$. This implies that a downward deviation in $\tilde{x}$ always strictly decreases the persuader's expected utility. Therefore, given that all the other persuaders choose $(\tilde{x}, 1)$, it is uniquely optimal for persuader $i$ to choose $(\tilde{x}, 1)$, as well.

## 6.6 Proof of Proposition 5

1. Recall that

$$
\tilde{x} = \frac{p - (1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{k}}\left(\frac{p}{1-p}\right)^{\frac{n-k}{k}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{k}}\left(\frac{p}{1-p}\right)^{\frac{n-k}{k}} - (1-p)}.
$$

When $k$ increases, $\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{k}}\left(\frac{p}{1-p}\right)^{\frac{n-k}{k}}$ strictly decreases, which means that the numerator in the above equation strictly increases and the denominator strictly decreases. As a result, $\tilde{x}$ strictly increases in $k$.

2. Let $k_1 < k_2$. Let $\tilde{x}_1$ and $\tilde{x}_2$ be the corresponding tests in the symmetric equilibria with acceptance fractions $\frac{k_1}{n}$ and $\frac{k_2}{n}$, respectively. Part 1 implies that $\tilde{x}_1 < \tilde{x}_2$. Let's compare the equilibrium outcomes from the perspective of the decision maker. In the first equilibrium, she picks the best action based on information from $n$ tests with condition probabilities $(\tilde{x}_1, 1)$. In the second equilibrium, she picks the best action based on information from $n$ tests with condition probabilities $(\tilde{x}_2, 1)$. Since $\tilde{x}_1 < \tilde{x}_2$, tests $(\tilde{x}_1, 1)$ generate strictly fewer false positive outcomes than $(\tilde{x}_2, 1)$, and the posterior belief distribution induced by $(\tilde{x}_1, 1)$ is a mean-preserving spread of the one induced by $(\tilde{x}_2, 1)$. Hence, the decision maker's expected utility is strictly higher in the equilibrium associated with $k_1$.

When $k = n$, the decision maker chooses $a_H$ only when all test results are positive. Proposition 4 implies that $U = \underline{U}$.

3-4. As $n \to \infty$, the actual fraction of positive results converges to the expected fraction, which is equal to $\Pr(positive|H) = p + \tilde{x}(1-p)$ when the state is $H$ and $\Pr(positive|L) = 1 - p + \tilde{x}p$ when the state is $L$. $\Pr(positive|H) > \Pr(positive|L)$ if and only if $\tilde{x} < 1$. Therefore, asymptotically, the decision maker can distinguish the two states by observing the fraction of positive results if and only if $\lim_{n\to\infty} \tilde{x}_n^{\alpha} < 1$. Recall that

$$\tilde{x}_n^{\alpha} = \frac{p - (1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{\alpha n}}\left(\frac{p}{1-p}\right)^{\frac{1-\alpha}{\alpha}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{\alpha n}}\left(\frac{p}{1-p}\right)^{\frac{1-\alpha}{\alpha}} - (1-p)}.$$

When $\alpha < 1$,

$$\lim_{n\to\infty} \tilde{x}_n^{\alpha} = \frac{2p-1}{p(1-p)}\left[\left(\frac{p}{1-p}\right)^{\frac{1}{\alpha}} - 1\right]^{-1} - \frac{1-p}{p} < 1.$$

Therefore, in the limit, the decision maker can always distinguish the two states and choose the action that exactly matches the true state. As a result, $\lim_{n\to\infty} U_n^{\alpha} = \overline{U}$ when $\alpha < 1$.

When $\alpha = 1$, Proposition 4 implies that $U_n^{\alpha} = \underline{U}$ always. In this case,

$$\tilde{x}_n^{\alpha=1} = \frac{p - (1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}} - (1-p)}, \tag{8}$$

which converges to 1 when $n \to \infty$. Asymptotically, the decision maker cannot distinguish the two states.

## 6.7 Proof of Proposition 6

When $\alpha < 1$, part 3 of Proposition 5 implies that the decision maker learns the true state, and she chooses $a_H$ if and only if the true state is $H$. Therefore, $V_n^\alpha$ converges to $\frac{1}{2}$, the ex ante probability of state $H$.

When $\alpha = 1$, everyone chooses the $\tilde{x}$ as defined by equation (8), and

$$\begin{aligned}
\Pr(a_H|H) = [\Pr(positive|H)]^n &= [p+(1-p)\tilde{x}]^n \\
&= \left[p+(1-p)\cdot\frac{p-(1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}-(1-p)}\right]^n \\
&\to \left(\frac{p_d}{1-p_d}\right)^{\frac{p-1}{2p-1}} \text{ as } n\to\infty.
\end{aligned}$$

$$\begin{aligned}
\Pr(a_H|L) = [\Pr(positive|L)]^n &= [(1-p)+p\tilde{x}]^n \\
&= \left[1-p+p\cdot\frac{p-(1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}-(1-p)}\right]^n \\
&\to \left(\frac{p_d}{1-p_d}\right)^{\frac{-p}{2p-1}} \text{ as } n\to\infty.
\end{aligned}$$

Hence,

$$V=\frac{1}{2}\cdot\Pr(a_H|H)+\frac{1}{2}\cdot\Pr(a_H|L)\to\frac{1}{2}\left[\left(\frac{p_d}{1-p_d}\right)^{\frac{p-1}{2p-1}}+\left(\frac{p_d}{1-p_d}\right)^{\frac{-p}{2p-1}}\right]\equiv V_\infty^{\alpha=1},$$

with

$$\frac{\partial V_\infty^{\alpha=1}}{\partial p_d}=\frac{\left(\frac{p_d}{1-p_d}\right)^{\frac{p-1}{2p-1}}\left(1+\frac{p}{p_d}-2p\right)}{2(1-2p)\,p_d\,(1-p_d)}<0,$$

$$\lim_{p_d \to \frac{1}{2}} V_\infty^{\alpha=1} = 1,$$

$$\lim_{p_d \to p} V_\infty^{\alpha=1} = \frac{1}{2} \cdot p^{\frac{-p}{2p-1}} (1-p)^{\frac{1-p}{2p-1}} < \frac{1}{2}.$$

$$\frac{\partial V_\infty^{\alpha=1}}{\partial p} = \frac{\left(\frac{p_d}{1-p_d}\right)^{\frac{-p}{2p-1}} \left[1 + \left(\frac{p_d}{1-p_d}\right)\right] \ln\left(\frac{p_d}{1-p_d}\right)}{2\left(2p-1\right)^2} > 0,$$

$$\lim_{p \to p_d} V_\infty^{\alpha=1} = \frac{1}{2} \cdot p_d^{\frac{-p_d}{2p_d-1}} (1-p_d)^{\frac{1-p_d}{2p_d-1}} < \frac{1}{2},$$

$$\lim_{p \to 1} V_\infty^{\alpha=1} = \frac{1}{2}\left(1 + \frac{1-p_d}{p_d}\right) > \frac{1}{2}.$$

Therefore, $\forall p_d \in \left(\frac{1}{2}, 1\right)$, there exist $\overline{p}$ s.t. $p \in (p_d, \overline{p})$ implies $V_\infty^{\alpha=1} < \frac{1}{2}$. Define region $B$ accordingly.

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