The Collateral Composition Channel

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Abstract

Wholesale financial markets reallocate deposits. Because of incentive problems, these flows are limited by endogenous collateral constraints. The *composition* of collateral matters. The use of inside collateral creates a "collateral pyramid": cash flows from one loan are pledged to secure another. Outside collateral, such as treasuries, is a foundation of, and stabilizes, the pyramid. Through collateral pyramids the financial sector sustains a large volume of reallocation across banks, but at the cost of systemic panics. During panics, the safe asset creation process stalls, the pyramid collapses, collateral becomes scarce. Markets are more fragile when loans are secured by inside collateral.

Keywords: Inside and Outside Collateral, Financial Fragility, Financial Crisis

JEL Class.: G01 – G10 – G20.

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1 Introduction

This paper studies the fragility of interbank markets. These markets facilitate the reallocation of funds within the banking sector: banks with relatively profitable opportunities borrow from other banks. These exchanges may include various market–financed transfers of assets between banks and take a variety of forms, including overnight funding markets, collateralized loan markets, etc.

Our analysis highlights the significance of collateral composition for the functioning of the interbank markets. We make a distinction between "outside collateral", such as the pledging of treasuries in a repo arrangement, and "inside collateral", such as the pledging of cash flows in the asset backed commercial paper market.[1] A key difference between the two types of collateral is that the cash flow from the assets underlying inside collateral is private information. This makes the pledgeability of this cash flow endogenous, and dependent on the degree of informational asymmetry and the quality of banks' assets. In turn, pledgeability determines how much safe assets (or collateral) can be created out of the cash flow, which affects the equilibrium outcome and the quality of the assets. The upshot is that markets are more fragile when they are secured by inside collateral than when they are secured by outside collateral.

Reflecting incentive problems in the interbank market, a bank's collateral determines its ability to borrow and thus the extent of the reallocation of savings from low to high quality banks. In equilibrium, the pledgeability of borrowers' cash flows itself depends, through adverse selection, on borrowers' quality. If lenders believe that borrowers are high quality, then borrowing constraints are lax, the demand for funding and the equilibrium interbank rate are high. Only high quality banks, those whose projects have a positive net present value (NPV), borrow funds. So, borrowers are indeed high quality.[2] Those complementarities are magnified when transactions are collateralized. High quality loans have high collateral value, which further relaxes borrowing constraints, raises demand, the equilibrium rate, and borrower quality.

Yet, for the same parameter values, there may exist a "collateral trap" in which lenders are pessimistic about the quality of borrowers. In this equilibrium, borrowing constraints are tight, borrowers demand little funding, and the equilibrium rate is low. As a consequence, most projects have positive NPV, even low quality banks borrow, and the sorting of borrowers is less efficient. The lack of information about borrowers' cash flows undermines their pledgeability. In a collateral trap, pledgeability is low and the volume of inside collateral is scaled down, which contributes to tightening the borrowing constraint. The resulting complementarity may generate multiple equilibria, including a collateral trap, characterized by (i) a reduction in the creation of inside collateral by banks, (ii) reduced inter-

[1] Note that treasuries may not necessarily be the only type of outside collateral or even may not always be considered as outside collateral, as would for example be the case when a sovereign issues debt in order to bail out its domestic banking sector.

[2] Boissay (2011) also features this type of complementarity but does not study collateralized lending transactions. As a consequence, that paper does not study the differentiated effects of collateral composition on financial fragility, which is the focus of this paper.

bank activity, (iii) resource mis–allocation, (iv) a recession. We provide a crisp characterization of the conditions on the economy's fundamentals for this type of multiplicity.

Whether a bank pledges one dollar worth of treasuries or one dollar worth of equally safe private asset has no effect on its borrowing capacity; but it is relevant for financial stability. In normal times, an economy in which banks finance themselves through private assets backed paper may thus be observationally identical to an economy where banks finance themselves through treasury repos and yet be intrinsically more fragile.[3] The fragility arises from the fact that the amount of safe assets (or collateral) that borrowers engineer out of their cash flow depends on the quality of their assets, and that lower asset quality may also be sustained as another equilibrium. This effect of collateral *composition* on financial stability is distinct from the usual collateral multiplier effect. The use of inside assets creates a type of "collateral pyramid" in that cash flows from one loan are pledged to secure another. Through collateral pyramids, the financial sector is able to raise the volume of pledgeable assets, and generates a collateral multiplier that resembles that in Kiyotaki and Moore (1997). But it does so at the cost of exposing the economy to systemic panics, during which the collateral creation process suddenly stalls, the collateral pyramid collapses, and the economy tanks. We refer to this latter mechanism as the "collateral composition channel".

Treasuries play a specific role in avoiding collateral traps. As the foundation of the collateral pyramid, they contribute to making the pyramid immune to beliefs and, therefore, less fragile. We show that there is a threshold for the share of treasuries in total pledgeable assets held by the banking sector, below which investors may panic and the interbank market freeze.

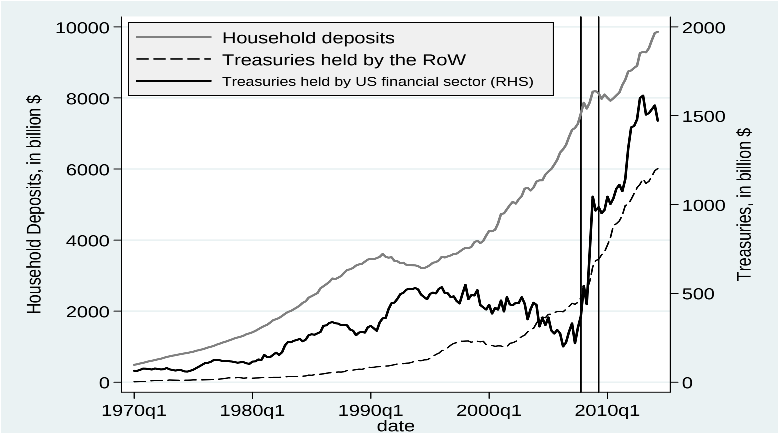
Link to Facts

The distinction between inside and outside collateral helps to understand why the 2007–8 financial crisis broke out despite the massive engineering of private pledgeable assets by the US banking sector. Figure 1 shows the long term evolution of the volume of treasuries held by the US financial sector and the rest of the world, against the evolution of US household deposits. Treasuries and deposits both increased from 1970 until 1995. After 1995, the growth of deposits and of the demand for treasuries by the rest of the world accelerated, while US banks reduced their holding of treasuries; until the start of the crisis. At that time, banks deposits were at a historical high, and treasuries held by the financial sector were at a historical low, reflecting a "crowding out effect" and a shortage of outside collateral. To finance their operations, US banks created safe assets out of their loans, which they used as collateral on wholesale funding markets in lieu of treasuries (see Figure 2 and Caballero (2009)).

Fact # 1: *The 2007–8 crisis occurred after a historical fall in US treasuries holdings by the US banks, and a change in the composition of banks' collateral —from treasuries to private assets.*

[3] In Definition 2 we are more precise about what we mean by financial "fragility". The comparison of such two economies will be key to our results, as this will allow us to control for the total amount of collateral available in the banking sector; this is discussed in Section 4.

Figure 1: US treasuries held by the financial sector and household deposits



Note: Total outstanding amount of US treasuries securities held by the US financial sector: *Source:* US Flows of Funds Accounts (Table L.209, series FL763061100.Q + FL753061103.Q + FL743061103.Q + FL633061105.Q + FL653061105.Q + FL663061105.Q). Household Deposits: total currency and deposits (including money market fund shares) by households and nonprofit organizations; *Source:* US Flows of Funds (Table L.100, series FL154000025.Q). Treasuries held by the Rest of the World: *Source:* US Flows of Funds Accounts (Table L.209, series FL263061105.Q). Vertical lines: Great Recession's troughs and peaks (NBER dates).

Our model suggests that this change in US banks' collateral composition (Fact #1) from outside toward inside collateral exposed the financial sector to a change in beliefs and, as a result, was a source of financial instability.

Fact # 2: *During the 2007-8 crisis, Asset Backed Security (ABS–ABCP) markets collapsed; in contrast, the markets secured by treasuries remained stable.*

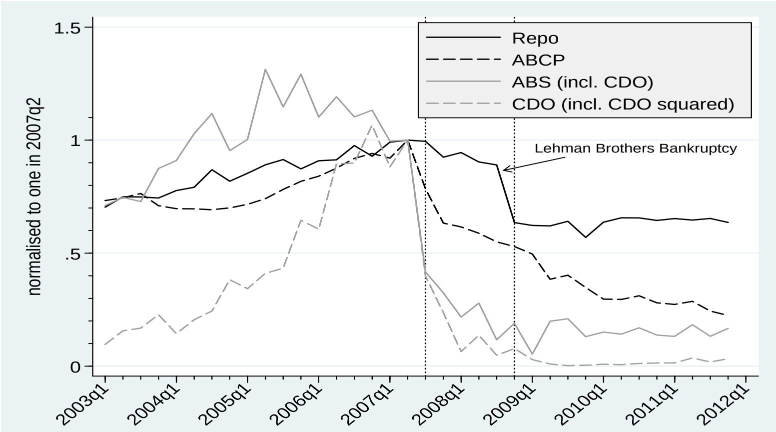
Moreover, the collateral trap in the model resembles the collapse in wholesale funding markets observed during the 2007–8 financial in the US (Fact #2). As the model predicts, the wholesale funding backed by inside collateral collapsed by more than that backed by outside collateral (Fact #2). This fact has been documented in a number of papers. For example, Copeland, Martin, and Walker (2011) show that the segments of the US repo market, such as tri-party repos and the bilateral repos, which represent more than two thirds of the US repo market, remained strikingly stable during the crisis. The reduction in repo transactions was notably limited compared to the reduction in ABCP–ABS transactions (see Figure 2).[4] The upshot is that funding markets secured by inside collateral were less stable than those secured by outside collateral.

Fact # 3: *During the crisis interbank rates fell abruptly, and faster than corporate loan rates.*

Figure 3 shows that interbank rates (here, the LIBOR and the Fed Fund rates) went up in the run–up to the crisis —to above 5% in 2007q2— and plummeted during the crisis, to below 0.25% in 2010q1. It also shows that those rates fell by more than corporate loan rates (plain black line). In this paper, we argue that this rise in the corporate loan spread is indicative of the impairment of the

[4] ABSs are securities whose income payments is derived from and "backed" by a specified pool of assets, like mortgages, credit card loans etc, which cannot be sold individually. Collateralized Debt Obligations (CDO) are structured financial products that pools together cash flow–generating assets and repackage those assets into discrete tranches that can be sold to investors. The maturity ABSs and CDOs is usually longer (above eighteen months) than that of repos and ABCPs, which does not exceed a quarter (overnight for the repos and less than six weeks for ABCPs). To compare the evolution of those markets, we report the quarterly issuances of ABS and CDOs together with the outstanding volumes (end–of–quarter) of repos and ABCPs.

Figure 2: Volume of Secured Loans, as a ratio to Households' Deposits

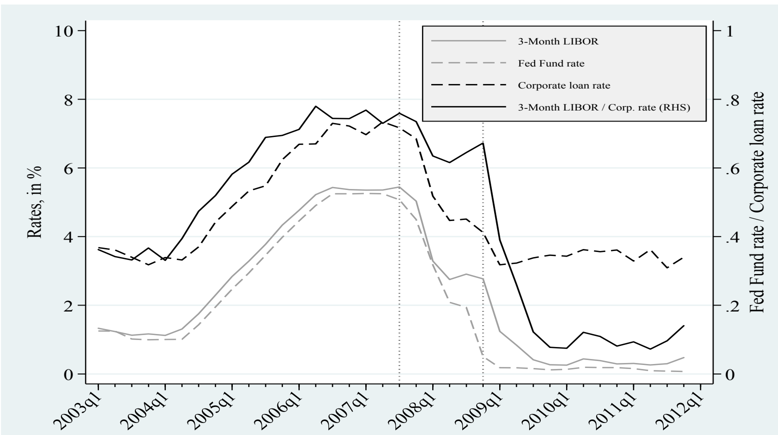


Note: Volumes of repos, ABCP, and ABS, divided by households deposits; the ratio is normalized to 1 in 2007q2. Repo and Fed Funds volumes: outstanding amounts borrowed by the financial sector (excluding foreign banks offices); *Source:* US Flows of Funds (Table L.207, series FL892150005.Q). ABCP: outstanding amounts; *Source:* Federal Reserve Bank of St Louis (FRED) database (series DTBSPCKAM). ABS and CDO: quarterly issuances; *Source:* Asset Backed Alert and Dealogic. Household Deposits: total currency and deposits (including money market fund shares) by households and nonprofit organizations; *Source:* US Flows of Funds (Table L.100, series FL154000025.Q). Vertical lines: Great Recession's troughs and peaks (NBER dates).

financial intermediation process, and of the mis–allocation of funds within the banking sector.

There are implications of this mis-allocation in terms of output and capital that match recent evidence on factor mis–allocation. For example, Foster, Grim, and Haltiwanger (2013) show that, in contrast to previous periods of low economic activity, the Great Recession did not correspond to a period of increased reallocation of factors of production. Using data for manufacturing firms in Spain, Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2015) also document that factor mis–allocation at the micro–level induced significant aggregate productivity losses over the period 1999–2012, and empirically link this pattern to financial frictions.

Figure 3: Nominal Interest Rates by Asset Classes



Note: Quarterly averages. The corporate loan rate is the average of commercial and industrial loan rates ; *Source:* Federal Reserve Board Survey of Terms of Business Lending. The 3-Month LIBOR rate and the Fed fund rates are from the Federal Reserve Bank of St Louis' FRED database (series usd3mtd156n and Fedfunds, respectively). Vertical lines: Great Recession's troughs and peaks (NBER dates).

Literature

Our terminology of inside and outside collateral is reminiscent of Holmstrom and Tirole (2011) and Holmstrom and Tirole (1998)'s distinction between inside (or private) and outside (or public) liquidity.[5] In our model, as in theirs, a complementarity emerges between inside and outside collateral. However, when they focus on the effects on economic efficiency, we instead focus on the effects of collateral composition on financial fragility.

Our model of strategic uncertainty in financial markets focuses on loans between financial entities, and on the characteristics of the assets used to secure those loans. It purposefully downplays the role of financial instability stemming from bank runs along the lines of Diamond and Dybvig (1983).[6] Indeed, sudden bank deposit withdrawals and capital outflows from the banking sector are not part of our analysis, as Figure 1 shows.[7]

As in Kiyotaki and Moore (1997) and the literature that follows (*e.g.* Benmelech and Bergman (2011), Benmelech and Bergman (2012), Chaney, Sraer, and Thesmar (2012)), collateral is central to our analysis as a means of coping with incentive problems. This literature focuses on total collateral value. Our collateral *composition* channel is distinct from —but not incompatible with— this collateral *value* channel. It emphasizes how the composition of collateral —given a total collateral value— determines the ability of banks to create safe assets out of future cash flows and lend to the real economy.

The complementarity between the two types of collateral in our model stands in contrast with Krishnamurthy and Vissing-Jorgensen (2015). In their model, banks provide safe assets to households and the rise in the supply of treasuries crowds out bank deposits via effects on the equilibrium price of treasuries. In contrast, we focus on the holdings of treasuries by collateral–constrained banks and on the role of such securities in securing banks' wholesale funding. As Krishnamurthy and Vissing-Jorgensen (2015), we too predict that, following an increase in treasury supply, the yield spread between risky loans and safe assets should go down (see also Krishnamurthy and Vissing-Jorgensen (2012)). But, in our case, the reduced spread reflects the lower shadow collateral value of safe assets for banks, not a deterioration of the non–pecuniary services of safe assets for households.

[5] Our concept of inside and outside collateral is however very different from Bolton, Santos, and Scheinkman (2011)'s, whose inside liquidity refers to own cash or reserves, and outside liquidity refers to the cash proceeds from the sales of assets to outside investors.

[6] See, for example, Krishnamurthy, Nagel, and Orlov (2014), Gorton and Metrick (2012), Covitz, Liang, and Suarez (2009), Copeland, Martin, and Walker (2011), Acharya and Schnabl (2010), and Fleming, Hrung, and Keane (2010) for arguments supporting this emphasis. Moreover, the runs observed on asset–backed commercial paper (ABCP) markets in 2007-8 were different from the traditional deposit–based bank runs, which Gorton (1988) showed to be mainly related to economic fundamentals. In particular, Covitz, Liang, and Suarez (2009) find that investors in ACBP markets ran from all types of programs, even from those with apparently solid fundamentals.

[7] The observation that the financial crisis in the US was not marked by a sudden withdrawal of external funds, and therefore could not be explained by a traditional deposit–based bank run has also been emphasized by Caballero (2009) and Shin (2010).

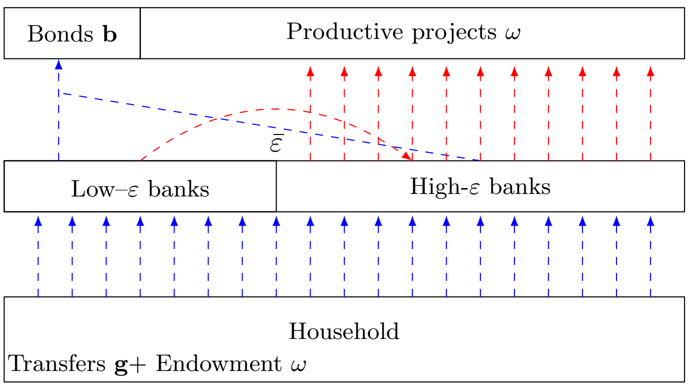
Roadmap

The paper proceeds as follows. Section 2 describes our theoretical framework. Section 3 describes the conditions under which financial markets may collapse. This section makes clear how the level of government debt influences the set of equilibria. We analyze the link between collateral composition and financial stability in Section 4. This discussion highlights the role of the composition of collateral for fragility. Section 5 explores the link between our model and the facts. The last section concludes.

2 Environment

We consider an economy populated by a government, a representative household, and a continuum of banks. All agents live one period. We present their choice problems and constraints in turn. The consequent flow of funds between households, banks and financial projects are illustrated in Figure 4.

Figure 4: Interbank Reallocation and Financial Intermediation



Note: Flows before (- - -)/after (- - -) the εs are realized.

In the end, the analysis reduces to the determination of a critical bank, denoted $\bar{\varepsilon}$, that partitions the banks into borrowers (high quality) and lenders (low quality) on the interbank market. In the equilibria, $\bar{\varepsilon}$ will determine both the volume and the quality of private collateral.

2.1 Government

The government finances expenditures $\mathbf{g}$ by borrowing $\mathbf{b} = \mathbf{g}$ from banks at rate r^b, and repays debt by raising taxes of $\mathbf{b}r^b$ on households at the end of the period. These expenditures are perfectly substitutable for private consumption in household utility. Given this and the presence of lump-sum taxes, and absent frictions in the intermediation process, Ricardian equivalence would hold and the equilibrium would be independent of $\mathbf{b}$.

2.2 Household

The representative household is endowed with ω units of the single good and receives a transfer of $\mathbf{g}$. Equivalently, the household values government spending. The household saves its entire income of $s \equiv \omega + \mathbf{g}$ through deposits into banks at the beginning of the period. At the end of the period the household consumes the return on deposits, sr^d, minus the taxes $\mathbf{b}r^b$.[8] For simplicity, assume linear utility over consumption, c:

$$u(c) = c = sr^d - \mathbf{b}r^b = \omega r^d + \mathbf{b}(r^d - r^b), \tag{1}$$

where we used the equality $s = \omega + \mathbf{b}$. From the above expression, the level of government debt matters for consumption if and only if $r^d \neq r^b$. As we explain later, in equilibria constructed for this economy, $r^d > r^b$, reflecting frictions in the financial system.

2.3 Banks

There is a continuum of mass one of banks collecting deposits s from the household. Each bank has access to productive, irreversible projects, to a storage technology, and to government bonds.

There are two stages of bank investments. In the first stage, banks are identical and use deposits to purchase government bonds, b, setting the remainder, $s - b$, aside for second stage investments. The return on government bonds, denoted by r^b, is determined in equilibrium. In the second stage, banks learn their productivity, ε, and become heterogenous. The εs are distributed over $[0, 1]$ with a continuously differentiable cumulative distribution function $G(\varepsilon)$, $\forall \varepsilon \in [0, 1]$. The projects of bank with productivity ε (for short, bank ε) yield gross unit return εR. Bank ε can then borrow funds up to ϕs from less productive banks and invest $s - b + \phi s$ into her projects; or she can lend $s - b$ to more productive banks. The quantity ϕ will be endogenously determined. The interbank market arises to allow banks to reallocate their funds once they have learned their productivity.[9] The return on the interbank loan market, denoted by r, is determined in equilibrium. In the second stage, banks may also store goods. Storage generates an exogenous return γR, with $\gamma < 1$, *i.e.*, storage is an inefficient technology. While the use of this technology may be one possible outcome in the equilibrium, the potential to store goods will in any case matter through its effects on borrowers' incentives.

The objective of a bank is to maximize her expected gross return on assets, π^b, with respect to her

[8] In the model, there will be no friction between banks and households. Therefore, there will be no material distinction between bank deposits and bank equity, and r^d may be equally interpreted as the return on deposits plus dividends.

[9] As noted earlier, our notion of "interbank market" encompasses all types of interbank transactions that facilitate the reallocation of funds among banks. Those may for instance include the trading of government bonds on a secondary market whereby less productive banks would purchase government bonds from more productive banks. Essentially, such secondary market transactions are similar to treasury repo market transactions whose "repurchase" leg would never be exercised. In Appendix 7.6, we show formally that, in our model, interbank transactions secured by the government bonds yield the same equilibrium outcome as interbank transactions through a secondary market for government bonds.

investment decisions:

$$\max_b \pi^b \equiv br^b + \int_0^1 \max_{\mathbb{1}} (1 - \mathbb{1}) (\varepsilon R(s - b + \phi s) - \phi sr) + \mathbb{1}(s - b) r dG(\varepsilon) \tag{2}$$

where $\mathbb{1}$ denotes the bank's decision to lend ($\mathbb{1} = 1$) or borrow ($\mathbb{1} = 0$) funds on the interbank market.[10] We first study the second stage choice of a bank over $\mathbb{1}$. Using this, we solve the optimization problem of the bank in the first stage and determine b.

2.3.1 Stage 2 Choices: Interbank Market Participation

Once the ε's are drawn, banks allocate their available funds, $s - b$, into interbank loans, into their own projects, or into storage. To fix ideas, we will refer to the banks that lend and to those that borrow on the interbank market as "lenders" and "borrowers", respectively.

There are four important features of bank trades in the interbank market. First, we assume that borrowers meet prospective lenders sequentially, and once. Second, the cash flows generated from storage are not seizable and, once she obtains a loan, a borrower can unilaterally decide to invest in storage instead of the productive projects she obtained the loan for. This gives the borrower the option to abscond and default, and creates a moral hazard problem *ex ante* similar to that described in Hart and Moore (1994). To deal with the borrower's incentive issues, lenders restrict borrowing and require the borrower to pledge assets as collateral. Third, a given lender is able to seize *ex post* all the cash flows from the productive projects she financed, and up to a fraction θ of the cash flows of the projects that someone else financed, with $\theta \leq 1$. Parameter θ reflects the lack of information of the lender about the borrower's investments she was not involved in, and more generally the limited pledgeability of a borrower's existing assets. The higher θ, the higher asset pledgeability.[11] Fourth, its realized value of ε is known only to the bank, which implies that, in equilibrium, lenders will be able to compute the expected value of ε for borrowers but contracts cannot be conditioned on bank specific variables.[12] As we shall see, high ε banks initially obtain a loan collateralized by their holding of government debt —by assumption, government debt is fully pledgeable. Once a project is undertaken, banks can meet other lenders and obtain additional funding by pledging the cash flows from this project.

[10] As we write (2), we anticipate —for convenience but without loss of generality— that bank ε either lends or borrows, and that in equilibrium $r \geq \gamma$, *i.e.* that storage will be used if and only if $r = \gamma$. Indeed, when $r < \gamma$, then there is no supply of interbank loans and r would not be an equilibrium rate —a contradiction; if $r > \gamma$, banks strictly prefer to lend than to store goods; only if $r = \gamma$ banks are indifferent.

[11] This assumption that the project has a higher value to the "initial financier" than to other creditors is borrowed from Diamond and Rajan (2000). For the time being we allow for $\theta = 1$; but later we will see that there is a threshold for θ, above which limited pledgeability does not affect the equilibrium outcome and that, in this case, the economy reaches the First Best. To make things interesting, we will eventually assume that θ is low enough so that limited pledgeability matters (see Assumption 1). Also notice that a borrower never defaults *ex post* provided that she invested into productive projects *ex ante*. That is, there is no moral hazard problem *ex post*. The only concern of a lender is therefore to make sure that her loans are properly used, *i.e.*, used to finance productive projects.

[12] We discuss this result in more detail in Appendix 7.8. In a frictionless world, the most productive bank, with $\varepsilon = 1$, would borrow an infinite amount of funds ($\phi = +\infty$) and be the only bank to initiate projects. All the other banks, with $\varepsilon < 1$, would be lenders.

The first order condition of the maximisation problem (2) with respect to $\mathbb{1}$ implies that bank ε will be a borrower ($\mathbb{1} = 0$) if and only if $\varepsilon \geq \bar{\varepsilon}$, where

$$\bar{\varepsilon} = \frac{r}{R} \tag{3}$$

is the bank, as indicted in Figure 4, that is indifferent between undertaking projects and lending on the interbank market. It also determines the productivity cut–off, above which banks undertake the productive projects. The higher $\bar{\varepsilon}$, the more productive the banks operating projects, and the more efficient the reallocation of savings within the banking sector. Accordingly, $\bar{\varepsilon}$ can be interpreted as a measure of financial efficiency.

The funds a bank can raise are limited due to (i) the bank's inability to commit herself to invest the funds into productive projects and (ii) the limited pledgeability of the cash flows of her past projects (or, equivalently, the "haircut" applied to those cash flows). In equilibrium, the borrowing limit ϕ is determined so that no borrower, including the least productive one, $\bar{\varepsilon}$, has an incentive to borrow, use her storage technology, and default.[13]

In this context, as elaborated below, a borrower raises interbank funds, ϕs, in multiple stages. Initially, a borrower only holds government bonds as pledgeable assets. So, she can only secure loans with bonds. Let $\phi^{(1)} s$ be the amount of those loans. The incentive constraint that prevents bank ε from borrowing funds and absconding is:

$$\gamma R \left(s - b + \phi^{(1)} s\right) \leq b r^b + \varepsilon R \left(s - b + \phi^{(1)} s\right) - \phi^{(1)} s r + \left(\phi - \phi^{(1)}\right) (\varepsilon R - r) s. \tag{4}$$

The left side is the payoff from absconding. If bank ε absconds, she earns $\gamma R(s - b + \phi^{(1)} s)$; but the returns on the pledged government bonds, $b r^b$, are lost, as the lenders have direct control over the government bonds pledged. By contrast, if she initiates her projects, she earns the return on government bonds plus the return on the project, $\varepsilon R \left(s - b + \phi^{(1)} s\right)$, net of the interbank loan repayment, $\phi^{(1)} s r$. This is indicated on the right side of (4). In addition, as we explain below, bank ε may use the future proceeds of her initial projects as collateral to lever additional funds, $\phi - \phi^{(1)}$, and earn a net unit return $\varepsilon R - r$ on those levered funds. Those additional returns are captured by the last term on the right side, $(\phi - \phi^{(1)})(\varepsilon R - r)$. Constraint (4) must hold for all borrowers $\varepsilon \geq \bar{\varepsilon}$, and binds for $\varepsilon = \bar{\varepsilon}$, so that (using (3)):

$$\phi^{(1)} s = \frac{1}{\gamma R} \left(b r^b + (s - b)(\bar{\varepsilon} - \gamma) R\right). \tag{5}$$

This expression shows that the debt capacity associated with outside collateral $\phi^{(1)} s$ can be seen as a (discounted) sum of pledgeable cash flows (the term in brackets), which consist of the return on government debt ($b r^b$) and the pledgeable return on projects ($(s - b)(\bar{\varepsilon} - \gamma) R$). The actual return of the projects for bank ε is $(s - b)\varepsilon R$, but only $(s - b)(\bar{\varepsilon} - \gamma) R < (s - b)\varepsilon R$ of it can be pledged.

[13] In Appendix 7.7, we show that no equilibrium with default exists.

The difference reflects both the cost of asymmetric information, $(\varepsilon - \bar{\varepsilon})(s-b)R$, and the cost of moral hazard, $-\gamma(s-b)R$.

Once the borrower gets funding $\phi^{(1)}s$, she invests $s - b + \phi^{(1)}s$ into productive projects. Those investments are observable, and the borrower can raise additional funding by pledging their future cash flows as collateral. Because prospective lenders have limited information about those investments, though, the second stage lenders can only seize a fraction θ of the cash flows *ex post*. So, for borrower ε, the seizable cash flows is $\theta\varepsilon R(s - b + \phi^{(1)}s)$. Moreover, because they depend on ε, those cash flows are subject to the asymmetric information problem and cannot be pledged as such, *i.e.*, they are "information sensitive" (see Gorton and Ordonez (2014)). Only the information insensitive part, $\theta\bar{\varepsilon}R(s - b + \phi^{(1)}s)$, is pledgeable.[14] Importantly, those cash flows are all the assets that the borrower can pledge at that stage, as no asset is being pledged multiple times. Let $\phi^{(2)}s$ be the borrowing limit in the second stage. This limit is such that borrower ε has no incentive to abscond on the loan, for all $\varepsilon \geq \bar{\varepsilon}$:

$$\begin{aligned}
&\gamma R\phi^{(2)}s + br^b + (\varepsilon - \theta\bar{\varepsilon})\, R\left(s - b + \phi^{(1)}s\right) - \phi^{(1)}sr \\
&\leq br^b + \varepsilon R\left(s - b + \left(\phi^{(1)} + \phi^{(2)}\right)s\right) - \left(\phi^{(1)} + \phi^{(2)}\right)sr + \left(\phi - \phi^{(1)} - \phi^{(2)}\right)(\varepsilon R - r)s. \qquad (6)
\end{aligned}$$

The left side is the total profit of borrower ε if she stores $\phi^{(2)}s$. In this case she gets the return from storage, $\gamma R\phi^{(2)}s$, defaults on the second stage lenders, but must forgo the fraction θ of the cash flows from the initial projects that she pledged as collateral, $\theta\bar{\varepsilon}R\left(s - b + \phi^{(1)}s\right)$. She also pays back the interbank loans contracted with the first stage lenders, $\phi^{(1)}sr$. If, in contrast, borrower ε invests $\phi^{(2)}s$ into new productive projects, then she earns the returns on those projects and on government bonds. The bank also takes into account that she will also be able to leverage upon the future proceeds of her second stage projects. This is indicated in the last term on the right hand side.

The features mentioned earlier about interbank trades are reflected in this incentive compatibility condition. First, the loans are made sequentially, and the projects are irreversible and observable. When the borrower finances a productive investment with the loan secured by government bonds, this investment is in place and is observed and hence used to obtain $\phi^{(2)}s$. Second, the decision whether or not to invest $\phi^{(2)}s$ into the storage technology has no impact on the first stage lenders. The reason is that, at that stage, the borrower cannot abscond and default on those lenders, as the latter have direct control over the government bonds pledged to them and can seize the unpledged cash flows from the first stage projects.[15]

The borrowing limit $\phi^{(2)}$ is determined such that the incentive compatibility condition (6) binds for

[14] Pledgeable and seizable cash flows coincide in the case of the marginal borrower, $\bar{\varepsilon}$.

[15] For the time being, we conjecture that (i) $br^b < r\phi^{(1)}s$, which means that the borrower's returns from the government bonds alone are insufficient to repay the first stage loan and (ii) that nonetheless $br^b + (\varepsilon - \theta\bar{\varepsilon})\, R\left(s - b + \phi^{(1)}s\right) > \phi^{(1)}sr$, which means that the total returns from the government bonds and the first stage projects (net of the cash flows pledged to the second stage lenders) are sufficient to repay the first stage loan in full. That is, the first stage lenders are always paid back. In Appendix 7.1, we verify that this is indeed always true in equilibrium.

$\varepsilon = \bar{\varepsilon}$, which implies (using (3) and (4)):

$$\phi^{(2)} = \frac{\theta\bar{\varepsilon}}{\gamma}\left(1 - \frac{b}{s} + \phi^{(1)}\right). \tag{7}$$

The term $\theta\bar{\varepsilon}/\gamma$ is the fraction of the project that can be pledged as collateral in subsequent funding transactions.

There is no reason for the process to stop. Indeed, once the borrower obtains $\phi^{(2)}s$, she invests into new projects, which she then can use to secure new loans, $\phi^{(3)}s$, and finance new productive projects, etc. In this way, the use of cash flows as collateral gives rise to a sequence of new collateralized loans and collateral creation inside the banking sector. This is why we refer to cash flows as "inside" collateral. Specifically, let $\phi^{(n)}s$ be the funds that the borrower can raise against the cash flows of the $(n-1)^{\text{th}}$ stage projects. Then, the corresponding n^{th} incentive constraint entails (for $n > 2$):

$$\phi^{(n)} = \frac{\theta\bar{\varepsilon}}{\gamma}\phi^{(n-1)}. \tag{8}$$

When $\theta \geq \gamma$, the collateral loss that borrower $\bar{\varepsilon}$ incurs when she stores funds exceeds the gain from storage. As a result, borrowers have no incentive to use the storage technology and the amount that they can raise, $\phi \equiv \phi^{(1)} + ... + \phi^{(+\infty)}$, ultimately goes to infinity. When $\theta < \gamma$, in contrast, the share of seizable cash flows is too small for a borrower to commit herself not to use her storage technology. In this case, lenders must limit borrowing to keep the borrower's incentives in check. Throughout the analysis we will proceed under the assumption of limited pledgeability:

Assumption 1. ***(Seizable Cash Flow)*** *$\theta < \gamma$.*

Under Assumption 1, the total amount of funding that a borrower can ultimately raise on the interbank market, ϕ, is equal to:[16]

$$\phi \equiv \phi^{(1)} + \phi^{(2)} + ... + \phi^{(+\infty)} = \left(\frac{b}{s}\frac{r^b - r}{\gamma R} + \frac{r}{\gamma R}\right)\frac{\gamma}{\gamma - \theta\bar{\varepsilon}} - 1 + \frac{b}{s}. \tag{9}$$

To be clear, ϕ is endogenous and depends on the equilibrium value of $\bar{\varepsilon}$. Further, ϕ depends on b both directly and through the influence of b on the equilibrium value of $\bar{\varepsilon}$.

Collateral Pyramid. The multiple stages of lending described above give rise to a "collateral pyramid". Figure 5 illustrates this. The foundation of the collateral pyramid (in dark colors) is made of assets created outside the financial sector, *i.e.* government bonds. The rest of the pyramid is made of collateral created inside the financial sector along the various lending stages. The size of the pyramid

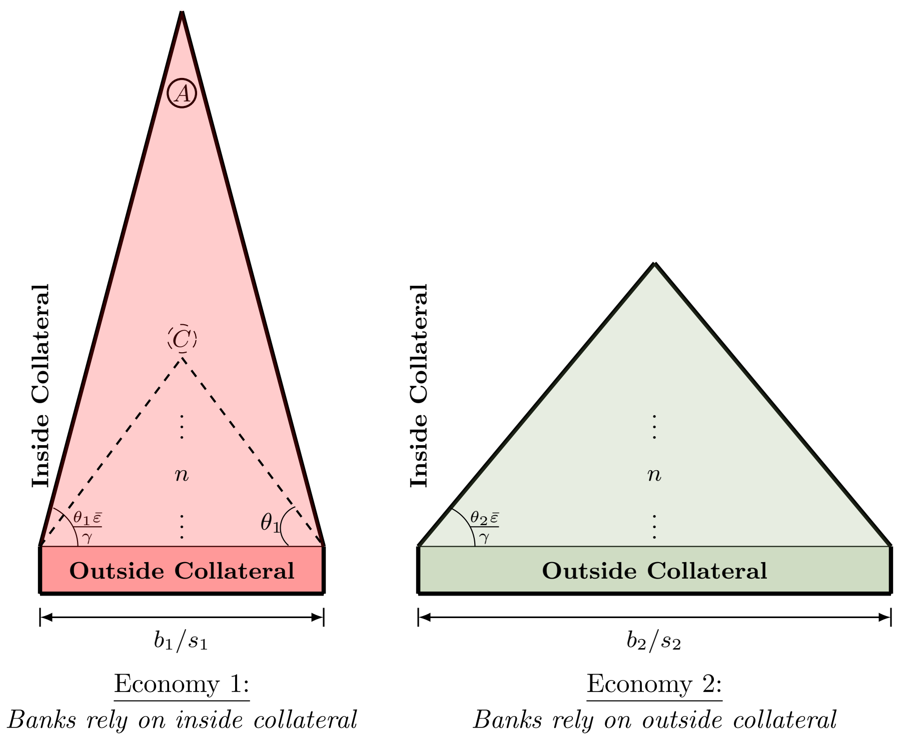
[16]Notice that the sequence of incentive compatibility constraints that emerges from the various lending stages can be re–written into one unique —but less intuitive— incentive compatibility constraint that takes into account those stages all at once. This incentive compatibility constraint writes $(\gamma - \theta\varepsilon)R(s - b + \phi s) \leq br^b + \varepsilon R(s - b + \phi s) - \phi sr$, where the term on the left hand side is the gain from storage, net of the loss of inside collateral. Indeed, the first term in parentheses, $\gamma - \theta\varepsilon$, is the unit gain from storage minus bank ε's unit loss on inside collateral. Accordingly, ϕ can also be derived from this constraint, for $\varepsilon = \bar{\varepsilon}$.

corresponds to the total volume of safe assets that can be pledged as collateral in interbank transactions. It is limited by two main factors.

First, it depends on the volume of government bonds held by banks relative to their deposits, b/s: all else equal, the larger this collateral base, the bigger the pyramid. Thus, inside collateral builds upon outside collateral.

Second, the volume of safe assets in the economy depends on the degree of pledgeability of the banks' cash flows, $\theta\bar{\varepsilon}/\gamma$. The degree of pledgeability plays a critical role. It determines how much inside collateral can be generated out of cash flows (*i.e.* a "collateral multiplier"). It has two components. There is an exogenous component, θ, which captures the technical ability of the banking sector to engineer safe assets out of cash flows. With a low θ, banks create little inside collateral; the pyramid is smaller. And there is an endogenous component, $\bar{\varepsilon}$, which captures both the quality of the borrowers and the degree of information asymmetry between borrowers and lenders. When $\bar{\varepsilon}$ is low, the distribution of borrowers is widespread, and as lenders have little information about whom they lend to, borrowers can credibly pledge only a small share of their cash flows. As we will see later, this endogenous component collapses during crises, and is the reason why the volume of inside collateral is much more fragile than that of outside collateral (our notion of financial fragility is defined in Definition 2).

Figure 5: Collateral Pyramid and Collateral Composition



The dependence of inside collateral on outside collateral stands in stark contrast with Krishnamurthy and Vissing-Jorgensen (2015), who view the two types of safe assets as substitutes. They too make a distinction between outside safe assets (treasuries) and inside safe assets (in their case, bank deposits). In their model, however, both types of safe assets are demanded by households for the non-pecuniary services those assets provide. When the supply and the yield of treasuries go down, households increase their demand for bank deposits. In contrast, in our case, safe assets are engineered and supplied by banks as a way to alleviate financing constraints in the interbank market. When the supply of treasuries goes down, banks have less outside collateral to secure wholesale funding, and counter–party fears on the interbank market rise, which reduces the pledgeability of cash flows and the creation of inside collateral. In our case, the two types of assets are therefore complements.

To recap, the first step is to form a diversified pool of cash flows. The next step is to slice the pool and to issue a safe security backed by the information insensitive slice of the pool. This funding then gives rise to new projects and cash flows, which will be later used as collateral to raise new funding, and so on.

Our model of the interbank market, which emphasizes the stages of the lending process, can be seen as a stylized representation of how the US shadow banking industry manufactured new financial products in the run–up to the 2008 financial crisis. As illustrated in Figure 2, banks have several sources of funding. They borrow through repos, which are backed by treasuries. In the model, this is captured by the first lending stage, $n = 1$. Banks also raise funding on ABCP and ABS markets, by repackaging risky loans into "safe" assets and issuing securities backed by those assets (see, *e.g.*, Coval, Jurek, and Stafford (2009) and Pozsard, Adrian, Ashcraft, and Boesky (2012)). In the model, this type of structured financing is captured by the subsequent lending stages, with $n \geq 2$.

2.3.2 Stage 1 Choices: Portfolio Choice

Ex ante banks maximize their expected gross return on assets, π^b, with respect to b. Given the optimal decisions on ϕ and $\mathbb{1}$ described above, (2) becomes:

$$\max_b \pi^b \equiv rs + b(r^b - r) + \left(1 - \frac{b}{s} + \phi\right) s \int_{\bar{\varepsilon}}^1 (\varepsilon R - r) dG(\varepsilon) \tag{10}$$

where ϕ is given by (9). The return on assets is the sum of the returns on government bonds, interbank loans, and projects, weighted by the share of those assets in banks' assets. The second and third terms reflect the excess return on governments bonds and projects over interbank loans. This optimization problem takes into account that, *ex post*, banks with $\varepsilon < \bar{\varepsilon}$ will be net lenders in the interbank market and banks with $\varepsilon \geq \bar{\varepsilon}$ will be net borrowers on the interbank market and invest into projects. Only the latter banks enjoy excess returns. Given (9), it is easy to see that the first order condition yields the no–arbitrage condition (11):

$$r^b = r \tag{11}$$

between government bonds and interbank loans. This no–arbitrage condition reflects banks' expected opportunity cost of holding government bonds. If the bank draws a low productivity *ex post* and lends funds on the interbank market, then her opportunity cost of having invested into government bonds is the return on interbank loans r. If on the contrary the bank draws a high productivity *ex post*, then her opportunity cost is the cost of borrowing on the interbank market, which is also r. Using (3), (9), and (11), we determine the limit to a bank's interbank funding:

$$\phi s = \frac{1}{\tilde{\gamma} R}\left(br^b + (s-b)(\bar{\varepsilon} - \tilde{\gamma})R\right) \tag{12}$$

where $\tilde{\gamma} \equiv \gamma - \theta\bar{\varepsilon}$. The comparison of (12) with (5) emphasizes that the cross–pledgeability of the assets, reflected in parameter θ, raises the effective share of a project that can be pledged, and reduces the cost of moral hazard to $-\tilde{\gamma}(s-b)R$. One can also re-arrange (12) as $\omega + \phi s = \bar{\varepsilon}(\omega + b)/(\gamma - \theta\bar{\varepsilon})$, which makes it clear that the total amount of lending by a high–ε bank (left hand side) depends on the equilibrium value of $\bar{\varepsilon}$. Equilibria with high values of $\bar{\varepsilon}$ entail a high volume of lending by the most productive banks, and thus feature an efficient allocation of the initial endowment ω.

Importantly, at the bank level, the demand for interbank loans monotonically increases with the productivity of the marginal borrower, $\bar{\varepsilon}$, and therefore (from (3)) with the interbank rate, r. Indeed, as r goes up, the borrowers with the lowest εs switch to lending. The banks remaining on the borrowers' side have higher εs, and less incentives to use the storage technology. This mitigates the asymmetric information problem, which in turn improves the pledgeability of borrowers' cash flows, and allows lenders to raise the borrowing limit, ϕ. We discuss the consequences of individual loan demands increasing with the interest rate later, when we derive the equilibrium of the interbank market.

2.3.3 Recap

Putting together the stage 2 borrowing constraints and the stage 1 portfolio choice of the banks leads to a set of conditions that must hold in *any* interbank market equilibrium. These conditions are stated in Lemma 1 given r, which we determine later, in the next section.

Lemma 1. *Given the interbank market rate r,*

- *The bank with productivity $\bar{\varepsilon}$ is indifferent between borrowing and lending, with $\bar{\varepsilon} = r/R$;*
- *An interior solution to bank optimization implies $r^b = r$;*
- *The total individual volume of interbank loans is $\phi = \bar{\varepsilon}/(\gamma - \theta\bar{\varepsilon}) - 1 + b/s$, of which:*
  - *$\phi^{(1)} = \bar{\varepsilon}/\gamma - 1 + b/s$ is backed by outside collateral;*
  - *$\phi - \phi^{(1)} = \theta\bar{\varepsilon}^2/\gamma(\gamma - \theta\bar{\varepsilon})$ is backed by inside collateral.*

Proof. The results follow directly from (3), (5), (11), and (12). □

3 Equilibrium Conditions

This section describes the set of equilibria. A sunspot variable is included in the construction of an equilibrium as there may be multiple equilibria in the interbank market. The sunspot only serves as an indicator of expectations; no significant economic decisions are taken prior to its realization.

3.1 Timing

The timing of the interactions between households, banks, and the government is as follows: (1) the household deposits s; (2) the sunspot determining beliefs and expectations is revealed; (3) banks allocate deposits across government bonds, b, and future investments, ω; (4) banks learn about the profitability of their projects, ε; and (5) banks decide to make either interbank loans, to invest into their projects, or store goods.

3.2 Market Clearing Conditions

There are four markets: government bonds, bank deposits, interbank loans, and consumption goods. We solve for the equilibria, given savings s and a stock of bonds $\mathbf{b}$. The interbank market equilibrium will determine r, and therefore (through (3)) $\bar{\varepsilon}$. Once this is known, the other components of the equilibria can be determined. The key to financial fragility comes from the multiple values of $\bar{\varepsilon}$ solving the interbank market clearing condition.

Government Bond Market. Given the supply of government bonds, $\mathbf{b}$, and the no–arbitrage condition (11), the market clears when banks' demand for government bonds is:

$$b = \mathbf{b}. \tag{13}$$

By our timing, this decision is made after the realization of the sunspot and hence given the rate of return on interbank loans and government debt. Given that $\mathbf{b} = \mathbf{g}$, the total amount of resources that banks invest into their projects, or store, in equilibrium is equal to $s - \mathbf{b} = \omega$, and is independent of $\mathbf{b}$.[17]

Bank Deposit Market. Banks are identical *ex ante*, compete for depositors, and break even *ex post*. In equilibrium, the return on bank deposits is given by (using (10), (11) and (12)):

$$r^d \equiv \frac{\pi^b}{s} = r\left(1 + \frac{1}{\gamma - \theta\bar{\varepsilon}} \int_{\bar{\varepsilon}}^{1} (\varepsilon - \bar{\varepsilon}) dG(\varepsilon)\right) > r. \tag{14}$$

Under Assumption 1, the deposit rate is larger than the interbank loan rate and the return on government bonds. The spread reflects two features of the model: (i) the value of government bonds

[17] Indeed, recall that government spending are not thrown away, but rebated lump sum to households.

as collateral and (ii) the value for a bank of being able to borrow on the interbank market *ex post*.[18] Both of these augmentations to the return on government debt arise in the event of high realizations of $\bar{\varepsilon}$. As demonstrated below, the spread responds to the level of debt, $\mathbf{b}$, through $\bar{\varepsilon}$.

Interestingly, if $\bar{\varepsilon} = 1$, which is the efficient allocation where all funds flows to the highest productivity bank, then $r^d = r = r^b$. In this case, there is no spread, and household consumption is independent of $\mathbf{b}$ (see (1)). As noted earlier, the frictions in the financial markets create a differential in returns and thus Ricardian equivalence does not hold.

Interbank Market. In equilibrium, banks with $\varepsilon \geq \bar{\varepsilon}$ demand funds ϕs. So the total demand for loans is $(1 - G(\bar{\varepsilon}))\phi(\omega + \mathbf{b})$ as $s = \omega + \mathbf{b}$. If $\bar{\varepsilon} > \gamma$, banks with $\varepsilon < \bar{\varepsilon}$ lend their idle resources, ω, in the interbank market, so that total supply is $G(\bar{\varepsilon})\omega$. When $\bar{\varepsilon} = \gamma$, banks are indifferent between lending to other banks storage, and so supply up to $G(\bar{\varepsilon})\omega$ on the interbank loan market. Notice that, in equilibrium, lenders must be indifferent as to whom they lend to, and when; so, the various stages of lending described in Section 2.3.1 are fully integrated. It follows that the market clearing condition is:

$$(1 - G(\bar{\varepsilon}))\phi(\omega + \mathbf{b}) \begin{cases} = G(\bar{\varepsilon})\omega & \text{if} \quad \bar{\varepsilon} > \gamma \qquad (a) \\ \\ \in [0, G(\bar{\varepsilon})\,\omega] & \text{if} \quad \bar{\varepsilon} = \gamma \qquad (b) \end{cases} \tag{15}$$

Returning to Figure 4, the interbank market flows are seen as the red line indicating the flows from low to high $\bar{\varepsilon}$ banks. Using (12) and (13), we re–write relation (15) as:

$$\underbrace{(1 - G(\bar{\varepsilon}))\frac{\bar{\varepsilon}}{\gamma - \theta\bar{\varepsilon}}}_{H(\bar{\varepsilon};\theta)}(\omega + \mathbf{b}) \begin{cases} = \omega & \text{if} \quad \bar{\varepsilon} > \gamma \qquad (a) \\ \\ \in [0, \omega] & \text{if} \quad \bar{\varepsilon} = \gamma \qquad (b) \end{cases} \tag{16}$$

This version of the interbank market clearing condition highlights the flow of funds into productive projects. The right side of this expression is banks' remaining resources after they purchased the government bonds, ω. As shown in Figure 4, the transfer of $\mathbf{g}$ to the households from the government is, in equilibrium, used by banks to purchase the government debt, $\mathbf{b}$. Public spending, therefore, does not affect the resources available for productive projects, ω. The left side of (16) is the total amount of resources actually invested into those projects. It highlights that only high ε banks ultimately invest. Total investment is thus the product of the mass of high productivity banks that invest, $1 - G(\bar{\varepsilon})$, and the size of each productive bank's project. In normal times, all resources ω are used productively, and total investment is equal to ω and independent of $\mathbf{b}$. That is, the volume of government spending does not affect investment. However, it affects the critical bank productivity ($\bar{\varepsilon}$) and thus the volume of lending through the role of government debt as collateral. The more government bonds, the more

[18] We are grateful to a referee for pointing out that the spread exists even if $\mathbf{b} = 0$, reflecting the option value of borrowing even if there is no outside collateral.

collateral available to banks, the better the functioning of the interbank market, and the better the allocation of funds toward the high–productivity banks.

Relation (16) is a convenient representation of the market clearing condition, because on its left side only the term $H(\bar{\varepsilon};\theta)$ depends on $\bar{\varepsilon}$. Intuitively, the term $H(\bar{\varepsilon};\theta)(\omega+\mathbf{b})$ is the funding need per bank, while ω on the right side of (16) is the supply of funding per bank. As we will see later, the richness of the model comes from the non–monotonicity — *i.e.* the hump–shape— of $H(\bar{\varepsilon};\theta)$. As $\bar{\varepsilon}$ increases, the fraction of banks who borrow falls, and their average quality goes up. Since higher quality banks have less incentives to abscond, the amount they can borrow increases. That is, from (12), ϕ is increasing in $\bar{\varepsilon}$. The non–monotonicity thus comes from the interaction of these extensive and intensive margins. Importantly, given $\bar{\varepsilon}$, the demand addressed to a given lender increases with the volume of collateral, that is, with the government bonds (outside collateral), $\mathbf{b}$, and pledgeable cash flows, θ (inside collateral).

Consumption Good Market. The consumption good market clears as aggregate output, y, is entirely consumed, which yields:

$$c = y \tag{17}$$

where aggregate output is given by:[19]

$$y = \begin{cases} \omega R \int_{\bar{\varepsilon}}^{1} \varepsilon \frac{\mathrm{d}G(\varepsilon)}{1-G(\bar{\varepsilon})} & \text{if} \quad \bar{\varepsilon} > \gamma \qquad (a) \\ \\ \frac{\omega+\mathbf{b}}{1-\theta} R \int_{\gamma}^{1} (\varepsilon-\gamma)\mathrm{d}G(\varepsilon) + \gamma R \omega & \text{if} \quad \bar{\varepsilon} = \gamma \qquad (b). \end{cases} \tag{18}$$

When $\bar{\varepsilon} > \gamma$, aggregate output is fully characterized by —and increases monotonically with— ω and $\bar{\varepsilon}$. The former is the amount of resources available for private investment, *i.e.* the size of the projects; the latter determines how those resources are allocated among banks, *i.e.* the overall productivity of the projects. When $\bar{\varepsilon} = \gamma$, aggregate output also depends on the fraction of ω that is invested in storage.

3.3 Equilibria

We discuss the types of equilibria that might arise and then give the conditions for their existence and co–existence. We will show that there are potentially multiple equilibria due to the asymmetric information and moral hazard of banks. For this discussion, we consider restrictions on $G(\cdot)$ so that those two frictions are meaningful:[20]

[19] *I.e.*, $y = \int_{\gamma}^{1} \varepsilon R\left(\omega + \phi(\omega+\mathbf{b})\right)\mathrm{d}G(\varepsilon) + \gamma R\left(\omega G(\gamma) - (1-G(\gamma))\phi(\omega+\mathbf{b})\right)$, where the first term is the output from the productive projects and the second term is the output from storage.

[20] These (mild) conditions are sufficient but not necessary. For example, all our results go through in the case of a uniform distribution —with $G(\bar{\varepsilon}) = \bar{\varepsilon}$, although for this distribution the second condition is satisfied only if $\gamma \leq 1/2$. To get the intuition behind Assumption 2, consider an extreme, trivial economy where those conditions are breached in that all banks draw ε below γ, so that $\gamma G'(\gamma)/(1-G(\gamma)) = +\infty$. Then, in such an economy, storage would be the best available technology, there would be no need for reallocation, and therefore no role for an interbank market.

Assumption 2. ***(Initial Mis–allocation)*** *$G''(\bar{\varepsilon}) \geq 0$, $\frac{\gamma G'(\gamma)}{1-G(\gamma)} \leq 1$, and $\frac{G'(1^-)}{1-G(1^-)} = +\infty$.*

Assumption 2 is a condition on the distribution of the εs. We require that there are enough high-ε banks in the economy for an interbank market to emerge or, put differently, that deposits are initially mis–allocated across banks. Under this assumption, $H(\bar{\varepsilon};\theta)$ is hump–shaped and reaches an interior maximum at $\bar{\varepsilon}^{\max}$, with:[21]

$$\bar{\varepsilon}^{\max} \equiv \arg\max_{\bar{\varepsilon}\in(\gamma,1)} H(\bar{\varepsilon};\theta). \tag{19}$$

The hump shape of $H(\bar{\varepsilon};\theta)$ reflects the ambiguous effect of $\bar{\varepsilon}$ on loan demand, which decreases with $\bar{\varepsilon}$ on the extensive margin but increases with $\bar{\varepsilon}$ on the intensive margin. As explained earlier, this latter unusual effect is due to the fact that borrowers' average quality goes up with $\bar{\varepsilon}$. The negative intensive margin effect dominates when $1 - G(\bar{\varepsilon})$ is large enough, that is, when $\bar{\varepsilon}$ is low. In this case, the loan demand curve bends backward.

Definition 1. ***(Equilibrium)*** *An equilibrium is a vector of returns (r, r^d, r^b), a cut–off level of ε, denoted $\bar{\varepsilon}$, government bond holdings, b, and a borrowing limit, ϕ, such that*

- *Ex ante, banks choose b to maximize their expected return on deposits;*
- *Ex post, banks with*
  - *$\varepsilon \geq \bar{\varepsilon}$ borrow in the interbank market up to the limit ϕ;*
  - *$\varepsilon < \bar{\varepsilon}$ either lend in the interbank market or store goods;*
- *ϕ is determined so that banks with $\varepsilon \geq \bar{\varepsilon}$ invest in productive projects rather than abscond;*
- *Beliefs about the distribution of types of banks borrowing in the interbank market are consistent with the equilibrium outcome;*
- *Markets for interbank loans, deposits and government bonds clear.*

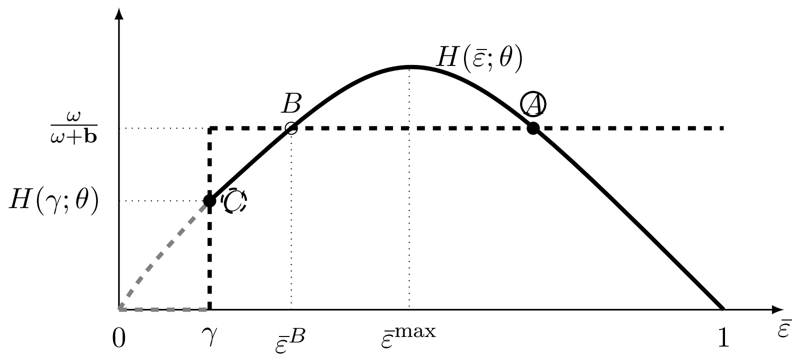
3.3.1 Normal Times

We first study outcomes which are solutions to (16a). These solutions become the basis for a subset of the equilibria. The right side of (16a) is predetermined and constant. Under Assumption 2, the left side is hump shaped —the exact shape will depend on the assumed form for $G(\cdot)$ as well as on parameters γ and θ. As a consequence, a solution $\bar{\varepsilon} > \gamma$ to (16a) does not always exist. Equilibria in the interbank loan market are shown in Figure 6. Since banks always have the outside option to get a return γR by storing goods, only the values $\bar{\varepsilon} \geq \gamma$ are relevant. The figure illustrates one solution to (16a) at point A (see also point A in Figure 5). This equilibrium determines a value of $\bar{\varepsilon} > \gamma$ such that banks with $\varepsilon \geq \bar{\varepsilon}$ borrow on the interbank market, and will be referred to as "normal times". In this type of equilibrium, no bank uses the storage technology, and the household endowment ω is entirely invested

[21] For a proof that $H(\bar{\varepsilon};\theta)$ is hump–shaped, see Appendix 7.2.

in productive projects. The volume of corporate loans does not depend on $\bar{\varepsilon}$, but the quality of those loans does. From the value of $\bar{\varepsilon}$, we can construct an equilibrium as follows. Use (3) to determine the interbank rate: $r = \bar{\varepsilon}R$. From the no-arbitrage condition, (11), $r^b = r$. Relations (9) and (12) yield $\phi^{(1)}$ and ϕ given $\bar{\varepsilon}$.

Figure 6: Multiple Equilibria in the Interbank Market



Since $H(\bar{\varepsilon};\theta)$ is hump–shaped, another interior solution (point B) can exist. Relative to A, the value of $\bar{\varepsilon}$ is lower, there are more banks borrowing but their borrowing constraint is tighter, loan quality is lower, and the intermediation process is less efficient. In the analysis that follows, we focus on the equilibrium at A as it is locally stable and thus has "reasonable" comparative statics. But, as we shall see, there is another equilibrium outcome other than B that is of interest.

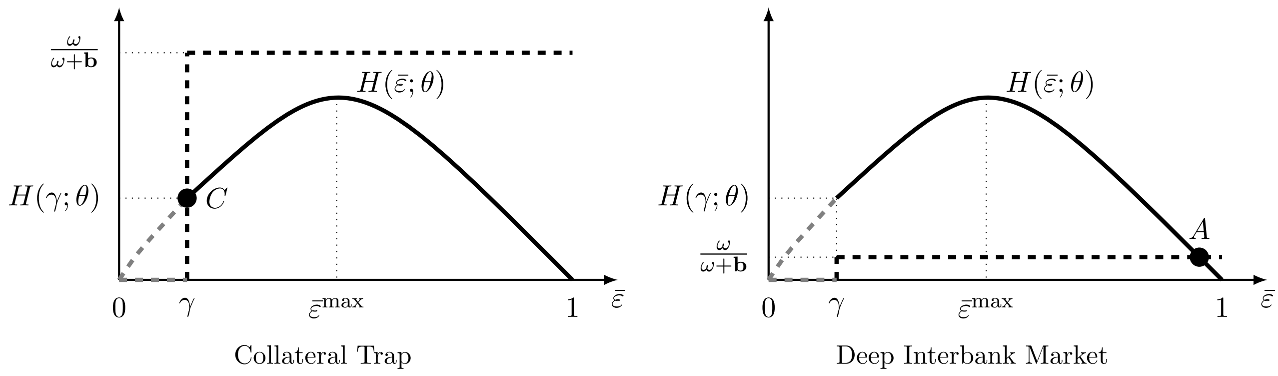
3.3.2 Crisis Times and the Collateral Trap

Another equilibrium, which we term a "collateral trap", can arise as a solution to (16b) with $\bar{\varepsilon} = \gamma$. This is a crisis state. There is an excess supply of loans on the interbank market, some banks inefficiently use the storage technology, and aggregate productivity and output are low. In Figure 6, this equilibrium is represented by point C (see also point C in Figure 5). At this point, the banks with $\varepsilon \leq \gamma$ are indifferent between lending to other banks and storing goods, and they are the only banks that supply interbank loans. The supply of interbank loans is infinitely elastic, as illustrated by the vertical dashed line. In a collateral trap, banks engineer little inside collateral, and the borrowing constraint holds tight: productive banks, who cannot distinguish themselves from other banks, are unable to commit themselves to undertake the productive projects, and therefore cannot raise more funding. Despite the low interbank rate, the demand for loans remains below supply, there is an excess supply of interbank funding, and the interbank market clears by having low productivity banks use the storage technology. This equilibrium exists if and only if (using (16b)):

$$H(\gamma;\theta) < \frac{\omega}{\omega + \mathbf{b}} \tag{20}$$

Though some banks store goods, interbank borrowing is still possible, *i.e.* $\phi > 0$, because banks can pledge as collateral both their government bonds as well as the cash flows of the projects that

Figure 7: Unique Equilibrium Configurations



are operated.[22] However, the volume of collateral is reduced compared to normal times, and so is the level of interbank activity. On the one hand, banks are limited in the creation of safe assets, because counter–party fears on the interbank market reduce the degree of pledgeability of the cash flows. On the other hand, counter–party fears are high because banks have little safe assets to pledge as collateral. Hence the collateral trap. In Figure 6, the collateral trap equilibrium in C co-exists with the normal times equilibrium in A. In this sense, the interbank market is susceptible to swings in confidence. It is natural to associate outcomes like A with optimism about the quality of borrowing banks, with high returns on interbank loans and relaxed borrowing restrictions. It is also natural to think of crises, like C, as the outcome of pessimism.

A and C are self–fulfilling equilibria. Assume for example that banks are pessimistic: they believe that unproductive banks demand interbank funding. Since unproductive banks are prone to absconding, lenders require borrowers to put more "skin in the game", and reduce ϕ. As every borrower demands less funding, the equilibrium interbank loan rate goes down, and the net present value of the unproductive banks' projects increases. Hence, unproductive banks do indeed demand interbank funding.

3.3.3 Dependence of a Collateral Trap on the Amount of Government Debt

This section highlights the dependence of the set of equilibria on the level of debt. Not all economies possess multiple equilibria. Other possible outcomes are illustrated in Figure 7, which shows cases of unique equilibria. In panel (a), there are no solutions to (16a), and there is a collateral trap equilibrium. In panel (b), by contrast, there is a solution to (16a) but not to (16b); in this case the normal times equilibrium is the only possible outcome. To describe the conditions of existence of a collateral trap and normal times equilibria, it is useful to define two critical levels of government debt, $\overline{b}(\theta)$ and $\underline{b}(\theta)$, as:

$$\frac{\overline{b}(\theta)}{\omega} = \frac{1 - H(\gamma;\theta)}{H(\gamma;\theta)} \tag{21}$$

[22] If $\mathbf{b} = 0$ (no outside collateral) and $\theta = 0$ (no inside collateral) then in a collateral trap $\phi = 0$ and there is no demand for interbank loans.

and

$$\frac{\underline{b}(\theta)}{\omega} = \frac{1 - H\left(\bar{\varepsilon}^{\max}; \theta\right)}{H\left(\bar{\varepsilon}^{\max}; \theta\right)} \tag{22}$$

with $\underline{b}(\theta) < \bar{b}(\theta)$. There are three regions for the level of public debt. In the upper dominance region, with $\mathbf{b} > \bar{b}(\theta)$, banks can use so much public debt as collateral that crises are ruled out, irrespective of banks' beliefs about borrowers' quality in the interbank market. Then there is a lower dominance region, with $\mathbf{b} < \underline{b}(\theta)$, where instead there is a shortfall of pledgable assets in the economy. The government did not issue enough debt to avoid the collateral trap. Finally, there is an intermediate region, where beliefs matter and sunspot equilibria arise; in this case, crises are self–fulfilling. Proposition 1 formalizes this characterization.

Proposition 1. ***(Equilibrium Regions)***

- *When $\boldsymbol{b} < \underline{b}(\theta)$, there is a unique collateral trap equilibrium, with $\bar{\varepsilon} = \gamma$;*
- *When $\boldsymbol{b} > \bar{b}(\theta)$, there is a unique normal times equilibrium, with $\bar{\varepsilon} > \bar{\varepsilon}^{max}$;*
- *When $\boldsymbol{b} \in [\underline{b}(\theta), \bar{b}(\theta)]$, there are sunspot equilibria, with either $\bar{\varepsilon} > \bar{\varepsilon}^{max}$ or $\bar{\varepsilon} = \gamma$.*

Proof. The results follow directly from the definition of $\bar{b}(\theta)$ and $\underline{b}(\theta)$ in (21) and (22). □

This proposition makes clear that the set of equilibria depend on the amount of government debt. Importantly, in normal times $\bar{\varepsilon}$ increases with $\mathbf{b}$. An increase in $\bar{\varepsilon}$ leads to an increase in ϕ ((12)) and in the spread between the deposit rate and the return on government debt is larger (from (11) and (14)). This is because government debt has more value as collateral when banks are more productive.

It is also easy to see that $H(\bar{\varepsilon}; \theta)$ increases with θ and the boundaries $\underline{b}(\theta)$ and $\bar{b}(\theta)$ both decrease with θ, which reflects the substitutability between inside and outside collateral noted earlier. When the economy is able to create safe assets on its own (*i.e.* θ is high), then the normal times equilibrium may arise even if banks hold few treasuries.

3.4 Comparing Outcomes: Collateral Traps versus Normal Times

One key feature of collateral traps is the fall in the returns on safe assets, r^b and r. The government bond yield goes down because, when collateral is scarce, the shadow (collateral) value those bonds increases. By arbitrage, the fall in the government bond yield is accompanied with a similar fall in the return of interbank loans, which in our model are as safe as government bonds.[23] This has important implications. First, at a lower r, a larger fraction of banks borrow in the interbank market, so the productivity cut–off $\bar{\varepsilon}$ goes down. Lower productivity banks that would lend to higher productivity banks during normal times actually borrow in a collateral trap. The aggregate demand for interbank

[23]Indeed, there is no risky asset traded, and no default, in equilibrium. Our model is therefore not equipped to account for the rise in interest rate spreads and yields of risky assets observed during the recent crisis. Instead, we focus on the variations in the return on safe assets and in the volumes of wholesale market transactions before and during the crisis, that we described in Section 1 (see also our discussion in Section 5).

loans increases on the extensive margin. Second, the fact that even low productivity banks demand funding augments the amplifies the asymmetry of information, raises counter-party fears, and therefore reduces the degree of pledgeability of cash flows. Hence, in a collateral trap, the collateral multiplier is low. There is also a collateral composition effect. Specifically, the volume of inside collateral collapses and the share of outside collateral in total collateral goes up. Essentially, the limit on the borrowing backed by outside collateral, $\phi^{(1)}$, is less sensitive than that on the borrowing backed by the cash flow from projects, $\phi - \phi^{(1)}$, because the latter is driven by the variation in the collateral multiplier. With lower quality banks borrowing against less collateral, banks' borrowing limits fall to avoid moral hazard. Countering the change on the extensive margin, there is ultimately a reduction in aggregate borrowing. In a collateral trap there is less activity in the interbank market, and some banks use the storage technology, which reduces aggregate productivity.

This fall in productivity has not only an overall negative real effect on the *levels* of output and consumption, but also a positive effect on the cross-sectional *dispersion* of banks' returns. The output and productivity implications are consistent with the evidence in Foster, Grim, and Haltiwanger (2013). To the extent the Great Recession contained more of a financial collapse than other recessions, the implications of our model, which attributes recessions to the collapse of the interbank market, are relevant.

Proposition 2. ***(Features of a Collateral Trap)*** *Relative to normal times, and all else equal, in a collateral trap:*

1. *The cut-off bank productivity, $\bar{\varepsilon}$, is lower;*
2. *The interbank loan rate, r, is lower;*
3. *The spread between the deposit rate and the government bond yield, $r^d - r^b$, (and therefore the shadow value of government bonds as collateral) is higher;*
4. *The spread between the government bond yields and the interbank loan rate, $r^b - r$, remains null;*
5. *Collateral becomes scarce and interbank market activity is lower:*
   a. *The volume of interbank transactions secured by outside collateral, $\phi^{(1)}$, is lower;*
   b. *The share of interbank transactions secured by inside collateral, $(\phi - \phi^{(1)})/\phi$, is lower;*
6. *Banks store goods, and not all deposits (net of government bonds) are used to finance productive projects;*
7. *Output and consumption fall;*
8. *The cross–sectional dispersion of bank productivity (the εs) is higher, and there is more information asymmetry.*

Proof. See Appendix 7.3. □

4 The Collateral Composition Channel

In this section we discuss the connection between the composition of banks' collateral and market fragility. Our aim is to show that the financial sector is more fragile when it relies on inside collateral than when it relies on outside collateral, given a total amount of collateral. Define financial fragility as follows:

Definition 2. ***(Financial Fragility)*** *A normal times equilibrium is fragile whenever the crisis times equilibrium co–exists with it. Financial fragility is measured by the distance between banks' actual holdings of government bonds, $\mathbf{b}$, and the upper bound of the multiple equilibrium region, $\overline{b}(\theta)$; the smaller this distance, the more fragile the economy.*

By Definition 2, a normal times equilibrium is fragile whenever, for given economic fundamentals, the mere swing in beliefs from optimism to pessimism triggers a crisis. Using this definition, equilibrium A in Figure 6 is fragile, whereas equilibrium A in Figure 7 is not.

Based on Definition 2, we also measure financial fragility as the distance between banks' actual holdings of government bonds, $\mathbf{b}$, and the upper bound of the multiple equilibrium region, $\overline{b}(\theta)$. This distance is negative when normal times are fragile, and increases with θ.

To isolate our collateral *composition* channel, we construct two economies, indexed by $i = 1, 2$, with $\theta_1 > \theta_2$, and such that the excess creation of inside collateral by banks in economy 1 exactly compensates for a lower amount of treasuries, relative to economy 2, $\mathbf{b}_1 < \mathbf{b}_2$ (see Appendix 7.4). The total volume and value of banks' collateral are the same in the two economies, but inside collateral represents a larger share of total collateral in economy 1 than in economy 2. Importantly, economies 1 and 2 are otherwise identical. The amount of resources available for private projects is the same: $\omega_1 = \omega_2 = \omega$. And we consider a normal times equilibrium, where $\bar{\varepsilon}$, r, r^d, r^b, productive investment, consumption, and output, are the same in the two economies.[24] We compare those two economies, as to the fragility of the normal times equilibrium.

Proposition 3. ***(Collateral Composition and Crisis Intensity)*** *All else equal, and conditional on the total amount of collateral, output and consumption collapse more during a crisis in economies that rely more on inside collateral.*

Proof. See Appendix 7.4. □

Proposition 3 emphasizes the effect of the composition of banks' collateral on the intensity of financial crises. Starting from the same normal times equilibrium, output and consumption fall by more in economy 1 than in economy 2 during a crisis. Importantly, these effects are independent of the standard asset price channel. Indeed, in the crisis the interest rate (or the collateral value) falls from

[24] Figure 5 illustrates the only difference between those economies: the collateral pyramid is taller but thinner in economy 1, compared to that in economy 2.

r^b to γR in both economies. During the crisis the collateral multipliers, $\theta_1\bar{\varepsilon}/\gamma$ and $\theta_2\bar{\varepsilon}/\gamma$, fall to θ_1 and θ_2, respectively. The creation of inside safe assets by banks recedes in both economies. But those variations are more relevant in economy 1, where a larger share of interbank transactions are secured by this type of assets. Hence the deeper recession in economy 1.

Proposition 4. ***(Collateral Composition and Financial Fragility)*** *All else equal, and conditional on the total amount of collateral, the financial sector is more fragile in economies that rely more on inside collateral.*

Proof. See Appendix 7.5. □

Proposition 4 means that economy 1 is more financially fragile —in the sense of Definition 2— than economy 2. In particular, a crisis equilibrium may co–exist with the normal times equilibrium in economy 1 when it does not exist in economy 2. This result is due to the endogenous nature of the safe asset creation process by banks, and to the strategic complementarities this process exacerbates. Banks create safe assets out of their projects, and finance new projects out of those safe assets. Through those complementarities, economy 1 is more exposed to swings in beliefs and panics. Economy 2 is less fragile because the volume of treasuries that banks use to secure their loans is independent from the productive projects' cash flows and other banks' beliefs about those cash flows. Importantly, this result is unrelated to the safety of the assets pledged as collateral. Indeed, in our model there is no risky asset in equilibrium and inside and outside assets are equally safe (see footnote 23).

5 Back to the Facts

We use our model to understand the link between the observed increase in savings and shortage of outside collateral (Fact #1) and the evolution in interbank market flows before and during the 2007–8 crisis (Fact #2). The model is structured around "normal" and "crisis" times. With this in mind, the pre-crisis period, roughly from 2003 to mid-2007, is viewed as "normal" times in the context of our model.[25] The crisis begins at that point.

Pre-Crisis. The rapid increase in bank deposits and the large demand for safe assets, notably for US treasuries, by the rest of the world (Figure 1) has been put forward as one of the key structural factors behind the crisis (Caballero and Krishnamurthy (2009)). Following these developments, the treasuries–to–deposit ratio of the US financial sector was in 2008 less than half of what it was in 2003, and the volume of US treasuries held by the US financial sector fell to a historical low. At the same time, the volume of loans secured by inside collateral rose to historical highs (Figure 2). One interpretation of

[25] "Normal times" here refer to a situation that looks normal in terms of equilibrium outcomes; not necessarily in terms of financial imbalances. In fact, as we discuss in this section, the pre–crisis period looked normal, while the substitution of inside collateral for outside collateral was sowing the seeds for a collateral trap. Similar observations have been made, for example, by Borio (2008).

those events is that, in response to the rest of the world's thirst for US safe assets, US banks engineered collateralized debt as substitutes to treasuries (Caballero (2009)).

From the perspective of our model, the increase in bank deposits is seen as an increase in ω and the fall in treasuries as a decrease in $\mathbf{b}$ (see Fact #1 and Figure 1). The rise in ABCP markets is seen as $\phi-\phi^{(1)}$ going up (Figure 2). Our interpretation of those events is that, by reducing US banks' collateral base $\mathbf{b}/(\omega+\mathbf{b})$, the crowding of US banks out of treasuries by the rest of the world made the US financial system more reliant on inside collateral, and more fragile (Proposition 4). The concomitant increase in deposits enabled banks to increase the volume of resources available to lending, ω, to generate more cash flows and create more inside collateral. These dynamics resulted in a change in the composition of banks' collateral, with an increase in inside collateral relative to outside collateral. From a financial stability perspective, this change is important, as wholesale funds secured by cash flows are sensitive to swings in beliefs. Based on our model, we argue that the reduction in the volume of US treasuries held by US banks drove the economy from the upper dominance region (with $\mathbf{b} > \bar{b}(\theta)$) into the multiple equilibrium or the collateral trap region (with $\mathbf{b} \leq \bar{b}(\theta)$); hence the crisis. Our result in Propositions 2 (item 7) and 3 is in line with the fact that the sudden stop in financial engineering was accompanied with an unusually severe recession.

Crisis: Falling into the Collateral Trap. On this basis, we interpret events post-2008 as a fall in confidence of lenders in the interbank market, made possible by the change in collateral composition. In our model, a crisis is defined by a dramatic fall in the productivity cut-off, $\bar{\varepsilon}$, which determines the borrowers and lenders in the interbank market. The implications of this switch from optimistic to pessimism are summarized in Proposition 2. During the crisis, the interbank loan rate, r, fell abruptly, consistent with Figure 3. Our model also predicts that the number of borrowers increases during a crisis. Due to the lack of data, such a prediction is difficult to verify but seems nonetheless consistent with the data described in Bräuning and Fecht (2017). Using a daily panel of overnight loans between German banks, they indeed find that the number of borrowing banks increased by around 10% during the crisis —both in absolute value and per lender, and that the change was statistically significant (Bräuning and Fecht (2017), Table 1, page 43). During the crisis, repo activity overall receded relatively little, to stabilize at its 2003 level, whereas ABS–CDO activity fell much more, by over 80% from its 2007 peak (Fact #2). We interpret those relative flows as a reduction in $(\phi - \phi^{(1)})/\phi$, as in Proposition 2 (item 5b).

6 Conclusion

In this paper, we explore how banks' holdings of outside collateral (like treasuries) affect banks' production of inside collateral, and how this collateral creation process affects the stability of the economy. We characterize a collateral *composition* channel.

Due to the presence of moral hazard and adverse selection, loans between heterogeneous banks have two main features. First, these loans are collateralized, either by treasuries (outside collateral) and/or cash flows (inside collateral).[26] Second, limits emerge endogenously on the amount banks can borrow in this market. Confidence is central to the functioning of the interbank market. When lenders are optimistic about the quality (productivity) of borrowing banks, quantity restrictions are relatively lax and interbank loan rates are high. In equilibrium, only high productivity banks borrow in the market and the reallocation of deposits across banks is relatively efficient.

But, for the same parameters, the economy may be stuck in a collateral trap. In this case, lenders are pessimistic about bank quality. The volume of inside collateral is low, borrowing restrictions are tight, and lending rates are low. Relatively low productivity banks borrow so that the interbank market is much less efficient in the reallocation of deposits. The reduction in interbank flows is more pronounced in loans backed by inside collateral than in loans backed by outside collateral. The disruption of financial reallocation in a collateral trap has real effects as well. The most profitable banks are unable to attract the flow of funds they would receive in normal times. The aggregate economy is less productive: output and consumption are lower.

Though the model is purposefully simple, structured to capture the interactions of inside and outside collateral, it is capable of matching some key features of the recent financial crisis. This includes the fall in the flow of funds financed by inside collateral relative to outside collateral, and the consequent fall in output. The model also uncovers a collateral *composition* transmission channel that is distinct from the traditional collateral value channel. During a crisis, the fall in output is indeed larger when the banking sector secures interbank transactions with inside collateral.

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[26] To be clear, in our model we abstracted from sovereign debt risk and considered treasuries as being safe assets.

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7 Appendix

7.1 Proof that, in Equilibrium, (i) $br^b < r\phi^{(1)}s$ and (ii) $br^b + (\varepsilon - \theta\bar{\varepsilon})\, R\left(s - b + \phi^{(1)}s\right) > \phi^{(1)}sr$

The inequality (i) is straightforward from (3), (5), and (11), which also imply:

$$br^b + (\varepsilon - \theta\bar{\varepsilon})R\left(s - b + \phi^{(1)}s\right) \geq br^b + (1-\theta)\bar{\varepsilon}R\left(s - b + \phi^{(1)}s\right) > \phi^{(1)}sr \Leftrightarrow 1 - \frac{\mathbf{b}}{s} + \phi^{(1)} < \frac{1}{\theta}.$$

Using Lemma 1 we re–write the above inequality as $\bar{\varepsilon} < \gamma/\theta$, which is always true under assumption 1. Hence inequality (ii) is satisfied in the equilibrium.

7.2 Proof that $H(\bar{\varepsilon})$ is Hump-shaped over Interval $[\gamma, 1]$

From the definition of $H(\bar{\varepsilon};\theta)$ in (16), one gets:

$$H'(\bar{\varepsilon};\theta) > 0 \Leftrightarrow \chi(\bar{\varepsilon}) \equiv \frac{1}{\bar{\varepsilon}}\left(1 - \frac{1 - G(\bar{\varepsilon})}{\bar{\varepsilon}G'(\bar{\varepsilon})}\right) < \frac{\theta}{\gamma}. \tag{23}$$

Under Assumption 2, it is easy to see that $H'(\gamma;\theta) > 0$ and $H'(1^-;\theta) < 0$. Therefore, $H(\bar{\varepsilon};\theta)$ reaches at least one maximum over interval $[\gamma, 1]$. It remains to show that this maximum is unique, *i.e.* that

$\chi'(\bar{\varepsilon}) > 0$ $\forall \bar{\varepsilon} \in [\gamma, 1]$: $\chi'(\bar{\varepsilon}) > 0 \Leftrightarrow 2G'(\bar{\varepsilon}) + \bar{\varepsilon}G''(\bar{\varepsilon}) > 0$, which is always true under Assumption 2. Hence the $H(\bar{\varepsilon};\theta)$ reaches one unique maximum in $\bar{\varepsilon}^{\max}$, with

$$\bar{\varepsilon}^{\max} = \chi^{-1}\left(\frac{\theta}{\gamma}\right). \tag{24}$$

7.3 Proof of Proposition 2

The proposition compares multiple equilibria A and C in Figure 6. The effects of the reduction in $\bar{\varepsilon}$ (item 1) so that banks use the storage technology (item 6) defines a collateral trap, as in Figure 6. As $\bar{\varepsilon} = r/R$, the interbank rate falls with $\bar{\varepsilon}$ (item 2), and from (3) and (14), it is easy to see that the spread between r^d and r goes up (item 3). Item 4 results from (11). The effects of a fall in $\bar{\varepsilon}$ on the volume of transaction secured by outside collateral (item 5a) and on collateral composition (item 5b) come directly from (12) and Lemma 1. Since aggregate output increases monotonically with $\bar{\varepsilon}$ (see (18)), output and consumption are lower (item 7). Item 8 follows from $\bar{\varepsilon}$ being reduced in a collateral trap.

7.4 Proof of Proposition 3

We want to prove that a crisis has differentiated effects on the economy, depending on whether banks initially rely on inside or outside collateral for interbank transactions. To do so, we compare two economies, indexed by $i = 1, 2$ and characterized initially by the same normal times equilibrium, and then study the effects of a crisis. In the normal times equilibrium, the two economies have the same degree of efficiency (*i.e.* $\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = \bar{\varepsilon}^* > \gamma$), the same productive investment ω, and the same output, $y_1(\bar{\varepsilon}^*) = y_2(\bar{\varepsilon}^*)$ (see (18a)). The only difference between economies 1 and 2 is that $\theta_1 > \theta_2$ and $\mathbf{b}_1 < \mathbf{b}_2$.

Given this, the condition $\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = \bar{\varepsilon}^*$ constrains $\mathbf{b}_1$ and $\mathbf{b}_2$ as follows (using (16)):

$$\frac{\mathbf{b}_i}{\omega + \mathbf{b}_i} = 1 - \frac{(1 - G(\bar{\varepsilon}^*))\bar{\varepsilon}^*}{\gamma - \theta_i \bar{\varepsilon}^*} = 1 - H(\bar{\varepsilon}^*; \theta_i) \tag{25}$$

for $i = 1, 2$, which in turn also implies:

$$\phi_1(\omega + \mathbf{b}_1) = \phi_2(\omega + \mathbf{b}_2). \tag{26}$$

In other terms, in the normal times equilibrium, output, consumption, interbank markets transactions, and bank efficiency are the same in economies 1 and 2. The two economies only differ in the way they reach this equilibrium: banks in economy 1 create and rely more on inside collateral than those of economy 2; economies 1 and 2 are otherwise "all–else–equal". Figure 5 is an example of collateral pyramids in economies 1 and 2. The pyramid of economy 1 is thinner and higher than that of economy 2, but the two pyramids overall have the same size.

To prove Proposition 3 we consider two comparable in which the normal time equilibrium co-exists with the crisis equilibrium (as in Figure 8), and show that during a crisis, output falls by more in economy 1 than in economy 2. Let $y_1(\gamma)$ and $y_2(\gamma)$ denote output in the crisis equilibrium of economy 1 and economy 2, respectively. Then we want to show that, $y_1(\gamma) - y_1(\bar{\varepsilon}^*) < y_2(\gamma) - y_2(\bar{\varepsilon}^*)$, and therefore that $y_1(\gamma) < y_2(\gamma)$. From (18b) and the definition of $H(\bar{\varepsilon};\theta)$ in (16), one gets:

$$y_1(\gamma) < y_2(\gamma) \Leftrightarrow \frac{\omega + \mathbf{b}_1}{1 - \theta_1} < \frac{\omega + \mathbf{b}_2}{1 - \theta_2} \Leftrightarrow (\omega + \mathbf{b}_1)H(\gamma; \theta_1) < (\omega + \mathbf{b}_2)H(\gamma; \theta_2),$$

which using condition (25) can be re–written as

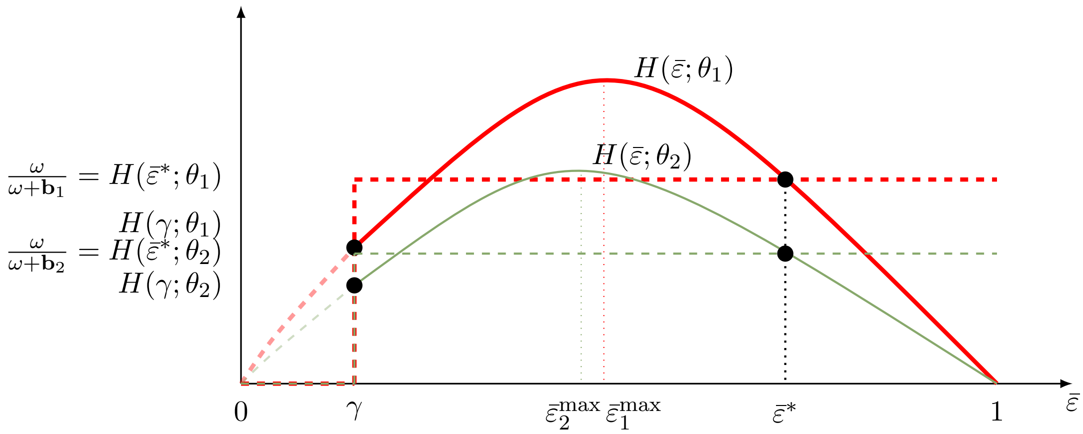
$$\frac{H(\gamma;\theta_2)}{H(\bar{\varepsilon}^*;\theta_2)} > \frac{H(\gamma;\theta_1)}{H(\bar{\varepsilon}^*;\theta_1)}$$

which in turn can be re–arranged as

$$\frac{\gamma-\theta_2\bar{\varepsilon}^*}{1-\theta_2} > \frac{\gamma-\theta_1\bar{\varepsilon}^*}{1-\theta_1}.$$

It is easy to see that the term $(\gamma-\theta\bar{\varepsilon}^*)/(1-\theta)$ monotonically decreases with θ, since by construction, $\bar{\varepsilon}^* > \gamma$. It follows that the above inequality holds if and only if $\theta_1 > \theta_2$, which is true. Hence the result $y_1(\gamma) - y_1(\bar{\varepsilon}^*) < y_2(\gamma) - y_2(\bar{\varepsilon}^*)$. For an illustration, see Figure 8.

Figure 8: "All–else–equal" Economies: Crisis Intensity



7.5 Proof of Proposition 4

Consider the all–else–equal economies 1 and 2 described in Appendix 7.4, with a normal times equilibrium characterized by $\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = \bar{\varepsilon}^*$ (with $\bar{\varepsilon}^* > \gamma$). We want to show that economy 1 is more fragile than economy 2 in the sense of Definition 2:

$$\mathbf{b}_2 - \bar{b}(\theta_2) > \mathbf{b}_1 - \bar{b}(\theta_1)$$

which can be re–written using (21) and (25) as:

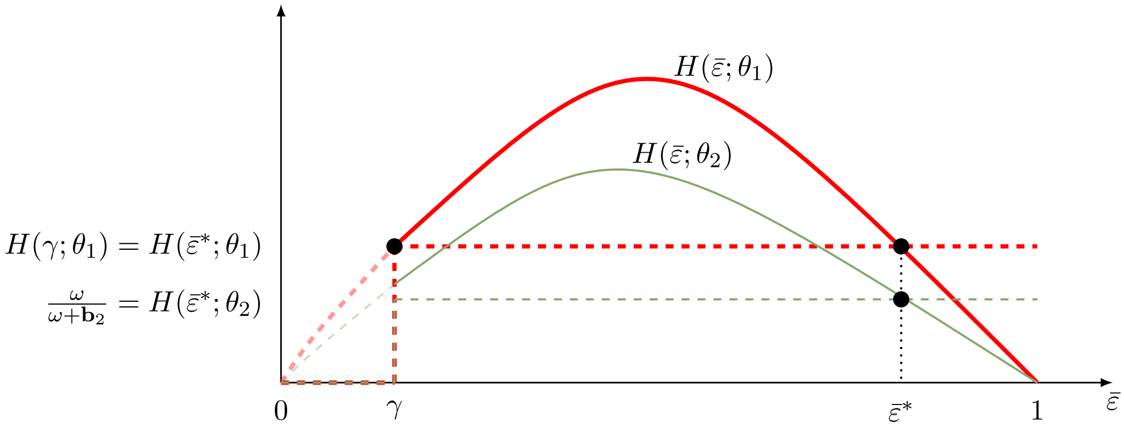
$$\frac{1-H(\bar{\varepsilon}^*;\theta_2)}{H(\bar{\varepsilon}^*;\theta_2)} - \frac{1-H(\gamma;\theta_2)}{H(\gamma;\theta_2)} > \frac{1-H(\bar{\varepsilon}^*;\theta_1)}{H(\bar{\varepsilon}^*;\theta_1)} - \frac{1-H(\gamma;\theta_1)}{H(\gamma;\theta_1)}.$$

Given that $H(\bar{\varepsilon}^*;\theta)$ increases in θ, and therefore $H(\bar{\varepsilon}^*;\theta_1) > H(\bar{\varepsilon}^*;\theta_2)$, a sufficient condition for the above inequality to hold is:

$$\frac{H(\gamma;\theta_2)}{H(\bar{\varepsilon}^*;\theta_2)} > \frac{H(\gamma;\theta_1)}{H(\bar{\varepsilon}^*;\theta_1)} \Leftrightarrow \theta_1 > \theta_2,$$

which is true. Hence the result. Figure 9 is an example of a case when economy 1 is fragile but economy 2 is not. (This representation is consistent with that in Figure 5.) In this example, $\bar{\varepsilon}^*$ satisfies $H(\bar{\varepsilon}^*;\theta_1) = H(\gamma;\theta_1)$, the crisis times equilibrium co–exists with the normal times equilibrium in economy 1, and it is easy to see (from the above inequality) that $H(\gamma;\theta_2) > H(\bar{\varepsilon}^*;\theta_2)$, *i.e.* that the normal times equilibrium is unique in economy 2.

Figure 9: "All–else–equal" Economies: Fragility



7.6 Equivalence between Interbank Transactions Secured by Treasuries and a Secondary Market for Treasuries

The reallocation of lending power across the heterogenous banks arises solely from interbank trades. Another way to facilitate lending by the high ε banks is to allow trades in outside assets once types are known. Typically, the high ε banks may sell their government bonds to free up resources and raise their productive investments. Below, we show that the two types of interbank trades yield the same equilibrium outcome.

Suppose there is *ex post* both an interbank market with trades at the interest rate r and a secondary market for treasuries with trades at unit price of q. For a lender, the opportunity cost of purchasing one unit of government bonds is the reduction in her loan supply by q units of cash. Since the opportunity cost is qr and the gain is r (using (11)), a lender will purchase government bonds only if $q \leq 1$. In contrast, for a borrower, the gain from selling one unit of government bonds is qr, whereas the opportunity cost is r, as we showed in Section 2.3.2. Hence, a borrower will sell government bonds only if $q \geq 1$. In equilibrium, $q = 1$, and banks are indifferent about their holding of government bonds.

It remains to show that the equilibrium of the interbank market, $\bar{\varepsilon}$, is not affected by banks trading government bonds. Let αb denote the amount of government bonds that borrowers retain on their balance sheet (and therefore $(1-\alpha)b$ the amount that they sell on the secondary market). Then the government bonds available for securing interbank loans is αb, and the resources available to them for investment is $s - \alpha b$, so that their borrowing limit is now given by (from (12)):

$$\phi = \frac{\bar{\varepsilon}}{\gamma - \theta\bar{\varepsilon}} - 1 + \frac{\alpha b}{s}.$$

Similarly, let βb be the amount of government bonds that a lender purchases on the secondary market. In the equilibrium of the secondary market for treasuries, aggregate demand adjusts to aggregate supply,

$$\beta = \frac{(1 - G(\bar{\varepsilon}))(1-\alpha)}{G(\bar{\varepsilon})}, \tag{27}$$

and in the equilibrium of the interbank market, the productivity cut–off $\bar{\varepsilon}$ adjusts so that the market clears,

$$(1-G(\bar{\varepsilon}))\phi s = G(\bar{\varepsilon})(s-(1+\beta)b).$$

Using (27), this latter condition yields

$$(1-G(\bar{\varepsilon}))\left(\frac{\bar{\varepsilon}}{\gamma-\theta\bar{\varepsilon}}-1+\frac{b}{s}\right)=G(\bar{\varepsilon})\left(1-\frac{b}{s}\right),$$

which corresponds to the market clearing condition (15). It follows that the equilibrium of the interbank market is not affected either by whether or not banks trade government bonds on a secondary market. Hence, the equivalence result.

7.7 Non–existence of an Equilibrium with Default

The purpose of this section is to prove the non–existence of an equilibrium with defaults. We prove this by contradiction. Assume that one borrower defaults in equilibrium. In the context of our model, this means that this borrower strictly prefers to borrow and divert the funds into storage rather than (i) invest the funds into her productive project or (2) lend to other bankers. Since interbank lending is a dominated strategy, no banker lends, which cannot be an equilibrium outcome. Hence a contradiction. To put it differently, the fact that we allow bankers to choose whether they lend or borrow on the interbank market —a specificity of our framework— rules out the existence of an equilibrium with default.

7.8 Non–existence of Revealing Financial Contracts

The purpose of this section is to prove that skillful borrowers cannot separate themselves from other borrowers by offering ε–specific loan contracts $(\phi(\varepsilon), r(\varepsilon))$, where $\phi(\varepsilon)$ is borrower ε's total borrowing limit, which the borrower commits to. Notice that this limit does not correspond to the loan per lender, as the borrower may raise funds from multiple lenders. On this basis, the proof consists of two parts. First, we notice that, in equilibrium, the lending rate $r(\varepsilon)$ must be the same across all borrowers. Otherwise, no lender would be willing to sign any contract earning less than the highest rate of return on offer; so, $r(\varepsilon)=r$ for all ε. Second, to be revealing, the menu of contracts $(\phi(\cdot), r)$ must be such that banker ε has no incentive to pick a contract intended for another banker $\tilde{\varepsilon}$. Therefore function $\phi(\cdot)$ must solve the following differential equation:

$$\varepsilon = \arg\max_{\tilde{\varepsilon}} \pi^b(\tilde{\varepsilon} \mid \varepsilon) \equiv br^b + \varepsilon R(s-b+\phi(\tilde{\varepsilon})s) - \phi(\tilde{\varepsilon})sr$$

for all borrowers $\varepsilon \geq r/R$. But since $\pi^b(\tilde{\varepsilon} \mid \varepsilon)$ monotonically increases with $\phi(\tilde{\varepsilon})$, (*i.e.* the Spence–Mirrlees single–crossing condition is not satisfied), there is no solution to the equation. Hence the result.