Should bank capital requirements be less risk-sensitive because of credit constraints?

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Abstract

We consider optimal capital requirements for banks' lending activities when the potential trade-off between financial stability and economic (productivity) growth is taken into account. Both sides of the trade-off are affected by banks' credit allocation, which in turn is affected by the risk weights used to set capital requirements on bank loans. We find that when firms are credit constrained, the optimal risk weights are flatter than those that are only set to safeguard against bank failures and their social costs. When risky borrowers are also more productive, the 'flattening' effect is amplified. A quantitative evaluation of the model using US corporate loan data suggests that the welfare cost of a purely risk-based rule may be small and equivalent to getting the level of capital requirements wrong by 1 percent.

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1 Introduction

"The current structure of the regulations may actually introduce biases against making (business) loans." - Anat Admati and Martin Hellwig 2013, p. 222

After the Global Financial Crisis of 2007-2009, banks' equity capital requirements have been considerably increased in order to reduce the likelihood of future crises. The requirements take the form of a minimum amount of equity required per risk-weighted assets of a bank, where the risk-weighting scheme has been largely kept unchanged. Only a relatively modest capital to assets (leverage ratio) requirement without any risk-weighting has been supplemented. Moreover, new elements such as a counter-cyclical adjustment to the minimum requirement have been introduced, and the largest banks (specifically, banks that are considered systemically important) have an extra requirement (see Basel Committee on Banking Supervision 2010).[1]

Both the optimal level of capital requirements as well as optimal risk weights are still much debated in the literature.[2] The question of the optimal level typically centers around the trade-off between reducing the likelihood of banking crises and possibly sacrificing short-term economic growth, as higher equity requirements may increase banks' funding costs and hence reduce bank lending in the economy.[3] However, this trade-off may also be affected by how credit is allocated across sectors and investment projects, and therefore the question of optimal risk weights also arises.[4]

The current capital regulation considers risk weights purely from the viewpoint of a bank's solvency or, more broadly, financial stability. In the current system, the risk weight on a corporate loan is determined by the loan's contri-

[1]It is also possible to determine an extra capital requirement on the basis of the systemic risk specific to a country's financial system as a whole. Further requirements on minimum loss absorbing capacity have been introduced, or are being considered, in the form of "bail-inable" debt which is in effect some form of contingent capital.

[2]See e.g. Dagher et al. 2016 and Martynova 2015 and the literature covered therein.

[3]See also Dagher et al. (2016); Mendicino et al. (2016) and Elenev et al. (2017) on the level of capital requirements and the need for a counter-cyclical buffer.

[4]Another important role of capital requirements is reducing systemic risk arising from contagion in financial networks (see Haldane 2009, and e.g. the literature review in section II.C of Dagher et al. 2016).

bution to the bank's overall loan portfolio risk.[5] However, after the crisis, the current risk-weighting system has been heavily criticized. This is because the crisis exposed problems with risk measurement and because risk weights based on risk measurement models are prone to be manipulated by banks (see e.g. Beltratti and Paladino 2016 and Berg et al. 2015). Subsequently, it has been argued that risk weights should be determined in a more robust manner, or even be replaced by a non-risk-weighted (but sufficiently stringent) leverage restriction (see e.g. Admati and Hellwig 2013). This would probably imply a flatter, if not an entirely flat, risk-weighting structure than the current one.[6] Moreover, Admati and Hellwig (2013) argue that the current risk weights may create a bias against traditional business loans, which would typically obtain a relatively high risk weight in the current system. This suggests that the current risk-weighting system may not be optimal from the viewpoint of economic growth.[7]

In this paper we study the optimal risk weights used in setting banks' capital requirements when the potential trade-off between financial stability and economic (productivity) growth is taken into account. Both sides of the trade-off are affected by credit allocation, which in turn is affected by the risk weights used to set banks' capital requirements. We find that the optimal risk weights may be flatter than those which are only set to buffer against banks' instability and its social costs. Hence, we provide an additional rationale for capital requirements which are less "risk-sensitive." The main message of the paper is that focusing solely on risk aspects when setting minimum capital requirements on (corporate) loans may lead to lost growth opportunities in the economy.

We build a simple model of banking where financial risks and economic rewards pose a trade-off. Entrepreneurs become borrowers and differ in terms of

[5]This is the case when a bank is allowed to use, subject to supervisory approval, the so called Internal Ratings Based Approach of the Basel rules for capital requirements. As a default option, typically smaller banks use a simpler risk-weighting system.

[6]In actuality, risk weights on corporate loans were flat in the first Basel Accord from 1998 (Basel I). The shift to model-based risk weights was introduced in the second Basel agreement in 2004 (Basel II), implemented e.g. in the EU in 2007.

[7]Representatives of the banking industry especially in Europe have also raised concerns that the increased capital requirements (together with the current risk-weighting system) may jeopardize sufficient lending to small and medium-sized enterprises, which are often seen as crucial to European economies. See Christian Clausen, president of the European Banking Federation, in Financial Times, 16 November 2014.

the productivity of their sector. Importantly, they are collateral constrained and can only borrow up to a fraction of the value of their investment project. This is a key imperfection in credit markets which drives our central result, and a notable difference with respect to the portfolio theoretic model underlying the current regulation, which implicitly assumes perfect markets. Further, banks specialize in lending to entrepreneurs from a given sector and face a sector-specific risk in their loan portfolios. Hence they are subject to a failure risk themselves. Consistent with the view that banks play a special role in facilitating economic activity, bank failures in our model generate pecuniary externalities which provide the impetus for capital regulation. Bank equity capital is scarce and hence the more costly source of financing for banks than deposits. Banks are competitive and hence pass on the cost of capital requirements to their borrowers. Loan demand responds accordingly such that capital requirements play a significant role in the allocation of bank credit across sectors.[8]

The main mechanism of the model works as follows. Banks pay the risk-free rate for deposits (because of deposit insurance) and do not internalize the social costs of their own failure. Second, productivity of entrepreneurial investment projects does not factor into banks' loan pricing problem because perfect competition among banks implies that they do not profit more from lending to more productive sectors. Further, entrepreneurs lever up to their borrowing constraint such that being more productive does not generate a cushion against negative shocks that can lead to their defaulting on their loans. Higher capital requirements can help *price* loans appropriately. In particular, capital requirements reduce (i) bank leverage which in turn reduces both the frequency and size of bank failures and (ii) borrower leverage by raising the cost of borrowing which tightens the collateral constraint, which further reduces the amount of borrowing per unit of net worth. The optimal risk weights trade off these two leverage effects - productive investment and bank failure.

Our main results can be characterized as follows. Note that in our model, there are two financial frictions. First, borrower collateral constraints distort

[8]See for instance Auer and Ongena (2016); De Jonghe et al. (2016); Gropp et al. (2016); Juelsrud and Wold (2017) and Celerier et al. (2017) for evidence of a shift in lending to safer borrowers in response to higher bank funding costs and in particular increases in required capital.

economic rewards from external financing. Here economic reward is the efficient allocation of credit. Absent bank failures, optimal policy will subsidize collateral-constrained entrepreneurs in productive sectors. Second, financial risk depends on the bank failure externality. When entrepreneurs in different sectors are equally productive, optimal policy will "tax" more heavily loan portfolios of the more risky sectors. We mimic the current risk-weighting scheme by imposing the constraint that all banks must have the same probability of failure. When comparing these to the unconstrained optimal risk weights we find the latter to be flatter, even if there are no productivity differences across sectors. The "flattening" result can be understood as follows: because borrowers are credit constrained, the risk weights that are needed to make banks which lend to high-risk sectors as safe as other banks will be so high that they reduce production in high-risk sectors too much. Hence, it is better to tolerate a higher probability of bank failures in high-risk sectors than in low-risk sectors and have a more even distribution of production across sectors. The flattening of risk weights is further amplified if risk and productivity across sectors are positively correlated. We will also discuss the robustness of these results.

To evaluate the quantitative importance of our results, we match key features of the model with US corporate loan data to assess the relative importance of the mis-allocation of credit induced by a purely risk-based risk weighting system. The purely risk-based approach is designed to mimic the Basel II risk-weighting scheme which has largely been unchanged in Basel III. We find that welfare losses from adopting this purely risk-based regulation tend to be small. Adopting such a scheme, at the right average level of capital requirements, generates a welfare loss that is equivalent to a policy with the right risk weights but the average level of capital requirements wrong by one percent.

Our model is closely related to Mendicino et al. (2016) who study the optimal level of dynamic bank capital requirements or both corporate loans and household mortgages. Our focus is on productivity and risk differences across corporate sectors and their effect on the optimal risk-weighted capital requirements per sector. In addition, we consider the financing constraint of firms, following the tradition of e.g. Kiyotaki and Moore (1997); Holmstrom and Tirole (1997); Bernanke and Gertler (1989) and Bernanke et al. (1999). Our emphasis on the importance of the composition of credit, due to differences in productivity and credit constraints, and the role that capital requirements play is related to Harris et al. (2017). In this sense, we are also related to the empirical literature on the impact of financial frictions on capital and credit mis-allocation and consequently output and productivity.[9] For instance, Gilchrist et al. (2013); Hassan et al. (2017) and Gopinath et al. (2017) show that differences in credit constraints, and credit allocation by banks, reduces productivity in the United States and in in Southern Europe. Martinez-Miera and Repullo (2017) also caution on the use of risk weights from a purely risk-based perspective as these may lead the riskiest borrowers to obtain credit from the shadow banking sector where monitoring is inefficiently lower. Our results formalize an additional argument for flatter risk weights, based on the trade-off between risk and productivity arising from the cross-sectional allocation of credit when borrowers are credit-constrained.

The paper is structured as follows. Section 2 presents the model and section 3 analyzes optimal capital requirements. Section 4 covers results from a quantitative evaluation of the model predictions and finally, Section 5 concludes. Proofs and some important extensions are provided in appendices.

2 Model

The key contribution of our analysis concerns the effect of risk-weighted capital requirements on the cross-sectional allocation of bank financing. To capture the trade-off between borrowing frictions on the one hand and societal pecuniary costs of bank failures on the other, we consider bank credit in a two-period model where borrower productivity varies across sectors. There is a continuum of competitive banks, and each bank specializes in lending to one sector, facing sector-specific loan portfolio risk. Agency frictions in the spirit of Gale and Hellwig (1985) motivate collateralized borrowing, which is limited by banks' valuations of collateral.[10]

For simplicity, we assume an actuarially fair deposit insurance scheme, which

[9] See Restuccia and Rogerson (2013, 2017) for a review of the literature.

[10] Agency costs also imply that a standard debt contract is the optimal mode of external financing in our setting. See as well Townsend (1979) and Kiyotaki and Moore (1997).

implies that banks have no excessive risk taking incentives arising from deposit insurance. However, banks prefer high leverage because bank equity is scarce and is hence assumed to bear a premium with respect to deposit financing. Moreover, if banks fail, society suffers pecuniary costs. This motivates bank capital requirements because banks do not internalize these costs (see also Gale and Ozgur, 2005 on why pecuniary externalities arising from bank failures may be appropriate as a motivation for capital requirements).[11]

2.1 Entrepreneurs

Consider a two-period economy populated by two sets of risk-neutral agents, entrepreneurs and bankers, who maximize old-age consumption of a numeraire good. First, entrepreneurs with a unit mass and indexed by i belong to a unit mass of sectors indexed by j. They are born with an endowment of the good e (equity) which, along with potential borrowing, they can invest into projects.

An entrepreneur's investment opportunity allows her to convert one unit of the numeraire today into A_j units of a specialized good tomorrow. After production she may then sell the product for price $\epsilon_{i,j}$ of the numeraire. This price is a random variable realized in period two and is log-normally distributed with a mean of one and a variance which may differ across sectors. Furthermore, in the case that the good is transferred to a banker (as collateral), the banker can convert it into $\theta_j < 1$ units of the numeraire. To prevent entrepreneurs from strategically defaulting and running away with the borrowed funds, banks limit lending up to their valuation of the specialized good which serves as a constraint on borrowing.

[11] Our two-period partial equilibrium setup ignores net worth dynamics. The focus on relative allocations also abstracts from the general equilibrium implications of various capital requirement schemes on the aggregate scarcity of bank equity, its pricing, and its consequences on the trade-offs we evaluate.

Entrepreneurs solve the following program,

$$\begin{array}{rcl}
\max\limits_{B_{i,j}} & & \mathbb{E}\left[\epsilon_{i,j}K_{i,j} - R_j^b B_{i,j}\right]^+ \\
\text{s.t.} & & \\
K_{i,j} & = & A_j(B_{i,j}+e) \\
R_j^b B_{i,j} & \leq & \theta_j K_{i,j} \\
log(\epsilon_{i,j}) & \sim i.i.d. & \mathcal{N}(-\sigma_j^2/2, \sigma_j^2)
\end{array}$$

where R_j^b is the loan rate set for loans in sector j.

The program above yields the following optimal size of borrowing and output,

$$B_j^* = \frac{\theta_j A_j}{R_j^b - \theta_j A_j} \quad \text{and} \quad K_j^* = \frac{A_j}{R_j^b - \theta_j A_j} R_j^b \tag{1}$$

where we have that the borrowing constraint is binding $R_j^b B_j^* = \theta_j K_j^*$ and we have normalized the initial net worth e of entrepreneurs to one. In turn, expected consumption is given by

$$\mathbb{E}\left[\epsilon_{i,j}K_{i,j} - R_j^b B_{i,j}\right]^+ = (1-\Phi_j)\left(\bar{\epsilon}_j^s - \theta_j\right) K_j^* \tag{2}$$

where $\Phi_j = Pr(\epsilon_{i,j} < \theta_j)$ is the probability of default and $\bar{\epsilon}_j^s = E[\epsilon_{i,j} | \epsilon_{i,j} \geq \theta_j]$ is the mean of the price shock conditional on not defaulting which reflects the gains from limited liability.

Key to our environment is that entrepreneurs (borrowers) need external financing and agency problems generate the need for borrowers to collateralize debt with future output. Since investment generates a specialized good which is of greater value (ex ante) to entrepreneurs, the size of borrowing is limited to a fraction of potential output from investment which reflects the banks' valuation of the collateral and the degree of specialization or tangibility of the output. Consequently, it is possible that investments in the most productive projects are severely limited due to financing constraints while other, less productive projects, may get external financing more easily.

2.2 Banks

The second set of agents are bankers. A continuum of retail bankers indexed by j fund their lending activities with bank equity e_j^b and deposits d_j. Each retail banker serves sector j and faces the threat of competitive entry for their entire loan portfolio. There is a single wholesale banker who holds all of bank equity and is willing to rent it at an expected return ρ to retail bankers. Similarly, a perfectly elastic supply of deposit funds is available to all retail bankers at a required return given by R^d.[12] We assume that $\rho > R^d$.[13] Deposit funds taken out by bankers are subject to a deposit insurance scheme such that R^d may be interpreted as the risk-free rate of return in our economy. The deposit insurance scheme guarantees the repayment of deposits in the case of a bank failure and charges an actuarially fair premium on surviving banks. The expected return on bank equity ρ_j is defined as net of this insurance premium. Finally, the share of lending financed with bank equity for each retail banker has to be greater than or equal to an exogenously set capital requirement κ_j.

Each retail bank in operation services a continuum of borrowers faced with idiosyncratic shocks. This implies that a constant fraction of the loan portfolio will default. Further, the binding collateral constraint implies that this fraction will also yield the same return to the bank as the non-defaulting borrowers. To incorporate risk, we include a loan portfolio shock ξ_j (as in Clerc et al., 2015; Mendicino et al., 2016) after borrower default has taken place. This may be interpreted as a reduced-form way of incorporating correlated risk of default of the individual loans (as a result of a systematic risk factor; see e.g. Gordy, 2003) or as a shock to the bankers' ability to extract the returns on her loan portfolio.

The portfolio shock is log-normally distributed with unit mean and independently distributed across sectors $log(\xi_j) \sim N(-\eta_j^2/2, \eta_j^2)$. When the shock is sufficiently large and negative, then the return on the portfolio is insufficient

[12] These deposits may be interpreted as savings from elsewhere in the economy (e.g. households). As typically assumed in the literature, see e.g. Repullo and Suarez (2004, 2013), deposit supply is perfectly elastic at the rate R^d.

[13] There are several reasons in the banking literature on why bank equity requires a higher return (see for instance Holmstrom and Tirole, 1997; Diamond and Rajan, 2000). Here we motivate it as primarily a way to capture the relative scarcity of bank equity and its effect on loan supply in a similar vein to Repullo and Suarez (2004) and Mendicino et al. (2016).

to cover the bank's liabilities and the bank itself goes into default.[14] Denote a bank's failure probability as Ψ_j.[15] This probability is completely determined by the riskiness of the bank's portfolio and its leverage (capital requirements),

$$\Psi_j \quad = \quad Pr(\xi_j R_j^b B_j < R^d d_j) \tag{3}$$

For simplicity, the variance of the portfolio shock is exogenous to individual borrower risk characteristics. Nevertheless, one should interpret portfolio risk as proportional to borrower risk. One may think for instance that the variance of the portfolio shock represents non-diversifiable risk (e.g. the correlation or the covariance across asset classes used to collateralize the loans) which may be increasing in individual borrower default probability Φ_j. An extension of the model along these lines is available in Appendix B.2 which demonstrates how portfolio risk (η_j) endogenously arises from entrepreneur risk (σ_j).

Given the threat of entry, the retail bankers' problem may be written as minimizing the loan rate,

$$\begin{aligned} \min R_j^b & \quad \text{s.t.} \\ \rho_j &\equiv \left(R_j^b B_j - R^d d_j\right)/e_j^b \geq \rho \\ e_j^b/B_j &\geq \kappa_j \end{aligned}$$

The constraints bind in equilibrium and yields the competitive loan rate,

$$R_j^b = R^d + \kappa_j(\rho_j - R^d) \tag{4}$$

Thus, we can rewrite the probability of bank failure as

$$\Psi_j \quad = \quad \Phi(\frac{\eta_j}{2} - \frac{\tilde{\kappa}_j}{\eta_j}) \tag{5}$$

where $\tilde{\kappa}_j \equiv log(1 + \frac{\kappa_j}{1-\kappa_j}\frac{\rho}{R^d})$ and $\Phi(\cdot)$ is the normal density.

The key friction motivating the need for capital requirements in our model

[14]See Clerc et al. (2015); Mendicino et al. (2016) for a similar modeling approach.

[15]See Appendix A for details on the relationship between our actuarially fair deposit insurance scheme and bank failure probabilities.

is that bank failures generate pecuniary externalities. One may interpret this as the fraction of bank failure costs which deposit insurance is unable to cover or correct. We assume that when a bank fails, and similar to Repullo and Suarez (2004), society as a whole suffers a cost proportional to the size of the bank's balance sheet, $\gamma R_j^b B_j$ where γ is a scale parameter.[16]

Finally, we define societal welfare as the sum of all expected consumption by entrepreneurs and payments by bankers (who make zero profits) less bank failure costs. A given sector's contribution to societal welfare is given by,

$$\begin{aligned} \mathbb{E}w_j &= \mathbb{E}c_j + \left[\rho e_j^b + R^d d_j\right] - \gamma \Psi_j R_j^b B_j \\ &= \mathbb{E}c_j + \theta_j K_j - \gamma \Psi_j R_j^b B_j \end{aligned}$$

where

$$\begin{aligned} \mathbb{E}c_j &= (1-\Phi_j)(\bar{\epsilon}^s - \theta_j)K_j \\ &= \left[1 + \Phi_j(\theta_j - \bar{\epsilon}^d)\right] K_j - \theta_j K_j \\ &= \tilde{K}_j - \theta_j K_j \end{aligned}$$

where $\bar{\epsilon}_j^d = E[\epsilon_{i,j} | \epsilon_{i,j} < \theta_j]$ is the mean of the price shock conditional on defaulting. Aggregate welfare is simply the sum of entrepreneurial output (in terms of the numeraire) net of bank failure costs,

$$W = \int \mathbb{E}w_j = \int \tilde{K}_j - \gamma \Psi_j R_j^b B_j$$

Here we have made the assumption that bank failure costs are linear in the size of bank failures. This is done to simplify the model and reflects a more micro-prudential (as against macro-prudential) interpretation of the cost of bank failures. One may think that, due to other amplification mechanisms, banking crises are likely to have non-linear costs.

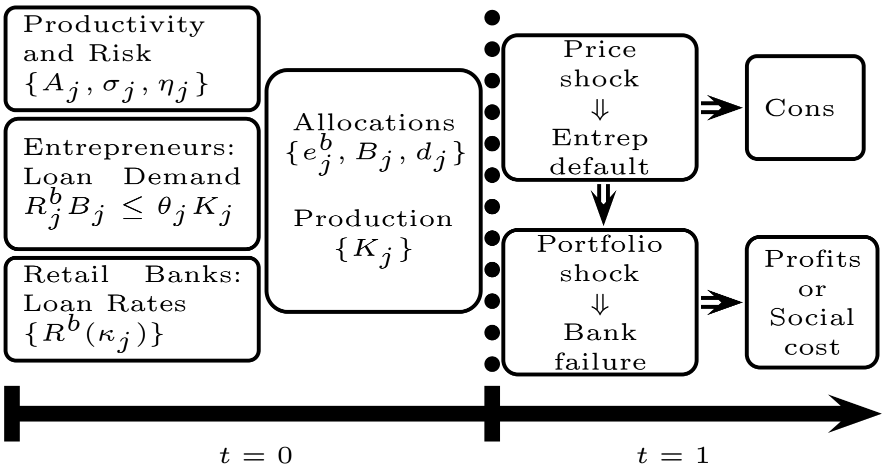
[16]See also Clerc et al. (2015); Mendicino et al. (2016) for similar formulations of the bank failure externality.

2.3 Timing

In the first period, each retail banker meets the set of entrepreneurs in her sector and makes a loan offer by posting a loan rate. Given loan rates, the solution to the entrepreneurs' problem yields loan demand. The retail banker then turns around and presents her loan portfolio to the wholesale banker and asks for bank equity as per capital requirements. Once loans are made the first period ends and production takes place.

In the second period, the entrepreneurs' price shock $\epsilon_{i,j}$ is realized and some repay while others default. In the latter case, retail banks appropriate the collateral which they then sell for θ_j. Finally, the bank portfolio shock ξ_j is realized and some banks fail. The sequence of choices and shock realizations are illustrated in Figure 1.

Figure 1: Timing



2.4 (Partial) Equilibrium

Given the set of parameters $\{A_j, \sigma_j, \theta_j, \eta_j, \kappa_j\}$ for all sectors and aggregate parameters $\{\rho, R^d\}$, equilibrium is defined as the set of choices $\{B_{i,j}, R^b_j, e^b_j, d_j\}$ for all entrepreneurs and sectors such that equations 1 and 4 hold and all constraints in the entrepreneurs' and banks' problems are binding.

3 Capital requirements

Clearly, the equilibrium allocation depends on the set of capital requirements. In this section, we first demonstrate that there is a need for capital regulation, proceed with the characterization of the set of optimal capital requirements, and compare these with simple rules which approximate the spirit of current regulation. Note that in practice, the capital requirement for a bank is determined as the minimum percentage of (equity) capital of the bank's risk-weighted assets. However, in our simple model we work directly with the capital requirement per loan in a given sector without having to specify the minimum percentage of capital and sector-specific risk weights separately. In short, two banks holding a portfolio of loans granted to the same sector have the same capital requirement in our model, but two banks lending to different sectors have a different capital requirement.

To simplify the analysis, from hereon and unless otherwise specified, we assume that all parameters other than bank portfolio risk (η_j) and entrepreneur productivity (A_j) are the same across sectors (i.e. $\theta_j = \theta$ and $\sigma_j = \sigma$).

3.1 Optimal capital requirements

Suppose a constrained social planner wants to maximize aggregate welfare by choosing a set of capital requirements for each sector. The optimal capital requirements chosen by the planner solves,

$$\begin{aligned} \max_{\{\kappa_j\}} \quad & \int \tilde{K}_j - \gamma \Psi_j R^b_j B_j \\ \text{s.t.} \quad & 0 \leq \kappa_j \leq 1 \quad \forall\, j \end{aligned}$$

where K_j,B_j, and R_j^b are given by equations 1 to 4.[17] An interior solution to the problem yields the following first-order condition for each sector,

$$\overbrace{(1+\Phi(\theta-\bar{\epsilon}^d))\frac{\partial K}{\partial \kappa}}^{\text{Entrep collateral constraint}} = \gamma\theta \overbrace{\left(K \underbrace{\frac{\partial \Psi}{\partial \kappa}}_{\text{Frequency}} + \Psi \underbrace{\frac{\partial K}{\partial \kappa}}_{\text{Size}} \right)}^{\text{Bank failure}} \tag{6}$$

The optimal capital requirement is determined by the following trade off: on the one hand, capital requirement affects the collateral constraint and thereby productive investment (left-hand side); on the other hand, it affects the bank failure externality (right-hand side).

A useful benchmark is the equilibrium under no capital regulation but with deposit insurance still in place. This leaves bank leverage unconstrained and in this case banks maximize the return on equity by holding as little of it as possible. In effect, as they compete to provide as low a lending rate as they can, banks maximize their own failure rate. Allocations are completely determined by relative productivity without regard for the bank failure externality. Given deposit insurance, this would be optimal only if bank failure costs are negligible yielding a corner solution for all sectors in the planner's problem.

When bank failures are socially costly, imposing capital requirements will bind and improve welfare. As intended, capital requirements deliver lower bank leverage, which directly reduces failure probabilities. However, doing so will raise the cost of borrowing by requiring loans to be partly financed by scarce (and thus more costly) bank equity (cf. Mendicino et al., 2016). In our model, loan demand is elastic. Borrower leverage is reduced with more costly borrowing. This also implies that the sector with a higher capital requirement is also relatively smaller in terms of the aggregate banking portfolio. All of these aspects are featured in equation 6.

[17] Recall that we are in the case where entrepreneurs from all sectors choose to invest in their projects.

3.2 Features of optimal capital requirements

We now characterize some features of the optimal capital requirements in terms of differences in borrower productivity and bank portfolio risk. First, note that all else equal banks with riskier portfolios are subject to higher capital requirements.

Lemma 1 (Capital requirements for risky and safe bank portfolios)**.** *For any two sectors with identical productivity but sector k exhibiting higher portfolio risk than sector j ($\eta_k^2 > \eta_j^2$), the optimal risk weight for bank k is higher than bank j whenever the resulting bank failure probabilities take reasonable values.*[18]

$$\eta_k^2 > \eta_j^2 \Rightarrow \kappa_k^* > \kappa_j^*$$

Second, when all sectors have the same portfolio risk, lending to more productive borrowers merit lower capital requirements.

Lemma 2 (Capital requirements by productivity)**.** *For any two otherwise identical sectors but with sector j more productive than sector k ($A_j > A_k$), the optimal risk weight for bank j is lower than bank k with a corresponding higher frequency of bank failure*

$$A_j > A_k \Rightarrow \kappa_j^* < \kappa_k^* \ , \ \Psi(\kappa_j^*) > \Psi(\kappa_k^*)$$

Combining both differential risk and productivity leads us to the following characterization of optimal capital requirements:

Proposition 1 (Capital requirements summary)**.** *Let sector 0 with characteristics $\{A_0, \eta_0\}$ have an optimal capital requirement given by κ_0^* and j be another sector which is otherwise identical to sector 0,*

- *Whenever $A_j > A_0$ and $\eta_0^2 > \eta_j^2$ then $\kappa_j^* < \kappa_0^*$*
- *In general, whenever $A_j > \underline{A}(\eta_j; \kappa_0^*)$ then $\kappa_j^* < \kappa_0^*$*

[18] The lemma holds for sufficiently low values of portfolio risk such that the resulting bank failure probabilities are not too large. For instance, this condition is always satisfied whenever bank failure probabilities are less than 16 percent. We assume this is the case for the rest of the paper. Details and proof for this lemma, as well as the subsequent lemmas and propositions, are in the appendix.

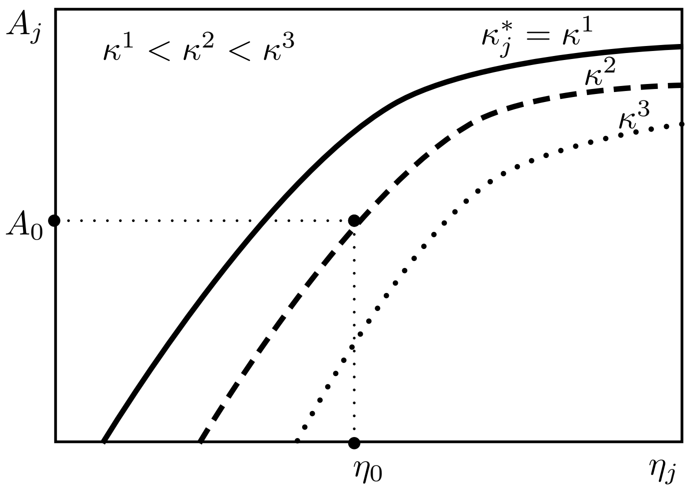
- *Conversely, whenever* $\eta_j < \bar{\eta}(A_j; \kappa_0^*)$ *then* $\kappa_j^* < \kappa_0^*$

where $\underline{A}(\eta_j; \kappa_0^*) : \kappa^*(\underline{A}, \eta_j) = \kappa_0^*$ *and* $\bar{\eta}(A_j; \kappa_0^*) : \kappa^*(A_j, \bar{\eta}) = \kappa_0^*$

The proposition above implies that depending on the distribution of risk and productivity across sectors in a given economy, capital requirements may even be non-monotonic in bank risk.

In Figure 2, we plot the set of sectors sorted by productivity (vertical axis) and risk (horizontal axis) and identify the combinations that yield the same optimal level of capital requirements.

Figure 2: Optimal capital requirements



As the figure indicates, the optimal capital requirements increase as productivity declines and risk increases (towards bottom right of the figure). Each line in the figure represents an iso-κ curve given by the function $\underline{A}(\eta_j; \kappa^*)$ (or $\bar{\eta}(A_j; \kappa^*)$). Thus, in an economy where the sectors exhibit a negative relation between productivity (A_j) and risk (η_j), the set of optimal capital requirements can be characterized as decreasing in productivity and increasing in risk. On the other hand, when the correlation is positive, a one-dimensional ordering is no longer possible. Consequently, current risk-weighted capital requirement schemes which focus only on risk factors need not coincide nor produce the same relative ordering as that implied by our model. We explore this possibility in the next subsection.

3.3 Optimal against purely risk-based capital requirements

We now consider a regulatory scheme which aims to capture the principle behind the current regulation of requiring more capital for riskier assets. In the spirit of this scheme, we consider a policy rule which sets capital requirements such that bank failure probabilities across sectors are equalized. We henceforth refer to this scheme as equal-bank failure probability capital requirements. [19]

Suppose a regulator wants to set $\Psi_j = \Psi_k = \Psi^*$

$$\max_{\Psi^*} \int K - \gamma \Psi^* R^b B$$
$$\text{s.t. } \tilde{\kappa}_j = \frac{\eta_j^2}{2} - \eta_j \Phi^{-1}(\Psi^*)$$

The resulting relative values of the equal-bank failure probability capital requirements depend only on the sectoral portfolio risk

$$\kappa_j = \frac{(exp(\tilde{\kappa}(\Psi^*)) - 1)R^d}{\rho + (exp(\tilde{\kappa}(\Psi^*)) - 1)R^d}$$

where Ψ^* solves: $\int_j -\eta \frac{\partial \kappa}{\partial \tilde{\kappa}} \left[-\gamma \theta K \frac{\partial \Psi}{\partial \kappa} + (1 + \Phi(\theta - \bar{\epsilon}^d) - \gamma \theta \Psi) \frac{\partial K}{\partial \kappa} \right] = 0$

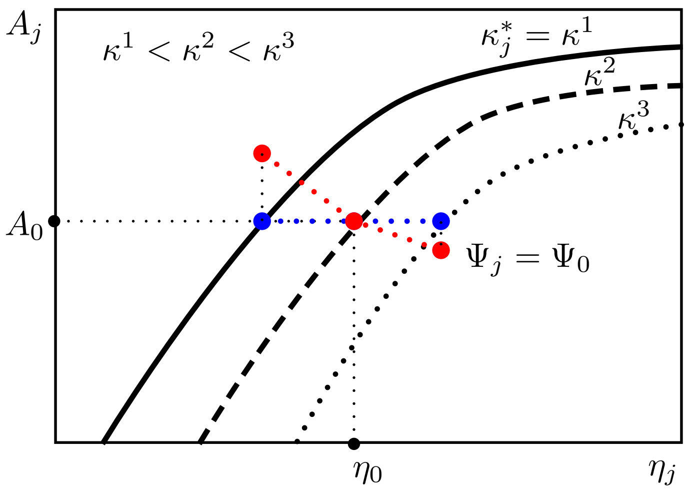
Proposition 2 (Equalizing bank failure probabilities)**.** *All else equal, a policy rule in which bank failure probabilities are equalized generates steeper than the optimal capital requirements: in other words, the requirement is too high for the risky sector and too low for the safe sector.*

Proposition 2 is illustrated in Figure 3. Consider three otherwise identical sectors but with different loan portfolio risks η. The optimal capital requirements are on three different curves representing $\kappa^* \in \{\kappa^1, \kappa^2, \kappa^3\}$ and highlighted by blue dots. The capital requirements imposed by the proposed policy rule are identified by the red dots which is the same as the optimal one for the benchmark

[19] The closest real-world counterpart in which this principle is applied is the Basel capital adequacy framework's Internal Ratings Based Approach. In its orginal form in the Basel II agreement, any bank with the permission to use it was to have a minimum amount of capital against the loan portfolio which covers loan losses with an annual 99.9% probability. If the minimum were fulfilled with equity capital and the bank's only assets were the loan portfolio, then this would translate into a 0.01% annual probability of the bank's failure.

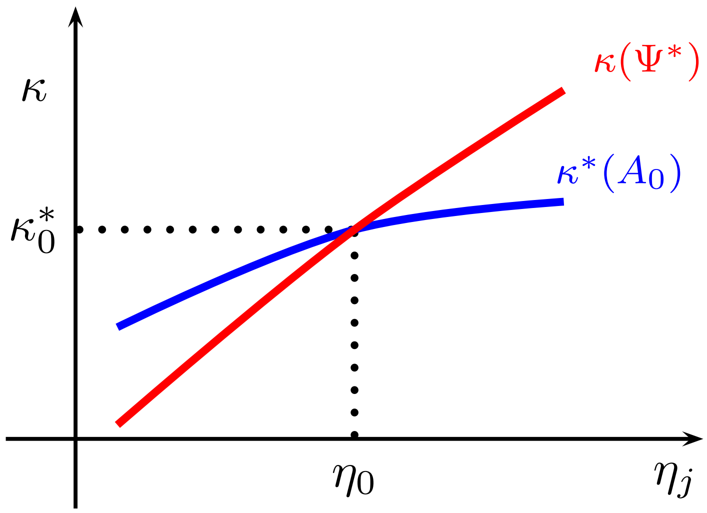
sector with risk η_0. For the lower risk sector (leftmost blue and red dots), the optimal capital requirement (κ^1) is higher than that imposed by the policy rule and in the opposite case, for the riskier sector with the optimal capital requirement κ^3, the proposed policy requires even more.

Figure 3: Equal bank failure capital requirements



The result that the optimal capital requirements do not equalize bank failure probabilities can be understood as follows. When a bank's capital requirement is raised, the effective borrowing constraint on its borrowers gets tighter and, as a result, the relative size of the borrower sector diminishes. Therefore it is optimal to set capital requirements on banks with riskier portfolios to a level where the riskier banks fail more often than the safer ones. Recall that, in this example, all sectors have the same productivity. Even so, it is not optimal to adjust capital requirements so as to equalize bank failure probabilities. If sectors have different productivities, the profile of optimal capital requirements is further affected by the relationship between risk and productivity. If the relationship is negative (i.e., high productivity sectors have low loan portfolio risk and vice versa), it is possible that optimal capital requirements coincide with or are even steeper with respect to loan portfolio risk than the current (equal-bank failure probability) capital requirements. On the other hand, if risk and productivity are positively correlated, then the "flattening" of optimal capital requirements is further amplified. Ultimately, the slope of the optimal capital requirements may be an empirical question, depending on the correlation between risk and productivity across sectors. Casual empirical evidence suggests that positive correlation between the two may well be relevant; higher productivity sectors often exhibit investments in new technologies which also imply higher risk taking.

Figure 4: Equal bank failure capital requirements



We further illustrate the "flattening" effect of optimal capital requirements (in the benchmark case of no productivity differences across sectors) with Figure 4, based on the above proposition, where sectoral portfolio risk (η**)** is on the horizontal axis, and capital requirements on the vertical axis. The blue line depicts the optimal capital requirements as a function of risk. The red line plots the capital requirements that arise from the policy rule where sector 0 is the benchmark sector.

3.4 Upward revision of bank failure risk

After the global financial crisis the current capital requirements based on the Basel framework have been increased considerably. The overall level of requirements has been increased but the risk weights have largely remained the same (except for the addition of a modest leverage ratio requirement, which effectively sets a floor to the lowest risk weights). Through the lens of our model, the impetus behind this move from the pre-crisis Basel II to the post-crisis Basel III is best interpreted as an upward revision in the perceived loan portfolio risk. In other words, although the actual crisis dynamics were complicated, the crisis revealed that bank asset risks were greater than had been thought. We next assess the regulatory reform against our model with the help of the following proposition.

Proposition 3 (Upward revision of portfolio risk)**.** *A proportional increase in portfolio risk across the board leads to higher capital requirements with more frequent bank failures and, if the increase is not too large, flatter capital requirements*

Suppose $\eta_j^{new} = c\eta_j \quad \forall : j$ *where* $c > 1$*. Then,*

- $\kappa_j^{*new} > \kappa_j^* \quad \forall\, j$
- $\Psi_j^{*new} > \Psi_j^* \quad \forall\, j$
- $\frac{\partial \kappa_j^{*new}}{\partial \eta_j} < \frac{\partial \kappa_j^*}{\partial \eta_j} \quad \forall\, j$ *if* c *is not too large.*

The first two parts of the proposition follow from Propositions 1 and 2 whereas the third result arises from concavity of the optimal capital requirements in portfolio risk (η).[20]

Proposition 3 suggests that, following a view that risks were previously underestimated (before the Global Financial Crisis), the reform to raise capital requirements across the board (post-crisis) should probably entail a relative flattening of risk weights as well. Note that the proposition compares the old capital requirement scheme with the new one assuming that both are optimal schemes. However, the previous (and current) risk-weighted capital regulation under the Basel framework is best characterized by the equal-bank failure probability capital requirement scheme introduced in the previous section. Consequently, if we want to implement the optimal capital requirement scheme starting from a 'sub-optimal' Basel II-type of regulatory framework and at the same time account for higher loan portfolio risks across the board, then by Propositions 2 and 3 the new set of capital requirements should be even flatter.

[20] As with the other propositions this result relies on the resulting bank failure probabilities to be sufficiently low (e.g. $\max(\Psi(\kappa^{*new})) < \Phi(-1) = 15.87\%$).

4 Quantitative evaluation

In this section, we match key features of the model to the data to evaluate the quantitative importance of credit mis-allocation arising from a purely risk-based risk-weighting scheme. To do so we use data on internal credit rating grades for commercial loans taken from the Federal Reserve Board survey of large banking organizations as used in Gordy (2000). The survey provides information in terms of shares and default probabilities across seven credit grade categories (using the S&P scale) in banks' commercial loan books. The shares and average (annualized) default probabilities are reported in Table 1.

Table 1: Commercial loans by credit grade

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**
Default	0.01	0.02	0.06	0.18	1.06	4.94	19.14
Share	3	5	13	29	35	12	3
Implied Risk (η)	0.025	0.034	0.052	0.078	0.143	0.219	0.275
Constant Productivity ($\bar{A}$)	1.44						
Variable Productivity (A)	0.99	1.14	1.29	1.44	1.59	1.74	1.89

We match the portfolio risk parameter η_j to these default probabilities using the internal ratings-based approach.[21] As the data does not provide a joint distribution of borrower risk and productivity, we consider two cases for the distribution of borrower productivities A_j. First, we assume that all borrowers are equally productive, $A_j = \bar{A}$, and calibrate $\bar{A}$ to match the average total asset to equity ratio of non-financial firms over the last two decades. Second, we assume that riskier borrowers are more productive than safer borrowers and set A_j to seven equally-spaced values such that the average is equal to $\bar{A}$ and that the riskiest borrower is 1.92 times more productive than the safest borrower matching the productivity dispersion estimates in Syverson (2004).[22] To match productivities to leverage, we have assumed that the borrowing constraint θ is

[21] Here we assume a correlation factor of 20 percent.

[22] See Syverson (2011) and Bartelsman and Wolf (2017) for a review of the literature on productivity dispersion in the US and Euro area. Hsieh and Klenow (2009) find productivity differences as high as a ratio of five for China and India.

the same across all borrowers and equal to 0.3 and the average productivity ($\bar{A}$) is 1.44. Next we set the deposit rate R^d and expected return on bank equity ρ equal to the average over the last two decades at two percent and 8.5 percent respectively. Finally, we consider several values for the cost of bank failures γ based on estimates from the literature which range from estimates based on cross-country evidence of small banking crises to estimates of the effects of the recent Global Financial Crisis. In particular we consider a conservative value which implies bank failure costs at approximately 15 percent of output, an intermediate value of 35 percent of output, and a high value of 95 percent of output.[23]

Table 2: Other parameters

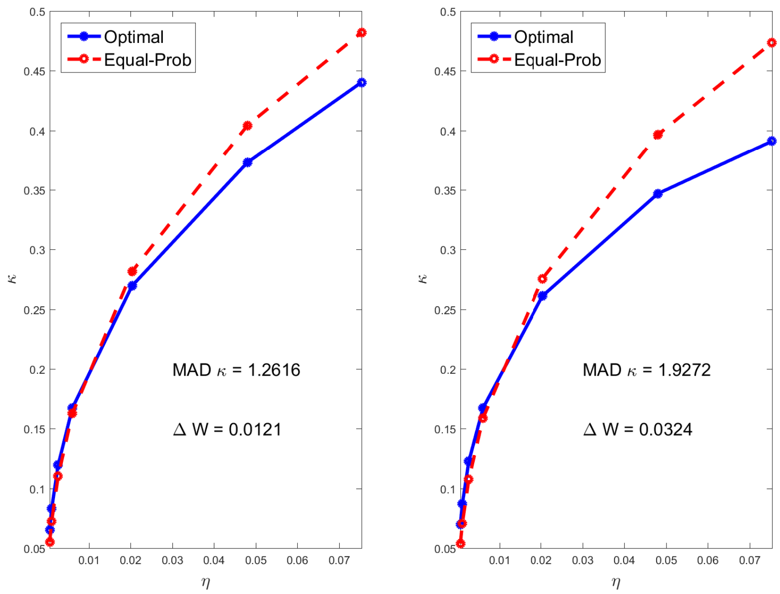
ρ	R^d	θ	$\bar{A}$	σ
1.085	1.02	0.3	1.44	1.43
Bank failure cost as % of output		15%	35%	95%
γ		0.50	1.17	3.20

4.1 Optimal vs Purely risk-based regulation

The following figure plots the set of optimal capital requirements against equal-default-probability requirements given the calibration at a cost of bank failures equal to 15 percent of output. The left plot has productivities constant while the right plot has the riskier borrowers more productive.

The calibration results are reported in the table below. As the table shows, the welfare cost of adopting the purely risk-based requirements appear to be quite modest at up to 0.03 percent change in welfare. This is equivalent to the welfare loss from a regulatory scheme which adopts the correct relative risk weights but gets the average level of capital requirements wrong by (plus or minus) one percent. As we have previously shown, this cost is increasing in the correlation between borrower risk and productivity. Also, the relative difference between the two regulatory schemes becomes smaller the larger the cost of bank failures. In addition, we also calculate the welfare cost of adopting a flat regula-

[23]See Hoggarth et al. (2002) who estimate a 15-20 percent fall in output in response to banking crises and Boyd and Heitz (2016) for estimates around 22-27 percent. On the extreme end of estimates, Andrew Haldane (in his 2010 address *The $100 billion question* at the Institute of Regulation and Risk) posits a cost of about 1 to 5 times GDP for systemic crises due to their persistent effects.



tory scheme where all borrowers are charged the same capital requirements equal to the average of the optimal set. This scheme, equivalent to a simple leverage ratio requirement on banks, generates marginally higher welfare losses an order of magnitude larger and up to 0.6 percent.

Table 3: Comparison of risk weighting schemes I

		Failure cost 15 %		**Failure cost 35 %**		**Failure cost 95 %**	
		Equal A	**25 % A Diff**	**Equal A**	**25 % A Diff**	**Equal A**	**25 % A Diff**
Equal Prob	**Bank Failure rate**	0.89	1.05	0.32	0.37	0.10	0.12
	MAD Requirements	1.26	1.93	0.98	1.50	0.77	1.19
	Welfare difference	0.01	0.03	0.01	0.02	0.01	0.02
Flat rate	**Leverage Ratio**	25.60	24.90	31.99	31.37	38.96	38.40
	MAD Requirements	8.19	8.14	10.37	11.07	13.87	15.09
	Welfare difference	0.33	0.25	0.48	0.40	0.64	0.56

The first three rows report results when comparing the purely risk-based requirements with the optimal. The first row reports the target bank failure rate under this policy. The second row reports the mean absolute difference (MAD) in effective capital requirements. The third row reports the Welfare loss in percentage terms. The fourth to sixth rows report differences with respect to a pure leverage ratio requirement. The fourth row reports the capital requirement for all sectors and the fifth and sixth rows are analogous to the second and third. The columns reflect different assumptions with regard to the bank failure costs (from 15 to 95 percent of output) and productivity differences across sectors.

Table 4: Comparison of risk weighting schemes II

		Failure cost 15 %		Failure cost 35 %		Failure cost 95 %	
		Equal A	**25 % A Diff**	**Equal A**	**25 % A Diff**	**Equal A**	**25 % A Diff**
Equal Prob	**Bank Failure rate**	1.06	1.24	0.38	0.43	0.12	0.13
	MAD Requirements	1.33	2.15	0.98	1.61	0.74	1.24
	Welfare difference	0.01	0.03	0.01	0.02	0.01	0.02
Level shift	**MAD Requirements**	0.16	0.20	0.16	0.18	0.20	0.18
	Welfare difference	0.01	0.02	0.01	0.02	0.02	0.02
Flat rate	**Leverage Ratio**	29.77	28.96	36.99	36.32	44.21	43.64
	MAD Requirements	8.99	8.87	10.67	11.53	13.93	15.37
	Welfare difference	0.33	0.24	0.47	0.38	0.61	0.53

The first three rows report results when comparing the purely risk-based requirements with the optimal. The first row reports the target bank failure rate under this policy. The second row reports the mean absolute difference (MAD) in effective capital requirements. The third row reports the Welfare loss in percentage terms. The fourth and fifth rows report differences with respect to a level increase in requirements whereas the sixth to eight rows are for the pure leverage ratio requirement. The columns reflect different assumptions with regard to the bank failure costs (from 15 to 95 percent of output) and productivity differences across sectors.

4.2 Response to increased risk

In a second exercise, we simulate a change that motivates the need for higher capital requirements by raising the riskiness of all borrowers by the same factor such that the average new optimal set of risk-weighted capital requirements is five percent higher much like the terminal requirement for common equity Tier 1 ratios following the implementation of Basel III requirements.[24]

We consider three schemes along with the new optimal set of requirements as a benchmark. These are (1) the new equal-default-probability requirements which sets a new constrained-optimal target Ψ^*, (2) a simple increase of five percent of the previous set of equal-default-probability requirements - a level shift, and (3) a flat rate regulatory regime equal to the average of the optimal set of capital requirements. The following table reports welfare losses for the various schemes.

We find that the increase in riskiness has led to a marginal increase in the optimal average bank failure rates and a negligible change in welfare across the various schemes. The current, purely risk-based, risk weighting scheme generates a difference in average effective capital requirements of 0.98-1.93 percent. This generates a credit spread (riskiest to safest sectors) which is 64 basis points higher than the one under the optimal set of risk weights and results in the riskiest sector

[24] This is done by raising the correlation factor from 20 to 30 percent in the calculation of η_j from the default probabilities in the data using the internal ratings-based approach.

borrowing 1.22 percent less than they otherwise would have.[25] Based on the results, the data suggests that given level of variation and risk across commercial loans, credit mis-allocation arising from purely risk-based risk-weighted capital regulation does not generate significant welfare losses.

5 Concluding Remarks

We have revisited the question of optimal capital requirements for banks from the viewpoint of risk weights. In the current bank capital regulation, a key principle is to set the general level of minimum capital requirements and the risk weights applied to bank loans in a way that restricts the bank's probability of failure to a desired maximum level which is equal for all banks. A risk weight should reflect a loan's contribution to the bank's loan portfolio risk. However, recent literature has suggested several reasons why purely risk-based capital requirements may not be optimal, and have therefore argued for less "risk-sensitive" capital requirements. The current risk-weighting system may be too prone to manipulation, it may spur excessive growth of the more lightly regulated "shadow banking" sector, and it may distort credit allocation away from the more productive sectors of the economy.

Our contribution is to show that the pure risk perspective of setting capital requirements can be too narrow from the viewpoint of optimal credit allocation. The optimal risk weights should also take into account borrowing constraints and possible productivity differences across borrower firms. We find that when firms face a borrowing constraint, the optimal capital requirements are flatter (as a function of risk) than those that mimic the current regulation in our model. This result obtains even if there are no productivity differences across sectors. When productivity and risk are positively correlated across sectors, the flattening effect is amplified. If the correlation between risk and productivity is negative, optimal risk weights could coincide with the current ones, or be even steeper. Casual empirical evidence suggests that positive correlation between risk and

[25] This is for the case where bank failure costs are at 15 percent of output and the riskiest sector is 1.92 times more productive than the safest sector.

productivity may well be the most relevant case. For example, higher productivity sectors often exhibit investments in new technologies which also imply higher risk taking.

As regards future research, our model has assumed competitive banks, which has an effect on how the cost of capital requirements is transmitted to borrower loan rates and hence credit allocation. An extension to the current setup could be to consider the effect of imperfect bank competition on credit allocation and hence optimal capital requirements. Second, in the current paper we have studied optimal capital requirements under the risk and productivity relationship, but have not considered differences in collateral constraints across firms. While this is a reasonable starting point, a further extension could be to consider differences in collateral constraints and how their interaction with risk and productivity affects the choice of optimal capital requirements. Third, one could consider social costs of bank failures, which are convex in bank size. In the model, all agents are risk neutral, so convex costs could be one way to introduce similar effects that would arise if some agents were risk averse. We conjecture that this could in actuality mitigate, possibly even reverse, the flattening result.

The qualitative results of our paper hopefully shed further light on policy-oriented discussions that have called for less risk-sensitive capital requirements, being motivated by concerns regarding business lending and economic growth. A next step would be to extend the model to a general equilibrium setting and calibrate it, in order to provide a more rigorous quantitative assessment of the optimal capital requirements vis-á-vis the actual regulations.

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A Deposit insurance

We assume deposit insurance to protect depositors in case of bank failure and financed by premia charged on banks. In this section we show that such a scheme generates enough funds to sufficiently insure deposits as well as preserve the determination of return on bank equity detailed in the main text. A deposit insurance fund is assumed to cover deposits made in retail banks by guaranteeing a return R^d in case of bank failure. The scheme is funded by insurance premia collected from surviving banks and the liquidation of assets from failed banks.

First, consider an insurance premium (s) such that the expected return on bank equity is still equal to ρ,

$$\begin{aligned} \rho &= \frac{(1-\Psi)}{\kappa}\left[\mathbb{E}[\xi]^{+}(\frac{R^B B}{B}) - (1-\kappa)R^d - \frac{s}{B}\right] \\ \Rightarrow & \\ \frac{s}{B} &= (\mathbb{E}[\xi]^{+} - 1)R^B - \frac{\Psi}{1-\Psi}\kappa\rho \end{aligned}$$

where $\mathbb{E}[\xi]^{+}$ and $\mathbb{E}[\xi]^{-}$ are conditional expectations of the portfolio shock in the case of bank failure and survival respectively. Note that specifying the premium in this way completely offsets the risk-taking motive that arises from limited liability. For the premium to be sufficient to cover the shortfall in the bank's asset value, we need

$$\begin{aligned} (1-\Psi)\frac{s}{B} &\geq \psi\left((1-\kappa)R^d - \mathbb{E}[\xi]^{-}R^B\right) \\ &\Rightarrow \\ 1 &\leq (1-\Psi)\mathbb{E}[\xi]^{+}\Psi\mathbb{E}[\xi]^{-} \\ 1 &= \mathbb{E}[\xi] \end{aligned}$$

This shows that an insurance premium s collected on surviving banks is sufficient both to make the expected return on bank capital equal to ρ and to finance the shortfall in asset value for failed banks needed in deposit insurance. For an economy with a continuum of sectors with potentially different rates of bank failure (Ψ) and collected premia, the above also guarantees that the deposit insurance fund breaks even in expectations. Clearly, since portfolio risk (η_j) are independent across sectors, if there is a continuum of banks (sectors) for each failure probability then the fund will always break even. More generally, if in a multi-period setup there exists an institution with sufficient reserves (or has government backing) from which it draws additional funds in times of shortfall and saves the excess in other times then such an institution would also be sustainable. We abstract from these details in our partial equilibrium setup.

In the absence of pecuniary costs and externalities, this insurance scheme makes bank failures irrelevant for social welfare. On the other hand, if there are additional costs to bank failure, in the bank failure resolution process or upkeep of the insurance fund for example, then such arguments would justify limiting the incidence of bank failure by imposing higher capital requirements. This is because these costs create a wedge between the expected value of the insurance fund from the premia collected and the needed bailout funds such that lump-sum taxation is necessary to balance the insurance funds' budget. In this case, the social planner must then evaluate the relative cost of preserving deposit insurance to taxing households. We assume that such costs are present and capital requirements are set to mitigate these costs.

B Extensions

B.1 Alternative policy objective

Consider a policy rule which jointly takes into account the probability of bank failure and the size of (societal) loss given default by maximizing aggregated risk-adjusted loan portfolios. In this case, the regulator recognizes that raising capital requirements will shrink the absolute size of that sector's bank's balance sheet.

$$\max_{\{\kappa\}} \int R^b B - \gamma \Psi R^b B$$
$$\text{s.t.} 0 \leq \kappa_j \leq 1$$

The solution to this problem yields a capital requirement in which maximizing risk-adjusted loan volume discounts the effect of the collateral constraint on the borrower.

$$\theta\frac{\partial K}{\partial \kappa} = \gamma\theta\left(K\frac{\partial \Psi}{\partial \kappa} + \Psi\frac{\partial K}{\partial \kappa}\right)$$

Note that the left-hand side of this condition is considerably smaller than the same condition in the main text. That is to say that this policy rule underweights the effect of capital requirements on entrepreneur output relative to the welfare-maximizing social planner's solution.

B.2 Endogenous portfolio risk

Consider now the case where the rest of the model is as before but the banker faces the same price shock as the borrower whenever it liquidates the assets of defaulting borrowers. Since the banker knows that she will be holding a portfolio of loans of which some may default, her participation constraint becomes

$$R_j^b B_{i,j} \leq \mathbb{E}[\epsilon_j^b]\theta_j K_{i,j} = \theta_j K_{i,j}$$

where $\epsilon_j^b \sim log\mathcal{N}(\sigma^2/2, \sigma^2)$ is the price that the banker faces when she sells the assets of all defaulting borrowers. Note that, although it has the same distribution, this shock is applied to all the assets that the bank has to liquidate and is independent to the shocks which would trigger individual entrepreneurs' default. Thus, the expected price the banker faces when selling such assets is uncorrelated with the actual incidence of default. The banker is willing to lend as much to the entrepreneur as what she would expect to get from selling the specialized good herself. Note that this is the same constraint as in the main text which results in the same equilibrium loan rates and levels of output and borrowing. What changes in this formulation of the model is that now a fraction of the banks' assets are going to be exposed to the same price shock that the borrowers are and, even in the absence of an additional portfolio shock, banks may fail. Consequently, we replaced the bank portfolio shock (ξ_j) in the main text with the price shock ϵ_j^b and note that the fraction $(1 - \Phi_j)$ of borrowers will repay their debt ($R_j^b B_j$) and only the fraction Φ who default will subject the banker to risk as she will collect $\epsilon_j^b \theta_j K_j$. Denote the fraction of deposits exposed to this risk after deducting repayments by borrowers with z_j,

$$\begin{aligned} z_j d_j &\equiv d_j - (1-\Phi_j) B_j \frac{R_j^b}{R^d} \\ &= d_j \left[1 - (1-\Phi_j) \left(1 + \frac{\kappa_j}{1-\kappa_j} \frac{\rho}{R^d} \right) \right] \\ &\Rightarrow \\ z_j &= 1 - (1-\Phi_j) exp(\tilde{\kappa_j}) \end{aligned}$$

Then, a bank fails when the risky fraction of its assets are insufficient to cover the fraction of deposits exposed to risk:

$$\begin{aligned} \Psi_j &= Pr(\Phi_j \epsilon_j^b \theta_j K_j \leq R^d d_j z_j) \\ &= Pr(\epsilon_j^b \leq \frac{z_j}{\Phi_j} \left[1 + \frac{\kappa_j}{1-\kappa_j} \frac{\rho}{R^d} \right]^{-1}) \\ &= Pr(log \epsilon_j^b \leq log(\frac{z_j}{\Phi_j}) - \tilde{\kappa}_j) \\ &= \Phi(\frac{\sigma_j}{2} - \frac{\tilde{\kappa}_j}{\sigma_j} + \frac{1}{\sigma_j} log(\frac{z_j}{\Phi_j})) \\ &= \Phi(\frac{\sigma_j}{2} - \frac{\tilde{\kappa}_j}{\sigma_j} + \frac{1}{\sigma_j} log(\frac{1}{\Phi_j} - \frac{(1-\Phi_j)}{\Phi_j} exp(\tilde{\kappa_j}))) \end{aligned}$$

Note that in this alternative formulation, there is a third term determining the likelihood of bank failure which captures the share of bank assets exposed to risk (defaulting borrowers) relative to the share of deposits exposed to risk. This third term depends on both the riskiness of the borrowers ($\Phi(\sigma)$) and the bank's leverage which amplifies the effects of both (or what would have been η for risk) in the setting of optimal capital requirements. On the one hand, a bank's failure probability from this alternative version of the model will be more sensitive to borrower risk than in the baseline model. On the other hand, capital requirements are also more effective in reducing the probability of bank failure in this alternative version.[26] On balance, we obtain qualitatively similar outcomes as in the main text.

As before, the optimal capital requirements trade off the cost of bank failure and the borrowers' credit constraint given by the following optimality condition:

$$\overbrace{(1+\Phi(\theta-\bar{\epsilon}^d))\frac{\partial K}{\partial \kappa}}^{\text{Entrep collateral constraint}} = \overbrace{\gamma\theta\left(K\underbrace{\frac{\partial \Psi}{\partial \kappa}}_{\text{Frequency}} + \Psi\underbrace{\frac{\partial K}{\partial \kappa}}_{\text{Size}}\right)}^{\text{Bank bankruptcy}}$$

This is the same condition in the main text with the subtle difference that the frequency of bank failures Ψ now has three terms (as outlined above) and the sensitivity of the frequency of bank failure to changes in the capital requirement is now given by

$$\frac{\partial \Psi}{\partial \kappa} = -\frac{\psi(\cdot)}{\sigma}\frac{\partial \tilde{\kappa}}{\partial \kappa}\left[1+\frac{1-\Phi}{z}exp(\tilde{\kappa})\right]$$

where in the main text we have $\frac{\partial \Psi}{\partial \kappa} = -\frac{\psi(\cdot)}{\sigma}\frac{\partial \tilde{\kappa}}{\partial \kappa}$. Here, since risk only affects a fraction of banks' assets, capital requirements are more effective in reducing the likelihood of bank failures which suggests that the level of optimal capital requirements under this alternative setup will be lower than those given in the main text (if η were equal to σ).

[26]Note as well that in this alternative version, it is sufficient to set $\kappa_j \geq \left[1+\frac{1-\Phi_j}{\Phi_j}\frac{\rho}{R^d}\right]^{-1}$ to guarantee that a bank never fails.

C Proofs

C.1 Proof of Proposition 1

We first prove the first two lemmas. Note that the first and second order conditions for optimality given an interior solution require that,

$$\begin{aligned} \frac{\partial w_j}{\partial \kappa_j^*} &= 0 && (7)\\ \frac{\partial^2 w_j}{\partial \kappa_j^{*2}} &< 0 && (8)\\ \text{where} & && (9)\\ w_j &= \left[1 + \Phi_j(\theta_j - \bar{\epsilon}^d) - \gamma\theta_j\Psi_j\right] K_j && (10) \end{aligned}$$

For any parameter x whenever $\frac{\partial^2 w_j}{\partial \kappa_j \partial x} > 0 \Rightarrow \partial\kappa^*/\partial x > 0$.[27] It is useful to note the following,

$$\begin{aligned} \frac{\partial K}{\partial \kappa} &= -\theta(\rho - R^d)\left[\frac{K}{R^b}\right]^2 < 0\\ \frac{\partial \Psi}{\partial \kappa} &= -\frac{\psi(\cdot)}{\eta}\left[\frac{\partial\tilde{\kappa}}{\partial\kappa}\right] < 0\\ \frac{\partial\tilde{\kappa}}{\partial\kappa} &= \frac{\rho}{(1-\kappa)R^b} > 0 \end{aligned}$$

where $\psi(\cdot) \equiv (2\pi)^{-\frac{1}{2}} exp(-(\frac{\eta}{2} - \frac{\tilde{\kappa}}{\eta})^2/2)$ is the normal pdf evaluated at the

[27]Implicit function theorem.

standardized value of the capital requirement. We now show that $\frac{\partial^2 w}{\partial\kappa\partial\eta} > 0$ since

$$\begin{aligned}
\frac{\partial^2 w_j}{\partial\kappa_j\partial\eta} &= -\gamma\theta\left[\frac{\partial K}{\partial\kappa}\frac{\partial\Psi}{\partial\eta} + K\frac{\partial^2\Psi}{\partial\kappa\partial\eta}\right] \\
&> 0 \quad \text{since} \\
\frac{\partial\Psi}{\partial\eta} &= \psi(\cdot)(\frac{1}{2} + \frac{\tilde{\kappa}}{\eta^2}) > 0 \\
\frac{\partial^2\Psi}{\partial\kappa\partial\eta} &= \frac{\partial\tilde{\kappa}}{\partial\kappa}\frac{\psi(\cdot)}{\eta^2}\left[1 + \frac{\eta^2}{4} - \frac{\tilde{\kappa}^2}{\eta^2}\right] \\
&< 0 \qquad \text{if} \quad \tilde{\kappa}^2 > \frac{\eta^4}{4} + \eta^2 \\
&\Leftrightarrow \\
\frac{\partial\kappa^*}{\partial\eta} &> 0
\end{aligned}$$

where $\tilde{\kappa}^2 > \frac{\eta^4}{4} + \eta^2$ is satisfied for reasonable (i.e. low) values of portfolio risk that generate low optimal failure probabilities $\Psi(\kappa^*)$. Let Ψ^* be the bank failure probability under the optimal capital requirement. Then,

$$\begin{aligned}
\tilde{\kappa}^2 &= \left(\frac{\eta^2}{2} - \eta\Phi^{-1}(\Psi^*)\right)^2 \\
&= \frac{\eta^4}{4} + \eta^2 + \eta^2\left[(\Phi^{-1}(\Psi^*))^2 - 1 - \eta\Phi^{-1}(\Psi^*)\right] \\
&> \frac{\eta^4}{4} + \eta^2 \quad \text{iff} \\
&\quad (\Phi^{-1}(\Psi^*))^2 - 1 - \eta\Phi^{-1}(\Psi^*) > 0 \\
&\Leftrightarrow \\
\Psi^* &< \Phi(\frac{\eta}{2} - \sqrt{\frac{\eta}{4} + 1})
\end{aligned}$$

where the last inequality is trivially satisfied when $\Psi^* < \Phi(-1) = 0.1587$. Note that this is a sufficient and not necessary condition for the lemma.

Similarly We now show that $\frac{\partial^2 w}{\partial\kappa\partial A} < 0$,

$$\begin{aligned}
\frac{\partial^2 w_j}{\partial\kappa_j \partial A} &= (R^b - \theta A)^{-2}\left[2\theta K_j(\rho - R^d)\left(\gamma\theta\Psi - (1+\Phi(\theta-\bar{\epsilon}^d))\right) - \gamma\theta R^{b2}\frac{\partial\Psi}{\partial\kappa}\right] \\
&< 0 \quad \text{iff} \\
& \quad \left[1+\Phi(\theta-\bar{\epsilon}^d) - \gamma\theta\Psi\right]\frac{\partial K}{\partial\kappa} < \frac{\gamma\theta K}{2}\frac{\partial\Psi}{\partial\kappa} \\
& \quad \frac{\partial w_j}{\kappa_j} = 0 < -\frac{\gamma\theta K}{2}\frac{\partial\Psi}{\partial\kappa} \\
&\Leftrightarrow \\
\frac{\partial\kappa^*}{\partial A} &< 0
\end{aligned}$$

Consequently, we can write the optimal capital requirement as a function of two arguments $\kappa^*(A,\eta)$ which is decreasing in A and increasing in η. The proof of Proposition 1 arises from defining the thresholds $\underline{A}(\eta_j;\kappa_0^*)$ such that $\kappa^*(\underline{A},\eta_j) = \kappa_0^*$ and $\bar{\eta}(A_j;\kappa_0^*)$ such that $\kappa^*(A_j,\bar{\eta}) = \kappa_0^*$.

Finally, an interior solution exists when,

$$\begin{aligned}
1+\Phi(\theta-\bar{\epsilon}^d) - \theta\gamma\Psi > 0 \quad \text{for some } \kappa \in [0\ 1] \quad &\Rightarrow \quad \kappa^* < 1 \\
\frac{\partial\Psi}{\partial\kappa} < 0 \quad &\Rightarrow \quad \kappa^* > 0
\end{aligned}$$

C.2 Proof of Proposition 2

The proof follows from the previous Proposition. The first lemma showed that $\kappa_j^* > \kappa_k^*$ whenever $\eta_j^2 > \eta_k^2$. Next we show that $\Psi(\kappa_j^*) > \Psi(\kappa_k^*)$

Let sector 0 have $\Psi(\kappa_0^*) = \Psi^*$. Consider now the equal bank failure probability capital requirement scheme where $\tilde{\kappa}_j^{epd} = \frac{\eta_j^2}{2} - \eta_j\Phi^{-1}(\Psi^*)$ and sector 0 be such that $\Psi(\tilde{\kappa}_0^{epd}) = \Psi^*$ where Ψ^* solves

$$\int_j -\eta\frac{\partial\kappa}{\partial\tilde{\kappa}}\left[-\gamma\theta K\frac{\partial\Psi}{\partial\kappa} + (1+\Phi(\theta-\bar{\epsilon}^d) - \gamma\theta\Psi)\frac{\partial K}{\partial\kappa}\right] = 0$$

That is, both the optimal capital requirement and the equal bank failure probability capital requirement scheme coincide for sector 0. Then, note that

$$\begin{aligned}
\tilde{\kappa}_j^{epd} - \frac{\eta_j^2}{2} &= \frac{\eta_j}{\eta_0}(\tilde{\kappa}_0^{epd} - \frac{\eta_0^2}{2}) \\
\text{and} & \\
\frac{\partial \tilde{\kappa}^{epd}}{\partial \eta} &= \frac{\eta_j}{2} + \frac{\tilde{\kappa}_j^{epd}}{\eta_j}
\end{aligned}$$

where the derivative assumes that $\frac{\partial \Psi^*}{\partial \eta_j} \to 0$ or a marginal increase in capital requirements for a given sector does not significantly change the target failure probability. On the other hand, consider the sensitivity of the optimal capital requirements to portfolio risk. Using the implicit function theorem,

$$\begin{aligned}
\frac{\partial \tilde{\kappa}^*}{\partial \eta} &= -\left[\frac{\partial^2 w_j}{\partial \tilde{\kappa}_j \partial \eta_j}\right]\left[\frac{\partial^2 w_j}{\partial \tilde{\kappa}_j^2}\right]^{-1} \\
\text{where} & \\
\frac{\partial^2 w}{\partial \tilde{\kappa} \partial \eta} &= \frac{\theta\gamma\psi(\cdot)}{\eta}\left[\frac{K}{\eta}\left(\frac{\tilde{\kappa}_j^2}{\eta^2} - \frac{\eta_j^2}{4} - 1\right) - \frac{\partial K}{\partial \tilde{\kappa}}(\frac{\eta_j}{2} + \frac{\tilde{\kappa}_j}{\eta_j})\right] \\
\text{thus} & \\
\frac{\partial \tilde{\kappa}^*}{\partial \eta} &= -\theta\gamma\frac{\psi(\cdot)}{\eta}\left[\left(\frac{\tilde{\kappa}_j^2}{\eta^2} - \frac{\eta_j^2}{4} - 1\right)\frac{K}{\eta} - (\frac{\eta_j}{2} + \frac{\tilde{\kappa}_j}{\eta_j})\left(\frac{\partial K}{\partial \tilde{\kappa}}\right)\right]\left[\frac{\partial^2 w_j}{\partial \tilde{\kappa}_j^2}\right]^{-1} \\
\text{where} & \\
\frac{\partial^2 w_j}{\partial \tilde{\kappa}_j^2} &= (1 + \Phi(\theta - \bar{\epsilon}^d) - \theta\gamma\Psi)\frac{\partial^2 K}{\partial \tilde{\kappa}^2} - 2\theta\gamma\frac{\partial K}{\partial \tilde{\kappa}}\frac{\partial \Psi}{\partial \tilde{\kappa}} - \theta\gamma K\frac{\partial^2 \Psi}{\partial \tilde{\kappa}^2}
\end{aligned}$$

Since $\frac{\partial w}{\partial K} > 0$ and $\frac{\partial K}{\partial \tilde{\kappa}} < 0$, it must be the case that

$$\frac{\partial \tilde{\kappa}^*}{\partial \eta} \leq \frac{\partial \tilde{\kappa}^*}{\partial \eta}\Big|_{\frac{\partial K}{\partial \tilde{\kappa}}=0}$$

That is, if investment is inelastic to capital requirements then the optimal capital requirement is more sensitive to portfolio risk. This hypothetical sensitivity is given by

$$
\begin{aligned}
\frac{\partial \tilde{\kappa}^*}{\partial \eta}\Big|_{\frac{\partial K}{\partial \tilde{\kappa}}=0} &= \left[-\frac{\theta\gamma K\psi(\cdot)}{\eta^2}\left(\frac{\tilde{\kappa}_j^2}{\eta^2}-\frac{\eta_j^2}{4}-1\right)\right]\left[\frac{\theta\gamma K\psi(\cdot)}{\eta^2}\left(\frac{\eta}{2}-\frac{\tilde{\kappa}}{\eta}\right)\right]^{-1} \\
&= \left[\left(\frac{\eta}{2}+\frac{\tilde{\kappa}}{\eta}\right)\left(\frac{\eta}{2}-\frac{\tilde{\kappa}}{\eta}\right)+1\right]\left[\left(\frac{\eta}{2}-\frac{\tilde{\kappa}}{\eta}\right)\right]^{-1} \\
&= \left(\frac{\eta}{2}+\frac{\tilde{\kappa}}{\eta}\right)+\frac{2\eta}{\eta^2-2\tilde{\kappa}} \\
&< \left(\frac{\eta}{2}+\frac{\partial\tilde{\kappa}}{\eta}\right)=\frac{\tilde{\kappa}^{epd}}{\partial \eta}
\end{aligned}
$$

where the last inequality follows from the assumption that $\tilde{\kappa}^2 > \frac{\eta^4}{4} + \eta^2$ (i.e. parameters are such that the optimal failure probabilities are sufficiently low). Thus we have shown that,

$$
\frac{\partial \tilde{\kappa}^*}{\partial \eta} \leq \frac{\partial \tilde{\kappa}^*}{\partial \eta}\Big|_{\frac{\partial K}{\partial \tilde{\kappa}}=0} < \frac{\tilde{\kappa}^{epd}}{\partial \eta}
$$

That is, the optimal capital requirements are less sensitive to increases in portfolio risk than the equal bank failure probability scheme. This implies that, all else equal, for $\eta_j > \eta_0$ we have $\kappa_0^* < \kappa_j^* < \kappa_j^{epd}$ and $\Psi(\kappa_j^*) > \Psi(\kappa_j^{epd})$.

Similarly, the capital requirement given by this policy scheme for a sector with a lower portfolio risk is lower than the optimal capital requirement hence the Proposition.

C.3 Proof of Proposition 3

In this scenario, we have that $\eta_j^{new} = c\eta_j \quad \forall\, j$ with $c > 1$. From Proposition 1, we know that higher portfolio risk leads to higher capital requirements,

$$
\kappa_j^{*new} > \kappa_j^* \quad \forall\, j
$$

From Proposition 2, we also know that the higher capital requirements will not completely offset the rise in failure probability such that,

$$
\Psi_j^{*new} > \Psi_j^* \quad \forall\, j
$$

We only need to show that the new and higher capital requirements are also relatively flatter than the previous set of capital requirements. Here, it is sufficient to show that the optimal capital requirements are (increasing and) concave in portfolio risk.

From Proposition 2, we know that $\frac{\partial \tilde{\kappa}^*}{\partial \eta} < \frac{\partial \tilde{\kappa}^{epd}}{\partial \eta}$. We now show that the equal bank failure probability capital requirement scheme is also concave in portfolio risk.

$$\begin{aligned}
\frac{\partial \tilde{\kappa}^{epd}}{\partial \eta} &= \left(\frac{\eta}{2} + \frac{\tilde{\kappa}^{epd}}{\eta}\right) \\
\Rightarrow & \\
\frac{\partial^2 \tilde{\kappa}^{epd}}{\partial \eta^2} &= \frac{1}{\eta}\left(\frac{\eta}{2} - \frac{\tilde{\kappa}^{epd}}{\eta}\right) \\
&< 0
\end{aligned}$$

where the last inequality holds for any bank failure probability less than one half (well above our previous assumption of $\Psi < \Phi(-1)$). Thus we have that (1) the equal bank failure probability capital requirement scheme is increasing and concave in portfolio risk (η) and (2) its slope is larger than that of the optimal capital requirement scheme. Thus, it must be the case that the optimal capital requirements are also concave in portfolio risk. Finally, given that the optimal capital requirements are concave in portfolio risk, a proportional increase in risk across the board would also lead to flatter requirements for so long as the increase does not lead to a violation of our assumption regarding the upper bound on bank failure probabilities: $max(\Psi(\kappa_j^{*new})) < \Phi(-1)$.