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The History of Economics and the Pre-History of Econophysics: Boltzmann versus the Marginalists

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ABSTRACT

Given the acknowledged influence of classical physics on the development of neoclassical economic theory, the emergence of the 'new' subject of econophysics in the mid-1990's poses a methodological and historical quandary. This paper explores this quandary by contrasting developments in the history of economics with the prehistory of econophysics. In particular, what role did physics play in the contributions by the marginalists to the subsequent development of neoclassical economics? And, what is the relationship to the ergodicity hypothesis introduced by L. Boltzmann that is essential to the history of statistical mechanics and the pre-history of econophysics? This paper addresses these questions by contrasting the methods and philosophy adopted from classical physics by Edgeworth and other marginalists with those of the ergodicity hypothesis introduced into late 19th century statistical mechanics by Boltzmann. An interpretation of ergodicity is provided that establishes a motivation for the emergence of econophysics during the 1990's. The relevance of ergodicity to the stochastic models of modern economics is also identified.

Keywords: Ergodicity; Ludwig Boltzmann; Econophysics; Neoclassical Economics; Marginal Revolution; Phenomenology.

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> "Rational mechanics gives us a first approximation to theory of the equilibrium and of the movements of bodies. In the same way the theories of Jevons, Walras, Marshall, Fisher, and others present us with a first approximation to the theory of economic phenomena."
> Vilfredo Pareto (1897, p.499).

1. Introduction

At least since Mirowski (1984), it has been widely recognized that important theoretical elements of neoclassical economics were adapted from mathematical concepts developed in 19th century classical physics. Much effort by historians of economic thought and others has been dedicated to exploring the validity and implications of "Mirowski's thesis", e.g., Carlson (1997); Ganley (1995); De Marchi (1993), especially the connection between the deterministic 'rational mechanics' approach of classical physics and the subsequent development of neoclassical economic theory (Mirowski 1988; Boumans 1993; Drakopoulos and Katselidis 2015).[1] This well traveled literature connecting classical physics with neoclassical economics is seemingly incongruent with the emergence of econophysics during the last decade of the twentieth century.[2] Econophysics is now a "recognized field" (Roehner 2002, p.23) within physics that involves the application of theoretical and empirical methods from statistical mechanics to the analysis of economic phenomena, e.g., Mantegna and Stanley (2000); Stanley et al. (2008); Jovanovic and Schinckus (2012); Schinckus and Jovanovic (2013); Sornette (2014); Ausloos et al. (2016). As such, the field of 'econophysics' encompasses contributions, appearing primarily in physics journals, on a number of topics that overlap with economics.

Surveys on contributions to econophysics reveal a number different threads, ranging from empirical studies on the distributional properties of financial data to theoretical studies of multi-agent order flow models, e.g., Chakraborti et al. (2011a,b). The common theme among these threads is the application of methods from statistical mechanics to traditional topics in the domain of economics. Significantly, this theme also captures a smattering of contributions by economists stretching back to the work of Pareto on the distribution of wealth. Is it possible that distinguishing 'econophysics' from 'economics' does not depend on identifying specific content or methodology but, rather, on the discipline of the scholarly journals publishing the research? Seeking to reveal more points of demarcation between econophysics and economics, this paper contrasts the history of economics with the prehistory of econophysics. In particular, the introduction into economics by Francis Ysidro Edgeworth (1845-1926) and other marginalists of mathematical methods adapted from classical physics is contrasted with the contemporaneous contribution of ergodicity to statistical mechanics by Ludwig Boltzmann (1844-1906). Tracing the complicated, subsequent evolution of stochastic concepts into economics reveals fundamental methodological differences between economics and econophysics arising from divergent uses of the ergodicity hypothesis to model economic phenomena.

As Mirowski (1989a,b, 1990, 1994) is at pains to emphasize, physical theory has evolved considerably from the deterministic classical approach which inspired neoclassical economics. In detailing historical developments in physics, Mirowski, Sornette (2014) and others are quick to jump from the mechanical determinism of energistics to Ising models and quantum mechanics to recent developments in chaos theory, overlooking the relevance of initial steps toward modeling the stochastic behavior of physical phenomena by Ludwig Boltzmann, James Maxwell (1831-1879) and Josiah Willard Gibbs (1839-1903). The introduction of the 'ergodicity hypothesis' by Boltzmann around the time that Mathematical Psychics (1881) by Edgeworth appeared is relevant to identifying and explaining divergence between the history of economics and the prehistory of econophysics. The emergence and development of stochastic concepts in econometrics initially adopted a form of ergodicity to generalize and empirically test the deterministic models of neoclassical economic theory. In this process, ergodicity permitted statistical techniques designed for repeatable experiments of the natural sciences to be extended to the non-experimental data of economic phenomena.

Early contributors to the econometrics project, such as Frisch, Tinbergen, Wold and Koopmans, featured training in physics or mathematical statistics, subjects where a variety of ergodic concepts are employed. Significantly, initial stochastic generalizations of the deterministic and static equilibrium models of neoclassical economic theory employed 'time reversible' ergodic probability models, especially the likelihood functions associated with (possibly transformed) stationary Gaussian error distributions. In contrast, from early ergodic models of Boltzmann to Ising models of interacting elements to fractals and chaos theory of Mandlebrot, statistical mechanics has employed a wider variety of non-linear, irreversible and chaotic stochastic models aimed at capturing key empirical characteristics of different physical problems at hand. These models often have a phenomenological motivation that varies substantively from the axiomatic basis for constrained optimization techniques that underpin neoclassical economic theory, restricting the straightforward adoption of econophysics models in mainstream economics. As Roehner (2002, p.20) observes:[3]

> From a methodological perspective, the strong emphasis econophysics has put on the search for regularities is probably one of its most important innovations. In contrast, for most economists a quantitative regularity is considered of no interest unless it can be interpreted in terms of agents' motivation and behavior and has a clearly defined theoretical status.

Overcoming the difficulties of adapting stochastic models developed for physical situations applicable to statistical mechanics to fit with the stochastic representation of axiomatic models for economic phenomena is a central problem confronting econophysicists seeking acceptance by mainstream economists, and vice versa.[4]

Schinckus (2010, p.3816) explores the positivist philosophical foundation of econophysics identifying the fundamental role of empirical observation: "The empiricist dimension is probably the first positivist feature of econophysics". Following McCauley (2004) and others, this concern with empiricism often focuses on the identification of macro-level statistical regularities that are characterized by scaling laws, such as those identified by Mandelbrot (1997) and Mandelbrot and Hudson (2004) for financial data. The ergodic hypothesis is an essential link facilitating this empirically driven ideal to confront the 'non-repeatable' experiments that characterize most observed economic and financial data. The quandary posed by having only a single observed *ex post* time path to estimate the distributional parameters for the ensemble of *ex ante* time paths is mirrored in the ergodicity hypothesis initially arising from the kinetic theory of gases. In contrast to the natural sciences, in general, and classical physics, in particular, the human sciences provide no assurance that *ex post* statistical regularity translates into *ex ante* forecasting accuracy creating conceptual difficulties for the types of 'time reversible' ergodic processes employed in econometrics. Exploring this complication highlights the usefulness of identifying the role of the 'ergodicity hypothesis' of Boltzmann in the subsequent evolution from the deterministic models of neoclassical economic theory to the stochastic models of modern economics.

2. From the Marginalists to Neoclassical Economics

Following Colander (2000) and many others, there is considerable confusion and debate about the time line and historical content of 'neoclassical economics'. Aspromourgos (1986) traces the etymology of the term to Veblen (1900) where it is used "to characterize the Marshallian version of marginalism". Recognizing that Veblen employed a "Darwinian" interpretation for the emergence of "neo-classical political economy", this etymology is consistent with neoclassical economics having "a positive, basic relationship to the earlier classical theory and some development beyond it" because "Marshall, more than any of the other marginalist founders, sought to present his theory as having a substantial continuity with classical economics". Building on previous contributions such as Hicks (1932), Stigler (1941) popularized the extension of neoclassical economics to include "all the marginalist founders". This interpretation is reflected in Colander (2000, p.133): "In many ways, the two books that tied up the loose ends and captured the essence of neoclassical economics, Hicks's Value and Capital (1939) and Samuelson's Foundations (1947), were culminating works — they put all the pieces of marginalism together. Important work thereafter was modern." This approach permits the emergence of "modern economics" from 'neoclassical economics' being connected to the transition from deterministic to stochastic models.

Presenting the marginalist founders as a cohesive group that led to the subsequent emergence of neoclassical economics disguises fundamental issues dividing the distinct marginalist approaches of Jevons, Walras, Menger, Edgeworth and Marshall, e.g. De Vroey (2012).[5] In the history of economic thought, such divisions have created difficulties in the search for a cohesive 'marginal revolution', e.g., Blaug (1972); Coats (1972); Hutchison (1972); Steedman (1997). These difficulties are compounded if the search for a cohesive and consistent connection with 19th century classical physics is introduced. Despite considerable variation in usage of 'marginalist' and 'neoclassical', it is still possible to identify certain relevant elements connecting seemingly disparate approaches. De Vroey (2012, p.772) captures one key element: "Equilibrium is at the core of both Marshall's and Walras' reasoning. Both of them adopted the stationary equilibrium conception in which equilibrium is defined as a state of rest." Samuelson (1947) extended this approach to include both static and dynamic theories. While static analysis "described equilibrium as resulting from economic agents solving maximization problems taking prices as parameters ... Samuelson's analysis of dynamics rested on the concept of stationary equilibrium, which holds that equilibrium constitutes a state of rest" (De Vroey and Duarte 2013, p.979). Lucas (1980, p.278) provides further elaboration of this point:

> The underlying idea seems to be taken from physics, as referring to a system 'at rest'. In economics, I suppose such a static equilibrium corresponds to a prediction as to how an economy would behave should external shocks remain fixed over a long period, so that households and firms would adjust to facing the same set of prices over and over again and attune their behavior accordingly.

Lucas attacked "the idea that an economic system is in any sense 'at rest' [as] simply an anachronism", advancing a 'modern' system that relies on "the equilibrium discipline, rational expectations and Walrasian microfoundations" (De Vroey and Duarte 2013, p.985).

Missing from much of the discussion surrounding the emergence of neoclassical economics from the marginalist contributions is a temporal perspective. Recognizing that the first marginalist steps involved the hedonic calculus of utility, Frank Knight (1931) captures the tenor of the times:

> The utility theory should be seen as the culmination, historically and logically, of the rationalistic and individualistic intellectual movement of which the competitive economic system itself is one aspect and modern science and technology are others. To its admirers it comes near to being the fulfilment of the eighteenth-century craving for a principle which would do for human conduct and society what Newtonian mechanics had done for the solar system.

The desire to see 'Darwinian' continuity in the development of economic thought represented in the marginalist contributions of Marshall masks an essential feature of the "new theories of economics" that separated classical political economists from a mathematical group of marginalists. Pareto (1897, p.490) provides an excellent illustration of the perspective from this subset:

> Rational mechanics gives us a first approximation to theory of the equilibrium and of the movements of bodies. In the same way the theories of Jevons, Walras, Marshall, Irving Fisher, and others present us with a first approximation to the full theory of economic phenomena. It must be clearly understood that it is only an approximation; it is similar to that just made in the case of the heavy body supposed to fall in a vacuum. Pure economics has no better way of expressing the concrete economic phenomenon than rational mechanics has for representing the concrete mechanical one. It is at this point that there is a place for mathematics. The problem of pure economics bears a striking likeness to that of rational mechanics. Now, in point of empirical fact, men have as yet not succeeded in treating the latter problem without the aid of mathematics. It therefore appears quite legitimate to appeal also to mathematics for assistance in the solution of the economic problem.

In recognizing Marshall, Pareto is apparently not averse to seeing marginalist contributions as an extension of classical political economy. It is the use of mathematics, and the associated connection to the developed mathematical theories of classical physics, that is the defining characteristic of the 'new theories of economics' introduced by those the marginalists that adopted the "mathematical" approach of the "new theories of economics", e.g., Fisher (1892).

Because the overwhelming theme in discussions of the early marginalists is the unifying notion of utility, the differing emphasis on mathematics can be overlooked, e.g., Colander (2007). It is not surprising that some of the early marginalists sought to emulate the profound advances that mathematics provided in classical physics. For example, Jevons in the "Mathematical Theory of Political Economy" (1874, p.482) observes: "Just as the gravitating force of a material body depends not alone upon the mass of that body, but upon the masses and relative positions and distances of the surrounding material bodies, so utility is an attraction between a wanting being and what is wanted." However, Jevons is primarily concerned with establishing the principles of equilibrium and utility, undermining the dominance of the classical political economy of Smith, Ricardo and Mill. Jevons is not concerned much with developing connections to classical physics. It is with Edgeworth and Mathematical Psychics: An essay on the application of mathematics to moral sciences (1881) where the most emphatic marginalist connecting classical physics with the application of mathematics to economics appears (Edgeworth 1881, p.v):

> An analogy is suggested between the Principles of Greatest Happiness, Utilitarian or Egoistic, which constitute the first principles of Ethics and Economics, and those Principles of Maximum Energy which are among the highest generalisations of Physics, and in virtue of which mathematical reasoning is applicable to physical phenomena quite as complex as human life.

Significantly, as reflected in Irving Fisher (1898), if the use of mathematics and the symbiotic connection with classical physics is emphasized, then the perception of a 'marginalist revolution' that begins with Jevons, Walras and Menger loses cohesiveness. From this perspective, the beginnings of a mathematical foundation for 'economic science' emulating classical physics can be traced to Cournot (1838).

That Antoine Augustin Cournot (1801-1877) went largely unnoticed by followers of the classical political economists is well known. As Irving Fisher, a student of Gibbs, observed (1898, p.119):

> Sixty years ago the mathematical treatise of Cournot was passed over in silence, if not contempt. To-day the equally mathematical work of Pareto is received with almost universal praise. In Cournot's time "mathematical economists" could be counted on one's fingers, or even thumbs.

Though receiving little or no recognition at the time, subsequent influence on the early marginalists is apparent; Cournot was closely studied by both Jevons and Walras; Marshall makes reference to Cournot's "genius" and exploits the marginal cost analysis developed by Cournot in extending the marginalist contribution beyond utility. Edgeworth makes abundant references to Cournot in <u>Mathematical Psychics</u>. However, even though the concept of a demand function was introduced by Cournot, without a developed connection to the use of marginal utility to derive the demand curve, the impact of Cournot on the 'marginal revolution' has been largely ignored.[6] In the process, the seminal role of Cournot in seeking a connection of political economy with mathematics and classical physics is also ignored. This connection is most apparent in Chapter II of the *Principes Mathématiques de la Théorie des Richesse* (1838) on ‘Changes in Value, Absolute and Relative’.

Excusing the understandable mistakes in a seminal, far reaching and ambitious project, the muted impact of Cournot and the *Principes Mathématiques* on the received history of the ‘marginal revolution’ can be traced to a decided difference in focus. Whereas the marginalists laid the foundation for the microeconomic theory of neoclassical economics, Cournot was largely concerned with the classical problem of wealth. Chapter I explicitly identifies wealth with value in exchange and not utility. Of particular relevance to the prehistory of econophysics, Chapter II deals with the values obtained in a system of commodities by making comparison to positions in a system of particles. In the fashion of classical physics, Cournot posits that the value of each commodity can be expressed relative to other commodities, in the same fashion that the position of each particle can be expressed by referencing the position of other particles. Addressing the identification of the numeraire, Cournot considers the relationship between a change in the relative values or positions, and the problem of determining which price has undergone an absolute change. Referencing a passage in the ‘Bible of classical physics’, the *Principia* of Isaac Newton, Cournot identifies “absolute space” as the background of mechanical motion, distinguishing this from“relative space” that is represented by a system of moving points.

3. Ludwig Boltzmann and the Origins of Ergodic Theory

Unlike the lack of contemporaneous recognition denied to Cournot, the contributions of Ludwig Boltzmann to statistical mechanics were fiercely resisted by a physics establishment wedded to the results of classical physics. Though much of the resistance at the time was associated with a reluctance to accept notions of atoms and molecules, it is the concept of ergodicity that sits at the intersection between the history of economics and the pre-history of econophysics. Initially developed in the context of gas molecules colliding in a glass cylinder or, alternatively, colliding billiard balls in a cube, Boltzmann hypothesized that the process of mechanical collision would lead to a state of maximum microscopic disorder that corresponds to maximum entropy and, as a consequence, macroscopic uniformity. To derive this result, Boltzmann introduced the ergodicity hypothesis to model the non-equilibrium velocity distributions for groups of molecules moving at the same speed and in the same direction. Put differently, in a world of mechanically colliding particles, Boltzmann derived the result that molecules moving "at the same speed and in the same direction" is "the most improbable case conceivable ... an infinitely improbable configuration of energy" (Boltzmann 1886, p.20). The ergodicity hypothesis permits the average properties of the empirically measurable macroscopic system, such as temperature, pressure and viscosity, to be theoretically determined from the infinite disorder of the microscopic system dynamics.

At a number of different levels, the results produced by Boltzmann were, and still are, profound. One consequence is the differing methodological approaches of modern economics and econophysics identified by Roehner (2002) and others. For example, while economic theorists strive to obtain the micro-foundations of macroeconomics or financial markets by modeling 'microeconomic equations of motion' based on the maximizing behavior of economic agents, the methods of statistical mechanics initially introduced by Boltzmann direct attention to identifying the stochastic properties of the macroscopic system. In this vein, econophysicts argue that even if the microeconomic foundations of a macro-system cannot initially be determined due to the complex interaction of relevant economic agents, there is still inherent value in determining the empirical properties of the 'global' macrosystem, such as the stochastic dynamics, correlation effects, self-organization, selfsimilarity or scaling properties.[7] At least since Koopmans (1947) critique of statistical business cycle models estimated by institutional economists, such an approach is often derided by economists as naive, 'measurement without theory'.[8]

In addition to methodological implications, the insights of Boltzmann capture philosophical differences between statistical mechanics – the foundation of econophysics – and the classical physics embedded in neoclassical economics. Inherited from Jevons, Walras, Edgeworth and other mathematical marginalists, neoclassical economic theory has essential features of mid-19th century physics: deterministic rational mechanics; conservation of energy; and the non-atomistic continuum view of matter that inspired the energetics movement later in the 19th century.[9] Jevons (1877, p.738-9) reflects the entrenched Laplacian determinism of these marginalists:

> We may safely accept as a satisfactory scientific hypothesis the doctrine so grandly put forth by Laplace, who asserted that a perfect knowledge of the universe, as it existed at any given moment, would give a perfect knowledge of what was to happen thenceforth and for ever after. Scientific inference is impossible, unless we may regard the present as the outcome of what is past, and the cause of what is to come. To the view of perfect intelligence nothing is uncertain.

The transition from rational to statistical mechanics inspired by Boltzmann transformed theoretical physics from the microscopic mechanistic models of Rudolf Clausius (1822-1888) and James Maxwell to the macroscopic probabilistic theories of Josiah Willard Gibbs and Albert Einstein (1879-1955).[10] With Boltzmann, the philosophical view that 'for perfect intelligence nothing is uncertain' is replaced by the view that 'perfect intelligence has created order from infinite uncertainty'

The ergodicity hypothesis has evolved considerably from Boltzmann and the kinetic theory of gases relevant to the early prehistory of econophysics. The <u>Encyclopedia of Mathematics</u> (2002) now defines ergodic theory as the "metric theory of dynamical systems. The branch of the theory of dynamical systems that studies systems with an invariant measure and related problems", e.g., Arnold

(1988). This modern definition implicitly identifies the birth of ergodic theory with proofs of the mean ergodic theorem by von Neumann (1932) and the pointwise ergodic theorem by Birkhoff (1931). These early proofs have had significant and varied impact in a wide range of subjects, including modern economics. Of particular relevance to econometrics, the notions of invariant measure and metric transitivity used in the proofs are fundamental to the measure theoretic foundation of modern probability theory (Doob 1953; Mackey 1974). Building on a seminal contribution to probability theory by Kolmogorov (1933), in the years immediately following it was recognized that the pointwise ergodic theorem generalizes the strong law of large numbers, an essential building block of many theoretical results in econometrics. Similarly, the equality of ensemble and time averages – the essence of the mean ergodic theorem – is necessary to the concept of a stationary stochastic process, another fundamental concept in econometrics.[11]

From the perspective of mathematics, modern statistical mechanics or systems theory, Birkhoff (1931) and von Neumann (1932) – not Boltzmann – are excellent starting points for a history of ergodic theory. Building on these ergodic theorems, subsequent developments in various fields have been dramatic though not typically of relevance to the study of economic phenomena. By introducing the ergodicity hypothesis to permit the theoretical phase space average to be interchanged with the empirically measurable time average, the less formally correct and rigorous contributions of Boltzmann do provide a more intuitive connection with the interactions of individuals forming prices in markets. Following the advances of Boltzmann and Gibbs, problems of subsequent interest in statistical mechanics involved quantum mechanics and Ising models, areas with limited applicability to economic phenomena. As a consequence, the formulation and solution of problems in modern statistical mechanics often have mathematical features which are inapplicable or unnecessary in economics. For example, in modern statistical mechanics, points in the phase space are often multi-dimensional functions representing the mechanical state of the system, leading to a group-theoretic interpretation of the ergodicity hypothesis, e.g., Nolte (2010). For purposes of contrasting modern economics with econophysics, the mathematical complexities of group theory are largely irrelevant and an alternative interpretation of ergodicity that captures the etymology and basic physical interpretation advanced by Boltzmann is more revealing.

Having descended from the deterministic rational mechanics of mid-19th century physics adopted by the marginalists, neoclassical economics culminated in defining works by Hicks (1939) and Samuelson (1947). These contributions do not capture the probabilistic approach to modeling systems initially introduced by Boltzmann and further clarified by Gibbs.[12] The time period from the mathematical contributions by Edgeworth to the culminating works of neoclassical economics coincided with significant advances in statistical mechanics that, unlike the ergodicity hypothesis, have limited applicability to economic phenomena. One profound advance involved the formulation of quantum mechanics by the likes of Max Born, Wolfgang Pauli, Werner Heisenberg and Erwin Schrödinger, with many path-breaking results appearing in a brief window from 1924-28. In addition, Ernst Ising produced the initial one-dimensional solution for the 'Ising model' of ferromagnetism in 1925, a model that was ultimately solved in the important two-dimensional phase transition case by Lars Onsager in 1944. Ergodic notions relevant to the prehistory of econophysics and the history of economics also evolved considerably. Mathematical problems associated with ergodic processes that confronted Boltzmann were subsequently solved by von Neumann and Birkhoff using tools introduced in a string of later contributions by the likes of the Ehrenfests and Cantor in set theory, Gibbs and Einstein in physics, and Lebesque in measure theory, with important extensions by Kolmogorov in probability theory and Weiner and Levy in stochastic processes. Some these advances have slowly acquired relevance for modern economics, some have not.

The kinetic gas model of Boltzmann is concerned with formulating dynamic properties of the stationary Maxwell distribution – the velocity distribution of gas molecules in thermal equilibrium. Starting in 1871, Boltzmann made the essential step of invoking the ergodicity hypothesis to determine the evolution equation for the distribution function.[13] The Maxwell distribution provides the probability for the relative number of molecules with velocities in a certain range. Using a mechanical model that involved molecular collision, Maxwell (1867) was able to demonstrate that, in thermal equilibrium, this distribution of molecular velocities was a 'stationary' distribution that would not change shape due to ongoing molecular collision. Boltzmann aimed to determine whether the Maxwell distribution would emerge in the limit whatever the initial state of the gas. In order to study the dynamics of the equilibrium distribution over time, Boltzmann introduced the probability distribution of the relative time a gas molecule has a velocity in a certain range while still retaining the notion of probability for velocities of a relative number of gas molecules. Under the ergodicity hypothesis, the average behavior of the macroscopic gas system, which can objectively be measured over time using empirical measures such as temperature and pressure, can be interchanged with the average value calculated from the ensemble of unobservable and highly complex microscopic molecular motions at a given point in time. In the words of Weiner (1939, p.1): "Both in the older Maxwell theory and in the later theory of Gibbs, it is necessary to make some sort of logical transition between the average behavior of all dynamical systems of a given family or ensemble, and the historical average of a single system."

The fundamental transformation in physics from rational to statistical mechanics inspired by

Boltzmann and Gibbs had little substantive impact on the subsequent development of neoclassical economic theory. Ergodic notions had little subsequent influence on modern economics until the appearance of discrete time contributions to empirical testing of neoclassical demand theory by Tinbergen, Wold and others during the interwar period. Ergodicity embedded in continuous time diffusion processes – a familiar theoretical construct in statistical mechanics – only enters modern economics with contributions to continuous time finance starting in the 1960's, culminating in the option pricing models of Black and Scholes (1973) and Merton (1973). The slow progress of ergodic notions initially into econometrics and, somewhat later, into economic theory has not overwhelmed the philosophical and methodological hold of classical physics. In particular, the methodology of rational mechanics associated with Laplacian determinism is well suited to the axiomatic formalization of microeconomic theory provided initially in the von Neumann and Morgenstern expected utility approach to modeling 'uncertainty' and the subsequent fascinations with game theory, rational expectations equilibrium and the Bourbaki inspired Arrow-Debreu general equilibrium theory, e.g., Weintraub (2002).

4. Evolution of Ergodicity in Economic Science

Using contributions of the marginalists as an initial reference point in the history of economic thought, the progression of ergodic notions into economics has been uneven and opaque. Ergodicity is an implicit assumption in the general application to the non-experimental data of economic phenomena of empirical methods developed for the experimental data of the natural sciences. Early applications appear initially in econometrics. Important contributors to early empirical estimation of economic theories, especially neoclassical demand theory, were strongly influenced by methods used in the natural sciences, especially physics, and mathematical statistics. These contributors include the first two winners of the Nobel prize in Economics – Ragnar Frisch (1895-1973) and Jan Tinbergen (1903-1994) – as well as the statistician *cum* econometrician Herman Wold (1908-1992).[14] The etymology for "econometrics" begins with Frisch (1926), which appeared the same year Frisch obtained a Ph.D. in mathematical statistics. Together with Irving Fisher, Frisch was responsible for the founding of the Econometric Society in 1930.[15] In the history of neoclassical economic theory, Frisch (1933) provided an important extension of work on commodity supply and demand curves initiated by Henry Schultz (1925, 1928). In addition to work on the measurement of utility, the relevance of Frisch to the evolution of neoclassical theory was described by R.G.D. Allen (1934): "The future progress of economic science depends largely upon the work of investigators, like Professor Frisch, who realise the importance of subjecting the concepts and conclusions of abstract economic theory to the test of statistical determination and verification".

Concern with the implications of estimating empirical relationships for non-experimental data is a common theme in early contributions to econometrics. This concern culminated with Wold (1969) where it is recognized that "experimental versus nonexperimental model building is a dualism that goes to the core of the scientific method." Wold (1969, p.372) summarizes received opinion at the time: "Recognition is due -- and perhaps overdue -- for nonexperimental analysis as an active partner to the method of controlled experiments, and for the integration of both experimental and non-experimental approaches in the scientific method." Building on Wold (1938) which presented the Wold decomposition theorem for (covariance) stationary time series, Wold and Jureen (1953) explicitly develops the ergodic foundations of demand theory, establishing the connection between ergodicity and stationarity for empirically testing neoclassical economic theories.[16] As such, this extends Wold (1949) where the theoretical connection is made only by referencing theorems in Wold

(1938). As a consequence, a relatively restricted form of ergodicity initially entered econometrics as a stationarity assumption imposed on the error term added to a deterministic model, permitting testing for the consistency of regression parameter estimates. To further regression hypothesis testing, it was natural to further assume the stationary error term had a Gaussian distribution.

Regression techniques translated readily to another consuming issue of the time: the estimation of macroeconomic models initiated by Tinbergen (1939). Unlike the estimation of commodity demand and supply functions central to neoclassical economic theory, these macroeconomic models were akin to 'complex systems' arising in statistical mechanics. As Wold (1949, p.4) observes:

> a serious obstacle in econometric analysis lies in the complicated structure of economic life, in the fact that if we wish to study the interdependence between two economic factors, we should pay regard to a great many other factors which are more or less correlated with the phenomena under analysis. To demonstrate the complications which may arise it is sufficient to recall J. Tinbergen's studies of business cycles; for the United States, his analysis takes into consideration 70 different economic factors and about 50 relations between them.

The analogy to complex systems theory references an important theme in econophysics (Roehner 2002, p.4; note 4 above). This empirically driven phenomenological approach to macroeconomics is decidedly different than the Laplacian determinism of neoclassical economics. Using economic theory to identify relevant endogenous and exogenous variables, the approach initiated by Tinbergen also differed methodologically from the measurement driven approach to business cycles of the institutional economists, such as Arthur Burns and W.C. Mitchell. Against this backdrop, the widest philosophical and methodological divide is between the empirically driven ideal of Tinbergen and the empirical skepticism of J.M. Keynes.

As Boumans (1993) details, Tinbergen studied mathematics and physics at the University of Leiden and produced a Ph.D. thesis in 1929 "*Minimumproblemen in de natuurkunde en de economie*" (Minimisation problems in Physics and Economics) that was strongly influenced by the physicist Paul

Ehrenfest, a doctoral student of Boltzmann. Yet, despite impressive training in methods of the natural sciences by key figures in the early history of econometrics, the lack of mathematical sophistication by the bulk of economists at the time contributed to unevenness in the acceptance, use and development of ergodic notions, e.g., Clark (1947). Reluctance to accept the methodological implications of phenomenology for establishing the validity of economic theory inspired fierce competition for intellectual attention space in economics captured in Keynes (1939) critical review of regression analysis in Tinbergen (1939) to test business cycle theories. The epistemology of one camp is set out by Keynes (1939, p.559):

> Prof. Tinbergen is obviously anxious not to claim too much. If only he is allowed to carry on, he is quite ready and happy at the end of it to go a long way towards admitting, with an engaging modesty, that the results probably have no value. The worst of him is that he is much more interested in getting on with the job than in spending time in deciding whether the job is worth getting on with. He so clearly prefers the mazes of arithmetic to the mazes of logic, that I must ask him to forgive the criticisms of one whose tastes in statistical theory have been, beginning many years ago, the other way round.

Striking at fundamental assumptions underpinning the use of time reversible (stationary) ergodic processes, Keynes questioned the validity of "passing from statistical description to inductive generalisation". This schism between alternative epistemologies persists to the present in the writings of Post Keynesian and other heterodox economists arguing against the use of 'reversible' stationary error terms to model the 'true uncertainty' of future events.

5. Phenomenology and Econophysics

This is not the place to provide a detailed examination of phenomenology, either in general or as applied to physics and economics.[17] Given the number of definitions for phenomenology that have been proposed, it is not clear that the same interpretation is used across subject fields.[18] Though the roots of phenomenology can be found in early Hindu and Buddist religious practice, it is generally accepted that as school of philosophy, phenomenology emerged with Edmund Husserl (1859-1938) and Martin Heidegger (1889-1976), the latter having an illustrious list of students including Hans-Georg Gadamer, Hannah Arendt and Herbert Marcuse. Gadamer was influential in making a distinction between the natural sciences, where knowledge is linear and cumulative, and the human sciences, where "the real problem that the human sciences present to thought is that one has not properly grasped the nature of the human sciences if one measures them by the yardstick of an increasing knowledge of regularity. The experience of the socio-historical world cannot be raised to a science by inductive procedure of the natural sciences" (Gadamer 1960, p.6). From this perspective, phenomenology argues against 'thinking' that the study of human interaction in markets can only be studied 'scientifically'.

Comparison of phenomenology in philosophy and the natural sciences, such as physics, is confusing. The philosophical interpretation of phenomenology maintains that the 'phenomena' of reality – objects and events – are perceived or understood in the 'first person' of the human consciousness, and are not independent of that consciousness. As such, phenomenology is the 'philosophy of being'. The connection with phenomenology in physics is that the search for understanding is primarily a process of discovery by looking and perceiving rather than making assumptions and logical deductions. Phenomenology in physics involves computing specific predictions of a theoretical model for comparison with experimental measurements, recognizing that there are constraints on available modes of measurement. In other words, the theoretical model is motivated by measurable quantities. An excellent example of phenomenology in physics is provided by the close collaboration between the theoretician Max Born and the experimentalist James Franck at Göttingen, starting in 1921, that led to important breakthroughs in quantum mechanics. As Im

(1996) observes: “Born's close collaboration with Frank was well suited to his research style: a formal and mathematical description of nature based upon plentiful observational data”. Theories that are concerned with quantities that are only logical and not measurable -- such as the utility function in neoclassical economics -- are not phenomenological. By allowing Boltzmann to connect the theoretical properties of unobservable molecular collisions with available measurements of temperature and pressure, the ergodicity hypothesis is representative of phenomenology in statistical mechanics.

It is generally accepted that the etymology for “econophysics” can be traced to the introduction of the term by Boston University physicist Harry Eugene Stanley at a conference on Statistical Physics held in Kolkata in 1995.[19] Widely recognized as a founder of econophysics, Stanley has produced numerous scholarly publications in the subject and served as editor of a leading physics journal publishing papers on the subject, Physica A. Given advances provided in other natural science fields where the application of physics produced useful results – such as geophysics, astrophysics and biophysics – applying the physics of statistical mechanics to the complex structure of markets, especially financial markets, seems a natural development.[20] A quote from Stanley reflects the tenor of econophysics: “Economics is a pure subject in statistical mechanics. It's not the case that one needs to master the field of economics to study this.” To this end, Stanley maintains that physics training provides ‘powerful mathematical tools, computer savvy, a facility in manipulating large sets of data, and an intuition for modeling and simplification. Such skills could bring new order into economics’.[21] However, if notions, tools and computer savvy essential to statistical mechanics have already gained widespread acceptance within modern economics, both before and after 1995, then how is econophysics going to ‘bring new order to economics’? Intellectual history can provide insights needed to answer such questions.

Given the general recognition of Boltzmann as a founder of statistical mechanics, and the acknowledgment by Stanley that econophysics is "a pure subject in statistical mechanics", tracing differences in specific applications of the ergodicity hypothesis has relevance to identifying the boundary between econophysics and modern economics. Casual inspection of scholarly outlets in econophysics reveals a wide variety of topics, including but not limited to: non-linear stochastic dynamics, with models involving chaos, Hurst exponents and stochastic volatility; scaling and application of fractal and multi-fractal methods; and, complex systems, random dynamic systems and network models. Jovanovic and Schinckus (2013, p.465) make the insightful observation that: "econophysicists have positioned themselves in theoretical niches that mathematicians and economists have barely investigated, or not investigated at all, because of the constraints of the theoretical framework." This observation makes a direct connection to the position of Rohener (2002) that: "for most economists a quantitative regularity is considered of no interest unless it can be interpreted in terms of agents' motivation and behavior and has a clearly defined theoretical status". The phenomenological approach to modeling the *ex ante* empirical properties of macroscopic variables of statistical mechanics and econophysics is concerned with generating statistical information from experimental or non-experimental observations on measurable variables of interest and employing a range of stochastic models to fit the empirical evidence.

The capital asset pricing model (CAPM) provides a useful illustration of the differing methodological approaches of econophysics and modern economics, e.g., Farmer and Geankopolos (2009). Commencing with theoretical derivation from 'microscopic' expected utility maximizing behavior of economic agents, the logical truth of the CAPM is then confronted with *ex post* empirical evidence for observed stock returns and interest rates. [22] As it turns out, the CAPM in various guises has not generated sufficiently credible *ex ante* performance due to instability in regression parameter estimates. By contrast, econophysicists have engaged in numerous attempts to devise stochastic models designed to fit the sample path properties of stock returns or stock prices, often without making substantive reference to the underlying microscopic motivations of economic agents. Instead of iterating the structure of the microscopic maximizing model to attempt a better *ex post* fit to the empirical evidence as in numerous contributions to the CAPM of financial economics, in econophysics weak *ex ante* performance of a stochastic model due, say, to random *ex ante* instability in the sample paths begs phenomenological questions such as: are there alternative stochastic models – ergodic or otherwise – that are a better fit to the *ex ante* properties of observed economic data? Is the random instability in the observed *ex ante* sample paths identified in stock return time series consistent with stochastic models that admit bifurcation of an ergodic process, e.g., Chiarella et al. (2008); Poitras and Heaney (2015)?

Confronted with non-experimental data for important economic variables, such as stock prices, wage rates, commodity prices, interest rates and the like, the dominant methodology of modern economics initially develops theoretical models based on maximizing behavior and then observes the fit to 'stylized empirical facts' associated with the endogenous model variables, where the theoretical models determine the stylized facts of interest. While comparison of 'stylized facts' with predictions of theories initially deduced directly from the 'first principles' of constrained maximizing behavior for individuals and firms is recommended practice in modern economics, such theories often have poor *ex ante* empirical performance resulting in an on-going horse race of specification searches. This has given impetus to a competing inherently inductive approach in mainstream econometrics, especially for macroeconomic and financial data. At least since Tinbergen (1939), when confronted with the problem of modeling complex ‘macroscopic’ economic variables, such as exchange rates or GNP or unemployment, where it is difficult or not possible to derive an empirically reliable theory from known neoclassical principles about the (microscopic) rational behavior of individuals and firms, a resolution of such problems has encouraged development of an inductive approach to empirical modeling in modern economics that is similar to the phenomenology of econophysics.

6. Inductive Reasoning in Modern Economics

Being a large tent, it is not surprisingly that modern economics has developed both deductive -- logical and theoretical -- and inductive -- empirical and phenomenological -- lines of inquiry. As such, phenomenology is not a distinguishing methodological feature separating econophysics from economics. However, stochastic models in modern economics have been heavily influenced by the empirical methods employed to test the theories of neoclassical economics that involve adding an ergodic, stationary, usually Gaussian, error term to a deterministic model and estimating a general linear model (GLM). Following the seminal contributions by Tinbergen, Wold and Koopmans, there were iterations and extensions of the GLM to deal with complications arising in empirical estimates that dominated subsequent work in econometrics, e.g., Dhrymes (1974) and Theil (1971). This led to application of generalized least squares estimation techniques that encompassed autocorrelated and heteroskedastic error terms. Employing L_2 vector space methods with combinations of stationary, i.e., independently, identically distributed (iid), error term distributions ensured these early stochastic models implicitly assumed a restricted form of ergodicity. The generalization of this discrete time estimation approach to the class of ARCH and GARCH error term models by Engle and Granger, e.g., Engle and Granger (1987), was of such significance that a Nobel memorial prize in economics was awarded for this contribution in 2003. By modeling the evolution of stationary error term volatility, this approach permitted a limited degree of non-linearity to be modeled providing a typically better *ex ante* fit to observed economic time series. Only recently has the ergodicity of the GARCH model and related methods been considered, e.g., Meitz and Saikkonen (2008).

Where similar methodologies are employed, it is not surprising that econophysics overlaps with modern economics. The emergence of ARCH, GARCH and related models was part of a general trend toward the use of inductive methods in economics, often employing discrete, linear time series methods to model transformed economic variables, e.g., Hendry (1995). In particular, at least since Dickey and Fuller (1979), it has been found that estimates of univariate time series models for many economic times series reveals evidence of 'non-stationarity', seemingly undermining applicability of the ergodicity hypothesis. A number of approaches have emerged to deal with this apparent empirical quandary.[23] In particular, transformation techniques for bivariate and multi-variate time series models have received considerable attention. Extension of the Box-Jenkins methodology led to the concept of economic time series being I(0) – stationary in the level – and I(1) – non-stationary in the level but stationary after first differencing. Two I(1) economic variables could be cointegrated if differencing the two series produced an I(0) process, e.g., Hendry (1995), permitting testing for 'causality' -- a research area overlapping with studies in econophysics. Extending early work on distributed lags, long memory processes have also been employed where the time series are subject to fractional differencing, another topic appearing in econophysics. Significantly, recent contributions on Markov switching processes and exponential smooth transition autoregressive processes in econometrics have demonstrated the "possibility that nonlinear ergodic processes can be misinterpreted as unit root nonstationary processes" (Kapetanios and Shin 2011, p.620).

The evolution of economics from contributions of the marginalists to deterministic models of neoclassical economics to stochastic models of modern economics has been incremental and disjointed. Recognizing that empirical limitations of ergodic processes have been recognized in statistical mechanics for almost a century, preference for linear models of equilibrium relationships with additive stationary error processes has restricted application of various ergodic and non-ergodic stochastic models employed in econophysics that have potential to capture more complex non-linear dynamics, e.g., multi-fractal models; chaos theory; truncated Levy processes; bifurcating processes. The conventional view of ergodicity in modern economics is reflected by Hendry (1995, p.100): "Whether economic reality is an ergodic process after suitable transformation is a deep issue" which is difficult to analyze rigorously. This opaque interpretation of ergodicity reflects limited recognition of the role that 'the ergodicity hypothesis' played in the evolution of modern economics. In contrast, the prehistory of econophysics begins with the ergodicity hypothesis arising from the physically transparent kinetic gas model. For Boltzmann, ergodicity permitted stochastic modeling of the unobserved complex microscopic interactions of individual gas molecules that had to obey the second law of thermodynamics, a concept that has limited application in economics.[24] The absence of such 'laws of natural science' in the human science of economics creates a quandary for the use of statistical techniques developed for experimental data.

Halmos (1949, p.1017) is a helpful starting point to sort out differing notions of ergodicity that arise in modern economics, econophysics and other subjects: "The ergodic theorem is a statement about a space, a function and a transformation". In mathematical terms, ergodicity or 'metric transitivity' is a property of 'indecomposable', measure preserving transformations. Because the transformation acts on points in the space, there is a fundamental connection to the method of measuring relationships such as distance or volume in the space. In von Neumann (1932) and Birkhoff (1931), this is accomplished using the notion of Lebesque measure: the admissible functions are either integrable (Birkhoff) or square integrable (von Neumann). In contrast to, say, statistical mechanics where spaces and functions account for the complex physical interaction of large numbers of particles, in modern economics the space can usually be specified in a mathematically convenient fashion. For example, in the case where there is a single random variable such as the Gaussian error term in a GLM equation, then the space is “superfluous” (Mackey 1974, p.182) as the random variable is completely described by the distribution. Multiple random variables can be handled by assuming the random variables are discrete with finite state spaces. In effect, conditions for an ‘invariant measure’ are often assumed in modern economics in order to focus attention on “finding and studying the invariant measures” (Arnold 1998, p.22) where, in the terminology of econometrics, the invariant measure usually corresponds to the (ergodic) stationary regression error distribution or the associated likelihood function.

The mean ergodic theorem of von Neumann (1932) provides an essential connection to the form of ergodicity hypothesis employed in modern economics. It is well known that, in the Hilbert and Banach spaces common to econometric work, the mean ergodic theorem corresponds to the strong law of large numbers. In econometric applications where strictly stationary distributions are assumed, as in the GLM and extensions, the relevant ergodic transformation, L^*, is the unit shift lag operator: $L^* \Psi[x(t)] = \Psi[L^* x(t)] = \Psi[x(t+1)]$; $[(L^*)^k]\ \Psi[x(t)] = \Psi[x(t+k)]$; and $\{(L^*)^{-k}\}\ \Psi[x(t)] = \Psi[x(t-k)]$ with k being an integer and $\Psi[x]$ the strictly stationary distribution for x that in the strictly stationary case is replicated at each t.[25] Significantly, this is a reversible transformation independent of initial time and state. Only the distance between observations is relevant. Because this transformation imposes strict stationarity on $\Psi[x]$, L^* will only work for a restricted number ergodic processes, such the Gaussian. The ergodic requirement that the transformation be measure preserving is weaker than the strict stationarity of the stochastic process required for L^*. The implications of the reversible transformation L^* are central to the criticisms of modern economic theory advanced by heterodox economists such as Paul Davidson (1991, p.331): "In an economic world governed entirely by ergodic processes ... economic relationships among variables are timeless, or ahistoric in the sense that the future is merely a statistical reflection of the past".[26]

The ergodicity hypothesis impacts modern economics through the use of reversible stochastic processes, thereby avoiding the complications associated with incorporating initial and boundary conditions for the state space into the distribution for $x(t)$. In order for $x(t)$ to be sufficiently similar to those for both $x(t+k)$ or $x(t-k)$, the transformation L^* has to be reversible and "ahistoric". Ergodicity differs substantively for Boltzmann where the concern is with demonstrating that the Maxwell distribution emerge in the limit as $t \div 4$ for systems with large numbers of particles. Whereas the reversible transformation avoids the need to consider initial and boundary conditions, for Boltzmann the limiting process for t requires that the system run long enough in order for the initial conditions to not impact the stationary distribution. Because the particle movements in a kinetic gas model are contained within an enclosed system, e.g., a sealed vertical glass tube, initial and boundary conditions impact the non-limiting solutions for the transition densities. As a consequence, the transition densities can be decomposed into a limiting stationary distribution and a power series of transient terms associated with initial and boundary conditions that impact the transition density (Poitras and Heaney 2015).

The distributional and other implications of initial and boundary conditions has been of interest in a number of subjects overlapping with modern economics and econophysics. In particular, results for the distributional implications of imposing regular boundaries and initial condition(s) on diffusion processes are representative of random systems theory which: "studies qualitative changes of the densities of invariant measures of the Markov semigroup generated by random dynamical systems induced by stochastic differential equations" (Crauel et al. 1999, p.27).[27] Because the initial condition of the system is explicitly recognized, ergodicity in these models takes a different form than that associated with the reversible unit shift transformation applied to stationary densities typically adopted in econometrics. The transition densities contain a transient term that is dependent on the initial condition of the system and boundaries imposed on the state space. Irreversibility can be introduced in various ways, such as by employing bifurcating multi-modal stationary densities, e.g., Cobb (1978, 1981); Cobb et al. (1983); Crauel and Fandolli (1998); Matz (1978); Veerstraeten (2004); Linetsky (2005). The implications of this result extend beyond comparison of stochastic models from modern economics and econophysics.[28]

7. Conclusion

The widely acknowledged influence of classical physics on marginalist contributions has had an enduring methodological and philosophical impact on subsequent development of the stochastic models of modern economics from the deterministic equilibrium models of neoclassical economics As such, emergence of the 'new' subject of econophysics in the mid-1990's begs an obvious question: if physics has exerted considerable and long lasting influence on economics, what is 'new' about econophysics? This paper addresses this question by tracing the prehistory of econophysics back to the origins of statistical mechanics and the seminal contributions of Ludwig Boltzmann. The evolution of an essential element of that prehistory – the ergodicity hypothesis – in the subsequent history of economics is identified and contrasted with the influence that the rational mechanics of classical physics exerted initially on the marginalists and, subsequently, on neoclassical economics and modern economics. Examination of this history reveals that the enduring philosophical influence of rational mechanics undermines the phenomenological use of various ergodic and non-ergodic processes in modern economics allowing econophysics to provide a distinct approach to modeling and predicting the stochastic behavior of economic phenomena, especially financial prices.

Consistent with the philosophy and methodology of rational mechanics inherited from the marginalists, through the evolution of empirical testing for neoclassical theories, modern economics generally proceeds by deriving a functional form from the maximizing behavior of economic agents and then conducts statistical inference by adding a unimodal, stationary, reversible, Gaussian error term – or some related transformation of such an error term. These stochastic models have a number of substantive limitations such as: model dimensionality that does not capture the stochastic complexity of economic variables; and, inability to incorporate or recognize the distributional impact of transient terms associated the imposition of initial and boundary conditions that can induce irreversible non-linearity in the stochastic process. From the early results in the prehistory of econophysics by Boltzmann to the fractal geometry of Mandelbrot, econophysics provides a wide variety of potential stochastic approaches to phenomenological modeling of non-experimental economic variables that may eventually prove to be more insightful and predictive than the relatively restrictive stochastic models employed in modern economics. However, until fundamental difficulties of applying experimental methods of the natural sciences to human behavior are overcome, the deterministic theories, deductive mathematical methodology and rationalist philosophy inherited from the marginalists will remain near the core of 'economic science'.

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NOTES

1. The ‘Mirowski thesis’ has been contentious, for a variety of reasons, e.g., Walker (1991); Varian (1991). Included in these reasons are a tendency to overstate the position and a lack of attention to specific detail. For example, Mirowski identifies the beginnings of neoclassical economics with the energistics approach to physics. From a history of physics perspective, this is not technically correct as energistics did not emerge in full force until after the 1870's when the so-called marginalist revolution commenced. More precisely, the marginalists were more influenced by results from rational mechanics which is a key component of the energistics approach. Cercignani (1998, p.202-9) discusses the connection between Boltzmann and the energetists of that time. Volovich (2011) details the differences between classical and quantum mechanics.

2. Though “to date, no history of econophysics has been produced”, Jovanovic and Schinckus (2012, 2013) detail many of the relevant contributions. Sornette (2014) also provides a helpful overview of historical developments in econophysics but makes numerous errors in describing the impact of physics on the history of economics. Of particular interest, Sornette (2014, p.3) makes the dubious unsupported statement: “In the second half of the 19th century, the microeconomists Francis Edgeworth and Alfred Marshall drew on the concept of macroequilibrium in gas, understood to be the result of the multitude of incessant micro-collisions of gas particles, which was developed by Clerk Maxwell and Ludwig Boltzmann. Edgeworth and Marshall thus developed the notion that the economy achieves an equilibrium state not unlike that described for gas.”

3. As illustrated by Sornette (2014, p.5), such perceptions are commonplace among econophysicists. Ausloos et al. (2016) detail difficulties with such broad generalizations and identify considerable areas of overlap between modern economics and econophysics.

4. To illustrate the methodology used in econophysics papers, consider the 10 econophysical papers appearing in the second Feb. issue of the 2013 (volume 392, n.4) of Physica A. The papers can be divided into studies of: scaling, chaos and fractals (2); quantum and complex systems (2); stable Paretian or truncated Levy processes (2); system dynamics (2); and, stochastic volatility (2). A common theme for a number of these articles is reflected in the comment: “Our model generates aggregate dynamics for key economic variables consistent with empirics.” By comparison, the 17 papers in the second November issue for 2017 (vol. 486) can be divided as: cluster analysis (2); networks (1); fractals, multifractals and chaos (2); stochastic volatility (1); Levy and stable Paretian processes (1); market phase transition (1); copulas (1); cross-correlation (5); evolutionary games (1); adaptive Market hypothesis (1); transfer entropy (1).

5. In addition, it ignores the importance of the “neoclassical synthesis” that appears in the third edition of the influential textboook Economics by Samuelson (1955). Just as the concept of ‘neoclassical economics’ evolved from Marshallian notions to include writings of all the marginalist founders, the neoclassical synthesis program of integrating Keynesian short run disequilibrium and Walrasian long run equilibrium analysis also evolved to achieve a “new neoclassical synthesis ... that bears little, if any, relation to the old synthesis” (De Vroey and Duarte 2013, p.966). Fundamental issues that separate the marginalism of Marshall, Edgeworth and Walras from later neoclassical economists reappear in the more recent debates in macroeconomics between ‘new classical

economists', real business cycle economists and the DSGE or new Keynesian economists.

6. Seminal is used somewhat loosely in connection with latter impact and method. There were various previous contributions that either sought a connection to physics or used mathematical methods to address issues relevant to economic or financial matters. For example, the identification of supply and demand with opposing physical forces did not originate with Cournot. An earlier interpretation can be found in Nicholas–Francois Canard (1801).

7. Chakraborti et al. (2011a,b) provide a useful overview.

8. Burns and Mitchell (1946) is the classic study of business cycles by institutional economists. Card (2011) is a recent example of the subtle implications of 'measurement without theory' and the difficulty of designing statistical measures of important theoretical macroeconomic variables such as unemployment and gross national production. The related early difficulties of the marginalists in 'measuring' utility has been carefully examined in Moscatti (2013) and Colander (2007). In contrast, much of literature in econophysics is concerned with modeling the stochastic behaviour of financial prices, including stock indexes, currencies and commodity futures, variables that have limited measurement issues.

9. In rational mechanics, once the initial positions of the particles of interest, e.g., molecules, are known, the mechanical model fully determines the future evolution of the system. This scientific and philosophical approach is often referred to as Laplacian determinism.

10. Boltzmann and Max Planck were vociferous opponents of energetics. The debate over energetics was part of a larger intellectual debate concerning determinism and reversibility. What Boltzmann, Planck and others had observed in statistical mechanics was that, even though the behavior of one or two molecules can be completely determined, it is not possible to generalize these mechanics to the describe the macroscopic motion of colliding molecules in large, complex systems, e.g., Brush (1983, esp. ch.II).

11. A number of different definitions of stationarity for a stochastic process are available. In L_2 space, which is conventional in econometrics, 'wide sense' stationary process has a constant mean and covariance function that depends only on the difference between observations so that the covariance function depends only on *t*. A 'strictly stationary' process has joint distribution functions that depend only on *t*. Strict stationarity implies wide sense stationarity but not the converse. Wide sense stationary processes are also referred to as 'covariance stationary' and 'weak-sense stationary' and 'stationary to order 2'. Working in L_2 space avoids significant complications associated with measurability and continuity of sample functions that are encountered with the stable Levy processes often adopted in econophysics, e.g., Jovanovic and Schinckus (2012).

12. As such, Boltzmann was part of the larger: "Second Scientific Revolution, associated with the theories of Darwin, Maxwell, Planck, Einstein, Heisenberg and Schrödinger, [which] substituted a world of process and chance whose ultimate philosophical meaning still remains obscure" (Brush (1983, p.79). This revolution superceded the: "First Scientific Revolution, dominated by the physical astronomy of Copernicus, Kepler, Galileo, and Newton, ... in which all changes are cyclic

and all motions are in principle determined by causal laws." The irreversibility and indeterminism of the Second Scientific Revolution replaces the reversibility and determinism of the First.

13. There are many interesting sources on these points which provide citations for the historical papers that are being discussed. Cercignani (1988, p.146-50) discusses the role of Maxwell and Boltzmann in the development of the ergodic hypothesis. Maxwell (1867) is identified as "perhaps the strongest statement in favour of the ergodic hypothesis". Brush (1976) has a detailed account of the development of the ergodic hypothesis. Gallavotti (1995) traces the etymology of "ergodic" to the 'ergode' in an 1884 paper by Boltzmann. More precisely, an ergode is shorthand for 'ergomonode' which is a 'monode with given energy' where a 'monode' can be either a single stationary distribution taken as an ensemble or a collection of such stationary distributions with some defined parameterization. The specific use is clear from the context. Boltzmann proved that an ergode is an equilibrium ensemble and, as such, provides a mechanical model consistent with the second law of thermodynamics. It is generally recognized that the modern usage of 'the ergodic hypothesis' originates with Ehrenfest (1911).

14. Paul Ehrenfest was the doctoral advisor for Tinbergen at the University of Leiden and, in turn, Boltzmann was the doctoral advisor for Ehrenfest at the University of Vienna. Tinbergen was the doctoral supervisor for Tjalling Koopmans. Wold studied mathematical statistics under Harald Cramer at the University of Stockholm, graduating in 1930.

15. From the Econometric society webpage: "The Econometric Society is the most prestigious learned society in the field of economics, with a world-wide membership. Its main object is to promote studies that aim at a unification of the theoretical-quantitative and empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking similar to that which has come to dominate in the natural sciences. It operates as a purely scientific organization, without any political, social, financial or nationalistic allegiance or bias." (http://www.econometricsociety.org/society.asp#history)

16. Dorfman (1953, p.540) describes the fundamental contribution to estimation of demand functions: "it follows from the Birkhoff-Khintchine ergodic theorem that the moments (i.e., averages, variances, covariances, etc.) calculated from a time series are consistent estimates of the moments of the population from which that time series is drawn. Classical regression analysis can be applied, but the standard error formulas have to be modified to allow for the increased instability caused by serial correlations."

17. Klump and Wörsdörfer (2011) provide a useful examination of relevant philosophical issues concerning phenomenology in economics. Madison (1994) provides earlier work on the connection between phenomology and economics.

18. The Encyclopedia of Phenomenology (Embree 1997) identifies seven different forms of phenomenology, none of which corresponds to the interpretations that have been given to the philosophy in economics and physics.

19. Jovanovic and Schinckus (2013) identify Stanley et al. (1996) as the first published contribution to econophysics.

20. In addition to Schinckus (2010) and Jovanovic and Schinckus (2012, 2013), see also Gingras and Schinckus (2012).

21. As quoted in Baker (1999, p.344).

22. It is well known that empirical estimations of the CAPM have a number of limitations. In particular, the model is defined using conditional expectations of returns. Empirical estimation requires assumptions about the use of statistical estimates derived from *ex post* data to determine such values. In addition, empirical representations for two essential variables – the risk free rate and the market portfolio – are not precisely defined.

23. Kapetanios and Shin (2011, p.620) capture the essence of this quandary: "Interest in the interface of nonstationarity and nonlinearity has been increasing in the econometric literature. The motivation for this development may be traced to the perceived possibility that nonlinear ergodic processes can be misinterpreted as unit root nonstationary processes. Furthermore, the inability of standard unit root tests to reject the null hypothesis of unit root for a large number of macroeconomic variables, which are supposed to be stationary according to economic theory, is another reason behind the increased interest."

24. The second law of thermodynamics is the universal law of increasing entropy – a measure of the randomness of molecular motion and the loss of energy to do work. First recognized in the early 19th century, the second law maintains that the entropy of an isolated system, not in equilibrium, will necessarily tend to increase over time. Entropy approaches a maximum value at thermal equilibrium. A number of attempts have been made to apply the entropy of information to problems in economics, with mixed success. In addition to the second law, physics now recognizes the zeroth law of thermodynamics that "any system approaches an equilibrium state" (Reed and Simon 1980, p.54). This implication of the second law for theories in economics was initially explored by Georgescu-Roegen (1971).

25. Dhyrmes (1971, p.1-29) discusses the algebra of the lag operator and numerous applications to distributed lag models in econometrics. In many cases, the $x(t)$ will be transformed, e.g., by differencing the observations in a time series or by taking logs, to achieve stationarity. Similarly, the $x(t)$ of interest could be an error term, endogenous variable or exogenous variable, depending on the context. As the Wold decomposition theorem demonstrates, the assumption on $x(t)$ of strict stationarity can be relaxed to covariance stationary without impacting properties of L^*.

26. As a consequence, the definition of ergodic theory in heterodox criticisms of mainstream economics lacks formal precision, e.g., the short term dependence of ergodic processes on initial conditions is not usually recognized. Ergodic theory is implicitly seen as another piece of the mathematical formalism inspired by Hilbert and Bourbaki and captured in the Arrow-Debreu general equilibrium model of mainstream economics.

27. The distinction between invariant and ergodic measures is fundamental. Recognizing a number of distinct definitions of ergodicity are available, following Medio (2005, p.70) the Birkhoff-Khinchin ergodic (BK) theorem for invariant measures can be used to demonstrate that ergodic measures are a class of invariant measures. More precisely, the BK theorem permits the limit of the time average to depend on initial conditions. In effect, the invariant measure is permitted to decompose into invariant 'sub-measures'. The physical interpretation of this restriction is that sample paths starting from a particular initial condition may only be able to access a part of the sample space, no matter how long the process is allowed to run. For an ergodic process, sample paths starting from any admissible initial condition will be able to 'fill the sample space', i.e., if the process is allowed to run long enough, the time average will not depend on the initial condition. Medio (2005, p.73) provides a useful example of an invariant measure that is not ergodic.

28. In the context of Hamiltonian dynamics, Percival (1987, p.131) outlines the limitations of statistical mechanics and ergodic processes in explaining a wide range of phenomena associated with chaotic motion: "The past few decades have seen a complete change of view, which affects almost all the practical applications. The motion of a conservative hamiltonian system is usually neither completely regular nor properly described by the methods of statistical mechanics. A typical system is mixed: it exhibits regular or chaotic motion for different initial conditions, and the transition between the two types of motion, as the initial conditions are varied, is complicated, subtle and beautiful ... the nature of the chaotic motion is still not fully understood ... it does not normally occupy the whole of an energy shell as required by the ergodic principle of traditional statistical mechanics. Far away from regular regions, the chaotic motion resembles a diffusion process, but close to them it does not". Because both statistical mechanics and the maximization models from financial economics and can be modeled using different Hamiltonian representations, this framework has the potential to provide an alternative approach to explaining fundamental theoretical differences between models in econophysics and economics in terms of the dimension of the system and the impact of initial and boundary conditions.