On the Theory of Exhaustible Resources: Ricardo vs. Hotelling

Heinz D. Kurz and *Neri Salvadori*

1. Introduction

While Piero Sraffa in his path-breaking book *Production of Commodities by Means of Commodities* (Sraffa 1960) had dealt with renewable resources, such as land of different qualities, which, following David Ricardo's lead, were taken to be possessed of "indestructible powers", he had not, at least not explicitly, tackled the case of exhaustible resources, such as mineral deposits or oil. More than twenty years later attempts were made to extend his classical approach to the theory of value and distribution to the case of exhaustible resources. Sergio Parrinello (1983) made a start, followed by Bertram Schefold (1989). This opened up a new field of research to those who adopted the approach under consideration and applied it to what Sraffa in the preparatory notes of his book called "wasting assets" (see Kurz and Salvadori, 2000: 290-93). Inspired by these works and motivated by the fact that the topic had not yet been exhausted, we also entered into a discussion of exhaustible resources in some of our contributions.

In a paper published in 2009 in a Festschrift in honour of Takashi Negishi (Kurz and Salvadori, 2009) we compared the approaches to exhaustible resources of David Ricardo and Harold Hotelling, because we felt that the distinctive features and genuine significance of the classical approach had yet to be established also with regard to the case of exhaustible resources. This was not an easy task in view of the fact that according to a widespread opinion the classical economists, Ricardo in particular, and those following in their tradition had nothing useful to say: they had not understood that working a mine was bound to exhaust it and therefore could not be treated in terms of Ricardo's theory of rent, which presupposed the inexhaustibility of the resource. The impression was close at hand that the classical approach was barren and irrelevant with regard to the case under consideration.

Scrutinizing the issue at hand, we arrived at the conviction that this common interpretation could not be sustained: Ricardo's theory was not barren an irrelevant – rather, it was different from the marginalist one. But wherein consisted the difference? And did the different theory focus on the problem at hand in a way that was both interesting and revealed aspects not covered by the marginalist theory? What was the relationship with the marginalist approach to the problem of exhaustible resources, represented by Harold Hotelling's model?

We argued that the fact that in the classical economists the famous Hotelling Rule is not yet to be found does not mean that their analyses are of necessity defective, incomplete or inferior. It rather means that their arguments relate to a world characterised by properties different from those contemplated by the Hotelling Rule. Moreover, the latter may be said to be implicit in Ricardo's analysis. What is missing is only an explicit reference to "royalties" as something distinct and different from profits.

The Hotelling Rule implies that in conditions of free competition the prices of resources *in situ* need to increase over time at a rate that is equal to the competitive rate of profits. This follows from the requirement that in such conditions the conservation of a resource is an economic activity, which ought to yield to the proprietors of deposits of the resource the same rate of profits as is obtained from any productive activity.[1] This seems in turn to imply that in the presence of exhaustible resources, and assuming a general framework of the analysis[2], *all* prices are bound to change over time. However, as will be shown below, this need not be the case.

In this paper we illustrate in terms of a numerical example our previous argument, which was designed to clarify the relationship between Ricardo's approach to the problem of mines in terms of differential rent theory and Hotelling's approach to the problem of exhaustible resources in terms of royalties. For this purpose we develop a simple model that allows us to incorporate both points of view and the underlying leading principles in a single scheme and discuss its mathematical properties. A main outcome of the analysis is a clear distinction between three different types of property income: rents, profits and royalties. As Ricardo stressed in his criticism of Adam Smith's doctrine (Smith 1976) with regard to the difference between the rent of land and profits, a clear

---

1 Empirical studies have shown that the Hotelling Rule does not perform all that well; see Krautkraemer (1998). Some of the reasons for this will become clear in the sequel. (Other important reasons, especially technical progress, will be mentioned only in passing.)

2 As is well known, Hotelling (1931) assumed a partial equilibrium framework in which the rate of interest (or profits) was given from outside the system. Ricardo's analysis may on the contrary be characterized as concerned with the system as a whole, in which the rate of profits will typically change as the scarcity of an exhaustible resource makes itself felt ever more severely as its exploitation proceeds.

distinction between the two is crucial, because as capital accumulates, the population grows and less and less fertile lands have to be cultivated the two component parts of the social surplus are typically affected differently: while rent rates increase, the competitive rate of profits falls.[3] In Ricardo land is treated as a renewable resource that is actually renewed all the time, that is, its quality does not deteriorate in the course of its utilization in the production of corn (or other crops). It thus differs markedly from exhaustible resources. The latter are gradually depleted each time parts of them are actually removed for productive (or consumptive) purposes from given stocks. However, Ricardo's finding that rents and profits move in different directions is corroborated with regard to different kinds of exhaustible resources (or, alternatively, differently fertile deposits of one such resource). As will be shown, royalties, which are a special kind of profits, may move in the opposite direction of rents. In well-specified circumstances this may imply that whereas the prices of exhaustible resources *in situ* are bound to change at a rate equal to the rate of profits, all other prices remain constant. This is so because the owners of deposits of resources receive both royalties and rents. The sum of royalties and rents for a given deposit may be constant even if royalties are changing, since rents are changing in equal amounts but in the opposite direction.

The composition of the paper is the following. In Section 2 we summarise the arguments of Adam Smith and David Ricardo on exhaustible resources. Section 3 deals briefly with Sraffa's treatment of the case in his preparatory notes to his 1960 book. Section 4 stresses the fact that the worlds Ricardo and the classical economists, old and new, and Hotelling contemplated in their analyses differ in important respects. Section 5 presents the model that serves as our work-horse for the following investigation and exemplifies the argument in terms of a numerical specification of the model. Section 6 provides a number of examples constructed in order to illustrate different possibilities as to whether the prices of produced commodities will, or will not change, as time goes by and some of the natural resources are actually gradually exhausted. Section 7 contains some concluding remarks.

2. Adam Smith and David Ricardo on exhaustible resources and mines

Ricardo develops an analysis of exhaustible resources in the context of a discussion of the difference between rent and profits. In the *Principles* he defines rent rigorously in the following way:

---

3 Ricardo took the soil to be possessed of "original and indestructible powers" (Ricardo, 1951: 67). For an elaboration of the classical theory of extensive and intensive rent, see Kurz and Salvadori (1995: chap. 10).

> Rent is that portion of the produce of the earth, *which is paid to the landlord for the use of the original and indestructible powers of the soil*. It is often, however, confounded with the interest and profit of capital, and, in popular language, the term is applied to whatever is annually paid by a farmer to his landlord. (*Works* I: 67; emphasis added)

Adam Smith, Ricardo goes on to argue, did not stick to a rigorously defined concept when using the word rent:

> He [Smith] tells us, that the demand for timber, and its consequent high price, in the more southern countries of Europe, caused a rent to be paid for forests in Norway, which could before afford no rent. Is it not, however, evident, that the person who paid what he thus calls rent, paid it in consideration of the valuable commodity which was then standing on the land, and that he actually repaid himself with a profit, by the sale of the timber? If, indeed, after the timber was removed, any compensation were paid to the landlord for the use of the land, for the purpose of growing timber or any other produce, with a view to future demand, such compensation might justly be called rent, because it would be paid for the productive powers of the land; *but in the case stated by Adam Smith, the compensation was paid for the liberty of removing and selling the timber, and not for the liberty of growing it* (Works I: 68; emphasis added).

Reference is clearly to WN I.xi.c.5. Ricardo's criticism extends to Smith's dicussion of coal mines and stone quarries:

> He [Smith] speaks also of the rent of coal mines, and of stone quarries, to which the same observation applies—that the compensation given for the mine or quarry, is paid for the value of the coal or stone which can be removed from them, and has no connection with the original and indestructible powers of the land. (*Works* I: 68)

In Ricardo's view the distinction between profits and rent is crucial, because as capital accumulates and the population grows the two component parts of the social surplus are typically affected differently and often move in different directions:

> This is a distinction of great importance, in an enquiry concerning rent and profits; for it is found, that *the laws which regulate the progress of rent, are widely different from those which regulate the progress of profits, and seldom operate in the same direction*. In all improved countries, that which is annually paid to the landlord, partaking of both characters, rent and profit, is sometimes kept stationary by the

> effects of opposing causes; at other times advances or recedes, as one or the other of these causes preponderates. In the future pages of this work, then, whenever I speak of the rent of land, I wish to be understood as speaking of that compensation, which is paid to the owner of land for the use of its original and indestructible powers (ibid: 68-9; emphasis added).

Hence what Smith called "rent" of coal mines or stone quarries is to Ricardo profits and not rent. But does Ricardo not contradict himself by giving Chapter 3 of the *Principles* the title "On the Rent of Mines"? Scrutiny shows that this is not so. Chapter 3 is actually devoted to the rent of mines precisely in the sense Ricardo intended. Each mine is typically subject to a capacity constraint that limits the amount of the coal or ore that can be extracted per unit of time. This constraint itself depends typically also on the amount already extracted. Effectual demand cannot be satisfied in the given circumstances by operating exclusively the most "fertile" mine, because the required rate of output in order to meet effectual demand cannot be generated in this way. Hence mines possessed of different "fertilities" are operated *simultaneously*. In such circumstances, Ricardo stresses, it is the "relative fertility of mines [which] determines the portion of their produce, which shall be paid for the rent of mines" (ibid: 330). Ricardo concludes that "the whole principle of rent is here … as applicable to land as it is to mines" (ibid: 330). When mines of different fertilities need to be wrought simultaneously, then this makes room for the emergence of (extensive) rents, exactly as in the case of the cultivation of different qualities of land. This is rent in the true sense of the word and has nothing whatsoever to do with what we nowadays call "royalties". What we call "royalties", Ricardo actually calls "profits".

Ricardo's use of the concept of profits for "the compensation ... paid for the liberty of removing and selling the timber" is not surprising: timber can be sown and grown again, it is clearly not an exhaustible resource, but a reproducible good, and to the extent to which it is used as a produced means of production it is capital. But the use of the word profits for the compensation paid for the liberty of removing and selling coal or stones may be surprising: coal cannot be reproduced by men, neither can stones. However, new coal pits can always be discovered and the cost of the search is equal to the value of the mine, a value that decreases with the amount of the resource that has been removed from it. In other words, Ricardo did not consider minerals and ores etc. as such as fully exhaustible in the foreseeable future. Both in Ricardo and in Smith we encounter time and again references to the finding of new deposits with no serious consideration given to the fact that such deposits, taken as a whole, are limited:

> In this search [for new mines] there seem to be no certain limits either to the possible success, or to the possible disappointment of human industry. In the course of a century or two, it is possible that new mines may be discovered more fertile than any that have ever yet been known; and it is just equally possible that the most fertile mine then known may be more barren than any that was wrought before the discovery of the mines of America (WN I.xi.m.21).

Modern theory has emphasized the Hotelling Rule. As it is typically presented, it concerns the fact that the price of a resource *in situ* need to increase over time at a rate that is equal to the competitive rate of profits. This rule is obviously related to the fact that royalties are a form of profit. Indeed it follows from the requirement that the conservation of a resource is an economic activity, which ought to yield to the proprietors of deposits of the resource the same rate of profits as is obtained from any productive activity. Ricardo did not elaborate what now is called the Hotelling Rule, but this seems mainly because the total exhaustion of certain resources was not yet considered a possibility worth studying.

The Hotelling Rule seems to imply that *all* prices need to change over time. But this is not obvious at all. Indeed, the increases in the prices of resources *in situ* over time according to the Hotelling Rule are entirely passed on to the prices of the extracted resources and, consequently, to the other prices, if the following assumptions hold:

(H1) The resource is available in a homogeneous quality and in an overall quantity that is limited and that at any moment of time is known with certainty.

(H2) The amount of the resource that can be extracted in a given period of time, a year, for example, is only constrained by the amount of it left over from the preceding period.

As mentioned above, Ricardo is concerned with the fact that each mine has a limited capacity of extraction and it is this fact that creates the rent that owners of mines can obtain. As clarified by Kurz and Salvadori (2009, 2011) even if Assumption (H1) holds the price of the extracted resource may be constant. The owners of deposits obtain both royalties and rents; in the course of time rents fall and royalties rise; and the sum of both may remain constant. If this condition is met, then the price changes of resources *in situ* will not affect any other prices in the economic system.

But there is something more to be said. As usual, Ricardo is "desirous only to elucidate the principle" at work (Works I: 121), as he stresses in another context, and therefore bases his argument on strong assumptions. These assumptions imply that the exhaustion of each and every single deposit of an exhaustible resource will nevertheless leave the prices of all produced commodities unaffected over time. We might go to the opposite extreme and postulate instead of assumptions (H1) and (H2) the following:

(R1) For each exhausted deposit of the resource another one with exactly the same characteristics is discovered and the cost of the search, in terms of labour and commodities, is always the same.

(R2) The working of each deposit is subject to a capacity constraint that limits the amount of the resource that can be extracted in a given period of time.

If assumption (R1) held true, even if Assumption (H2) holds, while each deposit would be exhaustible, the resource as such would not; and each deposit could in fact be treated as if it was a (reproducible) machine: the price of the new machine equals the cost of the search and the price of an old machine of age *t* equals the value of the deposit after *t* periods of utilization (see Kurz and Salvadori, 1995: 359-60). The price of the resource *in situ* would change as predicted by the Hotelling Rule, but the price of the extracted mineral would be constant over time. As mentioned above, if (H1) and (R2) apply, the changes of the prices of the resources *in situ* do not imply changes in the other prices.

Ricardo's approach to the problem of exhaustible resources and mines in terms of differential rents highlights the empirically important fact that the working of each single deposit of a resource is typically subject to a *capacity constraint*. Hence several deposits will have to be worked side by side, and if they are differently "fertile", differential rents will obtain. Ricardo's approach can easily be cross-bred with Hotelling's, giving rise to the familiar result that the prices of the resources *in situ* will rise at the competitive rate of profits, which in Ricardo is determined endogenously (whereas in Hotelling's original contribution it was given from the outside). This rise in these prices need not, however, affect the prices of the resources that have been extracted and thus the prices of commodities in whose production they enter.

3. Sraffa on exhaustible resources

In this section we recall some of the documents in Sraffa's papers in which he deals with the case of exhaustible resources.

The problem of exhaustible resources was on Sraffa's mind from an early time of his (re)constructive work, which started in the second half of the 1920s, and the problem remained there until his respective work culminated in the publication of his 1960 book. And Sraffa was also aware from an early time onwards that the problem caused difficulties for the long-period method.

In a note dated 25 March 1946 he dealt with the difference between the *physical real cost* approach to the problem of value and distribution, which he had endorsed, and the classical-Marxian *labour theory of value*. He actually insisted that the former was able to deal with exhaustible resources, whereas the latter was not:

> The difference between the "Physical real costs" and the Ricardo-Marxian theory of "labour costs" is that the first does, and the latter does not, include in them the natural resources that are used up in the course of production (such as coal, iron, exhaustion of land) – [Air, water etc. are not used up: as there is an unlimited supply, no subtraction can be made from ∞]. This is fundamental because it does away with "human energy" and such metaphysical things.

He added with regard to the natural ingredients of production:

> But how are we going to replace these natural things? There are 3 cases: a) they can be reproduced by labour (land properties, with manure and so on); b) they can be substituted by labour (coal by hydroelectric plant: or by spending in research and discovery of new sources and new methods of economising) c) they cannot be either reproduced nor substituted – and in this case they cannot find a place in a theory of <u>continuous</u> production and consumption: they are dynamical facts, that is a stock that is being gradually exhausted and cannot be renewed, and must ultimately lead to the destruction of the society. But this case does not satisfy our condition of a society that just manages to keep continuously alive. (Sraffa Papers, D3/12/42: 33)

In Sraffa's view exhaustible resources thus constitute "dynamical facts" which cannot be studied rigorously in a framework in which prices and income distribution do not change. A dynamic theory is needed. However, Sraffa was skeptical that a reliable dynamic theory could be elaborated. At any rate, the task was intrinsically difficult, as Sraffa emphasized in another note:

> It is "a fatal mistake" of some economists that they believe that by introducing complicated dynamic assumptions, they get nearer to the true reality; in fact they get further removed for two reasons: a) that the system is much more statical than we believe, and its "short periods" are very long, b) that the assumptions being too complicated it becomes impossible for the mind to grasp and dominate them – and thus it fails to realize the absurdity of the conclusions. (Sraffa Papers, D3/12/11: 33)

Interestingly, this warning did not prevent Sraffa from undertaking probing steps into the as yet unchartered territory. An echo of his respective attempts is actually to be found in the introductory passage of Chapter XI of his book, which is entitled "Land":

> Natural resources which are used in production, such as land and mineral deposits, and which being in short supply enable their oweners to obtain a rent, can be said to occupy among means of production a position equivalent to that of 'non-basics' among products. Being employed in production, but not themselves produced, they are the converse of commodities which, although produced, are not used in production. (Sraffa 1960: 74)

What the readers of *Production of Commodities by Means of Commodities* do not know is that up until the final proof stage of his book Sraffa kept a passage in the text of what is paragraph 91 of his book designed to deal with the specific character of wasting assets (as opposed to non-depletable natural resources including, by assumption, land).[4] The paragraph under consideration deals with machines of an obsolete type, and Sraffa draws a parallel between them and exhaustible resources. We first reproduce the paragraph as it has actually been published:

> Machines of an obsolete type which are still in use are similar to land in so far as they are employed as means of production, although not currently produced. The quasi-rent (if we may apply Marshall's term in a more restricted sense than he gave it) which is received for those fixed capital items which, having been in active use in the past, have now been superseded but are worth employing for what they can get, is determined precisely in the same way as the rent of land. And like land such obsolete instruments have the properties of non-basics and are excluded from the composition of the Standard product. (Sraffa 1960: 78)

The passage he deleted at the proof-reading stage reads:

> On the other hand, as in the case of other wasting assets (such as mineral deposits) the annual depletion must be taken into account, which gives rise to as many separate processes as are the years of the asset's prospective residual life, on the same general principle as was done in Chapter X for 'live' fixed capital. (Sraffa Papers: folder D3/12/96)

4. Different approaches to the problem of exhaustible resources

---

4 Sraffa drafted Chapter XI in January 1958. While this was not yet the final version, it came very close to it with the exception of the passage on wasting assets.

In modern interpretations of the Hotelling Rule it is typically assumed that the above two conditions are met: (H1) and (H2). In case one of these assumptions or both are not met, the Hotelling Rule has to be modified accordingly. The Rule portrays the bold case of a resource whose exhaustion is actually foreseeable with certainty. Alas, it does not fit (m)any cases in the real world![5] Yet it expresses an important principle at work that contributes to our understanding of what is going on in the real world. It can be objected that despite the fact that today we have a much clearer idea of what is still there of certain resources at a given moment of time and are possessed of much improved techniques to discover hitherto unknown deposits of resources, assumption (H1) is typically not met with regard to any single exhaustible resource. It is also not clear whether knowing precisely what is still there would mean much, because technical progress typically affects the economic importance of a resource. The discovery of new ways to use known substances as well as the discovery of the useful properties of hitherto unused substances may lead to substitution processes and in the extreme replace some given resource entirely by new ones. Also assumption (H2) is never strictly met. Typically, as Ricardo insisted, there are *capacity constraints* that limit the time rate of exploiting a deposit. These constraints are very often binding with regard to any single deposit of the resource, so that many deposits have to be exploited simultaneously in order to meet effectual demand.

We may therefore go to the opposite extreme and postulate the assumptions (R1) and (R2). The world contemplated by them is much closer to the one the classical economists experienced. As the evidence provided in Section 2 above and in Kurz and Salvadori (2009) shows, they were aware of the principal exhaustibility of some resources, but they did not think that this was an imminent problem. New deposits of such resources were discovered all the time as old ones were exhausted. In addition, technical progress continuously changed the conditions of production. John Stuart Mill expressed well the classical point of view in this regard. He argued that (i) the working of exhaustible resources is similar to the working of land (a resource that is taken to be inexhaustible); (ii) in both kinds of activities there are two antagonistic forces at work – diminishing returns and improvements (technical progress); (iii)

[5] Krautkraemer (1998) confirms this. He maintains, among other things: "For the most part, the implications of this basic Hotelling model have not been consistent with empirical studies of nonrenewable resource prices and *in situ* values" (p. 2066). "Other factors have overshadowed finite availability of the resource as determinants of the observed dynamic behavior of nonrenewable resource prices and *in situ* values" (p. 2087). And: "It does seem to be a recurring tendency *to overestimate the imminence of nonrenewable resource exhaustion*" (p. 2103; emphasis added).

the potential for such improvements is larger in the mining and other extraction processes than in agriculture (see Mill, 1965, p. 495).[6]

In this contribution we set aside technical progress. Nevertheless, the properties of a world that is subject to assumptions (R1) and (R2) are markedly different from those invoked in modern interpretations of the Hotelling Rule. Therefore it should come as no surprise that the Hotelling Rule appears to contradict Ricardo's view. However, the reason is not that one of the analyses is right and the other wrong, but that they deal with vastly different cases.

Intermediate cases between the two extremes can also be investigated: (H1) may be combined with (R2) or (H2) with (R1). In this paper (as well as in Kurz and Salvadori, 2009) we focus attention on the first possibility which combines Hotelling's assumption of given stocks of exhaustible resources with Ricardo's assumption that the exploitation of each stock is subject to a capacity constraint that limits the amount of the resource that can be extracted in a given period of time. In order to avoid the "end of the world" scenario, we add to assumption (H1) and (R2) the assumption that in addition to methods of production that use exhaustible resources there are methods that don't. These are known as "backstop" methods or techniques (see Kurz and Salvadori, 1995, p. 360).[7]

5. The model and its numerical specifications

The model employed in this paper is a simplified version of the model elaborated in Kurz and Salvadori (2009). One of its characteristic features is that it distinguishes between *production* processes (or methods), *extraction* processes and *conservation* processes. In order to put the features of the model in which we are interested into sharp relief, we assume that there is only a single consumption good, corn. Three methods of producing corn are known. It can be produced either

- by means of corn, extracted oil and labour;
- or by means of corn, extracted methane and labour;

6 Mill even contended boldly that the exhaustibility of each single resource is not really a problem: "the almost inevitable progress of human culture and improvement … *forbids us to consider* [*it*] *as probable*' (p. 496; emphases added). This is perhaps the strongest statement put forward in the history of political economy up until then that the exhaustion of natural resources (and, implicitly, the Hotelling Rule) need not concern us much: technical progress renders the problem of exhaustion improbable.

7 In the present context, a backstop process plays a role similar to that of a process paying no-rent (or using marginal land) in Ricardo's theory of extensive rent.

- or by means of corn and labour only.[8]

While each of the first two methods mentioned uses an exhaustible resource, oil or methane, the last one does not: it represents a backstop process. In Table 1, process (1) gives the last one, process (2) the one that uses oil and process (3) the one that uses methane. There are constant returns to scale with regard to each single process. Obviously, because of the backstop process the system is not doomed to extinction once all the available amounts of oil and methane happen to have been used up: it can survive without them – provided the backstop process is sufficiently productive, which we take for granted.[9] In this case only process (1) will be operated. If we take the wage rate w as the standard of value or numéraire, $w = 1$, then the price of corn in terms of labour (or, to use Adam Smith's concept, in terms of "labour commanded") will be

$$p_c = \frac{4}{3-r}$$

(where the subscript stands for "corn" (c)). This would be the *long-period* solution of the system.

Table 1

processes	material inputs					labour	products				
	corn	oil under the ground	methane under the ground	extracted oil	extracted methane		corn	oil under the ground	methane under the ground	extracted oil	extracted methane
(1)	1/4	-	-	-	-	1	1	-	-	-	-
(2)	1/10	-	-	1	-	1	1	-	-	-	-
(3)	1/4	-	-	-	1	1/10	1	-	-	-	-
(4)	-	1	-	-	-	c	-	-	-	1	
(5)	-	-	1	-	-	d	-	-	-	-	1
(6)	-	1	-	-	-	-	-	1	-	-	-
(7)	-	-	1	-	-	-	-	-	1	-	-

8 While in this paper we distinguish between different *types* of resources (oil and methane), readers can easily apply the argument elaborated to different "*fertilities*" of several fields or deposits of a single type of resource.

9 There is, of course, the possibility that the backstop process might become cost-minimising before all the oil and methane have been used up.

But what will be the prices during the transition toward the depletion of natural resouces? Obviously the input-output information given up until now is not sufficient to answer this question. We need a lot of additional pieces of information. First, we need to know which extraction processes are available in order to remove oil and methane from the ground. Secondly, we need to know how much of these resources is available at the beginning of our investigation, that is, at time 0. Third, we need to know which conservation processes are available. Fourth, we need to know whether and which capacity constraints apply to the extraction of oil and methane, respectively. Fifth, we need to know how much of the only consumption good, corn, is consumed per period. (The reference is to pure consumption and not to the use of corn as a means of production.) Finally, we need to know the amounts of produced inputs (including corn) available at time 0, since the analysis is not a long period one.

Clearly, depending on the whole set of data postulated, or givens, the model will generate different paths of quantities, prices and the distributive variables. Some of these will be illustrated in the following.

The extraction of oil and methane is represented by processes (4) and (5), respectively. It is for simplicity assumed that only labour is needed, and no other inputs. In order to extract one unit of oil c units of labour, and in order to extract one unit of methane d units of labour are required. (Varying the input magnitudes of labour per unit of output will be shown to generate different behaviours of the model.) Processes (6) and (7) give the conservation processes. It is for simplicity assumed that no costs are involved in keeping oil and methane in their *in situ* deposits.

Reflecting (H1), it is assumed that the total amount of oil available in oil fields is 2,500 units, and the total amount of methane in gas fields is 1,900 units, each expressed in its own technical unit. Reflecting (R2), if the extraction of oil and methane is subject to capacity constraints then it is assumed that no more than 400 units of oil and no more than 400 units of methane can be extracted per period (year).

We also have to specify the amount of corn that is consumed per year in addition to what is being used up, directly and indirectly, as a material input in its own production. It will be assumed that total (net) consumption amounts to 1,000 units (tons) of corn.

Finally we need to specify the amounts of commodities available on the ground at time 0 destined to be used as inputs. Further we need to know whether such commodities are perishable within a single period of time or partially perishable; in the latter case we would need to know also the rate at which they perish. This is so, since the analysis to be carried out is an intertemporal one. For instance, if there was only process (1), we know, as mentioned in the above, that the long-period price of corn in terms of labour is $p_c = 4/(3-r)$. But in an intertemporal analysis this is so only if the amount of corn available for production at time 0 happens to be equal to $1{,}000/3$ units, so that the corn produced is $4{,}000/3$ units, which means that one period later, at time 1, 1,000 units of corn are available for consumption and $1{,}000/3$ units are available for production. If the amount of corn available at time 0 happens instead to be larger than $1{,}000/3$, and corn perishes during a single period and therefore cannot be saved and carried over to the next period, there is at time 0 more corn available than what can be used. In competitive conditions the owners of corn will bid down the price of corn at time 0 to zero. As a consequence, the price of corn at time 1 will be unity, whatever is the rate of profits. It follows that the price of corn at time 2 will be

$$p_{c2} = \frac{5+r}{4}$$

and in general the price of corn at time *t* will be

$$p_{ct} = \frac{4}{3-r} - \frac{4}{3-r}\left[\frac{1+r}{4}\right]^t,$$

which effectively tends to $4/(3-r)$ as *t* approaches infinity.

In order not to confound the problem of arbitrarily given initial endowments and their implications for the quantity and price dynamics of the system under consideration with the problem of exhaustible resources, we assume throughout the following argument that the amounts of commodities available at time 0 for production are exactly those needed to have constant prices from time $\bar{t}$ onwards, whenever a time $\bar{t}$ with this property exists. (In the following it is shown that in the circumstances stated the property will always hold good.)

6. Examples

We may now construct a few numerical examples that illustrate different cases in which the prices of produced commodities will, or will not, change over time. The emphasis will be on cases in which the prices of commodities available on the ground may be constant over time even in the transition period. It will be seen that this is the result of the requirement, for which there exist different motivations, that the backstop process is employed.

(a) *The backstop technology is cost-minimizing since the beginning*

In the above we have seen that the backstop process may be used only after the natural resources have been exhausted and processes (1) and (2) can no longer be operated. However, there is also the (abstract) possibility of the backstop technology being cost minimising right from the beginning of our considerations. For a given rate of profits, r, it all depends on how the technical characteristics of the processes by means of which oil or methane are extracted *and* then used up in the production of corn, on the one hand, compare to the technical characteristics of the backstop process, on the other. Assuming given production processes (1) and (2), it all depends on extraction processes (4) and (5). A little calculation shows that if $c > 3/5(3-r)$ and if at the same time $d > 9/10(1+r)$, then neither oil nor methane would be extracted in order to be used in producing corn, because it would not be profitable to do so. The technology that extracts and employs oil and methane would be dominated by the backstop technology. In this case neither oil nor methane would be productive resources. The important lesson to be drawn from this little example is that whether some substance in the ground is, or is not, a resource *cannot generally be defined independently of the rate of profits and the technical alternatives that are available in the system*.

It goes without saying that in the case in which the strict inequality sign applies to only one of the labour coefficients, but not the other one, one of the substances will be a resource that can and in certain conditions will be extracted and then employed.

(b) *All production processes can be employed simultaneously*

Assume now that $c = 3/5(3-r)$ and $d = 9/10(1+r)$. In this case all three processes producing corn are equiprofitable and the price of corn has to meet the conditions defining the backstop process (see Section 3 above). In this case there is neither room for royalties on oil and methane nor for rents on oil and methane fields: the price of extracted oil at time t is given by

$$p_{eot} = c = \frac{3}{5(3-r)}$$

(the subscripts stand for “extracted” (e), “oil” (o) and "time” (t)), and the price of extracted methane is given by

$$p_{emt} = d = \frac{9}{10(1+r)}$$

(the subscript stands for “methane” (m)). Capitalists producing corn will be indifferent as to whether to produce corn by means of corn alone (and, of course, labour), or by means of corn and oil, or by means of corn and methane.

In the following we assume that

$$d = c < \frac{3}{5(3-r)} < \frac{9}{10(1+r)}.$$

Hence the two exhaustible resources can be expected to be actually exploited and used, and in order to do so either royalties on the resources or rents on the fields, or both, need to be paid. This brings us to a new set of examples.

(c) *Extraction with capacity constraints*

We now employ the assumptions mentioned in Section 3 above, namely, first, that the economy consumes (net) 1,000 units of corn per year and, second, that the extraction of oil and methane from given fields (containing a total of 2,500 and 1,900 units respectively) is subject to capacity constraints: only a maximum of 400 units of oil and a maximum of 400 units of methane can be extracted per year from the respective deposits.

Since the extraction of oil and methane is constrained, the owners of the oil or methane fields are able to get a rent (except when the extraction of oil or methane is smaller than 400 units per year). At the same time the capacity constraints have been chosen in such a way that it is impossible to supply the needed amount of corn only in terms of process (2) or only in terms of process (3) or even in terms of employing processes (2) and (3) conjointly. Without operating also backstop process (1) effectual (net) consumption demand could not be met. As a consequence

$$p_{ct+1} = (1+r)\frac{1}{4}p_{ct} + w_t \tag{1}$$

Hence if $p_{ct+1} = p_{ct}$ and $w_t = 1$, then $p_{ct+1} = p_{ct} = 4/(3-r)$. In case processes (2) and (3) are operated, then

$$p_{ct+1} = (1+r)\left[\frac{1}{10}p_{ct} + p_{eot}\right] + w_t \tag{2}$$

$$p_{ct+1} = (1+r)\left[\frac{1}{4}p_{ct} + p_{emt}\right] + \frac{1}{10}w_t \tag{3}$$

and, as a consequence,

$$p_{eot} = \frac{3}{5(3-r)} \quad \text{and} \quad p_{emt} = \frac{9}{10(1+r)}.$$

In this case some oil and methane need to be extracted and the rest conserved. Denoting the rent paid per unit of extracted oil (methane) by q_o (q_m), the corresponding price of oil (methane) by p_{eo} (p_{em}), and the price per unit of conserved or unextracted oil (methane) by p_{uo} (p_{um}), and keeping in mind that the extraction of oil and methane require the same amount of labour per unit of each resource ($c = d$), we have

$$p_{eot+1} = (1+r)\,p_{uot} + q_{ot} + cw_t \tag{4}$$

$$p_{emt+1} = (1+r)\,p_{umt} + q_{mt} + cw_t \tag{5}$$

$$p_{uot+1} = (1+r)\,p_{uot} \tag{6}$$

$$p_{umt+1} = (1+r)\,p_{umt} \tag{7}$$

Note that processes (6) and (7) are operated in any case, as long as the two resources have not been fully exhausted. Equations (6) and (7) imply that

$p_{uot} = (1+r)^t\, p_{uo0}$ and $p_{umt} = (1+r)^t\, p_{um0}$,

which is, of course, the Hotelling Rule. Further, $q_{m4} = 0$ and $q_{o6} = 0$, since for $t = 4$ only 300 units of methane are still available and no rent can be obtained on gas fields, and at $t = 6$ only 100 units of oil are still available and no rent can be obtained on oil fields. This implies that

$$p_{uo6} = \frac{3 - 5c(3-r)}{5(3-r)(1+r)} \quad \text{and} \quad p_{um4} = \frac{9 - 10c(1+r)}{10(1+r)^2};$$

and therefore

$$p_{uot} = \frac{3 - 5c(3-r)}{5(3-r)(1+r)^{7-t}} \quad \text{and} \quad p_{umt} = \frac{9 - 10c(1+r)}{10(1+r)^{5-t}}.$$

The important point to be emphasized is the following. Although the Hotelling Rule applies to the prices of the *in situ* stocks of the two resources, *the prices of the commodities above the ground, including extracted oil and methane, are constant*. This is so because the rent rates that result from the capacity constraints with regard to the extraction of the two resources change over time in a particular way: they start falling as soon as royalties on oil and methane start rising, and they do so in such a way that their fall just compensates the rise in royalties:

$$q_{ot} = \frac{\left[3 - 5c(3-r)\right]\left[1 - (1+r)^{6-t}\right]}{5(3-r)(1+r)^{6-t}}, \quad q_{mt} = \frac{\left[9 - 10c(1+r)\right]\left[(1+r)^{4-t} - 1\right]}{10(1+r)^{4-t}}.$$ [10]

[10] We may also contemplate the case of an *una tantum* technical progress in the extraction industries, for example, which reduces coefficients c and d. As is well known, in Ricardo's discussion in chapter 2 of the *Principles*, land-saving "improvements" tend to *reduce* the rents of land, given gross output levels (see Gehrke, Kurz and Salvadori, 2003). In our system above it is instead possible that technical progress may *increase* the rents obtained

(d) *No capacity constraint with regard to oil extraction*

In order to understand better the role played by capacity constraints on extraction, consider a case that is identical to the previous one, except that there is no capacity constraint limiting the extraction of oil, whereas there is one with regard to the extraction of methane.

This implies, first, that process (1) will not need to be activated at time 0: the entire effectual demand can be met without it. Further, no rent will be paid to the proprietors of oil fields. Hence, equations (2) and (6) hold, whereas instead of equation (4) we have

$$p_{eot+1} = (1+r)\, p_{uot} + cw_t \tag{8}$$

If $w_t = 1$, equations (2), (6), and (8) determine

$$p_{uot} = (1+r)^t\, p_{uo0}$$

$$p_{eot} = (1+r)^t\, p_{uo0} + c$$

$$p_{ct+1} = \frac{1+r}{10} p_{ct} + (1+r)^{t+1}\, p_{uo0} + (1+r)c + 1 \tag{9}$$

Equation (9) in turn determines

$$p_{ct} = A\left[\frac{1+r}{10}\right]^t + \frac{10}{9} p_{uo0}(1+r)^t + \frac{10\left[(1+r)c+1\right]}{9-r},$$

where A is a constant (to be ascertained) such that

$$p_{ct+1} \le (1+r)\frac{1}{4} p_{ct} + 1,$$

otherwise capitalists would prefer to use process (1) instead of process (2) to produce corn.

Let T be the time at which oil is almost entirely exhausted, that is, $T = 2$ in the case in which it is never profitable to produce corn with methane, i.e. use process (3). Otherwise either $T = 3$ or $T = 4$, in the case in which it is profitable to produce corn with methane for a positive number of years smaller or larger than 3, respectively, before oil is exhausted. Hence

$$p_{cT+1} = (1+r)\frac{1}{4} p_{cT} + 1$$

by owners of oil and methane fields as well as the royalties they are able to get. This will indeed be the case if the backstop process will have to be operated in order to meet effective demand even after the improvements in the extraction industries have taken place.

and therefore

$$A\left[\frac{1+r}{10}\right]^{T+1}+\frac{10}{9}p_{uo0}(1+r)^{T+1}+\frac{10\left[(1+r)c+1\right]}{9-r}=$$

$$=\frac{1+r}{4}\left\{A\left[\frac{1+r}{10}\right]^{T}+\frac{10}{9}p_{uo0}(1+r)^{T}+\frac{10\left[(1+r)c+1\right]}{9-r}\right\}+1.$$

That is,

$$A=\frac{5\cdot 10^{T+1}}{9}p_{uo0}+\frac{\left[5c(3-r)-1\right]10^{T+1}}{(9-r)(1+r)^{T}}$$

and

$$p_{ct}=\frac{10\left(1+5\cdot 10^{T-t}\right)}{9}(1+r)^{t}p_{uo0}+\frac{10}{9-r}\frac{c\left[5\cdot 10^{T-t}(3-r)+(1+r)^{T+1-t}\right]-10^{T-t}+(1+r)^{T-t}}{(1+r)^{T-t}}.$$

Note that only if

$$p_{uo0}=\frac{3(9-r)-15c(5-r)(3-r)}{5(3-r)(9-r)(1+r)^{T}}$$

$p_{cT+1}=4/(3-r)$ and the price of corn is constant for $t\geq T+1$.

It will be profitable to produce corn by methane for at least one year if

$$p_{ct+1}>(1+r)\left[\frac{1}{4}p_{ct}+c\right]+\frac{1}{10} \tag{10}$$

for some *t*. It is easily checked that inequality (10) holds if $c=0$ and $t=T$. Then there is a $\gamma>0$ such that inequality (10) holds for $0\leq c<\gamma$ and $0\geq t\geq T$. Let us assume that *c* is in this interval and therefore $T=4$. Moreover, equations (3) and (5) hold and $q_{m4}=0$. Since equation (7) always holds if methane is not exhausted,

$$p_{umt}=(1+r)^{t}p_{um0}.$$

This example shows that if the extraction capacity of one resource is not limited and therefore the backstop process does not need to be operated since the beginning, the interplay of rent and royalties is not sufficient to keep prices of corn and of extracted resources constant over time.

(e) *Higher capacity constraints with regard to oil and methane*

While the absence of any capacity constraint with regard to one resource and the presence of a constraint with regard to the other one refers already, in an extreme way, to the role of *differences in constraints* on the behaviour of the system, some further examples might contribute to our understanding of the phenomena at hand. Here it suffices to draw the attention to the following case. Assume that the two resources are subject to positive, but different capacity constraints: no more than 700 units of oil can be extracted per year and no more than 600 units of methane. In this case, the backstop process (1) will not be operated at time 0 and rent will be paid only on one of the two fields of exhaustible resources. The exercise is clear enough, and the interested readers can carry it out by themselves.

7. Concluding remarks

The paper provides a summary account of the classical approach to exhaustible natural resources, old and new, and compares it to the marginalist approach represented by the Hotelling model. It is argued that in well-specified circumstances the prices of produced commodities need not change, although the *in situ* prices of exhaustible resources that are actually exhausted over time will change. In order to show this, we start from the observation, to be found, for example, in the writings of the classical economists from Adam Smith to David Ricardo, that the exploitation of fields or deposits of exhaustible resources is typically subject to capacity constraints that limit the amount of the resource that can be extracted during a given time period from each field or deposit. As a consequence, several deposits of resources with different unit costs of extraction are exploited side by side. This implies that rents will have to be paid to the owners of fields that are more "fertile", to use Ricardo's expression in Chapter 3 of the *Principles*. Finally, if the backstop process is operated, this implies that the prices of commodities available on the ground are constant despite the fact that the prices of resources *in situ* vary over time according to the Hotelling Rule. This is made possible by an inverse movement of rents on the one hand and royalties on the other.

A little model in which corn can be produced by means of corn alone or by means of corn and oil or corn and methane, two exhaustible resources, is then elaborated and numerically specified. The findings of our analysis are illustrated by means of judiciously chosen examples. While the analytical structure of the model is quite simple, one gets a fairly rich typology of cases, each of which is associated with different time paths of the quantity of corn produced, the amounts of two kinds of exhaustible resources extracted or conserved in the ground, the prices of corn and the resources in and above the ground, and income distribution. The analysis allows one to distinguish clearly between three different types of property income: rents, profits and royalties. It also provides some reasons that help us to explain why the Hotelling Rule does not perform all that well in empirical studies.

References

Hotelling, H. (1931). The economics of exhaustible resources, *Journal of Political Economy*, 39, pp. 137-75.

Gehrke, C., Kurz, H. D. and Salvadori, N. (2003). Ricardo on agricultural improvements: a note, *Scottish Journal of Political Economy*, 50, pp. 291-6.

Krautkraemer, J. A. (1998). Nonrenewable resource scarcity, *Journal of Economic Literature*, 36(4), pp. 2065-2107.

Kurz, H. D. and Salvadori, N. (1995). *Theory of Production. A Long-Period Analysis*, Revised paperback edition 1997, Cambridge, U.K.: Cambridge University Press.

Kurz, H. D. and Salvadori, N. (2000). Economic dynamics in a simple model with exhaustible resources and a given real wage rate, *Structural Change and Economic Dynamics*, 11, pp. 167-79.

Kurz, H. D. and Salvadori, N. (2001). Classical economics and the problem of exhaustible resources, *Metroeconomica*, 52(3), pp. 282-96.

Kurz, H. D. and Salvadori, N. (2009). Ricardo on exhaustible resources, and the Hotelling Rule, in Aiko Ikeo and Heinz D. Kurz, eds, *The History of Economic Theory. Festschrift in Honour of Takashi Negishi*, London: Routledge, pp. 68-79.

Kurz, H. D. and Salvadori, N. (2011). Exhaustible resources. Rents, profits, royalties and prices, in V. Caspari (ed.), *The Evolution of Economic Theory: Essays in Honour of Bertram Schefold*, London: Routledge.

Mill, J. St. (1965). *Principles of Political Economy with Some of Their Applications to Social Philosophy*, 1st edn 1848, edited by J. M. Robson. Toronto: University of Toronto Press.

Parrinello, S. (1983). Exhaustible natural resources and the classical method of long-period equilibrium, in J. Kregel (ed.), *Distribution, Effective Demand and International Economic Relations*, London: Macmillan, pp. 186-99.

Ricardo D. (1951). *On the Principles of Political Economy and Taxation*, first edn 1817, in *The Works and Correspondence of David Ricardo*, edited by Piero Sraffa with the collaboration of Maurice H. Dobb, Vol. I, Cambridge: Cambridge University Press.

Schefold, B. (1989). *Mr Sraffa on Joint Production and Other Essays*, London: Macmillan.

Schefold, B. (1997). *Normal Prices, Technical Change and Accumulation*, London: Macmillan.

Smith A. (1976). *An Inquiry into the Nature and Causes of the Wealth of Nations*, first ed. 1776, Vol. II of *The Glasgow Edition of the Works and Correspondence of Adam Smith*, edited by R. H. Campbell, A. S. Skinner, and W. B. Todd, Oxford: Oxford University Press.