Endogenous Population Growth in a Macro Environmental Model

Faraz Farhidi*
Department of Economics
Georgia State University
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Abstract

I present a dynamic growth model that explicitly allows for the interaction between an economy and an environment. I allow for endogenous population growth, where population is affected by living standards and level of industrialization as well as natural resources, indirectly through production. I also incorporate a trade-off between non-renewable energy reserves and renewable resources. Running out of fossil fuel-based energy and endogenizing the population growth the growth rate of GDP per capita is lower under endogenous population scenario relative to exogenous population growth. In a decentralized model, firms conserve non-renewable energy for a shorter period, while do not fully internalize the negative externalities arise from utilizing of non-renewable energy, compared to the social planner approach. Imposing carbon-tax element on the energy producers would speed up the adaptation of the clean energy and sustain fossil fuel resources for a more extended period and would increase the individuals' total consumption in the long-run.

Keywords:

Fossil-fuel Energy, Renewable Energy, Population Growth, Endogenous Growth, Environmental Degradation

JEL Classification:

C61, O44, Q43, Q56

* Ph.D. candidate, email: ffarhidi1@gsu.edu

I. Introduction

While the Industrial Revolution allowed for the development of new schemes of utilizing fossil fuel resources that ultimately lead to economic growth (Stern, 2011), there is solid evidence that devastating effects of climate change – due to the use of fossil fuel-based energy – will take place unless major actions are taken immediately to transform our fossil fuel-based energy system into a non-fossil fuel-based system (Schwartzman, 2008). Predicting the economy's future growth path – while taking into consideration the effects of the environmental degradation – is of the utmost importance. In this regard, there are often two overlooked issues in the macro-environmental literature: Many models assume that population growth is exogenous and does not feedback on the environment. Second, most of the environmental approaches do not include the binding constraint of non-renewable resources into their model[1].

In this paper, I extend macro environmental framework by allowing for both non-renewable and renewable energy, and by endogenizing population growth, using both social planner and market-based approaches. The effect of population growth on economic activities is not clear based on different models and approaches. Hardin (1968) argued that to have a sustainable economy, population growth must be zero in order to keep our limited resources from being over-utilized. Meadows, et al. (1972) reported that the Earth's industrial capacity and the population would catastrophically decline if we continue the level of capital accumulation that Turner (2007) and Hall & Day (2009) showed. However, Grossman & Helpman (1991) and Aghion & Howitt (1998) claim that high population spurs technological change, which is the engine of economic growth (Romer, 1990; Jones, 2002). Building upon the existing endogenous population growth framework, I connect population growth with not only the living standards and level of industrialization but also with the adaptation of renewable energy resources. By solving the proposed model and making predictions based on the different energy adaptation scenarios, policy recommendations will be derived.

My work has two main contributions. First, moving away from the exogenous population growth as in the existing climate models, adjusting the framework proposed in the endogenous population studies such as Cigno (1981), Ehrlich and Lui (1997), Nerlove and Rault (1997), and Krutilla and

[1] Basically, there is no end point in their prediction.

Reuveny (2006); and adding a resource binding constraint to create a tradeoff between renewable and non-renewable energies. The second contribution is to modify a new model for technological progress in which new technology is a function of existed technology, number of researchers, and investment. Financing new advancement in technology is vital in the proposed setup which has been neglected from the previous works. In the model, energy is the primary factor in the production process, the same as technological progress, labor forces, and physical capital. Stiglitz (1974) explores the implications of introducing exhaustible natural resources. In his model, natural resources can make the system unstable, as an essential factor of production. Hartwick (1977), Nordhaus (1996, 2008), Popp (2004), Hassler & Krusell (2012), Krusell et al. (2016) and Kummel (2016) present similar models in which energy is considered a primary factor of production and is used to identify the impact of resource constraints on economic activity and the environment. However, the energy itself can be substituted by any other source of renewable energy. In the model setup, there are binding constraints for the resources – following Acemoglu et al. (2012) – which limit growth.

The proposed model assumes that while population is important for the growth path of the economy by providing labor force and researchers, it has an adverse impact on the economy due to the constraints of the environment and resources. Endogenizing the population growth, while including environmental erosion, results indicate that the growth rate in the economy would be slower in the endogenous population growth relative to the exogenous scenario. One of the reasons for such a different conclusion is that population leads to the economic growth through providing labor and researchers for the production process in both scenarios. In the endogenous case, however, there is feedback from environmental erosion on population, which diminishes the sources for future economic growth. Another important finding is that there would be a smooth transition in the economic activity, in the absence of fossil-fuel energy, and adapt the production process that relies entirely on using renewable energy as a primary energy factor if the population is considered exogenous. Comparing two modified approaches to solve the model, I show that in the market-based method the firms utilize intensively more fossil fuel, relative to the social planner approach. The rate of clean energy adaptation would be lower relative to the centralized method.

The paper is formatted as follows: The second section below reviews the existing literature which is connected to this research. Section three presents a theoretical model that can be used to verify the validity of the discussed questions in this research, with a following short section on solving the model and calibrating the parameters. Then, I propose a decentralized model which is closer to the current market structure in the developed countries. In the fifth section, the results of the social planner solution will be discussed for both exogenous and endogenous population growth, as well as a comparison between two different methods will be examined. Lastly, I introduce two policy recommendations to the market-based approach, and present a welfare analysis.

II. Literature Review

The existing literature in endogenous growth has focused on technology and rarely on population impacts; whereas the literature on environmental degradation, caused by utilizing fossil fuel energy, has relied mostly on exogenous technology and population growth as reviewed below. Neither literature yet has studied a comprehensive model in which the often-discussed elements have been fully addressed. Recent endogenous growth models, such as AK[3], R&D, and Schumpeterian growth models, explicitly allow for optimizing the technological process. In those models, both innovation and capital accumulation can determine the long-run growth rate. In the long run, the stock of ideas is proportional to the worldwide research effort, which in turn is proportional to the total population of innovating countries (Jones, 2002). Acemoglu et al. (2012) introduced environmental constraints into a growth model with competing applications of innovation. The fact that knowledge spillovers create positive externalities plays a crucial role in the ultimate cost of climate and technology policies (Fischer & Heutel, 2013).

Climate change engineered by human activity is a pure externality with global scope. The fossil-fuel use causes emissions of carbon dioxide into the atmosphere and results in global warming, thus imposing a cost that impacts not only all living humans but also future generations (Hassler & Krusell, 2012; Krusell et al. 2016). Mathiesen et al. (2011) revealed that utilizing renewable energy and more efficient conversion energy technologies can have positive socioeconomic impacts and lead to a potentially higher rate of employment and earnings. Fully renewable energy systems will be technically achievable soon and can be economically beneficial, compared to current energy systems. Tahvonen and Salo (2001) believed that there would be a smooth shift from non-renewable to renewable resources, and it causes a drop in the future economic growth

[3] AK model is one the first models which attempts to endogenize the economic growth by using a model in which output is a linear function of capital (Y=AK).

which is going to recover after some period. Sustainability of development depending on renewable resources has been confirmed by other researchers such as Li and Lofgren (2000) and Lund (2007).

Stiglitz (1974) explored the assumptions of introducing exhaustible natural resources, which can make a system unstable, as an essential factor of production with a constant rate of population growth. Later on, Kummel et al. (2002) presented a more advanced model, called KLEC, in which the combination of capital, labor, energy, and creativity produces a final good. Nordhaus (1977, 1994, 2000, 2008, 2011), Golosov et al. (2012), and Hassler and Krusell (2012) have pioneered the area by building integrated assessment models (vastly known as DICE and RICE) expanding neoclassical growth models. They augmented essentially with a set of climate equations mapping atmospheric carbon into temperature and energy sectors, allowing people to expend costly resources to limit emissions from a given amount of use of fossil fuels. There exists another line of literature (employed by Bernstein et al. 1999, Rutherford et al. 2009, and others) that explore the impacts of climate policies on the energy market and economy using Multi-Sector, Multi-Region Trade (MS-MRT) based on computable general equilibrium method (CGE). But the role of population in all of the mentioned models has been neglected.

Ehrlich & Ehrlich (1990) claimed that there is an issue with overpopulation in a region relative to its resources and the ability of the environment to sustain human activities. Recent issues such as climate change, the global decline in population growth rate, and the recent economic downturn have prompted renewed concerns about whether long-standing trajectories of the population and economic growth can continue (Brown et al. 2004). Meadows et al. (1972) stated that the earth's industrial capacity and population would catastrophically decline if we continue the level of capital accumulation that Turner (2007) and Hall and Day (2009) showed. Following Lee (1988), Kremer (1993) constructed an integrated model of population growth and technological change; the proposed empirical evidence supports his model that the growth rate of the world population has been proportional to the degree of population. These results are opposed by pioneering economists such as Becker and Barro (1989) and Acemoglu and Johnson (2007) who believed that population growth hurts income per capita.

Setting up a model of endogenous technological change that nests the Romer (1990) and the Jones (1995a) frameworks, Prettner (2013) introduced endogenous fertility decisions of households, caring for the number of kids they have, considering the associated costs. He indicated that underlying demographic processes play a vital role in characterizing the R&D intensity, and therefore, affect long-run economic growth contexts of industrialized countries. Nerlove & Rault (1997) modified the Solow-Swan model in 1956 by introducing a simple form of an endogenous population and showing that as income grows, fertility rate might not change because both birth and death rates fall, and physical and human capital per capita increase over time.

Cigno (1981) was the first one who argued that the assumption of a constant rate of population growth is implausible in an economy constrained by exhaustible resources and examined the implications of making the population growth rate a function of consumption and capital per capita. Fanti and Manfredi (2003) extended Solow’s model and accounted for the continuation of a delay in the process of employment, due to the age structure of the population. They also utilized the existence of a Malthusian relation between wage and fertility, to generate stable fluctuating growth paths. An interesting consequence of the presence of the endogenous population in their model is that population growth may eventually promote economic growth. Later on, Krutilla and Reuveny (2006) evaluated the dynamic effects of incorporating an endogenous process for population growth into a renewable resource-based growth model. Their model is abstract in the Macroeconomics sense since there is no capital accumulation and production process. In their model, renewable energy has only been used as a resource; thus, there is no trade-off between renewable and non-renewable resources. Moreover, they linked population to renewable resources where there is no limit on the non-renewable reserves. They, as did Stokey (1998) and Dasgupta and Maler (2000), reemphasized the urgency for the development of growth models that include both the environment and endogenous growth for human populations.

The models we have been discussing so far do not allow for the trade-off between non-renewable natural resources and renewable resources, or an endogeneity of population growth and technological progress. In the current research, to extend the environmental macro models, in the climate context such as DICE, my model specification includes endogenous population growth – based on the degree of industrialization and income level – as well as endogenous technological progress. As such, in the proposed framework, I am able to identify how endogenizing the model can affect the growth path of the economy, considering the environmental deterioration, and predict long-run growth with and without fossil fuel energy. In addition, the model will be calibrated based on not only the US data analysis, but also the empirical estimation derived from previous work.

III. Model and Solution Method

In this section, first, I plan to build up the model in the following sub-section. Then, I am going to disclose how values have been assigned to different parameters. In the third and fourth sub-sections, the method to solve the model has been explained.

III.A Constructing the model

There is a representative consumer in the model – consistent with the Ramsey-type models – with a utility function of a single commodity that is consumed at different points across time. The utility function includes a discounting factor to smooth consumption over time. The consumption good is delivered with an aggregate production function of technology, capital, labor, and energy, and it allows for the environmental degradation. Technological progress in clean energy, as well as the population growth, is endogenous in this model. Capital is accumulated in a standard Solow model, taking investment and consumption to be perfect substitutes.

The following model is a modified version of the Popp (2004) model[4], which is an extension of the DICE model itself by endogenizing the technological progress based on R&D models. I also endogenized population, according to the process in Cigno (1981). The other distinction between this model compared to the base model is the possibility of making a model stochastic by adding exogenous shock to the technological progress and discoveries the new resources. In the proposed model, households maximize their utility which is a function of consumption per capita, (Eq. 1) subjects to the income constraint (Eq. 3), in an infinite horizon.

$$\underbrace{\mathrm{Max}}_{C_t, K_{t+1}, TY_t, FE_t} = E_0 \sum_{t=0}^{\infty} \beta^t U(c(t)) \tag{1}$$

$$U\big(c(t)\big) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad c_t = {}^{C_t}/_{L_t} \tag{2}$$

[4] I will use the discrete model excluding the population in utility function according to Hassler & Krusell (2012).

In the equations above, U_t represents utility at time t, C_t is the total consumption, c_t is per capita consumption, L_t represents the total labor force in the market, β is a discount factor to represent the rate of time preference, and σ is the parameter for the risk attitude of the agent.

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t + CEX_t FE_t + TY_t \tag{3}$$

Equation three shows the income allocation in which CEX (the cost of the providing of fossil-fuel energy[5]) is derived endogenously in the model. In the above setting, part of the income (TY) finances the technology for the clean energy (AC). C is the total consumption, K is the physical capital, and FE represents for non-renewable[6] energy.

$$Y_t = ED_t[A_t K_t^{\alpha} PL_t^{1-\alpha-\gamma} E_t^{\gamma}] \tag{4}$$

$$E_t = [(CE_t)^{\rho} + FE_t^{\rho}]^{1/\rho} \tag{5}$$

$$CE_t = AC_t * CE \tag{6}$$

$$ED_t = 1 - ({FE_t}/{\varphi})^{\vartheta} \tag{7}$$

Here, I included the energy as another primary factor of production (Y_t), as did Krusell (2016). A_t is the technological progress, PL_t is the fraction of the labor force who participates in the production process – directly – and E_t is the energy input required in the production process as a primary factor. ED_t is the environmental deterioration constraint (or damage function), as a decreasing function of the non-renewable energy consumption (FE_t). φ is the normalizing factor to keep the negative impact of FE_t on the production less than one. Energy is another primary factor of production such as technology, physical capital, and labor. A key aspect here is that non-renewable energy resources are finite, unlike DICE-RICE models in which the fossil fuel supply is treated as inexhaustible (Nordhaus & Boyer, 2000). However, the renewable resources, based on the availability of the technology, are infinite. CE is the total available stock of clean energy in an area ready to use. However, we can only use part of the energy, based on technological advances, AC_t, to utilize it. The variables and parameters are listed and explained in Appendix A.

$$AC_{t+1} = AC_0 AC_t^{\theta}(TL_t TY_t)^{\omega} \tag{8}$$

[5] This cost is not exactly equivalent to the cost of extraction in Stiglitz (1976), as it is argued in Appendix D.
[6] Or we can consider it as fossil fuel energy.

Popp (2004) used an R&D based model (Jones, 1995) to endogenize the technological progress in his model. However, the production technology of the clean energy – which is utilized here – is the modified version of Jones (2002) in the way that TY has been added, which is the required resources for financing the technology, utilized recently by Farhidi (2017). TL is the effective research effort. AC_t is the required technology to utilize clean energy such as solar and wind. The economy consists of two types of labor: the researcher who produces a new idea, and the laborers who produce the final good as an output.

$$CEX_t = P_0 + P_1\left(\sum_{i=1}^{t} FE_i \Big/ \overline{FE}\right)^{P_2} \qquad \sum_{i=1}^{t} FE_i \leq \overline{FE} \tag{9}$$

Following the idea in Popp (2004), the cost of extraction of the fossil fuel energy (CEX) is the sum of the marginal cost of fuel extraction and a markup which includes any transaction costs and the difference between marginal costs of extraction and consumer prices (P_0), according to equation 9, in which $\overline{FE}$ is the total fossil fuel available to extract, and it is provided by nature. P_1 represents changes in marginal cost as the extraction changes, and P_2 shows the impact of the ratio of fossil fuel accumulation on the price level. When P_{FE} hits its maximum, $P_0 + P_1$, there would be no fossil fuel resources remaining to use.

$$A_{t+1} = (1 + \bar{A})A_t \text{[7]} \tag{10}$$

$$PL_t + TL_t = L_t \tag{11}$$

$$l_{PL} = \frac{PL_t}{L_t} \text{ and } l_{TL} = \frac{TL_t}{L_t} \tag{12 \& 13}$$

Technological progress for the production process (At) is considered exogenous. For labor force participation, we need to define two ratios (l_{PL} and l_{TL}), which are assumed to be constant over time; therefore, the distribution of the labor force does not change between two different sectors, which are shown in equations 12 and 13.

$$L_{t+1} = (1 + \bar{L})L_t \qquad \text{(If the population grows exogenously)} \tag{14}$$

$$L_{t+1} = L_t + L_0\left(Y_t / L_t\right)^{\varepsilon_1}\left(L_t / K_t\right)^{\varepsilon_2} \qquad \text{(If the population grows endogenously)} \tag{15}$$

[7] We can also consider the technological progress stochastic in the production process to capture any possible fluctuation later.

L_t is the level of population[8] in an economy. The main distinction of the presented model is built as follows. I endogenized the population growth which is directly retrieved from Cigno's (1981) model[9] by linking it to the environmental degradation through production; therefore, the constant population growth in equation 14 ($\bar{L}$) was replaced by the setup in equation 15. Therefore, I use equation 14 for the first specification of the model in which population grows exogenously; and then use equation 15 in the other model specification.

It must be noted that income plays an important role in population growth. Fertility theories proposed by Becker (Becker 1973; Becker et al. 1994) highlight the indirect influence of living standards within this framework. L_0 can be derived exogenously by the fact that population is a biological factor that grows exponentially. But, because of industrialization, the nature of this growth has varied over time. The rate of population growth is positively related to per capita consumption and inversely related to the degree of industrialization[10]. There are five choice variables in this model which are physical capital (K), fossil fuel energy (FE), utilizing the clean energy (AC), required resources for financing the clean technology (TY), and the consumption (C).

III.B Data Calibration

To calibrate the model's parameters, I assigned the previously used values – in the literature – to the parameters, and I estimated the ones which there are no values for, using real data. I used data from 1990-2012, mostly retrieved from the World Bank Data Center, for the different indices to calibrate the parameters using time series analysis for the US only. I also used environmental bio-capacity[11] – retrieved from the Global Footprint Network database – as a proxy for the environmental degradation. For the total energy (E_t), I included the country's total energy use, and

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[8] Population refers to the labor force in the current setup, not the total population of an economy

[9] Krutilla & Reuveny (2006) link the population only to renewable resources since their model does not include production process, capital accumulation, and non-renewable resources.

[10] Degree of industrialization is the capital-labor ratio. Based on Cigno (1981), industrialization and its concomitant, urbanization, have impacts on birth rates which is consistent with the intertemporal utility maximization. It is also consistent with the empirical observation that at low levels of industrialization the rate of population growth tends to move in the same direction as per capita consumption, while at high levels of industrialization it tends to move in the opposite direction.

[11] The bio-capacity has risen as one of the world's dominant measures of human demands on nature. It permits us to compute human pressure on the environment (e.g. if everyone lives the lifestyle of the average American, we would need at least four more planets). Environmental biocapacity thus focuses on whether the planet can keep up with our growing demands.

then I used renewable energy consumption as a percentage of total energy consumption to calculate FE_t and CE_t (as a proxy to get the required technology for utilizing the clean energy). More specifically, I used GDP inflation-adjusted for the total production, total gross capital inflation-adjusted using capital formation index, and calculating technological progress (A_t) – using methods developed in World Bank's 2008 report. World Bank provides the data for the total population, labor force participation, and the number of researchers in the R&D sector, the latter index utilized as a proxy for the number of researchers in clean energy production. For the technological progress for the clean energy, I used the total R&D spending in the US as a proxy.

For the value of β from the first equation, $\mathrm{Max}\,W = \sum_{t=0}^{T} \beta^t U(c(t))$, I used 0.96 for the yearly discount factor, which is commonly used in growth models. σ, the level of risk aversion in equation 2, $U\big(c(t)\big) = \frac{c_t^{1-\sigma}}{1-\sigma}$, is equal to 2. A higher (lower) value of σ corresponds to more (less) risk-averse agents can be used as well. Using the basic calibration from Krusell (2012), I used the parameters for equation 4 $\{Y_t = \mu_t[A_t K_t^{\alpha} P L_t^{1-\alpha-\gamma} E_t^{\gamma}]\}$ as follows: $\alpha = 0.27$, $\gamma = 0.04$.

I set the parameter ρ to 0.49 based on Popp's (2004) model. To estimate the equation 8 parameters ($CEX_t = P_0 + P_1(\sum_{i=1}^{t} FE_i / \overline{FE})^{P_2}$), I used the same values which are: P_0=276.29, P_1=700, and P_2=4. According to Popp (2004) [as in RICE model], I then scaled P_0 and P_1 by dividing them by hundred to fit into my calibration. To estimate the rest of the parameters, I used time series analysis which is fully explained in Appendix B (using Equations (B1), (B2) & (B4) in the Appendix). The summary of all of the above calibrations is shown in Table (1).

Table 1: Values of the parameters used in the model

Parameter	Value	Description
α	0.27	Capital share
γ	0.04	Energy share
ϑ	1.16	Fossil fuel impact on environment
θ	0.85	Clean energy technology impact on the new technology
ω	0.02	Researchers and financial impacts on the clean energy technology
ε_1	1.68	Income effect on the population growth
ε_2	2.16	Industrialization effect on the population growth
ρ	0.5	Substitution rate between clean energy and fossil fuel

III.C Solving the Growth Path (Exogenous population vs. endogenous)

To solve the model, first we can simplify the constraints by substituting the equation 6 into 5, and then substitute back the new one (total energy production) and 8 (environmental degradation) into the production function (Eq. 4), yielding equation C1 (Appendix C.1). Then, we substitute back the modified production function and the price for fossil fuel energy (Eq. 9) into the income allocation function (Eq. 3) to get equation C2. Then, substitute equations 12 and 13 into C1 and 8, respectively, for PL and TL, to get the two constraints (equations C3 and C4) for the Lagrangian. Now we can set up the Lagrangian, in which households are maximizing their utility over infinite time, for the base model in which the population growth is exogenous.

$$\mathcal{L} = E_0 \sum_{C_t, FE_t, TY_t, K_{t+1}, AC_{t+1}}^{t:1\to\infty} \Bigg[\beta^t \frac{{C_t/L_t}^{1-\sigma}}{1-\sigma} + \lambda_{1t} \Bigg\{ \left(1 - \frac{{FE_t}^{\vartheta}}{\varphi}\right) \left(A_t K_t^{\alpha} (l_{PL} L_t)^{1-\alpha-\gamma}\right) \Big((AC_t CE)^{\rho} + FE_t^{\rho} \Big)^{\gamma/\rho} - C_t - K_{t+1} + (1-\delta)K_t - \left(P_0 + P_1 \left(\sum_{i=1}^{t} FE_i \Big/ FE_0\right)^{P_2}\right) FE_t - TY_t \Bigg\} + \lambda_{2t}\Big\{AC_0 AC_t^{\theta} (l_{PL} L_t TY_t)^{\omega} - AC_{t+1}\Big\} + \lambda_{3t}\{(1+\bar{L})L_t - L_{t+1}\} + \lambda_{4t}\{(1+\bar{A})A_t - A_{t+1}\} + \lambda_{5t}\{\overline{FE} - \sum_{i=1}^{t} FE_i\} \Bigg] \tag{16}$$

There are four choice variables in the above functional setup: level of consumption, capital investment, investment in the technology of renewable energy resources, and the amount of fossil-fuel energy. The total stock of fossil-fuel is constant and a given. Solving the first-order conditions (F.O.Cs), we get the Euler equations from the F.O.Cs. The solving process is shown in Appendix C.1.

Considering the three equations for income allocation (Eq. 3), production (Eq. 4, including total energy consumption [Eq. 5], environmental degradation [Eq. 7], and technological production for renewable energy [Eq. 8]), and the Euler equations (Eq. C10, C11, C12, C13&C14), derived from the F.O.Cs, I can solve for this path using the actual values of the variables for the initial year (t=0) – which are shown in Table (2) – and then update the variables based on the above equations.

Therefore, I use forward iteration[12] to obtain next period values based on the previously driven values. Thus, there is an implicit uncertainty about the ending period of fossil fuel energy at the starting point[13]. To select these values, I used 2012 as a reference year, extracted the values for the U.S., and then normalized it by million. The amount of clean energy is set to be 8% of the total energy consumption.

Table 2: Initial values of the variables in the model

State variable	*K (mil \$)*	*PL (million)*	*TL (million)*	*FE (Gigawatthour)*	$\overline{FE}$ *(Gigawatthour)*
$T_0 = 2012$	1.8e+7	1.9e+2	1.2	17,680	2,205,000

The only issue we have to derive the growth path, using forward iteration is to define the value of C_0 which is demonstrated in the footnote[14]. Having the above values as initial conditions (and defining C_0 as it has been explained), we can compute the level of production from equation 4, the next period required technology for the clean energy from equation 8, and the cost of extracting the fossil fuel (CEX) from equation 9. Now, utilizing the budget constraint (equation 3), we can calculate the next period physical capital (K_{t+1}), knowing all values for the current (t=0) state.

[12] While it seems it might be the first time that the current method of forward iteration (by using the initial values and Euler equations) has been applied, it has been discussed in some cases such as DICE user manual, computational and algorithm aspects, by Nordhaus & Sctorc (2013) and Heer & Maussner (2009) in chapter fourth.
[13] Alternatively, I can guess the end period for running out of fossil fuel energy, and iterate it back to the initial point. Then, I can do the same process for different ending points to get the highest given utility; and compared the new results to the current ones.
[14] Since the understanding of solving this model might seem a bit confusing, alternatively, I can explain a simple Ramsey scenario (for a discrete time) in which environment, endogenous technology and population, and energy are dropped. Therefore, our Lagrangian gets the following form: $\mathcal{L} = \mathrm{E}_0 \sum_{C_t, K_{t+1}}^{t=1 \to T} [\beta^t \frac{{C_t/L_t}^{1-\sigma}}{1-\sigma} + \lambda_t\{Y_t - C_t - K_{t+1} + (1-\delta)K_t\}$. Solving the F.O.C we get: $C_{t+1} = C_t \left[(\beta[f'(K_{t+1}) + 1 - \delta])^{1/\sigma}(1+\bar{L})^{\sigma - 1/\sigma}\right]$. Now, to find the consumption path using my approach, we need the initial conditions such as C_0, K_0 and L_0. Since we cannot assign an initial value to C_0, we use the following procedure, just to derive the initial value for consumption, and then use the explained procedure in the main text to drive the growth path. We define a range of possible K_1 based on K_0 such as $0.5K_0 < K_1 < 1.5K_0$. Then, split the range into 100 possible values for K_1 and compute the corresponding utility for each of them. The one which maximizes the utility (of the household) would be our "K_1." Then, we can use the budget constraint to derive C_0. After that, we can use the formula for intertemporal consumption, to derive next period consumption and physical capital. Alternatively, we can derive the initial values using the steady state. Simply, set C_{t+1}, and K_{t+1} equal to C_t and K_t, and assign the values of C_{ss} and K_{ss} as the initial conditions. Having those we are able to derive the pathways for both consumption and physical capital by using the formula for the law of motion for consumption and budget constraint.

Now, we can update the labor force using equation 14 for the exogenous case. The next period technological progress in the production process (A) can be achieved from equation 10. Therefore, we can use equation C12 to get the required fossil fuel energy (FE) for the next period. At this time, we can use equation C10 (intertemporal consumption decision) to compute the level of consumption for the next period as well. Now, the only unknown variable for the next period would be the required resources for financing the clean energy technology (TY). Using the last Euler equation C13, we can calculate the amount of this element. Repeating the same process, we can update all values for each period moving forward.

It must be noted that the social planner is not predicting the growth path. The planner maximizes the utility every single period due to the existing resources at present. Therefore, the backward induction method has not been used since the exact time of depletion of natural resources is unknown. This form of set up is the real uncertainty of the model, implicitly implemented in the solving process. However, the issue of the uncertainty of discoveries or the exact time of running out of fossil fuel, in the starting point, has not been studied explicitly within this framework since the current setup is deterministic, not stochastic. It is also worth to mention that the social planner does not account for the nonrenewable resources constraint in the optimization problem in the beginning, but tries to deal with it while there are not enough resources left to utilize.

To solve the model for the endogenous population case, we need to change the last constraint of the Lagrangian by substituting equation 14 to 15. Therefore, we can rearrange the equation and substitute the production function (equation 4), and equation (11) to get the below equation:

$$L_{t+1} = L_0 \left(1 - \frac{FE_t{}^{\vartheta}}{\varphi}\right)^{\varepsilon_1} A_t^{\varepsilon_1} K_t^{\alpha\varepsilon_1-\varepsilon_2} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_tCE)^{\rho} + FE_t^{\rho}\right)^{\gamma\varepsilon_1/\rho} + L_t \qquad (17)$$

Changing the third constraint (equation above), we can set our updated Lagrangian for the endogenous population growth:

$$\mathcal{L} = E_0 \sum_{C_t,L_{t+1},FE_t,TY_t,K_{t+1},AC_{t+1}} [\beta^t \frac{{C_t/_{L_t}}^{1-\sigma}}{1-\sigma} + \lambda_{1t}\left\{\left(1 - \frac{FE_t{}^{\vartheta}}{\varphi}\right)(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma})\left((AC_tCE)^{\rho} +\right.\right.$$

$$\left.\left. FE_t^{\rho}\right)^{\gamma/\rho} - C_t - K_{t+1} + (1-\delta)K_t - \left(P_0 + P_1\left(\sum_{i=1}^{t} FE_i\Big/\overline{FE}\right)^{P_2}\right)FE_t - TY_t\right\} +$$

$$\lambda_{2t}\{AC_0AC_t^{\theta}(l_{TL}L_tTY_t)^{\omega} - AC_{t+1}\} + \lambda_{3t}\left\{L_0\left(1 - \right.\right.$$

$$\left.\frac{FE_t{}^{\vartheta}}{\varphi}\right)^{\varepsilon_1} A_t^{\varepsilon_1} K_t^{\alpha\varepsilon_1-\varepsilon_2} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_tCE)^{\rho} + FE_t^{\rho}\right)^{\gamma\varepsilon_1/\rho} + L_t - L_{t+1}\Big\} +$$

$$\lambda_{4t}\{(1+\overline{A})A_t - A_{t+1}\} + \lambda_{5t}\{\overline{FE} - \textstyle\sum_{i=1}^{t} FE_i\}] \tag{18}$$

Solving the first-order conditions, I can follow the same process as it has been done for the previous case to derive the Euler equations. Deriving the first-order conditions in the endogenous model is shown in Appendix C.2. Having the Euler equations beside the constraints, I am able to follow the same process in the exogenous population scenario to update the next period values with some minor adjustments. First, I am going to use equation 15 instead of 14 to update the next year's total labor force. And second, I need to solve equations C25, C27, and C28 simultaneously to get the next period values for C, FE, and TY.

IV. Market-based analysis

In this section, I plan to develop the decentralized approach based on Golosov et al. (2014). The distinction between the current model and the previous one is that firms pick the optimal level of both types of energy, and households receive a potential profit from their dividend in the energy sector. While individuals rent out their physical capital, firms decide what share needs to go to the production of final good, and which needs to invest in developing the required technology for producing clean energy. In the market-based approach, firms do not fully internalize the negative externalities risen from extracting and utilizing fossil fuel energy, as is the case in social planner framework.

Therefore, based on the deviation of the results in the market-based approach from the social planner, I can introduce a cost element – such as carbon tax – which would be included in the firms' profit function to capture negative externalities arising from environmental degradation. And in the next step, I can use this tax to finance the clean energy production, directly, without introducing the government section to see if the results converge with the social planner approach. If it does, I can propose a policy to promote the market approach analysis to mitigate the environmental problems in selecting the fossil fuel energy, without entering the government directly into the model.

IV.A Households

There is one representative household[15] for the whole economy who optimizes her utility based on her per capita consumption[16] bundle, subject to budget constraint 26:

$$\underbrace{\text{Max}}_{C_t, K_{t+1}} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \qquad c_t = {C_t}/{L_t} \qquad (19\&20)$$

$$C_t + K_{t+1} = w_t L_t + (1 + r_t) K_t + \pi_t \qquad (21)$$

In the above equation, w_t is the labor's wage, and π_t is the gained profit from energy sector. Wage is the same across all sectors of the economy which is perfectly mobile and substitutable labor. An individual can engage in two different sectors of the economy: First, in producing final good Y as PL; or, in developing new technology (AC) for producing clean energy. Either way, she earns the same compensation; therefore, I did not make any distinction in this section, but the firms can choose the final number. The household also compensates from renting her capital (K) to the market. She might receive some profit (π) from energy production sector as well.

Population grows according to equations which has been developed in the social planner approach. We can think about the fertility model in which households are choosing the next period population based on the income level and the industrialization intensity in an economy.

$$L_{t+1} = (1 + \bar{L}) L_t \qquad \text{(If the population grows exogenously)} \qquad (22)$$

$$L_{t+1} = L_t + L_0 ({Y_t}/{L_t})^{\varepsilon_1} ({L_t}/{K_t})^{\varepsilon_2} \qquad \text{(If the population grows endogenously)} \qquad (23)$$

IV.B Producers

There are two types of firms in our setup: the firms who produce final good (Y) – in the perfectly competitive market – for the consumption given the production frontier, and the intermediary firms who provide two types of energy (fossil fuel-based and clean energy) in which they may earn a positive profit. Since all the firms in each sector are identical with the same production frontier, for simplification in the model, we can assume there is a single firm in each category.

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[15] One can think of the continuum of households who are identical in any aspect and characteristics.
[16] To be consistent to the social planner approach, per capita consumption has been considered.

$$Y_t = ED[A_t KY_t^{\alpha} PL_t^{1-\alpha-\gamma} (CE_t^{\rho} + FE_t^{\rho})^{\gamma/\rho}] \tag{24}$$

$$E_t = [CE_t^{\rho} + FE_t^{\rho}]^{1/\rho}$$

The final good producers are solving their profit maximization by:

$$\underbrace{\text{Max}}_{KY_t, PL_t, CE_t, FE_t} \quad EDA_t KY_t^{\alpha} PL_t^{1-\alpha-\gamma} \left(CE_t^{\rho} + FE_t^{\rho}\right)^{\gamma/\rho} - w_t PL_t - (r_t + \delta) KY_t - P_{FEt} FE_t - P_{CEt} CE_t \tag{25}$$

In which P_{FE} is the price of fossil fuel energy, and P_{CE} is the price of clean energy. Damage function is also included to the production function, to be consistent to the planner approach for the comparability, however, the costs of pollution are not fully internalized by firms since ED is constant and does not depend on the rate of extraction of fossil fuel.

IV.C Energy producers

In this sector, the firms are producing energy subject to the below optimization process:

$$\underbrace{\text{Max}}_{TY_t, TL_t, AC_t, FE_t} \quad \beta^t \pi_t \tag{26}$$

in which $\pi_t = P_{FEt} FE_t + P_{CEt} CE_t - w_t TL_t - r_t TY_t - CEX_t FE_t$ (27)

$AC_{t+1} = AC_0 AC_t^{\theta} (TL_t TY_t)^{\omega}$ in which $CE_t = CE * AC_t$ (28 & 29)

To derive the cost of extraction of fossil fuel-based energy, we can use the previous setup from equation 9. The rest of the equations – for the technological progress and the population – are the same as the planner problem [(10), (11), (12) and (13)].

IV.D Solving the model

To solve this model, I plan to take advantage of the same framework that I have used in the social planner approach. Therefore, I am going to set up the Lagrangians for the household, as are shown in equation 30 and 31, and then solve the F.O.C.s for all the sectors (Appendix C.3). Having the Euler equations, along with the initial and market clearing conditions, I am able to set up the dynamic system of equations to derive the growth paths for the desirable variables.

$$\mathcal{L} = E_0 \sum_{C_t, K_{t+1}, L_{t+1}}^{t:1\to\infty} \left[\beta^t \frac{{C_t/L_t}^{1-\sigma}}{1-\sigma} + \lambda_{1t}\{Y_t - C_t - K_{t+1} + w_t L_t + (1+r_t)K_t + \pi_t\} + \lambda_{2t}\{(1+\bar{L})L_t - L_{t+1}\} \right] \quad \text{(Population is exogenous)} \tag{30}$$

$$\mathcal{L} = E_0 \sum_{C_t, K_{t+1}, L_{t+1}}^{t:1\to\infty} \left[\beta^t \frac{{C_t/L_t}^{1-\sigma}}{1-\sigma} + \lambda_{1t}\{Y_t - C_t - K_{t+1} + w_t L_t + (1+r_t)K_t + \pi_t\} + \lambda_{2t}\left\{L_t + L_0 ({Y_t/L_t})^{\varepsilon_1} ({L_t/K_t})^{\varepsilon_2} - L_{t+1}\right\} \right] \text{(Population is endogenous)} \tag{31}$$

It is worth to argue that one since firms do not fully internalize the negative externalities, the results in both social planner and market-based approaches are going to be different, as a fundamental distinction between the first best approach (social planner) and second best approach (market-based). There are also, at least, two other distinctions across these two setups. First, social planner chooses the optimal level of fossil fuel in each period; however households do not have that choice; firms select that level based on their expected profit, while there is no such a profit in social planner method. Second, households rent out the total capital and earn interest rate, and then, firms decide what portion of that should spend in clean energy, and what fraction should they invest in physical capital based on their optimality conditions. Whereas, in the other framework, social planner choose how much she should invest in physical capital and how much in clean energy. Thus, it is not the same process in decision making. As a result of these differences, one can see the lay of motion for consumption in planner solution (equation C34) is entirely different from the one in the decentralized model (equation C25). Therefore, the F.O.C.s and results should not be identical in both cases, fundamentally and computationally.

V. Results and Discussion

In this section, first, I am going to compare different exogenous growth rates in both social planner and market-based frameworks, then I plan to analyze the exogenous growth scenario to the endogenous one.

V.A Social planner solution (different exogenous growth rates)

The results are shown in Figure 1 when population grows exogenously with two different scenarios. In the first case, population grows by 0.02 percent every year. In the second one, it increases by 0.06 percent per year, and it matches US population growth to some extent. We can see this difference affects the economic growth per capita slightly, and it changes the period of utilization of fossil fuel energy (longer for the lower growth rate in population). Higher rate of population growth means more labor force and researcher, therefore, more primary factors of production. However, on the other hand, more resources are needed to be utilized as well. While economy produce more—thus it needs more energy and fossil fuel to use—in the higher population scenario, the economic growth per capita would be marginally lower because of the same argument.

V.B Social planner solution (exogenous versus endogenous growth)

The results are depicted in Figure 2. Assuming the population adjusts itself through income and level of industrialization – given the endogenous population growth that is shown in Figure 2, red lines – the economic growth per capita [17] would be slightly lower compared to the exogenous scenario, while population growth across two models are in the same range. The capital increases in both cases, but at a higher rate, after several periods, if the population grows exogenously.

The other finding is that by the time we are running out of the fossil-fuel energy, it takes a few periods to adapt the production process, entirely using renewable energy as a primary energy factor (therefore, there would be a delay in energy provision). While this transition does not affect the economy in this setup since—at the same time—there would be no adverse impact of fossil fuel utilization on the production process. Thus, a negative impulse from transforming to the full utilization of clean energy would be neutralized by a positive inclination of not having negative externalities in the economy.

Figure 2 shows us that if the population is considered exogenous, we conserve fossil fuel for a longer period (it runs out at time 92 which is not depicted), and it reaches the maximum point of

[17] In another attempt in Appendix E, to better match the projected growth with the US data over the next decades, I changed the capital share and reported the results

utilization later than in the endogenous scenario. If the population is tied to the income and level of industrialization, we utilize more fossil fuels and deplete non-renewable resources in a shorter period.

V.C Market-based solution (exogenous versus endogenous growth)

The results are depicted in Figure 3. The capital accumulation is higher in the exogenous scenario, as well as economic growth per capita. It is shown in Figure 2 that if the population is considered exogenous, we conserve fossil fuel for a shorter period, and it reaches the maximum point of utilization sooner than in the endogenous scenario. If the population is tied to the income and level of industrialization, we utilize fewer fossil fuels and deplete non-renewable resources in a more extended period. The result of fossil fuel utilization contradicts the previous comparison in the social planner approach. However, the economic growth per capita is higher in the exogenous scenario compared to the endogenous.

V.D Planner's problem vs. decentralized model (endogenous population)

Solving the model, the results show that the economy per capita would grow at a slightly lower rate in a centralized model relative to the decentralized while population grows with a lower rate in the latter framework; but ultimately, the economic growth per capita in both frameworks converge to the same amount. The firms accumulate more capital and invest less in clean energy in the market-based solution compared to the social planner approach. Also, the return on physical capital is higher than the return on energy in the production function and makes it more attractive for firms to invest in the capital, not the clean energy.

As is shown in Figure 4, in the planner's solution, the fossil fuel resources would have been exhausted in a longer time, and there would be a higher rate of adaptation of clean energy relative to the decentralized model. Despite the higher rate of energy consumption, the economic growth per capita is higher in the market-based solution because firms invest more on physical capital and use more fossil fuel compared to the planner who conserve fossil fuel for a longer period. The reason might be clear since firms do not adequately account for negative externalities arise from non-renewable resource utilization. The other finding is that the population[18] grows at a slightly higher rate in the planner's solution compared to the decentralized model, in earlier periods. With the current parameterization, population growth does not match existing rate in the United States. However, in another attempt, I captured the current trend in population growth using alternative calibration for equation 15 (endogenous population growth).

V.E Policy implication

In this section, I try to investigate the situations in which the government imposes a regulation to converge the results in the market-based approach with the social planner approach in regard to the clean energy adaptation. An intervening party can set a rule in which every year a certain percentage of the total income needs to finance the production of clean energy without any direct interference from the government, so there is no need to enter the government spending and budget into the model. To do that, I can simply utilize the following assignment in which financing the clean energy (TY) is not a choice variable as it was in the previous setup; instead, it is a policy regulated by the government (or social planner):

$$\mathrm{TY}_{t+1} = \mathrm{TY}_t * (1 + g_{\mathrm{TY}}) \text{ in which } g_{\mathrm{TY}} \text{ is the annual growth rate of TY} \tag{32}$$

In another attempt, I plan to propose two different methods (the second method is described in Appendix G) to include environmental erosion in the firms' cost-benefit analysis. To do that, I added an element of cost – which can be thought as a carbon-tax factor – to the firms' energy profit maximization process, to internalize the cost of degrading the environment. Here, I am going to use equation 6 [$\mathrm{ED}_t = 1 - ({}^{\mathrm{FE}_t}/_{\varphi})^{\vartheta}$] which states that the environment degrades as more fossil fuel is being used. Thus, the profit function 27 would be:

$$\pi_t = P_{\mathrm{FE}t}\mathrm{FE}_t + P_{\mathrm{CE}t}\mathrm{CE}_t - w_t\mathrm{TL}_t - r_t\mathrm{TY}_t - \mathrm{SC}_t \tag{33}$$

$$\text{Where: } \mathrm{SC}_t = P_{\mathrm{SC}t} * (1 - \mathrm{ED}_t) \rightarrow P_{\mathrm{SC}t} * ({}^{\mathrm{FE}_t}/_{\varphi})^{\vartheta} \tag{34}$$

In which P_{SC} is the price of eroding the environment and set by the social planner. Now, we can set the social cost in a way that the economic growth (or the total welfare) in the market-based

[18] One would question that the depicted population is not realistic for the US. In Appendix G, I try to argue such issues.

model would converge to the one in the social planner solution by making the firm's profit equal to zero. We might call that price the optimal Pigouvian taxation on carbon emission. The results are shown in Figures 5-A.

Figure 5-B indicates that imposing a carbon tax element on fossil fuel production can bring back the market-based approach to the social planner solution. Figure 5-C shows that imposing the environmental costs of utilizing fossil fuel energy can limit the production in a similar way to the social planner approach. However, it does not increase the investment in the adaptation of clean energy advancement. Also, it slows the utilization of fossil fuel energy but not in line with the social planner's solution. Figure 5-A shows the optimum tax ratio while fossil fuel resources are being used to produce energy.

Figure 5-A shows the fossil fuel utilization – per peta watt hour – and the dollar tax rate per kilowatt hour of energy production using fossil fuel resources (which is around one cent per kilowatt hour). As has been shown before, the results in both scenarios (endogenous vs. exogenous population growth) do not vary having both sources of energy, but it differs when running out of non-renewable energy. In order to have the optimal taxation policy on fossil fuel utilization, we need to impose a U-shape taxation system by which, in the beginning, the rate decreases as the firms use fossil fuels more intensively, and then increases

Given the results in Figure 5-C, by regulating the market – imposing the investment rule in clean energy production – we can limit production, but it slightly increases the utility of individuals in a way that converges the results to the social planner approach. However – as a tradeoff – it causes a slower future capital accumulation. The results for the first policy implication show that such a policy would be ineffective.

V.F Welfare analysis

Here, I intend to compare the effects of different model specifications (such as social planner vs. market-based approach) on the total welfare of the society, which can be seen as the utilitarian welfare function where all individuals have the same weight for the social planner over the horizon time discounted to the present value. Following Floden (2001), I am going to introduce the utilitarian welfare gain of model specification change as below:

$$W = \sum_{t=0}^{T} \beta^t U(c(t)) \tag{35}$$

Consider that the premium WG (compensating variation) can be thought of as the percent of consumption of individuals in economy B in each period, who need to be compensated in order to give up living in condition A, and move to economy B, which can be interpreted as the below equality:

$$\sum_{t=0}^{T} \beta^t U_A(c(t)) = \sum_{t=0}^{T} \beta^t U_B((1 + WG)c(t)) \tag{36}$$

Substituting the utility function, we are going to have:

$$\sum_{t=0}^{T} \beta^t \frac{c_{At}{}^{1-\sigma}}{1-\sigma} = \sum_{t=0}^{T} \beta^t \frac{(c_{Bt}*(1+WG))^{1-\sigma}}{1-\sigma} \tag{37}$$

Rearrange it for WG, and substitute back the welfare function, we get:

$$WG = \left(\frac{W_A}{W_B}\right)^{1/1-\sigma} - 1 \tag{38}$$

Using equation 38 – while W_A is the welfare in the social planner solution, and W_B is the welfare in the market-based approach – there is a loss in the welfare of the society of 0.0075 if we try to move away from the centralized model to the decentralized, if population grows endogenously. It means in order to maintain the same level of consumption in a social planner, we need to compensate the households about seven percent of their consumption in a decentralized model. This amount, for the exogenous case, is around the same amount. The results aligns with the previous findings in which social planner is the first best and market-based is the second, if firms do not fully internalize the negative externalities.

Undertaking the same process for the market-based model using different policies, we get the following results. Setting the time path for 85 years, there would be a negligible loss for no policy vs. policy-1. Applying the first policy which was setting a rule in which the firms need to increase the financing of the clean energy production by five percent annually[19]. Applying the second policy – which is the carbon-tax method – the gain would be more than five percent of consumption. Taxing the fossil fuel energy influence the welfare during the use of fossil fuel, slightly; at the same time, it affects the future welfare while the production process is utilizing one hundred percent clean energy as a resource. Therefore, by imposing a tax on fossil fuel consumption, we

[19] I can increase that to more than that, but it makes the model unstable after a few periods.

can improve the total utility of the households in the long-run. The summary of compensating variations across different models are reported in Table 3.

Table 3: Compensating variation* among different models;

Moving away	To	Gain(+)/loss(-)
Social planner exogenous**	Market-based exogenous	-7.2%
Social planner endogenous***	Market-based endogenous	-7.5%
Market-based endogenous	Regulation on energy investment	-0.9%
Market-based endogenous	Taxed on fossil fuel	+5.4

* The CV measures the percent of consumption of individuals in one economy, who need to be compensated in order to give up living in that economy and move to another
** Exogenous population growth
*** Endogenous population growth

Including endogeneity of population growth in any similar model shapes the future growth path is twofold. First, we are overestimating the future growth path with any scale since we have not considered the feedback loop from the system to population itself. Second, it is also worth to say that any in a decentralized economy, firms tend to utilize more of resources to produce more and ignore the negative externalities arise from a production process, which there should be policy (preferably a carbon-tax tool) to improve the society's welfare.

VI. Concluding remarks

I proposed a dynamic growth model that allows for the interaction between an economy and an environment, utilizing both a social planner framework and decentralized method. Having no fossil fuel left, treating population endogenously leads to slightly lower growth in the economy relative to the exogenous population growth during the hundred percent renewable energy utilization. This result is rational in a sense that when population grows exogenously, any changes in the income level of household does not affect the growth rate in population which is a primary factor of production, itself. However, when we tie the population to the income level and other factors of the model, using a feedback loop, then any fluctuation in those factors, directly impact the population growth, therefore economic growth as a result. Running out of fossil fuel energy would not cause a drop in economic growth, since it would neutralized by the positive impulse from removing the damage function arises from fossil fuel utilization. Therefore, there would be a smooth transition from using both sources of energy to just renewable energy.

In the market-based approach, firms tend to utilize fossil fuel energy in a shorter period and invest more in clean energy, as opposed to the social planner method. Implying carbon-tax element on the firms who produce the energy speeds up the adaptation of the clean energy, and increases the households' satisfaction due to the higher rate of consumption in the long-run, and recover the partial loss that has been imposed by moving away from the first best scenario.

The long-run economic growth per capita converges to 2% percent in the current setup in which there is an exogenous technology with the growth rate of 1.5%. This result is opposed the previous ones in a way that growth in an economy is proportionate to the growth in exogenous elements. Based on the current findings, it is essential to include the endogeneity of the population in an economy since it prohibits any overestimation in growth prediction. The developed framework also allows for distinguishing the gap between a social planner and a market-based approach, and pros and cons of each method regarding the projection of growth path.

I expect to expand an idea of entering energy consumption heterogeneity into the current setup based on the availability of resources and different marginal costs of producing energy. On the other hand, households might not have a unique preference toward energy exploitation that can affect their energy consumption. Considering these sources of heterogeneity, a follow-up paper might lead to a different conclusion than I have investigated so far, which can lead to different policy recommendations than those I have already made up.

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List of Figures in the main text:

Figure 1: Different growth rate in population for the social planner solution

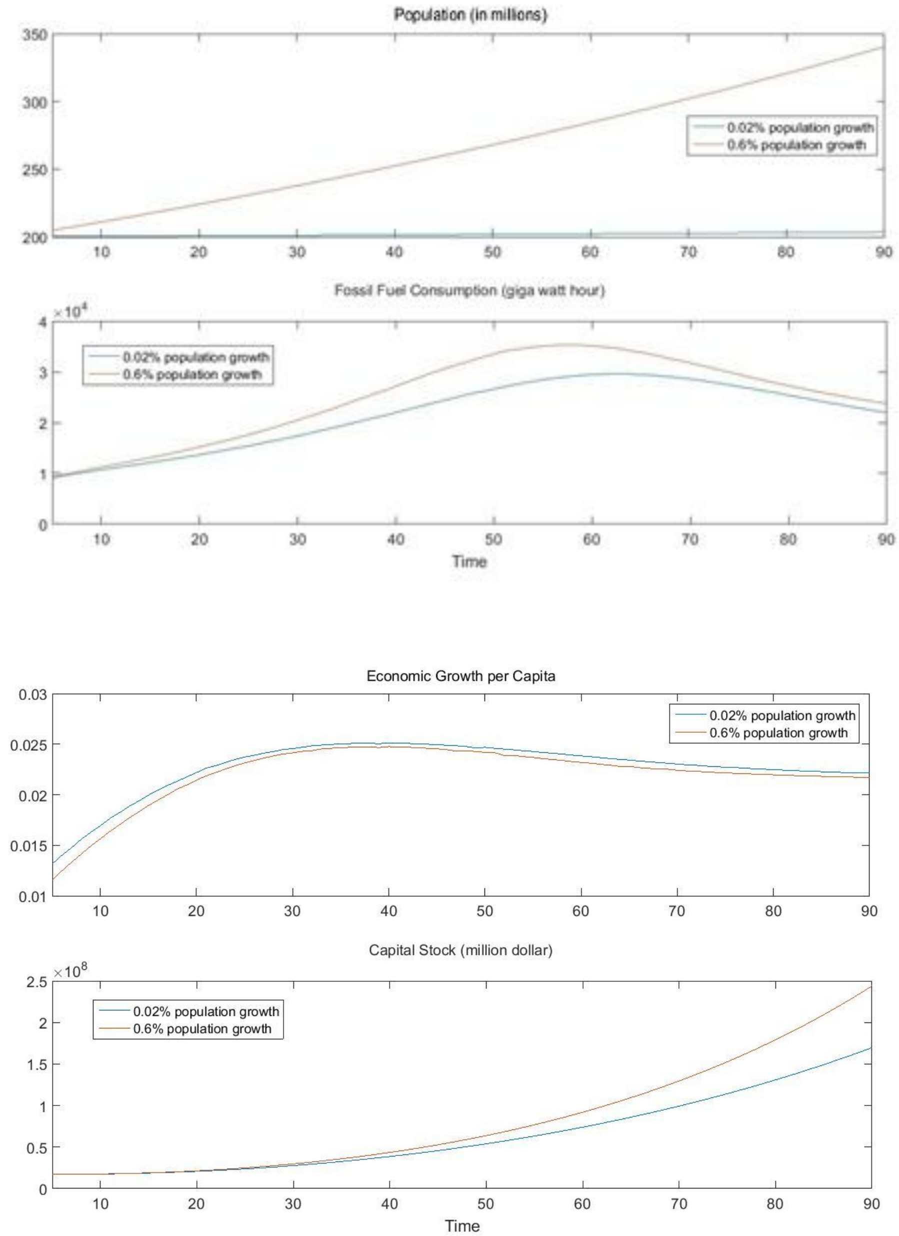

Figure 1 shows population growth (the US labor force), fossil fuel utilization, economic growth and capital accumulation while population grows by 0.02 % (blue lines) and 0.6% (red lines) per year. Population grows exogenously and the social planner solution has been applied for both cases.

Figure 2: Exogenous population growth versus endogenous for the social planner solution

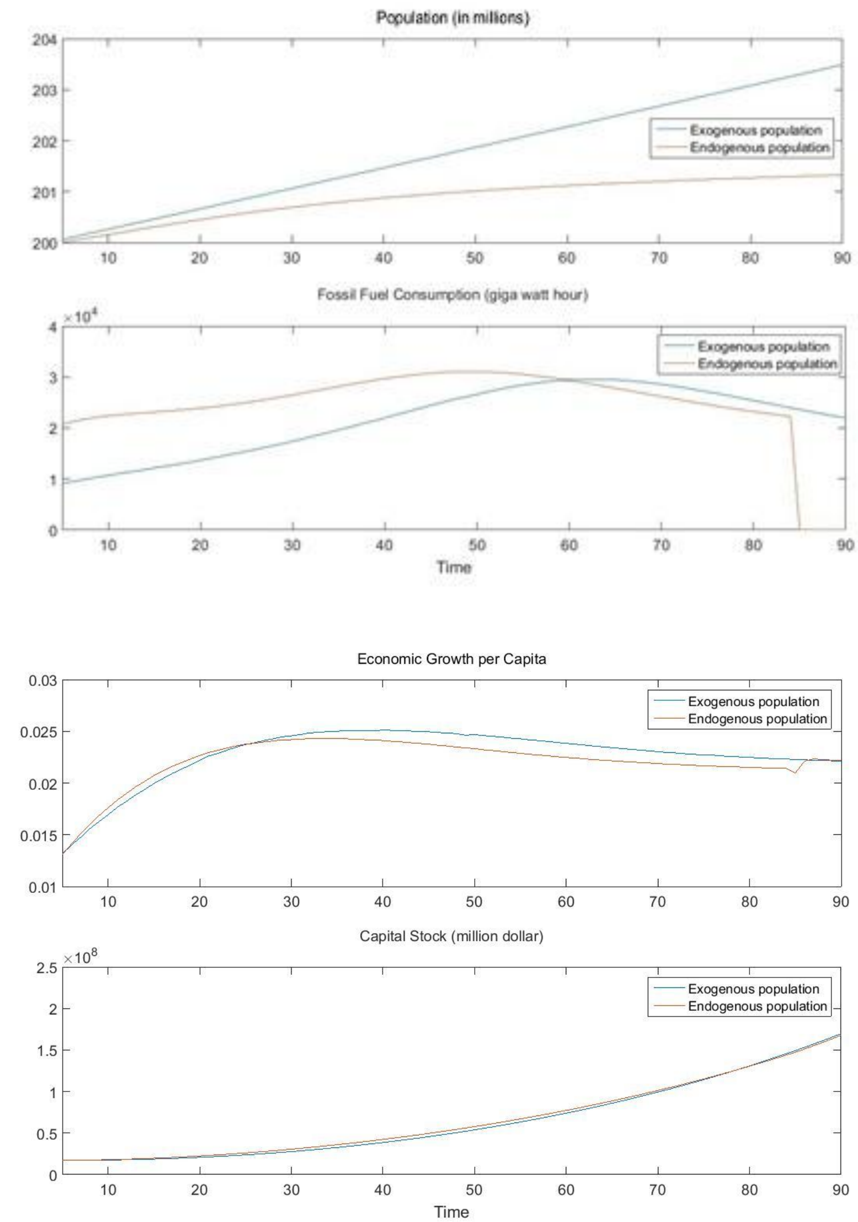

Figure 2 shows population growth (the US labor force), fossil fuel utilization, economic growth and capital accumulation while population grows endogenously (red lines), and when population growth is exogenous, and equals to 0.02% (blue lines). The social planner solution has been applied for both cases.

Figure 3: Exogenous population growth versus endogenous for the market-based approach

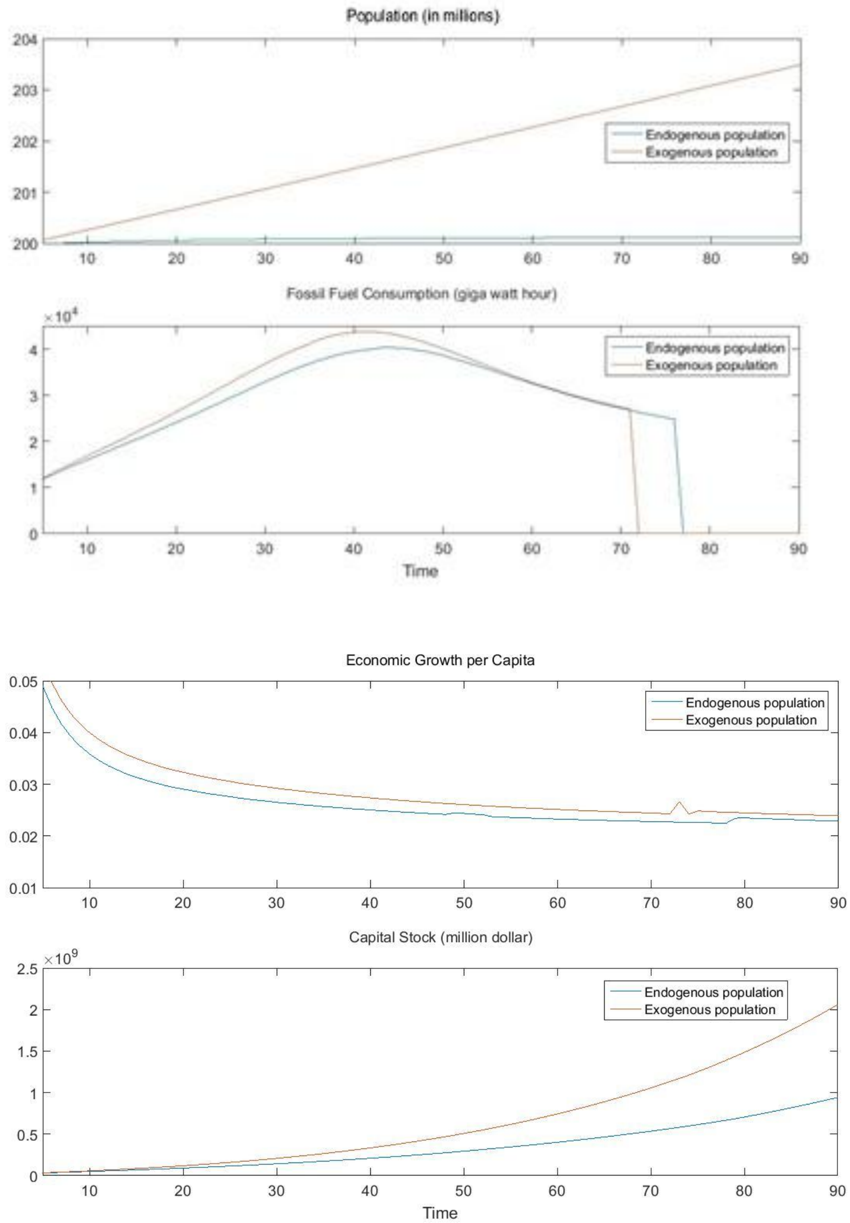

Figure 3 shows population growth (the US labor force), fossil fuel utilization, economic growth and capital accumulation while population grows endogenously (blue lines), and when population growth is exogenous, and equals to 0.02% (red lines). The market-based approach has been applied for both scenarios.

Figure 4: Endogenous population in the social planner versus the market-based approach

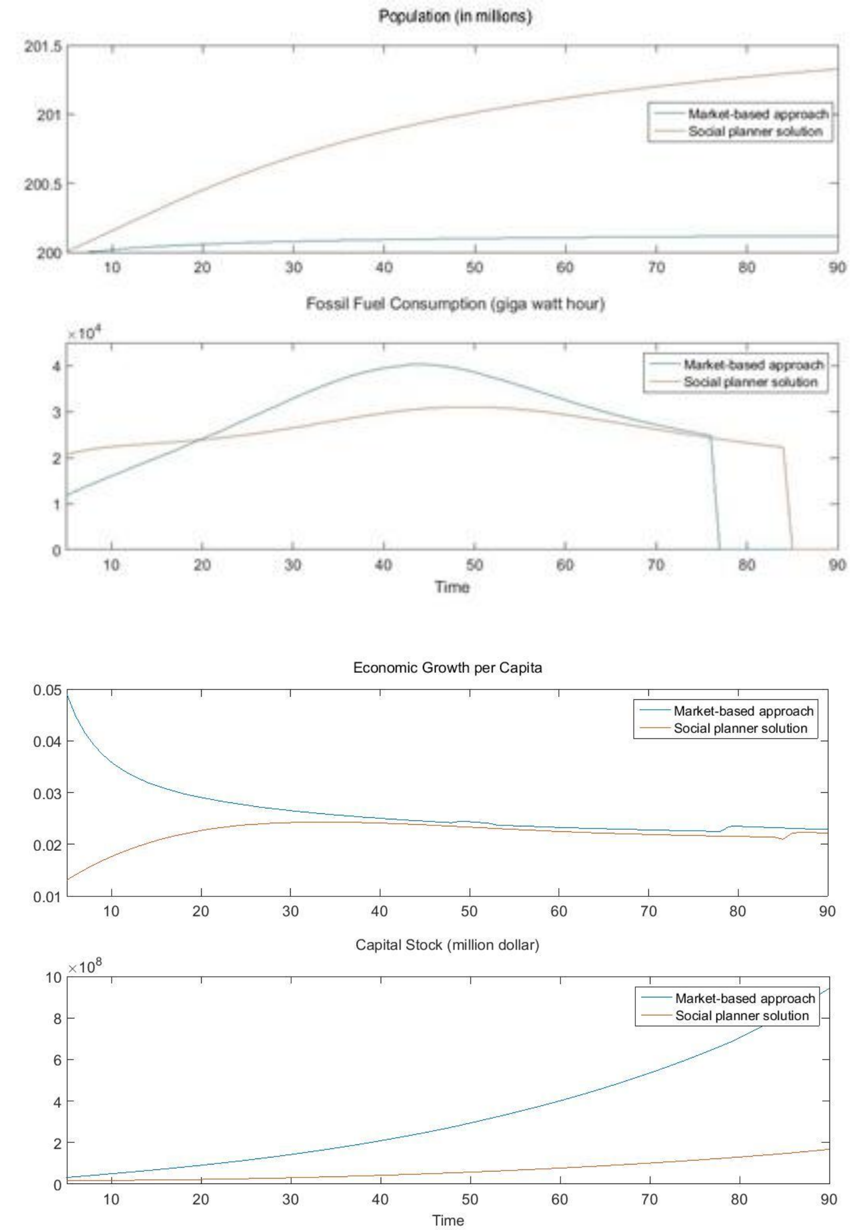

Figure 4 shows population growth (the US labor force), fossil fuel utilization, economic growth and capital accumulation for the market-based approach (blue lines) and social planner solution (red lines). The population grows endogenously in both cases.

Figure 5-A: Carbon tax element on the fossil fuel utilization for the market-based approach

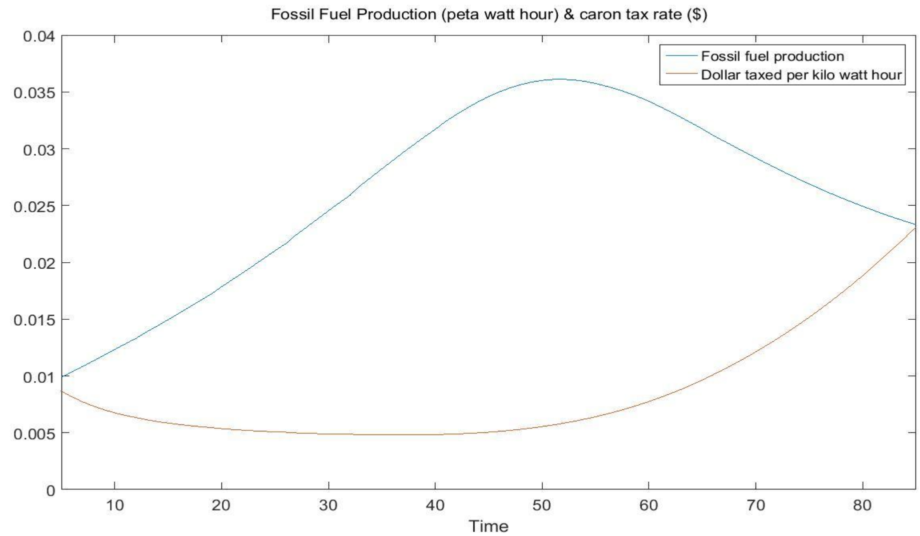

Figure 5-A shows the fossil fuel production (per peta watt hour) and the tax rate (per kilowatt hour of energy production using fossil fuel resources) given the intervention in the market for the endogenous population.

Figure 5-B: Comparison between the market-based approach, policy intervention on fossil fuel utilization, and social planner solution

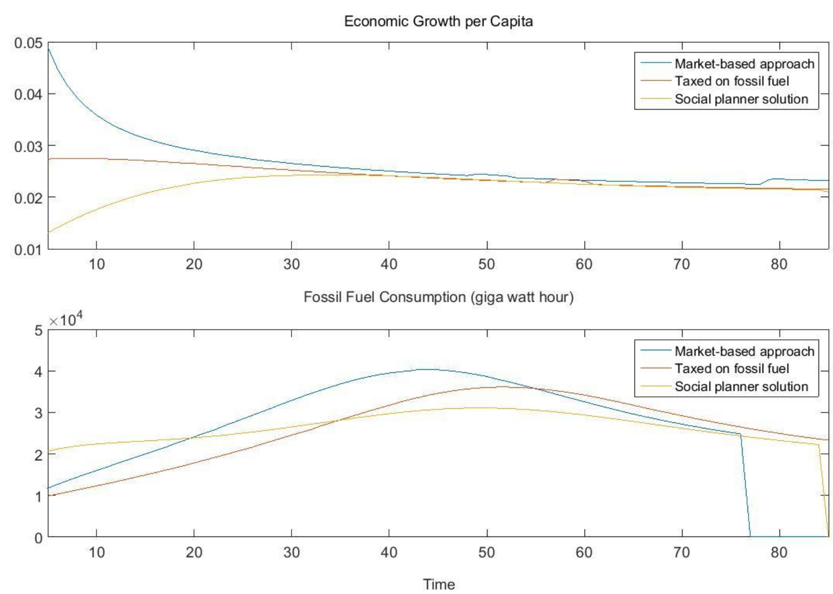

Figure 5-B shows economic growth and fossil fuel utilization when there is no intervention in the decentralized model (blue lines), while there exists a carbon-tax (red lines), and the social planner solution (yellow lines).

Figure 5-C: Comparison between base model and policy interventions on fossil fuel utilization

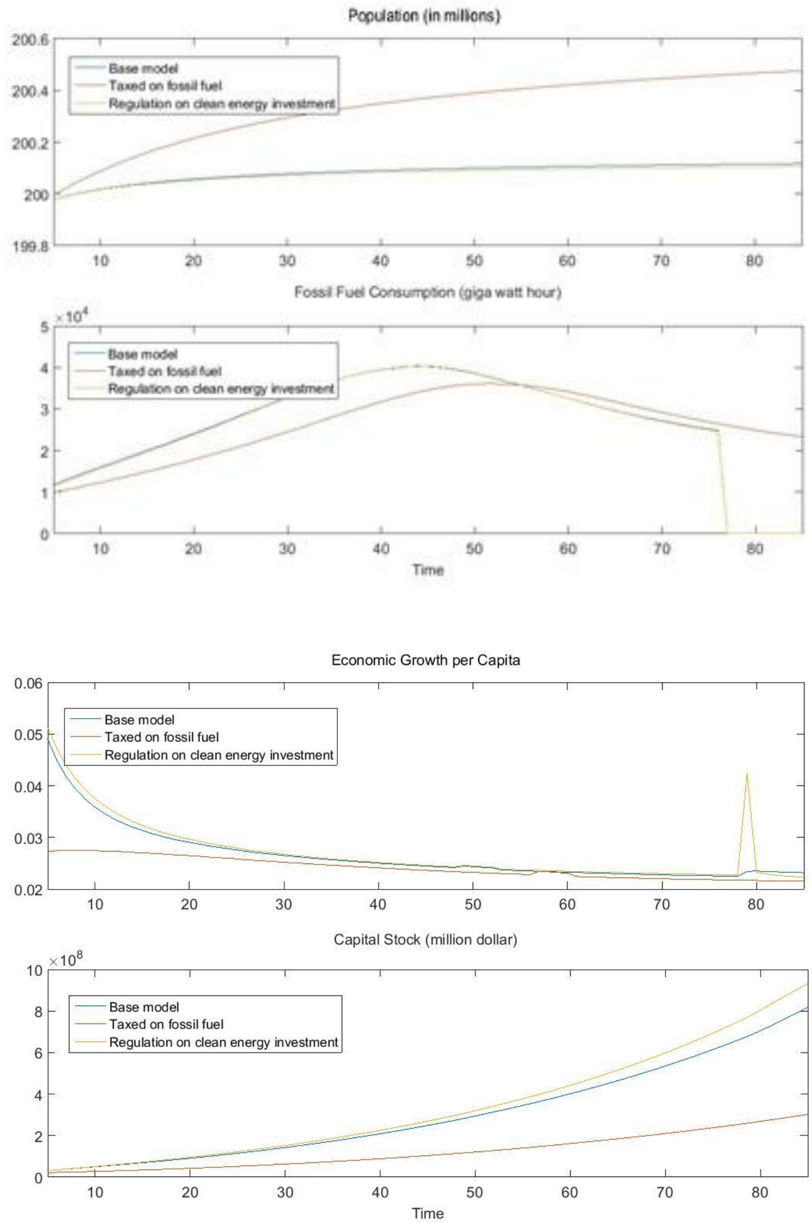

Figure 5-C shows population growth (the US labor force), fossil fuel utilization, economic growth and capital accumulation when there is no intervention in the market (blue lines), while there exists a carbon-tax (or tax on using fossil fuel energy) which decreases the profit in energy sectors to zero (red lines), and when government regulates the market by imposing a policy which requires firms to invest in clean energy production by 5% annually (yellow lines). All models are decentralized.

Appendix A (List of variables and parameters used in the model)

U: Utility function

C: Consumption

Y: Production/income

K: Physical capital

I: Investment in physical capital

L: Total number of laborers in the economy (the summation of workers and researchers)

PL: Number of workers available in the production process

TL: Number of researchers available in producing technology (Jones, 2002, uses 0.036*L)

A: Technological progress in the production function

A_C: Required Technology to utilize clean energy

N: total population

CE: Clean energy

FE: Fossil fuel energy

E: Total energy consumption

ED: Environmental degradation, or damage function

P_{FE}: Price of fossil fuel energy use

P_{AC}: Price of clean energy use

WG: Compensation due to a change in individuals' consumption

β: Discount factor (0.96 is used vastly in the Macro literature)

δ: Capital depreciation, 0.03 has been used as a value in this research

ξ: Fraction of the population in the labor force based on the BLS is 0.63

α: Capital share in the production function (the range between 0.27 ~ 0.33 are used vastly in the Macro literature)

σ: Level of risk aversion of the agent

γ: Energy share in the production function (Krusell [2012] used 0.04)

ρ: Substitution rate between clean energy and fossil fuel energy (Popp [2004] used 0.49 in his model)

υ: Impact of the fossil fuel energy consumption on the environmental degradation

φ: Normalizing factor to keep the negative impact of FE on the production less than one

ω: Impact of researchers on the production of clean energy technology (Jones [2002] used 0.015)

θ: Impact of the old clean energy technology on the new one (Jones [2002] used 0.94)

ε_1: Per capita income effect on the population growth (Cigno [1981] didn't use any value since it was purely a theory-based paper)

ε_2: Effect of the level of industrialization on the population growth

ε: Error term in stochastic shocks of technology in the production function which is normally distributed with the mean zero and standard deviation of σ

$\bar{A}$: Constant growth for technological progress in the production function

$\bar{L}$: Constant population growth (based on the average population growth in the US)

A_{C0}: Residuals in the equation explaining the technology for utilizing clean energy

Appendix B (Data calibration)

To estimate the parameters of equation 7, I used the time series for the US data. The main equation according to the model is: $\mathrm{ED_t} = 1 - \frac{\mathrm{FE_t}^{\vartheta}}{\varphi}$. Therefore, the estimating equation is given by:

$$\log(y_t) = \beta_1 + \beta_2 \log(\mathrm{FE_t}) + \varepsilon_t \quad \text{(B1)}$$

Where $y_t = 1 - \mathrm{ED_t}$, $\beta_2 = -\vartheta$ and $\beta_1 = \vartheta \log(\varphi)$, $\varepsilon_t = \rho\varepsilon_{t-1} + \epsilon_t$

The table below shows the results. The error terms are serially correlated. To estimate the above model, I used the generalized least-squares method to estimate the parameters in a linear regression analysis in which the errors are serially correlated. Specifically, the errors are assumed to follow a first-order autoregressive process. Based on the above estimation, we get the below values for the estimated parameters: $\vartheta = 1.162$ and $\varphi = 197738.7$

Table 4: Estimating the parameters in equation 7

LED	Coef	Std. Err.	t-stat	P > | t |
LEF	-1.1616	0.2812	-4.13	0.000
Cons	14.1654	3.0871	4.59	0.000
R-square	0.967			

LED = log of biocapacity index as a proxy for environmental degradation

LFE = log of fossil fuel energy production, trillion BTU

To derive the values of parameters in equation 8 [$AC_{t+1} = AC_0 AC_t^{\theta}(TL_t TY_t)^{\omega}$], we can use the following values for θ and ω based on Jones' (2002) calibration: $\theta = 0.94$ and $\omega = 0.015$. However, we changed the model by entering the interaction of financing the technology, therefore, it would be better to estimate it as follows:

$$\log(AC_{t+1}) = \log(AC_0) + \theta \log(AC_t) + \omega \log(TL_t * TY_t) + \epsilon_t \quad \text{(B2)}$$

Table 5: Estimating the parameters in equation 8

LACP	Coef	Std. Err.	t-stat	P > | t |
LAC	0.844	0.0203	41.49	0.000
LTYL	0.022	0.0045	4.90	0.000
LAC0 (Cons)	1.159	0.2855	4.06	0.001
R-square	0.957			

LACP= log of technology of clean energy utilization for the next period

LAC= log of technology of clean energy utilization

LTYL= log of the interaction between TL and TY (number of the researchers in the economy and the required resources to finance the technology)

Based on the above estimation, we get the below values for the estimated parameters: $\theta = 0.84$ and $\omega = 0.02$ which is close enough to what Jones used.

To estimate the equation 15 parameters, I need to use time series again. The main equation according to the model is: $L_{t+1} = L_t + L_0(Y_t/L_t)^{\varepsilon_1}(L_t/K_t)^{\varepsilon_2}$

Therefore, the estimating equation would be defined as follows:

$$\log(g_{L_t}) = \varepsilon_0 + \varepsilon_1 \log(Y_t/L_t) + \varepsilon_2 \log(L_t/K_t) + \varepsilon_t \quad \text{(B3)}$$

Where $\varepsilon_0 = \log(L_0)$ and g is the growth rate, rearrange the above equation for L, we get:

$$\log(g_{L_t}) = \varepsilon_0 + \varepsilon_1 \log(Y_t) + \varepsilon'_2 \log(K_t) + \varepsilon'' \log(L_t) + \varepsilon_t \quad \text{(B4)}$$

Where $\varepsilon'_2 = -\varepsilon_2$ and $\varepsilon'' = \varepsilon_2 - \varepsilon_1$

Table 6: Estimating the parameters in equation 15

LGN	Coef	Std. Err.	t-stat	P > | t |
LY	1.679	0.18	9.31	0.000
LK	-2.163	0.213	-10.16	0.000
LL	0.485	0.056	8.65	0.000
Cons	0.856	0.624	1.37	0.182
R-square	0.988			

LGN= log of population growth

LY= log of Y (GDP) (1.895)

LK= log of K (physical capital) (-2.11)

LL = log of L (labor force) (0.215)   Cons (17.354) (L0=3.4E+7)

Based on the above estimation, we get the below values for the estimated parameters:

$\varepsilon_1 = 1.679$, $\varepsilon_2 = 2.163$ and $L_0 = 2.353$.

Appendix C (Solving the F.O.C's for both social planner and market-based approaches)

Appendix C.1 (Solving the social planner's F.O.C [exogenous population])

$$Y_t = \left(1 - \frac{{FE_t}^{\vartheta}}{\varphi}\right)\left[A_t K_t^{\alpha} PL_t^{1-\alpha-\gamma}\left[(AC_t CE)^{\rho} + FE_t^{\rho}\right]^{\gamma/\rho}\right] \quad \text{(C1)}$$

$$\left(1 - \frac{{FE_t}^{\vartheta}}{\varphi}\right)\left(A_t K_t^{\alpha} PL_t^{1-\alpha-\gamma}\right)\left((AC_t CE)^{\rho} + FE_t^{\rho}\right)^{\gamma/\rho} = C_t + K_{t+1} - (1-\delta)K_t + \left(P_0 + P_1\left(\sum_{i=1}^{t} FE_i \Big/ \overline{FE}\right)^{P_2}\right) FE_t + TY_t \quad \text{(C2)}$$

$$\left(1-\frac{FE_t}{\varphi}^{\vartheta}\right)(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma})\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho}=C_t+K_{t+1}-(1-\delta)K_t+$$

$$\left(P_0+P_1\left(\sum_{i=1}^{t}FE_i\Big/\overline{FE}\right)^{P_2}\right)FE_t+TY_t \quad \text{(C3)}$$

$$AC_{t+1}=AC_0AC_t^{\theta}(l_{PL}L_tTY_t)^{\omega} \quad \text{(C4)}$$

Solving the Euler equations for the social planner when the population is exogenous

First-order conditions are:

$$\{C_t\}\colon \beta^t\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}=\lambda_{1t} \quad \text{(C5)}$$

$$\{K_{t+1}\}\colon \lambda_{1t+1}\{\alpha\left(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)(A_{t+1}K_{t+1}^{\alpha-1}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}+1-\delta\}=$$

$$\lambda_{1t} \quad \text{(C6)}$$

$$\{FE_t\}\colon \left(1-\frac{FE_t}{\varphi}^{\vartheta}\right)(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma})\gamma FE_t^{\rho-1}\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho-1}-$$

$$\frac{\vartheta}{\varphi}\frac{FE_t}{\varphi}^{\vartheta-1}(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma})\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho}-\left(P_0+P_1\left(\sum_{i=1}^{t}FE_i\Big/\overline{FE}\right)^{P_2}\right)-$$

$$\frac{P_1P_2}{\overline{FE}}\left(\sum_{i=1}^{t-1}FE_i\Big/\overline{FE}\right)^{P_2-1}FE_t=0 \quad \text{(C7)}$$

$$\{AC_{t+1}\}\colon \lambda_{2t+1}\{\theta AC_0AC_{t+1}^{\theta-1}(l_{TL}L_{t+1}TY_{t+1})^{\omega}\}+\lambda_{1t+1}\Big\{\Big(1-$$

$$\frac{FE_{t+1}}{\varphi}^{\vartheta}\Big)A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\gamma CE^{\rho}AC_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho-1}\Big\}=\lambda_{2t} \quad \text{(C8)}$$

$$\{TY_t\}\colon \lambda_{1t}=\lambda_{2t}[\omega AC_0AC_t^{\theta}(l_{TL}L_t)^{\omega}TY_t^{\omega-1}] \quad \text{(C9)}$$

Substituting the equation 58 and 62 (and the updated forms of them) in the above F.O.Cs, we can get the following Euler equations:

$$\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\{\alpha\left(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)(A_{t+1}K_{t+1}^{\alpha-1}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}+1-\delta\}=\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}$$

(C10)

And the above equation, when there is no fossil fuel energy left to use, will be:

$$\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\{\alpha(A_{t+1}K_{t+1}^{\alpha-1}(l_{PL}L_{t+1})^{1-\alpha-\gamma})(AC_{t+1}CE)^{\gamma}+1-\delta\}=\frac{C_t^{-\sigma}}{L_t^{1-\sigma}} \quad \text{(C11)}$$

$$\left(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)(A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\gamma FE_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho-1}=$$

$$\frac{\vartheta}{\varphi}\frac{FE_{t+1}}{\varphi}^{\vartheta-1}(A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}+\left(P_0+P_1\left(\sum_{i=1}^{t+1}FE_i\Big/\overline{FE}\right)^{P_2}\right)+$$

$$\frac{P_1P_2}{\overline{FE}}\left(\sum_{i=1}^{t}FE_i\Big/\overline{FE}\right)^{P_2-1}FE_{t+1} \tag{C12}$$

$$\frac{\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}}{\omega AC_0AC_{t+1}^{\theta}(l_{TL}L_{t+1})^{\omega}TY_{t+1}^{\omega-1}}\left\{\theta AC_0AC_{t+1}^{\theta-1}(l_{TL}L_{t+1}TY_{t+1})^{\omega}\right\}+\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\Bigg\{\Bigg(1-$$

$$\frac{FE_{t+1}}{\varphi}^{\vartheta}\Bigg)A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\gamma CE^{\rho}AC_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho-1}\Bigg\}=$$

$$\frac{\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}}{\omega AC_0AC_t^{\theta}(l_{TL}L_t)^{\omega}TY_t^{\omega-1}} \tag{C13}$$

And the above equation, when there is no fossil fuel energy left to use, will be:

$$\frac{\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}}{\omega AC_0AC_{t+1}^{\theta}(l_{TL}L_{t+1})^{\omega}TY_{t+1}^{\omega-1}}\left\{\theta AC_0AC_{t+1}^{\theta-1}(l_{TL}L_{t+1}TY_{t+1})^{\omega}\right\}+$$

$$\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\{A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\gamma CE(AC_{t+1}CE)^{\gamma-1}\}=\frac{\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}}{\omega AC_0AC_t^{\theta}(l_{TL}L_t)^{\omega}TY_t^{\omega-1}} \tag{C14}$$

Appendix C.2 (Solving the social planner's F.O.C [endogenous population])

Solving the Euler equations for the social planner when the population is endogenous

First-order conditions are:

$$\{C_t\}:\ \beta^t\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}=\lambda_{1t} \tag{C15}$$

$$\{L_{t+1}\}:-\beta^{t+1}C_{t+1}^{1-\sigma}L_{t+1}^{\sigma-2}+\lambda_{1t+1}\Bigg\{(1-\alpha-\gamma)\Bigg(1-$$

$$\frac{FE_{t+1}}{\varphi}^{\vartheta}\Bigg)(A_{t+1}K_{t+1}^{\alpha}(l_{PL})^{1-\alpha-\gamma}(L_{t+1})^{-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}\Bigg\}+$$

$$\lambda_{2t+1}\{\omega AC_0AC_{t+1}^{\theta}(l_{TL}TY_{t+1})^{\omega}L_{t+1}^{\omega-1}\}+\lambda_{3t+1}\Bigg\{L_0(\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1)\Bigg(1-$$

$$\frac{FE_{t+1}}{\varphi}^{\vartheta}\Bigg)^{\varepsilon_1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho}+1\Bigg\}=\lambda_{3t} \tag{C16}$$

$$\{K_{t+1}\}: \lambda_{1t+1}\left\{\left(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)\alpha(A_{t+1}K_{t+1}^{\alpha-1}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}+1-\delta\right\}+$$

$$\lambda_{3t+1}\{(\alpha\varepsilon_1-\varepsilon_2)L_0\left(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)^{\varepsilon_1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2-1}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_{t+1}CE)^{\rho}+\right.$$

$$\left.FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho}=\lambda_{1t} \tag{C17}$$

$$\{FE_t\}:\ \lambda_{1t}\left\{\left(1-\frac{FE_t}{\varphi}^{\vartheta}\right)(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma})\gamma FE_t^{\rho-1}\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho-1}-\right.$$

$$\frac{\vartheta}{\varphi}\frac{FE_t}{\varphi}^{\vartheta-1}(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma})\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho}-\left(P_0+P_1\left(\sum_{i=1}^{t}FE_i\Big/\overline{FE}\right)^{P_2}\right)-$$

$$\left.\frac{P_1P_2}{\overline{FE}}\left(\sum_{i=1}^{t-1}FE_i\Big/\overline{FE}\right)^{P_2-1}FE_t\right\}+\ \lambda_{3t}\{L_0\left(1-\right.$$

$$\left.\frac{FE_t}{\varphi}^{\vartheta}\right)^{\varepsilon_1}A_t^{\varepsilon_1}K_t^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\gamma\varepsilon_1FE_t^{\rho-1}\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma\varepsilon_1/\rho-1}-$$

$$L_0\frac{\vartheta\varepsilon_1}{\varphi}\frac{FE_t}{\varphi}^{\vartheta-1}\left(1-\frac{FE_t}{\varphi}^{\vartheta}\right)^{\varepsilon_1-1}A_t^{\varepsilon_1}K_t^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1+1}\left((AC_tCE)^{\rho}+\right.$$

$$\left.FE_t^{\rho}\right)^{\gamma\varepsilon_1/\rho}\}=0 \tag{C18}$$

$$\{AC_{t+1}\}: \lambda_{1t+1}\left\{\left(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\gamma CE^{\rho}AC_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho}+\right.\right.$$

$$\left.\left.FE_{t+1}^{\rho}\right)^{\gamma/\rho-1}\right\}+\lambda_{2t+1}\{\theta AC_0AC_{t+1}^{\theta-1}(l_{TL}L_{t+1}TY_{t+1})^{\omega}\}+\lambda_{3t+1}\{\gamma\varepsilon_1CE^{\rho}AC_{t+1}^{\rho-1}L_0\left(1-\right.$$

$$\left.\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)^{\varepsilon_1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho-1}\}=\lambda_{2t} \tag{C19}$$

$$\{TY_t\}: \lambda_{1t}=\lambda_{2t}[\omega AC_0AC_t^{\theta}(l_{TL}L_t)^{\omega}TY_t^{\omega-1}] \tag{C20}$$

Substituting the equations 68 and 73 (and the updated forms of them) in the above F.O.Cs, we can get the following equations:

$$-\beta^{t+1}C_{t+1}^{1-\sigma}L_{t+1}^{\sigma-2}+\ \beta^{t+1}\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\left\{(1-\alpha-\gamma)\left(1-\right.\right.$$

$$\left.\left.\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)(A_{t+1}K_{t+1}^{\alpha}(l_{PL})^{1-\alpha-\gamma}(L_{t+1})^{-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}\right\}+$$

$$\frac{\beta^{t+1}\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}}{\omega AC_0AC_{t+1}^{\theta}(l_{TL}L_{t+1})^{\omega}TY_{t+1}^{\omega-1}}\{\omega AC_0AC_{t+1}^{\theta}(l_{TL}TY_{t+1})^{\omega}L_{t+1}^{\omega-1}\}+\lambda_{3t+1}\left\{L_0(\varepsilon_2-\alpha\varepsilon_1-\right.$$

$$\gamma\varepsilon_1)\left(1-\frac{FE_{t+1}{}^{\vartheta}}{\varphi}\right)^{\varepsilon_1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho}+1\Big\}=\lambda_{3t}$$

(C21)

$$\beta^{t+1}\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\left\{\left(1-\frac{FE_{t+1}{}^{\vartheta}}{\varphi}\right)\alpha(A_{t+1}K_{t+1}^{\alpha-1}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}+1-\delta\right\}+$$

$$\lambda_{3t+1}\{(\alpha\varepsilon_1-\varepsilon_2)L_0\left(1-\frac{FE_{t+1}{}^{\vartheta}}{\varphi}\right)^{\varepsilon_1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2-1}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\big((AC_{t+1}CE)^{\rho}+$$

$$FE_{t+1}^{\rho}\big)^{\gamma\varepsilon_1/\rho}=\beta^{t}\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}\qquad\text{(C22)}$$

$$\beta^{t}\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}\left\{\left(1-\frac{FE_t{}^{\vartheta}}{\varphi}\right)(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma})\gamma FE_t^{\rho-1}\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho-1}-\right.$$

$$\frac{\vartheta}{\varphi}\frac{FE_t{}^{\vartheta-1}}{\varphi}(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma})\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho}-\left(P_0+P_1\left(\sum_{i=1}^{t}FE_i\Big/\overline{FE}\right)^{P_2}\right)-$$

$$\left.\frac{P_1P_2}{\overline{FE}}\left(\sum_{i=1}^{t-1}FE_i\Big/\overline{FE}\right)^{P_2-1}FE_t\right\}+\ \lambda_{3t}\{L_0\Big(1-$$

$$\frac{FE_t{}^{\vartheta}}{\varphi}\Big)^{\varepsilon_1}A_t^{\varepsilon_1}K_t^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\gamma\varepsilon_1FE_t^{\rho-1}\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma\varepsilon_1/\rho-1}-$$

$$L_0\frac{\vartheta\varepsilon_1}{\varphi}\frac{FE_t{}^{\vartheta-1}}{\varphi}\left(1-\frac{FE_t{}^{\vartheta}}{\varphi}\right)^{\varepsilon_1-1}A_t^{\varepsilon_1}K_t^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma\varepsilon_1/\rho}\}=0$$

(C23)

$$\beta^{t+1}\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\left\{\left(1-\frac{FE_{t+1}{}^{\vartheta}}{\varphi}\right)A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\gamma CE^{\rho}AC_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho-1}\right\}+$$

$$\frac{\beta^{t+1}\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}}{\omega AC_0AC_{t+1}^{\theta}(l_{TL}L_{t+1})^{\omega}TY_{t+1}^{\omega-1}}\{\theta AC_0AC_{t+1}^{\theta-1}(l_{TL}L_{t+1}TY_{t+1})^{\omega}\}+\lambda_{3t+1}\{\gamma\varepsilon_1CE^{\rho}AC_{t+1}^{\rho-1}L_0\Big(1-$$

$$\frac{FE_{t+1}{}^{\vartheta}}{\varphi}\Big)^{\varepsilon_1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho-1}\}=$$

$$\frac{\beta^{t}\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}}{\omega AC_0AC_t^{\theta}(l_{TL}L_t)^{\omega}TY_t^{\omega-1}}\qquad\text{(C24)}$$

Now, we can rearrange equation 75 for λ_{3t}, update it to get λ_{3t+1}, and replace them back into the equations 74, 75 and 77 to have our three Euler equations as follows:

$$-\beta C_{t+1}^{1-\sigma}L_{t+1}^{\sigma-2} + \beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\Big\{(1-\alpha-\gamma)\Big(1-$$

$$\frac{FE_{t+1}}{\varphi}^{\vartheta}\Big)\left(A_{t+1}K_{t+1}^{\alpha}(l_{PL})^{1-\alpha-\gamma}(L_{t+1})^{-\alpha-\gamma}\right)\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}\Big\} +$$

$$\frac{\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}}{\omega AC_0AC_{t+1}^{\theta}(l_{TL}L_{t+1})^{\omega}TY_{t+1}^{\omega-1}}\left\{\omega AC_0AC_{t+1}^{\theta}(l_{TL}TY_{t+1})^{\omega}L_{t+1}^{\omega-1}\right\} + \frac{AA}{BB} * \Big\{L_0(\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1)\Big(1-$$

$$\frac{FE_{t+1}}{\varphi}^{\vartheta}\Big)^{\varepsilon_1} A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho}+1\Big\} = \frac{CC}{DD} \qquad \text{(C25)}$$

$$AA{=}-\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\Big\{\Big(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\Big)\left(A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\right)\gamma FE_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho-1}-$$

$$\frac{\vartheta}{\varphi}\frac{FE_{t+1}}{\varphi}^{\vartheta-1}\left(A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\right)\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}-\left(P_0+P_1\left(\sum_{i=1}^{t+1}FE_i\Big/\overline{FE}\right)^{P_2}\right)-$$

$$\frac{P_1P_2}{\overline{FE}}\left(\sum_{i=1}^{t}FE_i\Big/\overline{FE}\right)^{P_2-1}FE_{t+1}\Big\}$$

$$BB{=}\Big\{L_0\Big(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\Big)^{\varepsilon_1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\gamma\varepsilon_1FE_{t+1}^{\rho-1}\big((AC_{t+1}CE)^{\rho}+$$

$$FE_{t+1}^{\rho}\big)^{\gamma\varepsilon_1/\rho-1}-L_0\frac{\vartheta\varepsilon_1}{\varphi}\frac{FE_{t+1}}{\varphi}^{\vartheta-1}\Big(1-$$

$$\frac{FE_{t+1}}{\varphi}^{\vartheta}\Big)^{\varepsilon_1-1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho}\Big\}$$

$$CC{=}-\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}\Big\{\Big(1-\frac{FE_t}{\varphi}^{\vartheta}\Big)\left(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma}\right)\gamma FE_t^{\rho-1}\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho-1}-$$

$$\frac{\vartheta}{\varphi}\frac{FE_t}{\varphi}^{\vartheta-1}\left(A_tK_t^{\alpha}(l_{PL}L_t)^{1-\alpha-\gamma}\right)\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma/\rho}-\left(P_0+P_1\left(\sum_{i=1}^{t}FE_i\Big/\overline{FE}\right)^{P_2}\right)-$$

$$\frac{P_1P_2}{\overline{FE}}\left(\sum_{i=1}^{t-1}FE_i\Big/\overline{FE}\right)^{P_2-1}FE_t\Big\}$$

$$DD{=}\{L_0\Big(1-\frac{FE_t}{\varphi}^{\vartheta}\Big)^{\varepsilon_1}A_t^{\varepsilon_1}K_t^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\gamma\varepsilon_1FE_t^{\rho-1}\left((AC_tCE)^{\rho}+FE_t^{\rho}\right)^{\gamma\varepsilon_1/\rho-1}-$$

$$L_0\frac{\vartheta\varepsilon_1}{\varphi}\frac{FE_t}{\varphi}^{\vartheta-1}\Big(1-\frac{FE_t}{\varphi}^{\vartheta}\Big)^{\varepsilon_1-1}A_t^{\varepsilon_1}K_t^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}$$

And the above equation, when there is no fossil fuel energy left to use, will be:

$$-\beta^2 C_{t+1}^{1-\sigma} L_{t+1}^{\sigma-2} + \beta^2 \frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}} \{(1-\alpha-\gamma)(A_{t+1}K_{t+1}^{\alpha}(l_{PL})^{1-\alpha-\gamma}(L_{t+1})^{-\alpha-\gamma})(AC_{t+1}CE)^{\gamma}\} +$$

$$\frac{\beta^2 \frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}}{\omega AC_0 AC_{t+1}^{\theta}(l_{TL}L_{t+1})^{\omega} TY_{t+1}^{\omega-1}} \{\omega AC_0 AC_{t+1}^{\theta}(l_{TL}TY_{t+1})^{\omega} L_{t+1}^{\omega-1}\} +$$

$$\frac{\beta \frac{C_t^{-\sigma}}{L_t^{1-\sigma}} - \beta^2 \frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}(\alpha(A_{t+1}K_{t+1}^{\alpha-1}(l_{PL}L_{t+1})^{1-\alpha-\gamma})(AC_{t+1}CE)^{\gamma}+1-\delta)}{(\alpha\varepsilon_1-\varepsilon_2)L_0 A_{t+1}^{\varepsilon_1} K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2-1} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}(AC_{t+1}CE)^{\gamma\varepsilon_1}} * \{L_0(\varepsilon_2 - \alpha\varepsilon_1 -$$

$$\gamma\varepsilon_1) A_{t+1}^{\varepsilon_1} K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1-1}(AC_{t+1}CE)^{\gamma\varepsilon_1} + 1\} =$$

$$\frac{\frac{C_{t-1}^{-\sigma}}{L_{t-1}^{1-\sigma}} - \beta \frac{C_t^{-\sigma}}{L_t^{1-\sigma}}(\alpha(A_t K_t^{\alpha-1}(l_{PL}L_t)^{1-\alpha-\gamma})(AC_t CE)^{\gamma}+1-\delta)}{(\alpha\varepsilon_1-\varepsilon_2)L_0 A_t^{\varepsilon_1} K_t^{\alpha\varepsilon_1-\varepsilon_2-1} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}(AC_t CE)^{\gamma\varepsilon_1}} \qquad \text{(C26)}$$

$$\beta \frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}} \left\{\left(1 - \frac{FE_{t+1}}{\varphi}^{\vartheta}\right) \alpha(A_{t+1}K_{t+1}^{\alpha-1}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho} + FE_{t+1}^{\rho}\right)^{\gamma/\rho} + 1 - \delta\right\} +$$

$$\left(\frac{EE}{FF}\right)\{(\alpha\varepsilon_1 - \varepsilon_2)L_0 \left(1 - \frac{FE_{t+1}}{\varphi}^{\vartheta}\right)^{\varepsilon_1} A_{t+1}^{\varepsilon_1} K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2-1} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_{t+1}CE)^{\rho} +\right.$$

$$\left.FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho} = \frac{C_t^{-\sigma}}{L_t^{1-\sigma}} \qquad \text{(C27)}$$

$$EE = -\beta \frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}} \left\{\left(1 - \frac{FE_{t+1}}{\varphi}^{\vartheta}\right)(A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\gamma FE_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho} + FE_{t+1}^{\rho}\right)^{\gamma/\rho-1} -\right.$$

$$\frac{\vartheta}{\varphi} \frac{FE_{t+1}}{\varphi}^{\vartheta-1} (A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho} + FE_{t+1}^{\rho}\right)^{\gamma/\rho} - \left(P_0 + P_1\left(\sum_{i=1}^{t+1} FE_i \Big/ \overline{FE}\right)^{P_2}\right) -$$

$$\left.\frac{P_1 P_2}{\overline{FE}}\left(\sum_{i=1}^{t} FE_i \Big/ \overline{FE}\right)^{P_2-1} FE_{t+1}\right\}$$

$$FF = \left\{L_0\left(1 - \frac{FE_{t+1}}{\varphi}^{\vartheta}\right)^{\varepsilon_1} A_{t+1}^{\varepsilon_1} K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1} \gamma\varepsilon_1 FE_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho} +\right.\right.$$

$$\left.FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho-1} - L_0 \frac{\vartheta\varepsilon_1}{\varphi} \frac{FE_{t+1}}{\varphi}^{\vartheta-1} \left(1 -\right.$$

$$\left.\left.\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)^{\varepsilon_1-1} A_{t+1}^{\varepsilon_1} K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_{t+1}CE)^{\rho} + FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho}\right\}$$

$$\beta \frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}} \left\{\left(1 - \frac{FE_{t+1}}{\varphi}^{\vartheta}\right) A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\gamma CE^{\rho} AC_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho} + FE_{t+1}^{\rho}\right)^{\gamma/\rho-1}\right\} +$$

$$\frac{\beta \frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}}{\omega AC_0 AC_{t+1}^{\theta}(l_{TL}L_{t+1})^{\omega} TY_{t+1}^{\omega-1}} \{\theta AC_0 AC_{t+1}^{\theta-1}(l_{TL}L_{t+1}TY_{t+1})^{\omega}\} + \left(\frac{GG}{HH}\right)\{\gamma\varepsilon_1 CE^{\rho} AC_{t+1}^{\rho-1} L_0 \left(1 -\right.$$

$$\left.\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)^{\varepsilon_1} A_{t+1}^{\varepsilon_1} K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2} l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma} L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho-1}\Big\} =$$

$$\frac{\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}}{\omega AC_0 AC_t^{\theta}(l_{TL}L_t)^{\omega}TY_t^{\omega-1}} \tag{C28}$$

$$GG=-\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\left\{\left(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)(A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\gamma FE_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho-1}-\right.$$

$$\frac{\vartheta}{\varphi}\frac{FE_{t+1}}{\varphi}^{\vartheta-1}(A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma})\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma/\rho}-\left(P_0+P_1\left(\sum_{i=1}^{t+1}FE_i\Big/\overline{FE}\right)^{P_2}\right)-$$

$$\left.\frac{P_1P_2}{\overline{FE}}\left(\sum_{i=1}^{t}FE_i\Big/\overline{FE}\right)^{P_2-1}FE_{t+1}\right\}$$

$$HH=\left\{L_0\left(1-\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)^{\varepsilon_1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\gamma\varepsilon_1 FE_{t+1}^{\rho-1}\left((AC_{t+1}CE)^{\rho}+\right.\right.$$

$$\left.FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho-1}-L_0\frac{\vartheta\varepsilon_1}{\varphi}\frac{FE_{t+1}}{\varphi}^{\vartheta-1}\left(1-\right.$$

$$\left.\left.\frac{FE_{t+1}}{\varphi}^{\vartheta}\right)^{\varepsilon_1-1}A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}\left((AC_{t+1}CE)^{\rho}+FE_{t+1}^{\rho}\right)^{\gamma\varepsilon_1/\rho}\right\}$$

And the above equation, when there is no fossil fuel energy left to use, will be:

$$\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}\{A_{t+1}K_{t+1}^{\alpha}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\gamma CE(AC_{t+1}CE)^{\gamma-1}\}+$$

$$\frac{\beta\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}}{\omega AC_0AC_{t+1}^{\theta}(l_{TL}L_{t+1})^{\omega}TY_{t+1}^{\omega-1}}\left\{\theta AC_0AC_{t+1}^{\theta-1}(l_{TL}L_{t+1}TY_{t+1})^{\omega}\right\}+$$

$$\frac{\beta\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}-\beta^2\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}(\alpha\left(A_{t+1}K_{t+1}^{\alpha-1}(l_{PL}L_{t+1})^{1-\alpha-\gamma}\right)(AC_{t+1}CE)^{\gamma}+1-\delta)}{(\alpha\varepsilon_1-\varepsilon_2)L_0A_{t+1}^{\varepsilon_1}K_{t+1}^{\alpha\varepsilon_1-\varepsilon_2-1}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_{t+1}^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}(AC_{t+1}CE)^{\gamma\varepsilon_1}}*$$

$$(\gamma\varepsilon_1L_0A_t^{\varepsilon_1}K_t^{\alpha\varepsilon_1-\varepsilon_2}l_{PL}^{\varepsilon_1-\alpha\varepsilon_1-\varepsilon_1\gamma}L_t^{\varepsilon_2-\alpha\varepsilon_1-\gamma\varepsilon_1}CE^{\gamma\varepsilon_1}AC_t^{\gamma\varepsilon_1-1})=\frac{\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}}{\omega AC_0AC_t^{\theta}(l_{TL}L_t)^{\omega}TY_t^{\omega-1}} \tag{C29}$$

Appendix C.3 (Solving the market-based F.O.C)

Solving the first-order conditions for the households, we get:

$$\{c_t\}\colon \beta^t\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}=\lambda_{1t} \tag{C30}$$

$$\{K_{t+1}\}\colon \lambda_{1t}=(1+r_{t+1})\lambda_{1t+1} \tag{C31}$$

$$\{L_{t+1}\}: \beta^{t+1}\frac{C_{t+1}^{1-\sigma}}{L_{t+1}^{2-\sigma}} = \lambda_{1t+1}w_{t+1} - \lambda_{2t} + \lambda_{2t+1}(1+\bar{L})$$

And when the population grows endogenously, we are going to have the below F.O.C:

$$\{K_{t+1}\}: \lambda_{1t} = (1+r_{t+1})\lambda_{1t+1} - \lambda_{2t+1}\varepsilon_2 L_0 L_{t+1}^{\varepsilon_2-\varepsilon_1} Y_{t+1}^{\varepsilon_1} K_{t+1}^{-\varepsilon_2-1} \tag{C32}$$

$$\{L_{t+1}\}: \beta^{t+1}\frac{C_{t+1}^{1-\sigma}}{L_{t+1}^{2-\sigma}} = \lambda_{1t+1}w_{t+1} - \lambda_{2t} + \lambda_{2t+1}(1 + L_0(\varepsilon_2-\varepsilon_1)Y_{t+1}^{\varepsilon_1}K_{t+1}^{-\varepsilon_2}L_{t+1}^{\varepsilon_2-\varepsilon_1-1}) \tag{C33}$$

Updating and substituting equations C30 in C31, C32, and C33 we derive the Euler equations for the households for both cases:

$$\frac{C_t^{-\sigma}}{L_t^{1-\sigma}} = \beta(1+r_{t+1})\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}} \qquad \text{(Population is exogenous)} \tag{C34}$$

$$\beta^2\frac{C_{t+1}^{1-\sigma}}{L_{t+1}^{2-\sigma}} = \beta^2\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}}w_{t+1} - \frac{\left[(1+r_t)\beta\frac{C_t^{-\sigma}}{L_t^{1-\sigma}} - \frac{C_{t-1}^{-\sigma}}{L_{t-1}^{1-\sigma}}\right]}{\left[\varepsilon_2 L_0 L_t^{\varepsilon_2-\varepsilon_1}Y_t^{\varepsilon_1}K_t^{-\varepsilon_2-1}\right]} + \frac{\left[(1+r_{t+1})\beta^2\frac{C_{t+1}^{-\sigma}}{L_{t+1}^{1-\sigma}} - \beta\frac{C_t^{-\sigma}}{L_t^{1-\sigma}}\right]}{\left[\varepsilon_2 L_0 L_{t+1}^{\varepsilon_2-\varepsilon_1}Y_{t+1}^{\varepsilon_1}K_{t+1}^{-\varepsilon_2-1}\right]}\left(1 + L_0(\varepsilon_2 - \varepsilon_1)Y_{t+1}^{\varepsilon_1}K_{t+1}^{-\varepsilon_2}L_{t+1}^{\varepsilon_2-\varepsilon_1-1}\right) \qquad \text{(Population is endogenous)} \tag{C35}$$

Solving the first-order conditions for the final good market, we have:

$$\{KY_t\}: \alpha EDA_t KY_t^{\alpha-1} PL_t^{1-\alpha-\gamma}\left(CE_t^{\rho} + FE_t^{\rho}\right)^{\gamma/\rho} = r_t + \delta \tag{C36}$$

$$\{PL_t\}: (1-\alpha-\gamma)EDA_t KY_t^{\alpha} PL_t^{-\alpha-\gamma}(CE_t^{\rho} + FE_t^{\rho})^{\gamma/\rho} = w_t \tag{C37}$$

$$\{FE_t\}: \gamma EDA_t KY_t^{\alpha} PL_t^{1-\alpha-\gamma} FE_t^{\rho-1}\left(CE_t^{\rho} + FE_t^{\rho}\right)^{\gamma/\rho-1} = P_{FEt} \tag{C38}$$

$$\{CE_t\}: \gamma EDA_t KY_t^{\alpha} PL_t^{1-\alpha-\gamma} CE_t^{\rho-1}(CE_t^{\rho} + FE_t^{\rho})^{\gamma/\rho-1} = P_{CEt} \tag{C39}$$

In the end, the F.O.C.s for the energy sector would be:

$$\{FE_t\}: P_0 + P_1\left(\sum_{i=1}^{t} FE_i \Big/ \overline{FE}\right)^{P_2} + \frac{P_1P_2}{\overline{FE}}\left(\sum_{i=1}^{t} FE_i \Big/ \overline{FE}\right)^{P_2-1} FE_t = P_{FEt} \tag{C40}$$

$$\{TL_t\}: \lambda_t \omega AC_0 AC_t^{\theta} TY_t (TL_t TY_t)^{\omega-1} = \beta^t w_t \tag{C41}$$

$$\{TY_t\}: \lambda_t \omega AC_0 AC_t^{\theta} TL_t (TL_t TY_t)^{\omega-1} = \beta^t r_t \tag{C42}$$

$$\{AC_{t+1}\}: \lambda_t - \lambda_{t+1} AC_0 AC_t^{\theta}(TL_t TY_t)^{\omega} = \beta^{t+1} CE * P_{CEt+1} \tag{C43}$$

Combining equations C41 and C42, we get:

$$\mathrm{TY_t} = \frac{\mathrm{w_t}}{\mathrm{r_t}}\mathrm{TL_t} \tag{C44}$$

Updating and substituting equation C44 in C43, we get another Euler equation for the Energy sector:

$$\frac{\mathrm{w_t}}{\omega \mathrm{AC_0 AC_t^{\theta} TY_t (TL_t TY_t)^{\omega-1}}} - \beta \mathrm{CE} * \mathrm{P_{CEt+1}} = \frac{\beta \mathrm{w_{t+1}}}{\omega \mathrm{AC_{t+1}^{\theta} TY_{t+1} (TL_{t+1} TY_{t+1})^{\omega-1}}} \mathrm{AC_t^{\theta} (TL_t TY_t)^{\omega}} \tag{C45}$$

Using equations C38 and C40, we can derive the price and the amount of fossil fuel energy.

Appendix D (Price of fossil fuel energy)

In this setup for simplicity, a social planner needs to provide non-renewable energy (while she owns it). Thus she needs to spend some of her resources to extract it. In the market-based approach, the energy sector (as a monopoly) owns the resources, but still needs to pay the extraction costs; and this cost is similar in both models (social planner and market-based). However, unlike Stiglitz assumption in which cost of extraction is decreasing over time, in this model, it is increasing over time since it would be harder to extract the fossil fuel in the bottom of a reservoir (and when there is less reserve remains in the reservoir) compared to the full reservoir. Another distinction of this model versus Stiglitz (or in general (or in general Hotelling setup) is that the objective in those models is that how should social planner utilize the exhaustible resources. Whereas in the current set up the purpose is how social planner should maximize household utility which is the consumption per capita. And consumption itself is a function of different investments. In the end, it is worth to say that if there is a strong and positive correlation between marginal cost and the price of fossil fuel, then the price of price fossil fuel has been increasing during the past decades while at the same time utilizing the energy has been increasing as well.

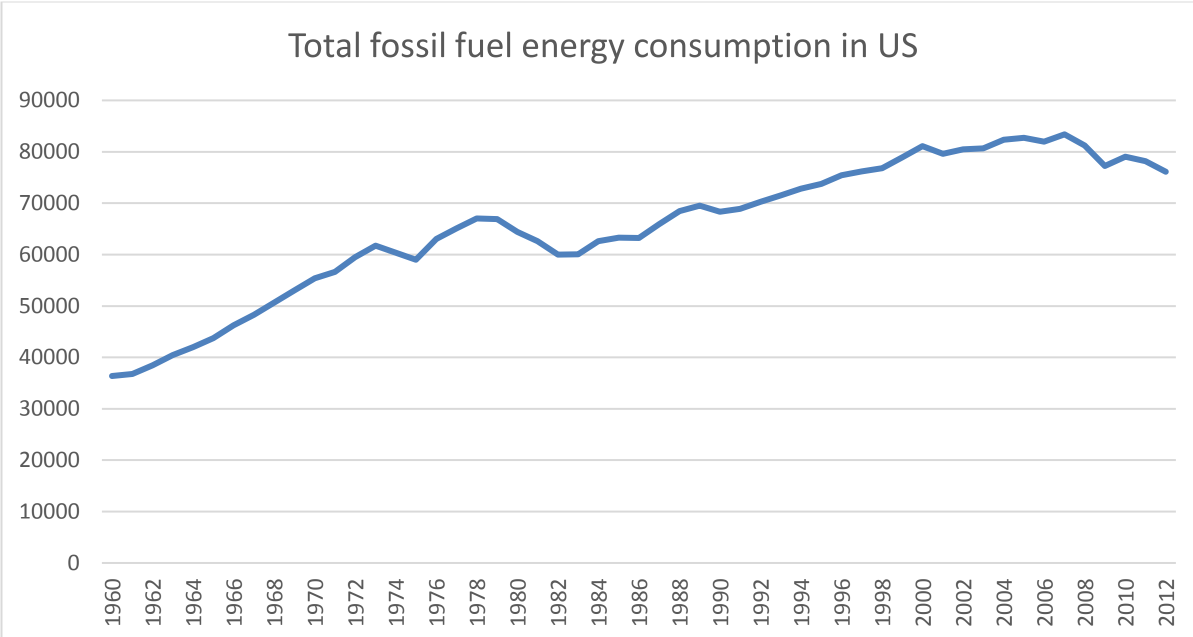

Figure 6: Total fossil fuel consumption in the US from 1960 – 2012. As it is shown, the rate of consumption/production has been increasing while the price of providing it has also been growing, in general.

Appendix E (Changing the capital share)

In this section, I want to investigate two simple cases as a sensitivity analysis. First, the capital changes from 0.27 to 0.21. Second, an extra element would be added to the income allocation equation (equation 3) to absorb the gap between the perfect income allocation of the model and the imperfect allocation of the real world (such as retirement, labor force participation which is not 100% and so on). Therefore, instead of equation 3, we will have:

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t + P_{FEt}FE_t + TY_t + Mis_t \tag{E1}$$

The results – for the social planner approach when the population grows endogenously – are depicted in Figure 7. We can see that the economic growth is lower when there is a misallocation in income, and population tends to grow even faster. However, population growth is slower when capital share decreases ad labor share increases.

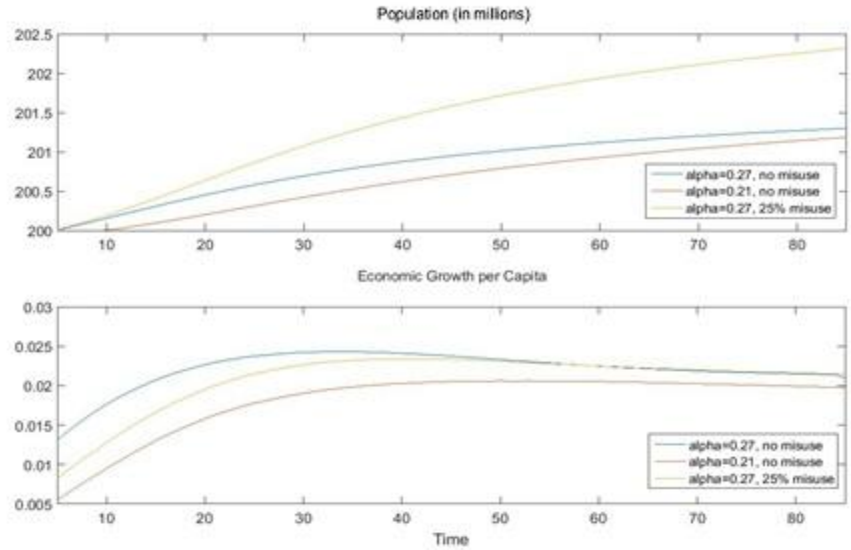

Figure 7 shows population growth and economic growth for three different cases. The blue lines show the base scenario when there is no misallocation of the resources and the capital share is 0.27. Red lines show the case in which labor share increases by 6 percent. And the yellow lines show the case that there exist 25% misallocation of the income. The social planner solution has been applied for all models.

Appendix F (Altering the population growth)

One would argue that in the US population grows with around one percent, whereas, in the proposed model it converges to zero. In an attempt, I used different parameterization for equation 15 ($L_{t+1} = L_t + L_0({}^{Y_t}/_{L_t})^{\varepsilon_1}({}^{L_t}/_{K_t})^{\varepsilon_2}$), to see if one percent growth rate in population is achievable using the current setup. As it is shown in Figure 7, population can grow faster in early stages; however, it tends to drop in the end. While the proposed model is well-fitted in the countries such as Western Europe and Japan, we need to change the value of the parameters in equation 15 ($\varepsilon_1 = 1.72 \rightarrow 1.8$, $\varepsilon_2 = 2.18 \rightarrow 2.1$ and $L_0 = 2.35 \rightarrow 12.35$) to capture the growth rate in population for US.

For the case of US, I can think of a plausible arguments. If we deduct US immigration rate (including immigrants' descendants, although they might be the US-born), the population growth would be much lower than the current rate. Although the counter argument would be, they still participate in the economy, however, they are not born in that economy but just brought in. The proposed model showed a high growth rate in early stages that we can think about the entrance of the immigrants with the high rate of population which tends to converge to its steady state.

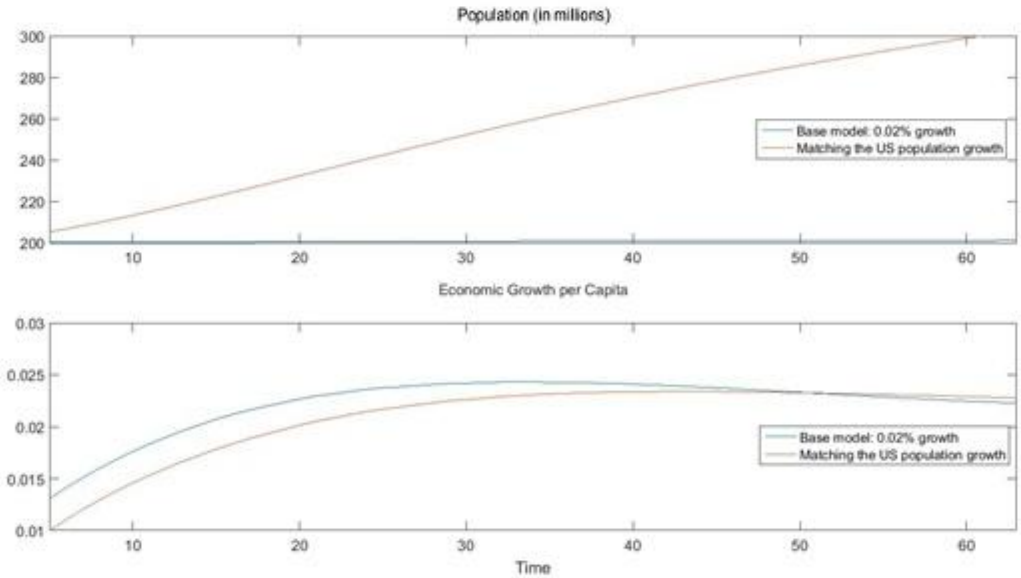

Figure 8 shows population and economic growth for two different scenarios of population growth. The blue lines show the base model scenario in which population grows at 0.02% rate. The red lines match the US population growth which is about 0.6 percent on average. The social planner solution has been applied for both models.

Appendix G (Assigning different utility function)

In another attempt, instead of exogenously imposing a social cost of using the fossil fuel-based energy, we can tweak the individuals' utility function in such a way that they evaluate the air quality (the environment in general) as another good. Under these conditions, the utility maximization process would be as below:

$$\mathrm{Max}\,W = E_0 \sum_{t=0}^{T} \beta^t \frac{(c_t^{\mu} ED_t^{1-\mu})^{1-\sigma}}{1-\sigma} \qquad c_t = {C_t}/{L_t} \qquad \text{(G1\&G2)}$$

The only difference in the above household maximization setting, compared to the social planner approach, is the idea arising from Rosen (1974) in which individuals evaluate the air quality as a commodity and add it to their consumption bundle accordingly. Since the households would profit from the firms, ultimately, air quality is endogenous within this setup. In the above setting, if individuals do not care for the environment, we can simply calibrate the value of μ to one. Thus, we get the same utility as we had before. Based on the degree of the individuals' awareness of the importance of the environment, this amount would be somewhere between zero and one. Updating equations C34 and C35 by including the environmental degradation we have:

$$\mathrm{ED_t}^{(1-\mu)(1-\sigma)}\mathrm{C}_t^{\mu-\mu\sigma-1}\mathrm{L}_t^{\mu\sigma-\mu} = \beta(1+r_{t+1})\mathrm{ED_{t+1}}^{(1-\mu)(1-\sigma)}\mathrm{C}_{t+1}^{\mu-\mu\sigma-1}\mathrm{L}_{t+1}^{\mu\sigma-\mu} \tag{G3}$$

$$\beta^2\mathrm{ED_t}^{(1-\mu)(1-\sigma)}\mathrm{C}_t^{\mu-\mu\sigma}\mathrm{L}_t^{\mu\sigma-\mu-1} = \beta^2\mathrm{ED_{t+1}}^{(1-\mu)(1-\sigma)}\mathrm{C}_{t+1}^{\mu-\mu\sigma-1}\mathrm{L}_{t+1}^{\mu\sigma-\mu}w_{t+1} - \frac{\left[(1+r_t)\beta\mathrm{ED_t}^{(1-\mu)(1-\sigma)}\mathrm{C}_t^{\mu-\mu\sigma-1}\mathrm{L}_t^{\mu\sigma-\mu}-\mathrm{ED_{t-1}}^{(1-\mu)(1-\sigma)}\mathrm{C}_{t-1}^{\mu-\mu\sigma-1}\mathrm{L}_{t-1}^{\mu\sigma-\mu}\right]}{\left[\varepsilon_2\mathrm{L}_0\mathrm{L}_t^{\varepsilon_2-\varepsilon_1}\mathrm{Y}_t^{\varepsilon_1}\mathrm{K}_t^{-\varepsilon_2-1}\right]} + \frac{\left[(1+r_{t+1})\beta^2\mathrm{ED_{t+1}}^{(1-\mu)(1-\sigma)}\mathrm{C}_{t+1}^{\mu-\mu\sigma-1}\mathrm{L}_{t+1}^{\mu\sigma-\mu}-\beta\mathrm{ED_t}^{(1-\mu)(1-\sigma)}\mathrm{C}_t^{\mu-\mu\sigma-1}\mathrm{L}_t^{\mu\sigma-\mu}\right]}{\left[\varepsilon_2\mathrm{L}_0\mathrm{L}_{t+1}^{\varepsilon_2-\varepsilon_1}\mathrm{Y}_{t+1}^{\varepsilon_1}\mathrm{K}_{t+1}^{-\varepsilon_2-1}\right]}\left(1+\mathrm{L}_0(\varepsilon_2-\varepsilon_1)\mathrm{Y}_{t+1}^{\varepsilon_1}\mathrm{K}_{t+1}^{-\varepsilon_2}\mathrm{L}_{t+1}^{\varepsilon_2-\varepsilon_1-1}\right) \tag{G4}$$

The results are summarized in Figure below. Households tend to consume less in the environmental friendly model compared to the others. However, it does not have any impact on fossil fuel production pattern.

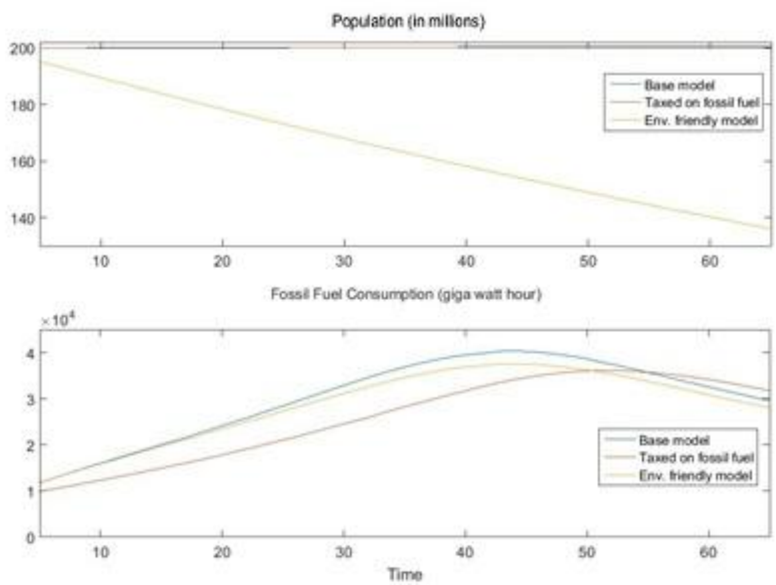

Figure 9 shows population growth and fossil fuel utilization for three different scenarios. The blue lines are for the base model. The red lines show the elements when there is an element of carbon tax. And the yellow lines show when individuals evaluate environment as another good in their utility maximization. All models are decentralized.