Reconsidering Prior Appropriative Water Rights in the Western United States with Heterogeneous Multiple Users and Risk Management

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Background & Objective

Water rights systems that provide *ex ante* rules for allocating stochastic water supplies differ in their methods of risk allocation, influencing *ex post* amounts of water received. The major system in the arid U.S. West, prior appropriations, is an **ordering scheme (OS)** that defines a water right with two parts: an *ex ante maximum* amount claimable (**water duty**), and a **priority status** for receiving that amount (Libecap 2011). In years when water is insufficient, OS water right holders may claim their water duty as long as higher (senior) priority holders receive their duties first. The probability of receiving a full duty increases with priority status. Alternatively, **Proportional sharing (PS)** defines a water right as a share of total supply (Bennett et al. 2000). Water right holders choose how many shares to own, and expected amount of water receivable and risk are determined jointly. All share holders receive at least some water when total water supply is very low. When water supplies are scarce, the OS tends to over-allocate water to higher priority users, while PS tends to over-allocate water to water right holders engaged in potentially lower value uses.

Bennet et al. (2000) propose a **Mixed Scheme (MS)** that combines features of OS and PS, in which water is allocated in intervals ordered by users' marginal benefits **(MB)**, from high to low. Within intervals, water is allocated using PS. Unlike PS, some lower value users may not receive water during low total supply. Theoretically, MS results in an efficient allocation, albeit with strong assumptions necessary for implementation.

We compare these systems to show that informational and administrative costs render MS infeasible for individual water rights. We analyze performance of OS and PS in terms of risk management under ***N*** heterogeneous users to show that ordering schemes approximate optimal allocations better than sharing schemes, and inefficiencies under PS increase with *N* and scarcity but does not under OS.

Notation and Model

***N*-user Model:** We generalize Bennett et al.'s (2000) two-user model to *N* individuals. Index i also represents the order of MB among users. By heterogeneity, $(\partial MB_i)/\partial w < (\partial MB_j)/\partial w, MB_i(0) > MB_j(0) \forall i > j$ (**Figure 1**). Under MS, user i faces $N-i+1$ water allocation intervals, with different proportional shares.

X = random total water supply, with PDF $f_x(.)$and CDF $F(.)$. x^g is total annual realized supply. w_i^{g*}= optimal water allocation for $X = x^g$. i indexes the order of marginal benefits and priority. Smaller i implies larger MB and higher priority. WR_i = user i's OS water duty. $A_i = \sum_{i=1}^{i-1} WR_i$ = water claimed by prior rights holders. w_O = water receivable under OS. A share is a proportion of total water supply, with 1 share defined as $1/N$. $S_{i,j}$ = user i's portion of water in the j^{th} interval in MS. MA^g = total water misallocation in water units under OS given x^g. AD_i = aggregate demand function for first i users. MB^g= marginal social benefit given x^g.

Water Management Criteria

Mean-Variance: For a given level of expected water receivable, a water rights portfolio with less variance is preferred.
Safety-First: Secure at least *C* amount of water receivable with probability *q*.

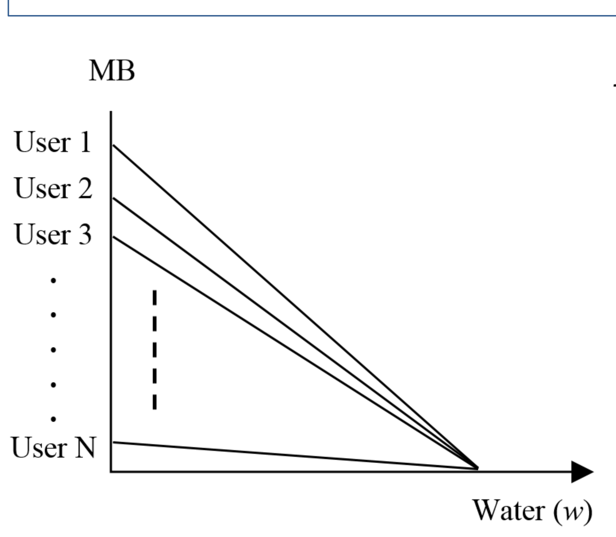

Figure 1. Marginal Benefits of *N* Users

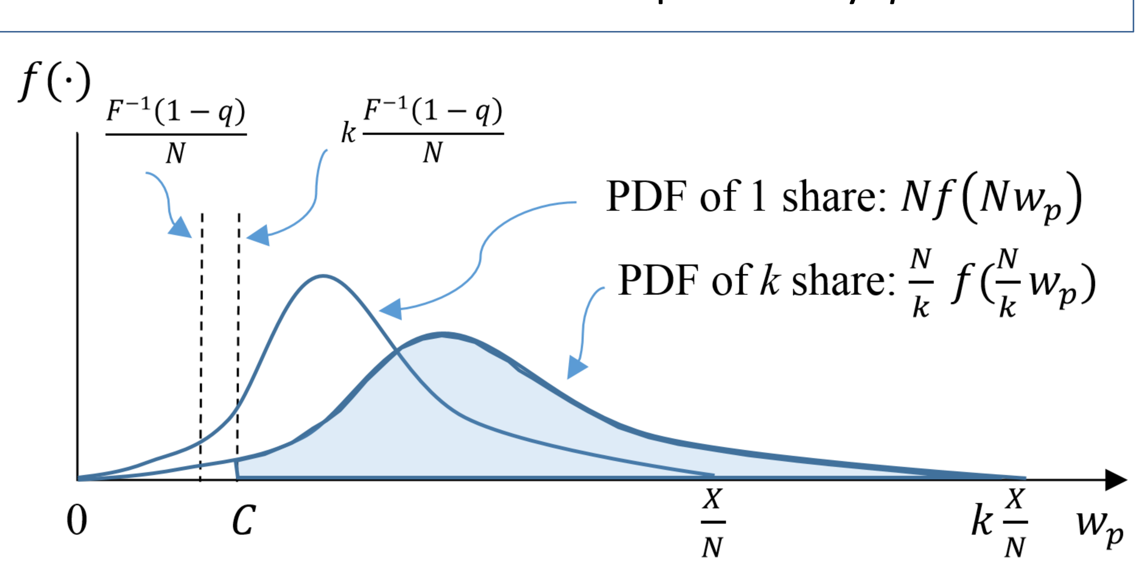

Figure 2. PDFs of Water Receivable for 1 and *k* shares

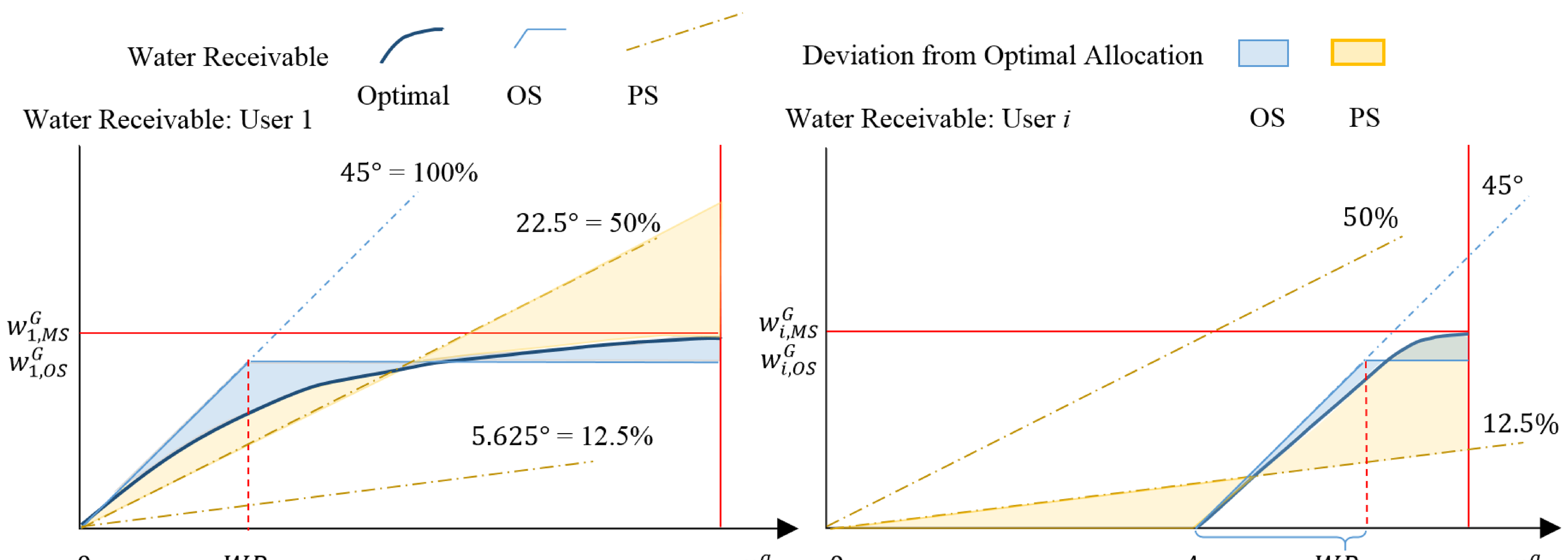

Figure 3. Deviation from Optimal Water Allocation under OS and PS: Highest Value User 1 and User *i*

Results

MS requires that a water authority knows the MB functions for all users; informational and administrative costs increase with the number of users. With MS, users share water within *N* allocation intervals with allocations across users uniquely determined for each interval. The authority calculates the sizes and number of intervals based on all *N* MB functions and total water supply and, under full information, determines up to $\sum_{i=1}^{N} N-(i-1) = (N^2+N)/2$ water allocations. Implementation costs and the administrative burden increase with *N*, number of users and their distinct MB functions.

Mean-Variance: For a given expected water receivable, a user can choose a smaller variance under OS than under PS by satisfying the distribution-free condition: $A_i \geq (N-1)WR_i$. Assume a desired mean $\bar{w}$, which can be obtained by 1 share in PS, or in an OS, through a combination of water duty and priority such that: $\bar{w}_{PS} = \frac{E(X)}{N} = \bar{w}_{OS} = \int_0^{WR_i} w_O f_x(w_O + A_i) dw_O + \int_{WR_i}^{\infty} WR_i f_x(w_O + A_i) dw_O$. It can be shown that a distribution-free condition for a smaller variance under OS is: $A_i \geq (N-1)WR_i$. Generalized to k shares, the condition is: $A_i \geq (N/k - 1)WR_i$. Under PS, variance cannot be chosen separately from a desired mean.

Safety-First: Implementing this criterion can result in over-acquisition of shares. Under PS, assume a desired share = 1. If the probability of receiving at least *C* amount of water is smaller than q, $1 - F(NC) < q$, the user can purchase k shares to the meet this criterion, where for $k > 1, 1 - \mathrm{F}(\frac{N}{k} * \frac{k}{N} F^{-1}(1-q)) = q$. Yet, this results in over-acquisition of water (**Figure 2**). Alternatively, under OS, a user can purchase a duty with the priority for desired mean and critical amount *C* with probability *q*.

PS over-allocates to lower value users. Over-allocation is increasing with N and scarcity. An optimal allocation implies that lower MB users may not receive water, depending on total water supply and relative MB. However, PS always allocates some water to low value users (**Figure 3**, right), resulting in welfare loss. Since the number of lower value users is non-decreasing with *N*, a larger *N* results in larger welfare loss. Since greater scarcity implies more users should optimally receive no water, welfare loss increases with water scarcity.

An OS better approximates optimal allocation relative to PS when N is large. **Figure 3** illustrates individual water allocation functions of total supply under MS, PS, and OS. When $\partial MB_N/\partial w \to 0$ and $\partial MB_i/\partial w \nrightarrow 0$, we have $S_{i,N} \longrightarrow 0$. For large *N*, the optimal allocation is approximated by a non-decreasing curve with slope → 0 at maximum water supply. This function under OS is a combination of a 45° line from A_i and a flat line after WR_i, while under PS it is simply a straight line starting from 0. An OS thus better approximates the MS curve for user *i*. Bennett et al.'s (2000) conclusion that PS better approximates MS is a special case in a 2-user model.

Ordering schemes over-allocate water to higher value users in scarcity. As $N \to \infty$, the extent of misallocation is limited by $A_i - AD_{i-1}(MB^{g*})$ for realized water x^g that falls in $A_{i+1} > x^g \geq A_i$, or by $x^g - AD_1\ (MB^{g*})$ if $x^g < A_1$. **Figure 4** shows a 4-user model under an OS. If $X = x^1$, the water quantity misallocated is $A_3 - AD_2(MB^{1*})$. Bounded from above, the misallocation increases with additional users. Aggregate demands AD_N for adding 1 and 2 users, each with MB_3, is shown by the two blue lines from ①, defined by $\max(AD_{i-1} = AD_i)$. As $N \to \infty$ (additional ∞ users all with MB_3), MA^1 is bounded by $A_3 -$ ①. ***Adding any number of users j with $MB_i(0) > MB_j(0)$ does not increase MA^g***, as in scarcity no water would be allocated to these lower value users under an OS. Adding higher value users still results in limited misallocation. Increasing scarcity from x^1 to x^2 results in a misallocation of $A_2 - AD_1(MB^{2*})$.

Conclusions and Implications

An OS allows heterogeneous water users to effectively choose variance separately from expected water receivable, while PS cannot. In the West where OS is common, many states allow trading of priority rights - resulting in unique water rights portfolios for heterogeneous users managing risk.

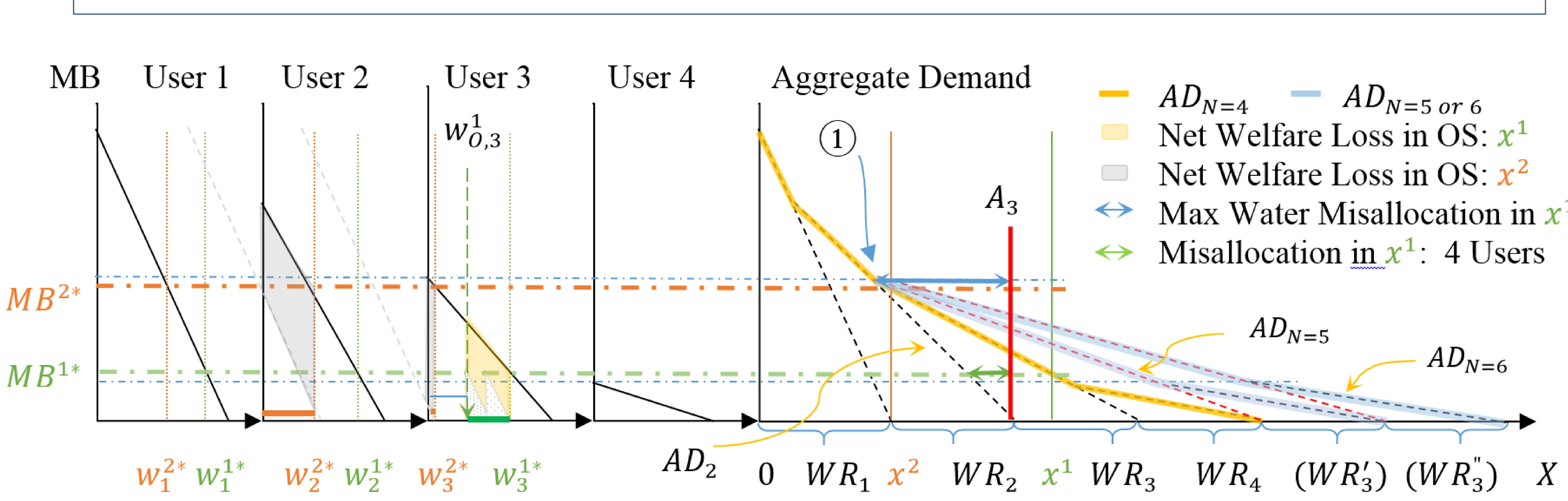

Figure 4. Welfare Loss under an Ordering Scheme

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